Response of Materials to Various Shock Loading Conditions Generated by Plate Impact Experiments

Gifford W. Plume IV
University of Rhode Island, giffer500@yahoo.com

Follow this and additional works at: https://digitalcommons.uri.edu/oa_diss

Recommended Citation
https://digitalcommons.uri.edu/oa_diss/42

This Dissertation is brought to you for free and open access by DigitalCommons@URI. It has been accepted for inclusion in Open Access Dissertations by an authorized administrator of DigitalCommons@URI. For more information, please contact digitalcommons@etal.uri.edu.
RESPONSE OF MATERIALS TO VARIOUS SHOCK LOADING CONDITIONS GENERATED BY PLATE IMPACT EXPERIMENTS

BY

GIFFORD W. PLUME IV

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN MECHANICAL ENGINEERING AND APPLIED MECHANICS

UNIVERSITY OF RHODE ISLAND 2013
DOCTOR OF PHILOSOPHY DISSERTATION

BY

GIFFORD W. PLUME IV

APPROVED:

Thesis Committee:

Major Professor: Dr. Carl-Ernst Rousseau

Dr. Arun Shukla

Dr. David Taggart

Dr. Michael Greenfield

Nasser H. Zawia
DEAN OF THE GRADUATE SCHOOL

UNIVERSITY OF RHODE ISLAND
2013
ABSTRACT

The response of materials to shock loading has been investigated through use of a plate impact experimental technique. A single stage gas gun was utilized to drive projectiles to velocities between 50-500 m/s, facilitating investigations into low to moderate shock loading conditions. Temporal records of the dynamic events were captured with the use of commercial manganin stress gauges that were embedded within layers of test material.

Within this thesis, there is a bimodal theme. The first portion of this thesis investigated the spall fracture of cast irons with varying microstructure. Although the study of the spall fracture of materials is a common theme in literature, there exists a noteworthy scarcity of research specific to cast iron. Given that cast iron is one of the most widely utilized materials in engineering structures, this research was pursued in an effort to characterize its strength and identify the fracture mechanisms and kinetics associated with its failure process. The second portion of this thesis involved the development of a new technique that could be utilized to generate multiple Hugoniot states in a single experiment. Generation of a material’s Hugoniot is a fundamental theme in shock wave studies because it allows researchers to determine all mechanical and thermodynamic properties associated with dynamic loading conditions. Traditionally, the locus of points necessary to construct a material’s Hugoniot is obtained through a rigorous series of experiments, where each test produces a single data set. By considering the shock wave processes associated with layered plates, a
new method was developed that will significantly reduce the process of obtaining material Hugoniots.

Within the study of the spall fracture of cast iron, experiments were designed to induce an extreme tensile state within test samples from the interaction of decompression waves. The dynamic fracture strength, known as spall strength, was determined from temporal records of the stress evolution inside the samples. In order to encompass a vast majority of castings typical to industry, five separate cast irons were tested. Four of these castings consisted of gray cast iron with graphite in flake form, where three were classified as Type VII A2 and the other contained a bimodal distribution of Type VII A4 and VII D8. The fifth casting consisted of ductile cast iron with graphite in nodular form, classified as Type I with an average of 200 nodules per square millimeter of size class 5. The spall strength for the Type VII A2 gray cast irons was found to vary 40-370 MPa, and the additional gray cast iron was found to vary between 410-490 MPa. The spall strength of the ductile cast iron was found to fall within the range of 0.94-1.2 GPa. It was shown that the spall strength is linked to the damage level at the spall plane, where an increased amount of tensile stress is required to generate higher levels of damage. Post mortem analysis was performed on recovered samples in order to establish a relationship between microstructure and the fracture mechanisms of the failure process. This study has identified the graphite phase as the primary factor governing the spall fracture of cast irons, where crack nucleation is directly correlated to the debonding of graphite from the metal matrix. It has been noted that the average length of graphite found within a casting is linked to the material’s strength, where strength has been shown to increase as a function of
decreasing length. The morphology, and mean free path of graphite precipitates, has been shown to further govern the subsequent coalescence of initiated cracks to form a complete fracture plane. In cases where graphite spacing is large, an increased amount of energy is required to complete the fracture process. A secondary factor governing the spall fracture of cast irons has been linked to the microstructure of the metal matrix. It has been noted that pearlite will yield higher spall strengths in cast irons than free ferrite.

Within the second portion of this thesis, an experimental approach was developed to induce shock reflections in a low impedance inner-layer embedded within a high impedance bulk structure. By capturing temporal records of the stress evolution at each side of the inner-layer, step-like loading profiles were obtained that allowed for the capture of multiple Hugoniot states. The mathematical framework employed in this technique utilized the classical Rankine-Hugoniot equations in the method of impedance matching, where either the bulk material (case 1) or inner-layer (case 2) was required to have a known Hugoniot. Validation of the new technique was performed by testing well classified materials in order to facilitate comparison of the Hugoniots generated from the method with published data found in literature. For the first case, where the Hugoniot of the bulk material is known, the Hugoniot Ring-Up Method (HRUM) was shown to accurately generate states along the Hugoniot of the inner-layer, where the number of states acquired is directly linked to the experimental design. Factors including the wave velocities in the materials, input pulse duration (controlled by the thickness and wave velocity of the impactor), thickness of the inner-layer, and diameter of the test samples (arrival of the radial release) affect the number
of states that can be generated from a single experiment. Experiments employing 6061 aluminum and polycarbonate, respectively, as the bulk material and inner-layer, accurately generated six Hugoniot states for polycarbonate. Additionally, experiments employing A572 grade 50 structural steel as the bulk material were able to accurately generate ten Hugoniot states of the polycarbonate inner-layer. In these experiments, the method was extended to generate a Hugoniot equation defining the material response of the inner-layer within the domain encompassed by the specific test. Through comparison of these experimentally determined equations to the real Hugoniot of polycarbonate, it has been shown that a single HRUM experiment can yield an accurate Hugoniot for the inner-layer. For the second case, when the Hugoniot of the inner-layer is known, the HRUM failed to accurately generate states along the Hugoniot of the bulk material. Thus, the HRUM requires significant improvements before it can be used in this application. In light of these shortcomings, a procedure utilizing over-deterministic methodology has been proposed, that may allow future researchers to extend application of the HRUM to the case of determining the Hugoniot of the bulk material.
ACKNOWLEDGEMENTS

I would like to begin by extending my sincere gratitude to my advising professor, Dr. Carl-Ernst Rousseau, for giving me this chance to pursue my PhD at the University of Rhode Island (URI). I could not have imagined a better adviser, in terms of his perpetual commitment to my research and willingness to help in any way possible. I feel extremely fortunate that I was given the freedom to choose my research directions, and was able to run experiments that may not have directly pertained to the requirements specified by funding grants. Without this freedom, I could not have imagined the possibility of developing the new Hugoniot Ring-Up Method (HRUM), which may prove entirely useful to continued research in the field of shock waves.

To all of my committee members, Dr. Arun Shukla, Dr. David Taggart, Dr. Michael Greenfield, Dr. Donna Meyer, and Dr. Otto Gregory, I deeply appreciate the time and consideration that you have all provided me. Within all of your classes, I have been consistently inspired to improve my understanding of mechanics from the unwavering enthusiasm shown by all of you. Dr. Shukla, although I am not one of your students, our relationship has made me feel otherwise, where you have always made time for me, and have continued to provide me with guidance and advice. If it were not for your suggestions within my comprehensive exam, I likely would never have challenged myself to develop the HRUM. I would additionally like to thank you for continuing to extend any resource within you power to aid in my research. Dr. Taggart, I am grateful for the assistance that you provided during my finite element modeling attempts with Abaqus. Dr. Greenfield, your enthusiasm and constant desire
to tackle problems from all sectors of engineering is completely inspirational. Dr. Meyer, I feel indebted to you for inspiring me to pursue a graduate degree, where if I had not known you, I would probably have settled with my bachelor’s degree and continued to industry. I am especially grateful for our many discussions, and you allowing me to come to your office for guidance at any time. Dr. Gregory, thank you for introducing me to microscopy, I do not think my research into the spall fracture of cast iron could have ended in anything but failure without your respected teachings. I deeply appreciate your willingness to extend the use of your SEM to my investigations. I would also like to acknowledge Dr. Hamouda Ghonem for his continued support of my endeavors, and allowing me to utilize any resources within his lab that could benefit my research. Additionally, I would like to acknowledge the rest of the faculty within the engineering department for the well prepared lectures that I received while completing my course work at URI.

Special thanks are due to Joseph Gomez, for his guidance and advice in the machine shop. Although my continued studies were primary focused on engineering, I am also leaving URI with an apprenticeship in machining. Within my time here, Joe continued to teach me skills and trade tricks that could only have been learned from a lifetime in the field. I would like to acknowledge James Byrnes and Robert D’Ambrosca for the technical support provided to me while at URI. I am grateful for the clerical assistance provided to me by Jen Cerullo. I would like to acknowledge the assistance provided by Annie Jones, and Jean McCullough within the Dean’s office. Additionally, I would like to thank the rest of the staff within the Department of Mechanical, Industrial and Systems Engineering (MCISE).
I would like to acknowledge my family and friends for continuing to support me in my studies. I would especially like to thank my dad for the inspiration that he has provided me all of these years. I would like to acknowledge Krys for her role as a motherly figure in my life for the past twenty years. I am grateful for the unwavering support given to me by my better half, Heather, and the fact that she was willing to put her life on hold while I finished my degree at URI. Special thanks are due to Gerralyn Perry for providing the driving force for me to continue my education, and helping me to resolve issues associated with the university’s frustrating e-campus system. I would like to acknowledge one of my close friends, Nathaniel Gardner, for our special relationship, where our heated debates and study sessions may have been quite alarming to outside observers. I am grateful for the productive design discussions that I have had with William Visser. I would like to thank Miguel Goñi for his inquisitive mind, constantly keeping me on my toes, and therefore challenging my understanding of mechanics. To the rest of my fellow graduate students, I am grateful to have spent the last several years working amongst you.

Finally, I would like to acknowledge that the research presented in this thesis was made possible by the funding provided by the United States Department of Energy, and Department of Homeland Security.
# TABLE OF CONTENTS

ABSTRACT ............................................................................................................................. ii

ACKNOWLEDGEMENTS ........................................................................................................ vi

TABLE OF CONTENTS .......................................................................................................... ix

LIST OF TABLES ................................................................................................................... xiv

LIST OF FIGURES ................................................................................................................ xv

CHAPTER 1: INTRODUCTION ............................................................................................... 1
  1.1 BACKGROUND .................................................................................................................. 1
  1.2 MOTIVATION .................................................................................................................... 4
  1.3 OUTLINE .......................................................................................................................... 6
  1.4 REFERENCES .................................................................................................................. 9

CHAPTER 2: THEORETICAL CONSIDERATIONS .............................................................. 16
  2.1 CONSERVATION RELATIONS ...................................................................................... 17
  2.2 SHOCK WAVES ............................................................................................................. 20
  2.3 THERMODYNAMIC CONSIDERATIONS ...................................................................... 24
  2.4 PLATE IMPACT EXPERIMENTAL TECHNIQUE .......................................................... 26
  2.5 SPALL FRACTURE CONSIDERATIONS ....................................................................... 30
  2.6 SHOCK WAVES IN LAYERED PLATES ......................................................................... 41
  2.7 RADIAL RELEASE WAVES ............................................................................................ 50
  2.8 REFERENCES ................................................................................................................ 53
4.3.2 Sample Preparation ................................................................. 103
4.3.3 Methods of Graphite Classification ................................. 105
4.3.4 Application of Graphite Classification to the Studied Materials ... 109
4.3.5 Identification of Additional Phases ................................... 114
4.4 REFERENCES .......................................................... 131

CHAPTER 5: SPALL FRACTURE OF CAST IRON ......................... 134
5.1 EXPERIMENTAL DESIGN ................................................ 134
5.2 RESULTS FOR THE CAST IRONS STUDIED .......................... 142
5.3 THE SPALL STRENGTH OF 6061 ALUMINUM ......................... 151
5.4 CORRELATION OF RESULTS TO MICROSTRUCTURE .............. 154
5.4.1 Initiation of Failure in Ductile Cast Iron ..................... 155
5.4.2 Initiation of Failure in Gray Cast Iron ....................... 159
5.4.3 Graphite-Matrix Bond Strength ............................... 163
5.4.4 The Effect of Graphite Size on Spall Strength .................. 165
5.4.5 Mechanisms Governing the Completion of Failure ............. 166
5.5 CONCLUDING REMARKS .............................................. 173
5.6 REFERENCES .......................................................... 174

CHAPTER 6: GENERATING A MATERIAL’S HUGONIOT .............. 178
6.1 HRUM WITH 6061 ALUMINUM AND POLYCARBONATE .................... 178
6.1.1 Experimental Design .................................................. 179
A.2 PROCEDURE FOR USING THE SYSTEM ................................................................. 237

A.2.1 Initial Setup ........................................................................................................ 237

A.2.2 Gas Gun Loading Procedure ............................................................................ 239

A.2.3 Final Setup Procedure ...................................................................................... 240

A.2.4 Firing Procedure ............................................................................................... 241

A.2.5 Completion of an Experiment .......................................................................... 242

APPENDIX B: MATLAB CODES FOR THE HRUM ................................................. 244

B.1 SOLVING FOR THE INNER-LAYER ................................................................. 244

B.2 SOLVING FOR THE BULK MATERIAL .............................................................. 246

B.2.1 Assuming a linear relationship between $U_s$ and $u_p$ .................................. 246

B.2.2 Assuming a polynomial relationship between $U_s$ and $u_p$ ......................... 248

BIBLIOGRAPHY ......................................................................................................... 251
## LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table 4.1.</strong> Densities and wave speeds of the materials studied</td>
<td>96</td>
</tr>
<tr>
<td><strong>Table 4.2.</strong> Acoustic impedances of the materials studied</td>
<td>97</td>
</tr>
<tr>
<td><strong>Table 4.3.</strong> Elastic constants of the materials studied</td>
<td>98</td>
</tr>
<tr>
<td><strong>Table 4.4.</strong> Chemical composition of gray cast irons expressed as weight percentage</td>
<td>102</td>
</tr>
<tr>
<td><strong>Table 4.5.</strong> Summary of graphite volume fractions for the cast irons studied</td>
<td>114</td>
</tr>
<tr>
<td><strong>Table 4.6.</strong> Recommended number of test points as a function of visually estimated area fractions recreated from ASTM E562</td>
<td>128</td>
</tr>
<tr>
<td><strong>Table 4.7.</strong> Volume fractions of phases identified within the cast irons studied</td>
<td>129</td>
</tr>
<tr>
<td><strong>Table 5.1.</strong> Summary of results from the spall fracture experiments on cast iron</td>
<td>145</td>
</tr>
<tr>
<td><strong>Table 5.2.</strong> Summary of results from the spall fracture experiments on 6061 Aluminum</td>
<td>154</td>
</tr>
<tr>
<td><strong>Table 6.1.</strong> Experimentally determined Hugoniot states of polycarbonate</td>
<td>192</td>
</tr>
<tr>
<td><strong>Table 6.2.</strong> Experimentally determined Hugoniot states of 6061 aluminum</td>
<td>198</td>
</tr>
<tr>
<td><strong>Table 6.3.</strong> Hugoniot states for A572 grade 50 structural steel</td>
<td>205</td>
</tr>
<tr>
<td><strong>Table 6.4.</strong> Experimentally determined Hugoniot states of polycarbonate</td>
<td>211</td>
</tr>
<tr>
<td><strong>Table 6.5.</strong> Experimentally determined Hugoniot states of steel</td>
<td>217</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Figure 2.1.</strong> Generation of a compression pulse by the impact of a flyer plate on a target; (a) Time-distance diagram; (b) Stress-Particle Velocity plot; (c) Pressure profiles; (d) Radial-release and edge effects</td>
<td>29</td>
</tr>
<tr>
<td><strong>Figure 2.2.</strong> Decomposition of a spall experiment; (a) Time-distance diagram; (b) Stress-Particle Velocity plot; (c) Stress transmitted to the window</td>
<td>32</td>
</tr>
<tr>
<td><strong>Figure 2.3.</strong> Stress-Particle velocity plot for cast iron and polycarbonate where the solid lines represent stress generated from a known Hugoniot in eq. (2.17) and the dashed lines represent stress calculated by the acoustic approximation in eq. (2.21)</td>
<td>40</td>
</tr>
<tr>
<td><strong>Figure 2.4.</strong> Wave processes associated with an incident pulse acting on a thin layer in an otherwise homogenous structure; (a) t-X diagram when the inner layer is of lower impedance; (b) $\sigma$-$u_p$ diagram associated with part (a); (c) t-X diagram when the inner layer is of higher impedance; (d) $\sigma$-$u_p$ diagram associated with part (c)</td>
<td>43</td>
</tr>
<tr>
<td><strong>Figure 2.5.</strong> Construction of the inner-layer’s principal Hugoniot from an experiment where the Hugoniot of the bulk plate is known, generated with published Hugoniot data for 6061 Aluminum and Polycarbonate</td>
<td>47</td>
</tr>
</tbody>
</table>
Figure 2.6. Radial release waves associated with a symmetrical impact............. 53
Figure 3.1. The plate impact apparatus .............................................................. 58
Figure 3.2. The inside of the vacuum chamber...................................................... 62
Figure 3.3. Test sample assembled to the sample mount....................................... 65
Figure 3.4. Sectional view of the apparatus demonstrating the relative position
of the velocity traps .......................................................................................... 70
Figure 3.5. Manganin stress gauge; (a) Partial section view showing the active
grid and kapton insulation; (b) Final gauge assembly ..................................... 77
Figure 3.6. Calibration curve for 50 Ω manganin gauges manufactured by
Dynasen® ......................................................................................................... 78
Figure 3.7. Projectile assembly; (a) Actual projectile; (b) SolidWorks section
view.................................................................................................................. 85
Figure 3.8. Sample assembly; (a) Spall fracture sample; (b) HRUM sample .......... 88
Figure 4.1. The pulse-echo ultrasonic method....................................................... 93
Figure 4.2. ASTM A247 classification of types of graphite in cast iron ............108
Figure 4.3. ASTM A247 classification of distributions of graphite in cast iron.... 109
Figure 4.4. Optical micrographs showing graphite structures in polished
samples at 100x magnification. (a) Cast Iron A; (b) Cast Iron B;
(c) Cast Iron C; (d) Cast Iron D; (e) Ductile Cast Iron................................. 110
Figure 4.5. Example showing transformation of an original color micrograph
into a grayscale and then a binary image for use in determining
the area fraction of the graphite...................................................................... 114
Figure 4.6. Optical micrographs of cast iron A etched with 4% nital; (a) 200x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G; Steadite as S; Manganese sulfite as MS.

Figure 4.7. Optical micrographs of cast iron B etched with 4% nital; (a) 200x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G; Steadite as S; Manganese sulfite as MS.

Figure 4.8. Optical micrographs of cast iron C etched with 4% nital; (a) 200x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G; Manganese sulfite as MS.

Figure 4.9. Optical micrographs of cast iron D etched with 4% nital; (a) 400x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G; Manganese sulfite as MS.

Figure 4.10. Optical micrographs of ductile cast iron etched with 4% nital; (a) 200x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G.

Figure 4.11. Optical micrographs of steadite colonies in cast iron A viewed at 1000x magnification; (a) Polished and tint etched with Murakami reagent; (b) Pre-etched with 4% nital before tint etching with Murakami’s reagent. Iron phosphide is denoted as IP; Ferrite as F; Pearlite as P; Voids as V.
**Figure 4.12.** Optical micrographs of steadite colonies in cast iron B viewed at 1000x magnification; (a) Polished and tint etched with Murakami reagent; (b) Pre-etched with 4% nital before tint etching with Murakami’s reagent. Iron phosphide is denoted as IP; Ferrite as F; Pearlite as P. 125

**Figure 4.13.** Optical micrographs of steadite colonies in cast iron C viewed at 1000x magnification; (a) Polished and tint etched with Murakami reagent; (b) Pre-etched with 4% nital before tint etching with Murakami’s reagent. Iron phosphide is denoted as IP; Ferrite as F; Pearlite as P; Voids as V. 126

**Figure 4.14.** Rejected cast iron samples demonstrating porosity; (a) cast iron B; (b) cast iron C. 130

**Figure 5.1.** Time-Distance diagram for the cast iron spall experiments. 137

**Figure 5.2.** Transmitted stress profile for an experiment on ductile cast iron impacted at 300m/s representative of a model test. 138

**Figure 5.3.** Stress-particle velocity plot demonstrating the response of the manganin stress gauges used in the subsequent study. 140

**Figure 5.4.** Recorded stress profiles transmitted to the polycarbonate window. (a) Cast Iron A; (b) Cast Iron B; (c) Cast Iron C; (d) Cast Iron D; (e) Ductile Cast Iron. 143

**Figure 5.5.** Example of a recovered sample cut to expose the spall plane. 146

**Figure 5.6.** Summary of spall strengths determined for the five cast irons studied. 148
Figure 5.7. Spall strength versus unloading rate for the five cast irons studied ..... 150

Figure 5.8. Recorded stress profiles transmitted to the polycarbonate window for two spall fracture experiments on 6061 Aluminum............................ 153

Figure 5.9. SEM micrographs of the spall plane in the ductile cast iron sample impacted at 190 m/s; (a) 75x magnification; (b) 200x magnification of boxed region in (a). Graphite is denoted as G; Cracks as C; Polishing pullout as P .................................................. 156

Figure 5.10. SEM micrographs of the spall plane in the ductile cast iron sample impacted at 300 m/s; (a) 70x magnification; (b) 150x magnification of boxed region in (a). Graphite is denoted as G; Cracks as C; Microcracks as C_M; Evidence of graphite debonding from the matrix is denoted as D ................................................................. 157

Figure 5.11. SEM micrograph taken at 250x magnification of the spall plane in the ductile cast iron sample impacted at 300 m/s. Graphite is denoted as G; Evidence of graphite debonding from the matrix is denoted as D ................................................................. 158

Figure 5.12. SEM micrograph taken at 500x magnification of the spall plane in the cast iron D sample impacted at 120 m/s. Graphite is denoted as G; Evidence of graphite debonding from the matrix is denoted as D ................................................................. 160
**Figure 5.13.** SEM micrographs taken at 500x magnification of the spall plane in the cast iron D sample impacted at 120 m/s; (a) Backscattered electrons in combo mode; (b) Secondary electrons. Graphite is denoted as G; Cracks as C; Initiation of graphite debonding as $D_I$; Completely debonded graphite as $D_c$.

**Figure 5.14.** Spall strengths plotted as a function of average graphite size class.

**Figure 5.15.** SEM micrograph taken at 100x magnification of the spall plane in the cast iron D sample impacted at 120 m/s. Graphite with type A distribution is denoted as $G_A$; Graphite with type $D$ distribution is denoted as $G_D$; Cracks are denoted C. The direction of the acting tensile stress (T) is indicated on the left of the micrograph.

**Figure 5.16.** SEM micrograph taken at 200x magnification of the spall plane in the cast iron D sample impacted at 190 m/s. The spall plane is denoted as SP; The region containing Type VII $A_4$ Graphite is denoted as $G_A$; The region containing Type VII $D_8$ Graphite is denoted as $G_D$.

**Figure 5.17.** SEM micrographs adjacent to the spall plane in the cast iron D sample impacted at 190 m/s; (a) 100x magnification; (b) 1000x magnification of boxed region in (a). The spall plane is denoted as SP; Cracks as C; Type VII $A_4$ graphite as $G_A$; Type VII $D_8$ graphite as $G_D$. 
Figure 6.1. Time-distance diagram for the HRUM experiments utilizing 6061 aluminum as the bulk material and polycarbonate as the inner-layer ................................................................. 181

Figure 6.2. The Hugoniot of 6061 aluminum in stress-particle velocity space ...... 184

Figure 6.3. The Hugoniot of polycarbonate in stress-particle velocity space........ 185

Figure 6.4. Stress records from the first test on aluminum/polycarbonate with an inner-layer thickness of 1.27 mm and an impact velocity of 320 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges ........................................................................................................ 187

Figure 6.5. Stress records from the second test on aluminum/polycarbonate with an inner-layer thickness of 0.838 mm and an impact velocity of 326 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges ........................................................................................................ 188

Figure 6.6. Stress records from the third test on aluminum/polycarbonate with an inner-layer thickness of 0.508 mm and an impact velocity of 325 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges ........................................................................................................ 189

Figure 6.7. Stress records from the fourth test on aluminum/polycarbonate with an inner-layer thickness of 0.508 mm and an impact velocity of 326 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges ........................................................................................................ 190
Figure 6.8. Stress-particle velocity plot of polycarbonate’s Hugoniot comparing the shocked states obtained through use of the HRUM to published data ................................................................. 193

Figure 6.9. Shock velocity plotted as a function of particle velocity; (a) Test 1; (b) Test 2; (c) Test 3; (d) Test 4 ................................................................. 195

Figure 6.10. Deviation of calculated stresses utilizing the Hugoniot equations obtained from the HRUM from the real Hugoniot of polycarbonate, plotted as a function of particle velocity ....................... 197

Figure 6.11. Stress-particle velocity plot of 6061 aluminum’s Hugoniot comparing the shocked states obtained through use of the HRUM to published data ................................................................. 199

Figure 6.12. Time-distance diagram for the HRUM experiments utilizing steel as the bulk material and polycarbonate as the inner-layer ......................... 203

Figure 6.13. Five experiments conducted on A572 grade 50 structural steel demonstrating traditional methods of Hugoniot determination; (a) Stress histories of the experiments; (b) Stress-particle velocity plot of the experiments ................................................................. 205

Figure 6.14. The Hugoniot of steel in stress-particle velocity space .................. 207

Figure 6.15. Stress records from the first test on steel/polycarbonate with an inner-layer thickness of 0.508 mm and an impact velocity of 300 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges ................................................................. 209
Figure 6.16. Stress records from the second test on steel/polycarbonate with an inner-layer thickness of 0.508 mm and an impact velocity of 304 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges .................................................. 210

Figure 6.17. Stress-particle velocity plot of polycarbonate’s Hugoniot comparing the shocked states obtained through use of the HRUM to published data .................................................. 212

Figure 6.18. Shock velocity plotted as a function of particle velocity; (a) Test 1; (b) Test 2 .................................................. 213

Figure 6.19. Deviation of the calculated stresses utilizing the Hugoniot equations obtained from the HRUM from the real Hugoniot of polycarbonate, plotted as a function of particle velocity ................. 214

Figure 6.20. Deviation of the calculated stresses utilizing either a linear fit or a polynomial fit to the shock velocity-particle velocity data obtained from test 1 .................................................. 216

Figure 6.21. Stress-particle velocity plot of steel’s Hugoniot comparing the shocked states obtained through use of the HRUM to published data .................................................. 218

Figure 7.1. Stress-particle velocity plots generated from the analysis of the data from test 1 on the steel/polycarbonate configuration; (a) Determining the Hugoniot of the inner-layer; (b) Determining the Hugoniot of the bulk material .................................................. 227
Figure 7.2. Stress-particle velocity plot demonstrating the material response associated with extreme shocked states ................................................ 229

Figure 7.3. Stress-particle velocity plot depicting the first phase of the suggested improved analysis ................................................. 231

Figure 7.4. Stress record captured by the front gauge in test 1 of the steel/polycarbonate configuration ............................................... 232

Figure A.1. Important components of the gas gun; (a) Hardware used to connect the vessel to the helium tank; (b) gas gun pressure vessel ...... 235

Figure A.2. Important components of the vacuum chamber; (a) Depiction of the gas gun side of the chamber; (b) Location of the diagnostic systems inside the chamber; (c) Keyence velocity trap control unit located on the door side of the chamber; (d) The vacuum pump assembly ................................................................. 236

Figure A.3. Additional components of the vacuum chamber; (a) Vacuum dump valve located on the opposite side of the chamber from the gas gun; (b) Power protection unit located under the vacuum chamber ........................................................................................................... 237

Figure A.4. The soft recovery setup; (a) Example of clay packing within the catch box; (b) Final position and assembly for the soft recovery setup ................................................................................................................ 238

Figure A.5. Final vacuum chamber setup; (a) Installation of the safety shield; (b) Closing and latching of the vacuum chamber door ................. 241

Figure A.6. Final firing procedure .................................................................................................................. 242
CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

The response of materials to shock loading events is a common theme in literature. There exist numerous books detailing the theory required for the fundamental understanding and interpretation of waves in condensed matter [1-6]. Understanding of the fundamentals outlined in these books is necessary before one is able to pursue research in the field of shock physics. Studies in the area have been extended to analytical, numerical, and experimental approaches. The main goal of these investigations is to develop methods for predicting the effects of explosions, high-velocity collisions, and various other intense dynamic loading events on materials and structures. The true goal of this research field will be achieved when computer models can accurately simulate these processes of interest. Accuracy of these simulations depends on developments of thermo-physical equations of state, and therefore requires experimental investigations into idealized loading situations. For the focus of this thesis, an experimental approach will be utilized in the investigation of shocked states of given materials. The aim of this research is to provide validating data to aid in the development of an overall understanding of material states and failure mechanisms in the study of shock waves in condensed matter.
Shock waves are typically generated in a laboratory setting by impact of a flyer plate on a target [7, 8], detonation of explosive charges [9, 10], or laser ablation [11, 12]. The wave evolution inside a test sample is commonly measured by either capturing a temporal record of the free surface velocity or stress profile. In the case of free surface velocity measurements, the most commonly employed method is the use of the Velocity Interferometer System for Any Reflector (VISAR) [13-15]. This method utilizes an interferometer to measure the minute Doppler shift in light frequency of a laser beam as it is reflected from a moving sample surface. Within recent years a new technique for particle velocity measurement known as Photonic Doppler Velocimetry (PDV) has been developed at Lawrence Livermore National Laboratory (LLNL) [16-18]. For stress measurements, sensors are typically embedded within layers of test material or mounted between the back surface of a test sample and a low impedance window. Sensors used to measure shock-loaded samples include piezoresistance gauges, dielectric gauges, and ferroelectric gauges. In all cases, gauge output is carefully calibrated so that it may be transformed to an assumed stress value. The most commonly used gauges in literature are manganin piezoresistive gauges [19-22] and PVDF ferroelectric gauges [23-26].

Shock waves are generated when loading is sufficient to initiate plastic flow. The threshold compressive stress associated with this condition is referred to as the Hugoniot Elastic Limit (HEL). Investigations into the HEL of different materials are common in literature [27-34]. Researchers have investigated the mechanisms of failure associated with this phenomenon [27-29] and frequently sought a relationship between the HEL and the yield strength [30-33].
Shock waves are large amplitude disturbances across which the stress, density, and other physical properties change in a discontinuous manner. The relationship between the parameters of state reached by shock compression is known as the Hugoniot of a material. Construction of a material’s Hugoniot is fundamental in the development of an Equation of State (EOS) and is a common theme in shock studies [34-39]. In work by Rosenberg, et al., it has been shown that by varying the impact velocity and capturing temporal records of the shock amplitude with use of in situ manganin gauges, the Hugoniot curve can be generated for materials of interest [34-36]. With the use of multiple gauges embedded at known distances within a sample, the shock velocity can be determined as a function of the shock induced state [37-39].

One problem of particular interest in the investigation of high rate events is the study of dynamic fracture strength or spall strength. Spallation is a dynamic material failure mode that occurs when tensile stresses are generated by the interaction of two decompression waves. The failure mechanism of spallation has been widely investigated since it was first identified by Hopkinson in 1914 [40-46]. The study of spallation requires active measurements of the dynamic time-dependent stress or particle velocity. Equations relating the observed wave forms to a critical failure stress have been well established in literature [47, 48]. The spall strength of different materials has been studied as a function of pulse duration [49, 50], strain rate [49, 51], temperature [50, 52], and loading history [53].

In the study of spallation, it is important to point out that the material failure process is strongly influenced by microstructure. Active measurements of a material’s spall strength alone can not completely describe the process. The occurrence of spall
involves the nucleation, growth, and eventual coalescence of an array of cracks initiated at the spall plane. It is commonly noted in literature that pre-existing defects within a material’s microstructure often serve as damage nucleation sites under the action of dynamic tensile loading [3]. Second phase particles in an otherwise homogeneous material, grain boundaries, microcracks, and voids are examples of such defects that can strongly influence the mechanisms involved in dynamic fracture. It has been widely accepted in literature that passive measurements such as post mortem examinations of the recovered samples are necessary to gain a complete understanding of the fracture mechanisms and kinetics of the material failure process [54-62].

1.2 MOTIVATION

Generation of a material’s Hugoniot is a fundamental theme in literature, and has been the focus of countless researchers in the field of shock physics. A material’s Hugoniot, coupled with a valid thermodynamic EOS, allows researchers to accurately develop models to predict all states of matter associated with dynamic loading. Traditionally, the locus of points necessary to construct a material’s Hugoniot is obtained through a rigorous series of experiments, where each test produces a single data set. Often, an experimental study into the dynamic response of a material, such as the study of spall fracture, requires a well defined Hugoniot in the subsequent analysis of the results. In such a situation, the primary objective of the study will undergo significant delays if a researcher has to begin by initially determining the material’s Hugoniot. Thus, there is a strong necessity to develop a method that can significantly reduce the process of obtaining Hugoniots.
In literature, there are limited approaches that can be used to obtain more than one Hugoniot state from a single experiment. In work by Brown, a novel technique was developed utilizing converging shock waves, which has been shown to significantly extend the pressure ranges achievable from any specific test apparatus [63-66]. An interesting aspect of this experimental technique is that it inherently creates a gradient of pressures and particle velocities across the sample’s free surface. Theoretically, if one was able to obtain quality spatial and time resolution of the particle velocity or pressure across this gradient, a large range of a material’s Hugoniot could be obtained from a single experiment. In this work, Brown utilized the Optically Recording Velocity Interferometer System (ORVIS) to obtain full field particle velocity information, in an attempt to capture an entire Hugoniot from a single experiment [63]. However the case, the ORVIS requires significant developments to be utilized, and these experiments lacked the spatial resolution to accurately capture the Hugoniot states. Within the thesis of Brown, it was shown that multi-point VISAR experiments could be used to obtain multiple states of a material’s Hugoniot, where each probe would capture a separate data set [66]. Although promising, the method developed by Brown violates conditions of one-dimensional strain, and therefore requires completed analysis and expensive diagnostics to be employed. Thus, despite these significant developments, there still remains a necessity to develop a method that can obtain multiple states along a material’s Hugoniot from a single experiment which can universally be applied using the classical Rankine-Hugoniot equations widely employed in shock wave research.
The study of spall fracture of materials is an important focus within shock physics. Often, man-made structures are designed with an internal cavity to house either people or items (e.g. vehicles, shelters, tunnels, pipes, etc.), and therefore these structures inherently possess a geometry that will facilitate spallation under dynamic loading conditions. One problem of particular significance is the spall fracture of cast iron, where there is a noteworthy scarcity of research on the topic published in literature. Cast iron is one of the oldest cast ferrous products traditionally chosen for many engineering applications. Despite the development of advanced engineering materials, cast iron remains widely in use today due to its low cost, easy castability, relatively good machinability, and wide range of achievable mechanical properties [67-70]. In light of the material’s continued use, there is a strong necessity to investigate its response to dynamic fracture in order to quantify its spall strength and identify the fracture mechanisms and kinetics respective to its failure process.

1.3 OUTLINE

The structure of this thesis is as follows: Following this introduction (Chapter 1), Chapter 2 will present the fundamental theoretical considerations to the extent necessary for subsequent discussion of the dynamic experiments conducted in this thesis. It will begin by outlining the conservation relations for wave propagation considering the assumption of one-dimensional motion in a compressible continuous media, which in turn underlie all subsequent solutions. Next, the classical analysis of shock waves, using the well known Rankine-Hugoniot equations will be addressed. Following this, some thermodynamic relations that allow for a complete
characterization of a material’s equation of state (EOS) will briefly be discussed. An outline of the generation of shock waves with the use of the plate impact experimental technique will be presented. The experimental technique used to study the spall fracture of cast irons will be outlined, and theory pertaining to the experiments conducted in this thesis will be described. The consideration of the shock wave processes associated with layered plates that will directly pertain to a new method proposed to generate multiple Hugoniot data points in a single experiment will be addressed. Chapter 2 will conclude with concerns associated with radial release waves in the experiments.

Chapter 3 will address the details pertaining to the experimental technique used in this thesis to study the dynamic response of materials. It will begin by introducing the hardware involved in the plate impact apparatus. A complete description of the relevant diagnostics, including velocity and stress measurement techniques will be given. Details associated with projectile design and sample fabrication will also be included.

Chapter 4 will outline the measurements and calculations of the initial state parameters for the materials studied. These materials will include five different cast irons, polycarbonate, A572 Grade 50 structural steel, and 6061 aluminum. Details pertaining to the determination of the initial state parameters, including density and initial wave speeds, will be presented in this chapter. Following this, the acoustic impedances and dynamic elastic constants constructed from the experimentally determined wave speeds will be presented. Chapter 4 will conclude with an
investigation into the as-received microstructures of the five cast irons studied in the spall fracture experiments.

Chapter 5 will discuss the results of the spall fracture experiments conducted on cast iron. It will begin by outlining the experimental design, which will include the choice of sample dimensions and the subsequent results expected. A summary of the results from the spall fracture experiments on the five cast irons studied will be given. Spurred by the unexpected nature of the results found for these cast irons, a small investigation into the spall strength of 6061 Aluminum will be included to address concerns associated with the technique employed and equations utilized for the calculation of spall strengths in this thesis. The chapter will conclude by correlating the spall strengths of the cast irons to their respective microstructures.

Chapter 6 will address the results from the Hugoniot ring-up method (HRUM), proposed to obtain multiple Hugoniot states from a single experiment. The HRUM will be employed in two configurations, utilizing either 6061 Aluminum or A572 grade 50 structural steel as the bulk material, while employing polycarbonate as the inner-layer. In both cases the method will first be used to determine the Hugoniot of the inner-layer from the known Hugoniot of the bulk material. Conversely, the method will then be employed to generate the Hugoniot of the bulk material from the known Hugoniot of the inner-layer. In all cases, validation of the new technique will be accomplished through comparison of the Hugoniots generated from the method with published data found in literature.

Chapter 7 will summarize the key points found in this thesis. It will include conclusions from the spall fracture study on cast iron. Following this, key notes from
the HRUM experiments will be provided. The chapter will present several ideas for future work that could further extend the usefulness of the HRUM technique.

Within the Appendices, supplementary information is provided that may prove useful to future researchers. Appendix A contains a detailed procedure for utilizing the gas gun apparatus. Appendix B contains the Matlab codes that were used in the HRUM experiments to solve for the unknown particle velocities through the method of impedance matching. The code found in Appendix B.1 can be utilized to determine the Hugoniot of a low impedance inner-layer with knowledge of the Hugoniot of the high impedance bulk material. Conversely, the codes found in Appendix B.2 can be utilized to determine the Hugoniot of the bulk material from the known Hugoniot of the inner-layer. It is assumed that the relationship between shock velocity and particle velocity are defined by either a linear or polynomial functions respectively, in the codes found in appendices B.2.1 and B.2.2.

1.4 REFERENCES


CHAPTER 2

THEORETICAL CONSIDERATIONS

This chapter presents fundamental theoretical considerations to the extent necessary for subsequent discussion of the dynamic experiments conducted in this thesis. A comprehensive account detailing the theory required for the understanding and interpretation of waves in condensed matter can be found in text books by Meyers (1994); Davison, Grady, and Shahinpoor (1996); Antourn, et al. (2003); Zhernokletov and Glushak (2006); Kanel, Razorenov, and Fortov (2004) [1-5]. This chapter will begin by outlining the conservation relations for wave propagation considering the assumption of one-dimensional motion in a compressible continuous media, which in turn underlie all subsequent solutions. Next, the classical analysis of shock waves with use of the well known Rankine-Hugoniot equations will be addressed. Following this, some thermodynamic relations that allow for a complete characterization of a material’s equation of state (EOS) will briefly be discussed. An outline of the generation of shock waves with the use of the plate impact experimental technique will be presented. The experimental technique used to study the spall fracture of cast irons will be outlined, and theory pertaining to the experiments conducted in this thesis will be described. The consideration of the shock wave processes associated with layered plates that will directly pertain to the new method proposed to generate
multiple Hugoniot data points in a single experiment will be addressed. This chapter will conclude with concerns associated with radial release waves in the experiments.

2.1 CONSERVATION RELATIONS

The theoretical analysis of stress wave and shock wave propagation typically begins with use of the classic equations of motion. These equations, commonly known as the conservation equations, are universal equations that can be applied to all materials that satisfy the underlying assumptions of continuum mechanics. Since experimental conditions within the laboratory typically employ measurements of stress waves where the sensor employed is fixed relative to the material, the conservation equations are almost exclusively derived in Lagrangian coordinates. Also key to experimental conditions is the concept of uniaxial strain, where experiments involving the normal impact of plane parallel surfaces are assumed to produce a one-dimensional state of strain. The conservation equations developed in Lagrangian coordinates under the assumption of one-dimensional motion for the conservation of mass, momentum, and energy, respectively, take the form:

\[ \frac{\partial u_p}{\partial X} = \rho_o \frac{\partial V}{\partial t} \]  
\[ \frac{\partial \sigma_{11}}{\partial X} = -\rho_o \frac{\partial u_p}{\partial t} \]  
\[ \frac{\partial E}{\partial t} = -\sigma_{11} \frac{\partial V}{\partial t} \]

where \( X \) is the Lagrangian position of a particle, \( t \) is time, \( u_p \) is the particle velocity, \( \rho_o \) is the initial density, \( V \) is the specific volume, \( \sigma_{11} \) is the Cauchy stress in the direction
of wave propagation taken to be positive in compression, and \( E \) is the specific internal energy.

It should be noted that these equations form an incomplete system and alone, do not permit solutions to wave propagation problems. It is easily observed that the number of unknown independent variables including \( u_p, \rho, \sigma_{11}, \) and \( E \) exceeds the number of equations by one variable. In order to form a mathematical solution, an additional constitutive relation referred to as an equation of state (EOS) is often introduced. Commonplace in shock wave studies, the response of a material is assumed to be adiabatic in that the system is assumed to have zero heat flux \( (Q = 0) \). In this case the additional EOS is typically employed to generate a relationship between internal energy \( (E) \), pressure \( (\sigma) \), and specific volume \( (V) \) in the form:

\[
E = E(\sigma, V) \tag{2.4}
\]

In some cases the assumption of an adiabatic process is unacceptable, violating the laws of thermodynamics. For these situations the EOS is typically founded to relate stress to kinematical and thermodynamic variables. Often such an EOS will specify one thermodynamic quantity as a function of two other quantities taking the form:

\[
E = E(V, S) \tag{2.5}
\]

where \( S \) is the system’s entropy. For the case of this thesis, the assumption of an adiabatic process will be utilized, giving rise to the use of an EOS of the form found in equation (2.4). Specifics related to the use of an EOS of the form found in equation (2.4) will be outlined in section 2.3 where thermodynamic considerations are addressed.
There exist two standard solution procedures for solving problems in wave propagation. The first procedure involves the use of numerical techniques such as the finite difference method or finite element method to reduce the differential conservation relations to a set of ordinary algebraic equations. The second procedure employs the theory of characteristics to generate this reduction. For the purpose of this thesis the method of characteristics will briefly be outlined because its mathematical procedure is closely related to the wave motion and therefore aids in the understanding and interpretation of waves in the subsequent studies. Characteristics represent trajectories in time-distance space across which stress waves propagate. Important to the fundamental understanding of characteristics is the Lagrangian sound velocity \( a \), which can be related to the velocity \( c \) in the laboratory coordinate system by the equation:

\[
a = \frac{\rho}{\rho_0} c = \frac{\rho}{\rho_0} \sqrt{\frac{\partial \sigma}{\partial \rho}} \quad (2.6)
\]

where \( \sigma \) is the stress in the direction of wave propagation, and the notation \(( \ )_s\) indicates that the derivative is taken along the isentrope, a path of constant entropy. For isentropic flow the characteristics \( C^+ \) and \( C^- \) respectively represent the trajectories of perturbations in the positive and negative directions governed by the equation:

\[
\frac{\partial X}{\partial t} = \pm a 
\quad (2.7)
\]

where the variation of the material state along these characteristics in the time-distance space is described by the set of differential equations:

\[
\frac{du}{ds} + \frac{1}{\rho_0 a} \frac{d\sigma}{ds} = 0 \quad \text{along} \ C^+ \text{ characteristics} \quad (2.8)
\]
\[ \frac{du}{ds} - \frac{1}{\rho_o a} \frac{d\sigma}{ds} = 0 \quad \text{along C- characteristics} \quad (2.9) \]

and the integrals of these equations are the Riemann integrals:

\[
\begin{align*}
\sigma &= u_o - \int_{\sigma_o}^{\sigma} \frac{d\sigma}{\rho_o a} \quad \text{along C+ characteristics} \quad (2.10) \\
\sigma &= u_o + \int_{\sigma_o}^{\sigma} \frac{d\sigma}{\rho_o a} \quad \text{along C- characteristics} \quad (2.11)
\end{align*}
\]

where \( u_o \) and \( \sigma_o \) are integration constants.

A flow in which all disturbances propagate in the same direction is frequently termed a simple or progressive wave. For a simple wave, the states along characteristics in the direction of wave propagation remain constant, while all states along any other path in \( X-t \) space are described by a function \( u(\sigma) \) or \( \sigma(u) \) corresponding to the Riemann integrals. When all characteristics originate at a single point in the \( X-t \) plane, the wave is referred to as a centered simple wave. The slope of the Riemann invariant which refers to trajectories in the stress-particle velocity plane defines a material’s dynamic impedance written as:

\[
\frac{d\sigma}{du} = \pm \rho_o a \quad (2.12)
\]

### 2.2 SHOCK WAVES

Shock waves can be generated when a material is impacted at a rate sufficient to initiate plastic flow. The threshold compressive stress associated with this condition for a material in a state of uniaxial strain is referred to as the Hugoniot Elastic Limit (HEL). Once generated, shock waves are large amplitude disturbances
across which the stress, density, and other physical properties are assumed to change in a discontinuous manner. The relationship between the parameters of state reached by shock compression is known as the Hugoniot of a material. The Hugoniot is a locus of points describing the high-stress states for which mass, momentum and energy are conserved. The Rankine-Hugoniot equations represent the most commonly used form of equations (2.1), (2.2), and (2.3) in shock physics and express these conservation laws for an assumed hydrodynamic shock wave as:

\[ V = V_0 \frac{U_s - (u_p - u_0)}{U_s} \quad (2.13) \]

\[ \sigma = \sigma_0 + \rho_0 U_s (u_p - u_0) \quad (2.14) \]

\[ E = E_0 + \frac{1}{2} (\sigma + \sigma_0) (V_0 - V) \quad (2.15) \]

where \( V, \sigma, E, \) and \( u_p \) are respectively the end states of specific volume, pressure, energy, and particle velocity achieved from the passage of a shock wave [1-5]. \( U_s \) is the shock front velocity relative to the initial material state of specific volume \( (V_0) \), density \( (\rho_0) \), pressure \( (\sigma_0) \), particle velocity \( (u_0) \), and energy \( (E_0) \). Taking note of equations (2.13), (2.14), and (2.15), it can be seen that there are five unknown parameters associated with shock loading which include \( U_s, u_p, \sigma, V, \) and \( E \). If any two of these five parameters are known, then the jump conditions represented by the Rankine-Hugoniot equations allow for the complete characterization of the shocked state. By measuring a series of shocked states for a given material, a locus of points can be obtained that is representative of that material’s Hugoniot. Hugoniots of a material can be utilized from any initial state. However, when the initial state is the
undeformed material at ambient temperature and pressure, the locus of points is referred to as the principal Hugoniot.

It is often convenient to express a material’s principal Hugoniot in terms of a relationship between the particle velocity \( u_p \) and the shock velocity \( U_s \) in the form:

\[
U_s = c_b + su_p + Qu_p^2
\]  

(2.16)

where \( c_b \) is the sound velocity at near zero pressure corresponding to the initial equilibrium bulk compressibility of the medium, and the terms \( s \) and \( Q \) are fit parameters. For most materials, it is commonly accepted in literature that the linear form of equation (2.16) is sufficient to express a material’s Hugoniot, where \( Q \) is equal to zero, and the dimensionless material constant \( s \) is typically in the range of 1-2 [1-5]. Once values of \( c_b \), \( s \), and \( Q \) are determined for a material, its respective Hugoniot can be completely defined by equation (2.16). With a fully defined Hugoniot it becomes possible to determine all state parameters of a shocked material with the specification of any single parameter and use of equations (2.13), (2.14), and (2.15).

The Rankine-Hugoniot equations allow a material’s Hugoniot to be depicted graphically by plotting any two state parameters as functions of one another. Typically researchers will plot a material’s Hugoniot in the stress-particle velocity or stress-specific volume planes depending on their theoretical background. The stress-particle velocity plane is extremely useful to researchers in the field of solid mechanics because the slope of the resulting plot is representative of a material’s dynamic impedance. This is immediately evident when one considers a manipulated form of the Rankine-Hugoniot conservation of momentum equation (2.14) written as:
\[(\sigma - \sigma_o) = \rho_0 U_s (u_p - u_o) = \rho_o \left(c_b + su_p + Q u_p^2 \Gamma (u_p - u_o)\right)\]  

(2.17)

where a jump in stress can be determined by a jump in particle velocity multiplied by the product of a material’s density and shock velocity which in turn is defined as a material’s dynamic impedance. If the material’s Hugoniot is well represented by equation (2.16) then substitution for \(U_s\) yields the final form found on the right side of equation (2.17). As a result, when an incident wave falls subject on an interface of two different materials, observing the Hugoniots of these materials in the \(\sigma-u_p\) plane allows researchers to graphically visualize reflected and transmitted stress amplitudes required for the continuity of an interface. Researchers with a thermodynamics background, on the other hand, may choose to represent a material’s Hugoniot in the stress-specific volume plane due to its higher sensitivity to phase changes and other associated thermodynamic mechanisms. Since this thesis focuses primarily on solid mechanics, the stress-particle velocity plots prove the most useful and therefore will be used in the proceeding discussions.

Within the field of shock-physics, the velocity of sound has typically been shown to increase with increasing pressure for most materials as a result of the densification of the material. It therefore can be noted that shock waves propagate with a velocity that is supersonic with respect to the undisturbed material, however subsonic with respect to the shocked material state. As a result of this, the rise time of a shock wave will typically decrease with an increase in propagation distance which explains their stability. The release from a shocked state is accomplished by an unloading wave which is often referred to as a rarefaction fan. Rarefaction waves travel at the velocity of sound within the shocked material state. As portions of these
waves relieve the material from its shocked state the sound velocity decreases causing the rarefaction waves to diverge during their propagation, giving rise to the term “rarefaction fan”. Rarefaction fans are typically idealized as an infinite series of weak shocks that progressively release a material from its shocked state. In contrast to shock loading, which follows a material’s Hugoniot, the process of unloading is assumed to be isentropic and therefore follow a material’s isentrope. An isentrope is a series of states of stress, energy, density, temperature, and entropy along which the entropy is constant. The Hugoniot of a material has been shown to provide a good approximation of an isentrope for weak shocks. However, in the case of strong shocks where shock-wave compression is accompanied by an increase in entropy and irreversible heating of the material, this is no longer the case. Nevertheless, experiments show that in the pressure-particle velocity plane, the release isentrope of many materials deviates from the Hugoniot by no more than 3% for pressures up to 50 GPa [1-5]. For the analysis of the experiments found within this thesis it will be assumed that the unloading and loading paths coincide, following the Hugoniot which can be considered quasi-isentropic. This is a reasonable assumption considering that 50 GPa is an entire order of magnitude greater than the maximum shock pressures achieved in the present studies.

2.3 THERMODYNAMIC CONSIDERATIONS

For the case presented in this thesis, the Hugoniot is sufficient to describe and relate the state parameters associated with loading and unloading caused by shock waves. However, in order to conduct investigations into processes taking place off the
Hugoniot, a complete description of the thermodynamic EOS is required. The Mie-Grüneisen EOS is the most commonly selected equation of state used in literature to solve for processes following paths such as the isentrope or even the isotherm (a series of states along which the temperature remains constant typical of quasi-static loading) [3, 4, 6-11]. The Grüneisen parameter is a fundamental thermodynamic derivative that can be expressed as:

$$\gamma = V \left( \frac{\partial P}{\partial E} \right)_V = -\frac{V}{T} \left( \frac{\partial T}{\partial V} \right)_s = -V \frac{\partial \left[ \ln \left( \frac{T}{T_R} \right) \right]}{\partial V}$$ \hspace{1cm} (2.18)

where $P$ denotes pressure, $E$ is internal energy, $T$ is the temperature, $V$ is specific volume, $S$ is the entropy, and the subscript $R$ denotes a reference state. Often it is assumed that $\gamma$ is a function of only $V$, and therefore it can be shown that the pressure and specific internal energy at a given volume can be related to the Hugoniot by:

$$P(V) - P^H(V) = \frac{\gamma(V)}{V} \left[ E(V) - E^H(V) \right]$$ \hspace{1cm} (2.19)

where the superscript $H$ refers to the Hugoniot. Equation (2.19) is generally referred to as the Mie-Grüneisen P-V-E equation of state. The Hugoniot can be used in conjunction with this EOS to solve for processes through all possible paths within the multi-dimensional hyperspace created by the state parameters. For the purpose of this thesis, detailed use of equation (2.19) will not be outlined. Rather, the reader is advised to refer to the literature previously cited for a more in depth discussion of the topic [3, 4, 6-11]. A good example of its use in research can also be found in work by Mosenfelder, et al., in which they investigated the thermodynamic properties of MgSiO$_3$ through global inversion of shock and static compression data [12].
shock states is additionally exemplified in the Dissertation of Brown where he studied high pressure regimes in solids using a novel mach reflection technique [13].

In addition to the Mie-Grünesien EOS there exist numerous thermodynamic frameworks that have been proposed for use in the study of shock loading of materials. Obviously, the extreme states of matter exposed by shock loading experiments provide an exciting field for researchers with a thermo-mechanics background and therefore treatment of these experiments is not limited to researchers in the solid mechanics field. Most of these works base their approach on the first and second laws of thermodynamics. In work by Maugin, and Berezovski, they suggest a need for distinguishing between internal and free energies of a system and they outline a thermomechanical framework that can be used under the adiabatic assumptions common to the treatment of shock waves [14]. It seems noteworthy to point out work by O’Reilly and Varadi where they suggest a modification of the classical Clausius-Duhem inequality, representative of the second law of thermodynamics, in order to provide a jump equation for the calculation of entropy in a system undergoing shock loading or phase transitions [15]. In addition, jump equations for entropy as a function of pressure as well as a complete thermomechanical framework for the treatment of shock waves in both gasses and solids can be readily found in literature, e.g., Bradley [16].

2.4 PLATE IMPACT EXPERIMENTAL TECHNIQUE

Plane shock waves can be generated in condensed matter by impacting a test sample with a flyer plate at a velocity sufficient to exceed the HEL. For visualization
of this process refer to Figure 2.1. Part (a) of Figure 2.1 depicts a time-distance diagram in Eulerian coordinates constructed from the method of characteristics for the wave process proceeding from impact of a flyer plate on a target plate. The characteristics represented by solid lines are compressive waves and the dotted characteristics are representative of unloading waves. Although the analysis of shock waves is performed in Lagrangian coordinates, Eulerian coordinates were chosen for this figure in order to allow visualization of the resulting motion of the impacted sample in relation to the stress wave propagation within. Part (b) of Figure 2.1 contains a generalized stress-particle velocity plot for this event. Part (c) of Figure 2.1 depicts pressure profiles at times $t_1$ and $t_2$ correlated with the time-distance diagram where the dotted line represents the initial position of the discontinuities. Finally, part (d) contains a schematic of an impactor and a target that depicts the concepts of radial release and dispersion.

Initially we will assume that the target is at a state of rest with zero stress and the impactor is at a state of zero stress traveling at some rate $U_i$. Upon impact, continuity of stress and particle velocity must be achieved at the impact face yielding a simple centered compressive wave of C- and C+ trajectories propagating respectively into the impactor and target. In stress-particle velocity space C- waves are depicted as negatively sloped curves and C+ waves are depicted as positively sloped curves. This is better visualized when one considers that the C- compression wave decelerates the impactor to an equilibrium state, while the C+ compression wave accelerates the target from rest to an equilibrium state. Since a material’s Hugoniot defines all possible values of stress and particle velocity, the equilibrium state after impact is achieved at
the intersection of the flyer and target Hugoniots, as depicted in part (b) of Figure 2.1. If the impactor is of the same material as the target, then due to symmetry of the Hugoniots, the particle velocity of the shock compressed material is exactly half of the initial impactor velocity. If the impactor is “harder” than the target (i.e. of higher impedance) the resulting equilibrium will achieve a greater particle velocity and stress than the former case as depicted by the dotted line H.I. on part (b) of Figure 2.1. Conversely, the dotted line marked L.I. depicts the resulting equilibrium when the impactor is “softer” or of lower impedance than the target. It can be noted that by varying the impact velocity or impacting a target with different impactor materials of known Hugoniot, a series of data sets can be obtained and the Hugoniot of the target can be generated.

When a compressive pulse reflects from a free surface the stress goes to zero and particle velocity doubles. The resulting reflection of such an interaction in the case of the impactor in part (a) of Figure 2.1 is a release wave known as a rarefaction fan. This fan traveling along the C+ characteristic further decelerates the impactor to state o in part (b) of Figure 2.1. As previously stated, the rarefaction fan travels at the sound velocity in shock compressed matter, while the initial shock wave is subsonic to the compressed matter behind its front, which results in the rarefaction wave overtaking the shock front at some distance. This mechanism will cause attenuation in the initial shock wave as depicted in the pressure profiles found in part (c) of Figure 2.1 which correlate to \( t_1 \) and \( t_2 \) in the time-distance diagram depicted in part (a).

By observing the impact in Eulerian coordinates in the time-distance diagram the densification of the material caused by the shock loading is easily noted. The
initial impact causes the front face of the target plate to move in the C+ direction while the back face or free surface of the target remains stationary. The free surface of the target will continue to remain stationary until the arrival of the initial compressive front.

Figure 2.1. Generation of a compression pulse by the impact of a flyer plate on a target; (a) Time-distance diagram; (b) Stress-Particle Velocity plot; (c) Pressure profiles; (d) Radial-release and edge effects.

Fundamental to the analysis of plate impact experiments is the assumption of one-dimensional motion of the medium during the period of time required for the measurements to be conducted. This condition is violated by radial release waves that propagate inward from the edges of the impactor and the target, as depicted in part (d) of Figure 2.1. Also depicted is the concept of dispersion when the impactor is smaller than the sample. In order to achieve acceptable time durations of one-dimensional
strain, impactors and targets with large diameter-to-thickness ratios are typically chosen. For the purpose of this thesis complications associated with dispersion were eliminated by specifying that the impactor and samples have the same diameters.

2.5 SPALL FRACTURE CONSIDERATIONS

Spall fracture, commonly referred to as spalling, is a failure mechanism that involves the interaction of release waves, within a sample, that creates a critical tensile state that invokes failure. For visualization of this process, refer to Figure 2.2, in which a time-distance plot (a), stress-particle velocity plot (b), and transmitted stress profile (c) are depicted for the case of a symmetrical impact of a flyer plate on a sample backed by a low impedance window through which stress transmissions can be measured. Similar to Figure 2.1, the time distance diagram in Figure 2.2 is constructed in Eulerian coordinates to allow for visualization of sample movement in space. In the current case, symmetrical signifies that the contact surface and material of the impactor are the same as those of the sample.

At the point of impact, compressive waves are generated in the C- and C+ directions respectively in the impactor and sample, resulting in state 1 on the stress-particle velocity plane. When the forward moving compressive pulse arrives at the low impedance window, the assumption of continuity at the interface between the sample and window can only be achieved through intersection of their respective Hugoniot in the $\sigma-u_p$ plane. This assumption requires a C- tensile release wave of magnitude $(\sigma_1-\sigma_2)$ to be generated in the sample and a C+ compressive wave of magnitude $\sigma_2$ to be generated in the window, resulting in state 2. This compressive
transmission to the window is associated with $\sigma_{\text{max}}$ in the stress record found in part (c) of Figure 2.2. Meanwhile the initial C- compressive wave in the impactor reflects off the free surface as a decompression wave traveling in the C+ direction. Its transmission into the sample completely brings the impactor to state 3. Within the sample the C+ and C- release waves interact creating some critical tension signified by state 4. State 4 is observed at the sample window interface as state 5 through a C+ tensile release of the window from state 2 to 5, and a C- compressive reloading of the sample from state 4 to 5. Once this critical tension is achieved, a spall plane may initiate. The near-instantaneous release of tension from the generation of the spall plane causes a compressive pulse to propagate back to the sample-window interface, reloading the window from state 5 associated with $\sigma_{\text{min}}$ in the stress profile found in part (c) of Figure 2.2. The drop in stress from states 2 to 5 associated with $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$ in the stress profile is often termed the pullback signal.

Looking at the Figure 2.2 it should be noted that state 5 is depicted with a star. This star helps to differentiate this state from others due to the fact that the characteristic related to state 5 on the time-distance diagram is not depicted. As previously noted, rarefaction waves are idealized as an infinite number of weak shocks. Obviously it would be impossible to draw an infinite number of characteristics representative of the rarefaction wave, however, the divergence of the unloading front is depicted by the region encompassed within the two dotted characteristics that serve to represent the rarefaction fan. It should therefore be noted that state 5, marked with a star, is representative of the final portion of the rarefaction fan that reached the target-window interface before the reloading by the C+.
compressive front. The reloading associated with the C+ compression characteristic adjacent to state 5 in the time-distance diagram causes the subsequent second plateau realized at the stress gauge, and can be visualized by following the sample Hugoniot through states 6 and 7 depicted in the stress-particle velocity plot.

![Diagram showing the decomposition of a spall experiment](image)

**Figure 2.2.** Decomposition of a spall experiment; (a) Time-distance diagram; (b) Stress-Particle Velocity plot; (c) Stress transmitted to the window.

It should be noted, when referring to Figure 2.2, that all experimental methods involved in measuring the dynamic tension of a sample associated with state 4 are indirect. This is an obvious factor when one considers the impossibility of introducing a sensor into a sample without inherently influencing its resistance to tensile stress. In light of this, the dynamic tension that represents a material’s spall strength is indirectly determined from temporal records of either the stress or particle velocity that are
respectively measured at either the interface between the sample’s back face and a low impedance window or the sample’s free surface. The most commonly used equation in literature, employed for this determination, takes the acoustic approach and relates measured states 2 and 5 to state 4 in the form:

\[ \sigma_{sp} = \frac{1}{2} \rho_o c_o \Delta u_{fs} \]  

(2.20)

where \( \sigma_{sp} \) is the spall strength indicative of the critical tension that invoked failure (state 4), \( \rho_o \) is the initial density, \( c_o \) is the initial wave speed, \( \Delta u_{fs} \) is the velocity pullback associated with the difference in particle velocities of state 2 and state 5 [3]. Equation (2.20) is typically employed when particle velocity measurements are conducted in the experiments and is derived for the case to which there is no window backing the sample, and therefore, stresses associated with states 2 and 5 are zero. In the case of particle velocity and stress measurements that incorporate a low impedance window, equation (2.20) must be rederived to incorporate complications associated with the sample-window interface.

Since the manipulated form of equation (2.20) will exclusively be used in the subsequent study of spall fracture, let us briefly show how it can be derived in accordance to the state numbers assigned in Figure 2.2. We will begin this derivation with the assumption that transmitted stress is measured and therefore stresses at states 2 and 5 are known and respectively correlate to \( \sigma_{max} \) and \( \sigma_{min} \) as observed in the transmitted stress signal found in part (c) of Figure 2.2. Within the acoustic approach, wave velocity is assumed to be independent of stress and particle velocity, resulting in a special form of equation (2.17) given as:

\[ (\sigma - \sigma_o) = \rho c (u_p - u_o) = Z(u_p - u_o) \]  

(2.21)
where stress ($\sigma$) is a function of particle velocity ($u_p$), density ($\rho$), and constant wave speed ($c$). The product of density and wave speed represents a material’s acoustic impedance ($Z$). Equation (2.21) allows us to begin by investigating the transmitted stress signals to the window. The window is initially loaded from state 0 to state 2 through a C+ compression wave giving rise to the jump condition:

$$ (\sigma_2 - \sigma_0) = Z_w (u_2 - u_0) $$  \hspace{1cm} (2.22)

where the subscript “w” refers to the acoustic impedance of the window material since we are following paths along its acoustically simplified Hugoniot in $\sigma$-$u_p$ space as depicted in part (b) of Figure 2.2. Similarly the jump from state 2 to state 5 is accomplished through a C+ unloading wave transmitted into the window giving rise to the jump condition:

$$ (\sigma_5 - \sigma_2) = Z_w (u_5 - u_2) $$  \hspace{1cm} (2.23)

which defines the pullback signal associated with the drop from $\sigma_{\text{max}}$ to $\sigma_{\text{min}}$ in the transmitted stress profile found in part (c) of Figure 2.2. Equation (2.22) can be simplified noting that $\sigma_0$ and $u_0$ are representative of the undisturbed material state and therefore are equal to zero. Setting these terms to zero and applying a simple manipulation to equation (2.22) allows us to write:

$$ u_2 = \frac{\sigma_2}{Z_w} $$  \hspace{1cm} (2.24)

where the unknown particle velocity at state 2 can be determined in terms of the measured stress. Equation (2.24) can be substituted into equation (2.23) to write the unknown particle velocity at state 5 in terms of the measured stress as:

$$ u_5 = \frac{\sigma_5}{Z_w} $$  \hspace{1cm} (2.25)
Equation (2.21) allows the jumps from states 2 to 4 and from states 4 to 5 within the sample material to be defined as:

\[ (\sigma_4 - \sigma_2) = Z_s (u_4 - u_2) \]  
(2.26)

\[ (\sigma_5 - \sigma_4) = -Z_s (u_5 - u_4) \]  
(2.27)

where the subscript “s” refers to the acoustic impedance of the sample material since we are following paths along its respective curve in the \( \sigma-u_p \) plane depicted in part (b) of Figure 2.2. The negative sign that arises in equation (2.27) is easily understood when one considers that we are following the path from state 4 to state 5 in the \( \sigma-u_p \) plane (negative sloped path) which is associated with the C- recompression of the sample plate. Equations (2.24-2.27) now form a system that allow for determination of the dynamic tension \( \sigma_4 \) as a function of either measured stresses or measured particle velocities at states 2 and 5. Since the assumption is that stresses at these states are known, the derivation will be completed to determine the stress at state 4 as a function of the measured stresses. By substitution of equations (2.24) and (2.25) respectively into equations (2.26) and (2.27), the unknown particle velocities associated with states 2 and 5 can be eliminated giving rise to equations of the following form:

\[ (\sigma_4 - \sigma_2) = Z_s \left( u_4 - \frac{\sigma_2}{Z_w} \right) \]  
(2.28)

\[ (\sigma_5 - \sigma_4) = -Z_s \left( \frac{\sigma_5}{Z_w} - u_4 \right) \]  
(2.29)

Finally equations (2.28) and (2.29) can be combined to eliminate the unknown parameter associated with the particle velocity at state 4 giving rise to the equation:
where a simple rearrangement yields:

\[
\sigma_4 = \frac{1}{2} \sigma_5 \left( 1 + \frac{Z_s}{Z_w} \right) + \frac{1}{2} \sigma_2 \left( 1 - \frac{Z_s}{Z_w} \right) \tag{2.31}
\]

thus yielding an equation to approximate the stress at state 4 from known transmitted stresses at states 2 and 5. Had particle velocity been measured instead of stress, equation (2.31) can be manipulated with use of equations (2.24) and (2.25) to show:

\[
\sigma_4 = \frac{1}{2} u_5 (Z_w + Z_s) + \frac{1}{2} u_2 (Z_w - Z_s) \tag{2.32}
\]

and, in the case of no window, \(Z_w\) becomes zero, and equation (2.32) is equivalent to equation (2.20). Thus, we have a method for determining spall strength that is consistent with literature. By replacing the subscripts 2, 4, and 5 we can write equation (2.31) in a form that can easily be utilized considering the transmitted stress pulse depicted in part (c) of Figure 2.2, which gives rise to:

\[
\sigma_{sp} = \frac{1}{2} \sigma_{\text{min}} \left( 1 + \frac{Z_s}{Z_w} \right) + \frac{1}{2} \sigma_{\text{max}} \left( 1 - \frac{Z_s}{Z_w} \right) \tag{2.33}
\]

where \(\sigma_{\text{max}}\) is the magnitude of the initial compressive wave (state 2), \(\sigma_{\text{min}}\) is the magnitude of the minimum pullback signal (state 5), and the subscript “sp” is amended in place of state 4 to indicate the calculation of spall strength. As previously noted, the specimen and window impedances can be calculated by the product of their respective initial densities and appropriate wave speeds. Special care must be exercised when calculating these impedances in regards to choosing the appropriate wave speed. If the material response preceding spall is elastic, a longitudinal elastic
velocity should be used. On the other hand, if the response is hydrodynamic, a bulk wave speed must be used. It has been widely recognized that when elastic waves are important to the analysis of a hydrodynamic event, an equivalent wave speed can be utilized. This equivalent wave speed, proposed by Romanchenko and Stepanov, is the harmonic mean of the longitudinal and bulk wave speeds [17].

Within this thesis, stress measurements incorporating a low impedance window were exclusively used in the study of spall fracture and therefore equation (2.33) was utilized. Since spall strengths determined in this thesis were the primary consideration of the dynamic fracture experiments, it seems important to point out that there is a wide acceptance and extensive use of either equation (2.20) or its manipulated form (2.33) in literature [17-30]. When observing equations (2.20) and (2.33) one can quickly note a trivial shortcoming associated with their use. Each of these equations will output a value for spall strength even if the tested sample is free from failure and experiences complete unloading. This shortcoming is easily addressed through discerning use of the equations, where they are only applied to solve for spall strength when a notable pull-back signal is observed. However, researchers such as Church, et al. [31], have tried to address concerns with the calculation of a “false spall strength” by suggesting an alternative approach where the pull-back signal is interpreted as the rise from \( \sigma_{\text{min}} \) to the second plateau observed in part (c) of Figure 2.2. Obviously the subsequent indirect assumption of state 4 becomes much more complicated when utilizing states 5 and 7, as can be noted in part (b) of Figure 2.2. For this reason, the proposed approach has seen limited use in
literature, and in many cases has been shown to provide a significant underestimate of the stress associated with state 4 [20,26].

In work by Gathers it is pointed out that a much better approximation of the critical tension associated with state 4 can be accomplished if the material’s Hugoniot is known [32]. The proposed method deviates from the acoustic approach by using the known Hugoniot of the material to determine state 4 from the experimentally determined states 2 and 5, as observed in Figure 2.2. In this method the Hugoniot is assumed to be quasi-isentropic and therefore provide a valid representation of the release isentrope. It is also assumed that the material’s Hugoniot is well represented by the linear relationship between shock and particle velocities, where \( Q \) is equal to zero in equation (2.16). Application of the technique developed by Gathers to situations where low impedance windows are utilized can be found in work by Chen [29]. The key deviation from the acoustic approach relies on the specification of the relationship between stress and particle velocity, where the acoustic approach utilizes equation (2.21) and the hydrodynamic approach utilizes equation (2.17).

It should be pointed out that the Hugoniots of the specific cast irons studied in this thesis were not known. Therefore, the acoustic method was utilized to approximate the spall strength from the temporal stress records. Derivation of an equation for spall strength utilizing the Hugoniot as suggested by Gathers would follow the same process presented in equations (2.22-2.31) with the use of equation (2.17) replacing (2.21). Let us briefly consider the error associated with the variation of the two methods by utilizing published Hugoniots for cast iron and polycarbonate found in work performed at the Los Alamos National Laboratory (LANL) [33].
Figure 2.3 contains a stress-particle velocity plot generated from the data obtained. The data sets obtained from LANL are depicted as open diamonds and squares, respectively, for cast iron and polycarbonate. The solid lines in Figure 2.3 represent the Hugoniots of the two materials as calculated from equation (2.17). The dotted lines are representative of the acoustic approach where the bulk wave speed was utilized in conjunction with initial density in equation (2.21). Noting the difference between the dotted lines and the respective solid lines, it is quite clear that the acoustic approach will provide significant deviations from the Hugoniot when stress and particle velocity are high. It should be pointed out that the maximum velocity impact related to the testing of the cast irons in this thesis was 300 m/s, and is therefore depicted by the dotted line originating from $u_i = 300$. Figure 2.3 clearly illustrates that variation between the acoustic approach and the use of known Hugoniots is quite minimal within the testing range of the spall fracture experiments found within this thesis. Thus it has been shown that in the absence of known Hugoniots of the two materials, the acoustic approach is valid for use in the determination of spall strengths in the subsequent study.

Important to the study of spall fracture of materials is the ability to determine strain rate from the temporal stress or particle velocity records. It is widely recognized in literature that the spall strengths of many materials exhibit a notable dependence on the strain rate associated with unloading [3]. Strain rates can be estimated from stress records through use of the commonly employed equation:

$$\dot{\varepsilon} = \frac{du}{dx} = \frac{1}{\rho_c c} \frac{d\sigma}{dx} = \frac{1}{\rho_c c^2} \frac{d\sigma}{dt}$$ (2.34)
where \( \rho_o \) is the initial density, \( c \) is an appropriate wave speed, and the derivative of stress with respect to time is the slope associated with the rise time or fall time of a recorded stress signal [17, 20, 22, 29, 30, 34]. Similar to the case of equation (2.33) special consideration must be taken when choosing an appropriate wave speed.

![Stress-Particle velocity plot for cast iron and polycarbonate](image)

**Figure 2.3.** Stress-Particle velocity plot for cast iron and polycarbonate where the solid lines represent stress generated from a known Hugoniot in eq. (2.17) and the dashed lines represent stress calculated by the acoustic approximation in eq. (2.21).

In work by Grady, a relationship between the spall strength and tensile strain rate is suggested for brittle materials through use of a parameter associated with fracture energy [35]. To the best of the author’s knowledge, the use of this relationship in literature is quite limited and only can be found in works by Paris, et. al, where they manipulated it to investigate fracture energy of ceramics as a function of experimentally determined spall strengths [20]. Despite its scarcity of use, this relation will be employed here to investigate fracture energy in the form:
where $\gamma$ is the fracture energy per unit square, $\sigma_{sp}$ is the experimentally determined spall strength, $\rho_o$ is the initial density, $c_l$ is the longitudinal sound velocity, and the strain rate is associated to the unloading rate that invoked failure determined from the descending part of the pull-back signal depicted in part (c) of Figure 2.2.

### 2.6 SHOCK WAVES IN LAYERED PLATES

Generation of a material’s Hugoniot is a fundamental theme in literature and has fueled the focus of countless researchers in the field of shock physics. A material’s Hugoniot, coupled with a valid thermodynamic EOS allows researchers to accurately develop models to predict all states of matter associated with dynamic loading. As noted in section 2.5, a well defined Hugoniot can also help eliminate errors associated with the indirect method utilized to study spall fracture. Generally, obtaining the locus of points necessary to construct a material’s Hugoniot is a tedious process where a separate experiment is typically required for each data set. This process can be significantly reduced if we consider the shock wave processes associated with layered plates.

In order to begin let us consider the shock wave processes associated with an incident pulse acting on a thin layer in an otherwise homogenous structure. For visualization purposes, refer to Figure 2.4 in which time-distance and stress-particle velocity diagrams associated with this process can be found. Parts (a) and (b) respectively depict the time-distance and stress-particle velocity diagrams for the case.
where the thin inner-layer is of low impedance in regards to the rest of the structure. Likewise, parts (c) and (d) respectively depict the time-distance and stress-particle velocity diagrams for the case when the thin inner-layer is of high impedance in relation to the rest of the structure. In regards to Figure 2.4, B.M. is used to denote bulk material represented by the bulk material Hugoniot in the $\sigma-u_p$ plane and I.L. is used to denote inner-layer represented by the inner-layer Hugoniot in the $\sigma-u_p$ plane. In the time-distance diagrams solid line characteristics represent compressive waves while dotted line characteristics represent unloading waves.

Since the time-distance and associated stress-particle velocity diagrams found in sections 2.4 and 2.5 were extensively described, it will be assumed that a general understanding of the process associated with a thin layer sandwiched between two thick plates can be obtained from observation of Figure 2.4, and therefore a complete description of the process will not be included. In both cases the plates experience a ring-up period where the magnitude of the initial incident pulse is achieved through multiple reverberations within the thin inner-layer. The time of this ring-up period is directly associated with the magnitude of the initial pulse, variation of the two Hugoniot, and the thickness of the inner-layer. It should be noted when looking at Figure 2.4 that the approach to the final state follows a completely different path in the stress-particle velocity plane in regards to the two separate cases. For the case where the inner-layer has lower impedance than the plates, the final state is obtained through compressive reflections within the inner-layer that result in a step-like loading profile. In contrast, when the inner-layer has higher impedance than the plates, the final state
is obtained through combined compressive and tensile reflections within the inner-layer that give rise to a damped oscillatory loading profile.

Figure 2.4. Wave processes associated with an incident pulse acting on a thin layer in an otherwise homogenous structure; (a) t-X diagram when the inner layer is of lower impedance; (b) $\sigma-u_p$ diagram associated with part (a); (c) t-X diagram when the inner layer is of higher impedance; (d) $\sigma-u_p$ diagram associated with part (c).

In regards to capturing a material’s Hugoniot through use of these processes, it is quite obvious that the case for which the inner-layer is of lower impedance than the plates proves much more advantageous. Easily noted when observing parts (a) and (b) of Figure 2.4, this case allows a much wider region of the Hugoniot curves of the two respective materials to be investigated. With the exception of the first C- unloading wave generated in the front plate, the ring-up to final state is accomplished entirely by
compression waves. This is especially important because the assumption previously stated that the Hugoniot and isentrope coincide only needs to be applied once when the first plate is unloaded from state 0’ to state 1. For these reasons, the case for which the inner-layer is of lower impedance than the plate materials will exclusively be used in the subsequent Hugoniot Ring-up Method (HRUM).

Introduction of in-situ stress gauges to both sides of the inner-layer allows for the capture of temporal stress records associated with the ring-up to equilibrium. In the current case, these records will demonstrate the step-like loading profile associated with the process. Compressive steps observed on the front gauge will directly relate to the stress at states 1, 3, and 5, while steps observed by the back gauge will directly relate to the stress at states 2, 4, and 6. As previously noted, the Rankine-Hugoniot equations form a complete system when two state parameters are specified. There are two ways in which this requirement can be satisfied. First, the shock velocity within the inner-layer could be obtained as a function of shock amplitude by comparison of the temporal records of the two gauges and relating the time lag between the observed steps to the thickness of the inner-layer. This method was set aside in favor of the more promising method of impedance matching, where it will be required that either the inner-layer material or bulk material has a known Hugoniot.

Let us first consider the simplified case for which the Hugoniot of the inner-layer is constructed from the known Hugoniot of the bulk plate. In the stress-particle velocity diagram depicted in part (b) of Figure 2.4 it can be noted that the ring-up associated with the back gauge follows the principal Hugoniot of the base plate, while the ring-up of the front gauge follows the negatively sloped Hugoniot drawn from the
impact velocity. Assuming that the Hugoniot of the bulk plate is well represented by
the linear form of equation (2.16), the Rankine-Hugoniot conservation of momentum
equation (2.14) can be used to solve for the particle velocity associated with the
measured stresses at each state. For the case of the back gauge following the principal
Hugoniot of the bulk plate, the particle velocity of each successive step can be solved
in terms of the measured stress in the form:

\[ \sigma_n - \sigma_{n-2} = \rho_{n-2}^{BP} \left( c_{b}^{BP} + s_{n}^{BP} u_n \right) \left( u_n - u_{n-2} \right) \] (2.36)

where the superscript \( BP \) refers to properties associated with the bulk plate Hugoniot.

In terms of Figure 2.4(b) the subscript \( n \) can be replaced with states 2, 4, or 6. The
solution of equation (2.36) requires an iterative process where each additional jump in
states (e.g., states 2-4, and 4-6) is solved in terms of the solution of equation (2.36)
from the previous state jump. When solving for state 2 from state 0, the initial density
associated with state 0 is utilized. In contrast, solution of state 4 from state 2 takes the
assumption that state 2 is the initial state in the jump equation, therefore requiring the
determination of the density associated with state 2. Likewise, any additional jumps
require the solution of the density at the previous state, which will in turn represent the
density of the defined initial state in the jump equation (2.36). Through manipulation
of the Rankine-Hugoniot conservation of mass equation (2.13), the density at states 2,
4, and 6 can be solved in terms of the density at the previous state in the form:

\[ \rho_n = \rho_{n-2} \frac{c_{b}^{BP} + s_{n}^{BP} u_n}{(c_{b}^{BP} + s_{n}^{BP} u_n) - (u_n - u_{n-2})} \] (2.37)

where each solution requires the particle velocity associated with the state to be
generated from the preceding solution of equation (2.36).
Likewise the particle velocity associated with states 1, 3, and 5 can be solved in terms of the measured stress from the front gauge in the form:

\[
\sigma_n - \sigma_{n-2} = -\rho_{n-2}^{BP} \left( c_b^{BP} + s^{BP} \left( u_i - u_n \right) \right) (u_n - u_{n-2})
\]  

(2.38)

where \( u_i \) refers to the impact velocity and the density can be solved in the form:

\[
\rho_n = \rho_{n-2} \frac{c_b^{BP} + s^{BP} (u_i - u_n)}{(c_b^{BP} + s^{BP} (u_i - u_n)) + (u_n - u_{n-2})}
\]

(2.39)

while taking note that the density associated with state -1 in Figure 2.4 (b) is assumed to be the initial density (\( \rho_0 \)). Equations (2.36)-(2.38) now allow for the determination of particle velocity given measured stress, which in turn permit the solution of any additional state parameters of interest with use of the Rankine-Hugoniot jump equations.

Before we continue to the second case where the bulk plate Hugoniot is determined from the known Hugoniot of the inner-layer let us consider how the obtained data sets can be transformed in order to construct the principal Hugoniot of the inner-layer. Figure 2.5 depicts a stress-particle velocity diagram for an experiment where the known bulk plate is 6061 aluminum and the unknown inner-layer is polycarbonate constructed from published Hugoniot data of the two materials [33]. The impact velocity is around 300 m/s and each gauge is shown to receive three steps of loading as in Figure 2.4 (b). The dotted horizontal lines in Figure 2.5 provide a visual example of the translation of the obtained data sets to allow for the construction of the principal Hugoniot of the polycarbonate inner-layer. Also exemplified in Figure 2.5 is the large domain that the polycarbonate Hugoniot was able to be investigated with only a 300 m/s impact velocity. Had conventional methods been employed
where the polycarbonate would be symmetrically impacted, impact velocity would have to be greater than 1000 m/s to achieve a similar range of the material’s Hugoniot.

Let us now address the more complicated case where the Hugoniot of the bulk plate is constructed from a known Hugoniot of the inner-layer. Referring back to part (b) of Figure 2.4, we will first follow the inner-layer’s Hugoniot from state 0 to state 1 with use of the Rankine-Hugoniot conservation of momentum equation (2.14) under the assumption that the Hugoniot is well represented by the linear form of equation (2.16). The particle velocity at state 1 can be solved in terms of the measured stress in the form:

\[
\sigma_1 - \sigma_0 = \rho_o u \left( \frac{c_s u + s u_1}{u_1 - u_0} \right) \]

Figure 2.5. Construction of the inner-layer’s principal Hugoniot from an experiment where the Hugoniot of the bulk plate is known, generated with published Hugoniot data for 6061 Aluminum and Polycarbonate.
where the superscript $IL$ refers to properties associated with the inner-layer’s Hugoniot. Solving for the particle velocity at state 2 in terms of the stress gives rise to the form:

$$\sigma_2 - \sigma_1 = -\rho_1^{IL} \left(c_b^{IL} u_1^{IL} + s^{IL} (u_1 + (u_1 - u_2)) \right) (u_2 - u_1)$$  \hspace{1cm} (2.41)

where the density at state 1 ($\rho_1$) can be solved in terms of the density at state 0 ($\rho_0$) with use of the Rankine-Hugoniot conservation of mass equation (2.13) in the form:

$$\rho_1 = \rho_0 \frac{c_b^{IL} + s^{IL} u_1}{(c_b^{IL} + s^{IL} u_1) - (u_1 - u_0)}$$  \hspace{1cm} (2.42)

Likewise investigation of state 3 yields:

$$\sigma_3 - \sigma_2 = \rho_2^{IL} \left(c_b^{IL} u_1^{IL} + s^{IL} (u_1 + (u_1 - u_2) + (u_3 - u_2)) \right) (u_3 - u_2)$$  \hspace{1cm} (2.43)

where the density at state 2 ($\rho_2$) can be solved in terms of the density at state 1 ($\rho_1$) in the form:

$$\rho_2 = \rho_1 \frac{c_b^{IL} + s^{IL} (u_1 + (u_1 - u_2))}{(c_b^{IL} + s^{IL} (u_1 + (u_1 - u_2)) + (u_2 - u_1)}$$  \hspace{1cm} (2.44)

In a similar manner each additional state can be investigated in terms of the measured stress and calculated particle velocity and density from the previous state. Noting the pattern of equations (2.40)-(2.44) one can write a generic equation in the form:

$$\sigma_n - \sigma_{n-1} = (-1)^{n-1} \rho_n^{IL} \left[c_b^{IL} + s^{IL} \left(\sum_{i=1}^{n-1} \left((-1)^{i-1} 2u_i\right) + (-1)^n u_n\right)\right] (u_n - u_{n-1})$$  \hspace{1cm} (2.45)

which can be used to solve for the particle velocity at any state number ($n$) associated with the case where the Hugoniot of the inner-layer is known. For use with equation (2.45), a generic equation for the calculation of density can be expressed in the form:
\[ \rho_n = \rho_{n-1} \frac{c_b^{IL} + s^{IL} \left( \sum_{i=1}^{n-1} (-1)^{i-1} 2u_i \right) - (-1)^n u_n}{c_b^{IL} + s^{IL} \left( \sum_{i=1}^{n-1} (-1)^{i-1} 2u_i \right) - (-1)^n u_n} \]  

(2.46)

There are some fundamental guidelines that need to be addressed before utilizing the HRUM to construct a material’s Hugoniot. The first guideline is that the incident compressive pulse duration be sufficiently long to enable adequate ring-up of the inner-layer before the arrival of the subsequent incident unloading pulse. This requirement can be satisfied by using a thick impactor. In light of the discussion of spall fracture found in section 2.5, the initial C- unloading of the front plate could result in spallation if it is allowed to couple with the C+ unloading of the impactor. In order to address this concern, the front plate and impactor should be of the same thickness. Additionally, concerns about the reflected unloading of the back plate can be addressed by specifying that the back plate be at least the same thickness as the front plate (thicker is likely more advantageous). Finally the thickness of the inner-layer must be chosen to balance the two concerns that loading steps captured by the stress gauges are clearly identifiable and radial release waves do not present themselves during the ring-up period. The concern about radial release waves presenting themselves during the ring-up period is quite applicable considering that the necessity of a thick front face deviates from the typical requirement of large diameter to thickness ratios previously noted. Initial knowledge of the wave speeds of the bulk and inner-layer materials is imperative in order to construct a time-distance diagram of the designed experiment that will help in the balance of these concerns.
Before concluding, let us revisit Figure 2.4. Although the case associated with the inner-layer having lower impedance than the plates will be exclusively used in the HRUM, both scenarios can present themselves when utilizing in-situ gauges to conduct stress measurements. Obviously, introduction of a thin layer in an otherwise homogenous material will directly affect the rise-time and associated strain rate of the incident shock wave. The ring-up period that the gauge undergoes in its approach to equilibrium will cause a delay in its response time. With knowledge of the Hugoniots of the in-situ gauge and test materials coupled with an assumed shock amplitude, the number of reverberations required to reach equilibrium can be determined through construction of a stress-particle velocity plot representative of the specific experiment. Once the number of reverberations is determined, knowledge of the gauge thickness and shock velocity within will allow for determination of the delay in a stress gauge’s response time. Once determined, a more accurate assumption of the strain-rate associated with the incident shock wave can be obtained by subtracting the response time from the time increment related to the slope of the recorded stress signal in equation (2.34).

2.7 RADIAL RELEASE WAVES

It should be apparent by now that radial release waves generate significant complications in shock wave experiments. In addition to violating the initial assumptions of one-dimensional motion, they become increasingly detrimental to the interpretation of stress signals captured with the use of imbedded gauges. The primary diagnostic used in this thesis to capture wave profiles will be stress gauges, and
subsequently it is extremely important to be able to predict the arrival time of radial release waves to the measurement location. Noted at the end of section 2.4, researchers typically chose samples with large diameter-to-thickness ratios in order to avoid complications associated with these waves [1-5]. In literature, simple equations exist to calculate minimum ratios for samples that have been shown to minimize this concern [4]. However, these equations fall short in actually predicting the arrival time of the radial release wave to the measurement location. An equation to predict the arrival time of the radial release wave as a function of the propagation distance into the sample is increasingly important in the design of the HRUM experiments, because these experiments require significantly thick impactors and front faces. This obligation creates a delicate balance between capturing the associated ring-up steps while not violating the assumption of one-dimensional motion. Obviously larger diameter samples and impactors could easily satisfy this balance, however, maximum diameters are usually limited by the experimental apparatus, and therefore fixed at some value associated with the diameter of the gun barrel (as is the case with plate-impact experiments).

Time-distance diagrams, as presented in the previous sections, are the most valuable tool for researchers attempting to design a plate impact experiment. To the best of the author’s knowledge, there exists no equation to predict the arrival time of radial release waves to a measurement location that can facilitate the construction of a time-distance diagram. In order to address this dilemma, let us consider a schematic of the impact of an impactor on a target found in Figure 2.6. The impactor and target have identical surface areas, where their diameters are equal to 2r. We will assume
that the wave diagnostic employed in this test is a distance of \( r \) from the edge of the target at a depth of \( D \) from the impact face. The initial compressive pulse generated upon impact is denoted \( S \), while the spherically expanding radial release waves generated from the edge of the impactor/target are denoted \( R \). Calculation of the arrival time of the radial release waves can be achieved through use of Pythagorean theorem. The transit distance \((D_T)\) that the radial release waves undergo on their approach to the diagnostic location can be generically written as a function of \( D \) and \( r \) in the form:

\[
D_T = \sqrt{D^2 + r^2}
\]  

(2.47)

and the subsequent arrival time \((t)\) of the radial release waves can simply be calculated in the form:

\[
t = \frac{D_T}{c} = \frac{\sqrt{D^2 + r^2}}{c}
\]  

(2.48)

where \( c \) is an appropriate wave speed associated with the experiment. In the case of shock wave experiments \( U_s \) can be utilized in place of \( c \). For experiments involving elastic wave propagation, the longitudinal wave speed \((c_l)\) can be used. By fixing \( r \) to a value associated with the diameter of the test sample, equation (2.48) can be used to determine the arrival time of the radial release wave at any distance within the target.

Thus, an equation to determine the arrival time of radial release waves for the case of symmetrical impacts is achieved that can directly lend itself to the design of experiments, allowing the construction of time-distance diagrams.
2.8 REFERENCES


CHAPTER 3

EXPERIMENTAL METHOD

This chapter discusses the details pertaining to the experimental technique used to study the dynamic response of materials. It will begin by introducing the hardware involved in the plate impact apparatus. A complete description of the relevant diagnostics including velocity and stress measurement techniques will be given. Details associated with projectile design and sample fabrication will also be included. Extensive modifications have been made to the existing apparatus and it is therefore relevant to refer future researchers planning to utilize the system to Appendix A, where a detailed procedure for using the apparatus can be found. This procedure, supplemented with the key points addressed in this chapter, will enable continued investigations into the dynamic response of materials at the University of Rhode Island (URI) in the Failure Characterization and Optical Laboratory (FCOL).

3.1 PLATE IMPACT APPARATUS

The hardware associated with the plate impact apparatus found in the FCOL consists of a single stage gas gun and a vacuum test chamber. The gas gun is used to drive projectiles to sufficient velocities to produce shock waves upon impact. The vacuum chamber houses the stationary test sample and relevant diagnostics. An overall depiction of the system can be found in Figure 3.1.
3.1.1 Single Stage Gas Gun

The gas gun system consists of a pressure vessel capable of driving projectiles to velocities in the range of 50-500 m/s. It employs a regulator, with an upper-threshold of 600 psi, to pressurize the vessel with ultra high purity helium gas. Important to the firing process, the vessel utilizes an inner separation connected to a push rod that seals off the front of the system. While the vessel is pressurized, the two sides of the separation receive equal distribution of pressure. The minimum pressure that will allow the front of the system to seal is in the range of 3-5 psi, giving rise to a lower end velocity around 50 m/s. Firing is accomplished by releasing the pressure from the back of the system, which in turn pulls the separation connected to the push rod toward the back of the vessel. This process allows the pressure associated with the front of the separation to release forward into the gun barrel, providing the driving force for the projectile.

The gas gun was initially designed for use with either a one or two inch inner diameter barrel. However the case, the two inch barrel was exclusively used in the work found within this thesis. The reason for this is quite apparent if we remember
the suggestion found at the end of section 2.4, where it was pointed out that longer time durations of one-dimensional strain can be achieved when impactors and targets have large diameter-to-thickness ratios. The barrel length is about seven feet long. Its inside surface is free from defects and assumed to have a constant diameter, as a result of an internal honing process. Although the barrel is called two inch, the honing process created an actual internal diameter of 2.0325”. For the purpose of this thesis, the length of the barrel will be assumed to be sufficient in order to provide gradual acceleration of the projectile. This is a key factor when trying to utilize the Rankine-Hugoniot jump equations. As noted in section 2.4, the typical assumption of the initial impact condition is that the impactor is at a state of zero stress traveling at some known velocity. Obviously, any launch force applied to the projectile will impart some compressive waves within the impactor. However, it is widely accepted in literature that these forces become negligible when a long gun barrel is used to allow for a gradual acceleration of the projectile [1]. Furthermore, any stress waves initiated within the impactor should be drastically overshadowed by the resulting shock waves generated upon impact, which will typically be several orders of magnitude greater.

3.1.2 Vacuum Test Chamber

The ability to conduct experiments within a vacuum is an important aspect to many of the underlying assumptions outlined in chapter 2. The largest factor addressed by the use of a vacuum test apparatus is the elimination of any air shock that would otherwise be generated when the projectile is driven down the barrel of the gun. Impact of this air shock on the test sample would complicate the Rankine-Hugoniot analysis because it would violate the assumption that the test sample is initially at a
state of rest ($\sigma = 0, u_p = 0$). Another detrimental effect associated with this air shock is that it will typically dislodge the test sample from its holder. This consequence will result in a non-planer impact condition, thus, violating the assumption of a one-dimensional state of strain. Intuitively, one could suggest that the sample be held more firmly. However, a key assumption taken in many experiments is that the sample is “freely supported”. That is not to say that plate impact experiments are never conducted under fixed sample conditions. However, unconstrained conditions are overwhelmingly used in literature.

Early research in the FCOL, conducting experiments without a vacuum chamber, addressed concerns with the air shock by separating the gas gun from the sample with a distance sufficient to allow for the air shock’s dissipation. In these investigations, the end of the gas gun was often at least two feet from the test sample. Although this solution likely eliminated any concerns associated with the air shock, it consequently reduced the repeatability of the system. This solution required meticulous alignment of the gun to the front face of the sample, typically resulting in hours of time spent in the process. Even with these meticulous alignment procedures, planarity of impact was often inconsistent. The likelihood of this is easily envisioned when one considers that the path of a projectile would decline due to the acting forces of gravity. Typically this drop is not global in regards to the projectile, because of its unbalanced nature. The projectile often has more mass in the front due to the additional presence of the impactor. Consequently, gravity causes a change in pitch, where the front of the projectile will dip downward. Obviously, the entire analysis presented in the theory found in chapter 2 will be violated when planarity is an issue.
(violation of one-dimensional strain). Thus, non-planar impacts were always omitted from results.

With the addition of a vacuum test chamber, concerns about the air shock are addressed. Thus, impacts can be accomplished where the projectile experiences no free flight. In terms of free flight, impact conditions can be generated while the projectile is still guided by the end of the gun barrel. By allowing the projectile to impact the test sample while it is still partially within the gun barrel, the meticulous alignment procedure previously discussed is eliminated, yielding a drastic improvement in the system’s repeatability, and a reduction in the experimental setup time.

Vacuum chambers associated with plate impact experiments are not very complicated systems. They are essentially an additional vessel, capable of holding vacuum pressure, which is attached to the end of a gas gun system. Many laboratories with extensive budgets seek to build test chambers capable of achieving ultra-high vacuum pressures in the range of $10^{-7}$ to $10^{-9}$ torr. These chambers require multiple vacuum pumps, coupled with an extremely well sealed system, to achieve their targeted pressure range. The author was initially motivated to build such a system in light of its common appearance in literature, however, private communications with Dr. Proud, of the University of Cambridge [2], who has published countless works within the field of shock physics and plate impact, helped shed light on the subject. The result of this interaction was the disclosure that only a rough vacuum is necessary to eliminate concerns of the air shock associated with plate impact experiments. Many other useful suggestions were also provided during this discussion, including the use
of sticky-tack (putty used to mount posters to walls), which was revealed as a useful last-ditch sealant that can be employed to salvage an experiment when the chamber fails to hold vacuum. In light of this meeting, the vacuum chamber developed for the subsequent studies found in this thesis, can be considered “rough”, were tests are conducted in the range of 10 torr.

The key components associated with the vacuum test chamber include a vacuum pump, a gauge capable of monitoring the vacuum pressure, a test sample holder, a system for capturing the impacted sample, and ports to allow diagnostic equipment to be employed. Figure 3.2 contains a depiction of the inside of the vacuum chamber where key components can be visualized.

Figure 3.2. The inside of the vacuum chamber.
3.1.2.1 Vacuum Pump and Gauge

Vacuum is achieved in the system through use of a dry scroll pump manufactured by Varian\textsuperscript{®}. This pump is capable of generating vacuum pressures to a minimum of $5.0 \times 10^{-2}$ torr. Vacuum pressures are monitored through use of a vacuum gauge, also manufactured by Varian\textsuperscript{®}, which is capable of measuring absolute pressures in torr, mbar, or kPa. An important aspect associated with these two products is the presence of overpressure, where 1 atmosphere is the maximum pressure that either can witness without damage. In order to address this concern, ball valves are employed to seal the pump and gauge diaphragms off from the system before the subsequent firing of the gas gun. In addition to damaging the pump and pressure gauge, the overpressure associated with the highly pressurized helium charged vessel, can wreak havoc on the chamber’s seals and diagnostic equipment. This concern is addressed by incorporating a large “blow-off” valve into the system. Given its size and weight, this valve was designed to release minimum gauge pressures (referenced against ambient air pressure) of one psi.

3.1.2.2 Test Sample Mount

The test sample mount incorporated into the vacuum chamber consists of a holding plate, mounted to stand-offs, which are connected to a seal plate that is threaded to the end of the gas gun barrel. The sample mounting plate and seal plate are constructed out of stainless steel, while the standoffs are polycarbonate. This design was chosen to minimize the transmission of vibrations (associated with firing the gun) to the sample. Utilizing the acoustic approach discussed in chapter 2, it can easily be shown that any incident vibrations will be reduced to 1\% of their initial
amplitude, as a result of the impedance mismatch between the two plates and the polycarbonate stand-offs. Additionally, samples tested in this thesis required extra stand-off rings before they could be mounted to the holder. These rings were constructed from gray PVC. Thus, it can be noted that vibrations are additionally reduced to 0.1% of their incident magnitude.

Stand-off rings were utilized to mount test samples in an effort to eliminate dispersion considerations within the experiments. As previously discussed in section 2.4, this can be accomplished by using impactors and samples of the same diameter. Since an impactor is typically carried by a sabot, which in turn completes the projectile package, it should be apparent that the edges of the sabot would strike any holder that directly contacted a test sample having the same diameter as the impactor. Early research in the FCOL addressed this concern by impacting samples of larger diameter than the projectile package. However the case, the resulting dispersion associated with this geometry often generated complications in the interpretation of results.

In order to assume a “freely supported” test sample, stand-off rings were mounted to the samples with the use of eight small drops of super glue. Eight additional drops were used to mount the stand-off ring to the test holder. Planarity of impact is strongly dependent on the mounting process of the test sample. Thus, it is strongly advised that special consideration be taken in manufacturing of the stand-off rings, to ensure that their bored section is perfectly perpendicular to their face. Additionally, when mounting a sample package to the test ring, a precision ground table should be used, where the front face of the sample and front face of the stand off
ring are placed flat against the table, and glue is applied to the back interface. Before mounting the test sample to the sample holder, all glue from previous experiments should be meticulously removed in order to ensure that the sample sits perfectly concentric in the holder. It should be pointed out that the time required for the glue to dry is a limiting factor associated with the number of experiments that can be conducted in a single day. In order to achieve rapid dry times, the author has recognized that a minimal amount of glue is needed. Many different glues and epoxies were utilized in preliminary studies. However, Gorilla® super glue proved the most useful in terms of bond strength and drying times. Figure 3.3 contains an image of a sample mounted to the test holder.

![Test Sample](image)

Figure 3.3. Test sample assembled to the sample mount.

### 3.1.2.3 Soft Recovery Method

With the sample holder discussed, let us briefly address the sample recovery method. In order to minimize damage to the vacuum chamber, it is imperative to incorporate a method for capturing the impacted sample. When post-mortem
in investigations are not required, this method can simply employ thick plates of steel. In
the study of spall fracture, it was important to perform post mortem analysis of the
subsequent damage inflicted by the acting tensile waves. In order to aid in these
investigations, a soft recovery method was developed. This method utilizes a catch
box, filled with clay, mounted to a linear track with a foam shock absorber. The
premise of this scheme is to gradually decelerate the impacted sample without causing
additional damage. The clay is utilized to allow for a gradual transfer of momentum
to the catch box. The linear track and foam bumper are used to dissipate the resulting
kinetic energy imparted on the box by capturing the test sample. Polyurethane foam
was chosen for the bumper material, due to its ability to return to its original shape
after compression, and maintain its resilience after repeated use. It should be noted
that foam was chosen instead of commercial shock absorbers due to the inherent
difficulties with their use in a vacuum. It should be immediately apparent that the
slow off-gassing of air trapped in the foam and clay, will significantly affect the
achievable vacuum pressures of the system. Had ultra-high vacuum pressures been
pursued, the soft recovery method should be moved outside the vacuum chamber. In
this configuration, a thin Mylar diaphragm could be employed to seal off the back of
the chamber, while still allowing for the impacted sample to project out.

3.1.2.4 Projectile Loading

An additional concern associated with utilizing a vacuum test chamber is the
tendency of the projectile to be pulled down the barrel of the gun as a result of the
small amount of air trapped behind it. This concern was addressed by pumping
vacuum on both sides of the projectile. An added advantage of this design is the
ability to utilize the vacuum pump to load the projectile into the gun barrel. Previous investigations accomplished this by employing rudimentary techniques common to muzzle loaders and cannons, using a push rod to load the projectile.

3.1.2.5 Vacuum Through-Ports and Grounding

In order to bring the diagnostic equipment into the vacuum chamber, six fiber-optic and six electrical through ports are employed. Four of the six fiber-optic through ports were designed and made at URI, while the other two were obtained through commercial sources. All six electrical through ports were designed and manufactured at URI.

Before we continue to the relevant diagnostics, the importance of a well-grounded system should be addressed. Electromagnetic frequencies (EMF) and static discharge represent two specific examples of interference that can significantly affect a test apparatus. These interferences often result in pre-mature triggering of diagnostics, and can therefore generate “wasted” experiments. In order to address this concern, grounding wire is connected to numerous points throughout the plate impact apparatus. These wires are all tied into a central copper hub that is subsequently connected to the ground associated with the wall outlet. A simple check with a multimeter demonstrates that there is less than 0.1 $\Omega$ resistance between any surface in the apparatus and the ground connection. Finally, it should be noted that a completely separate circuit is used to feed power to the diagnostics in order to avoid any electrical noise associated with the grounding method. Obviously, at some point these ground sources coincide, considering that each circuit receives electricity from the same power plant. However the case, there is more than 6 $\Omega$ resistance between the ground
source used by the diagnostics and that of the system grounding loop. Thus, we can consider that these two circuits are sufficiently separated.

3.2 DIAGNOSTICS

Up to this point, the hardware associated with the plate impact apparatus has been discussed. Before getting into the relevant diagnostics of the system, we will briefly revisit the concern of electrical noise affecting experiments. Within the solid mechanics department at URI, there exist numerous test apparatus that all inherently impart noise into the building’s power supply. The vacuum pump, used to evacuate the test chamber, provides a perfect example of this circumstance, where the wall voltage experiences a disruption from its normal AC sine wave as the pump is running. For this reason, all important diagnostics are connected to the building’s power supply through a power protection unit. This unit is an AC voltage regulator that utilizes isobar AC spike and line noise filters to generate a perfect sine wave power signal. Finally, it should be noted that power is supplied to the “noisy” vacuum pump from a different circuit than the one used for the diagnostics.

3.2.1 Recording Devices

Temporal records of the experiments are obtained with the use of oscilloscopes. Two, four channel oscilloscopes are required to collect all the diagnostic data obtained from the velocity measurement and stress measurement schemes. The spall fracture study on cast iron utilized two TDS 3014B digital phosphor oscilloscopes manufactured by Tektronix®. These scopes have a 100MHz bandwidth, maximum sample rate of 1.25GS/s, and are limited to capturing 10,000
data sets. The time scale of these scopes was consistently set to 40$\mu$s/division in order to capture all required data. This time scale limited the sample rate to 25MS/s, resulting in data sets being obtained every 40nanoseconds.

An update to the FCOL included the purchase of a new oscilloscope, which replaced one of the two previously used in the study of the spall fracture of cast iron. This scope, also manufactured by Tektronix® (DPO 3054), represents a drastic improvement in data acquisition. The new scope has a 500MHz bandwidth, maximum sample rate of 2.5GS/s, and five-million data set limit. Unlike the previous scopes, the time scale of this scope could be set to 200$\mu$s/division, while still utilizing the maximum sample rate of 2.5GS/s, and therefore capturing data every 0.4nanoseconds. In cases where a smaller time scale is utilized, it should be noted that if the maximum sample rate is specified, coupled with a specification of the maximum sample quantity, the scope will automatically collect data that extends beyond its screen. A quick calculation can show that the scope, used to its limitations, will collect data over a period of two milliseconds. Capturing five million data sets provides some interesting problems when trying to import the data into Microsoft Office Excel. Initially, a Matlab program was written to strategically analyze the data and output only the required portion to Excel. However, the time associated with running this program was quite frustrating. It should be noted that the scope has built-in gating features that allow a user to selectively save only a portion of data from the entire collection. When using the gating feature, along with the scope set at its maximum sample rate, 12.8 $\mu$s is the maximum time duration one can use in Excel to generate
plots. This is easily calculated by noting that plots in Excel are limited to 32,000 data sets.

3.2.2 Velocity Measurement

The apparatus uses two separate systems to measure impact velocity. The technique employed by these two systems involves the interruption of through-beams. Each system consists of two light sources and two receivers. As the projectile travels towards the target, it will sequentially interrupt the through-beams. Velocity is inferred by dividing the distance between the receivers by the time between interruptions. Figure 3.4 contains a sectional view of the apparatus, demonstrating the relative position of the velocity traps in regards to the gun barrel and test sample. The distance between the through-beams is one inch for both of the velocity measurement systems.

![Sectional view of the apparatus demonstrating the relative position of the velocity traps.](image)

**Figure 3.4.** Sectional view of the apparatus demonstrating the relative position of the velocity traps.
The first system employs components manufactured by Keyence®. It uses two FS-M1H units, capable of sending and receiving a signal. The light source of these units is red LED. These units have a maximum response time of 20 µs. They output a positive 12 V when their respective beams are broken.

The second system employs components manufactured by ThorLabs®. The receiver end of this system uses high speed photo detectors (DET10A). In contrast to the previous system, these detectors employ photodiodes, and therefore output voltage as a function of incident light. When the beam is broken, the voltage outputted by these detectors drops to zero. The maximum response time of these detectors is 1 ns. This time can only be achieved by applying a 50Ω termination, when bringing the signal to the oscilloscope. Light is carried to the detectors with the use of 600 µm fiber-optic cables. The light source of this system employs 650 nm, 7 mW, red laser diodes (L650P007). As such, special care should be taken when using this system to avoid eye damage.

Before we continue, let us briefly address the use of the two systems. The Keyence® system is by far the more robust of the two, requiring very little maintenance between tests. However, its relatively slow response time can yield significant errors in velocity measurements. In contrast, the ThorLabs® system provides the most accurate velocity measurements. Unfortunately this benefit comes at the expense of an extremely finicky system. The relative low acceptance angle of the employed fiber optic cables requires that incident light must be almost perpendicular to the fibers for the system to work. This condition typically requires hours of fine tuning of the laser diodes. Once properly aligned, the system produces
two additional velocity measurements. By considering that 600 µm fibers are employed, the time it takes for each detector to drop from their saturated voltage to zero can be correlated to the diameter of the fibers (600µm/∆t). When attaining velocity measurements, this process should only be used to confirm the velocity calculated from the distance between detectors and time between interruptions. The most valuable information will come from the ThorLabs® system. The measurements obtained from the Keyence® system should only be used to confirm the velocity measured from the ThorLabs® system, or for cases that the later fails to perform.

3.2.3 Stress Measurement

Stress measurements are made with the use of commercial manganin gauges. Manganin is an alloy composed of approximately 84% copper, 12% manganese, and 4% nickel. Manganin has a large piezoresistance coefficient, where changes in pressure generate large variations in resistance. Conversely, Manganin has a low temperature resistivity coefficient, and therefore temperature has little effect on its resistance. In light of these properties, manganin is an ideal material for use in shock wave studies because the change in resistance caused by shock compression is much larger than the corresponding change in resistance caused by shock heating. For longitudinal stress measurements, manganin transducers are embedded in samples such that the active gauge element is normal to the direction of wave propagation. The manganin element is typically insulated from the sample by thin layers of Kapton, Mylar, Teflon, or epoxy. In application, a constant electrical current is supplied to the gauge. The passage of a shock pulse through the gauge plane creates a rise in
pressure, which generates an increase in resistance, in turn yielding a rise in the output voltage.

To ensure that manganin gauges respond to pressure acting normal to their active grid, a thin manganin foil is typically employed. Thus, the dimension of the gauge in the direction of the shock front is negligible. In comparison, the dimension of the gauge normal to the shock front is relatively large. In literature, there exist numerous analytical accounts into the elastic-plastic response of manganin gauges, validating the assumption that their response is governed by pressures acting normal to the active gauge element [3-6]. These accounts rely on the assumption of a one-dimensional state of strain, acting normal to the plane of the gauge. In situations where this is not the case, the sensor will respond to both the stress and the strain acting along the gauge plane. When dimensional changes in the gauge are not negligible (2-D/3D strain), independent strain measurements are required to allow for the separation of the stress component from the measured change of the gauge. Hence, it is imperative to generate planar loading conditions, to evoke a state of one-dimensional strain in samples, when manganin sensors are the primary diagnostic tool.

Normal stress is determined from the fractional change in resistance of the active grid through use of a calibration equation. These relationships, which relate change in resistance to normal stress, have been widely established in literature [7-14]. The procedure for calibrating manganin gauges involves controlled, uniaxial strain, shock wave experiments in well-classified materials. In these experiments, the fractional change in resistance is correlated to the stress at the gauge plane, which is typically calculated using the Rankine-Hugoniot conservation of momentum, equation
(2.14). By repeating this procedure at several stress levels, a relationship can be developed to define the stress component normal to the gauge as a function of the fractional change in resistance.

Manganin remains linear elastic up to pressures of 1.5 GPa [7]. Beyond this range, there is a hysteresis in the resistance of the gauge, where it will no longer return to its initial resistance after unloading. The hysteresis of the gauge has been frequently attributed to strain hardening effects, where shock compression generates an increasing concentration of defects in the gauge material [1, 15]. The residual resistance associated with hysteresis is relatively small, usually below 2% to 2.5%. For pressures below 7 GPa, the residual increment of the resistance is nearly proportional to the peak pressure [15]. Numerous researchers have proposed hysteresis-corrected calibration methods to address this condition [16-17]. These works are consistent in the assumption that the hysteresis only affects the ability of a gauge to return to its initial resistance after unloading. Additionally, it has been shown in work by Rosenberg that hysteresis does not affect a gauge’s ability to accurately quantify multiple-shock loadings [18]. In this work, graded impactors were utilized to induce stepped loading profiles on samples. Through comparison between the amplitudes of these multiple-shock events to those of single-shock events of the same final amplitudes, it was proven that the response of the gauge is equal in both scenarios. Considering that the HRUM will generate reshocking conditions within the gauges, this work by Rosenberg provides validation of employing manganin stress gauges as the primary diagnostics in these experiments.
Before we continue to the specific transducers employed in this thesis, let us briefly address some additional observations common to manganin stress gauges. As the pressure rises within a material, atomic realignments associated with phase changes can be generated. This mechanism is extremely important in regards to the insulation material chosen for the gauge package. At high pressures, phase changes in many insulation materials result in the loss of their insulation properties. Consequently, the manganin gauge will fail by a fast short-circuiting effect, where the current is able to bridge the legs of the gauge. For example, Kapton insulation has been shown to transition to a conductor at pressures above 9 GPa [19]. This consequence can be addressed by choosing insulators with a high shock induced threshold pressure, such as Teflon [17, 20]. Another approach has been to utilize manganin gauges with an initial resistance that is several orders of magnitude lower than the insulating material [21].

In addition to shunting of manganin gauges, some key observations have been made in regards to using manganin transducers in conducting targets. In these applications, the response of the gauge can have a superimposed electrical ringing around breaks in slope. This ringing will typically exhibit an exponential decay, where the gauge signal will quickly equalize to the level associated with the stress magnitude. Inspired by the increased observation of this ringing phenomenon in many laboratories where newer oscilloscopes operating at GHz have been purchased, the gauge response was modeled by utilizing a system of electrical equations, in works by Bourne [22]. In this work, it was demonstrated that the electrical ringing is caused by
capacitive linkage between the gauge and the target surface, through the dielectric sheet used in mounting, and the inductance of the gauge used for measurement.

The stress gauges utilized in the proceeding investigations are $50 \Omega$ manganin gauges manufactured by Dyansen® (MN4-50-EK). They have an active grid of 0.15” by 0.15”, and an overall length of 3”. The thickness of the manganin foil is 0.0004”. To insulate the manganin from the environment, it is embedded within 0.001” thick Kapton sheets. The applicable pressure range of these gauges is between 0.1-10 GPa. A constant-voltage excitation is applied to the gauges with use of piezoresistive pulse power supply, manufactured by Dyansen® (CK2-50/0.050-300). This power supply is capable of simultaneously pulsing two gauges. For each channel, it has a continuously adjustable charge voltage from 30 to 300 volts. The pulse duration can be varied between 5-1500 µs. When utilizing $50 \Omega$ gauges, each channel uses a Wheatstone bridge, in a quarter-bridge arrangement, where one of the legs is completed by the gauge. Impedance matched, $50 \Omega$ coaxial cables are used to transfer voltage between the power supply and the gauges. The fastest measurements can be obtained when utilizing the $75 \Omega$, unattenuated bridge output. Thus, $75 \Omega$ coaxial cables are utilized to transfer the output to the oscilloscope, while applying a $75 \Omega$ termination at the oscilloscope. The power supply can be triggered with an input voltage from 6-100 volts. Referring back to the velocity measurement section, the Keyence® detectors generate a voltage within that range. Depending on the velocity of impact, either the first or second detector can be employed to trigger the gauge power supply (low velocities must use the final detector). The reader is advised to refer to the
The stress gauges require some initial preparation before they can be utilized in
the experimental investigations. The first preparation step requires about 16 inches of
strain gauge wire to be soldered to the leads of the gauge. To ensure that the soldered
connections do not break during handling and sample fabrication, a strain relief
method should be employed. This can be accomplished by utilizing a thin strip of
Mylar, where the gauge and wire are taped to the strip on both sides of the solder joint.
Additionally, the final package should be wrapped with electrical tape to avoid gauge
shunting, in the event of contact with a metallic surface. Figure 3.5 contains images of
the employed gauges. Part (a) of the figure contains a partial section view where the
active grid and kapton insulation of a gauge can be noted. Part (b) of the figure
depicts a gauge assembly, which is ready to be embedded within a test sample.

![Manganin stress gauge](image)

**Figure 3.5.** Manganin stress gauge; (a) Partial section view showing the active grid
and kapton insulation; (b) Final gauge assembly.

### 3.2.3.1 Calibration

As previously noted, there exist numerous relationships proposed in literature
to determine the normal stress from a manganin gauge’s change in resistance [7-14].
The gauges manufactured by Dyansen® are unique in that they have been accurately
characterized over a range of stress from 0-12.5 GPa [23]. From these tests, the relationship between stress in the longitudinal direction ($\sigma_x$), and relative change in resistance ($\Delta R/R_o$), can be written as:

$$\sigma_x = 5\left(\frac{\Delta R}{R_o}\right) - 0.052\left(\frac{\Delta R}{R_o}\right)^2 + 0.00002\left(\frac{\Delta R}{R_o}\right)^4$$

(3.1)

where the relative change in resistance is measured in percent. $\Delta R$ refers to the change in a gauge’s resistance, while $R_o$ is the initial resistance of the gauge. Figure 3.6 contains a plot of this equation, which will be used in the proceeding calibration procedure.

**Figure 3.6.** Calibration curve for 50 $\Omega$ manganin gauges manufactured by Dynasen®.

Before every experiment, each gauge requires calibration. This procedure allows for the conversion of the voltage received by the scope into a percent resistance change of the gauge. The calibration procedure begins by estimating the maximum stress that the gauge will witness. This can be accomplished by assuming an impact
velocity for the experiment. If the experiment is symmetrical (same material for both impactor and sample), the acoustic approximation can be used to estimate the impact stress from equation (2.21), where the resulting particle velocity is half of the impact velocity. For most experiments, this impact stress should represent the maximum stress achieved by the experiment. However, if unique geometries are employed, additional theoretical considerations should be used to estimate the maximum stress that the gauge will witness. With an appropriate maximum stress, Figure 3.6 can be used to determine the relative change in resistance that the gauge will experience.

Next, the initial resistance \((R_o)\) of the stress gauge needs to be measured with a multimeter. From this measurement, the maximum change in resistance that the gauge will undergo \((\Delta R)\), can be calculated from the percentage attained from Figure 3.6, in the form:

\[
\Delta R = \frac{\left(\frac{\Delta R}{R_o}\right)R_o}{100}
\]  

(3.2)

where the maximum resistance the gauge will achieve is \((\Delta R+R_o)\).

Now, all the required values that will be used in the subsequent calibration have been acquired. The calibration of the gauges should be performed immediately before the experiment, once the oscilloscopes and power supply are activated. Settings for the voltage scale of the oscilloscope and the power supply should be chosen before generating the calibration parameters. For the investigations found in this thesis, the power supply was set to apply a voltage of 100 V for a duration of 350 \(\mu\)s. Additionally, the 75 \(\Omega\), unattenuated bridge output was exclusively used. Without much experience with the system, choice of voltage scale of the oscilloscope may
require repetition of the procedure. Essentially, the proper scale will yield a maximum pulse that occupies 50% of the total voltage window (this will allow larger stresses than anticipated to be captured).

A decade resistance box is used to conduct the calibration, where the coaxial cable employed to supply power to the gauge is connected to the resistance box. Using the same multimeter that was employed to measure the resistance of the gauge, the box is set to simulate the gauge’s initial resistance. At this point, the location of the resistance measurements must be chosen. For the investigations found in this thesis, all measurements were made at the upper end of the coaxial cables (point where they attach to the power supply). Thus, any resistance of the cables was taken into account in these investigations. The bridge of the power supply is next balanced to the initial value of the gauge. It has been noted that the specific power supply, found in the FCOL, produces the best balance at -002. Once balanced, the unit is fired. The voltage increase on the oscilloscope (\( V_1 \)), associated with the balanced gauge, needs to be recorded. Next the resistance box is set to the anticipated maximum resistance the gauge will achieve in the experiment (\( \Delta R + R_o \)). The power supply is again fired, and the voltage increase on the oscilloscope (\( V_2 \)) is recorded. If this final step produces a voltage that is unacceptably close to the maximum range of the scope, the procedure should be repeated while using a larger voltage scale.

The initial calibration procedure is now complete. Once the sample is mounted in the chamber, and all associated connections with the gauge are made, a final check should be employed. Since resistance measurements in this thesis were made at the upper end of the coaxial cables, this final check was used to confirm that the initial
“upper-end” resistance used in the calibration, matches that of the actual gauge/cable assembly. In cases where agreement is not achieved, the calibration procedure is revisited until there is consistency in the measured initial resistances.

3.2.3.2 Data Reduction

After the completion of an experiment, the values obtained from the calibration procedure are used to transform the voltages into a relative change in resistance. Due to slight variations in the applied circuitry of the power supply, the supplied voltage typically exhibits either a decline or increase through the duration of the pulse. Additionally, it should be apparent from the determination of $V_I$ that the initial voltage is not exactly zero. Thus, the signal needs to be corrected so that the initial voltage yields a flat-line at zero. This can be accomplished by applying a linear curve fit to the voltage data obtained a few microseconds before the arrival of the stress profile. From this curve fit, the data can be balanced so that voltage starts from zero ($V_B$), in the form:

$$V_B = V_{\text{measured}} - mt - a$$

(3.3)

where $V_{\text{measured}}$ is the actual voltage output of the oscilloscope, $t$ is the time associated with each data set, $m$ is the slope of the curve fit, and $a$ is the y-intercept of the curve fit. Once accomplished, the voltage can be transformed into a relative change in resistance. The first step utilizes the parameters generated in the calibration procedure in the form:

$$K_{75} = \left[ \frac{V_2 - V_1}{\left( \frac{R_3}{R_2} \right) - \left( \frac{R_3}{R_0 + \Delta R + R_1} \right)} \right]$$

(3.4)
where $R_f$ the resistance of the three additional legs of the Whetstone bridge, specified as 86.6 Ω. $V_I, V_2, R_o,$ and $\Delta R$ are the parameters obtained from the calibration procedure. From this calculation, the voltage can be transformed into a relative change in resistance in the form:

$$\left(\frac{\Delta R}{R_o}\right) = \left[\frac{R_1}{R_o} \left(\frac{1}{\left(\frac{R_1}{R_o} + \frac{V_B}{K_{75}}\right)}\right) - 1 - \frac{R_1}{R_o}\right] \times 100 \quad (3.5)$$

For all investigations found in this thesis, the resultant stress was calculated by using this relative change in resistance ($\Delta R/R_o$), in equation (3.1).

### 3.3 PROJECTILE DESIGN AND SAMPLE FABRICATION

The following sections will address details pertaining to the projectile and sample configurations utilized in this thesis. First, the projectile assembly will be outlined. Following this, details associated with sample fabrication will be discussed. The dimensions of the samples and impactors utilized in the subsequent studies will be outlined in their respective chapters.

#### 3.3.1 Projectile Design

The projectile assembly consists of an impactor and sabot. The impactor is the fundamental tool used in plate impact studies to evoke shock waves in samples. As noted in chapter 2, the thickness of the impactor controls the loading duration of an experiment. Additionally, its impedance and velocity will control the amplitude of the shock loading. The sabot serves as the delivery system, carrying the impactor to its targeted destination. Although the impactor is fundamental to the experimental design
of plate impact investigations, the geometry of the sabot can additionally influence the outcome of experiments. Thus, much time and effort has been spent considering an array of sabot designs. These designs included the use of Teflon, polyethylene, foam, ABS, and PVC. Additionally, these various designs utilized multiple support methods for carrying the impactor, including free back, fixed back, and foam backed.

An essential aspect associated with sabots is their ability to thwart blow-by. Blow-by occurs when a sabot fails to seal off the high pressure driving gas from the test chamber. For similar reasons addressed in regards to air shocks, blow-by can be detrimental to an experiment. Preliminary designs addressed this concern by ensuring that the outer-diameter of the sabot closely matched the inner-diameter of the gun barrel. In these situations, the large contact area between the sabot and the gun barrel required the use of materials with low coefficients of friction. Teflon was initially chosen to satisfy this condition. However, its relatively high density and extremely high cost made it unpractical for the current investigations. Ultra high density polyethylene was the next suitable choice to satisfy the requirements of a low coefficient of friction. Although this material is relatively low in cost and density, its high coefficient of thermal expansion made it extremely difficult to achieve the tolerances required to thwart blow-by. In light of these difficulties, the concern about blow-by was addressed by incorporation of o-rings into the sabot design. By doing so, the choice of sabot material was no longer restricted by requirements of low coefficients of friction. Thus, gray PVC was ultimately chosen due to its excellent machinability, low density, and low cost.
An additional aspect associated with sabot design is the method by which the impactor is supported. To ensure a complete unloading of the shocked state, the impactor should be freely supported. Although this condition is easy to conceptualize, it is relatively difficult to attain in application. It should be noted that the planarity of impact is directly influenced by mounting of the impactor to the sabot. Thus, in order to ensure that the impactor’s front face is perpendicular to the length of the sabot, a small support surface is needed. If this support surface is large, flexural waves will be activated within the impactor as a result of the arrival of the first C-compressive wave to its back surface. Conversely, if this support surface is negligible, it will be difficult to ensure that accurate planarity is achieved at the impactor-specimen interface.

Let us consider the sabot that was used in the proceeding investigations. The length of the PVC sabot was three inches. It employed two o-rings (75 Viton, size 224), 0.5” and 0.25” respectively, for the front and back. The groves for these specific o-rings were 0.150” wide. The diameter of these groves will directly affect both the sealing capabilities and attainable velocity of the tests. Through trial and error, it was found that a diameter of 1.764” provided a good balance between sealing capabilities and attainable velocity. In order to minimize friction, o-rings were lubricated with white lithium grease containing PTFE (FUCHS® Renolt ST-80). Considering that o-ring were utilized to seal the barrel, the outer diameter of the sabot is not crucial, and typically was in the range of 1.8-1.95”. In order to minimize weight, the front of the sabot was bored to a diameter of 1.5” and a depth of 2.75”. Additionally, to ensure a freely supported impactor, the front was bored to a diameter of 1.672” and depth of 0.4”. Considering that 45 mm (1.772”) diameter impactors were exclusively utilized,
this process resulted in a 0.05” support ring around the back edge of the impactor. In order to accept the impactor, a final boring process to a diameter of 1.772” was applied to the front of the sabots. The depth of this final bore was chosen so that there is at least a 75% inset of the impactor. The final assembly consists of mounting the impactor to the sabot. The same glue utilized to mount the sample was employed in this process. Figure 3.7 contains a depiction of the projectile assembly. Part (a) is a picture of an actual projectile. Part (b) is a SolidWorks assembly of the package, where a sectional view can be observed.

Figure 3.7. Projectile assembly; (a) Actual projectile; (b) SolidWorks section view.

3.3.2 Sample Fabrication

Sample preparation consisted of the initial manufacture of the test specimen, followed by a lay-up procedure. The manufacture of the samples was very important to the experiments. In this thesis, all test samples and impactors were constructed out
of 45 mm diameter cylinders. Thus, manufacture of these components was conducted with use of a lathe. In order to ensure minimal microstructure damage, a constant coolant flood was utilized. Additionally, for the case of the cast iron and steel samples, an additional grinding process was used to ensure that the sample/impactor faces were parallel. The lay-up procedure was used to embed the employed stress gauges within layers of the test sample.

Assuming that all components of the test samples are manufactured, let us consider the lay-up procedure. Bonding was accomplished with the use of a low viscosity, two part, Buehler® epoxy. These parts consist of an EpoThin® epoxy resin (20-8140-128) and an EpoThin® epoxy hardener (20-8142-064). The epoxy was applied to the hardener with a weight ratio of 5:1.95. Bonding was conducted atop a metal plate coated with release agent (Buehler® Release Agent 20-8185-016). The first step of the procedure was to rough all bonding surfaces with 240 grit sand paper. Next, the release agent was applied to the metal plate, to ensure that the sample assembly could be removed after bonding. Once the release agent was completely dry (about 5 min), the epoxy and hardener were mixed at the prescribed ratio. For the samples used in the proceeding investigations, 20 g of epoxy and 7.8 g of hardener were utilized. A wooden tongue depressor was utilized to mix the two parts for a period of five minutes. Once properly mixed, the solution was placed in a vacuum bell jar to evacuate any trapped air bubbles. About ten minutes after the initial mixing stage, the solution was ready to apply to the samples. The back plate of the sample was placed on the metal bonding plate. Next, a sufficient quantity of the epoxy solution was poured across the top surface of the back plate. Following this, the stress
gauge was mounted, ensuring that its active grid is located at the center of the sample. An additional amount of epoxy was applied to the top of the stress gauge. For single gauge experiments, the sample’s front face was placed on top of the gauge and back plate. In cases of a two gauge configuration, the preceding steps were repeated until all gauges were embedded within the sample layers. The final step utilized a 300g mass to establish a bonding pressure within the sample assembly. During this step, much of the applied epoxy would be expelled from the sides of the sample assembly. Thus, special care was exercised to ensure that the sample’s layers did not slide around on one another. Intuitively one might address this concern by providing an additional ring to maintain concentric conditions. However, this is not a good solution, because it will not permit the epoxy to expel from the sides. Thus, this solution would result in an unacceptable layer thickness associated with the gauge plane (gauge response time will be slow due to the ring-up of a thick layer). In light of these shortcomings, radial sliding was addressed by providing three line contacts in the axial direction around the diameter of the sample (0°, 120°, 240°), using small scraps of angle stock. Additionally, the excess epoxy was wiped from the sides of the sample. Once completed, the assembly was allowed to cure for a duration of twenty-four hours. After this time, the 300g mass was removed, and the sample assembly was removed from the metal base plate.

It should be pointed out that epoxy continues to cure for years after it is mixed. During this process, its mechanical properties exhibit significant variations. These changes in properties are the most noticeable in the first few weeks. Thus, all samples tested in this thesis were allowed at least a two week cure duration. In order to
facilitate comparison between tests, the cure duration was consistent for all samples from each specific investigation. A few days prior to the experiments, the stand-off rings were assembled to the sample packages, as described in section 3.1.2.2. Figure 3.8 contains examples of the assembled samples utilized in the proceeding investigations. Parts (a) and (b) respectively depict a spall fracture sample and a HRUM sample.

![Sample assembly](image)

**Figure 3.8.** Sample assembly; (a) Spall fracture sample; (b) HRUM sample.

### 3.4 REFERENCES


CHAPTER 4

MATERIALS STUDIED

This chapter outlines the measurements and calculations of the initial state parameters for the materials studied. These materials include five different cast irons, polycarbonate, A572 Grade 50 structural steel, and 6061 aluminum. Presented in this chapter are details pertaining to the determination of the initial state parameters including density and initial wave speeds. Following this, the acoustic impedances and dynamic elastic constants constructed from the experimentally determined wave speeds are presented. This chapter concludes with an investigation into the as-received microstructures of the five cast irons studied in the spall fracture experiments.

4.1 INITIAL STATE PARAMETERS

In order to begin a shock study on a material of interest, determination of the initial state parameters is necessary. These parameters include the initial density and wave speeds of a material within elastic pressure regimes. Taking note of the Rankine-Hugoniot equations previously presented in chapter 2, it can be observed that the initial density \( \rho_0 \) and bulk wave speed \( c_b \) become fundamental parameters in the EOS analysis. Knowledge of the initial wave speeds is of particular importance in the design of plate impact experiments. Accurately characterized wave speeds allow for
the construction of time-distance diagrams, which prove useful in understanding wave interactions and allow for the interpretation of an experiment’s temporal stress record.

4.1.1 Density Determination

Densities of the materials studied were determined by dividing the measured mass of cylindrical test samples by their respectively calculated volumes. Masses were measured to the nearest tenth of a gram using a digital scale. In order to minimize errors associated with mass measurement samples were chosen so that their total mass was roughly eighty percent of the maximum 400g limit specified for the scale. This guideline also maximized the volume of the test samples which in turn helped to minimize errors associated with volume calculations. Volumes were calculated from measurements of the thickness and diameter of the test samples. A micrometer was utilized to measure the thickness and diameter of test samples to the nearest ten-thousandth of an inch, which relates to the nearest 2.5 micron within the metric system. Densities calculated for the materials studied are summarized in Table 4.1.

4.1.2 Wave Speed Determination

Wave speeds of the materials studied were determined with use of ultrasonic contact transducers employed in a pulse echo mode. This method, first developed in 1964 by researchers Carnevale, Lynnworth, and Larson, has been shown to produce longitudinal and shear-wave velocities with errors less than 1% [1]. Since its establishment in 1964, the method has gained wide acceptance in literature and has subsequently been adopted into the ASTM standards for non-destructive testing of materials as ASTM E494-05 [2]. The basic principle of the pulse echo mode with use
of contact transducers can be visualized in Figure 4.1. A contact transducer which is capable of sending and receiving a signal is utilized to send a weak amplitude stress wave into a sample. The wave reflects off the sample’s free surface and returns to the transducer where it is partially transmitted. The remaining portion of the wave continues reflecting between the transducer face and free surface, producing echoes of the original signal each time it returns to the transducer face. The wave speed can therefore be calculated from temporal records of the echoes by dividing twice the thickness (T) of the sample by the transit time (Δt) between echoes. Due to timing delays caused by initial contact issues with the transducer and sample face, it has been pointed out in literature that a significant reduction in error can be achieved by utilizing a transit time between multiple echoes instead of the time between the initial pulse and first echo [3].

![Diagram of the pulse-echo ultrasonic method](image)

**Figure 4.1.** The pulse-echo ultrasonic method.

The ultrasonic equipment utilized in the determination of sound velocities was manufactured by Panametrics®, a subdivision of Olympus®. For the task of longitudinal wave speed measurements direct contact transducers V103-RM and V126-RM were utilized, which respectively outputted central frequencies of 1 MHz and 5 MHz. Transverse wave speeds were determined with the use of direct contact transducers V153-RM and V154-RM which respectively outputted central frequencies...
of 1 MHz and 2.5 MHz. Transducers were excited with use of a pulser/receiver model 5058PR. This unit was designed for use in both pulse-echo and through transmission testing modes and is capable of controlling the repetition rate, dampening, and pulse height of the original transducer signal. With the use of attenuation controls, gain settings, and low and high pass frequency filters, signals received by the transducer can be analyzed, and subsequently, echoes can be determined from an initially noisy data set. All signals generated and received by the pulser/receiver were sent to an oscilloscope, allowing a temporal record of the echoes to be generated.

A general set of guidelines and a loose procedure can be adopted for determining wave speeds with the use of the ultrasonic equipment, which can help alleviate difficulties in signal identification and processing. For the determination of wave speeds in this thesis, cylindrical samples were utilized with the requirement that the diameter of the sample be at least four times its thickness. This guideline helps eliminate the possibility of reflections from the sides, and allows the researcher to assume that received signals are representative of echoes that are returning from the sample’s free surface. In order to aid in the identification of the echoes, the test samples should have a sufficient thickness so that there is not a coupling of waves at the transducer/sample interface. Samples tested in this thesis typically had dimensions of about 10 mm thickness and 45 mm diameter. Couplants were utilized between the transducers and the front faces of the samples in order to eliminate contact gaps. Couplants D and SWC, manufactured by Panametrics®, were used respectively for the longitudinal and transverse transducers. Given the mode of vibration of a longitudinal transducer, it can be noted that accurate results could be obtained with use of a water
couplant or even no couplant at all. The transverse transducer, on the other hand, required a high viscosity couplant to aid in the transfer of the shear pulse into the sample.

An effective procedure for setting up the pulser/receiver can be adopted using the following outline. First, settings related to the pulser portion of the unit are specified in order to attain a good input pulse. Typically settings for the pulser portion of the unit included setting the repetition rate to 500 Hz, the dampening to 50 Ω, and pulse amplitude to 400 V. The low and high pass frequency filters found on the receiver portion of the unit were next set to match the specific frequency output of the transducer utilized. This step often helped to eliminate much of the noise from the signal. Ideally the gain was maintained at 40 dB. However in instances where multiple echoes were not achievable, the gain was increased to 60 dB. If adjustments in the gain did not produce the desired result, a thinner sample was chosen for the test. As a final step, adjustments were made to the attenuation settings in order to allow identification of the echoes from possible noise still present. The minimum attenuation required to produce smooth signal free gaps between echoes was chosen. Once properly dialed, the output of the pulser/receiver should be consistent with the signal depicted in Figure 4.1.

Wave speeds for the materials studied in this thesis, as determined through use of the pulse-echo ultrasonic method are presented in Table 4.1. Also included in this table are calculations of the bulk and equivalent elastic wave speeds. The bulk wave speed is especially important in the study of shock waves and represents the initial plastic wave speed within a hydrodynamic media. The bulk wave speed of a material
can be calculated from the longitudinal \((c_l)\) and shear \((c_s)\) wave speeds by use of the equation:

\[
c_b = \sqrt{\frac{c_l^2}{3} - \frac{4}{3}c_s^2}
\]  

(4.1)

It has been widely recognized that when elastic waves are important to the analysis of a hydrodynamic event, an equivalent wave speed can be utilized. This equivalent wave speed, proposed by Romanchenko and Stepanov [4], is the harmonic mean of the longitudinal and bulk wave speeds, given by:

\[
c_e = \frac{2c_lc_b}{c_l + c_b}
\]  

(4.2)

<table>
<thead>
<tr>
<th>Material</th>
<th>(\rho_o) (kg/m(^3))</th>
<th>(c_l) (m/sec)</th>
<th>(c_s) (m/sec)</th>
<th>(c_b) (m/sec)</th>
<th>(c_e) (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast Iron A</td>
<td>7090</td>
<td>4190</td>
<td>2510</td>
<td>3030</td>
<td>3520</td>
</tr>
<tr>
<td>Cast Iron B</td>
<td>7130</td>
<td>4690</td>
<td>2600</td>
<td>3600</td>
<td>4080</td>
</tr>
<tr>
<td>Cast Iron C</td>
<td>7070</td>
<td>4550</td>
<td>2520</td>
<td>3500</td>
<td>3960</td>
</tr>
<tr>
<td>Cast Iron D</td>
<td>7690</td>
<td>4670</td>
<td>2650</td>
<td>3530</td>
<td>4020</td>
</tr>
<tr>
<td>Ductile Cast Iron</td>
<td>6970</td>
<td>5590</td>
<td>3090</td>
<td>4300</td>
<td>4860</td>
</tr>
<tr>
<td>Polycarbonate</td>
<td>1178</td>
<td>2260</td>
<td>910</td>
<td>2000</td>
<td>2120</td>
</tr>
<tr>
<td>A572 Grade 50 Structural Steel</td>
<td>7814</td>
<td>5940</td>
<td>3260</td>
<td>4590</td>
<td>5180</td>
</tr>
<tr>
<td>6061 Aluminum</td>
<td>2703</td>
<td>6420</td>
<td>3130</td>
<td>5310</td>
<td>5810</td>
</tr>
</tbody>
</table>

4.1.3 Acoustic Impedance

An important parameter in the study of dynamic events and wave interactions is a material’s acoustic impedance \((Z)\). Acoustic impedance is defined as the product of a material’s wave speed \((c)\) and density \((\rho)\). Properly defined, this parameter allows researchers to predict transmission and reflection ratios based on the interaction of an incident stress wave with an interface of two different materials [5]. Table 4.2 contains calculated impedances for the materials studied where the elastic and
hydrodynamic impedances were generated respectively using the longitudinal and bulk wave speeds.

**Table 4.2. Acoustic impedances of the materials studied.**

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic Impedance ([Z_e]) (Kg m(^{-2}) s(^{-1}) x 10(^6))</th>
<th>Hydrodynamic Impedance ([Z_h]) (Kg m(^{-2}) s(^{-1}) x 10(^6))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast Iron A</td>
<td>29.71</td>
<td>21.48</td>
</tr>
<tr>
<td>Cast Iron B</td>
<td>33.44</td>
<td>25.67</td>
</tr>
<tr>
<td>Cast Iron C</td>
<td>32.17</td>
<td>24.75</td>
</tr>
<tr>
<td>Cast Iron D</td>
<td>35.91</td>
<td>27.15</td>
</tr>
<tr>
<td>Ductile Cast Iron</td>
<td>38.96</td>
<td>29.97</td>
</tr>
<tr>
<td>Polycarbonate</td>
<td>2.66</td>
<td>2.36</td>
</tr>
<tr>
<td>A572 Grade 50 Structural Steel</td>
<td>46.42</td>
<td>35.87</td>
</tr>
<tr>
<td>6061 Aluminum</td>
<td>17.35</td>
<td>14.35</td>
</tr>
</tbody>
</table>

It must be pointed out that although the Hugoniot of a material provides a much more accurate depiction of a material’s impedance in the high pressure regime, knowledge of acoustic impedance is useful in the preliminary design and interpretation of experiments. One can use impedance relationships to determine the reflected \((\sigma_R)\) and transmitted \((\sigma_T)\) stress amplitudes of a normal incident wave traveling from material A into material B, respectively, with use of the well known equations:

\[
\sigma_R = \frac{Z_B - Z_A}{Z_B + Z_A} \sigma_I \tag{4.3}
\]

\[
\sigma_T = \frac{2Z_B}{Z_B + Z_A} \sigma_I \tag{4.4}
\]

where \(Z_A\) and \(Z_B\) are the respective impedances of material A and B and \(\sigma_I\) is the incident stress amplitude [2,5-7].

### 4.2 Dynamic Elastic Constants

The determination of longitudinal and transverse sound velocities coupled with determined densities makes it possible to approximately calculate the elastic constants
for the materials studied. These constants determined in this way are often termed “dynamic” and have been noted in literature to differ from the ones determined through quasi-static testing methods [8-12]. There are many explanations proposed in literature to explain these discrepancies including strain amplitude effects and mechanisms related to viscoelastic behavior [8,9]. Although these constants are of little use in the experimental study of shock physics they provide a valuable resource in subsequent modeling attempts of the experiments and are therefore presented in Table 4.3.

Table 4.3. Elastic constants of the materials studied.

<table>
<thead>
<tr>
<th>Material</th>
<th>Poisson's Ratio [ν]</th>
<th>Young's Modulus [E] (Gpa)</th>
<th>Shear Modulus [µ] (Gpa)</th>
<th>Bulk Modulus [K] (Gpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast Iron A</td>
<td>0.22</td>
<td>109</td>
<td>45</td>
<td>65</td>
</tr>
<tr>
<td>Cast Iron B</td>
<td>0.28</td>
<td>123</td>
<td>48</td>
<td>93</td>
</tr>
<tr>
<td>Cast Iron C</td>
<td>0.28</td>
<td>115</td>
<td>45</td>
<td>87</td>
</tr>
<tr>
<td>Cast Iron D</td>
<td>0.26</td>
<td>136</td>
<td>54</td>
<td>96</td>
</tr>
<tr>
<td>Ductile Cast Iron</td>
<td>0.28</td>
<td>170</td>
<td>67</td>
<td>129</td>
</tr>
<tr>
<td>Polycarbonate</td>
<td>0.40</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>A572 Grade 50 Structural Steel</td>
<td>0.28</td>
<td>213</td>
<td>83</td>
<td>165</td>
</tr>
<tr>
<td>6061 Aluminum</td>
<td>0.34</td>
<td>71</td>
<td>26</td>
<td>76</td>
</tr>
</tbody>
</table>

Found in Table 4.3 are Poisson’s Ratio (ν), Young’s Modulus (E), Shear Modulus (µ), and the Bulk Modulus (K) calculated from the determined density (ρ), longitudinal wave speed (c_l), and shear wave speed (c_s) for the materials studied through the well known relations [2,6,7]:

\[
\nu = \frac{1-2\left(\frac{c_s}{c_l}\right)^2}{2-2\left(\frac{c_s}{c_l}\right)^2} \tag{4.5}
\]

\[
E = \frac{\rho c_s^2 \left(3c_l^2 - 4c_s^2\right)}{c_l^2 - c_s^2} \tag{4.6}
\]

\[
\mu = \rho c_s^2 \tag{4.7}
\]
\[ K = \rho \left( c_i^2 - \frac{4}{3} c_s^2 \right) \]  

(4.8)

4.3 MICROSTRUCTURE OF CAST IRON

An important aspect in the study of spall fracture is the ability to relate the observed mechanical behavior to the evolution of damage at the microstructural level. This aspect requires detailed characterization of the microstructure both before and after the spall experiment. It is widely accepted in literature that pre-existing defects within a material’s microstructure often serve as damage nucleation sites under the action of dynamic tensile loading [13]. Second phase particles in an otherwise homogeneous material, grain boundaries, microcracks, and voids are examples of such defects that can strongly influence the mechanisms involved in dynamic fracture. In order to begin a study into the spall strength of a material it is imperative to utilize microscopy techniques to identify and quantify the presence of initial defects.

Within this thesis the spall fracture of five separate cast irons was studied. These castings include four separate gray cast irons and one ductile cast iron representative of typical castings produced in industrial foundries. Cast iron is one of the oldest cast ferrous products traditionally chosen for many different engineering applications. Despite the development of advanced engineering materials, cast iron remains widely in use today due to its low cost, easy castability, relatively good machinability, and wide range of achievable mechanical properties [14]. For differentiation from steel and cast steel, cast iron is defined as a cast alloy with a minimum carbon content of 2.03% which ensures the solidification of the final phase with a eutectic transformation [15].
Cast iron is generally viewed as a composite material consisting of precipitated graphite particles in a solid metal matrix. Graphite is a relatively soft phase composed of free carbon with a hexagonal crystal structure. Although graphite has been shown to significantly reduce the strength of cast irons, the material owes its relatively good machinability, damping properties, and reduced shrinkage to the existence of this phase. Components commonly found within the metal matrix include austenite, ferrite, cementite, pearlite, martensite, steadite, and ledeburite. Austenite, known as \( \gamma \)-iron, is a metastable phase consisting of a solid solution of carbon and iron with a face-centered cubic crystal structure. Ferrite, known as \( \alpha \)-iron, is a soft phase consisting of a body-centered cubic crystal structure. Ferrite contributes to cast iron’s ductility while also causing a notable depreciation in its mechanical strength. Cementite (Fe\(_3\)C), commonly referred to as iron-carbide, is a hard intermetallic phase consisting of a complex orthorhombic crystal structure. Cementite is associated with increased wear resistance and reduced machinability. Pearlite is a metastable lamellar aggregate of alternating layers of ferrite and cementite. This phase contributes to increased mechanical strength in cast irons. Martensite is a generic term for microstructures that form by diffusionless transformation where the parent and product phases have a specific crystallographic relationship. Martensite in cast iron is typically characterized as a hard metastable phase. Steadite is representative of the phosphorous eutectic iron-phosphide (Fe\(_3\)P). It typically consists of either a ternary eutectic of ferrite, iron phosphide and cementite or a pseudobinary eutectic of ferrite and iron phosphide. Steadite is related to hard and brittle behavior within cast irons.
Ledeburite is a massive eutectic phase composed of cementite and austenite associated with high hardness and good wear resistance.

The properties of cast iron are strongly dependent on the specific microstructure within. This microstructure is inconsistent across all materials classified as cast iron and is influenced by factors including chemical composition, inoculation, solidification rate, and cooling rate [16]. It is widely observed that the overall mechanical properties of cast iron can be greatly altered by small changes in microstructure. Generally cast iron classification is based on the type, distribution, and size of the graphite phase within the microstructure [15-20]. Gray cast iron defines irons that contain graphite in the form of flakes that are dispersed in either a pearlitic or ferritic matrix. On the other hand ductile cast irons contain graphite in the form of nodules or spherulites that are typically surrounded by free ferrite in a matrix of pearlitic.

In the proceeding sections the chemical composition of the four gray cast irons will be defined and related to their resulting microstructure development. Details pertaining to sample preparation of the cast irons for use in optical microscopy will be outlined. Micrographic observations obtained through use of a reflection light microscope will be presented. The five cast irons studied will be classified, first based on the graphite phase, then based on additional phases present within the metal matrix.

4.3.1 Chemical Composition of Gray Cast Irons Studied

The chemical composition of the four gray cast irons was investigated with use of gamma spectroscopy conducted at the Rhode Island Nuclear Science Center (RINSC). The goal of this investigation was to determine the weight percentages of
carbon, silicon, and phosphorous so that the irons could be properly classified as either hypoeutectic, eutectic, or hypereutectic. This definition is based on the carbon equivalent value, which is the total carbon content plus one-third the sum of the silicon and phosphorous contents. Cast iron with a carbon equivalent of approximately 4.3 solidifies as a eutectic. When the carbon equivalent is greater than 4.3 the cast iron is said to be hypereutectic, however when it is less than this value the iron is classified as a hypoeutectic [15]. Another goal of this investigation was to identify possible alloying agents utilized by the respective foundries and help to relate their presence to key components within the microstructure. Normal elements found in castings typically include iron, carbon, silicon, manganese, phosphorus, and sulfur. Alloyed iron is used to designate castings containing additions of elements like chromium, nickel, molybdenum, or copper [18]. The role that these elements play is often critical in the development of graphite. Elemental additions often fall into two categories: graphitizing elements and carbide promoting elements [21-22]. Graphitizing elements tend to promote the carbon-carbon bond leading to the generation of graphite while carbide promoting elements tend to promote the carbon-iron bond to produce cementite. The results of the gamma spectroscopy are presented in Table 4.4 as weight percentages of the elements found. Errors to these percentages are roughly on the order of ±1%. The term “Trace” is used to specify the presence of elements that were difficult to quantify with the testing equipment.

<table>
<thead>
<tr>
<th>Cast Iron</th>
<th>Fe</th>
<th>W</th>
<th>Ba</th>
<th>As</th>
<th>Cu</th>
<th>Mn</th>
<th>Si</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>92</td>
<td>trace</td>
<td>0</td>
<td>trace</td>
<td>trace</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>97</td>
<td>trace</td>
<td>0</td>
<td>trace</td>
<td>trace</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>95</td>
<td>trace</td>
<td>0</td>
<td>trace</td>
<td>trace</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>96</td>
<td>trace</td>
<td>trace</td>
<td>trace</td>
<td>0.01</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
Unfortunately, determination of the carbon and phosphorous content was not possible with use of the current equipment at the RINSC. Noteworthy in the results found in Table 4.4 is the presence of trace quantities of copper in all four gray cast irons. Copper is a common alloying element that acts as a mild strengthener to cast irons, acting to break up massive cementite particles that would have contributed to poor mechanical properties and brittle behavior [18]. It seems noteworthy that the composition of the four materials contained a balance of graphitizing and carbide promoting elements. Copper, Silicon, and Barium fall into the class of graphitizing elements, promoting the formation of graphite. Tungsten and Manganese on the other hand have been shown to promote carbide formation [18].

4.3.2 Sample Preparation

Sample preparation is one of the most important factors involved in utilizing a reflection light microscope to view microstructure. By following guidelines outlined in ASTM standard E3-11, reasonably clear micrographs for most metallographic samples should be attainable [23]. In the case of gray cast iron, special care must be taken in order to retain the relatively soft graphite phase. It has been noted that the use of standard polishing techniques will significantly damage the graphite phase [15, 20]. For the initial grinding phase with SiC papers it has been pointed out that the graphite will be eroded away if water is utilized. It has also been shown that the use of grinding papers past their optimum life will result in graphite being ripped from the sample surface [15, 20]. In light of these findings, samples were prepared by using SiC papers at about 300 rpm on a Buehler polishing and grinding station in a sequence of, first, 120 grit followed by a final grinding with 240 grit. In order to avoid staining,
methanol was used instead of water to clean the sample between steps. Careful consideration was taken between each step to clean hands and all other areas in an effort to avoid contamination of the proceeding step. After grinding, a three step polishing process was employed utilizing diamond paste, which has been suggested for the retention of graphite [15, 20]. Samples were polished using a sequence of 9 μm, 3μm, and 1μm diamond pastes lubricated with METADI at about 150 rpm on a polishing wheel mounted to the same Buehler polishing station used previously. Again, methanol was utilized to clean the samples between steps. After the cast iron samples were polished, they were viewed under the optical microscope to analyze the graphite phase before etching.

After adequate viewing of the graphite phase, samples were etched to reveal additional phases present within the matrix microstructure. Chemical etchants work on the principle of corrosive processes, where high energy areas such as grain boundaries within the microstructure are attacked, providing relief to the otherwise polished surface. These surface pits created through etching provide a black and white contrast of the microstructure when viewed in optical microscopes. The most commonly used chemical etchant in microstructural studies on cast iron is 4% nital which is composed of a solution of 0.8 mL Nitric acid and 19.2 mL of methanol [15, 20]. Exposure time to the etchant was determined though trial and error to be optimized at 10 seconds. If etched for only five seconds, grain boundaries remained incomplete under inspection, however surface pitting and staining were observed when 15 seconds was used. After the 10 second etch, methanol was used to flush etchant off of samples before they were blown dry with a hairdryer.
Although chemical etching allowed for clear identification of ferrite, cementite, and pearlite colonies, it fell short in the identification of steadite. Steadite is clearly observed in cast iron when etched with nital, however, with only black and white contrast it is easily misinterpreted as transformed Lebedurite which takes the structure of cementite particles within a ferrite matrix. In order to resolve this issue and allow for proper identification of steadite, selective color etching was utilized. In contrast to chemical etchants, reagents referred to as tint enchant typically are used to deposit a thin transparent film on a phase of interest. For the case of steadite identification, the standard version of Murakami’s reagent was employed to color iron phosphide dark yellow to brown. This reagent consists of a solution of 10g potassium ferricyanide, 10 g potassium hydroxide, and 100 mL distilled water. Tinting is accomplished by submerging samples within the reagent maintained at 50 °C for three minutes. Upon completion of the three minutes, samples were first washed under running water, followed by methanol, before being blown dry with a hair dryer.

4.3.3 Methods of Graphite Classification

One of the most important features used in defining cast irons is the structure, distribution, and relative volume fraction of the graphite phase. To conduct this classification it is convenient to refer to ASTM standard A247-10 which outlines the standard test method for evaluation of the microstructure of graphite in iron castings [24]. Taken from this standard are two figures which help initially classify the type and distribution of graphite. Figure 4.2 contains seven different shapes of graphite used to identify the graphite phase. Figure 4.3 contains five different distributions of graphite found in cast irons. Graphite types I through VI are commonly found within
ductile cast irons. Ideally cast, ductile irons should consist mostly of type I graphite although the misshapen nodules found in type II have been shown to result in negligible adverse effects on the casting’s mechanical properties [24]. Type VII is the standard flake form of graphite found in gray cast irons. For standard engineering applications requiring good mechanical properties, it is widely accepted that graphite forms I, II, and VII are the most desirable, giving rise to the vast use of ductile and gray cast irons in industry [15-16,24].

In order to continue, it seems important to outline the formation of graphite in cast irons as knowledge of this process will become increasingly important in the subsequent study of the spall fracture of the material. As molten iron cools, graphite precipitates out of solution and nucleates naturally because the solubility limit of carbon decreases with decreasing temperature. In order to control the shape, size, and distribution of the graphite precipitates it is common place in industrial foundries to introduce additional elements to the molten iron immediately before casting. These additions, known as inoculants, have been shown to greatly increase the number of nuclei available for graphite precipitation [21]. Uninoculated iron is characterized by poor control of the graphite morphology where castings contain inconsistent graphite distributions with uncontrolled size and shape. Important to the inoculation process is the concept of fade which refers to the loss in effectiveness of inoculants over time. This process is easily understood by considering the commonly used graphite promoter ferrosilicon. In the case of ferrosilicon, the melting point is close to 1210 °C which is far below typical casting temperatures in the range of 1350-1400 °C [21]. It can therefore be noted that if the molten metal is held for an extended period of time
after inoculation, inoculant particles will have completely melted and dispersed, losing the effective nucleation sites for graphite precipitation. While the inoculation particles are in the process of melting within the molten metal they create pockets of undercooling with respect to graphite’s equilibrium eutectic temperature, creating a strong driving force for the precipitation of graphite.

In the case of gray cast irons, the inoculation process tends to produce a system of silica-rich oxide bifilms that serve as nucleating points for the precipitating graphite [21]. Within a well inoculated casting, graphite flakes typically grow in regions ahead of the solidification front on the oxide substrates in suspension and therefore form in the melt prior to the appearance of austenite. Eventually as cooling persists the graphite becomes incorporated into the solid, frozen in place by the growth of austenite dendrites that fill the spaces between already grown graphite lamellae. It is interesting to point out that in the case of flake graphite with a type A distribution, the eutectic cell is said to have a continuous graphite skeleton created by primary growth of graphite in the liquid metal [15, 19]. When inoculation is insufficient to provide adequate nucleation sites for graphite precipitation within the molten metal a continuous growth process is excited where there is a coupled eutectic growth of graphite and austenite. Unlike the primary growth of graphite within the molten metal, during coupled growth processes graphite flakes have to continually realign their growth directions because of the intrusion of neighboring austenite crystals in their growth space [21]. Because the growth direction of graphite is mainly parallel to the basal (0001) plane, graphite crystals have to develop faults to allow a change in growth direction. The rosette formation found in type B distribution is an example of
a coupled growth process characteristic of thin-walled castings where each rosette group of graphite is representative of one eutectic cell. With increased cooling rates graphite growth is further hindered by the advancing solidification front of austenite, resulting in finer flakes with either a dendritic or interdendritic distribution as noted respectively in types D and E. It has been pointed out in literature that cast irons with types D and E graphite distributions contain high defect densities and should be avoided for use in most engineering applications [21-22]. Because type A graphite distributions are associated with the primary growth of graphite, free from the constraints imposed by solidifying austenite, the crystal structure of the precipitated graphite is relatively perfect, containing the lowest density of faults. Type B graphite distributions are often representative of an intermediate case between the two extremes.

Figure 4.2. ASTM A247 classification of types of graphite in cast iron [24].
Foundry techniques used to produce ductile cast irons typically involve additions of magnesium, which act to break up the bifilms created within the inoculation process [21-22]. The dispersed bifilms serve as separate nucleation points for graphite precipitation where graphite growth tends to encapsulate each separate nucleus. In contrast to gray cast irons, graphite precipitation in ductile cast irons occurs as separate precipitates within the molten metal, where each nodule can be considered as a separate eutectic cell. During solidification, graphite spheroids initially develop in the liquid before developing a shell of austenite. As austenite continues to grow, spheroids become incorporated into the matrix via advancing austenite dendrites.

**4.3.4 Application of Graphite Classification to the Studied Materials**

In order to carry out the initial classification of the shape and distribution of graphite within the cast iron samples a reflection light microscope was utilized in bright field to capture images at 100x magnification. A sample of micrographs from each cast iron studied can be found in Figure 4.4 where the dark components are the graphite and the light component is the metal matrix.
Figure 4.4. Optical Micrographs showing graphite structures in polished samples at 100x magnification. (a) Cast Iron A; (b) Cast Iron B; (c) Cast Iron C; (d) Cast Iron D; (e) Ductile Cast Iron.

Looking at Figure 4.4 (a-d), it can immediately be observed that cast irons A, B, C, and D are indeed classified as gray cast irons composed of type VII graphite flakes. The ductile cast iron on the other hand is comprised of type I graphite nodules. The absence of types II-VI graphite in the ductile cast iron suggests extremely good
foundry techniques in its production and is indicative of little to no fade of the inoculants before casting. It seems noteworthy to point out that amongst the gray cast irons studied, cast irons A, B, and C all contained similar morphology in terms of the graphite shape, size, and distribution. Castings A, B, and C were predominately comprised of randomly oriented type A distributions while some rosettes classified as type B were also observed. In light of the preceding discussion about graphite formation, the type A distribution of these irons suggests that their respective foundries utilized good inoculation techniques which allowed for the primary growth of the graphite within the molten metal to dominate. Additionally, the large size of these flakes suggests relatively slow cooling rates through graphite’s eutectic temperature range [15]. It can be hypothesized that the coupled growth process indicated by the rosette distributions may have been associated with areas near the casting walls where significant undercooling was likely present. In contrast to cast irons A, B, and C, the morphology of the graphite within cast iron D was very different. Within the microstructure of cast iron D, graphite distributions of type A, B, and D can all be observed. The relatively small flake size, coupled with distributions B, and D suggest that significantly high cooling rates were utilized in the respective foundry. During solidification it seems logical to hypothesize that significant temperature gradients were present within the casting. In areas that contained a high degree of undercooling, small flakes dominated, taking on a dendritic distribution due to the advancing solidification of austenite.

In order to quantify the amount of graphite present in each casting it was decided to utilize quantitative stereology techniques to estimate the area fraction and
in turn volume fraction of graphite. Areal analysis conducted in this manner was first developed by Delesse in 1848, in which it was shown that it is statistically accurate to assume that the area percent of a phase on a 2-D plane is equal to its volume percent in 3-D [25]. The statistical accuracy of this method has since gained wide acceptance in literature and is subsequently outlined in the ASTM book of standards as E562 [26]. A manual point count method could be employed for this method, however it was decided to utilize numerical techniques to aid in accuracy and alleviate effort, allowing for multiple images to be analyzed and averaged for each iron tested. For this task a program was developed in Matlab to first convert micrographs to grayscale images. Advanced thresholding algorithms built into Matlab were employed to convert the grayscale images into binary images. An example of this process can be found in Figure 4.5. Once the image was converted into a binary image (white=1, black=0), the task of determining the area fraction was reduced to tallying up the total number of zero value pixels and dividing by the total pixel count in the image. Five separate randomly selected micrographs were analyzed for each cast iron and averaged to produce an average volume percent of graphite for each cast iron studied. A summary of these results can be found in Table 4.5. Also found in Table 4.5 is a measurement of the longest serpentine length of graphite within the four gray cast irons and the largest nodule diameter for the ductile cast iron found within the five micrographs of each respective casting.

Based on the maximum graphite length/diameter found for the cast irons studied final classification of the size class for each can be achieved based on ASTM standard A247 [24]. Cast irons A, B, and C all fall into graphite size class 1 based on
the longest serpentine length of graphite observed. It should be pointed out however that the average length of graphite found in castings A, B, and C fall into the range of 500 µm which would subsequently place these castings in size class 2. Cast iron D would initially fall into size class 3 based on the largest graphite observed. In reality there is a bimodal distribution of graphite sizes found in cast iron D where average size classes can be related to their respective distributions. The average size of the graphite found for distributions A and B fall in the range of 100 µm which in turn relates to size class 4. In comparison, graphite size within the type D distribution of cast iron D was much smaller, averaging in the range of only 10 µm, thus giving rise to size class 8. In light of the discussion of graphite formation previously presented, and more specifically the competition between primary growth and coupled eutectic growth, the varying sizes observed, coupled with their respective distributions, is easily understood. The ductile cast iron studied would initially be classified as size class 4 based on the largest diameter observed. However the true average size of roughly 50 µm would place this cast iron in size class 5.

Before continuing, let us take a moment to summarize the results obtained through graphite classification in accordance with ASTM standard A247 [24]. Although the presence of type B distributions was noted in castings A, B, and C, the majority of the graphite distribution fell into type A. It will therefore be stated that the graphite found in cast irons A, B, and C can all be classified as Type VII A2. Cast Iron D will be generally classified as a bimodal distribution of graphite types VII A4 and VII D8. The ductile cast iron studied will be classified as Type I with an average of 200 nodules per square millimeter of size class 5.
Figure 4.5. Example showing transformation of an original color micrograph into a grayscale and then a binary image for use in determining the area fraction of the graphite.

Table 4.5. Summary of graphite volume fractions for the cast irons studied.

<table>
<thead>
<tr>
<th>Material</th>
<th>Trial</th>
<th>Volume Fraction of Graphite</th>
<th>Average Percent of Graphite</th>
<th>Longest Graphite Length (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast Iron A</td>
<td>1</td>
<td>0.099</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.097</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.107</td>
<td>10.3%</td>
<td>1178</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.117</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.094</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cast Iron B</td>
<td>1</td>
<td>0.113</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.134</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.117</td>
<td>11.7%</td>
<td>1350</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.106</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.114</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cast Iron C</td>
<td>1</td>
<td>0.105</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.115</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.121</td>
<td>10.8%</td>
<td>1024</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.107</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.094</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cast Iron D</td>
<td>1</td>
<td>0.131</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.133</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.131</td>
<td>13.1%</td>
<td>237</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.136</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ductile Cast Iron</td>
<td>1</td>
<td>0.142</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.131</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.115</td>
<td>12.2%</td>
<td>138</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.117</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.107</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.3.5 Identification of Additional Phases

After the classification of the cast irons based on graphite morphology was completed, details pertaining to the microstructure of the metal matrix were
investigated through use of chemical and tint etching as discussed in section 4.3.2. Figures 4.6, 4.7, 4.8, 4.9, and 4.10 contain sample micrographs which exemplify the phases revealed within the metal matrix with the use of 4% nital etching respectively for the gray cast irons A, B, C, and D as well as the ductile cast iron studied. In order to facilitate a comparison of the microstructure between castings, the micrographs found in part (b) of each respective figure are consistently 600x magnification. For the five cast irons studied, phases of graphite, pearlite, and ferrite were all observed and respectively denoted G, P, and F within the figures. As noted earlier, pearlite is a laminar structure of alternating layers of ferrite and cementite. While observing the pearlite within the micrographs it should be noted that the dark lamella are the cementite while the light lamella are the ferrite. In addition to these phases, the gray cast irons studied all possessed small slate colored inclusions throughout their matrix microstructure. In literature these inclusions are frequently related to precipitated manganese sulfide, which in the case of low volume fractions, have been noted to have negligible effects on the resulting mechanical properties of cast iron [14, 15, 18]. In light of the results obtained through gamma spectroscopy of the gray cast irons where manganese was noted in all four samples, it seems reasonable to consider these inclusions to be precipitated manganese sulfide and they are therefore denoted MS within the respective micrographs. Within gray cast irons A, B, and C, an additional phase was observed and later identified as steadite with the use of tint etching techniques and is thus denoted as S within the micrographs. Figures 4.11, 4.12, and 4.13 contain micrographs of steadite colonies observed at 1000x magnification respectively for cast irons A, B, and C. The effective use of Murakami’s reagent on
both polished and pre-etched samples is respectively presented in parts (a) and (b) of each figure. Within these figures, iron phosphide is denoted as IP, ferrite as F, and pearlite as P. Dark components adjacent to the colonies are easily misinterpreted as graphite, however imaging in dark field proved that they were voids and therefore denoted as V.

Taking note of the micrographs presented, it is interesting to point out that cast irons A and C contain similar microstructures suggesting consistent foundry techniques in their production, likely indicating that they were actually produced in the same foundry. The metal matrix in these castings is primarily comprised of ferrite, pearlite, and steadite. The ferrite phase is predominately found in the neighborhood of graphite precipitates, which is easily anticipated due to the concept of microsegregation [15]. This same concept is exemplified by the bulls eye structure found within the ductile cast iron where graphite nodules are surrounded by free ferrite in a matrix of pearlite.

In order to continue, it seems necessary to describe how the final metal matrix forms in a cooling cast iron. As cooling cast iron approaches ambient temperatures from its initial casting temperature the austenite phase that previously solidified eventually falls below its stable eutectoid temperature. At this point it becomes unstable and decomposes into pearlite through a process of diffusion where rejected carbon atoms combine with iron atoms to form cementite, leaving behind areas of pure ferrite. When the diffusional distance is relatively small, or when cooling through this eutectoid temperature range is excessively slow the carbon locked in the austenite is able to migrate to neighboring graphite flakes, exciting further growth of the graphite
phase and leaving behind free ferrite in the neighboring region. When the diffusional distance to a neighboring graphite phase is relatively large, and slow cooling is still present, large rearrangement of atoms is possible resulting in a coarse pearlite structure with increased inter-laminar spacing.

In light of this discussion, the large ferrite grains in combination with the coarse pearlite structure suggest that significantly slow cooling rates were utilized on castings A and C. This hypothesis is strengthened by the partial spheroidization of cementite lamella found within many of the pearlite colonies for these castings [17]. In comparison to castings A and C, cast iron B, which contained very similar graphite morphology, seems to have been more rapidly cooled through the final eutectoid temperature range, resulting in smaller inter-laminar spacing within the pearlite and minimal free ferrite. Cast iron D seems to exemplify an extreme case of this scenario exhibiting virtually no free ferrite coupled with a fine pearlite structure. This seems reasonable considering that cast iron D has already been linked to high cooling rates through its eutectic temperature range based on the relatively small graphite flakes found within.

It seems noteworthy to point out the area marked P* found within part (b) of figure 4.9. This area can easily be misinterpreted as ferrite upon initial observation, however, imaging the area at 1000x magnification proved that it is actually partially etched pearlite. This is quite reasonable considering the commonly noted sensitivity of nital to the crystallographic orientation of pearlite grains in literature [15]. In cases where pearlite is very fine, nital etching tends to leave white, unetched areas within the respective phase. A more uniform etch of the pearlitic structure could have been
achieved with use of 4% picral, which is composed of a solution of 4g picric acid and 100 mL of ethanol. Working with picric acid imposes many safety hazards to the laboratory setting. In addition to being toxic to the user, it tends to be extremely explosive and possesses a high sensitivity to shock, heat, and friction. For the sake of safety picral was avoided, and the related investigations discontinued.

It is noteworthy to discuss the steadite colonies found within castings A, B, and C. These colonies typically took on an inter-granular distribution that is easily understood considering iron phosphide’s tendency to segregate from the solidifying graphite and austenite phases. It should be pointed out that the eutectic steadite is the last to solidify and is therefore representative of the final areas of molten metal within castings [14, 15, 22]. As steadite solidifies it often can draw material from thin sections to feed thick sections, consequently resulting in microscopic shrinkage voids in the thin sections [22]. It is interesting to note that the steadite colonies found in cast irons A and C possessed similar shape and structure, further suggesting the concept that they were likely manufactured within the same foundry. Additionally the relative high phosphorous content associated with the formation of steadite in the gray cast irons A, B, and C suggest that they were likely manufactured with the use of either foreign or southern USA mined iron ores [18]. On the other hand, the lack of steadite in cast iron D and the ductile cast iron suggest that they were likely produced with either northern USA mined ores or scrap from steel manufacture [18].
Figure 4.6. Optical micrographs of cast iron A etched with 4% nital; (a) 200x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G; Steadite as S; Manganese sulfite as MS.
Figure 4.7. Optical micrographs of cast iron B etched with 4% nital; (a) 200x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G; Steadite as S; Manganese sulfite as MS.
Figure 4.8. Optical micrographs of cast iron C etched with 4% nital; (a) 200x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G; Steadite as S; Manganese sulfate as MS.
Figure 4.9. Optical micrographs of cast iron D etched with 4% nital; (a) 400x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G; Manganese sulfite as MS.
Figure 4.10. Optical micrographs of ductile cast iron etched with 4% nital; (a) 200x magnification; (b) 600x magnification. Ferrite is denoted as F; Pearlite as P; Graphite as G.
Figure 4.11. Optical micrographs of steadite colonies in cast iron A viewed at 1000x magnification; (a) Polished and tint etched with Murakami reagent; (b) Pre-etched with 4% nital before tint etching with Murakami’s reagent. Iron phosphide is denoted as IP; Ferrite as F; Pearlite as P; Voids as V.
Figure 4.12. Optical micrographs of steadite colonies in cast iron B viewed at 1000x magnification; (a) Polished and tint etched with Murakami reagent; (b) Pre-etched with 4% nital before tint etching with Murakami’s reagent. Iron phosphide is denoted as IP; Ferrite as F; Pearlite as P.
Figure 4.13. Optical micrographs of steadite colonies in cast iron C viewed at 1000x magnification; (a) Polished and tint etched with Murakami reagent; (b) Pre-etched with 4% nital before tint etching with Murakami’s reagent. Iron phosphide is denoted as IP; Ferrite as F; Pearlite as P; Voids as V.
The relative volume fractions of the phases identified within the micrographs of the respective cast irons was investigated with use of a manual point count method. This method is conveniently outlined in ASTM standard E562 [26]. The method involves superimposing of a grid on a micrograph, typically achieved by drawing fine perpendicular crossing lines where the points analyzed are the intersection of the lines. Points that fall within a phase of interest are counted as a 1, and points that fall within an interface of that phase and the matrix are counted as ½. The area fraction and thus volume fraction of the phase are determined by dividing the sum of points counted by the total number of points within the test grid. It seems appropriate to outline some of the key guidelines found within the standard that have been shown to optimize the competing factors involving time spent counting and accuracy of the statistical information obtained. Key to the method is the choice of grid size, which involves an initial visual estimate of the area fraction. The standard suggests the number of test points \( P_T \) that should be utilized on a micrograph based on the visual estimate of the area fraction as summarized in Table 4.6. Once an appropriate number of test points are chosen, the magnification of the micrograph should be addressed. It is proposed that the magnification be as low as possible to adequately resolve the microstructure without resulting in adjacent grid points overlaying a single constituent feature. This guideline typically requires the grid spacing to be roughly double the average size of the phase investigated. As a final note, it is strongly advised that micrographs be selected without bias and that multiple rounds of this method be employed on non-overlapping micrographs of the sample.
Table 4.6. Recommended number of test points as a function of visually estimated area fractions recreated from ASTM E562 [25].

<table>
<thead>
<tr>
<th>Visual Area Fraction Estimate</th>
<th>$P_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 2%</td>
<td>400</td>
</tr>
<tr>
<td>2 - 5%</td>
<td>100</td>
</tr>
<tr>
<td>5 - 10%</td>
<td>49</td>
</tr>
<tr>
<td>10 - 20%</td>
<td>25</td>
</tr>
<tr>
<td>&gt; 20%</td>
<td>16</td>
</tr>
</tbody>
</table>

In accordance to the guidelines proposed in ASTM standard E562, volume fractions of ferrite, pearlite, and steadite were determined from five randomly selected micrographs of each respective cast iron. It was decided to omit determination of the volume fraction of manganese sulfide observed in cast irons A, B, C, and D due to its relatively low volume fraction and its link to negligible effects on the resulting mechanical properties of the materials. In the case of gray cast irons A, B, and C, the point count method was employed to determine the volume fractions of ferrite and steadite, and with knowledge of the volume fraction of graphite previously presented the fraction of pearlite was calculated assuming it completed the total of 100%. For determination of ferrite percentages in castings A and C a 49 point grid was utilized on micrographs taken at 100x magnification. In the case of cast iron B, a 400 point grid was utilized on micrographs taken at 100x magnification. For determination of steadite percentages 400 point grids were used for castings A and C while 100 point grids were used for casting B on micrographs taken at 100x magnification. In the case of gray cast iron D, the method was employed to determine the percentage of ferrite, and subsequently, the pearlite percentage was calculated in a similar fashion as castings A, B, and C. For the determination of ferrite percentage in cast iron D a 400 point grid was utilized on micrographs taken at 200x magnification. Due to the relatively low volume fraction of pearlite in comparison to ferrite in the ductile cast
iron, an opposite procedure to that used for cast iron D was adopted where the method was employed to estimate the percentage of pearlite. In this case a 25 point grid was utilized on micrographs taken at 100x magnification. A summary of the average volume fractions of phases found within the metal matrix, as determined for the cast irons studied, can be found in Table 4.7. In order to visualize the 100% total used to calculate the primary phase, the volume fractions of graphite previously presented in Table 4.5 are also presented in Table 4.7.

Table 4.7. Volume fractions of phases identified within the cast irons studied.

<table>
<thead>
<tr>
<th>Material</th>
<th>Volume Fraction (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ferrite</td>
</tr>
<tr>
<td>Cast Iron A</td>
<td>14.5</td>
</tr>
<tr>
<td>Cast Iron B</td>
<td>0.4</td>
</tr>
<tr>
<td>Cast Iron C</td>
<td>15.3</td>
</tr>
<tr>
<td>Cast Iron D</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>Ductile Cast Iron</td>
<td>60.3</td>
</tr>
</tbody>
</table>

In addition to the various microstructural phases identified and quantified for the cast irons studied it seems important to point out that castings B and C also contained significant macroscopic porosity. This porosity was difficult to quantify due to its irregular distribution within the respective castings. It generally took on a segregated distribution where high percentages were found in isolated areas of the castings. Due to the spherical shape of this porosity, its existence can very likely be explained as gas entrapment during the solidification of the irons [17]. In light of the porosity’s irregular distribution, it can be expected that an array of values will likely be found in the experimental study into the respective spall strengths for castings B and C due to the porosity serving as additional nucleating sites for damage. In order to
visualize the magnitude of this porosity, Figure 4.14 contains images of two rejected test samples, where parts (a) and (b) are respectively cast irons B and C. To understand the scale of the image, it should be pointed out that the diameter of these samples was 45 mm.

![Rejected cast iron samples demonstrating porosity](image)

**Figure 4.14.** Rejected cast iron samples demonstrating porosity; (a) cast iron B; (b) cast iron C.

In light of the results obtained in the microstructural investigation of the five cast irons it is important to point out some key observations that can be used to relate their respective spall strengths to mechanisms associated with their inherent microstructure. Obviously, cast iron is a highly heterogeneous material containing a vast array of microstructural features that inherently result in a multitude of mechanical properties. The mixed microstructure often creates competing mechanisms associated with mechanical properties where the interrelation and overlapping of different effects makes discerning a specific microstructural feature’s influence nontrivial at best. However true, by studying five different microstructures some key links between the microstructure of cast iron and spall strength can be
investigated through systematic relation of the various strengths to the respective microstructures. The effect of graphite shape can be investigated by comparison of spall strengths found for the gray cast irons to those of the ductile iron. On the other hand, the effect of graphite size on the strength of gray cast irons can be investigated by comparing strength results from castings A, B, and C to those of casting D. Finally, the role that the microstructure of the metal matrix plays can additionally be investigated by comparison of results from castings A and C to those of casting B. By conducting these comparisons a complete understanding of the effects of mechanisms associated with microstructure on the spall fracture of cast iron can be facilitated.

4.4 REFERENCES


CHAPTER 5

SPALL FRACTURE OF CAST IRON

This chapter discusses the results of the spall fracture experiments conducted on cast iron. This chapter will begin by outlining the experimental design, which will include the choice of sample dimensions and the subsequent results expected. A summary of the results from the spall fracture experiments on the five cast irons studied will be given. Spurred by the unexpected nature of the results found for these cast irons, a small investigation into the spall strength of 6061 Aluminum will be included to address concerns associated with the technique employed, and equations utilized, for the calculation of spall strengths in this thesis. The chapter will conclude by correlating the spall strengths of the cast irons to their respective microstructures.

5.1 EXPERIMENTAL DESIGN

As pointed out in chapters 2 and 4, the initial design of any plate impact experiment should begin with the construction of a time-distance diagram that is representative of the planned test. Figure 5.1 contains a time-distance diagram for the intended experiments on cast iron, constructed under the assumption of an elastic-plastic material response. In order to induce a spall plane in the center of the test specimen, the impactor thickness was chosen so that it was exactly half of the specimen thickness. The impactor and specimen are 45 mm in diameter and
respectively 5mm and 10mm thick. The low impedance polycarbonate window has a thickness of 19.05 mm (0.75”). Within the cast iron, elastic waves are representative of the stress magnitude associated with the HEL, and are depicted as dotted characteristic lines that travel at an assumed longitudinal wave speed of 4400 m/s. With the exception of the rarefaction waves, plastic waves, which are depicted as solid characteristic lines, travel at an assumed bulk wave speed of 3320 m/s in the cast iron. The elastic and plastic waves within the polycarbonate window respectively travel at 2260 m/s and 2000 m/s, as determined in section 4.1.2, and summarized in Table 4.1. When referring back to Table 4.1, it can be noted that specific wave speeds within the various cast irons ranged both faster and slower than the assumed speeds used in the construction of this time-distance diagram. However the case, the general experimental design is still adequately represented by this figure. The impactor thickness of 5mm can be noted to induce an elastic loading duration of about 2 µs and a maximum stress plateau of about 1.5µs. Figure 5.1 demonstrates that the intended window thickness is sufficient to allow for complete capture of the experimental stress record before any transmitted waves reflect from its free surface. The shaded trapezoid found near the spall plane is used to indicate the location of the tensile build up in time-distance space linked with the interaction of the C+ and C- decompression waves. The manganin gauge used to capture stresses transmitted into the polycarbonate window is shown to reside between the interface of the back of the specimen and front of the low impedance window.

The cutoff time associated with the violation of conditions of one-dimensional strain was estimated with the use of equation (2.48). The distance of the stress gauge
from the edge of the target \((r)\) was specified as 20 mm, under the assumption that the active grid of the stress gauge was \(5 \times 5\) mm and the gauge was centered within the 45 mm diameter target plate. By taking \(D\) to be 10 mm, and assuming the process to be represented by the bulk wave speed of 3320 m/s, the cutoff time was estimated to be about 6.72 \(\mu\)s after impact. In Figure 5.1, it can be noted that this cutoff tends to run dangerously close to the anticipated arrival time of the pullback signal, subsequently, any second plateau (state 7 in Figure 2.2) would likely be difficult to capture. This is not a great concern, considering that the use of equation (2.33) in the determination of spall strength only requires the capture of the maximum and minimum stress magnitudes associated with the first pull back signal. Obviously, if second or even third stress plateaus were desired, impactors and specimen with smaller thicknesses could have been utilized while maintaining the same 45 mm diameters. Adjacent research into the spall strengths of the same materials employed much larger diameter samples, and was therefore able to capture longer time durations for measurements while respectively employing impactor and target thickness of 6mm and 12mm. Because the test apparatus utilized in this adjacent research was limited in its minimum impact velocity, a major goal of the current study was to generate spall strengths as a function of low to moderate shock loading associated with impact velocities under 300 m/s, which could facilitate the generation of a model to encompass all tests. The limitations on diameter imposed by the test apparatus used in this study required the reduction of the dimensions employed in the adjacent research to 5mm and 10mm respectively. This reduction was mutually agreed to provide the desired validating points for the construction of the model, while minimizing
complications associated with the variation of spall strength as a function of pulse duration.

Figure 5.1. Time-Distance diagram for the cast iron spall experiments.

In order to illustrate the usefulness of time-distance diagrams in the interpretation of experimentally captured stress profiles, let us consider a model test of an elastic-plastic material response found in Figure 5.2, which directly relates to the time-distance diagram in Figure 5.1. Figure 5.2 contains a stress profile transmitted to the polycarbonate window for an experiment on ductile cast iron impacted at 300 m/s. Clearly labeled in this figure are the maximum and minimum stresses associated with the pullback signal. Through comparison between the two figures, the knee associated with the initial rise of the compressive pulse is immediately identified as the HEL of the material. With the use of equation (4.4), the magnitude of the HEL in the cast iron (\(\sigma_I\)), can be determined from the transmitted HEL signal in the gauge record (\(\sigma_T\)),

137
utilizing the elastic impedances for the two respective materials, summarized in Table 4.2. For the current case, the HEL in the ductile cast iron is about 1 GPa. Also noteworthy in the transmitted stress pulse associated with an elastic-plastic response is the kink in the unloading portion of the pulse. Again, the time-distance diagram allows a researcher to interpret this as the transition from elastic to plastic unloading. The magnitude of the initial portion associated with the elastic unloading is on the order of twice the HEL, which is in agreement with literature [1], further supporting the claim that this artifact is indeed associated with the elastic unloading.

Figure 5.2. Transmitted stress profile for an experiment on ductile cast iron impacted at 300m/s representative of a model test.

Before we continue to the major results of the experiments let us briefly address some final concerns linked to the use of manganin stress gauges in the present study. As noted in section 3.2.3, it has been shown in literature that manganin
remains linear elastic up to pressures of 1.5 GPa [2]. The resulting gauge hysteresis
from exceeding this range would present an obvious problem when trying to
determine the magnitude of the minimum stress associated with the pullback signal.
Errors linked to this determination would in turn affect the determined spall strengths
with the use of equation (2.33). In all spall fracture experiments conducted within this
thesis, it should therefore be noted that transmitted stresses to the polycarbonate
windows never exceeded 1.5 GPa, thus reducing errors and associated complications
in the determination of the pullback signal magnitude.

In order to accurately calculate strain rates from the experimental stress records
with use of equation (2.34), we must address the response time of the manganin
gauges. As pointed out in chapter 2.6, this can be accomplished through construction
of a stress-particle velocity plot representative of the specific experiment. It should be
noted that polycarbonate was chosen as the window material because its Hugoniot
closely matches that of the stress gauge’s kapton insulation and epoxy used to embed
the gauge between the two layers. Thus we will utilize polycarbonate’s Hugoniot to
represent the insulating layers and epoxy of the gauge package. The maximum
response time associated with the ring-up of the manganin foil is directly correlated to
the maximum stress within the cast iron. We will therefore address the maximum
response time by considering an impact velocity of 300 m/s, which was the highest
velocity used in the subsequent study. There is a lack of Hugoniot data for manganin
in literature. However, considering that the alloy is 84% copper, the Hugoniot of
copper is frequently used in research to approximate the Hugoniot of manganin gauges
[3]. Figure 5.3 contains a stress-particle velocity plot for the extreme case of the
subsequent study, constructed with published Hugoniot's of copper, cast iron, and polycarbonate found in literature [4].

\[ \text{Figure 5.3. Stress-particle velocity plot demonstrating the response of the manganin stress gauges used in the subsequent study.} \]

The symmetrical impact at 300 m/s will result in state 1 within the cast iron. Transmission of this stress into the front layers of the gauge package (epoxy and kapton) will result in state 2. The stress transmitted to the gauge layers will subsequently bring the manganin foil to state 3 upon transmission. The seven states indicated with the five pointed stars represent the subsequent transmission of stress to the back layers of the gauge package and polycarbonate window. Because the gauge foil is embedded between layers of comparatively low impedance, the response of the manganin foil will achieve its final magnitude through damped oscillatory loading, as pointed out in section 2.6. Noting that the manganin foil is 10 µm thick, and assuming
the longitudinal velocity for copper to be 4760 m/s, oscillations in the recorded stress signal will present themselves with a period of 4 ns. Figure 5.3 demonstrates that in addition to the first transmission into the back layers (state 1’), 99% of the stress at state 2 can be achieved through 6 reverberations within the gauge foil. Utilizing the longitudinal wave speed for copper, the final ring up of the stress gauge to 99% of the stress at state 2 will be achieved in 27 ns. Thus, 27 ns represents the maximum response time of the manganin gauges associated with the subsequent experiments, where lower velocity impacts would result in faster response times.

In terms of applying a correction factor associated with the response time of the stress gauge to attain a more accurate estimate of strain rate with use of equation (2.34), the current design does not require the full 27 ns. Due to the fact that the manganin foil is embedded within low impedance layers, the final state is achieved through a decaying oscillatory loading profile. Referring back to Figure 5.3, it can be noted that the gauge will initially overshoot the stress at state 2. Since strain rate is established by taking the slope of the transitional portion of a stress profile, the initial rise of the gauge record will be almost instantaneous. The delay in the rise of the gauge, for the case of Figure 5.3, can be calculated by assuming that the first C+ compression of the foil must completely pass through its thickness in order to achieve equilibrium, subsequently resulting in a delay of 2 ns. Thus, in 2 ns, the stress record will have risen from zero stress to an overshot value associated with state 3. For the preceding experiments, a response correct strain rate can be achieved with use of equation (2.34), by measuring the slope of the stress transition, and removing 2 ns from the time portion ($d\sigma/(dt-2\text{ns})$).
5.2 RESULTS FOR THE CAST IRONS STUDIED

Histories of the transmitted stress to the polycarbonate windows from experiments on the five cast irons studied can be found in Figure 5.4. A total of four experiments were conducted for each casting which covered an impact velocity range of 120-300 m/s, and subsequently resulted in low to moderate shock loading conditions within the materials. For comparative purposes, the four tests for each casting are overlaid to produce a single stress-time plot, where parts (a), (b), (c), (d), and (e), respectively, correspond to transmitted stress records for the gray cast irons A, B, C, D, and the ductile cast iron studied. Utilizing the elastic wave speeds for each particular casting, summarized in Table 4.2, the recorded stress profiles are shifted in the time axis so that impact can be noted to occur at time zero seconds. In order to facilitate comparison between the experiments on the five materials, the stress axis and time axis are consistent within the plots found in parts (a), (b), (c), (d), and (e) of Figure 5.4.

Referring to Figure 5.4, it can be seen that the elastic-plastic model used in the construction of the time-distance diagram found in Figure 5.1 only applies to the ductile cast iron studied. This material clearly exhibited coupled elastic-plastic response in the transmitted gauge records, where a HEL is easily perceived in its loading path. Tests on the gray cast irons exhibited loading profiles with a smooth rise representing purely plastic material response, where any elastic precursor associated with the HEL is overdriven by the shock wave front. Thus, the time-distance diagram in Figure 5.1 can be amended for these experiments by omitting the consideration of the dotted characteristics that serve to represent elastic waves.
Figure 5.4. Recorded stress profiles transmitted to the polycarbonate window. (a) Cast Iron A; (b) Cast Iron B; (c) Cast Iron C; (d) Cast Iron D; (e) Ductile Cast Iron.
Table 5.1 contains a summary of the twenty experiments conducted on the various cast irons. The impact velocities were determined according to the method described in section 3.2.2. The impact stress ($\sigma_{\text{IMPACT}}$) was calculated utilizing the acoustic approach in equation (2.21), under the assumption of a symmetrical impact, where the resulting particle velocity is half of the impact velocity. For each test, the respective material densities and longitudinal wave speeds summarized in Table 4.3 were used to estimate the impact stress. The strain rate of the initial compressive pulse (loading rate) and the unloading rate were calculated with use of equation (2.34). These calculations required measurement of the slopes of the shock wave portion and the unloading portion of the stress profile. Under the assumption of continuity of the interface, the strain rate of the incident stress within the cast iron is equivalent to the strain rate of the stress transmitted to the polycarbonate window. Therefore, these slopes could be measured directly from the transmitted gauge records, where calculation of the strain rate in equation (2.34), utilized the density and longitudinal wave speed of the polycarbonate window. In order to attain better estimations of strain rate, the response time of the stress gauges was taken into account, as described in the preceding section. The maximum ($\sigma_{\text{max}}$) and minimum ($\sigma_{\text{min}}$) stresses were directly determined from the experimental stress records in accordance with the labels found in Figure 5.2. The magnitude of the pullback signal ($\Delta\sigma$) is representative of the difference between the maximum and minimum stresses. Spall strengths were calculated utilizing equation (2.33), where a hydrodynamic material response was assumed, and therefore respective bulk wave speeds were used in the calculation of impedances. The fracture energy ($\gamma$) was determined utilizing equation (2.35).
Table 5.1. Summary of results from the spall fracture experiments on cast iron.

<table>
<thead>
<tr>
<th>Material</th>
<th>Impact Velocity (m/s)</th>
<th>Loading Rate (s⁻¹)</th>
<th>Unloading Rate (s⁻¹)</th>
<th>σ_max (GPa)</th>
<th>σ_min (GPa)</th>
<th>Δσ (GPa)</th>
<th>σ_sp (GPa)</th>
<th>γ (J/m²)</th>
<th>Damage Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast Iron A</td>
<td>270</td>
<td>4.01</td>
<td>8.0E+05</td>
<td>0.73</td>
<td>0.56</td>
<td>0.17</td>
<td>0.13</td>
<td>1.8</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>240</td>
<td>3.56</td>
<td>8.0E+05</td>
<td>0.67</td>
<td>0.50</td>
<td>0.17</td>
<td>0.19</td>
<td>5.4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>190</td>
<td>2.82</td>
<td>4.0E+05</td>
<td>0.46</td>
<td>0.36</td>
<td>0.10</td>
<td>0.05</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>130</td>
<td>1.93</td>
<td>2.0E+05</td>
<td>0.26</td>
<td>0.19</td>
<td>0.07</td>
<td>0.09</td>
<td>2.1</td>
<td>3</td>
</tr>
<tr>
<td>Cast Iron B</td>
<td>270</td>
<td>4.51</td>
<td>6.0E+05</td>
<td>0.71</td>
<td>0.53</td>
<td>0.18</td>
<td>0.37</td>
<td>24.5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>240</td>
<td>4.01</td>
<td>5.0E+05</td>
<td>0.60</td>
<td>0.46</td>
<td>0.14</td>
<td>0.22</td>
<td>8.3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>3.01</td>
<td>3.0E+05</td>
<td>0.40</td>
<td>0.31</td>
<td>0.09</td>
<td>0.15</td>
<td>4.5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>2.09</td>
<td>2.0E+05</td>
<td>0.26</td>
<td>0.20</td>
<td>0.06</td>
<td>0.08</td>
<td>1.1</td>
<td>3</td>
</tr>
<tr>
<td>Cast Iron C</td>
<td>270</td>
<td>4.34</td>
<td>6.0E+05</td>
<td>0.66</td>
<td>0.51</td>
<td>0.15</td>
<td>0.21</td>
<td>7.1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>230</td>
<td>3.70</td>
<td>5.0E+05</td>
<td>0.48</td>
<td>0.34</td>
<td>0.14</td>
<td>0.30</td>
<td>25.7</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>210</td>
<td>3.38</td>
<td>3.0E+05</td>
<td>0.45</td>
<td>0.37</td>
<td>0.09</td>
<td>0.04</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>130</td>
<td>2.09</td>
<td>1.0E+05</td>
<td>0.21</td>
<td>0.16</td>
<td>0.05</td>
<td>0.08</td>
<td>1.7</td>
<td>3</td>
</tr>
<tr>
<td>Cast Iron D</td>
<td>265</td>
<td>4.76</td>
<td>5.0E+05</td>
<td>0.61</td>
<td>0.44</td>
<td>0.17</td>
<td>0.45</td>
<td>41.1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>230</td>
<td>4.13</td>
<td>5.0E+05</td>
<td>0.57</td>
<td>0.40</td>
<td>0.17</td>
<td>0.49</td>
<td>62.9</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>190</td>
<td>3.41</td>
<td>3.0E+05</td>
<td>0.44</td>
<td>0.31</td>
<td>0.13</td>
<td>0.37</td>
<td>38.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>2.15</td>
<td>2.0E+05</td>
<td>0.28</td>
<td>0.21</td>
<td>0.11</td>
<td>0.41</td>
<td>81.2</td>
<td>3</td>
</tr>
<tr>
<td>Ductile Cast Iron</td>
<td>300</td>
<td>5.84</td>
<td>9.0E+04</td>
<td>0.79</td>
<td>0.5</td>
<td>0.29</td>
<td>1.20</td>
<td>492.7</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>210</td>
<td>4.09</td>
<td>5.0E+04</td>
<td>0.48</td>
<td>0.29</td>
<td>0.19</td>
<td>0.82</td>
<td>275.3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>210</td>
<td>4.09</td>
<td>5.0E+04</td>
<td>0.50</td>
<td>0.32</td>
<td>0.21</td>
<td>0.94</td>
<td>448.2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>190</td>
<td>3.70</td>
<td>8.0E+04</td>
<td>0.43</td>
<td>0.23</td>
<td>0.2</td>
<td>0.94</td>
<td>248.8</td>
<td>3</td>
</tr>
</tbody>
</table>

In addition to the variables described, Table 5.1 also contains a classification of the respective damage level associated with the spall zone for each experiment. In order to facilitate a meaningful post mortem investigation into the damage level of the spall plane as a function of the spall strength, without the consideration of additional damage inflicted from capturing the samples, impacted samples from the experiments were captured utilizing the soft recovery system discussed in chapter 3. Recovered samples were cut through their cross-section to expose the spall zone, as shown in Fig. 5.5. For this process, a Buehler precision diamond cutoff saw was utilized where it was determined that a blade speed of 900 RPM and a sample load of 100 g provided a good balance between cut time, while minimizing additional microstructural damage. In order to further minimize microstructure damage related to excess heat build up, cutting was conducted with a constant coolant application to the blade. Although cut samples exhibited a mirror-like surface to the naked eye, under magnification,
striations from the cutting process were identifiable. To address this issue, grinding and polishing were conducted on the cut surfaces of the samples following the same guidelines and procedures discussed in section 4.3.2. Due to the presence of cracks associated with the spall plane, cleaning between steps was accomplished by an additional process utilizing an ultrasonic vibrator with methanol. A vacuum chamber was utilized to evaporate the methanol from the cracks, and this cleaning process was repeated multiple times in order to minimize contamination of proceeding polishing steps. Once polished, the damage level of the spall planes was investigated with the use of a reflection light optical microscope.

![Figure 5.5. Example of a recovered sample cut to expose the spall plane.](image)

The degree of failure found in Table 5.1 was characterized in accordance with some generally accepted quantitative measure of the level of damage proposed in literature [5]. In literature, damage is typically classified by five levels ranging from complete spall fracture to microscopic total integrity of the sample. Damage level 1 consists of complete spall fracture, where there is the presence of a main crack across the whole sample section, and all structural integrity is lost. Damage level 2 denotes partial macroscopic failure, where there is the presence of separate macrocracks in the same section. Damage level 3 characterizes intensive microfailure, where there is the presence of a large number of isolated or merged microcracks in the spall zone.
Damage level 4 consists of weak microfailure, where microcracks are separate and scattered in distribution. Finally, damage level 5 is used to classify total microscopic integrity of the sample, where there is an absence of microcracks in the section when observed at 1000x magnification. Often in literature, damage levels 2, 3, and 4 are combined to form a single level, thus, a “rougher” estimation of the degree of damage is attained by considering only 3 levels [6].

Figure 5.6 contains a summary of the spall strengths determined as a function of their respective material, where the range of values determined for each material is encompassed within an open box. The shaded regions found for the gray cast irons are used to denote cases where the sample underwent complete spall fracture, defined by damage level 1 in Table 5.1. For the case of the ductile cast iron studied, no experiments were able to generate complete spall fracture, and in turn the material continued to maintain partial structural integrity. Immediately apparent when considering Figure 5.6 is the fact that the highest spall strengths were almost exclusively found for cases where the material exhibited complete fracture. This is easily anticipated and in good agreement with literature, where it has been shown that determined spall strengths represent underestimations when damage is low and overestimations when complete fracture exists [6]. Spall strengths determined from the low severity impacts can be directly related to the energy required to initiate cracks within the material. The range of values from the low damage to high damage levels can be directly correlated to the increased energy required to further coalesce initiated cracks into a complete fracture plane. The data set marked by a star for cast iron C represents a violation of this condition, where the material exhibited complete fracture
while yielding an extremely low spall strength. This experiment can be addressed if we consider the random distribution of voids found within this casting, as noted in chapter 4. It seems reasonable to hypothesize that this test cannot be compared to the rest because it likely contained an array of these voids at the spall plane, which allowed for spall fracture to occur with minimal energy. Confirmation of this hypothesis is difficult, considering that the material exhibited a complete loss of structural integrity, and post mortem analysis of the damage plane was therefore not possible.

![Figure 5.6. Summary of spall strengths determined for the five cast irons studied.](image)

Although there is a scarcity of studies into the spall strength of cast iron published in literature, let us consider the agreement of the above values with studies
conducted on pure iron. The spall strength of pure iron has typically been shown to fall in the range of 1-2 GPa [7-9]. The determined spall strengths for the ductile cast iron seem in good agreement with these findings, however, the gray cast irons were almost an order of magnitude lower. It seems alarming to consider that the additional graphite phase in these castings can result in such a significant reduction in the strength of iron.

It is frequently noted in literature that the spall strength of most materials exhibits a strong relationship with the unloading strain rate [10-15]. In the case of metals, spall strength is typically shown to display a power law dependence on the unloading rate, where increased values of spall strength are associated with increased unloading rates. Let us consider this relationship for the cast irons studied by referring to Figure 5.7 in which spall strengths are plotted as a function of the respective unloading rates. In general, the data sets in Figure 5.7 do not permit a very clear interpretation of the spall strength as a function of unloading rate, however, there is a distinguishable increase in spall strength observed for increasing unloading rates, which is shown by the solid, dashed, and dotted arrows respectively for the ductile cast iron, cast iron D, and castings A-C. Applying a power law curve fit to the data sets respective of the gray cast irons A, C, and D, or the ductile cast iron, would exhibit significant deviations from the experimental data. A power law curve fit could be applied to the data set of cast iron B, but four experiments would not facilitate a conclusive trend.

Before we try to correlate the experimentally determined strengths for the cast irons to their inherent microstructure, it seems important to validate the experimental
method used in the current study. Obviously, the varying nature and lack of a distinguishable relationship between spall strength and unloading rate can be related to the concept that the microstructure within specific castings is inconsistent, therefore varying throughout each casting. This consideration would imply that each separate test on a specific casting is not actually a test of the same material, thus giving rise to the lack of trend in the strength-unloading rate plot. Although the values of spall strength for the gray cast irons seem alarmingly low, this trend can be answered in terms of the addition of graphite flakes to the iron matrix. Regardless, we will first validate the method before turning to microstructure to shed light on the current investigation.

![Figure 5.7. Spall strength versus unloading rate for the five cast irons studied.](image)
5.3 THE SPALL STRENGTH OF 6061 ALUMINUM

In order to validate the experimental method, let us move away from heterogeneous materials such as cast iron and consider the spall strength of a well classified homogenous material. One of the most widely studied materials of such a class in literature is aluminum [10,13,15-21]. Testing at room temperature, within the range of shock conditions achievable with the current apparatus, the strength of aluminum has been shown to vary between 1-1.6 GPa. To validate the test method employed in the previous spall fracture experiments, the same approach used to study cast iron will be adopted to investigate the spall strength of 6061 aluminum.

Two experiments were conducted on aluminum, targeting the same velocity of 350 m/s, in order to demonstrate repeatability of the method and to facilitate comparison with published results. The impactor and target specimen were 45 mm in diameter and respectively 4 mm and 8 mm thick. The same polycarbonate window thickness used in the cast iron study was employed. Figure 5.8 contains histories of the transmitted stress to the polycarbonate windows for the two experiments conducted on aluminum. Similar to the cast iron plots, the stress profiles have been shifted in time so that impact can be noted to occur at time zero seconds. A summary of these results can be found in Table 5.2, in which values were determined as previously discussed in section 5.2. The spall strengths determined for the two experiments were 1.37 GPa and 1.38 GPa, which fall within the range of values published in literature.

Taking note of Figure 5.8, the evolution of the gas gun apparatus is exemplified, where repeatability has been significantly increased, and it is therefore
difficult to discern that two plots reside on the same figure. For the case of the chosen
dimensions for these tests, the arrival time of the radial release is around 4.5 µs after
impact, subsequently allowing the second stress plateau (state 7) to be clearly captured
in the recorded stress profiles. The stress profiles exhibit an elastic-plastic material
response similar to the tested ductile cast iron, where the material’s HEL is easily
identifiable. In comparison to the tests on cast iron, the noted clarity of the transmitted
stress signal was attained with use of the newly purchased oscilloscope described in
chapter 3. This oscilloscope allowed for the capture of a distinguishable ringing, with
a period of 80 ns, at the top of the compressive stress profile. This ringing can easily
be misinterpreted as an artifact associated with the response of the employed stress
gauges. However, construction of a stress-particle velocity diagram, similar to that
found in Figure 5.3, disproves that assumption. In literature it has been shown that
this ringing is actually associated with capacitive linkage between the gauge and the
target surface, through the dielectric sheet used in mounting, and the inductance of the
gauge used for measurement [3]. For similar stress gauges to those used in the present
study, the ringing period was determined to be 80 ns, which is in good agreement with
the ringing observed in Figure 5.8.

In literature, the spall strength of aluminum has been shown to be well
represented by a power law fit when plotted as function of the unloading rate. Since
unloading rate was determined for the two experiments, further validation of the spall
fracture method can be achieved by calculating spall strength from this commonly
accepted power fit. For the case of 1100 aluminum, spall strength can be determined
from the unloading rate in the form:
where the values 0.635 and 0.059 are parameters generated from the curve fit of a multitude of data sets [8]. Utilizing the experimentally determined unloading rate of $2.3 \times 10^5\; s^{-1}$ in equation (5.1), results in a calculated spall strength of 1.32 GPa. This value is within 5\% of the spall strength values determined utilizing the current method, found in Table 5.2. Considering that the fit parameters used in this calculation are for a different aluminum, it is reasonable to state that the experimental method employed can accurately output values of spall strength that are consistent with literature. Thus, validation of the method has been shown, and microstructure remains the only tool left to investigate the spall strengths of the cast irons studied.

\[
\sigma_{sp} = 0.635 (\dot{\varepsilon})^{0.059} 
\]  
\hspace{1cm} (5.1)

Figure 5.8. Recorded stress profiles transmitted to the polycarbonate window for two spall fracture experiments on 6061 Aluminum.
Table 5.2. Summary of results from spall fracture experiments on 6061 Aluminum

<table>
<thead>
<tr>
<th>Material</th>
<th>Impact Velocity (m/s)</th>
<th>$\sigma_{\text{IMPACT}}$ (GPa)</th>
<th>Loading Rate (s$^{-1}$)</th>
<th>Unloading Rate (s$^{-1}$)</th>
<th>$\sigma_{\text{max}}$ (GPa)</th>
<th>$\sigma_{\text{min}}$ (GPa)</th>
<th>$\Delta\sigma$ (GPa)</th>
<th>$\sigma_{sp}$ (GPa)</th>
<th>$\gamma$ (J/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6061 Al</td>
<td>350</td>
<td>3.03</td>
<td>2.2E+06</td>
<td>2.3E+05</td>
<td>0.85</td>
<td>0.22</td>
<td>0.63</td>
<td>1.37</td>
<td>1211</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>3.03</td>
<td>1.8E+06</td>
<td>2.3E+05</td>
<td>0.86</td>
<td>0.23</td>
<td>0.64</td>
<td>1.38</td>
<td>1270</td>
</tr>
</tbody>
</table>

5.4 CORRELATION OF RESULTS TO MICROSTRUCTURE

Previously addressed in chapter 4.3, the spall fracture of materials is strongly tied to their inherent microstructure. It is widely accepted in literature that pre-existing defects within a material’s microstructure often serve as damage nucleation sites under the action of dynamic tensile loading [8]. Second phase particles in an otherwise homogeneous material, grain boundaries, microcracks, and voids are examples of such defects that can strongly influence the mechanisms involved in dynamic fracture. In the case of cast iron, it is commonly accepted in literature that, under quasi-static test conditions, the additional graphite phase found within the metal matrix has the most significant effect on the resulting strength of the material [22-24]. In terms of spall fracture, this common observation seems quite relevant. It should be noted that optical micrographs captured while conducting the damage classification discussed in section 5.2 indicated some connection between the initiation of cracks and the graphite phase. Within these optical micrographs, it appeared as if crack nucleation was caused by the debonding of the graphite phase from the metal matrix. Limited by the wavelength of visible light, micrographs captured utilizing a reflection light optical microscope fell short in providing the resolution and depth of field required to confirm this hypothesis.
In order to confirm the hypothesis that the initiation of spall fracture in cast iron is tied to the debonding of the graphite phase from the metal matrix, it was decided to utilize a scanning electron microscope (SEM) to capture micrographs of the spall zones. Where visible light has wavelengths ranging 400-700 nm, the wavelength of electrons is on the order of 0.005nm, in turn, facilitating the capture of micrographs with higher resolution and depth of field. This SEM investigation was conducted within Dr. Otto Gregory’s SST Surface Characterization Laboratory under the supervision of his lab manager Michael Platek.

5.4.1 Initiation of Failure in Ductile Cast Iron

Figure 5.9 contains SEM micrographs, captured with the use of backscattered electrons in combo mode, of the spall zone in the ductile cast iron sample that was impacted at 190 m/s. Part (a) of Figure 5.9 contains a micrograph taken at 75x magnification. Part (b) is a close up of the boxed region depicted in the micrograph from part (a) taken at 200x magnification. Within the figure, graphite nodules are denoted as G, and the crack is denoted as C. An example of a graphite nodule pullout associated with the grinding and polishing preparation of the sample is denoted as P. Figure 5.10 contains SEM micrographs, captured with the use of backscattered electrons in combo mode, of the spall plane in the ductile cast iron sample that was impacted at 300 m/s. Part (a) of Figure 5.10 contains a micrograph taken at 70x magnification. Part (b) of Figure 5.10 is a close up of the boxed region depicted in the micrograph from part (a) taken at 250x magnification. Within the figure, graphite nodules are denoted as G, cracks are denoted as C, small scale microcracks are denoted as $C_M$, and evidence of the debonding of graphite is denoted as D.
Figure 5.9. SEM micrographs of the spall plane in the ductile cast iron sample impacted at 190 m/s; (a) 75x magnification; (b) 200x magnification of boxed region in (a). Graphite is denoted as G; Cracks as C; Polishing pullout as P.
Figure 5.10. SEM micrographs of the spall plane in the ductile cast iron sample impacted at 300 m/s; (a) 70x magnification; (b) 150x magnification of boxed region in (a). Graphite is denoted as G; Cracks as C; Microcracks as $C_M$; Evidence of graphite debonding from the matrix is denoted as D.
Figure 5.11 contains a final SEM micrograph of the spall plane in the ductile cast iron sample that was impacted at 300m/s. This micrograph was taken at 250x magnification with the use of backscattered electrons in combo mode. Graphite nodules are denoted as G, and evidence of the debonding of graphite is denoted as D.

![SEM micrograph](image)

**Figure 5.11.** SEM micrograph taken at 250x magnification of the spall plane in the ductile cast iron sample impacted at 300 m/s. Graphite is denoted as G; Evidence of graphite debonding from the matrix is denoted as D.

Referring to Figures 5.9-5.11, it can be observed that the fracture of ductile cast iron involves the debonding of the graphite nodules from the metal matrix. This mechanism is difficult to observe on the two-dimensional plane created by the preparation of these samples, thus requiring careful observation to discern the effect. In Figure 5.9 (a), the crack adjacent to the right arrow originating from the designation C, can be noted to propagate along two graphite nodules. The path followed by this crack provides some indication of a looping mechanism associated with particle debonding. Similarly, the looping mechanism is indicated by the small crack observed
adjacent to the lower specified graphite nodule in Figure 5.11. The areas specified as D within Figure 5.11 demonstrate completely debonded nodules that can also be observed on the two-dimensional plane. The microcracks, marked $C_M$ in Figure 5.10 (a), provide an example of crack initiation due to the debonding of graphite from the matrix.

Although these examples provide some validating evidence of graphite debonding, the true advantage of utilizing an SEM in the analysis of material fracture is realized when considering the fracture surfaces deep within the two-dimensional plane. The areas denoted D within Figure 5.10 demonstrate cases of particle debonding that are indicated by the cup-like topography observed on the fracture surface. Taking note of Figure 5.10 (b), the fracture surface exhibits a multitude of cup-like structures that yield a fracture plane with a sponge appearance. These micrographs confirm the hypothesis that the weak bonds between the graphite and metal matrix provide nucleation points for the generation of spall fracture. This finding is complemented by the associated difficulties in the metallurgical preparation of cast iron samples, previously addressed in section 4.3.2, where it was noted that graphite is easily pulled from the matrix during grinding and polishing processes.

**5.4.2 Initiation of Failure in Gray Cast Iron**

Moving away from ductile cast iron, let us now address the failure of gray cast irons. Figure 5.12 contains an SEM micrograph, taken at 500x magnification with the use of backscattered electrons, of the spall plane in the cast iron D sample impacted at 120 m/s. Within this figure, graphite is denoted as G and examples of debonding are denoted D. Figure 5.13 contains two micrographs, taken at 500x magnification, of the
spall plane in the cast iron D sample impacted at 120 m/s. Part (a) was captured with the use of backscattered electrons in combo mode, while part (b) was captured with the use of secondary electrons. Within these micrographs, G is used to denote graphite, cracks are denoted as C, and D_i and D_C respectively denote initiation and completion of graphite debonding.

Referring to Figures 5.12 and 5.13, it can be noted that the fracture of gray cast iron involves the debonding of the graphite flakes from the metal matrix. Unlike the case of the ductile cast iron, this observation is not as easily facilitated by considering the fracture surface deep within the spall plane. In the case of this material, confirmation of graphite debonding was primarily generated by considering the two-dimensional plane created by sectioning the sample. Taking note of Figure 5.12, two examples of graphite debonding observed on the two-dimensional plane are presented.

![Figure 5.12. SEM micrograph taken at 500x magnification of the spall plane in the cast iron D sample impacted at 120 m/s. Graphite is denoted as G; Evidence of graphite debonding from the matrix is denoted as D.](image-url)
Figure 5.13. SEM micrographs taken at 500x magnification of the spall plane in the cast iron D sample impacted at 120 m/s; (a) Backscattered electrons in combo mode; (b) Secondary electrons. Graphite is denoted as G; Cracks as C; Initiation of graphite debonding as D_1; Completely debonded graphite as D_c.
While conducting the SEM investigation into the spall planes of the cast irons, the use of secondary electrons in the acquisition of micrographs was initially rejected due to their high sensitivity to contamination. It should be noted that the metallurgical preparation of the impacted samples was rather difficult due to the presence of large cracks associated with the spall plane. These cracks tended to capture fibers from the polishing pads used in the final stages of preparation. Due to the low atomic weight of these fibers, they were almost invisible when utilizing backscattered electrons, resulting in relatively clear micrographs of the cast iron below. However the case, secondary electrons tended to provided excellent resolution and contrast of inclined surfaces. This trend is due to the fact that steeper surfaces, in terms of their angle away from the normal of the incident beam, allow for the escape of more secondary electrons, thus yielding an increase in image brightness. Figure 5.13 provides a visual example of the variation in imaging techniques, clearly demonstrating the advantages of secondary electrons for the current investigation. With the use of secondary electrons, the initiation of graphite debonding is quickly identified, where it may have been overlooked when only considering the backscattered image in part (a). Similarly, the completely debonded graphite flake, found on the right of the micrograph in Figure 5.13 (b), could have easily been misinterpreted as part of the crack associated with the spall plane.

The current SEM investigation into the fracture surfaces of the tested cast irons has indeed confirmed the hypothesis that dynamic failure of the material is linked to the debonding of graphite from the metal matrix. The concept that fracture processes within cast irons are linked to the debonding of graphite from the metal matrix is well
supported in literature. In work by Voigt and Holmgren, crack initiation and propagation studies on gray cast irons demonstrated that crack propagation typically involved the decoherence of graphite from the matrix [25]. Similarly, work by Mohammed, et al., further supports that fracture initiation in cast irons is linked to the interface between graphite and the metal matrix [26].

5.4.3 Graphite-Matrix Bond Strength

Referring back to Figure 5.6, let us now consider the lower bound spall strengths respective of each specific casting, to represent the tensile stress required to nucleate cracks within the materials through the mechanism of graphite debonding. In doing so, there remains the necessity to seek an explanation as to why the ductile cast iron outperformed the gray cast irons in terms of initial strength. If we consider crack nucleation to be directly connected to the bond strength of the graphite phase to the metal matrix, it is immediately apparent that the graphite nodules in the ductile casting possess higher bond strengths than the flakes in the gray cast irons. The explanation to this observation lies in the foundry techniques used in the production of cast irons. Taking note of the differences in the eutectic growth of nodular and flake graphite, presented in section 4.3.3, a hypothesis for this trend can quickly be facilitated. In this chapter it was noted that Type A flake graphite is typically produced with an inoculation process that produces a system of silica-rich oxide bifilms that serve as nucleating points for the precipitating graphite. For gray cast irons, graphite flakes have been noted to grow along the oxide bifilms. The final solid can thus be noted to contain a thin layer of oxygen and silicon, linked to these bifilms, which will reside between the precipitated graphite flakes and the metal matrix. In contrast, the nodules
associated with ductile cast iron are generated by additions of magnesium that act to break up the bifilms created within the initial inoculation process. For ductile cast irons, graphite nodules have been noted to encapsulate each separate nucleus associated with the dispersed bifilms, thus, there remains no layer separating the precipitated graphite nodules from the metal matrix.

In order to utilize this observation to describe the increased bond strength associated with the ductile cast iron, it must be shown that the bifilms act as the decohering agent in regards to the flake graphite. While conducting the SEM investigation, energy-dispersive X-ray spectroscopy (EDS) was performed on the fracture surfaces of the samples in order to investigate the presence of elements that could be linked to these bifilms. In the current case, this investigation was not able to confirm the presence of bifilms because the associated metallurgical preparation likely erased all evidence.

In order to address this issue, let us evoke a summary of the experiments conducted by Johnson and Smart, presented by Campbell [27], where cast iron samples were carefully fractured and observed in high vacuum. In this research, sophisticated Auger analysis was used to prove that two or three atomic layers of oxygen and sulfur were present on fracture surfaces of gray irons that were adjacent to graphite flakes. In contrast, the hollows in the fracture surfaces of ductile cast irons contained no evidence of bifilms. In work by Campbell, it is proposed that the principal cause of reduced mechanical properties in all cases of non-spheroidal forms of cast iron is the presence of various kinds of oxide bifilms that act as cracks [27]. In this work, it is noted that ductile iron has increased tensile strength due to the absence
of oxide bifilms, not because of its spherical graphite morphology. Additionally, in a
book by Campbell, it is noted that ductile cast irons can fail disastrously by the brittle
“plate fracture” mechanism in cases where poor foundry techniques do not
successfully disperse the initial bifilms [28]. In these cases, it is suggested that cracks
propagate along trapped bifilms, which serve as nucleation points for the brittle
failure. In light of these findings, the significantly low spall strengths found for the
gray cast irons are attributed to the presence of bifilms.

5.4.4 The Effect of Graphite Size on Spall Strength

Although there is a scarcity of spall fracture experiments conducted on cast
irons in literature, some key findings from the quasi-static strength of various castings
may be applicable to the current study. In literature it is frequently noted that the
average length of graphite has a direct correlation with the strength [24]. Generally a
casting will have increased strength as the average length of graphite decreases.
Applying this observation to the current study, refer to Figure 5.14, where the spall
strengths of the cast irons are plotted as a function of their respective average graphite
size class determined in section 4.3.4. Although cast iron D contained a bimodal
distribution of graphite types VII A4 and VII D8, we will utilize the size class
associated with the type A distribution in this figure. Remembering that the lower
numbered size classes are associated with the largest graphite flakes, Figure 5.14
indicates that the strength of the material, in terms of dynamic fracture, does increase
with decreasing graphite length. This observation is useful in describing the increased
spall strength of cast iron D in comparison to castings A-C.
Figure 5.14. Spall strengths plotted as a function of average graphite size class.

5.4.5 Mechanisms Governing the Completion of Failure

Let us now consider the claim that one can relate the range of strengths from the low damage to high damage levels to the increased energy required to coalesce initiated cracks into a complete fracture plane. This process would require the cracks nucleated at the graphite/matrix interface to propagate through the metal matrix. Thus, the strength of the matrix will become an important factor governing the completion of spall fracture. In literature, it is frequently noted that pearlitic matrix microstructures result in some of the highest strength cast irons [22, 24, 29]. Although ferritic microstructures have been linked to increased ductility, they have been shown to reduce a casting’s strength [22]. These findings are in good agreement with the results of the current study. In terms of cast iron B, the wide range of values can be
linked to the increased strength of its inherent matrix microstructure. Although the base line values from Castings A-C are in good agreement with the conclusion that strength is tied to the length of the graphite (which was essentially the same between the castings), the upper values associated with casting B outperformed those of casting A and C. This comparison demonstrates the concept that when graphite morphology is consistent, pearlite microstructures will yield higher strength cast irons.

In order to address differences between the results found for castings A and C, some shortcomings associated with the metallurgical study presented in chapter 4 should be noted. Obviously, if these castings contained similar matrix microstructures, as observed in the metallurgical study, one would expect similar results for the two respective materials. In addition, the porosity observed in casting C should have resulted in lower strength values than those found for casting A. In contrast to cast iron D and the ductile cast iron, there existed a limited supply of the materials respectively denoted cast irons A-C. The metallurgical study into the as-received microstructures of these castings was performed after conducting the spall experiments. Only scraps from sample manufacture and rejected samples were available to perform this metallurgical study, which were likely not representative of the bulk castings. Adjacent research into the microstructure of these materials was in good agreement with the current study in terms of the microstructure of cast irons B and C. In the case of casting A, this adjacent research observed a matrix microstructure that was almost entirely ferritic. Since the adjacent metallurgical study on this material was performed at multiple points within the casting, it can be assumed that these findings better represent the microstructure of the bulk material. If we
consider that the matrix microstructure of cast iron A is essentially ferrite, then the small range of strengths for this material are in good agreement with observations common to literature. After crack initiation, minimal energy was required to propagate cracks through the matrix to completely fracture cast iron A. Under the assumption that the spall strength marked with a star in Figure 5.6 is representative of a case where porosity reduced the strength of cast iron C, we can investigate the range of this material by considering the remaining three data sets. In which case, the range found for cast iron C indicates an intermediate case between the strength of a ferritic matrix (cast iron A) and that of a pearlitic matrix (cast iron B).

When taking note of Figure 5.6, it is immediately apparent that the small range of strength values found for cast iron D violates some of the above conclusions. In Chapter 4, the matrix microstructure of cast iron D was shown to be completely pearlitic. Furthermore, the inter-laminar spacing within the pearlite colonies was noted to be much smaller than that found in cast irons A-C. In literature, it is frequently noted that the strength of pearlite increases with decreasing inter-laminar spacing [30]. In light of these points, the spall strength of cast iron D should have increased significantly as a function of damage level. However the case, we must remember that graphite morphology is the primary mechanism governing fracture processes within cast irons. In order to address this concern, let us consider some additional SEM micrographs of the spall zone within cast iron D. Figure 5.15 contains a SEM micrograph taken at 100x magnification, captured with the use of secondary electrons, of the cast iron D sample that was impacted at 120 m/s. Within the micrograph, graphite with types A and D distributions are respectively denoted as $G_A$.
and $G_D$, $C$ is used to denote cracks, and the direction of the acting tensile stress (T) is indicated on the left of the micrograph. Figure 5.16 contains a SEM micrograph taken at 200x magnification, captured with the use of backscattered electrons in combo mode, of the spall plane in the cast iron D sample that was impacted at 190 m/s. Within this micrograph, the spall plane is denoted SP, and a region of Type VII A4 graphite is specified as $G_A$, while a region of Type VII D8 graphite is specified as $G_D$. Figure 5.17 contains two additional SEM micrographs, captured with the use of backscattered electrons in combo mode, of the same sample imaged in Figure 5.16. Part (a) of Figure 5.17 contains a micrograph taken at 100x magnification, while Part (b) is a close up of the boxed region depicted in part (a), taken at 1000x magnification. It should be noted that Figure 5.17 images locations remote from the major spall plane, where the spall plane (SP) in part (a), is the same location marked SP in Figure 5.16. Within Figure 5.17, graphite with types $A$ and $D$ distributions are respectively denoted as $G_A$ and $G_D$, and $C$ is used to denote cracks.

Referring to Figures 5.15-5.17 it is apparent that the bimodal distribution of graphite classes in cast iron D played a large role in its response to spall fracture. The crack specified in Figure 5.15 can be noted to propagate along a dendrite of Type VII D8 graphite. Intuitively one would expect cracks to propagate in a direction normal to the acting tensile stresses, however, in the case of this crack, propagation almost occurred in the same direction as the tensile loading. This phenomenon indicates that the dendrite of fine graphite growth resulted in a significant loss of mechanical strength, thus encouraging the crack to follow this preferential direction. The decreased mechanical strength of regions with Type VII D8 graphite is further
exemplified by considering Figure 5.16. Noted in the region of Type VII $A_4$ graphite, the fracture mechanism primarily involved the debonding of graphite from the matrix. Although this region demonstrated an array of cracks, structural integrity was maintained by the relatively large areas of the metal matrix. In contrast, the region of Type VII $D_8$ graphite displayed complete loss of structural integrity, possessing a spall plane with an opening mode on the order of 250 µm.

![Figure 5.15. SEM micrograph taken at 100x magnification of the spall plane in the cast iron D sample impacted at 120 m/s. Graphite with type A distribution is denoted as $G_A$; Graphite with type D distribution is denoted as $G_D$; Cracks are denoted C. The direction of the acting tensile stress (T) is indicated on the left of the micrograph.](image)

Figure 5.17 demonstrates that cracks tend to follow preferential directions within regions of Type VII $D_8$ graphite. Considering that these micrographs are taken below the major spall plane, it should be noted that cracks generated within are not associated with the maximum tensile stresses of the experiment. It is interesting that the regions of Type VII $D_8$ graphite allow cracks to propagate for large distances under relatively low tensile stresses. This observation can be addressed by
considering the concept of mean free path. In cases where the relative spacing between graphite flakes is small, cracks can bridge between flakes without requiring much propagation distance through the metal matrix. This mechanism would inherently require less energy than the case where cracks have to propagate through relative large regions of the metal matrix. It has already been concluded that graphite is the weakest link in terms of the mechanical strength of cast iron, serving as nucleation points for cracks. It has also been noted that an increased amount of energy is typically required to coalesce initiated cracks into a complete spall plane. Therefore, it seems quite reasonable that cracks will follow preferential directions that require less energy in the process of completing the material’s fracture. In light of this conclusion, the relatively small range of spall strengths found for cast iron D can be considered a direct result of the Type VII D8 graphite found within.

Figure 5.16. SEM micrograph taken at 200x magnification of the spall plane in the cast iron D sample impacted at 190 m/s. The spall plane is denoted as SP; The region containing Type VII A4 Graphite is denoted as GA; The region containing Type VII D8 Graphite is denoted as GD.
Figure 5.17. SEM micrographs adjacent to the spall plane in the cast iron D sample impacted at 190 m/s; (a) 100x magnification; (b) 1000x magnification of boxed region in (a). The spall plane is denoted as SP; Cracks as C; Type VII A4 graphite as $G_A$; Type VII D8 graphite as $G_D$. 
The concept that the mean free path between graphite inclusions is linked to the associated energy required to complete the spall fracture of cast iron can additionally be applied to the case of the ductile casting. In terms of matrix microstructure, this material contained a considerable amount of free ferrite, which would be expected to reduce the materials strength. However the case, the relatively large spacing of the graphite nodules required initiated cracks to propagate great distances through the matrix in order to complete the spall plane. Referring back to Figure 5.6, it is immediately noted that this material contained the widest range of spall strengths. If one considers that complete fracture was not achieved in any of the experiments, it is easy to anticipate that the upper bound will increase until it encompasses cases of complete spall fracture. Thus it can be concluded that, in addition to the composition of the metal matrix, the energy required to complete fracture in cast irons is directly tied to the propagation distance within the metal matrix.

5.5 CONCLUDING REMARKS

Cast iron is a highly heterogeneous material, where the mixed microstructure results in competing mechanisms associated with mechanical properties. In addition, the microstructure within any specific casting can exhibit large variations from mold wall to core. Unlike the case of homogenous materials such as aluminum, this inherent shortcoming yields complications when trying to discern data trends from small batches of experiments. However the case, by studying five different cast irons, some key conclusions can be drawn from the investigation into the spall fracture of the
material. The importance of metallurgical investigations in the study of spall fracture of materials is exemplified by this investigation, where conclusions could not have been drawn entirely form the experimental results.

This study has identified the graphite phase as the primary factor governing the spall fracture of cast irons, where crack nucleation is directly correlated to the debonding of graphite from the metal matrix. It has been noted that the average length of graphite found within a casting is linked to the material’s strength, where strength has been shown to increase as a function of decreasing length. The morphology, and mean free path of graphite precipitates further govern the subsequent coalescence of initiated cracks to form a complete fracture plane. In cases where graphite spacing is large, an increased level of energy is required to complete the fracture process. A secondary factor governing the spall fracture of cast irons has been linked to the microstructure of the metal matrix. It has been noted that pearlite will yield higher mechanical strengths in cast irons than free ferrite.

5.6 REFERENCES


CHAPTER 6

GENERATING A MATERIAL’S HUGONIOT

This chapter investigates the application of the Hugoniot ring-up method (HRUM) to generate Hugoniots of materials of interest. Within the chapter, the HRUM will be utilized in two configurations. The first configuration will employ 6061 aluminum as the bulk material while utilizing polycarbonate as the low impedance inner-layer. Following this, A572 grade 50 structural steel and polycarbonate will be respectively employed as the bulk material and inner-layer. In all cases, comparison of the resulting Hugoniot states obtained from the HRUM to published data found in literature will be conducted in order to provide validating evidence supporting the accuracy of the proposed method.

6.1 HRUM WITH 6061 ALUMINUM AND POLYCARBONATE

6061 aluminum and polycarbonate are two materials that are widely studied in literature. Thus, there is a plethora of published data respective to the two material’s Hugoniots that will facilitate validation of the HRUM. One concern associated with this method is that the Hugoniot of a thin-layer of polymer will deviate from its bulk material response. This concern has been addressed in literature, where it has been shown that submicron films of PMMA have the same material response to shock loading as macroscopic samples [1].
This section will begin by introducing the experimental design, considering a time-distance diagram of the planned experiments. Next, the Hugoniot parameters that will be used in the subsequent analysis will be outlined. Following this the experimental results will be presented. From these results, the HRUM will be employed to first determine the Hugoniot of the polycarbonate inner-layer with use of the known Hugoniot of the aluminum bulk material. Conversely, the method will next be utilized to determine the Hugoniot of the aluminum bulk material from the known Hugoniot of the polycarbonate inner-layer. In both cases, validation of the HRUM will be achieved through comparison of the Hugoniot states obtained to published data found in literature. In cases where there is good agreement with the published data, it will be shown how one can obtain an equation that defines the material’s Hugoniot within the domain encompassed by the specific test. Additionally, the accuracy of these experimentally obtained Hugoniot equations will be investigated through comparison with published Hugoniots of the respective materials.

### 6.1.1 Experimental Design

In order to begin, let us consider a time-distance plot of the planned experiments depicted in Figure 6.1. This time-distance diagram was constructed under the assumption of a plastic response, therefore all characteristics were generated with the respective bulk wave speeds of the materials. The impactor and test sample are both 45 mm in diameter. In order to prevent any spall fracture conditions in the aluminum front face, its thickness was chosen so that it matched that of the impactor. The thickness of the front face and impactor are 7.62 mm (0.3”). The thickness of the polycarbonate inner-layer varied between 1.27 mm (0.05”) and 0.508 mm (0.02”),
where tests 1, 2, 3, and 4 respectively employed polycarbonate thicknesses of 1.27 mm, 0.838 mm (0.033”), 0.508 mm, and 0.508 mm. In all cases, stress gauges were embedded between the two interfaces of the polycarbonate inner-layer and the bulk material. Not completely depicted in Figure 6.1, the aluminum back face of the sample was chosen to be 12.7 mm (0.5”) to ensure that no reflections from its free surface would reach the stress gauge locations during the measurement period of the experiments.

Within Figure 6.1, compressive waves and release waves are respectively drawn with solid and dotted characteristic lines. The radial release wave, indicative of the violation of one-dimensional strain, was constructed with use of equation (2.48), while utilizing the bulk wave velocity of aluminum, found in Table 4.1. The chosen impactor thickness can be noted to induce a loading of duration 2.9 µs. Figure 6.1 clearly demonstrates complications associated with radial release waves when employing samples with relatively small diameter to thickness ratios. Taking note of the respective arrival times of the C+ unloading and the radial release to the polycarbonate inner-layer, the chosen thickness of the impactor is shown to maximize the ring-up period achievable with 45 mm diameter samples.

Due to the varying thickness of the polycarbonate inner-layer utilized in the four tests, the stress reflections within the inner-layer are not depicted in Figure 6.1. Using the bulk wave speed of 2000 m/s for polycarbonate, the layer thickness employed in test 1 can be noted to result in compressive reflections in the inner-layer every 0.635 µs. Therefore, jumps in the stress observed at each interface will initially occur every 1.27µs. It should be noted that the shock velocity within the
polycarbonate layer will increase for each subsequent reflection as a result of the increasing pressure within, thus, the time lag between stress jumps will decrease with increasing state numbers. In order to address this condition, the first polycarbonate thickness was chosen to ensure that these stress jumps would be sufficiently long to accurately resolve the states from the stress records. For test 2, the polycarbonate layer thickness was chosen to produce compressive reflections every 0.42 µs, resulting in initial time lags of 0.84 µs between stress jumps at each interface. Likewise, the polycarbonate layer thickness chosen for tests 3 and 4 can be noted to produce compressive reflections every 0.25 µs, and therefore, lag times between stress jumps at each interface will initially occur every 0.51 µs.

![Time-distance diagram for the HRUM experiments utilizing 6061 aluminum as the bulk material and polycarbonate as the inner-layer.](image)

**Figure 6.1.** Time-distance diagram for the HRUM experiments utilizing 6061 aluminum as the bulk material and polycarbonate as the inner-layer.
6.1.2 The Hugoniot Parameters Utilized

Before we continue to the experiments and subsequent analysis, we must obtain the Hugoniot parameters of the employed materials. To do so, let us consider the published data for 6061 Aluminum and polycarbonate found in work performed at LANL [2]. In the case of aluminum, the relationship between shock velocity ($U_s$) and particle velocity ($u_p$) was shown to be well represented by the linear form of equation (2.16) written as:

$$U_s = 5350 + 1.34 u_p$$  \hspace{1cm} (6.1)

where the bulk wave speed ($c_b$) is 5350 m/s, the dimensionless fit parameter $s$ is 1.34, and the second order fit parameter ($Q$) is equal to zero. Utilizing equation (6.1) in equation (2.17), with an assumed density ($\rho_o$) of 2703 Kg/m$^3$, yields the Hugoniot of 6061 aluminum in stress-particle velocity space. The agreement of this Hugoniot equation to the published data for 6061 aluminum can be investigated by considering the stress-particle velocity plot found in Figure 6.2. Within this plot, the published data is represented by closed diamonds and the line represents the Hugoniot calculated with equation (2.17). The agreement between the published data and the calculated Hugoniot demonstrates that we have adequate fit parameters for aluminum to use in the subsequent analysis. Since aluminum is well defined by a linear fit between shock velocity and particle velocity, equations (2.36-2.39) can be utilized to determine the Hugoniot of the polycarbonate inner-layer from that of the aluminum bulk material.

Contrary to the published data on Aluminum, no fit equation was readily available to define the relationship between shock velocity and particle velocity for polycarbonate. The published data included all the necessary state parameters,
generated over a range of particle velocities between 0-5200 m/s. In order to obtain a useable fit equation, the published shock velocities were plotted as a function of the respective particle velocities. Curve fits of the data were forced through the assumed bulk velocity of 2000 m/s, which was determined in chapter 4. Within the range of particle velocities of 0-1000 m/s the relationship between shock velocity ($U_s$) and particle velocity ($u_p$) can be defined by a linear fit written in the form:

$$U_s = 2000 + 2.122u_p$$  \hspace{1cm} (6.2)

where the bulk wave speed ($c_b$) is 2000 m/s, the dimensionless fit parameter $s$ is 2.122, and the second order fit parameter ($Q$) is equal to zero. Extending the range of particle velocities to 4000 m/s requires a polynomial fit in the form:

$$U_s = 2000 + 2.1612u_p - 0.0002u_p^2$$  \hspace{1cm} (6.3)

where the bulk wave speed ($c_b$) is 2000 m/s, the dimensionless fit parameter $s$ is 2.1612, and the second order fit parameter ($Q$) is equal -0.0002 s/m. Additionally, extending the range of particle velocities to encompass the entire domain of the published data requires a third order polynomial fit. Considering that it is very unlikely that the current experiments will excite the higher domain, the third order fit is not necessary for the subsequent analysis. Utilizing either equations (6.2) or (6.3) in equation (2.17), with an assumed density ($\rho_o$) of 1193 Kg/m$^3$, yields the Hugoniot of polycarbonate in stress-particle velocity space. Let us investigate which shock velocity fit is needed for the subsequent analysis by considering the agreement of these Hugoniot equations to the published data for polycarbonate in the stress-particle velocity plot found in Figure 6.3. Within this plot, the published data are represented by closed diamonds. The dotted and solid lines respectively represent the Hugoniot
calculated with equation (2.17) using either the linear or polynomial fit between shock velocity and particle velocity. Clearly illustrated in this figure is the fact that the Hugoniot equation obtained using the linear fit significantly diverges from the published data when particle velocities exceed 1000 m/s. Conversely, the Hugoniot equation obtained utilizing the polynomial fit provides good agreement with the published data through the entire range of particle velocities shown. Thus, the polynomial fit found in equation (6.3) will be utilized in the analysis to define shock velocity as a function of particle velocity.

![Figure 6.2. The Hugoniot of 6061 aluminum in stress-particle velocity space.](image)

Since we are adopting a polynomial fit between the shock velocity and particle velocity for polycarbonate, equations (2.45-2.46) must be modified to conduct the
subsequent analysis into the Hugoniot of aluminum. In order to construct the 
Hugoniot of aluminum from the known Hugoniot of the polycarbonate inner-layer, 
equation (2.45) can be amended to include the polynomial fit in the form:

$$ \sigma_n - \sigma_{n-1} = (-1)^{n-1} \rho_{n-1} \left( c_b^{IL} + s^{IL} u_{\text{principal}, n} + Q^{IL} u_{\text{principal}, n} \right) \left( u_n - u_{n-1} \right) $$  \hspace{1cm} (6.4)

where the term $u_{\text{principal}, n}$ can be written as:

$$ u_{\text{principal}, n} = \sum_{i=1}^{n-1} \left( (-1)^{i-1} 2u_i \right) - (-1)^n u_n $$  \hspace{1cm} (6.5)

and the calculation of density in equation (2.46) can be modified to the form:

$$ \rho_n = \rho_{n-1} \left( c_b^{IL} + s^{IL} u_{\text{principal}, n} + Q^{IL} u_{\text{principal}, n} \right) \left( u_n - u_{n-1} \right) $$  \hspace{1cm} (6.6)

![Diagram](image_url)

**Figure 6.3.** The Hugoniot of polycarbonate in stress-particle velocity space.
6.1.3 Experimental Results

Histories of the stress evolution within the polycarbonate inner-layers for tests 1, 2, 3, and 4 can be respectively found in Figures 6.4, 6.5, 6.6, and 6.7. Parts (a) and (b) of each figure respectively contain the stress records from the front and back gauges. In part (c) of each figure there is a coupled stress record, where the records from the front and back gauges have been overlain to produce a single plot. The determined stress at states 1, 3, and 5, of the front stress record, are clearly labeled in part (a) of the figures. Likewise, the determined stress at states 2, 4, and 6, of the back stress record, are labeled in part (b) of the figures. In all four cases, the HEL of the aluminum is easily noted in the initial loading profile and is indicated in part (c) of the figures.

When considering Figures 6.4-6.7, it is clearly evident that the thickness of the polycarbonate inner-layer directly affects the time lag between the observed state jumps at each gauge. The stress records found in Figure 6.4, for the 1.27 mm polycarbonate inner-layer, provide relatively long time durations at each stress jump. Thus, determining the stress magnitude at each state for this test was relatively straightforward. In contrast, the stress records found in Figures 6.6-6.7, for the 0.508 mm polycarbonate inner-layer, possessed much shorter time durations at each stress jump, often resulting in no clear plateau. In these cases, the stress at each state was estimated by taking the midpoint between the partially sloping signals of each stress plateau. Also noteworthy when considering these stress profiles, is the fact that the final state jumps occur more rapidly than the initial ones. This is due to the increasing shock velocity within the polycarbonate inner-layer.
Figure 6.4. Stress records from the first test on aluminum/polycarbonate with an inner-layer thickness of 1.27 mm and an impact velocity of 320 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges.
Figure 6.5. Stress records from the second test on aluminum/polycarbonate with an inner-layer thickness of 0.838 mm and an impact velocity of 326 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges.
Figure 6.6. Stress records from the third test on aluminum/polycarbonate with an inner-layer thickness of 0.508 mm and an impact velocity of 325 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges.
Figure 6.7. Stress records from the fourth test on aluminum/polycarbonate with an inner-layer thickness of 0.508 mm and an impact velocity of 326 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges.
6.1.4 Generating the Hugoniot of the Polycarbonate Inner-Layer

From the experimental results, we now have the stress magnitude at states 1-6, for the stress-particle velocity plot found in Figure 2.4 (b). Equations (2.36-2.39) can now be utilized to solve for the particle velocities at these states utilizing the Hugoniot parameters of aluminum summarized in section 6.1.2. To conduct this analysis, a program was written in Matlab, which can be found in Appendix B.1. A summary of the results from this analysis can be found in Table 6.1. Within the table, the test number, inner-layer thickness measured in mm, impact velocity in m/s, state number, stress ($\sigma$) in GPa, and particle velocity ($u_p$) in m/s are presented. In order to construct the principal Hugoniot of the polycarbonate inner-layer from the calculated particle velocities at each state, equation (6.5) was utilized. These results are tabulated in the column marked principal $u_p$ in Table 6.1. Additionally, the shock velocities in the polycarbonate inner-layer at each state were calculated with use of the Rankine-Hugoniot conservation of momentum equation (2.14), where the principal particle velocities and density of polycarbonate were utilized.

In order to provide validation of the HRUM in the current application, Figure 6.8 contains a stress-particle velocity plot of the obtained results compared to published data. Within this figure, the published data for polycarbonate is depicted by closed diamonds. The Hugoniot states obtained from tests 1, 2, 3, and 4 are respectively depicted by open squares, triangles, circles, and diamonds. The solid line in Figure 6.8 depicts the Hugoniot curve for polycarbonate, constructed with use of equation (2.17) and the respective parameters outlined in section 6.1.2. The agreement between the experimental obtained Hugoniot states and the Hugoniot curve
of polycarbonate clearly demonstrates that the HRUM can be successfully employed to generate states along the Hugoniot of an inner-layer embedded within a well classified bulk material. It is noteworthy to point out that the error between the published data and the Hugoniot curve of polycarbonate is on the same order as and even exceeds that of the data obtained through the current method. The published data were obtained through carefully controlled single shock experiments, where as the data sets in the HRUM were obtained through re-shocking of the inner-layer. The agreement between the states obtained through traditional methods to those obtained utilizing the HRUM demonstrates that materials follow the same path (Hugoniot) through stress-particle velocity space in either cases of single shock or re-shock events.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Inner-layer Thickness (mm)</th>
<th>Impact Velocity (m/s)</th>
<th>State #</th>
<th>$\sigma$ (GPa)</th>
<th>$u_p$ (m/s)</th>
<th>Principal $u_p$ (m/s)</th>
<th>$U_S$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.27</td>
<td>320</td>
<td>1</td>
<td>0.78</td>
<td>266</td>
<td>266</td>
<td>2486</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.5</td>
<td>102</td>
<td>431</td>
<td>2955</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>1.9</td>
<td>192</td>
<td>521</td>
<td>3098</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>2.14</td>
<td>144</td>
<td>568</td>
<td>3197</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>2.26</td>
<td>168</td>
<td>592</td>
<td>3239</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>2.32</td>
<td>156</td>
<td>604</td>
<td>3258</td>
</tr>
<tr>
<td>2</td>
<td>0.838</td>
<td>326</td>
<td>1</td>
<td>0.78</td>
<td>272</td>
<td>272</td>
<td>2431</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.5</td>
<td>102</td>
<td>443</td>
<td>2875</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>1.95</td>
<td>194</td>
<td>535</td>
<td>3092</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>2.16</td>
<td>145</td>
<td>584</td>
<td>3138</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>2.35</td>
<td>168</td>
<td>607</td>
<td>3285</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>2.4</td>
<td>161</td>
<td>614</td>
<td>3316</td>
</tr>
<tr>
<td>3</td>
<td>0.508</td>
<td>325</td>
<td>1</td>
<td>0.8</td>
<td>270</td>
<td>270</td>
<td>2515</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.5</td>
<td>102</td>
<td>438</td>
<td>2906</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>1.95</td>
<td>193</td>
<td>530</td>
<td>3125</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>2.18</td>
<td>147</td>
<td>576</td>
<td>3211</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>2.3</td>
<td>170</td>
<td>600</td>
<td>3253</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>2.37</td>
<td>159</td>
<td>612</td>
<td>3290</td>
</tr>
<tr>
<td>4</td>
<td>0.508</td>
<td>326</td>
<td>1</td>
<td>0.85</td>
<td>268</td>
<td>268</td>
<td>2696</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.5</td>
<td>102</td>
<td>433</td>
<td>2938</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>1.9</td>
<td>198</td>
<td>529</td>
<td>3048</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>2.15</td>
<td>145</td>
<td>582</td>
<td>3135</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>2.35</td>
<td>168</td>
<td>606</td>
<td>3294</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>2.4</td>
<td>161</td>
<td>613</td>
<td>3325</td>
</tr>
</tbody>
</table>
Figure 6.8. Stress-particle velocity plot of polycarbonate’s Hugoniot comparing the shocked states obtained through use of the HRUM to published data.

In light of the good agreement with the obtained Hugoniot states to published data for polycarbonate, let us expand the method by considering how one can obtain a Hugoniot equation from the data. This can be accomplished through a number of ways. Without much effort, one could apply a curve fit to the data plotted in the stress-particle velocity plane. When trying to fit data in this manner, the curve fit equation should be at least a second order polynomial. This can easily be understood by considering equation (2.17), which defines the relationship between stress and particle velocity. In cases where the relationship between shock velocity and particle velocity is defined by a linear function, the second order polynomial fit would provide
an equation that is consistent with equation (2.17) when $Q = 0$. Depending on the material and range of the data, one may likely opt for a third order polynomial fit which would represent the case when $Q$ is not equal to zero. However appealing this fit method may seem, we must consider that the true usefulness of the HRUM will be achieved when it can be applied to unknown materials. In these applications, the appropriate order polynomial fit to apply to the data may not be readily apparent.

In light of this consideration, a better approach would be to try to define a relationship between shock velocity and particle velocity. Taking note of equation (2.16), a curve fit to shock velocity plotted as a function of particle velocity can be forced through an experimentally determined bulk velocity. Additionally, the linear relationship between shock velocity and particle velocity, commonly noted from many materials, would be more easily recognized in shock velocity-particle velocity space. Let us consider this approach by referring to Figure 6.9 in which shock velocities are plotted as a function of particle velocities generated from the four experiments. Parts (a), (b), (c), and (d) of Figure 6.9 correspond to the data obtained from tests 1, 2, 3, and 4, respectively. Within this figure the experimental data sets are depicted with open diamonds, and curve fits of the data are shown with the solid lines. The second order polynomial fit equations for each test are depicted in the lower right corner of each plot. It should be noted that in all cases, the curve fits were forced through the bulk wave speed for polycarbonate, determined in chapter 4. It is quite apparent than none of these fit equations directly match the Hugoniot of polycarbonate defined by equation (6.3). This is reasonable considering that the parameters for $Q$ and $s$ in equation (6.3) were generated with data covering a much larger range of particle
velocities. Thus, it is extremely important to point out that any curve fit of obtained data should be limited to the range covered by the experiment. In other words, one should never extend analysis to higher states of stress/particle velocity from a fit equation obtained from low stress states. This concept is easily observed by considering the variation between the linear \( U_s \) fit and polynomial \( U_s \) fit depicted in Figure 6.3.

![Figure 6.9](image)

**Figure 6.9.** Shock velocity plotted as a function of particle velocity; (a) Test 1; (b) Test 2; (c) Test 3; (d) Test 4.
Before we continue, let us investigate the accuracy of the obtained Hugoniot equations for polycarbonate shown in Figure 6.9. Typically experimental investigations will utilize a material’s Hugoniot to define the relationship between stress and particle velocity. We can therefore utilize the generated fit equations to define shock velocity in equation (2.17), while also employing the density for polycarbonate. Figure 6.10 demonstrates the deviation of the obtained polycarbonate Hugoniots from that published in literature. In this figure, the error in calculated stress is investigated as a function of particle velocity for tests 1, 2, 3, and 4. Also included is the error that would be achieved if the acoustic approximation with the bulk wave speed in equation (2.21) was used. Clearly illustrated in Figure 6.10 is the vast improvement a single HRUM experiment can yield over utilizing the acoustic approximation in the analysis of results. In all cases the error in calculated stress never exceeds 5% for the Hugoniots obtained utilizing the HRUM. The average errors for tests one, two, three, and four, are respectively 0.6%, 3.1%, 1%, and 1.8% over the range of particle velocities encompassed within the test. In contrast, the acoustic approximation accumulates an average error of 22.4% and significantly diverges from the Hugoniot of polycarbonate to errors of 38% for particle velocities of 600 m/s. Considering that the Hugoniots for polycarbonate were obtained from single HRUM experiments, it is likely that grouping multiple experiments into a data set will further reduce deviations from the real Hugoniot of the material. Regardless, even a single experiment will yield significant improvements in the analysis of results, where researchers would typically have chosen the acoustic approximation when the material under study does not have a well defined Hugoniot.
6.1.5 Generating the Hugoniot of the 6061 Aluminum Bulk Material

Let us now consider application of the HRUM to determine the Hugoniot of the bulk material from the known Hugoniot of the polycarbonate inner-layer. Returning to the experimental results, we have the stress magnitude at states 1-6 for the stress-particle velocity plot found in Figure 2.4 (b). Equations (6.4-6.6) can be utilized to solve for the particle velocities at these states utilizing the Hugoniot parameters of polycarbonate summarized in section 6.1.2. Similar to the previous case, this analysis was conducted utilizing a Matlab program, which can be found in
Appendix B.2.2. A summary of the results from this analysis can be found in Table 6.2. Within the table, the test number, inner-layer thickness measured in mm, impact velocity in m/s, state number, stress (σ) in GPa, and particle velocity (u_p) in m/s are outlined. For states 1, 3, and 5 the principal Hugoniot of the aluminum bulk material was determined from the calculated particle velocities at each state by subtracting these particle velocities from the impact velocity. States 2, 4, and 6 did not require this transformation due to the fact that they followed the principal Hugoniot of the aluminum bulk material, as noted in Figure 2.4 (b).

<table>
<thead>
<tr>
<th>Test #</th>
<th>Inner-layer Thickness (mm)</th>
<th>Impact Velocity (m/s)</th>
<th>State #</th>
<th>σ (GPa)</th>
<th>u_p (m/s)</th>
<th>Principal u_p (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.27</td>
<td>320</td>
<td>1</td>
<td>0.78</td>
<td>256</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.5</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>1.9</td>
<td>162</td>
<td>158</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>2.14</td>
<td>111</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>2.26</td>
<td>136</td>
<td>184</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>2.32</td>
<td>124</td>
<td>124</td>
</tr>
<tr>
<td>2</td>
<td>0.838</td>
<td>326</td>
<td>1</td>
<td>0.78</td>
<td>256</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.5</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>1.95</td>
<td>173</td>
<td>153</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>2.16</td>
<td>128</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>2.35</td>
<td>167</td>
<td>159</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>2.4</td>
<td>157</td>
<td>157</td>
</tr>
<tr>
<td>3</td>
<td>0.508</td>
<td>325</td>
<td>1</td>
<td>0.8</td>
<td>261</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.5</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>1.95</td>
<td>183</td>
<td>142</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>2.18</td>
<td>135</td>
<td>135</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>2.3</td>
<td>160</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>2.37</td>
<td>146</td>
<td>146</td>
</tr>
<tr>
<td>4</td>
<td>0.508</td>
<td>326</td>
<td>1</td>
<td>0.85</td>
<td>275</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.5</td>
<td>110</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>1.9</td>
<td>200</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>2.15</td>
<td>147</td>
<td>147</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>2.35</td>
<td>187</td>
<td>139</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>2.4</td>
<td>177</td>
<td>177</td>
</tr>
</tbody>
</table>

Figure 6.11 contains a stress-particle velocity plot of the obtained results compared to published data. Within this figure, the published data for 6061 aluminum
is depicted by closed diamonds. The Hugoniot states obtained from tests 1, 2, 3, and 4 are respectively depicted by open squares, triangles, circles, and diamonds. The solid line in Figure 6.11 depicts the Hugoniot curve for aluminum, constructed with use of equation (2.17), while utilizing the respective parameters outlined in section 6.1.2. The dashed line in Figure 6.11 depicts the acoustic approximation, constructed with the bulk wave speed in equation (2.21).

Figure 6.11. Stress-particle velocity plot of 6061 aluminum’s Hugoniot comparing the shocked states obtained through use of the HRUM to published data.

Taking note of Figure 6.11, it can be observed that there is a relatively poor agreement between the experimentally determined states and the published Hugoniot for aluminum. The initial states seem to follow the curve, however with increasing state numbers, the outputted data tends to fan out, where the material’s Hugoniot cuts
through the midline of the spread data sets. This increasing error linked to increasing state numbers is likely the result of the mathematical framework employed in the analysis. Each additional state jump is determined with the input of the calculated particle velocities from the previous states. It can therefore be noted that if an error presents itself, it will be magnified in each subsequent step.

Within the domain covered by these tests, Figure 6.11 demonstrates that the acoustic approximation would provide a better representation of the Hugoniot curve for aluminum. Contrary to the analysis of the inner-layer, a very small portion of the Hugoniot of aluminum could be investigated through this method. This observation is consistent with traditional methods of Hugoniot determination, where the impact velocity provides a limiting threshold on the achievable range of the experiment. Due to the lack of agreement of the obtained Hugoniot states to the real Hugoniot of aluminum, we will not try to output a Hugoniot curve from these experiments. However the case, if one was interested in obtaining the Hugoniot of aluminum, they could very easily employ it as an inner-layer embedded in a higher impedance material like steel or tungsten. This configuration would allow for a much larger domain of the curve to be investigated and would utilize the validated analysis from the preceding section.

6.2 HRUM WITH STEEL AND POLYCARBONATE

In contrast to aluminum, the wave speed within steel is relatively slow, thus allowing for a longer ring-up period within the inner-layer while minimizing concerns associated with the arrival of radial release waves. Additionally, the impedance of
steel is much higher than that of aluminum. This will facilitate the investigation into a larger range of the Hugoniot of the polycarbonate inner-layer, under the same impact velocities employed in the preceding configuration. These observations make steel a perfect candidate for further validation of the HRUM.

This section will begin by introducing the experimental design considering a time-distance diagram of the planned experiments. In order to establish the Hugoniot parameters for the specific steel employed, a small investigation demonstrating the conventional method of obtaining a material’s Hugoniot will be included. Following this, the experimental results will be presented. From these results, the HRUM will be employed to first determine the Hugoniot of the polycarbonate inner-layer with use of the known Hugoniot of the steel bulk material. Conversely, the method will next be utilized to determine the Hugoniot of the steel bulk material from the known Hugoniot of the polycarbonate inner-layer. In both cases, comparison of the resulting Hugoniot states obtained from the HRUM to published data found in literature will be conducted in order to provide validating evidence supporting the accuracy of the proposed method. In cases where there is good agreement with the published data, it will be shown how one can obtain an equation to define the material’s Hugoniot within the domain encompassed by the specific test. Additionally, the accuracy of these experimentally obtained Hugoniot equations will be investigated through comparison with published Hugoniots of the respective materials.

6.2.1 Experimental Design

In order to begin, let us consider a time-distance plot of the planned experiments depicted in Figure 6.12. Similar to the preceding section, this time-
distance diagram was constructed under the assumption of a plastic response and therefore all characteristics were generated with the respective bulk wave speeds of the materials. The impactor and test sample are both 45 mm in diameter. In order to prevent any spall fracture conditions in the steel front face, its thickness was chosen so that it matched that of the impactor. The thickness of the front face and impactor are 5.08 mm (0.2”). The thickness of the polycarbonate inner-layer is 0.508 mm. Stress gauges are embedded between the two interfaces of the polycarbonate inner-layer and the bulk material. Not completely depicted in Figure 6.12, the steel back face of the sample was chosen to be 7.62 mm (0.3”) to ensure that no reflections from its free surface would reach the stress gauge locations during the measurement period of the experiments.

Within Figure 6.12, compressive waves and release waves are respectively drawn with solid and dotted characteristic lines. The radial release wave, indicative of the violation of one-dimensional strain, was constructed with use of equation (2.48) while utilizing the bulk wave velocity of steel, found in Table 4.1. The chosen impactor thickness can be noted to induce a loading duration of 2.2 µs. Contrary to the preceding configuration, the arrival of the radial release to the polycarbonate inner-layer occurs much later in time than the arrival of the C+ unloading wave. In the current configuration, the impactor thickness could be increased substantially to maximize the ring-up period within the polycarbonate inner-layer. However the case, the goal of this study was to maximize the achievable impact velocity, thus allowing for a sufficiently large range of polycarbonate’s Hugoniot to be investigated. Taking note of the high density of steel, a large impactor would significantly increase the
mass of the projectile, therefore decreasing the achievable impact velocity of the experiments.

Since the experiments conducted on this configuration all utilized the same polycarbonate inner-layer thickness, the wave reflections within have been included in Figure 6.12. These reflections were all generated with the use of the bulk wave speed of 2000 m/s, for polycarbonate. As previously stated, the wave speed will increase with increasing state numbers, thus it should be noted that Figure 6.12 provides a rough estimate of the event. From this time-distance diagram, it can be noted that five states can be captured at each interface. Using the bulk wave speed of 2000 m/s for polycarbonate, the layer thickness employed will result in compressive reflections in the inner-layer every 0.25 µs. Therefore, jumps in the stress observed at each interface will initially occur every 0.51µs.

![Figure 6.12. Time-distance diagram for the HRUM experiments utilizing steel as the bulk material and polycarbonate as the inner-layer.](image-url)
6.2.2 Obtaining The Hugoniot Parameters For Steel

In order to continue, we must acquire the Hugoniot parameters for the steel employed in this configuration. Although there are numerous Hugoniots for steel published in literature, there is a lack of data specific to A572 Grade 50 Structural Steel. However, we had previously tested this specific steel over a wide range of impact velocities. Some of the resulting data were published by Visser, et al. [3]. Within this investigation, five symmetrical plate impacts were performed on the steel. Stress histories were captured with the use of manganin gauges embedded within layers of the steel.

Although the goal of that research was not intended to generate the Hugoniot of the material, these experiments possessed an excellent configuration to attain Hugoniot states through traditional methods. Since the impact conditions utilized the same material impactor and test sample, symmetry of the Hugoniots would result in particle velocities that are exactly half of the impact velocities. Additionally, due to the fact that the stress gauges were embedded within the steel targets, the experimental stress records would capture an adequate representation of the pressure associated with these particle velocities. In order to continue, let us consider the recorded stress profiles (a) and associated stress-particle velocity plot (b) of the experiments, found in Figure 6.13. For ease of grayscale viewing, it can be noted that the legend in Figure 6.13 is arranged in the same order as the profiles, where Ui refers to the impact velocity. A summary of the five tests can be found in Table 6.3. Within this table, the impact velocity in m/s, stress in GPa, and particle velocity in m/s are outlined for the
five experiments. Additionally, the shock velocities were calculated with use of equation (2.14), assuming an initial density of 7814 Kg/m$^3$.

\[
\sigma = \frac{U_p}{U_i} \times 2.33 \times 10^{-2}
\]

**Figure 6.13.** Five experiments conducted on A572 grade 50 structural steel demonstrating traditional methods of Hugoniot determination; (a) Stress histories of the experiments; (b) Stress-particle velocity plot of the experiments.

**Table 6.3.** Hugoniot states for A572 grade 50 structural steel.

<table>
<thead>
<tr>
<th>Impact Velocity (m/s)</th>
<th>$\sigma$ (Gpa)</th>
<th>$u_p$ (m/s)</th>
<th>$U_s$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>241</td>
<td>4.2</td>
<td>120.5</td>
<td>4461</td>
</tr>
<tr>
<td>382</td>
<td>7</td>
<td>191</td>
<td>4690</td>
</tr>
<tr>
<td>405</td>
<td>7.33</td>
<td>202.5</td>
<td>4632</td>
</tr>
<tr>
<td>418</td>
<td>7.52</td>
<td>209</td>
<td>4605</td>
</tr>
<tr>
<td>477</td>
<td>8.82</td>
<td>238.5</td>
<td>4733</td>
</tr>
</tbody>
</table>

Let us now compare these experimentally determined Hugoniot states with a published equation for steel found in literature. In work performed at Lawrence Livermore National Laboratory, it has been shown that the relationship between shock velocity and particle velocity for 4340 Steel can be defined as:

\[
U_s = 4578 + 1.33u_p
\]
where the bulk wave speed \((c_b)\) is 4578 m/s, the dimensionless fit parameter \(s\) is 1.33 and the second order fit parameter \((Q)\) is equal to zero [4]. Utilizing equation (6.7) in equation (2.17), with an assumed density \((\rho_o)\) of 7810 Kg/m\(^3\), yields the Hugoniot of 4340 steel in stress-particle velocity space. Let us compare this Hugoniot equation with the experimentally determined data found in Table 6.3 by means of the stress-particle velocity plot shown in Figure 6.14. Within this figure, the experimental data for A572 grade 50 structural steel is displayed as open squares, and the solid line represents the Hugoniot of 4340 Steel.

Taking note of Figure 6.14, there is a good agreement between the Hugoniot of 4340 steel and the experimentally determined Hugoniot states for A572 grade 50 structural steel. As previously stated, the states of stress and particle velocity are limited to the range of the experimental data obtained. It is very likely that the Hugoniot of 4340 steel will deviate from that of our steel when the stress and particle velocities increase. Considering that the structural steel studied has a low carbon content, it is possible that a better representation could be achieved by utilizing the Hugoniot of pure iron. Iron’s Hugoniot can be noted to match steel’s Hugoniot until about 10 GPa. After this point, the curve of pure iron exhibits lower impedance than that of steel. However the case, the experiments conducted on the structural steel represent some of the highest impact velocities achieved with the current test apparatus. Thus, any tests within the current study will likely not exceed the range of agreement between the Hugoniot of 4340 steel and the Hugoniot states for the structural steel. We will therefore utilize the fit parameters for 4340 steel in equations...
(2.36-2.39), when trying to determine the Hugoniot of the polycarbonate inner-layer from the experiments.

![Graph showing stress-particle velocity relationship for steel Hugoniot and A572 Grade 50 Structural Steel.](image)

**Figure 6.14.** The Hugoniot of steel in stress-particle velocity space.

### 6.2.3 Experimental Results

Two experiments were conducted on the current configuration targeting impact velocities of 300 m/s. Histories of the stress evolution within the polycarbonate inner-layers for two tests can be found in Figures 6.15 and 6.16. Parts (a) and (b) of each figure, respectively contain the stress records from the front and back gauges. In part (c) of each figure there is a coupled stress record, where the records from the front and back gauges have been overlain to produce a single plot. The determined stress at
states 1, 3, 5, 7, and 9, of the front stress record, are clearly labeled in part (a) of the figures. Likewise, the determined stress at states 2, 4, 6, 8, and 10, of the back stress record, are labeled in part (b) of the figures. In both cases, the HEL of the steel is easily noted in the initial loading profile, and is indicated in part (c) of the figures.

In contrast to the experiments on aluminum, the experimental stress records for the steel allowed for the capture of four additional states. This is likely due to the reduced concern associated with the arrival of the radial release to the measurement location. The arrival time of the radial release for the aluminum experiments, depicted in Figure 6.1, was determined under the consideration that the stress gauges were perfectly centered within the test samples. In reality, it is very likely that the location of these gauges may have varied a few millimeters from center. In which case, the arrival of the radial release to the gauge location may be slightly sooner than depicted in Figure 6.12. Thus, any additional states were likely lost to the violation of one-dimensional strain within the stress gauge, associated with the arrival of the radial release.

Additionally, the relatively high impedance of steel allowed for peak pressures of about 5.5 GPa to be achieved. In contrast, the tests on the aluminum configuration only achieved peak pressures of about 2.5 GPa. Thus, experiments on the current configuration excited a much larger range of the Hugoniot of polycarbonate than the preceding configuration.
Figure 6.15. Stress records from the first test on steel/polycarbonate with an inner-layer thickness of 0.508 mm and an impact velocity of 300 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges.
Figure 6.16. Stress records from the second test on steel/polycarbonate with an inner-layer thickness of 0.508 mm and an impact velocity of 304 m/s; (a) Front gauge; (b) Back gauge; (c) Coupled plot of both gauges.
6.2.4 Generating the Hugoniot of the Polycarbonate Inner-Layer

From the experimental results, we now have the stress magnitude at states 1-10, of the ring-up of the polycarbonate inner-layer. Equations (2.36-2.39) can now be utilized to solve for the particle velocities at these states utilizing the Hugoniot parameters for steel summarized in section 6.2.2. This analysis was conducted utilizing a similar Matlab code to the one found in Appendix B.1. A summary of the results from this analysis can be found in Table 6.4. All parameters found in this table were determined as previously noted in section 6.1.4.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Inner-layer Thickness (mm)</th>
<th>Impact Velocity (m/s)</th>
<th>State #</th>
<th>σ (GPa)</th>
<th>u_p (m/s)</th>
<th>Principal u_p (m/s)</th>
<th>U_S (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.508</td>
<td>300</td>
<td>1</td>
<td>0.8</td>
<td>278</td>
<td>278</td>
<td>2445</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.75</td>
<td>48</td>
<td>507</td>
<td>2929</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>2.7</td>
<td>226</td>
<td>685</td>
<td>3346</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>3.5</td>
<td>95</td>
<td>816</td>
<td>3643</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>4.1</td>
<td>189</td>
<td>909</td>
<td>3829</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>4.6</td>
<td>124</td>
<td>973</td>
<td>4013</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>4.95</td>
<td>166</td>
<td>1015</td>
<td>4140</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>5.15</td>
<td>139</td>
<td>1042</td>
<td>4194</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>5.3</td>
<td>157</td>
<td>1061</td>
<td>4242</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>5.4</td>
<td>145</td>
<td>1072</td>
<td>4274</td>
</tr>
</tbody>
</table>

| 2      | 0.508                     | 304                   | 1       | 0.8     | 282        | 282                 | 2410     |
|        |                           |                       | 2       | 1.75    | 48         | 515                 | 2883     |
|        |                           |                       | 3       | 2.75    | 229        | 696                 | 3356     |
|        |                           |                       | 4       | 3.6     | 98         | 826                 | 3699     |
|        |                           |                       | 5       | 4.1     | 193        | 921                 | 3780     |
|        |                           |                       | 6       | 4.7     | 127        | 986                 | 4045     |
|        |                           |                       | 7       | 5       | 169        | 1028                | 4127     |
|        |                           |                       | 8       | 5.25    | 141        | 1056                | 4221     |
|        |                           |                       | 9       | 5.35    | 160        | 1074                | 4228     |
|        |                           |                       | 10      | 5.5     | 148        | 1086                | 4298     |

Figure 6.17 contains a stress-particle velocity plot of the obtained results compared to published data for polycarbonate. Within this figure, the published data is depicted by closed diamonds. The Hugoniot states obtained from tests 1 and 2 are respectively depicted by open squares and triangles. The solid line in Figure 6.17
depicts the Hugoniot curve for polycarbonate, constructed by utilizing the respective parameters outlined in section 6.1.2 in equation (2.17).

![Stress-particle velocity plot](image.png)

**Figure 6.17.** Stress-particle velocity plot of polycarbonate’s Hugoniot comparing the shocked states obtained through use of the HRUM to published data.

Taking note of Figure 6.17, there is a good agreement between the experimentally determined Hugoniot states and the published Hugoniot for polycarbonate. We will therefore continue the analysis to determine an appropriate Hugoniot equation that can be applied to the regime covered by these experiments. Similar to the preceding analysis, we will attempt to establish a relationship between shock velocity and particle velocity. Figure 6.18 contains plots of the shock velocity versus particle velocity for the two tests conducted on the steel/polycarbonate
configuration. Tests 1 and 2 are respectively found in parts (a) and (b) of Figure 6.18. The experimental data sets are depicted with open diamonds, and the applied curve fits are shown by solid lines. The equations for the second order polynomial fits applied to the data are found within each plot. It should be noted that these fits were forced through the bulk velocity of 2000 m/s, for polycarbonate.

\[ y = 0.0005x^2 + 1.5742x + 2000 \]

\[ y = 0.0006x^2 + 1.4714x + 2000 \]

![Figure 6.18. Shock velocity plotted as a function of particle velocity; (a) Test 1; (b) Test 2.](image)

Let us now investigate the accuracy of the obtained Hugoniot equations for polycarbonate, found in Figure 6.18. We will utilize the relationships between shock velocity and particle velocity in equation (2.17) while additionally using the density for polycarbonate. Figure 6.19 demonstrates the deviation of the obtained Hugoniot from the published Hugoniot for polycarbonate. In this figure, the error in calculated stress is investigated as a function of particle velocity for the two tests. Also included is the error that that would result from the use of the acoustic approximation with the bulk wave velocity in equation (2.21). Similar to the preceding discussion, Figure 6.19 clearly illustrates the vast improvement that a single HRUM experiment can yield.
over utilizing the acoustic approximation in the analysis of results. The average errors for tests one and two are respectively 2.8% and 3.3%. In contrast, the acoustic approximation results in an average error of 33.7%.

**Figure 6.19.** Deviation of the calculated stresses utilizing the Hugoniot equations obtained from the HRUM from the real Hugoniot of polycarbonate, plotted as a function of particle velocity.

Before we continue let us revisit the shock velocity-particle velocity plots found in Figure 6.18. The data in these figures could be represented by a linear curve fit. This is also the case for the data contained in Figure 6.9, for the aluminum/polycarbonate configuration. It should be reiterated that a linear fit was used to describe polycarbonate’s relationship between shock velocity and particle
velocity for particle velocities up to 1000 m/s. Let us consider the average error obtained from applying a linear fit to the data from test 1. In this case, the linear fit equation between shock velocity and particle velocity can be written as:

\[ U_s = 2000 + 2.064u_p \]  

(6.8)

where the bulk wave speed \((c_b)\) is 2000 m/s, the dimensionless fit parameter \(s\) is 2.064, and the second order fit parameter \((Q)\) is equal to zero. This equation closely resembles the linear fit equation (6.2), previously presented for polycarbonate. Utilizing this fit equation yields a reduction in the error to an average value of 1.1% for calculated stresses over the range of particle velocities up to 1100 m/s. Figure 6.20 shows the deviation of the newly obtained Hugoniot equation from the published Hugoniot for polycarbonate. In this figure, the error in calculated stress is investigated as a function of particle velocity. For comparative purposes, the Hugoniot equations determined from the second order fit and the linear fit of the data from test 1 are both included in this figure.

It should be noted that in the current investigation, there was prior knowledge of the Hugoniot of polycarbonate. Due to this knowledge, a second order curve fit was used when trying to establish a relationship between shock velocity and particle velocity. If this was not the case, the linear trend of the data found in Figures 6.9 and 6.18 would likely have led to the use of linear fits. Thus, in the absence of knowledge of a material’s Hugoniot, it is very likely that a researcher will achieve a better fit to the true Hugoniot of a material within the specific range tested. Regardless, even with less accurate fit equations, the HRUM still provides a vast improvement over utilization of the acoustic approximation in the analysis of experimental results.
Figure 6.20. Deviation of the calculated stresses utilizing either a linear fit or a polynomial fit to the shock velocity-particle velocity data obtained from test 1.

6.2.5 Generating the Hugoniot of the Steel Bulk Material

Let us now consider application of the HRUM to determine the Hugoniot of the bulk material from the known Hugoniot of the polycarbonate inner-layer. Returning to the experimental results, we have the stress magnitude at states 1-10. Equations (6.4-6.6) can be utilized to solve for the particle velocities at these states utilizing the Hugoniot parameters of polycarbonate summarized in section 6.1.2. Similar to the previous case, this analysis was conducted utilizing a Matlab program, which can be found in Appendix B.2.2. A summary of the results from this analysis can be found in Table 6.5. All parameters found in this table were determined as previously noted in section 6.1.5.
Table 6.5. Experimentally determined Hugoniot states of steel.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Inner-layer Thickness (mm)</th>
<th>Impact Velocity (m/s)</th>
<th>State #</th>
<th>$\sigma$ (GPa)</th>
<th>$u_p$ (m/s)</th>
<th>Principal $u_p$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.508</td>
<td>300</td>
<td>1</td>
<td>0.8</td>
<td>261</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.75</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>2.7</td>
<td>218</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>3.5</td>
<td>79</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>4.1</td>
<td>174</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>4.6</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>4.95</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>5.15</td>
<td>122</td>
<td>122</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>5.3</td>
<td>143</td>
<td>157</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>5.4</td>
<td>129</td>
<td>129</td>
</tr>
<tr>
<td>2</td>
<td>0.508</td>
<td>304</td>
<td>1</td>
<td>0.8</td>
<td>261</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.75</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>2.75</td>
<td>227</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>3.6</td>
<td>81</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>4.1</td>
<td>160</td>
<td>144</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>4.7</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>5</td>
<td>114</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>5.25</td>
<td>79</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>5.35</td>
<td>93</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>5.5</td>
<td>73</td>
<td>73</td>
</tr>
</tbody>
</table>

Figure 6.21 contains a stress-particle velocity plot of the obtained results compared to published data. Within this figure, the data obtained from traditional methods is depicted by closed diamonds. The Hugoniot states obtained from tests 1 and 2 are respectively depicted by open squares and triangles. The solid line in Figure 6.8 depicts the Hugoniot curve for 4340 Steel, constructed with use of equation (2.17), while utilizing the respective parameters outlined in section 6.2.2. The dashed line in Figure 6.11 depicts the acoustic approximation, constructed with the bulk wave speed and density of 4340 steel, in equation (2.21).

Similar to the case of aluminum, the HRUM fails to provide an accurate representation of the Hugoniot states of the bulk material. Within the domain covered by these tests, Figure 6.21 demonstrates that the acoustic approximation would provide a better representation of the Hugoniot curve for steel. Thus, a researcher...
should likely opt to utilize the acoustic approximation, rather than going through the effort to try to utilize the HRUM in this manner.

Figure 6.21. Stress-particle velocity plot of steel’s Hugoniot comparing the shocked states obtained through use of the HRUM to published data.

6.3 CONCLUDING REMARKS

An experimental approach was developed to induce shock reflections in a low impedance inner-layer embedded within a high impedance bulk structure. By capturing temporal records of the stress evolution at each side of the inner-layer, step-like loading profiles were obtained that allowed for the capture of multiple Hugoniot states. The mathematical framework employed in this technique utilized the classical Rankine-Hugoniot equations in the method of impedance matching, where either the
bulk material (case 1) or inner-layer (case 2) was required to have a known Hugoniot. Validation of the new technique was performed by testing well classified materials in order to facilitate comparison of the Hugoniots generated from the method with published data found in literature. For the first case, when the Hugoniot of the bulk material is known, the HRUM was shown to accurately generate states along the Hugoniot of the inner-layer, where the number of states acquired is directly linked to the experimental design. Factors including the wave velocities in the materials, input pulse duration (controlled by the thickness and wave velocity of the impactor), thickness of the inner-layer, and diameter of the test samples (arrival of the radial release) affect the number of states that can be generated from a single experiment. Experiments employing 6061 aluminum and polycarbonate, respectively, as the bulk material and inner-layer, accurately generated six Hugoniot states for polycarbonate. Additionally, experiments employing A572 grade 50 structural steel as the bulk material were able to accurately generate ten Hugoniot states of the polycarbonate inner-layer. In these experiments, the method was extended to generate a Hugoniot equation defining the material response of the inner-layer within the domain encompassed by the specific test. Through comparison of these experimentally determined equations to the real Hugoniot of polycarbonate, it has been shown that a single HRUM experiment can yield an accurate Hugoniot for the inner-layer. For the second case, when the Hugoniot of the inner-layer is known, the HRUM failed to accurately generate states along the Hugoniot of the bulk material. Thus, the HRUM requires significant improvements before it can be used in this application. In light of these shortcomings, a procedure utilizing over-deterministic methodology will be
proposed in the next chapter. This procedure may allow future researchers to extend application of the HRUM to the case of determining the Hugoniot of the bulk material.

6.4 REFERENCES


CHAPTER 7

CONCLUDING REMARKS

The response of materials to shock loading has been investigated through use of a plate impact experimental technique. A single stage gas gun was utilized to drive projectiles to velocities between 50-500 m/s, facilitating investigations into low to moderate shock loading conditions. Within this thesis, there was a bimodal theme. The first portion of this thesis investigated the spall fracture of cast irons with varying microstructure. Although the study of the spall fracture of materials is a common theme in literature, there exists a noteworthy lack of research specific to cast iron. Given that cast iron is one of the most widely utilized materials for engineering structures, this research was pursued in an effort to characterize its strength and identify the fracture mechanisms and kinetics associated with its failure process. The second portion of this thesis involved the development of a new technique that could be utilized to generate multiple Hugoniot states in a single experiment. Generation of a material’s Hugoniot is a fundamental theme in shock wave studies because it allows researchers to determine all mechanical and thermodynamic properties associated with dynamic loading conditions. Traditionally, the locus of points necessary to construct a material’s Hugoniot is obtained through a rigorous series of experiments, where each test produces a single data set. By considering the shock wave processes associated
with layered plates, a new method was developed that will significantly reduce the process of obtaining material Hugoniots.

**7.1 SPALL FRACTURE OF CAST IRON**

Within this thesis it has been shown that cast iron is a highly heterogeneous material, where the mixed microstructure results in competing mechanisms associated with mechanical properties. In addition, the microstructure within any specific casting can exhibit large variations from mold wall to core. Unlike the case of homogenous materials such as aluminum, this inherent shortcoming yields complications when trying to discern data trends from small batches of experiments. However the case, by studying five different cast irons, some key conclusions can be drawn from the investigation into the spall strength of the material. The importance of metallurgical investigations in the study of spall fracture of materials is exemplified by this investigation, where conclusions could not have been made purely from the experimental results.

The first conclusion that can be immediately drawn from the spall fracture study is that experimentally determined spall strengths are directly linked to the damage level at the spall plane, where it has been shown that spall strength is proportional to damage level. We can thus consider the lower bound spall strengths to represent the minimum stresses required to nucleate cracks within a material. Conversely, the variation between the lower and upper bound spall strengths can be noted to represent the increased amount of energy required to coalesce initiated cracks.
into a complete fracture plane. Additionally, it has been shown that these upper bound spall strengths are excited by increased unloading rates within the material.

Through metallurgical investigations, this study has identified the graphite phase as the primary factor governing the spall fracture of cast irons, where crack nucleation is directly correlated to the debonding of graphite from the metal matrix. It has been noted that the average length of graphite found within a casting is linked to the material’s strength, where strength has been shown to be inversely proportional to graphite length. The morphology, and mean free path of graphite precipitates further govern the subsequent coalescence of initiated cracks to form a complete fracture plane. In cases where graphite spacing is large, an increased amount of energy is required to complete the fracture process. A secondary factor governing the spall fracture of cast irons has been linked to the microstructure of the metal matrix. It has been noted that pearlite will yield higher mechanical strengths in cast irons than free ferrite.

7.2 HRUM EXPERIMENTS

Within the second portion of this thesis, an experimental approach was developed to induce shock reflections in a low impedance inner-layer embedded within a high impedance bulk structure. By capturing temporal records of the stress evolution at each side of the inner-layer, step-like loading profiles were obtained, which allowed for the capture of multiple Hugoniot states from a single experiment. The mathematical framework employed in this technique utilized the classical Rankine-Hugoniot equations in the method of impedance matching, where either the
inner-layer or bulk material was required to have a known Hugoniot. Validation of the new technique was performed by testing well classified materials in order to facilitate comparison of the Hugoniots generated from the method with published data found in literature.

The HRUM has been utilized in two configurations including the use of either aluminum or steel as the bulk material and polycarbonate as the inner layer. In the case of the aluminum/polycarbonate configuration, it has been shown that the HRUM can accurately generate at least six Hugoniot states for the polycarbonate inner-layer from a single experiment. Conversely, it has been shown that the HRUM can accurately generate at least ten Hugoniot states for the polycarbonate inner-layer while employing the steel/polycarbonate configuration. In these cases, the method was extended to demonstrate how a researcher can generate a Hugoniot equation for the inner-layer from any single HRUM experiment. Through comparison of these experimentally determined equations to the real Hugoniot of polycarbonate, it has been shown that the HRUM provides a significant improvement over utilization of an acoustic approximation, and thus accurately defines a material’s Hugoniot within the regime covered by the experiment.

Nevertheless, the HRUM requires some improvements before it can be utilized to solve for the Hugoniot of the bulk material from the Hugoniot of the inner-layer. The major shortcoming of this application is likely due to the error associated with determining the stress magnitude at each state. It is interesting that the first method of determining the Hugoniot of the inner-layer is less influenced by these errors in stress. In order to understand the reason for this observation, let us briefly consider the
variation in the mathematical framework employed in the two methods. When determining the Hugoniot of the inner-layer, the experimentally determined stress magnitudes are placed on the Hugoniot of the bulk material to determine the particle velocities. By doing so, the method is less influenced by a singular error source. For example, if there was an error in the measured stress at state 1, the analysis will likely only produce an error in the calculated particle velocity respective of the inner-layer at state 1. Continuing the analysis, state 3 can still output an accurate particle velocity respective of the inner-layer, if the measured stress is free from error. In contrast, when utilizing the HRUM to construct the Hugoniot of the bulk material, the Hugoniot of the inner-layer is utilized to determine the respective particle velocities by zigzagging through stress-particle velocity space. Within the mathematical framework of this method, any error in a measured stress will influence all subsequent solutions. For example let us consider that there is an error in the measured stress at state 1. This will result in an error in the determined particle velocity at state 1. Since all additional states utilize this particle velocity, the error will magnify as the analysis marches through the stress states, resulting in a divergence from the real Hugoniot of the bulk material.

Figure 7.1 contains two outputted stress-particle velocity diagrams from the analysis of the results generated from the first test on the steel/polycarbonate configuration. Part (a) of the figure demonstrates the Matlab output when utilizing the HRUM to solve for the Hugoniot of the inner-layer. Part (b) of the figure demonstrates the Matlab output when utilizing the HRUM to solve for the Hugoniot of the bulk material. Within both figures, the same stresses summarized in Table 6.4
were utilized. Within the figure, the data sets marked (*) represent the outputted principal Hugoniot states determined from the respective methods.

Referring to Figure 7.1 (a), the relative simplicity of the analysis respective to constructing the Hugoniot of an inner-layer is exemplified. In this case, knowledge of the Hugoniot of the bulk material allows us to specify a domain within which the solution must fall. For the even numbered states, we are able to utilize the bulk material’s principal Hugoniot. For the odd numbered states, we are able to utilize the negatively sloped Hugoniot of the bulk material, drawn from the measured impact velocity of the test. This method allows us to follow two paths through stress-particle velocity space, thus reducing the compounding of errors as the numbers associated to the sequential states increase. In contrast, constructing the Hugoniot of the bulk material becomes much more complicated. In this case, with the exception of state 1, we are not able to specify a domain within which the solution must fall. While marching through states, each subsequent solution relies on the solutions from the preceding states. Thus, the governing path created by the Hugoniot of the inner-layer becomes unrealistic as the error compounds within the analysis. To exemplify this claim, let us consider two lines (not shown in the figure) that would respectively connect the even numbered states of the back stress record and odd numbered states of the front stress record. These lines would respectively represent the Bulk material’s principal Hugoniot and negative Hugoniot drawn from the impact velocity, as shown in part (a). It can be noted that the line corresponding to the principal Hugoniot would follow a path of higher impedance than that of the negatively sloped line. In reality,
these Hugoniot lines should be symmetrical about the vertical plane passing through their intersection point.

Figure 7.1. Stress-particle velocity plots generated from the analysis of the data from test 1 on the steel/polycarbonate configuration; (a) Determining the Hugoniot of the inner-layer; (b) Determining the Hugoniot of the bulk material.
7.3 FUTURE WORK

There are a few improvements that can be made to the HRUM that will likely extend the application of the technique. Although the HRUM has been shown to accurately output Hugoniot states for an inner-layer, within the work found in this thesis, some additional considerations are necessary to apply the method to extreme shocked states. Thus, we will briefly consider a modification of the technique that will allow for application to these high pressure regimes. In regards to generating the Hugoniot of the bulk material, it has been shown that the HRUM needs some significant improvements. In light of this, some recommendations will be provided, that can be used to improve the accuracy of the determined Hugoniot states respective of the bulk material from HRUM experiments.

7.3.1 Extending The HRUM To Extreme Pressures

One minor shortcoming of the HRUM involves the first analysis, where the Hugoniot of the inner-layer is determined from the known Hugoniot of the bulk material. It should be reiterated that the determination of the particle velocities from the stress states of the front gauge record are made under the assumption that the Hugoniot and isentrope coincide. This was a valid assumption for the domain of pressures excited by the investigations found within this thesis. However the case, application of the method to extreme pressures would require an additional solution of the Mie-Grüneisen P-V-E equation of state (2.19). This solution would accurately determine the particle velocity at state 1 by following the bulk material’s isentrope from the impact state to state 1, respective of the initial unloading of the front face of the bulk material. Once this particle velocity is determined, the material’s negatively
sloped Hugoniot can be drawn through this respective data point. The particle velocity found when this Hugoniot crosses zero stress would become the new input parameter for $u_i$ in equations (2.38) and (2.39). For visualization of this process refer to the stress-particle velocity plot found in Figure 7.2. In this figure, it is assumed that the impact state is at such a magnitude that the Hugoniot and isentropic unloading no longer coincide. Thus, the initial unloading of the front face of the bulk material from the impact state to state 1 must follow the isentrope, as depicted by the dotted line. The subsequent reloading of the front face to states 3, 5, 6, etc. would in turn follow the negatively sloped Hugoniot drawn through state 1. The new $u_i$ that should be used in equations (2.38-2.39) is indicated in the figure.

![Figure 7.2. Stress-particle velocity plot demonstrating the material response associated with extreme shocked states.](image-url)
The preceding discussion only need be applied to analysis of stresses associated with the front face of the inner-layer. Stress records captured by the back gauge will follow the principal Hugoniot of the bulk material. Thus, if a researcher does not wish to apply the preceding analysis, high shocked states can still be investigated by employing a single gauge at the back face of the inner-layer. In which case, equations (2.36-2.37) can be utilized in the analysis.

7.3.2 Determining The Hugoniot Of The Bulk Material

There are major shortcomings associated with utilizing the HRUM to construct the Hugoniot of the bulk material. Future work involving over-deterministic methodology such as the minimization process associated with the least squares method may improve the accuracy of the HRUM. Let us briefly consider how over-deterministic methodology could be applied to facilitate application of the HRUM in the construction of Hugoniot states of the bulk material. Typically an experimentalist conducting research in the field of shock physics will begin an investigation into a material by initially determining the longitudinal, shear, and bulk wave speeds, as described in chapter 4. The bulk velocity, coupled with the initial density, allows a researcher to determine the initial impedance of a material. In the stress-particle velocity space, it can be noted that a material’s Hugoniot will deviate from the initially calculated impedances, where at increased particle velocities and pressures, the Hugoniot will follow a path of higher impedance. Keeping this in mind, one could begin the analysis by specifying an area of stresses and particle velocities that cannot be outputted by the HRUM analysis. For visualization of this concept, refer to the stress particle velocity plot found in Figure 7.3. Within this figure, the acoustic
approximation can be generated utilizing equation (2.21), while employing the bulk velocity as $c$. We will assume an impact velocity of $u_i$, as indicated within the figure. The shaded area within Figure 7.3 represents the region that will initially be excluded from the analysis, constructed by connecting the C+ and C- impedances determined from the acoustic approximation. For visualization purposes, the material’s Hugoniot has been drawn into the figure. However, when conducting the analysis we must remember that this Hugoniot will be unknown.

Figure 7.3. Stress-particle velocity plot depicting the first phase of the suggested improved analysis.

The second phase of this over-deterministic methodology will involve the determination of the stress at each state. Within this thesis, it was commonly noted
that the plateau at each stress jump was partially sloping. In the analysis, the stress at each level was taken as the midpoint of these partially sloping stress steps. Use of an over-deterministic methodology would allow a researcher to input a range of stress magnitudes for each step. In this case, a researcher could specify that the stress at a specific state will fall within the range of the minimum and maximum stresses observed at each plateau. For visualization purposes, let us consider the front gauge record from test 1 on the steel/polycarbonate configuration, found in Figure 7.4. In contrast to Figure 6.15, a range is now used to specify the magnitude of stress at each state.

![Figure 7.4. Stress record captured by the front gauge in test 1 of the steel/polycarbonate configuration.](image-url)
The final phase of the analysis would utilize over-deterministic methods to converge on the most accurate stress associated with each state found within the range specified. This process would additionally require either equations (2.45-2.46) or (6.4-6.5), respectively for cases where the inner-layer exhibits either a linear or a polynomial relationship between shock velocity and particle velocity. Each iteration of the analysis should end by confirming that there is consistency between the slopes of the C- and C+ Hugoniot of the bulk material. Thus, the shortcomings of the method outlined in section 7.2 (depicted in Figure 7.1(b)), can be systematically addressed. It is very likely that adopting this procedure will allow for the extension of the HRUM to applications where the Hugoniot of the bulk material is constructed form the known Hugoniot of the inner-layer.

Before we conclude, it should be pointed out that in most situations the HRUM can be employed in the validated configuration. For many materials, a researcher will be able to find a material of higher impedance to utilize as the bulk material. For example, steel’s Hugoniot could be investigated by employing it as the inner-layer, where tungsten could be utilized as the bulk material. Thus, even without validation of the HRUM for applications of constructing the bulk material’s Hugoniot, the method remains a useful tool for continued investigations into the shock response of materials. It has been shown that a single HRUM experiment can accurately construct a useable Hugoniot equation for a material of interest embedded within a well documented bulk material of higher impedance.
APPENDIX A

THE PLATE IMPACT APPARATUS

A.1 INTRODUCTION TO THE SYSTEM

Before utilizing the plate impact apparatus to conduct an actual experiment, it is imperative to become familiar with all of the important components of the system. Additionally, users of the system should consult the manuals for the Tektronix® oscilloscopes, the Keyence® detectors, the ThorLabs® detectors, and the Dynasen® stress gauge power supply. Knowledge of the operation and working requirements of all of the diagnostic systems will be useful in troubleshooting problems encountered while trying to conduct an experiment. Users should initially refer to Chapter 3, where a general introduction to the apparatus is presented. Within Chapter 3, many of the important components have already been discussed. This section will provide some supplementary figures of important components that were not depicted in the figures found in Chapter 3.

Figure A.1 depicts the important components associated with the gas gun. Part (a) of the figure shows the hardware utilized to pressurize the gun’s vessel. The Regulator can be set to any desired pressure between 0-600 psi. It contains two pressure gauges that allow for measurement of the pressure in the helium tank and the pressure delivered to the vessel. The fill valve on the helium tank is used to release helium into the vessel. If the vessel is over-pressurized, the bleed valve can be used to
slowly release helium from the vessel. Part (b) of the figure shows the hardware involved in the gas gun’s pressure vessel. The pressure within the vessel should be monitored with use of the pressure gauge depicted in part (b) of the figure. The isolation valve is used to equalize the pressure on both sides of the internal separation (described in chapter 3). If this valve is closed, the two sides of the internal separation will be isolated. The fire valve is used to release the pressure from the back separation, allowing the gun to fire. Finally, the projectile loading valve is used to supply vacuum pressure to the rear of the projectile.

Figure A.1. Important components of the gas gun; (a) Hardware used to connect the vessel to the helium tank; (b) gas gun pressure vessel.

Figure A.2 depicts the important components of the vacuum test chamber. Part (a) of the figure shows the relative position of the vacuum gauge, vacuum pressure sensor, vacuum valve, voltage through-ports for the stress gauges, and the ON/OFF switches for the ThorLabs® velocity diagnostics. Part (b) depicts the inside of the vacuum chamber where the voltage through-ports for the stress gauges, and the Keyence®/ThorLabs® velocity traps can be noted. Part (c) of the figure shows the relative position of the control units for the Keyence® velocity traps. It should be noted that the system is functioning properly when both the red and green LEDs are illuminated. Part (d) of the figure contains a picture of the vacuum pump assembly,
where the location of the ON/OFF switch can be noted. In order to prevent debris from entering the pump a filter is employed. This filter should be inspected as part of the routine maintenance of the system.

![Diagram of vacuum chamber components](image)

**Figure A.2.** Important components of the vacuum chamber; (a) Depiction of the gas gun side of the chamber; (b) Location of the diagnostic systems inside the chamber; (c) Keyence velocity trap control unit located on the door side of the chamber; (d) The vacuum pump assembly.

Figure A.3 depicts some additional components of the vacuum chamber that are located on the opposite side of the chamber from the gas gun connection. Part (a) of the figure shows the vacuum dump valve. For experiments involving low driving pressures, this valve can be opened to release the vacuum pressure from the system after an experiment. Part (b) of the figure shows the location of the power protection unit that is used to feed power to all the diagnostics.
Figure A.3. Additional components of the vacuum chamber; (a) Vacuum dump valve located on the opposite side of the chamber from the gas gun; (b) Power protection unit located under the vacuum chamber.

A.2 PROCEDURE FOR USING THE SYSTEM

Once the experimentalist becomes familiar with all of the components and diagnostics of the system, the apparatus can be used to conduct an experiment. This section outlines a procedure for using the apparatus. It will begin with the assumption that the user has prepared the projectile assembly, as discussed in section 3.3.1. Additionally, the sample should already be fabricated and mounted to the sample holder (outlined in Chapter 3).

A.2.1 Initial Setup

When setting up for an experiment, the oscilloscopes and stress gauge power supply should first be turned on. These devices require several minutes to warm-up. Thus, it is important to allow at least a ten minute warm-up period, before conducting the stress gauge calibration procedure (discussed in section 3.2.2.1). Before every experiment, all debris from previous experiments should be removed from the system. After the system is sufficiently clean, the clay within the catch box should be packed. Ideally there should be about 2-2.5” of clay, solidly packed into the back of the box.
Additionally, a free brick of clay should be positioned in the center of the box. For visualization purposes, Figure A.4 depicts the soft recovery method. Part (a) of this figure provides an example of how the clay should be packed within the catch box. Part (b) of the figure depicts the final position of the catch box before an experiment. This final assembly will employ foam bumpers on the front and back of the box, as noted in Figure A.4 (b). At this point in the procedure, the soft recovery system does not yet need to be set up according to part (b).

![Figure A.4. The soft recovery setup; (a) Example of clay packing within the catch box; (b) Final position and assembly for the soft recovery setup.](image-url)

Next, the velocity traps should be tested to ensure that they are operating correctly. The Keyence® traps can be checked by inspecting the diagnostic lights found in part (c) of Figure A.2. After turning on the photodiodes and laser diodes, the alignment of the ThorLabs® velocity traps can be checked by inspecting the output voltage on the oscilloscope. Additionally, the trigger settings for both oscilloscopes can be confirmed by passing an object through the beams. If properly set, this action should trigger both oscilloscopes. Once there is confirmation that the velocity system is working properly, the laser diodes can be turned off. This will allow the user to
work within the chamber, without fear of eye damage (laser-safety glasses, found on the green cart, should be worn if working in the chamber while the diodes are on).

### A.2.2 Gas Gun Loading Procedure

After the initial setup is complete, the gas gun can be loaded. The first step requires the pressure vessel to be pressurized to about 5 psi. To accomplish this, the firing valve must be in the closed position, and the isolation valve must be in the open position. Next, the fill valve on the helium tank is slowly opened and closed. This will allow the pressure vessel to pressurize to a level sufficient to engage the front seal plate. Once this is accomplished, the projectile assembly can be loaded into the system. First grease must be applied to the o-rings of the projectile. Next the projectile can be inserted into the front of the gun barrel. To ensure that the sabot does not impact the pressure vessel end of the barrel (this will typically dislodge the impactor from the sabot), care should be exercised when employing the vacuum pump to load the projectile. Thus, when the vacuum pump is running, the projectile loading valve and vacuum valve should be closed. The vacuum pressure should be brought down to the $10^{-1}$ torr range. Once this is accomplished, the projectile loading valve can be slowly opened. This will gradually pull the projectile down the barrel. This process typically takes three iterations. Once the projectile is in position at the pressure vessel end of the gun barrel, the vacuum pump can be activated with the projectile loading valve fully open. This will allow for the seal of the sabot to be checked. A good sabot should be able to hold vacuum pressure once the pump is turned off. Until noted, the projectile loading valve should now be left in the open position.
A.2.3 Final Setup Procedure

After the gun is loaded, the stress gauges must be calibrated in accordance with the procedure found in section 3.2.2.1. There are many triggering options that can be chosen for this calibration procedure. However, the velocity traps utilized to trigger the diagnostics are the best choice because it will allow further confirmation that they are functioning properly. Once calibration is complete, the sample mount can be attached to the four polycarbonate standoffs. Additionally, the lead wires of the gauge(s) must be connected to the voltage through ports on the inside of the chamber. On the outside of the vacuum chamber, the coaxial cables used to connect the stress gauge(s) to the power supply should be connected to the voltage through ports. At this point, an upper end resistance measurement should be made to confirm that the calibration parameters match the resistance of the actual gauge/cable assembly (noted in section 3.2.2.1).

Once the sample is mounted, and the calibration procedure is confirmed, the catch box can be moved into position. With the foam bumpers attached, the final position of the soft recovery setup should match the picture found in Figure A.4 (b). Next the polycarbonate safety shield is attached to the chamber using the four mounting bolts. Once this is done, the door of the chamber can be closed and the door latches locked. For visualization purposes, Figure A.5 contains a depiction of these final steps. Part (a) shows the safety shield and mounting locations. Part (b) shows the closed door and door latches in the locked position.

At this point the vacuum chamber is ready to be evacuated. With the vacuum dump valve closed and the vacuum valve open, the vacuum pump can be turned on
(the projectile loading valve should still be open). The system will typically take about 10 to 20 minutes to achieve a 10 torr vacuum pressure.

During the time that the vacuum chamber is being evacuated, the pressure vessel can be pressurized to the desired level. Again, ensuring that the isolation valve is open, the fill valve can be slowly opened to allow helium to pressurize the gas gun’s pressure vessel. If the pressure vessel is pressurized higher than the desired level, the bleed valve can be used to slowly leak helium from the vessel. Once this is done, the calibration parameters can be inputted into the computer to facilitate a quick analysis of the stress signal after the experiment (if time permits, while waiting for the vacuum chamber to achieve the desired pressure).

![Figure A.5. Final vacuum chamber setup; (a) Installation of the safety shield; (b) Closing and latching of the vacuum chamber door.](image)

**A.2.4 Firing Procedure**

Once the vacuum chamber and the gun’s pressure vessel are at the desired pressures, the gas gun can be fired utilizing the following procedure. Figure A.6 contains a picture of the gas gun apparatus where the proceeding steps can be visualized. First the laser diodes should be turned on (also confirm the photo diodes are on and the LEDs on the Keyence® control are illuminated). Next the vacuum pump should be turned off. The system will begin to loose vacuum pressure after this
step. Therefore the rest of the procedure should be conducted in a timely fashion. The next step is to confirm that the oscilloscopes and power supply are ready to accept a trigger source. Once confirmed, the firing process is accomplished by addressing the four valves. Typically, this final stage of these numerous steps is accomplished as a count down to fire (3, 2, 1, fire). First the vacuum valve is closed. Then the projectile loading valve is closed. The isolation valve is next closed. Finally the gun is fired by rapidly opening the fire valve.

![Figure A.6. Final firing procedure.](image)

**A.2.5 Completion of an Experiment**

Immediately after firing the gun, the data on the oscilloscopes should be saved. Since capture of the stress records is essential, these should be saved first. Once complete, the impact velocity can be determined by using the cursors on the oscilloscopes to measure the time between interruptions for each system. Next, the vacuum door latches can be opened. If the door remains shut, the vacuum dump valve can be used to bring the system back to atmospheric pressure. The safety shield can be removed employing the suction cup found on the green cart. Once this is removed,
the sample can be recovered from the catch box. Finally, stress is determined from the experimental data through the data reduction procedure discussed in section 3.2.2.2.
APPENDIX B

MATLAB CODES FOR THE HRUM

B.1 SOLVING FOR THE INNER-LAYER

This Matlab code can be utilized in the HRUM to solve for the Hugoniot of the inner-layer from the known Hugoniot of the bulk material. It is assumed that the Hugoniot of the bulk material is well defined by the linear form of equation (2.16), thus, \( U_s = c_b + s u_p \). Equations (2.36-2.39) are utilized in this code. The Matlab code is as follows:

```matlab
clear all; clc
opts = optimset('fsolve');
opts = optimset(opts, 'TolFun', 1e-12);

%This program determines the particle velocities from the known Hugoniot of the Bulk Material (Outer Layer) when \( U_s = c_b + s u_p \)

%Enter the Hugoniot data for the Bulk Material:
cb=5310; %Bulk velocity (m/s)
s=1.3626; %Linear s parameter
p0=2703; %Density (Kg/m^3)
uimp=325; %Impact velocity (m/s)

%Enter the stress in Pa measured at states -1,1,3,5 from the front gauge
SF=[0 0.8*10^9 1.95*10^9 2.3*10^9];
%Enter the stress in Pa measured at states 0,2,4,6 from the back gauge
SB=[0 1.5*10^9 2.18*10^9 2.37*10^9];

%Along the Front Gauge:
ui=uimp;
p=p0;
for i=1:3
    u135(i)=fsolve(@(u) SF(i+1)+SF(i)+p*(cb+s*(uimp-u))*(u-u1), uimp, opts);
    uo=u135(i);
    Us135(i)=(cb+s*uo);
    po=p*us135(i)./(Us135(i)-(uo-uo));
end

%Along the Back Gauge:
uo=0;
p=p0;
for i=1:3
    u246(i)=fsolve(@(u) SB(i+1)+SB(i)+p*(cb+s*(uimp-u))*(u-u1), uimp, opts);
    uo=ui;
    u1=u246(i);
    Us246(i)=(cb+s*ui);
    po=p*us246(i)./(Us246(i)-(ui-uo));
end
```

244
% Plotting the Experiment:
ufrontplate=[0:.01:uimp/2]; stressfrontplate=p0.*(cb+s.*ufrontplate).*ufrontplate;
ubackplate=[uimp:0:.01:uimp/2]; stressbackplate=p0.*(cb+s.*(uimp-ufrontplate)).*ufrontplate;
uiinnerlayer=[0 uinnerlayer(1) uinnerlayer(2) uinnerlayer(3) uinnerlayer(4) uinnerlayer(5) uinnerlayer(6) uinnerlayer(7)];stressinnerlayer=[0 SF(2) SB(2) SF(3) SB(3) SF(4) SB(4) SF(5) SB(5) SF(6) SB(6) SF(7) SB(7)];
plot(ufrontplate,stressfrontplate*10^(-9), 'b', 'linewidth',2); hold on;
plot(ubackplate,stressbackplate*10^(-9), 'b', 'linewidth',2); hold on;
plot(uiinnerlayer,stressinnerlayer*10^(-9), 'r*', 'linewidth',2);
xlabel('Particle Velocity (m/s)');
ylabel('Stress (GPa)');

% Transformation of up for the Principal Hugoniot of the Inner Layer:
uprincipal=[0 uiinnerlayer(2)];
for i=1:5
uprincipal(i+2)=uprincipal(i+1)+abs(uiinnerlayer(i+1));
end
plot(uprincipal,stressinnerlayer*10^(-9), 'r*' );

Optimization terminated: norm of relative change in X is less than max(options.TolX^2,eps) and sum-of-squares of function values is less than sqrt(options.TolFun).
Optimization terminated: first-order optimality is less than options.TolFun.
Optimization terminated: norm of relative change in X is less than max(options.TolX^2,eps) and sum-of-squares of function values is less than sqrt(options.TolFun).
Optimization terminated: norm of relative change in X is less than max(options.TolX^2,eps) and sum-of-squares of function values is less than sqrt(options.TolFun).
Optimization terminated: norm of relative change in X is less than max(options.TolX^2,eps) and sum-of-squares of function values is less than sqrt(options.TolFun).
Optimization terminated: norm of relative change in X is less than max(options.TolX^2,eps) and sum-of-squares of function values is less than sqrt(options.TolFun).

u135 =
270.0374  193.3243  170.4332
u246 =
101.8465  146.6519  159.0314
uprincipal =
Columns 1 through 6
0 270.0374 438.2283 529.7061 576.3785 600.1598
Column 7
611.5616
B.2 SOLVING FOR THE BULK MATERIAL

B.2.1 Assuming a linear relationship between $U_s$ and $u_p$

This Matlab code can be utilized in the HRUM to solve for the Hugoniot of the bulk material from the known Hugoniot of the inner-layer. It is assumed that the Hugoniot of the inner-layer is well defined by the linear form of equation (2.16), thus, $U_S = c_b + s u_p$. Equations (2.45-2.46) are used in this code. The Matlab code is as follows:

```matlab
clear all; clc
opts = optimset('fsolve');
opts = optimset(opts, 'TolFun', 1e-12);
%This program determines the particle velocities from the known Hugoniot of
%the Inner Layer when Up=cb+sup

%Enter the hugoniot data for the Inner Layer:
cb=2000; %Bulk velocity (m/s)
s=2.122; %linear s parameter
p0=1193; %Density(Kg/m^3)
```

![Graph showing stress vs. particle velocity](image.png)
uimp=325; %Impact velocity (m/s)

% Enter the stress in Pa measured at states 0, 1, 2, 3, 4, 5, 6
Stress=[0 0.8*10^9 1.5*10^9 1.95*10^9 2.18*10^9 2.3*10^9 2.37*10^9];

ui=0;
p=p0;
iteration=0;
for i=1:6
    u(i)=fsolve(@(u) u(i)-Stress(i+1)+Stress(i)+(Stress(i+1)+Stress(i)+(Stress(i+1)+Stress(i)+(Stress(i+1)+Stress(i)+(Stress(i+1)+Stress(i)))/*(u-ui)), uimp, opts);
    u0=u(i);
    ui=u(i);
    iteration=iteration+abs(ui-u0);
    Us(i)=(cb+s*(iteration));
    p=p0*Us(i)./(Us(i)-(abs(ui-u0)))
end

u

% Plotting the Experiment:
plot([0 u],Stress*10^9), 'linewidth',2); hold on;
xlabel('Particle Velocity (m/s)')
ylabel('Stress (GPa)')

% Transformation of up for the Principal Hugoniot of the Bulk Material:

u_principal=[0 uimp u(1) u(2) uimp u(3) u(4) uimp u(5) u(6)]
plot(u_principal,Stress*10^9,'r*')

Optimization terminated: first-order optimality is less than options.TolFun.
Optimization terminated: first-order optimality is less than options.TolFun.
Optimization terminated: first-order optimality is less than options.TolFun.
Optimization terminated: norm of relative change in X is less than max(options.TolX^2,eps) and sum-of-squares of function values is less than sqrt(options.TolFun).
Optimization terminated: first-order optimality is less than options.TolFun.
Optimization terminated: norm of relative change in X is less than max(options.TolX^2,eps) and sum-of-squares of function values is less than sqrt(options.TolFun).

u =
    262.2942   83.0116  183.8786  135.5477  159.9993  145.9678

u_principal =
Columns 1 through 6
     0   62.7058   83.0116  141.1214  135.5477  165.0007
Column 7
    145.9678
B.2.2 Assuming a polynomial relationship between $U_s$ and $u_p$

This Matlab code can be utilized in the HRUM to solve for the Hugoniot of the bulk material from the known Hugoniot of the inner-layer. It is assumed that the Hugoniot of the inner-layer is well defined by the polynomial form of equation (2.16), thus, $U_s = c_b + s u_p + Q u_p^2$. Equations (6.4-6.6) are used in this code. The Matlab code is as follows:

```matlab
clear all; clc
opts = optimset('fsolve');
opts = optimset(opts, 'TolFun', 1e-12);

% This program determines the particle velocities from the known Hugoniot of
% the Inner Layer when $U_s = c_b + s u_p + Q u_p^2$
% Enter the Hugoniot data for the Inner Layer:
% $c_b = 2000$ m/s
% $s = 2.1612$ linear $s$ parameter
% $Q = -0.0002$ polynomial $Q$ parameter
% $p_0 = 1193$ kg/m$^3$
% $u_{imp} = 325$ m/s
% Enter the stress in Pa measured at states 0,1,2,3,4,5,6
```

---

248
Stress=[0 0.8*10^9 1.5*10^9 1.95*10^9 2.18*10^9 2.3*10^9 2.37*10^9];
ui=0;
p=p0;
iteration=0;
for i=1:6
    u(i)=fsolve(@(u) u(i)=fsolve(@(u) u(i)=fsolve(@(u) u(i)=fsolve(@(u)-- --Stress(i+1)+Stress(i)+( Stress(i+1)+Stress(i+( ... 1)*p*(cb+s*(itteration+abs(u 1)*p*(cb+s*(itteration+abs(u-- --ui)+Q*(itteration+abs(u ui)+Q*(itteration+abs(u ui)+Q*(itteration+abs(u ui)+Q*(itteration+abs(u-- --ui))))*(u ui))))*(u ui))))*(u-- --ui), uimp, opts);
    uo=ui;
    iteration=iteration+abs(ui itteration=itteration+abs(ui itteration=itteration+abs(ui itteration=itteration+abs(ui-- --uo);
    po=po*Us(i)./(Us(i) =po*Us(i)./(Us(i) =po*Us(i)./(Us(i) =po*Us(i)./(Us(i)-- --(abs(ui (abs(ui (abs(ui (abs(ui-- --uo))); uo))); uo)); uo));
end

%Plotting the Experiment:
plot([0 u],Stress*10^( plot([0 u],Stress*10^( plot([0 u],Stress*10^( plot([0 u],Stress*10^(-- --9), ' 9), ' 9), ' ... --b*', 'linewidth',2); hold on; b*', 'linewidth',2); hold on; b*', 'linewidth',2); hold on; b*', 'linewidth',2); hold on;
xlabel('Particle Velocity (m/s)') xlabel('Particle Velocity (m/s)') xlabel('Particle Velocity (m/s)') xlabel('Particle Velocity (m/s)')
ylabel('Stress (GPa)')

%Transformation of up for the Principal Hugoniot of the Bulk Material:
ua_principal=
plot(u_principal,Stress*10^( plot(u_principal,Stress*10^( plot(u_principal,Stress*10^( plot(u_principal,Stress*10^(-- --9),'r*') 9),'r*') 9),'r*') 9),'r*') 9),'r*') 9),'r*')

Optimization terminated: first-order optimality is less than options.TolFun.
Optimization terminated: first-order optimality is less than options.TolFun.
Optimization terminated: first-order optimality is less than options.TolFun.
Optimization terminated: norm of relative change in X is less than max(options.TolX^2,eps) and sum-of-squares of function values is less than sqrt(options.TolFun).
Optimization terminated: norm of relative change in X is less than max(options.TolX^2,eps) and sum-of-squares of function values is less than sqrt(options.TolFun).
Optimization terminated: first-order optimality is less than options.TolFun.

u =
    261.4426   82.8632  183.3261  135.1874  159.5420  145.5662

u_principal =
Columns 1 through 6
    0  63.5574  82.8632  141.6739  135.1874  165.4580
Column 7
    145.5662


Proud, W., Private Communications, IMPLAST Conference, Renaissance Providence Hotel, Providence, RI, October 12, 2010.


