Müller Replies: The simulation data reported in the preceding Comment[1] are very significant, indeed unprecedented in extent for classical spin dynamics. In relation to my own simulation study,[2] which was much more limited in computational power, the data reported by Gerling and Landau demonstrate that the spin-autocorrelation function $\langle \mathbf{S}_l(t)\cdot\mathbf{S}_l(0)\rangle$ in the log-log plot is still somewhat curved beyond the point I figured it was. This was not recognizable in the data shown in Ref. 2. Consequently, my estimate of the characteristic exponent, $\alpha=0.609$, was definitely somewhat high. However, I do not think that there is substantial evidence for a crossover to $\alpha=0.5$ further out on the long-time tail of $\langle \mathbf{S}_l(t)\cdot\mathbf{S}_l(0)\rangle$.

From my own analysis[3] of the data shown in Fig. 1 of the Comment I conclude that for the time interval on which $\ln\langle E_l(t)E_l(0)\rangle$ is consistent with a straight line of slope $\alpha=0.5$, the function $\ln\langle \mathbf{S}_l(t)\cdot\mathbf{S}_l(0)\rangle$ is as convincingly consistent with a straight line of slope $\alpha\simeq0.58$, still in good agreement with the NMR proton spin-relaxation measurements on tetramethyl ammonium manganese trichloride. There is no compelling evidence that the true asymptotic behavior of the spin-autocorrelation function sets in significantly later than that of the energy-autocorrelation function. On the other hand, the conclusion proposed by Gerling and Landau, which is based on a detailed regression analysis, cannot be dismissed entirely and should be heeded as a possibility. However, in my opinion, any trend of persistent curvature in the data line at $Jt>100$, which one may still suspect to be present, is well within the noise level. Whatever the final word on the true long-time asymptotic behavior of $\langle \mathbf{S}_l(t)\cdot\mathbf{S}_l(0)\rangle$ will be, the extent of the anomalous long-time tail over an interval in excess of $Jt=200$ makes it relevant for the interpretation of experiments such as those quoted in Ref. 2.

In order to further illustrate the anomalous character of the spin-autocorrelation function discussed here, consider three variants of the classical 1D Heisenberg model:

$$H=-\sum_l J_{l,l+1}\mathbf{S}_l\cdot\mathbf{S}_{l+1}\,,$$

with (i) uniform exchange coupling $[J_{l,l+1}=1]$, (ii) alternating exchange coupling $[J_{l,l+1}=(-1)^l]$, and (iii) random exchange coupling $[J_{l,l+1}=\pm1]$. Note that all three models have the same rotational symmetry (in spin space), guaranteeing the conservation law necessary for spin diffusion, but have different translational symmetries, which are likely to influence the spin-diffusion process. Figure 1 shows (in log-log plot) simulation data for the spin-autocorrelation function $\langle \mathbf{S}_l(t)\cdot\mathbf{S}_l(0)\rangle$ of the three models. The anomalous behavior ($\alpha\simeq0.58$) shows up only in the model with uniform exchange. The other two models exhibit long-time tails which are consistent (within statistical uncertainties) with standard spin-diffusion theory ($\alpha=0.5$).

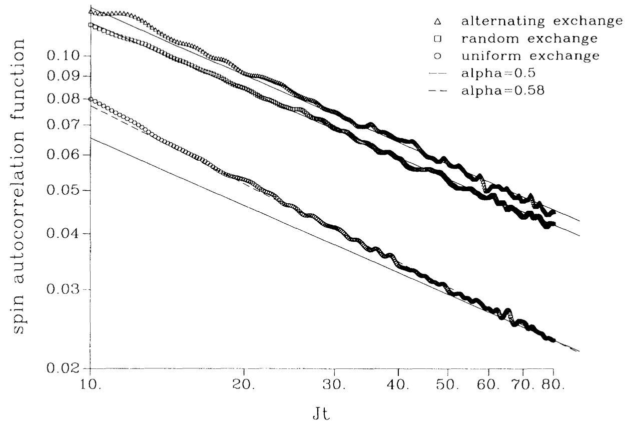

FIG. 1. Log-log plot of the spin-autocorrelation function $\langle \mathbf{S}_l(t)\cdot\mathbf{S}_l(0)\rangle$ for the three variants of the 1D classical Heisenberg model described in the text. The simulation was performed as described in Ref. 2 for systems of $N=120$ spins. The three curves (top to bottom) represent data averaged over $K=4088$, 4895, and 15850 random initial conditions, respectively. In each of the 4985 runs for the random exchange model, the exchange constants $J_{l,l+1}=\pm1$ were also randomly picked.

Finally, I should like to point out that anomalous transport properties are not altogether unexpected in low-dimensional systems. They do occur, e.g., in $d\le2$ models for incompressible viscous fluids,[4] but then they fail to make their appearance in a semimacroscopic model for classical spin systems with O(3) symmetry.[5]

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