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Possible Tests on the Verification of and Departure from \(1S_0 \rightarrow 3S_1\) Radiative Transition in Thermal \(n-p\) Capture*  

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Tests are suggested to verify the hypothesis that thermal \(n-p\) capture proceeds via the transition \(1S_0 \rightarrow 3S_1\) and that approximately 10\% of the observed capture cross section is due to interaction effects. It is shown that measurements of the \(\gamma\) polarization effect using polarized neutrons and protons are capable of testing the predictions of the hypothesis to within 1\%.  

The nuclear two-body problem illustrates perhaps in the most direct way the complexity of nuclear forces. At small values of the relative momenta between the nucleons, the two-body interaction is assumed to be rather insensitive to the detailed nature of the nucleon-nucleon interaction. Such an assumption has been the basis of treating the low-energy \(n-p\) capture and its counterpart, the photodisintegration of the deuteron in the so-called zero-range approximation. (The approximation implies the use of the asymptotic values of the initial- and final-state wave function.) Noyes\(^1\) has analyzed the present status. The conclusion that persists is that the theoretical calculations are approximately 10\% lower than the measured value of the thermal \(n-p\) capture cross section \(a_n\) of 334 mb.\(^2\) A possible explanation first suggested by Austern and Rost\(^3\) in terms of certain unaccounted-for elementary-particle currents and labeled as "interaction effect" is assumed to be the cause of the discrepancy.  

Briefly the customary view point is to assume \(S\)-wave neutron capture and that the capture proceeds via the transition \(1S_0 \rightarrow 3S_1\). The transition is characterized as a magnetic dipole isospin-flip and even-\(G\)-parity transition. The operator responsible for the transition is the isovector nucleon magnetic-moment operator. Theoretical calculations using the Bethe-Longmire\(^4\) approximation yield a value of 305 mb. Various efforts have been made to include contributions due to possible \(\pi-\pi\), \(w+\rho\), and \(\eta-\pi\) vertices. The net result of such calculations is an increase of 10 mb with most of it resulting from the \(\pi-\pi\) vertex.\(^5\)  

In a recent article Noyes\(^6\) concludes that the nucleon-nucleon scattering experiments below 10 MeV are consistent with an \(n-p\) effective range \(r_{np} = 2.73 \pm 0.03\) F. If the discrepancy between the observed and calculated value of \(a_n\) were to be explained by a downward revision of \(r_{np}\), then its value must be less than 2.4 F.\(^7\) (A decrease in \(r_{np}\) increases the value of \(a_n\).) Noye's analysis indicates such a possibility to be highly unlikely. Lately it has been suggested by Malik and Sailor\(^8\) and independently by Breit and Rustgi\(^9\) that a \(3S_1 \rightarrow 3S_1\) transition may be the source of discrepancy. It was further suggested that observations of polarization of the \(\gamma\) rays produced in the capture of polarized neutrons by polarized protons can test the presence of such a transition. Breit and Rustgi, using density-matrix description of the beam and target, examine in detail the asymmetry in the angular distribution and the degree of polarization of the capture \(\gamma\) rays. Their calculations show that for a geometric-mean target-beam polarization \(f = (f_n f_p)^{1/2}\), where \(f_n\) and \(f_p\) are the neutron and proton polarization, respectively, of 0.954 to 0.577, asymmetries in the angular distribution ranging from 23 to 1.96\% may be found. These results are obtained on the assumption that 9\% of the capture occurs via the transition \(5S_1 \rightarrow 3S_1\). In terms of practical possibilities, even the lower value of \(f\) is difficult to achieve except in the case of dynamic polarization. The targets used for dynamic polarization include nuclei with large absorption cross sections, making the observation more difficult.  

The purpose of this comment is to suggest a means of establishing departures from the basic underlying hypothesis of the capture process; namely, that the capture occurs solely via the transition \(1S_0 \rightarrow 3S_1\). The essential content of the proposed method is a measurement of the spin dependence of the capture \(\gamma\) intensities. It amounts to testing whether the assumed orthogonality of \(5S_1\) continuum and the \(3S_1\) part of the ground state of deuteron is strictly valid and/or \(p\)-wave capture is likely. Presented below are the theoretical estimates of spin-dependent cross sections to-
gether with the possibility of their observation.

Assume that the neutron beam of polarization \( f_n \) is incident on a proton target of polarization \( f_p \). The quantities \( f_n \) and \( f_p \) are defined as

\[
f_n = \frac{w_+ - w_-}{w_+ + w_-},
\]

where \( w_+ \) are the beam intensities corresponding to the neutrons with \( m_+ \) (the magnetic quantum number) = \( \pm \frac{1}{2} \), respectively, and

\[
f_p = \frac{v_+ - v_-}{v_+ + v_-},
\]

where \( v_+ \) are the target nuclei with magnetic quantum numbers \( \pm \frac{1}{2} \), respectively, in the case of protons. The extent of nuclear polarization is determined by the hyperfine constant via the expression

\[
f_n = \frac{2I + 1}{2I} \coth \left( \frac{2I + 1}{2I} \frac{a}{2I} \right) - \coth \left( \frac{a}{2I} \right),
\]

where \( a = \mu_B H/kT, \) \( \mu_B \) is the ground-state magnetic moment of the target nuclei at the temperature \( T \) in the field of \( H \) gauss, and \( k \) is the Boltzmann constant.

The capture cross section \( \sigma_+ (t^+) \) and \( \sigma_0 (t^+) \) for neutrons polarized parallel and antiparallel to the proton polarization, respectively, can be obtained and for \( f_n = 1 \) are given by

\[
\sigma_+ (t^+) = \frac{I + 1}{2I + 1} \sigma_{n,γ}^0 + \frac{I}{2I + 1} \sigma_{n,γ}^0 \sigma_0 + \frac{I}{2I + 1} f_n (\sigma_{n,γ}^1 - \sigma_{n,γ}^0),
\]

\[
\sigma_0 (t^+) = \frac{I + 1}{2I + 1} \sigma_{n,γ}^0 + \frac{I}{2I + 1} \sigma_{n,γ}^0 \sigma_0 - \frac{I}{2I + 1} f_n (\sigma_{n,γ}^1 - \sigma_{n,γ}^0),
\]

where \( \sigma_{n,γ}^0 \) and \( \sigma_{n,γ}^0 \) are the capture cross sections corresponding to the compound state \( J = I + \frac{1}{2} \) and \( J = I - \frac{1}{2} \), respectively. The corresponding total cross sections \( \sigma_+ \) are

\[
\sigma_+ = \sigma_T (1 + \rho f_n)
\]

and

\[
\sigma_0 = \sigma_T (1 - \rho f_n),
\]

where \( \rho = -\frac{1}{I+1} \sigma_0 / \sigma_T \), with \( \sigma_+ \) and \( \sigma_0 \) values of the total cross sections corresponding to the compound state \( J = I + \frac{1}{2} \), respectively.

Having given the expressions for all the necessary cross sections we proceed to evaluate the difference in the intensity of the 2.24-MeV \( γ \) ray (produced as a result of neutron capture) for parallel and antiparallel spin orientation of the neutrons and protons. In the evaluation of these intensities account must be taken of the variations in the beam intensities \( w_+ \) at different depths inside the sample. It is easily shown that \( w_+ \) satisfy the following differential equations:

\[
dw_+ = \left[ -w_+ \sigma_+ (1 + \rho f_n) - D_+ w_+ + D_- w_- \right] dx,
\]

\[
dw_- = \left[ -w_- \sigma_0 (1 - \rho f_n) + D_+ w_+ - D_- w_- \right] dx.
\]

The Eqs. (7a) and (7b) take into account the different values of the incoherent scattering cross sections for the parallel and antiparallel neutron-proton spin orientations and leading to the neutron spin-flip in the zero-magnetic-moment quantum number state of the triplet state. The spin-flip probability thus depends upon the value \( a \), where 1 and 0 refer to the scattering lengths for the triplet and the singlet state. The factors \( D_+ \) and \( D_- \) are the values of the depolarization factors corresponding to the parallel and antiparallel spin orientations. The Eqs. (7a) and (7b) require numerical integration. If, however, we assume that \( D_+ \) can be replaced by the average value \( D = \frac{1}{2}(D_+ - D_-) \) and that the target is thin (i.e., the neutrons scattered more than once, escape the target, and do not

FIG. 1. Differences in the values of the variations of the \( γ \) polarization effect" corresponding to proton polarizations \( f_n = 0.125 \) and \( f_n = 0.275 \) as a function of \( \xi (\sigma_+ / \sigma_0) \) and the ZrH\(_{1.85}\) target thickness. All differences are measured from the value of \( \Delta \xi \) for \( \xi = 0 \).
contribute appreciably to the observed $\gamma$-ray intensities) Eqs. (7a) and (7b) can be solved. The result is

$$w_i^0 = e^{-\alpha x} \left[ w_i^0 \left[ \cosh(\kappa x) \pm \tau \sinh(\kappa x) \right] + w_i^0 \nu \sinh(\kappa x) \right],$$

where

$$\alpha = n \sigma_\tau + D, \quad \kappa = \sqrt{n^2 \sigma_\tau^2 + \rho^2 + D^2}, \quad \tau = n \sigma_\tau \rho / \kappa, \quad \nu = D / \kappa, \quad \rho = \rho f_k.$$\n
The intensities of the two components of the incident neutron beam are $w_i^0$ and $w_j^0$, and $n$ is the number of target nuclei per cm$^3$.

The intensity of $\gamma$ rays (2.224-MeV photons) by a strip of thickness $dx$ located at distance $x$ from the entrance surface of the sample can be expressed as

$$dN_\gamma = n dx \left[ \psi_i \left[ w_i (1 + \psi) + w_i (1 - \psi) \right] + \psi_j \left[ w_j (1 + \psi) + w_j (1 - \psi) \right] \right].$$

After inserting values of $w_i$, $w_j$, and after somewhat lengthy operations, Eq. (10) can be inte-

expression for $\epsilon$:

$$\epsilon = \frac{1 + \phi}{2} \int_0^\infty \frac{(3 \xi + 1) r G_2 + (1 - \xi) f_k G_1 - \nu G_2}{(3 \xi + 1) r G_2 + (1 - \xi) f_k G_1 - \nu G_2}.$$\n
We note that $\epsilon = 0$ as $f_k^0$ and/or $f_k$ goes to zero. However, for nonzero values of $f_k^0$ and $f_k$, $\epsilon$ does not become zero even when $\xi = 0$. This residual effect is due to the difference in the available neutrons with proper spin orientations for capture in the singlet state. One would therefore measure $\Delta \epsilon$ (variation of $\epsilon$) as a function of nuclear (proton) polarization.

In Fig. 1, the differences of $\Delta \epsilon$ calculated for the two values of $f_k = 0.125$ and 0.275 have been plotted as a function of target thickness for various values of $\xi$, starting from $\Delta \epsilon$ for $\xi = 0$ as the base. We have taken for the present purposes a target of ZrH$_{12.5}$. Cross-section measurements indicate that the hydrogen in the compound remains atomic$^{12}$ and can therefore be polarized because of the proton magnetic moment. A number of other hydrogen compounds also appear promising.$^{13}$ The decrease in $\Delta \epsilon$'s versus $t$ corre-

grated to yield

$$N_\gamma = \frac{1}{2} \left\{ \left[ (G_1 + (\nu + \tau f_k^0 G_2)(3 \xi + 1) \right] - f_k^0 G_1 - \tau f_k G_2 \right\}.$$\n
(11)

where

$$G_1 = F_1 + F_2, \quad G_2 = F_1 - F_2, \quad \xi = \sigma_\gamma / \sigma_0.$$\n
and

$$F_1 = \int_0^\infty e^{-\alpha t} dt, \quad F_2 = \int_0^\infty e^{-\alpha t} dt.$$\n
$f_k^0 = (w_i^0 - w_j^0)/(w_i^0 + w_j^0)$ is the incident-neutron-beam polarization. If now the incident-beam-neutron spins are flipped with a flipping efficiency $\phi$, an equation similar to Eq. (11) can be obtained.

The result is

$$N'_\gamma = \frac{1}{2} \left\{ \left[ (G_1 + (\nu + \tau f_k^0 G_2)(3 \xi + 1) \right] - (1 - \xi) f_k^0 G_1 - \tau f_k G_2 \right\}.$$\n
(12)

A quantity $\epsilon$, which we designate as the "$\gamma$ polarization effect," similar to the definition of polarization can then be defined as

$$\epsilon = \frac{N_\gamma - N'_\gamma}{N_\gamma + N'_\gamma}.$$\n
Using Eqs. (11) and (12), we obtain the following size of the effect. For an actual experimental circumstance the competing requirement of a statistically significant number of photons must balance the obvious conclusion of Fig. 1 that the thinnest possible sample yields the maximum ef-

The neutron flux available at the High Flux Beam Reactor and the techniques of nuclear polarization (either static or dynamic) make such an experiment a practical possibility. Calculations show that measurements corresponding to $\xi = 0.01$ ($\sigma = 3$ mb) are possible.

We conclude this note by reemphasizing that an observation of nonzero effect implies two possible causes. They are: (1) possible capture via the transition $^3S_1 - ^3S_1$, and (2) $P$-wave capture from continuum $p$ state to the deuteron ground state. To distinguish between the two possibilities, measurements of $\gamma$-ray polarization and angular distribution can be made. The capture in the $^3S_1$-
tinuum state would require, as indicated earlier, the nonorthogonality of the $^3\Sigma$ continuum and the $^3\Sigma_d$ deuteron ground state. Formation of the $P$ wave would appear to require significant departure from the present theory of direct interactions. In any case, possible experimental verification of all the neutron capture occurring via the continuum singlet state should clearly demonstrate that the mechanism leading to the so-called interaction effect must be sorted out in detail or, failing that, a fresh approach may be needed to deal with the electromagnetic phenomenon associated with the two-body nuclear problem.

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