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An Experimental Determination of Hydrodynamic Masses and Mechanical Impedances

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AN EXPERIMENTAL DETERMINATION OF HYDRODYNAMIC MASSES AND MECHANICAL IMPEDANCES

by

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ABSTRACT

The study of the forces acting on a body due to the body's motion through a fluid is facilitated by the introduction of a "hydrodynamic" mass. ie. The mass of fluid that appears to be carried by the body as the body accelerates in the fluid. If the body's motion is periodic, the relation between the hydrodynamic forces acting on the body and the body velocities is described by a mechanical or acoustical impedance.

Hydrodynamic masses have been computed from ideal fluid theory for mathematically "easy" shapes--spheres, circular discs, etc. There are three methods available for the computation of hydrodynamic mass; the impedance approach, the kinetic energy method, and Darwin's "drift" method. Each of these methods is presented in appendices.

Mechanical impedances have been computed for a very limited number of shapes. The mechanical impedance is computed directly by integration of pressures over the body (the impedance approach). An alternate method of impedance computation is the computation of hydrodynamic mass from one of the above methods. The computation of damping constants follows from viscous flow theory.

Because of the difficulty encountered when the computation of hydrodynamic mass or mechanical impedance is attempted for an irregular body, it becomes necessary to determine hydrodynamic masses and mechanical impedances experimentally for bodies of irregular shape. This thesis presents the results of an extensive experimental investigation into hydrodynamic masses and mechanical impedances for many bodies of complex shape.

Three techniques were employed for these measurements. The relative merits of each are discussed. A table is presented that compiles hydrodynamic mass factors from the literature and from this study for many different bodies. Other tables included show mechanical impedances for different bodies. Mechanical impedances are not available in the literature.

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I INTRODUCTION

When a submerged body is accelerated in a fluid, the resulting motion cannot be described mathematically unless the effect of the fluid acting on the body is taken into account. In a real fluid, this reaction force has two components; a force due to the mass of entrained fluid which varies directly with acceleration, and a force due to viscosity which varies directly with velocity. If the body is accelerated and decelerated in simple harmonic motion, the forces can be represented by two orthogonal vectors rotating at a speed ω, the frequency of the motion. The vector sum of the two forces divided by the magnitude of the velocity vector is termed the acoustic impedance. The reactive componet of the impedance divided by the angular frequency is termed the hydrodynamic mass. This represents the mass of fluid carried by the body as the body is accelerated. The sum of the body's mass and the hydrodynamic mass is called the virtual mass.

There are three methods of computing the hydrodynamic mass of a body. One method is to integrate the increase in pressure due to the motion of the body over the surface of the body. Solution of the resulting force integral is accomplished by means of Bessel functions. The solution is composed of two orthogonal components. Following usual mathematical notation, the component in the direction of the velocity phasor is termed "real"; the component in the direction of the acceleration phasor is termed "imaginary". Division of the force by the magnitude of the velocity vector yields the acoustic impedance. Division of the imaginary component of the impedance by the angular frequency yields the hydrodynamic mass. This method is commonly used by acousticians; see Appendix B for a typical example.

The "traditional" method of solution for hydrodynamic mass has been to compute the increase in kinetic energy of an ideal fluid due to the motion of the body. This approach points out the directional properties of hydrodynamic mass . The hydrodynamic mass for a body is different for different directions of motion. The kinetic energy due to a body having six degrees of freedom is:

$$T = \frac{1}{2} \sum_{i=1}^{6} \sum_{j=1}^{6} A_{ij} u_i u_j$$

where Aij, the hydrodynamic masses, are given by:

$$A_{ij} = \rho \int_S \phi_i n_j \, dS = \rho \int_S \phi_j n_i \, dS \qquad i,j = 1, \cdots, 6$$

Thus, it is seen that a complete description of the hydrodynamic masses and moments of inertia for a body is given by a 6 by 6 matrix. See Appendix C for a typical solution utilizing the kinetic energy approach.

The most recent interpretation of hydrodynamic mass has been put forth by Sir Charles Darwin in his "drift" concept. An infinite thin plane of fluid is assummed to lie normal to the direction of motion of the body. After the body has passed through this plane, the shape of the formerly plane surface is described by considering the displacement or "drift" of each fluid particle. It is shown that the mass of fluid enclosed by the original plane and the deformed plane is equal to the hydrodynamic mass. See Appendix D for a typical solution utilizing the drift concept.

Although there are three general methods of calculating the hydrodynamic mass for a body accelerating in a given direction, they all become exceedingly difficult as the shape of the body deviates from a mathematically "easy" shape. For bodies that are mathematically difficult, the most practical method of obtaining hydrodynamic mass data for a given motion is by experimentation. To date, most of the experimental work has been done for a relatively limited range of shapes.

II REVIEW OF LITERATURE

The effect of the fluid medium acting on a submerged body was first considered by Bessel (1.) in 1828 while studying the motion of pendulums. He found that if he assummed that the mass of the pendulum increased when placed in a fluid, the experimental results would agree with theory. Green and Stokes (2.) developed an exact mathematical interpretation for the hydrodynamic mass of a sphere in 1833.

The theoretical aspects of hydrodynamic mass have been discussed extensively by Lamb (3.), Munk (4.) and Birkhoff (5.). Exact values for a sphere, a sphere in close proximity to another sphere, a circular cylinder of infinite length, a flat strip of infinite length, a circular disc, and an ellipsoid are given by Lamb (3.). Munk (4.) has also calculated hydrodynamic masses for ellipsoids and spheres. Munk considered the case of an ellipsoid of neglible thickness--i.e. an elliptical disc. Zahm (6.) has also computed hydrodynamic masses for ellipsoids.

An excellent discussion is presented by Birkhoff (5.) utilizing tensor notation and the concept of an inertial Lagrangian system. Recently, Sir Charles Darwin (7.) has suggested the drift concept of hydrodynamic mass.

Hydrodynamic masses for two-dimensional bodies have been computed by Wendel (8.) using the Schwartz-Christofell method. Bryson (9.) has extended this work using the hodograph method.

Brahmig (10.) describes the natural frequency test method which he employed to investigate the hydrodynamic mass of circular discs. Other experimental work, referred to by Wendel (8.) and Brahmig (10.), include investigations by Hirsh (11.) for spherical balloons; Cook (12.) for spherical mine cases impacting in water; Pabst (13.) for rectangular plates; Koch (14.) for rectangular sections; Lewis (15.) for ship hull sections; Moullin and Browne (16.) for prismatic bars; and Dimpker and Holstein (17.) for wedges, cylinders and cubes at the surface of the fluid.

Hydrodynamic masses of bodies oscillating at a free surface have been investigated theoretically by Landweber and Macagno (18.), (19.), (20.). Goodman and Sargent (21.) have set forth a method for the calculation of hydrodynamic masses of three dimensional bodies.

Although some of the investigators referenced have computed damping characteristics as well as the hydrodynamic mass of a body; there has been no specific study of the mechanical impedance of a body. The closest that one can come to this is to refer to an acoustics text, for example Kinsler and Frey (22.).

Inspection of available material indicates that enough hydrodynamic mass data has been obtained to verify the theoretical analysis. In general, hydrodynamic masses have been obtained theoretically for mathematically "easy" shapes. Experimental work has also been done for these simpler shapes in order to verify the theory. A definite need exists in this area of hydrodynamics. The need is to extend the experimental work to include complex shapes for which the hydrodynamic mass cannot be calculated. Also, investigations into mechanical impedances for various bodies is called for.

III THE INVESTIGATION

A. Object

The object of this thesis is to present the results of an extensive experimental program in which the hydrodynamic masses and impedances of several bodies were determined. Other objectives of this study were to determine the best method of experimentally determining hydrodynamic mass; to determine the effect of frequency on hydrodynamic mass and to determine the effect of displacement amplitude on hydrodynamic mass.

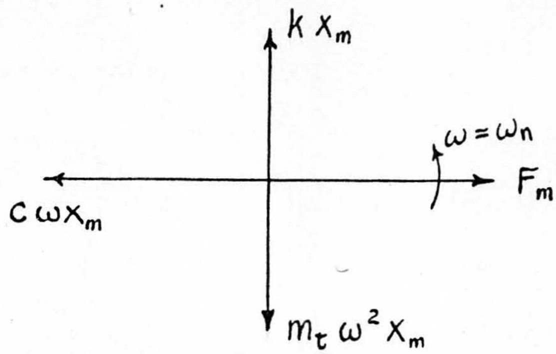
B. Test Methods

Three methods were employed to experimentally determine the hydrodynamic mass of a body for a given motion. The most obvious method was to give the immersed body an acceleration and to measure the force required to produce this acceleration. The total mass can be found from:

$$m_t = \frac{F}{A}.$$

subtracting the mass of the body from m_t yields the hydrodynamic mass if the body is immersed in an ideal fluid. However, in a real fluid, viscous forces are present that must be accounted for. Because a certain amount of time is required for the boundary layer to build up. This method should yield dependable results if the data is taken as the motion first starts. Because this method is non-oscillatory, impedances cannot be determined.

The second method used was the natural frequency method. It is commonly known that a simple spring-mass system being driven at its natural frequency will show the following vector relations:



At resonance, the driving force, F_m, balances the frictional force, $c\omega X_m$, and the inertia force, $m_t \omega^2 X_m$, balances the spring force, kX_m. Thus the hydrodynamic mass can be determined by measuring the spring constant, the natural frequency with the body immersed, and the in vacuo mass of the system. Hence,

$$m_t = \frac{k}{\omega_n^2}$$

and

$$m_h = m_t - m_b$$

An alternate set of measurements would be to measure the natural frequency of the system in a vacuum and in the fluid. Hence,

$$m_h = \frac{k}{\omega_{n_f}^2} - \frac{k}{\omega_{n_v}^2}$$

The principal advantage of the natural frequency method is that the hydrodynamic mass can be determined without considering the effect of damping. Also, the damping constant can be calculated by equating the driving force and the damping force.

$$c\,\omega X_m = F_m$$

$$c = \frac{F_m}{\omega X_m} = \sqrt{\frac{m_t k}{(Tr^2 - 1)}}$$

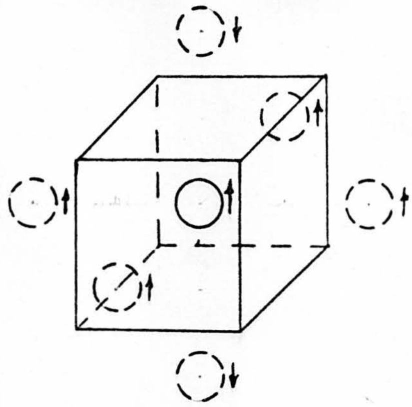
With both damping and hydrodynamic mass available, the impedance can be calculated. The major difficulty associated with the natural frequency method is that for a given spring-mass combination, there is but one frequency. Consequently, a great number of springs and masses are required to obtain data over a wide range of frequencies.

The third method employed to measure impedance and hydrodynamic masses is similar to the first method, in that forces and accelerations are measured directly. This method involves measurement of the forces and accelerations as the test body is mechanically oscillated in the fluid. A phase angle is also observed, thus the force can be resolved into its resistive and reactive components. The reactive component can be divided by the acceleration yielding the total mass below the load sensing element. When the mass of the body is subtracted, the remainder is the hydrodynamic mass. The impedance is determined by dividing the force by the velocity:

$$Z = \frac{F}{V} = \frac{F\omega}{A}$$

This method allows measurement over a wide range of frequencies and amplitudes.

It is well known that the hydrodynamic mass is influenced by the presence of boundaries in the fluid medium. It is not well known however, how to correct for the presence of the boundaries. In principle the effect of the boundaries can be accounted for by the image method. An image is placed on the "other" side of each boundary and is given a motion similar to that of the real body such that the normal velocities at the boundaries are equal to zero.



However, if one is capable of solving this problem; the simple problem of computing the hydrodynamic mass of the body could also be solved, and there would be no need for experimentation at all! The boundaries can be accounted for approximately by calibrating the test tank with a sphere and a disc then applying the correction to bodies of similar shape.

C. Apparatus

1. Design of the Equipment

The design requirements for the experimental apparatus were as follows:

1. The equipment must be capable of being used for the three test methods discussed.
2. The frequency range of the oscillating equipment should be from 0.0 to 2.0 c.p.s.
3. The amplitude range of the oscillating equipment should allow for total displacement to diameter ratios from 0 to 1.
4. The test body will be mounted on the end of a shaft such that the effect of the shaft on the body's hydrodynamic mass is small.
5. The apparatus should utilize available equipment and instrumentation.

The test tank used had the dimensions 20'X20'X7' and was filled to a depth of 5'. It had concrete walls and floor; and was located one floor level below the labratory area.

Three test bodies were available at the start of this study, a 12 inch diameter sphere; an 18 inch diameter sphere and a 12 inch by 24 inch ellipsoid. These bodies were constructed from white pine and were designed to be mounted on the end of a 1 inch diameter shaft. Ideally, the driving shaft and test body should have as little mass as possible in order to increase the accuracy of the determined hydrodynamic mass. However, if the mass of the body is less than that of an equal volume of water, the body and shaft are very difficult to handle if the body is being held under the water by the shaft. (The system's most unstable position.) Also, the cost of construction and availability of materials must be considered. With these requirements in mind, it was decided that in general wooden bodies would be satisfactory. If a certain method of testing required a neutrally buoyant body, a wooden body could be easily drilled and filled with lead.

Because the access hole in the test tank is 2 ft. square, the maximum model size must be 2 ft. The maximum frequency and the -1.00 to + 1.50 G range of the available accelerometer indicated that the maximum vertical dimension of the body should be 6.1 inches. However, at lower frequencies, the displacement to diameter ratio requirement could be met with a larger body. Also, the accuracy of the data increases as the body size increases thus a 12 inch maximum vertical size was decided upon.

To measure forces at the body, a load sensor was designed. The device had to measure axial loads from - 50 to + 50 lbs. and be capable of supporting the body if the shaft is held horizontally. (Bending loads) The final decision incorporated a 1 1/16" O.D. nylon cylinder with 1/32" walls. A four gage bridge was used utilizing strain gages. The system is therefore temperature compensated and not influenced by bending loads.

The 6 inch displacement amplitude along with the requirement to make the motion for the driven mode as close to simple harmonic motion as possible, dictated the connecting rod length and thus; the test frame height. The final design is shown in figures 1 and 2.

The design power for a 12 inch diameter sphere being operated at maximum frequency could not be obtained with the larger sphere if it were used, a half horsepower electric motor was used. The motor's speed was varied with a "variac".

The signals from the accelerometer and from the load sensor were recorded on a two-channel recording oscillograph. Design drawings of the apparatus and its component parts are on file in the Mechanical Engineering Department at the University of Rhode Island.

2. Use of the Equipment

The free translation tests are made by allowing the body and vertically guided shaft to either float to the surface or sink, depending whether or not the body-shaft combination was buoyant.

The natural frequency tests are made by connecting the body-shaft combination to a spring, the lower end of which is oscillated through a 1/2 inch amplitude. The motor's speed was varied until resonance occured. When the body-shaft combination was buoyant, only one spring was used (Mode I). When the body-shaft combination was only slightly buoyant or non-buoyant, two or more springs were used; one connected to the top of the test frame in parallel with the driven spring. (Mode II)

The forced oscillation tests were conducted with the connecting rod connected to the driving shaft and to one of five possible crank-pin locations in the flywheel.

(There are five amplitudes available--2, 3, 4, 5, and 6 inch.) The motor speed was varied and forces and accelerations were measured.

D. Test Proceedure

The first series of tests using the free translation method were conducted with a 2:1 Ellipsoid as the test body. The 24 by 12 inch wooden body was forced underwater by pushing on the shaft and was released suddenly such that it could "bob" to the surface. A "strobotach" flashed every tenth of a second giving multiple exposures on a time exposed film. A displacement vs. time diagram could be constructed from the multiple exposure photograph.

The second series of tests conducted with the free translation method used the strain-gaged load sensor and the accelerometer. After calibrating both instruments, the body and vertically guided shaft were allowed to sink or float depending on the buoyancy of the body. The forces and acceleration were recorded.

The buoyancy of the test body dictated which spring arrangement would be used for the natural frequency test method. If there was sufficient buoyant force to give the spring a steady-state displacement greater than the displacement amplitude, Mode I of testing was utilized. If the body sank or was neutrally buoyant, Mode II was used. With the springs attached, motor speed was varied until resonance occured. At resonance, the natural frequency was measured by recording the accelerations on the recorder, and the total displacement was measured by sighting across the top of the guide shaft onto a ruler. Other marked data on the strip of recorder paper were the weight of the body-shaft-accelerometer combination, the submergence of the body, the springs used, the mode of spring set-up, and other information pertaining to the particular run.

The electric motor was reversed on its mount and the sprocket used to chain drive the 5.8:1 gear reducer was removed. The pulley and vee belt were assembled and the connecting rod was connected to the guide shaft. The test frame could then be used for the forced oscillation method of testing. The amplitude is selected and the crank-pin is located in the proper hole. The variac is adjusted until the motor is at the desired speed and the recorder is switched on, recording accelerations and forces. Other data marked on the recorder paper are the body weight, submergence, the gain of the preamplifiers and other pertinent information. The gain is needed so that the correct calibration is used to read the recorded data.

E. Test Accuracy

Considering the worst run during the free translation tests, it was estimated that the force could be read within $\pm$ 3.9% and the acceleration within $\pm$ 4.5%. This gives a possible error in the experimental hydrodynamic mass of $\pm$ 12.9%.

The data runs most likely to have large errors for the natural frequency method of testing are when the hydrodynamic mass is small in relation to the mass of the body and shaft and when the resonant frequency is low. Run number 124 would yield hydrodynamic mass within $\pm$ 10.1% assuming the frequency can be read within 1/2 cycle and that the mass of the body and shaft can be determined within 0.5%. The usual data run, with the same input accuracy, yields hydrodynamic mass values good for $\pm$ 3.1%. The acoustic resistance or damping values are accurate to $\pm$ 3.0%. Thus mechanical impedance values are good to $\pm$ 3.0%.

The accuracy of the data obtained during the forced oscillation tests is comparable to that obtained during the free translation tests. This is so because the readings were made with the same devices and with the same accuracy.

A significant factor influencing the accuracy of these tests is the size of the testing tank. Boundary effects were accounted for by calibrating the tank with a sphere and a circular disc. The calibration for the sphere was applied to all "three-dimensional" bodies and the calibration for the circular disc was applied to all discs and flat plates. However, the calibration for a sphere is only valid for a sphere and is not absolutely correct for a body of some other shape. Thus, errors are introduced because the boundary effects cannot be accounted for exactly.

F. Results of Tests

1. Free Translation Tests--The results of the free translation hydrodynamic mass tests for a 2:1 ellipsoid are shown in Table 1. The test method used is the first described under "Test Procedure." Hydrodynamic mass factors are based upon the mass of the displaced volume filled with water. A sample of the graphical data reduction method is shown in Figure 3.

Table 2 exhibits the results of free translation tests for a sphere. These tests were conducted using the second test method. Again, the hydrodynamic mass factor is based upon the mass of the displaced volume filled with water.

Results of free translation tests using the second test method for a circular disc are shown in Table 3. The hydrodynamic mass factor is based upon the theoretical hydrodynamic mass, A sample oscillograph trace for the disc is shown on Figure 4.

2. Natural Frequency Tests--Thirty-three bodies were used for the 160 natural frequency, hydrodynamic mass data runs. These test bodies are described in Table 4. The data obtained with these bodies are shown in Table 5.

Tabulated results of the natural frequency tests are shown in Table 6. Hydrodynamic mass factors for the various bodies are based upon the following:

ellipsoid--mass of displaced water
ellipsoid with wings--mass of displaced water of ellipsoid only.
sphere--mass of displaced water
disc and plates--theoretical hydrodynamic mass of circular discs
I-beam--mass of displaced water of a circular cylinder of the same width
streamlined bodies (bodies no. 23, 24, 25, and 26)--refer to Appendix G.
parallelepipeds--mass of displaced water

Resistance (friction) values have been corrected to remove resistance due to the driving shaft and friction in the test frame assembly. The resistance for the test frame assembly was determined by the log decrement method after tests without a body and is shown on Figure 5 for different shafts.

Displacement to diameter ratios are the ratios of the total distance travelled in a half cycle (twice the displacement amplitude) to the minimum horizontal diameter of the body. Submergence to diameter ratios are the ratios of the distance from the surface of the water to the vertical geometric center of the body to the minimum horizontal diameter of the body.

The dimensionless frequency $\omega R/c$ is used in preference to the more common hydrodynamic dimensionless frequency $\omega L/\nu$ because the phenomonon involved is acoustic in nature. The characteristic diameter used is the minimum diameter in the horizontal plane through the body's C. of G. The sound velocity in water has been assumed to be 5000 ft./sec. for ease of calculation.

Least mean squares plots of hydrodynamic mass vs. frequency and displacement are shown on Figures 6 and 7 respectively for a 2:1 ellipsoid with 20% wings (the area of the wings is equal to 20% of the area of the elliptical section.) The mean submergence to diameter ratio for these tests was 2.0.

The impedance factor listed in the tables is based upon the same characteristic mass as the hydrodynamic mass factor. To compute the mechanical impedance, one would multiply the impedance factor by the product of the angular freguency and the characteristic mass (usually the mass of displaced fluid).

Hydrodynamic mass factors and mechanical (acoustical) impedance factors are listed in Table 7 for a 2:1 ellipsoid with and without wings. The hydrodynamic mass factors have been extrapolated to infinite submergence on the assumption that the theoretical hydrodynamic mass factor for a 2:1 ellipsoid is correct. The phase angle listed in all tables is the angle between the resistance (damping) component and the impedance vector. Values of hydrodynamic mass factors from Table 7 are plotted on Figure 8.

Results of the natural frequency tests on spheres have been reduced to the mean values listed in Table 8. The mean hydrodynamic mass factors at various submergence to diameter ratios are plotted on Figure 9. This curve was used to calibrate the tank.

Figure 10 shows the effect of submergence on the hydrodynamic mass factor of 6 inch diameter, circular steel discs. The mean displacement to diameter ratio for these data is 0.55. The mean dimensionless frequency for these data is 11.49×10^{-3}. A circular disc of fir plywood was used to calibrate the tank for the other plywood discs and plates. The results of these runs, along with the runs for the other discs and plates are listed in Table 9. End effects for rectangular flat plates are shown on Figure 8.

An I-beam section (body no. 22) was tested. Its hydrodynamic mass factor and impedance factor are listed in Table 10.

Four typical towed bodies were tested for hydrodynamic mass and mechanical impedance in vertical translation. These bodies are described in Figures 13 through 19 inclusively. The results of these tests are shown in Table 11. Appendix G contains the characteristic masses to be used with the hydrodynamic mass factors and impedance factors listed.

Table 12 and Figure 12 display results of tests on parallelepipeds.

3. Forced Oscillation Tests--Figure 20 shows typical data from the forced oscillation tests of a sphere. Results of these tests for spheres are listed in Table 13.

A sample data trace for a circular disc is shown in Figure 21. Results for a circular disc are listed in Table 14.

IV ANALYSIS AND DISCUSSION OF RESULTS

A. Hydrodynamic Mass

The first method used in the free translation tests did not give reliable results. The accuracy of the results is poor because the graphical differentiation process is inherently inaccurate. The hydrodynamic mass factors for a 2:1 ellipsoid as found by this method are, on the average, higher than the theoretical values.

Free translation tests of a sphere yielded hydrodynamic mass factors 310% over theoretical. This occured because the accelerometer did not have the required accuracy at low acceleration amplitudes. The accelerometer was calibrated at high acceleration amplitudes and the calibration was extrapolated to lower acceleration levels. Evidently this process was in error.

Free translation tests of a circular disc however, yielded results within 10.7% of theoretical. Larger forces and accelerations were experienced on these runs.

One of the objectives was to determine the effect of frequency on hydrodynamic mass. Unfortunately, the testing did not cover a wide enough frequency range to completely fulfill this objective. The test runs for the 2:1 ellipsoid with 20% wings cover the widest frequency range. Figure 6 shows a least-mean squares plot of this data which indicates an increase in hydrodynamic mass with increasing frequency. The points are too scattered to construct any curve other than a least mean squares curve. The increase with increasing frequency is noted for the hydrodynamic masses; and, to a lesser degree, for the mechanical impedance for all other bodies. This observed increase is contrary to what one expects if the variation of hydrodynamic mass with frequency for a circular disc is considered. Theoretically, the hydrodynamic mass decreases with increasing frequency for a circular disc.

The effect of displacement amplitude on hydrodynamic mass is shown in Figure 7. Again, the trend is for increased hydrodynamic mass with increased displacement amplitude. This same general trend is observed with the other bodies. A possible explanation of this effect is that the increased velocity amplitude (if the frequency is the same) for a greater amplitude causes an increase in the boundary layer thickness on the body. Because the hydrodynamic mass is a function of the body sized cubed, a small increase in the body's effective size will cause a significant increase in the body's hydrodynamic mass. A very rough calculation, using the mean displacement boundary layer thickness for a flat plate moving at the root-mean-square velocity of the ellipsoid, indicates an increase in hydrodynamic mass of 15% for displacement to diameter ratios of 0.4. This same explanation may apply to the increase of hydrodynamic mass with frequency. Again, the velocity amplitude increases with increasing frequency; thus, the boundary layer thickness would increase.

The increase computed was 15%, the increase observed was in the order of 20 to 30%; thus, the above hypothesis is a possible explanation of the observed increase.

Results of tests on an ellipsoid with wings yielded reasonable results. Theory and common sense predict an increase in hydrodynamic mass as the wing area increases. One would also expect a non-linear curve because hydrodynamic mass is not an additive property.

Figure 9, the effect of submergence on the hydrodynamic mass of spheres, was used to calibrate the test tank. The free surface causes an increase as it is approached. However, as the body begins to emerge from the fluid, the hydrodynamic mass decreases.

The effect of the free surface was seen to have different effects on bodies of different materials. This may be explained by the following hypothesis:

The mode of energy transfer involved in this process of accelerating a body in a fluid is sonic. A pressure wave is created which propogates away from the body. True that the sound is of extremely low frequency and intensity but it is sound. As the body moves toward the surface (any surface) it generates a pressure wave which propogates away from the body at the speed of sound in the fluid. As the sound wave strikes the boundary, some of its energy is transmitted through the boundary, and some is reflected depending upon the acoustic impedance mis-match at the boundary.

The reflected wave travels from the boundary to the body. If the acoustic impedance of the body is close to that of the fluid in which it is immersed, the reflected sound wave passes through the body with little or no effect on the body. (This was observed with the plywood circular disc.--mis-match $\approx$ 0.10x10⁶ RAYLS) If the impedance mis-match is great however, the body "see's" a pressure increase on the side toward the boundary. This pressure increase is added to that due to the body's acceleration. In other words, the acoustic or mechanical impedance of the body is increased; thus, its hydrodynamic mass is increased. (The steel disc at the same submergence to diameter ratio as the plywood disc has a much greater increase in hydrodynamic mass. Mis-match $\approx$ 45×10^6 RAYLS

This reasoning leads to the conclusion that if the test body had the same acoustic impedance as the fluid, there would be no surface effects!

Tests conducted with plywood discs and flat plates used the circular disc as a calibration standard. The tank surfaces caused a 7.9% increase in hydrodynamic mass for the disc. This increase was accounted for to obtain hydrodynamic masses at infinite submergence for the other bodies. The hydrodynamic masses for the elliptical discs were found to be within 4.0% of their theoretical values using this calibration, thus it is concluded that these results were quite reliable .

An I-beam section when tested yielded hydrodynamic mass factors of reasonable values. If one extrapolated from the hydrodynamic mass factor of a rectangular section, an hydrodynamic mass factor of the observed magnitude would be obtained.

Table 11 lists hydrodynamic mass factors for various towed bodies. These results are regarded as significant because hydrodynamic mass factors for bodies of this nature cannot be found in the literature.

Table 12 and Figure 12 show the variation of hydrodynamic mass factor with body depth to width ratio for parallepipeds. Again, this is not available in the literature.

Forced oscillation test results for spheres and circular discs are displayed in Tables 13 and 14 respectively. As with the free translation tests, the acceleration levels attained were to low to give reliable results. The results can be used comparitively however to observe an increase in hydrodynamic mass with increasing displacement amplitude. This manner of testing shows the greatest promise as a test method to determine the effects of frequency and displacement amplitude on hydrodynamic mass and impedance. With a lower range accelerometer, better results would have been obtained. Upon completion of the tests with a circular disc, the strain-gaged load sensor flooded and further testing with this method could not be undertaken.

Limits for the Use of Appendix G--The hydrodynamic mass factors found experimentally during this study, along with hydrodynamic mass computations available in the literature are listed in Appendix G to facilitate usage. Because many of these factors have been obtained from ideal fluid theory, and because the testing during this study has been done for low frequencies, these hydrodynamic mass factors are only valid for zero frequency. Also, the displacement amplitude will influence the factors listed. Other facts to be considered when reference is made to Appendix G is that the values listed for a given body are only valid for vertical motion of that body as shown. Also, one should consider frictional forces (resistive terms) which are not listed.

B. Mechanical Impedances

Because the accuracy of the results for mechanical impedances is less than that for hydrodynamic masses, the impedances listed are to be regarded as order of magnitude figures only. Also, the effects of the free surface or of boundaries on mechanical impedance are not known. Thus, the mean values listed are valid only for the submergences and frequencies shown.

V CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

As illustrated in the text of this thesis, the major objective of this study has been accomplished. The hydrodynamic mass of a body is a measurable quantity and although difficult to measure, can be determined accurately if the correct experimental techniques are used. The natural frequency method of testing was found to be the most reliable. The other methods would have been equally reliable if the proper instrumentation were used. The natural frequency method was used to experimentally determine hydrodynamic masses for many bodies of mathematically difficult shapes.

The study has shown a definite effect on hydrodynamic mass due to frequency of oscillation and due to displacement amplitude. A possible explanation of these effects has been presented.

Mechanical impedances have been investigated but further investigation is called for to extend the scope of these investigations.

B. Recommendations

The values listed in Appendix G should be added to such that the table would include hydrodynamic masses for other directions of translation. Also, a study of rotational hydrodynamic masses should be undertaken. Finally, the cross-coupled hydrodynamic mass factors for motions involving rotatation and translation should be studied. Although much of this can be approached through the use of stability derivatives, stability derivatives break down when a body undergoes gross oscillations with complete cross-coupling.

The effect of frequency on hydrodynamic mass and on mechanical impedance should be investigated more extensively. A study of the mechanical impedances at various submergences is needed.

VI SUMMARY

The major objective has been accomplished. Hydrodynamic masses were measured with a reasonable degree of accuracy for many bodies using the natural frequency test method. The objective of determining the dependency of hydrodynamic mass on frequency was partially fulfilled. Over the narrow frequency range in which the tests were conducted, the hydrodynamic mass appeared to increase with increasing frequency. An extension of this study would involve testing over a greater range of frequencies.

Another objective, that of determining the effect of displacement amplitude on hydrodynamic mass, has been accomplished. The hydrodynamic mass appeared to increase with increasing displacement. This effect, along with the narrow range frequency dependancy, is explained by the hypothesis presented.

The tables presented contain hydrodynamic mass factors and mechanical impedance factors for various bodies. Some of these factors cannot be found in the literature.

An appendix is presented which summarizes the majority of hydrodynamic mass factors available including the results of this study.

VII ACKNOWLEDGEMENTS

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Appendix A

Nomenclature

A - body acceleration

a - radius of a circular disc, cylinder or a sphere

A_{ij} - hydrodynamic mass dyadic

c - velocity of sound in a fluid medium

C - damping constant

D - drift volume

e - 2.71828

f_r - reaction force on a piston

F - force

F_m - magnitude of a force phaser

g - gravitation constant, 32.2 ft./sec.

i,j - direction indices

j - imaginary number $\sqrt{-1}$

k - spring constant, wave number

K - hydrodynamic mass factor

K' - impedance factor

L - characteristic length

M_b - body mass in vacuum

M_h - hydrodynamic mass

M_t - total mass in motion

n - unit vector normal to a surface area S

p - pressure

P - location of a point in space

r - spherical radius to a point in space

r' - spherical radius from a point in space to an elemental area on the face of a piston

R_r - radiation resistance

S - surface area

t - time

T - kinetic energy of a fluid field

T_r - transmissibility, ratio of output displacement amplitude to input displacement amplitude

u - fluid velocity

U - body velocity

U_o - body velocity amplitude

V - body velocity

X - coordinate axis

Xm - magnitude of displacement phasor

X_r - radiation reactance

y - coordinate axis

Y - stream function

z - coordinate axis

Z - mechanical impedance

θ - angular dimension

ν - kinematic viscosity

π - 3.1416

ρ, ρ_o - mean fluid density

σ - radius to an elemental area on a piston

ϕ - velocity potential

ψ - angular dimension

ω - angular frequency of a simple harmonic motion

ω_n - natural frequency

ω_{nf} - natural frequency in the fluid

ω_{nv} - natural frequency in a vacuum

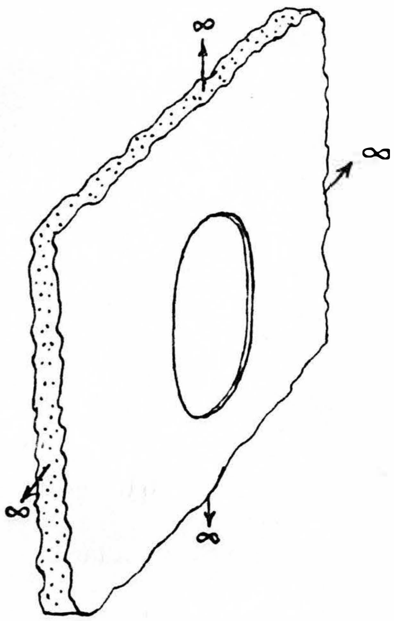
Appendix B

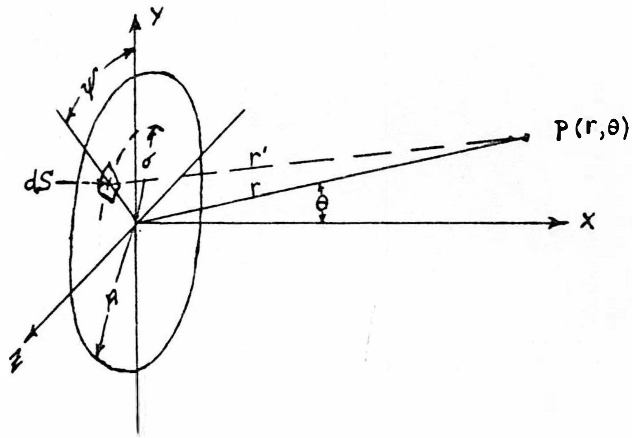
Computation of Hydrodynamic Mass

By Direct Integration of Pressures

The following analysis is discussed in greater detail in reference 22, Chapter 7.

Consider a rigid circular piston mounted in an infinite baffle.





Assume that the piston is moving with simple harmonic motion. The pressure produced at any point by the piston is the sum of the pressures that would be produced at the point by an equivalent assembly of simple sources.

For a simple source:

$$p = \frac{j\rho_0 c k A^2 U_0}{r} e^{j(\omega t - kr)}.$$

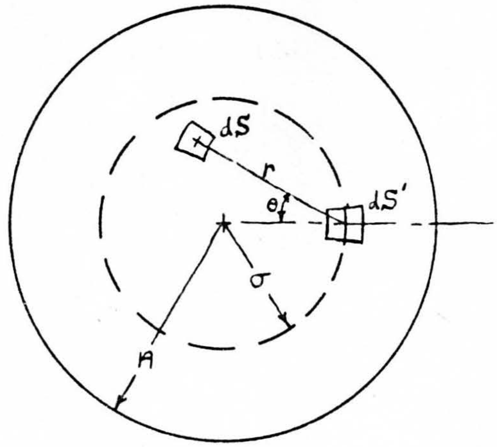
Thus each element of area dS contributes an element of pressure dp given by:

$$dp = \frac{j\rho_0 c k}{2\pi r'} (\vec{U} \cdot d\vec{S})\, e^{j(\omega t - kr')}$$

Since the motion of every surface element is normal to the surface of the piston, the scalar product $\vec{U} \cdot d\vec{S}$ is $U_0\, dS$.

Thus: $dp = \frac{j\rho_0 c k}{2\pi r'} U_0\, dS\, e^{j(\omega t - kr')}$

To determine the effect of the medium on the piston, consider the pressures acting on the face of the piston induced by the motion of the piston.



Let dp be the increment increase in pressure that the motion of dS produces in the medium at a point adjacent to some other element of area of the piston, dS'. The total increase in pressure in the medium adjacent to dS' can be found by integrating

$$dp = \frac{j\rho_0 cK}{2\pi r'} U_0 \, dS \, e^{j(\omega t - Kr')}$$

over the surface of the piston.

$$p = \iint \frac{j\rho_0 cK}{2\pi r} U_0 \, e^{j(\omega t - Kr)} \, dS$$

where r is as shown

The total reaction force acting on the piston is

$$f_r = -\iint p \, dS'$$

$$f_r = -\frac{j\rho_0 cK}{2\pi} U_0 \, e^{j\omega t} \iint dS' \iint \frac{e^{-jKr}}{r} \, dS$$

The reaction force acting on an element dS' due to the motion of dS is the same as the force acting on dS due to the motion of dS', so that the ultimate result of the double integration indicated in the previous equation is exactly twice that which would be obtained if the limits of integration were so chosen as to include the force due to each pair of elements only once.

Let σ be the radical distance of element dS' from the center of the piston. Then we may ensure that each pair of elements is used only once by integrating with respect to dS only over the area of the piston that is included within a concentric circle of radius σ. Let θ be as shown and let $r\,d\theta\,dr$ represent an element of area dS. The maximum distance, in the direction θ, from dS' to any point within the circle of radius σ is $2\sigma\cos\theta$. The entire area within this circle will be covered if we integrate r from 0 to $2\sigma\cos\theta$, and θ from $-\pi/2$ to $+\pi/2$. The integration of dS' is now to be extended over the entire surface of the piston. Let $dS' = \sigma\,d\sigma\,d\psi$ and then integrate ψ from 0 to 2π and σ from 0 to A. Thus:

$$f_r = -\frac{j\rho_0 ck}{\pi} U_0 e^{j\omega t} \int_0^{A} \sigma\,d\sigma \int_0^{2\pi} d\psi \int_{-\pi/2}^{\pi/2} d\theta \int_0^{2\sigma\cos\theta} e^{-jkr}\,dr$$

carrying out the integration,

$$f_r = -\rho_0 c\pi A^2 U_0 e^{j\omega t}\left[R_1(2kA) + jX_1(2kA)\right]$$

where: $$R_1(2kA) = \frac{(2kA)^2}{2\cdot 4} - \frac{(2kA)^4}{2\cdot 4^2\cdot 6} + \frac{(2kA)^6}{2\cdot 4^2\cdot 6^2\cdot 8} - \cdots\cdots$$

$$X_1(2kA) = \frac{4}{\pi}\left(\frac{(2kA)}{3} - \frac{(2kA)^3}{3^2\cdot 5} + \frac{(2kA)^5}{3^2\cdot 5^2\cdot 7} - \cdots\cdots\right)$$

The acoustic impedance is the force exerted by the piston on the medium divided by the piston velocity, hence,

$$Z_r = \frac{-f_r}{U_0 e^{j\omega t}} = \rho_0 c \pi a^2 \left[R_1(2ka) + jX_1(2ka) \right]$$

consequently, the radiation resistance is:

$$R_r = \rho_0 c \pi a^2 R_1(2ka)$$

and reactance is:

$$X_r = \rho_0 c \pi a^2 X_1(2ka)$$

The radiation reactance is always positive, and its effect is therefore equivalent to adding to the actual mass of the piston an additional mass.

(the hydrodynamic mass)

$$m_h = \frac{X_r}{\omega} = \pi a^2 \rho_0 \frac{X_1(2ka)}{k}$$

as $\omega \longrightarrow 0$, $X_1(2ka) \longrightarrow \frac{4}{\pi} \frac{2ka}{3}$

Therefore, for low frequencies the hydrodynamic mass of a piston or a circular disc is:

$$m_h = \pi a^2 \rho_0 \frac{4 \cdot 2 \cdot k \cdot a}{\pi \cdot k \cdot 3} = \frac{8}{3} \rho_0 a^3$$

Appendix C

Computation of Hydrodynamic Mass by Kinetic Energy Method

The kinetic energy of a flow is:

$$2T = -\rho \int_S \phi \frac{\partial \phi}{\partial n} dS \quad ;$$

it is also equal to:

$$2T = m_h U^2$$

Thus, the hydrodynamic mass is equal to:

$$m_h = \frac{2T}{U^2} = \frac{-\rho \int_S \phi \frac{\partial \phi}{\partial n} dS}{U^2}$$

For a circular cylinder,

$$\phi = -\frac{U A^2}{r} \cos\theta$$

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial r} = \frac{U A^2}{r^2} \cos\theta$$

Thus the hydrodynamic mass per unit length for a circular cylinder is:

$$m_h = \frac{\rho A^2 U^2 \int_0^{2\pi} \cos^2\theta \, d\theta}{U^2} = \rho \pi A^2$$

(Refer to references 3, 4, and 8 to see this solution in greater detail.)

Appendix D

Computation of Hydrodynamic Mass

By Use of Darwin's Drift Concept

Consider a circular cylinder travelling to the right in the plus X direction. For a velocity of unity, the velocity potential is

$$\phi = X + \frac{A^2 X}{r^2}$$

The flow is described by:

$$\frac{dx}{dt} = -1 + A^2\left(\frac{X^2 - Y^2}{r^4}\right) \quad ; \quad \frac{dy}{dt} = A^2\left(\frac{2XY}{r^4}\right)$$

The integral of these equations is the stream function which may be written

$$y\left(1 - A^2/r^2\right) = Y$$

so that the constant Y corresponds to the initial and final position of the streamline with reference to the central line of motion.

The quantity required, the drift, is the total displacement of a particle in the X direction, referred to axes in which the infinite parts of fluid are at rest, thus; transforming such that the polar coordinate becomes the independant variable.

$$X = \int_{-\infty}^{\infty} (\dot{X} + 1)\, dt = \int_0^{\pi} \frac{A^2 \cos 2\theta \, d\theta}{\sqrt{Y^2 + 4A^2 \sin^2\theta}}$$

The drift volume, the volume enclosed by the initial and final positions of an infinite wall of fluid normal to the direction of motion, is given by:

$$D = \int_{-\infty}^{\infty} X \, dY$$

Solving this yields: $D = \pi A^2$

Thus, the mass of the drift volume, $\rho \pi A^2$, is equal to the hydrodynamic mass of the cylinder as obtained from the kinetic energy method. (Appendix C)

Dawrin has also used this approach to calculate the hydrodynamic mass of a sphere. The hydrodynamic mass calculated by means of the drift concept is in exact agreement with the hydrodynamic mass computed from the kinetic energy method.

Appendix E

Computer Programs

For Data Reduction

The data taken were reduced by means of computation on an IBM 1620 computer. The following program in FORGO II was used.

```
1.  READ, AIRM, S, FREQ, TR, DISPM, DIA
    WN = FREQ* .10472
    DDRAT = TR/(2.0*DIA)
    HM = (S/WN**2.0)—AIRM
    HMF = HM/DISPM
    C = SQRTF(AIRM*S/(TR* *2.0—1.0))
    Z = HM*WN
    F = FREQ/60.0
    PUNCH, F, DDRAT, HM, HMF
    PUNCH, C, Z
    GØ TØ 1
    STØP
    END
```

total oscillating mass in air	spring constant	frequency in water	transmissibility	mass of displaced water	characterist diameter of body

The result cards yield the following:

-on the first card-

frequency	displacement to diameter ratio	hydrodynamic mass	hydrodynamic mass factor

-on the second card-

resistance	reactance

For future use, this program should be re-written to include computation of the mechanical impedance, mechanical impedance factor dimensionless frequency and phase angle. If the same number of runs were made for each body tested, the program could include a least mean squares computation of the curve to fit the data.

Appendix F

Sample Calculations

1. Free Translation Tests

A. Data taken by multiple-exposure photograph of the free translation of a 2:1 ellipsoid. (Refer to Figure No. 3)

static buoyant force $B = 15.65$ lbs

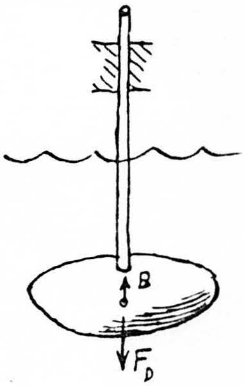
When the body has moved 5.0 inches, the velocity is 21.2 in/sec. and the acceleration is 50 in/sec²

Mass of body, $m_b = 1.055$ lb-sec²/ft.

Mass of shaft, $m_{sh} = 0.524$ lb-sec²/ft

Mass of displaced water, $m_{wd} = 2.027$ lb-sec²/ft

Drag coefficient of ellipsoid $C_D = 0.6$



$$\Sigma F_v = m_T A_v$$

$$B - F_D = (m_b + m_{sh} + m_h) A$$

$$m_h = \frac{(B - F_D)}{A} - (m_b + m_{sh})$$

hydrodynamic mass factor $K = m_h / m_{wd}$

$$K = \frac{\frac{(B - F_D)}{A} - (m_b + m_{sh})}{m_{wd}}$$

$$F_D = C_D \frac{\rho}{2} A V^2$$

$$A = \pi a b = 3.14 \cdot \frac{(1\ \text{ft.})}{2} \cdot \frac{(2\ \text{ft.})}{2} = 1.57\ \text{ft}^2$$

$$F_D = 0.6 \left(\frac{1.99\ \frac{\text{lb-sec}^2}{\text{ft}^4}}{2} \right) (1.57\ \text{ft}^2) \left(\frac{21.2\ \frac{\text{in}}{\text{sec}}}{12\ \frac{\text{in}}{\text{ft}}} \right)^2 = 2.95\ \text{lbs}$$

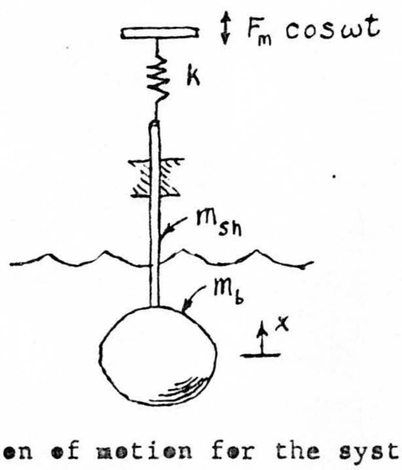
$$K = \frac{\dfrac{(15.65\ \text{lbs} - 2.95\ \text{lbs})\ 12\ \frac{\text{in}}{\text{ft}}}{50.0\ \frac{\text{in}}{\text{sec}^2}} - \left(1.055\ \frac{\text{lb-sec}^2}{\text{ft}} + 0.524\ \frac{\text{lb-sec}^2}{\text{ft}}\right)}{2.027\ \frac{\text{lb-sec}^2}{\text{ft}}}$$

$$= 0.723$$

B. Data taken from oscillograph recording of forces and accelerations for a 12 inch diameter sphere.

The calculations are made in the same manner as described in Part A. However, forces and accelerations are taken from the trace at the start of the motion to eliminate drag forces.

2. Natural Frequency Tests



The equation of motion for the system when the body is immersed is:

$$(m_b + m_{sh} + m_h)\frac{d^2x}{dt^2} + C\frac{dx}{dt} + Kx = F_m \cos \omega t$$

If the system oscillates with simple harmonic motion, at resonance:

$$(m_b + m_{sh} + m_h)\,\omega_{nf}^2 X_m = K X_m \quad \text{and} \quad C\,\omega_{nf} X_m = F_m$$

Thus, $m_h = \dfrac{K}{\omega_{nf}^2} - (m_b + m_{sh})$

And $C = \dfrac{F_m}{\omega_{nf} X_m}$

Consider run no. 41 (See Table 5)

Total oscillating weight 43.80 lbs

Spring Constant 7.758 lb/in

Resonant Frequency 49.50 cyc/min

Displacement Amplitude X2 5.75 in

Submergence 24.0 in

Body Diameter 12.0 in

displacement to diameter ratio $\dfrac{5.75 \text{ in}}{12.0 \text{ in}} = 0.48$

submergence to diameter ratio $\dfrac{24.0 \text{ in}}{12.0 \text{ in}} = 2.0$

dimensionless frequency

$$\frac{\left(49.5 \frac{\text{cyc}}{\text{min}}\right)\left(\frac{2\pi}{60} \frac{\text{rad/cyc}}{\text{sec/min}}\right)(1 \text{ ft})}{5000 \text{ ft/sec}} = 1.0341 \times 10^{-3}$$

hydrodynamic mass

$$m_h = \frac{7.758 \text{ lb/in}}{\left[\left(49.5 \frac{\text{cyc}}{\text{min}}\right)\left(\frac{2\pi}{60} \frac{\text{rad/cyc}}{\text{sec/min}}\right)\right]^2} - \frac{43.8 \text{ lbs}}{386 \text{ in/sec}^2}$$

$$= 0.17516 \ \frac{\text{lb-sec}^2}{\text{in}}$$

mass of displaced water

$$m_{wd} = \rho \frac{4}{3} \pi a b^2 = \left(.934 \times 10^{-4} \frac{\text{lb-sec}^2}{\text{in}^4}\right) \frac{4}{3} \pi (12 \text{ in})(6 \text{ in})^2$$

$$= 0.1690 \ \frac{\text{lb-sec}^2}{\text{in}}$$

hydrodynamic mass factor

$$K = \frac{m_h}{m_{wd}} = \frac{.17516}{.1690} = 1.0358$$

reactance

$$X_r = m_h \omega = (.17516 \ \tfrac{lb\text{-}sec^2}{in})(49.5 \ \tfrac{cyc}{min})\left(\frac{2\pi \ \tfrac{rad}{cyc}}{60 \ \tfrac{sec}{min}}\right)$$

$$= 0.90821 \ \tfrac{lb\text{-}sec}{in.}$$

resistance--not being able to measure the force directly, we can use the relation:

$$Tr = \frac{F_b}{F_s} = \frac{x_b}{x_s} = \frac{\sqrt{1+(2\,c/c_c)^2}}{\sqrt{(2\,c/c_c)^2}}$$

$$Tr^2 = \frac{1+(2\,c/c_c)^2}{(2\,c/c_c)^2} = \frac{1}{(2\,c/c_c)^2} + 1$$

$$Tr^2 = \frac{C_c^2}{4C^2} + 1$$

but $C_c = 2K/\omega_n$ Thus;

$$Tr^2 = \frac{4K^2}{4\omega_n^2 C^2} + 1$$

$$C^2 = \frac{K^2}{\omega_n^2 (Tr^2 - 1)}$$

$$C = \frac{K}{\omega_n \sqrt{Tr^2 - 1}}$$

Thus the total damping of the system

$$C_t = \frac{7.758 \ \tfrac{lb}{in}}{(49.5 \ \tfrac{cyc}{min})\left(\frac{2\pi \ \tfrac{rad}{cyc}}{60 \ \tfrac{sec}{min}}\right)\sqrt{\left(\frac{5.75 \ in}{1 \ in}\right)^2 - 1}}$$

$$C_r = 0.2638 \ \frac{\text{lb-sec}}{\text{in}}$$

The damping of the body alone is

damping of machinery & shaft $= 0.1177 \ \frac{\text{lb-sec}}{\text{in}}$

$$C = 0.2638 \ \frac{\text{lb-sec}}{\text{in}} - 0.1177 \ \frac{\text{lb-sec}}{\text{in}}$$

$$= 0.14603 \ \frac{\text{lb-sec}}{\text{in}}$$

The magnitude of the impedance is

$$Z = \sqrt{R_r^2 + X_r^2} = \sqrt{(.14603)^2 + (.90821)^2}$$

$$= 0.920 \ \frac{\text{lb-sec}}{\text{in}}$$

The angle that it leads the velocity is

$$\tan\theta = \frac{.90821}{.14603} = 6.21$$

$$\theta = 80.86^\circ .$$

The impedance factor is

$$K' = \frac{Z/\omega}{m_{wd}} = \frac{0.920 \ \frac{\text{lb-sec}}{\text{in}}}{\left(49.5 \ \frac{\text{cyc}}{\text{min}}\right)\left(\frac{2\pi \ \frac{\text{rad}}{\text{cyc}}}{60 \ \frac{\text{sec}}{\text{min}}}\right)\left(.1690 \ \frac{\text{lb-sec}^2}{\text{in}}\right)}$$

$$= 1.051$$

3. Forced Oscillation Tests

Force amplitude 12.60 lbs

Acceleration amplitude 0.45 G

Frequency 69.5 cyc/min.

Amplitude 2.0 in

Phase angle 10.90°

The total impedance $Z = \frac{F}{A/\omega}$

$$Z = \frac{12.60 \text{ lbs}}{.45\,(386\ \text{in/sec}^2) \Big/ \left(69.5\ \frac{\text{cyc}}{\text{min}}\right)\left(\frac{2\pi}{60}\ \frac{\text{rad/cyc}}{\text{sec/min}}\right)}$$

$$= 0.526\ \frac{\text{lb-sec}}{\text{in}}$$

The reactive component is:

$$X = .526\ \frac{\text{lb-sec}}{\text{in}}\ \cos 10.9^\circ = 0.517\ \frac{\text{lb-sec}}{\text{in}}$$

The resistive component is:

$$R = .526\ \frac{\text{lb-sec}}{\text{in}}\ \sin 10.9^\circ = 0.0995\ \frac{\text{lb-sec}}{\text{in}}$$

Hydrodynamic mass

$$m_h = \frac{X}{\omega} = \frac{.517\ \frac{\text{lb-sec}}{\text{in}}}{\left(69.5\ \frac{\text{cyc}}{\text{min}}\right)\left(\frac{2\pi}{60}\ \frac{\text{rad/cyc}}{\text{sec/min}}\right)}$$

$$= 0.071\ \frac{\text{lb-sec}^2}{\text{in}}$$

Appendix G

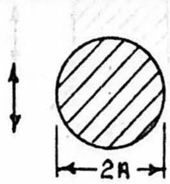
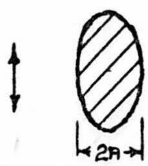
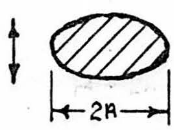
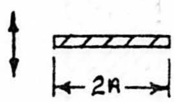
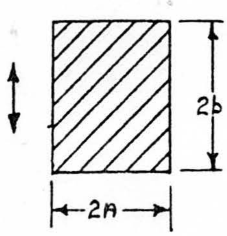
A Summary of Hydrodynamic

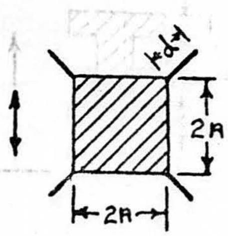
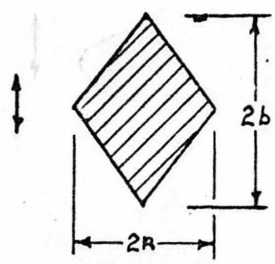
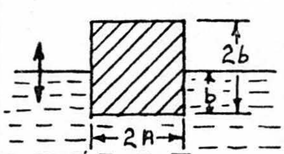
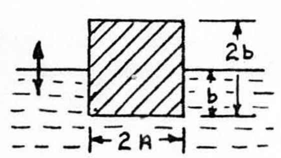
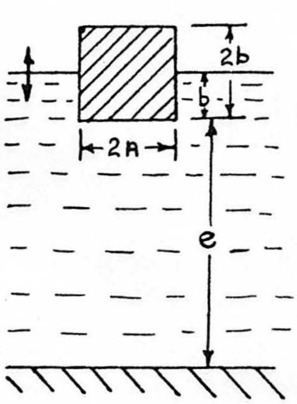
Mass Factors

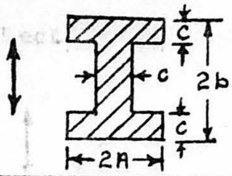
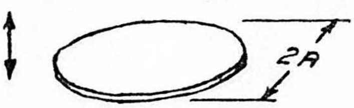
NOTATION FOR APPENDIX G

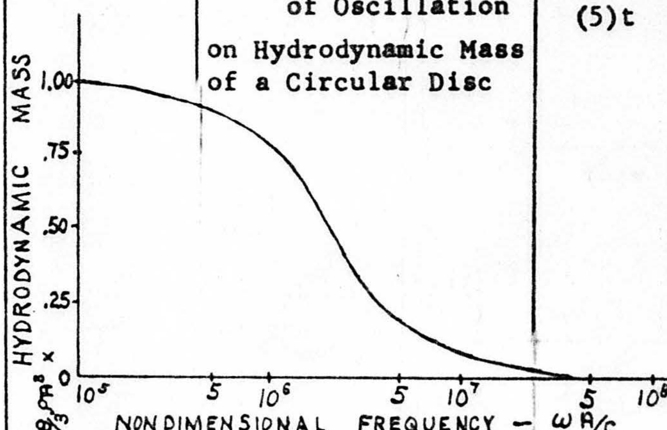
A	area	L^2
a,b,c,d,l	dimensions of bodies	L
e	distance from boundary to bottom of body	L
e	experimental	
F	force	F
i,j	indices used in tensor notation	
k	wave number	$1/L$
K	hydrodynamic mass factor	
m	mass	FT^2/L
m_h	hydrodynamic mass	FT^2/L
m_v	virtual mass	FT^2/L
N	ratio of wing area to area of body section	
π	3.1416	
ρ	density of fluid in which the body is immersed	FT^2/L^4
ϕ	velocity potential	
φ	normalized velocity potential	
S	distance from free surface to center of body	L
t	theoretical	

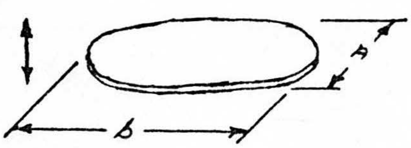
TWO DIMENSIONAL BODIES

Section Through Body	Translational Direction	Hydrodynamic Mass per Unit Length	Source
	Vertical	$m_h = 1\pi\rho a^2$	(4)t
	Vertical	$m_h = 1\pi\rho a^2$	(4)t
	Vertical	$m_h = 1\pi\rho a^2$	(4)t
	Vertical	$m_h = 1\pi\rho a^2$	(4)t, (6)e
 $a/b = \infty$	Vertical	$m_h = 1\pi\rho a^2$	(4)t
$a/b = 10$		$m_h = 1.14\pi\rho a^2$	(4)t
$a/b = 5$		$m_h = 1.21\pi\rho a^2$	(4)t
$a/b = 2$		$m_h = 1.36\pi\rho a^2$	(4)t
$a/b = 1$		$m_h = 1.51\pi\rho a^2$	(4)t
$a/b = 1/2$		$m_h = 1.70\pi\rho a^2$	(4)t
$a/b = 1/5$		$m_h = 1.98\pi\rho a^2$	(4)t
$a/b = 1/10$		$m_h = 2.23\pi\rho a^2$	(4)t

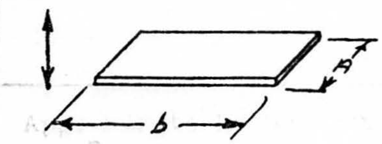
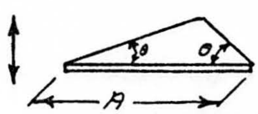
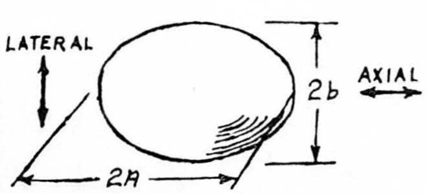
Section Through Body		Translational Direction	Hydrodynamic Mass per Unit Length	Source
	d/a=.05	Vertical	$m_h = 1.61 \pi \rho a^2$	(4)t
	d/a=.10		$m_h = 1.72 \pi \rho a^2$	(4)t
	d/a=.25		$m_h = 2.19 \pi \rho a^2$	(4)t
	a/b= 2	Vertical	$m_h = .85 \pi \rho a^2$	(4)t
	a/b= 1		$m_h = .76 \pi \rho a^2$	(4)t
	a/b=1/2		$m_h = .67 \pi \rho a^2$	(4)t
	a/b=1/5		$m_h = .61 \pi \rho a^2$	(4)t
	a/b= 1	Vertical (normal to free surface)	$m_h = .75 \pi \rho a^2$	(4)t
	a/b =1	Horizontal (parallel to free surface)	$m_h = .25 \pi \rho a^2$	(4)t
	a/b= 1; e/b=∞	Vertical (normal to free surface)	$m_h = .75 \pi \rho a^2$	(4)t
	e/b=2.6		$m_h = .83 \pi \rho a^2$	(4)t
	e/b=1.8		$m_h = .89 \pi \rho a^2$	(4)t
	e/b=1.5		$m_h \; 1.00 \pi \rho a^2$	(4)t
	e/b=.5		$m_h \; 1.35 \pi \rho a^2$	(4)t
	e/b=.25		$m_h \; 2.00 \pi \rho a^2$	(4)t

Body Shape	Translational Direction	Hydrodynamic Mass	Source
	Vertical	$m_h = 2.11\,\pi\,\rho\,a^2$	(6)e
THREE DIMENSIONAL BODIES			
1. Flat Plates Circular Disc 	Vertical	$m_h = \frac{8}{3}\,\rho\,a^3$ Effect of Frequency of Oscillation on Hydrodynamic Mass of a Circular Disc	(1)t, (5)t (5)t



Body Shape	Translational Direction	Hydrodynamic Mass	Source
Elliptical Disc 	As Shown	$m_h = Kba^2\,\frac{\pi}{6}\,\rho$	(7)t

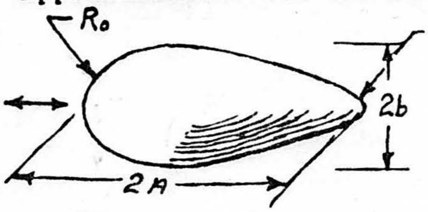
b/a	K
∞	1.00
14.3	.991
12.75	.987
10.43	.985
9.57	.983
8.19	.978
7.00	.972
6.00	.964
5.02	.952
4.00	.933
3.00	.900
2.00	.826
1.50	.748
1.00	.637

Body Shape	Translational Direction	Hydrodynamic Mass	Source
Rectangular Plates 	Vertical	$m_h = K \pi \rho \frac{a^2}{4} b$ b/a — K 1.0 — .478 1.5 — .680 2.0 — .840 2.5 — .953 3.0 — 1.00 3.5 — 1.00 4.0 — 1.00 ∞ — 1.00	(6)e
Triangular Plates 	Vertical	$m_h = \frac{\rho}{3} a^3 \frac{(\mathrm{TAN}\theta)^{3/2}}{(\pi)}$	(6)e
3. Bodies of Revolution **Spheres** 	Vertical	$m_h = \frac{2}{3} \pi \rho a^3$	(1)t, (2)t
Ellipsoids 	Vertical a/b 1.00 1.50 2.00 2.51 2.99 3.99 4.99 6.01 6.97	$m_h = K \cdot \frac{4}{3} \pi \rho ab^2$ K for Axial Motion — K for Lateral Motion .500 — .500 .305 — .621 .209 — .702 .156 — .763 .122 — .803 .082 — .860 .059 — .895 .045 — .918 .036 — .933	(1)t

Body Shape	Translational Direction	Hydrodynamic Mass	Source
Ellipsoids (continued)		a/b K axial K lateral 8.01 .029 .945 9.02 .024 .954 9.97 .021 .960 0 1.000	

a/b	K axial	K lateral
8.01	.029	.945
9.02	.024	.954
9.97	.021	.960
	0	1.000

Approximate Method for Elongated Bodies of Revolution.



$$m_h = K_1 \rho V = Ke \left[1+17.0(Cp-2/3)^2 + 2.49(M-1/2)^2 + .283\left[(r_o-1/2)^2 + (r_1-1/2)^2\right]\right]$$

where; K_1 - Hydrodynamic Mass coefficient for axil motion

Ke - Hydrodynamic Mass Coefficient for axil motion of an ellipsoid of the same ratio of a/b

V - Volume of body

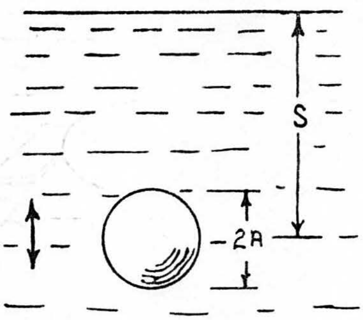
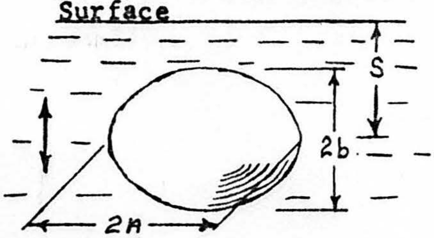
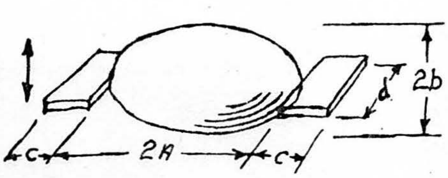
C_p - Prismatic coefficient $= \dfrac{4\,V}{b^2(2a)}$

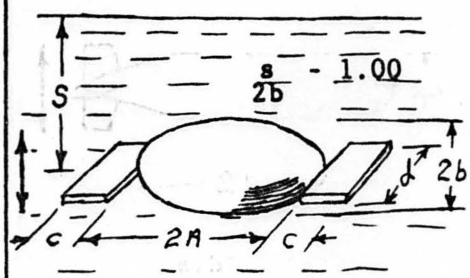
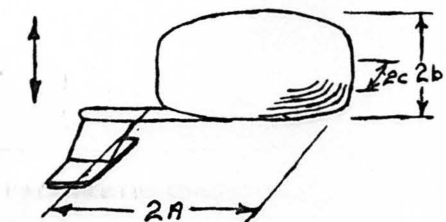
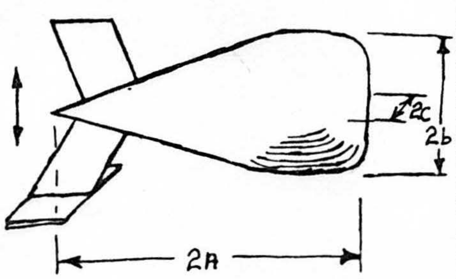
M - Nondimensional abscissa $X_{m/1}$ corresponding to maximum ordinate

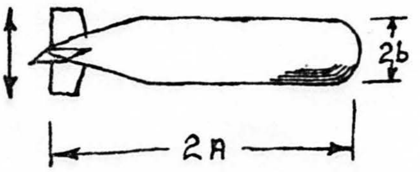
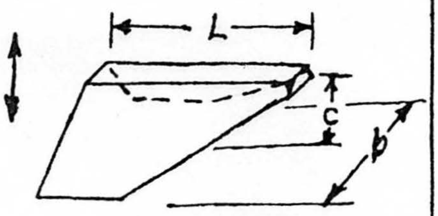
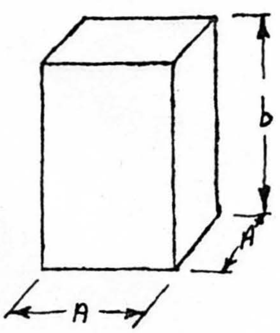
r_o, r_1 - Dimensionless radii of curvature at nose and tail

$$r_o = \frac{Ro\ (2a)}{b^2} \qquad r_1 = \frac{R1\ (2a)}{b^2}$$

Body Shape	Translational Direction	Hydrodynamic Mass	Source
	Lateral Motion	Munk has shown that the hydrodynamic mass of an elongated body of revolution can be reasonably approximated by the product of the density of the fluid, the volume of the body and the k - factor for an ellipsoid of the same a/b ratio.	

Body Shape	Translational Direction	Hydrodynamic Mass	Source
Sphere Near a Free Surface 	Vertical	$m_h = K\frac{2}{3}\pi\rho a^3$ s/2a — K 0 — .50 .5 — .88 1.0 — 1.08 1.5 — 1.16 2.0 — 1.18 2.5 — 1.18 3.0 — 1.16 3.5 — 1.12 4.0 — 1.04 4.5 — 1.00	(6)e
Ellipsoid Near a Free Surface 	Vertical	$m_h = K\cdot\frac{4}{3}\pi\rho ab^2$ $a/b = 2.00$ s/2b — K 1.00 — .913 2.00 — .905	(6)e
2. <u>Bodies of Arbitrary Shape</u> Ellipsoid with Attached Rectangular Flat Plates 	Vertical	$m_h = K\cdot\frac{4}{3}\pi\rho ab^2$ $a/b = 2.00;\quad c=b$ $c.d = N\pi ab$ N — K 0 — .7024 .20 — .8150 .30 — 1.0240 .40 — 1.1500 .50 — 1.2370	(6)e

Body Shape	Translational Direction	Hydrodynamic Mass	Source
Ellipsoid with Attached Rectangular Flat Plates Near a Free Surface.  	Vertical	$m_h = K \cdot \frac{4}{3} \pi \rho ab^2$ $a/b = 2.00; \quad c = b$ $c.d = N \pi ab$ N K 0 .9130 .20 1.0354 .30 1.3010 .40 1.4610 .50 1.5706	(6)e
Streamlined Body  $\frac{a}{b}$ - 2.38 $\frac{a}{c}$ - 2.11	Vertical	$m_h = 1.124\rho\left[\frac{4}{3}\pi ad^2\right]$ $d = \frac{c+b}{2}$	(6)e
Area of Horizontal "Tail" =25%	of Area of	Body Maximum Horizont	al Section.
Streamlined Body  	Vertical	$m_h = .672\rho\left[\frac{4}{3}\pi\ ad^2\right]$ $d = \frac{c+b}{2}$	(6)e
Area of Horizontal "Tail" = 20%	of Area of	Body Maximum Horizontal	Section.

Body Shape	Translational Direction	Hydrodynamic Mass	Source
"Torpedo" Type Body	Vertical	$m_h = .818\,\pi\rho b^2(2a)$	(6)e
			
$a/b = 5.0$			
Area of Horizontal "Tail" =	10% of Area	of Body Maximum Horizontal Section.	
V-Fin Type Body	Vertical	$m_h = .3975\,\rho L^3$	(6)e
			
$\frac{l}{b} = 1.0$ $\quad \frac{l}{c} = 2.0$			
Parallelepipeds	Vertical	$m_h = K\rho\, a^2 b$	(6)e
		b/a: 1, 2, 3, 4, 5, 6, 7, 10; K: 2.32, .86, .62, .47, .37, .29, .22, .10	

b/a	K
1	2.32
2	.86
3	.62
4	.47
5	.37
6	.29
7	.22
10	.10

REFERENCES AND SOURCES OF DATA

For Appendix G

1. H. Lamb, "Hydrodynamics," Cambridge University Press, 1932

2. G. Birkhoff, "Hydrodynamics," Princeton University Press, 1960

3. R. Bramig, "Experimental Determination of the Hydrodynamic Increase in Mass in Oscillating Bodies," DTMB Translation 118

4. K. Wendel, "Hydrodynamic Masses & Hydrodynamic Moments of Inertia," DTMB Translation 260

5. Kinsler-Frev, "Fundamentals of Acoustics," John Wiley & Sons, Inc., 1962

6. K. Patton, "An Experimental Investigation of Hydrodynamic Mass and Mechanical Impedances," Thesis, University of Rhode Island, 1964

7. M. Munk, " Fluid Mechanics, Part II," Aerodynamic Theory, Vol. I, edited by W. Durand, Dover Publications Inc., 1963

8. L. Landweber, "Motion of Immersed and Floating Bodies," Handbook of Fluid Dynamics, edited by V. Streeter, McGraw-Hill Book Company, Inc., 1961

TABLES

TABLE 1

RESULTS OF FREE TRANSLATION TESTS

OF A 2:1 ELLIPSOID

run no.	displacement in.	hydrodynamic mass factor	hydrodynamic * mass factor corrected for submergence
1	1.7	0.980	0.906
	5.0	0.723	0.723
	10.0	0.369	0.475
2	3.5	1.359	1.300
	5.0	1.280	1.280
	10.0	0.909	1.170
3	3.9	1.613	1.551
	5.0	1.520	1.520
	10.0	1.118	1.439
Average Values			
	3.03		1.252
	5.0		1.174
	10.0		1.028

* Theoretical hydrodynamic mass factor - 0.7024

TABLE 2

RESULTS OF FREE TRANSLATION

TESTS OF A SPHERE

run	hydrodynamic mass factor	hydrodynamic mass * factor corrected for submergence
1	0.686	0.636
2	2.540	2.350
3	3.230	2.990
4	2.400	2.221
average		2.049

* Theoretical hydrodynamic mass factor - 0.5

TABLE 3

RESULTS OF FREE TRANSLATION

TESTS OF A CIRCULAR DISC

run	hydrodynamic mass factor	hydrodynamic mass * factor corrected for submergence
1	0.753	0.699
2	1.260	1.170
3	1.157	1.071
4	0.580	0.540
5	1.180	1.095
6	0.845	0.784
average		0.893

* Theoretical hydrodynamic mass factor - 1.0

TABLE 4

DESCRIPTION OF BODIES USED

IN HYDRODYNAMIC MASS TESTS

Body Number	Description	Characteristic Dimensions-inches X (horizontal)	Y (horizontal)	Z (vertical)	Material
1	2:1 ellipsoid	24.0	12.0	12.0	soft pine
2	sphere	18.0	18.0	18.0	soft pine
3	2:1 ellipsoid with 1"thick pine "wings" attached to either end total "wing" area 20% of area of ellipsoid section	36.0	12.0	12.0	soft pine
4	2:1 ellipsoid with "wings" "wing" area 30% of ellipsoid section	36.0	12.0	12.0	soft pine
5	2:1 ellipsoid with "wings" "wing" area 40% of ellipsoid section	36.0	12.0	12.0	soft pine
6	2:1 ellipsoid with "wings" "wing" area 50% of ellipsoid section	36.0	12.0	12.0	soft pine
7	circular disc	6.0	6.0	0.25	steel
8	circular disc	12.0	12.0	0.375	fir plywood

Table 4--Description of Bodies Used in Hydrodynamic Mass Tests

(continued-2)

Body Number	Description	(horizontal)	(horizontal)	(vertical)	Material
9	square plate	10.625	10.625	0.375	fir plywood
10	1.5:1 rectangular plate	13.000	8.75	0.375	fir plywood
11	2:1 rectangular plate	15.00	7.50	0.375	fir plywood
12	2.5:1 rectangular plate	16.75	6.75	0.375	fir plywood
13	3:1 rectangular plate	18.375	6.125	0.375	fir plywood
14	60°-60°-60° triangular plate	16.1875	14.0	0.375	fir plywood
15	45°-90°-45^{0} triangular plate	21.25	10.625	0.375	fir plywood
16	1.5:1 elliptical disc	14.75	9.75	0.375	fir plywood
17	2:1 elliptical disc	17.0	8.5	0.375	fir plywood
18	4:1 rectangular plate	20.0	5.0	0.375	fir plywood
19	5:1 rectangular plate	25.0	5.0	0.375	fir plywood
20	sphere	12.0	12.0	12.0	soft pine
21	sphere - hollow, free flooding (rubber ball)	4.0	4.0	4.0	soft rubber
22	I--beam 2"wide, 3 1/2" deep web and flange thickness 3/4"	24.0	2.0	3.5	soft pine

Table 4--Description of Bodies Used in Hydrodynamic Mass Tests

(continued-3)

Body Number	Description	(horizontal)	(horizontal)	(vertical)	Material
23	Streamlined Body (refer to figure no. 13)	14.25	4.0	6.5	mahogony
24	Streamlined Body (refer to figure no. 14)	20.75	6.0	7.0	soft pine
25	"torpedo" type body (refer to figure no. 16)	20.0	4.0	4.0	soft pine
26	"V"-fin type body (refer to figure no. 18)	12.3125	12.0	5.625	soft pine
27	cube 1:1:1	3.5	3.5	3.5	fir
28	parallelepiped 1:1:2	3.5	3.5	7.0	fir
29	parallelepiped 1:1:3	3.5	3.5	10.5	fir
30	parallelepiped 1:1:4	3.5	3.5	14.0	fir
31	parallelepiped 1:1:5	3.5	3.5	17.5	fir
32	parallelepiped 1:1:6	3.5	3.5	21.0	fir
33	parallelepiped 1:1:7	3.5	3.5	24.5	fir

TABLE 5

HYDRODYNAMIC MASS DATA

NATURAL FREQUENCY TESTS

run number	body number	total weight oscillating lbs.	spring constant lb/in	suspension method	natural frequency cyc/min	displacement amplitude X 2 in.	submergence to center of body in.
1	1	52.50	3.2000	I	32.00	8.75	12.0
2	1	52.50	3.6400	I	35.00	8.00	12.0
3	1	52.50	6.4000	I	43.00	10.13	12.0
4	2	87.20	6.3780	I	36.00	6.75	15.0
5	2	87.20	7.7580	I	41.00	6.50	15.0
6	2	87.20	8.2320	I	40.50	7.00	15.0
7	2	87.20	8.8070	I	47.50	7.25	15.0
8	3	53.00	6.3780	I	42.00	6.00	12.0
9	3	53.00	3.2000	I	33.00	3.88	12.0
10	3	53.00	1.8850	I	29.50	1.50	12.0
11	3	53.00	3.6400	I	34.00	5.25	12.0
12	3	53.00	6.2602	I	45.00	6.75	12.0
13	3	53.00	6.3780	I	40.00	5.38	12.0
14	3	53.00	7.7580	I	47.00	6.00	12.0
15	3	53.00	3.2000	I	31.50	5.25	12.0
16	3	53.00	6.3780	II	40.00	3.25	12.0
17	3	53.00	1.8850	I	23.50	3.50	12.0
18	3	53.00	9.4427	II	49.00	4.50	12.0
19	4	53.88	6.2602	I	41.00	5.75	12.0
20	4	53.88	6.3780	I	39.00	5.38	12.0
21	4	53.88	7.7580	I	44.00	5.75	12.0
22	4	53.88	3.2000	I	29.50	4.13	12.0
23	4	53.88	6.3780	II	39.00	3.63	12.0
24	4	53.88	9.4427	II	48.00	3.75	12.0
25	5	54.02	6.2602	I	40.00	4.63	12.0

Table 5--Hydrodynamic Mass Data, Natural Frequency Tests

(continued-2)

run number	body number	total weight oscillating lbs.	spring constant lb/in	suspension method	natural frequency cyc/min	displacement amplitude X 2 in.	submergence to center of body in.
26	5	54.02	6.3780	I	38.50	4.88	12.0
27	5	54.02	7.7580	I	42.00	4.38	12.0
28	5	54.02	3.2000	I	28.00	4.19	12.0
29	5	54.02	6.3780	II	38.00	3.00	12.0
30	5	54.02	9.4427	II	46.50	4.00	12.0
31	6	54.29	6.2602	I	38.25	4.63	12.0
32	6	54.29	6.3780	I	38.00	4.75	12.0
33	6	54.29	7.7580	I	40.50	4.88	12.0
34	6	54.29	3.2000	I	27.50	3.69	12.0
35	6	54.29	6.3780	II	37.25	2.69	12.0
36	6	54.29	9.4427	II	45.75	3.88	12.0
37	1	43.80	6.2602	I	46.00	7.50	24.0
38	1	43.80	9.4427	II	57.50	6.88	24.0
39	1	43.80	3.2000	I	35.50	----	24.0
40	1	43.80	6.3780	I	45.50	8.75	24.0
41	1	43.80	7.7580	I	49.50	5.75	24.0
42	7	18.70	6.2602	II	95.00	2.75	30.0
43	7	18.70	4.9682	II	92.00	1.69	30.0
44	7	18.70	4.9682	II	85.50	2.19	24.0
45	7	18.70	6.2602	II	92.00	2.81	24.0
46	7	18.97	4.9682	II	91.00	4.25	36.0
47	7	18.97	6.2602	II	94.00	4.56	36.0
48	7	18.52	6.2602	II	98.00	4.50	28.3
49	7	25.06	6.2602	II	82.50	3.75	29.0
50	7	18.13	6.2602	II	96.50	4.50	20.5
51	7	24.82	6.2602	II	86.00	4.38	21.5
52	7	24.82	4.9682	II	83.00	2.75	19.0
53	7	18.13	4.9682	II	86.00	4.13	17.0

Table 5--Hydrodynamic Mass Data, Natural Frequency Tests

(continued-3)

run number	body number	total weight oscillating lbs.	spring constant lb/in	suspension method	natural frequency cyc/min	displacement amplitude X 2 in.	submergence to center of body in.
54	7	17.94	6.2602	II	102.50	2.63	18.0
55	7	17.94	6.2602	II	98.00	2.69	12.0
56	7	24.58	6.2602	II	86.00	2.63	13.0
57	7	24.58	6.2602	II	88.50	3.00	9.0
58	7	17.94	6.2602	II	101.00	2.25	9.5
59	8	22.76	6.2602	II	69.00	1.88	23.5
60	8	22.76	4.9682	II	60.00	1.19	24.0
61	8	22.76	8.1452	II	82.00	1.56	22.0
62	8	22.76	9.5550	II	88.00	1.63	22.0
63	9	22.76	9.5550	II	91.00	1.38	24.0
64	9	22.76	6.2602	II	74.00	1.63	24.0
65	9	22.76	4.9682	II	68.50	0.94	25.3
66	9	22.76	8.1452	II	83.00	1.56	23.0
67	10	22.76	6.2602	II	68.50	1.88	25.3
68	10	22.76	8.1452	II	76.00	1.56	23.8
69	10	22.76	4.9682	II	60.00	1.25	26.5
70	10	22.76	9.5550	II	84.00	1.38	24.3
71	11	22.76	9.5550	II	87.00	1.44	24.3
72	11	22.76	6.2602	II	67.50	1.75	24.0
73	11	22.76	8.1452	II	86.50	1.31	23.3
74	11	22.76	4.9682	II	66.50	1.19	26.0
75	12	22.76	4.9682	II	66.00	1.19	26.0
76	12	22.76	8.1452	II	76.00	1.44	24.3
77	12	22.76	6.2602	II	72.50	1.63	24.0
78	12	22.76	9.5550	II	83.00	1.38	24.0
79	13	22.76	9.5550	II	83.50	1.31	24.0
80	13	22.76	6.2602	II	70.50	1.69	24.8

Table 5--Hydrodynamic Mass Data, Natural Frequency Tests

(continued-4)

run number	body number	total weight oscillating lbs.	spring constant lb/in	suspension method	natural frequency cyc/min	displacement amplitude X 2 in.	submergence to center of body in.
81	13	22.76	4.9682	II	64.00	1.00	26.3
82	13	22.76	8.1452	II	79.00	1.31	23.4
83	14	22.76	8.1452	II	79.00	1.44	23.5
84	14	22.76	6.2602	II	70.00	1.75	24.0
85	14	22.76	4.9682	II	66.00	1.06	26.4
86	14	22.76	9.5550	II	83.00	1.25	23.0
87	15	22.76	9.5550	II	83.00	1.13	23.0
88	15	22.76	6.2602	II	72.00	1.75	25.0
89	15	22.76	8.1452	II	76.00	1.44	24.0
90	15	22.76	4.9682	II	63.00	1.00	26.8
91	16	22.76	4.9682	II	65.00	1.25	24.0
92	16	22.76	6.2602	II	69.25	1.50	24.0
93	16	22.76	8.1452	II	77.70	1.50	24.0
94	16	22.76	9.5550	II	85.50	1.25	24.0
95	17	22.76	4.9682	II	66.00	1.13	24.0
96	17	22.76	6.2602	II	69.50	1.88	24.0
97	17	22.76	8.1452	II	77.25	1.50	24.0
98	17	22.76	9.5550	II	88.50	1.31	24.0
99	18	22.76	4.9682	II	71.00	1.19	24.0
100	18	22.76	6.2602	II	78.00	1.75	24.0
101	18	22.76	8.1452	II	82.00	1.38	24.0
102	18	22.76	9.5550	II	91.50	1.25	24.0
103	19	22.76	4.9682	II	69.00	0.94	24.0
104	19	22.76	6.2602	II	72.00	1.50	24.0
105	19	22.76	8.1452	II	79.75	1.25	24.0
106	19	22.76	9.5550	II	87.38	1.13	24.0
107	20	38.26	6.2602	II	62.00	----	24.0
108	20	38.26	6.3595	II	62.50	8.50	24.0

Table 5--Hydrodynamic Mass Data, Natural Frequency Tests

(continued-5)

run number	body number	total weight oscillating lbs.	spring constant lb/in	suspension method	natural frequency cyc/min	displacement amplitude X 2 in.	submergence to center of body in.
109	20	38.26	8.1452	II	68.00	----	24.0
110	20	38.26	9.5550	II	76.25	7.38	24.0
111	20	38.26	6.2602	II	62.00	8.50	18.0
112	20	38.26	6.3595	II	63.00	8.63	18.0
113	20	38.26	9.5550	II	76.50	6.75	18.0
114	20	38.26	6.2602	II	59.75	4.63	12.0
115	20	38.26	8.1452	II	69.25	5.88	12.0
116	20	38.26	9.5550	II	77.75	4.50	12.0
117	21	20.17	4.9682	II	90.75	8.13	14.0
118	21	20.17	6.2602	II	97.00	4.88	12.0
119	21	20.17	9.5550	II	141.00	----	10.3
120	22	22.91	4.9682	II	81.00	1.63	22.0
121	22	22.91	6.2602	II	86.50	2.75	21.0
122	22	22.91	8.1452	II	100.00	2.25	21.0
123	22	22.91	9.4427	II	108.00	0.94	20.5
124	23	21.49	4.9682	II	85.50	2.25	23.0
125	23	21.49	6.2602	II	91.00	5.13	26.0
126	23	21.49	6.2602	II	90.50	4.88	22.0
127	23	21.49	8.1452	II	107.00	4.25	22.0
128	24	25.76	4.9682	II	69.50	2.88	20.3
129	24	25.76	6.2602	II	74.50	4.13	19.5
130	24	25.76	8.1452	II	84.50	3.38	18.5
131	24	25.76	9.5550	II	93.00	2.38	17.3
132	25	22.16	4.9682	II	77.50	3.50	22.5
133	25	22.16	6.2602	II	88.00	4.75	22.0
134	25	22.16	8.1452	II	99.00	4.13	21.0
135	25	22.16	9.5550	II	105.75	3.38	18.3

Table 5--Hydrodynamic Mass Data, Natural Frequency Tests

(continued-6)

run number	body number	total weight oscillating lbs.	spring constant lb/in	suspension method	natural frequency cyc/min	displacement amplitude X 2 in.	submergence to center of body in.
136	26	21.22	4.9682	II	62.50	1.50	22.5
137	26	21.22	6.2602	II	68.00	2.13	21.5
138	26	21.22	8.1452	II	74.50	1.75	20.0
139	26	21.22	9.5550	II	84.00	1.38	17.0
140	27	19.90	4.9682	II	90.00	4.00	21.8
141	27	19.90	6.2602	II	96.50	5.38	21.0
142	27	19.90	8.1452	II	109.00	3.00	19.5
143	28	20.83	8.1452	II	111.00	4.88	21.5
144	28	20.83	6.2602	II	95.50	5.63	21.5
145	28	20.83	4.9682	II	87.75	4.88	21.3
146	29	21.68	4.9682	II	84.50	4.50	25.8
147	29	21.68	8.1452	II	108.00	5.00	23.0
148	29	21.68	6.2602	II	94.00	5.63	23.5
149	30	22.49	6.2602	II	91.75	5.63	25.3
150	30	22.49	8.1452	II	106.00	4.63	25.0
151	30	22.49	4.9682	II	86.00	4.50	27.0
152	31	23.32	4.9682	II	81.00	4.38	28.3
153	31	23.32	8.1452	II	104.00	4.88	26.3
154	31	23.32	6.2602	II	90.00	5.50	27.3
155	32	24.17	6.2602	II	88.75	5.63	27.0
156	32	24.17	8.1452	II	102.00	5.13	26.5
157	32	24.17	4.9682	II	84.00	4.50	28.0
158	33	24.87	4.9682	II	83.00	4.38	30.8
159	33	24.87	8.1452	II	104.50	5.25	28.3
160	33	24.87	6.2602	II	85.75	5.25	29.3

TABLE 6

RESULTS

NATURAL FREQUENCY TESTS

run number	displacement to diameter ratio	submergence to diameter ratio	dimensionless frequency	hydrodynamic mass lb-sec^2/in	resistance lb-sec/in	reactance lb/sec/in	hydrodyanmic mass factor-K
1	0.73	1.00	8.03	0.14880	0.05904	0.49878	0.8799
2	0.67	1.00	8.79	0.13481	0.07179	0.49423	0.7972
3	0.85	1.00	10.80	0.17945	0.07575	0.80830	1.0612
4	0.37	0.83	13.56	0.22251	0.16300	0.83908	0.7796
5	0.36	0.83	15.44	0.19460	0.18932	0.83578	0.6819
6	0.39	0.83	15.25	0.23139	0.18002	0.98165	0.8108
7	0.40	0.83	17.90	0.12974	0.17962	0.64554	0.4546
8	0.50	1.00	10.54	0.19232	0.14127	0.84610	1.1373
9	0.32	1.00	8.29	0.13060	0.16014	0.45146	0.7723
10	0.12	1.00	7.40	0.06021	0.43801	0.18604	0.3560
11	0.44	1.00	8.54	0.14977	0.12027	0.53340	0.8857
12	0.56	1.00	11.30	0.14454	0.12198	0.68135	0.8548
13	0.45	1.00	10.04	0.22609	0.16208	0.94733	1.3370
14	0.50	1.00	11.80	0.18287	0.15754	0.90032	1.0814
15	0.44	1.00	7.91	0.15671	0.11171	0.51710	0.9268
16	0.27	1.00	10.04	0.22609	0.15611	0.94733	1.3370
17	0.29	1.00	5.90	0.17388	0.13477	0.42802	1.0282
18	0.37	1.00	12.30	0.22122	0.18309	1.13550	1.3082
19	0.48	1.00	10.30	0.20010	0.14807	0.85938	1.1833
20	0.45	1.00	9.79	0.24286	0.16163	0.99215	1.4362
21	0.48	1.00	11.05	0.22590	0.16674	1.0412	1.3359
22	0.34	1.00	7.41	0.19582	0.14998	0.60510	1.1580
23	0.30	1.00	9.79	0.24286	0.13593	0.99215	1.4362
24	0.31	1.00	12.05	0.23421	0.23570	1.17760	1.3851
25	0.39	1.00	10.04	0.21658	0.19047	0.90748	1.2808

Table 6--Results, Natural Frequency Tests

(continued-2)

run number	displacement to diameter ratio	submergence to diameter ratio	dimensionless frequency	hydrodynamic mass lb-sec^2/in	resistance lb-sec/in	reactance lb-sec/in	hydrodyanmic mass factor-K
26	0.41	1.00	9.67	0.25215	0.18120	1.01690	1.4911
27	0.36	1.00	10.54	0.26081	0.22784	1.1475	1.5424
28	0.35	1.00	7.03	0.23199	0.14775	0.68041	1.3719
29	0.25	1.00	9.55	0.26254	0.17604	1.04500	1.5526
30	0.33	1.00	11.69	0.25800	0.21639	1.25670	1.5257
31	0.39	1.00	9.60	0.24936	0.19091	0.99910	1.4746
32	0.40	1.00	9.54	0.26194	0.18708	1.04270	1.5490
33	0.41	1.00	10.18	0.29045	0.20204	1.23220	1.7176
34	0.31	1.00	6.90	0.24504	0.17213	0.70585	1.4491
35	0.22	1.00	99.35	0.27831	0.20522	1.08600	1.6458
36	0.32	1.00	11.49	0.27056	0.22647	1.29660	1.6000
37	0.63	2.00	11.54	0.15623	0.09373	0.75278	0.9239
38	0.57	2.00	14.43	0.14689	0.09556	0.88472	0.8686
39	----	2.00	8.92	0.11801	-------	0.43884	0.6979
40	0.73	2.00	11.41	0.16737	0.07822	0.79772	0.9898
41	0.48	2.00	12.41	0.17516	0.14603	0.90821	1.0358
42	0.46	5.00	11.91	0.01482	0.10935	0.14745	2.2016
43	0.28	5.00	11.53	0.00510	0.13031	0.04910	0.7571
44	0.37	4.00	10.71	0.01354	0.08832	0.12125	2.0116
45	0.47	4.00	11.53	0.01901	0.10719	0.18317	2.8242
46	0.71	6.00	11.41	0.00558	0.11233	0.05317	0.8287
47	0.76	6.00	11.79	0.01547	0.10494	0.15232	2.2985
48	0.75	4.72	12.30	0.01406	0.10714	0.11709	1.6948
49	0.63	4.83	10.34	0.01883	0.15850	0.16268	2.7971
50	0.75	3.42	12.10	0.01427	0.10751	0.14421	2.1199
51	0.73	3.58	10.80	0.01274	0.13273	0.11477	1.8931
52	0.46	3.16	10.41	0.00133	0.20502	0.01153	0.1969
53	0.69	2.83	10.80	0.01422	0.10539	0.12810	2.1129

Table 6--Results, Natural Frequency Tests

(continued-3)

run number	displacement to diameter ratio	submergence to diameter ratio	dimensionless frequency	hydrodynamic mass lb-sec^2/in	resistance lb-sec/in	reactance lb-sec/in	hydrodynamic mass factor-K
54	0.44	3.00	12.88	0.00780	0.11736	0.08379	1.1596
55	0.45	2.00	12.30	0.01291	0.11445	0.13248	1.9176
56	0.44	2.16	10.80	0.01354	0.14101	0.12198	2.0120
57	0.50	1.50	11.10	0.00924	0.11738	0.08569	1.3735
58	0.38	1.58	12.68	0.00942	0.15022	0.09976	1.4011
59	0.16	1.96	17.32	0.06088	0.22260	0.44006	1.1296
60	0.10	2.00	15.07	0.06682	0.26901	0.41998	1.2398
61	0.13	1.83	20.60	0.05145	0.22487	0.44192	0.9545
62	0.14	1.83	22.08	0.05349	0.17989	0.49316	0.9926
63	0.15	2.26	20.22	0.04621	0.23208	0.44046	0.8573
64	0.13	2.26	16.46	0.04524	0.28950	0.35066	0.8393
65	0.15	2.38	15.23	0.03755	0.48236	0.26941	0.6966
66	0.15	2.16	18.46	0.04881	0.22466	0.42432	0.9055
67	0.21	2.89	12.53	0.06264	0.22230	0.44947	1.1622
68	0.18	2.72	13.90	0.06957	0.22445	0.55374	1.2907
69	0.14	3.03	10.99	0.06682	0.24179	0.41998	1.2398
70	0.16	2.78	15.37	0.06446	0.23119	0.56722	1.1960
71	0.19	3.24	13.64	0.05610	0.21706	0.51125	1.0408
72	0.23	3.20	10.59	0.06627	0.25199	0.46857	1.2295
73	0.17	3.10	13.58	0.04026	0.30205	0.36480	0.7469
74	0.16	3.20	10.42	0.04344	0.26901	0.30258	0.8059
75	0.18	3.85	9.32	0.04499	0.26901	0.31107	0.8348
76	0.21	3.60	10.72	0.06957	0.25876	0.55384	1.2907
77	0.24	3.56	10.22	0.04959	0.28950	0.37663	0.9201
78	0.20	3.55	11.71	0.06656	0.23399	0.57865	1.2348
79	0.21	3.92	11.06	0.06596	0.25028	0.57681	1.2235
80	0.28	4.05	9.03	0.05584	0.26904	0.41237	1.0360

Table 6--Results, Natural Frequency Tests

(continued-4)

run number	displacement to diameter ratio	submergence to diameter ratio	dimensionless frequency	hydrodynamic mass $lb\text{-}sec^2/in$	resistance lb-sec/in	reactance lb-sec/in	hydrodynamic mass factor-K
81	0.16	4.30	8.20	0.05159	0.39777	0.34588	0.9572
82	0.21	3.82	10.12	0.05999	0.30205	0.49646	1.1130
83	0.10	1.68	23.14	0.05999	0.25846	0.49646	1.1130
84	0.13	1.72	20.50	0.05749	0.25199	0.42151	1.0665
85	0.07	1.89	19.32	0.04499	0.33197	0.31107	0.8348
86	0.09	1.72	24.30	0.06747	0.27235	0.58648	1.2515
87	0.11	2.16	18.48	0.06746	0.32722	0.58648	1.2515
88	0.16	2.35	16.01	0.05111	0.25177	0.38544	0.9482
89	0.14	2.26	16.91	0.06957	0.25967	0.55384	1.2907
90	0.09	2.52	14.02	0.05513	0.39772	0.36382	1.0228
91	0.13	2.46	13.26	0.04822	0.24166	0.32830	0.8946
92	0.15	2.46	14.11	0.06002	0.34075	0.43539	1.1136
93	0.15	2.46	15.85	0.06401	0.23950	0.52095	1.1875
94	0.13	2.46	17.43	0.06017	0.27213	0.53890	1.1164
95	0.13	2.82	11.73	0.04499	0.30032	0.31107	0.8348
96	0.22	2.82	12.36	0.05917	0.22260	0.43074	1.0977
97	0.18	2.82	13.74	0.06544	0.23899	0.52956	1.2142
98	0.15	2.82	15.73	0.05223	0.25080	0.48421	0.9691
99	0.24	4.80	7.42	0.03087	0.26472	0.22959	0.5727
100	0.35	4.80	8.15	0.03483	0.25199	0.28455	0.6461
101	0.28	4.80	8.56	0.05145	0.27825	0.44192	0.9545
102	0.25	4.80	9.56	0.04506	0.27213	0.43190	0.8360
103	0.19	4.80	7.21	0.03615	0.48266	0.26130	0.6707
104	0.30	4.80	7.52	0.05111	0.34075	0.38544	0.9482
105	0.25	4.80	8.34	0.05777	0.33198	0.48257	1.0717
106	0.23	4.80	9.13	0.05511	0.32700	0.50443	1.0225
107	----	2.00	15.57	0.04922	-------	0.31967	1.1645
108	0.71	2.00	15.70	0.04917	0.03149	0.32193	1.1633

Table 6--Results, Natural Frequency Tests

(continued-5)

run number	displacement to diameter ratio	submergence to diameter ratio	dimensionless frequency	hydrodynamic mass $lb\text{-}sec^2/in$	resistance lb-sec/in	reactance lb-sec/in	hydrodynamic mass factor-K
109	----	2.00	17.07	0.06134	-------	0.43691	1.4511
110	0.62	2.00	19.20	0.05058	0.02773	0.40396	1.1962
111	0.71	1.50	15.57	0.04922	0.03322	0.31967	1.1645
112	0.72	1.50	15.80	0.04683	0.02935	0.30902	1.1078
113	0.56	1.50	19.20	0.04959	0.03078	0.39746	1.1734
114	0.39	1.00	15.00	0.06061	0.08173	0.37935	1.4339
115	0.49	1.00	17.40	0.05559	0.09100	0.40328	1.3152
116	0.38	1.00	19.53	0.04485	0.06189	0.36530	1.0611
117	2.03	3.50	7.60	0.00274	0.01095	0.02604	0.5834
118	1.22	3.00	11.80	0.00839	0.05315	0.08527	1.7874
119	----	2.58	6.78	0.00844	-------	0.12471	1.7985
120	0.82	11.00	3.39	0.00962	0.15690	0.08155	1.4563
121	1.37	10.50	3.62	0.01685	0.12526	0.15269	2.5532
122	1.12	10.50	4.19	0.01483	0.14881	0.15537	2.2473
123	0.47	10.24	4.52	0.01438	0.41618	0.16268	2.1788
124	0.56	5.75	7.15	0.00494	0.09379	0.04423	0.5858
125	1.28	6.50	7.62	0.01897	0.04912	0.11340	1.4129
126	1.22	5.50	7.57	0.01266	0.05377	0.12001	1.5018
127	1.06	5.50	8.95	0.00784	0.05318	0.08784	0.9297
128	0.48	3.38	8.73	0.02730	0.0732	0.1987	0.637
129	0.69	3.25	9.35	0.03640	0.0762	0.2840	0.849
130	0.56	3.08	10.60	0.03730	0.0819	0.3302	0.870
131	0.40	2.87	11.69	0.03420	0.1164	0.3327	0.798
132	0.88	5.63	6.49	0.01760	0.0491	0.1432	0.750
133	1.19	5.50	7.36	0.02000	0.0562	0.1800	0.851
134	1.03	5.25	8.29	0.01840	0.0557	0.1909	0.784
135	0.85	4.56	8.85	0.02090	0.0658	0.2305	0.890

Table 6--Results, Natural Frequency Tests

(continued-6)

run number	displacement to diameter ratio	submergence to diameter ratio	dimensionless frequency	hydrodynamic mass lb-sec^2/in	resistance lb-sec/in	reactance lb-sec/in	hydrodynamic mass factor-K
136	0.13	1.87	15.69	0.06090	0.1634	0.3984	0.376
137	0.18	1.79	17.07	0.06860	0.1771	0.4892	0.425
138	0.15	1.67	18.70	0.07900	0.1839	0.6156	0.490
139	0.13	1.42	21.08	0.06880	0.2270	0.6045	0.426
140	1.14	6.21	6.59	0.00453	0.0375	0.0426	1.129
141	1.54	6.00	7.06	0.00999	0.0437	0.1007	2.486
142	0.75	5.57	7.98	0.01103	0.0694	0.1260	2.753
143	1.39	6.14	8.13	0.00650	0.0418	0.0754	0.810
144	1.61	6.14	7.00	0.00890	0.0422	0.0889	1.110
145	1.39	6.08	6.43	0.00500	0.0281	0.0459	0.623
146	1.29	7.35	6.19	0.00736	0.0329	0.0650	0.612
147	1.43	6.57	7.91	0.00748	0.0418	0.0845	0.621
148	1.61	6.71	6.89	0.00868	0.0379	0.0854	0.721
149	1.61	7.22	6.72	0.00985	0.0541	0.0945	0.614
150	1.32	7.14	7.76	0.00814	0.0476	0.0902	0.507
151	1.29	7.71	6.30	0.00320	0.0338	0.0287	0.199
152	1.25	8.07	5.93	0.00880	0.0362	0.0745	0.431
153	1.39	7.50	7.61	0.00833	0.0450	0.0906	0.409
154	1.57	7.79	6.59	0.01000	0.0468	0.0941	0.490
155	1.61	7.71	6.50	0.01023	0.0461	0.0951	0.054
156	1.47	7.57	7.46	0.00895	0.0429	0.0955	0.126
157	1.29	8.00	6.15	0.00165	0.0356	0.0145	0.477
158	1.25	8.79	6.08	0.00150	0.0380	0.0130	0.425
159	1.50	8.07	7.65	0.00355	0.0423	0.0388	0.372
160	1.50	8.36	6.28	0.01340	0.0542	0.1201	0.069

TABLE 7

HYDRODYNAMIC MASS AND MECHANICAL IMPEDANCE

FOR A 2:1 ELLIPSOID WITH AND WITHOUT "WINGS"

Mean Displacement To Diameter Ratio - 0.42

Mean Dimensionless Frequency - 12.24 $X10^{-3}$

I. Submergence To Diameter Ratio - 1.0

wing area (as a percentage of ellipse section)	hydrodynamic mass factor (based on ellipsoid without wings)	impedance factor (based on ellipsoid without wings)	phase angle
0%	0.892	0.953	83.20°
20%	1.037	1.077	73.00°
30%	1.298	1.346	79.81°
40%	1.461	1.492	79.21°
50%	1.572	1.604	79.22°

II. Submergence To Diameter Ratio - ∞

wing area (as a percentage of ellipse section)	hydrodynamic mass factor (based on ellipsoid without wings)
0%	0.702
20%	0.815
30%	1.024
40%	1.150
50%	1.237

TABLE 8

HYDRODYNAMIC MASS AND MECHANICAL IMPEDANCE

FOR A SPHERE

Mean Displacement To Diameter Ratio - 0.67

Mean Dimensionless Frequency - 15.20 $X10^{-3}$

Mean Submergence To Diametet Ratio - 1.64

hydrodynamic mass factor	impedance factor	phase angle
0.632	0.630	80.87^{o}

TABLE 9

HYDRODYNAMIC MASS AND MECHANICAL IMPEDANCE

FOR DISCS AND PLATES

Mean Displacement To Diameter Ratio - 0.17

Mean Dimensionless Frequency - 15.46 $X10^{-3}$

Mean Submergence To Diameter Ratio - 2.83

Body Shape	Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
Circular Disc	1.079	1.218	63.39^{o}
1:5:1 Elliptical Disc	0.841	0.994	58.55^{o}
2:1 Elliptical Disc	0.922	1.080	59.30^{o}
Square Plate	0.502	0.687	51.00^{o}
1.5:1 Rectangular Plate	0.912	0.977	65.01^{o}
2:1 Rectangular Plate	0.826	0.988	56.85^{o}
2.5:1 Rectangular Plate	1.035	1.215	58.63^{o}
3:1 Rectangular Plate	1.148	1.422	55.78^{o}
4:1 Rectangular Plate	1.043	1.417	51.24^{o}
5:1 Rectangular Plate	1.092	1.524	47.37^{o}
60^{o}-60^{o}-60^{o} Triangular Plate	1.069	1.277	57.48^{o}
45^{o}-90^{o}-45^{o} Triangular Plate	1.130	1.381	56.24^{o}

TABLE 10

HYDRODYNAMIC MASS AND MECHANICAL IMPEDANCE

FOR AN I-BEAM TYPE SECTION

Mean Displacement To Diameter Ratio - 0.945

Mean Dimensionless Frequency - 3.93 $X10^{-3}$

Mean Submergence To Diameter Ratio - 10.56

hydrodynamic mass factor	impedance factor	phase angle
2.110	3.893	36.25°

TABLE 11

HYDRODYNAMIC MASS AND MECHANICAL IMPEDANCE

FOR FOUR TYPICAL TOWED BODIES

Body No.	Mean Displacement To Diameter Ratio	Mean Dimensionless Frequency $X10^3$	Mean Submergence To Diameter Ratio	Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
23	1.03	7.82	5.81	1.316	1.379	54.13°
24	0.53	10.09	3.15	.787	.827	72.88°
25	0.99	7.75	5.24	.820	.852	72.87°
26	0.15	18.14	1.69	.429	.456	70.12°

TABLE 12

HYDRODYNAMIC MASS AND MECHANICAL IMPEDANCE

FOR PARALLELEPIPEDS OF SQUARE SECTION

Mean Displacement To Diameter Ratio - 1.46

Mean Dimensionless Frequency - 7.99 $X10^{-3}$

Mean Submergence To Diameter Ratio - 7.54

parallepiped height to width ratio	hydrodynamic mass factor	impedance factor	phase angle
1	2.123	2.452	58.77^{o}
2	0.848	0.960	61.38^{o}
3	0.651	0.724	64.21^{o}
4	0.440	0.528	54.23^{o}
5	0.443	0.494	63.73^{o}
6	0.289	0.399	43.96^{o}
7	0.219	0.292	42.38^{o}

TABLE 13

RESULTS OF FORCED OSCILLATION TESTS

FOR A SPHERE

Mean Submergence To Diameter Ratio - 2.208

Mean Dimensionless Frequency - 13.06 $X10^{-3}$

Displacement To Diameter Ratio - .333

Hydrodynamic Mass Factor	Impedance Factor	Phase Angle
0.715	1.020	75.06^{o}

TABLE 14

RESULTS OF FORCED OSCILLATION TESTS

FOR A CIRCULAR DISC

Mean Submergence To Diameter Ratio - 2.0

Mean Dimensionless Frequency - 7.09 $X10^{-3}$

Displacement To Diameter Ratio	Hydrodynamic Mass Factor
0.333	1.498
0.500	1.757
0.666	2.059

Mean Impedance Factor	Phase Angle
3.333	68.39^{o}

ILLUSTRATIONS

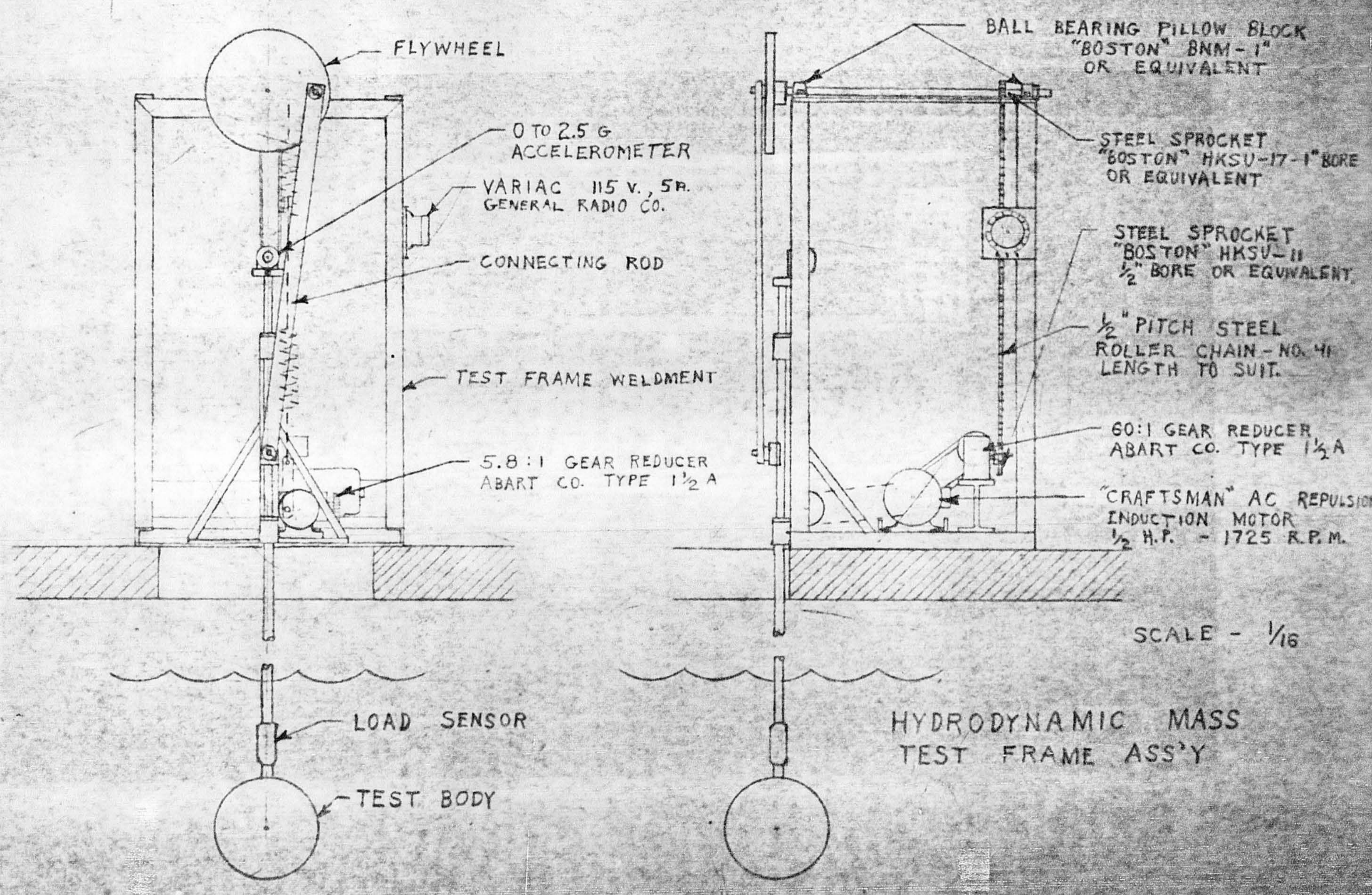

FLYWHEEL
0 TO 2.5 G
ACCELEROMETER
VARIAC 115 V., 5A.
GENERAL RADIO CO.
CONNECTING ROD
TEST FRAME WELDMENT
5.8 : 1 GEAR REDUCER
ABART CO. TYPE 1½ A
LOAD SENSOR
TEST BODY
BALL BEARING PILLOW BLOCK
"BOSTON" BNM-1"
OR EQUIVALENT
STEEL SPROCKET
"BOSTON" HKSU-17-1" BORE
OR EQUIVALENT
STEEL SPROCKET
"BOSTON" HKSU-11
½" BORE OR EQUIVALENT
½" PITCH STEEL
ROLLER CHAIN - NO. 41
LENGTH TO SUIT.
60:1 GEAR REDUCER
ABART CO. TYPE 1½ A
"CRAFTSMAN" AC REPULSION
INDUCTION MOTOR
½ H.P. - 1725 R.P.M.
SCALE - 1/16
HYDRODYNAMIC MASS
TEST FRAME ASS'Y

FIG. No 1

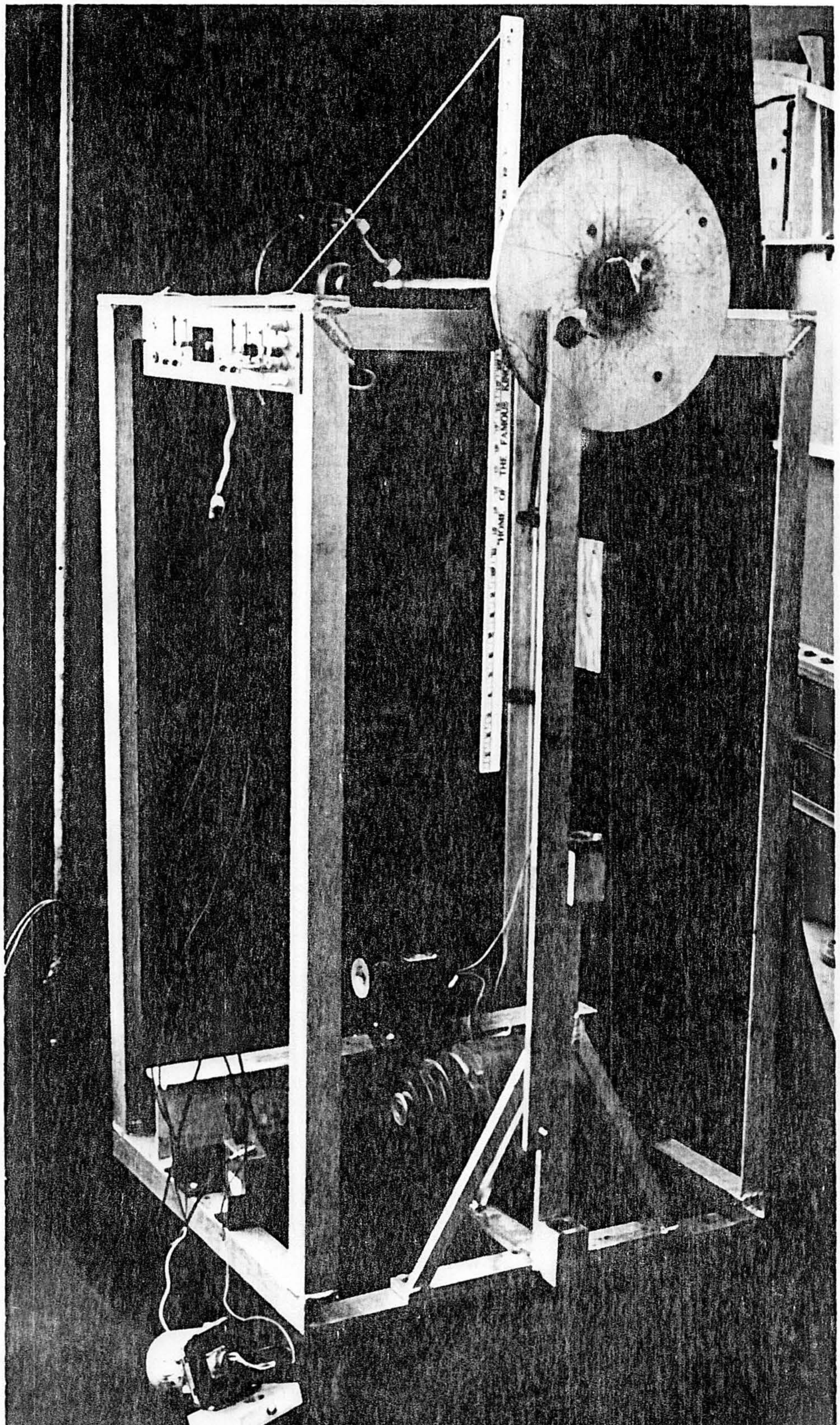
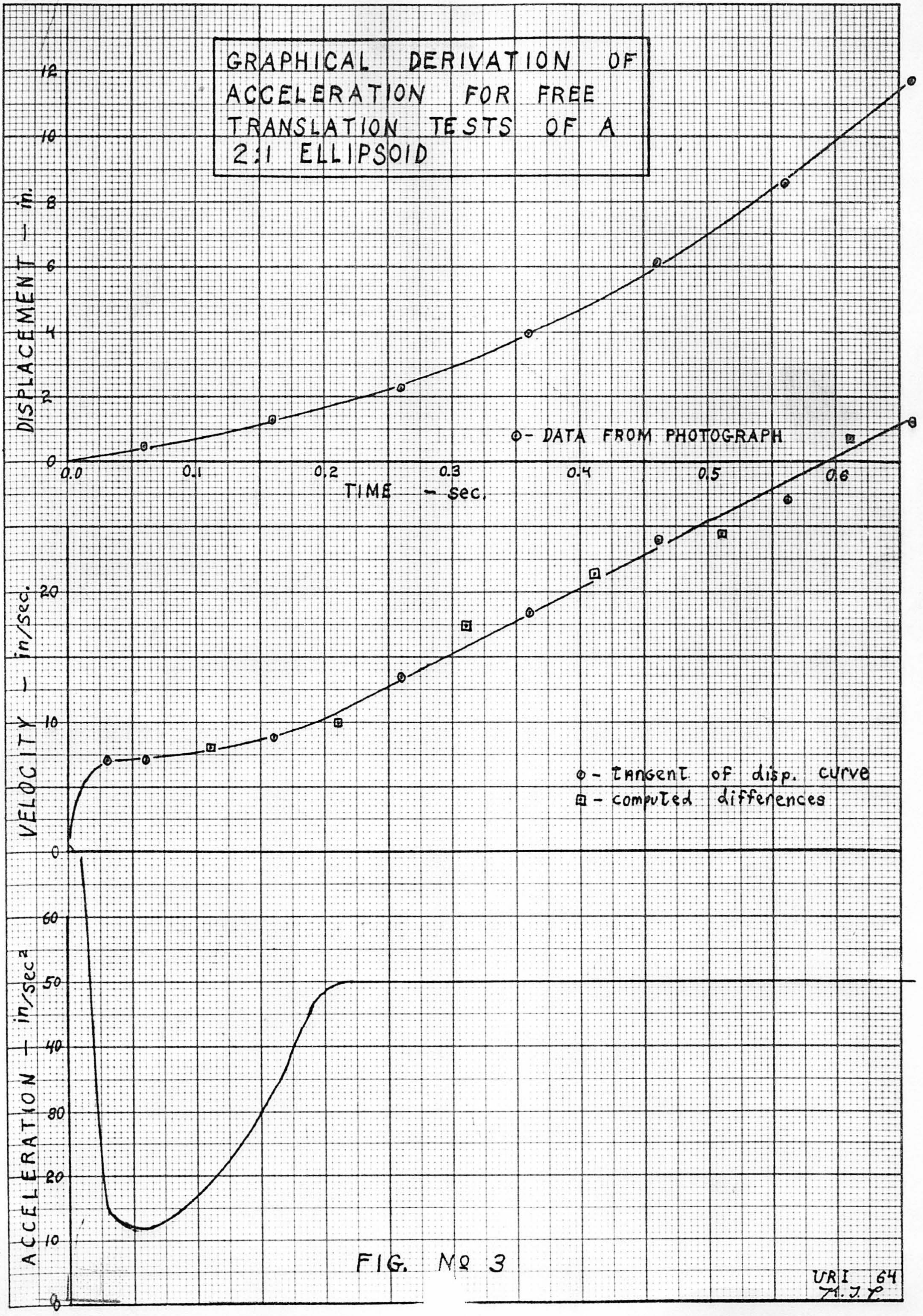


FIG. Nº 2



GRAPHICAL DERIVATION OF ACCELERATION FOR FREE TRANSLATION TESTS OF A 2:1 ELLIPSOID
DISPLACEMENT — in.
0
2
4
6
8
10
12
0.0
0.1
0.2
0.3
0.4
0.5
0.6
TIME — sec.
- DATA FROM PHOTOGRAPH
VELOCITY — in/sec.
0
10
20
- tangent of disp. curve
- computed differences
ACCELERATION — in/sec²
0
10
20
30
40
50
60
URI 64
M.J.L.

FIG. № 3

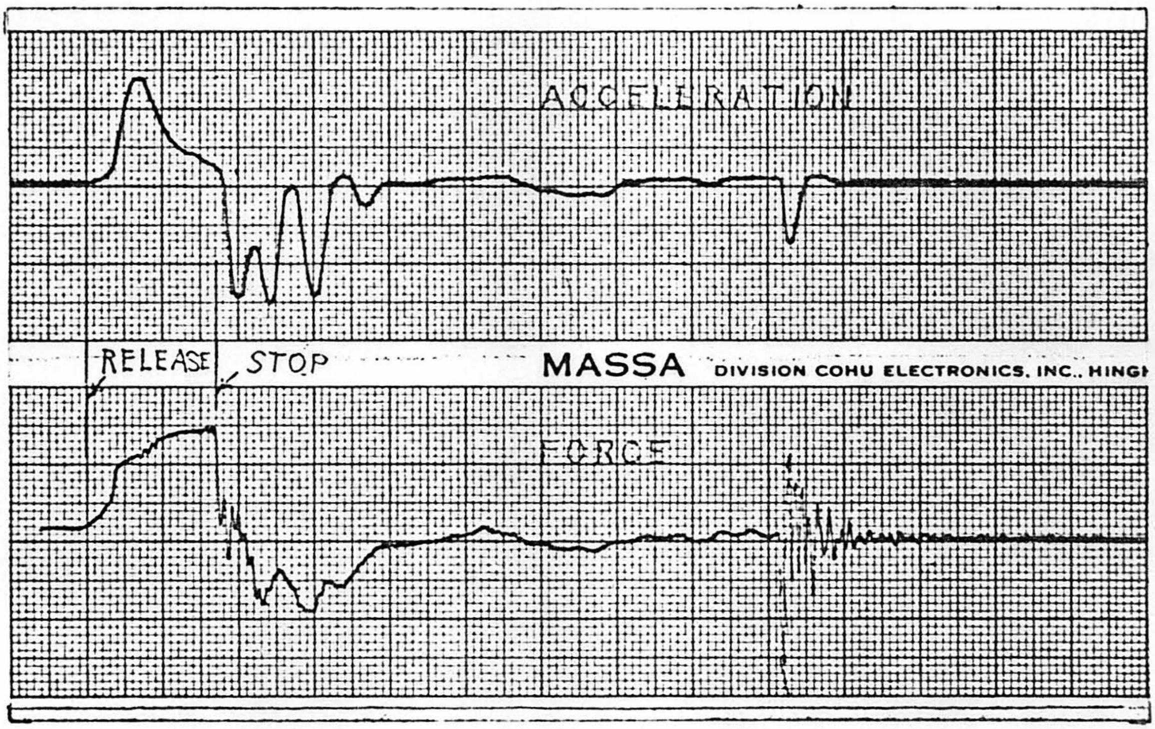


FIGURE 4

FREE TRANSLATION TEST OF A CIRCULAR DISC

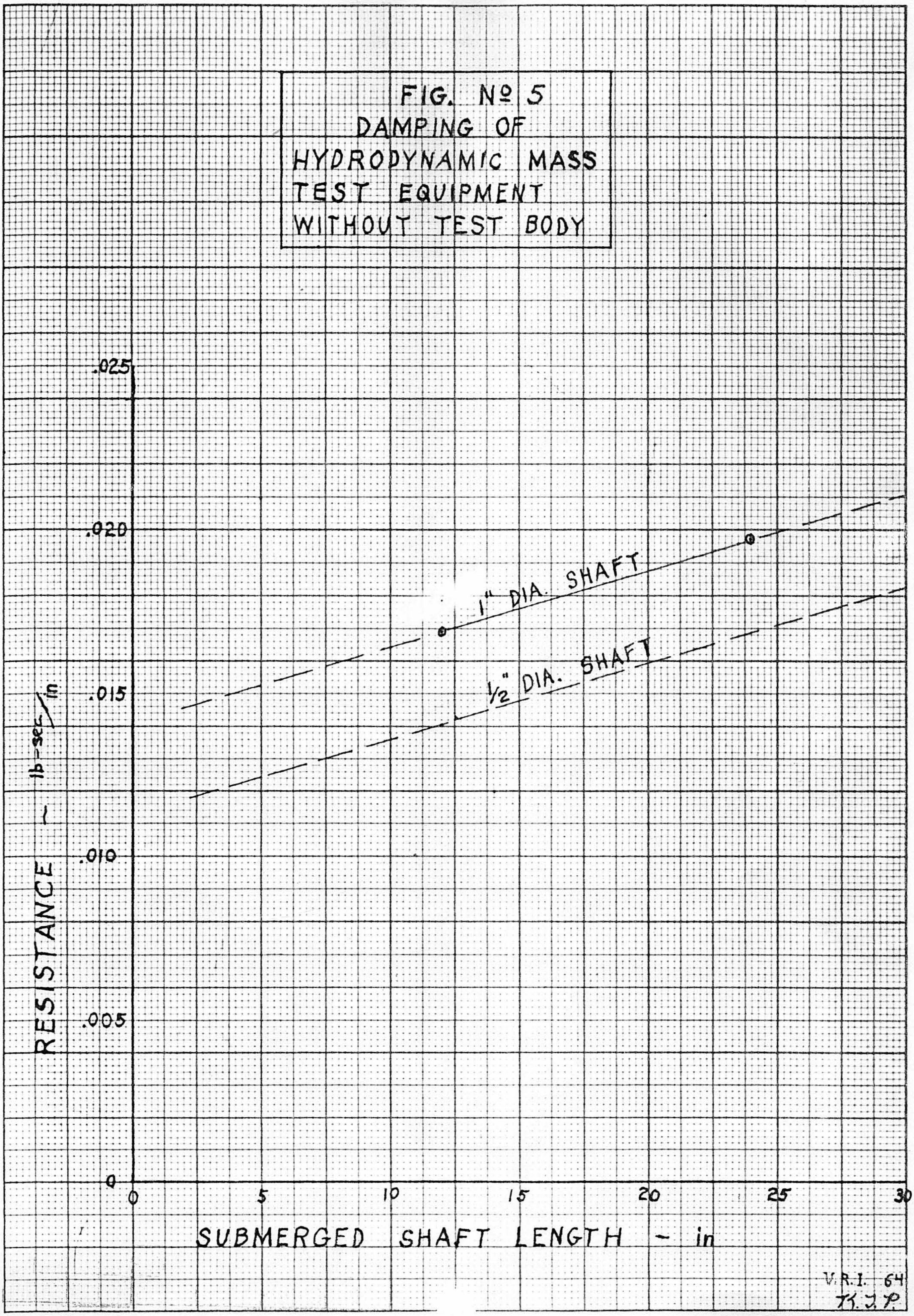
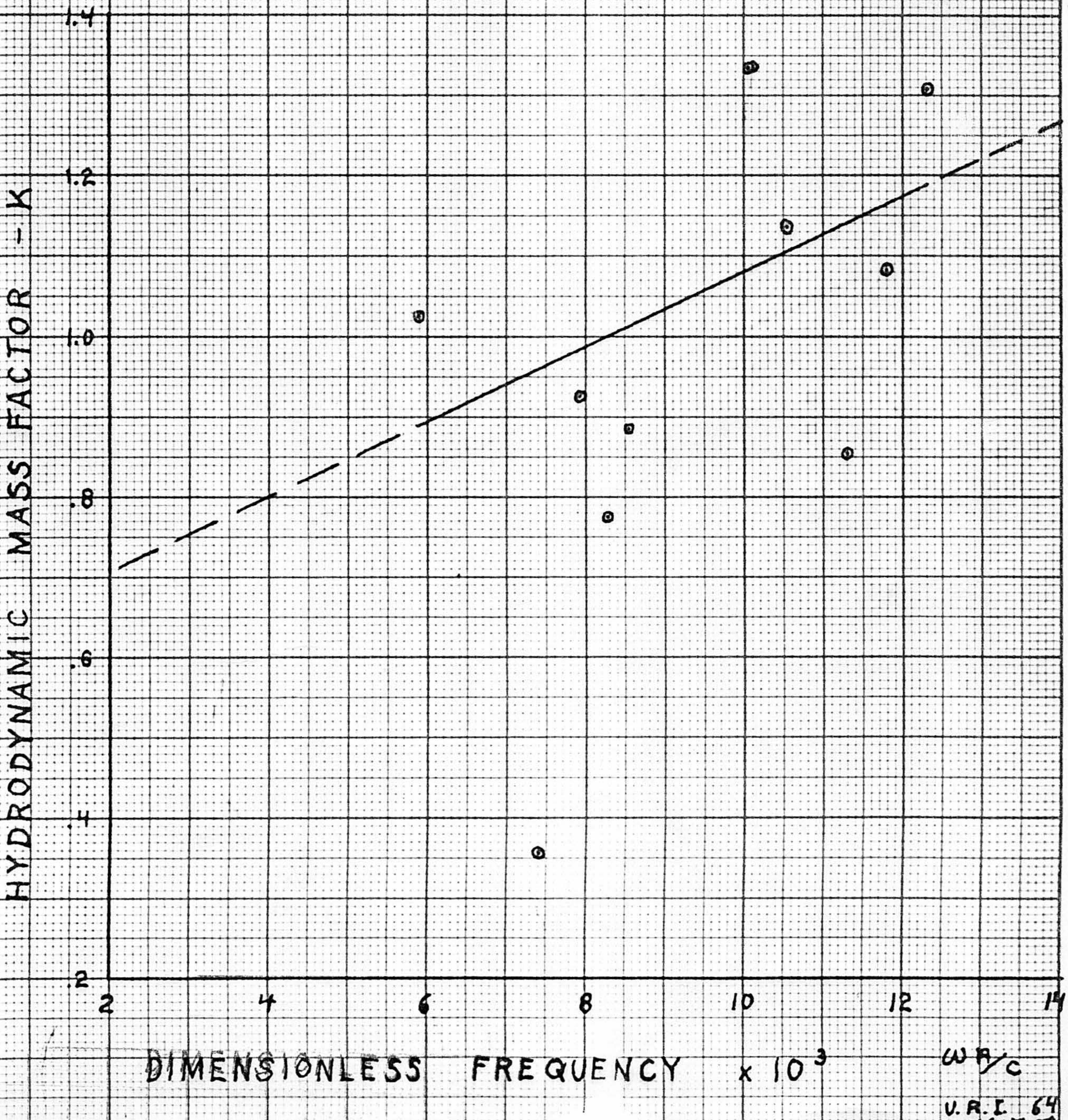


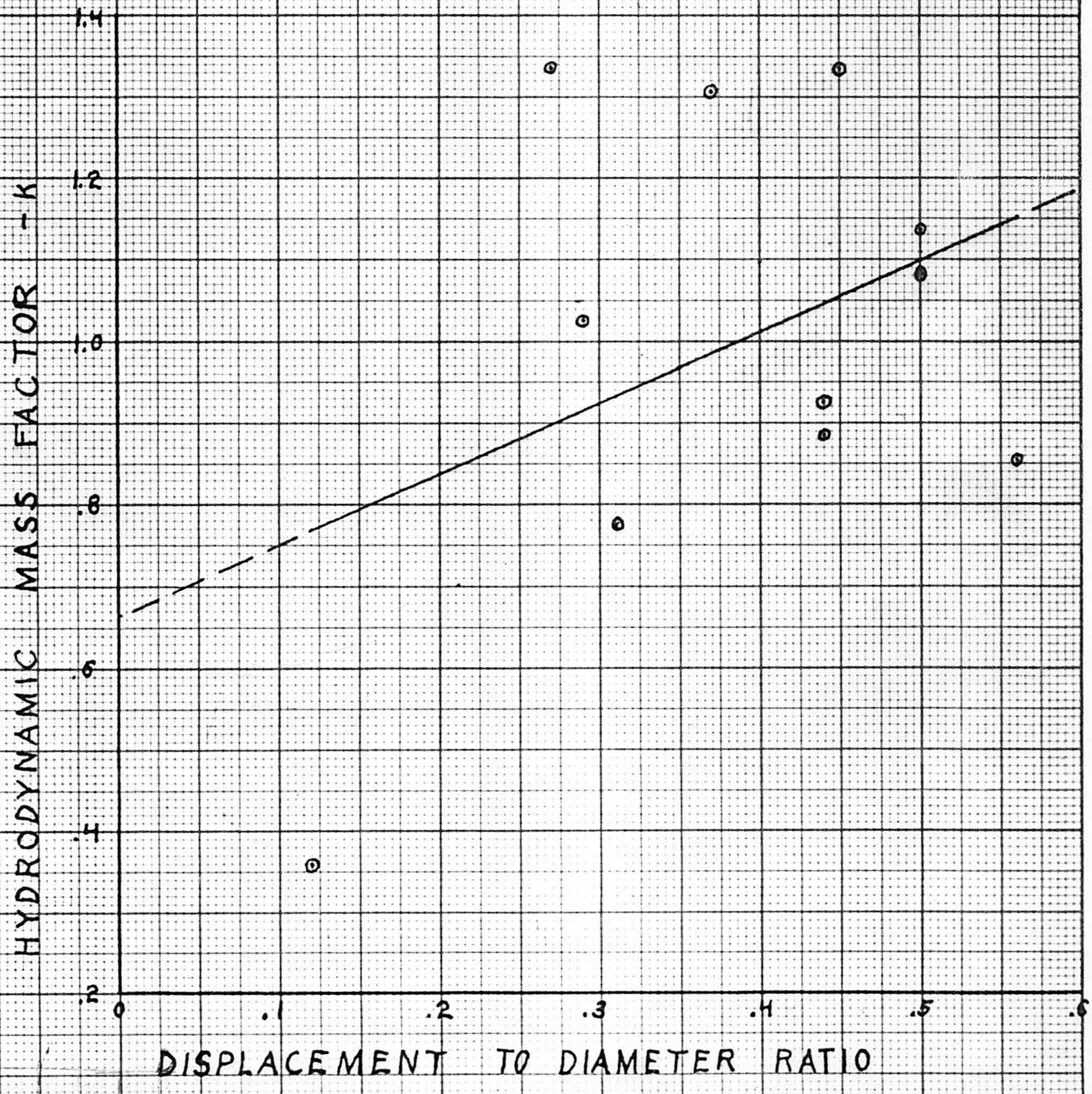
FIG. № 5
DAMPING OF
HYDRODYNAMIC MASS
TEST EQUIPMENT
WITHOUT TEST BODY
1" DIA. SHAFT
1/2" DIA. SHAFT
.025
.020
.015
.010
.005
0
0
5
10
15
20
25
30
RESISTANCE ~ lb-sec/in
SUBMERGED SHAFT LENGTH ~ in
V.R.I. 64
K.J.P.

FIG. Nº 6
LEAST MEAN SQUARES PLOT
of
HYDRODYNAMIC MASS FACTOR
vs.
FREQUENCY
for A
2:1 ELLIPSOID WITH 20% "WINGS"



V.R.I. 64
K.J.P.

FIG. № 7
LEAST MEAN SQUARES PLOT
of
HYDRODYNAMIC MASS FACTOR
VS.
DISPLACEMENT TO DIAMETER
RATIO
for A
2:1 ELLIPSOID WITH 20% "WINGS"



U.R.I. 64
K.J.P.

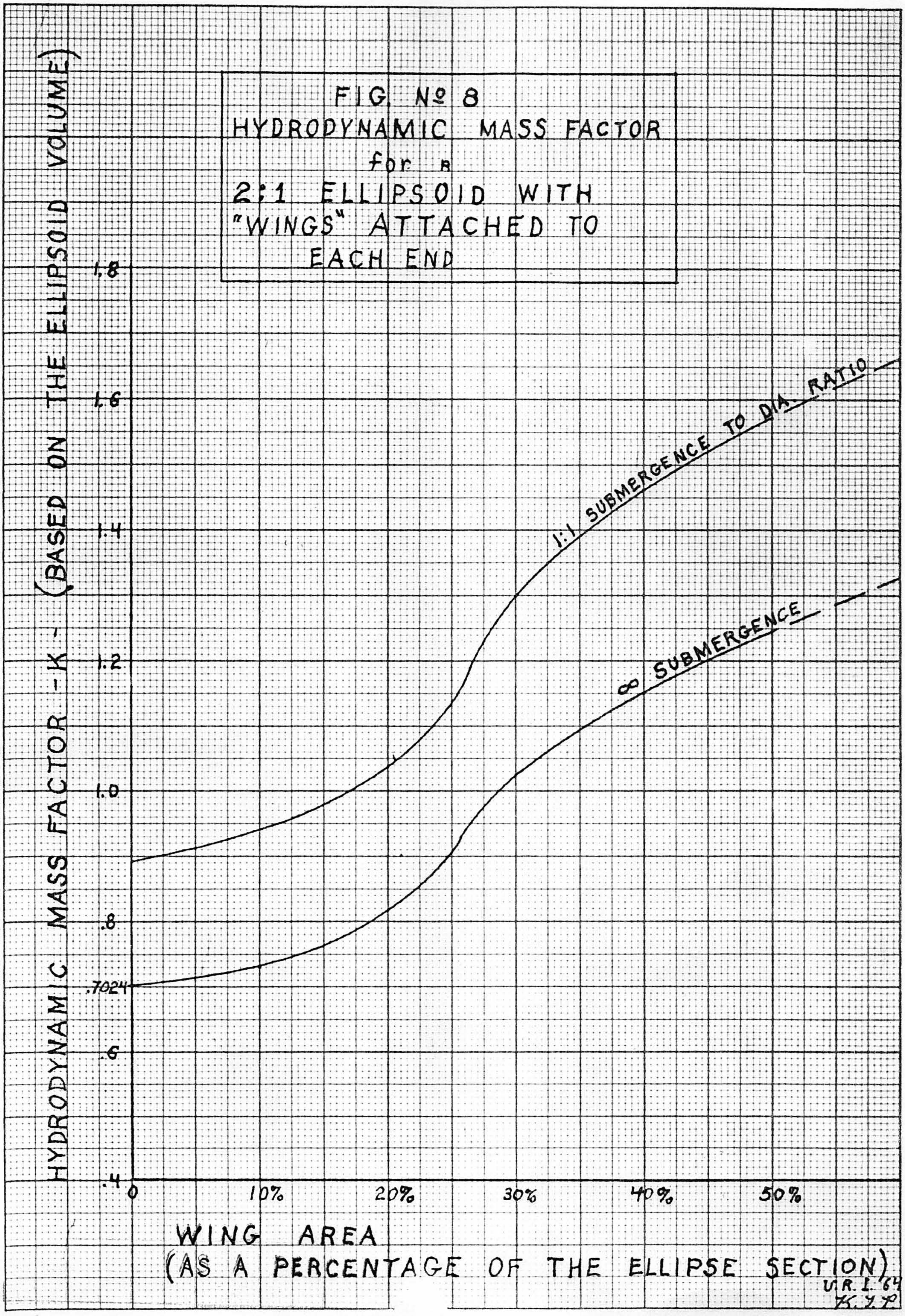


FIG. № 8
HYDRODYNAMIC MASS FACTOR
for a
2:1 ELLIPSOID WITH
"WINGS" ATTACHED TO
EACH END
HYDRODYNAMIC MASS FACTOR - K - (BASED ON THE ELLIPSOID VOLUME)
1.8
1.6
1.4
1.2
1.0
.8
.7024
.6
.4
0
10%
20%
30%
40%
50%
1:1 SUBMERGENCE TO DIA. RATIO
∞ SUBMERGENCE
WING AREA
(AS A PERCENTAGE OF THE ELLIPSE SECTION)
U.R.I. '64
K.T.P.

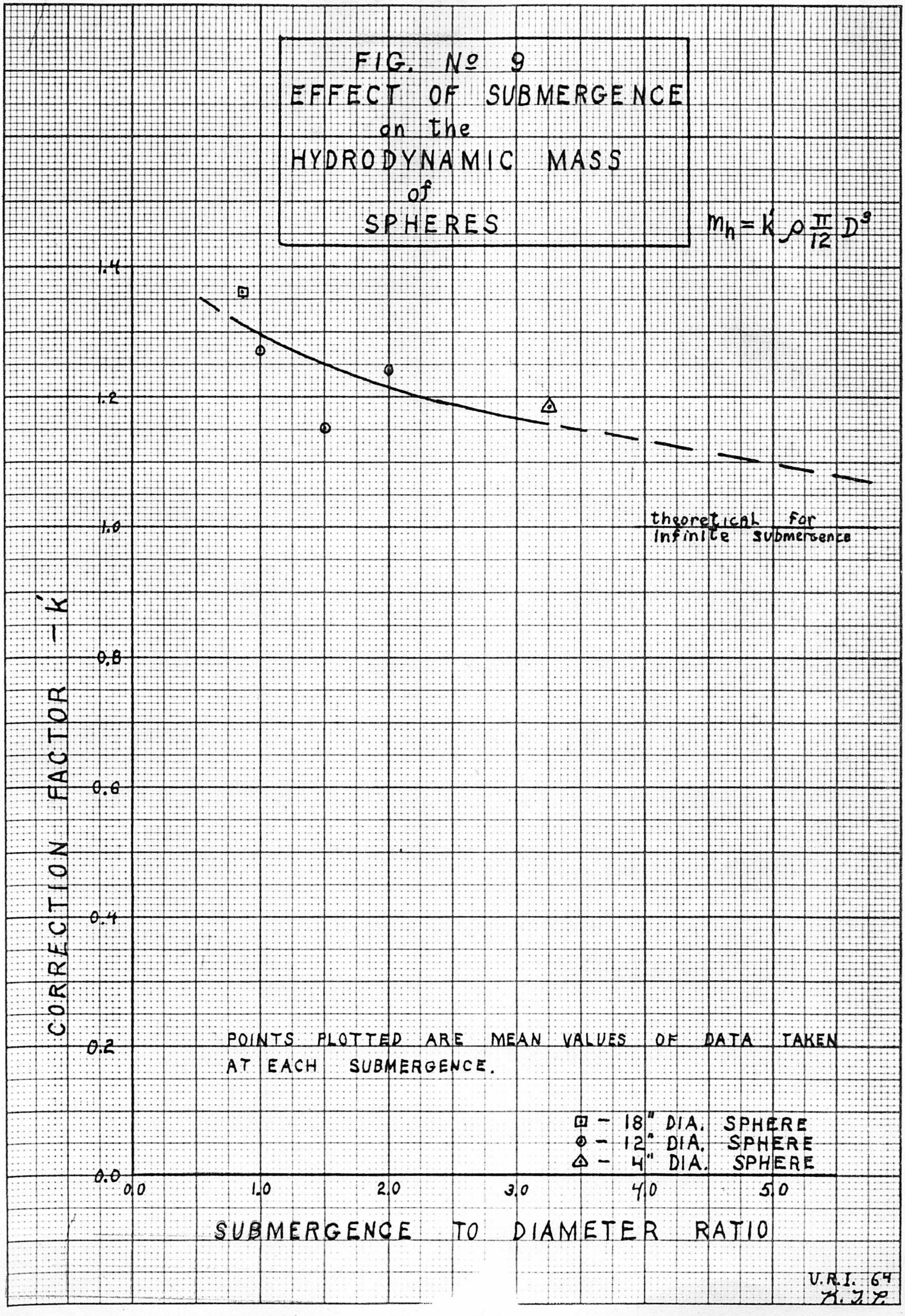


FIG. No 9
EFFECT OF SUBMERGENCE on the HYDRODYNAMIC MASS of SPHERES
$m_h = k' \rho \frac{\pi}{12} D^3$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
1.0
2.0
3.0
4.0
5.0
theoretical for infinite submergence
CORRECTION FACTOR $-k'$
POINTS PLOTTED ARE MEAN VALUES OF DATA TAKEN AT EACH SUBMERGENCE.
□ – 18" DIA. SPHERE
⊙ – 12" DIA. SPHERE
△ – 4" DIA. SPHERE
SUBMERGENCE TO DIAMETER RATIO
U.R.I. 64
K.J.P.

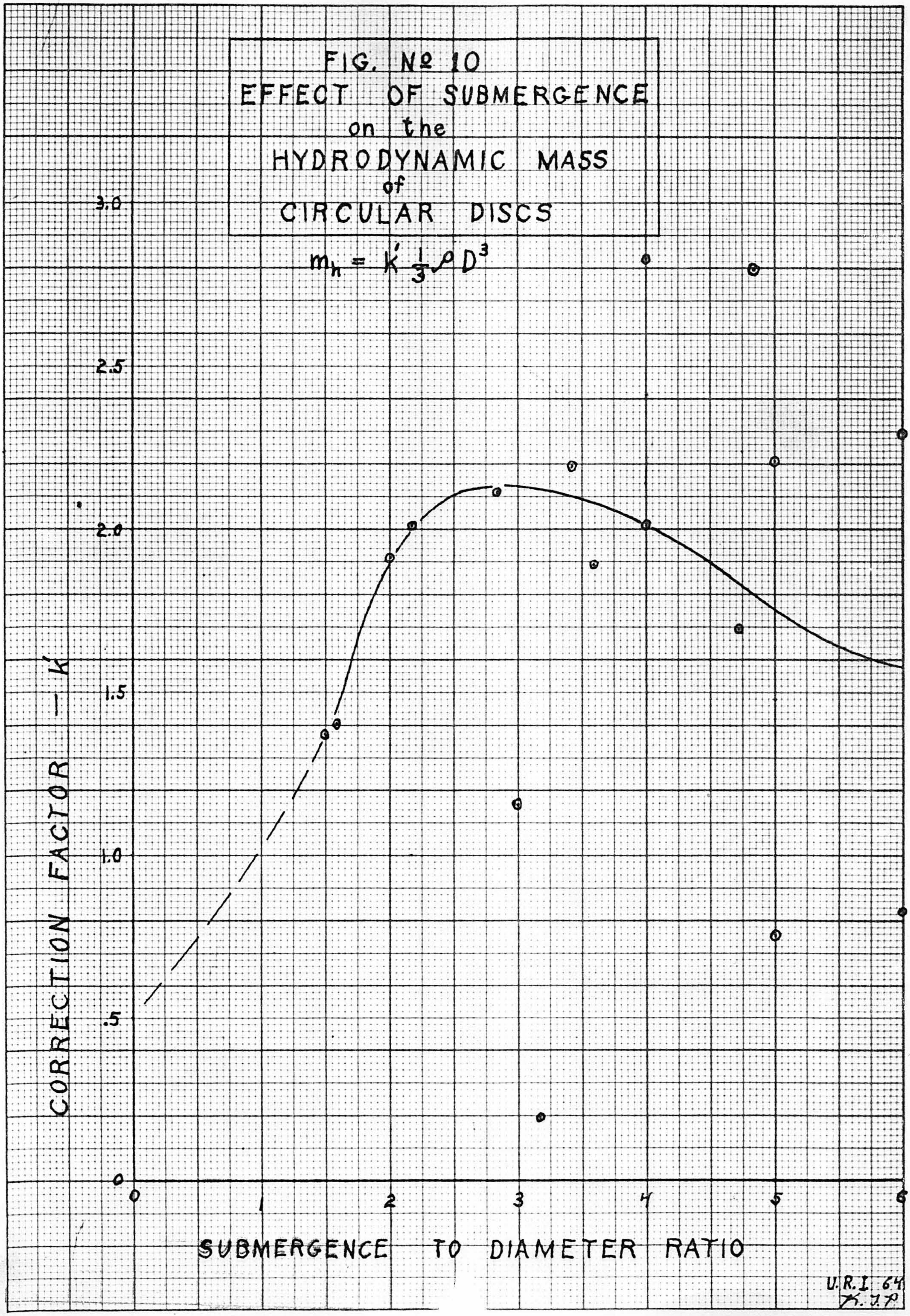
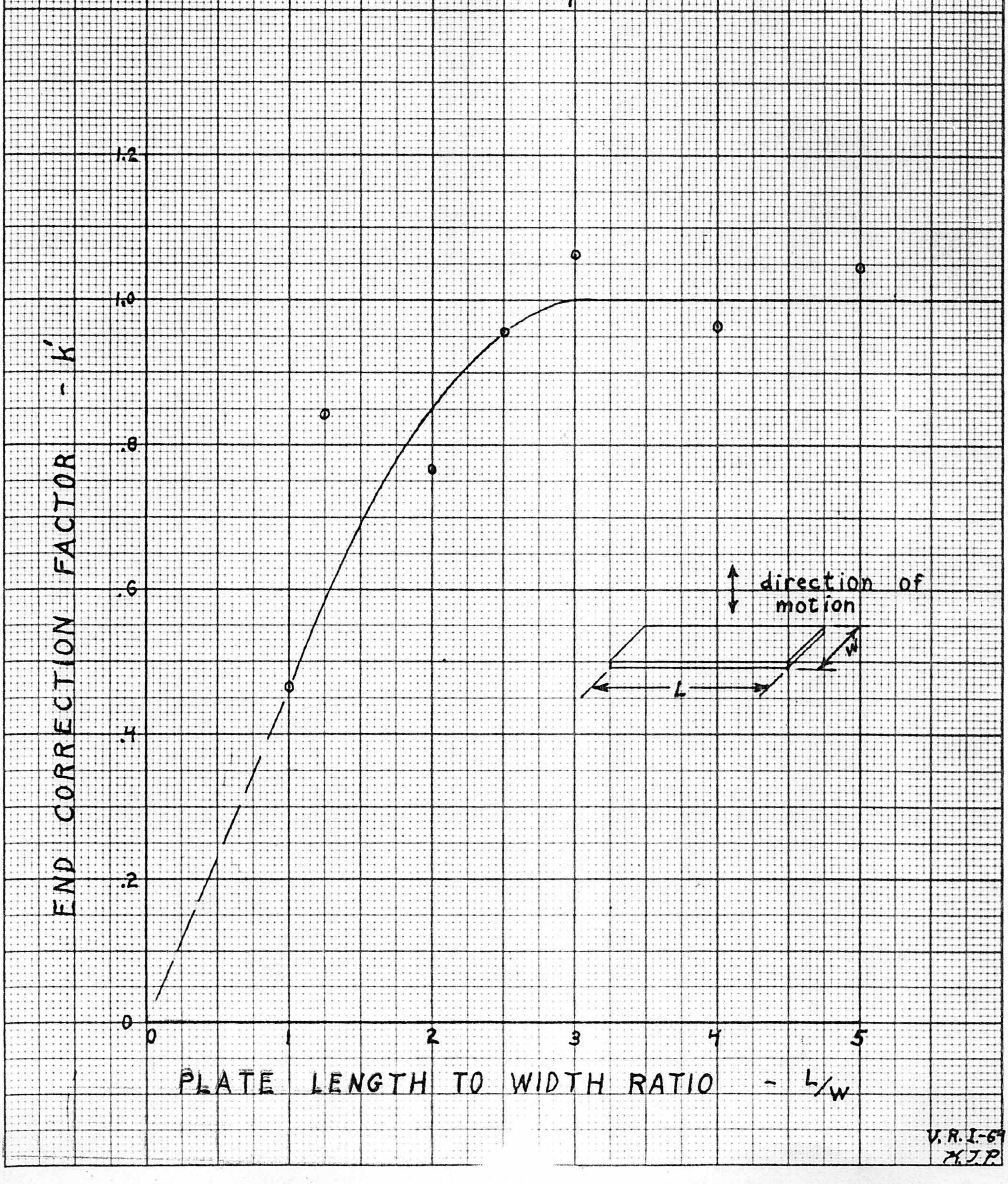


FIG. № 10
EFFECT OF SUBMERGENCE on the HYDRODYNAMIC MASS of CIRCULAR DISCS
$m_h = K' \frac{1}{3} \rho D^3$
3.0
2.5
2.0
1.5
1.0
.5
0
CORRECTION FACTOR — K'
0
1
2
3
4
5
6
SUBMERGENCE TO DIAMETER RATIO
U.R.I. 64
K.T.P.

FIG. № 11
HYDRODYNAMIC MASS
END CORRECTION FACTOR
for
RECTANGULAR FLAT PLATES

$$m_h = k' \rho \pi \frac{W^2}{4} L$$



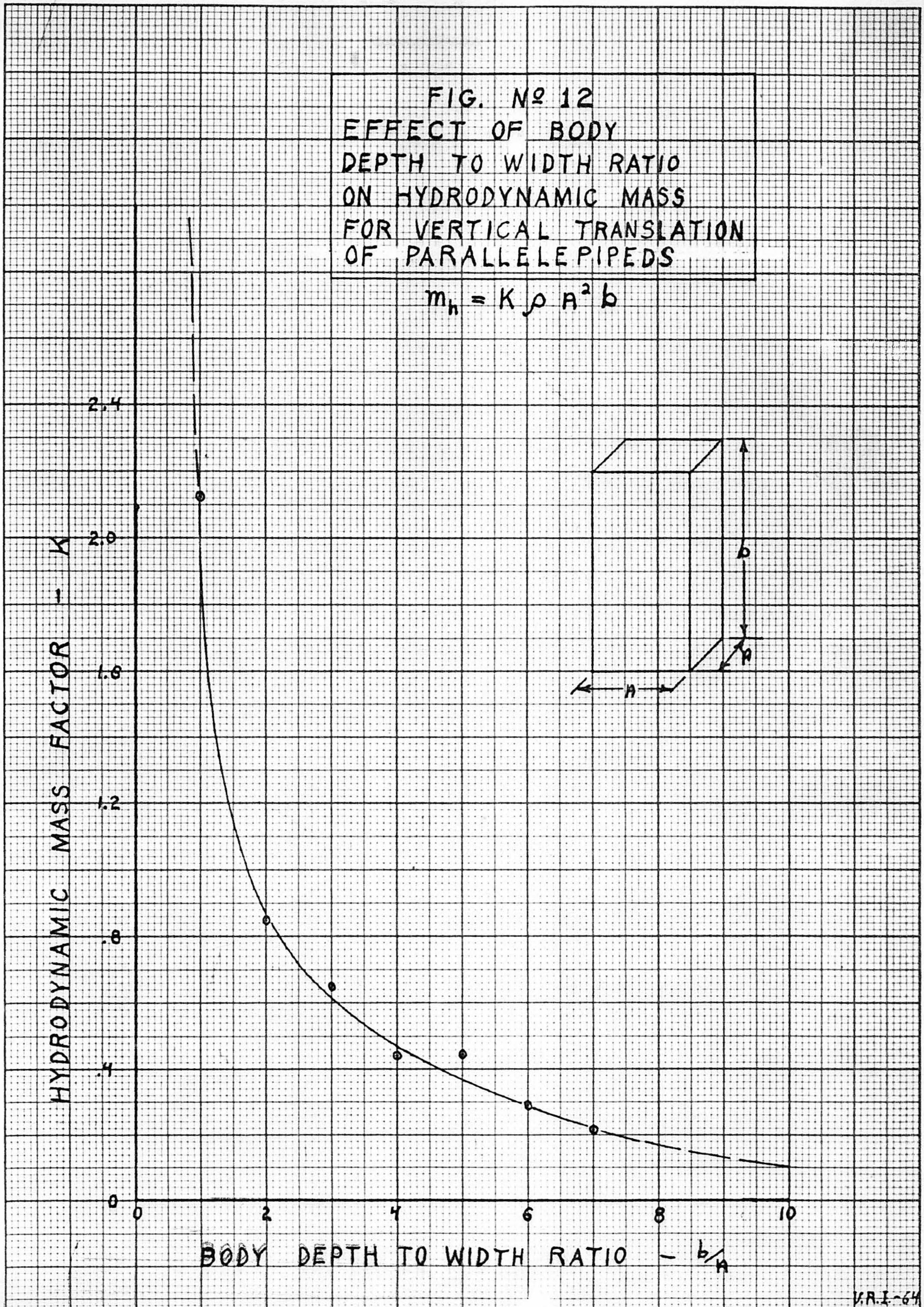
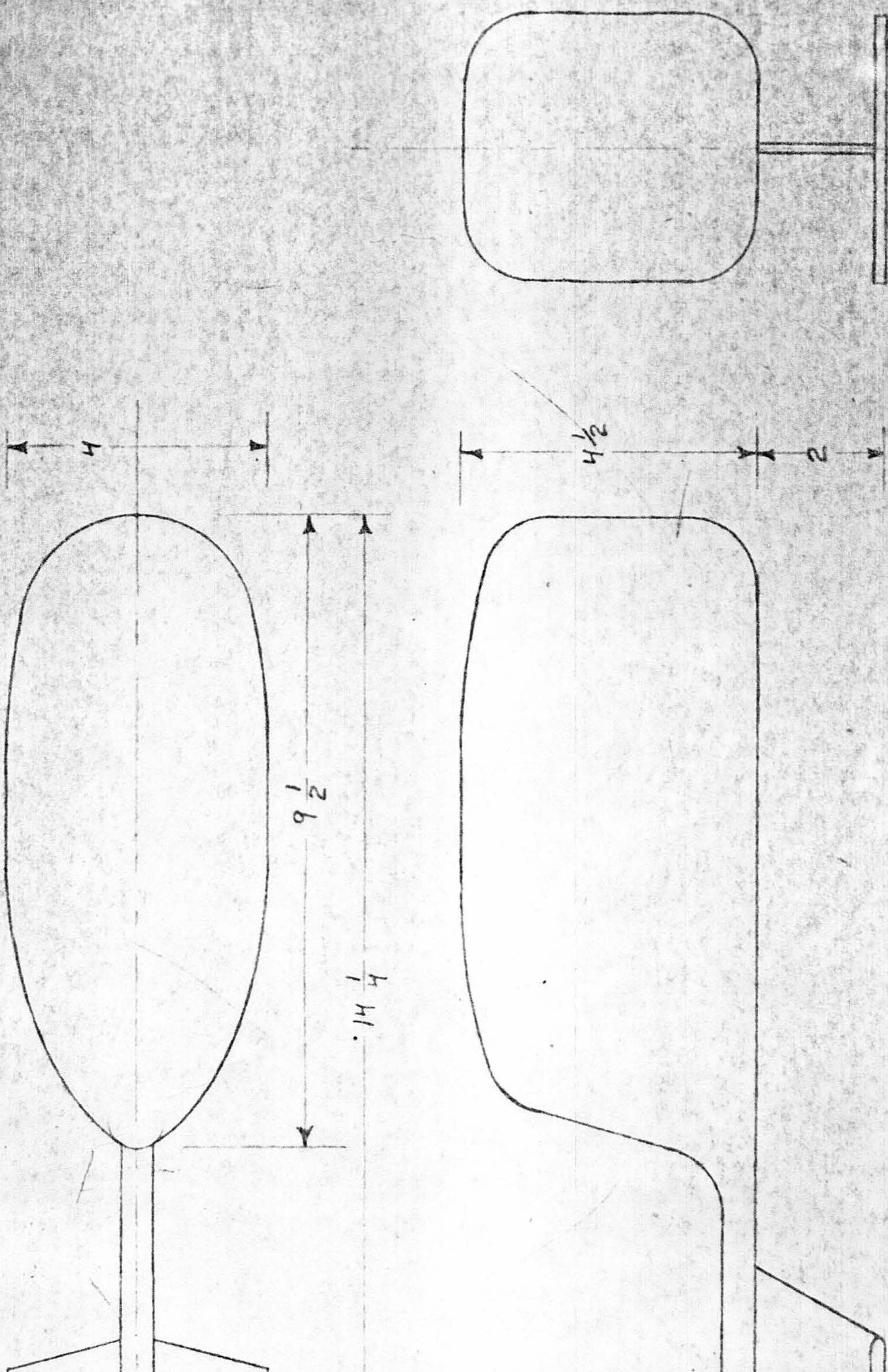


FIG. № 12
EFFECT OF BODY
DEPTH TO WIDTH RATIO
ON HYDRODYNAMIC MASS
FOR VERTICAL TRANSLATION
OF PARALLELEPIPEDS
$m_h = K \rho A^2 b$
HYDRODYNAMIC MASS FACTOR - K
BODY DEPTH TO WIDTH RATIO - b/A
2.4
2.0
1.6
1.2
.8
.4
0
0
2
4
6
8
10
b
A
A
V.R.I.-64
K.J.P.



SCALE – 1/2.66

STREAMLINED BODY
(BODY No 23)

FIG. N° 13

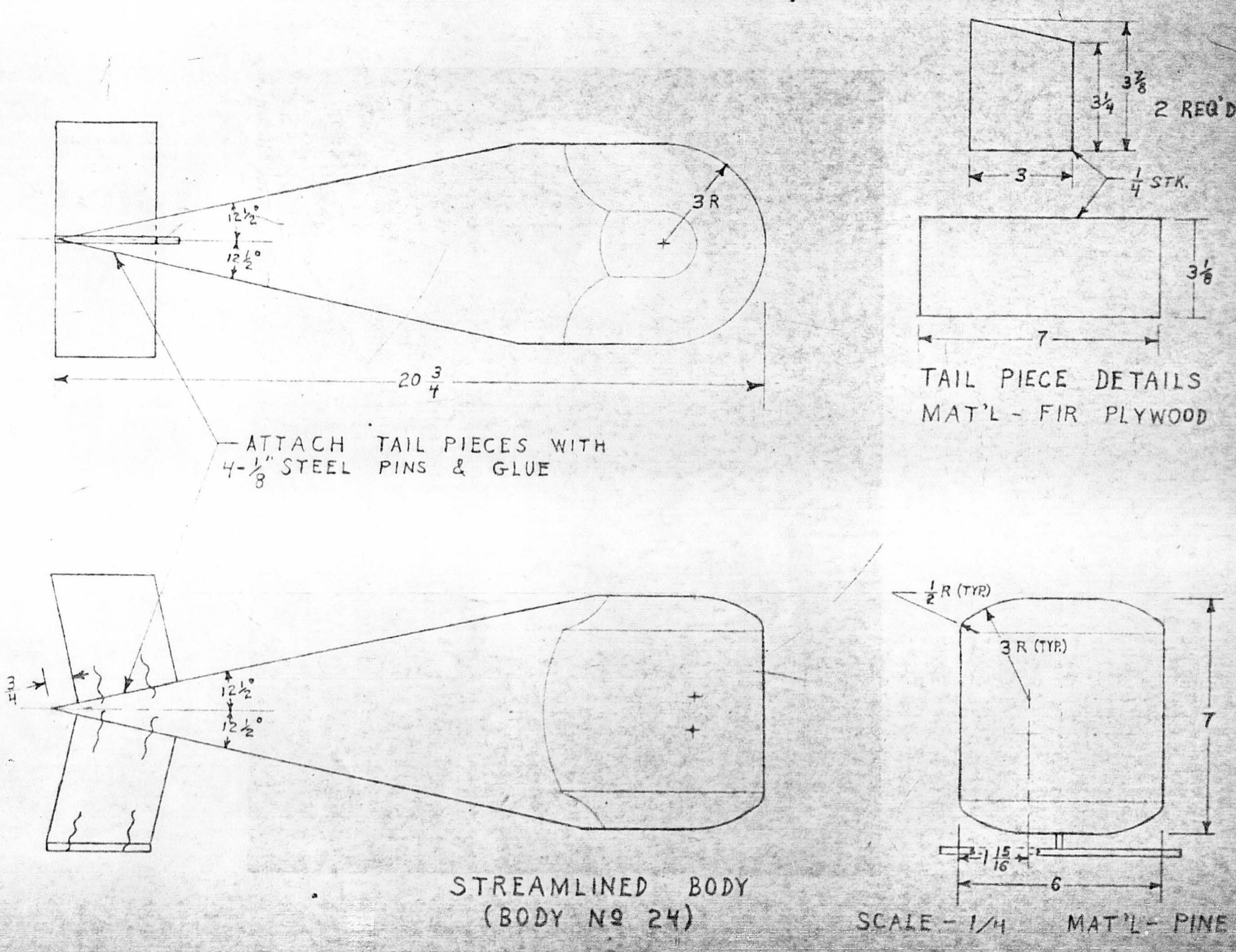

2 REQ'D
3
1/4 STK.
3 1/4
3 7/8
3 1/8
7
TAIL PIECE DETAILS
MAT'L - FIR PLYWOOD
3 R
12 1/2°
12 1/2°
20 3/4
ATTACH TAIL PIECES WITH
4 - 1/8" STEEL PINS & GLUE
3/4
12 1/2°
12 1/2°
1/2 R (TYP.)
3 R (TYP.)
7
1 15/16
6
STREAMLINED BODY
(BODY № 24)
SCALE - 1/4
MAT'L - PINE

FIG. № 14

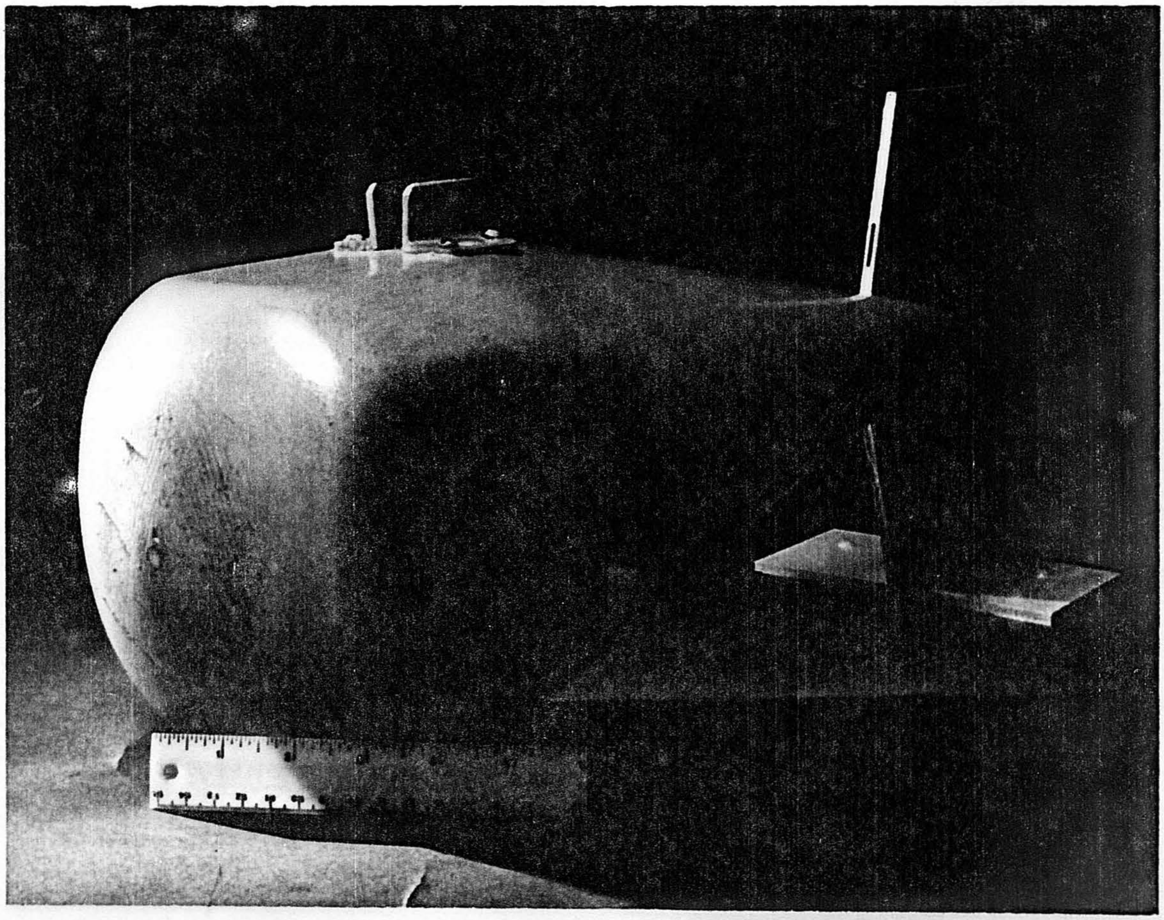


FIG. Nº 15

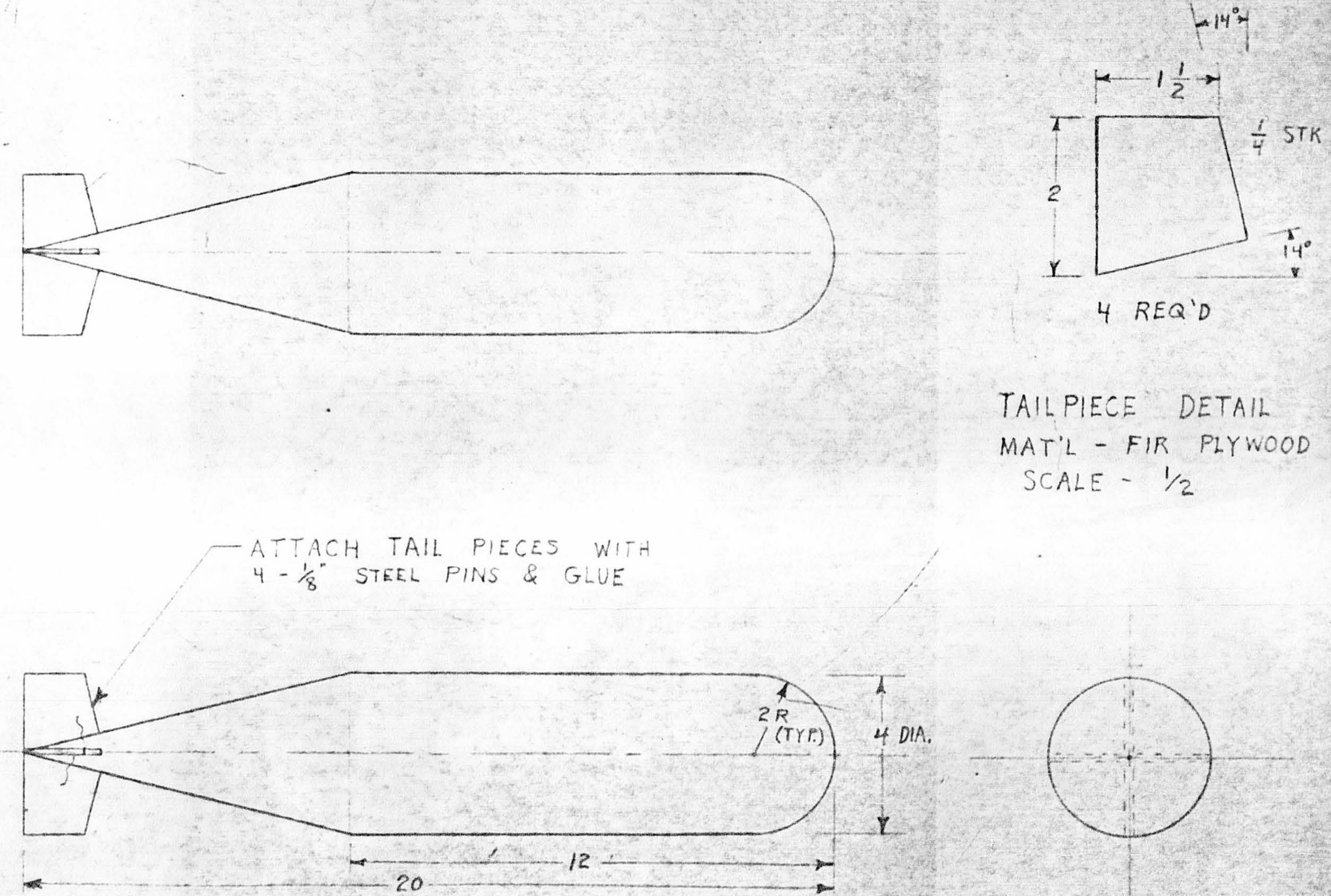


FIG. Nº 16

TORPEDO BODY
(BODY Nº 25)

SCALE - 1/4 MAT'L - PINE

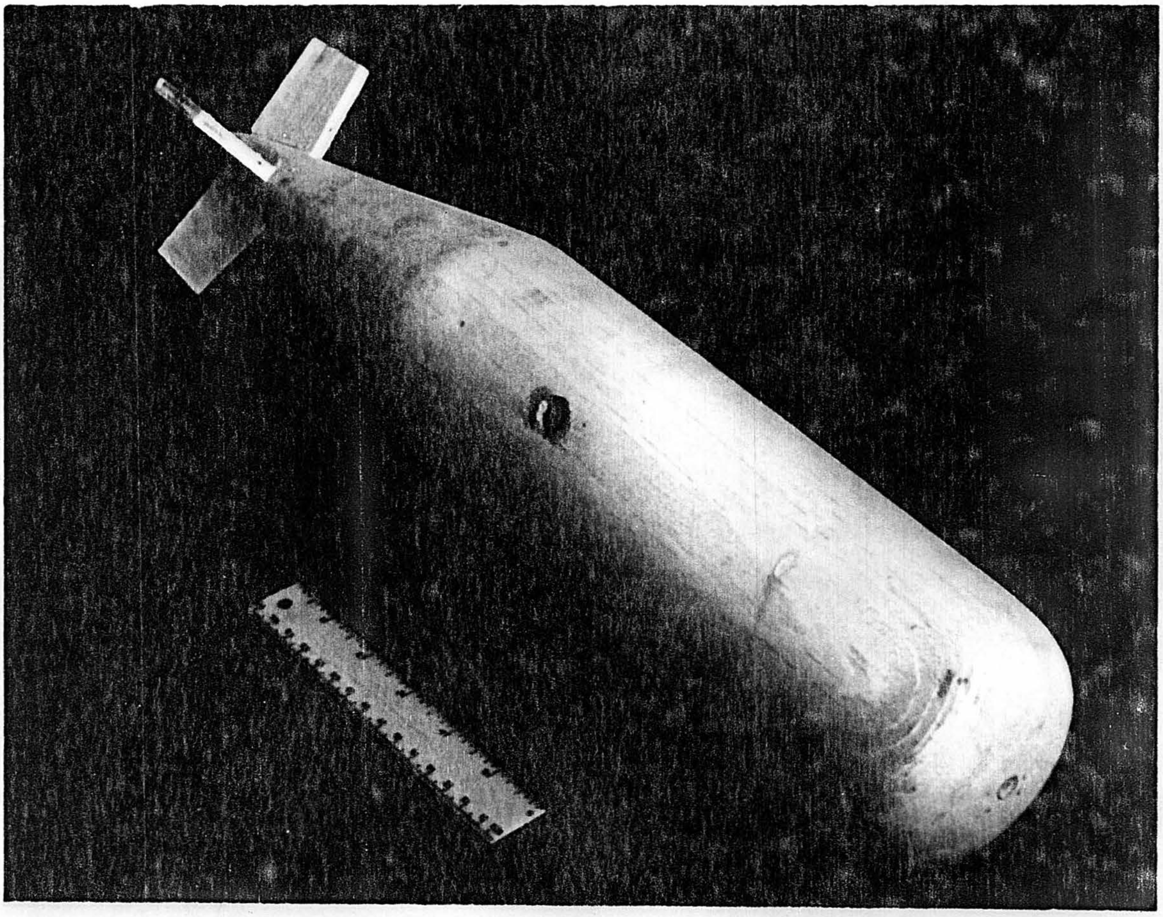
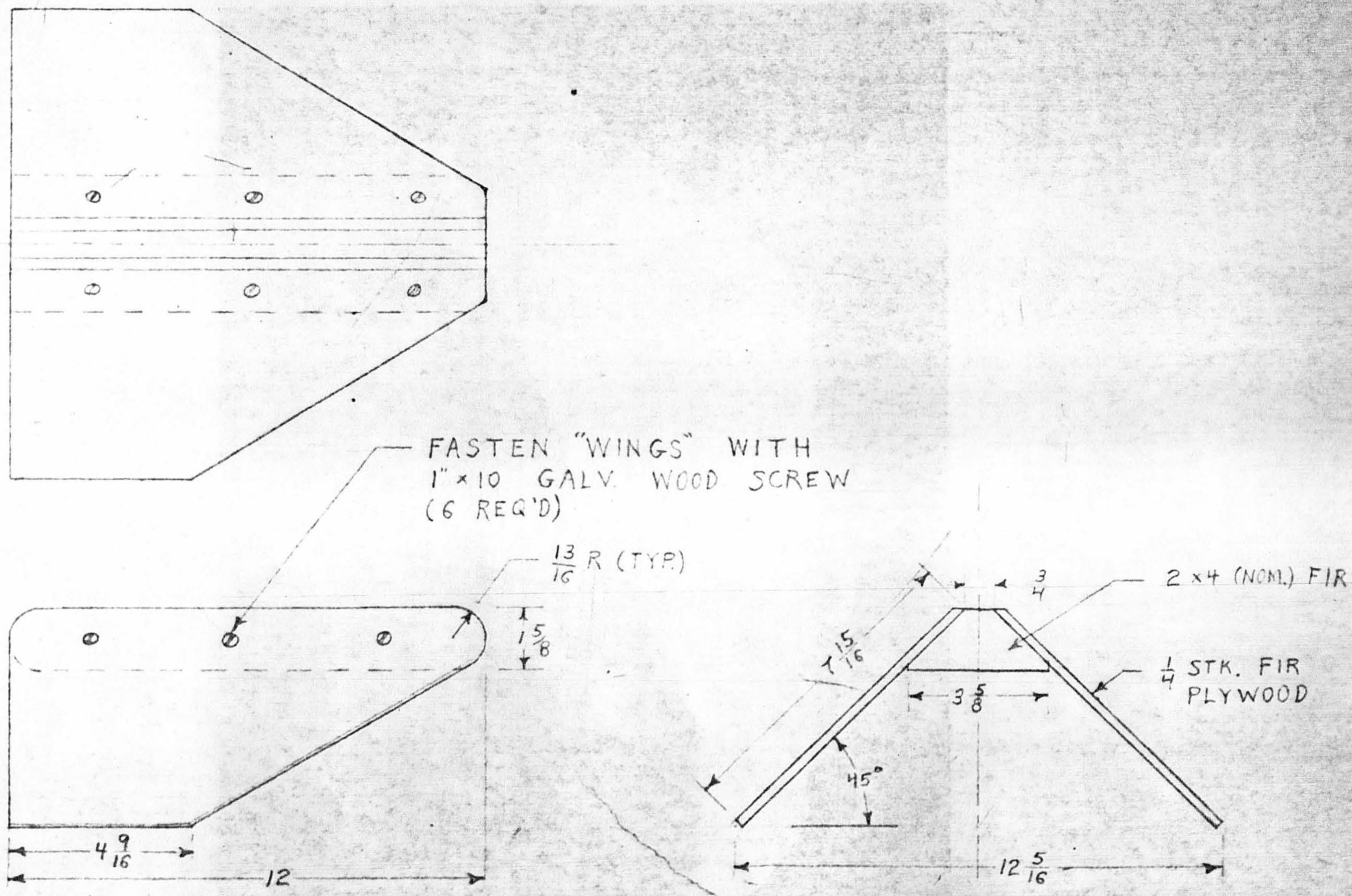


FIG. № 17



"V"-FIN BODY
BODY No 26

FIG No 18

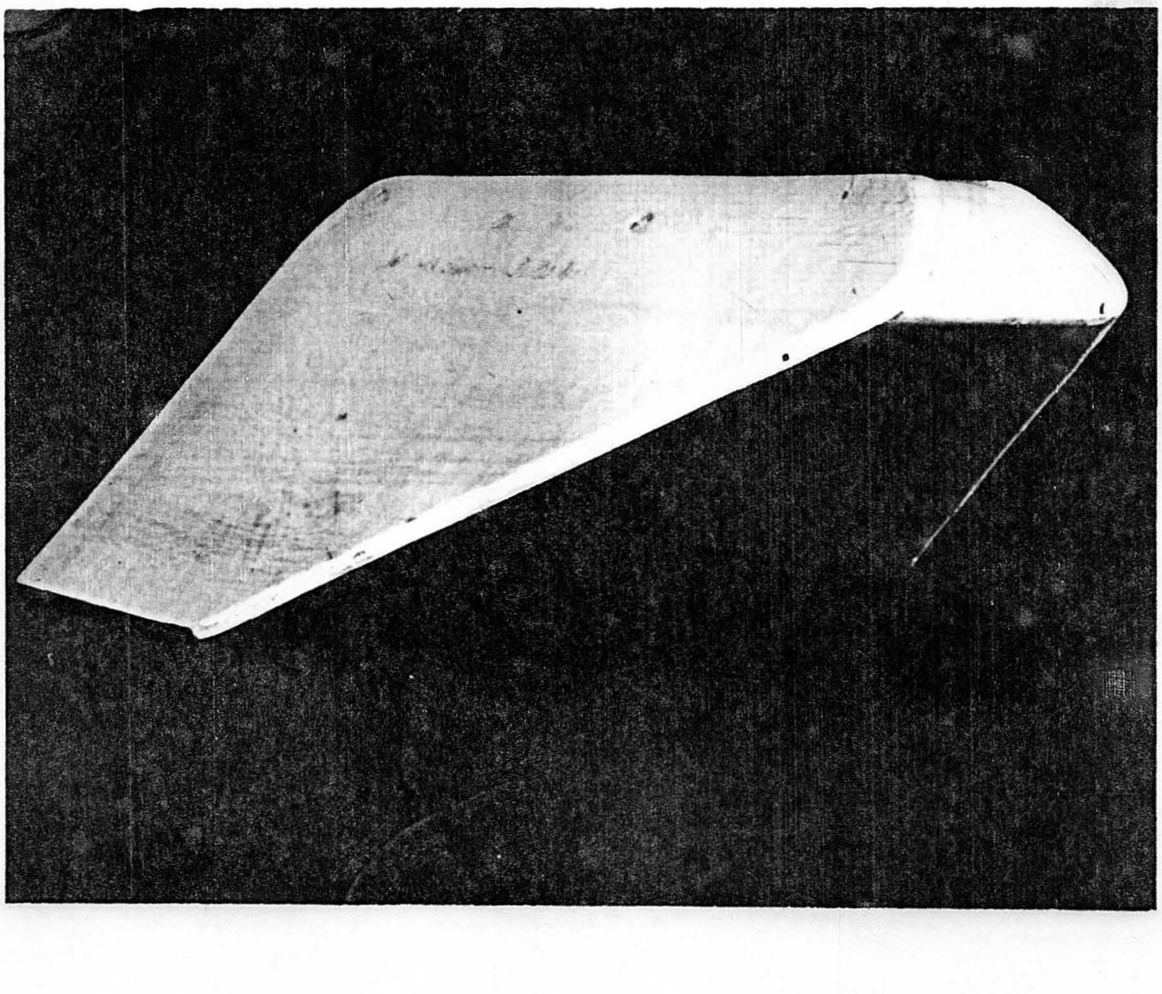


FIG. Nº 19

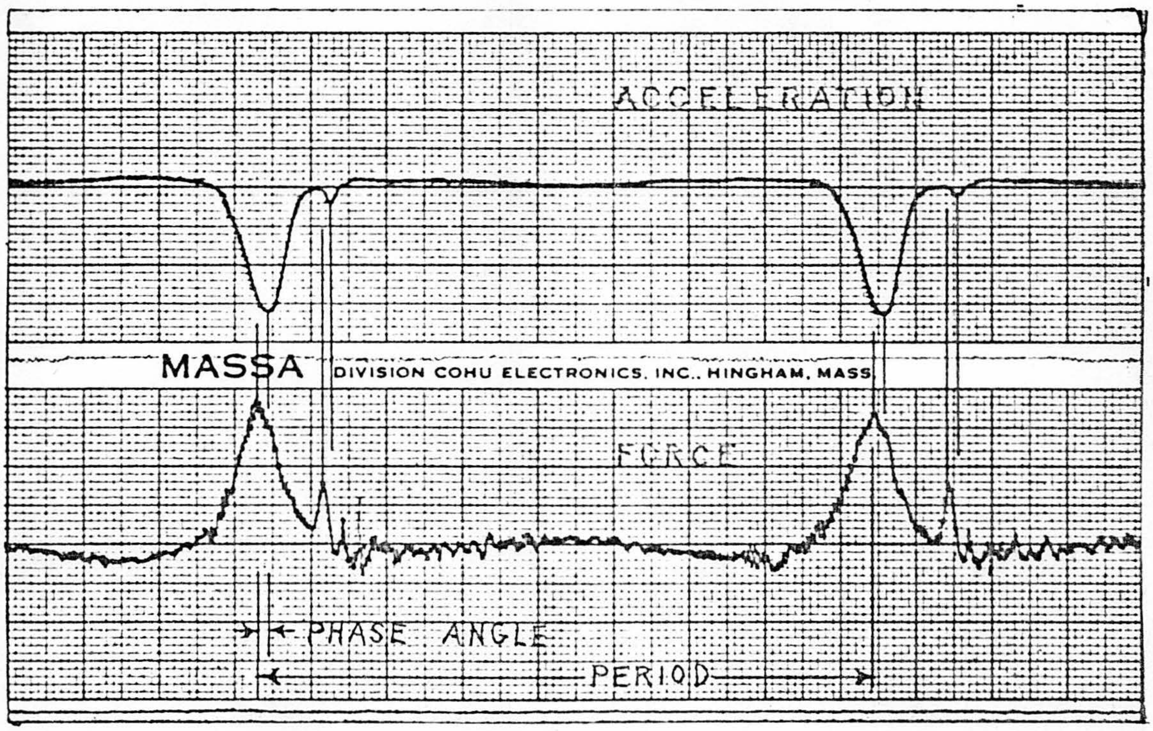


FIGURE 20

FORCED OSCILLATION TESTS OF A SPHERE

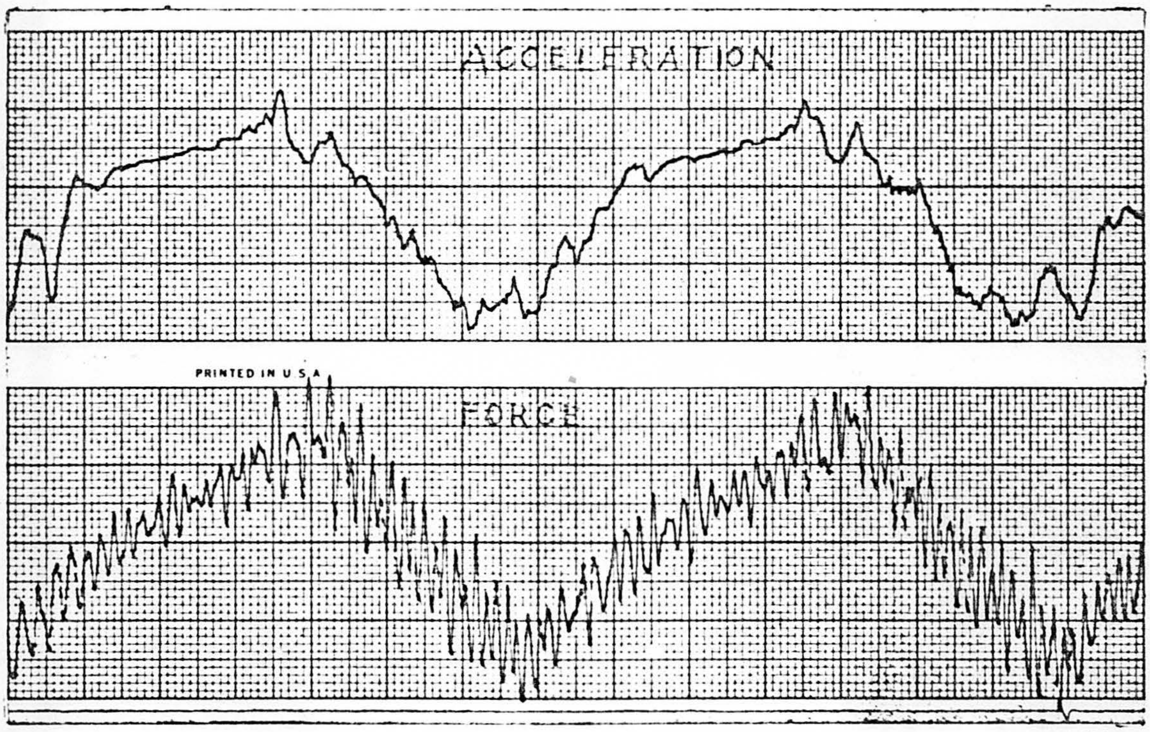


FIGURE 21

FORCED OSCILLATION TESTS OF A CIRCULAR DISC