APPENDIX B: GAMS INPUT CODING

TEST INPUT

```
Sets
   l     external suppliers   / A*D /
   m      producers            / 1*3 /
   n      markets              / 1*5 /
   r      recipes              / 1*7 / ;

alias (m,mp)    ;

Parameters
 S(l)  capacity of external supplier l in UNITS
 /  A    30000
    B    30000
    C    30000
    D    30000 /

 D(n)  demand at market n in UNITS
 /  1       500
    2       500
    3       500
    4       500
    5       500 /

 wi(l)  weighted cost of shipping l in DOLLARS PER UNIT
 /  A  1
    B  1
    C  1
    D  1   /

 wo(n)  weighted cost of shipping n in DOLLARS PER UNIT
 /  1  1
    2  1
    3  1
    4  1
```

5 1 /;

Table dx(l,m) distance from supplier l to producer m in MILES

	1	2	3
A	2	2	2
B	2	2	2
C	2	2	2
D	2	2	2 ;

Table dy(m,mp) distance from producer m to producer mp in MILES

	1	2	3
1	0	2	2
2	2	0	2
3	2	2	0 ;

Table dz(m,n) distance from producer m to market n in MILES

	1	2	3	4	5
1	2	2	2	2	2
2	2	2	2	2	2
3	2	2	2	2	2 ;

Table hrm(r,m) recipes r used at each producer m

	1	2	3
1	1	0	0
2	1	0	0
3	0	1	0
4	0	1	0
5	0	1	0
6	0	0	1
7	0	0	1 ;

Table a(r,l) amount of input l used in recipe r in UNITS

	A	B	C	D
1	4	5	0	0
2	0	3	1.5	0
3	1	0	0	4
4	3.5	5.5	0	0

5 0 0 1 2

6 0 0 2 6

7 0.5 0 1 0 ;

Table bz(r,n) amount of output n produced by recipe r in UNITS

	1	2	3	4	5
1	1.0	0.0	0.0	0.0	0.0
2	0.0	4.2	0.0	0.0	0.0
3	0.0	0.0	3.0	0.0	0.0
4	1.8	0.0	0.0	0.0	0.0
5	0.0	1.0	0.0	2.0	0.0
6	0.0	0.0	0.5	1.5	0.0
7	0.0	0.0	0.0	0.5	0.5 ;

Table by(r,l) amount of input l produced by recipe r in UNITS

	A	B	C	D
1	0.0	0.0	0.0	2.0
2	0.0	0.0	0.0	1.5
3	0.0	0.0	0.0	0.0
4	0.0	0.0	2.0	0.0
5	0.0	0.0	0.5	0.0
6	0.0	2.0	0.0	0.0
7	0.0	1.0	0.0	0.0 ;

Parameter cx(l,m) cost of transporting input l from supplier l to producer m in DOLLARS PER UNIT;

cx(l,m) = dx(l,m) * wi(l) ;

Parameter cy(l,m,mp) cost of transporting input l from producer m to producer mp in DOLLARS PER UNIT;

cy(l,m,mp) = dy(m,mp) * wi(l) ;

Parameter cz(m,n) cost of transporting output n from producer m to market n in DOLLARS PER UNIT;

cz(m,n) = dz(m,n) * wo(n) ;

Variables

x(l,m) flow quantities from supplier to producer in UNITS

```
  y(l,m,mp) flow quantities between producers in UNITS
  z(m,n)    flow quantities from producer to market in UNITS
  h(r)      rate of production of recipe r at producer m in MULTIPLES OF r
  f         total cost in DOLLARS ;

Positive variable x;
Positive variable y;
Positive variable z;
Positive variable h;

Equations
cost            define objective function
supply(l)       observe supply limit at supplier l
demand(n)       satisfy or exceed demand at market n
recinputs(l,m)  satisfy material balance between inputs received at m and inputs used in recipes at
m
rpo(m,n)        satisfy that recipe product outputs from each producer equals products received by
markets
rbo(l,m)        producer m must produce an amount of each byproduct output equal to or greater
than the amount flowing from m to mp as inputs   ;

cost ..             f =e= sum((l,m), cx(l,m)*x(l,m))+ sum((l,m,mp), cy(l,m,mp)*y(l,m,mp))+
sum((m,n), cz(m,n)*z(m,n))   ;

supply(l) ..        sum(m, x(l,m))  =l=  s(l)   ;

demand(n) ..        sum(m, z(m,n))  =g=  d(n)   ;

recinputs (l,m) ..   x(l,m) + sum(mp, y(l,mp,m)) =e= sum(r, h(r)*a(r,l)$(hrm(r,m) gt 0))   ;

rpo(m,n) ..  sum(r, h(r)*bz(r,n)$(hrm(r,m) gt 0)) =e= z(m,n)   ;

rbo(l,m) ..   sum(r, h(r)*by(r,l)$(hrm(r,m) gt 0)) =g= sum(mp, y(l,m,mp))   ;

option limrow = 15, limcol = 30 ;
  MODEL NETWORK /ALL/ ;
  SOLVE NETWORK USING LP MINIMIZING f ;
display x.l,h.l,y.l,z.l,x.m,h.m,y.m,z.m ;
```

PROBLEM 1 INPUT

```
Sets
   l     external suppliers   / A*D /
   m      producers            / 1*3 /
   n      markets              / 1*5 /
   r      recipes              / 1*7 / ;

alias (m,mp)    ;

Parameters
 S(l)  capacity of external supplier l in UNITS
 /   A    2000
     B    2000
     C    2000
     D    2000 /

 D(n)  demand at market n in UNITS
 /   1        500
     2        500
     3        500
     4        500
     5        500 /

 wi(l)  weighted cost of shipping l in DOLLARS PER UNIT
 /   A  4
     B  5
     C  6
     D  7   /

 wo(n)  weighted cost of shipping n in DOLLARS PER UNIT
 /   1  1
     2  1
     3  1
     4  1
     5  1     /;

Table dx(l,m)   distance from supplier l to producer m in MILES
```

```
     1    2    3
A    4    6    10
B    6    4    8
C    8    4    6
D    10   6    4  ;
```

Table dy(m,mp) distance from producer m to producer mp in MILES

```
    1    2    3
1   0    1    2
2   1    0    1
3   2    1    0  ;
```

Table dz(m,n) distance from producer m to market n in MILES

```
     1    2    3    4    5
1    5    6    7    8    9
2    7    6    5    6    7
3    9    8    7    6    5  ;
```

Table hrm(r,m) recipes r used at each producer m

```
     1  2  3
1    1  0  0
2    1  0  0
3    0  1  0
4    0  1  0
5    0  1  0
6    0  0  1
7    0  0  1  ;
```

Table a(r,l) amount of input l used in recipe r in UNITS

```
        A       B       C       D
1       4       5       0       0
2       0       3       1.5     0
3       1       0       0       4
4       3.5     5.5     0       0
5       0       0       1       2
6       0       0       2       6
7       0.5     0       1       0     ;
```

Table bz(r,n) amount of output n produced by recipe r in UNITS

	1	2	3	4	5
1	1.0	0.0	0.0	0.0	0.0
2	0.0	4.2	0.0	0.0	0.0
3	0.0	0.0	3.0	0.0	0.0
4	1.8	0.0	0.0	0.0	0.0
5	0.0	1.0	0.0	2.0	0.0
6	0.0	0.0	0.5	1.5	0.0
7	0.0	0.0	0.0	0.5	0.5 ;

Table by(r,l) amount of input l produced by recipe r in UNITS

	A	B	C	D
1	0.0	0.0	0.0	2.0
2	0.0	0.0	0.0	1.5
3	0.0	0.0	0.0	0.0
4	0.0	0.0	2.0	0.0
5	0.0	0.0	0.5	0.0
6	0.0	2.0	0.0	0.0
7	0.0	1.0	0.0	0.0 ;

Parameter cx(l,m) cost of transporting input l from supplier l to producer m in DOLLARS PER UNIT;

cx(l,m) = dx(l,m) * wi(l) ;

Parameter cy(l,m,mp) cost of transporting input l from producer m to producer mp in DOLLARS PER UNIT;

cy(l,m,mp) = dy(m,mp) * wi(l) ;

Parameter cz(m,n) cost of transporting output n from producer m to market n in DOLLARS PER UNIT;

cz(m,n) = dz(m,n) * wo(n) ;

Variables

x(l,m) flow quantities from supplier to producer in UNITS

y(l,m,mp) flow quantities between producers in UNITS

z(m,n) flow quantities from producer to market in UNITS

h(r) rate of production of recipe r at producer m in MULTIPLES OF r

```
 f        total cost in DOLLARS ;

Positive variable x;
Positive variable y;
Positive variable z;
Positive variable h;

Equations
cost           define objective function
supply(l)      observe supply limit at supplier l
demand(n)      satisfy or exceed demand at market n
recinputs(l,m)  satisfy material balance between inputs received at m and inputs used in recipes at
m
recoutputs(m,n) satisfy that product outputs from each producer equals products received by
markets
recbyprod(l,m)  producer m must generate an amount of byproduct type l equal or greater than the
amount of l used by producers mp as inputs ;


cost ..            f =e= sum((l,m), cx(l,m)*x(l,m))+ sum((l,m,mp), cy(l,m,mp)*y(l,m,mp))+
sum((m,n), cz(m,n)*z(m,n))   ;

supply(l) ..       sum(m, x(l,m))  =l=  s(l)   ;

demand(n) ..       sum(m, z(m,n))  =g=  d(n)   ;

recinputs (l,m) ..  x(l,m) + sum(mp, y(l,mp,m)) =e= sum(r, h(r)*a(r,l)$(hrm(r,m) gt 0))   ;

recoutputs(m,n) ..  sum(r, h(r)*bz(r,n)$(hrm(r,m) gt 0)) =e= z(m,n)   ;

recbyprod(l,m) ..   sum(r, h(r)*by(r,l)$(hrm(r,m) gt 0)) =g= sum(mp, y(l,m,mp))   ;


option limrow = 15, limcol = 30 ;
 MODEL NETWORK /ALL/ ;
 SOLVE NETWORK USING LP MINIMIZING f ;
display x.l,h.l,y.l,z.l,x.m,h.m,y.m,z.m ;
```

PROBLEM 2 INPUT

```
Sets
   l     external suppliers   / A*D /
   m      producers            / 1*5 /
   n      markets              / 1*5 /
   r      recipes              / 1*11 / ;

alias (m,mp)    ;

Parameters
 S(l)  capacity of external supplier l in UNITS
 /   A    2000
     B    2000
     C    2000
     D    2000 /

 D(n)  demand at market n in UNITS
 /   1        500
     2        500
     3        500
     4        500
     5        500 /

 wi(l)  weighted cost of shipping l in DOLLARS PER UNIT
 /   A  4
     B  5
     C  6
     D  7   /

 wo(n)  weighted cost of shipping n in DOLLARS PER UNIT
 /   1  1
     2  1
     3  1
     4  1
     5  1     /;

Table dx(l,m)   distance from supplier l to producer m in MILES
```

	1	2	3	4	5
A	4	6	10	5	12
B	6	4	8	6	9
C	8	4	6	9	6
D	10	6	4	12	5 ;

Table dy(m,mp) distance from producer m to producer mp in MILES

	1	2	3	4	5
1	0	1	2	0.5	3
2	1	0	1	2	2
3	2	1	0	3	0.5
4	0.5	2	3	0	4
5	3	2	0.5	4	0 ;

Table dz(m,n) distance from producer m to market n in MILES

	1	2	3	4	5
1	5	6	7	8	9
2	7	6	5	6	7
3	9	8	7	6	5
4	6	7	8	9	10
5	10	9	8	7	6 ;

Table hrm(r,m) recipes r used at each producer m

	1	2	3	4	5
1	1	0	0	0	0
2	1	0	0	0	0
3	0	1	0	0	0
4	0	1	0	0	0
5	0	1	0	0	0
6	0	0	1	0	0
7	0	0	1	0	0
8	0	0	0	1	0
9	0	0	0	1	0
10	0	0	0	0	1
11	0	0	0	0	1 ;

*producer 4 and 5 each have two recipes they can use

*producer 4 can use recipes 8 and 9

*producer 5 can use recipes 10 and 11

Table a(r,l) amount of input l used in recipe r in UNITS

	A	B	C	D
1	4	5	0	0
2	0	3	1.5	0
3	1	0	0	4
4	3.5	5.5	0	0
5	0	0	1	2
6	0	0	2	6
7	0.5	0	1	0
8	4	5	0	0
9	4	5	0	0
10	1	0	1	1
11	0	0	5	0 ;

*recipes 8 and 9 both have the same input coefficient as recipe 1

*recall that recipe 1 is not used in the problem 1 solution

*recipe 10 requires small amounts of all inputs except B

*recipe 11 requires only large amounts of input C

Table bz(r,n) amount of output n produced by recipe r in UNITS

	1	2	3	4	5
1	1.0	0.0	0.0	0.0	0.0
2	0.0	4.2	0.0	0.0	0.0
3	0.0	0.0	3.0	0.0	0.0
4	1.8	0.0	0.0	0.0	0.0
5	0.0	1.0	0.0	2.0	0.0
6	0.0	0.0	0.5	1.5	0.0
7	0.0	0.0	0.0	0.5	0.5
8	2.0	0.0	0.0	0.0	0.0
9	1.0	0.0	0.0	0.0	0.0
10	0.0	0.0	3.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.5 ;

*recipe 8 produces twice the amount of the same product as recipe 1

*recipe 9 produces an equal amount of the same product as recipe 1

*recipe 10 produces an equal amount of product 3 as recipe 3

*recipe 11 produces an equal amount of product 5 as recipe 7

Table by(r,l) amount of input l produced by recipe r in UNITS

	A	B	C	D
1	0.0	0.0	0.0	2.0
2	0.0	0.0	0.0	1.5
3	0.0	0.0	0.0	0.0
4	0.0	0.0	2.0	0.0
5	0.0	0.0	0.5	0.0
6	0.0	2.0	0.0	0.0
7	0.0	1.0	0.0	0.0
8	0.0	0.0	0.0	2.0
9	0.5	2.0	0.0	1.0
10	0.0	0.0	0.0	0.0
11	0.5	0.0	0.0	0.0 ;

*recipe 8 produces the same by-products as recipe 1

*recipe 9 produces more by-products than recipe 1

*recipe 10 produces the no by-products, like recipe 3

*recipe 11 produces only a small amount of by-product A

*Producer 4 competes with producer 1, with recipe 8 and 9

* where one recipe is more productive, and the other has more utility

* for other producers

*Producer 5 competes with producers 2 and 3, with recipe 10 and 11

* where recipe 10 produces product 3 from a less but more varied inputs

* and recipe 11 produces the same product outputs as recipe 3, and produces

* only by-product A

Parameter cx(l,m) cost of transporting input l from supplier l to producer m in DOLLARS PER UNIT;

cx(l,m) = dx(l,m) * wi(l) ;

Parameter cy(l,m,mp) cost of transporting input l from producer m to producer mp in DOLLARS PER UNIT;

cy(l,m,mp) = dy(m,mp) * wi(l) ;

Parameter cz(m,n) cost of transporting output n from producer m to market n in DOLLARS PER UNIT;

```
cz(m,n) = dz(m,n) * wo(n)   ;

Variables
 x(l,m)    flow quantities from supplier to producer in UNITS
 y(l,m,mp) flow quantities between producers in UNITS
 z(m,n)    flow quantities from producer to market in UNITS
 h(r)      rate of production of recipe r at producer m in MULTIPLES OF r
 f         total cost in DOLLARS ;

Positive variable x;
Positive variable y;
Positive variable z;
Positive variable h;

Equations
cost           define objective function
supply(l)      observe supply limit at supplier l
demand(n)      satisfy or exceed demand at market n
recinputs(l,m)  satisfy material balance between inputs received at m and inputs used in recipes at
m
recoutputs(m,n) satisfy that product outputs from each producer equals products received by
markets
recbyprod(l,m)  producer m must generate an amount of byproduct type l equal or greater than the
amount of l used by producers mp as inputs ;


cost ..            f =e= sum((l,m), cx(l,m)*x(l,m))+ sum((l,m,mp), cy(l,m,mp)*y(l,m,mp))+
sum((m,n), cz(m,n)*z(m,n))   ;

supply(l) ..       sum(m, x(l,m))  =l=  s(l)   ;

demand(n) ..       sum(m, z(m,n))  =g=  d(n)   ;

recinputs (l,m) ..  x(l,m) + sum(mp, y(l,mp,m)) =e= sum(r, h(r)*a(r,l)$(hrm(r,m) gt 0))   ;

recoutputs(m,n) ..  sum(r, h(r)*bz(r,n)$(hrm(r,m) gt 0)) =e= z(m,n)   ;

recbyprod(l,m) ..   sum(r, h(r)*by(r,l)$(hrm(r,m) gt 0)) =g= sum(mp, y(l,m,mp))   ;
```

```
option limrow = 30, limcol = 30 ;
  MODEL NETWORK /ALL/ ;
  SOLVE NETWORK USING LP MINIMIZING f ;
display x.l,h.l,y.l,z.l,x.m,h.m,y.m,z.m ;
```

SPECIAL CASE 1 ENVIRONMENT

```
Sets
   l     external suppliers   / A*D /
   m      producers            / 1*5 /
   n      markets              / 1*5 /
   r      recipes              / 1*11 / ;

alias (m,mp)    ;

Parameters
 S(l)  capacity of external supplier l in UNITS
 /   A    2000
     B    2000
     C    2000
     D    2000 /

 D(n)  demand at market n in UNITS
 /   1        500
     2        500
     3        500
     4        500
     5        500 /

 wi(l)  weighted cost of shipping l in DOLLARS PER UNIT
  /   A  4
     B  5
     C  6
     D  7   /

 wo(n)  weighted cost of shipping n in DOLLARS PER UNIT
 /   1  1
     2  1
     3  1
     4  1
     5  1     /;

Table dx(l,m)   distance from supplier l to producer m in MILES
```

	1	2	3	4	5
A	4	6	10	5	12
B	6	4	8	6	9
C	8	4	6	9	6
D	10	6	4	12	5 ;

Table dy(m,mp) distance from producer m to producer mp in MILES

	1	2	3	4	5
1	0	1	2	0.5	3
2	1	0	1	2	2
3	2	1	0	3	0.5
4	0.5	2	3	0	4
5	3	2	0.5	4	0 ;

Table dz(m,n) distance from producer m to market n in MILES

	1	2	3	4	5
1	5	6	7	8	9
2	7	6	5	6	7
3	9	8	7	6	5
4	6	7	8	9	10
5	10	9	8	7	6 ;

Table hrm(r,m) recipes r used at each producer m

	1	2	3	4	5
1	1	0	0	0	0
2	1	0	0	0	0
3	0	1	0	0	0
4	0	1	0	0	0
5	0	1	0	0	0
6	0	0	1	0	0
7	0	0	1	0	0
8	0	0	0	1	0
9	0	0	0	1	0
10	0	0	0	0	1
11	0	0	0	0	1 ;

Table a(r,l) amount of input l used in recipe r in UNITS

	A	B	C	D
1	4	5	0	0
2	0	3	1.5	0
3	1	0	0	4
4	3.5	5.5	0	0
5	0	0	1	2
6	0	0	2	6
7	0.5	0	1	0
8	4	5	0	0
9	4	5	0	0
10	1	0	1	1
11	0	0	5	0 ;

Table bz(r,n) amount of output n produced by recipe r in UNITS

	1	2	3	4	5
1	1.0	0.0	0.0	0.0	0.0
2	0.0	4.2	0.0	0.0	0.0
3	0.0	0.0	3.0	0.0	0.0
4	1.8	0.0	0.0	0.0	0.0
5	0.0	1.0	0.0	2.0	0.0
6	0.0	0.0	0.5	1.5	0.0
7	0.0	0.0	0.0	0.5	0.5
8	2.0	0.0	0.0	0.0	0.0
9	1.0	0.0	0.0	0.0	0.0
10	0.0	0.0	3.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.5 ;

Table by(r,l) amount of input l produced by recipe r in UNITS

	A	B	C	D
1	0.0	0.0	0.0	2.0
2	0.0	0.0	0.0	1.5
3	0.0	0.0	0.0	0.0
4	0.0	0.0	2.0	0.0
5	0.0	0.0	0.5	0.0
6	0.0	2.0	0.0	0.0
7	0.0	1.0	0.0	0.0
8	0.0	0.0	0.0	2.0

```
9  0.5 2.0 0.0 1.0
10 0.0 0.0 0.0 0.0
11 0.5 0.0 0.0 0.0 ;

Parameter cx(l,m)   cost of transporting input l from supplier l to producer m in DOLLARS PER UNIT;
  cx(l,m) = dx(l,m) * wi(l)  ;

Parameter cy(l,m,mp)   cost of transporting input l from producer m to producer mp in DOLLARS PER UNIT;
  cy(l,m,mp) = dy(m,mp) * wi(l)   ;

Parameter cz(m,n)   cost of transporting output n from producer m to market n in DOLLARS PER UNIT;
  cz(m,n) = dz(m,n) * wo(n)   ;

Parameter bpw(l) environmental cost of disposing of material l
/ A 10
  B 11
  C 12
  D 13 /    ;

Scalar VMT virgin material tax
/ 200 /;

Variables
  x(l,m)    flow quantities from supplier to producer in UNITS
  y(l,m,mp) flow quantities between producers in UNITS
  z(m,n)    flow quantities from producer to market in UNITS
  h(r)      rate of production of recipe r at producer m in MULTIPLES OF r
  f         total cost in DOLLARS
  env       environmental cost in DOLLARS;

Positive variable x;
Positive variable y;
Positive variable z;
Positive variable h;
```

```
Equations
cost            define objective function
supply(l)       observe supply limit at supplier l
demand(n)       satisfy or exceed demand at market n
recinputs(l,m)  satisfy material balance between inputs received at m and inputs used in recipes at m
recoutputs(m,n) satisfy that product outputs from each producer equals products received by markets
recbyprod(l,m)  producer m must produce an amount of byproduct type l equal to or greater than the amount of l used by producers mp as inputs
envcost         environemental cost for unused byproduct disposed of by each producer m instead of shipping to other producers mp            ;


cost ..             f =e= env + sum((l,m), cx(l,m)*x(l,m))+ sum((l,m,mp), cy(l,m,mp)*y(l,m,mp))+ sum((m,n), cz(m,n)*z(m,n))   ;


supply(l) ..        sum(m, x(l,m))  =l=  s(l)   ;


demand(n) ..        sum(m, z(m,n))  =g=  d(n)   ;


recinputs (l,m) ..   x(l,m) + sum(mp, y(l,mp,m)) =e= sum(r, h(r)*a(r,l)$(hrm(r,m) gt 0))   ;


recoutputs(m,n) ..  sum(r, h(r)*bz(r,n)$(hrm(r,m) gt 0)) =e= z(m,n)   ;


recbyprod(l,m) ..   sum(r, h(r)*by(r,l)$(hrm(r,m) gt 0)) =g= sum(mp, y(l,m,mp))   ;


envcost ..          env =e= sum((l,m), bpw(l)*(sum(r, h(r)*by(r,l)$(hrm(r,m) gt 0)) - sum(mp, y(l,m,mp))))+ sum((l,m), vmt*x(l,m))  ;


option limrow = 15, limcol = 30 ;
 MODEL NETWORK /ALL/ ;
 SOLVE NETWORK USING LP MINIMIZING f ;
```

APPENDIX C: GAMS OUTPUT

TEST OUTPUT

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G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m

C o m p i l a t i o n

```
 1  Sets
 2     l    external suppliers   / A*D /
 3     m     producers           / 1*3 /
 4     n     markets             / 1*5 /
 5     r     recipes             / 1*7 / ;
 6
 7  alias (m,mp)    ;
 8
 9  Parameters
10    S(l)  capacity of external supplier l in UNITS
11     /   A    30000
12         B    30000
13         C    30000
14         D    30000 /
15
16    D(n)  demand at market n in UNITS
17     /   1       500
18         2       500
19         3       500
20         4       500
21         5       500 /
22
23    wi(l)  weighted cost of shipping l in DOLLARS PER UNIT
24     /   A  1
25         B  1
26         C  1
27         D  1   /
28
```

```
29   wo(n)  weighted cost of shipping n in DOLLARS PER UNIT
30    /   1  1
31        2  1
32        3  1
33        4  1
34        5  1    /;
35
36  Table dx(l,m)   distance from supplier l to producer m in MILES
37        1    2    3
38   A    2    2    2
39   B    2    2    2
40   C    2    2    2
41   D    2    2    2  ;
42
43  Table dy(m,mp)   distance from producer m to producer mp in MILES
44      1    2    3
45   1  0    2    2
46   2  2    0    2
47   3  2    2    0  ;
48
49  Table dz(m,n)   distance from producer m to market n in MILES
50        1    2    3    4    5
51   1    2    2    2    2    2
52   2    2    2    2    2    2
53   3    2    2    2    2    2   ;
54
55  Table hrm(r,m) recipes r used at each producer m
56       1  2  3
57   1   1  0  0
58   2   1  0  0
59   3   0  1  0
60   4   0  1  0
61   5   0  1  0
62   6   0  0  1
63   7   0  0  1  ;
64
65  Table a(r,l)   amount of input l used in recipe r in UNITS
```

```
66          A       B       C       D
67  1       4       5       0       0
68  2       0       3       1.5     0
69  3       1       0       0       4
70  4       3.5     5.5     0       0
71  5       0       0       1       2
72  6       0       0       2       6
73  7       0.5     0       1       0     ;
74
75  Table bz(r,n)   amount of output n produced by recipe r in UNITS
76          1   2   3   4   5
77  1       1.0 0.0 0.0 0.0 0.0
78  2       0.0 4.2 0.0 0.0 0.0
79  3       0.0 0.0 3.0 0.0 0.0
80  4       1.8 0.0 0.0 0.0 0.0
81  5       0.0 1.0 0.0 2.0 0.0
82  6       0.0 0.0 0.5 1.5 0.0
83  7       0.0 0.0 0.0 0.5 0.5 ;
84
85  Table by(r,l)   amount of input l produced by recipe r in UNITS
86    A   B   C   D
87  1  0.0 0.0 0.0 2.0
88  2  0.0 0.0 0.0 1.5
89  3  0.0 0.0 0.0 0.0
90  4  0.0 0.0 2.0 0.0
91  5  0.0 0.0 0.5 0.0
92  6  0.0 2.0 0.0 0.0
93  7  0.0 1.0 0.0 0.0                  ;
94
95  Parameter cx(l,m)   cost of transporting input l from supplier l to produc
    er m in DOLLARS PER UNIT;
96    cx(l,m) = dx(l,m) * wi(l)  ;
97
98  Parameter cy(l,m,mp)   cost of transporting input l from producer m to pro
    ducer mp in DOLLARS PER UNIT;
99    cy(l,m,mp) = dy(m,mp) * wi(l)   ;
100
```

```
101  Parameter cz(m,n)   cost of transporting output n from producer m to marke
     t n in DOLLARS PER UNIT;
102    cz(m,n) = dz(m,n) * wo(n)   ;
103
104  Variables
105    x(l,m)    flow quantities from supplier to producer in UNITS
106    y(l,m,mp) flow quantities between producers in UNITS
107    z(m,n)    flow quantities from producer to market in UNITS
108    h(r)      rate of production of recipe r at producer m in MULTIPLES OF r
109    f         total cost in DOLLARS ;
110
111  Positive variable x;
112  Positive variable y;
113  Positive variable z;
114  Positive variable h;
115
116  Equations
117  cost            define objective function
118  supply(l)       observe supply limit at supplier l
119  demand(n)       satisfy or exceed demand at market n
120  recinputs(l,m)  satisfy material balance between inputs received at m and
     inputs used in recipes at m
121  rpo(m,n)        satisfy that recipe product outputs from each producer equ
     als products received by markets
122  rbo(l,m)        producer m must produce an amount of each byproduct output
      equal to or greater than the amount flowing from m to mp as inputs   ;
123
124  cost ..             f =e= sum((l,m), cx(l,m)*x(l,m))+ sum((l,m,mp), cy(l,m
     ,mp)*y(l,m,mp))+ sum((m,n), cz(m,n)*z(m,n))   ;
125
126  supply(l) ..        sum(m, x(l,m))  =l=  s(l)   ;
127
128  demand(n) ..        sum(m, z(m,n))  =g=  d(n)   ;
129
130  recinputs (l,m) ..   x(l,m) + sum(mp, y(l,mp,m)) =e= sum(r, h(r)*a(r,l)$(h
     rm(r,m) gt 0))   ;
131
```

```
132  rpo(m,n) ..  sum(r, h(r)*bz(r,n)$(hrm(r,m) gt 0)) =e= z(m,n)   ;
133
134  rbo(l,m) ..   sum(r, h(r)*by(r,l)$(hrm(r,m) gt 0)) =g= sum(mp, y(l,m,mp))
     ;
135
136
137
138  option limrow = 15, limcol = 30 ;
139    MODEL NETWORK /ALL/ ;
140    SOLVE NETWORK USING LP MINIMIZING f ;
141  display x.l,h.l,y.l,z.l,x.m,h.m,y.m,z.m ;
142
```

COMPILATION TIME = 0.000 SECONDS 3 Mb WEX239-239 Nov 9, 2012

---- cost =E= define objective function

cost.. - 2*x(A,1) - 2*x(A,2) - 2*x(A,3) - 2*x(B,1) - 2*x(B,2) - 2*x(B,3)

- 2*x(C,1) - 2*x(C,2) - 2*x(C,3) - 2*x(D,1) - 2*x(D,2) - 2*x(D,3)

- 2*y(A,1,2) - 2*y(A,1,3) - 2*y(A,2,1) - 2*y(A,2,3) - 2*y(A,3,1)

- 2*y(A,3,2) - 2*y(B,1,2) - 2*y(B,1,3) - 2*y(B,2,1) - 2*y(B,2,3)

- 2*y(B,3,1) - 2*y(B,3,2) - 2*y(C,1,2) - 2*y(C,1,3) - 2*y(C,2,1)

- 2*y(C,2,3) - 2*y(C,3,1) - 2*y(C,3,2) - 2*y(D,1,2) - 2*y(D,1,3)

- 2*y(D,2,1) - 2*y(D,2,3) - 2*y(D,3,1) - 2*y(D,3,2) - 2*z(1,1) - 2*z(1,2)

- 2*z(1,3) - 2*z(1,4) - 2*z(1,5) - 2*z(2,1) - 2*z(2,2) - 2*z(2,3)

- 2*z(2,4) - 2*z(2,5) - 2*z(3,1) - 2*z(3,2) - 2*z(3,3) - 2*z(3,4)

- 2*z(3,5) + f =E= 0 ; (LHS = 0)

---- supply =L= observe supply limit at supplier l

supply(A).. x(A,1) + x(A,2) + x(A,3) =L= 30000 ; (LHS = 0)

supply(B).. x(B,1) + x(B,2) + x(B,3) =L= 30000 ; (LHS = 0)

supply(C).. x(C,1) + x(C,2) + x(C,3) =L= 30000 ; (LHS = 0)

supply(D).. x(D,1) + x(D,2) + x(D,3) =L= 30000 ; (LHS = 0)

---- demand =G= satisfy or exceed demand at market n

demand(1).. z(1,1) + z(2,1) + z(3,1) =G= 500 ; (LHS = 0, INFES = 500 ****)

demand(2).. z(1,2) + z(2,2) + z(3,2) =G= 500 ; (LHS = 0, INFES = 500 ****)

demand(3).. z(1,3) + z(2,3) + z(3,3) =G= 500 ; (LHS = 0, INFES = 500 ****)

demand(4).. z(1,4) + z(2,4) + z(3,4) =G= 500 ; (LHS = 0, INFES = 500 ****)

demand(5).. z(1,5) + z(2,5) + z(3,5) =G= 500 ; (LHS = 0, INFES = 500 ****)

---- recinputs =E= satisfy material balance between inputs received at m and i
nputs used in recipes at m

recinputs(A,1).. x(A,1) + y(A,1,1) + y(A,2,1) + y(A,3,1) - 4*h(1) =E= 0 ;

(LHS = 0)

recinputs(A,2).. x(A,2) + y(A,1,2) + y(A,2,2) + y(A,3,2) - h(3) - 3.5*h(4) =E=
0 ; (LHS = 0)

recinputs(A,3).. x(A,3) + y(A,1,3) + y(A,2,3) + y(A,3,3) - 0.5*h(7) =E= 0 ;

(LHS = 0)

recinputs(B,1).. x(B,1) + y(B,1,1) + y(B,2,1) + y(B,3,1) - 5*h(1) - 3*h(2) =E=
0 ; (LHS = 0)

recinputs(B,2).. x(B,2) + y(B,1,2) + y(B,2,2) + y(B,3,2) - 5.5*h(4) =E= 0 ;

(LHS = 0)

recinputs(B,3).. x(B,3) + y(B,1,3) + y(B,2,3) + y(B,3,3) =E= 0 ; (LHS = 0)

recinputs(C,1).. x(C,1) + y(C,1,1) + y(C,2,1) + y(C,3,1) - 1.5*h(2) =E= 0 ;

(LHS = 0)

recinputs(C,2).. x(C,2) + y(C,1,2) + y(C,2,2) + y(C,3,2) - h(5) =E= 0 ;

(LHS = 0)

recinputs(C,3).. x(C,3) + y(C,1,3) + y(C,2,3) + y(C,3,3) - 2*h(6) - h(7) =E= 0 ; (LHS = 0)

recinputs(D,1).. x(D,1) + y(D,1,1) + y(D,2,1) + y(D,3,1) =E= 0 ; (LHS = 0)

recinputs(D,2).. x(D,2) + y(D,1,2) + y(D,2,2) + y(D,3,2) - 4*h(3) - 2*h(5) =E= 0 ; (LHS = 0)

recinputs(D,3).. x(D,3) + y(D,1,3) + y(D,2,3) + y(D,3,3) - 6*h(6) =E= 0 ;

(LHS = 0)

---- rpo =E= satisfy that recipe product outputs from each producer equals products received by markets

rpo(1,1).. - z(1,1) + h(1) =E= 0 ; (LHS = 0)

rpo(1,2).. - z(1,2) + 4.2*h(2) =E= 0 ; (LHS = 0)

rpo(1,3).. - z(1,3) =E= 0 ; (LHS = 0)

rpo(1,4).. - z(1,4) =E= 0 ; (LHS = 0)

rpo(1,5).. - z(1,5) =E= 0 ; (LHS = 0)

rpo(2,1).. - z(2,1) + 1.8*h(4) =E= 0 ; (LHS = 0)

rpo(2,2).. - z(2,2) + h(5) =E= 0 ; (LHS = 0)

rpo(2,3).. - z(2,3) + 3*h(3) =E= 0 ; (LHS = 0)

rpo(2,4).. - z(2,4) + 2*h(5) =E= 0 ; (LHS = 0)

rpo(2,5).. - z(2,5) =E= 0 ; (LHS = 0)

rpo(3,1).. - z(3,1) =E= 0 ; (LHS = 0)

rpo(3,2).. - z(3,2) =E= 0 ; (LHS = 0)

rpo(3,3).. - z(3,3) + 0.5*h(6) =E= 0 ; (LHS = 0)

rpo(3,4).. - z(3,4) + 1.5*h(6) + 0.5*h(7) =E= 0 ; (LHS = 0)

rpo(3,5).. - z(3,5) + 0.5*h(7) =E= 0 ; (LHS = 0)

---- rbo =G= producer m must produce an amount of each byproduct output equal
to or greater than the amount flowing from m to mp as inputs

rbo(A,1).. - y(A,1,1) - y(A,1,2) - y(A,1,3) =G= 0 ; (LHS = 0)

rbo(A,2).. - y(A,2,1) - y(A,2,2) - y(A,2,3) =G= 0 ; (LHS = 0)

rbo(A,3).. - y(A,3,1) - y(A,3,2) - y(A,3,3) =G= 0 ; (LHS = 0)

rbo(B,1).. - y(B,1,1) - y(B,1,2) - y(B,1,3) =G= 0 ; (LHS = 0)

rbo(B,2).. - y(B,2,1) - y(B,2,2) - y(B,2,3) =G= 0 ; (LHS = 0)

rbo(B,3).. - y(B,3,1) - y(B,3,2) - y(B,3,3) + 2*h(6) + h(7) =G= 0 ; (LHS = 0)

rbo(C,1).. - y(C,1,1) - y(C,1,2) - y(C,1,3) =G= 0 ; (LHS = 0)

rbo(C,2).. - y(C,2,1) - y(C,2,2) - y(C,2,3) + 2*h(4) + 0.5*h(5) =G= 0 ;

(LHS = 0)

rbo(C,3).. - y(C,3,1) - y(C,3,2) - y(C,3,3) =G= 0 ; (LHS = 0)

rbo(D,1).. - y(D,1,1) - y(D,1,2) - y(D,1,3) + 2*h(1) + 1.5*h(2) =G= 0 ;

(LHS = 0)

rbo(D,2).. - y(D,2,1) - y(D,2,2) - y(D,2,3) =G= 0 ; (LHS = 0)

rbo(D,3).. - y(D,3,1) - y(D,3,2) - y(D,3,3) =G= 0 ; (LHS = 0)

---- x flow quantities from supplier to producer in UNITS

x(A,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 supply(A)
1 recinputs(A,1)

x(A,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 supply(A)
1 recinputs(A,2)

x(A,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 supply(A)
1 recinputs(A,3)

x(B,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 supply(B)
1 recinputs(B,1)

x(B,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 supply(B)
1 recinputs(B,2)

x(B,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 supply(B)

1 recinputs(B,3)

x(C,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 supply(C)

1 recinputs(C,1)

x(C,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 supply(C)

1 recinputs(C,2)

x(C,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 supply(C)

1 recinputs(C,3)

x(D,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 supply(D)

1 recinputs(D,1)

x(D,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 supply(D)

1 recinputs(D,2)

x(D,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 supply(D)

1 recinputs(D,3)

---- y flow quantities between producers in UNITS

y(A,1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,1)

-1 rbo(A,1)

y(A,1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(A,2)

-1 rbo(A,1)

y(A,1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(A,3)

-1 rbo(A,1)

y(A,2,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(A,1)

-1 rbo(A,2)

y(A,2,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,2)

-1 rbo(A,2)

y(A,2,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(A,3)

-1 rbo(A,2)

y(A,3,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(A,1)

-1 rbo(A,3)

y(A,3,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(A,2)

-1 rbo(A,3)

y(A,3,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,3)

-1 rbo(A,3)

y(B,1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(B,1)

-1 rbo(B,1)

y(B,1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(B,2)

-1 rbo(B,1)

y(B,1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(B,3)

-1 rbo(B,1)

y(B,2,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 recinputs(B,1)
-1 rbo(B,2)

y(B,2,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
1 recinputs(B,2)
-1 rbo(B,2)

y(B,2,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 recinputs(B,3)
-1 rbo(B,2)

y(B,3,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 recinputs(B,1)
-1 rbo(B,3)

y(B,3,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 recinputs(B,2)
-1 rbo(B,3)

y(B,3,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
1 recinputs(B,3)
-1 rbo(B,3)

y(C,1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(C,1)

-1 rbo(C,1)

y(C,1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(C,2)

-1 rbo(C,1)

y(C,1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(C,3)

-1 rbo(C,1)

y(C,2,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(C,1)

-1 rbo(C,2)

y(C,2,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(C,2)

-1 rbo(C,2)

y(C,2,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(C,3)

-1 rbo(C,2)

y(C,3,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(C,1)

```
      -1       rbo(C,3)

y(C,3,2)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
      -2       cost
       1       recinputs(C,2)
      -1       rbo(C,3)

y(C,3,3)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
       1       recinputs(C,3)
      -1       rbo(C,3)

y(D,1,1)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
       1       recinputs(D,1)
      -1       rbo(D,1)

y(D,1,2)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
      -2       cost
       1       recinputs(D,2)
      -1       rbo(D,1)

y(D,1,3)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
      -2       cost
       1       recinputs(D,3)
      -1       rbo(D,1)

REMAINING 6 ENTRIES SKIPPED

---- z  flow quantities from producer to market in UNITS

z(1,1)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
      -2       cost
```

```
      1       demand(1)
     -1       rpo(1,1)

z(1,2)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -2       cost
      1       demand(2)
     -1       rpo(1,2)

z(1,3)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -2       cost
      1       demand(3)
     -1       rpo(1,3)

z(1,4)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -2       cost
      1       demand(4)
     -1       rpo(1,4)

z(1,5)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -2       cost
      1       demand(5)
     -1       rpo(1,5)

z(2,1)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -2       cost
      1       demand(1)
     -1       rpo(2,1)

z(2,2)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -2       cost
      1       demand(2)
```

-1 rpo(2,2)

z(2,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 demand(3)

-1 rpo(2,3)

z(2,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 demand(4)

-1 rpo(2,4)

z(2,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 demand(5)

-1 rpo(2,5)

z(3,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 demand(1)

-1 rpo(3,1)

z(3,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 demand(2)

-1 rpo(3,2)

z(3,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 demand(3)

-1 rpo(3,3)

z(3,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 demand(4)

-1 rpo(3,4)

z(3,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 demand(5)

-1 rpo(3,5)

---- h rate of production of recipe r at producer m in MULTIPLES OF r

h(1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 recinputs(A,1)

-5 recinputs(B,1)

1 rpo(1,1)

2 rbo(D,1)

h(2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-3 recinputs(B,1)

-1.5 recinputs(C,1)

4.2 rpo(1,2)

1.5 rbo(D,1)

h(3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-1 recinputs(A,2)

-4 recinputs(D,2)

3 rpo(2,3)

h(4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-3.5 recinputs(A,2)

-5.5 recinputs(B,2)

1.8 rpo(2,1)

2 rbo(C,2)

h(5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-1 recinputs(C,2)

-2 recinputs(D,2)

1 rpo(2,2)

2 rpo(2,4)

0.5 rbo(C,2)

h(6)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 recinputs(C,3)

-6 recinputs(D,3)

0.5 rpo(3,3)

1.5 rpo(3,4)

2 rbo(B,3)

h(7)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-0.5 recinputs(A,3)

-1 recinputs(C,3)

0.5 rpo(3,4)

0.5 rpo(3,5)

1 rbo(B,3)

---- f total cost in DOLLARS

f

(.LO, .L, .UP, .M = -INF, 0, +INF, 0)

1 cost

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
Model Statistics SOLVE NETWORK Using LP From line 140

MODEL STATISTICS

BLOCKS OF EQUATIONS	6	SINGLE EQUATIONS	49
BLOCKS OF VARIABLES	5	SINGLE VARIABLES	71
NON ZERO ELEMENTS	208		

GENERATION TIME = 4.587 SECONDS 4 Mb WEX239-239 Nov 9, 2012

EXECUTION TIME = 4.665 SECONDS 4 Mb WEX239-239 Nov 9, 2012

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
Solution Report SOLVE NETWORK Using LP From line 140

S O L V E S U M M A R Y

MODEL NETWORK OBJECTIVE f
TYPE LP DIRECTION MINIMIZE
SOLVER CPLEX FROM LINE 140

**** SOLVER STATUS 1 Normal Completion
**** MODEL STATUS 1 Optimal
**** OBJECTIVE VALUE 15738.0952

RESOURCE USAGE, LIMIT 0.016 1000.000
ITERATION COUNT, LIMIT 8 2000000000

IBM ILOG CPLEX Jul 4, 2012 23.9.5 WEX 36376.36401 WEI x86_64/MS Windows
Cplex 12.4.0.1

LP status(1): optimal
Optimal solution found.
Objective : 15738.095238

LOWER LEVEL UPPER MARGINAL

---- EQU cost . . . 1.000

cost define objective function

---- EQU supply observe supply limit at supplier l

LOWER LEVEL UPPER MARGINAL

A -INF 1638.889 30000.000 .

B -INF 1884.921 30000.000 .
C -INF 1178.571 30000.000 .
D -INF 488.095 30000.000 .

---- EQU demand satisfy or exceed demand at market n

LOWER LEVEL UPPER MARGINAL

1 500.000 500.000 +INF 12.000
2 500.000 500.000 +INF 4.143
3 500.000 500.000 +INF 5.333
4 500.000 500.000 +INF .
5 500.000 500.000 +INF 10.000

---- EQU recinputs satisfy material balance between inputs received at m and in puts used in recipes at m

LOWER LEVEL UPPER MARGINAL

A.1 . . . 2.000
A.2 . . . 2.000
A.3 . . . 2.000
B.1 . . . 2.000
B.2 . . . 2.000
B.3 . . . .
C.1 . . . 2.000
C.2 . . . -5.857
C.3 . . . 2.000
D.1 . . . .
D.2 . . . 2.000
D.3 . . . -0.889

---- EQU rpo satisfy that recipe product outputs from each producer equals prod ucts received by markets

LOWER LEVEL UPPER MARGINAL

1.1	.	.	.	10.000
1.2	.	.	.	2.143
1.3	.	.	.	3.333
1.4	.	.	.	.
1.5	.	.	.	8.000
2.1	.	.	.	10.000
2.2	.	.	.	2.143
2.3	.	.	.	3.333
2.4	.	.	.	-2.000
2.5	.	.	.	8.000
3.1	.	.	.	10.000
3.2	.	.	.	2.143
3.3	.	.	.	3.333
3.4	.	.	.	-2.000
3.5	.	.	.	8.000

---- EQU rbo producer m must produce an amount of each byproduct output equal t
o or greater than the amount flowing from m to mp as inputs

	LOWER	LEVEL	UPPER	MARGINAL
A.1	.	.	+INF	2.000
A.2	.	.	+INF	2.000
A.3	.	.	+INF	2.000
B.1	.	.	+INF	2.000
B.2	.	.	+INF	2.000
B.3	.	1000.000	+INF	.
C.1	.	.	+INF	2.000
C.2	.	555.556	+INF	.
C.3	.	.	+INF	2.000
D.1	.	.	+INF	EPS
D.2	.	.	+INF	2.000
D.3	.	.	+INF	.

---- VAR x flow quantities from supplier to producer in UNITS

	LOWER	LEVEL	UPPER	MARGINAL

A.1	.	.	+INF	.
A.2	.	1138.889	+INF	.
A.3	.	500.000	+INF	.
B.1	.	357.143	+INF	.
B.2	.	1527.778	+INF	.
B.3	.	.	+INF	2.000
C.1	.	178.571	+INF	.
C.2	.	.	+INF	7.857
C.3	.	1000.000	+INF	.
D.1	.	.	+INF	2.000
D.2	.	488.095	+INF	.
D.3	.	.	+INF	2.889

---- VAR y flow quantities between producers in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
A.1.1	.	.	+INF	.
A.1.2	.	.	+INF	2.000
A.1.3	.	.	+INF	2.000
A.2.1	.	.	+INF	2.000
A.2.2	.	.	+INF	.
A.2.3	.	.	+INF	2.000
A.3.1	.	.	+INF	2.000
A.3.2	.	.	+INF	2.000
A.3.3	.	.	+INF	.
B.1.1	.	.	+INF	.
B.1.2	.	.	+INF	2.000
B.1.3	.	.	+INF	4.000
B.2.1	.	.	+INF	2.000
B.2.2	.	.	+INF	.
B.2.3	.	.	+INF	4.000
B.3.1	.	.	+INF	EPS
B.3.2	.	.	+INF	EPS
B.3.3	.	.	+INF	EPS
C.1.1	.	.	+INF	.

C.1.2	.	.	+INF	9.857
C.1.3	.	.	+INF	2.000
C.2.1	.	.	+INF	EPS
C.2.2	.	.	+INF	5.857
C.2.3	.	.	+INF	EPS
C.3.1	.	.	+INF	2.000
C.3.2	.	.	+INF	9.857
C.3.3	.	.	+INF	.
D.1.1	.	.	+INF	EPS
D.1.2	.	178.571	+INF	.
D.1.3	.	.	+INF	2.889
D.2.1	.	.	+INF	4.000
D.2.2	.	.	+INF	.
D.2.3	.	.	+INF	4.889
D.3.1	.	.	+INF	2.000
D.3.2	.	.	+INF	EPS
D.3.3	.	.	+INF	0.889

---- VAR z flow quantities from producer to market in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
1.1	.	.	+INF	.
1.2	.	500.000	+INF	.
1.3	.	.	+INF	.
1.4	.	.	+INF	2.000
1.5	.	.	+INF	.
2.1	.	500.000	+INF	.
2.2	.	.	+INF	.
2.3	.	500.000	+INF	.
2.4	.	.	+INF	.
2.5	.	.	+INF	.
3.1	.	.	+INF	.
3.2	.	.	+INF	.
3.3	.	.	+INF	.
3.4	.	500.000	+INF	.
3.5	.	500.000	+INF	.

---- VAR h rate of production of recipe r at producer m in MULTIPLES OF r

	LOWER	LEVEL	UPPER	MARGINAL
1	.	.	+INF	8.000
2	.	119.048	+INF	.
3	.	166.667	+INF	.
4	.	277.778	+INF	.
5	.	.	+INF	.
6	.	.	+INF	.
7	.	1000.000	+INF	.

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR f	-INF	15738.095	+INF	.

f total cost in DOLLARS

**** REPORT SUMMARY : 0 NONOPT
0 INFEASIBLE
0 UNBOUNDED

---- 141 VARIABLE x.L flow quantities from supplier to producer in UNITS

	1	2	3
A		1138.889	500.000
B	357.143	1527.778	
C	178.571		1000.000
D		488.095	

---- 141 VARIABLE h.L rate of production of recipe r at producer m in MULTIPLES OF r

2 119.048, 3 166.667, 4 277.778, 7 1000.000

---- 141 VARIABLE y.L flow quantities between producers in UNITS

	2
D.1	178.571

---- 141 VARIABLE z.L flow quantities from producer to market in UNITS

	1	2	3	4	5
1		500.000			
2	500.000		500.000		
3				500.000	500.000

---- 141 VARIABLE x.M flow quantities from supplier to producer in UNITS

	1	2	3
B		2.000	
C		7.857	
D	2.000		2.889

---- 141 VARIABLE h.M rate of production of recipe r at producer m in MULTIP
LES OF r

1	8.000

---- 141 VARIABLE y.M flow quantities between producers in UNITS

	1	2	3
A.1		2.000	2.000
A.2	2.000		2.000
A.3	2.000	2.000	
B.1		2.000	4.000
B.2	2.000		4.000
B.3	EPS	EPS	EPS
C.1		9.857	2.000
C.2	EPS	5.857	EPS
C.3	2.000	9.857	
D.1	EPS		2.889
D.2	4.000		4.889
D.3	2.000	EPS	0.889

---- 141 VARIABLE z.M flow quantities from producer to market in UNITS

1 2.000

EXECUTION TIME = 0.063 SECONDS 3 Mb WEX239-239 Nov 9, 2012

USER: David G. Taggart G121115:1510AP-WIN
University of Rhode Island, Mechanical, Industrial & SystemsDC445
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**** FILE SUMMARY

Input \\psf\Home\Documents\gamsdir\projdir\Final Models For Thesis\Final Te
st.gms
Output \\psf\Home\Documents\gamsdir\projdir\Final Test.lst

PROBLEM 1 OUTPUT

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
C o m p i l a t i o n

```
 1  Sets
 2     l     external suppliers   / A*D /
 3     m     producers            / 1*3 /
 4     n     markets              / 1*5 /
 5     r     recipes              / 1*7 / ;
 6
 7  alias (m,mp)    ;
 8
 9  Parameters
10    S(l)  capacity of external supplier l in UNITS
11     /   A    2000
12         B    2000
13         C    2000
14         D    2000 /
15
16    D(n)  demand at market n in UNITS
17     /   1       500
18         2       500
19         3       500
20         4       500
21         5       500 /
22
23    wi(l)  weighted cost of shipping l in DOLLARS PER UNIT
24     /   A  4
25         B  5
26         C  6
27         D  7   /
28
29    wo(n)  weighted cost of shipping n in DOLLARS PER UNIT
30     /   1  1
31         2  1
32         3  1
```

```
33        4  1
34        5  1     /;
35
36  Table dx(l,m)   distance from supplier l to producer m in MILES
37         1    2    3
38    A    4    6    10
39    B    6    4    8
40    C    8    4    6
41    D    10   6    4  ;
42
43  Table dy(m,mp)   distance from producer m to producer mp in MILES
44       1    2    3
45    1  0    1    2
46    2  1    0    1
47    3  2    1    0  ;
48
49  Table dz(m,n)   distance from producer m to market n in MILES
50         1    2    3    4    5
51    1    5    6    7    8    9
52    2    7    6    5    6    7
53    3    9    8    7    6    5  ;
54
55  Table hrm(r,m) recipes r used at each producer m
56         1  2  3
57    1    1  0  0
58    2    1  0  0
59    3    0  1  0
60    4    0  1  0
61    5    0  1  0
62    6    0  0  1
63    7    0  0  1  ;
64
65  Table a(r,l)   amount of input l used in recipe r in UNITS
66           A        B        C        D
67  1        4        5        0        0
68  2        0        3        1.5      0
69  3        1        0        0        4
```

```
 4       3.5      5.5      0        0
 5       0        0        1        2
 6       0        0        2        6
 7       0.5      0        1        0     ;

Table bz(r,n)   amount of output n produced by recipe r in UNITS
           1   2   3   4   5
 1         1.0 0.0 0.0 0.0 0.0
 2         0.0 4.2 0.0 0.0 0.0
 3         0.0 0.0 3.0 0.0 0.0
 4         1.8 0.0 0.0 0.0 0.0
 5         0.0 1.0 0.0 2.0 0.0
 6         0.0 0.0 0.5 1.5 0.0
 7         0.0 0.0 0.0 0.5 0.5 ;

Table by(r,l)   amount of input l produced by recipe r in UNITS
    A   B   C   D
 1  0.0 0.0 0.0 2.0
 2  0.0 0.0 0.0 1.5
 3  0.0 0.0 0.0 0.0
 4  0.0 0.0 2.0 0.0
 5  0.0 0.0 0.5 0.0
 6  0.0 2.0 0.0 0.0
 7  0.0 1.0 0.0 0.0                  ;

Parameter cx(l,m)   cost of transporting input l from supplier l to produc
   er m in DOLLARS PER UNIT;
   cx(l,m) = dx(l,m) * wi(l)  ;

Parameter cy(l,m,mp)   cost of transporting input l from producer m to pro
   ducer mp in DOLLARS PER UNIT;
   cy(l,m,mp) = dy(m,mp) * wi(l)   ;

Parameter cz(m,n)   cost of transporting output n from producer m to marke
   t n in DOLLARS PER UNIT;
   cz(m,n) = dz(m,n) * wo(n)   ;
```

```
104  Variables
105    x(l,m)    flow quantities from supplier to producer in UNITS
106    y(l,m,mp) flow quantities between producers in UNITS
107    z(m,n)    flow quantities from producer to market in UNITS
108    h(r)      rate of production of recipe r at producer m in MULTIPLES OF r
109    f         total cost in DOLLARS ;
110
111  Positive variable x;
112  Positive variable y;
113  Positive variable z;
114  Positive variable h;
115
116  Equations
117  cost            define objective function
118  supply(l)       observe supply limit at supplier l
119  demand(n)       satisfy or exceed demand at market n
120  recinputs(l,m)  satisfy material balance between inputs received at m and
     inputs used in recipes at m
121  recoutputs(m,n) satisfy that product outputs from each producer equals pro
     ducts received by markets
122  recbyprod(l,m)  producer m must generate an amount of byproduct type l equ
     al or greater than the amount of l used by producers mp as inputs ;
123
124
125  cost ..             f =e= sum((l,m), cx(l,m)*x(l,m))+ sum((l,m,mp), cy(l,m
     ,mp)*y(l,m,mp))+ sum((m,n), cz(m,n)*z(m,n))   ;
126
127  supply(l) ..        sum(m, x(l,m))  =l=  s(l)   ;
128
129  demand(n) ..        sum(m, z(m,n))  =g=  d(n)   ;
130
131  recinputs (l,m) ..  x(l,m) + sum(mp, y(l,mp,m)) =e= sum(r, h(r)*a(r,l)$(hr
     m(r,m) gt 0))   ;
132
133  recoutputs(m,n) ..  sum(r, h(r)*bz(r,n)$(hrm(r,m) gt 0)) =e= z(m,n)   ;
134
135  recbyprod(l,m) ..   sum(r, h(r)*by(r,l)$(hrm(r,m) gt 0)) =g= sum(mp, y(l,m
```

```
    ,mp))   ;
136
137
138  option limrow = 15, limcol = 30 ;
139    MODEL NETWORK /ALL/ ;
140    SOLVE NETWORK USING LP MINIMIZING f ;
141  display x.l,h.l,y.l,z.l,x.m,h.m,y.m,z.m ;
142
```

COMPILATION TIME = 0.000 SECONDS 3 Mb WEX239-239 Nov 9, 2012

---- cost =E= define objective function

cost.. - 16*x(A,1) - 24*x(A,2) - 40*x(A,3) - 30*x(B,1) - 20*x(B,2) - 40*x(B,3)

- 48*x(C,1) - 24*x(C,2) - 36*x(C,3) - 70*x(D,1) - 42*x(D,2) - 28*x(D,3)

- 4*y(A,1,2) - 8*y(A,1,3) - 4*y(A,2,1) - 4*y(A,2,3) - 8*y(A,3,1)

- 4*y(A,3,2) - 5*y(B,1,2) - 10*y(B,1,3) - 5*y(B,2,1) - 5*y(B,2,3)

- 10*y(B,3,1) - 5*y(B,3,2) - 6*y(C,1,2) - 12*y(C,1,3) - 6*y(C,2,1)

- 6*y(C,2,3) - 12*y(C,3,1) - 6*y(C,3,2) - 7*y(D,1,2) - 14*y(D,1,3)

- 7*y(D,2,1) - 7*y(D,2,3) - 14*y(D,3,1) - 7*y(D,3,2) - 5*z(1,1) - 6*z(1,2)

- 7*z(1,3) - 8*z(1,4) - 9*z(1,5) - 7*z(2,1) - 6*z(2,2) - 5*z(2,3)

- 6*z(2,4) - 7*z(2,5) - 9*z(3,1) - 8*z(3,2) - 7*z(3,3) - 6*z(3,4)

- 5*z(3,5) + f =E= 0 ; (LHS = 0)

---- supply =L= observe supply limit at supplier l

supply(A).. x(A,1) + x(A,2) + x(A,3) =L= 2000 ; (LHS = 0)

supply(B).. x(B,1) + x(B,2) + x(B,3) =L= 2000 ; (LHS = 0)

supply(C).. x(C,1) + x(C,2) + x(C,3) =L= 2000 ; (LHS = 0)

supply(D).. x(D,1) + x(D,2) + x(D,3) =L= 2000 ; (LHS = 0)

---- demand =G= satisfy or exceed demand at market n

demand(1).. z(1,1) + z(2,1) + z(3,1) =G= 500 ; (LHS = 0, INFES = 500 ****)

demand(2).. z(1,2) + z(2,2) + z(3,2) =G= 500 ; (LHS = 0, INFES = 500 ****)

demand(3).. z(1,3) + z(2,3) + z(3,3) =G= 500 ; (LHS = 0, INFES = 500 ****)

demand(4).. z(1,4) + z(2,4) + z(3,4) =G= 500 ; (LHS = 0, INFES = 500 ****)

demand(5).. z(1,5) + z(2,5) + z(3,5) =G= 500 ; (LHS = 0, INFES = 500 ****)

---- recinputs =E= satisfy material balance between inputs received at m and i
                nputs used in recipes at m

recinputs(A,1).. x(A,1) + y(A,1,1) + y(A,2,1) + y(A,3,1) - 4*h(1) =E= 0 ;

     (LHS = 0)

recinputs(A,2).. x(A,2) + y(A,1,2) + y(A,2,2) + y(A,3,2) - h(3) - 3.5*h(4) =E=
    0 ; (LHS = 0)

recinputs(A,3).. x(A,3) + y(A,1,3) + y(A,2,3) + y(A,3,3) - 0.5*h(7) =E= 0 ;

     (LHS = 0)

recinputs(B,1).. x(B,1) + y(B,1,1) + y(B,2,1) + y(B,3,1) - 5*h(1) - 3*h(2) =E=
    0 ; (LHS = 0)

recinputs(B,2).. x(B,2) + y(B,1,2) + y(B,2,2) + y(B,3,2) - 5.5*h(4) =E= 0 ;

     (LHS = 0)

recinputs(B,3).. x(B,3) + y(B,1,3) + y(B,2,3) + y(B,3,3) =E= 0 ; (LHS = 0)

recinputs(C,1).. x(C,1) + y(C,1,1) + y(C,2,1) + y(C,3,1) - 1.5*h(2) =E= 0 ;

(LHS = 0)

recinputs(C,2).. x(C,2) + y(C,1,2) + y(C,2,2) + y(C,3,2) - h(5) =E= 0 ;

(LHS = 0)

recinputs(C,3).. x(C,3) + y(C,1,3) + y(C,2,3) + y(C,3,3) - 2*h(6) - h(7) =E= 0 ; (LHS = 0)

recinputs(D,1).. x(D,1) + y(D,1,1) + y(D,2,1) + y(D,3,1) =E= 0 ; (LHS = 0)

recinputs(D,2).. x(D,2) + y(D,1,2) + y(D,2,2) + y(D,3,2) - 4*h(3) - 2*h(5) =E= 0 ; (LHS = 0)

recinputs(D,3).. x(D,3) + y(D,1,3) + y(D,2,3) + y(D,3,3) - 6*h(6) =E= 0 ;

(LHS = 0)

---- recoutputs =E= satisfy that product outputs from each producer equals pro
ducts received by markets

recoutputs(1,1).. - z(1,1) + h(1) =E= 0 ; (LHS = 0)

recoutputs(1,2).. - z(1,2) + 4.2*h(2) =E= 0 ; (LHS = 0)

recoutputs(1,3).. - z(1,3) =E= 0 ; (LHS = 0)

recoutputs(1,4).. - z(1,4) =E= 0 ; (LHS = 0)

recoutputs(1,5).. - z(1,5) =E= 0 ; (LHS = 0)

recoutputs(2,1).. - z(2,1) + 1.8*h(4) =E= 0 ; (LHS = 0)

recoutputs(2,2).. - z(2,2) + h(5) =E= 0 ; (LHS = 0)

recoutputs(2,3).. - z(2,3) + 3*h(3) =E= 0 ; (LHS = 0)

recoutputs(2,4).. - z(2,4) + 2*h(5) =E= 0 ; (LHS = 0)

recoutputs(2,5).. - z(2,5) =E= 0 ; (LHS = 0)

recoutputs(3,1).. - z(3,1) =E= 0 ; (LHS = 0)

recoutputs(3,2).. - z(3,2) =E= 0 ; (LHS = 0)

recoutputs(3,3).. - z(3,3) + 0.5*h(6) =E= 0 ; (LHS = 0)

recoutputs(3,4).. - z(3,4) + 1.5*h(6) + 0.5*h(7) =E= 0 ; (LHS = 0)

recoutputs(3,5).. - z(3,5) + 0.5*h(7) =E= 0 ; (LHS = 0)

---- recbyprod =G= producer m must generate an amount of byproduct type l equa
l or greater than the amount of l used by producers mp as i
nputs

recbyprod(A,1).. - y(A,1,1) - y(A,1,2) - y(A,1,3) =G= 0 ; (LHS = 0)

recbyprod(A,2).. - y(A,2,1) - y(A,2,2) - y(A,2,3) =G= 0 ; (LHS = 0)

recbyprod(A,3).. - y(A,3,1) - y(A,3,2) - y(A,3,3) =G= 0 ; (LHS = 0)

recbyprod(B,1).. - y(B,1,1) - y(B,1,2) - y(B,1,3) =G= 0 ; (LHS = 0)

recbyprod(B,2).. - y(B,2,1) - y(B,2,2) - y(B,2,3) =G= 0 ; (LHS = 0)

recbyprod(B,3).. - y(B,3,1) - y(B,3,2) - y(B,3,3) + 2*h(6) + h(7) =G= 0 ;

(LHS = 0)

recbyprod(C,1).. - y(C,1,1) - y(C,1,2) - y(C,1,3) =G= 0 ; (LHS = 0)

recbyprod(C,2).. - y(C,2,1) - y(C,2,2) - y(C,2,3) + 2*h(4) + 0.5*h(5) =G= 0 ;

(LHS = 0)

recbyprod(C,3).. - y(C,3,1) - y(C,3,2) - y(C,3,3) =G= 0 ; (LHS = 0)

recbyprod(D,1).. - y(D,1,1) - y(D,1,2) - y(D,1,3) + 2*h(1) + 1.5*h(2) =G= 0 ;

(LHS = 0)

recbyprod(D,2).. - y(D,2,1) - y(D,2,2) - y(D,2,3) =G= 0 ; (LHS = 0)

recbyprod(D,3).. - y(D,3,1) - y(D,3,2) - y(D,3,3) =G= 0 ; (LHS = 0)

---- x flow quantities from supplier to producer in UNITS

x(A,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-16 cost
1 supply(A)
1 recinputs(A,1)

x(A,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-24 cost
1 supply(A)
1 recinputs(A,2)

x(A,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-40 cost
1 supply(A)
1 recinputs(A,3)

x(B,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-30 cost
1 supply(B)
1 recinputs(B,1)

x(B,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-20 cost
1 supply(B)
1 recinputs(B,2)

x(B,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-40 cost

1 supply(B)

1 recinputs(B,3)

x(C,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-48 cost

1 supply(C)

1 recinputs(C,1)

x(C,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-24 cost

1 supply(C)

1 recinputs(C,2)

x(C,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-36 cost

1 supply(C)

1 recinputs(C,3)

x(D,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-70 cost

1 supply(D)

1 recinputs(D,1)

x(D,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-42 cost

1 supply(D)

1 recinputs(D,2)

x(D,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-28 cost

1 supply(D)

1 recinputs(D,3)

---- y flow quantities between producers in UNITS

y(A,1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,1)

-1 recbyprod(A,1)

y(A,1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 cost

1 recinputs(A,2)

-1 recbyprod(A,1)

y(A,1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 recinputs(A,3)

-1 recbyprod(A,1)

y(A,2,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 cost

1 recinputs(A,1)

-1 recbyprod(A,2)

y(A,2,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,2)

-1 recbyprod(A,2)

y(A,2,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 cost

1 recinputs(A,3)

-1 recbyprod(A,2)

y(A,3,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 recinputs(A,1)

-1 recbyprod(A,3)

y(A,3,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 cost

1 recinputs(A,2)

-1 recbyprod(A,3)

y(A,3,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,3)

-1 recbyprod(A,3)

y(B,1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(B,1)

-1 recbyprod(B,1)

y(B,1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 cost

1 recinputs(B,2)

-1 recbyprod(B,1)

y(B,1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-10 cost

1 recinputs(B,3)

-1 recbyprod(B,1)

y(B,2,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-5 cost
1 recinputs(B,1)
-1 recbyprod(B,2)

y(B,2,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
1 recinputs(B,2)
-1 recbyprod(B,2)

y(B,2,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-5 cost
1 recinputs(B,3)
-1 recbyprod(B,2)

y(B,3,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-10 cost
1 recinputs(B,1)
-1 recbyprod(B,3)

y(B,3,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-5 cost
1 recinputs(B,2)
-1 recbyprod(B,3)

y(B,3,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
1 recinputs(B,3)
-1 recbyprod(B,3)

y(C,1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(C,1)

-1 recbyprod(C,1)

y(C,1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 recinputs(C,2)

-1 recbyprod(C,1)

y(C,1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-12 cost

1 recinputs(C,3)

-1 recbyprod(C,1)

y(C,2,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 recinputs(C,1)

-1 recbyprod(C,2)

y(C,2,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(C,2)

-1 recbyprod(C,2)

y(C,2,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 recinputs(C,3)

-1 recbyprod(C,2)

y(C,3,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-12 cost

1 recinputs(C,1)

```
      -1       recbyprod(C,3)

y(C,3,2)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
      -6       cost
       1       recinputs(C,2)
      -1       recbyprod(C,3)

y(C,3,3)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
       1       recinputs(C,3)
      -1       recbyprod(C,3)

y(D,1,1)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
       1       recinputs(D,1)
      -1       recbyprod(D,1)

y(D,1,2)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
      -7       cost
       1       recinputs(D,2)
      -1       recbyprod(D,1)

y(D,1,3)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -14       cost
       1       recinputs(D,3)
      -1       recbyprod(D,1)

REMAINING 6 ENTRIES SKIPPED

---- z  flow quantities from producer to market in UNITS

z(1,1)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
      -5       cost
```

1 demand(1)
-1 recoutputs(1,1)

z(1,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-6 cost
1 demand(2)
-1 recoutputs(1,2)

z(1,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-7 cost
1 demand(3)
-1 recoutputs(1,3)

z(1,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-8 cost
1 demand(4)
-1 recoutputs(1,4)

z(1,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-9 cost
1 demand(5)
-1 recoutputs(1,5)

z(2,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-7 cost
1 demand(1)
-1 recoutputs(2,1)

z(2,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-6 cost
1 demand(2)

-1 recoutputs(2,2)

z(2,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 cost

1 demand(3)

-1 recoutputs(2,3)

z(2,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(4)

-1 recoutputs(2,4)

z(2,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(5)

-1 recoutputs(2,5)

z(3,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-9 cost

1 demand(1)

-1 recoutputs(3,1)

z(3,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 demand(2)

-1 recoutputs(3,2)

z(3,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(3)

-1 recoutputs(3,3)

z(3,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(4)

-1 recoutputs(3,4)

z(3,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 cost

1 demand(5)

-1 recoutputs(3,5)

---- h rate of production of recipe r at producer m in MULTIPLES OF r

h(1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 recinputs(A,1)

-5 recinputs(B,1)

1 recoutputs(1,1)

2 recbyprod(D,1)

h(2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-3 recinputs(B,1)

-1.5 recinputs(C,1)

4.2 recoutputs(1,2)

1.5 recbyprod(D,1)

h(3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-1 recinputs(A,2)

-4 recinputs(D,2)

3 recoutputs(2,3)

h(4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-3.5 recinputs(A,2)

-5.5 recinputs(B,2)

1.8 recoutputs(2,1)

2 recbyprod(C,2)

h(5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-1 recinputs(C,2)

-2 recinputs(D,2)

1 recoutputs(2,2)

2 recoutputs(2,4)

0.5 recbyprod(C,2)

h(6)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 recinputs(C,3)

-6 recinputs(D,3)

0.5 recoutputs(3,3)

1.5 recoutputs(3,4)

2 recbyprod(B,3)

h(7)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-0.5 recinputs(A,3)

-1 recinputs(C,3)

0.5 recoutputs(3,4)

0.5 recoutputs(3,5)

1 recbyprod(B,3)

---- f total cost in DOLLARS

f

(.LO, .L, .UP, .M = -INF, 0, +INF, 0)

1 cost

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
Model Statistics SOLVE NETWORK Using LP From line 140

MODEL STATISTICS

BLOCKS OF EQUATIONS	6	SINGLE EQUATIONS	49
BLOCKS OF VARIABLES	5	SINGLE VARIABLES	71
NON ZERO ELEMENTS	208		

GENERATION TIME = 0.156 SECONDS 4 Mb WEX239-239 Nov 9, 2012

EXECUTION TIME = 0.171 SECONDS 4 Mb WEX239-239 Nov 9, 2012

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
Solution Report SOLVE NETWORK Using LP From line 140

S O L V E S U M M A R Y

MODEL NETWORK OBJECTIVE f
TYPE LP DIRECTION MINIMIZE
SOLVER CPLEX FROM LINE 140

**** SOLVER STATUS 1 Normal Completion
**** MODEL STATUS 1 Optimal
**** OBJECTIVE VALUE 133829.3651

RESOURCE USAGE, LIMIT 0.000 1000.000
ITERATION COUNT, LIMIT 10 2000000000

IBM ILOG CPLEX Jul 4, 2012 23.9.5 WEX 36376.36401 WEI x86_64/MS Windows
Cplex 12.4.0.1

LP status(1): optimal
Optimal solution found.
Objective : 133829.365079

LOWER LEVEL UPPER MARGINAL

---- EQU cost . . . 1.000

cost define objective function

---- EQU supply observe supply limit at supplier l

LOWER LEVEL UPPER MARGINAL

A -INF 1638.889 2000.000 .

B -INF 884.921 2000.000 .
C -INF 623.016 2000.000 .
D -INF 488.095 2000.000 .

---- EQU demand satisfy or exceed demand at market n

LOWER LEVEL UPPER MARGINAL

1 500.000 500.000 +INF 81.444
2 500.000 500.000 +INF 24.214
3 500.000 500.000 +INF 69.000
4 500.000 500.000 +INF .
5 500.000 500.000 +INF 93.000

---- EQU recinputs satisfy material balance between inputs received at m and in puts used in recipes at m

LOWER LEVEL UPPER MARGINAL

A.1 . . . 16.000
A.2 . . . 24.000
A.3 . . . 40.000
B.1 . . . 25.000
B.2 . . . 20.000
B.3 . . . .
C.1 . . . 36.000
C.2 . . . -62.786
C.3 . . . 36.000
D.1 . . . .
D.2 . . . 42.000
D.3 . . . -3.333

---- EQU recoutputs satisfy that product outputs from each producer equals prod ucts received by markets

LOWER LEVEL UPPER MARGINAL

1.1	.	.	.	76.444
1.2	.	.	.	18.214
1.3	.	.	.	62.000
1.4	.	.	.	.
1.5	.	.	.	84.000
2.1	.	.	.	74.444
2.2	.	.	.	18.214
2.3	.	.	.	64.000
2.4	.	.	.	-6.000
2.5	.	.	.	86.000
3.1	.	.	.	72.444
3.2	.	.	.	16.214
3.3	.	.	.	62.000
3.4	.	.	.	-6.000
3.5	.	.	.	88.000

---- EQU recbyprod producer m must generate an amount of byproduct type l equal or greater than the amount of l used by producers mp as inpu ts

	LOWER	LEVEL	UPPER	MARGINAL
A.1	.	.	+INF	32.000
A.2	.	.	+INF	36.000
A.3	.	.	+INF	40.000
B.1	.	.	+INF	25.000
B.2	.	.	+INF	20.000
B.3	.	.	+INF	15.000
C.1	.	.	+INF	36.000
C.2	.	.	+INF	30.000
C.3	.	.	+INF	36.000
D.1	.	.	+INF	35.000
D.2	.	.	+INF	42.000
D.3	.	.	+INF	35.000

---- VAR x flow quantities from supplier to producer in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
A.1	.	.	+INF	.
A.2	.	1138.889	+INF	.
A.3	.	500.000	+INF	.
B.1	.	.	+INF	5.000
B.2	.	884.921	+INF	.
B.3	.	.	+INF	40.000
C.1	.	.	+INF	12.000
C.2	.	.	+INF	86.786
C.3	.	623.016	+INF	.
D.1	.	.	+INF	70.000
D.2	.	488.095	+INF	.
D.3	.	.	+INF	31.333

---- VAR y flow quantities between producers in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
A.1.1	.	.	+INF	16.000
A.1.2	.	.	+INF	12.000
A.1.3	.	.	+INF	.
A.2.1	.	.	+INF	24.000
A.2.2	.	.	+INF	12.000
A.2.3	.	.	+INF	.
A.3.1	.	.	+INF	32.000
A.3.2	.	.	+INF	20.000
A.3.3	.	.	+INF	.
B.1.1	.	.	+INF	.
B.1.2	.	.	+INF	10.000
B.1.3	.	.	+INF	35.000
B.2.1	.	.	+INF	.
B.2.2	.	.	+INF	EPS
B.2.3	.	.	+INF	25.000
B.3.1	.	357.143	+INF	.
B.3.2	.	642.857	+INF	.
B.3.3	.	.	+INF	15.000

C.1.1	.	.	+INF	.
C.1.2	.	.	+INF	104.786
C.1.3	.	.	+INF	12.000
C.2.1	.	178.571	+INF	.
C.2.2	.	.	+INF	92.786
C.2.3	.	376.984	+INF	.
C.3.1	.	.	+INF	12.000
C.3.2	.	.	+INF	104.786
C.3.3	.	.	+INF	.
D.1.1	.	.	+INF	35.000
D.1.2	.	178.571	+INF	.
D.1.3	.	.	+INF	52.333
D.2.1	.	.	+INF	49.000
D.2.2	.	.	+INF	.
D.2.3	.	.	+INF	52.333
D.3.1	.	.	+INF	49.000
D.3.2	.	.	+INF	.
D.3.3	.	.	+INF	38.333

---- VAR z flow quantities from producer to market in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
1.1	.	.	+INF	.
1.2	.	500.000	+INF	.
1.3	.	.	+INF	.
1.4	.	.	+INF	8.000
1.5	.	.	+INF	.
2.1	.	500.000	+INF	.
2.2	.	.	+INF	.
2.3	.	500.000	+INF	.
2.4	.	.	+INF	.
2.5	.	.	+INF	.
3.1	.	.	+INF	.
3.2	.	.	+INF	.
3.3	.	.	+INF	.
3.4	.	500.000	+INF	.

3.5 . 500.000 +INF .

---- VAR h rate of production of recipe r at producer m in MULTIPLES OF r

LOWER LEVEL UPPER MARGINAL

1 . . +INF 42.556
2 . 119.048 +INF .
3 . 166.667 +INF .
4 . 277.778 +INF .
5 . . +INF .
6 . . +INF .
7 . 1000.000 +INF .

LOWER LEVEL UPPER MARGINAL

---- VAR f -INF 1.3383E+5 +INF .

f total cost in DOLLARS

**** REPORT SUMMARY : 0 NONOPT
0 INFEASIBLE
0 UNBOUNDED

---- 141 VARIABLE x.L flow quantities from supplier to producer in UNITS

	2	3
A	1138.889	500.000
B	884.921	
C		623.016
D	488.095	

---- 141 VARIABLE h.L rate of production of recipe r at producer m in MULTIPLES OF r

2 119.048, 3 166.667, 4 277.778, 7 1000.000

---- 141 VARIABLE y.L flow quantities between producers in UNITS

	1	2	3
B.3	357.143	642.857	
C.2	178.571		376.984
D.1		178.571	

---- 141 VARIABLE z.L flow quantities from producer to market in UNITS

	1	2	3	4	5
1		500.000			
2	500.000		500.000		
3				500.000	500.000

---- 141 VARIABLE x.M flow quantities from supplier to producer in UNITS

	1	2	3
B	5.000		40.000
C	12.000	86.786	
D	70.000		31.333

---- 141 VARIABLE h.M rate of production of recipe r at producer m in MULTIP
LES OF r

1	42.556

---- 141 VARIABLE y.M flow quantities between producers in UNITS

	1	2	3
A.1	16.000	12.000	
A.2	24.000	12.000	
A.3	32.000	20.000	
B.1		10.000	35.000
B.2		EPS	25.000
B.3			15.000
C.1		104.786	12.000
C.2		92.786	
C.3	12.000	104.786	
D.1	35.000		52.333
D.2	49.000		52.333
D.3	49.000		38.333

---- 141 VARIABLE z.M flow quantities from producer to market in UNITS

4

1 8.000

EXECUTION TIME = 0.093 SECONDS 3 Mb WEX239-239 Nov 9, 2012

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**** FILE SUMMARY

Input \\psf\Home\Documents\gamsdir\projdir\Final Models For Thesis\Problem 1.gms
Output \\psf\Home\Documents\gamsdir\projdir\Problem 1.lst

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
C o m p i l a t i o n

```
 1  Sets
 2     l    external suppliers   / A*D /
 3     m     producers           / 1*5 /
 4     n     markets             / 1*5 /
 5     r     recipes             / 1*11 / ;
 6
 7  alias (m,mp)    ;
 8
 9  Parameters
10    S(l)  capacity of external supplier l in UNITS
11     /   A    2000
12         B    2000
13         C    2000
14         D    2000 /
15
16    D(n)  demand at market n in UNITS
17     /   1       500
18         2       500
19         3       500
20         4       500
21         5       500 /
22
23    wi(l)  weighted cost of shipping l in DOLLARS PER UNIT
24     /   A  4
25         B  5
26         C  6
27         D  7   /
28
29    wo(n)  weighted cost of shipping n in DOLLARS PER UNIT
30     /   1  1
31         2  1
32         3  1
```

```
33        4  1
34        5  1     /;
35
36  Table dx(l,m)   distance from supplier l to producer m in MILES
37         1    2    3    4    5
38    A    4    6    10   5    12
39    B    6    4    8    6    9
40    C    8    4    6    9    6
41    D    10   6    4    12   5 ;
42
43  Table dy(m,mp)   distance from producer m to producer mp in MILES
44       1    2    3    4    5
45    1  0    1    2    0.5  3
46    2  1    0    1    2    2
47    3  2    1    0    3    0.5
48    4  0.5  2    3    0    4
49    5  3    2    0.5  4    0 ;
50
51  Table dz(m,n)   distance from producer m to market n in MILES
52         1    2    3    4    5
53    1    5    6    7    8    9
54    2    7    6    5    6    7
55    3    9    8    7    6    5
56    4    6    7    8    9    10
57    5    10   9    8    7    6 ;
58
59  Table hrm(r,m) recipes r used at each producer m
60        1  2  3  4  5
61    1   1  0  0  0  0
62    2   1  0  0  0  0
63    3   0  1  0  0  0
64    4   0  1  0  0  0
65    5   0  1  0  0  0
66    6   0  0  1  0  0
67    7   0  0  1  0  0
68    8   0  0  0  1  0
69    9   0  0  0  1  0
```

```
   10  0  0  0  0  1
   11  0  0  0  0  1 ;
*producer 4 and 5 each have two recipes they can use
*producer 4 can use recipes 8 and 9
*producer 5 can use recipes 10 and 11

Table a(r,l)   amount of input l used in recipe r in UNITS
          A       B       C       D
1         4       5       0       0
2         0       3       1.5     0
3         1       0       0       4
4         3.5     5.5     0       0
5         0       0       1       2
6         0       0       2       6
7         0.5     0       1       0
8         4       5       0       0
9         4       5       0       0
10        1       0       1       1
11        0       0       5       0  ;
*recipes 8 and 9 both have the same input coefficient as recipe 1
*recall that recipe 1 is not used in the problem 1 solution
*recipe 10 requires small amounts of all inputs except B
*recipe 11 requires only large amounts of input C

Table bz(r,n)   amount of output n produced by recipe r in UNITS
          1   2   3   4   5
1         1.0 0.0 0.0 0.0 0.0
2         0.0 4.2 0.0 0.0 0.0
3         0.0 0.0 3.0 0.0 0.0
4         1.8 0.0 0.0 0.0 0.0
5         0.0 1.0 0.0 2.0 0.0
6         0.0 0.0 0.5 1.5 0.0
7         0.0 0.0 0.0 0.5 0.5
8         2.0 0.0 0.0 0.0 0.0
9         1.0 0.0 0.0 0.0 0.0
10        0.0 0.0 3.0 0.0 0.0
11        0.0 0.0 0.0 0.0 0.5 ;
```

```
107  *recipe 8 produces twice the amount of the same product as recipe 1
108  *recipe 9 produces an equal amount of the same product as recipe 1
109  *recipe 10 produces an equal amount of product 3 as recipe 3
110  *recipe 11 produces an equal amount of product 5 as recipe 7
111
112  Table by(r,l)   amount of input l produced by recipe r in UNITS
113     A   B   C   D
114  1  0.0 0.0 0.0 2.0
115  2  0.0 0.0 0.0 1.5
116  3  0.0 0.0 0.0 0.0
117  4  0.0 0.0 2.0 0.0
118  5  0.0 0.0 0.5 0.0
119  6  0.0 2.0 0.0 0.0
120  7  0.0 1.0 0.0 0.0
121  8  0.0 0.0 0.0 2.0
122  9  0.5 2.0 0.0 1.0
123  10 0.0 0.0 0.0 0.0
124  11 0.5 0.0 0.0 0.0 ;
125
126  *recipe 8 produces the same by-products as recipe 1
127  *recipe 9 produces more by-products than recipe 1
128  *recipe 10 produces the no by-products, like recipe 3
129  *recipe 11 produces only a small amount of by-product A
130
131  *Producer 4 competes with producer 1, with recipe 8 and 9
132  *   where one recipe is more productive, and the other has more utility
133  *   for other producers
134  *Producer 5 competes with producers 2 and 3, with recipe 10 and 11
135  *   where recipe 10 produces product 3 from a less but more varied inputs
136  *   and recipe 11 produces the same product outputs as recipe 3, and produ
     ces
137  *   only by-product A
138
139  Parameter cx(l,m)   cost of transporting input l from supplier l to produc
     er m in DOLLARS PER UNIT;
140    cx(l,m) = dx(l,m) * wi(l)  ;
141
```

```
142  Parameter cy(l,m,mp)   cost of transporting input l from producer m to pro
     ducer mp in DOLLARS PER UNIT;
143    cy(l,m,mp) = dy(m,mp) * wi(l)   ;
144
145  Parameter cz(m,n)   cost of transporting output n from producer m to marke
     t n in DOLLARS PER UNIT;
146    cz(m,n) = dz(m,n) * wo(n)   ;
147
148  Variables
149    x(l,m)    flow quantities from supplier to producer in UNITS
150    y(l,m,mp) flow quantities between producers in UNITS
151    z(m,n)    flow quantities from producer to market in UNITS
152    h(r)      rate of production of recipe r at producer m in MULTIPLES OF r
153    f         total cost in DOLLARS ;
154
155  Positive variable x;
156  Positive variable y;
157  Positive variable z;
158  Positive variable h;
159
160  Equations
161  cost            define objective function
162  supply(l)       observe supply limit at supplier l
163  demand(n)       satisfy or exceed demand at market n
164  recinputs(l,m)  satisfy material balance between inputs received at m and
     inputs used in recipes at m
165  recoutputs(m,n) satisfy that product outputs from each producer equals pro
     ducts received by markets
166  recbyprod(l,m)  producer m must generate an amount of byproduct type l equ
     al or greater than the amount of l used by producers mp as inputs ;
167
168
169  cost ..             f =e= sum((l,m), cx(l,m)*x(l,m))+ sum((l,m,mp), cy(l,m
     ,mp)*y(l,m,mp))+ sum((m,n), cz(m,n)*z(m,n))   ;
170
171  supply(l) ..        sum(m, x(l,m))  =l=  s(l)   ;
172
```

```
173  demand(n) ..        sum(m, z(m,n))  =g=  d(n)   ;
174
175  recinputs (l,m) ..  x(l,m) + sum(mp, y(l,mp,m)) =e= sum(r, h(r)*a(r,l)$(hr
     m(r,m) gt 0))   ;
176
177  recoutputs(m,n) ..  sum(r, h(r)*bz(r,n)$(hrm(r,m) gt 0)) =e= z(m,n)   ;
178
179  recbyprod(l,m) ..   sum(r, h(r)*by(r,l)$(hrm(r,m) gt 0)) =g= sum(mp, y(l,m
     ,mp))   ;
180
181
182  option limrow = 30, limcol = 30 ;
183    MODEL NETWORK /ALL/ ;
184    SOLVE NETWORK USING LP MINIMIZING f ;
185  display x.l,h.l,y.l,z.l,x.m,h.m,y.m,z.m ;


COMPILATION TIME     =        0.000 SECONDS      3 Mb  WEX239-239 Nov  9, 2012
```

---- cost =E= define objective function

cost.. - 16*x(A,1) - 24*x(A,2) - 40*x(A,3) - 20*x(A,4) - 48*x(A,5) - 30*x(B,1)

- 20*x(B,2) - 40*x(B,3) - 30*x(B,4) - 45*x(B,5) - 48*x(C,1) - 24*x(C,2)

- 36*x(C,3) - 54*x(C,4) - 36*x(C,5) - 70*x(D,1) - 42*x(D,2) - 28*x(D,3)

- 84*x(D,4) - 35*x(D,5) - 4*y(A,1,2) - 8*y(A,1,3) - 2*y(A,1,4)

- 12*y(A,1,5) - 4*y(A,2,1) - 4*y(A,2,3) - 8*y(A,2,4) - 8*y(A,2,5)

- 8*y(A,3,1) - 4*y(A,3,2) - 12*y(A,3,4) - 2*y(A,3,5) - 2*y(A,4,1)

- 8*y(A,4,2) - 12*y(A,4,3) - 16*y(A,4,5) - 12*y(A,5,1) - 8*y(A,5,2)

- 2*y(A,5,3) - 16*y(A,5,4) - 5*y(B,1,2) - 10*y(B,1,3) - 2.5*y(B,1,4)

- 15*y(B,1,5) - 5*y(B,2,1) - 5*y(B,2,3) - 10*y(B,2,4) - 10*y(B,2,5)

- 10*y(B,3,1) - 5*y(B,3,2) - 15*y(B,3,4) - 2.5*y(B,3,5) - 2.5*y(B,4,1)

- 10*y(B,4,2) - 15*y(B,4,3) - 20*y(B,4,5) - 15*y(B,5,1) - 10*y(B,5,2)

- 2.5*y(B,5,3) - 20*y(B,5,4) - 6*y(C,1,2) - 12*y(C,1,3) - 3*y(C,1,4)

- 18*y(C,1,5) - 6*y(C,2,1) - 6*y(C,2,3) - 12*y(C,2,4) - 12*y(C,2,5)

- 12*y(C,3,1) - 6*y(C,3,2) - 18*y(C,3,4) - 3*y(C,3,5) - 3*y(C,4,1)

- 12*y(C,4,2) - 18*y(C,4,3) - 24*y(C,4,5) - 18*y(C,5,1) - 12*y(C,5,2)

- 3*y(C,5,3) - 24*y(C,5,4) - 7*y(D,1,2) - 14*y(D,1,3) - 3.5*y(D,1,4)

- 21*y(D,1,5) - 7*y(D,2,1) - 7*y(D,2,3) - 14*y(D,2,4) - 14*y(D,2,5)

- 14*y(D,3,1) - 7*y(D,3,2) - 21*y(D,3,4) - 3.5*y(D,3,5) - 3.5*y(D,4,1)

- 14*y(D,4,2) - 21*y(D,4,3) - 28*y(D,4,5) - 21*y(D,5,1) - 14*y(D,5,2)

- 3.5*y(D,5,3) - 28*y(D,5,4) - 5*z(1,1) - 6*z(1,2) - 7*z(1,3) - 8*z(1,4)

- 9*z(1,5) - 7*z(2,1) - 6*z(2,2) - 5*z(2,3) - 6*z(2,4) - 7*z(2,5)

- 9*z(3,1) - 8*z(3,2) - 7*z(3,3) - 6*z(3,4) - 5*z(3,5) - 6*z(4,1)

- 7*z(4,2) - 8*z(4,3) - 9*z(4,4) - 10*z(4,5) - 10*z(5,1) - 9*z(5,2)

- 8*z(5,3) - 7*z(5,4) - 6*z(5,5) + f =E= 0 ; (LHS = 0)

---- supply =L= observe supply limit at supplier l

supply(A).. x(A,1) + x(A,2) + x(A,3) + x(A,4) + x(A,5) =L= 2000 ; (LHS = 0)

supply(B).. x(B,1) + x(B,2) + x(B,3) + x(B,4) + x(B,5) =L= 2000 ; (LHS = 0)

supply(C).. x(C,1) + x(C,2) + x(C,3) + x(C,4) + x(C,5) =L= 2000 ; (LHS = 0)

supply(D).. x(D,1) + x(D,2) + x(D,3) + x(D,4) + x(D,5) =L= 2000 ; (LHS = 0)

---- demand =G= satisfy or exceed demand at market n

demand(1).. z(1,1) + z(2,1) + z(3,1) + z(4,1) + z(5,1) =G= 500 ;

     (LHS = 0, INFES = 500 ****)

demand(2).. z(1,2) + z(2,2) + z(3,2) + z(4,2) + z(5,2) =G= 500 ;

(LHS = 0, INFES = 500 ****)

demand(3).. z(1,3) + z(2,3) + z(3,3) + z(4,3) + z(5,3) =G= 500 ;

(LHS = 0, INFES = 500 ****)

demand(4).. z(1,4) + z(2,4) + z(3,4) + z(4,4) + z(5,4) =G= 500 ;

(LHS = 0, INFES = 500 ****)

demand(5).. z(1,5) + z(2,5) + z(3,5) + z(4,5) + z(5,5) =G= 500 ;

(LHS = 0, INFES = 500 ****)

---- recinputs =E= satisfy material balance between inputs received at m and i
nputs used in recipes at m

recinputs(A,1).. x(A,1) + y(A,1,1) + y(A,2,1) + y(A,3,1) + y(A,4,1) + y(A,5,1)

- 4*h(1) =E= 0 ; (LHS = 0)

recinputs(A,2).. x(A,2) + y(A,1,2) + y(A,2,2) + y(A,3,2) + y(A,4,2) + y(A,5,2)

- h(3) - 3.5*h(4) =E= 0 ; (LHS = 0)

recinputs(A,3).. x(A,3) + y(A,1,3) + y(A,2,3) + y(A,3,3) + y(A,4,3) + y(A,5,3)

- 0.5*h(7) =E= 0 ; (LHS = 0)

recinputs(A,4).. x(A,4) + y(A,1,4) + y(A,2,4) + y(A,3,4) + y(A,4,4) + y(A,5,4)

- 4*h(8) - 4*h(9) =E= 0 ; (LHS = 0)

recinputs(A,5).. x(A,5) + y(A,1,5) + y(A,2,5) + y(A,3,5) + y(A,4,5) + y(A,5,5)

- h(10) =E= 0 ; (LHS = 0)

recinputs(B,1).. x(B,1) + y(B,1,1) + y(B,2,1) + y(B,3,1) + y(B,4,1) + y(B,5,1)

- 5*h(1) - 3*h(2) =E= 0 ; (LHS = 0)

recinputs(B,2).. x(B,2) + y(B,1,2) + y(B,2,2) + y(B,3,2) + y(B,4,2) + y(B,5,2)

- 5.5*h(4) =E= 0 ; (LHS = 0)

recinputs(B,3).. x(B,3) + y(B,1,3) + y(B,2,3) + y(B,3,3) + y(B,4,3) + y(B,5,3)
=E= 0 ; (LHS = 0)

recinputs(B,4).. x(B,4) + y(B,1,4) + y(B,2,4) + y(B,3,4) + y(B,4,4) + y(B,5,4)

- 5*h(8) - 5*h(9) =E= 0 ; (LHS = 0)

recinputs(B,5).. x(B,5) + y(B,1,5) + y(B,2,5) + y(B,3,5) + y(B,4,5) + y(B,5,5)
=E= 0 ; (LHS = 0)

recinputs(C,1).. x(C,1) + y(C,1,1) + y(C,2,1) + y(C,3,1) + y(C,4,1) + y(C,5,1)

- 1.5*h(2) =E= 0 ; (LHS = 0)

recinputs(C,2).. x(C,2) + y(C,1,2) + y(C,2,2) + y(C,3,2) + y(C,4,2) + y(C,5,2)

- h(5) =E= 0 ; (LHS = 0)

recinputs(C,3).. x(C,3) + y(C,1,3) + y(C,2,3) + y(C,3,3) + y(C,4,3) + y(C,5,3)

- 2*h(6) - h(7) =E= 0 ; (LHS = 0)

recinputs(C,4).. x(C,4) + y(C,1,4) + y(C,2,4) + y(C,3,4) + y(C,4,4) + y(C,5,4)
=E= 0 ; (LHS = 0)

recinputs(C,5).. x(C,5) + y(C,1,5) + y(C,2,5) + y(C,3,5) + y(C,4,5) + y(C,5,5)

- h(10) - 5*h(11) =E= 0 ; (LHS = 0)

recinputs(D,1).. x(D,1) + y(D,1,1) + y(D,2,1) + y(D,3,1) + y(D,4,1) + y(D,5,1)
=E= 0 ; (LHS = 0)

recinputs(D,2).. x(D,2) + y(D,1,2) + y(D,2,2) + y(D,3,2) + y(D,4,2) + y(D,5,2)

- 4*h(3) - 2*h(5) =E= 0 ; (LHS = 0)

recinputs(D,3).. x(D,3) + y(D,1,3) + y(D,2,3) + y(D,3,3) + y(D,4,3) + y(D,5,3)

- 6*h(6) =E= 0 ; (LHS = 0)

recinputs(D,4).. x(D,4) + y(D,1,4) + y(D,2,4) + y(D,3,4) + y(D,4,4) + y(D,5,4)
=E= 0 ; (LHS = 0)

recinputs(D,5).. x(D,5) + y(D,1,5) + y(D,2,5) + y(D,3,5) + y(D,4,5) + y(D,5,5)

- h(10) =E= 0 ; (LHS = 0)

---- recoutputs =E= satisfy that product outputs from each producer equals pro
ducts received by markets

recoutputs(1,1).. - z(1,1) + h(1) =E= 0 ; (LHS = 0)

recoutputs(1,2).. - z(1,2) + 4.2*h(2) =E= 0 ; (LHS = 0)

recoutputs(1,3).. - z(1,3) =E= 0 ; (LHS = 0)

recoutputs(1,4).. - z(1,4) =E= 0 ; (LHS = 0)

recoutputs(1,5).. - z(1,5) =E= 0 ; (LHS = 0)

recoutputs(2,1).. - z(2,1) + 1.8*h(4) =E= 0 ; (LHS = 0)

recoutputs(2,2).. - z(2,2) + h(5) =E= 0 ; (LHS = 0)

recoutputs(2,3).. - z(2,3) + 3*h(3) =E= 0 ; (LHS = 0)

recoutputs(2,4).. - z(2,4) + 2*h(5) =E= 0 ; (LHS = 0)

recoutputs(2,5).. - z(2,5) =E= 0 ; (LHS = 0)

recoutputs(3,1).. - z(3,1) =E= 0 ; (LHS = 0)

recoutputs(3,2).. - z(3,2) =E= 0 ; (LHS = 0)

recoutputs(3,3).. - z(3,3) + 0.5*h(6) =E= 0 ; (LHS = 0)

recoutputs(3,4).. - z(3,4) + 1.5*h(6) + 0.5*h(7) =E= 0 ; (LHS = 0)

recoutputs(3,5).. - z(3,5) + 0.5*h(7) =E= 0 ; (LHS = 0)

recoutputs(4,1).. - z(4,1) + 2*h(8) + h(9) =E= 0 ; (LHS = 0)

recoutputs(4,2).. - z(4,2) =E= 0 ; (LHS = 0)

recoutputs(4,3).. - z(4,3) =E= 0 ; (LHS = 0)

recoutputs(4,4).. - z(4,4) =E= 0 ; (LHS = 0)

recoutputs(4,5).. - z(4,5) =E= 0 ; (LHS = 0)

recoutputs(5,1).. - z(5,1) =E= 0 ; (LHS = 0)

recoutputs(5,2).. - z(5,2) =E= 0 ; (LHS = 0)

recoutputs(5,3).. - z(5,3) + 3*h(10) =E= 0 ; (LHS = 0)

recoutputs(5,4).. - z(5,4) =E= 0 ; (LHS = 0)

recoutputs(5,5).. - z(5,5) + 0.5*h(11) =E= 0 ; (LHS = 0)

---- recbyprod =G= producer m must generate an amount of byproduct type l equa
                l or greater than the amount of l used by producers mp as i
                nputs

recbyprod(A,1).. - y(A,1,1) - y(A,1,2) - y(A,1,3) - y(A,1,4) - y(A,1,5) =G= 0 ;

     (LHS = 0)

recbyprod(A,2).. - y(A,2,1) - y(A,2,2) - y(A,2,3) - y(A,2,4) - y(A,2,5) =G= 0 ;

     (LHS = 0)

recbyprod(A,3).. - y(A,3,1) - y(A,3,2) - y(A,3,3) - y(A,3,4) - y(A,3,5) =G= 0 ;

     (LHS = 0)

recbyprod(A,4).. - y(A,4,1) - y(A,4,2) - y(A,4,3) - y(A,4,4) - y(A,4,5)

     + 0.5*h(9) =G= 0 ; (LHS = 0)

recbyprod(A,5).. - y(A,5,1) - y(A,5,2) - y(A,5,3) - y(A,5,4) - y(A,5,5)

     + 0.5*h(11) =G= 0 ; (LHS = 0)

recbyprod(B,1).. - y(B,1,1) - y(B,1,2) - y(B,1,3) - y(B,1,4) - y(B,1,5) =G= 0 ;

     (LHS = 0)

recbyprod(B,2).. - y(B,2,1) - y(B,2,2) - y(B,2,3) - y(B,2,4) - y(B,2,5) =G= 0 ;

     (LHS = 0)

recbyprod(B,3).. - y(B,3,1) - y(B,3,2) - y(B,3,3) - y(B,3,4) - y(B,3,5)

     + 2*h(6) + h(7) =G= 0 ; (LHS = 0)

recbyprod(B,4).. - y(B,4,1) - y(B,4,2) - y(B,4,3) - y(B,4,4) - y(B,4,5)

+ 2*h(9) =G= 0 ; (LHS = 0)

recbyprod(B,5).. - y(B,5,1) - y(B,5,2) - y(B,5,3) - y(B,5,4) - y(B,5,5) =G= 0 ;

(LHS = 0)

recbyprod(C,1).. - y(C,1,1) - y(C,1,2) - y(C,1,3) - y(C,1,4) - y(C,1,5) =G= 0 ;

(LHS = 0)

recbyprod(C,2).. - y(C,2,1) - y(C,2,2) - y(C,2,3) - y(C,2,4) - y(C,2,5)

+ 2*h(4) + 0.5*h(5) =G= 0 ; (LHS = 0)

recbyprod(C,3).. - y(C,3,1) - y(C,3,2) - y(C,3,3) - y(C,3,4) - y(C,3,5) =G= 0 ;

(LHS = 0)

recbyprod(C,4).. - y(C,4,1) - y(C,4,2) - y(C,4,3) - y(C,4,4) - y(C,4,5) =G= 0 ;

(LHS = 0)

recbyprod(C,5).. - y(C,5,1) - y(C,5,2) - y(C,5,3) - y(C,5,4) - y(C,5,5) =G= 0 ;

(LHS = 0)

recbyprod(D,1).. - y(D,1,1) - y(D,1,2) - y(D,1,3) - y(D,1,4) - y(D,1,5)

+ 2*h(1) + 1.5*h(2) =G= 0 ; (LHS = 0)

recbyprod(D,2).. - y(D,2,1) - y(D,2,2) - y(D,2,3) - y(D,2,4) - y(D,2,5) =G= 0 ;

(LHS = 0)

recbyprod(D,3).. - y(D,3,1) - y(D,3,2) - y(D,3,3) - y(D,3,4) - y(D,3,5) =G= 0 ;

(LHS = 0)

recbyprod(D,4).. - y(D,4,1) - y(D,4,2) - y(D,4,3) - y(D,4,4) - y(D,4,5)

+ 2*h(8) + h(9) =G= 0 ; (LHS = 0)

recbyprod(D,5).. - y(D,5,1) - y(D,5,2) - y(D,5,3) - y(D,5,4) - y(D,5,5) =G= 0 ;

(LHS = 0)

---- x flow quantities from supplier to producer in UNITS

x(A,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-16 cost
1 supply(A)
1 recinputs(A,1)

x(A,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-24 cost
1 supply(A)
1 recinputs(A,2)

x(A,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-40 cost
1 supply(A)
1 recinputs(A,3)

x(A,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-20 cost
1 supply(A)
1 recinputs(A,4)

x(A,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-48 cost
1 supply(A)
1 recinputs(A,5)

x(B,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-30 cost

1 supply(B)

1 recinputs(B,1)

x(B,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-20 cost

1 supply(B)

1 recinputs(B,2)

x(B,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-40 cost

1 supply(B)

1 recinputs(B,3)

x(B,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-30 cost

1 supply(B)

1 recinputs(B,4)

x(B,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-45 cost

1 supply(B)

1 recinputs(B,5)

x(C,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-48 cost

1 supply(C)

1 recinputs(C,1)

x(C,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-24 cost

1 supply(C)

1 recinputs(C,2)

x(C,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-36 cost

1 supply(C)

1 recinputs(C,3)

x(C,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-54 cost

1 supply(C)

1 recinputs(C,4)

x(C,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-36 cost

1 supply(C)

1 recinputs(C,5)

x(D,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-70 cost

1 supply(D)

1 recinputs(D,1)

x(D,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-42 cost

1 supply(D)

1 recinputs(D,2)

x(D,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-28 cost
1 supply(D)
1 recinputs(D,3)

x(D,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-84 cost
1 supply(D)
1 recinputs(D,4)

x(D,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-35 cost
1 supply(D)
1 recinputs(D,5)

---- y flow quantities between producers in UNITS

y(A,1,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
1 recinputs(A,1)
-1 recbyprod(A,1)

y(A,1,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-4 cost
1 recinputs(A,2)
-1 recbyprod(A,1)

y(A,1,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-8 cost
1 recinputs(A,3)
-1 recbyprod(A,1)

y(A,1,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost
1 recinputs(A,4)
-1 recbyprod(A,1)

y(A,1,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-12 cost
1 recinputs(A,5)
-1 recbyprod(A,1)

y(A,2,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 cost
1 recinputs(A,1)
-1 recbyprod(A,2)

y(A,2,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,2)
-1 recbyprod(A,2)

y(A,2,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 cost
1 recinputs(A,3)
-1 recbyprod(A,2)

y(A,2,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost
1 recinputs(A,4)
-1 recbyprod(A,2)

y(A,2,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 recinputs(A,5)
-1 recbyprod(A,2)

y(A,3,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-8 cost
1 recinputs(A,1)
-1 recbyprod(A,3)

y(A,3,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-4 cost
1 recinputs(A,2)
-1 recbyprod(A,3)

y(A,3,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
1 recinputs(A,3)
-1 recbyprod(A,3)

y(A,3,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-12 cost
1 recinputs(A,4)
-1 recbyprod(A,3)

y(A,3,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 recinputs(A,5)
-1 recbyprod(A,3)

y(A,4,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 recinputs(A,1)
-1 recbyprod(A,4)

y(A,4,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 recinputs(A,2)

-1 recbyprod(A,4)

y(A,4,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-12 cost

1 recinputs(A,3)

-1 recbyprod(A,4)

y(A,4,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,4)

-1 recbyprod(A,4)

y(A,4,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-16 cost

1 recinputs(A,5)

-1 recbyprod(A,4)

y(A,5,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-12 cost

1 recinputs(A,1)

-1 recbyprod(A,5)

y(A,5,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 recinputs(A,2)

-1 recbyprod(A,5)

y(A,5,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(A,3)

-1 recbyprod(A,5)

y(A,5,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-16 cost

1 recinputs(A,4)

-1 recbyprod(A,5)

y(A,5,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,5)

-1 recbyprod(A,5)

y(B,1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(B,1)

-1 recbyprod(B,1)

y(B,1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 cost

1 recinputs(B,2)

-1 recbyprod(B,1)

y(B,1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-10 cost

1 recinputs(B,3)

-1 recbyprod(B,1)

y(B,1,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2.5 cost

1 recinputs(B,4)

-1 recbyprod(B,1)

y(B,1,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-15 cost

1 recinputs(B,5)

-1 recbyprod(B,1)

REMAINING 70 ENTRIES SKIPPED

---- z flow quantities from producer to market in UNITS

z(1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 cost

1 demand(1)

-1 recoutputs(1,1)

z(1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(2)

-1 recoutputs(1,2)

z(1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(3)

-1 recoutputs(1,3)

z(1,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 demand(4)

-1 recoutputs(1,4)

z(1,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-9 cost

1 demand(5)

-1 recoutputs(1,5)

z(2,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(1)

-1 recoutputs(2,1)

z(2,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(2)

-1 recoutputs(2,2)

z(2,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 cost

1 demand(3)

-1 recoutputs(2,3)

z(2,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(4)

-1 recoutputs(2,4)

z(2,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(5)

-1 recoutputs(2,5)

z(3,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-9 cost
1 demand(1)
-1 recoutputs(3,1)

z(3,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-8 cost
1 demand(2)
-1 recoutputs(3,2)

z(3,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-7 cost
1 demand(3)
-1 recoutputs(3,3)

z(3,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-6 cost
1 demand(4)
-1 recoutputs(3,4)

z(3,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-5 cost
1 demand(5)
-1 recoutputs(3,5)

z(4,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-6 cost
1 demand(1)
-1 recoutputs(4,1)

z(4,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-7 cost

1 demand(2)
-1 recoutputs(4,2)

z(4,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-8 cost
1 demand(3)
-1 recoutputs(4,3)

z(4,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-9 cost
1 demand(4)
-1 recoutputs(4,4)

z(4,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-10 cost
1 demand(5)
-1 recoutputs(4,5)

z(5,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-10 cost
1 demand(1)
-1 recoutputs(5,1)

z(5,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-9 cost
1 demand(2)
-1 recoutputs(5,2)

z(5,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-8 cost
1 demand(3)

-1 recoutputs(5,3)

z(5,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(4)

-1 recoutputs(5,4)

z(5,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(5)

-1 recoutputs(5,5)

---- h rate of production of recipe r at producer m in MULTIPLES OF r

h(1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 recinputs(A,1)

-5 recinputs(B,1)

1 recoutputs(1,1)

2 recbyprod(D,1)

h(2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-3 recinputs(B,1)

-1.5 recinputs(C,1)

4.2 recoutputs(1,2)

1.5 recbyprod(D,1)

h(3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-1 recinputs(A,2)

-4 recinputs(D,2)

3 recoutputs(2,3)

h(4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-3.5 recinputs(A,2)

-5.5 recinputs(B,2)

1.8 recoutputs(2,1)

2 recbyprod(C,2)

h(5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-1 recinputs(C,2)

-2 recinputs(D,2)

1 recoutputs(2,2)

2 recoutputs(2,4)

0.5 recbyprod(C,2)

h(6)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 recinputs(C,3)

-6 recinputs(D,3)

0.5 recoutputs(3,3)

1.5 recoutputs(3,4)

2 recbyprod(B,3)

h(7)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-0.5 recinputs(A,3)

-1 recinputs(C,3)

0.5 recoutputs(3,4)

0.5 recoutputs(3,5)

1 recbyprod(B,3)

h(8)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 recinputs(A,4)

-5 recinputs(B,4)

2 recoutputs(4,1)

2 recbyprod(D,4)

h(9)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 recinputs(A,4)

-5 recinputs(B,4)

1 recoutputs(4,1)

0.5 recbyprod(A,4)

2 recbyprod(B,4)

1 recbyprod(D,4)

h(10)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-1 recinputs(A,5)

-1 recinputs(C,5)

-1 recinputs(D,5)

3 recoutputs(5,3)

h(11)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 recinputs(C,5)

0.5 recoutputs(5,5)

0.5 recbyprod(A,5)

---- f total cost in DOLLARS

f

(.LO, .L, .UP, .M = -INF, 0, +INF, 0)

1 cost

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
Model Statistics SOLVE NETWORK Using LP From line 184

MODEL STATISTICS

BLOCKS OF EQUATIONS	6	SINGLE EQUATIONS	75
BLOCKS OF VARIABLES	5	SINGLE VARIABLES	157
NON ZERO ELEMENTS	463		

GENERATION TIME = 0.016 SECONDS 4 Mb WEX239-239 Nov 9, 2012

EXECUTION TIME = 0.016 SECONDS 4 Mb WEX239-239 Nov 9, 2012

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
Solution Report SOLVE NETWORK Using LP From line 184

S O L V E S U M M A R Y

MODEL NETWORK OBJECTIVE f
TYPE LP DIRECTION MINIMIZE
SOLVER CPLEX FROM LINE 184

**** SOLVER STATUS 1 Normal Completion
**** MODEL STATUS 1 Optimal
**** OBJECTIVE VALUE 126019.8413

RESOURCE USAGE, LIMIT 0.031 1000.000
ITERATION COUNT, LIMIT 23 2000000000

IBM ILOG CPLEX Jul 4, 2012 23.9.5 WEX 36376.36401 WEI x86_64/MS Windows
Cplex 12.4.0.1

LP status(1): optimal
Optimal solution found.
Objective : 126019.841270

LOWER LEVEL UPPER MARGINAL

---- EQU cost . . . 1.000

cost define objective function

---- EQU supply observe supply limit at supplier l

LOWER LEVEL UPPER MARGINAL

A -INF 1638.889 2000.000 .

B	-INF	884.921	2000.000	.
C	-INF	745.040	2000.000	.
D	-INF	122.024	2000.000	.

---- EQU demand satisfy or exceed demand at market n

	LOWER	LEVEL	UPPER	MARGINAL
1	500.000	500.000	+INF	81.444
2	500.000	500.000	+INF	29.929
3	500.000	500.000	+INF	47.667
4	500.000	500.000	+INF	.
5	500.000	500.000	+INF	93.000

---- EQU recinputs satisfy material balance between inputs received at m and in
puts used in recipes at m

	LOWER	LEVEL	UPPER	MARGINAL
A.1	.	.	.	-2.639
A.2	.	.	.	24.000
A.3	.	.	.	40.000
A.4	.	.	.	20.000
A.5	.	.	.	48.000
B.1	.	.	.	25.000
B.2	.	.	.	20.000
B.3	.	.	.	.
B.4	.	.	.	18.978
B.5	.	.	.	.
C.1	.	.	.	36.000
C.2	.	.	.	-25.071
C.3	.	.	.	36.000
C.4	.	.	.	.
C.5	.	.	.	36.000
D.1	.	.	.	.
D.2	.	.	.	26.000
D.3	.	.	.	-5.111

D.4 . . . .

D.5 . . . 35.000

---- EQU recoutputs satisfy that product outputs from each producer equals prod
ucts received by markets

	LOWER	LEVEL	UPPER	MARGINAL
1.1	.	.	.	76.444
1.2	.	.	.	23.929
1.3	.	.	.	40.667
1.4	.	.	.	.
1.5	.	.	.	84.000
2.1	.	.	.	74.444
2.2	.	.	.	23.929
2.3	.	.	.	42.667
2.4	.	.	.	-6.000
2.5	.	.	.	86.000
3.1	.	.	.	72.444
3.2	.	.	.	21.929
3.3	.	.	.	40.667
3.4	.	.	.	-6.000
3.5	.	.	.	88.000
4.1	.	.	.	75.444
4.2	.	.	.	22.929
4.3	.	.	.	39.667
4.4	.	.	.	.
4.5	.	.	.	83.000
5.1	.	.	.	71.444
5.2	.	.	.	20.929
5.3	.	.	.	39.667
5.4	.	.	.	.
5.5	.	.	.	87.000

---- EQU recbyprod producer m must generate an amount of byproduct type l equal
or greater than the amount of l used by producers mp as inpu
ts

	LOWER	LEVEL	UPPER	MARGINAL
A.1	.	.	+INF	36.000
A.2	.	.	+INF	40.000
A.3	.	.	+INF	46.000
A.4	.	.	+INF	32.000
A.5	.	.	+INF	273.000
B.1	.	.	+INF	25.000
B.2	.	.	+INF	20.000
B.3	.	.	+INF	15.000
B.4	.	.	+INF	22.500
B.5	.	.	+INF	10.000
C.1	.	.	+INF	36.000
C.2	.	.	+INF	30.000
C.3	.	.	+INF	36.000
C.4	.	.	+INF	33.000
C.5	.	.	+INF	36.000
D.1	.	.	+INF	19.000
D.2	.	.	+INF	26.000
D.3	.	.	+INF	31.500
D.4	.	.	+INF	12.000
D.5	.	.	+INF	35.000

---- VAR x flow quantities from supplier to producer in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
A.1	.	.	+INF	18.639
A.2	.	1016.865	+INF	.
A.3	.	500.000	+INF	.
A.4	.	.	+INF	.
A.5	.	122.024	+INF	.
B.1	.	.	+INF	5.000
B.2	.	884.921	+INF	.
B.3	.	.	+INF	40.000
B.4	.	.	+INF	11.022

B.5	.	.	+INF	45.000
C.1	.	.	+INF	12.000
C.2	.	.	+INF	49.071
C.3	.	623.016	+INF	.
C.4	.	.	+INF	54.000
C.5	.	122.024	+INF	.
D.1	.	.	+INF	70.000
D.2	.	.	+INF	16.000
D.3	.	.	+INF	33.111
D.4	.	.	+INF	84.000
D.5	.	122.024	+INF	.

---- VAR y flow quantities between producers in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
A.1.1	.	.	+INF	38.639
A.1.2	.	.	+INF	16.000
A.1.3	.	.	+INF	4.000
A.1.4	.	.	+INF	18.000
A.1.5	.	.	+INF	.
A.2.1	.	.	+INF	46.639
A.2.2	.	.	+INF	16.000
A.2.3	.	.	+INF	4.000
A.2.4	.	.	+INF	28.000
A.2.5	.	.	+INF	.
A.3.1	.	.	+INF	56.639
A.3.2	.	.	+INF	26.000
A.3.3	.	.	+INF	6.000
A.3.4	.	.	+INF	38.000
A.3.5	.	.	+INF	.
A.4.1	.	.	+INF	36.639
A.4.2	.	.	+INF	16.000
A.4.3	.	.	+INF	4.000
A.4.4	.	.	+INF	12.000
A.4.5	.	.	+INF	.
A.5.1	.	.	+INF	287.639

A.5.2	.	.	+INF	257.000
A.5.3	.	.	+INF	235.000
A.5.4	.	.	+INF	269.000
A.5.5	.	.	+INF	225.000
B.1.1	.	.	+INF	.
B.1.2	.	.	+INF	10.000
B.1.3	.	.	+INF	35.000
B.1.4	.	.	+INF	8.522
B.1.5	.	.	+INF	40.000
B.2.1	.	.	+INF	.
B.2.2	.	.	+INF	EPS
B.2.3	.	.	+INF	25.000
B.2.4	.	.	+INF	11.022
B.2.5	.	.	+INF	30.000
B.3.1	.	357.143	+INF	.
B.3.2	.	642.857	+INF	.
B.3.3	.	.	+INF	15.000
B.3.4	.	.	+INF	11.022
B.3.5	.	.	+INF	17.500
B.4.1	.	.	+INF	.
B.4.2	.	.	+INF	12.500
B.4.3	.	.	+INF	37.500
B.4.4	.	.	+INF	3.522
B.4.5	.	.	+INF	42.500
B.5.1	.	.	+INF	.
B.5.2	.	.	+INF	EPS
B.5.3	.	.	+INF	12.500
B.5.4	.	.	+INF	11.022
B.5.5	.	.	+INF	10.000
C.1.1	.	.	+INF	.
C.1.2	.	.	+INF	67.071
C.1.3	.	.	+INF	12.000
C.1.4	.	.	+INF	39.000
C.1.5	.	.	+INF	18.000
C.2.1	.	178.571	+INF	.
C.2.2	.	.	+INF	55.071
C.2.3	.	376.984	+INF	.

C.2.4	.	.	+INF	42.000
C.2.5	.	.	+INF	6.000
C.3.1	.	.	+INF	12.000
C.3.2	.	.	+INF	67.071
C.3.3	.	.	+INF	.
C.3.4	.	.	+INF	54.000
C.3.5	.	.	+INF	3.000
C.4.1	.	.	+INF	.
C.4.2	.	.	+INF	70.071
C.4.3	.	.	+INF	15.000
C.4.4	.	.	+INF	33.000
C.4.5	.	.	+INF	21.000
C.5.1	.	.	+INF	18.000
C.5.2	.	.	+INF	73.071
C.5.3	.	.	+INF	3.000
C.5.4	.	.	+INF	60.000
C.5.5	.	.	+INF	.
D.1.1	.	.	+INF	19.000
D.1.2	.	178.571	+INF	.
D.1.3	.	.	+INF	38.111
D.1.4	.	.	+INF	22.500
D.1.5	.	.	+INF	5.000
D.2.1	.	.	+INF	33.000
D.2.2	.	.	+INF	.
D.2.3	.	.	+INF	38.111
D.2.4	.	.	+INF	40.000
D.2.5	.	.	+INF	5.000
D.3.1	.	.	+INF	45.500
D.3.2	.	.	+INF	12.500
D.3.3	.	.	+INF	36.611
D.3.4	.	.	+INF	52.500
D.3.5	.	.	+INF	.
D.4.1	.	.	+INF	15.500
D.4.2	.	.	+INF	.
D.4.3	.	.	+INF	38.111
D.4.4	.	.	+INF	12.000
D.4.5	.	.	+INF	5.000

D.5.1 . . +INF 56.000

D.5.2 . . +INF 23.000

D.5.3 . . +INF 43.611

D.5.4 . . +INF 63.000

D.5.5 . . +INF .

---- VAR z flow quantities from producer to market in UNITS

LOWER LEVEL UPPER MARGINAL

1.1 . . +INF .

1.2 . 500.000 +INF .

1.3 . . +INF .

1.4 . . +INF 8.000

1.5 . . +INF .

2.1 . 500.000 +INF .

2.2 . . +INF .

2.3 . 133.929 +INF .

2.4 . . +INF .

2.5 . . +INF .

3.1 . . +INF .

3.2 . . +INF .

3.3 . . +INF .

3.4 . 500.000 +INF .

3.5 . 500.000 +INF .

4.1 . . +INF .

4.2 . . +INF .

4.3 . . +INF .

4.4 . . +INF 9.000

4.5 . . +INF .

5.1 . . +INF .

5.2 . . +INF .

5.3 . 366.071 +INF .

5.4 . . +INF 7.000

5.5 . . +INF .

---- VAR h rate of production of recipe r at producer m in MULTIPLES OF r

```
     LOWER     LEVEL     UPPER    MARGINAL

1      .         .       +INF       .
2      .       119.048   +INF       .
3      .        44.643   +INF       .
4      .       277.778   +INF       .
5      .         .       +INF       .
6      .         .       +INF       .
7      .      1000.000   +INF       .
8      .         .       +INF       .
9      .         .       +INF     26.444
10     .       122.024   +INF       .
11     .         .       +INF       .

                 LOWER     LEVEL     UPPER    MARGINAL

---- VAR f             -INF  1.2602E+5     +INF       .

  f  total cost in DOLLARS


**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
```

---- 185 VARIABLE x.L flow quantities from supplier to producer in UNITS

	2	3	5
A	1016.865	500.000	122.024
B	884.921		
C		623.016	122.024
D			122.024

---- 185 VARIABLE h.L rate of production of recipe r at producer m in MULTIPLES OF r

2 119.048, 3 44.643, 4 277.778, 7 1000.000, 10 122.024

---- 185 VARIABLE y.L flow quantities between producers in UNITS

	1	2	3
B.3	357.143	642.857	
C.2	178.571		376.984
D.1		178.571	

---- 185 VARIABLE z.L flow quantities from producer to market in UNITS

	1	2	3	4	5
1		500.000			
2	500.000		133.929		
3			500.000	500.000	

5 366.071

---- 185 VARIABLE x.M flow quantities from supplier to producer in UNITS

	1	2	3	4	5
A	18.639				
B	5.000		40.000	11.022	45.000
C	12.000	49.071		54.000	
D	70.000	16.000	33.111	84.000	

---- 185 VARIABLE h.M rate of production of recipe r at producer m in MULTIP
LES OF r

9 26.444

---- 185 VARIABLE y.M flow quantities between producers in UNITS

	1	2	3	4	5
A.1	38.639	16.000	4.000	18.000	
A.2	46.639	16.000	4.000	28.000	
A.3	56.639	26.000	6.000	38.000	
A.4	36.639	16.000	4.000	12.000	
A.5	287.639	257.000	235.000	269.000	225.000
B.1		10.000	35.000	8.522	40.000
B.2		EPS	25.000	11.022	30.000
B.3			15.000	11.022	17.500
B.4		12.500	37.500	3.522	42.500
B.5		EPS	12.500	11.022	10.000
C.1		67.071	12.000	39.000	18.000
C.2		55.071		42.000	6.000
C.3	12.000	67.071		54.000	3.000
C.4		70.071	15.000	33.000	21.000

C.5	18.000	73.071	3.000	60.000
D.1	19.000	38.111	22.500	5.000
D.2	33.000	38.111	40.000	5.000
D.3	45.500	12.500	36.611	52.500
D.4	15.500	38.111	12.000	5.000
D.5	56.000	23.000	43.611	63.000

---- 185 VARIABLE z.M flow quantities from producer to market in UNITS

4

1	8.000
4	9.000
5	7.000

EXECUTION TIME = 0.031 SECONDS 3 Mb WEX239-239 Nov 9, 2012

USER: David G. Taggart G121115:1510AP-WIN
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**** FILE SUMMARY

Input \\psf\Home\Documents\gamsdir\projdir\Final Models For Thesis\Problem 2.gms
Output \\psf\Home\Documents\gamsdir\projdir\Problem 2.lst

SPECIAL CASE 1 OUTPUT

General Algebraic Modeling System
Compilation

```
 1  Sets
 2     l    external suppliers   / A*D /
 3     m     producers           / 1*5 /
 4     n     markets             / 1*5 /
 5     r     recipes            / 1*11 / ;
 6
 7  alias (m,mp)    ;
 8
 9  Parameters
10    S(l)  capacity of external supplier l in UNITS
11     /   A    2000
12         B    2000
13         C    2000
14         D    2000 /
15
16    D(n)  demand at market n in UNITS
17     /   1       500
18         2       500
19         3       500
20         4       500
21         5       500 /
22
23    wi(l)  weighted cost of shipping l in DOLLARS PER UNIT
24      /   A  4
25         B  5
26         C  6
27         D  7   /
28
29    wo(n)  weighted cost of shipping n in DOLLARS PER UNIT
30     /   1  1
31         2  1
32         3  1
```

```
33        4  1
34        5  1     /;
35
36  Table dx(l,m)   distance from supplier l to producer m in MILES
37         1    2    3    4    5
38    A    4    6    10   5    12
39    B    6    4    8    6    9
40    C    8    4    6    9    6
41    D    10   6    4    12   5 ;
42
43  Table dy(m,mp)   distance from producer m to producer mp in MILES
44       1    2    3    4    5
45    1  0    1    2    0.5  3
46    2  1    0    1    2    2
47    3  2    1    0    3    0.5
48    4  0.5  2    3    0    4
49    5  3    2    0.5  4    0 ;
50
51  Table dz(m,n)   distance from producer m to market n in MILES
52         1    2    3    4    5
53    1    5    6    7    8    9
54    2    7    6    5    6    7
55    3    9    8    7    6    5
56    4    6    7    8    9    10
57    5    10   9    8    7    6 ;
58
59  Table hrm(r,m) recipes r used at each producer m
60        1  2  3  4  5
61    1   1  0  0  0  0
62    2   1  0  0  0  0
63    3   0  1  0  0  0
64    4   0  1  0  0  0
65    5   0  1  0  0  0
66    6   0  0  1  0  0
67    7   0  0  1  0  0
68    8   0  0  0  1  0
69    9   0  0  0  1  0
```

```
10  0  0  0  0  1
11  0  0  0  0  1 ;

Table a(r,l)   amount of input l used in recipe r in UNITS
          A       B       C       D
1         4       5       0       0
2         0       3       1.5     0
3         1       0       0       4
4         3.5     5.5     0       0
5         0       0       1       2
6         0       0       2       6
7         0.5     0       1       0
8         4       5       0       0
9         4       5       0       0
10        1       0       1       1
11        0       0       5       0  ;

Table bz(r,n)   amount of output n produced by recipe r in UNITS
          1   2   3   4   5
1         1.0 0.0 0.0 0.0 0.0
2         0.0 4.2 0.0 0.0 0.0
3         0.0 0.0 3.0 0.0 0.0
4         1.8 0.0 0.0 0.0 0.0
5         0.0 1.0 0.0 2.0 0.0
6         0.0 0.0 0.5 1.5 0.0
7         0.0 0.0 0.0 0.5 0.5
8         2.0 0.0 0.0 0.0 0.0
9         1.0 0.0 0.0 0.0 0.0
10        0.0 0.0 3.0 0.0 0.0
11        0.0 0.0 0.0 0.0 0.5 ;

Table by(r,l)   amount of input l produced by recipe r in UNITS
     A   B   C   D
1  0.0 0.0 0.0 2.0
2  0.0 0.0 0.0 1.5
3  0.0 0.0 0.0 0.0
4  0.0 0.0 2.0 0.0
```

```
107  5  0.0 0.0 0.5 0.0
108  6  0.0 2.0 0.0 0.0
109  7  0.0 1.0 0.0 0.0
110  8  0.0 0.0 0.0 2.0
111  9  0.5 2.0 0.0 1.0
112  10 0.0 0.0 0.0 0.0
113  11 0.5 0.0 0.0 0.0 ;
114
115  Parameter cx(l,m)   cost of transporting input l from supplier l to produc
     er m in DOLLARS PER UNIT;
116    cx(l,m) = dx(l,m) * wi(l)  ;
117
118  Parameter cy(l,m,mp)   cost of transporting input l from producer m to pro
     ducer mp in DOLLARS PER UNIT;
119    cy(l,m,mp) = dy(m,mp) * wi(l)   ;
120
121  Parameter cz(m,n)   cost of transporting output n from producer m to marke
     t n in DOLLARS PER UNIT;
122    cz(m,n) = dz(m,n) * wo(n)   ;
123
124  Parameter bpw(l) environmental cost of disposing of material l
125  / A 10
126    B 11
127    C 12
128    D 13 /     ;
129
130  Scalar VMT virgin material tax
131  / 200 /;
132
133  Variables
134    x(l,m)    flow quantities from supplier to producer in UNITS
135    y(l,m,mp) flow quantities between producers in UNITS
136    z(m,n)    flow quantities from producer to market in UNITS
137    h(r)      rate of production of recipe r at producer m in MULTIPLES OF r
138    f         total cost in DOLLARS
139    env       environmental cost in DOLLARS;
140
```

```
141  Positive variable x;
142  Positive variable y;
143  Positive variable z;
144  Positive variable h;
145
146  Equations
147  cost            define objective function
148  supply(l)       observe supply limit at supplier l
149  demand(n)       satisfy or exceed demand at market n
150  recinputs(l,m)  satisfy material balance between inputs received at m and
     inputs used in recipes at m
151  recoutputs(m,n) satisfy that product outputs from each producer equals pro
     ducts received by markets
152  recbyprod(l,m)  producer m must produce an amount of byproduct type l equa
     l to or greater than the amount of l used by producers mp as inputs
153  envcost         environemental cost for unused byproduct disposed of by ea
     ch producer m instead of shipping to other producers mp            ;
154
155  cost ..             f =e= env + sum((l,m), cx(l,m)*x(l,m))+ sum((l,m,mp),
     cy(l,m,mp)*y(l,m,mp))+ sum((m,n), cz(m,n)*z(m,n))   ;
156
157  supply(l) ..        sum(m, x(l,m))  =l=  s(l)   ;
158
159  demand(n) ..        sum(m, z(m,n))  =g=  d(n)   ;
160
161  recinputs (l,m) ..   x(l,m) + sum(mp, y(l,mp,m)) =e= sum(r, h(r)*a(r,l)$(h
     rm(r,m) gt 0))   ;
162
163  recoutputs(m,n) ..  sum(r, h(r)*bz(r,n)$(hrm(r,m) gt 0)) =e= z(m,n)   ;
164
165  recbyprod(l,m) ..   sum(r, h(r)*by(r,l)$(hrm(r,m) gt 0)) =g= sum(mp, y(l,m
     ,mp))   ;
166
167  envcost ..          env =e= sum((l,m), bpw(l)*(sum(r, h(r)*by(r,l)$(hrm(r,
     m) gt 0)) - sum(mp, y(l,m,mp))))+ sum((l,m), vmt*x(l,m))  ;
168
169  option limrow = 15, limcol = 30 ;
```

```
170    MODEL NETWORK /ALL/ ;
171    SOLVE NETWORK USING LP MINIMIZING f ;
172
```

COMPILATION TIME = 0.000 SECONDS 3 Mb WEX239-239 Nov 9, 2012

---- cost =E= define objective function

cost.. - 16*x(A,1) - 24*x(A,2) - 40*x(A,3) - 20*x(A,4) - 48*x(A,5) - 30*x(B,1)

- 20*x(B,2) - 40*x(B,3) - 30*x(B,4) - 45*x(B,5) - 48*x(C,1) - 24*x(C,2)

- 36*x(C,3) - 54*x(C,4) - 36*x(C,5) - 70*x(D,1) - 42*x(D,2) - 28*x(D,3)

- 84*x(D,4) - 35*x(D,5) - 4*y(A,1,2) - 8*y(A,1,3) - 2*y(A,1,4)

- 12*y(A,1,5) - 4*y(A,2,1) - 4*y(A,2,3) - 8*y(A,2,4) - 8*y(A,2,5)

- 8*y(A,3,1) - 4*y(A,3,2) - 12*y(A,3,4) - 2*y(A,3,5) - 2*y(A,4,1)

- 8*y(A,4,2) - 12*y(A,4,3) - 16*y(A,4,5) - 12*y(A,5,1) - 8*y(A,5,2)

- 2*y(A,5,3) - 16*y(A,5,4) - 5*y(B,1,2) - 10*y(B,1,3) - 2.5*y(B,1,4)

- 15*y(B,1,5) - 5*y(B,2,1) - 5*y(B,2,3) - 10*y(B,2,4) - 10*y(B,2,5)

- 10*y(B,3,1) - 5*y(B,3,2) - 15*y(B,3,4) - 2.5*y(B,3,5) - 2.5*y(B,4,1)

- 10*y(B,4,2) - 15*y(B,4,3) - 20*y(B,4,5) - 15*y(B,5,1) - 10*y(B,5,2)

- 2.5*y(B,5,3) - 20*y(B,5,4) - 6*y(C,1,2) - 12*y(C,1,3) - 3*y(C,1,4)

- 18*y(C,1,5) - 6*y(C,2,1) - 6*y(C,2,3) - 12*y(C,2,4) - 12*y(C,2,5)

- 12*y(C,3,1) - 6*y(C,3,2) - 18*y(C,3,4) - 3*y(C,3,5) - 3*y(C,4,1)

- 12*y(C,4,2) - 18*y(C,4,3) - 24*y(C,4,5) - 18*y(C,5,1) - 12*y(C,5,2)

- 3*y(C,5,3) - 24*y(C,5,4) - 7*y(D,1,2) - 14*y(D,1,3) - 3.5*y(D,1,4)

- 21*y(D,1,5) - 7*y(D,2,1) - 7*y(D,2,3) - 14*y(D,2,4) - 14*y(D,2,5)

- 14*y(D,3,1) - 7*y(D,3,2) - 21*y(D,3,4) - 3.5*y(D,3,5) - 3.5*y(D,4,1)

- 14*y(D,4,2) - 21*y(D,4,3) - 28*y(D,4,5) - 21*y(D,5,1) - 14*y(D,5,2)

- 3.5*y(D,5,3) - 28*y(D,5,4) - 5*z(1,1) - 6*z(1,2) - 7*z(1,3) - 8*z(1,4)

- 9*z(1,5) - 7*z(2,1) - 6*z(2,2) - 5*z(2,3) - 6*z(2,4) - 7*z(2,5)

- 9*z(3,1) - 8*z(3,2) - 7*z(3,3) - 6*z(3,4) - 5*z(3,5) - 6*z(4,1)

- 7*z(4,2) - 8*z(4,3) - 9*z(4,4) - 10*z(4,5) - 10*z(5,1) - 9*z(5,2)

- 8*z(5,3) - 7*z(5,4) - 6*z(5,5) + f - env =E= 0 ; (LHS = 0)

---- supply =L= observe supply limit at supplier l

supply(A).. x(A,1) + x(A,2) + x(A,3) + x(A,4) + x(A,5) =L= 2000 ; (LHS = 0)

supply(B).. x(B,1) + x(B,2) + x(B,3) + x(B,4) + x(B,5) =L= 2000 ; (LHS = 0)

supply(C).. x(C,1) + x(C,2) + x(C,3) + x(C,4) + x(C,5) =L= 2000 ; (LHS = 0)

supply(D).. x(D,1) + x(D,2) + x(D,3) + x(D,4) + x(D,5) =L= 2000 ; (LHS = 0)

---- demand =G= satisfy or exceed demand at market n

demand(1).. z(1,1) + z(2,1) + z(3,1) + z(4,1) + z(5,1) =G= 500 ;

     (LHS = 0, INFES = 500 ****)

demand(2).. z(1,2) + z(2,2) + z(3,2) + z(4,2) + z(5,2) =G= 500 ;

(LHS = 0, INFES = 500 ****)

demand(3).. z(1,3) + z(2,3) + z(3,3) + z(4,3) + z(5,3) =G= 500 ;

(LHS = 0, INFES = 500 ****)

demand(4).. z(1,4) + z(2,4) + z(3,4) + z(4,4) + z(5,4) =G= 500 ;

(LHS = 0, INFES = 500 ****)

demand(5).. z(1,5) + z(2,5) + z(3,5) + z(4,5) + z(5,5) =G= 500 ;

(LHS = 0, INFES = 500 ****)

---- recinputs =E= satisfy material balance between inputs received at m and i
nputs used in recipes at m

recinputs(A,1).. x(A,1) + y(A,1,1) + y(A,2,1) + y(A,3,1) + y(A,4,1) + y(A,5,1)

- 4*h(1) =E= 0 ; (LHS = 0)

recinputs(A,2).. x(A,2) + y(A,1,2) + y(A,2,2) + y(A,3,2) + y(A,4,2) + y(A,5,2)

- h(3) - 3.5*h(4) =E= 0 ; (LHS = 0)

recinputs(A,3).. x(A,3) + y(A,1,3) + y(A,2,3) + y(A,3,3) + y(A,4,3) + y(A,5,3)

- 0.5*h(7) =E= 0 ; (LHS = 0)

recinputs(A,4).. x(A,4) + y(A,1,4) + y(A,2,4) + y(A,3,4) + y(A,4,4) + y(A,5,4)

- 4*h(8) - 4*h(9) =E= 0 ; (LHS = 0)

recinputs(A,5).. x(A,5) + y(A,1,5) + y(A,2,5) + y(A,3,5) + y(A,4,5) + y(A,5,5)

- h(10) =E= 0 ; (LHS = 0)

recinputs(B,1).. x(B,1) + y(B,1,1) + y(B,2,1) + y(B,3,1) + y(B,4,1) + y(B,5,1)

- 5*h(1) - 3*h(2) =E= 0 ; (LHS = 0)

recinputs(B,2).. x(B,2) + y(B,1,2) + y(B,2,2) + y(B,3,2) + y(B,4,2) + y(B,5,2)

- 5.5*h(4) =E= 0 ; (LHS = 0)

recinputs(B,3).. x(B,3) + y(B,1,3) + y(B,2,3) + y(B,3,3) + y(B,4,3) + y(B,5,3)
=E= 0 ; (LHS = 0)

recinputs(B,4).. x(B,4) + y(B,1,4) + y(B,2,4) + y(B,3,4) + y(B,4,4) + y(B,5,4)

- 5*h(8) - 5*h(9) =E= 0 ; (LHS = 0)

recinputs(B,5).. x(B,5) + y(B,1,5) + y(B,2,5) + y(B,3,5) + y(B,4,5) + y(B,5,5)
=E= 0 ; (LHS = 0)

recinputs(C,1).. x(C,1) + y(C,1,1) + y(C,2,1) + y(C,3,1) + y(C,4,1) + y(C,5,1)

- 1.5*h(2) =E= 0 ; (LHS = 0)

recinputs(C,2).. x(C,2) + y(C,1,2) + y(C,2,2) + y(C,3,2) + y(C,4,2) + y(C,5,2)

- h(5) =E= 0 ; (LHS = 0)

recinputs(C,3).. x(C,3) + y(C,1,3) + y(C,2,3) + y(C,3,3) + y(C,4,3) + y(C,5,3)

- 2*h(6) - h(7) =E= 0 ; (LHS = 0)

recinputs(C,4).. x(C,4) + y(C,1,4) + y(C,2,4) + y(C,3,4) + y(C,4,4) + y(C,5,4)
=E= 0 ; (LHS = 0)

recinputs(C,5).. x(C,5) + y(C,1,5) + y(C,2,5) + y(C,3,5) + y(C,4,5) + y(C,5,5)

- h(10) - 5*h(11) =E= 0 ; (LHS = 0)

REMAINING 5 ENTRIES SKIPPED

---- recoutputs =E= satisfy that product outputs from each producer equals pro
ducts received by markets

recoutputs(1,1).. - z(1,1) + h(1) =E= 0 ; (LHS = 0)

recoutputs(1,2).. - z(1,2) + 4.2*h(2) =E= 0 ; (LHS = 0)

recoutputs(1,3).. - z(1,3) =E= 0 ; (LHS = 0)

recoutputs(1,4).. - z(1,4) =E= 0 ; (LHS = 0)

recoutputs(1,5).. - z(1,5) =E= 0 ; (LHS = 0)

recoutputs(2,1).. - z(2,1) + 1.8*h(4) =E= 0 ; (LHS = 0)

recoutputs(2,2).. - z(2,2) + h(5) =E= 0 ; (LHS = 0)

recoutputs(2,3).. - z(2,3) + 3*h(3) =E= 0 ; (LHS = 0)

recoutputs(2,4).. - z(2,4) + 2*h(5) =E= 0 ; (LHS = 0)

recoutputs(2,5).. - z(2,5) =E= 0 ; (LHS = 0)

recoutputs(3,1).. - z(3,1) =E= 0 ; (LHS = 0)

recoutputs(3,2).. - z(3,2) =E= 0 ; (LHS = 0)

recoutputs(3,3).. - z(3,3) + 0.5*h(6) =E= 0 ; (LHS = 0)

recoutputs(3,4).. - z(3,4) + 1.5*h(6) + 0.5*h(7) =E= 0 ; (LHS = 0)

recoutputs(3,5).. - z(3,5) + 0.5*h(7) =E= 0 ; (LHS = 0)

REMAINING 10 ENTRIES SKIPPED

---- recbyprod =G= producer m must produce an amount of byproduct type l equal
to or greater than the amount of l used by producers mp as
inputs

recbyprod(A,1).. - y(A,1,1) - y(A,1,2) - y(A,1,3) - y(A,1,4) - y(A,1,5) =G= 0 ;

(LHS = 0)

recbyprod(A,2).. - y(A,2,1) - y(A,2,2) - y(A,2,3) - y(A,2,4) - y(A,2,5) =G= 0 ;

(LHS = 0)

recbyprod(A,3).. - y(A,3,1) - y(A,3,2) - y(A,3,3) - y(A,3,4) - y(A,3,5) =G= 0 ;

(LHS = 0)

recbyprod(A,4).. - y(A,4,1) - y(A,4,2) - y(A,4,3) - y(A,4,4) - y(A,4,5)

+ 0.5*h(9) =G= 0 ; (LHS = 0)

recbyprod(A,5).. - y(A,5,1) - y(A,5,2) - y(A,5,3) - y(A,5,4) - y(A,5,5)

+ 0.5*h(11) =G= 0 ; (LHS = 0)

recbyprod(B,1).. - y(B,1,1) - y(B,1,2) - y(B,1,3) - y(B,1,4) - y(B,1,5) =G= 0 ;

(LHS = 0)

recbyprod(B,2).. - y(B,2,1) - y(B,2,2) - y(B,2,3) - y(B,2,4) - y(B,2,5) =G= 0 ;

(LHS = 0)

recbyprod(B,3).. - y(B,3,1) - y(B,3,2) - y(B,3,3) - y(B,3,4) - y(B,3,5)

+ 2*h(6) + h(7) =G= 0 ; (LHS = 0)

recbyprod(B,4).. - y(B,4,1) - y(B,4,2) - y(B,4,3) - y(B,4,4) - y(B,4,5)

+ 2*h(9) =G= 0 ; (LHS = 0)

recbyprod(B,5).. - y(B,5,1) - y(B,5,2) - y(B,5,3) - y(B,5,4) - y(B,5,5) =G= 0 ;

(LHS = 0)

recbyprod(C,1).. - y(C,1,1) - y(C,1,2) - y(C,1,3) - y(C,1,4) - y(C,1,5) =G= 0 ;

(LHS = 0)

recbyprod(C,2).. - y(C,2,1) - y(C,2,2) - y(C,2,3) - y(C,2,4) - y(C,2,5)

+ 2*h(4) + 0.5*h(5) =G= 0 ; (LHS = 0)

recbyprod(C,3).. - y(C,3,1) - y(C,3,2) - y(C,3,3) - y(C,3,4) - y(C,3,5) =G= 0 ;

(LHS = 0)

recbyprod(C,4).. - y(C,4,1) - y(C,4,2) - y(C,4,3) - y(C,4,4) - y(C,4,5) =G= 0 ;

(LHS = 0)

recbyprod(C,5).. - y(C,5,1) - y(C,5,2) - y(C,5,3) - y(C,5,4) - y(C,5,5) =G= 0 ;

(LHS = 0)

REMAINING 5 ENTRIES SKIPPED

---- envcost =E= environemental cost for unused byproduct disposed of by each producer m instead of shipping to other producers mp

envcost.. - 200*x(A,1) - 200*x(A,2) - 200*x(A,3) - 200*x(A,4) - 200*x(A,5)

- 200*x(B,1) - 200*x(B,2) - 200*x(B,3) - 200*x(B,4) - 200*x(B,5)

- 200*x(C,1) - 200*x(C,2) - 200*x(C,3) - 200*x(C,4) - 200*x(C,5)

- 200*x(D,1) - 200*x(D,2) - 200*x(D,3) - 200*x(D,4) - 200*x(D,5)

+ 10*y(A,1,1) + 10*y(A,1,2) + 10*y(A,1,3) + 10*y(A,1,4) + 10*y(A,1,5)

+ 10*y(A,2,1) + 10*y(A,2,2) + 10*y(A,2,3) + 10*y(A,2,4) + 10*y(A,2,5)

+ 10*y(A,3,1) + 10*y(A,3,2) + 10*y(A,3,3) + 10*y(A,3,4) + 10*y(A,3,5)

+ 10*y(A,4,1) + 10*y(A,4,2) + 10*y(A,4,3) + 10*y(A,4,4) + 10*y(A,4,5)

+ 10*y(A,5,1) + 10*y(A,5,2) + 10*y(A,5,3) + 10*y(A,5,4) + 10*y(A,5,5)

+ 11*y(B,1,1) + 11*y(B,1,2) + 11*y(B,1,3) + 11*y(B,1,4) + 11*y(B,1,5)

+ 11*y(B,2,1) + 11*y(B,2,2) + 11*y(B,2,3) + 11*y(B,2,4) + 11*y(B,2,5)

+ 11*y(B,3,1) + 11*y(B,3,2) + 11*y(B,3,3) + 11*y(B,3,4) + 11*y(B,3,5)

+ 11*y(B,4,1) + 11*y(B,4,2) + 11*y(B,4,3) + 11*y(B,4,4) + 11*y(B,4,5)

+ 11*y(B,5,1) + 11*y(B,5,2) + 11*y(B,5,3) + 11*y(B,5,4) + 11*y(B,5,5)

+ 12*y(C,1,1) + 12*y(C,1,2) + 12*y(C,1,3) + 12*y(C,1,4) + 12*y(C,1,5)

+ 12*y(C,2,1) + 12*y(C,2,2) + 12*y(C,2,3) + 12*y(C,2,4) + 12*y(C,2,5)

+ 12*y(C,3,1) + 12*y(C,3,2) + 12*y(C,3,3) + 12*y(C,3,4) + 12*y(C,3,5)

+ 12*y(C,4,1) + 12*y(C,4,2) + 12*y(C,4,3) + 12*y(C,4,4) + 12*y(C,4,5)

+ 12*y(C,5,1) + 12*y(C,5,2) + 12*y(C,5,3) + 12*y(C,5,4) + 12*y(C,5,5)

+ 13*y(D,1,1) + 13*y(D,1,2) + 13*y(D,1,3) + 13*y(D,1,4) + 13*y(D,1,5)

+ 13*y(D,2,1) + 13*y(D,2,2) + 13*y(D,2,3) + 13*y(D,2,4) + 13*y(D,2,5)

+ 13*y(D,3,1) + 13*y(D,3,2) + 13*y(D,3,3) + 13*y(D,3,4) + 13*y(D,3,5)

+ 13*y(D,4,1) + 13*y(D,4,2) + 13*y(D,4,3) + 13*y(D,4,4) + 13*y(D,4,5)

+ 13*y(D,5,1) + 13*y(D,5,2) + 13*y(D,5,3) + 13*y(D,5,4) + 13*y(D,5,5)

- 26*h(1) - 19.5*h(2) - 24*h(4) - 6*h(5) - 22*h(6) - 11*h(7) - 26*h(8)

- 40*h(9) - 5*h(11) + env =E= 0 ; (LHS = 0)

---- x flow quantities from supplier to producer in UNITS

x(A,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-16 cost
1 supply(A)
1 recinputs(A,1)
-200 envcost

x(A,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-24 cost
1 supply(A)
1 recinputs(A,2)
-200 envcost

x(A,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-40 cost
1 supply(A)
1 recinputs(A,3)
-200 envcost

x(A,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-20 cost
1 supply(A)
1 recinputs(A,4)
-200 envcost

x(A,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)

```
    -48      cost
      1      supply(A)
      1      recinputs(A,5)
   -200      envcost

x(B,1)
            (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -30      cost
      1      supply(B)
      1      recinputs(B,1)
   -200      envcost

x(B,2)
            (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -20      cost
      1      supply(B)
      1      recinputs(B,2)
   -200      envcost

x(B,3)
            (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -40      cost
      1      supply(B)
      1      recinputs(B,3)
   -200      envcost

x(B,4)
            (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -30      cost
      1      supply(B)
      1      recinputs(B,4)
   -200      envcost

x(B,5)
            (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -45      cost
      1      supply(B)
```

```
      1      recinputs(B,5)
   -200       envcost

x(C,1)
              (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -48       cost
      1      supply(C)
      1      recinputs(C,1)
   -200       envcost

x(C,2)
              (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -24       cost
      1      supply(C)
      1      recinputs(C,2)
   -200       envcost

x(C,3)
              (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -36       cost
      1      supply(C)
      1      recinputs(C,3)
   -200       envcost

x(C,4)
              (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -54       cost
      1      supply(C)
      1      recinputs(C,4)
   -200       envcost

x(C,5)
              (.LO, .L, .UP, .M = 0, 0, +INF, 0)
    -36       cost
      1      supply(C)
      1      recinputs(C,5)
   -200       envcost
```

x(D,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-70 cost

1 supply(D)

1 recinputs(D,1)

-200 envcost

x(D,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-42 cost

1 supply(D)

1 recinputs(D,2)

-200 envcost

x(D,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-28 cost

1 supply(D)

1 recinputs(D,3)

-200 envcost

x(D,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-84 cost

1 supply(D)

1 recinputs(D,4)

-200 envcost

x(D,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-35 cost

1 supply(D)

1 recinputs(D,5)

-200 envcost

---- y flow quantities between producers in UNITS

y(A,1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,1)

-1 recbyprod(A,1)

10 envcost

y(A,1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 cost

1 recinputs(A,2)

-1 recbyprod(A,1)

10 envcost

y(A,1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 recinputs(A,3)

-1 recbyprod(A,1)

10 envcost

y(A,1,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 cost

1 recinputs(A,4)

-1 recbyprod(A,1)

10 envcost

y(A,1,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-12 cost

1 recinputs(A,5)

-1 recbyprod(A,1)

10 envcost

y(A,2,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 cost
1 recinputs(A,1)
-1 recbyprod(A,2)
10 envcost

y(A,2,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,2)
-1 recbyprod(A,2)
10 envcost

y(A,2,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-4 cost
1 recinputs(A,3)
-1 recbyprod(A,2)
10 envcost

y(A,2,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost
1 recinputs(A,4)
-1 recbyprod(A,2)
10 envcost

y(A,2,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost
1 recinputs(A,5)
-1 recbyprod(A,2)
10 envcost

y(A,3,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost
1 recinputs(A,1)

-1 recbyprod(A,3)
10 envcost

y(A,3,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-4 cost
1 recinputs(A,2)
-1 recbyprod(A,3)
10 envcost

y(A,3,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
1 recinputs(A,3)
-1 recbyprod(A,3)
10 envcost

y(A,3,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-12 cost
1 recinputs(A,4)
-1 recbyprod(A,3)
10 envcost

y(A,3,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 recinputs(A,5)
-1 recbyprod(A,3)
10 envcost

y(A,4,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 recinputs(A,1)
-1 recbyprod(A,4)
10 envcost

y(A,4,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 recinputs(A,2)

-1 recbyprod(A,4)

10 envcost

y(A,4,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-12 cost

1 recinputs(A,3)

-1 recbyprod(A,4)

10 envcost

y(A,4,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

1 recinputs(A,4)

-1 recbyprod(A,4)

10 envcost

y(A,4,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-16 cost

1 recinputs(A,5)

-1 recbyprod(A,4)

10 envcost

y(A,5,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-12 cost

1 recinputs(A,1)

-1 recbyprod(A,5)

10 envcost

y(A,5,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 recinputs(A,2)
-1 recbyprod(A,5)
10 envcost

y(A,5,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-2 cost
1 recinputs(A,3)
-1 recbyprod(A,5)
10 envcost

y(A,5,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-16 cost
1 recinputs(A,4)
-1 recbyprod(A,5)
10 envcost

y(A,5,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
1 recinputs(A,5)
-1 recbyprod(A,5)
10 envcost

y(B,1,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
1 recinputs(B,1)
-1 recbyprod(B,1)
11 envcost

y(B,1,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-5 cost
1 recinputs(B,2)
-1 recbyprod(B,1)
11 envcost

y(B,1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-10 cost

1 recinputs(B,3)

-1 recbyprod(B,1)

11 envcost

y(B,1,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2.5 cost

1 recinputs(B,4)

-1 recbyprod(B,1)

11 envcost

y(B,1,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-15 cost

1 recinputs(B,5)

-1 recbyprod(B,1)

11 envcost

REMAINING 70 ENTRIES SKIPPED

---- z flow quantities from producer to market in UNITS

z(1,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 cost

1 demand(1)

-1 recoutputs(1,1)

z(1,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(2)

-1 recoutputs(1,2)

z(1,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(3)

-1 recoutputs(1,3)

z(1,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 demand(4)

-1 recoutputs(1,4)

z(1,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-9 cost

1 demand(5)

-1 recoutputs(1,5)

z(2,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(1)

-1 recoutputs(2,1)

z(2,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(2)

-1 recoutputs(2,2)

z(2,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 cost

1 demand(3)

-1 recoutputs(2,3)

z(2,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(4)

-1 recoutputs(2,4)

z(2,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(5)

-1 recoutputs(2,5)

z(3,1)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-9 cost

1 demand(1)

-1 recoutputs(3,1)

z(3,2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-8 cost

1 demand(2)

-1 recoutputs(3,2)

z(3,3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-7 cost

1 demand(3)

-1 recoutputs(3,3)

z(3,4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-6 cost

1 demand(4)

-1 recoutputs(3,4)

z(3,5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 cost
1 demand(5)
-1 recoutputs(3,5)

z(4,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-6 cost
1 demand(1)
-1 recoutputs(4,1)

z(4,2)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-7 cost
1 demand(2)
-1 recoutputs(4,2)

z(4,3)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-8 cost
1 demand(3)
-1 recoutputs(4,3)

z(4,4)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-9 cost
1 demand(4)
-1 recoutputs(4,4)

z(4,5)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-10 cost
1 demand(5)
-1 recoutputs(4,5)

z(5,1)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-10 cost

```
      1       demand(1)
     -1       recoutputs(5,1)

z(5,2)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -9       cost
      1       demand(2)
     -1       recoutputs(5,2)

z(5,3)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -8       cost
      1       demand(3)
     -1       recoutputs(5,3)

z(5,4)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -7       cost
      1       demand(4)
     -1       recoutputs(5,4)

z(5,5)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -6       cost
      1       demand(5)
     -1       recoutputs(5,5)


---- h  rate of production of recipe r at producer m in MULTIPLES OF r

h(1)
                (.LO, .L, .UP, .M = 0, 0, +INF, 0)
     -4       recinputs(A,1)
     -5       recinputs(B,1)
      1       recoutputs(1,1)
      2       recbyprod(D,1)
    -26       envcost
```

h(2)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-3 recinputs(B,1)

-1.5 recinputs(C,1)

4.2 recoutputs(1,2)

1.5 recbyprod(D,1)

-19.5 envcost

h(3)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-1 recinputs(A,2)

-4 recinputs(D,2)

3 recoutputs(2,3)

h(4)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-3.5 recinputs(A,2)

-5.5 recinputs(B,2)

1.8 recoutputs(2,1)

2 recbyprod(C,2)

-24 envcost

h(5)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-1 recinputs(C,2)

-2 recinputs(D,2)

1 recoutputs(2,2)

2 recoutputs(2,4)

0.5 recbyprod(C,2)

-6 envcost

h(6)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-2 recinputs(C,3)

-6 recinputs(D,3)

0.5 recoutputs(3,3)

1.5 recoutputs(3,4)
2 recbyprod(B,3)
-22 envcost

h(7)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-0.5 recinputs(A,3)
-1 recinputs(C,3)
0.5 recoutputs(3,4)
0.5 recoutputs(3,5)
1 recbyprod(B,3)
-11 envcost

h(8)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-4 recinputs(A,4)
-5 recinputs(B,4)
2 recoutputs(4,1)
2 recbyprod(D,4)
-26 envcost

h(9)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-4 recinputs(A,4)
-5 recinputs(B,4)
1 recoutputs(4,1)
0.5 recbyprod(A,4)
2 recbyprod(B,4)
1 recbyprod(D,4)
-40 envcost

h(10)
(.LO, .L, .UP, .M = 0, 0, +INF, 0)
-1 recinputs(A,5)
-1 recinputs(C,5)
-1 recinputs(D,5)
3 recoutputs(5,3)

h(11)

(.LO, .L, .UP, .M = 0, 0, +INF, 0)

-5 recinputs(C,5)

0.5 recoutputs(5,5)

0.5 recbyprod(A,5)

-5 envcost

---- f total cost in DOLLARS

f

(.LO, .L, .UP, .M = -INF, 0, +INF, 0)

1 cost

---- env environmental cost in DOLLARS

env

(.LO, .L, .UP, .M = -INF, 0, +INF, 0)

-1 cost

1 envcost

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
Model Statistics SOLVE NETWORK Using LP From line 171

MODEL STATISTICS

BLOCKS OF EQUATIONS	7	SINGLE EQUATIONS	76
BLOCKS OF VARIABLES	6	SINGLE VARIABLES	158
NON ZERO ELEMENTS	594		

GENERATION TIME = 0.016 SECONDS 4 Mb WEX239-239 Nov 9, 2012

EXECUTION TIME = 0.016 SECONDS 4 Mb WEX239-239 Nov 9, 2012

G e n e r a l A l g e b r a i c M o d e l i n g S y s t e m
Solution Report SOLVE NETWORK Using LP From line 171

S O L V E S U M M A R Y

MODEL NETWORK OBJECTIVE f
TYPE LP DIRECTION MINIMIZE
SOLVER CPLEX FROM LINE 171

**** SOLVER STATUS 1 Normal Completion
**** MODEL STATUS 1 Optimal
**** OBJECTIVE VALUE 788738.0952

RESOURCE USAGE, LIMIT 0.000 1000.000
ITERATION COUNT, LIMIT 33 2000000000

IBM ILOG CPLEX Jul 4, 2012 23.9.5 WEX 36376.36401 WEI x86_64/MS Windows
Cplex 12.4.0.1

LP status(1): optimal
Optimal solution found.
Objective : 788738.095238

LOWER LEVEL UPPER MARGINAL

---- EQU cost . . . 1.000

cost define objective function

---- EQU supply observe supply limit at supplier l

LOWER LEVEL UPPER MARGINAL

A -INF 1638.889 2000.000 .

B -INF 884.921 2000.000 .

C -INF 785.714 2000.000 .

D -INF . 2000.000 .

---- EQU demand satisfy or exceed demand at market n

	LOWER	LEVEL	UPPER	MARGINAL
1	500.000	500.000	+INF	859.222
2	500.000	500.000	+INF	219.810
3	500.000	500.000	+INF	205.444
4	500.000	500.000	+INF	.
5	500.000	500.000	+INF	293.000

---- EQU recinputs satisfy material balance between inputs received at m and in puts used in recipes at m

	LOWER	LEVEL	UPPER	MARGINAL
A.1	.	.	.	-24.028
A.2	.	.	.	224.000
A.3	.	.	.	240.000
A.4	.	.	.	220.000
A.5	.	.	.	248.000
B.1	.	.	.	225.000
B.2	.	.	.	220.000
B.3	.	.	.	.
B.4	.	.	.	197.422
B.5	.	.	.	.
C.1	.	.	.	236.000
C.2	.	.	.	128.143
C.3	.	.	.	236.000
C.4	.	.	.	.
C.5	.	.	.	236.000
D.1	.	.	.	.
D.2	.	.	.	94.333
D.3	.	.	.	8.037

D.4 . . . .

D.5 . . . 108.333

---- EQU recoutputs satisfy that product outputs from each producer equals prod
ucts received by markets

	LOWER	LEVEL	UPPER	MARGINAL
1.1	.	.	.	854.222
1.2	.	.	.	213.810
1.3	.	.	.	198.444
1.4	.	.	.	.
1.5	.	.	.	284.000
2.1	.	.	.	852.222
2.2	.	.	.	213.810
2.3	.	.	.	200.444
2.4	.	.	.	-6.000
2.5	.	.	.	286.000
3.1	.	.	.	850.222
3.2	.	.	.	211.810
3.3	.	.	.	198.444
3.4	.	.	.	-6.000
3.5	.	.	.	288.000
4.1	.	.	.	853.222
4.2	.	.	.	212.810
4.3	.	.	.	197.444
4.4	.	.	.	.
4.5	.	.	.	283.000
5.1	.	.	.	849.222
5.2	.	.	.	210.810
5.3	.	.	.	197.444
5.4	.	.	.	.
5.5	.	.	.	287.000

---- EQU recbyprod producer m must produce an amount of byproduct type l equal
to or greater than the amount of l used by producers mp as i
nputs

	LOWER	LEVEL	UPPER	MARGINAL
A.1	.	.	+INF	246.000
A.2	.	.	+INF	250.000
A.3	.	.	+INF	256.000
A.4	.	.	+INF	242.000
A.5	.	.	+INF	2083.000
B.1	.	.	+INF	236.000
B.2	.	.	+INF	231.000
B.3	.	.	+INF	226.000
B.4	.	.	+INF	419.778
B.5	.	.	+INF	221.000
C.1	.	.	+INF	248.000
C.2	.	.	+INF	242.000
C.3	.	.	+INF	248.000
C.4	.	.	+INF	245.000
C.5	.	.	+INF	248.000
D.1	.	.	+INF	100.333
D.2	.	.	+INF	107.333
D.3	.	.	+INF	117.833
D.4	.	.	+INF	93.333
D.5	.	.	+INF	121.333

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU envcost	.	.	.	1.000

envcost environemental cost for unused byproduct disposed of by each producer m instead of shipping to other producers mp

---- VAR x flow quantities from supplier to producer in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
A.1	.	.	+INF	240.028
A.2	.	976.190	+INF	.

A.3	.	500.000	+INF	.
A.4	.	.	+INF	.
A.5	.	162.698	+INF	.
B.1	.	.	+INF	5.000
B.2	.	884.921	+INF	.
B.3	.	.	+INF	240.000
B.4	.	.	+INF	32.578
B.5	.	.	+INF	245.000
C.1	.	.	+INF	12.000
C.2	.	.	+INF	95.857
C.3	.	623.016	+INF	.
C.4	.	.	+INF	254.000
C.5	.	162.698	+INF	.
D.1	.	.	+INF	270.000
D.2	.	.	+INF	147.667
D.3	.	.	+INF	219.963
D.4	.	.	+INF	284.000
D.5	.	.	+INF	126.667

---- VAR y flow quantities between producers in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
A.1.1	.	.	+INF	260.028
A.1.2	.	.	+INF	16.000
A.1.3	.	.	+INF	4.000
A.1.4	.	.	+INF	18.000
A.1.5	.	.	+INF	.
A.2.1	.	.	+INF	268.028
A.2.2	.	.	+INF	16.000
A.2.3	.	.	+INF	4.000
A.2.4	.	.	+INF	28.000
A.2.5	.	.	+INF	.
A.3.1	.	.	+INF	278.028
A.3.2	.	.	+INF	26.000
A.3.3	.	.	+INF	6.000
A.3.4	.	.	+INF	38.000

A.3.5	.	.	+INF	.
A.4.1	.	.	+INF	258.028
A.4.2	.	.	+INF	16.000
A.4.3	.	.	+INF	4.000
A.4.4	.	.	+INF	12.000
A.4.5	.	.	+INF	.
A.5.1	.	.	+INF	2109.028
A.5.2	.	.	+INF	1857.000
A.5.3	.	.	+INF	1835.000
A.5.4	.	.	+INF	1869.000
A.5.5	.	.	+INF	1825.000
B.1.1	.	.	+INF	.
B.1.2	.	.	+INF	10.000
B.1.3	.	.	+INF	235.000
B.1.4	.	.	+INF	30.078
B.1.5	.	.	+INF	240.000
B.2.1	.	.	+INF	EPS
B.2.2	.	.	+INF	.
B.2.3	.	.	+INF	225.000
B.2.4	.	.	+INF	32.578
B.2.5	.	.	+INF	230.000
B.3.1	.	357.143	+INF	.
B.3.2	.	642.857	+INF	.
B.3.3	.	.	+INF	215.000
B.3.4	.	.	+INF	32.578
B.3.5	.	.	+INF	217.500
B.4.1	.	.	+INF	186.278
B.4.2	.	.	+INF	198.778
B.4.3	.	.	+INF	423.778
B.4.4	.	.	+INF	211.356
B.4.5	.	.	+INF	428.778
B.5.1	.	.	+INF	EPS
B.5.2	.	.	+INF	.
B.5.3	.	.	+INF	212.500
B.5.4	.	.	+INF	32.578
B.5.5	.	.	+INF	210.000
C.1.1	.	.	+INF	.

C.1.2	.	.	+INF	113.857
C.1.3	.	.	+INF	12.000
C.1.4	.	.	+INF	239.000
C.1.5	.	.	+INF	18.000
C.2.1	.	178.571	+INF	.
C.2.2	.	.	+INF	101.857
C.2.3	.	376.984	+INF	.
C.2.4	.	.	+INF	242.000
C.2.5	.	.	+INF	6.000
C.3.1	.	.	+INF	12.000
C.3.2	.	.	+INF	113.857
C.3.3	.	.	+INF	.
C.3.4	.	.	+INF	254.000
C.3.5	.	.	+INF	3.000
C.4.1	.	.	+INF	.
C.4.2	.	.	+INF	116.857
C.4.3	.	.	+INF	15.000
C.4.4	.	.	+INF	233.000
C.4.5	.	.	+INF	21.000
C.5.1	.	.	+INF	18.000
C.5.2	.	.	+INF	119.857
C.5.3	.	.	+INF	3.000
C.5.4	.	.	+INF	260.000
C.5.5	.	.	+INF	.
D.1.1	.	.	+INF	87.333
D.1.2	.	15.873	+INF	.
D.1.3	.	.	+INF	93.296
D.1.4	.	.	+INF	90.833
D.1.5	.	162.698	+INF	.
D.2.1	.	.	+INF	101.333
D.2.2	.	.	+INF	.
D.2.3	.	.	+INF	93.296
D.2.4	.	.	+INF	108.333
D.2.5	.	.	+INF	EPS
D.3.1	.	.	+INF	118.833
D.3.2	.	.	+INF	17.500
D.3.3	.	.	+INF	96.796

D.3.4	.	.	+INF	125.833
D.3.5	.	.	+INF	.
D.4.1	.	.	+INF	83.833
D.4.2	.	.	+INF	.
D.4.3	.	.	+INF	93.296
D.4.4	.	.	+INF	80.333
D.4.5	.	.	+INF	EPS
D.5.1	.	.	+INF	129.333
D.5.2	.	.	+INF	28.000
D.5.3	.	.	+INF	103.796
D.5.4	.	.	+INF	136.333
D.5.5	.	.	+INF	.

---- VAR z flow quantities from producer to market in UNITS

	LOWER	LEVEL	UPPER	MARGINAL
1.1	.	.	+INF	.
1.2	.	500.000	+INF	.
1.3	.	.	+INF	.
1.4	.	.	+INF	8.000
1.5	.	.	+INF	.
2.1	.	500.000	+INF	.
2.2	.	.	+INF	.
2.3	.	11.905	+INF	.
2.4	.	.	+INF	.
2.5	.	.	+INF	.
3.1	.	.	+INF	.
3.2	.	.	+INF	.
3.3	.	.	+INF	.
3.4	.	500.000	+INF	.
3.5	.	500.000	+INF	.
4.1	.	.	+INF	.
4.2	.	.	+INF	.
4.3	.	.	+INF	.
4.4	.	.	+INF	9.000
4.5	.	.	+INF	.

5.1 . . +INF .

5.2 . . +INF .

5.3 . 488.095 +INF .

5.4 . . +INF 7.000

5.5 . . +INF .

---- VAR h rate of production of recipe r at producer m in MULTIPLES OF r

LOWER LEVEL UPPER MARGINAL

1 . . +INF .

2 . 119.048 +INF .

3 . 3.968 +INF .

4 . 277.778 +INF .

5 . . +INF .

6 . . +INF .

7 . 1000.000 +INF .

8 . . +INF .

9 . . +INF .

10 . 162.698 +INF .

11 . . +INF .

LOWER LEVEL UPPER MARGINAL

---- VAR f -INF 7.8874E+5 +INF .

---- VAR env -INF 6.6190E+5 +INF .

f total cost in DOLLARS

env environmental cost in DOLLARS

**** REPORT SUMMARY : 0 NONOPT

0 INFEASIBLE

0 UNBOUNDED

EXECUTION TIME = 0.000 SECONDS 2 Mb WEX239-239 Nov 9, 2012

USER: David G. Taggart G121115:1510AP-WIN
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**** FILE SUMMARY

Input \\psf\Home\Documents\gamsdir\projdir\Final Models For Thesis\Special Case Environment Cost.gms
Output \\psf\Home\Documents\gamsdir\projdir\Special Case Environment Cost.lst