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A Simulation Study of Groundwater Withdrawals from the Upper Pawcatuck River Basin

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A SIMULATION STUDY OF GROUNDWATER WITHDRAWALS FROM THE UPPER PAWCATUCK RIVER BASIN

BY

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ABSTRACT

This study examined the risk of low flow under conditions of extensive groundwater withdrawals in the Upper Pawcatuck River basin.

The streamflow record of the Pawcatuck River at Wood River Junction from 1941-1968 was used in this study. The contribution of groundwater to streamflow (termed baseflow) was estimated from these records. There is a constant gravity drain or baseflow recession occurring in the groundwater reservoir. A recession constant relating the recession to an exponential decay process was found using the streamflow record. The recharge of the aquifer, resulting from the infiltration of precipitation to the aquifer, was estimated for every month in the record.

A simulation model of the stream was then developed using recharge as a random variable and the recession equation as a deterministic component. Recharge was generated from empirical distributions on a monthly basis. The effect of pumping from wells near the stream (stream depletion) was found using Jenkins' model of an idealized stream-aquifer system.

The output of the simulation model under conditions of no pumping was compared with the historical records in order to validate the model. A search program was then used

in conjunction with the simulation model in order to find the maximum withdrawal possible subject to a constraint on the maximum number of mean flows below a set minimum flow. An additional constraint was necessary to restrict the allowable range of the pumping rates. The simulation was then altered to reflect a more realistic situation: 3 wells with different stream depletion factors and fixed pumping rates. The combination of wells which would maintain a certain annual supply of water and which would deplete the stream the least was found. These results were related to the 1, 7, and 30 day minimum flow.

This study did not present any new safe yield figure or single optimal pumping plan. Instead, the study demonstrated the effects of time and location of pumping on the risk of low flow in the Pawcatuck River Basin. It is possible that a more elaborate model could be used to determine the safe yield of the basin.

ACKNOWLEDGEMENTS

I would like to express my graditude and appreciation to Dr. Lawing and Dr. Kelly, who provided countless hours of assistance throughout this study.

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I. Introduction

1.1 Objective

This study will examine the supply of groundwater in the Upper Pawcatuck River Basin and evaluate alternative policies for the withdrawal of water. The Upper Pawcatuck River Basin is approximately 70 square miles and located in southeastern Rhode Island. In 1966, the U. S. Geological Survey reported in Water-Supply Paper #1821 (1) (termed Water Resources Report in this study) that the basin would yield a total of 25.6 million gallons per day (39.4 cubic feet per second).[1] Extensive field tests were done to support this estimate. Later reports (2,3,4) reduced this figure to 8- 10 mgd (12.3- 15.4 cfs) due to concern that extensive pumping would severely lower the streamflow. It is essential that the safe yield be established in order to properly plan for the future. Antak (5) concluded that if the U.S.G.S. estimate of 25.6 mgd is correct, then plans for the construction of Big River Impounding Reservoir might be postponed or cancelled.

Water may be withdrawn from either surface or groundwater sources. Surface water withdrawals from lakes and rivers may require costly treatment due to pollutants in the water. Groundwater will require far less treatment due to the natural filtration of the groundwater basin or aquifer,

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[1]One million gallons per day (mgd) = 1.54 cubic feet per sec. (cfs).

except in some areas where there is high concentration of manganese (1). However, there will be additional pumping costs for groundwater withdrawals. It is common practice to place wells near streams to minimize the drawdown or distance the well must lift the water. Pumping near a stream will draw the water from the stream, so there is "stream depletion" or a lowering of the flow in the stream. This study will be limited to groundwater withdrawals where depletion is the major problem.

1.2 Description of the Upper Pawcatuck Basin

The Upper Pawcatuck River Basin is located in the south-central part of Rhode Island and includes a major portion of Exeter, West Greenwich, East Greenwich, Richmond, North Kingstown and Charlestown (Figure 1.1). The basin is approximately 15 miles long and 7 miles wide with a total drainage area of 70 square miles.

The principal river in the basin is the Pawcatuck which is fed by two tributaries, the Chipuxet and the Usquepaug-Queen River (Figure 1.2). The Chipuxet River flows through Worden's Pond while the Usquepaug-Queen River flows through the Great Swamp before they join to form the Pawcatuck River. Many small ponds are located in the basin, including Wordens, Yawgoo, Barbers, Hundred Acre, Larkins and Tucker, as shown in Figure 1.2. Streamflow is measured in the Chipuxet at West Kingston. The Pawcatuck River is measured at Kenyon in the upper basin and at Wood River Junction and Westerly in the lower basin.

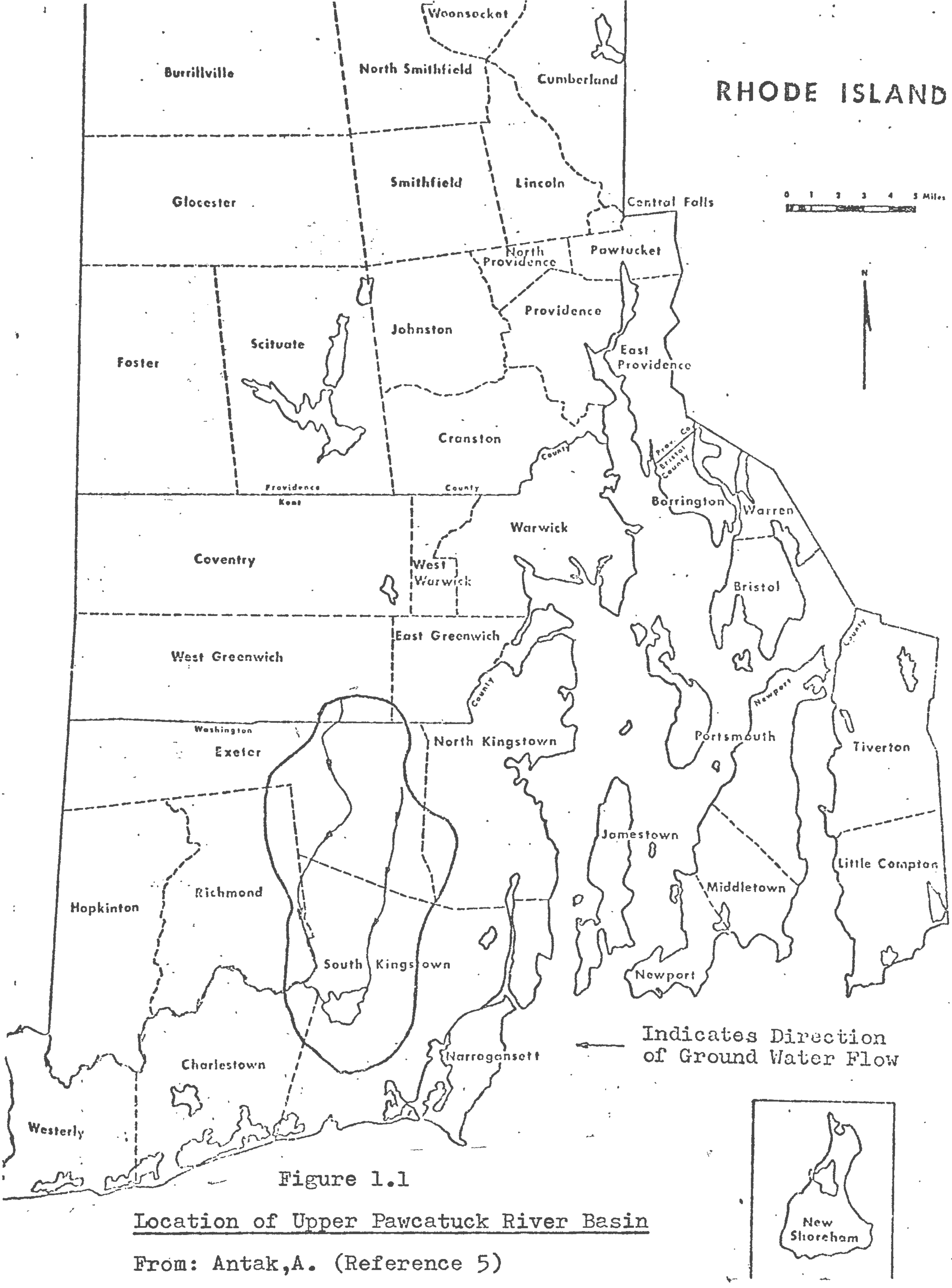


Figure 1.1

Location of Upper Pawcatuck River Basin

From: Antak,A. (Reference 5)

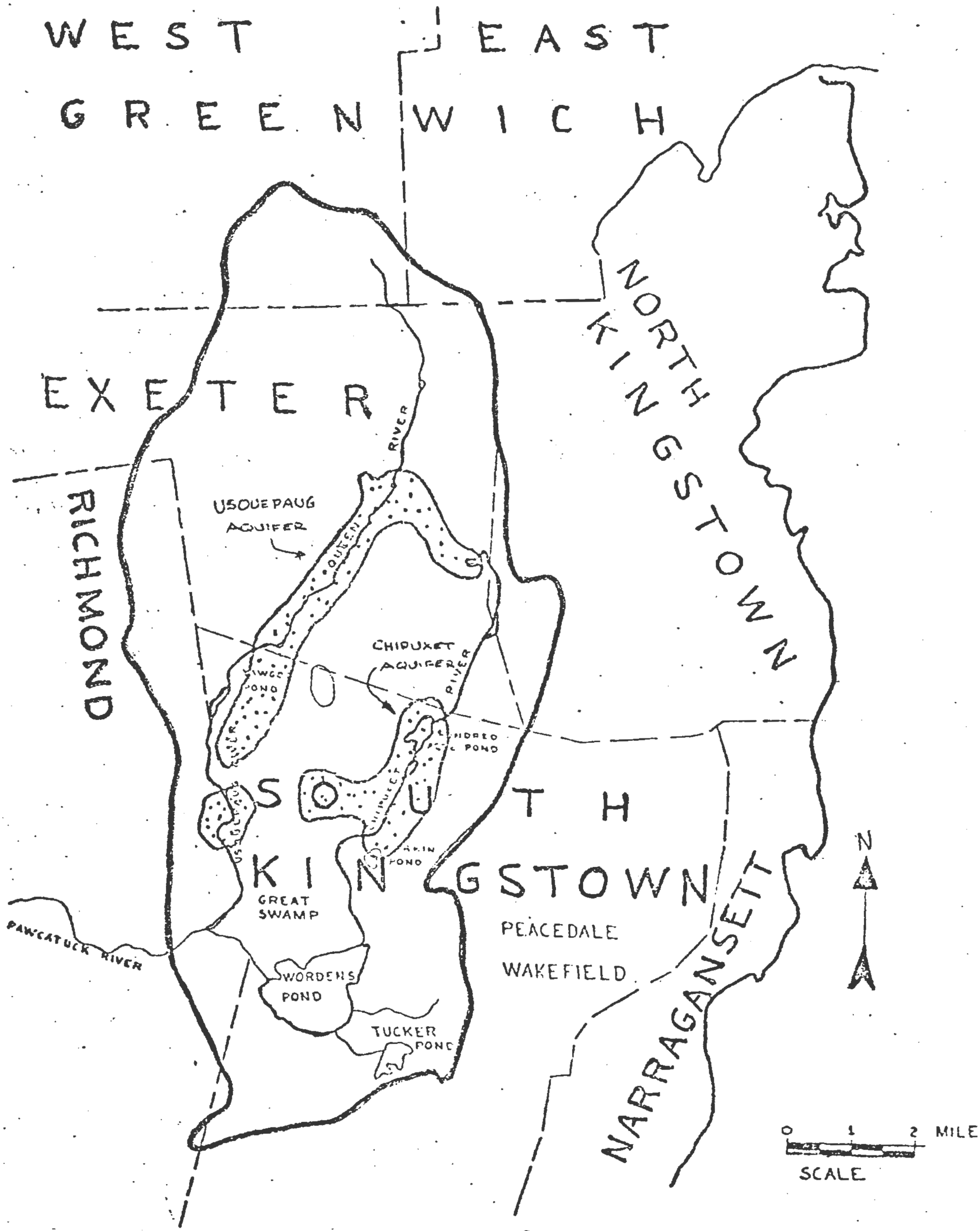


Figure 1.2 Upper Pawcatuck River Basin

From: Antak, A., Economic Planning Considerations of Groundwater Pollution for the Upper Pawcatuck Basin (5)

The Upper Pawcatuck consists of glacially rounded hills and flat valleys. Low rounded hills are found in the northern part of the basin, while the southern part is basically flat and swampy and forms a plain of 90-100 feet above sea level. The southern boundary of the basin consists of a belt of low hills and ridges known as the Charlestown morriane (1).

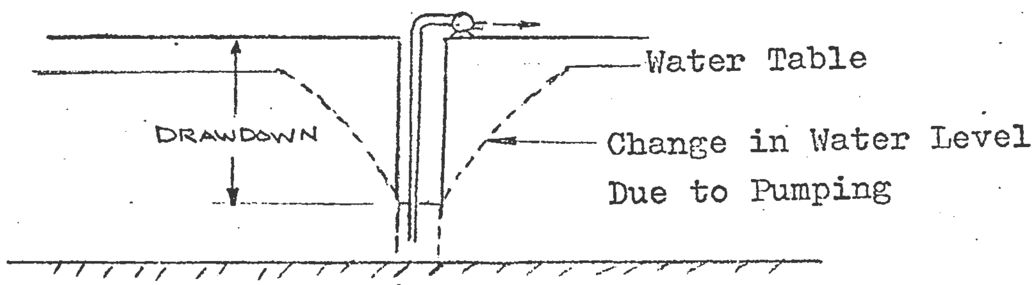
1.3 Groundwater Reservoir Properties

In order for a groundwater reservoir or "aquifer" to be suitable for extensive withdrawals, the groundwater must be able to travel through the aquifer without excessive resistance and there must be a sufficient supply of groundwater. Aquifers not only store water, but also transmit it from one place to another in response to hydraulic gradients. One measure of the ability of an aquifer to transmit water is permeability. Permeability, K $[gpd/ft^2]$, is defined as the flow of water through a cross sectional area of an aquifer under the driving force of a unit hydraulic gradient.

An initial estimate of the permeability can be made by examining the distribution of soil grain sizes. The Water Resources Report found that the unconsolidated deposits from the central part of the Chipuxet and Usquepaug-Queen River valley formed stratified layers of sand and gravel. These deposits had a high sorting coefficient and therefore a high coefficient of permeability.

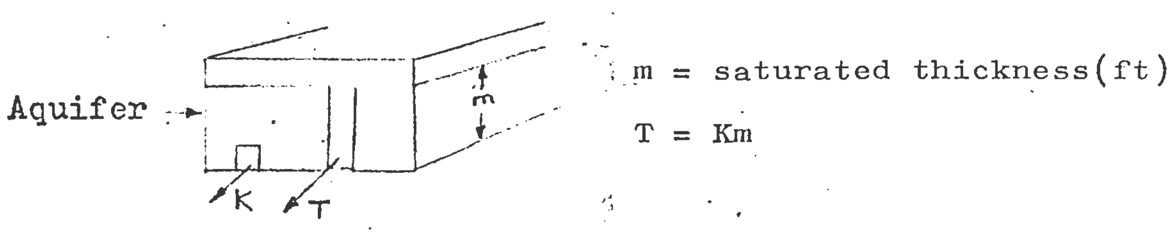
To obtain a more accurate estimate of the aquifer's properties, extensive pumping tests were made in the basin. The variable measured in these tests was "drawdown" which is the distance from the ground (or any datum) to the water level in the well. The pumping of a well draws groundwater from the aquifer, which will lower the water level in the well, as shown in Figure 1.3. The pumping test is used to determine the transmissivity and storativity of an aquifer. The transmissivity, T [gpd/ft], is the rate at which water flows through a vertical strip, one foot wide, extending through the saturated thickness of the aquifer under the driving force of a unit hydraulic gradient. For uniform

Figure 1.3 Drawdown in a Pumped Well



permeability and constant saturated thickness (conditions which are not actually met in the aquifer, but can be assumed to be approximately correct), the transmissivity is equal to the permeability times the saturated thickness, m (ft.) as shown in Figure 1.4.

Figure 1.4 Relationship Between the Transmissivity and Permeability



Storativity, S, is a measure of the ability of the aquifer to expand and contract its structure due to pressure of the groundwater. It is expressed as the ratio of the volume of water to the volume of the aquifer and it is dimensionless. The specific yield is defined as the water removed from a volume of the aquifer under the force of gravity. Since the aquifers of the Pawcatuck basin are unconfirmed, the specific yield would equal the storativity of the aquifer.

Under constant pumping, it is possible to find the aquifer's transmissivity using Thiem's equation (6) once equilibrium conditions have been reached, using the drawdown in nearby observation wells. However, it may take 10 days or more to achieve equilibrium conditions, so it would be an expensive and time-consuming test. The U.S.G.S. (7) determined estimates of T and S using the transient behavior of groundwater to pumping, which meant that the test could be completed in less than 48 hours. The Theis non-equilibrium curve and the matching point method (8) was used to find estimates of T and S for 9 wells. Estimates of transmissivity were also obtained for all 16 wells using the specific capacities of the wells. The specific capacity is the quantity of water a well yield (gpm) per foot of drawdown (ft). The specific capacity was used to make initial estimates of T and then dividing T by the saturated thickness, the permeability was computed.

It was determined by the U.S.G.S. that the central part of the Chipuxet River and the Usquepaug-Queen River valleys had permeabilities of 1,000 gpd/ft^2 or more, on the basis of field and lab tests. The report assumed that the maximum drawdown would be 3/4 the saturated thickness. On this basis, the Water Resources Report estimates that properly constructed wells in this aquifer would yield 700-2000 gpm (1-2.9 mgd or 1.54-4.47 cfs).

1.4 Availability of Groundwater Within the Basin

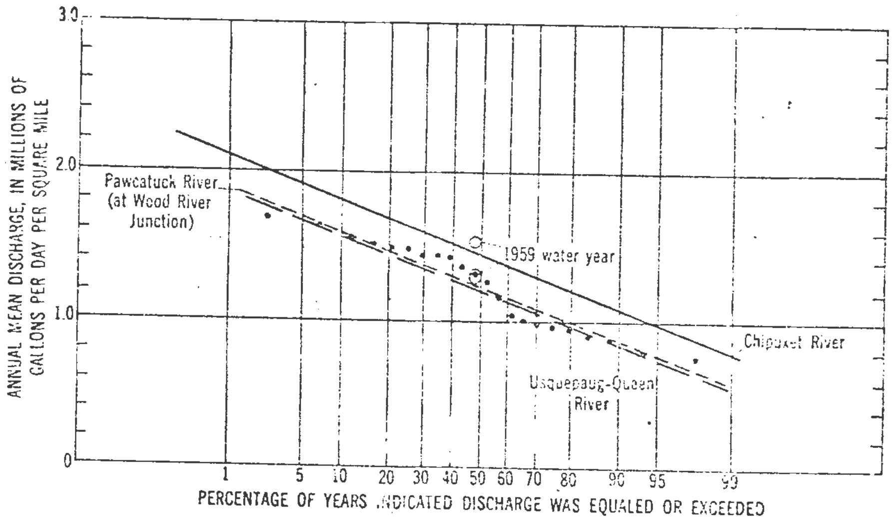
The average yearly rainfall from 1889-1962 at Kingston was 48" (1). If rainfall is considered to be uniformly distributed over the basin, then 48" of rainfall over 70 square miles (total area) for one year is the equivalent to a constant flow of 247 cfs. The losses from the basin are due to evapotranspiration which the Water Resources Report estimated a mean yearly total of 24" on the basis of air temperature records at Kingston. This leaves 24" of rainfall or 123.5 cfs deposited on the basin yearly. More accurate estimates of evapotranspiration can be made on the basis of well data and streamflow records.

The meteorological records are a good indicator of the abundance of water in the basin. It is possible to use these records to supplement limited streamflow records. On a yearly basis, there is good correlation between rainfall and streamflow. However, on a daily or monthly basis, the relationship becomes more complex, because the effect of rainfall in one month is dependent on the previous month.

There has been considerable research in this area to relate rainfall and streamflow. They range from simple empirical equations to extremely complex computer simulations (20,11).

The streamflow records available to the Water Resources Report were sufficient to evaluate the supply of groundwater and correlations with precipitation were unnecessary. The Upper Pawcatuck records were available from 1941-1962 for the 2 years in common, 1958 and 1959. The cumulative distribution of annual mean flows for the Pawcatuck River at Wood River Junction is shown in Figure 1.5. The results are extended to the Chipuxet and Usquepaug-Queen River by comparison of mean annual flows. The individual probabilities of the annual mean flows were computed by ranking since the sample size is small (n=22).

Figure 1.5 <u>Cumulative Distribution of Annual Mean Flow</u>[1]



[1] Figure 1.5 obtained from Figure 17 of the Water Resources Report, (1), page 45.

In assessing the availability of groundwater, the essential variable is not the average annual mean flow (the equivalent of a grand mean), but the mean annual flow which is expected to occur very rarely or which has a low probability of occurring. In the report, the criterion used to establish the yield of the basin was the mean annual flow, A, for which there is a .05 probability that a mean annual flow would occur that would be equal to or less than this flow or: $P\left\{ \overline{Q} \leq A \right\} \leq .05$. This criterion is termed a 20-year flow, because the expected number of years before a mean annual flow would equal or be less than this flow is 20 years. This assumes that the mean annual flows are independent random variables.

The results of the Water Resources Report are shown in Table 1.1, which is the same as Table 6 in the Report, except the values for the Pawcatuck River have also been

Table 1.1 Annual Mean Flow[2]

	Area (mi^2)	10-Yr Flow (mgd/mi^2)	20-Yr Flow (mgd/mi^2)	Baseflow[1] 10-Yr Flow mgd (cfs)	Baseflow[1] 20-Yr Flow mgd (cfs)
Chipuxet	9.9	1.08	.97	7.4 (11.4)	6.4 (9.9)
Usquepaug	36.0	.86	.76	22.0 (33.9)	19.0 (29.3)
Pawcatuck	100.0	.90*	.80*	63.0 (97.0)	56.0 (86.2)

* estimated from graph

1) Assuming 70% of Streamflow is baseflow

[2]Table 1.1 obtained from Table 6 of the Water Resources Report (1), page 44.

estimated from the graph. The report used the 20 year mean annual baseflow in determining the groundwater available for withdrawal from the Usquepaug-Queen and Chipuxet River aquifers, respectively (Table 1.1). After considering the potential infiltration of streamflow through its bed and toward the well and the storage capacity of the aquifer, the report concluded that 26.2 and 13.2 cfs (17 mgd and 8.6 mgd) could be withdrawn from the Usquepaug-Queen and Chipuxet River aquifer, respectively, for a total withdrawal of 39.4 cfs or 25.6 mgd.

1.5 Available Data

Streamflow and well level data are measured by the U.S. Geological Survey and reported in the Water Supply Papers (8,9). The Upper Pawcatuck River is continuously monitored at Kenyon, Wood River Junction and Westerly, which are tributaries of the Pawcatuck. The entire daily streamflow record of the Pawcatuck, from 1941 to 1968, is stored on computer tape and available from U.S.G.S. regional office in Boston. Records after 1968 are available from the Washington Office. There are 8 observation wells distributed throughout the Pawcatuck basin. The U.S. Geological Survey has measured the wells once a month, usually in the last week of the month, since 1955. These records are available in the Water Supply Papers (8) for the period 1955 to 1972.

The Water Resources Board desired more detailed data on the basin for their study. As such, streamflow was recorded on the Chipuxet River and Usquepaug-Queen River, the two main tributaries of the Pawcatuck River for the period February 1, 1958 to July 6, 1960. Also, the number of observation wells was increased from 8 to 16 and observations were taken twice monthly. The data is contained in a report by Allen et al. (7). In 1973, the Water Resources Board started to record these rivers again on a daily basis. Currently, records are available from September 14, 1973 to September 30, 1974.

The streamflow is gaged by measuring the height of streamflow from an arbitrary datum, then it is converted to actual discharge in cubic feet per second by a rating curve (21). The quality of the daily discharge is rated "excellent" if 95% of the discharges are within 5% of the true values, "good" if they are within 10%, "fair" if they are within 15%, and below 15% "poor." Most records of the Pawcatuck River at Wood River Junction were rated excellent, some were rated good. The U.S. Geological Survey also reported some regulation of streamflow at low flow due to powerplants and mills. There is no indication in the record as to the time and degree the stream was regulated. However, after plotting the streamflow data of the Pawcatuck River at Wood River Junction, the regular releases of mills at low flow can be identified. The years 1957, 1965, and 1968 had considerable regulation.

Climatological data is gathered by the National Oceanic and Atmospheric Administration National Weather Service with the regional offices in Warwick, Rhode Island. The Kingston Weather Station has been collecting data on daily rainfall, evaporation, air temperature, relative humidity, and ground temperature since 1889. However, only records from 1941 to 1973 were available for this study (10). These records are not available in computer readable form.

The Water Resources Board recorded precipitation and evaporation at three additional sites from 1958 to 1959 (7). These additional sites helped to measure the change in climatic conditions within the basin.

1.6 Use of Operations Research

Operating research was developed to make the most effective use of scarce resources. It is a very broad field utilizing many different techniques, including simulation and mathematical programming. There has been a rapid growth in the use of simulation and mathematical programming in the development of water resources.

Simulation of River Basins

The use of simulation in the analysis of water resource systems began on a large scale with the Lehigh River Basin project (11). The four year study, conducted by the Harvard Water Program, was to determine the best development of water resources within a basin. It was a complex system, involving six reservoirs for supply and many different uses of water, including irrigation, recreational use, municipal and industrial supply and hydro-electric power generation.

Also, the use of reservoirs in flood prevention was considered. The complexities of the system made an analytical solution impractical, so a computer simulation model was used. There were 42 decision variables in the final model, which allowed the planner to vary the sizes of reservoirs, the capacities of power plants, the amount of water diverted from one source to another, the acceptable water levels and the quality of the streamflow. The decision variables were related to cost or profit. It was infeasible to find a global optimal solution, so the program randomly selects 20 trial designs and find the best three designs.

The Lehigh River basin project was a key beginning point for the application of operations research in water resource development. In 1962, the Harvard Water Program published <u>The Design of Water Resource Systems</u>, which was a massive effort to combine the disciplines of economics, operations research and engineering in the overall planning of a water supply system (12).

A simulation program requires a streamflow generator, that is, a routine that will produce numbers similar to the historical flow record. The historical record can be used directly, but many simulations require a record longer than the historical record. There has been extensive research into the development and use of statistical streamflow generators (13,14).

Mathematical Programming

The techniques of mathematical programming have been useful in solving models involving the withdrawal of surface and groundwater from a basin. These techniques include linear, integer, dynamic, stochastic and non-linear programming.

Taylor (15) used linear programming to find the optimum withdrawal rates of surface and groundwater in order to minimize the depletion of the stream. The constraints to the model are: (1) the total pumpage must be equal to the demand for each month and (2) the total volume withdrawn from the aquifer must be less than a specified limit. The model was applied to the Arkansas River valley in southeastern Colorado for the two most critical months, July and August. A sensitivity analysis showed that the pumping in July was very critical.

Dracup (16) utilized a form of linear programming called parametric linear programming in allocating water from various sources to particular users. The model is essentially a transshipment model, where the cost of transporting a unit of water from a particular source to a destination has a unique and known value. The sources of water are: external surface water, basin surface water, basin ground water, and wastewater. The destination costs are: municipal and industrial use, agricultural use and recharge of basin. The costs are for pumping, treatment and storage. In the case of external water source, there is also a purchase price.

The model was used to compute the optimal schedule of withdrawals from 1965 to 1995 for the San Gabriel Valley in Southern California.

Hughes (17) formulated the decisions concerning the capacity of wells, treatment plants, impounding reservoirs and distribution system as a mixed integer problem. The model recognizes that well pumps and pipe sizes are available in finite number of sizes and as such are integer variables. The objective function is to minimize the total costs which includes both the initial construction costs and operating costs. While no applications are presented in the article, the model appears to be quite realistic and useful in planning an overall water supply system.

Nieswand (18) used chance - constraint linear programming to find the optimal schedule of withdrawals from both surface and groundwater sources. The objective of the model is to maximize the total withdrawal from surface and groundwater sources while maintaining a minimum allowable overflow. The monthly streamflow was considered a random variable with a log-normal distribution. The model used chance-constraints to limit the risk of low flow. The model was successfully applied to Mullica River basin in New Jersey.

Domenico (19) used dynamic programming to find the optimum schedule of withdrawals from surface and groundwater sources over a three year period. The model assumes that there is a considerable lowering of the water table in proportion to the groundwater pumped. This would increase the pumping costs. The model becomes a sequential allocation problem because previous decisions to pump from the groundwater sources increase the cost in pumping in the next period. While no actual application was presented, a numerical problem was solved.

Many other models have been suggested in the literature. Where there are many different users of water and a scarcity of water resources exist, it is critical that the best allocation of these resources be made. In the western states, most notably Colorado and California, there has been extensive study in the optimum allocation of water resources. The water system differs somewhat in the humid northeastern states in that: (1) there are few "multiple use" water resources projects, in that most water is for municipal and industrial use and not as many irrigation or hydroelectric projects as in the West. (2) There is, at least for the present, no great scarcity of water resources in the East. The groundwater resources in the East are largely underdeveloped, while in parts of the West, they are heavily mined to the point where the aquifer is constantly being dewatered.

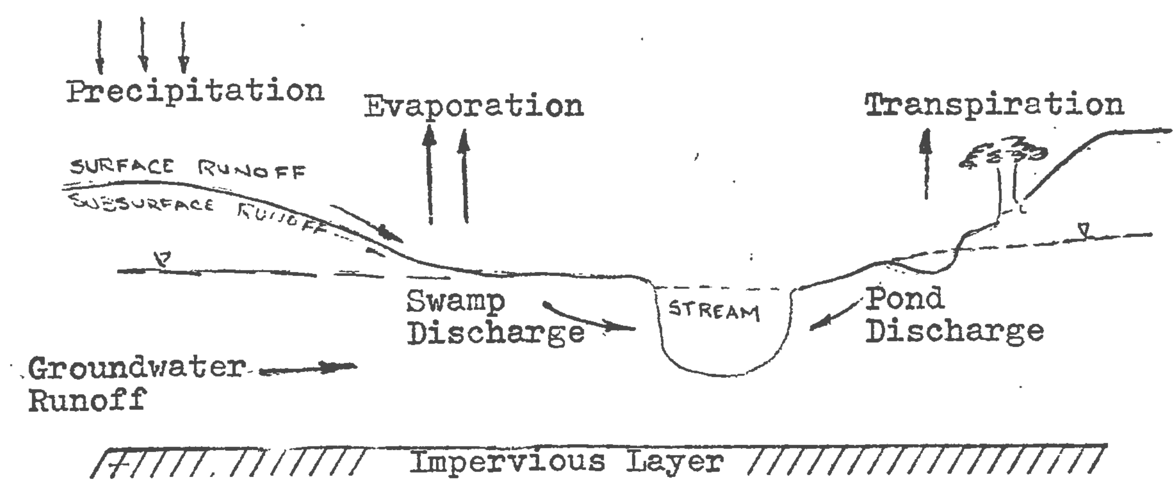
II. Streamflow Analysis

2.1 Hydrological Cycle

Figure 2.1 shows a simplified hydrological cycle. The cycle involves the circulation of water as precipitation, then surface and groundwater runoff, then finally returning to the atmosphere through evaporation and transpiration. It is possible to isolate certain parts of the cycle for study. For example, the meteorologist is concerned with evaporation-precipitation relationships, while the hydrologist would study the rainfall-streamflow relationships (1,2).

The total flow entering the stream is termed basin run-off, which is composed of surface, subsurface and groundwater runoff. The total flow leaving the basin is evaporation from free surfaces (lakes, streams and swamps) and from the ground moisture, plus transpiration from vegetation. Combined, these losses are termed "evapotranspiration." The surface

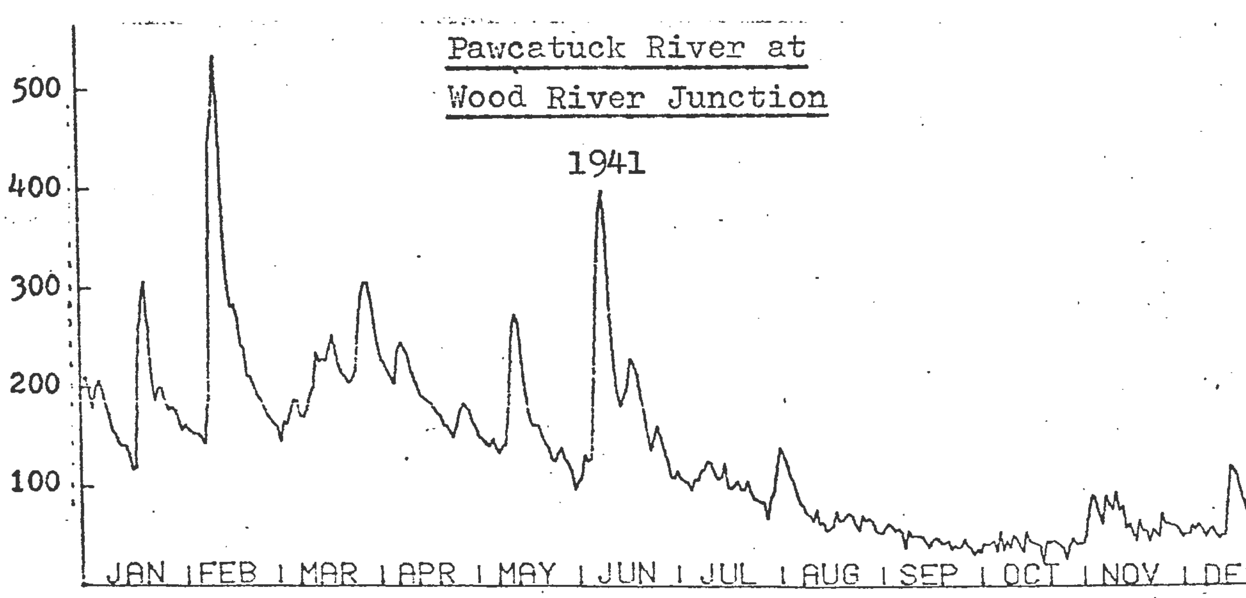
Figure 2.1 Hydrological Cycle



and subsurface flows enter the stream almost immediately after the start of the storm. A portion of precipitation will slowly infiltrate downwards to the groundwater reservoir or aquifer and then travel towards the stream as groundwater runoff or baseflow. The reaction of groundwater to a storm is more lagged and less responsive than surface runoff.

The effects of the different inputs to the stream become more nearly apparent after examining the streamflow record. A typical record of the hydrograph is presented below. The sharp peaks represent the surface runoff contribution and the underlying cyclical trend is due to groundwater runoff. The groundwater runoff or baseflow is important because it is considered the dependable portion of stream flow.

Figure 2.2 Typical Streamflow Record



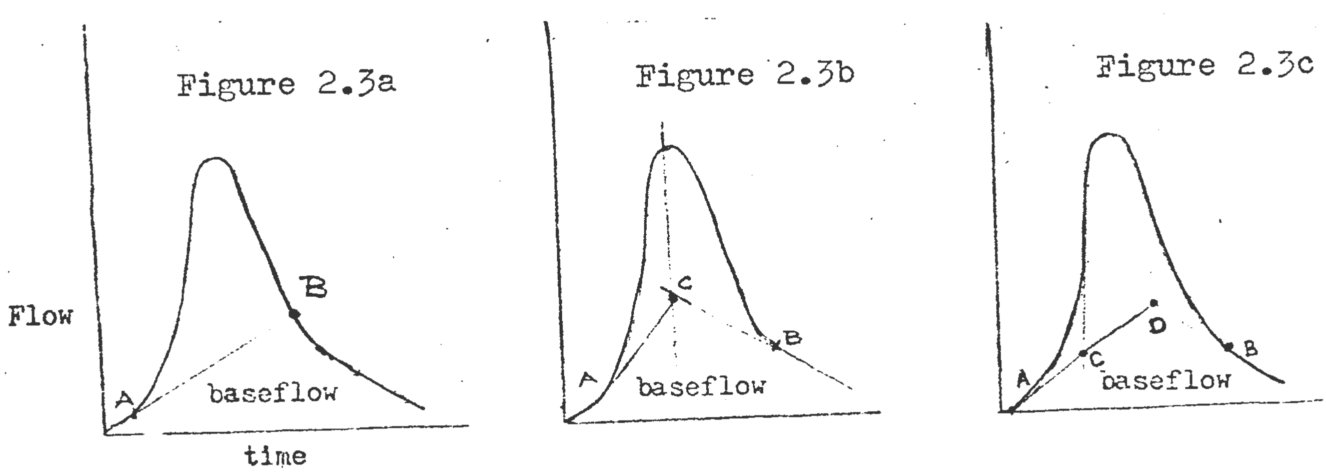
2.2 Baseflow Separation

During long periods of no precipitation, it can be assumed that the stream receives all its water from groundwater. However, it is apparent from the typical streamflow record that long periods with no rainfall are infrequent. More often, the stream is in the process of recovering from the effects of one storm when a second one occurs. During the winter months, it becomes even more difficult to identify the groundwater or baseflow component due to the slow melting of ice and snow. There have been many methods devised to separate the baseflow component, either on a daily basis or monthly basis. The best method to use depends on the amount and accuracy of the data and the need for precise estimates. Three methods will be discussed.

<u>Graphical Methods</u>

Every textbook seems to have slightly different methods for graphical separation of the hydrograph. This study will

Figure 2.3 <u>Graphical Baseflow Separation</u>



discuss only three common procedures (1,2).

The three methods are shown in Figures 2.3a, b, c. Figure 2.3a assumes the flood hydrograph is symmetrical and point B is where the curve departs from symmetry. Point A is where the hydrograph first begins to rise. The first method, Figure a, assumes that the rise from point A to B is uniform. In the second method, (Figure b), a tangent line is extended back from point B to the center line, point C, and then connected with A. The third method (Figure c) uses tangent lines from points A and B to the vertical lines drawn from the inflection points on the curve. Points C and D are then connected. The lines thus formed represent baseflow. These methods require a well-formed single-peaked flood hydrograph. The methods require a certain amount of judgment and therefore are open to human error. Also, these methods would be quite tedious to use with large amounts of data.

Baseflow Separation Using Well Data

The level of the groundwater table can be measured using observation wells to provide a good guide in the separation of baseflow. In the summer and fall months, the loss of groundwater due to evapotranspiration results in a base flow that is always lower than would be predicted using well level data (5).

The Water Resources Report (4) separated the baseflow on a daily basis for the Upper Pawcatuck River at Kenyon and the Chipuxet River at West Kingston for the period October 1958 to September 1959. The average well level from 16 observation wells distributed throughout the basin was used in the separation of baseflow.

The fitting of well data under the hydrograph requires a certain subjective judgment in correcting for the evapotranspiration and other effects. This method does offer an improvement over static techniques presented in the previous section, but it would still be difficult to implement on a computer. Also, a limited amount of data is available.

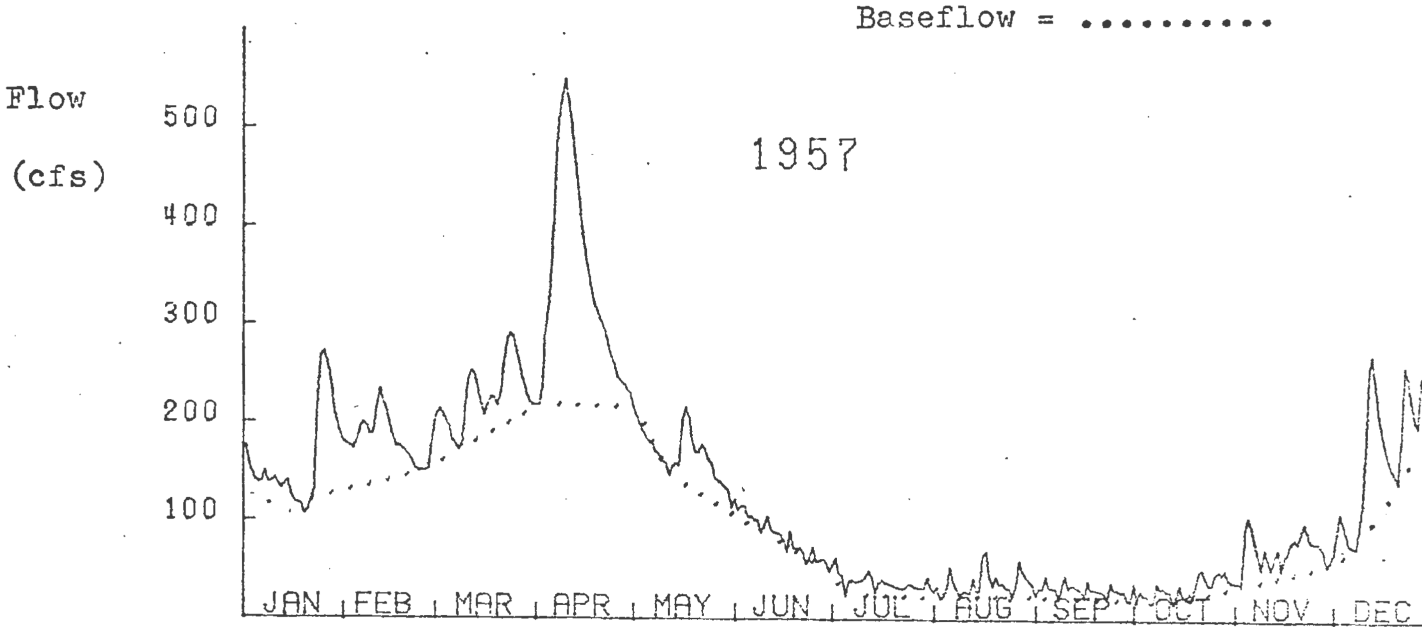
Interval Method

There are 28 years (1941-1968) of Pawcatuck River daily streamflow records on computer tape. It is therefore necessary to find a method that would separate the baseflow component "automatically" - meaning a method that did not rely on additional information or the intuition of a person. One method which has been used successfully is Fourier Series analysis which could identify the underlying cyclical pattern (11). It was found that it was possible to extract the cyclical pattern without becoming involved in time series analysis. The "interval method" was devised for this study for the simple and efficient separation of baseflow.

It is important to find the expected duration (days) of the flood hydrograph in order to implement the interval method. Linsley reported that the time a flood hydrograph takes to recover (that is, from peak flow to baseflow conditions), is proportional to the drainage area (2). He found that as a rough guide, the recovery time, N (days), is found by $N = A_d^{.2}$, where A_d is the drainage area in square miles. Since the Pawcatuck basin at Wood River Junction has 100 square miles of drainage, N = 2.5 days. Linsley also states that there may be large departures from his equation and values for N can be found by inspecting the hydrograph. By inspection of the Pawcatuck River record, it was estimated that it took 1-3 days for the flood hydrograph to reach a peak and 1-15 days to recover to baseflow conditions.

For the interval method, it will be assumed that baseflow conditions are present at least one day in any 20 day interval. The record is divided into 20 day intervals and the minimum streamflow and the day on which the minimum streamflow occurred is found. The baseflow for any day can be found by interpolating between the two minimum points. This method was implemented on the computer to separate all data of the Upper Pawcatuck River at Wood River Junction from 1941 to 1968. Sample plots and mean monthly baseflows are shown in Figure 2.3 and Table 2.1, respectively. Graphs of every year are in the Appendix.

Figure 2.4 Baseflow Separation Using Interval Method



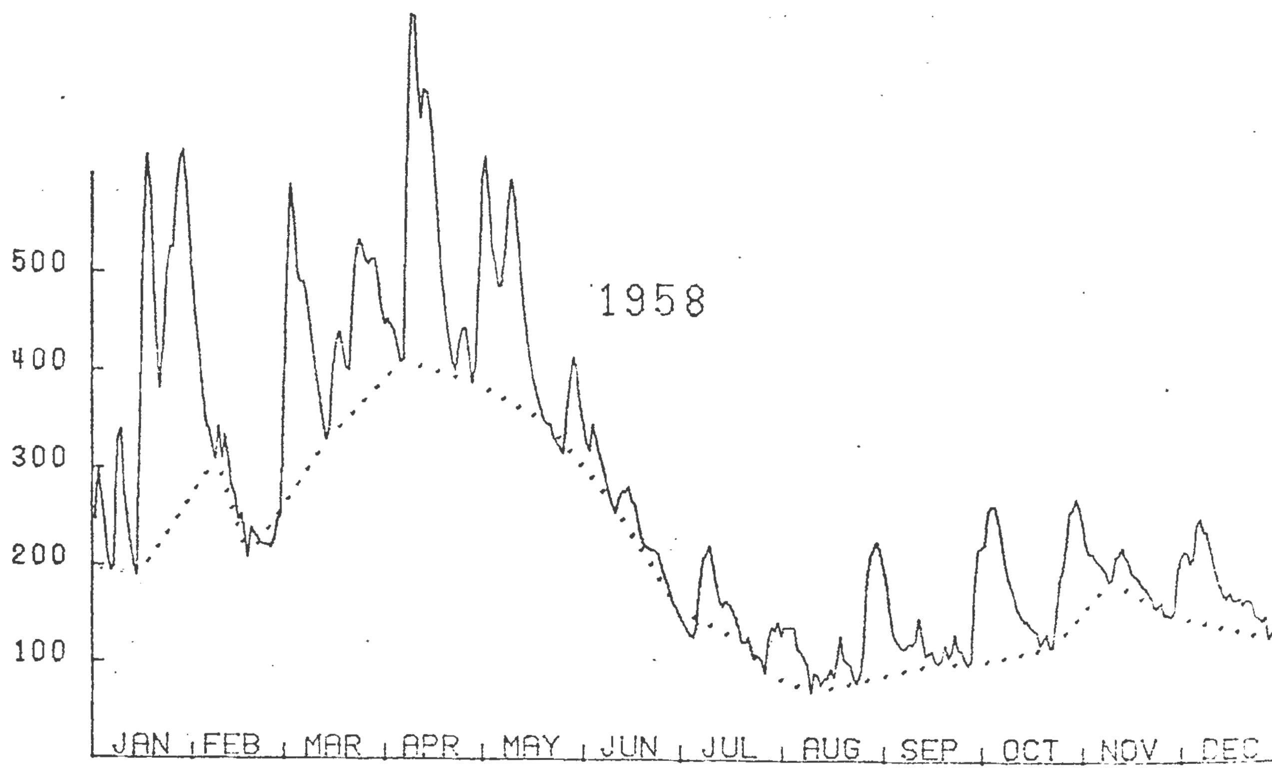


Table 2.1

BASEFLOW IN THE PAWCATUCK RIVER AT WOOD RIVER JUNCTION

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1941	116.20	141.20	175.80	170.80	117.30	100.50	88.90	55.40	35.20	26.70	41.60	49.90
1942	62.80	157.80	218.80	217.70	126.90	78.10	58.30	43.60	41.90	47.10	78.20	148.80
1943	217.90	232.00	225.10	194.70	197.00	142.80	65.00	41.20	31.90	39.40	62.90	47.00
1944	54.80	67.40	147.70	191.20	145.40	84.80	55.70	37.40	30.90	40.00	95.10	198.60
1945	270.10	181.80	281.80	212.90	184.80	136.30	73.30	52.00	37.90	34.30	81.40	171.50
1946	167.00	230.70	238.50	192.70	179.50	158.60	86.60	164.70	119.70	86.40	70.10	65.40
1947	91.40	119.20	153.70	199.00	218.30	130.00	82.60	59.20	31.80	28.40	80.10	95.90
1948	120.10	160.40	329.60	386.00	287.50	304.70	142.00	66.90	41.50	38.80	61.00	73.50
1949	161.50	237.40	256.20	245.20	184.50	91.00	40.50	23.30	25.20	24.80	27.60	46.10
1950	86.20	155.40	202.20	221.50	178.60	125.60	71.70	41.30	28.50	27.70	33.30	70.40
1951	153.90	231.80	256.50	263.30	175.30	130.80	65.50	39.90	33.60	37.00	99.60	137.70
1952	224.60	270.50	264.00	239.30	183.40	140.80	64.40	94.60	78.30	41.20	41.10	67.00
1953	164.80	307.90	381.70	452.70	329.50	138.50	70.60	55.90	40.30	45.20	85.90	189.60
1954	174.40	184.10	198.40	207.90	234.80	145.40	72.30	76.20	129.00	161.20	184.60	237.80
1955	243.50	214.60	272.50	213.10	163.40	112.20	68.80	65.80	86.60	186.60	294.30	228.40
1956	168.80	273.40	359.10	336.00	236.10	133.50	81.80	64.20	43.00	47.10	60.30	96.60
1957	72.70	142.60	188.90	217.00	143.50	75.10	25.10	23.50	21.80	24.00	46.50	102.90
1958	200.00	258.60	336.50	398.30	348.70	227.80	122.40	79.10	98.10	120.10	168.10	136.80
1959	136.50	152.20	332.40	304.50	202.00	127.20	106.40	77.00	60.90	52.90	75.00	162.20
1960	180.70	241.20	263.80	249.40	191.60	129.00	72.00	40.50	38.40	54.80	89.10	94.30
1961	186.00	166.70	314.80	308.30	282.80	196.10	98.90	60.50	70.90	141.90	148.70	170.60
1962	208.90	201.50	292.00	259.10	177.50	112.90	85.40	52.00	41.00	108.50	207.30	222.60
1963	202.20	218.50	222.20	195.00	180.50	143.60	69.70	48.90	32.70	31.50	63.60	94.80
1964	235.10	251.20	228.30	329.40	257.30	97.00	54.20	31.90	19.80	29.80	35.60	77.90
1965	128.20	155.10	220.80	171.40	152.70	89.30	34.60	18.00	19.40	24.50	21.50	28.30
1966	36.30	78.70	184.00	126.40	113.40	120.20	63.70	26.30	29/20	35.10	61.10	78.20
1967	116.60	104:00	166.10	226.50	229.40	183.20	141.90	96.30	55.00	50.20	90.00	196.50
1968	208.90	201.40	195.80	248.20	162.10	147.30	98.00	47.70	23.80	26.50	59.70	112.90

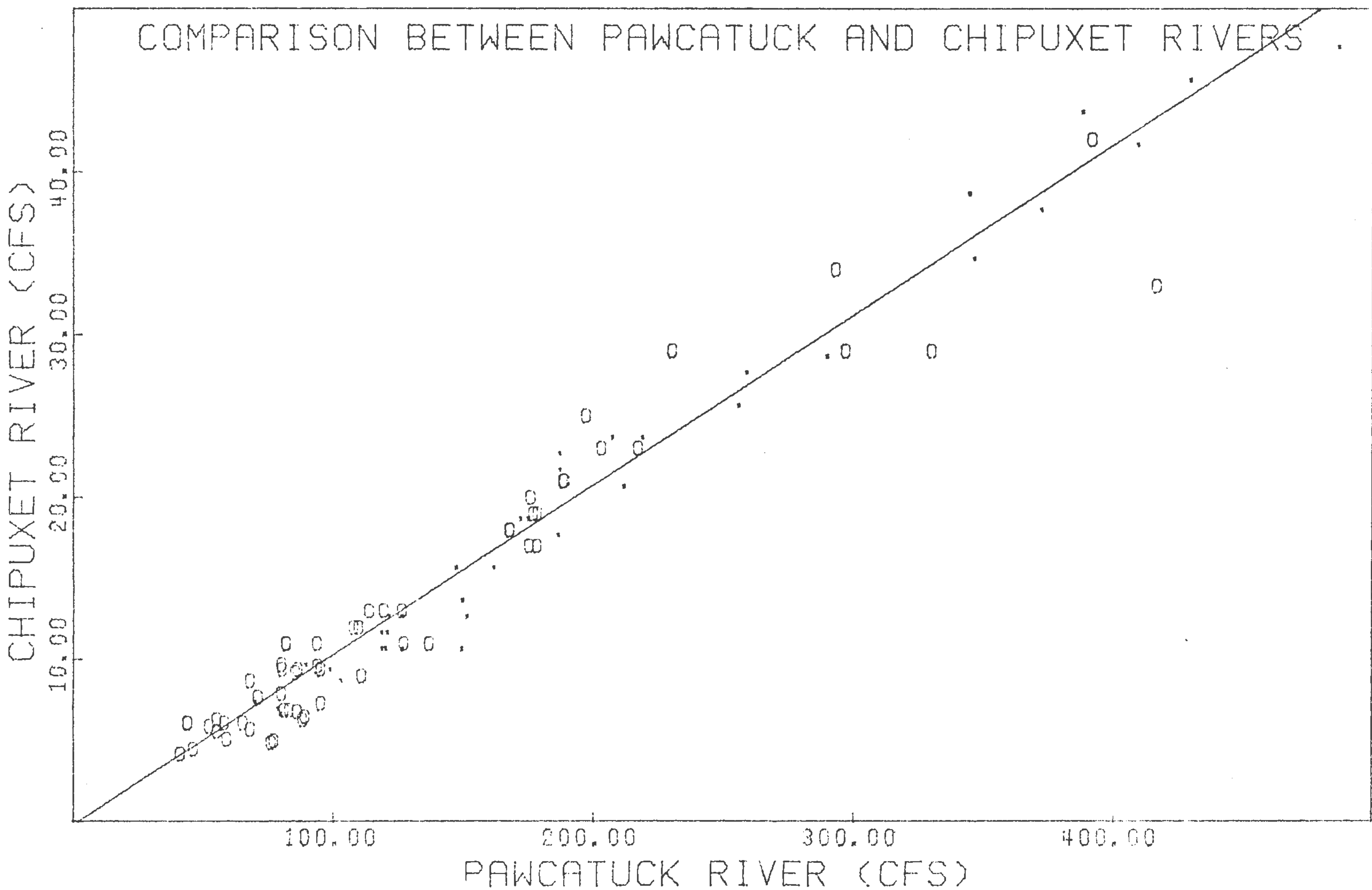
2.3 Comparison Between the Chipuxet and Pawcatuck River

The Chipuxet and the Pawcatuck River have very similar streamflow records. This is natural since the Pawcatuck River receives about 8-10% of its flow from the Chipuxet River. Since the Pawcatuck and the Chipuxet Rivers share a common groundwater basin, the ratio of their baseflow is expected to equal the ratio of their drainage areas. The Chipuxet has a drainage area of 9.9 square miles, while the Pawcatuck at Wood River Junction has 100 square miles of drainage. Therefore, it is expected that the baseflow of the Chipuxet is approximately 9.9% that of the Pawcatuck.

Only the lowest streamflow within a 10 day period for the Pawcatuck and Chipuxet Rivers for the period 1958-1960 and 1973-1974 for the months of April through December were used, so our relation would be based on low flow measurements in both rivers. The relation found by least squares regression was: $C = .104P - .18$ where C is the baseflow (cfs) in the Chipuxet and P is the baseflow (cfs) in the Pawcatuck (Figure 2.5). The F test was performed at $\alpha = .05$ to test if there is a linear relationship between C and P. The test statistic was well outside the critical region, so we reject the null hypothesis that no significant linear relation exists. Analysis of variance and original data are in the Appendix.

The relationship is important because it enables us to extend the record of the Chipuxet River. Several researchers

Figure 2.5


COMPARISON BETWEEN PAWCATUCK AND CHIPUXET RIVERS
CHIPUXET RIVER (CFS)
10.00
20.00
30.00
40.00
100.00
200.00
300.00
400.00
PAWCATUCK RIVER (CFS)

have investigated the reliability of estimates of means and standard deviations when the sample has both historical and derived data (6). It was found that the effective length of the streamflow record is extended if the correlation coefficient is greater than .8. The correlation between the Pawcatuck and Chipuxet Rivers was .959.

There has been an increase in withdrawals from wells near the Chipuxet River in recent years (as much as 1 mgd or 1.54 cfs), which could affect the relationship between the Chipuxet and Pawcatuck Rivers. However, the graph does not reveal any significant difference (Figure 2.5, 1973-1974 flows are circled).

2.4 Baseflow Recession

There is a constant gravity drain or groundwater recession occurring in the basin. The streamflow increases in the late fall and the winter months because the rate of recharge exceeds the rate of loss from the groundwater recession. The recession becomes apparent in the spring and summer months when there is little or no recharge of the aquifer. If we assume that the resistance to the movement of water within an aquifer is constant, then the outflow, Q, is proportional to the volume of water stored in the aquifer, S, or $Q = KS$. By continuity:

Rate In - Rate Out = Change in Storage

$$- Q = \frac{dS}{dt} = \frac{1}{K}\frac{dQ}{dt}$$

Therefore: $Q = Qo \exp(-Kt)$

This means that the baseflow should recede exponentially, if there is uniform resistance throughout the basin. This is a valid assumption if the basin is small and the aquifer is simple.

Meybloom (7) developed essentially the same equation, except using base 10: $Q = K_1\ 10^{-t/k_2}$. The recession constant, k_2, is equal to 2.3/K and $K_1 = Q_o$. Meybloom's equation may be easier to apply since k_2 can be read directly from the semi-log plot of the hydrograph and represents the number of days for a 10-fold decrease in the hydrograph to occur.

Singh (8) suggested other forms of the recession curve due to changes in the transmissivity is continuous, the recession curve could be fitted to the empirical form, $Q = Q_o \exp(-k^n t)$. Singh also showed that the changes in the recession rates could be represented by a composite curve consisting of several different recession rates at different streamflow levels. While there seems to be considerable discussion of the theory of baseflow recession, methods to analytically identify the recession constant seem to be lacking. The recession is most apparent in the summer and fall months when there is little or no recharge of the aquifer. When the hydrograph is plotted on semi-log paper (Figure 2.6) with streamflow on the log scale, the baseflow plots as a straight line. The slope of the recession line would be 2.3 times that of the recession constant (conversion to natural log scale). The driest years would yield the best estimates of groundwater recession. Arbitrarily, it was decided that the dry years would include all years less than 30" of rainfall between April and December. These years are: 1949, 1950, 1951, 1964, 1965, and 1968. The points used to identify the baseflow are the mininum 20 day flows discussed in Section 2.2. The natural log of the baseflow was obtained so our model became: $\ln Q = \ln Q_o - kt$. Linear regression was used to find parameters Q_o and k. The predicted recession line is shown in Figure 2.6. The average k was found to be .01635 days^{-1} or the equivalent constant using Meybloom's equation is 140.67 days.

There is considerable error in computing a recession constant. There is a continuous loss during the summer and fall months due to evapotranspiration which is quite variable over this period. This loss is directly related to the mean monthly air temperature and the depth of the groundwater. Some researchers (9) have concluded that there should be a summer recession curve which reflects both the loss in groundwater due to the gravity drain and evaporation, and a winter recession curve, which accounts for only losses due to the gravity drain. The winter recession curve could be obtained from studying the change in well level. Comparing the two recession curves would yield an estimate for the evapotranspiration which occurred.

There is additional error or inconsistency with the equation when the aquifer is not simple or homogenous. There is a possibility that a more thorough investigation of the recession in the Upper Pawcatuck may reveal a slight curve at low flows due to change in the transmissivity of the aquifer. At low flow, some of the contributions from swamp and pond discharges may be lost because these sources have dried up. Also, the tributaries leading into the Pawcatuck may dry up. There is also human error in identifying the recession curve. In the humid Northeast, very few days can be considered completely baseflow. It is possible that time series analysis could be useful in analytically computing the recession, although these methods are beyond the scope of this study.

The recession curve is directly related to the aquifer's properties, namely the hydraulic diffusivity, T/S. Rorabaugh (10) developed an equation which expresses the relationship between the slope of the recession curve and transmissivity: $\frac{T}{L^2S} = \frac{.933}{k_2}$ where k_2 equals the recession constant as expressed in Meybloom's recession equation and L is the average distance from the stream to the hydraulic divide. From topographical maps, the distance from the stream to the till areas, which are fairly impermeable deposits surrounding the aquifer, is about 1500 feet, so using $L = 1500$ ft. and $K_2 = 140.67$ days, the hydraulic diffusivity equals 14,994 ft^2/day (112,000 gpd/ft^2). If we assume the specific yield equals .2 throughout the aquifer, then

$T = 2998.9\ ft^2/day$ or 22,431.7 gpd/ft. Estimates of L could also be found by using the relation:

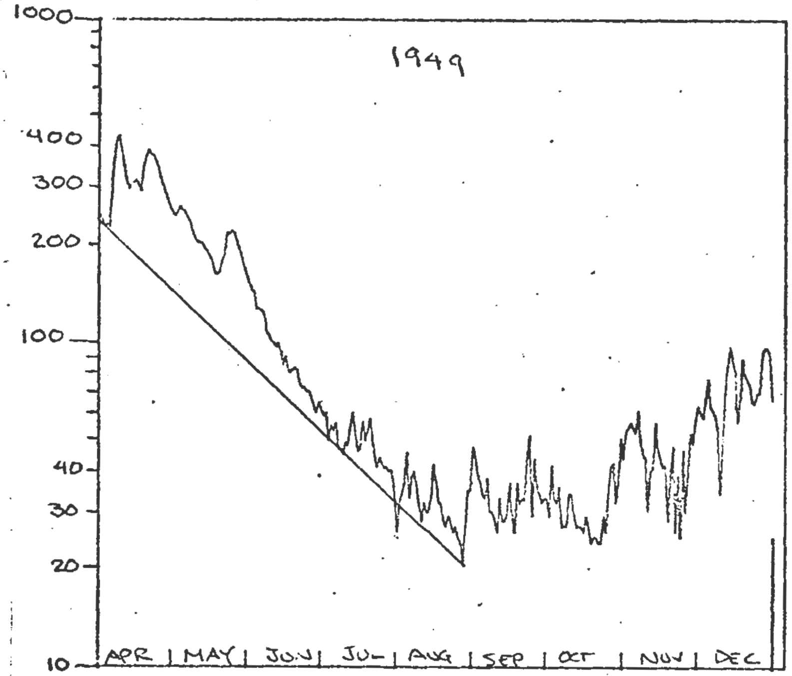
$L = \frac{\text{Basin Area}}{2* \text{ Stream Length}}$. The transmissivity computed is an areal estimate for the entire upper basin, which is quite different from the transmissivity obtained from the pumping tests which were specific to the relatively small areas they affected. The coefficients of transmissibilities from the pumping tests ranged from 30,000 gpd/ft to 200,000 gpd/ft. It is reasonable that the pumping tests should give considerably higher estimates of transmissivity, since they were performed in the highly permeable deposits in the Chipuxet and Usquepaug-Queen River valley.

The average T or T/S can be useful as an initial indication of the properties of an aquifer (12). However, the average values cannot be used to estimate the potential yield of a well. The average T could be used as the minimum T, in that it represents the transmissivity at the hydraulic divide. This will be useful in Section 3.3 in which the location of the well in relation to stream depletion will be discussed.

Figure 2.6

Recession Constants from Semi-Log Plots

Flow
(cfs)



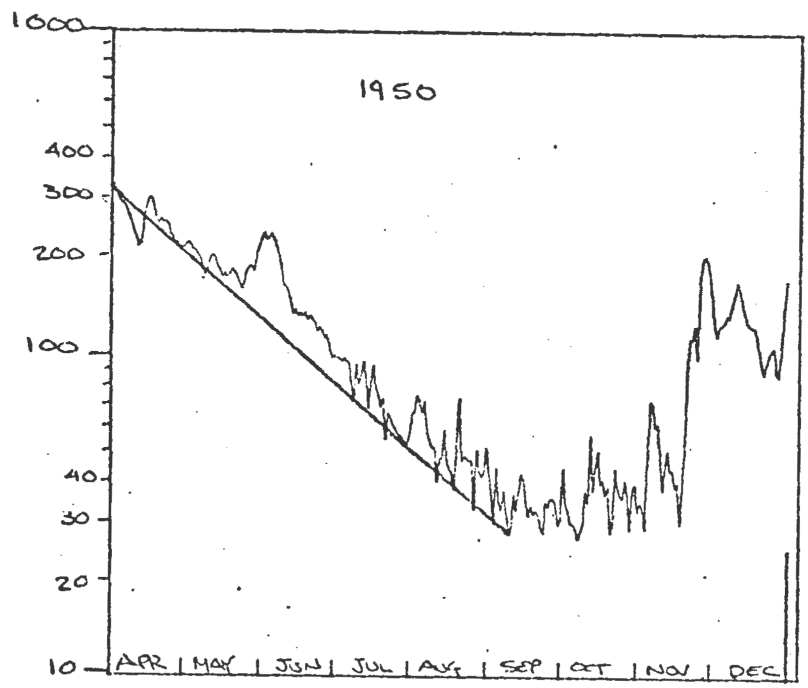
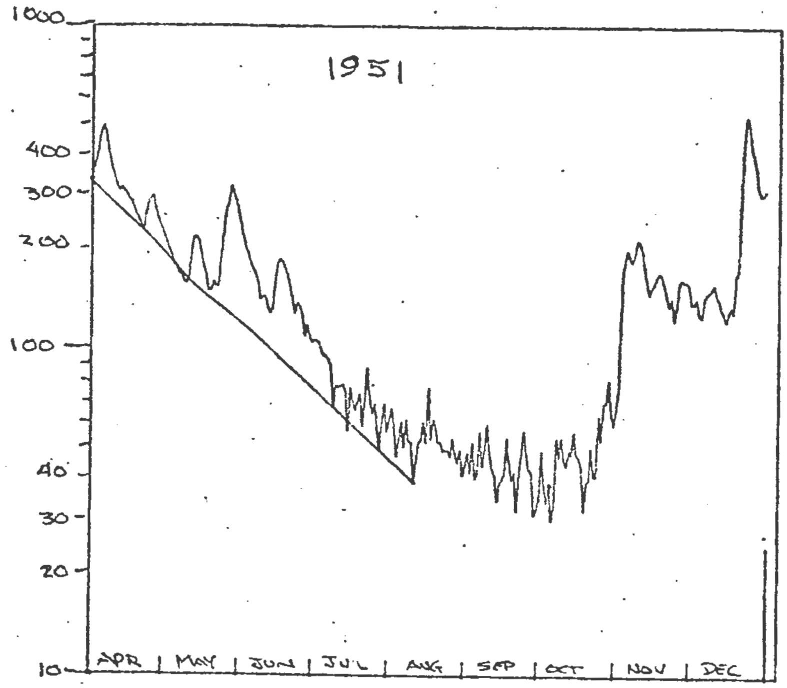


Figure 2.6 (continue)



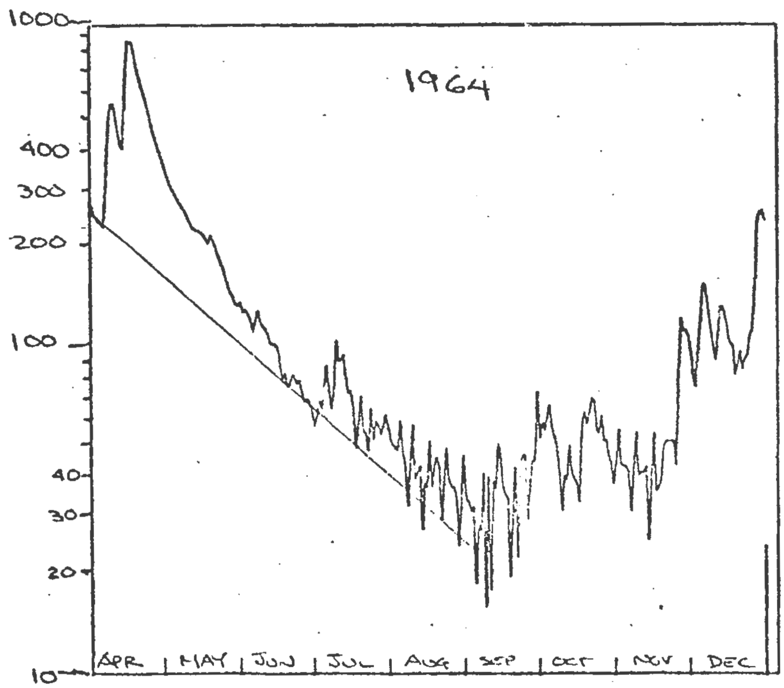
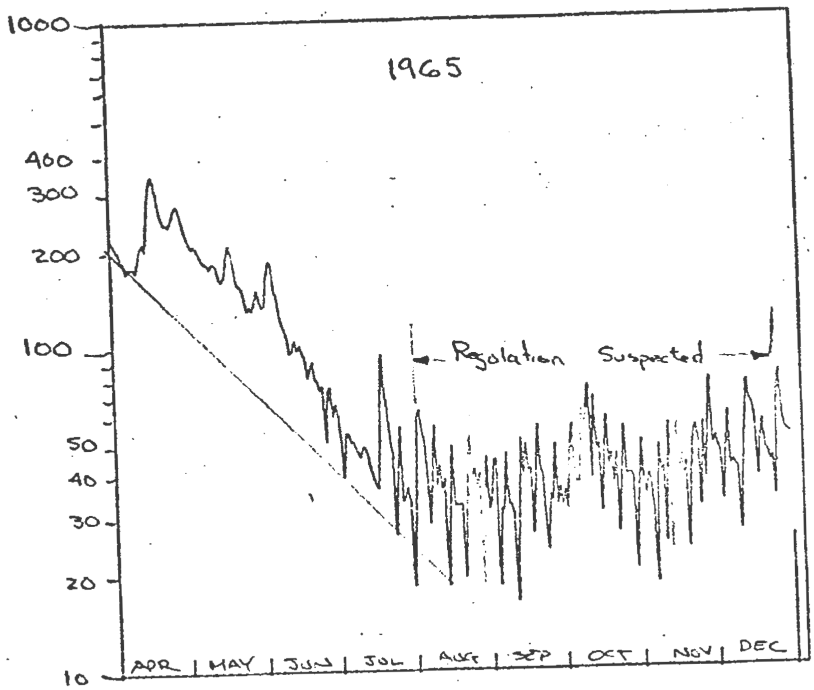


Figure 2.6 (continue)



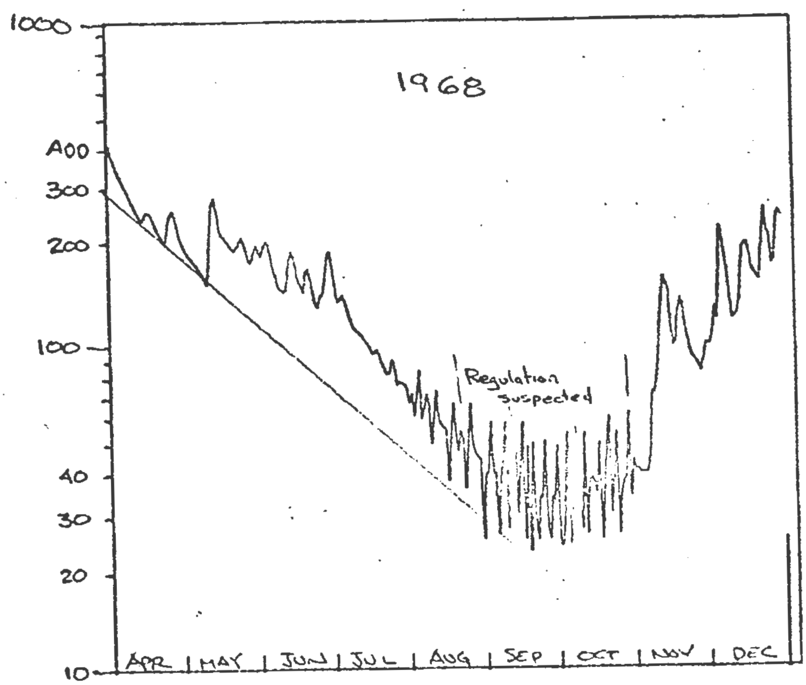


Table 2.2

Recession Constants

Year	Precipitation Apr- Dec, inch.	Air Temperature Annual Mean, ^{o}F	Baseflow Recession $Days^{-1}$
1949	23.66	50.90	$17.41 \cdot 10^{-3}$
1950	23.66	48.60	14.24
1951	30.43	50.10	14.82
1964	28.76	48.70	19.08
1965	21.94	48.00	17.34
1968	30.89	48.50	14.81

Average $K = 16.35 \cdot 10^{-3}$

2.5 Volume of Recharge

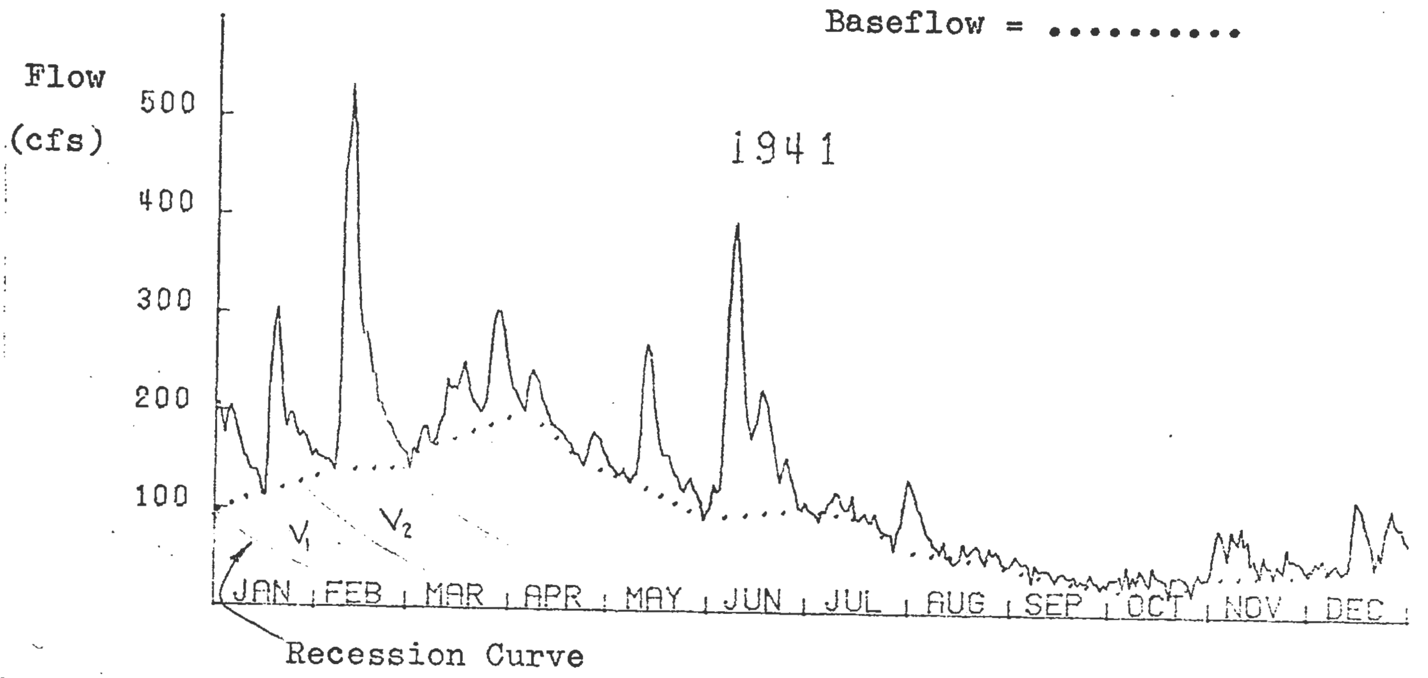
Recharge is the amount of water that infiltrates through the soil and into the aquifer. Some have related recharge with the change in mean monthly water levels measured from a series of observation wells distributed throughout the basin (13). This approach was not used since our well level data was limited.

Others (14) have related recharge to the precipitation and evapotranspiration. This can be done when streamflow records are poor or unavilable. However, it is possible to obtain estimates of recharge directly from the hydrograph, as demonstrated in Meybloom's article (7).

Figure 2.7 shows the technique used in this study to find the volume of recharge per month. The volume of recharge occurring in January, V_1, would equal the area under the baseflow line and between the recession curve beginning in January and the recession curve beginning in February. The calculation of V_1 required first summing the area between the baseflow line and the lower recession curve and then computing the area between the two recession lines beginning at the end of the month, which is equal to $Q_1/k - Q_2/k$ (k is the recession constant), as shown in Figure 2.7. There were months where the flow declined at a rate which exceeded the recession rate and a negative value for recharge was found. This is reasonable, since there is considerable evapotranspiration in the summer and fall months

Figure 2.7

Estimation of the Volume of Recharge



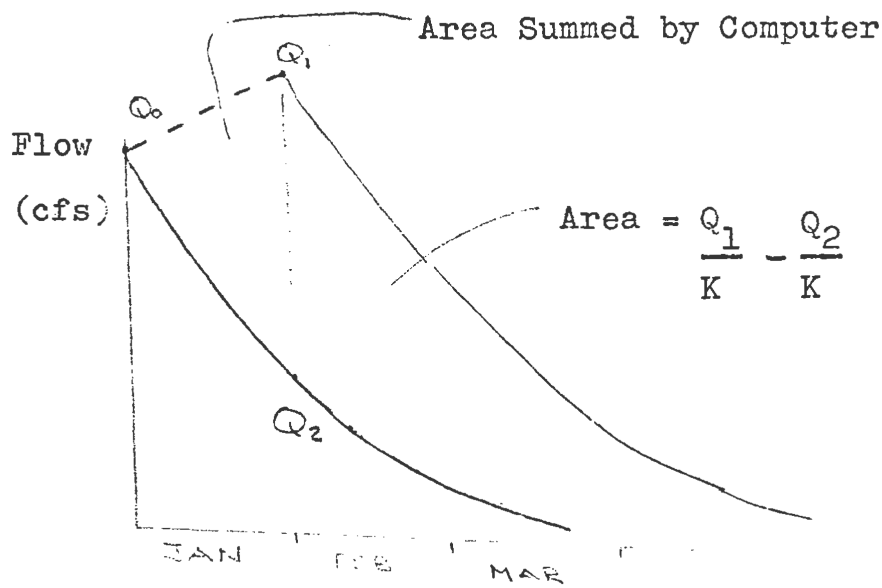


Table 3.1

RECHARGE (INCHES) FROM 1941 TO 1968

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1941	1.01	1.68	3.23	0.67	0.36	1.33	0.10	0.26	-0.02	0.58	0.58	0.86
1942	1.74	3.79	3.59	0.46	0.19	0.21	0.30	0.35	0.41	1.24	1.58	3.65
1943	2.56	3.70	1.71	2.04	1.96	-0.53	-0.04	0.14	0.23	1.11	0.70	0.25
1944	1.06	1.12	4.20	1.87	0.16	0.03	-0.00	0.25	0.63	0.41	3.48	4.17
1945	1.77	3.28	4.21	0.51	1.69	-0.01	-0.04	0.34	0.16	0.49	2.90	3.84
1946	3.33	1.85	3.10	0.33	3.10	0.03	-0.08	4.17	-0.28	0.73	0.51	0.91
1947	2.04	1.63	2.45	3.82	1.29	-0.21	0.38	-0.10	0.23	0.65	2.08	1.02
1948	2.18	4.09	7.65	1.48	6.07	-0.45	-0.94	-0.02	0.07	0.73	1.16	1.59
1949	3.92	4.43	2.01	3.36	-0.64	-0.46	-0.19	0.16	0.36	0.32	0.35	1.48
1950	1.93	2.70	3.95	2.09	0.64	0.36	-0.19	-0.00	0.23	0.35	0.79	1.97
1951	3.95	4.05	3.68	1.45	0.20	0.56	-0.34	0.18	0.27	1.06	2.59	2.74
1952	4.78	2.65	3.25	0.85	1.81	-0.29	-0.12	2.75	-0.63	0.25	0.73	1.67
1953	5.36	5.33	6.70	5.38	-1.04	-1.13	0.29	0.31	0.21	1.32	1.92	5.47
1954	1.14	1.56	2.58	3.70	1.17	-0.25	-0.02	1.51	3.14	1.44	3.79	3.03
1955	1.55	4.76	2.34	0.72	1.01	0.21	-0.00	1.28	1.68	6.26	3.44	-0.70
1956	3.34	5.67	4.85	2.70	-0.42	-0.06	0.43	0.01	0.18	0.98	1.15	1.66
1957	1.84	2.11	3.51	2.43	-0.55	-0.43	-0.09	0.28	0.16	0.61	1.10	3.16
1958	5.20	2.47	6.79	4.27	2.19	-0.62	-0.16	1.02	1.43	2.59	1.72	1.16
1959	1.57	2.73	8.00	0.86	0.33	0.38	0.89	0.14	0.37	0.85	2.19	3.08
1960	2.41	4.14	3.05	2.41	0.59	-0.40	0.23	-0.09	0.79	1.58	1.01	1.90
1961	2.46	4.05	5.34	3.33	1.73	0.10	-0.12	0.15	1.81	3.05	1.75	2.58
1962	2.81	2.78	4.78	1.04	0.02	0.76	0.05	0.26	0.69	3.86	3.81	1.77
1963	2.63	2.25	2.68	0.90	2.53	-0.32	-0.06	0.30	0.12	0.93	1.18	1.94
1964	6.54	1.28	3.27	7.17	-2.09	-0.54	0.16	-0.16	0.34	0.41	0.99	1.90
1965	1.96	3.53	2.44	1.15	1.05	-0.74	-0.23	0.20	0.35	0.24	0.30	0.54
1966	0.58	2.87	3.11	-0.01	1.82	0.55	-0.38	-0.07	0.73	0.68	1.13	1.42
1967	1.52	1.44	4.29	2.70	2.45	0.83	0.45	0.42	-0.04	0.85	3.09	3.46
1968	3.27	0.46	4.23	1.37	1.20	0.85	-0.03	-0.37	0.20	0.54	1.81	2.08

which might exceed the amount of precipitation that infiltrates to the aquifer.

2.6 Hydrological Budget

The hydrological cycle, as discussed in Section 2.1, is a continuous dynamic process and as such, it would be difficult to account for all inputs and outputs to the system on a daily basis. However, it is possible to make approximate estimates of the inputs and outputs to the basin on a monthly and yearly basis.

The Upper Pawcatuck basin can be considered a "closed" system, in that the streamflow originates in the basin and the flow out of the basin as groundwater and subsurface water is negligible. It will be assumed that the precipitation is uniformly distributed over the entire basin. The balance equation may then be written:

$$P - SR - GW - \Delta S - ET = 0$$

where P = precipitation, SR = surface runoff, GW = groundwater flow, ΔS = change in groundwater storage and ET = evapotranspiration losses. The groundwater recharge, computed in Section 2.5, would equal GW + ΔS. So the balance equation may be written:

$$P - SR - RECH - ET = 0$$

where RECH = GW + ΔS. The balance or "budget" for the basin will be computed on a volumetric basis and then converted to depth in inches over the drainage area at Wood River Junction (100 square miles).[1] By using units of inches, similar basins may be compared.

The hydrological budget was computed using the precipitation records from the Kingston station and recharge estimates found in Section 2.5. The surface flow was computed by subtracting the baseflow estimates (Section 2.2) from the total flow (Appendix A-6). The total evapotranspiration for each month was computed from the balance equation. The complete hydrological budget on a monthly basis for 1941-1968 is found in Appendix A-7. A statistical summary of the budget is shown in Table 2.4.

On a yearly basis, the change in groundwater storage may be assumed negligible, so the total annual volume of recharge would be approximately equal to the total volume of baseflow. In Table 2.5, the hydrological budget is presented using baseflow instead of recharge for purposes of comparison with a 1956 study (15). The 1956 study considered streamflow records of the Pawcatuck River from 1945 to 1954 at Wood River Junction. It can be seen that the estimates from this study are reasonably close to those in the 1956 study.

The Water Resources Report also formulated an extensive budget for the Chipuxet, Usquepaug-Queen, and the Pawcatuck Rivers for the period October 1958 to September 1959.

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[1]One inch of water over 1 square mile = 26.88 cfs - day or 2.32×10^6 cubic feet.

The baseflow component was extracted in the Report using the average well level from 16 wells in the basin. The baseflow estimates in this study are in close agreement with those in the Water Resources Report.

There is, however, considerable error in the monthly hydrological budget (in Appendix A-7). In some winter months, there is a negative loss of water, indicating a gain in water. This may be explained by melting snow and ice from the previous month or by an error involved in the estimation of surface and recharge components. The budget is obviously in error for the years 1950, 1951, and 1968 where November has the highest evapotranspiration.

It is possible to refine the techniques used in this chapter to eliminate obvious errors. Well level and air temperature data could be correlated with baseflow and evapotranspiration estimates. Possibly, the conceptual models could be used to make more exact estimates of the hydrological budget (15).

Table 2.4

Hydrological Budget

	1941	1942	1943	1944	1945	1946	1947
Precip.	35.02	48.71	31.76	39.48	42.85	39.24	41.68
Surface	5.06	7.32	4.74	6.56	5.62	7.66	5.78
Rech.	10.64	17.51	13.83	17.38	19.14	17.70	15.29
Losses	19.32	23.88	13.18	15.53	18.09	13.88	20.60

	1948	1949	1950	1951	1952	1953	1954
Precip.	42.73	35.85	35.41	42.65	44.96	61.12	53.82
Surface	7.60	4.09	3.59	5.64	6.64	11.52	10.24
Rech.	23.61	15.11	14.81	20.39	17.69	30.14	22.81
Losses	11.52	16.65	17.01	16.62	20.62	19.46	20.77

	1955	1956	1957	1958	1959	1960	1961
Precip.	50.40	45.63	34.00	53.99	44.84	46.38	53.15
Surface	8.77	6.26	5.29	9.27	5.16	5.24	10.09
Rech,	22.55	20.69	14.14	28.07	21.41	17.62	26.24
Losses	19.08	18.67	14.57	16.66	18.27	23.52	16.82

	1962	1963	1964	1965	1966	1967	1968
Precip.	49.61	42.04	40.45	30.69	38.54	50.61	47.34
Surface	9.10	5.85	5.14	4.34	3.90	6.44	8.39
Rech.	22.65	15.09	19.27	10.79	12.43	21.43	15.61
Losses	17.86	21.10	16.04	15.56	22.21	22.71	23.33

Table 2.4 Hydrological Budget (continue)

	Mean	Minimum	Maximum	Std. Deviation
Rainfall	43.68	30.69	61.12	7.396
Surface Flow	6.62	3.59	11.52	2.091
Recharge	18.72	10.642	30.14	4.903
Losses	18.34	11.52	23.88	3.245

Table 2.5

Comparison of Hydrological Budget With Other Studies

	This Study		1956 Report	
	Inches	Percent	Inches	Percent
Precipitation	43.7		48.0	
Total Runoff	26.0	60%[1]	24.0	50%
Baseflow	19.4	75%[2]	17.0	71%
Surface flow	6.6	25%	7.0	29%
Evapotran. and Other Losses	17.6	40%	24.0	50%

1) Percent of Total Rainfall

2) Percent of Total Streamflow

Comparison of Baseflow Estimates from Water Resources Report (Inches)

	1958			1959		
	Oct	Nov	Dec	Jan	Feb	Mar
This Study[1]	1.33	2.02	1.67	1.55	1.47	4.20
Water Res. Report[2]	1.48	2.13	1.85	1.49	1.27	2.75

	1959					
	Apr	May	Jun	Jul	Aug	Sep
This Study	3.41	2.14	1.33	1.11	.69	.46
Water Res. Report	3.20	2.07	1.32	1.38	.81	.50

1) Pawcatuck R. at Wood River Junction

2) Pawcatuck R. at Kenyon

III. Simulation of Streamflow

3.1 Need for Simulation

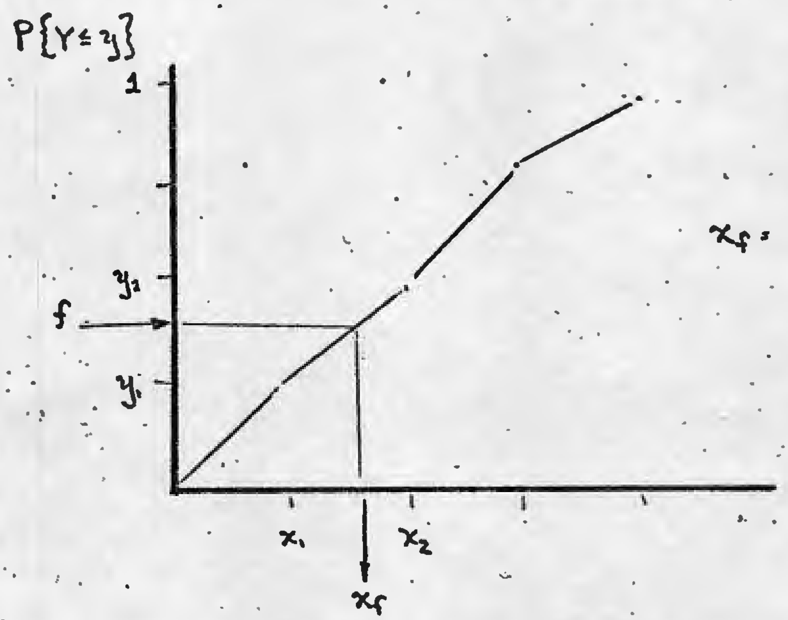
Simulation is a process which "duplicates the essence of a system or activity without attaining reality itself" (1). Engineers use simulation because it is often the only method which can effectively deal with the complexities of a large system. A simulation is also "uncheckable" in that there is no direct check of the correctness of the results. Therefore, one must be very cautious in checking the logic and coding of the program. It is possible to make spot checks of certain results and thereby demonstrate the validity of the model.

The simulation model formulated in this study will be very simple in comparison to the Lehigh Model, mentioned in Section 1.6 and others in the literature (2,3). In our model, there is a loss of water within the aquifer due to baseflow recession and a gain in water due to recharge of the aquifer. The amount of recharge is considered a random variable and will be generated on a monthly basis. The decision variable is the monthly pumping rate.

3.2 Generation of Recharge

The cumulative distribution of recharge, presented in Table 3.1 will be used to generate recharge. For each month, a random variable, f, is generated between 0 and 1. This random variable corresponds to a probability on the cumulative distribution (Figure 3.1). The inches of recharge can then be found through interpolation. The recharge is then converted to a volume (in cfs-days) of recharge. The generation of recharge from discrete distributions was adequate for our simulation. There are, however, other ways to generate random variables, such as fitting the frequency distribution to a probability distribution.

Figure 3.1 Generating Random Numbers from a Discrete Distribution



$$x_f = x_1 - \left[\frac{y_1 - f}{y_1 - y_2}\right](x_1 - x_2)$$

3.3 Pumping Program

A well placed near a stream will deplete the stream in proportion to the pumping rate. The rate at which water leaves the stream due to pumping is termed "streamflow depletion." There is a lag between the beginning of pumping and the start of streamflow depletion. Also, if the pumping

Figure 3.2 Frequency Distribution of Recharge

(28 Years)

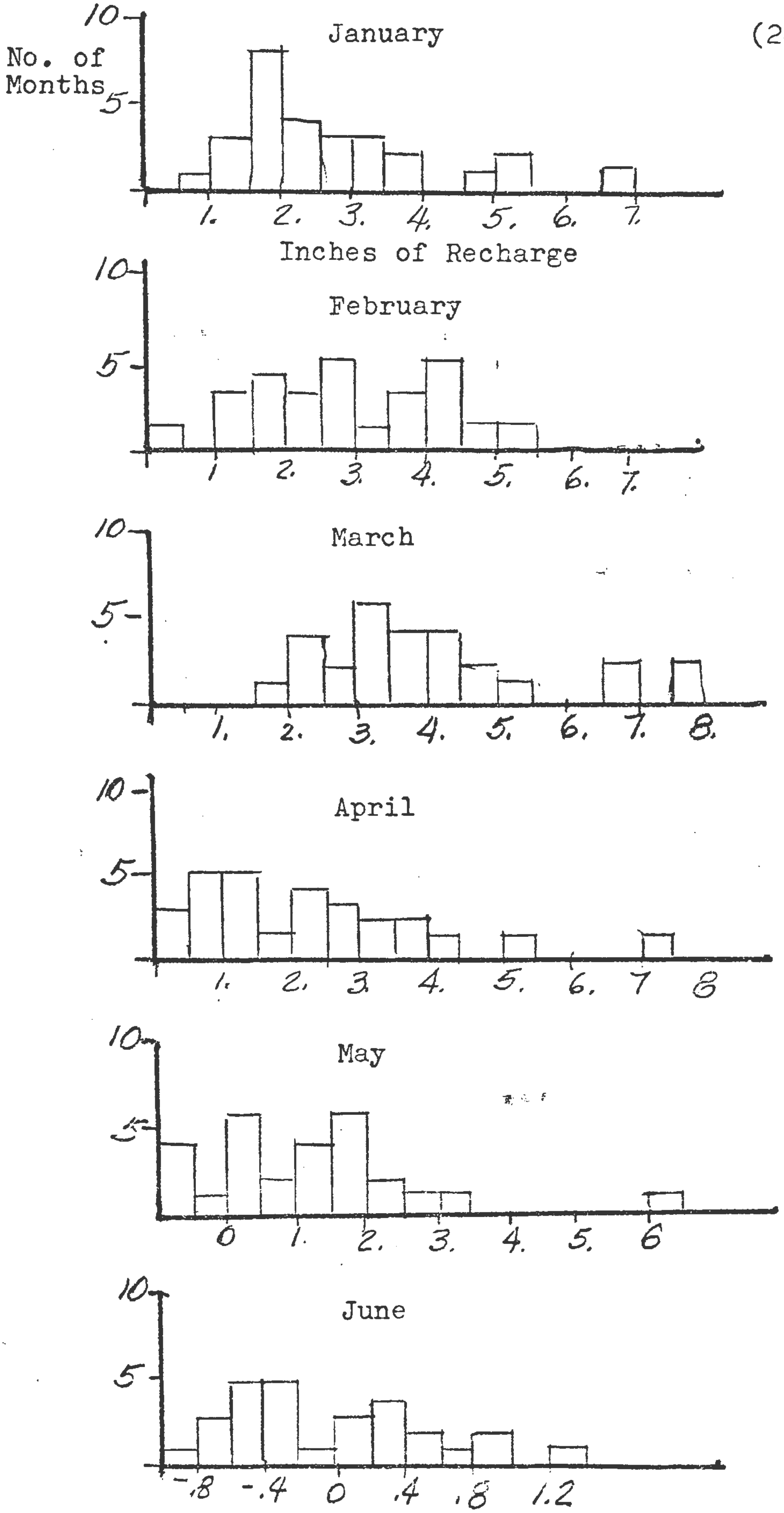


Fig 3.2 (cont.)

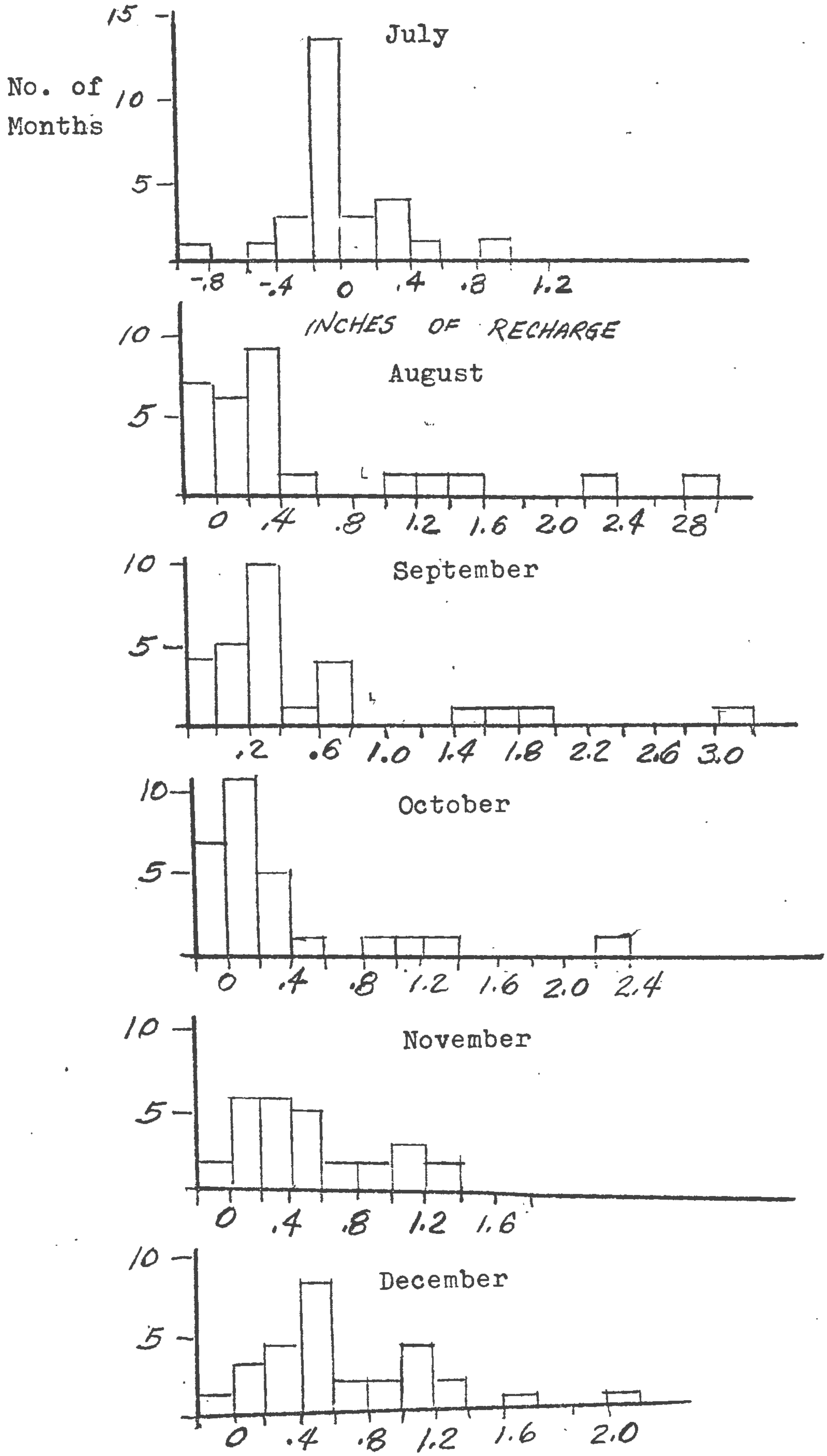

No. of Months
15
10
5
July
-.8 -.4 0 .4 .8 1.2
INCHES OF RECHARGE
10
5
August
0 .4 .8 1.2 1.6 2.0 2.4 28
10
5
September
.2 .6 1.0 1.4 1.8 2.2 2.6 3.0
10
5
October
0 .4 .8 1.2 1.6 2.0 2.4
10
5
November
0 .4 .8 1.2 1.6
10
5
December
0 .4 .8 1.2 1.6 2.0

Table 3.1 **Frequency Distribution of Recharge**

JAN		FEB		MAR		APR	
P	A	P	A	P	A	P	A
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.500	0.500	1.000	0.500	0.250	0.500	3.000	0.500
1.000	1.000	2.500	1.000	0.500	1.000	8.000	1.000
4.000	1.500	4.000	1.500	0.750	1.500	13.000	1.500
12.000	2.000	8.000	2.000	1.000	2.000	14.000	2.000
16.000	2.500	11.000	2.500	5.000	2.500	18.000	2.500
19.000	3.000	16.000	3.000	7.000	3.000	21.000	3.000
22.000	3.500	17.000	3.500	13.000	3.500	23.000	3.500
24.000	4.000	20.000	4.000	17.000	4.000	25.000	4.000
24.500	4.500	25.000	4.500	21.000	4.500	26.000	4.500
25.000	5.000	26.000	5.000	23.000	5.000	26.500	5.000
27.000	5.500	27.000	5.500	24.000	5.500	27.000	5.500
27.300	6.000	28.000	6.000	24.670	6.000	27.250	6.000
27.660	6.500	0.0	0.0	25.330	6.500	27.500	6.500
28.000	7.000	0.0	0.0	26.000	7.000	27.750	7.000
0.0	0.0	0.0	0.0	27.000	7.500	28.000	7.500
0.0	0.0	0.0	0.0	28.000	8.000	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

MAY		JUN		JUL		AUG	
P	A	P	A	P	A	P	A
0.0	-1.000	0.0	-1.000	0.0	-1.000	0.0	-0.200
4.000	-0.500	1.000	-0.800	1.000	-0.800	7.000	0.0
5.000	0.0	4.000	-0.600	1.500	-0.600	13.000	0.200
11.000	1.500	9.000	-0.400	2.000	-0.400	22.000	0.400
13.000	1.000	14.000	-0.200	5.000	-0.200	23.000	0.600
17.000	1.500	15.000	0.0	19.000	0.0	23.300	0.800
23.000	2.000	18.000	0.200	22.000	0.200	23.600	1.000
25.000	2.500	22.000	0.400	26.000	0.400	24.000	1.200
26.000	3.000	24.000	0.600	27.000	0.600	25.000	1.400
27.000	3.500	25.000	0.800	27.500	0.800	26.000	1.600
27.160	4.000	27.000	1.000	28.000	1.000	26.160	1.800
27.330	4.500	27.500	1.200	0.0	0.0	26.330	2.000
27.500	5.000	28.000	1.400	0.0	0.0	26.500	2.200
27.660	5.500	0.0	0.0	0.0	0.0	26.660	2.400
27.840	6.000	0.0	0.0	0.0	0.0	26.840	2.600
28.000	6.500	0.0	0.0	0.0	0.0	27.000	2.800
0.0	0.0	0.0	0.0	0.0	0.0	27.500	3.000
0.0	0.0	0.0	0.0	0.0	0.0	28.000	3.200

Frequency Distribution of Recharge (continue)

SEP		OCT		NOV		DEC	
P	A	P	A	P	A	P	A
0.0	-0.200	0.0	0.0	0.0	0.0	0.0	0.0
4.000	0.0	7.000	0.500	2.000	0.500	1.000	0.500
9.000	0.200	18.000	1.000	8.000	1.000	4.000	1.000
19.000	0.400	23.000	1.500	14.000	1.500	8.000	1.500
20.000	0.600	24.000	2.000	19.000	2.000	16.000	2.000
24.000	0.800	24.500	2.500	21.000	2.500	18.000	2.500
24.250	1.000	25.000	3.000	23.000	3.000	20.000	3.000
24.500	1.200	26.000	3.500	26.000	3.500	24.000	3.500
24.750	1.400	27.000	4.000	28.000	4.000	26.000	4.000
25.000	1.600	27.200	4.500	0.0	0.0	26.500	4.500
26.000	1.800	27.400	5.000	0.0	0.0	27.000	5.000
27.000	2.000	27.600	5.500	0.0	0.0	27.300	5.500
27.160	2.200	27.800	6.000	0.0	0.0	27.600	6.000
27.330	2.400	28.000	6.500	0.0	0.0	28.000	6.500
27.500	2.600	0.0	0.0	0.0	0.0	0.0	0.0
27.660	2.800	0.0	0.0	0.0	0.0	0.0	0.0
27.840	3.000	0.0	0.0	0.0	0.0	0.0	0.0
28.000	3.200	0.0	0.0	0.0	0.0	0.0	0.0

A = Flow (cfs)

P = Frequency Count of Flows Less than A was stopped or the rate changed, there would be a lag in the response of the stream depletion rate. This lagged response is proportional to the hydraulic diffusivity (T/S) and inversely proportional to the distance a well is from the stream.

Jenkins (4) used a simplified stream-aquifer system to reduce depletion effect to a mathematical model. The assumptions of his system were: (1) aquifer is isotropic, homogeneous and semi-infinite in extent; (2) stream is straight and fully penetrates the aquifer; (3) well is open and fully penetrates the aquifer; (4) water is released immediately from storage; (5) transmissivity does not change with time; (6) the temperature of the stream is assumed constant and to be the same as the groundwater; (7) pumping is steady during any period; (8) flat water table. Assumptions 1 through 4 are the conditions for perfect hydraulic conductance between the stream and aquifer. Jenkins solved this system, relating pumpage to depletion:

$$\frac{sd_t}{qw_t} = \frac{2}{\pi}\int_0^{\sqrt{\frac{SDF}{2t}}} e^{-u^2}\,du \equiv 1 - \mathrm{erf}\left[\sqrt{\frac{SDF}{4t}}\right]$$

where: SDF = stream depletion factor

$$= a^2/(T/S)$$

Notation

a = distance from stream to well (ft.)	t = Time (days)
T = Transmissivity (ft^2/day)	gw_t = Pumping rate (cfs) during t
S = Storativity	
sd_t = Streamflow depletion (cfs)	

Jenkins' model relates the ratio of the stream depletion rate to the groundwater withdrawals for a specific stream depletion factor (SDF). The SDF is directly proportional to the square of the distance from the stream to the well and inversely proportional to the hydraulic diffusivity.

The SDF at the hydraulic divide may be computed using the recession constant found in Section 2.4. Since $\frac{T}{L^2 S} = \frac{.933}{K_2} = \frac{1}{SDF}$, where L = distance from stream to hydraulic divide (ft.) and K_2 = recession constant (140.67 days as computed in Section 2.4), the SDF for a well at the hydraulic divide equals 150.67 days. This represents an upper limit of the SDF for wells in the basin. The effects of the various levels of SDF, is shown in Figure 3.3. It can be seen that at higher levels of SDF there is a slower response of the stream to pumping. Physically, this means that wells which are located far away from the stream or pump from aquifers with low hydraulic diffusivity, would take a long time for their withdrawals to affect streamflow.

When the pumping rate is stopped or changed, there is also a lag in the response. This is computed using Jenkins equation by adding or subtracting the net increase or decrease. When pumping is stopped, the time to recover is proportional to the SDF, as shown in Figure 3.4. The distance from the well to the stream in Figures 3.3 and 3.4 would be for an aquifer with T=100,000 GPD/ft^2 and S = .2.

As previously stated, Jenkins' model assumes a stream-aquifer system coupled by a perfect hydraulic conductor, which the well penetrates completely. As shown in Figure 3.5, our system departs considerably from the model. However, the model is still applicable. Since our system deviates from the assumptions in the model, a well close to the stream may produce stream depletion rates similar to what Jenkins' model would predict at a very distant well. This means that the SDF can be evaluated from pumping tests instead of using a SDF computed from the distance to the stream and the hydraulic diffusivity of an aquifer. It is also possible to obtain the appropriate SDF through numerical modelling of the basin, by treating the aquifer as a distributed system.

Figure 3.5 Comparison Between Ideal and Actual Aquifers

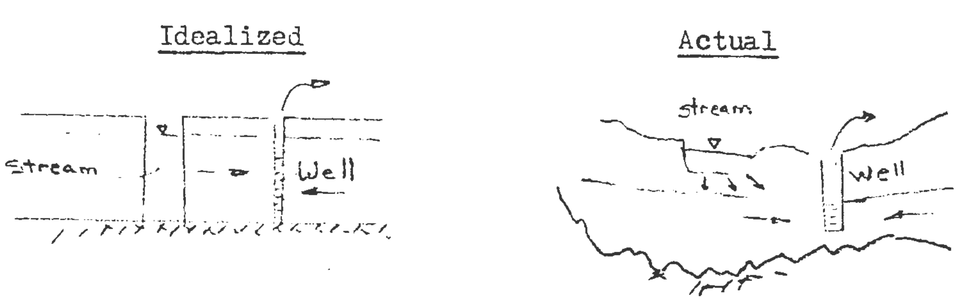


Table 3.3

Effects of Stream Depletion Factors (SDF) in Relation to the Stream Depletion

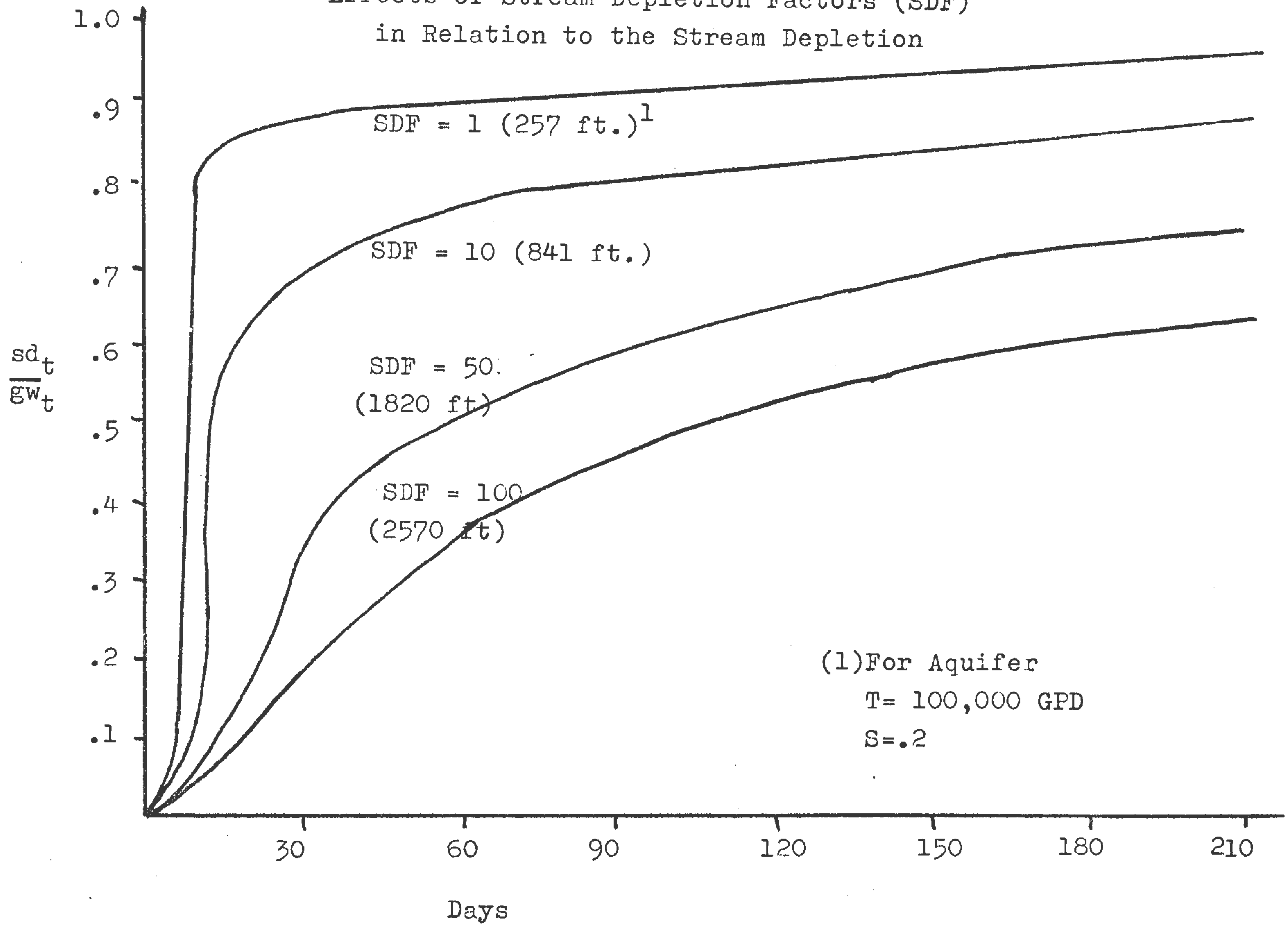
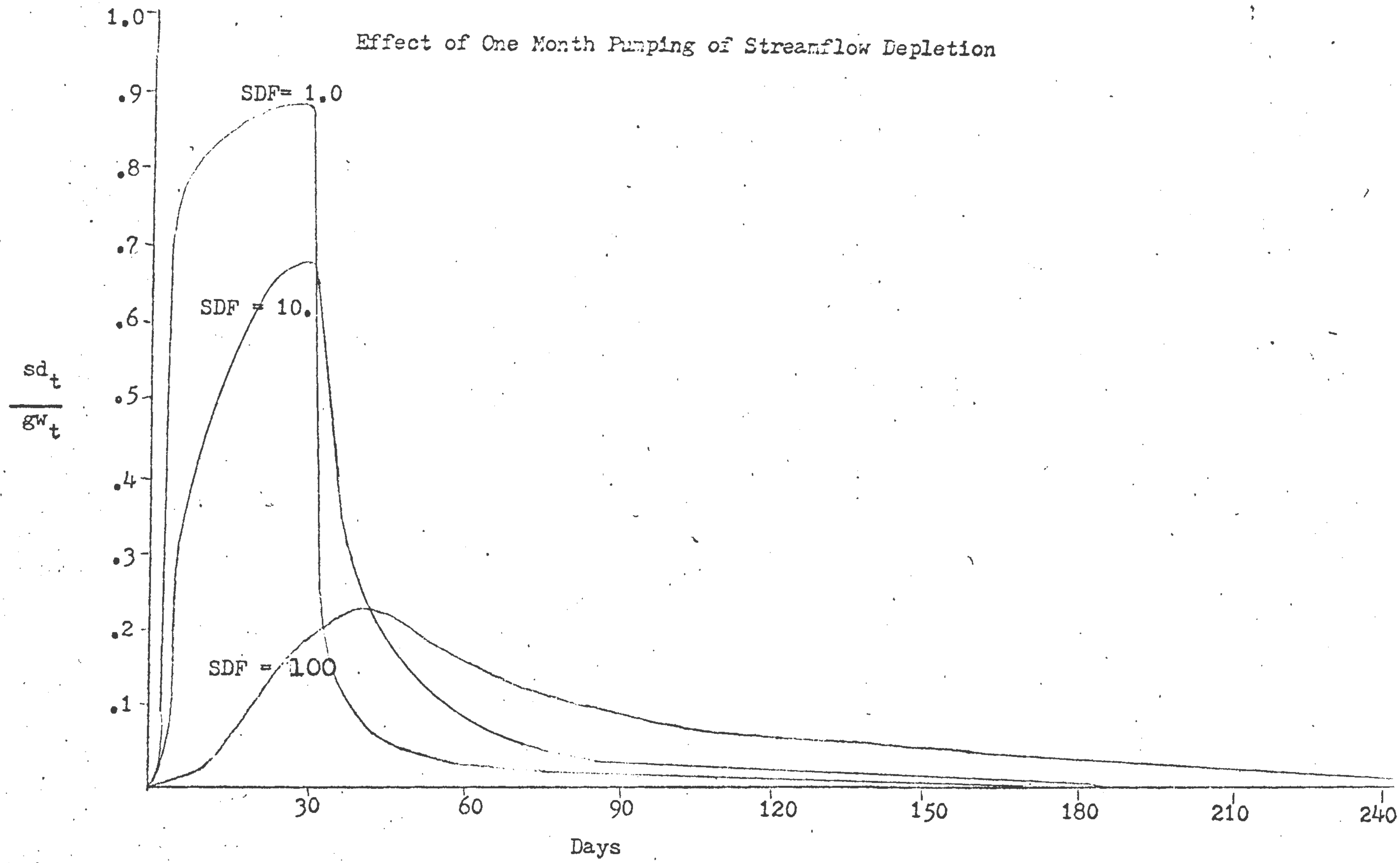


Figure 3.4


Effect of One Month Pumping of Streamflow Depletion
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
SDF= 1.0
SDF = 10.
SDF = 100
30
60
90
120
150
180
210
240
Days

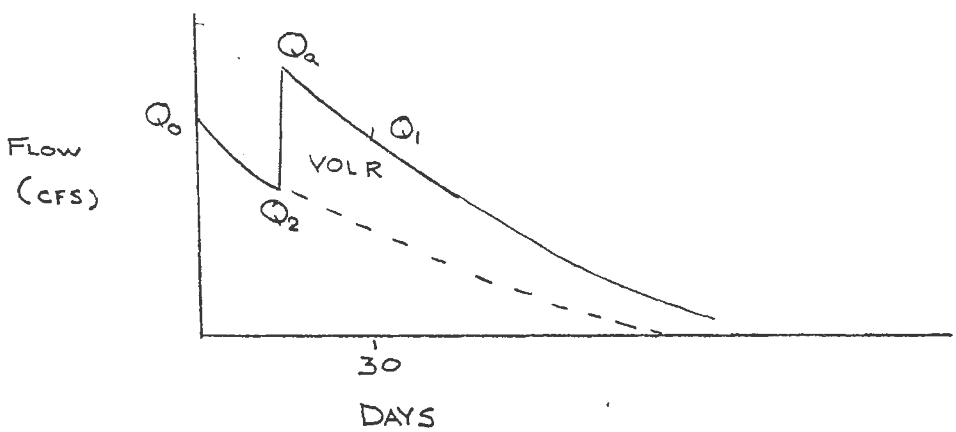
Jenkins' model assumes that eventually all the water held in storage will flow into the stream. The further the well is from the stream, the more delayed the response of the stream to pumping. Jenkins model assumes that even for very distant wells, there is still some depletion effect. In reality, the depletion effects of pumping are within a finite radius of the well. The aquifer is also finite in capacity, so in winter months the recharge may be sufficient to make up for water withdrawn during summer months and fill the aquifer to capacity. As a result, the more distant wells may have no effect on the stream. It is possible that the model could be modified to account for these discrepancies.

The model is based on daily pumping rates. However, it would be very inefficient to simulate every day. Instead, the month is divided into 10 day intervals. The monthly depletion is found through averaging these evaluations.

3.4 Generation of Streamflow

In each month, there is a gain in baseflow due to recharge and a loss due to streamflow depletion and recession. The simulation part of the computer program is shown in Figure 3.6. The average stream depletion for each month is computed earlier in the program and is stored in array D1. The program calls subroutine RECH to obtain the volume of recharge for each month, V. The stream depletion, D, is multiplied by 30 days to convert it to a monthly volume (cfs-days) and then subtracted from V to obtain the total volume change, VOLR. The volume change will be added in the middle of the month (15^{th} day). The streamflow recedes using the recession constant to Q_2 (Figure 3.6) then the net recharge is added, bringing it up to Q_a and the streamflow recedes again to the end of the month, Q_1. The average monthly flow is found by averaging Qo and Q_1. The method used to estimate recharge is different from the method to generate recharge, but the results should be almost exactly the same.

Figure 3.6 Generation of Streamflow



$$VOLR = \frac{Q_a}{K} - \frac{Q_2}{K}$$

$$Q_a = VOLR * K + Q_2$$

$$Q_1 = Q_a * e^{-15K}$$

$$Q_{ave} = (Q_1 + Q_2)/2.$$

$$Q_o = Q_1$$

Program Statements generate monthly flows from recharge

3.5 Validation of Simulation Model

As stated in Section 3.1, there is no direct check of the simulation results. However, it is expected that the simulation of streamflow with no pumping would give flows whose distributions are close to the historical record. The distribution of flows is shown in Figure 3.7. The Kolmorgov-Smirnov test was used (5) to test the hypothesis that the two samples are from the same distribution. The test is a non-parametric test, whose test statistic is the maximum difference between the cumulative distribution of the two samples. The critical region is $1.36 \sqrt{(m+n)/m*n}$ where m and n are the sample sizes. The maximum difference is .053 and the critical region with m = 336, n = 240, is .115. Since the test statistic is well within the critical region, the hypothesis that the two samples are from the same distribution is accepted.

The Kolmorgov test does not validate the program; it simply states there is not sufficient evidence to conclude that the distributions are not the same. The simulated and historical distributions in Figure 3.7 are very similar, but not exactly identical. The simulated record was consistently higher by 5-10 cfs (obtained by shifting the curves until they matched), until 160-170 cfs, where they became practically identical. This discrepancy does warrant further study of the simulation model. It is possible that the simulation estimates are closer to the true distribution of flows. The period of the historical flows (1941-1968) had an average rainfall of 43.8" while the long term average from 1889 is 48.3".

The random component of the simulation is recharge. It was assumed that the recharge is an independent random variable, but in reality there is some correlation between successive months. This would be a possible source of the discrepancy between our model and the historical values.

The simulation model was run with different initial random numbers (called the "seed" of the generator) to test for the variations within the model. The results (Table 3.8) show that there is a considerable range in the low end of the frequency table. The 10% flow or the flow which 10% of the monthly mean flows are equal to or less than, varies from 41.43 - 46.60 cfs, while the 20% flow varies from 57.18 - 68.85 cfs. One possible way to reduce this error is to run the program for a longer period. However, the computer time increases exponentially with longer simulation runs, so our simulation period will be limited to relatively short periods (20 years seemed adequate).

Figure 3.7

<u>Comparison Between Historical and</u>

<u>Simulated Streamflows</u>

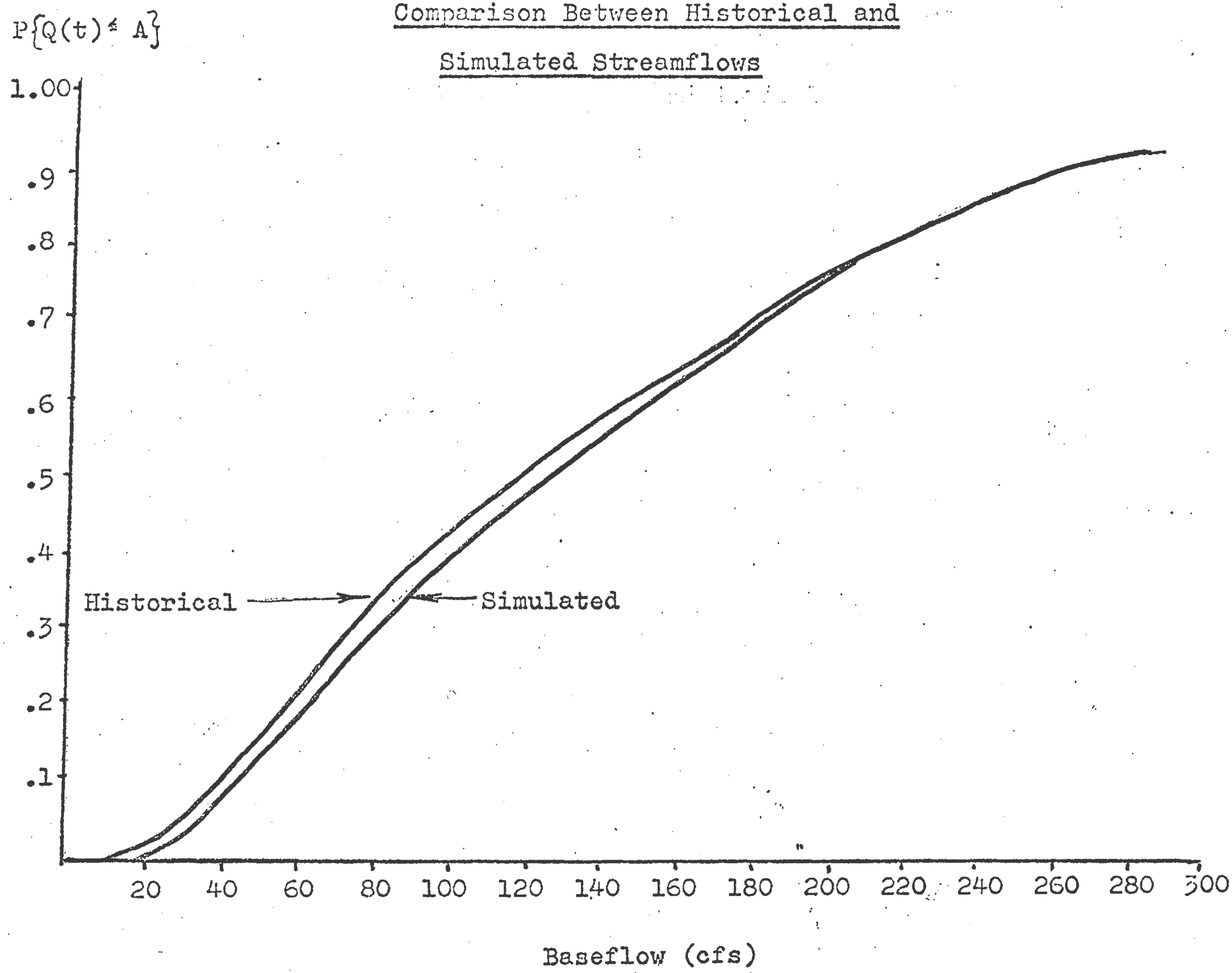


Table 3.2

Use of Different Random Numbers in Simulation

100 Year Simulation

Less Than:	(1)	(2)	(3)	(4)	(5)	Mean
0 Cfs	.002	.0	.0	.001	.001	.0008
10	.002	.004	.0	.002	.007	.003
20	.008	.012	.007	.011	.017	.011
30	.046	.034	.022	.036	.034	.034
40	.091	.070	.064	.092	.074	.078
50	.154	.125	.118	.144	.127	.134
60	.218	.188	.169	.205	.194	.225
70	.258	.237	.219	.257	.246	.243

$P(Q(t) \leq A)$	Flows (cfs) (1)	(2)	(3)	(4)	(5)	Mean
.10	41.43	45.45	46.60	41.50	44.90	43.90
.20	57.18	62.44	66.20	59.20	66.85	62.77

Random Numbers

(1) 25

(2) 375567695

(3) 1974000271

(4) -100241457

(5) -1109923249

IV. Optimal Schedule of Groundwater Withdrawals

4.1 Optimization Model

The simulation program can be used simply as a forecasting tool, in that, given a schedule of monthly withdrawals, the program can predict the risk of low flow. This would be helpful to an administrator in planning to meet current demands.

In planning for the future, it would be necessary to know the maximum withdrawal possible without excessive risk of low flows. In the optimization model, the monthly pumping rates gw(t), t = 1, 2, ... 12, become decision variables. The withdrawals from groundwater will produce a certain lowering of streamflow or stream depletion, sd(t), during time period t. The constraint to the model is that the streamflow must not be excessively low for some month. This could be expressed: $Q(t) - sd(t) \leq A$. However, streamflow is a random variable and as such a chance constraint is appropriate:

$$P\left\{Q(t) - sd(t) \leq A\right\} \leq \alpha \quad \text{for all } t$$

or the probability that the mean monthly streamflow minus stream depletion is less than A must be less than α. To complete the model, the stream depletion rate is a function of the present and all previous pumping rates, as explained in Section 3.3. This would be expressed:

$$sd(t) = f(gw(t), gw(t-1), \ldots)$$

The optimization problem involves a non-linear constraint and a chance constraint and as such is not easily solved. Nieswand showed (1) that an analytical solution is possible for this model and a conjunctive model using chance-constraint linear programming. An alternative to an analytical solution is to search for an optimal pumping schedule through a selective trial and error technique. The search technique is very useful when the model may be changed often to relate different situations.

4.2 Methods of Evaluating the Risk of Low Flow

The assessment of low flow in a stream is complicated by the fact that the measure of low flow appears to be multi-dimensional. One is interested in the magnitude, duration and the expected recurrence interval of a low flow. However, these dimensions are interrelated, in that, one would expect an extremely low flow to have a long duration and a long recurrence interval.

One measure of low flow is the flow duration curve. This is the cumulative distribution of the annual, monthly and daily mean flows. Since the supply of groundwater is of importance, the baseflow duration curve was found (Figure 4.1 , values in Appendix) for the annual and monthly baseflow. From this curve, the 10 and 20 year annual mean baseflow were found to be 94 cfs and 86.2 cfs, respectively. This is in close agreement with values estimated in the Water Resources

Report's annual mean flow duration curve (Figure 1.5), which by assuming 70% of the total flow is baseflow, would yield 97 cfs and 86.2 cfs, respectively for the 10 and 20 year annual flows. The cumulative distribution of daily total flows is also shown in Figure 4.1 (values for curve in Appendix). The curve is in close agreement with values from the Water Resource Report curve (Figure 13 in the Report). The difference between these estimates is due to the different time periods used (our study used 1941-1968; Water Resources Report 1942-1962) and human error in reading the graph in the Report.

Another measure of low flow, which has been used in water quality studies, is the "7 day minimum flow" (2). This is the lowest value in a running average of 7 consecutive daily flows in a year. The 1 and 30 day minimum flows, which are also useful statistics, are similarly defined. The 1, 7 and 30 day minimum flows are shown in Table 4.1 and the cumulative distribution in Figure 4.2. These were obtained from the historical record of total flow. Since the simulation program was designed to generate mean monthly baseflows, it would not be possible to obtain 1, 7 and 30 day minimum flows directly from the simulation output. However, a comparable statistic which can be generated in the simulation program is the minimum monthly baseflow for each year. The minimum baseflow was obtained from the monthly mean baseflows (Table 2.1) and these values are shown in

Figure 4.1

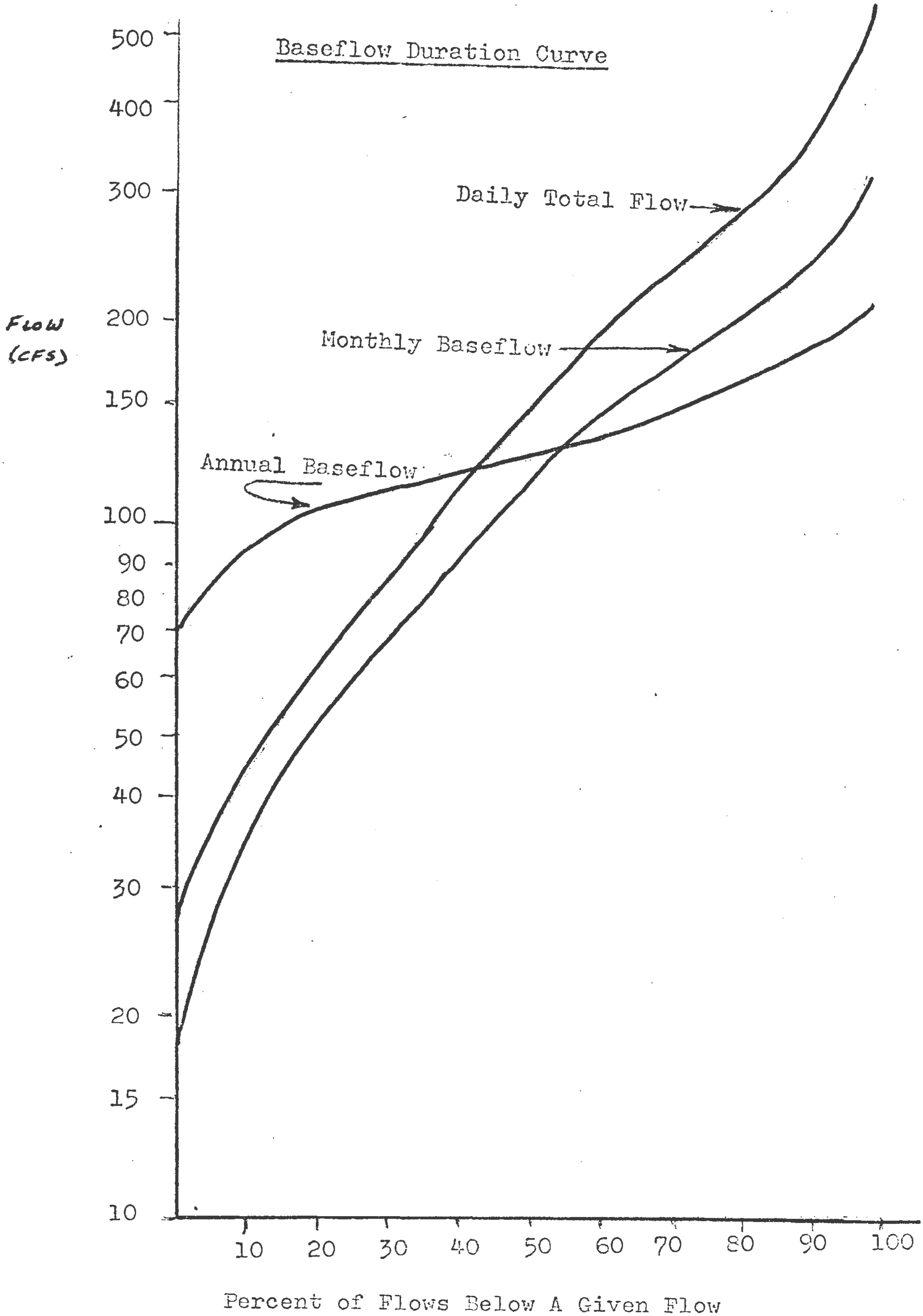

Baseflow Duration Curve
Daily Total Flow
Monthly Baseflow
Annual Baseflow
FLOW (CFS)
500
400
300
200
150
100
90
80
70
60
50
40
30
20
15
10
10
20
30
40
50
60
70
80
90
100
Percent of Flows Below A Given Flow

Table 4.2 for each year. The cumulative distribution of these flows is given in Figure 4.2. It can be seen from Figure 4.2 that the minimum baseflow is between the 1 and 7 day minimum flows. This is reasonable since the baseflow was separated by interpolation between the minimum daily flows in a 20 day interval. During periods of low flow, generally September to November, the baseflow changes very little and is near the minimum of the year. The minimum monthly baseflow can be related to the 1,7 and 30 day minimum flows by use of the ratios, as shown in Table 4.3. The ratio of minimum monthly baseflow to the 1, 7 and 30 day minimum flows is 1.23 , .945 and .815, respectively.

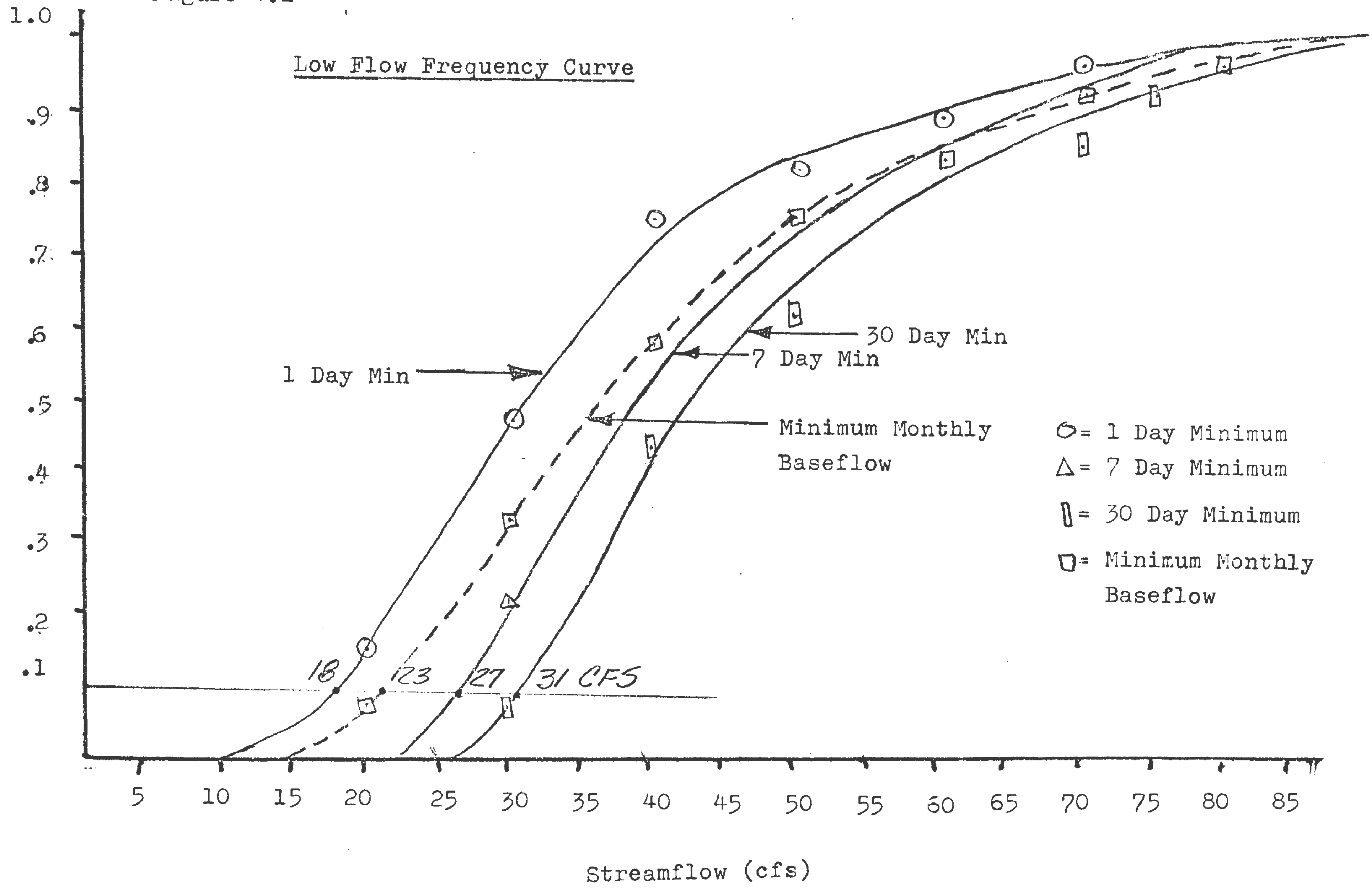
Measures of low flow are closely related. The years which have the lowest 1 day minimum would also be likely to have a very low 7 day minimum and that year would also contribute more months to the tail of the baseflow duration curve. Figure 4.3 shows the years 1949, 1957, 1964, 1965 and 1968 are common to the 1 and 7 day minimum flows below a probability of .2 and these years also have months with mean monthly baseflows below the .05 limit. Also, it was found that these years had rainfall from April to December of less than 31" (within the 25% percentile). The profile of a 10 year low flow (expected recurrence once in ten years) is shown in Figure 4.3.

Table 4.1

Low Flow Frequency

	1 Day Annual Minimum			7 Day Minimum			30 Day Minimum		
Year	Month	Day	Flow (cfs)	Month	Day	Flow	Month	Day	Flow
1941	10	19	20.00	9	26	34.14	9	20	37.43
1942	10	17	39.00	10	11	48.29	9	18	52.60
1943	9	26	29.00	10	5	32.00	9	12	34.60
1944	9	12	22.00	9	6	26.43	8	15	32.40
1945	10	13	33.00	10	18	34.43	9	25	36.27
1946	12	15	59.00	12	11	61.43	11	21	68.83
1947	10	11	15.00	10	9	28.71	9	30	35.13
1948	10	7	32.00	10	4	34.14	9	24	36.77
1949	8	28	21.00	10	19	25.29	9	27	29.80
1950	10	8	27.00	10	4	29.57	9	10	33.27
1951	10	6	30.00	9	30	36.71	9	8	41.87
1952	7	26	37.00	7	26	44.14	10	24	52.30
1953	10	3	36.00	10	17	46.14	9	26	49.60
1954	7	31	62.00	7	16	73.00	7	10	77.33
1955	8	7	50.00	8	3	61.14	7	13	72.17
1956	10	5	37.00	9	9	50.00	9	7	53.47
1957	10	5	19.00	9	30	25.57	9	19	28.00
1958	8	9	71.00	8	9	85.71	7	26	108.23
1959	9	30	44.00	9	30	44.57	9	24	55.97
1960	9	3	27.00	9	3	34.00	8	13	44.43
1961	8	19	50.00	8	14	60.43	7	23	81.37
1962	9	22	34.00	9	11	43.86	8	28	52.33
1963	10	26	26.00	10	22	40.14	8	30	46.67
1964	9	9	16.00	9	5	27.57	8	25	34.53
1965	9	11	16.00	9	6	30.71	8	14	33.27
1966	9	3	20.00	8	23	36.71	8	5	40.47
1967	9	23	45.00	9	22	55.00	8	31	67.43
1968	9	19	23.00	9	26	30.86	9	17	36.37

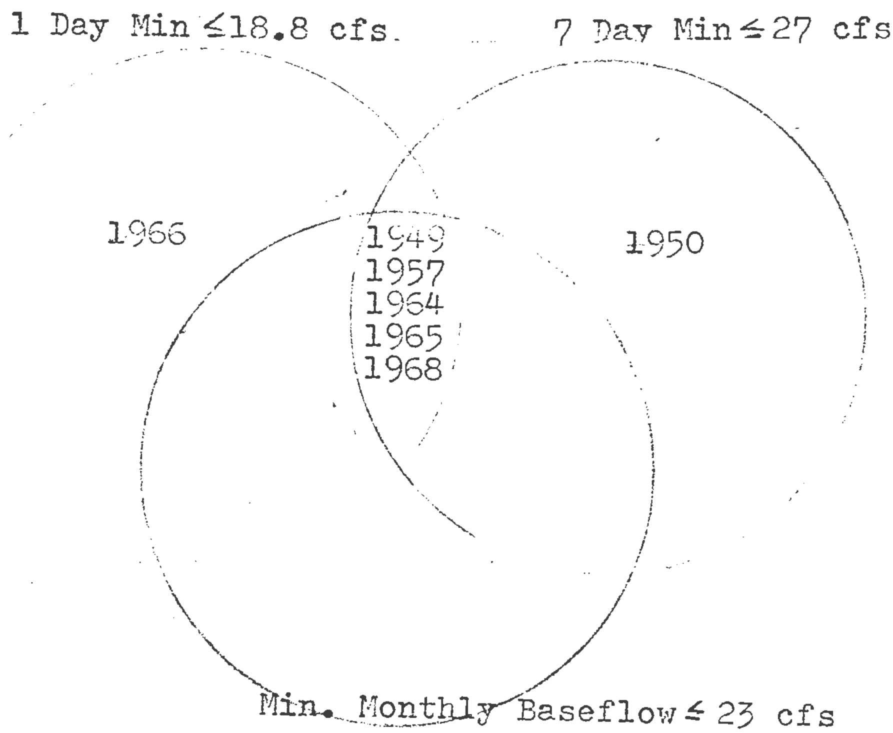
Figure 4.2



Low Flow Frequency Curve
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1 Day Min
30 Day Min
7 Day Min
Minimum Monthly Baseflow
= 1 Day Minimum
= 7 Day Minimum
= 30 Day Minimum
= Minimum Monthly Baseflow
18
23
27
31 CFS
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
Streamflow (cfs)

Figure 4.3

Relationship Between 1, 7 and 30 Day Minimum Flows and Minimum Monthly Baseflow



Profile of a 10 Year Low Flow

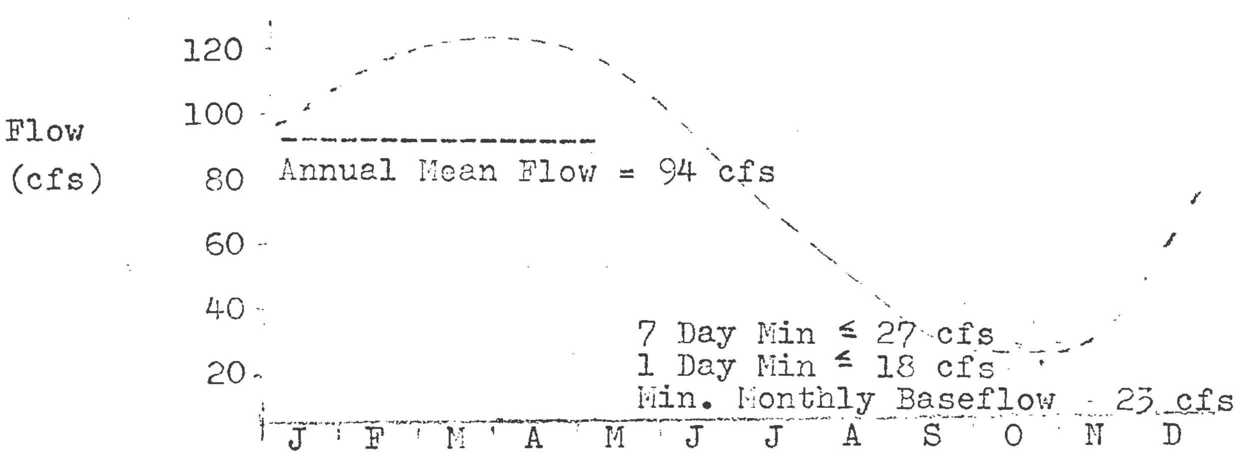


Table 4.2

Comparison of Minimum Monthly Baseflow to 1, 7, 30 Day Minimum Flows

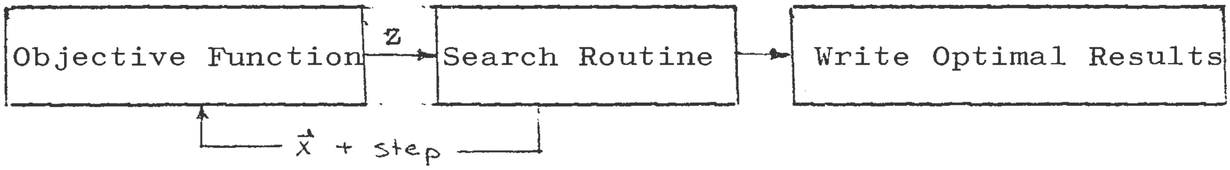
Year	Minimum Monthly Baseflow	Ratio: Min. Mon. Baseflow/ 1, 7, 30 Day Min. 1 Day	7 Day	30 Day
1941	26.70	1.34	.782	.713
1942	41.90	1.07	.868	.797
1943	31.90	1.10	.997	.922
1944	30.90	1.40	1.170	.954
1945	34.30	1.04	1.004	.946
1946	65.40	1.11	1.065	.950
1947	28.40	1.89	.989	.808
1948	38.80	1.21	1.136	1.055
1949	23.30	1.11	.921	.782
1950	27.70	1.03	.937	.833
1951	33.60	1.12	.915	.803
1952	41.10	1.17	.931	.786
1953	40.30	1.12	.873	.812
1954	72.30	1.16	.990	.935
1955	65.80	1.32	1.076	.914
1956	43.00	1.16	.860	.804
1957	21.80	1.14	.853	.779
1958	79.10	1.11	.930	.730
1959	52.90	1.20	1.187	.945
1960	40.50	1.50	1.191	.911
1961	60.50	1.21	1.001	.744
1962	41.00	1.21	.935	.783
1963	31.50	1.21	.785	.675
1964	29.80	1.86	1.081	.863
1965	18.00	1.13	.586	.541
1966	26.30	1.31	.716	.650
1967	50.20	1.12	.913	.744
1968	23.80	1.03	.771	.654
	Mean Ratio:	1.23	.945	.815
	Std Dev. :	.214	.144	.115

4.3 Search Techniques

A search technique is like mountain climbing in the dark, only in this case, the mountain climber (i.e., the computer) has a very short memory of where it's been. Methods are classified as derivative-free and gradient methods. The gradient method requires the function and its derivative, while the derivative-free methods require only function evaluations. In general, one would expect gradient methods to be more efficient due to the added information provided. The gradients may be evaluated numerically, however this would cause some problem as the gradients near the vicinity of the optimum become extremely small (3). We will use derivative-free multivariant search techniques to solve the model.

The search program finds the optimal value by evaluating the objective function at different points until an optimum is found. The decision variables are incremented or decreased a certain "step" and the change inthe objective function is measured. If no improvement is found by moving in any direction, the program assumes that it has found the optimum point. If the surface is not unimodel, it is possible the search will end at a local optimal solution. There is no way to guarantee the success of a search routine. If there is some doubt that the optimal solution is true, the routine could be run again using a different initial point and then one could see if the results are the same.

Figure 4.4. General Search Technique



The flowchart of the optimization program is shown in Figure 4.5. The search program varies the decision variables in the objective function until an optimal solution is found. Our objective function must be changed to force the search program into a feasible region. A penalty is assessed for violation of a constraint and subtracted from the objective function. The model becomes:

$$\text{Maximize} \quad Z = \sum_t gw_t - \text{penalty}$$

$$\text{where:} \quad \text{penalty} = 100 * (P\ Q'(t) \leq A)$$

$$\text{if } P\left\{Q'(t) \leq A\right\} \geq \alpha$$

$$= 0; \text{ Otherwise}$$

<u>Notation</u>

$Q'(t)$ = Monthly baseflow (cfs) after pumping

A = Limit assigned in program (cfs)

α = Limit assigned in program (cfs)

$P\left\{Q'(t) \leq A\right\}$ = Estimated in Simulation Program

An estimate of $P\left\{Q'(t) \leq A\right\}$ is found in the simulation program by counting the monthly flows less than A and dividing by the total number of months simulated. The form of the penalty function is somewhat arbitrary and alternative forms are discussed in the literature (4). If the penalty is too small with respect to the violation, then convergence toward an optimum would be too slow. If the penalty is too high, there is the possibility the search program would increase its step size and skip over the maximum point. The risk of low flow could be put in terms of damage costs, and the penalty function could be based on economic loss.

4.4 Search Program

Many search techniques are available in the literature (3). In this study, a derivative-free search routine, SDRMIN, was used (5). The routine would be regarded as a slow but safe routine in that it advances to an optimum solution slowly in comparison to gradient methods. It would be considered a safe method in that it continually reports its progress, making debugging and restarts easier.

The flowcharts of the overall search routine and the exploratory section are shown in Figure 4.6 and 4.7. Initially, the user must supply: (1) the initial values of the decision variables, X(i), i = 1,... N; (2) the number of decision variables, N; (3) the minimum and maximum values for the decision variables, XMIN(i) and XMAX(i); (4) the maximum number of objective function evaluations, MAXTRY. The initial step size is assumed to be 10% of the allowable range, unless specified. The initial evaluation of the

Figure 4.5

Optimization Program Flowchart

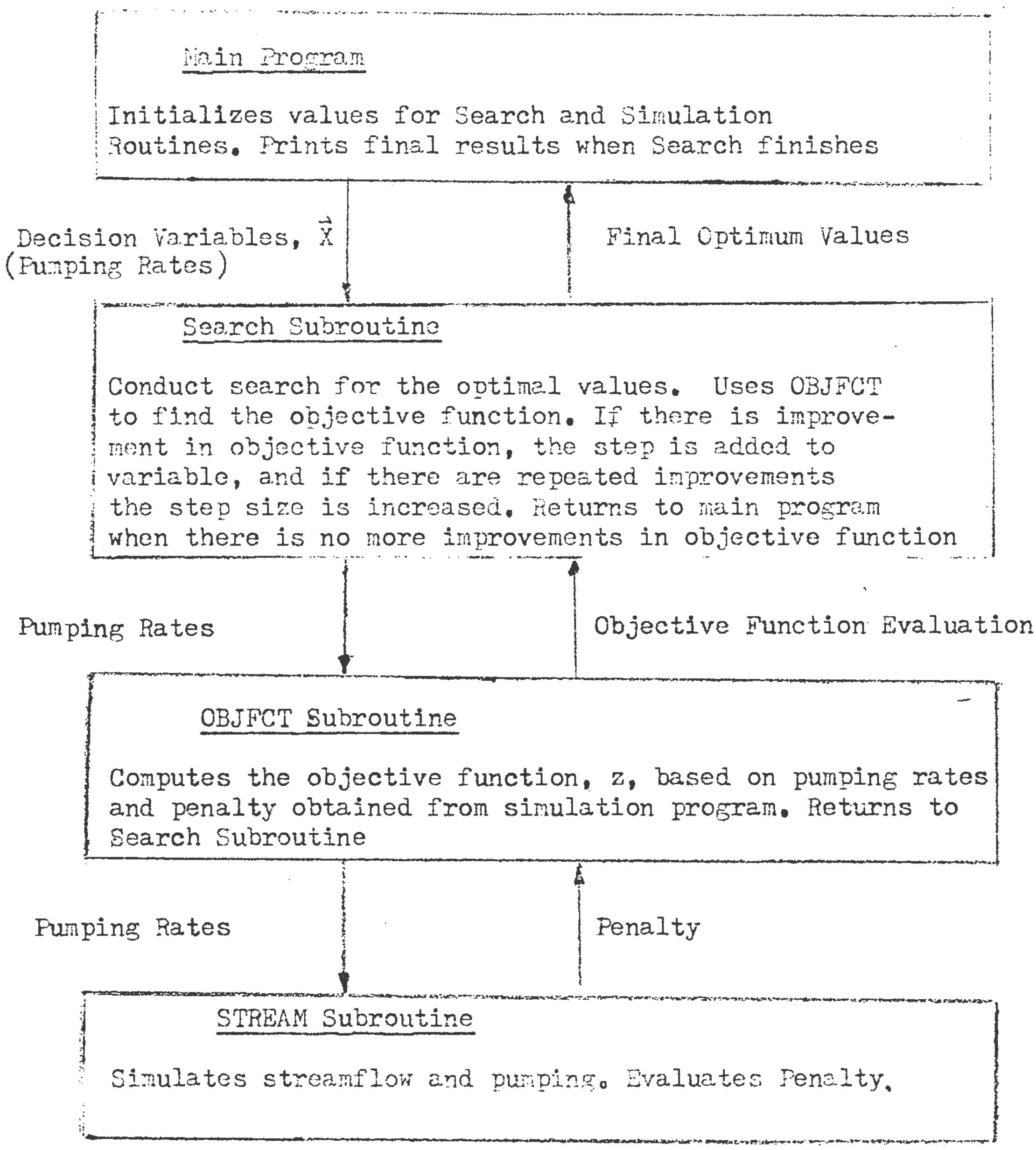


Figure 4.6

Flow Chart of Subroutine SDRMIN

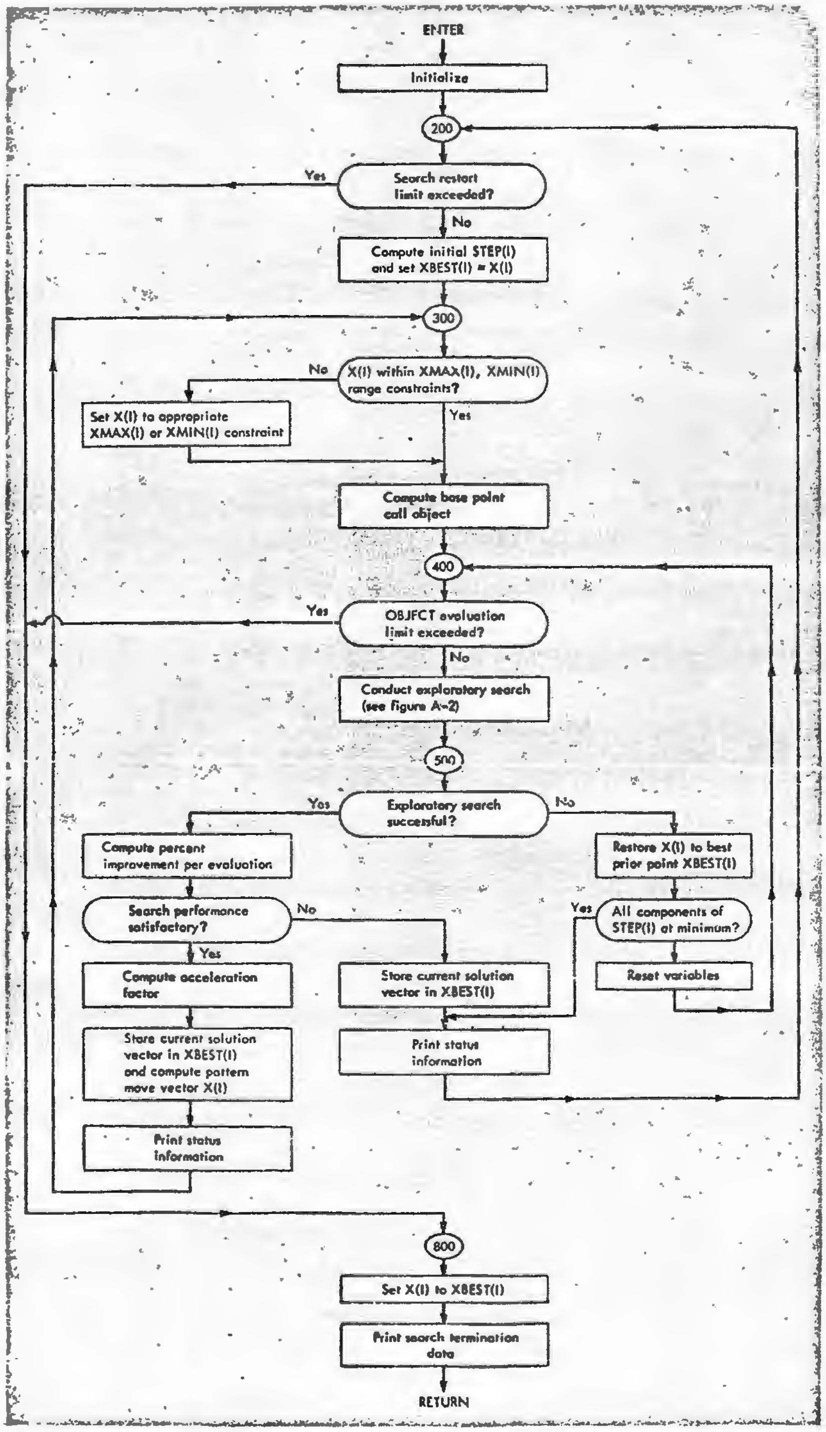
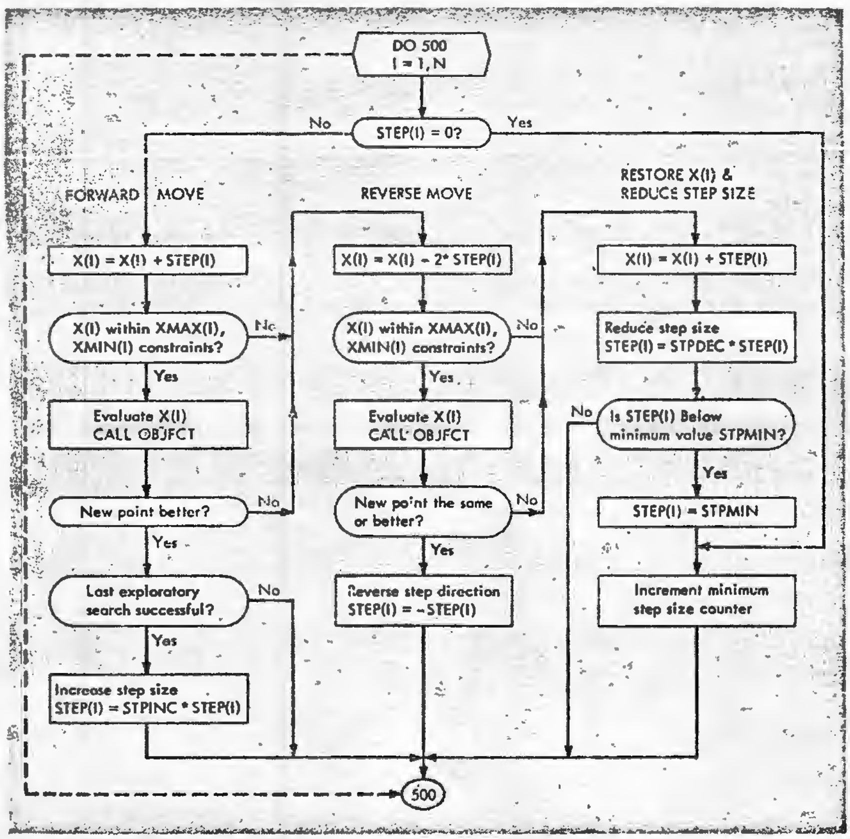


Figure 4.7

Flow Chart of Exploratory Search Section of Subroutine SDRMIN



objective function establishes a "base point" by which improvements can be judged. The decision variables are incremented by a "step" one at a time and then evaluated in the objective function. If there is any improvement, the new value is retained. However, if there is no improvement, a "reverse move" is tried (Figure 4.7), where the step is substracted from the variable. If this fails to make any improvement, the variable is restored to its original value. If there is no improvement in any variable, this would indicate an optimal point. To insure that this point is an optimum, the program is restarted at another point to see if the same optimum is found. In order to accelerate the search process, the step size will be incremented when there is an improvement resulting from two consecutive moves.

There is no guarantee that the search program will find an optimal solution,and not a local maximum. The bounds may be placed so the optimal solution is above the maximum allowed. Time is also a critical factor, as each evaluation of the objective function means the simulation program must be run.

4.5 Optimum Pumping Policies

Optimum is used here only with respect to the model and while efforts were made to have the model reflect reality, nevertheless, the model greatly simplifies the complexities of the stream-aquifer system. Also, the "optimum policies" may be rather difficult to implement. Any withdrawal plan must be consistent with existing pumping capacities, treatment, storage and distribution system and the demand for water. The primary objective in this study is to show the maximum possible development given a certain level of risk of stream depletion.

Constant Pumping

Under constant pumping, the rate at which water is withdrawn will eventually become equal to the rate at which the stream is depleted using Jenkins' model. Therefore, if 10 cfs is continuously withdrawn, the daily streamflow would be lowered by 10 cfs. The monthly and yearly mean baseflow and the minimum monthly baseflows would also be lowered by 10 cfs. This is the equivalent of shifting the minimum flow curve (Figure 4.2) and the baseflow duration curve (Figure 4.1) to the left by 10 cfs. Pumping 10 cfs would lower the 7-day, 10-year flow from 27 cfs to 17 cfs; and the 30-day 10-year flow from 31 cfs to 21 cfs. Also, the monthly mean baseflow occurring 10% or less would shift this value from 41.1 to 31.1 cfs. Continuous pumping simulations were useful in validating the operation of the pumping routine.

Variable Pumping - Single Stream Depletion Factor

If we are allowed to change the pumping rate each month, the maximum allowable withdrawals in a year is not immediately obvious. An absolute maximum would be to pump until the stream is dry, which would equal the total recharge. This is a rather extreme policy. A more reasonable approach is to set a constraint based on the results of continuous pumping and determine if there is an improvement under variable pumping.

Under constant pumping of 10 cfs, the monthly baseflow duration curve at the 10% limit would shift from 41.1 cfs to 31.1 cfs. The constraint used in our optimization model is:

$$P\left\{ Q(t) \leq 31.1 \right\} \leq .10$$

or the probability that the monthly mean baseflow is less than 31.1 cfs is less than .10. A penalty will be assessed if more than 10% of the months have baseflows of less than 31.1 cfs. A second constraint was found necessary to limit the range of pumping rates. The constraint limited the minimum pumping rate to 75% of the maximum or:

$$\text{Min}\left\{ gw_t \right\} \geq .75\ \text{Max}\left\{ gw_t \right\}$$

It is possible that the range constraint could be related to the available water storage in the basin. It is assumed that the wells do not interfere and there is a single SDF. The simulation program will simulate 20 years of streamflow, and report the violations to the search routine. It was found that with 12 decision variables (12 monthly pumping rates), the search routine required over 300 objective function evaluations and took over 5 minutes to complete. This problem was overcome by grouping the monthly pumping rates into 4 groups: Jan.-Mar., Apr.-June, July-Sept., Oct.-Dec., so the program will need only to determine four variables. The lower constraint on the decision variables, XMIN, was set at 5 cfs and the upper limit, XMAX, was set at 30 cfs. The program was run for SDF equal to 1, 50, 100 and range constraint of .75, .50 and .25, as shown in Table 2.3. A well placed far from the stream would mean that there would be considerable delay between the start of pumping and the depletion of the stream, and so more water can be withdrawn without lowering the stream below a critical level. This is reflected in the results in Table 4.3, where the maximum withdrawal of 150.09 cfs-month occurs at SDF = 100. When the range constraint is relaxed, the advantage of a slow response becomes a disadvantage in that lowering the pumping rates in the Summer and Fall months when streamflow is critical, will not result in an immediate lowering of stream depletion. From Table 2.3, the maximum withdrawal at a range constraint of .25 (wide range) was 165.84 cfs-month, which occurred at SDF = 1 (near the stream).

<u>Variable Pumping</u> - Multiple Stream Depletion Factors

It is not realistic to assume that the wells within the basin will have the same SDF. There may be some high capacity wells close to the stream which means a low SDF which would cause a fast response from the stream. The wells close to the stream could pump during the winter, when the streamflow is high and more distant pumps could be started

Table 4.3 Optimum Withdrawal Schedule - Single SDF

Constraints: $P\{Q(t) \le 31.1\} \le .10$

$Min(gw_t) \ge .75\ Max(gw_t)$

SDF	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec	Total
1	11.48 cfs	9.94	9.07	11.56	126.18 cfs-mo.
10	9.73	9.72	9.72	10.18	118.08
50	12.05	9.44	9.44	11.95	128.54
100	13.14	10.71	13.06	13.13	150.09

$Min(gw_t) \ge .50\ Max(gw_t)$

SDF	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec	Total
1	16.47	9.44	8.06	17.89	155.58
50	16.06	9.20	8.00	11.21	133.41
100	16.00	9.20	8.24	10.00	130.32

$Min(gw_t) \ge .25\ Max(gw_t)$

SDF	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec	Total
1	19.90	9.34	8.06	17.98	165.84
50	16.06	9.20	8.00	11.21	133.41
100	16.00	9.20	8.24	10.01	130.35

in the summer and fall months when the streamflow is critically low. The effects of the more distant pumps would not be felt on the stream immediately. The proposed strategy is to pump from the distant wells from July to September on the assumption that the stream depletion rate will not reach its maximum until several months after the start of pumping. By that time, in November or December, there should be sufficient amount of recharge.

To test this strategy, the model was modified to include three wells:

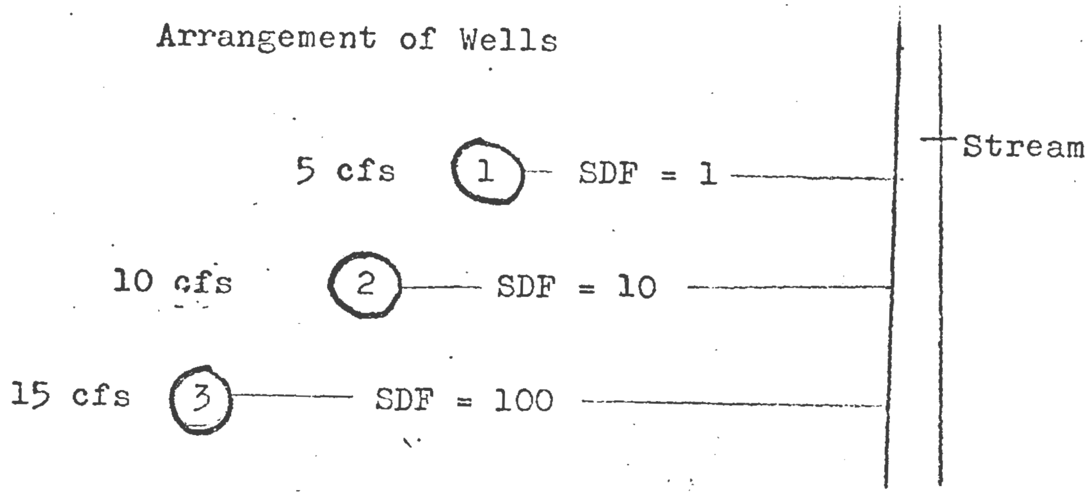
Well 1: Pumping Capacity = 15 cfs, SDF = 1 -- fast response, near stream

Well 2: Pumping Capacity = 10 cfs, SDF = 10 -- medium response, between Well 1 and 3

Well 3: Pumping Capacity = 5 cfs, SDF = 100 -- slow response, distant from stream

The arrangement is shown in Figure 4.8. It is assumed that the pumps do not interact with each other. Typical simulation runs are shown in Figure 4.9. Since the pumps have fixed pumping rates, the problem is not how much to pump, but which pumps should be kept on and which pumps should be turned off. After some initial runs, it became obvious that the low flows were most sensitive to the pumping in period 3, July to September. Therefore, the pumping rate for period 3 was set at 5 cfs. Then the three remaining periods were allowed to vary from 10 to 30 cfs. (10 cfs

Figure 4.8


Arrangement of Wells
Stream
5 cfs
1
SDF = 1
10 cfs
2
SDF = 10
15 cfs
3
SDF = 100

would be pumped from Well 2, 30 cfs would be pumped using all three wells.) Using monthly baseflow statistics, the best pumping schedules were selected, as shown in Table 4.3. For certain pumping schedules, the minimum monthly baseflow was found and its frequency distribution was plotted as shown in Figure 4.10. It was assumed that the ratio of the minimum monthly baseflow to the 1 and 7 day minimum flows and the annual mean flow obtained in Section 4.2 is valid under pumping conditions. The 10 and 20 year minimum flows were computed from the minimum monthly baseflow as shown in Table 4.5.

The pumping schedules presented may in some way appear intuitive, in that someone who has had experience in setting pumping rates could obtain the same results without the use of simulation and search techniques. However, as a system becomes larger, intuition becomes poorer. Certainly, in the early planning stages, it would be essential to find the correct location of the wells. The result shows the advantage of placing some wells near the stream and others more distant.

A more complex system might include more wells at different SDF and a certain capacity to store the withdrawn water. The objective would be to supply a constant volume of water each month by selecting the wells which would be least likely to lower streamflow to which levels. The relationship between the water storage and the location and time of the groundwater withdrawals could be analyzed.

Figure 4.7

Typical Output from Simulation

TOTAL PUMPED 180.00000

SIMULATE FOR 20 YEARS

STREAM DEPLETION FACTOR IN WELL NO. 1 100.0000
STREAM DEPLETION FACTOR IN WELL NO. 2 10.0000
STREAM DEPLETION FACTOR IN WELL NO. 3 1.0000

PUMPING SCHEDULE:		JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
WELL NO.	1	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	0.0	0.0	0.0
WELL NO.	2	10.0	10.0	10.0	10.0	10.0	10.0	0.0	0.0	0.0	10.0	10.0	10.0
WELL NO.	3	0.0	0.0	0.0	15.0	15.0	15.0	0.0	0.0	0.0	0.0	0.0	0.0

MONTH	FLOW (CFS)	AVER. DEP.	WELL NO 1	WELL NO 2	WELL NO 3
1	148.6165	11.1224	2.4696	8.4295	0.2232
2	196.2252	11.5326	2.6661	8.6686	0.1980
3	237.8537	11.8876	2.9101	8.7993	0.1781
4	229.1791	23.7281	3.0992	8.8907	11.7383
5	177.2999	25.6385	3.2420	8.9597	13.4368
6	115.8839	26.1264	3.3526	9.0143	13.7595
7	63.1577	10.2612	3.4409	4.4613	2.3590
8	40.2408	6.8568	3.5133	2.5805	0.7631
9	39.8433	6.0603	3.5738	1.9732	0.5133
10	49.6813	9.7990	3.1756	6.2306	0.3928
11	70.8900	11.0512	2.8083	7.9226	0.3203
12	104.9957	11.1018	2.4325	8.3977	0.2715
TOTAL		165.1660	36.684	84.328	44.154

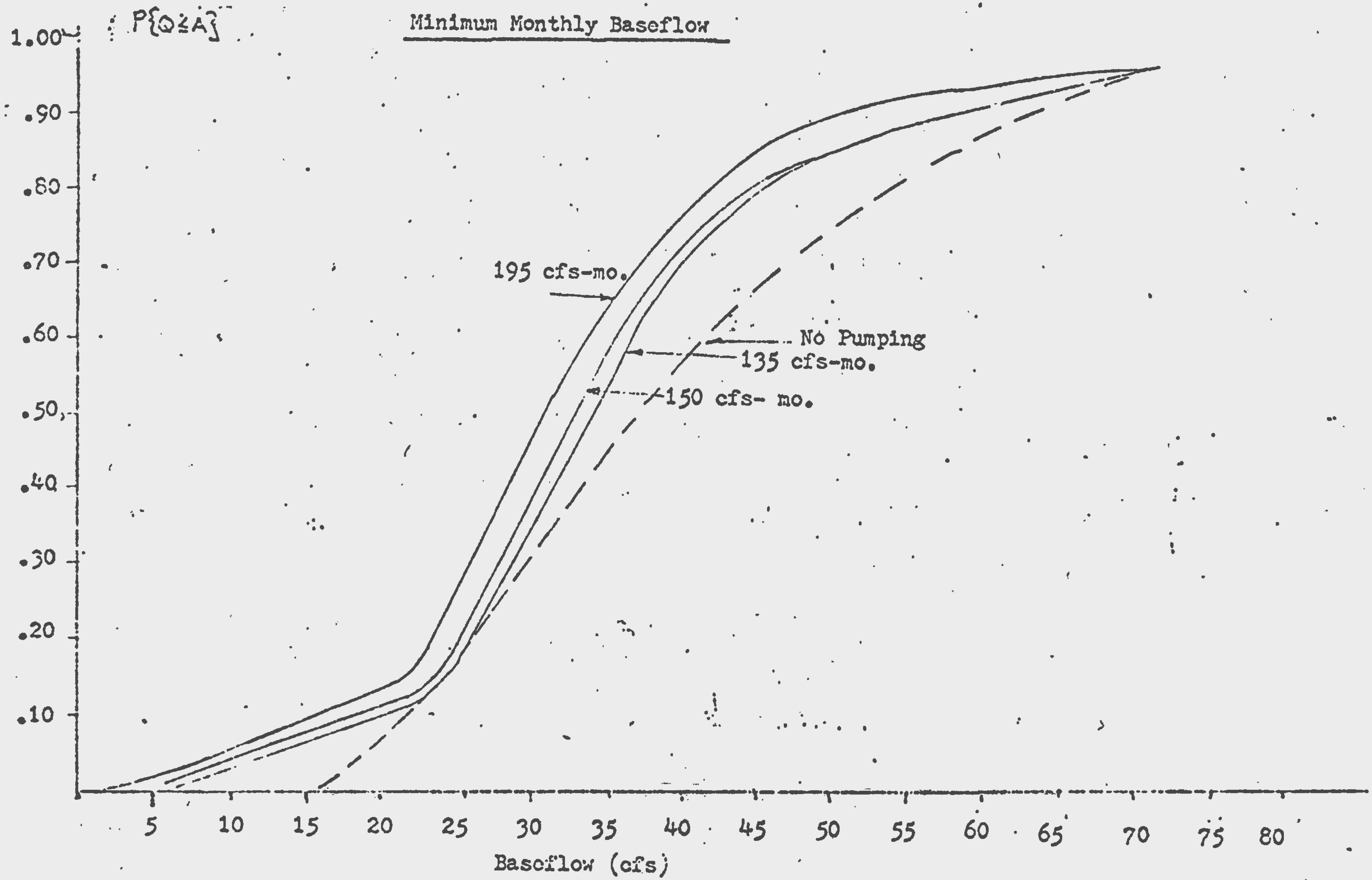
INTERVAL :	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL	CUM TOTAL	FRACTION
-10 - 0	0	0	0	0	0	0	0	0	0	0	0	0	0.0	0.0	0.0
0 - 10	0	0	0	0	0	0	0	0	0	0	1	0	1.00	1.00	0.004
10 - 20	0	0	0	0	0	0	0	3	2	2	0	0	7.00	8.00	0.033
20 - 30	0	0	0	0	0	0	1	4	4	6	1	0	16.00	24.00	0.100
30 - 40	0	1	0	0	0	0	3	4	7	3	1	1	20.00	44.00	0.183
40 - 50	0	0	0	0	0	0	4	3	2	3	1	1	14.00	58.00	0.242
50 - 60	1	0	0	0	0	1	2	3	3	0	4	1	15.00	73.00	0.304
60 - 70	0	0	0	0	0	1	4	2	0	2	7	0	16.00	89.00	0.371
70 - 80	0	0	0	0	0	3	1	1	2	1	2	1	11.00	100.00	0.417
80 - 90	1	0	0	0	0	2	1	0	0	1	0	1	6.00	106.00	0.442

Table 4.4

Optimum Pumping Schedules

Total Pumped	Well No.	Jan- Mar	Apr- Jun	Jul- Sep	Oct- Dec
135 cfs-mo.	(1)	5	5	5	0
	(2)	10	10	0	10
	(3)	0	0	0	0
150 cfs-mo.	(1)	5	5	5	5
	(2)	10	10	0	10
	(3)	0	0	0	0
165 cfs-mo	(1)	5	5	5	5
	(2)	10	10	0	0
	(3)	0	0	0	15
195 cfs-mo.	(1)	5	5	5	0
	(2)	0	10	0	10
	(3)	0	15	0	15

Figure 4.10



$P\{Q \leq A\}$
Minimum Monthly Baseflow
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
195 cfs-mo.
No Pumping
135 cfs-mo.
150 cfs- mo.
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
Baseflow (cfs)

Table 4.5

Changes in Minimum Flows Under Pumping

Total Withdrawn (cfs-month)	Min. Monthly Baseflow	1 Day Min Flow (1.230)[1]	7 Day Min Flow (.945)	30 Day Min (.815)
10 Year Recurrence Interval				
No Pumping	23.00 cfs	18.80[2] cfs	27.00 cfs	31.00cfs
135 cfs-mo.	20.00	16.30	21.16	24.54
150 cfs-mo.	17.00	13.82	17.99	20.85
195 cfs-mo.	15.50	12.62	16.40	19.02
20 Year Recurrence Interval				
No Pumping	19.00	15.49	20.11	23.31
135 cfs-mo.	13.00	10.60	13.76	15.96
150 cfs-mo.	11.00	8.94	11.64	13.50
195 cfs-mo.	8.00	6.50	8.46	9.82

1) Ratios: Min. Monthly Baseflow/1, 7, 30 Day Min Flows from Table 4.2
2) From Historical Record (Figure 4.2)

V. Conclusions and Recommendations

5.1 Conclusions

No single pumping plan was proposed as an optimal schedule. The maximum withdrawal possible is dependent upon the location of the wells and the schedule of the withdrawals.

This study initially simulated the effects of a single well system with a fixed SDF (stream depletion factor). A constraint was placed on the occurrence of low flow and the allowable range of withdrawal rates. The range constraint could be set to reflect the available storage for the withdrawn water. A series of simulation runs were conducted at various levels of SDF and a search program found the maximal annual withdrawal possible for each SDF. It was found that at a narrow range, the maximum withdrawal possible can be achieved from wells very distant from streams. However, when a large range is allowed, the maximum withdrawal occurred at wells near the stream. The advantage to pumping from wells distant from the stream is that stream depletion will be considerably delayed. However, this advantage becomes a disadvantage when the pumping rate is lowered so there is a delay in the change in stream depletion.

The study then examined a more realistic situation: 3 wells with different SDF and fixed withdrawal rates. The decision variable then becomes the best combination of pumps in each period to satisfy a fixed demand. It was found that the third period, July-September, was most critical and only withdrawals from the most distant well could be made.

It is hoped that this study will provide a basic framework and statistical base for future studies. In analyzing the streamflow record, the interval method was devised to separate the baseflow component in the Pawcatuck River at Wood River Junction. By traditional methods, it would be a tedious process to analyze several years of the streamflow record. However, by using the computer, the separation of streamflow components and subsequent statistical analysis of 28 years of streamflow record was easily computed. The Computer Plotter also proved to be an invaluable tool in the baseflow separation and the recession curve. It was demonstrated that the Pawcatuck River was a good reference in that it can be compared with the baseflow in smaller tributaries.

There are many improvements possible within the model. The following changes are possible:

(1) A more refined method could be used to extract the baseflow, possibly using time-series analysis well level data.

(2) The recession line, presented in Figure 2.6, may be considered a two piece line, where the lower end would reflect the lower transmissivity at lower depths of the aquifer.

(3) The aquifer storage may be assumed to be finite and the recharge above a certain level would not go into the aquifer.

(4) The Jenkins model for pumping could be replaced by another relationship obtained from field data.

(5) The grouping of months in the search program could be changed, where the most critical months, Sept.-Nov. would be in one group.

(6) The effect of pumping when the stream is dry could be studied and the program could be modified to account for this.

(7) The model could be modified to include some storage capacity for withdrawn water. Various levels of storage could be investigated.

It is possible to use other techniques for generating streamflow, such as "Fiering's" method, which generates monthly flows using serially correlated random number generator (1).

5.2 Recommendations

It is difficult to recommend specific areas of research for the future without full information on current studies and the needs of the area. Extensive studies are being conducted on the upper and lower basin, but unfortunately the results were not made available for this study. It seems that operations research studies often take a back burner position to more traditional studies in water resources engineering. However, judging from the literature, the use of operations research is rapidly growing.

Extensive research has been conducted in the conjunctive use of ground and surface water to satisfy an expected demand. This is a complex problem and simply an analysis of the demand for water might be a complete study in itself. There is a definite need to know accurately how much above the expected demand a water system should be designed for. This problem has been studied in an economic context by Hoeh (2).

Another possible area of study is the further analysis of streamflow data utilizing the computer. The methods of time series analysis have been shown to be an effective means of analyzing extensive streamflow records (3). It is possible that a more precise measure of the occurrence of floods and droughts could result from the study. Also, a "regional analysis" could be performed (4), where the common characteristics from a series of basins are compared in order to extend the size of the record, hence increase the reliability of the records.

The use of operations research may be extended into the overall planning of water resources in a large area. It could be determined which areas are in most critical demand and what sharing of water resources are possible. The sharing of many different sources of water to satisfy many users has been formulated as a network problem (5).

The success of any project will depend ultimately on the ability to be flexible in the application of operations research.

BIBLIOGRAPHY

Chapter I

1. Allen, W.B. and Brackley, R. The Availability of Groundwater in the Upper Pawcatuck Basin. Geological Survey: Water Supply Paper No. 1821, Washington: U.S. Geological Survey, 1966.

2. Preliminary Plan for Public Water Supply and Distribution. Providence, R.I., Statewide Comprehensive Transportation and Land Use Planning Program, September, 1969.

3. Metcalf and Eddy. Report to the Water Resources Coordinating Board of R.I. on the Development Plan for Water Supply Resources in Rhode Island. Boston: Metcalf and Eddy, August 27, 1967.

4. Report of the Governor's Task Force on Water Resource Planning. Providence, R.I.: Governor's Task Force on Water Resource Planning, 1969.

5. Antak, A. Economic Planning Consideration of Groundwater Pollution for the Upper Pawcatuck River Basin. M.S. Thesis, University of Rhode Island, 1970.

6. Babbit, H.E. and J.J. Doland. Water Supply Engineering. McGraw Hill: N.Y., N.Y., 1967.

7. Allen, W.B. Geohydrological Data for the Upper Pawcatuck River Basin. R.I. Geological Bulletin No. 13, Providence: U.S. Geological Survey, 1963.

8. Groundwater Levels - Northeastern States. Water Supply Papers 1537, 1782, 1977, 2140, Washington, U.S.G.S., 1956-1972.

9. Surface Water Records - Northeastern States. Water Supply Papers 1900 (Index to all years 1941-1968).

10. Climatological Records. National Oceanic and Atmospheric Administration, Washington, D.C.

11. Hufschmidt and Fiering. Simulation Techniques for Design of Water-Resource System. Harvard Press, Mass., 1966.

12. Maass, et al. Design of Water-Resource Systems. Harvard University Press, Mass., 1962.

13. Payne, K. and W.R. Neuman. "Daily Streamflow Simulation." Journal of the Hydraulics Division. ASCE, Vol. 95, HY4, Proc. Paper. 6665, July 1969, pp. 1165-1179.

14. Quimpo, R.G. "Stochastic Analysis of Daily River Flows." Journal of Hydraulics Division, ASCE, Vol. 94, No. HY1, Proc. Paper 5719, Jan. 1969, pp. 43-57.

15. Taylor, O.J. "Optimization of Conjunctive Use of Water in Stream-Aquifer System Using Linear Programming." Geological Survey Research. U.S. Geological Survey Professional Paper 700-C, 1970, pp. c218-c221.

16. Dracup, J.A. "Optimum Use of a Groundwater and Surface Water System: A Parametric Linear Programming Approach." Technical Paper 6-24, Water Resources Center Contribution No. 107, University of California, Berkeley.

17. Hughes, T.C. and Calvin Clyde. "Municipal Water Planning - A Mixed Integer Approach." Journal of Hydraulics Division, Vol. 99, HY11, Procedure Paper 10131, November 1974, pp. 2079-2083.

18. Niewsand, G.H. The Conjunctive Use of Surface and Groundwaters in the Mullica River Basin: A Chance Constraint Linear Programming Approach. Ph.D. Thesis, Rutgers University, New Brunswick, N.J., 1967.

19. Domenico, Patrick A. Concepts and Models in Groundwater Hydrology. McGraw Hill: New York, 1972.

Chapter II

1. Bruce, J.H. and R.H. Clark. Introduction to Hydrometeorology. Pergamon Press: Oxford, 1966.

2. Linsley, R.K. Water Resources Engineering. McGraw Hill: New York, 1972.

3. Gray, D. Handbook on the Principles of Hydrology. Water Information Center, Port Washington: N.Y., 1970.

4. Allen, W.B. and R. Brackley. Availability of Groundwater in the Upper Pawcatuck Basin. U.S.G.S. Water Supply Paper 1821, U.S.G.S.: Washington, 1963.

5. Walton, W. Groundwater Resources Evaluation. McGraw Hill: New York, 1970.

6. Hardison, G.H. and M.F. Moss. "Accuracy of Low-Flow Characteristics Estimated by Correlation of Baseflow Measurements." Manual of Hydrology, Part 2, Low-Flow Techniques, USGS Water Supply Paper 1542-B.

7. Meybloom, P. "Estimating Groundwater Recharge from Stream Hydrograph." American Geophysical Research, Vol. 66, No. 4, April 1961.

8. Singh, K.P. and J. Stall. "Derivation of Recession Curves and Parameters." Water Resources Research, Vol. 7, No. 2, Apr. 1971.

9. Olmsted, E.H. and Hely, A.G. "Relationship Between Ground and Surface Waters in Brandywine, Penn." USGS Professional Paper 417-A.

10. Rorabaug, M.L. "Exploration of Methods of Relating Groundwater to Surface Water - Columbia River Basin." USGS Open File Report, 1966.

11. Quimpo, R. "Stochastic Analysis of Daily River Flows." Journal of the Hydraulics Division, ASCE, Vol. 94, HY1, Jan. 1968.

12. Trainer, F.W. and Frank Watkins. "Use of Baseflow Runoff Curves to Determine Areal Transmissivity in the Upper Potomac River Basin." Journal of Research, USGS, Vol. 2, No. 1, Feb. 1974.

13. Rassmussen, W.C. and G.E. Andreason. "Hydrological Budget of the Beaverdam Creek, Maryland." U.S.G.S. Water Supply Paper No. 1472, U.S.G.S.: Washington,D.C.

14. Rorabaug, M.I. "Prediction of Groundwater Levels from Rainfall and Temperature Correlations." Transactions of American Geophysical Union, Vol. 37, No. 14, Aug. 1956.

15. Bierschenk, W. Groundwater Resources of the Kingston Quadrangle, Rhode Island. Rhode Island Development Council. Geological Bulletin, 1956.

Chapter III

1. R.L. Ackoff. Progress in Operations Research. John Wiley, New York, 1971.

2. Burns, A.W. "Ipwich River Basin Study." New England Water Works Association, Vol. 88, No. 4, Dec. 1973.

3. James and Lee. Economics of Water Resources Planning. McGraw Hill, N.Y., 1971.

4. Jenkins, C.T. "Computation of Rate and Volume of Stream Depletion by Wells." Techniques of Water Resources Investigation of the USGS, Chap. D1, Book 4, USGS, Washington, 1970.

5. Conover, W.J. Practical Non-Parametric Statistics, John Wiley, N.Y., 1961.

Chapter IV

1. Nieswand, G.H. "The Conjunctive Use of Surface and Ground Waters in the Mullica River Basin, New Jersey: A Chance Constraint Linear Programming Approach." Ph.D. Thesis, Rutgers University, New Brunswick, N.J. June, 1970.

2. Chow, V. Handbook of Applied Hydrology. McGraw Hill, N.Y., 1964.

3. Kuester and Mize. Optimization Techniques with Fortran. McGraw Hill: NY, 1973.

4. Wilde, D.J. and C.S. Beightler. Foundations of Optimization. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1967.

5. Buffa and Taubert. Production-Inventory Systems. Irwin, Hometown, Illinois, 1966.

Chapter V

1. Fiering, M.B. Streamflow Synthesis. Harvard Press, Cambridge, Mass., 1967.

2. Hoeh, Robert. Water Supply Planning in Rhode Island. M.S. Thesis, University of Rhode Island, Kingston, R.I., 1972.

3. Quimpo, R. "Stochastic Analysis of Daily River Flows." Journal of the Hydraulics Division, ASCE, Vol. 94, HY1, January 1968.

4. Tasker, G.D. "Combining Estimates of Low Flow Characteristics of Streams in Massachusetts and Rhode Island." Journal of Research, Vol. 3, No. 1, Jan-Feb. 1975.

5. Hamdain, A.S. and D.D. Meredith. "Screening Model for Conjunctive Use of Water Systems." Journal of the Hydraulic Division, ASCE, Vol. 101, HY10, Oct. 1975.

APPENDIX

APPENDIX A

DATA REPORT

A-1 Data Used in Comparison of Chipuxet and Pawcatuck R. at Wood River Junction- 1958, 1959, 1973, 1974

A-2 Comparison Between Flow Duration Curve in this Study and Curve in Water Resources Report

A-3 Frequency Distribution of Daily Flows

A-4 Comparison Between Historical and Simulated Streamflow

A-5 Analysis of Variance for Relationship Between Chipuxet and Pawcatuck

A-6 Mean Monthly Streamflow- Pawcatuck River at Wood River Junction

A-7 Hydrological Budget

A-1

Data Used in Comparison of Chipuxet and Pawcatuck River at Wood River J.

1958

		Chip.	Paw.
APR	5	42.0	410.0 *
APR	20	46.0	430.0 *
APR	27	44.0	389.0
MAY	6	48.0	488.0
MAY	20	39.0	346.0
MAY	30	35.0	348.0
JUN	10	26.0	257.0
JUN	20	24.0	220.0
JUN	30	16.0	149.0
JUL	4	13.0	128.0
JUL	20	13.0	123.0
JUL	25	10.0	91.0
AUG	10	7.7	71.0
AUG	12	7.3	80.0
AUG	24	7.0	81.0
SEP	5	11.0	121.0
SEP	15	9.0	104.0
SEP	26	9.7	100.0
OCT	10	19.0	173.0
OCT	20	12.0	122.0
OCT	21	11.0	120.0
NOV	8	22.0	188.0
NOV	20	19.0	176.0
NOV	26	14.0	151.0
DEC	10	24.0	208.0
DEC	20	16.0	163.0
DEC	29	11.0	128.0

1959

		Chip.	Paw.
APR	10	38.0	373.0
APR	19	29.0	291.0
APR	26	28.0	260.0
MAY	10	21.0	213.0
MAY	18	18.0	188.0
MAY	30	11.0	151.0
JUN	1	13.0	153.0
JUN	12	12.0	120.0
JUN	30	23.0	188.0
JUL	9	12.0	110.0
JUL	12	21.0	188.0
JUL	31	13.0	114.0
AUG	8	11.0	94.0
AUG	20	9.7	80.0
AUG	28	7.7	71.0
SEP	10	8.7	68.0
SEP	15	6.9	83.0
SEP	28	6.1	44.0
OCT	7	6.1	44.0
OCT	20	6.3	55.0
OCT	22	5.9	52.0
NOV	6	9.4	80.0
NOV	16	9.4	86.0
NOV	23	9.2	85.0
DEC	6	13.0	119.0
DEC	11	19.0	178.0
DEC	26	17.0	178.0

1960

		Chip.	Paw.
APR	3	42.0	391.0
APR	20	34.0	293.0
APR	30	29.0	230.0
MAY	8	25.0	197.0
MAY	20	23.0	203.0
MAY	31	20.0	176.0
JUN	3	18.0	168.0
JUN	17	12.0	108.0
JUN	30	11.0	82.0

1973

		Chip.	Paw.
SEP	14	6.9	81.0
SEP	22	9.4	95.0
OCT	10	6.8	86.0
OCT	20	5.0	77.0
OCT	23	4.9	76.0
NOV	10	7.3	95.0
NOV	20	6.5	89.0
NOV	21	6.3	88.0
DEC	7	13.0	126.0
DEC	14	17.0	175.0
DEC	26	33.0	416.0

1974

		Chip.	Paw.
APR	8	29.0	330.0
APR	20	29.0	297.0
APR	30	23.0	217.0
MAY	9	21.0	189.0
MAY	20	19.0	177.0
MAY	27	18.0	168.0
JUN	10	11.0	137.0
JUN	14	9.0	111.0
JUN	30	11.0	127.0
JUL	10	9.6	94.0
JUL	15	7.9	80.0
JUL	30	5.7	68.0
AUG	6	5.1	59.0
AUG	20	4.5	46.0
AUG	22	4.2	41.0
SEP	1	6.1	65.0
SEP	20	5.6	55.0
SEP	27	6.1	58.0

* not shown on figure

Comparison Between Flow Duration Curve in this Study and Curve in the Water Resources Report

(1) Daily Flow Duration (Figure 4.1 in this study and Figure 13 in Water Resources Report)

$P\{Q \leq A\}$	A (from Report)	A (from this study)
.05	38 cfs	32 cfs
.10	45 cfs	45 cfs
.20	62 cfs	63 cfs
.30	88 cfs	86 cfs
.40	110 cfs	118 cfs
.50	150 cfs	150 cfs

(2) Annual Flow Duration (Figure 4.1 in this study and Figure 17 in Water Resources Report; it was assumed that 70% is baseflow)

$P\{Q \leq A\}$	A (from Report)	A (from study)
.05	86.24 cfs	86 cfs
.10	89.47 cfs	94 cfs
.20	107.80 cfs	106 cfs
.30	118.58 cfs	117 cfs
.40	127.20 cfs	126 cfs
.50	131.51 cfs	134 cfs

Note: The difference between values from this study and the ones obtained from the Report are due to: (1) the different periods used, (the Report used 1942- 62, this study used 1941- 1968) and (2) human error from reading graphs in the Report.

A-3

Frequency Distribution of Daily Flows (Total Flow)

Flows Less Than:	No of Flows	Probability (Cumulative)
30. cfs	188.	0.0184
60.	1872.	0.1830
90.	3273.	0.3200
120.	4267.	0.4172
150.	5123.	0.5009
180.	5913.	0.5782
210.	6685.	0.6537
240.	7361.	0.7198
270.	7996.	0.7819
300.	8459.	0.8271
330.	8819.	0.8623
360.	9095.	0.8893
390.	9339.	0.9132
420.	9561.	0.9349
450.	9701.	0.9486
480.	9815.	0.9597
510.	9901.	0.9681
540.	9981.	0.9759
570.	10036.	0.9813
600.	10089.	0.9865
630.	10116.	0.9891
660.	10136.	0.9911
690.	10157.	0.9932
720.	10170.	0.9944
750.	10184.	0.9958
780.	10197.	0.9971
810.	10203.	0.9977
840.	10203.	0.9977
870.	10209.	0.9982
900.	10212.	0.9985
930.	10214.	0.9987
960.	10215.	0.9988
990.	10216.	0.9989
1020.	10219.	0.9992
1050.	10220.	0.9993
1080.	10220.	0.9993
1110.	10222.	0.9995
1140.	10222.	0.9995
1170.	10223.	0.9996
1200.	10223.	0.9996

Comparison Between Historical and Simulated Streamflow -- No Pumping

Less Than:	Simulated	Historical
0 cfs	0.0	0.0
10	.002	0.0
20	.008	.008
30	.046	.068
40	.091	.131
50	.154	.199
60	.218	.241
70	.258	.300
80	.306	.360 * Maximum Difference .054
90	.355	.407
100	.407	.449
110	.450	.464
120.	.487	.523
130	.523	.547
140	.563	.592
150	.607	.616
160	.650	.654
170	.680	.684
180	.717	.726
190	.751	.762
200	.790	.791
210	.814	.818
220	.833	.851

Comparison Between Historical and Simulated (Continue)

	Simulated	Historical
230 cfs	.856	.881
240	.881	.895
250	.896	.913
260	.912	.922
270	.930	.934
280	.943	.943
290	.970	.961
310	.977	.964
320	.986	.974
330	.992	.982
340	.995	.985
400	.999	.999

A-5

Analysis of Variance for Relationship Developed in Section 2.3

Source	d.f.	Sum of Squares	Mean Square	F
Attributable to Regression	1	10204.371	10204.371	2155.67
Deviation from Regression	91	430.770	4.734	
Total	92	10635.141		

1) Test $H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

2) Set $\alpha = .05$

3) Critical Region: $F_{c_{(1,91)}} \cong 3.95$

4) Since $F > F_c$, The null hypothesis is rejected.

A-6 PAWCATUCK RIVER AT WOOD RIVER JUNCTION (CFS)

		JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1941	0	183.74	243.00	218.10	190.93	155.48	191.07	100.71	70.94	42.27	38.71	63.63	74.58
1942	0	96.16	250.00	444.19	241.43	136.71	83.57	81.29	98.87	58.40	72.97	111.14	261.00
1943	0	278.52	320.04	319.19	229.07	261.55	142.37	75.58	61.55	37.93	53.10	84.37	58.81
1944	0	98.81	103.86	220.19	267.17	175.52	119.43	67.61	42.16	73.57	57.61	163.53	347.10
1945	0	293.74	227.68	430.16	223.67	258.23	143.83	80.87	64.32	41.03	39.29	95.10	324.06
1946	0	384.87	305.04	334.71	205.63	217.26	235.77	97.68	274.65	132.13	99.97	75.73	83.16
1947	0	135.29	134.86	264.26	289.30	289.13	155.17	101.68	70.06	49.57	38.26	150.13	130.03
1948	0	147.10	278.55	529.00	462.97	410.68	394.27	130.42	69.48	41.07	46.71	86.87	95.77
1949	0	214.84	325.96	299.90	322.97	207.77	94.77	49.35	31.68	34.70	31.10	44.83	72.13
1950	0	102.23	185.89	257.29	263.30	190.19	157.60	76.94	53.03	35.23	36.77	81.10	124.10
1951	0	202.55	331.39	296.58	321.77	205.84	157.67	75.97	53.06	44.07	48.29	164.37	228.71
1952	0	311.35	359.79	421.45	256.03	243.48	147.00	68.61	189.48	82.57	63.35	58.73	102.39
1953	0	303.39	424.64	598.32	606.90	374.03	136.90	97.23	95.06	59.37	60.16	158.87	379.61
1954	0	217.74	225.43	256.71	280.97	347.68	152.83	83.10	144.58	374.43	176.35	273.63	389.90
1955	0	305.16	314.36	332.77	233.23	191.81	122.40	84.48	194.42	133.10	331.58	470.87	221.84
1956	0	245.71	377.90	399.58	431.20	249.61	157.50	138.71	70.55	54.77	71.94	91.53	172.03
1957	0	162.45	182.79	227.68	345.43	166.19	82.33	38.16	38.10	32.13	34.45	72.87	175.03
1958	0	398.42	288.46	461.32	523.47	436.90	248.93	147.19	124.84	130.27	192.45	190.87	181.61
1959	0	168.77	202.96	417.87	349.57	208.23	186.87	170.39	92.97	71.77	71.61	100.23	210.39
1960	0	211.55	321.48	317.87	363.00	221.97	125.97	86.23	55.42	73.90	75.81	118.37	142.45
1961	0	209.77	227.00	393.23	480.57	377.90	263.20	112.00	93.06	248.83	254.48	184.60	206.00
1962	0	453.23	219.89	443.45	352.90	181.74	151.40	90.45	63.03	54.57	191.94	306.67	274.68
1963	0	252.16	291.61	373.32	204.63	234.55	166.27	80.32	69.48	48.67	49.19	104.30	152.61
1964	0	270.35	308.00	294.97	518.13	229.58	98.53	68.97	42.84	36.90	52.58	55.87	131.10
1965	0	154.90	248.14	287.29	230.87	172.10	102.73	46.84	38.23	35.83	44.19	42.23	49.77
1966	0	60.10	142.14	247.16	129.53	167.65	147.80	64.61	42.03	47.00	56.10	106.10	92.13
1967	0	132.00	123.14	297.26	309.07	354.16	244.13	158.26	114.68	68.03	81.87	102.03	247.77
1968	0	271.16	271.72	589.06	274.07	202.10	168.00	104.26	55.94	40.10	39.42	91.93	176.77

A-7 Hydrological Budget 1941- 1968

1941

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	3.75	1.56	3.27	1.99	2.48	6.83	4.15	1.87	0.35	2.19	3.75	2.83	35.02
SURFACE	0.75	1.14	0.47	0.22	0.43	1.01	0.13	0.17	0.08	0.13	0.25	0.28	5.06
RECHARGE	1.01	1.68	3.23	0.67	0.36	1.33	0.10	0.26	-0.02	0.58	0.58	0.86	10.64
EVAP	1.98	-1.26	-0.43	1.10	1.70	4.49	3.92	1.43	0.29	1.47	2.92	1.70	19.32

1942

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	3.61	4.60	7.10	0.72	1.72	2.65	4.26	6.00	2.21	4.27	5.19	6.38	48.71
SURFACE	0.37	1.03	2.52	0.26	0.11	0.06	0.26	0.62	0.18	0.29	0.37	1.25	7.32
RECHARGE	1.74	3.79	3.59	0.46	0.19	0.21	0.30	0.35	0.41	1.24	1.58	3.65	17.51
EVAP	1.49	-0.21	0.99	-0.00	1.42	2.38	3.71	5.03	1.61	2.74	3.25	1.48	23.88

1943

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	3.43	2.10	2.60	3.33	3.61	1.40	2.69	2.63	1.20	4.57	2.66	1.54	31.76
SURFACE	0.68	0.98	1.05	0.38	0.72	-0.00	0.12	0.23	0.07	0.15	0.24	0.13	4.74
RECHARGE	2.56	3.70	1.71	2.04	1.96	-0.53	-0.04	0.14	0.23	1.11	0.70	0.25	13.83
EVAP	0.19	-2.58	-0.16	0.91	0.92	1.93	2.61	2.26	0.90	3.31	1.72	1.15	13.18

1944

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	2.03	1.82	5.61	3.32	0.76	2.10	0.67	1.82	6.10	2.46	9.90	2.89	39.48
SURFACE	0.49	0.41	0.81	0.85	0.34	0.39	0.13	0.05	0.48	0.20	0.76	1.66	6.56
RECHARGE	1.06	1.12	4.20	1.87	0.16	0.03	-0.00	0.25	0.63	0.41	3.48	4.17	17.38
EVAP	0.48	0.30	0.60	0.60	0.26	1.68	0.54	1.52	4.99	1.85	5.66	-2.94	15.54

1945

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	3.14	3.18	2.28	2.59	4.86	2.84	1.22	3.83	1.25	2.33	8.16	7.17	42.85
SURFACE	0.26	0.51	1.66	0.12	0.82	0.08	0.08	0.14	0.03	0.06	0.15	1.70	5.62
RECHARGE	1.77	3.28	4.21	0.51	1.69	-0.01	-0.04	0.34	0.16	0.49	2.90	3.84	19.14
EVAP	1.10	-0.62	-3.58	1.96	2.35	2.77	1.17	3.35	1.06	1.79	5.11	1.63	18.09

1946

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	4.47	2.60	1.73	2.53	4.77	2.83	2.04	11.12	2.24	0.57	0.96	3.38	39.24
SURFACE	2.43	0.83	1.07	0.14	0.42	0.86	0.12	1.23	0.14	0.15	0.06	0.20	7.66
RECHARGE	3.33	1.85	3.10	0.33	3.10	0.03	-0.08	4.17	-0.28	0.73	0.51	0.91	17.70
EVAP	-1.29	-0.08	-2.44	2.06	1.25	1.94	2.00	5.72	2.38	-0.31	0.38	2.27	13.88

1947

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	3.41	0.67	3.33	5.26	4.42	3.31	4.79	1.62	2.35	3.60	6.12	2.80	41.68
SURFACE	0.49	0.17	1.23	1.01	0.79	0.28	0.31	0.12	0.20	0.11	0.78	0.38	5.78
RECHARGE	2.04	1.63	2.45	3.82	1.29	-0.21	0.38	-0.10	0.23	0.65	2.08	1.02	15.29
EVAP	0.88	-1.14	-0.35	0.43	2.34	3.24	4.20	1.60	1.92	2.84	3.25	1.40	20.60

1948

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	5.79	2.77	4.43	4.20	8.61	2.37	1.87	0.79	0.96	4.22	4.52	2.20	42.73
SURFACE	0.30	1.32	2.23	0.86	1.37	1.00	-0.13	0.03	-0.00	0.09	0.29	0.25	7.60
RECHARGE	2.18	4.09	7.65	1.48	6.07	-0.45	-0.94	-0.02	0.07	0.73	1.16	1.59	23.61
EVAP	3.31	-2.64	-5.45	1.86	1.17	1.82	2.94	0.78	0.89	3.40	3.08	0.36	11.52

1949

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	5.22	4.39	2.58	4.57	2.63	0.04	1.89	2.51	4.02	1.60	3.40	3.00	35.85
SURFACE	0.60	0.99	0.49	0.87	0.26	0.04	0.10	0.09	0.11	0.07	0.19	0.29	4.09
RECHARGE	3.92	4.43	2.01	3.36	-0.64	-0.46	-0.19	0.16	0.36	0.32	0.35	1.48	15.11
EVAP	0.70	-1.03	0.09	0.34	3.01	0.45	1.98	2.25	3.56	1.21	2.86	1.23	16.65

1950

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	3.55	3.97	3.46	2.14	3.02	2.00	1.18	2.94	1.28	1.56	6.76	3.55	35.41
SURFACE	0.18	0.34	0.61	0.47	0.13	0.36	0.06	0.13	0.08	0.10	0.53	0.60	3.59
RECHARGE	1.93	2.70	3.95	2.09	0.64	0.36	-0.19	-0.00	0.23	0.35	0.77	1.97	14.81
EVAP	1.45	0.93	-1.11	-0.42	2.25	1.28	1.31	2.81	0.97	1.11	5.44	0.98	17.01

1951

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	4.07	3.67	4.48	2.81	3.90	2.38	1.05	3.55	2.55	2.60	6.84	4.75	42.65
SURFACE	0.54	1.11	0.45	0.65	0.34	0.30	0.12	0.15	0.12	0.13	0.72	1.02	5.64
RECHARGE	3.95	4.05	3.68	1.45	0.20	0.56	-0.34	0.18	0.27	1.06	2.59	2.74	20.39
EVAP	-0.42	-1.49	0.35	0.70	3.36	1.52	1.27	3.22	2.17	1.42	3.53	0.99	16.62

1952

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL
RAIN	5.02	3.60	4.76	3.06	4.18	2.43	0.43	13.56	1.17	0.88	1.86	4.00	44.95
SURFACE	0.97	1.00	1.76	0.19	0.67	0.07	0.05	1.06	0.05	0.25	0.20	0.39	6.64
RECHARGE	4.78	2.65	3.25	0.85	1.81	-0.29	-0.12	2.75	-0.63	0.25	0.73	1.67	17.69
EVAP	-0.73	-0.05	-0.25	2.03	1.69	2.65	0.51	9.76	1.75	0.38	0.93	1.94	20.62