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02. Probability: Intuition - Ambiguity - Absurdity - Puzzles

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Abstract
Part two of course materials for Nonequilibrium Statistical Physics (Physics 626), taught by Gerhard Müller at the University of Rhode Island. Entries listed in the table of contents, but not shown in the document, exist only in handwritten form. Documents will be updated periodically as more entries become presentable.

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[nex2] Pick the winning die!

The six faces of four dice A, B, C, D have these numbers on them:

A	1	1	5	5	5	5
B	4	4	4	4	4	4
C	3	3	3	3	7	7
D	2	2	2	6	6	6

You and your opponent each pick one die to use throughout the game. In each round you both roll, and the high number wins. Which is the best die to have? Should you pick first or let your opponent pick first? What is your probability of winning? Give a detailed explanation.

Solution:

[nex4] Educated guess

A railroad company numbers its locomotives in order, $1, 2, \ldots, N$.
(a) One day, you see a locomotive, and its number is 60. What is your best guess for the total number N of locomotives which the company owns?
(b) On the following days, you see four more locomotives, all with numbers smaller than 60. What is your best guess for N based on this additional information?
Describe your reasoning carefully and in detail.

Solution:

[nex82] Coincident birthdays

(a) In a group of n children, what is the probability $P(n)$ that at least two kids have their birthday on the same day of the year? For simplicity assume that every year has 365 days. (b) What is the minimum size of the group for which that probability exceeds 10%, 50%, 90%? Write down your own guess for part (b) before you calculate the answer.

Solution:

[nex11] Win the new car or take a goat!

A contestant in the game show hosted by Monty Hall faces three closed doors. Behind two of the doors are goats and behind the third is a new car. The contestant chooses a door without opening it. Monty Hall, who knows which door hides the car, then opens one of the other doors to reveal a goat. Now the contestant is given a second opportunity to guess the door which hides the car. Is it to the contestant's advantage to switch doors, to stick to the original choice, or does it make no difference?
The question was correctly answered by the newspaper columnist Marylin vos Savant in her weekly column. Her solution stimulated thousands of letters, many from professors of mathematics and statistics, that claimed that her answer was incorrect.

Solution:

[nex13] Three-cornered duel.

The Vicomte de Morcerf, the Baron Danglars and the Count of Monte Christo (characters from the novel by Alexandre Dumas) meet at dawn in a Paris park to fight a three-cornered pistol duel. The Vicomte's chance of hitting the target is 30% and the Baron's chance 50%. The Count never misses. This information is known to all three. They are to fire at their choice of target in succession in the order Vicomte/Count/Baron, cyclically until only one man is left standing. Once a man has been hit he loses further turns and is no longer shot at.
The Vicomte begins. What should be his strategy? Calculate the Vicomte's survival probability (a) if he hits the Baron in his first shot, (b) if he misses in his first shot, and (c) if he hits the Count in his first shot.

Solution:

[nex74] Bad luck: waiting for the worst

Harry's share of bad luck on any given day m is measured by a random number $0 < X_m < 1$. How many days, on average, does Harry have to wait, until his luck is worse than yesterday's ($m = 0$)?

Hint: Calculate the probabilities for respite on days $m - 1$ and m. The probability P_m for the worst luck occurring on day m is the difference. Check the normalization of P_m and calculate $\langle m \rangle$, which turns out to be pretty good news.

Solution:

[nex12] Random quadratic equations

What is the probability that the quadratic equation $x^2 + 2bx + c = 0$ has real roots if b and c are chosen randomly from the real numbers?
Demonstrate that this question has no unique answer. The instruction, "choose b and c randomly from the real numbers", needs to be further specified. Calculate two distinct answers that follow the instructions to the letter.

Solution:

[nex84] Crossing a river

Four Chadian friends wish to cross the Chari river at night over a narrow and treacherous bridge. They have one flashlight. Gougouma, Ndakta, Maïtaïna, and Kaïssebo require 1, 2, 5, and 10 minutes, respectively, to walk across the bridge. The bridge supports not more than two persons simultaneously. The use of the flashlight is essential on every trip. The duration of each trip is dictated by the slower person. Find the shortest time in which the four friends can make it to the other side of the river. Describe the sequence of trips that minimizes the total time.

Solution:

[nex124] Combinatorics of poker hands

The game of poker is played with a deck of 52 cards: 4 suits ($\clubsuit, \diamondsuit, \heartsuit, \spadesuit$) with 13 ranks each (2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A). The table below states the number of hands of five cards from nine different types and the probability of each type. Explain how you arrive at each number in the middle column.

type of hand	number	probability
straight flush	40	0.000015
four of a kind	624	0.000240
full house	3744	0.001441
flush	5108	0.001965
straight	10,200	0.003925
three of a kind	54,912	0.021129
two pairs	123,552	0.047539
one pair	1,098,240	0.422569
nothing	1,302,540	0.501177
total	2,598,960	1.000000

A *straight flush* has five cards in sequence, all from the same suit. A *straight* has five cards of sequential rank, not all from the same suit. A *full house* has three of one kind (rank) and a pair of a different rank. A *flush* has five cards of the same suit but not all in sequence. In a *straight* (*flush* or not) the ace (A) can be the highest or the lowest card in rank.

Solution:

[nex125] Know your odds!

In the year 1654 the Chevalier de Méré complained to Blaise Pascal that mathematics does not deal with questions of everyday life, which, for the Chevalier, meant gambling. De Méré did well financially by betting that he wins if at least one 6 shows in 4 rolls of one die (original version). His continued success had the consequence that nobody would bet against him any longer. Therefore, he offered the following modified version: he wins if at least one double-6 shows in 24 rolls of two dice. The Chevalier reasoned that in both versions the ratio between the number of throws and the number of possible outcomes is the same, 4/6 and 24/36, respectively, and a single outcome is desirable. Therefore, he concluded, the chances of winning should be the same as well. However, de Méré started to lose money heavily with the modifid version. "My dear friend Blaise, please explain!"

Solution: