E3. Previous Unit Exams 3

Gerhard Müller
University of Rhode Island, gmuller@uri.edu

Creative Commons License

This work is licensed under a Creative Commons Attribution-Noncommercial-Share Alike 4.0 License.

Follow this and additional works at: http://digitalcommons.uri.edu/elementary_physics_2

Abstract

Previous unit exams of course materials for Elementary Physics II (PHY 204), taught by Gerhard Müller at the University of Rhode Island. Documents will be updated periodically as more entries become presentable.

Some of the slides contain figures from the textbook, Paul A. Tipler and Gene Mosca. Physics for Scientists and Engineers, 5th/6th editions. The copyright to these figures is owned by W.H. Freeman. We acknowledge permission from W.H. Freeman to use them on this course web page. The textbook figures are not to be used or copied for any purpose outside this class without direct permission from W.H. Freeman.

Recommended Citation

http://digitalcommons.uri.edu/elementary_physics_2/1

This Course Material is brought to you for free and open access by the Physics Course Materials at DigitalCommons@URI. It has been accepted for inclusion in PHY 204: Elementary Physics II by an authorized administrator of DigitalCommons@URI. For more information, please contact digitalcommons@etal.uri.edu.
An infinitely long straight current of magnitude $I = 6\text{A}$ is directed into the plane $(\otimes)$ and located a distance $d = 0.4\text{m}$ from the coordinate origin (somewhere on the dashed circle). The magnetic field $\vec{B}$ generated by this current is in the negative $y$-direction as shown.

(a) Find the magnitude $B$ of the magnetic field.
(b) Mark the location of the position of the current $\otimes$ on the dashed circle.
An infinitely long straight current of magnitude $I = 6\text{A}$ is directed into the plane (⊗) and located a distance $d = 0.4\text{m}$ from the coordinate origin (somewhere on the dashed circle). The magnetic field $\vec{B}$ generated by this current is in the negative $y$-direction as shown.

(a) Find the magnitude $B$ of the magnetic field.

(b) Mark the location of the position of the current ⊗ on the dashed circle.

Solution:

(a) $B = \frac{\mu_0 I}{2\pi d} = 3\mu\text{T}$. 
An infinitely long straight current of magnitude $I = 6\, \text{A}$ is directed into the plane (⊗) and located a distance $d = 0.4\, \text{m}$ from the coordinate origin (somewhere on the dashed circle). The magnetic field $\vec{B}$ generated by this current is in the negative $y$-direction as shown.

(a) Find the magnitude $B$ of the magnetic field.

(b) Mark the location of the position of the current $⊗$ on the dashed circle.

Solution:

(a) $B = \frac{\mu_0}{2\pi} \frac{I}{d} = 3\mu\, \text{T}$.

(b) Position of current $⊗$ is at $y = 0$, $x = -0.4\, \text{m}$.
In the circuit shown we close the switch $S$ at time $t = 0$. Find the current $I$ through the battery and the voltage $V_L$ across the inductor

(a) immediately after the switch has been closed,
(b) a very long time later.
In the circuit shown we close the switch $S$ at time $t = 0$. Find the current $I$ through the battery and the voltage $V_L$ across the inductor

(a) immediately after the switch has been closed,
(b) a very long time later.

Solution:

(a) $I = \frac{12V}{2\Omega + 4\Omega + 2\Omega} = 1.5\text{A}$, \quad $V_L = (4\Omega)(1.5\text{A}) = 6\text{V}$.
In the circuit shown we close the switch $S$ at time $t = 0$. Find the current $I$ through the battery and the voltage $V_L$ across the inductor

(a) immediately after the switch has been closed,
(b) a very long time later.

**Solution:**

(a) $I = \frac{12V}{2\Omega + 4\Omega + 2\Omega} = 1.5A$, $V_L = (4\Omega)(1.5A) = 6V$.

(b) $I = \frac{12V}{2\Omega + 2\Omega} = 3A$, $V_L = 0$. 
At time $t = 0$ the capacitor is charged to $Q_{max} = 3\mu C$ and the current is instantaneously zero.

(a) How much energy is stored in the capacitor at time $t = 0$?
(b) At what time $t_1$ does the current reach its maximum value?
(c) How much energy is stored in the inductor at time $t_1$?

$L = 40\text{mH}$

$C = 9\mu\text{F}$
At time $t = 0$ the capacitor is charged to $Q_{max} = 3\mu\text{C}$ and the current is instantaneously zero.

(a) How much energy is stored in the capacitor at time $t = 0$?

(b) At what time $t_1$ does the current reach its maximum value?

(c) How much energy is stored in the inductor at time $t_1$?

Solution:

(a) $U_C = \frac{Q_{max}^2}{2C} = 0.5\mu\text{J}$. 

\[
\begin{align*}
L &= 40\text{mH} \\
C &= 9\mu\text{F}
\end{align*}
\]
At time $t = 0$ the capacitor is charged to $Q_{\text{max}} = 3\mu\text{C}$ and the current is instantaneously zero.

(a) How much energy is stored in the capacitor at time $t = 0$?

(b) At what time $t_1$ does the current reach its maximum value?

(c) How much energy is stored in the inductor at time $t_1$?

Solution:

(a) $U_C = \frac{Q_{\text{max}}^2}{2C} = 0.5\mu\text{J}$.

(b) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC} = 3.77\text{ms}$, \quad $t_1 = \frac{T}{4} = 0.942\text{ms}$. 

$L = 40\text{mH}$

$C = 9\mu\text{F}$
At time $t = 0$ the capacitor is charged to $Q_{\text{max}} = 3\mu\text{C}$ and the current is instantaneously zero.

(a) How much energy is stored in the capacitor at time $t = 0$?

(b) At what time $t_1$ does the current reach its maximum value?

(c) How much energy is stored in the inductor at time $t_1$?

Solution:

(a) $U_C = \frac{Q_{\text{max}}^2}{2C} = 0.5\mu\text{J}$.

(b) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC} = 3.77\text{ms}$, \quad $t_1 = \frac{T}{4} = 0.942\text{ms}$.

(c) $U_L = U_C = 0.5\mu\text{J}$ \quad (energy conservation.)
Consider the circuit shown. The ac voltage supplied is $E = E_{max} \cos(\omega t)$ with $E_{max} = 170\, \text{V}$ and $\omega = 377\, \text{rad/s}$.

(a) What is the maximum value $I_{max}$ of the current?

(b) What is the emf $E(t)$ at $t = 0.01\, \text{s}$?

(c) What is the current $I(t)$ at $t = 0.01\, \text{s}$?

\[ 16\, \mu\text{F} \]
Consider the circuit shown. The ac voltage supplied is \( E = E_{\text{max}} \cos(\omega t) \) with \( E_{\text{max}} = 170 \text{V} \) and \( \omega = 377 \text{rad/s} \).

(a) What is the maximum value \( I_{\text{max}} \) of the current?

(b) What is the emf \( E(t) \) at \( t = 0.01 \text{s} \)?

(c) What is the current \( I(t) \) at \( t = 0.01 \text{s} \)?

Solution:

(a) \( I_{\text{max}} = \frac{E_{\text{max}}}{X_C} = E_{\text{max}} \omega C = 1.03 \text{A} \).
Consider the circuit shown. The ac voltage supplied is \( \mathcal{E} = \mathcal{E}_{max} \cos(\omega t) \) with \( \mathcal{E}_{max} = 170 \text{V} \) and \( \omega = 377 \text{rad/s} \).

(a) What is the maximum value \( I_{max} \) of the current?
(b) What is the emf \( \mathcal{E}(t) \) at \( t = 0.01 \text{s} \)?
(c) What is the current \( I(t) \) at \( t = 0.01 \text{s} \)?

**Solution:**

(a) \( I_{max} = \frac{\mathcal{E}_{max}}{X_C} = \mathcal{E}_{max} \omega C = 1.03 \text{A} \).

(b) \( \mathcal{E} = (170 \text{V}) \cos(3.77 \text{rad}) = (170 \text{V})(-0.809) = -138 \text{V} \).
Consider the circuit shown. The ac voltage supplied is $E = E_{max} \cos(\omega t)$ with $E_{max} = 170\text{V}$ and $\omega = 377\text{rad/s}$.

(a) What is the maximum value $I_{max}$ of the current?
(b) What is the emf $E(t)$ at $t = 0.01\text{s}$?
(c) What is the current $I(t)$ at $t = 0.01\text{s}$?

Solution:

(a) $I_{max} = \frac{E_{max}}{X_C} = E_{max}\omega C = 1.03\text{A}$.

(b) $E = (170\text{V})\cos(3.77\text{rad}) = (170\text{V})(-0.809) = -138\text{V}$.

(c) $I = E_{max}\omega C \cos(3.77\text{rad} + \pi/2) = (1.03\text{A})(0.588) = 0.605\text{A}$.
A proton \((m = 1.67 \times 10^{-27} \text{ kg}, \, q = 1.60 \times 10^{-19} \text{ C})\) with velocity \(v = 3.7 \times 10^4 \text{ m/s}\) enters a region of magnetic field \(B\) directed perpendicular to the plane of the sheet. The field bends the path of the proton into a semicircle of radius \(r = 19\text{ cm}\) as shown.

(a) Find the force necessary to keep the proton moving on the circle

(b) Find the direction (\(\odot\) or \(\otimes\)) and the magnitude of the magnetic field \(B\) that provides this force.

(c) Find the time \(t\) it takes the proton to complete the semicircular motion.
A proton \((m = 1.67 \times 10^{-27}\text{kg}, q = 1.60 \times 10^{-19}\text{C})\) with velocity \(v = 3.7 \times 10^4\text{m/s}\) enters a region of magnetic field \(B\) directed perpendicular to the plane of the sheet. The field bends the path of the proton into a semicircle of radius \(r = 19\text{cm}\) as shown.

(a) Find the force necessary to keep the proton moving on the circle

(b) Find the direction (⊙ or ⊗) and the magnitude of the magnetic field \(B\) that provides this force.

(c) Find the time \(t\) it takes the proton to complete the semicircular motion.

Solution:

(a) \(F = \frac{mv^2}{r} = 1.20 \times 10^{-17}\text{N}.\)
A proton \((m = 1.67 \times 10^{-27} \text{kg}, \ q = 1.60 \times 10^{-19} \text{C})\) with velocity \(v = 3.7 \times 10^4 \text{m/s}\) enters a region of magnetic field \(B\) directed perpendicular to the plane of the sheet. The field bends the path of the proton into a semicircle of radius \(r = 19\text{cm}\) as shown.

(a) Find the force necessary to keep the proton moving on the circle

(b) Find the direction (⊙ or ⊗) and the magnitude of the magnetic field \(B\) that provides this force.

(c) Find the time \(t\) it takes the proton to complete the semicircular motion.

Solution:

(a) \(F = \frac{mv^2}{r} = 1.20 \times 10^{-17} \text{N}\).

(b) \(F = qvB \Rightarrow B = \frac{F}{qv} = 1.71 \times 10^{-3} \text{T}\). ⊗
A proton \( m = 1.67 \times 10^{-27} \text{kg}, \ q = 1.60 \times 10^{-19} \text{C} \) with velocity \( v = 3.7 \times 10^4 \text{m/s} \) enters a region of magnetic field \( B \) directed perpendicular to the plane of the sheet. The field bends the path of the proton into a semicircle of radius \( r = 19 \text{cm} \) as shown.

(a) Find the force necessary to keep the proton moving on the circle

(b) Find the direction (\( \odot \) or \( \otimes \)) and the magnitude of the magnetic field \( B \) that provides this force.

(c) Find the time \( t \) it takes the proton to complete the semicircular motion.

Solution:

(a) \[ F = \frac{mv^2}{r} = 1.20 \times 10^{-17} \text{N}. \]

(b) \[ F = qvB \quad \Rightarrow \quad B = \frac{F}{qv} = 1.71 \times 10^{-3} \text{T.} \quad \otimes \]

(c) \[ vt = \pi r \quad \Rightarrow \quad t = \frac{\pi r}{v} = 1.61 \times 10^{-5} \text{s.} \]
Consider two infinitely long, straight wires with currents of equal magnitude $I_1 = I_2 = 5\, \text{A}$ in the directions shown. Find the direction (in/out) and the magnitude of the magnetic fields $B_1$ and $B_2$ at the points marked in the graph.
Consider two infinitely long, straight wires with currents of equal magnitude \( I_1 = I_2 = 5 \text{A} \) in the directions shown.

Find the direction (in/out) and the magnitude of the magnetic fields \( B_1 \) and \( B_2 \) at the points marked in the graph.

**Solution:**

\[ B_1 = \frac{\mu_0}{2\pi} \left( \frac{5 \text{A}}{4\text{m}} - \frac{5 \text{A}}{4\text{m}} \right) = 0 \] (no direction).
Consider two infinitely long, straight wires with currents of equal magnitude $I_1 = I_2 = 5\, \text{A}$ in the directions shown.
Find the direction (in/out) and the magnitude of the magnetic fields $B_1$ and $B_2$ at the points marked in the graph.

Solution:

- $B_1 = \frac{\mu_0}{2\pi} \left( \frac{5\, \text{A}}{4\, \text{m}} - \frac{5\, \text{A}}{4\, \text{m}} \right) = 0$ (no direction).
- $B_2 = \frac{\mu_0}{2\pi} \left( \frac{5\, \text{A}}{2\, \text{m}} - \frac{5\, \text{A}}{4\, \text{m}} \right) = 0.25\, \mu\text{T}$ (out of plane).
A conducting loop in the shape of a square with area \( A = 4 \text{m}^2 \) and resistance \( R = 5 \Omega \) is placed in the \( yz \)-plane as shown. A time-dependent magnetic field \( \mathbf{B} = B_x \hat{\mathbf{i}} \) is present. The dependence of \( B_x \) on time is shown graphically.

(a) Find the magnetic flux \( \Phi_B \) through the loop at time \( t = 0 \).

(b) Find magnitude and direction (cw/ccw) of the induced current \( I \) at time \( t = 2 \text{s} \).
A conducting loop in the shape of a square with area $A = 4\text{m}^2$ and resistance $R = 5\Omega$ is placed in the $yz$-plane as shown. A time-dependent magnetic field $\mathbf{B} = B_x \hat{i}$ is present. The dependence of $B_x$ on time is shown graphically.

(a) Find the magnetic flux $\Phi_B$ through the loop at time $t = 0$.
(b) Find magnitude and direction (cw/ccw) of the induced current $I$ at time $t = 2\text{s}$.

Choice of area vector: $\odot/\otimes \Rightarrow$ positive direction = ccw/cw.

(a) $\Phi_B = \pm(1\text{T})(4\text{m}^2) = \pm4\text{Tm}^2$. 
A conducting loop in the shape of a square with area $A = 4\text{m}^2$ and resistance $R = 5\Omega$ is placed in the $yz$-plane as shown. A time-dependent magnetic field $\mathbf{B} = B_x \hat{i}$ is present. The dependence of $B_x$ on time is shown graphically.

(a) Find the magnetic flux $\Phi_B$ through the loop at time $t = 0$.
(b) Find magnitude and direction (cw/ccw) of the induced current $I$ at time $t = 2\text{s}$.

Choice of area vector: $\bigcirc / \otimes \Rightarrow$ positive direction = ccw/cw.

(a) $\Phi_B = \pm (1\text{T})(4\text{m}^2) = \pm 4\text{Tm}^2$.

(b) $\frac{d\Phi_B}{dt} = \pm (0.5\text{T}/\text{s})(4\text{m}^2) = \pm 2\text{V} \Rightarrow \mathcal{E} = -\frac{d\Phi_B}{dt} = \mp 2\text{V}$. 

$\Rightarrow I = \frac{\mathcal{E}}{R} = \mp \frac{2\text{V}}{5\Omega} = \mp 0.4\text{A}$ (cw).
In the circuit shown the switch $S$ is initially open. Find the current $I$ through the battery
(a) while the switch is open,
(b) immediately after the switch has been closed,
(c) a very long time later.
In the circuit shown the switch $S$ is initially open. Find the current $I$ through the battery
(a) while the switch is open, (b) immediately after the switch has been closed, (c) a very long time later.

\[ I = \frac{12V}{2\Omega + 3\Omega + 6\Omega} = 1.09A. \]
In the circuit shown the switch \( S \) is initially open. Find the current \( I \) through the battery
(a) while the switch is open,
(b) immediately after the switch has been closed,
(c) a very long time later.

\[ I = \frac{12V}{2\Omega + 3\Omega + 6\Omega} = 1.09A. \]

\[ I = \frac{12V}{2\Omega + 3\Omega + 6\Omega} = 1.09A. \]
In the circuit shown the switch $S$ is initially open. Find the current $I$ through the battery
(a) while the switch is open,
(b) immediately after the switch has been closed,
(c) a very long time later.

(a) $I = \frac{12V}{2\Omega + 3\Omega + 6\Omega} = 1.09A$.

(b) $I = \frac{12V}{2\Omega + 3\Omega + 6\Omega} = 1.09A$.

(c) $I = \frac{12V}{2\Omega + 3\Omega} = 2.4A$. 
Consider the circuit shown. The ac voltage supplied is $E = E_{max} \cos(\omega t)$ with $E_{max} = 170\text{V}$ and $\omega = 377\text{rad/s}$.

(a) What is the maximum value $I_{max}$ of the current?
(b) What is the emf $E$ at $t = 0.02\text{s}$?
(c) What is the current $I$ at $t = 0.02\text{s}$?
Consider the circuit shown. The *ac* voltage supplied is \( E = E_{max} \cos(\omega t) \) with \( E_{max} = 170\text{V} \) and \( \omega = 377\text{rad/s} \).

(a) What is the maximum value \( I_{max} \) of the current?
(b) What is the emf \( E \) at \( t = 0.02\text{s} \)?
(c) What is the current \( I \) at \( t = 0.02\text{s} \)?

(a) \[ I_{max} = \frac{E_{max}}{X_L} = \frac{E_{max}}{\omega L} = \frac{170\text{V}}{11.3\Omega} = 15.0\text{A}. \]
Consider the circuit shown. The ac voltage supplied is \( E = E_{\text{max}} \cos(\omega t) \) with \( E_{\text{max}} = 170\text{V} \) and \( \omega = 377\text{rad/s} \).

(a) What is the maximum value \( I_{\text{max}} \) of the current?

(b) What is the emf \( E \) at \( t = 0.02\text{s} \)?

(c) What is the current \( I \) at \( t = 0.02\text{s} \)?

(a) \( I_{\text{max}} = \frac{E_{\text{max}}}{X_L} = \frac{E_{\text{max}}}{\omega L} = \frac{170\text{V}}{11.3\Omega} = 15.0\text{A} \).

(b) \( E = E_{\text{max}} \cos(7.54\text{rad}) = (170\text{V})(0.309) = 52.5\text{V} \).
Consider the circuit shown. The ac voltage supplied is $E = E_{max} \cos(\omega t)$ with $E_{max} = 170\text{V}$ and $\omega = 377\text{rad/s}$.

(a) What is the maximum value $I_{max}$ of the current?
(b) What is the emf $E$ at $t = 0.02\text{s}$?
(c) What is the current $I$ at $t = 0.02\text{s}$?

\(L = 30\text{mH}\)

(a) $I_{max} = \frac{E_{max}}{X_L} = \frac{E_{max}}{\omega L} = \frac{170\text{V}}{11.3\Omega} = 15.0\text{A}$.
(b) $E = E_{max} \cos(7.54\text{rad}) = (170\text{V})(0.309) = 52.5\text{V}$.
(c) $I = I_{max} \cos(7.54\text{rad} - \pi/2) = (15.0\text{A})(0.951) = 14.3\text{A}$. 
Consider a rectangular conducting loop in the $xy$-plane with a counterclockwise current $I = 7 \text{A}$ in a uniform magnetic field $\vec{B} = 3 \text{T} \hat{i}$.

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side $ab$ of the rectangle.

(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.
Consider a rectangular conducting loop in the $xy$-plane with a counterclockwise current $I = 7\text{A}$ in a uniform magnetic field $\vec{B} = 3\text{T}\hat{i}$.

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side $ab$ of the rectangle.

(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.

Solution:

(a) $\vec{\mu} = (7\text{A})(45\text{m}^2)\hat{k} = 315\text{Am}^2\hat{k}$. 
Consider a rectangular conducting loop in the $xy$-plane with a counterclockwise current $I = 7\text{A}$ in a uniform magnetic field $\vec{B} = 3\text{T} \hat{i}$.

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side $ab$ of the rectangle.

(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.

Solution:

(a) $\vec{\mu} = (7\text{A})(45\text{m}^2)\hat{k} = 315\text{Am}^2\hat{k}$.

(b) $\vec{F} = I\vec{L} \times \vec{B} = (7\text{A})(5\text{m}\hat{j}) \times (3\text{T}\hat{i}) = -105\text{N}\hat{k}$.
Consider a rectangular conducting loop in the $xy$-plane with a counterclockwise current $I = 7A$ in a uniform magnetic field $\vec{B} = 3T\hat{i}$.

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side $ab$ of the rectangle.

(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.

Solution:

(a) $\vec{\mu} = (7A)(45m^2)\hat{k} = 315Am^2\hat{k}$.

(b) $\vec{F} = I\vec{L} \times \vec{B} = (7A)(5m\hat{j}) \times (3T\hat{i}) = -105N\hat{k}$.

(c) $\vec{\tau} = \vec{\mu} \times \vec{B} = (315Am^2\hat{k}) \times (3T\hat{i}) = 945Nm\hat{j}$.
Consider two very long, straight wires with currents $I_1 = 6\, \text{A}$ at $x = 1\, \text{m}$ and $I_3 = 3\, \text{A}$ at $x = 3\, \text{m}$ in the directions shown. Find magnitude and direction (up/down) of the magnetic field
(a) $B_0$ at $x = 0$,
(b) $B_2$ at $x = 2\, \text{m}$,
(c) $B_4$ at $x = 4\, \text{m}$. 
Consider two very long, straight wires with currents $I_1 = 6\,\text{A}$ at $x = 1\,\text{m}$ and $I_3 = 3\,\text{A}$ at $x = 3\,\text{m}$ in the directions shown. Find magnitude and direction (up/down) of the magnetic field

(a) $B_0$ at $x = 0$,
(b) $B_2$ at $x = 2\,\text{m}$,
(c) $B_4$ at $x = 4\,\text{m}$.

**Solution:**

(a) $B_0 = -\frac{\mu_0 (6\,\text{A})}{2\pi (1\,\text{m})} + \frac{\mu_0 (3\,\text{A})}{2\pi (3\,\text{m})} = -1.2\mu T + 0.2\mu T = -1.0\mu T$ (down),
Consider two very long, straight wires with currents $I_1 = 6A$ at $x = 1m$ and $I_3 = 3A$ at $x = 3m$ in the directions shown. Find magnitude and direction (up/down) of the magnetic field

(a) $B_0$ at $x = 0$,
(b) $B_2$ at $x = 2m$,
(c) $B_4$ at $x = 4m$.

Solution:

(a) $B_0 = -\frac{\mu_0(6A)}{2\pi(1m)} + \frac{\mu_0(3A)}{2\pi(3m)} = -1.2\mu T + 0.2\mu T = -1.0\mu T$ (down),

(b) $B_2 = \frac{\mu_0(6A)}{2\pi(1m)} + \frac{\mu_0(3A)}{2\pi(1m)} = 1.2\mu T + 0.6\mu T = 1.8\mu T$ (up),
Consider two very long, straight wires with currents $I_1 = 6\text{A}$ at $x = 1\text{m}$ and $I_3 = 3\text{A}$ at $x = 3\text{m}$ in the directions shown. Find magnitude and direction (up/down) of the magnetic field

(a) $B_0$ at $x = 0$,
(b) $B_2$ at $x = 2\text{m}$,
(c) $B_4$ at $x = 4\text{m}$.

Solution:

(a) $B_0 = -\frac{\mu_0 (6\text{A})}{2\pi (1\text{m})} + \frac{\mu_0 (3\text{A})}{2\pi (3\text{m})} = -1.2\mu \text{T} + 0.2\mu \text{T} = -1.0\mu \text{T}$ (down),

(b) $B_2 = \frac{\mu_0 (6\text{A})}{2\pi (1\text{m})} + \frac{\mu_0 (3\text{A})}{2\pi (1\text{m})} = 1.2\mu \text{T} + 0.6\mu \text{T} = 1.8\mu \text{T}$ (up),

(c) $B_4 = \frac{\mu_0 (6\text{A})}{2\pi (3\text{m})} - \frac{\mu_0 (3\text{A})}{2\pi (1\text{m})} = 0.4\mu \text{T} - 0.6\mu \text{T} = -0.2\mu \text{T}$ (down).
A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown.

(a) Find the magnetic flux \( \Phi_B \) through the frame at the instant shown.
(b) Find the induced emf \( \mathcal{E} \) at the instant shown.
(c) Find the direction (cw/ccw) of the induced current.

\[ v = 4 \text{m/s} \]

\[ B = 5 \text{T} \]
A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown.

(a) Find the magnetic flux $\Phi_B$ through the frame at the instant shown.
(b) Find the induced emf $\mathcal{E}$ at the instant shown.
(c) Find the direction (cw/ccw) of the induced current.

Solution:

(a) $\Phi_B = \vec{A} \cdot \vec{B} = \pm (20m^2)(5T) = \pm 100\text{Wb}$. 
A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown.

(a) Find the magnetic flux $\Phi_B$ through the frame at the instant shown.
(b) Find the induced emf $\mathcal{E}$ at the instant shown.
(c) Find the direction (cw/ccw) of the induced current.

**Solution:**

(a) $\Phi_B = \vec{A} \cdot \vec{B} = \pm (20 \text{m}^2)(5 \text{T}) = \pm 100 \text{Wb}.$

(b) $\mathcal{E} = -\frac{d\Phi_B}{dt} = \pm (5 \text{T})(2 \text{m})(4 \text{m/s}) = \pm 40 \text{V}.$
A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown. 
(a) Find the magnetic flux $\Phi_B$ through the frame at the instant shown. 
(b) Find the induced emf $\mathcal{E}$ at the instant shown. 
(c) Find the direction (cw/ccw) of the induced current.

Solution:

(a) $\Phi_B = \vec{A} \cdot \vec{B} = \pm (20 \text{m}^2)(5 \text{T}) = \pm 100 \text{Wb}$. 
(b) $\mathcal{E} = -\frac{d\Phi_B}{dt} = \pm (5 \text{T})(2 \text{m})(4 \text{m/s}) = \pm 40 \text{V}$. 
(c) clockwise.
Consider two circular currents $I_1 = 3\, \text{A}$ at radius $r_1 = 2\, \text{m}$ and $I_2 = 5\, \text{A}$ at radius $r_2 = 4\, \text{m}$ in the directions shown.

(a) Find magnitude $B$ and direction ($\bigodot$, $\bigotimes$) of the resultant magnetic field at the center.

(b) Find magnitude $\mu$ and direction ($\bigodot$, $\bigotimes$) of the magnetic dipole moment generated by the two currents.
Consider two circular currents $I_1 = 3\,\text{A}$ at radius $r_1 = 2\,\text{m}$ and $I_2 = 5\,\text{A}$ at radius $r_2 = 4\,\text{m}$ in the directions shown.

(a) Find magnitude $B$ and direction $(\bigcirc, \bigotimes)$ of the resultant magnetic field at the center.

(b) Find magnitude $\mu$ and direction $(\bigcirc, \bigotimes)$ of the magnetic dipole moment generated by the two currents.

Solution:

(a) $B = \frac{\mu_0 (3\,\text{A})}{2(2\,\text{m})} - \frac{\mu_0 (5\,\text{A})}{2(4\,\text{m})} = (9.42 - 7.85) \times 10^{-7}\,\text{T}$

$\Rightarrow B = 1.57 \times 10^{-7}\,\text{T} \, \bigotimes$
Consider two circular currents $I_1 = 3A$ at radius $r_1 = 2m$ and $I_2 = 5A$ at radius $r_2 = 4m$ in the
directions shown.
(a) Find magnitude $B$ and direction ($\odot$, $\otimes$) of the resultant magnetic field at the center.
(b) Find magnitude $\mu$ and direction ($\odot$, $\otimes$) of the magnetic dipole moment generated by the two
currents.

Solution:

(a) $B = \frac{\mu_0 (3A)}{2(2m)} - \frac{\mu_0 (5A)}{2(4m)} = (9.42 - 7.85) \times 10^{-7}T$
$\Rightarrow B = 1.57 \times 10^{-7}T \otimes$

(b) $\mu = \pi (4m)^2 (5A) - \pi (2m)^2 (3A) = (251 - 38)Am^2$
$\Rightarrow \mu = 213Am^2 \odot$
(a) Consider a solid wire of radius $R = 3\text{mm}$.
Find magnitude $I$ and direction (in/out) that produces a magnetic field $B = 7 \mu T$ at radius $r = 8\text{mm}$.

(b) Consider a hollow cable with inner radius $R_{int} = 3\text{mm}$ and outer radius $R_{ext} = 5\text{mm}$.
A current $I_{out} = 0.9\text{A}$ is directed out of the plane.
Find direction (up/down) and magnitude $B_2, B_6$ of the magnetic field at radius $r_2 = 2\text{mm}$ and $r_6 = 6\text{mm}$, respectively.
(a) Consider a solid wire of radius $R = 3\text{mm}$.
Find magnitude $I$ and direction (in/out) that produces a magnetic field $B = 7\mu\text{T}$ at radius $r = 8\text{mm}$.
(b) Consider a hollow cable with inner radius $R_{int} = 3\text{mm}$ and outer radius $R_{ext} = 5\text{mm}$.
A current $I_{out} = 0.9\text{A}$ is directed out of the plane.
Find direction (up/down) and magnitude $B_2$, $B_6$ of the magnetic field at radius $r_2 = 2\text{mm}$ and $r_6 = 6\text{mm}$, respectively.

Solution:

(a) $7\mu\text{T} = \frac{\mu_0 I}{2\pi(8\text{mm})} \Rightarrow I = 0.28\text{A} \quad (\text{out}).$
(a) Consider a solid wire of radius $R = 3\text{mm}$.
Find magnitude $I$ and direction (in/out) that produces a magnetic field $B = 7\mu T$ at radius $r = 8\text{mm}$.
(b) Consider a hollow cable with inner radius $R_{\text{int}} = 3\text{mm}$ and outer radius $R_{\text{ext}} = 5\text{mm}$.
A current $I_{\text{out}} = 0.9\text{A}$ is directed out of the plane.
Find direction (up/down) and magnitude $B_2, B_6$ of the magnetic field at radius $r_2 = 2\text{mm}$ and $r_6 = 6\text{mm}$, respectively.

Solution:

(a) $7\mu T = \frac{\mu_0 I}{2\pi (8\text{mm})} \Rightarrow I = 0.28\text{A} \quad \text{(out)}$.
(b) $B_2 = 0$, $B_6 = \frac{\mu_0 (0.9\text{A})}{2\pi (6\text{mm})} = 30\mu T \quad \text{(up)}$. 
A circular wire of radius $r = 2.5\text{m}$ and resistance $R = 4.8\Omega$ is placed in the $yz$-plane as shown. A time-dependent magnetic field $\mathbf{B} = B_x \hat{i}$ is present. The dependence of $B_x$ on time is shown graphically.

(a) Find the magnitude $|\Phi_B^{(1)}|$ and $|\Phi_B^{(3)}|$ of the magnetic flux through the circle at times $t = 1\text{s}$ and $t = 3\text{s}$, respectively.

(b) Find magnitude $I_1$, $I_3$ and direction (cw/ccw) of the induced current at times $t = 1\text{s}$ and $t = 3\text{s}$, respectively.
A circular wire of radius \( r = 2.5 \text{ m} \) and resistance \( R = 4.8 \Omega \) is placed in the \( yz \)-plane as shown. A time-dependent magnetic field \( B = B_x \hat{i} \) is present. The dependence of \( B_x \) on time is shown graphically.

(a) Find the magnitude \( |\Phi_B(1)| \) and \( |\Phi_B(3)| \) of the magnetic flux through the circle at times \( t = 1 \text{s} \) and \( t = 3 \text{s} \), respectively.

(b) Find magnitude \( I_1 \), \( I_3 \) and direction (cw/ccw) of the induced current at times \( t = 1 \text{s} \) and \( t = 3 \text{s} \), respectively.

**Solution:**

(a) \( |\Phi_B(1)| = \pi (2.5 \text{ m})^2 (2 \text{T}) = 39.3 \text{ Wb} \),

\( |\Phi_B(3)| = \pi (2.5 \text{ m})^2 (1 \text{T}) = 19.6 \text{ Wb} \).

(b) \( \left| \frac{d\Phi_B(1)}{dt} \right| = 0 \Rightarrow I_1 = 0 \),

\( \left| \frac{d\Phi_B(3)}{dt} \right| = \pi (2.5 \text{ m})^2 (-1 \text{T/s}) = 19.6 \text{ V} \Rightarrow I_3 = \frac{19.6 \text{ V}}{4.8 \Omega} = 4.1 \text{ A} \) (ccw).
A triangular conducting loop in the $yz$-plane with a counterclockwise current $I = 3\, \text{A}$ is free to rotate about the axis $PQ$. A uniform magnetic field $\vec{B} = 0.5\, \text{T}\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle.
(b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle.
(c) Find the magnetic force $\vec{F}_H$ (magnitude and direction) acting on the long side (hypotenuse) of the triangle.
(d) Find the force $\vec{F}_R$ (magnitude and direction) that must be applied to the corner $R$ to keep the triangle from rotating.
A triangular conducting loop in the $yz$-plane with a counterclockwise current $I = 3 \text{A}$ is free to rotate about the axis $PQ$. A uniform magnetic field $\vec{B} = 0.5 \text{T} \hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle. 
(b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle.
(c) Find the magnetic force $\vec{F}_H$ (magnitude and direction) acting on the long side (hypotenuse) of the triangle.
(d) Find the force $\vec{F}_R$ (magnitude and direction) that must be applied to the corner $R$ to keep the triangle from rotating.

Solution:

(a) $\vec{\mu} = (3 \text{A})(32 \text{m}^2) \hat{i} = 96 \text{Am}^2 \hat{i}$. 

![Diagram of the triangular conducting loop with a counterclockwise current, a magnetic field, and forces and torques indicated]
A triangular conducting loop in the $yz$-plane with a counterclockwise current $I = 3A$ is free to rotate about the axis $PQ$. A uniform magnetic field $\vec{B} = 0.5T\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle. 
(b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle. 
(c) Find the magnetic force $\vec{F}_H$ (magnitude and direction) acting on the long side (hypotenuse) of the triangle. 
(d) Find the force $\vec{F}_R$ (magnitude and direction) that must be applied to the corner $R$ to keep the triangle from rotating.

**Solution:**

(a) $\vec{\mu} = (3A)(32m^2)\hat{i} = 96Am^2\hat{i}$. 
(b) $\vec{\tau} = \vec{\mu} \times \vec{B} = (96Am^2\hat{i}) \times (0.5T\hat{k}) = -48Nm\hat{j}$. 

![Diagram of triangular loop with magnetic field and forces]
A triangular conducting loop in the $yz$-plane with a counterclockwise current $I = 3A$ is free to rotate about the axis $PQ$. A uniform magnetic field $\vec{B} = 0.5T\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle. (b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle. (c) Find the magnetic force $\vec{F}_H$ (magnitude and direction) acting on the long side (hypotenuse) of the triangle. (d) Find the force $\vec{F}_R$ (magnitude and direction) that must be applied to the corner $R$ to keep the triangle from rotating.

Solution:

(a) $\vec{\mu} = (3A)(32m^2)\hat{i} = 96Am^2\hat{i}$.

(b) $\vec{\tau} = \vec{\mu} \times \vec{B} = (96Am^2\hat{i}) \times (0.5T\hat{k}) = -48Nm\hat{j}$.

(c) $F_H = (3A)(8\sqrt{2}m)(0.5T)(\sin 45°) = 12N \quad \odot$. 

Diagram:

- Triangle with sides labeled 8m,
- Magnetic field $\vec{B}$ pointing upwards,
- Current $I$ entering the plane from the bottom.
- Forces and moments indicated at various points.
A triangular conducting loop in the $yz$-plane with a counterclockwise current $I = 3A$ is free to rotate about the axis $PQ$. A uniform magnetic field $\vec{B} = 0.5T\hat{k}$ is present. (a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the triangle. (b) Find the magnetic torque $\vec{\tau}$ (magnitude and direction) acting on the triangle. (c) Find the magnetic force $\vec{F}_H$ (magnitude and direction) acting on the long side (hypotenuse) of the triangle. (d) Find the force $\vec{F}_R$ (magnitude and direction) that must be applied to the corner $R$ to keep the triangle from rotating.

**Solution:**

(a) $\vec{\mu} = (3A)(32m^2)\hat{i} = 96Am^2\hat{i}$.

(b) $\vec{\tau} = \vec{\mu} \times \vec{B} = (96Am^2\hat{i}) \times (0.5T\hat{k}) = -48Nm\hat{j}$.

(c) $F_H = (3A)(8\sqrt{2}m)(0.5T)(\sin 45^\circ) = 12N$ \(\odot\).

(d) $(-8m\hat{k}) \times \vec{F}_R = -\vec{\tau} = 48Nm\hat{j} \Rightarrow \vec{F}_R = -6N\hat{i}$. 
Two semi-infinite straight wires are connected to a curved wire in the form of a full circle, quarter circle, or half circle of radius $R = 1\text{m}$ in four different configurations. A current $I = 1\text{A}$ flows in the directions shown. Find magnitude $B_a, B_b, B_c, B_d$ and direction ($\bigodot/\bigotimes$) of the magnetic field thus generated at the points $a, b, c, d$. 
Two semi-infinite straight wires are connected to a curved wire in the form of a full circle, quarter circle, or half circle of radius $R = 1$ m in four different configurations. A current $I = 1$ A flows in the directions shown. Find magnitude $B_a$, $B_b$, $B_c$, $B_d$ and direction ($\circlearrowright$/$\circlearrowleft$) of the magnetic field thus generated at the points $a$, $b$, $c$, $d$.

**Solution:**

$$B_a = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R} \right| = |100\text{nT} + 628\text{nT} + 100\text{nT}| = 828\text{nT} \circlearrowright$$
Two semi-infinite straight wires are connected to a curved wire in the form of a full circle, quarter circle, or half circle of radius $R = 1\text{m}$ in four different configurations. A current $I = 1\text{A}$ flows in the directions shown. Find magnitude $B_a, B_b, B_c, B_d$ and direction ($\bigcirc/\bigotimes$) of the magnetic field thus generated at the points $a, b, c, d$.

Solution:

$$B_a = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R} \right| = |100nT + 628nT + 100nT| = 828nT \quad \bigotimes$$

$$B_b = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4\pi R} - \frac{\mu_0 I}{4\pi R} \right| = |100nT + 314nT - 100nT| = 314nT \quad \bigotimes$$
Two semi-infinite straight wires are connected to a curved wire in the form of a full circle, quarter circle, or half circle of radius $R = 1\text{ m}$ in four different configurations. A current $I = 1\text{ A}$ flows in the directions shown. Find magnitude $B_a, B_b, B_c, B_d$ and direction (⊙/⊗) of the magnetic field thus generated at the points $a, b, c, d$. 

Solution:

\[
B_a = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R} \right| = 100\text{nT} + 2 \times 628\text{nT} + 100\text{nT} = 828\text{nT} \quad \otimes
\]

\[
B_b = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R} - \frac{\mu_0 I}{4\pi R} \right| = 100\text{nT} + 3 \times 314\text{nT} - 100\text{nT} = 314\text{nT} \quad \otimes
\]

\[
B_c = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{8R} + 0 \right| = 100\text{nT} + \frac{1}{8} \times 157\text{nT} = 257\text{nT} \quad \otimes
\]
Two semi-infinite straight wires are connected to a curved wire in the form of a full circle, quarter circle, or half circle of radius $R = 1\text{m}$ in four different configurations. A current $I = 1\text{A}$ flows in the directions shown. Find magnitude $B_a, B_b, B_c, B_d$ and direction ($\bigcirc/\bigotimes$) of the magnetic field thus generated at the points $a, b, c, d$.

Solution:

$$B_a = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R} \right| = |100\text{nT} + 628\text{nT} + 100\text{nT}| = 828\text{nT} \bigotimes$$

$$B_b = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R} - \frac{\mu_0 I}{4\pi R} \right| = |100\text{nT} + 314\text{nT} - 100\text{nT}| = 314\text{nT} \bigotimes$$

$$B_c = \left| \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{8R} + 0 \right| = |100\text{nT} + 157\text{nT}| = 257\text{nT} \bigotimes$$

$$B_d = \left| \frac{\mu_0 I}{4\pi R} - \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R} \right| = |100\text{nT} - 628\text{nT} + 100\text{nT}| = 428\text{nT} \bigcirc$$
A pair of rails are connected by two mobile rods. A uniform magnetic field $B$ directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is $R = 0.2\Omega$ in each case. Find magnitude $I$ and direction (cw/ccw) of the induced current in each case.
A pair of rails are connected by two mobile rods. A uniform magnetic field \( B \) directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is \( R = 0.2\Omega \) in each case. Find magnitude \( I \) and direction (cw/ccw) of the induced current in each case.

**Solution:**

\[
(a) \ |\mathcal{E}| = (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}, \quad I = \frac{8.4\text{V}}{0.2\Omega} = 42\text{A} \quad \text{ccw}
\]
A pair of rails are connected by two mobile rods. A uniform magnetic field $B$ directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is $R = 0.2\ \Omega$ in each case. Find magnitude $I$ and direction (cw/ccw) of the induced current in each case.

**Solution:**

(a) $|\mathcal{E}| = (3\text{ m/s})(0.7\text{T})(4\text{ m}) = 8.4\text{ V}, \quad I = \frac{8.4\text{ V}}{0.2\Omega} = 42\text{ A} \quad \text{ccw}$

(b) $|\mathcal{E}| = (5\text{ m/s})(0.7\text{T})(4\text{ m}) = 14\text{ V}, \quad I = \frac{14\text{ V}}{0.2\Omega} = 70\text{ A} \quad \text{cw}$
A pair of rails are connected by two mobile rods. A uniform magnetic field \( B \) directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is \( R = 0.2\Omega \) in each case. Find magnitude \( I \) and direction (cw/ccw) of the induced current in each case.

\[ |\mathcal{E}| = (\text{velocity})(B)(\text{length}) \]

**Solution:**

(a) \[ |\mathcal{E}| = (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}, \quad I = \frac{8.4\text{V}}{0.2\Omega} = 42\text{A} \quad \text{ccw} \]

(b) \[ |\mathcal{E}| = (5\text{m/s})(0.7\text{T})(4\text{m}) = 14\text{V}, \quad I = \frac{14\text{V}}{0.2\Omega} = 70\text{A} \quad \text{cw} \]

(c) \[ |\mathcal{E}| = (5\text{m/s} - 3\text{m/s})(0.7\text{T})(4\text{m}) = 5.6\text{V}, \quad I = \frac{5.6\text{V}}{0.2\Omega} = 28\text{A} \quad \text{cw} \]
A pair of rails are connected by two mobile rods. A uniform magnetic field $B$ directed into the plane is present. In the situations (a), (b), (c), (d), one or both rods move at constant velocity as shown. The resistance of the conducting loop is $R = 0.2\Omega$ in each case. Find magnitude $I$ and direction (cw/ccw) of the induced current in each case.

Solution:

(a) $|\mathcal{E}| = (3\text{m/s})(0.7\text{T})(4\text{m}) = 8.4\text{V}$, $I = \frac{8.4\text{V}}{0.2\Omega} = 42\text{A}$ ccw

(b) $|\mathcal{E}| = (5\text{m/s})(0.7\text{T})(4\text{m}) = 14\text{V}$, $I = \frac{14\text{V}}{0.2\Omega} = 70\text{A}$ cw

(c) $|\mathcal{E}| = (5\text{m/s} - 3\text{m/s})(0.7\text{T})(4\text{m}) = 5.6\text{V}$, $I = \frac{5.6\text{V}}{0.2\Omega} = 28\text{A}$ cw

(d) $|\mathcal{E}| = (5\text{m/s} + 3\text{m/s})(0.7\text{T})(4\text{m}) = 22.4\text{V}$, $I = \frac{22.4\text{V}}{0.2\Omega} = 112\text{A}$ ccw
(a) Two very long straight wires carry currents as shown. A cube with edges of length 8cm serves as scaffold. Find the magnetic field at point $P$ in the form $\mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ with $B_x, B_y, B_z$ in SI units.

(b) Two circular currents of radius 5cm, one in the $xy$-lane and the other in the $yz$-plane, carry currents as shown. Both circles are centered at point $O$. Find the magnetic field at point $O$ in the form $\mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ with $B_x, B_y, B_z$ in SI units.
(a) Two very long straight wires carry currents as shown. A cube with edges of length 8cm serves as scaffold. Find the magnetic field at point $P$ in the form $\mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ with $B_x, B_y, B_z$ in SI units.

(b) Two circular currents of radius 5cm, one in the $xy$-lane and the other in the $yz$-plane, carry currents as shown. Both circles are centered at point $O$. Find the magnetic field at point $O$ in the form $\mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ with $B_x, B_y, B_z$ in SI units.

Solution:

(a) $B_x = 0, \quad B_y = \frac{\mu_0 (2A)}{2\pi (0.08m)} = 5 \mu T, \quad B_z = \frac{\mu_0 (3A)}{2\pi (0.08m)} = 7.5 \mu T$. 

4/11/2015 [tsl415 – 19/42]
Unit Exam III: Problem #1 (Spring ’11)

(a) Two very long straight wires carry currents as shown. A cube with edges of length 8cm serves as scaffold. Find the magnetic field at point $P$ in the form $\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$ with $B_x, B_y, B_z$ in SI units.

(b) Two circular currents of radius 5cm, one in the $xy$-lane and the other in the $yz$-plane, carry currents as shown. Both circles are centered at point $O$. Find the magnetic field at point $O$ in the form $\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$ with $B_x, B_y, B_z$ in SI units.

Solution:

(a) $B_x = 0, \quad B_y = \frac{\mu_0 (2A)}{2\pi (0.08m)} = 5\mu T, \quad B_z = \frac{\mu_0 (3A)}{2\pi (0.08m)} = 7.5\mu T.$

(b) $B_x = \frac{\mu_0 (2A)}{2(0.05m)} = 25.1\mu T, \quad B_y = 0, \quad B_z = -\frac{\mu_0 (3A)}{2(0.05m)} = -37.7\mu T.$
The coaxial cable shown has surfaces at radii 1mm, 3mm, and 5mm. The magnetic field is the same at radii 2mm and 6mm, namely $B = 7\mu T$ in the direction shown.

(a) Find magnitude (in SI units) and direction (in/out) of the current $I_{\text{int}}$ flowing through the inner conductor.

(b) Find magnitude (in SI units) and direction (in/out) of the current $I_{\text{ext}}$ flowing through the outer conductor.
The coaxial cable shown has surfaces at radii 1mm, 3mm, and 5mm. The magnetic field is the same at radii 2mm and 6mm, namely \( B = 7 \mu \text{T} \) in the direction shown.

(a) Find magnitude (in SI units) and direction (in/out) of the current \( I_{\text{int}} \) flowing through the inner conductor.

(b) Find magnitude (in SI units) and direction (in/out) of the current \( I_{\text{ext}} \) flowing through the outer conductor.

Solution:

\[
(7 \mu \text{T})(2\pi)(0.002 \text{m}) = \mu_0 I_{\text{int}} \quad \Rightarrow \quad I_{\text{int}} = 0.07 \text{A} \quad \text{(out)}
\]
The coaxial cable shown has surfaces at radii 1mm, 3mm, and 5mm. The magnetic field is the same at radii 2mm and 6mm, namely $\mathbf{B} = 7 \mu \text{T}$ in the direction shown.

(a) Find magnitude (in SI units) and direction (in/out) of the current $I_{\text{int}}$ flowing through the inner conductor.

(b) Find magnitude (in SI units) and direction (in/out) of the current $I_{\text{ext}}$ flowing through the outer conductor.

Solution:

(a) $(7 \mu \text{T})(2\pi)(0.002\text{m}) = \mu_0 I_{\text{int}}$ $\Rightarrow$ $I_{\text{int}} = 0.07\text{A}$ (out)

(b) $(7 \mu \text{T})(2\pi)(0.006\text{m}) = \mu_0 (I_{\text{int}} + I_{\text{ext}})$ $\Rightarrow$ $I_{\text{int}} + I_{\text{ext}} = 0.21\text{A}$ (out)

$\Rightarrow I_{\text{ext}} = 0.14\text{A}$ (out)
A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown. The rod moves at constant velocity.
(a) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame at the instant shown.
(b) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame two seconds later.
Write magnitudes only (in SI units), no directions.
A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown. The rod moves at constant velocity.

(a) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame at the instant shown.

(b) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame two seconds later.

Write magnitudes only (in SI units), no directions.

Solution:

(a) $\Phi_B = (20m^2)(3T) = 60Wb$, $\mathcal{E} = (2m/s)(3T)(2m) = 12V$. 
A conducting frame with a moving conducting rod is located in a uniform magnetic field as shown. The rod moves at constant velocity.

(a) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame at the instant shown.

(b) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame two seconds later.

Write magnitudes only (in SI units), no directions.

Solution:

(a) $\Phi_B = (20m^2)(3T) = 60\text{Wb}$, \quad $\mathcal{E} = (2m/s)(3T)(2m) = 12\text{V}$.

(b) $\Phi_B = (8m^2)(3T) = 24\text{Wb}$, \quad $\mathcal{E} = (2m/s)(3T)(4m) = 24\text{V}$. 
In the circuit shown we close the switch $S$ at time $t = 0$. Find the current $I_L$ through the inductor and the voltage $V_6$ across the $6\Omega$-resistor
(a) immediately after the switch has been closed,
(b) a very long time later.
In the circuit shown we close the switch $S$ at time $t = 0$. Find the current $I_L$ through the inductor and the voltage $V_6$ across the $6\Omega$-resistor
(a) immediately after the switch has been closed,
(b) a very long time later.

Solution:

(a) $I_L = 0$, $I_6 = \frac{12V}{10\Omega} = 1.2A$, $V_6 = (6\Omega)(1.2A) = 7.2V$. 
In the circuit shown we close the switch $S$ at time $t = 0$. Find the current $I_L$ through the inductor and the voltage $V_6$ across the 6Ω-resistor
(a) immediately after the switch has been closed,
(b) a very long time later.

Solution:

(a) $I_L = 0$, $I_6 = \frac{12V}{10\Omega} = 1.2A$, $V_6 = (6\Omega)(1.2A) = 7.2V$.
(b) $I_L = \frac{12V}{4\Omega} = 3A$, $V_6 = 0$. 
At time $t = 0$ the capacitor is charged to $Q_{\text{max}} = 4 \mu \text{C}$ and the switch is being closed. The charge on the capacitor begins to decrease and the current through the inductor begins to increase.

(a) At what time $t_1$ is the capacitor discharged for the first time?
(b) At what time $t_2$ has the current through the inductor returned to zero for the first time?
(c) What is the maximum energy stored in the capacitor at any time?
(d) What is the maximum energy stored in the inductor at any time?
At time $t = 0$ the capacitor is charged to $Q_{max} = 4\mu C$ and the switch is being closed. The charge on the capacitor begins to decrease and the current through the inductor begins to increase.

(a) At what time $t_1$ is the capacitor discharged for the first time?
(b) At what time $t_2$ has the current through the inductor returned to zero for the first time?
(c) What is the maximum energy stored in the capacitor at any time?
(d) What is the maximum energy stored in the inductor at any time?

**Solution:**

(a) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC} = 2.43\text{ms}$, \[ t_1 = \frac{T}{4} = 0.608\text{ms}. \]

(b) $t_2 = \frac{T}{2} = 1.22\text{ms}$.

(c) $U_{Cmax} = \frac{Q_{max}^2}{2C} = 1.6\mu J$.

(d) $U_{Lmax} = U_{Cmax} = 1.6\mu J$ (energy conservation.)
The ac voltage supplied in the circuit shown is $\mathcal{E} = \mathcal{E}_{max} \cos(\omega t)$ with $\mathcal{E}_{max} = 170$V and $\omega = 377$rad/s.

(a) What is the maximum value $I_{max}$ of the current?
(b) What is the emf $\mathcal{E}(t)$ at $t = 5$ms?
(c) What is the current $I(t)$ at $t = 5$ms?
(d) What is the power transfer $P(t)$ between ac source and device at $t = 5$ms?

$L = 40$mH
The ac voltage supplied in the circuit shown is $E = E_{max} \cos(\omega t)$ with $E_{max} = 170$ V and $\omega = 377$ rad/s.

(a) What is the maximum value $I_{max}$ of the current?
(b) What is the emf $E(t)$ at $t = 5$ ms?
(c) What is the current $I(t)$ at $t = 5$ ms?
(d) What is the power transfer $P(t)$ between ac source and device at $t = 5$ ms?

\[
L = 40 \text{ mH}
\]

Solution:

(a) $I_{max} = \frac{E_{max}}{\omega L} = \frac{170 \text{ V}}{(377 \text{ rad/s})(40 \text{ mH})} = 11.3 \text{ A}.$

(b) $E = (170 \text{ V}) \cos(1.885 \text{ rad}) = (170 \text{ V})(-0.309) = -52.5 \text{ V}.$

(c) $I = (11.3 \text{ A}) \cos(1.885 \text{ rad} - \pi/2) = (11.3 \text{ A}) \cos(0.314) = (11.3 \text{ A})(0.951) = 10.7 \text{ A}.$

(d) $P = EI = (-52.5 \text{ V})(10.7 \text{ A}) = -562 \text{ W}.$
In a region of uniform magnetic field $\mathbf{B} = 5 \text{mT} \hat{i}$, a proton $(m = 1.67 \times 10^{-27} \text{kg}, \, q = 1.60 \times 10^{-19} \text{C})$ is launched with velocity $\mathbf{v}_0 = 4000 \text{m/s} \hat{k}$.

(a) Calculate the magnitude $F$ of the magnetic force that keeps the proton on a circular path.

(b) Calculate the radius $r$ of the circular path.

(c) Calculate the time $T$ it takes the proton to go around that circle once.

(d) Sketch the circular path of the proton in the graph.
In a region of uniform magnetic field $\mathbf{B} = 5 \text{mT} \hat{i}$, a proton ($m = 1.67 \times 10^{-27} \text{kg}, \ q = 1.60 \times 10^{-19} \text{C}$) is launched with velocity $\mathbf{v}_0 = 4000 \text{m/s} \hat{k}$.

(a) Calculate the magnitude $F$ of the magnetic force that keeps the proton on a circular path.
(b) Calculate the radius $r$ of the circular path.
(c) Calculate the time $T$ it takes the proton to go around that circle once.
(d) Sketch the circular path of the proton in the graph.

Solution:

(a) $F = qv_0 B = 3.2 \times 10^{-18} \text{N}$. 
In a region of uniform magnetic field $B = 5 \text{mT} \hat{i}$, a proton $(m = 1.67 \times 10^{-27} \text{kg}, \ q = 1.60 \times 10^{-19} \text{C})$ is launched with velocity $v_0 = 4000 \text{m/s} \hat{k}$.

(a) Calculate the magnitude $F$ of the magnetic force that keeps the proton on a circular path.
(b) Calculate the radius $r$ of the circular path.
(c) Calculate the time $T$ it takes the proton to go around that circle once.
(d) Sketch the circular path of the proton in the graph.

**Solution:**

(a) $F = qv_0B = 3.2 \times 10^{-18} \text{N}$.

(b) $\frac{mv_0^2}{r} = qv_0B \ \Rightarrow \ r = \frac{mv_0}{qB} = 8.35 \text{mm}$. 
In a region of uniform magnetic field $B = 5\text{mT}\hat{i}$, a proton 
($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) is launched with velocity $v_0 = 4000\text{m/s}\hat{k}$.
(a) Calculate the magnitude $F$ of the magnetic force that keeps the proton on a circular path.
(b) Calculate the radius $r$ of the circular path.
(c) Calculate the time $T$ it takes the proton to go around that circle once.
(d) Sketch the circular path of the proton in the graph.

Solution:

(a) $F = qv_0B = 3.2 \times 10^{-18}\text{N}$.
(b) $\frac{mv_0^2}{r} = qv_0B \implies r = \frac{mv_0}{qB} = 8.35\text{mm}$.
(c) $T = \frac{2\pi r}{v_0} = \frac{2\pi m}{qB} = 13.1\mu\text{s}$.
In a region of uniform magnetic field $B = 5 \text{mT} \hat{i}$, a proton 
($m = 1.67 \times 10^{-27} \text{kg}$, $q = 1.60 \times 10^{-19} \text{C}$) is launched with velocity $v_0 = 4000 \text{m/s} \hat{k}$.

(a) Calculate the magnitude $F$ of the magnetic force that keeps the proton on a circular path.

(b) Calculate the radius $r$ of the circular path.

(c) Calculate the time $T$ it takes the proton to go around that circle once.

(d) Sketch the circular path of the proton in the graph.

**Solution:**

(a) $F = qv_0 B = 3.2 \times 10^{-18} \text{N}$.

(b) $\frac{mv_0^2}{r} = qv_0 B \quad \Rightarrow \quad r = \frac{mv_0^2}{qB} = 8.35 \text{mm}$.

(c) $T = \frac{2\pi r}{v_0} = \frac{2\pi m}{qB} = 13.1 \mu\text{s}$.

(d) Center of circle to the right of proton’s initial position (cw motion).
(a) Two very long straight wires positioned in the $xy$-plane carry electric currents $I_1$, $I_2$ as shown. Calculate magnitude ($B_1$, $B_2$) and direction ($\odot$, $\otimes$) of the magnetic field produced by each current at the origin of the coordinate system.

(b) A conducting loop of radius $r = 3\text{cm}$ placed in the $xy$-plane carries a current $I_3 = 0.7\text{A}$ in the direction shown. Find direction and magnitude of the torque $\vec{\tau}$ acting on the loop if it is placed in a magnetic field $\vec{B} = 5\text{mT}\hat{i}$. 
(a) Two very long straight wires positioned in the \(xy\)-plane carry electric currents \(I_1, I_2\) as shown. Calculate magnitude \((B_1, B_2)\) and direction \((\odot, \otimes)\) of the magnetic field produced by each current at the origin of the coordinate system.

(b) A conducting loop of radius \(r = 3\text{cm}\) placed in the \(xy\)-plane carries a current \(I_3 = 0.7\text{A}\) in the direction shown. Find direction and magnitude of the torque \(\vec{\tau}\) acting on the loop if it is placed in a magnetic field \(\vec{B} = 5\text{mT}\hat{i}\).

**Solution:**

(a) \(B_1 = \frac{\mu_0(3\text{A})}{2\pi(2\text{cm})} = 30\mu\text{T}\). \(\odot\)

\(B_2 = \frac{\mu_0(5\text{A})}{2\pi(1.41\text{cm})} = 70.9\mu\text{T}\). \(\odot\)
(a) Two very long straight wires positioned in the $xy$-plane carry electric currents $I_1, I_2$ as shown. Calculate magnitude ($B_1, B_2$) and direction ($\bigodot, \bigotimes$) of the magnetic field produced by each current at the origin of the coordinate system.

(b) A conducting loop of radius $r = 3\text{cm}$ placed in the $xy$-plane carries a current $I_3 = 0.7\text{A}$ in the direction shown. Find direction and magnitude of the torque $\vec{\tau}$ acting on the loop if it is placed in a magnetic field $\mathbf{B} = 5\text{mT}\hat{i}$.

Solution:

(a) $B_1 = \frac{\mu_0 (3\text{A})}{2\pi (2\text{cm})} = 30\mu\text{T}. \quad \bigodot \quad B_2 = \frac{\mu_0 (5\text{A})}{2\pi (1.41\text{cm})} = 70.9\mu\text{T}. \quad \bigotimes$

(b) $\vec{\mu} = \pi (3\text{cm})^2 (0.7\text{A})\hat{k} = 1.98 \times 10^{-3} \text{Am}^2\hat{k} \quad \Rightarrow \quad \vec{\tau} = \vec{\mu} \times \mathbf{B} = 9.90 \times 10^{-6} \text{Nm}\hat{j}.$
The coaxial cable shown in cross section has surfaces at radii 1mm, 3mm, and 5mm. Equal currents flow through both conductors: $I_{int} = I_{ext} = 0.03\text{A}$ (out). Find direction ($\uparrow$, $\downarrow$) and magnitude ($B_1$, $B_3$, $B_5$, $B_7$) of the magnetic field at the four radii indicated ($\circ$).
The coaxial cable shown in cross section has surfaces at radii 1mm, 3mm, and 5mm. Equal currents flow through both conductors: \( I_{int} = I_{ext} = 0.03A \) \( \odot \) (out). Find direction (↑, ↓) and magnitude \((B_1, B_3, B_5, B_7)\) of the magnetic field at the four radii indicated (●).

**Solution:**

\[
2\pi(1\text{mm})B_1 = \mu_0(0.03A) \quad \Rightarrow \quad B_1 = 6\mu T \quad \uparrow
\]
The coaxial cable shown in cross section has surfaces at radii 1mm, 3mm, and 5mm. Equal currents flow through both conductors: \( I_{int} = I_{ext} = 0.03\, \text{A} \) (out). Find direction (↑, ↓) and magnitude \((B_1, B_3, B_5, B_7)\) of the magnetic field at the four radii indicated (●).

Solution:

\[
2\pi(1\, \text{mm})B_1 = \mu_0(0.03\, \text{A}) \quad \Rightarrow \quad B_1 = 6\, \mu\text{T} \quad \uparrow
\]

\[
2\pi(3\, \text{mm})B_1 = \mu_0(0.03\, \text{A}) \quad \Rightarrow \quad B_1 = 2\, \mu\text{T} \quad \uparrow
\]
The coaxial cable shown in cross section has surfaces at radii 1mm, 3mm, and 5mm. Equal currents flow through both conductors: $I_{int} = I_{ext} = 0.03A$ (out). Find direction ($\uparrow$, $\downarrow$) and magnitude ($B_1$, $B_3$, $B_5$, $B_7$) of the magnetic field at the four radii indicated ($\bullet$).

Solution:

\[
2\pi(1\text{mm})B_1 = \mu_0(0.03\text{A}) \Rightarrow B_1 = 6\mu\text{T} \uparrow
\]
\[
2\pi(3\text{mm})B_1 = \mu_0(0.03\text{A}) \Rightarrow B_1 = 2\mu\text{T} \uparrow
\]
\[
2\pi(5\text{mm})B_1 = \mu_0(0.06\text{A}) \Rightarrow B_1 = 2.4\mu\text{T} \uparrow
\]
The coaxial cable shown in cross section has surfaces at radii 1mm, 3mm, and 5mm. Equal currents flow through both conductors: \( I_{int} = I_{ext} = 0.03\text{A} \odot \text{out} \). Find direction (↑, ↓) and magnitude \((B_1, B_3, B_5, B_7)\) of the magnetic field at the four radii indicated (●).

**Solution:**

\[
2\pi (1\text{mm}) B_1 = \mu_0 (0.03\text{A}) \implies B_1 = 6\mu T \uparrow \\
2\pi (3\text{mm}) B_1 = \mu_0 (0.03\text{A}) \implies B_1 = 2\mu T \uparrow \\
2\pi (5\text{mm}) B_1 = \mu_0 (0.06\text{A}) \implies B_1 = 2.4\mu T \uparrow \\
2\pi (7\text{mm}) B_1 = \mu_0 (0.06\text{A}) \implies B_1 = 1.71\mu T \uparrow \\
\]
In a region of uniform magnetic field $\mathbf{B}$ a proton ($m = 1.67 \times 10^{-27}\text{kg}, \ q = 1.60 \times 10^{-19}\text{C}$) experiences a force $\mathbf{F} = 9.0 \times 10^{-19}\text{N} \hat{i}$ as it passes through point $P$ with velocity $\mathbf{v}_0 = 3000\text{m/s} \hat{j}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius $r$ of the circular path.

(c) Locate the center $C$ of the circular path in the coordinate system on the page.
In a region of uniform magnetic field $\mathbf{B}$ a proton ($m = 1.67 \times 10^{-27}\text{kg}$, $q = 1.60 \times 10^{-19}\text{C}$) experiences a force $\mathbf{F} = 9.0 \times 10^{-19}\text{N}\hat{i}$ as it passes through point $P$ with velocity $\mathbf{v}_0 = 3000\text{m/s}\hat{j}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).
(b) Calculate the radius $r$ of the circular path.
(c) Locate the center $C$ of the circular path in the coordinate system on the page.

Solution:

(a) $B = \frac{F}{qv_0} = 1.88 \times 10^{-3}\text{T}$, $\hat{i} = \hat{j} \times \hat{k}$

$\Rightarrow \mathbf{B} = 1.88 \times 10^{-3}\text{T}\hat{k}.$

(b) $F = \frac{mv_0^2}{r} = qv_0B$

$\Rightarrow r = \frac{mv_0^2}{F} = \frac{mv_0}{qB} = 1.67\text{cm}$.

(c) $C = 4.67\text{cm}\hat{i} + 3.00\text{cm}\hat{j}$. 
In a region of uniform magnetic field $\mathbf{B}$ a proton ($m = 1.67 \times 10^{-27}$ kg, $q = 1.60 \times 10^{-19}$ C) experiences a force $\mathbf{F} = 8.0 \times 10^{-19} \text{ N} \hat{i}$ as it passes through point $P$ with velocity $\mathbf{v}_0 = 2000 \text{ m/s} \hat{k}$ on a circular path. 

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).  
(b) Calculate the radius $r$ of the circular path.  
(c) Locate the center $C$ of the circular path in the coordinate system on the page.
In a region of uniform magnetic field $\mathbf{B}$ a proton ($m = 1.67 \times 10^{-27} \text{kg}, \ q = 1.60 \times 10^{-19} \text{C}$) experiences a force $\mathbf{F} = 8.0 \times 10^{-19} \text{N} \hat{i}$ as it passes through point $P$ with velocity $\mathbf{v}_0 = 2000 \text{m/s} \hat{k}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).
(b) Calculate the radius $r$ of the circular path.
(c) Locate the center $C$ of the circular path in the coordinate system on the page.

Solution:

(a) $B = \frac{F}{qv_0} = 2.50 \times 10^{-3} \text{T}$, \quad $\hat{i} = \hat{k} \times (-\hat{j})$

$\Rightarrow \mathbf{B} = -2.50 \times 10^{-3} \text{T} \hat{j}.$
In a region of uniform magnetic field $B$ a proton ($m = 1.67 \times 10^{-27}$ kg, $q = 1.60 \times 10^{-19}$ C) experiences a force $F = 8.0 \times 10^{-19}$ N $\hat{i}$ as it passes through point $P$ with velocity $v_0 = 2000$ m/s $\hat{k}$ on a circular path.

(a) Find the magnetic field $B$ (magnitude and direction).
(b) Calculate the radius $r$ of the circular path.
(c) Locate the center $C$ of the circular path in the coordinate system on the page.

Solution:

(a) $B = \frac{F}{qv_0} = 2.50 \times 10^{-3}$ T, $\hat{i} = \hat{k} \times (-\hat{j})$

$\Rightarrow B = -2.50 \times 10^{-3}$ T $\hat{j}$.

(b) $F = \frac{mv_0^2}{r} = qv_0 B$

$\Rightarrow r = \frac{mv_0^2}{F} = \frac{mv_0}{qB} = 0.835$ cm.
In a region of uniform magnetic field $\mathbf{B}$ a proton ($m = 1.67 \times 10^{-27}\text{kg},\ q = 1.60 \times 10^{-19}\text{C}$) experiences a force $\mathbf{F} = 8.0 \times 10^{-19}\text{N}\hat{i}$ as it passes through point $P$ with velocity $\mathbf{v}_0 = 2000\text{m/s}\hat{k}$ on a circular path.

(a) Find the magnetic field $\mathbf{B}$ (magnitude and direction).

(b) Calculate the radius $r$ of the circular path.

(c) Locate the center $C$ of the circular path in the coordinate system on the page.

Solution:

(a) $B = \frac{F}{qv_0} = 2.50 \times 10^{-3}\text{T},\ \hat{i} = \hat{k} \times (-\hat{j})$

$\Rightarrow \mathbf{B} = -2.50 \times 10^{-3}\text{T}\hat{j}$.

(b) $F = \frac{mv_0^2}{r} = qv_0B$

$\Rightarrow r = \frac{mv_0^2}{F} = \frac{mv_0}{qB} = 0.835\text{cm}$.

(c) $C = 3.84\text{cm}\hat{i} + 3.00\text{cm}\hat{k}$.
A very long, straight wire is positioned along the $x$-axis and a circular wire of 1.5 cm radius in the $yz$ plane with its center $P$ on the $z$-axis as shown. Both wires carry a current $I = 0.6 \text{ A}$ in the directions shown.

(a) Find the magnetic field $\mathbf{B}_c$ (magnitude and direction) generated at point $P$ by the current in the circular wire.

(b) Find the magnetic field $\mathbf{B}_s$ (magnitude and direction) generated at point $P$ by the current in the straight wire.

(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the circular current.
A very long, straight wire is positioned along the $x$-axis and a circular wire of 1.5cm radius in the $yz$ plane with its center $P$ on the $z$-axis as shown. Both wires carry a current $I = 0.6A$ in the directions shown.

(a) Find the magnetic field $\mathbf{B}_c$ (magnitude and direction) generated at point $P$ by the current in the circular wire.

(b) Find the magnetic field $\mathbf{B}_s$ (magnitude and direction) generated at point $P$ by the current in the straight wire.

(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the circular current.

Solution:

(a) $\mathbf{B}_c = \frac{\mu_0 (0.6A)}{2(0.015m)} (-\hat{i}) = -2.51 \times 10^{-5} \text{T} \hat{i}$.

(b) $\mathbf{B}_s = \frac{\mu_0 (0.6A)}{2\pi (0.03m)} (-\hat{j}) = -4.00 \times 10^{-6} \text{T} \hat{j}$.

(c) $\vec{\mu} = \pi (0.015mm)^2 (0.6A)(-\hat{i}) = -4.24 \times 10^{-4} \text{Am}^2 \hat{i}$. 
A very long straight wire is positioned along the $x$-axis and a circular wire of 2.0 cm radius in the $yz$ plane with its center $P$ on the $y$-axis as shown. Both wires carry a current $I = 0.5 \text{ A}$ in the directions shown.

(a) Find the magnetic field $\mathbf{B}_c$ (magnitude and direction) generated at point $P$ by the current in the circular wire.

(b) Find the magnetic field $\mathbf{B}_s$ (magnitude and direction) generated at point $P$ by the current in the straight wire.

(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the circular current.
A very long straight wire is positioned along the $x$-axis and a circular wire of 2.0cm radius in the $yz$ plane with its center $P$ on the $y$-axis as shown. Both wires carry a current $I = 0.5A$ in the directions shown.

(a) Find the magnetic field $B_c$ (magnitude and direction) generated at point $P$ by the current in the circular wire.

(b) Find the magnetic field $B_s$ (magnitude and direction) generated at point $P$ by the current in the straight wire.

(c) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the circular current.

Solution:

(a) $B_c = \frac{\mu_0 (0.5A)}{2(0.02m)} \hat{i} = 1.57 \times 10^{-5} \text{T} \hat{i}$.

(b) $B_s = \frac{\mu_0 (0.5A)}{2\pi (0.035m)} (-\hat{k}) = -2.86 \times 10^{-6} \text{T} \hat{k}$.

(c) $\vec{\mu} = \pi (0.02m)^2 (0.5A) \hat{i} = 6.28 \times 10^{-4} \text{Am}^2 \hat{i}$. 
Consider a wire with a resistance per unit length of $1\,\Omega/cm$ bent into a rectangular loop and placed into the $yz$-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (2\mathbf{i} + 1\mathbf{j} + 3\mathbf{k})t\,T/s$, where $t$ is the time in seconds.

(a) Find the magnetic flux $\Phi_B$ through the rectangle at time $t = 2s$.
(b) Find magnitude and direction (cw/ccw) of the induced EMF $\mathcal{E}$ around the rectangle at time $t = 2s$.
(c) Infer the induced current $I$ from the induced EMF.
Consider a wire with a resistance per unit length of $1\Omega/cm$ bent into a rectangular loop and placed into the $yz$-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (2\hat{i} + 1\hat{j} + 3\hat{k})T/s$, where $t$ is the time in seconds.

(a) Find the magnetic flux $\Phi_B$ through the rectangle at time $t = 2s$.
(b) Find magnitude and direction (cw/ccw) of the induced EMF $\mathcal{E}$ around the rectangle at time $t = 2s$.
(c) Infer the induced current $I$ from the induced EMF.

Solution:

(a) $\Phi_B = \pm(4\text{cm})(3\text{cm})(2T/s)(2s) = \pm4.8 \times 10^{-3}\text{Wb}$
Consider a wire with a resistance per unit length of $1\,\Omega/cm$ bent into a rectangular loop and placed into the $yz$-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (2\hat{i} + 1\hat{j} + 3\hat{k})t\,T/s$, where $t$ is the time in seconds.

(a) Find the magnetic flux $\Phi_B$ through the rectangle at time $t = 2s$.

(b) Find magnitude and direction (cw/ccw) of the induced EMF $\mathcal{E}$ around the rectangle at time $t = 2s$.

(c) Infer the induced current $I$ from the induced EMF.

**Solution:**

(a) $\Phi_B = \pm(4\,\text{cm})(3\,\text{cm})(2\,\text{T}/s)(2\,\text{s}) = \pm4.8 \times 10^{-3}\,\text{Wb}$

(b) $\mathcal{E} = \mp(4\,\text{cm})(3\,\text{cm})(2\,\text{T}/s) = \mp2.4\,\text{mV}$ (cw)
Consider a wire with a resistance per unit length of $1\Omega/cm$ bent into a rectangular loop and placed into the $yz$-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (2\hat{i} + 1\hat{j} + 3\hat{k})t \text{T/s}$, where $t$ is the time in seconds.

(a) Find the magnetic flux $\Phi_B$ through the rectangle at time $t = 2s$.

(b) Find magnitude and direction (cw/ccw) of the induced EMF $\mathcal{E}$ around the rectangle at time $t = 2s$.

(c) Infer the induced current $I$ from the induced EMF.

Solution:

(a) $\Phi_B = \pm(4 \text{cm})(3 \text{cm})(2 \text{T/s})(2s) = \pm 4.8 \times 10^{-3} \text{Wb}$

(b) $\mathcal{E} = \mp(4 \text{cm})(3 \text{cm})(2 \text{T/s}) = \mp 2.4 \text{mV}$ (cw)

(c) $I = \frac{2.4 \text{mV}}{(1\Omega/cm)(14\text{cm})} = 0.171 \text{mA}$
Consider a wire with a resistance per unit length of $1 \Omega/cm$ bent into a rectangular loop and placed into the $yz$-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (3\hat{i} + 1\hat{j} + 2\hat{k})t T/s$, where $t$ is the time in seconds.

(a) Find the magnetic flux $\Phi_B$ through the rectangle at time $t = 2s$.

(b) Find magnitude and direction (cw/ccw) of the induced EMF $\mathcal{E}$ around the rectangle at time $t = 2s$.

(c) Infer the induced current $I$ from the induced EMF.
Consider a wire with a resistance per unit length of $1\Omega/cm$ bent into a rectangular loop and placed into the $yz$-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (3\hat{i} + 1\hat{j} + 2\hat{k})t T/s$, where $t$ is the time in seconds.
(a) Find the magnetic flux $\Phi_B$ through the rectangle at time $t = 2s$.
(b) Find magnitude and direction (cw/ccw) of the induced EMF $\mathcal{E}$ around the rectangle at time $t = 2s$.
(c) Infer the induced current $I$ from the induced EMF.

Solution:

(a) $\Phi_B = \pm (5cm)(3cm)(3T/s)(2s) = \pm 9.0 \times 10^{-3}Wb$
Consider a wire with a resistance per unit length of $1 \Omega/cm$ bent into a rectangular loop and placed into the $yz$-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (3\mathbf{i} + 1\mathbf{j} + 2\mathbf{k})t \text{T/s}$, where $t$ is the time in seconds.

(a) Find the magnetic flux $\Phi_B$ through the rectangle at time $t = 2s$.
(b) Find magnitude and direction (cw/ccw) of the induced EMF $\mathcal{E}$ around the rectangle at time $t = 2s$.
(c) Infer the induced current $I$ from the induced EMF.

Solution:

(a) $\Phi_B = \pm (5\text{cm})(3\text{cm})(3 \text{T/s})(2\text{s}) = \pm 9.0 \times 10^{-3} \text{Wb}$

(b) $\mathcal{E} = \mp (5\text{cm})(3\text{cm})(3 \text{T/s}) = \mp 4.5 \text{mV} \quad (\text{cw})$
Consider a wire with a resistance per unit length of $1 \Omega/cm$ bent into a rectangular loop and placed into the $yz$-plane as shown. The magnetic field in the entire region is uniform and increases from zero as follows: $\mathbf{B} = (3\hat{i} + 1\hat{j} + 2\hat{k})tT/s$, where $t$ is the time in seconds.

(a) Find the magnetic flux $\Phi_B$ through the rectangle at time $t = 2s$.
(b) Find magnitude and direction (cw/ccw) of the induced EMF $\mathcal{E}$ around the rectangle at time $t = 2s$.
(c) Infer the induced current $I$ from the induced EMF.

Solution:

(a) $\Phi_B = \pm (5cm)(3cm)(3T/s)(2s) = \pm 9.0 \times 10^{-3} \text{Wb}$
(b) $\mathcal{E} = \mp (5cm)(3cm)(3T/s) = \mp 4.5 \text{mV}$ (cw)
(c) $I = \frac{4.5 \text{mV}}{(1\Omega/cm)(16cm)} = 0.281 \text{mA}$
A counterclockwise current $I = 1.7 \text{A}$ [$I = 1.3 \text{A}$] is flowing through the conducting rectangular frame shown in a region of magnetic field $\mathbf{B} = 6 \text{mT} \hat{j}$ [$\mathbf{B} = 6 \text{mT} \hat{k}$].

(a) Find the force $\mathbf{F}_{bc}$ [$\mathbf{F}_{ab}$] (magnitude and direction) acting on side $bc$ [$ab$] of the rectangle.
(b) Find the magnetic moment $\mathbf{\mu}$ (magnitude and direction) of the current loop.
(c) Find the torque $\mathbf{\tau}$ (magnitude and direction) acting on the current loop.
A counterclockwise current $I = 1.7A$ [$I = 1.3A$] is flowing through the conducting rectangular frame shown in a region of magnetic field $B = 6mT\hat{j}$ [$B = 6mT\hat{k}$].

(a) Find the force $\mathbf{F}_{bc}$ [$\mathbf{F}_{ab}$] (magnitude and direction) acting on side $bc$ [$ab$] of the rectangle.

(b) Find the magnetic moment $\mathbf{\mu}$ (magnitude and direction) of the current loop.

(c) Find the torque $\mathbf{\tau}$ (magnitude and direction) acting on the current loop.

**Solution:**

(a) $\mathbf{F}_{bc} = (1.7A)(3cm\hat{k}) \times (6mT\hat{j}) = -3.06 \times 10^{-4}N\hat{i}$

$\mathbf{F}_{ab} = (1.3A)(2cm\hat{j}) \times (6mT\hat{k}) = 1.56 \times 10^{-4}N\hat{i}$

(b) $\mathbf{\mu} = [(2cm)(3cm)\hat{i}](1.7A) = 1.02 \times 10^{-3}Am^2\hat{i}$

$\mathbf{\mu} = [(2cm)(3cm)\hat{i}](1.3A) = 7.8 \times 10^{-4}Am^2\hat{i}$

(c) $\mathbf{\tau} = (1.02 \times 10^{-3}Am^2\hat{i}) \times (6mT\hat{j}) = 6.12 \times 10^{-6}Nm\hat{k}$

$\mathbf{\tau} = (7.8 \times 10^{-4}Am^2\hat{i}) \times (6mT\hat{k}) = -4.68 \times 10^{-6}Nm\hat{j}$
(a) Find the magnetic field $\mathbf{B}_a$ (magnitude and direction) generated by the three long, straight currents $I_1 = I_2 = I_3 = 1.8\text{mA}$ [$2.7\text{mA}$] in the directions shown.

(b) Find the magnetic field $\mathbf{B}_b$ (magnitude and direction) generated by the two circular currents $I_5 = I_6 = 1.5\text{mA}$ [$2.5\text{mA}$] in the directions shown.
(a) Find the magnetic field $B_a$ (magnitude and direction) generated by the three long, straight currents $I_1 = I_2 = I_3 = 1.8\text{mA} [2.7\text{mA}]$ in the directions shown.

(b) Find the magnetic field $B_b$ (magnitude and direction) generated by the two circular currents $I_5 = I_6 = 1.5\text{mA} [2.5\text{mA}]$ in the directions shown.

Solution:

(a) $B_a = \frac{\mu_0(1.8\text{mA})}{2\pi(9\text{cm})} = 4 \times 10^{-9}\text{T} \quad \text{(directed ←)}$

$[B_a = \frac{\mu_0(2.7\text{mA})}{2\pi(9\text{cm})} = 6 \times 10^{-9}\text{T} \quad \text{(directed ←)}]$

(b) $B_b = \frac{\mu_0(1.5\text{mA})}{2(4\text{cm})} - \frac{\mu_0(1.5\text{mA})}{2(8\text{cm})} = 1.18 \times 10^{-8}\text{T} \quad \text{(directed ⊗)}$

$[B_b = \frac{\mu_0(2.5\text{mA})}{2(4\text{cm})} - \frac{\mu_0(2.5\text{mA})}{2(8\text{cm})} = 1.96 \times 10^{-8}\text{T} \quad \text{(directed ⊗)}]$
Consider a region of uniform magnetic field $\mathbf{B} = (3\hat{i} + 2\hat{j} + 1\hat{k}) \text{mT}$ [$\mathbf{B} = (2\hat{i} + 3\hat{j} + 1\hat{k}) \text{mT}$]. A conducting rod slides along conducting rails in the $yz$-plane as shown. The rails are connected on the right. The clock is set to $t = 0$ at the instant shown.

(a) Find the magnetic flux $\Phi_B$ through the conducting loop at $t = 0$.
(b) Find the magnetic flux $\Phi_B$ through the conducting loop at $t = 1\text{s}$.
(c) Find the induced EMF.
(d) Find the direction (cw/ccw) of the induced current.
Consider a region of uniform magnetic field $\mathbf{B} = (3\hat{i} + 2\hat{j} + 1\hat{k})\text{mT}$ [$\mathbf{B} = (2\hat{i} + 3\hat{j} + 1\hat{k})\text{mT}$]. A conducting rod slides along conducting rails in the $yz$-plane as shown. The rails are connected on the right. The clock is set to $t = 0$ at the instant shown.

(a) Find the magnetic flux $\Phi_B$ through the conducting loop at $t = 0$.
(b) Find the magnetic flux $\Phi_B$ through the conducting loop at $t = 1\text{s}$.
(c) Find the induced EMF.
(d) Find the direction (cw/ccw) of the induced current.

Solution:

(a) $\Phi_B = (3\text{cm})(2\text{cm})(3\text{mT}) = 1.8 \times 10^{-6}\text{Wb}$
  
  [$\Phi_B = (3\text{cm})(2\text{cm})(2\text{mT}) = 1.2 \times 10^{-6}\text{Wb}$]

(b) $\Phi_B = (4\text{cm})(2\text{cm})(3\text{mT}) = 2.4 \times 10^{-6}\text{Wb}$
  
  [$\Phi_B = (4\text{cm})(2\text{cm})(2\text{mT}) = 1.6 \times 10^{-6}\text{Wb}$]

(c) $\mathcal{E} = (1\text{cm/s})(3\text{mT})(2\text{cm}) = 6 \times 10^{-7}\text{V}$
  
  [$\mathcal{E} = (1\text{cm/s})(2\text{mT})(2\text{cm}) = 4 \times 10^{-7}\text{V}$]

(d) cw [cw]
Consider two infinitely long, straight wires with currents $I_a = 7\, \text{A}$, $I_b = 9\, \text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $B_1$, $B_2$, $B_3$ at the points marked in the graph.
Consider two infinitely long, straight wires with currents $I_a = 7\text{A}$, $I_b = 9\text{A}$ in the directions shown. Find direction (in/out) and magnitude of the magnetic fields $B_1$, $B_2$, $B_3$ at the points marked in the graph.

Solution:

- Convention used: out = positive, in = negative
- $B_1 = \frac{\mu_0}{2\pi} \left( \frac{7\text{A}}{6\text{m}} - \frac{9\text{A}}{3\text{m}} \right) = -0.367\mu\text{T}$ (in).
- $B_2 = \frac{\mu_0}{2\pi} \left( \frac{7\text{A}}{3\text{m}} - \frac{9\text{A}}{3\text{m}} \right) = -0.133\mu\text{T}$ (in).
- $B_3 = \frac{\mu_0}{2\pi} \left( \frac{7\text{A}}{3\text{m}} - \frac{9\text{A}}{6\text{m}} \right) = +0.167\mu\text{T}$ (out).
Consider the (piecewise rectangular) conducting loop in the $xy$-plane as shown with a counterclockwise current $I = 4\, \text{A}$ in a uniform magnetic field $\vec{B} = 2\text{T}\hat{j}$.

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side $ab$ of the rectangle.

(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.
Consider the (piecewise rectangular) conducting loop in the $xy$-plane as shown with a counterclockwise current $I = 4\, \text{A}$ in a uniform magnetic field $\vec{B} = 2\, \text{T}\hat{j}$.

(a) Find the magnetic moment $\vec{\mu}$ (magnitude and direction) of the loop.

(b) Find the force $\vec{F}$ (magnitude and direction) acting on the side $ab$ of the rectangle.

(c) Find the torque $\vec{\tau}$ (magnitude and direction) acting on the loop.

**Solution:**

(a) $\vec{\mu} = (4\, \text{A})(75\, \text{m}^2)\hat{k} = 300\, \text{Am}^2\hat{k}$.

(b) $\vec{F} = I\vec{L} \times \vec{B} = (4\, \text{A})(10\, \text{m}\hat{i}) \times (2\, \text{T}\hat{j}) = 80\, \text{N}\hat{k}$.

(c) $\vec{\tau} = \vec{\mu} \times \vec{B} = (300\, \text{Am}^2\hat{k}) \times (2\, \text{T}\hat{j}) = -600\, \text{Nm}\hat{i}$
A conducting frame with a moving conducting rod is located in a uniform magnetic field directed out of the plane as shown. The rod moves at constant velocity.

(a) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame at the instant shown.

(b) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame two seconds later.

Write magnitudes only (in SI units), no directions.

$B = 5 \text{T}$

$v = 2 \text{m/s}$
A conducting frame with a moving conducting rod is located in a uniform magnetic field directed out of the plane as shown. The rod moves at constant velocity.

(a) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame at the instant shown.
(b) Find the magnetic flux $\Phi_B$ through the frame and the induced emf $\mathcal{E}$ around the frame two seconds later.
Write magnitudes only (in SI units), no directions.

**Solution:**

(a) $\Phi_B = (16m^2)(5T) = 80\text{Wb}$, \hspace{1cm} $\mathcal{E} = (2\text{m/s})(5T)(4m) = 40\text{V}$.

(b) $\Phi_B = (4m^2)(5T) = 20\text{Wb}$, \hspace{1cm} $\mathcal{E} = (2\text{m/s})(5T)(2m) = 20\text{V}$. 
A clockwise current $I = 2.1\text{A}$ is flowing around the conducting triangular frame shown in a region of uniform magnetic field $\vec{B} = -3\text{mT}\hat{j}$.

(a) Find the force $\vec{F}_{ab}$ acting on side $ab$ of the triangle.

(b) Find the force $\vec{F}_{bc}$ acting on side $bc$ of the triangle.

(c) Find the magnetic moment $\vec{\mu}$ of the current loop.

(d) Find the torque $\vec{\tau}$ acting on the current loop.

Remember that vectors have components or magnitude and direction.
A clockwise current \( I = 2.1\text{A} \) is flowing around the conducting triangular frame shown in a region of uniform magnetic field \( \vec{B} = -3\text{mT}\hat{j} \).

(a) Find the force \( \vec{F}_{ab} \) acting on side \( ab \) of the triangle.
(b) Find the force \( \vec{F}_{bc} \) acting on side \( bc \) of the triangle.
(c) Find the magnetic moment \( \vec{\mu} \) of the current loop.
(d) Find the torque \( \vec{\tau} \) acting on the current loop.

Remember that vectors have components or magnitude and direction.

**Solution:**

(a) \( \vec{F}_{ab} = (2.1\text{A})(-2\text{cm}\hat{k}) \times (-3\text{mT}\hat{j}) = -1.26 \times 10^{-4}\text{N}\hat{i} \).

(b) \( \vec{F}_{bc} = 0 \).

(c) \( \vec{\mu} = \left[-\frac{1}{2}(2\text{cm})(2\text{cm})\hat{i}\right](2.1\text{A}) = -4.2 \times 10^{-4}\text{Am}^2\hat{i} \).

(d) \( \vec{\tau} = (-4.2 \times 10^{-4}\text{Am}^2\hat{i}) \times (-3\text{mT}\hat{j}) = 1.26 \times 10^{-6}\text{Nm}\hat{k} \).
Consider four long, straight currents in four different configurations. All currents are $I = 4\, \text{mA}$ in the directions shown (⊗ = in, ⊙ = out). Find the magnitude (in SI units) and the direction (←, →, ↑, ↓) of the magnetic fields $B_1, B_2, B_3, B_4$ generated at the points 1, . . . , 4, respectively.
Consider four long, straight currents in four different configurations. All currents are \( I = 4\text{mA} \) in the directions shown (⊗ = in, ⊙ = out). Find the magnitude (in SI units) and the direction (←, →, ↑, ↓) of the magnetic fields \( B_1, B_2, B_3, B_4 \) generated at the points 1, ..., 4, respectively.

\[
B_1 = 2 \frac{\mu_0(4\text{mA})}{2\pi(3\text{cm})} = 5.33 \times 10^{-8}\text{T} \quad \text{(directed \( \downarrow \)).}
\]

\[
B_2 = 0 \quad \text{(no direction).}
\]

\[
B_3 = 2 \frac{\mu_0(4\text{mA})}{2\pi(2\text{cm})} = 8.00 \times 10^{-8}\text{T} \quad \text{(directed \( \rightarrow \)).}
\]

\[
B_4 = 0 \quad \text{(no direction).}
\]
A wire shaped into a triangle has resistance $R = 3.5\Omega$ and is placed in the $yz$-plane as shown. A uniform time-dependent magnetic field $B = B_x(t)\hat{i}$ is present. The dependence of $B_x$ on time is shown graphically.

(a) Find magnitude $|\Phi_B^{(1)}|$ and $|\Phi_B^{(4)}|$ of the magnetic flux through the triangle at times $t = 1\,\text{s}$ and $t = 4\,\text{s}$, respectively.

(b) Find magnitude $I_1, I_4$ and direction (cw/ccw) of the induced current at times $t = 1\,\text{s}$ and $t = 4\,\text{s}$, respectively.
A wire shaped into a triangle has resistance $R = 3.5\, \Omega$ and is placed in the $yz$-plane as shown. A uniform time-dependent magnetic field $\mathbf{B} = B_x(t) \hat{i}$ is present. The dependence of $B_x$ on time is shown graphically.

(a) Find magnitude $|\Phi_B^{(1)}|$ and $|\Phi_B^{(4)}|$ of the magnetic flux through the triangle at times $t = 1\, \text{s}$ and $t = 4\, \text{s}$, respectively.

(b) Find magnitude $I_1$, $I_4$ and direction (cw/ccw) of the induced current at times $t = 1\, \text{s}$ and $t = 4\, \text{s}$, respectively.

Solution:

(a) $|\Phi_B^{(1)}| = |(2\, \text{m}^2)(-2\, \text{T})| = 4.0\, \text{Wb}$,

$|\Phi_B^{(4)}| = |(2\, \text{m}^2)(0\, \text{T})| = 0$.

(b) $\left| d\Phi_B^{(1)} \right| = \left| A \frac{dB}{dt} \right| = |(2\, \text{m}^2)(0\, \text{T}/\text{s})| = 0$

$\Rightarrow I_1 = 0$,

$\left| d\Phi_B^{(4)} \right| = \left| A \frac{dB}{dt} \right| = |(2\, \text{m}^2)(1\, \text{T}/\text{s})| = 2.0\, \text{V}$

$\Rightarrow I_4 = \frac{2.0\, \text{V}}{3.5\, \Omega} = 0.571\, \text{A} \quad (\text{cw})$. 