1987

An Empirical Comparison of Factor, Image, Component, and Scale Scores

Joseph L. Fava
University of Rhode Island

Follow this and additional works at: <https://digitalcommons.uri.edu/theses>

Recommended Citation

Fava, Joseph L., "An Empirical Comparison of Factor, Image, Component, and Scale Scores" (1987). *Open Access Master's Theses*. Paper 1561.
<https://digitalcommons.uri.edu/theses/1561>

This Thesis is brought to you for free and open access by DigitalCommons@URI. It has been accepted for inclusion in Open Access Master's Theses by an authorized administrator of DigitalCommons@URI. For more information, please contact digitalcommons-group@uri.edu.

AN EMPIRICAL COMPARISON OF FACTOR,
IMAGE, COMPONENT, AND SCALE SCORES

BY
JOSEPH L. FAVA

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS
IN
PSYCHOLOGY

UNIVERSITY OF RHODE ISLAND

1987

Abstract

The relationships among principal component, image component, three types of factor scores, and a scale score method were examined. The scores were compared over several levels of variables (p), component saturations (a_{ij}), sample sizes (N), variable to component ratios (p/m), and factor pattern rotations. Scores were compared on the same (convergent) components, and also on different (divergent) components. These comparisons indicated that there were virtually no differences among score methods, although comparisons within score methods, i.e. between component scores and among factor scores, generally indicated slightly higher relationships than did comparisons between the respective score methods. Also, the scale score method, while correlating the lowest, was still very representative of the component and factor scores. When scores did depart from each other, it usually occurred in one of the conditions of low component saturation, low sample size, low p/m ratio, or a combination of these conditions.

Acknowledgements

I would like to thank the members of my Master's committee: Jim Prochaska, Roger Peck, and especially Wayne Velicer. As my major professor, Wayne provided extensive stimulation, support, and encouragement throughout the course of this study.

I would also like to thank Terri Hodson and Elaine Taylor for their help as they patiently typed their way through several revisions of this manuscript.

Finally, I would like to thank my family and friends for their encouragement in the pursuit of this achievement. I would like to note a special thanks, and dedicate this thesis to my deceased mother, who instilled within me a work ethic and sense of endeavor which has accompanied me throughout my life.

Table of Contents

Abstract	ii
Acknowledgements	iii
Table of Contents	iv
List of Tables	v
Introduction	1
Method	10
Results	23
Discussion	39
References	44
APPENDIX A	48
APPENDIX B	53
APPENDIX C	58
APPENDIX D	63
APPENDIX E	68
APPENDIX F	73
APPENDIX G	78
APPENDIX H	81
APPENDIX I	84
APPENDIX J	87

List of Tables

TABLE	PAGE
1. Comparable p/m ratios, where p =variables, and m =components	12
2. Population Pattern Matrix, where $p=36$, $m=6$, and $a_{ij}=.8$	15
3. Variances and Covariances of Factor Score Estimates	17
4. Covariances between Principal Component Scores and Image Component Scores with Factor Score Estimates	19
5. Covariances between Scale Scores and Factor, Image Component, and Principal Component Scores	21
6. Mean Correlations among Convergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, $p=72$	26
7. Mean Correlations among Convergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, $p=36$	27
8. Mean Correlations among Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, $p=72$	30
9. Mean Correlations among Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, $p=36$	31
10. Mean Correlations among Divergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, $p=72$	33
11. Mean Correlations among Divergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, $p=36$	34
12. Mean Correlations among Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, $p=72$	36

13.	Mean Correlations among Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p=36.	37
A-1.	Standard Deviations of the Mean Correlations among the Convergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, p=72	49
A-2.	Standard Deviations of the Mean Correlations among the Convergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, p=36	50
A-3.	Standard Deviations of the Mean Correlations among the Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p=72	51
A-4.	Standard Deviations of the Mean Correlations among the Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p=36	52
B-1.	Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, p=72	54
B-2.	Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, p=36	55
B-3.	Standard Deviations of the Mean Correlations among the Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p=72	56
B-4.	Standard Deviations of the Mean Correlations among the Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p=36	57

C-1. Mean Correlations among Convergent Components for Principal Component, Image Component, and Factor Scores: Promax Position, p=72	59
C-2. Mean Correlations among Convergent Components for Principal Component, Image Component, and Factor Scores: Promax Position, p=36	60
C-3. Mean Correlations among Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=72	61
C-4. Mean Correlations among Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=36	62
D-1. Standard Deviations of the Mean Correlations among the Convergent Components for Principal Component, Image Component, and Factor Scores: Promax Position, p=72	64
D-2. Standard Deviations of the Mean Correlations among the Convergent Components for Principal Component, Image Component, and Factor Scores: Promax Position, p=36	65
D-3. Standard Deviations of the Mean Correlations among the Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=72	66
D-4. Standard Deviations of the Mean Correlations among the Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=36	67
E-1. Mean Correlations among Divergent Components for Principal Component, Image Component, and Factor Scores: Promax Position, p=72	69
E-2. Mean Correlations among Divergent Components for Principal Component, Image Component, and Factor Scores: Promax Position, p=36	70

E-3.	Mean Correlations among Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=72	71
E-4.	Mean Correlations among Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=36	72
F-1.	Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Promax Position, p=72	74
F-2.	Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Promax Position, p=36	75
F-3.	Standard Deviations of the Mean Correlations among the Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=72	76
F-4.	Standard Deviations of the Mean Correlations among the Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=36	77
G-1.	Mean Correlations among Convergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, p=72	79
G-2.	Mean Correlations among Convergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, p=36	80
H-1.	Standard Deviations of the Mean Correlations among the Convergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, p=72	82
H-2.	Standard Deviations of the Mean Correlations among the Convergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, p=36	83

I-1. Mean Correlations among Divergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, $p=72$	85
I-2. Mean Correlations among Divergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, $p=36$	86
J-1. Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, $p=72$	88
J-2. Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, $p=36$	89

Introduction

Factor analysis and component analysis, which includes principal component analysis (Hotelling, 1933) and image component analysis (Guttman, 1953), are competing methods used in data reduction. Both factor analysis and component analysis serve the same two broad purposes. The first purpose concerns the question of pattern interpretation, whereby the factor pattern is studied to determine which variables are related to each other. By studying these relationships, it is possible to replace the original p variables by m new variables, such that m is less than p . Specifically, these m new variables are called components or factors depending on the method, but these terms can be used interchangeably for purposes of discussion. Previous research (Velicer, 1977; Velicer, Peacock, & Jackson, 1982; Velicer & Fava, 1987a, 1987b) has indicated that there is very little practical difference between the resulting factors derived from the methods of principal component analysis, factor analysis, and image component analysis.

The second purpose concerns a question of parsimony and relates to the reduction of the p original variables to m new variables. In this case we are replacing the p observed scores by m new scores, such that m is less than p . And in this sense, the use of component and factor scores allows a convenient form of data summary in place of the original scores.

An obvious problem raised by this second purpose concerns possible differences that may exist among various alternative score methods. These score methods include: 1) two types of component scores, i.e.,

principal component and image component scores, 2) as many as five types of factor scores (Harris, 1967), and 3) scale scores, the most common of which is a unit-weighted scale score in which variables that load above a certain value on a component are scored as one (1), and variables that load below that certain value are scored as zero (0).

A major purpose of this study is to compare the various types of score methods. Scores will be compared within methods and between methods. Within method comparisons will compare the various factor score methods to each other, and will compare the principal component score method with the image component method. Between method comparisons will compare factor scores to component scores, factor scores to the scale score, and component scores to the scale score.

A second purpose of this study will be to provide some groundwork for future research. It is expected that the correlation technique used to compare the various score methods will lend itself to a future study which will compare the scores of components based on a 'true' population pattern with scores based on overextracted and underextracted components of the same population pattern.

The factor analysis model follows the form

$$R = AA' + U^2 \quad (1)$$

where R is the $p \times p$ correlation matrix of the variables in the sample, A is the $p \times m$ factor pattern, and U^2 is the $p \times p$ diagonal matrix of unique variances. The principal component analysis model follows the form

$$R = AA' \quad (2)$$

where R and A are defined as above. The image component analysis model follows the form

$$S^{-1}RS^{-1} = AA' \quad (3)$$

where R and A are defined as above, and

$$S^2 = \text{diag}^{-1}(R^{-1}). \quad (4)$$

Principal component scores are calculated by the formula

$$F_c = YL_c D_c^{-1} \quad (5)$$

where Y is the $\underline{n} \times \underline{p}$ matrix of normalized deviation scores, such that

$$Y'Y = R \quad (6)$$

where R is the $\underline{p} \times \underline{p}$ correlation matrix of the variables in the sample, L_c is a $\underline{p} \times \underline{m}$ matrix of eigen vectors from an eigen decomposition of R , and D_c^2 is a diagonal matrix containing the first $\underline{m}$ eigen roots of the eigen decomposition of R . Image component scores are calculated by the formula

$$F_I = YS^{-1}L_I D_I^{-1} \quad (7)$$

where Y is defined as above, S^2 is defined by formula 4, L_I is a $p \times m$ matrix of eigen vectors from an eigen decomposition of $S^{-1}RS^{-1}$, and D_I^2 is a diagonal matrix containing the first m eigen roots of the eigen decomposition of $S^{-1}RS^{-1}$.

Formulas for factor scores are, by contrast, not nearly as straightforward, and the result has been the creation of various factor score estimates. This issue is a direct result of the factor indeterminacy problem (Guttman, 1955; Schonemann & Wang, 1972), in which the factors of a factor analysis cannot be uniquely defined. Consequently, factor scores cannot be uniquely defined either, and, in fact, an infinite number of sets of factor scores could be computed for each factor analysis following Guttman's (1955) method. In point of fact, however, all the factor score estimates in common use are " . . . well-defined and are simply linear composites of the observed variables, weighted according to some rationale" (Velicer, 1976, p.151).

Five methods of computing factor score estimates, discussed by Harris (1967), are

$$F_1 = YR^{-1}A \quad (8)$$

$$F_2 = YU^{-2}A(A'U^{-2}A)^{-1} \quad (9)$$

$$F_3 = YU^{-2}A(A'U^{-2}RU^{-2}A)^{-\frac{1}{2}} \quad (10)$$

$$F_4 = YA(A'A)^{-1} \quad (11)$$

$$F_5 = YA \quad (12)$$

where F_1 is an $n \times m$ matrix of factor score estimates, Y is an $n \times p$ matrix of normalized deviation scores following the relationship of formula 6, R is a $p \times p$ correlation matrix of the variables in the sample, A is a $p \times m$ factor pattern, and U^2 is a $p \times p$ diagonal matrix of unique variances.

F_1 , F_2 , and F_3 are widely-used, competing methods of computing factor score estimates that will be examined in this study. Each of these methods contains some, but not all, of the four properties that are desired in factor scores, in the context of the orthogonal factor model, as noted by McDonald and Burr (1967). These properties include: 1) a high correlation between the constructed factor score and the true factor score that is being estimated, 2) orthogonality between the constructed factor scores, 3) univocality of constructed factor scores, i.e., each constructed factor score should be uncorrelated with each non-corresponding true factor score, and 4) constructed factor scores as conditionally unbiased estimators of their true factor scores.

F_1 , which is a regression estimate (Thurstone, 1935; Thomson, 1936), while generally a popular method, is only imbued with property 1 above. F_2 , which minimizes the sum of squares of the residuals

(Bartlett, 1937), is imbued with properties 1, 3, and 4 above. F_3 (Anderson & Rubin, 1956) also minimizes the sum of squares of the residuals, but is constrained in such a way that it satisfies properties 1 and 2 above. While F_1 may appear the weakest in that it only satisfies property 1, it also yields higher correlations than F_2 between the constructed factor scores and the true factor scores that it attempts to estimate (McDonald & Burr, 1967). And in general, the arguments persist over which of the four properties desired in factor scores are the most important, and hence which of the methods of computing factor score estimates is the 'best'.

The two factor score estimates that will not be examined in this study are F_4 and F_5 . F_4 is known as the direct method or the method of ideal variables, and assumes that the unique part of the score is not essential to obtaining a good approximation of the true factor score (Horn, 1965). F_5 was a common early technique used before computers were widely available to eliminate the need of matrix inversion (Horn, 1965). It has been noted by Velicer (1976) that both F_4 and F_5 "... make unnecessary assumptions" (p.151).

The unit-weighted scale score is calculated by the formula

$$U_c = YP_{vf}(P'_{vf}P_{vf})^{-1} \quad (13)$$

where Y is an $n \times p$ matrix of normalized deviation scores following the relationship of formula 6, and P_{vf} is a salience pattern matrix based on the varimax-rotated and promax-rotated principal

component patterns, where 1 replaces all loadings greater than or equal to .30, -1 replaces all loadings less than or equal to -.30, and 0 replaces all loadings greater than -.30 and less than .30. This formula is based on formula 12.1.5, as given by Gorsuch (1983), and is useful for the case where one assumes the variables are perfectly measured with no unique variance, as is the case for unit-weighted component scores.

A previous study by Velicer (1976) explored the issue of differences between principal component, image component, and factor scores F_1 , F_2 , and F_3 . In his study, Velicer derived scores from 9 classic data sets often employed in the literature, which had been systematically analyzed by others (Joreskog, 1967; Schonemann & Wang, 1972) in the past, and found no practical differences among these different types of scores. Because these data sets were so well analyzed their characteristics were well-known; however, because the data sets were real data sets, Velicer's (1976) study can only be thought of as covering a very specific range of situations. This present study expands on and further explores the validity of Velicer's (1976) results by using simulated data sets that have known population parameters and which cover a wider range of potential situations. From these simulated data sets, principal component scores, the 3 types of factor scores, i.e., F_1 , F_2 , and F_3 , image component scores, and also scale scores are calculated and compared. This study offers a more objective interpretation of results, despite its idealized data structures.

The inclusion of a unit-weighted scale score lends an important addition to the scope of this study. Unit-weighted scale scores are

in common use, are quite parsimonious and simple to calculate, and have been recommended for many years in the literature (Trites & Sells, 1955; Horn, 1965; Schweiker, 1967). Unit-weighted scores have also been criticized by some (Glass & Maguire, 1966) as an incorrect method of computing factor scores, which may lead to substantial intercorrelations of factor scores and which may not correlate highly with the correctly computed scores. Mosely and Klett (1964) have also noted substantial intercorrelations of factor scores when unit-weights are applied to factor scores. One rejoinder (Wackwitz & Horn, 1971) notes that it is reasonable to assume that some samples from a population of uncorrelated factor scores may indeed be correlated. It has also been noted that unit-weighted scale scores take little advantage of chance influences affecting their weight matrices and thus should help to control for shrinkage of reliability in cross-validation (Horn, 1965; Schweiker, 1967). A Monte Carlo study conducted by Morris (1979) strongly suggests that the use of unit-weighted, and unique unit-weighted, factor and component scores as independent variables with regression procedures is superior to the use of sample-specific regression weights of variables for purposes of replicability and cross-validation of regression equations. For these reasons, and especially considering their great potential use in the investigation of issues concerning reliability and validity, the relationship of scale scores to actual principal component, image component, and factor scores deserves to be further studied. In this present study, the issue of cross-validation, using scale, principal component, image component, and factor scores, is not addressed.

There are several factors which can potentially affect the degree of relationship among the various score methods and which will be manipulated in this study. These factors include the number of variables in the data set, the number of components, the variable to component ratio, the factor loadings (component saturation), the sample size, and the rotational position of the factor pattern.

Based on previous research (Velicer, 1976), it is hypothesized that for well-defined factor patterns there will be little or no practical differences between principal component, image component, and the three types of factor scores, derived respectively from principal component analysis (Hotelling, 1933), an image component analysis variant (Harris, 1962, 1967), and the algorithm by Browne (1968) for maximum likelihood factor analysis. It is expected that the unit-weighted salience score, which is commonly used in most scale score situations, and which represents a most parsimonious application of scale scoring, will not correlate as highly with the other types of scores as the other types of scores will correlate among themselves. The range of the correlational relationships involving the unit-weighted score is one of the topics of investigation of this study.

Method

In an attempt to approximate possibilities that might typically confront the applied researcher, several levels of each factor of importance were sampled. While the conditions that were sampled were necessarily somewhat idealized and did not correspond precisely to real world data sets, the computer simulated data structures allowed the retention of control over population parameters and resulted in more objective comparisons among methods for obtaining scores.

The factors that were manipulated included sample size (N), number of components (m), component saturation (a_{ij}), the number of variables (p), and the number of variables to component ratio (p/m).

The number of variables was represented in this study under two conditions, $p = 36$ and $p = 72$. These values can be considered as representing small and moderately-sized data sets in an applied situation (Zwick & Velicer, 1982, 1986). Based on related research (Guadagnoli & Velicer, in press; Velicer & Fava, 1987a, 1987b) it was not felt that this variable would have a strong effect. However, the exact conditions of this study, i.e., comparing principal component, factor, image component, and scale scores, had not been previously examined, and it was possible that the number of variables would affect score comparisons.

Previous research (Guadagnoli & Velicer, in press) suggested that the variable to component ratio (p/m) was crucial to component stability. Also, studies by Velicer and Fava (1987a; 1987b), which investigated the effects of variable sampling on principal component analysis, image component analysis, and factor analysis, found that

the p/m ratio was important. As it seemed likely that stable scores depended on stable components, the p/m ratio was a factor of concern.

The number of components represented in this study was 3, 6, 9, 12, and 18. Components (m) and variables (p) were matched so as to create p/m ratios of 12, 6, and 4 for each variable condition. Table 1 displays the p/m ratios being compared. Twelve variables per factor can be thought of as representing a very strong factor, 6 variables per factor a moderately strong factor, and 4 variables per factor as representing a near minimum for factor replicability, where 3 variables per factor would be considered the minimum for factor replicability (Anderson & Rubin, 1956; Rindskopf, 1984).

It should be noted that because of rotational indeterminacy, the same components, generated from a principal component analysis, a factor analysis, and an image component analysis of the same sample matrix, can end up in different positions after a varimax rotation. A permutation matrix (Velicer, 1974, 1976, 1977; Guadagnoli & Velicer, in press) was used to match the respective factor patterns generated by each statistical technique in order to properly compare them. However, under certain conditions, such as low factor loadings and a low p/m ratio, the permutation matrix sometimes failed to match the components accurately. This inability to match and compare components accurately was especially true in some pilot research for this study, under the condition of $p/m = 3$, previously noted as the minimum number of variables for factor replicability (Anderson & Rubin, 1956; Rindskopf, 1984). This was also the main reason that $p/m = 4$ was chosen as the lowest level of that factor to be manipulated.

Table 1

Comparable p/m ratios, where p=variables, and m=components

p	<u>m</u>	ratio
36	3	12:1
72	6	12:1
36	6	6:1
72	12	6:1
36	9	4:1
72	18	4:1

Factor loadings (component saturation) were examined at values of .4, .6, and .8. This represented a very strong loading (.8), a moderate loading (.6), and a borderline limit for an acceptable loading (.4). These loading values were chosen for their representativeness of typical values encountered in real life situations. Also they allowed a degree of control over the stability of the component pattern, as high loadings produce the most stable component patterns, while low loadings produce less stable component patterns (Guadagnoli & Velicer, in press).

Sample size was manipulated under conditions of $N = 80, 160, 240,$ and 480. For most multivariate studies, a sample size of 80 is considered to be fairly low and may produce unstable results. Sample sizes in the range of 160 to 240 are thought to produce stable results for many multivariate purposes (Boosma, 1982; Velicer et al, 1982; Guadagnoli & Velicer, in press). And a sample size of 480 is thought to produce very stable results in most situations.

Also represented were the 6 different methods of producing scores, i.e., the principal component score, the image component score, 3 types of factor scores, and the scale score based on the varimax-rotated and promax-rotated principal component patterns. The scores produced by these 6 methods were compared after each of 3 different rotations of their basic pattern matrix. The first comparison took place in the varimax-rotated (Kaiser, 1958) position. The second comparison occurred in the promax-rotated (Henderson & White, 1964) position. And the third comparison was achieved after a procrustes rotation (Hurley & Cattell, 1962) using the varimax component pattern as the target pattern. Unit-weighted scores were

not compared after a procrustes rotation as this was not a situation where unit-weighted scores would logically be applied. In this comparison of the 6 different methods of creating scores there were 15 correlation matrices that were calculated after each of the first two rotations, and 10 correlation matrices that were calculated after the third rotation, for a total of 40 correlation matrices after all three rotations.

The actual data matrices that were analyzed were constructed in the following manner. First, population matrices were constructed for the different p , m , and a_{ij} combinations using a procedure previously employed by Zwick and Velicer (1982), Velicer, Peacock, and Jackson (1982), and Guadagnoli and Velicer (in press). This involved creating a population pattern matrix (A) for each possible p , m , and a_{ij} combination. Table 2 gives an example of an A matrix, when $p = 36$, $m = 6$, and $a_{ij} = .8$. This $p \times m$ matrix, A , was then post-multiplied by its transpose, thus creating a new matrix R^* ($p \times p$), where

$$R^* = AA'. \quad (14)$$

The final population matrix, R , was then created by replacing the diagonal elements of R^* with unities. Secondly, each created population matrix R was input to a computer program by Montanelli (1975). Montanelli's program added random error to each created

Table 2
Population Pattern Matrix, where $p=36$, $m=6$, and $a_{ij}=.8$

		Components (m)					
		1	2	3	4	5	6
Variables (p)	1	.80	.00	.00	.00	.00	.00
	2	.80	.00	.00	.00	.00	.00
	3	.80	.00	.00	.00	.00	.00
	4	.80	.00	.00	.00	.00	.00
	5	.80	.00	.00	.00	.00	.00
	6	.80	.00	.00	.00	.00	.00
	7	.00	.80	.00	.00	.00	.00
	8	.00	.80	.00	.00	.00	.00
	9	.00	.80	.00	.00	.00	.00
	10	.00	.80	.00	.00	.00	.00
	11	.00	.80	.00	.00	.00	.00
	12	.00	.80	.00	.00	.00	.00
	13	.00	.00	.80	.00	.00	.00
	14	.00	.00	.80	.00	.00	.00
	15	.00	.00	.80	.00	.00	.00
	16	.00	.00	.80	.00	.00	.00
	17	.00	.00	.80	.00	.00	.00
	18	.00	.00	.80	.00	.00	.00
	19	.00	.00	.00	.80	.00	.00
	20	.00	.00	.00	.80	.00	.00
	21	.00	.00	.00	.80	.00	.00
	22	.00	.00	.00	.80	.00	.00
	23	.00	.00	.00	.80	.00	.00
	24	.00	.00	.00	.80	.00	.00
	25	.00	.00	.00	.00	.80	.00
	26	.00	.00	.00	.00	.80	.00
	27	.00	.00	.00	.00	.80	.00
	28	.00	.00	.00	.00	.80	.00
	29	.00	.00	.00	.00	.80	.00
	30	.00	.00	.00	.00	.80	.00
	31	.00	.00	.00	.00	.00	.80
	32	.00	.00	.00	.00	.00	.80
	33	.00	.00	.00	.00	.00	.80
	34	.00	.00	.00	.00	.00	.80
	35	.00	.00	.00	.00	.00	.80
	36	.00	.00	.00	.00	.00	.80

population matrix R , such that R was transformed to an approximate R with

$$\hat{R} = R + \underline{e}_{ij} \quad (15)$$

where $\underline{e}_{ij}$ represents random error added to each cell in the population matrix R , where $i \neq j$, and unities are maintained in the diagonal ($i=j$) elements of $\hat{R}$. Five sample correlation matrices were generated from each created population matrix for each different sample size, for each p , m , and $\underline{a}_{ij}$ combination.

These sample correlation matrices were then analyzed by principal component analysis, image component analysis, and maximum likelihood factor analysis. After these analyses the various factor patterns were rotated, and the principal component, image component, factor, and scale scores were correlated.

In comparing the various factor score estimates this study does not deal with actual scores, rather it uses the variances and covariances between the estimates in the manner of Harris (1967) and McDonald and Burr (1967). This allows the formation of correlations between the estimates using the formula

$$R_{ij} = \text{diag}^{-1/2}(\underline{F}_i' \underline{F}_i)(\underline{F}_i' \underline{F}_j) \text{diag}^{-1/2}(\underline{F}_j' \underline{F}_j) \quad (16)$$

where $i = 1, 2, 3$ and $j = 1, 2, 3$ with $i \neq j$. See Table 3 for the within variance-covariance matrices that are the basis of the correlations between factor score estimates.

Table 3
Variances and Covariances of Factor Score Estimates

	F_1	F_2	F_3
F_1'	$A'R^{-1}A$	I	$(A'U^{-2}RU^{-2}A)^{-\frac{1}{2}}(A'U^{-2}A)$
F_2'	$(A'U^{-2}A)^{-1}(A'U^{-2}RU^{-2}A)$	$(A'U^{-2}RU^{-2}A)^{-\frac{1}{2}}(A'U^{-2}RU^{-2}A)$	$(A'U^{-2}A)^{-1}$
F_3'			I

Also, since

$$F_I' F_I = I \quad (17)$$

and

$$F_c' F_c = I \quad (18)$$

the correlations between the factor scores and the image component score, and the correlations between the factor scores and the principal component score follow the respective formulas

$$R_{Ij} = (F_I' F_j) \text{diag}^{-1/2} (F_j' F_j) \quad (19)$$

and

$$R_{Cj} = (F_c' F_j) \text{diag}^{-1/2} (F_j' F_j) \quad (20)$$

where $j = 1, 2, 3$. The covariances between the image component score and the principal component score with the factor scores are given in Table 4.

The correlation between the image component score and the principal component score are found by the relation

$$R_{IC} = F_c' F_I \quad (21)$$

Table 4
Covariances between Principal Component Scores and Image Component
Scores with Factor Score Estimates.

	F_1	F_2	F_3
F'_C	$D_C^{-1} L'_C A$	$D_C^{-1} L'_C R U^{-2} A (A' U^{-2} A)^{-1}$	$D_C^{-1} L'_C R U^{-2} A (A' U^{-2} R U^{-2} A)^{-1/2}$
F'_I	$D_I^{-1} L'_I S^{-1} A$	$D_I^{-1} L'_I S^{-1} R U^{-2} A (A' U^{-2} A)^{-1}$	$D_I^{-1} L'_I S^{-1} R U^{-2} A (A' U^{-2} R U^{-2} A)^{-1/2}$

The correlations for the scale score with the factor scores follow the relationship of formula 16, the correlation for the scale score with the image component score follows the relationship of formula 19, and the correlation for the scale score with the principal component score follows the relationship of formula 20, generating respectively

$$R_{Uj} = \text{diag}^{-1/2}(U_c' U_c) (U_c' F_j) \text{diag}^{-1/2}(F_j' F_j) \quad (22)$$

where $j = 1, 2, 3$, and

$$R_{UI} = (F_I' U_c) \text{diag}^{-1/2}(U_c' U_c) \quad (23)$$

and

$$R_{UC} = (F_c' U_c) \text{diag}^{-1/2}(U_c' U_c) \quad (24)$$

The covariances between the scale score and the factor scores, the scale score and the image component score, and the scale score and the principal component score are set forth in Table 5.

Table 5
Covariances between Scale Scores and Factor, Image Component, and
Principal Component Scores

	U_c'
F_1	$(P_{vf}'P_{vf})^{-1}P_{vf}'A$
F_2	$(P_{vf}'P_{vf})^{-1}P_{vf}'RU^{-2}A(A'U^{-2}A)^{-1}$
F_3	$(P_{vf}'P_{vf})^{-1}P_{vf}'RU^{-2}A(A'U^{-2}RU^{-2}A)^{-1/2}$
F_c	$(P_{vf}'P_{vf})^{-1}P_{vf}'RL_cD_c^{-1}$
F_I	$(P_{vf}'P_{vf})^{-1}P_{vf}'RS^{-1}L_ID_I^{-1}$

Two final summary statistics were also calculated. One statistic was

$$\text{Tr}(\mathbf{R})/m \quad (25)$$

which is equal to the average correlation between different methods for the same component (factor) scores. This statistic measures the convergence of principal component, image component, factor, and scale scores when they are measuring the same factor. The second statistic was

$$\left(\sum_{i=1}^m \sum_{j=1}^m |r_{ij}| - \sum_{i=1}^m r_{ii} \right) / m(m-1) \quad (26)$$

which is equal to the average correlation among the off-diagonal (different) component scores between different methods. This statistic measures the divergence among the various scores when they are measuring different factors.

In addition, means and standard deviations were calculated on each set of 5 sample summary correlations within each cell of the design. These means and standard deviations were calculated for both the convergent and divergent summary correlations.

Results

The mean of the correlations and the standard deviation of the mean of the correlations among the various scores were computed. The primary summary statistics are contained in Tables 6 to 13. Further summary statistics are compiled in the tables in Appendices A to J. Certain conventions are followed in these tables and will be stated here once only to avoid unnecessary repetition.

For some cells of the $p=72$ condition, the analysis could not be completed because of the prohibitively high (>50) number of sample matrices that would have had to have been generated in order to get 5 'good' sample matrices from which it was possible to accurately match components between methods. The correct matching of components was essential for accurate correlations. Dashed lines are used to indicate incomplete cells. The second convention concerns the underlining of certain values in the tables. Correlations among convergent components are underlined when the correlations are .9995 or higher to indicate values that could not logically be rounded up to 1.00. In the case of those tables that measure the mean of the correlations among the divergent components, an underlined value represents correlations of less than .0005, which could not be rounded up to .001. The underlined values in the cells which compare factor scores F1 and F2 are indeed .000 over all cases because of the mathematical relationship between these scores. Tables which summarize standard deviations are contained in the appendices, and for

these tables underlined values represent standard deviations that were less than .0005 that could not be rounded up to .001.

Naming conventions used in the column headings of the tables are as follows: 1) $p/\underline{m}$ indicates the p (number of variables) to $\underline{m}$ (number of components) ratio; 2) the notation $\underline{a}_{ij}$ indicates the population factor loading condition; 3) the letter $\underline{N}$ represents the sample size for the particular cell condition; and 4) C, I, F1, F2, F3, and U stand, respectively, for the principal component score, the image component score, the Thomson (1936) factor score estimate, the Bartlett (1937) factor score estimate, the Anderson and Rubin (1956) factor score estimate, and the scale score, such that any combination of 2 abbreviations for score names represents a comparison condition between those 2 scores.

Tables 6 to 13 present information on comparisons that occurred with the factor patterns in the orthogonal varimax position. The original population patterns that were used as the basis for the sample matrices were also orthogonal in nature. Consequently, comparisons of scores in the other rotational positions, i.e., promax position and procrustes position, added minimal information beyond that attained by the comparisons of scores in the varimax position. Thus, the summary information for the score comparisons in the other rotational positions, as well as for all standard deviation information, since it added little of significance, is contained in the appendices and will not be discussed in detail. In the summary of results discussed below concerning Tables 6 to 13, various collapsed

cell means were calculated. Individual cell means that were above .9995 were rounded down to .999 and values that were below .0005 were rounded up to .001 (except for the true .000 values between F1 and F2) in the calculation of mean averages. This convention results in slightly more conservative estimates of the actual convergency and divergency of score relationships. Note also that the lower case k values, used in parentheses below, represent the number of cells, rather than the number of samples, that contributed to the mean values. Each tabled cell value is itself a reduction of the individual components of each of the five samples representing that cell.

Tables 6 and 7 present the summary statistics of each cell of the design when $p = 72$ and $p = 36$, respectively, for the mean of the correlations among principal component, image component, and the three factor scores for the case when the scores are measuring the same component in the varimax position. The collapsed mean value of the 330 correlations representing the $p = 72$ condition was .982, while the collapsed mean value of the 360 correlations representing the $p = 36$ condition was .978. The difference was very slight, indicating no effect for variable size. Further comparisons of values contained in these two tables reflect the collapse across the data points of both tables. The collapsed mean value of the 690 correlations represented in Tables 6 and 7 was .980. Collapsing across p , the effect of the variable to component (p/m) ratio was examined. The average mean values for $p/m = 12, 6$, and 4 , respectively, were .988 ($k = 240$), .978 ($k = 230$), and .974 ($k = 220$). The effect was thus minimal for the

Table 6

Mean Correlations among Convergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, p = 72

<u>p/m</u>	<u>a_{ij}</u>	<u>N</u>	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	999	999	999	994	994	994	994	999	999	999
12	.8	160	999	999	999	999	999	999	999	999	999	999
12	.8	240	999	999	999	999	999	999	999	999	999	999
12	.8	480	999	999	999	999	999	999	999	999	999	999
12	.6	80	997	998	998	980	981	977	977	992	992	999
12	.6	160	999	999	999	999	999	996	996	996	996	999
12	.6	240	999	999	999	999	999	996	997	996	996	999
12	.6	480	999	999	999	999	999	999	999	998	999	999
12	.4	80	953	942	994	777	797	768	770	946	947	999
12	.4	160	996	987	987	996	995	978	979	973	974	999
12	.4	240	998	986	986	998	998	980	980	975	976	999
12	.4	480	999	994	994	999	999	991	991	988	989	999
6	.8	80	997	999	999	988	990	987	988	996	996	999
6	.8	160	999	999	999	999	999	999	999	998	999	999
6	.8	240	999	999	999	999	999	999	999	999	999	999
6	.8	480	999	999	999	999	999	999	999	999	999	999
6	.6	80	978	989	990	886	892	880	884	980	981	999
6	.6	160	996	998	998	997	998	993	994	990	991	999
6	.6	240	998	998	999	999	999	995	996	993	994	999
6	.6	480	999	999	999	999	999	997	998	995	996	999
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	887	893	897	937	924	919	921	950	953	999
6	.4	240	938	938	941	959	967	955	957	954	958	999
6	.4	480	991	987	989	999	995	983	984	969	973	999
4	.8	80	988	998	999	982	981	982	983	989	990	999
4	.8	160	998	999	999	998	999	998	998	996	997	999
4	.8	240	999	999	999	999	999	999	999	997	998	999
4	.8	480	999	999	999	999	999	999	999	999	999	999
4	.6	80	920	948	950	874	871	882	885	963	966	999
4	.6	160	976	988	990	994	984	986	988	976	979	999
4	.6	240	990	996	997	998	995	993	994	982	985	999
4	.6	480	997	998	999	999	999	997	997	992	994	999
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	891	919	923	987	910	924	928	923	929	998
4	.4	480	927	937	940	976	949	950	953	948	953	998

Note. The decimal point is omitted before correlations.

Table 7

Mean Correlations among Convergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, p = 36

p/m	a_{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	999	999	999	999	999	998	998	998	998	999
12	.8	160	999	999	999	999	999	999	999	999	999	999
12	.8	480	999	999	999	999	999	999	999	999	999	999
12	.8	480	999	999	999	999	999	999	999	999	999	999
12	.6	80	998	997	997	998	998	991	991	991	991	999
12	.6	160	999	998	998	999	999	996	996	995	995	999
12	.6	240	999	999	999	999	999	998	998	997	997	999
12	.6	480	999	999	999	999	999	999	999	999	999	999
12	.4	80	953	889	892	987	969	881	887	877	885	997
12	.4	160	997	977	978	998	998	967	967	962	963	999
12	.4	240	999	989	989	999	999	984	985	981	981	999
12	.4	480	999	993	993	999	999	990	991	989	989	999
6	.8	80	998	999	999	998	999	997	998	997	997	999
6	.8	160	999	999	999	999	999	999	999	999	999	999
6	.8	240	999	999	999	999	999	999	999	999	999	999
6	.8	480	999	999	999	999	999	999	999	999	999	999
6	.6	80	992	995	996	993	995	983	984	979	981	999
6	.6	160	997	997	998	999	999	993	994	989	990	999
6	.6	240	998	998	998	999	999	996	996	993	994	999
6	.6	480	999	999	999	999	999	998	998	996	997	999
6	.4	80	782	797	807	934	817	800	808	836	844	996
6	.4	160	928	912	919	986	934	894	901	877	888	995
6	.4	240	979	958	963	998	985	952	957	925	932	997
6	.4	480	996	988	989	999	998	984	985	973	975	999
4	.8	80	996	999	999	998	998	997	997	993	994	999
4	.8	160	998	999	999	999	999	998	999	996	997	999
4	.8	240	999	999	999	999	999	999	999	998	998	999
4	.8	480	999	999	999	999	999	999	999	999	999	999
4	.6	80	951	976	977	964	975	973	974	952	954	999
4	.6	160	987	995	996	997	992	988	989	973	976	999
4	.6	240	995	997	998	999	998	994	995	986	988	999
4	.6	480	998	998	999	999	999	997	997	992	993	999
4	.4	80	832	888	893	955	837	854	860	868	874	997
4	.4	160	886	918	922	958	909	921	926	885	890	998
4	.4	240	894	906	912	957	923	913	918	887	895	997
4	.4	480	982	979	981	999	986	974	976	946	951	998

Note. The decimal point is omitted before correlations.

p/m ratio. Collapsing across p , the effect of loading (a_{ij}) was examined. The average mean values for $a_{ij} = .8, .6, \text{ and } .4$, respectively, were $.998$ ($k = 240$), $.989$ ($k = 240$), and $.949$ ($k = 210$). The $a_{ij} = .4$ condition was visibly different, although still quite high, from the other two loading conditions. Collapsing across p , the effect of sample size (N) was examined. The average mean values for $N = 80, 160, 240, \text{ and } 480$, respectively, were $.958$ ($k = 160$), $.982$ ($k = 170$), $.985$ ($k = 180$), and $.993$ ($k = 180$). The $N = 80$ condition was most different, although still quite high, from the other three sample size conditions.

Using the collapsed Tables 6 and 7, the various specific score comparisons were examined. Comparing within the component score types, the average mean correlation between principal component and image component scores was $.986$ ($k = 69$). Comparing within the factor score types, the average mean correlation between F1 and F2 was $.975$ ($k = 69$), between F1 and F3 was $.977$ ($k = 69$), and between F2 and F3 was $.999$ ($k = 69$). The overall mean correlation among the three types of factor scores was $.983$ ($k = 207$). Comparing between principal component and factor score methods, the average mean correlation between C and F1 was $.979$ ($k = 69$), between C and F2 was $.980$ ($k = 69$), and between C and F3 was $.982$ ($k = 69$). The overall mean correlation between principal component and the three factor score methods was $.980$ ($k = 207$). Comparing between image component and factor score methods, the average mean correlation between I and F1 was $.978$ ($k = 69$), between I and F2 was $.972$ ($k = 69$), and between I

and F3 was .974 ($\underline{k} = 69$). The overall mean correlation between image component and the three factor score methods was .975 ($\underline{k} = 207$). With the exception of the correlation between F2 and F3, which was near unity, all of the above correlations within and between score methods approach .98, with differences essentially in the third decimal place.

Tables 8 and 9 present the summary statistics of each cell of the design when $p = 72$ and $p = 36$, respectively, for the mean of the correlations of the scale score with principal component, image component, and the three factor scores for the case when the scores are measuring the same component in the varimax position. The collapsed mean value of the 165 correlations representing the $p = 72$ condition was .954, while the collapsed mean value of the 180 correlations representing the $p = 36$ condition was .963. This difference was very small, indicating no practical effect for variable size. Further comparisons of values contained in these two tables reflect the collapse across the data points of both tables. The collapsed mean value of the 345 correlations represented in Tables 8 and 9 was .959. Collapsing across p , the effect of the $p/\underline{m}$ ratio was examined. The average mean values for $p/\underline{m} = 12, 6, \text{ and } 4$, respectively, were .982 ($\underline{k} = 120$), .954 ($\underline{k} = 115$), and .939 ($\underline{k} = 110$). There seemed to be an effect, although values are all quite high, for the three levels of the $p/\underline{m}$ ratio, within the context of scale scores. Collapsing across p , the effect of $\underline{a}_{ij}$ was examined. The average mean values for $\underline{a}_{ij} = .8, .6, \text{ and } .4$, respectively, were .991 ($\underline{k} = 120$), .973 ($\underline{k} = 120$), and .906 ($\underline{k} = 105$). The $\underline{a}_{ij} = .4$ condition was most different, although the value was still quite

Table 8
Mean Correlations among Convergent Components for Scale Scores with
Principal Component, Image Component, and Factor Scores: Varimax
Position, p = 72

<u>p/m</u>	<u>a_{ij}</u>	<u>N</u>	<u>U</u> <u>C</u>	<u>U</u> <u>I</u>	<u>U</u> <u>F1</u>	<u>U</u> <u>F2</u>	<u>U</u> <u>F3</u>
12	.8	80	992	986	992	992	992
12	.8	160	995	994	995	995	995
12	.8	240	997	997	997	997	997
12	.8	480	999	998	999	998	999
12	.6	80	983	965	980	980	982
12	.6	160	992	990	991	990	992
12	.6	240	994	993	993	993	994
12	.6	480	997	997	997	997	997
12	.4	80	944	743	904	888	892
12	.4	160	960	953	952	949	952
12	.4	240	970	967	967	958	960
12	.4	480	985	983	983	982	984
6	.8	80	979	968	978	975	978
6	.8	160	990	989	990	988	990
6	.8	240	994	994	994	993	994
6	.8	480	997	997	997	997	997
6	.6	80	943	848	927	932	937
6	.6	160	974	970	972	969	973
6	.6	240	982	981	982	978	982
6	.6	480	991	990	991	989	991
6	.4	80	---	---	---	---	---
6	.4	160	882	842	802	797	803
6	.4	240	892	863	855	842	849
6	.4	480	939	936	929	925	931
4	.8	80	963	948	958	956	962
4	.8	160	984	982	984	980	983
4	.8	240	987	987	989	984	987
4	.8	480	994	994	995	993	994
4	.6	80	906	819	855	867	874
4	.6	160	941	937	930	927	936
4	.6	240	962	960	958	952	961
4	.6	480	984	983	984	980	984
4	.4	80	---	---	---	---	---
4	.4	160	---	---	---	---	---
4	.4	240	847	838	791	794	802
4	.4	480	877	859	831	827	835

Note. The decimal point is omitted before correlations.

Table 9

Mean Correlations among Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p = 36

<u>p/m</u>	<u>a_{ij}</u>	<u>N</u>	<u>U</u> <u>C</u>	<u>U</u> <u>I</u>	<u>U</u> <u>F1</u>	<u>U</u> <u>F2</u>	<u>U</u> <u>F3</u>
12	.8	80	997	995	996	996	997
12	.8	160	998	998	998	998	998
12	.8	240	999	998	998	998	999
12	.8	480	999	999	999	999	999
12	.6	80	995	991	991	994	994
12	.6	160	997	995	995	996	997
12	.6	240	998	997	997	997	998
12	.6	480	999	999	999	999	999
12	.4	80	968	953	919	860	865
12	.4	160	978	973	972	959	960
12	.4	240	988	986	984	984	984
12	.4	480	994	993	992	992	993
6	.8	80	990	988	989	988	990
6	.8	160	996	995	995	995	996
6	.8	240	997	996	997	996	997
6	.8	480	999	998	998	998	999
6	.6	80	974	965	965	968	972
6	.6	160	988	986	986	985	987
6	.6	240	992	991	991	991	992
6	.6	480	996	995	995	995	996
6	.4	80	924	861	731	734	747
6	.4	160	935	918	872	849	871
6	.4	240	950	946	931	906	914
6	.4	480	973	971	968	967	970
4	.8	80	982	980	981	978	982
4	.8	160	993	992	992	991	993
4	.8	240	995	995	995	994	995
4	.8	480	998	997	997	997	998
4	.6	80	942	912	903	921	926
4	.6	160	975	971	965	966	973
4	.6	240	986	984	982	981	985
4	.6	480	992	991	990	989	992
4	.4	80	915	883	783	833	839
4	.4	160	904	874	815	834	840
4	.4	240	892	862	825	829	837
4	.4	480	952	949	934	936	942

Note. The decimal point is omitted before correlations.

high. Collapsing across p , the effect of N was examined. The average mean values for $N = 80, 160, 240$, and 480 , respectively, were $.936$ ($k = 80$), $.958$ ($k = 85$), $.961$ ($k = 90$), and $.978$ ($k = 90$). The $N = 80$ condition was most different, although still quite high, from the other three sample size conditions.

Using the collapsed Tables 8 and 9, the various specific score comparisons were examined. The comparison between U and C yielded an average mean correlation of $.971$ ($k = 69$). The comparison between U and I yielded an average mean correlation of $.959$ ($k = 69$). The comparisons between the scale and factor score types yielded an average mean correlation between U and F1 of $.955$ ($k = 69$), between U and F2 of $.953$ ($k = 69$), and between U and F3 of $.957$ ($k = 69$). The overall mean correlation of the scale and factor score types was $.955$ ($k = 207$). The only real difference in the above score comparisons, between U and C, was probably due to U being built on the principal component pattern. For all essential purposes the correlations are very close and quite high.

Tables 10 and 11 present the summary statistics of each cell of the design when $p = 72$ and $p = 36$, respectively, for the mean of the correlations among principal component, image component, and the three factor scores for the case when the scores are measuring different components in the varimax position. The correlational patterns represented in these tables are very low, and are nearly mirror images of Tables 6 and 7. The overall mean value for the 690 data points in these two tables was $.011$. Slight differences were noted on cells representing the $a_{ij} = .4$ condition and the $N = 80$ condition; however the mean correlational values representing these two conditions,

Table 10

Mean Correlations among Divergent Components for Principal Component,
Image Component, and Factor Scores: Varimax Position, $p = 72$

p/m	a_{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	002	002	000	001	003	002	001	000	002	002
12	.8	160	002	002	000	000	002	002	000	000	002	002
12	.8	240	001	001	000	000	001	001	000	000	001	001
12	.8	480	001	001	000	000	001	001	000	000	001	001
12	.6	80	006	006	002	006	009	005	006	000	006	006
12	.6	160	004	004	000	000	004	004	001	000	004	004
12	.6	240	004	004	000	000	004	004	000	000	004	004
12	.6	480	003	003	000	000	003	003	000	000	003	003
12	.4	80	032	034	034	105	101	097	099	000	011	012
12	.4	160	010	010	005	004	010	009	006	000	009	010
12	.4	240	009	009	003	001	009	009	003	000	010	010
12	.4	480	008	008	001	000	008	008	001	000	008	008
6	.8	80	004	004	001	002	005	003	002	000	004	004
6	.8	160	003	003	000	000	003	003	000	000	003	003
6	.8	240	002	002	000	000	002	002	000	000	002	002
6	.8	480	002	002	000	000	002	002	000	000	002	002
6	.6	80	014	013	011	037	041	038	039	000	008	008
6	.6	160	007	007	002	001	007	007	002	000	007	007
6	.6	240	007	007	001	001	006	007	001	000	007	007
6	.6	480	005	006	000	000	005	005	000	000	005	006
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	065	065	065	046	055	055	055	000	011	011
6	.4	240	036	036	034	025	024	024	022	000	012	013
6	.4	480	013	013	006	002	012	012	005	000	012	012
4	.8	80	007	006	003	006	009	007	006	000	005	005
4	.8	160	004	004	001	001	004	004	001	000	004	004
4	.8	240	004	004	000	000	004	004	000	000	004	004
4	.8	480	003	003	000	000	003	003	000	000	003	003
4	.6	80	036	038	037	047	045	045	045	000	008	008
4	.6	160	015	014	011	006	013	012	009	000	009	009
4	.6	240	010	010	004	002	009	009	003	000	009	009
4	.6	480	007	007	001	000	007	007	001	000	007	007
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	048	050	048	018	044	045	044	000	011	012
4	.4	480	037	036	034	018	031	030	028	000	012	012

Note. The decimal point is omitted before correlations.

Table 11

Mean Correlations among Divergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, p = 36

<u>p/m</u>	<u>a_{ij}</u>	<u>N</u>	<u>C</u> F1	<u>C</u> F2	<u>C</u> F3	<u>C</u> I	<u>I</u> F1	<u>I</u> F2	<u>I</u> F3	<u>F1</u> F2	<u>F1</u> F3	<u>F2</u> F3
12	.8	80	002	002	000	000	002	002	000	000	002	002
12	.8	160	002	002	000	000	002	002	000	000	002	002
12	.8	240	002	002	000	000	002	002	000	000	002	002
12	.8	480	001	001	000	000	001	001	000	000	001	001
12	.6	80	005	005	001	001	005	005	002	000	005	006
12	.6	160	004	005	000	000	004	004	000	000	004	004
12	.6	240	004	004	000	000	004	004	000	000	004	004
12	.6	480	002	002	000	000	002	002	000	000	002	002
12	.4	80	075	067	072	027	057	050	056	000	034	042
12	.4	160	010	010	004	003	009	009	005	000	012	012
12	.4	240	008	008	002	001	008	008	002	000	008	008
12	.4	480	008	008	001	000	008	008	001	000	008	009
6	.8	80	004	004	000	001	004	004	001	000	004	004
6	.8	160	003	003	000	000	003	003	000	000	003	003
6	.8	240	003	003	000	000	003	003	000	000	003	003
6	.8	480	002	002	000	000	002	002	000	000	002	002
6	.6	80	012	012	005	005	011	011	005	000	011	011
6	.6	160	009	009	001	001	008	008	001	000	009	009
6	.6	240	007	007	000	000	007	007	001	000	007	007
6	.6	480	006	006	000	000	006	006	000	000	006	006
6	.4	80	113	126	123	069	113	115	117	000	025	031
6	.4	160	042	042	041	019	044	044	042	000	028	034
6	.4	240	019	022	020	004	018	020	019	000	021	025
6	.4	480	014	014	003	001	014	014	003	000	014	014
4	.8	80	007	007	001	001	007	007	001	000	007	007
4	.8	160	004	004	000	000	004	004	000	000	004	004
4	.8	240	003	003	000	000	003	003	000	000	003	003
4	.8	480	003	003	000	000	003	003	000	000	003	003
4	.6	80	028	028	025	023	021	022	018	000	012	013
4	.6	160	013	013	004	002	012	012	004	000	012	013
4	.6	240	010	010	001	001	010	010	001	000	010	010
4	.6	480	008	008	001	000	008	008	001	000	008	008
4	.4	80	080	087	085	049	088	090	089	000	017	020
4	.4	160	061	064	061	034	054	056	053	000	017	020
4	.4	240	059	061	058	036	051	054	050	000	018	022
4	.4	480	020	021	010	002	019	020	010	000	016	017

Note. The decimal point is omitted before correlations.

collapsed across p , were still below .03. Also, in examining the various specific score comparisons in these two tables, in the manner of previous analyses performed on Tables 6 and 7, no sets of score types correlated above .016.

Tables 12 and 13 present the summary statistics of each cell of the design when $p = 72$ and $p = 36$, respectively, for the mean of the correlations of the scale score with principal component, image component, and the three factor scores for the case when the scores are measuring different components in the varimax position. The correlational patterns represented in these tables are quite low, and are nearly mirror images of Tables 8 and 9. The overall mean value for the 345 data points in these two tables was .037. Paralleling the results of the analyses on Tables 10 and 11, slight differences were noted on cells representing the $a_{ij} = .4$ condition and the $N = 80$ condition. However, the mean correlational values representing these two conditions, collapsed across p , were still below .06. Also, in examining the various specific score comparisons in these tables, in the manner of previous analyses performed on Tables 8 and 9, there were no discernible differences. The low correlation occurred between U and C and was .034 ($k = 69$), while the high correlation occurred between U and F1 and was .042 ($k = 69$).

Standard deviations computed on the cell values represented in Tables 6 to 9 are contained in Appendix A, while standard deviations computed on the cell values represented in Tables 10 to 13 are contained in Appendix B. The standard deviations are very low in virtually all cases, suggesting the stability of the observed correlational patterns.

Table 12

Mean Correlations among Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p = 72

<u>p/m</u>	<u>a_{ij}</u>	<u>N</u>	U C	U I	U F1	U F2	U F3
12	.8	80	041	042	043	039	041
12	.8	160	034	034	036	033	034
12	.8	240	024	024	025	023	024
12	.8	480	018	018	019	017	018
12	.6	80	049	051	055	044	049
12	.6	160	032	032	036	028	032
12	.6	240	029	029	033	025	029
12	.6	480	021	021	023	018	021
12	.4	80	048	104	063	059	062
12	.4	160	034	036	042	028	035
12	.4	240	027	028	035	020	028
12	.4	480	021	021	028	013	021
6	.8	80	044	045	047	041	044
6	.8	160	031	031	034	029	031
6	.8	240	024	024	026	022	024
6	.8	480	017	017	018	015	017
6	.6	80	051	064	057	047	052
6	.6	160	034	034	041	028	034
6	.6	240	028	028	035	022	028
6	.6	480	020	020	025	015	020
6	.4	80	---	---	---	---	---
6	.4	160	052	065	074	070	074
6	.4	240	045	050	055	048	053
6	.4	480	028	028	038	022	029
4	.8	80	046	048	050	042	046
4	.8	160	032	032	035	028	032
4	.8	240	028	028	032	025	028
4	.8	480	018	018	021	016	018
4	.6	80	057	069	067	063	066
4	.6	160	040	040	047	034	041
4	.6	240	031	031	039	023	031
4	.6	480	019	019	026	013	019
4	.4	80	---	---	---	---	---
4	.4	160	---	---	---	---	---
4	.4	240	050	052	059	060	063
4	.4	480	039	043	050	045	049

Note. The decimal point is omitted before correlations.

Table 13

Mean Correlations among Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p = 36

<u>p/m</u>	<u>a_{ij}</u>	<u>N</u>	<u>U</u> <u>C</u>	<u>U</u> <u>I</u>	<u>U</u> <u>F1</u>	<u>U</u> <u>F2</u>	<u>U</u> <u>F3</u>
12	.8	80	046	046	048	044	046
12	.8	160	036	036	037	034	036
12	.8	240	032	032	034	031	032
12	.8	480	019	019	020	018	019
12	.6	80	038	038	043	033	038
12	.6	160	032	032	036	027	032
12	.6	240	028	028	033	024	029
12	.6	480	014	014	016	012	014
12	.4	80	041	056	100	072	090
12	.4	160	031	031	039	022	031
12	.4	240	027	028	035	018	026
12	.4	480	021	021	029	012	021
6	.8	80	048	048	052	044	048
6	.8	160	032	032	035	029	032
6	.8	240	027	027	029	025	027
6	.8	480	018	018	020	016	018
6	.6	80	047	048	055	041	048
6	.6	160	033	034	041	025	033
6	.6	240	026	026	033	019	026
6	.6	480	020	020	025	014	020
6	.4	80	064	095	109	116	119
6	.4	160	045	052	066	054	070
6	.4	240	038	039	051	036	050
6	.4	480	025	025	038	014	025
4	.8	80	048	048	053	042	048
4	.8	160	029	029	033	025	029
4	.8	240	024	024	027	021	024
4	.8	480	018	018	021	016	018
4	.6	80	055	061	067	055	060
4	.6	160	036	036	046	026	036
4	.6	240	028	028	037	018	028
4	.6	480	019	019	026	012	019
4	.4	80	068	079	093	099	100
4	.4	160	059	072	080	080	082
4	.4	240	050	060	067	064	068
4	.4	480	029	030	043	024	032

Note. The decimal point is omitted before correlations.

All of the comparisons discussed above were performed in the varimax rotational position. The other rotational positions examined were the promax position and the procrustes position. Little information was added by these rotational positions. Emerging patterns did reveal: 1) correlational values were usually just a little lower for convergent components and just a little higher for divergent components when measured in the promax position as compared to the varimax position; 2) standard deviations were usually just a little higher in the promax position as compared to the varimax position; 3) correlational values were often just a little higher for convergent components and just a little lower for divergent components when measured in the procrustes position as compared to the varimax position; 4) standard deviations were usually just a little lower for the procrustes position as compared to the varimax position; and 5) differences in nearly all cases in the comparisons between the varimax position and the other rotational positions occurred in the third decimal place.

Appendices C to F contain tables of summary information on the promax rotational position, while Appendices G to J contain tables of summary information on the procrustes rotational position.

Discussion

This study investigated the relationships among the principal component score, the image component score, the regression factor score estimate (Thurstone, 1935; Thomson, 1936), the Bartlett (1937) factor score estimate, and the Anderson and Rubin (1956) factor score estimate, and a scale score method based on the principal component factor pattern. Conditions that were manipulated included the number of variables (p), component saturation (a_{ij}), sample size (N), the p/m ratio, where m is the number of components, and the rotation of the basic component pattern.

Perhaps the most striking finding of this study was the strength of the relationships among the various score methods. Across all the conditions of this study the average correlation coefficient among the convergent components, excluding the scale score relationships, was .98. This indicates virtual indistinguishability among methods for all practical purposes. When the component and factor score methods did begin to depart from unity, it was most likely to occur in one of the problematic conditions of low component loading ($a_{ij} = .4$), low sample size ($N = 80$), or a combination of these two conditions. This finding was not unexpected as previous related research (Guadagnoli & Velicer, in press; Velicer & Fava, 1987a, 1987b) found greater departures from the known population values when component saturation, sample size, or the p/m ratio were low.

The overall performance of the scale score method when compared with the other score methods was also quite remarkable. While the

scale score method did correlate lower with the other score methods than the other score methods correlated among themselves, this degradation was small and the average correlational value was still .96. The scale score method appeared to be a little more sensitive to the previously identified problematic conditions of low component saturation, low sample size, and low p/m ratio. Interactions of the problem areas also caused a further departure from unity for the scale score relationships.

Within method comparisons revealed that component score methods were more closely related to each other than to the factor score methods, while two of the factor score methods (F2 and F3) were near unity in their relationship to each other.

Between method comparisons revealed a surprise result. Although the difference was in the 3rd decimal place, the principal component score correlated more highly with factor scores F2 and F3 than did F1. The image component score, however, was slightly less correlated with all the factor scores, than the factor scores were correlated among themselves. The scale score method was slightly more correlated to the principal component score method, but as the scale score was based on the principal component factor pattern this was not unusual. The scale score method was approximately equally related to the image component and the factor score methods, differing only in the third decimal place.

The essential sameness of the score methods was reinforced by the extremely low correlations among the divergent components for the different score methods. This is important since for scores of the

different methods to be essentially the same we would not only want scores on the same components to correlate highly, but we would want scores on different components to be essentially uncorrelated as they would be within each of the separate methods. The scale score method, although slightly more correlated, still performed quite well on this criterion of low relatedness among divergent components.

This study also suggests that past research (Velicer, 1976) may well have underestimated the relationship among the various score estimates, especially for the varimax position. There are several possible reasons for this past underestimation. First, it is possible that the data sets employed, because they were 'real world' data sets, did not have an accurate number of components derived from them. It is possible that the number of components derived from these data sets represented an overextraction (or an underextraction) of the 'true' number of components. Such a case could easily have occurred as past efforts to determine the proper number of components to retain did not have the advantage of simulation studies (Zwick & Velicer, 1982, 1986) to guide them in the choice of the most accurate methods to use in the determination of the number of components to retain. Correlating scores based on the wrong number of components may possibly have produced correlations lower than if the 'true' number of components had formed the basis of the scores correlated. Second, it is possible that the components in these data sets were not truly orthogonal, and in such a case one might expect that a varimax rotation, which is an orthogonal rotation, would not accurately portray the component values and could lead to degradation of the 'true' correlation values. In

the present study, an orthogonal population pattern was used and the correlation values found after the varimax rotation were extremely high. They were higher, in fact, than those values attained after the promax rotation, which would be more suited to a situation with correlated components, and they were almost identical to those achieved after a procrustes rotation, often being slightly smaller only in the 3rd or 4th decimal place. Conversely, it seems likely, though it remains uninvestigated, that a correlated component population pattern used to underly the various scores would show higher correlation values among the score methods after a rotation such as promax, rather than a varimax rotation, and that in such a case the correlation values after a varimax rotation might be substantially lower than 'true' correlation values based on the proper rotation. And third, it is possible that some of these data sets were more representative of some of the problem areas of low saturation, low sample size, or low p/m ratio, which could, as was noted in this present study, lower the correlations among score methods.

The present study does have its own limitations. One prominent limitation is the use of ideal variable patterns, in which variables only loaded highly on one component, for the underlying population patterns. It remains to be investigated what the effects on the various score relationships would be if complex variables (variables which load highly on more than one factor) are used in the underlying population factor pattern and/or if the underlying population pattern is oblique. Given these limitations, this present study still represents the most extensive study of its type, given the range of

conditions examined across p , a_{ij} , N , p/m ratio, score types, and methods of rotation.

Finally, the present study has several implications for applied researchers. First, the overall results suggest there is no practical difference between the score methods of principal component analysis, image component analysis, and factor analysis. This is especially true when one avoids the problem areas of low variable loadings, low sample size, and low p/m ratio. This finding is consistent with a body of research (Velicer, 1974, 1976, 1977; Velicer, Peacock, & Jackson, 1982; Velicer & Fava, 1987a, 1987b) in the area, which has continued to find little or no difference between the methods of component and factor analysis. When one does, however, consider the computational efficiency of component analysis, the problem of factor indeterminacy (Schonemann & Wang, 1972), and the great number of boundary cases and the frequent nonconvergence that may be encountered in factor analysis (Velicer & Fava, 1987a, 1987b), the case for principal component analysis as an alternative to factor analysis remains strong. Second, this study lends strong support for the continued use of scale scores in many applied situations. They have been shown to be very representative of the actual component score, and they also represent a most parsimonious and simple form of scoring. Although the scale score method used in this study was based on principal component analysis, it seems highly probable, given the strong relationship among all the score methods, that a scale score based on image component or factor analysis would also perform in a highly representative fashion of the true component score.

References

- Anderson, T. W. & Rubin, H. (1956). Statistical inference in factor analysis. In J. Neyman (Ed.), Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability (pp. 111-150). Berkeley: University of California Press.
- Bartlett, M. S. (1937). The statistical conception of mental factors. British Journal of Psychology, 28, 97-104.
- Boosma, A. (1982). The robustness of LISREL against small sample sizes in factor analysis models. In K. G. Joreskog and H. Wold (Eds.), Systems under indirect observation (Part 1) (pp. 149-173). Amsterdam: North Holland.
- Browne, M. W. (1968). A comparison of factor analytic techniques. Psychometrika, 33, 267-334.
- Glass, G. V. & Maguire, T. O. (1966). Abuses of factor scores. American Educational Research Journal, 4, 297-304.
- Guadagnoli, E. & Velicer, W. F. (in press). The relationship of sample size to the stability of component patterns. Psychological Bulletin.
- Gorsuch, R. L. (1983). Factor Analysis (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Guttman, L. (1953). Image theory for the structure of quantitative variates. Psychometrika, 18, 277-296.
- Guttman, L. (1955). The determinancy of factor score matrices with implications for five other basic problems of common-factor theory. British Journal of Statistical Psychology, 8, 65-81.
- Harris, C. W. (1962). Some Rao-Guttman relationships. Psychometrika, 27, 247-263.

- Harris, C. W. (1967). On factors and factor scores. Psychometrika, 32, 363-379.
- Hendrickson, A. E., & White, P. O. (1964). Promax: A quick method for rotation to oblique simple structure. British Journal of Statistical Psychology, 17, 65-70.
- Horn, J. L. (1965). An empirical comparison of methods for estimating factor scores. Educational and Psychological Measurement, 25, 195-197.
- Hotelling, H. (1933). Analysis of a complex of statistical variables into principal components. Journal of Educational Psychology, 24, 417-441, 498-520.
- Hurley, J., & Cattell, R. B. (1962). The procrustes program: Producing direct rotation to test a hypothesized factor structure. Behavioral Science, 7, 258-262.
- Joreskog, K. G. (1967). Some contributions to maximum likelihood factor analysis. Psychometrika, 32, 443-482.
- Kaiser, H. F. (1958). The varimax criterion for analytic rotation in factor analysis. Psychometrika, 23, 187-200.
- McDonald, R. P. & Burr, E. J. (1967). A comparison of four methods of constructing factor scores. Psychometrika, 32, 381-401.
- Montanelli, R. G., Jr. (1975). A computer program to generate sample correlation and covariance matrices. Educational and Psychological Measurement, 35, 195-197.
- Morris, J. D. (1979). A comparison of regression prediction accuracy on several types of factor scores. American Educational Research Journal, 16, 17-24.

- Moseley, E. C. & Klett, C. J. (1964). An empirical comparison of factor scoring methods. Psychological Reports, 14, 179-184.
- Rindskopf, D. (1984). Structural equation models: Empirical identification, Heywood cases, and related problems. Sociological Methods & Research, 13, 109-119.
- Schonemann, P. H. & Wang, M. (1972). Some new results on factor indeterminacy. Psychometrika, 37, 61-91.
- Schweiker, R. F. (1967). Factor scores aren't sacred: Comments on "Abuses of factor scores". American Educational Research Journal, 4, 168-170.
- Thomson, G. (1936). Some points of mathematical technique in the factorial analysis of ability. Journal of Educational Psychology, 27, 37-54.
- Thurstone, L. L. (1935). The vectors of mind. Chicago: University of Chicago Press.
- Trites, D. K. & Sells, S. B. (1955). A note on alternative methods for estimating factor scores. Journal of Applied Psychology, 39, 455-456.
- Velicer, W. F. (1974). A comparison of the stability of factor analysis, principal component analysis, and rescaled image analysis. Educational and Psychological Measurement, 34, 563-572.
- Velicer, W. F. (1976). The relation between factor score estimates, image scores, and principal component scores. Educational and Psychological Measurement, 36, 149-159.
- Velicer, W. F. (1977). An empirical comparison of the similarity of principal component, image, and factor patterns. Multivariate Behavioral Research, 12, 3-22.

- Velicer, W. F., & Fava, J. L. (1987a). An evaluation of the effects of variable sampling on component, image, and factor analysis. Multivariate Behavioral Research, 22, 193-209.
- Velicer, W. F., & Fava, J. L. (1987b). The effects of variable and subject sampling on factor pattern recovery. Manuscript submitted for publication.
- Velicer, W. F., Peacock, A. C., & Jackson, D. N. (1982). A comparison of component and factor patterns: A monte carlo approach. Multivariate Behavioral Research, 17, 371-388.
- Wackwitz, J. H. & Horn, J. L. (1971). On obtaining the best estimates of factor scores within an ideal simple structure. Multivariate Behavioral Research, 6, 389-408.
- Zwick, W. R. & Velicer, W. F. (1982). Factors influencing four rules for determining the number of components to retain. Multivariate Behavioral Research, 17, 253-269.
- Zwick, W. R., & Velicer, W. F. (1986). A comparison of five rules for determining the number of components to retain. Psychological Bulletin, 99, 432-442.

APPENDIX A

Table A-1

Standard Deviations of the Mean Correlations among the Convergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position. p=72

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	001	001	001	003	003	003	003	001	001	001
12	.8	160	001	001	001	001	001	000	001	001	001	001
12	.8	240	001	001	001	001	001	001	001	001	001	001
12	.8	480	001	001	001	001	001	001	001	001	001	001
12	.6	80	001	001	001	007	006	007	007	003	002	001
12	.6	160	001	001	001	001	000	001	001	001	001	001
12	.6	240	001	001	001	001	001	001	001	001	001	001
12	.6	480	001	001	001	001	001	001	001	001	001	001
12	.4	80	067	077	074	087	098	098	098	018	017	001
12	.4	160	003	004	004	001	002	006	006	008	008	001
12	.4	240	001	006	006	001	001	007	007	008	008	001
12	.4	480	001	002	001	001	001	002	002	002	002	001
6	.8	80	001	001	001	005	005	005	005	001	001	001
6	.8	160	000	001	001	001	001	001	001	001	001	001
6	.8	240	001	001	001	001	001	001	001	001	001	001
6	.8	480	001	001	001	001	001	001	001	001	001	001
6	.6	80	013	008	008	041	034	034	033	003	003	001
6	.6	160	001	001	001	001	001	002	001	001	001	001
6	.6	240	001	001	001	001	001	001	001	002	001	001
6	.6	480	001	001	001	001	001	001	001	002	001	001
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	046	053	051	038	036	032	032	014	014	001
6	.4	240	059	063	060	048	044	045	043	026	024	001
6	.4	480	005	005	005	001	003	005	005	008	007	001
4	.8	80	006	002	001	011	010	012	011	004	004	001
4	.8	160	001	001	001	001	001	001	001	001	001	001
4	.8	240	001	001	001	001	001	001	001	000	001	001
4	.8	480	001	001	001	001	001	001	001	001	001	001
4	.6	80	017	019	018	048	038	042	042	006	006	001
4	.6	160	011	007	006	002	009	005	005	006	006	001
4	.6	240	002	001	001	001	002	002	002	004	004	001
4	.6	480	001	001	001	001	001	001	001	001	001	001
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	035	025	025	006	030	028	027	029	029	001
4	.4	480	024	027	026	034	021	022	021	010	010	001

Note. The decimal point is omitted from tabled values.

Table A-2

Standard Deviations of the Mean Correlations among the
Convergent Components for Principal Component, Image
Component, and Factor Scores: Varimax Position, p=36

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	001	001	001	001	001	001	001	001	001	001
12	.8	160	001	001	001	001	001	001	001	001	001	001
12	.8	240	001	001	001	001	001	001	001	001	001	001
12	.8	480	001	001	001	001	001	001	001	001	001	001
12	.6	80	001	001	002	001	001	002	002	003	003	001
12	.6	160	001	001	001	001	001	001	001	001	001	001
12	.6	240	001	001	001	001	001	001	001	001	001	001
12	.6	480	001	001	001	001	000	001	001	001	001	001
12	.4	80	088	129	126	008	049	096	090	096	086	005
12	.4	160	001	008	008	001	001	011	011	013	012	001
12	.4	240	001	005	005	001	001	006	006	007	007	001
12	.4	480	001	004	004	001	001	004	004	006	005	001
6	.8	80	001	001	001	001	001	001	001	001	001	001
6	.8	160	000	001	001	001	001	001	001	001	001	001
6	.8	240	001	001	001	001	001	001	001	001	001	001
6	.8	480	001	001	001	001	001	001	001	001	001	001
6	.6	80	002	002	002	002	003	003	003	003	003	001
6	.6	160	001	001	001	001	001	002	002	003	002	001
6	.6	240	001	001	001	001	001	002	002	002	002	001
6	.6	480	001	001	001	001	001	001	001	001	001	001
6	.4	80	090	094	090	041	061	055	053	033	032	001
6	.4	160	059	068	067	017	064	086	082	064	057	004
6	.4	240	028	030	026	002	020	029	025	044	040	003
6	.4	480	002	003	003	001	001	004	004	007	007	001
4	.8	80	001	001	001	001	001	001	001	002	002	001
4	.8	160	001	001	001	001	001	001	001	002	002	001
4	.8	240	001	001	001	001	001	001	001	001	001	001
4	.8	480	001	001	001	001	001	001	001	001	001	001
4	.6	80	027	021	021	047	015	010	010	010	010	001
4	.6	160	008	002	002	001	008	003	003	008	007	001
4	.6	240	003	001	001	001	001	002	002	005	005	001
4	.6	480	001	001	001	001	001	002	002	003	003	001
4	.4	80	050	034	034	032	083	075	074	046	045	001
4	.4	160	044	043	040	077	039	035	032	035	035	001
4	.4	240	039	034	033	073	024	018	018	027	026	001
4	.4	480	016	008	007	001	015	008	008	017	015	001

Note. The decimal point is omitted from tabled values.

Table A-3

Standard Deviations of the Mean Correlations among the Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p=72

p/m	a _{ij}	N	U C	U I	U F1	U F2	U F3
12	.8	80	002	003	002	002	002
12	.8	160	001	002	002	002	002
12	.8	240	002	001	001	001	001
12	.8	480	001	001	001	001	001
12	.6	80	004	009	004	005	004
12	.6	160	002	002	002	002	002
12	.6	240	001	001	001	001	001
12	.6	480	001	001	001	001	001
12	.4	80	006	088	059	063	060
12	.4	160	008	009	013	012	012
12	.4	240	003	004	004	011	012
12	.4	480	002	002	002	002	002
6	.8	80	001	005	002	001	001
6	.8	160	002	002	001	002	002
6	.8	240	001	001	001	001	001
6	.8	480	001	001	001	001	001
6	.6	80	006	033	014	010	010
6	.6	160	003	004	003	004	003
6	.6	240	002	002	002	002	002
6	.6	480	001	001	001	001	001
6	.4	80	---	---	---	---	---
6	.4	160	011	033	044	049	047
6	.4	240	014	042	048	054	053
6	.4	480	012	012	014	023	020
4	.8	80	004	012	006	005	004
4	.8	160	001	001	001	001	001
4	.8	240	002	002	002	002	002
4	.8	480	001	001	001	001	001
4	.6	80	010	046	021	017	018
4	.6	160	009	008	012	010	009
4	.6	240	004	005	007	006	005
4	.6	480	001	002	002	002	002
4	.4	80	---	---	---	---	---
4	.4	160	---	---	---	---	---
4	.4	240	003	010	019	016	017
4	.4	480	020	034	029	034	034

Note. The decimal point is omitted from tabled values.

Table A-4

Standard Deviations of the Mean Correlations among the
Convergent Components for Scale Scores with Principal
Component, Image Component, and Factor Scores:
Varimax Position, p=36

---

p/m	a _{1j}	N	U C	U I	U F1	U F2	U F3
12	.8	80	002	002	002	002	002
12	.8	160	001	001	002	002	001
12	.8	240	001	001	001	001	001
12	.8	480	001	001	001	001	001
12	.6	80	002	002	002	002	002
12	.6	160	002	001	001	002	001
12	.6	240	001	001	001	001	001
12	.6	480	001	001	000	001	001
12	.4	80	015	017	083	120	117
12	.4	160	003	004	004	013	013
12	.4	240	003	003	004	009	008
12	.4	480	002	002	002	003	002
6	.8	80	004	004	003	004	004
6	.8	160	001	001	002	002	002
6	.8	240	001	001	001	002	001
6	.8	480	001	001	001	001	001
6	.6	80	006	008	006	008	007
6	.6	160	002	002	002	003	002
6	.6	240	001	001	002	001	001
6	.6	480	002	002	001	002	002
6	.4	80	017	051	102	086	083
6	.4	160	009	029	056	074	073
6	.4	240	011	013	031	034	031
6	.4	480	009	009	009	010	009
4	.8	80	003	003	003	004	003
4	.8	160	001	001	001	001	001
4	.8	240	001	001	001	001	001
4	.8	480	001	001	001	001	001
4	.6	80	012	049	029	026	026
4	.6	160	005	005	008	008	005
4	.6	240	002	003	004	003	002
4	.6	480	002	002	003	002	002
4	.4	80	007	035	052	034	035
4	.4	160	006	059	029	041	039
4	.4	240	021	050	031	039	041
4	.4	480	009	009	018	014	013

Note. The decimal point is omitted from tabled values.

APPENDIX B

Table B-1

Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position. p=72

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	000	000	000	000	000	000	000	000	000	000
12	.8	160	000	000	000	000	000	000	000	000	000	000
12	.8	240	000	000	000	000	000	000	000	000	000	000
12	.8	480	000	000	000	000	000	000	000	000	000	000
12	.6	80	001	001	001	002	003	001	002	000	001	002
12	.6	160	001	001	000	000	001	001	000	000	001	001
12	.6	240	001	001	000	000	001	001	000	000	001	001
12	.6	480	000	000	000	000	000	000	000	000	000	000
12	.4	80	021	028	030	031	036	033	033	000	003	004
12	.4	160	003	003	004	002	002	002	002	000	001	001
12	.4	240	001	001	002	000	002	002	002	000	001	002
12	.4	480	001	002	000	000	001	002	000	000	002	002
6	.8	80	000	000	000	001	001	000	001	000	000	000
6	.8	160	000	000	000	000	000	000	000	000	000	000
6	.8	240	000	000	000	000	000	000	000	000	000	000
6	.8	480	000	000	000	000	000	000	000	000	000	000
6	.6	80	003	002	003	010	008	009	009	000	001	001
6	.6	160	001	001	000	000	001	001	000	000	001	001
6	.6	240	001	001	000	000	001	001	000	000	001	001
6	.6	480	000	000	000	000	000	000	000	000	000	000
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	019	019	020	020	018	019	019	000	002	002
6	.4	240	023	025	027	024	018	018	021	000	002	003
6	.4	480	004	004	005	001	003	003	004	000	002	002
4	.8	80	002	001	002	003	003	002	003	000	000	000
4	.8	160	000	000	000	000	000	000	000	000	000	000
4	.8	240	000	000	000	000	000	000	000	000	000	000
4	.8	480	000	000	000	000	000	000	000	000	000	000
4	.6	80	006	006	006	011	008	009	009	000	001	001
4	.6	160	004	004	004	002	003	003	004	000	001	001
4	.6	240	002	001	001	000	001	001	001	000	001	001
4	.6	480	001	001	000	000	001	001	000	000	001	001
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	008	009	010	007	009	009	010	000	001	001
4	.4	480	010	011	011	014	008	008	009	000	001	001

Note. The decimal point is omitted from tabled values.

Table B-2

Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Varimax Position, p=36

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	001	001	000	000	001	001	000	000	001	001
12	.8	160	001	001	000	000	001	001	000	000	001	001
12	.8	240	001	001	000	000	001	001	000	000	001	001
12	.8	480	001	001	000	000	001	001	000	000	001	001
12	.6	80	002	002	000	000	002	002	001	000	001	002
12	.6	160	001	001	000	000	001	001	000	000	001	001
12	.6	240	002	002	000	000	002	002	000	000	002	002
12	.6	480	001	001	000	000	001	001	000	000	001	001
12	.4	80	123	099	117	040	090	071	083	000	025	039
12	.4	160	003	004	002	002	003	004	002	000	006	006
12	.4	240	004	004	001	000	004	004	001	000	004	004
12	.4	480	003	003	000	000	003	003	001	000	004	004
6	.8	80	001	001	000	000	001	001	000	000	001	001
6	.8	160	000	000	000	000	000	000	000	000	000	000
6	.8	240	001	001	000	000	001	001	000	000	001	001
6	.8	480	000	000	000	000	000	000	000	000	000	000
6	.6	80	003	003	002	001	004	003	002	000	003	003
6	.6	160	002	002	000	000	002	002	000	000	002	002
6	.6	240	001	001	000	000	001	001	000	000	001	001
6	.6	480	001	001	000	000	001	001	000	000	001	001
6	.4	80	029	035	032	046	029	033	030	000	003	004
6	.4	160	023	020	028	021	021	021	029	000	008	013
6	.4	240	008	015	018	002	007	011	015	000	007	010
6	.4	480	002	002	001	000	001	002	001	000	002	002
4	.8	80	001	001	001	000	001	001	000	000	001	001
4	.8	160	001	001	000	000	001	001	000	000	001	001
4	.8	240	000	000	000	000	000	000	000	000	000	000
4	.8	480	000	000	000	000	000	000	000	000	000	000
4	.6	80	015	016	016	023	010	011	011	000	002	002
4	.6	160	003	003	002	001	003	003	002	000	002	003
4	.6	240	001	001	000	000	001	001	000	000	001	001
4	.6	480	001	001	000	000	001	001	000	000	001	001
4	.4	80	019	020	021	026	031	033	034	000	002	003
4	.4	160	022	025	024	038	017	020	019	000	001	002
4	.4	240	026	027	030	040	024	025	026	000	001	002
4	.4	480	008	010	011	001	008	010	011	000	002	002

Note. The decimal point is omitted from tabled values.

Table B-3

Standard Deviations of the Mean Correlations among the Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position. p=72

p/m	a _{ij}	N	U C	U I	U F1	U F2	U F3
12	.8	80	007	007	007	007	007
12	.8	160	004	004	004	004	004
12	.8	240	004	004	005	004	004
12	.8	480	004	004	004	004	004
12	.6	80	006	007	007	005	006
12	.6	160	005	005	006	005	005
12	.6	240	003	003	004	003	003
12	.6	480	003	003	004	003	003
12	.4	80	004	027	015	021	022
12	.4	160	004	005	004	003	004
12	.4	240	005	004	005	005	002
12	.4	480	004	004	006	002	004
6	.8	80	002	002	002	002	002
6	.8	160	003	003	003	003	003
6	.8	240	002	002	002	002	002
6	.8	480	002	002	002	002	002
6	.6	80	005	002	006	006	006
6	.6	160	003	003	004	003	003
6	.6	240	003	003	003	002	003
6	.6	480	001	001	001	001	001
6	.4	80	---	---	---	---	---
6	.4	160	004	015	014	013	013
6	.4	240	005	008	006	011	010
6	.4	480	005	004	005	006	007
4	.8	80	003	004	004	003	004
4	.8	160	001	001	002	001	001
4	.8	240	002	002	002	002	002
4	.8	480	001	001	001	001	001
4	.6	80	003	005	003	003	004
4	.6	160	003	002	003	002	002
4	.6	240	003	003	004	003	003
4	.6	480	001	001	002	001	001
4	.4	80	---	---	---	---	---
4	.4	160	---	---	---	---	---
4	.4	240	002	002	003	004	004
4	.4	480	004	009	005	008	007

Note. The decimal point is omitted from tabled values.

Table B-4

Standard Deviations of the Mean Correlations among the Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Varimax Position, p=36

p/m	R_{ij}	N	U C	U I	U F1	U F2	U F3
12	.8	80	015	015	016	014	015
12	.8	160	015	015	016	014	015
12	.8	240	009	009	009	008	009
12	.8	480	012	012	012	011	012
12	.6	80	016	016	017	014	015
12	.6	160	008	008	009	007	008
12	.6	240	012	012	014	010	012
12	.6	480	005	005	006	005	006
12	.4	80	015	027	109	088	097
12	.4	160	009	009	013	006	007
12	.4	240	014	014	018	009	013
12	.4	480	009	009	013	005	009
6	.8	80	009	009	010	008	009
6	.8	160	005	005	005	004	005
6	.8	240	003	003	004	003	003
6	.8	480	003	003	003	002	003
6	.6	80	009	010	011	010	010
6	.6	160	006	006	008	004	006
6	.6	240	003	002	003	002	003
6	.6	480	005	005	006	004	005
6	.4	80	011	031	025	028	027
6	.4	160	006	010	016	016	016
6	.4	240	010	009	011	018	019
6	.4	480	006	006	007	004	005
4	.8	80	005	005	006	004	005
4	.8	160	003	003	003	003	003
4	.8	240	002	002	002	002	002
4	.8	480	002	002	003	002	002
4	.6	80	005	013	009	012	011
4	.6	160	007	007	010	005	007
4	.6	240	004	004	005	003	004
4	.6	480	002	002	003	001	002
4	.4	80	006	018	016	015	014
4	.4	160	007	028	019	021	020
4	.4	240	010	020	013	022	018
4	.4	480	008	008	008	012	012

Note. The decimal point is omitted from tabled values.

APPENDIX C

Table C-1

Mean Correlations among Convergent Components for
Principal Component, Image Component, and Factor Scores:
Promax Position, p=72

<u>p/m</u>	<u>a_{ij}</u>	<u>N</u>	<u>C</u> <u>F1</u>	<u>C</u> <u>F2</u>	<u>C</u> <u>F3</u>	<u>C</u> <u>I</u>	<u>I</u> <u>F1</u>	<u>I</u> <u>F2</u>	<u>I</u> <u>F3</u>	<u>F1</u> <u>F2</u>	<u>F1</u> <u>F2</u>	<u>F2</u> <u>F3</u>
12	.8	80	993	993	<u>999</u>	994	987	986	994	971	992	993
12	.8	160	995	995	<u>999</u>	999	995	994	999	980	995	995
12	.8	240	997	997	<u>999</u>	<u>999</u>	997	996	999	989	997	997
12	.8	480	998	998	<u>999</u>	<u>999</u>	999	998	<u>999</u>	994	998	998
12	.6	80	990	989	998	980	974	967	977	960	984	991
12	.6	160	994	994	999	999	994	991	996	976	991	995
12	.6	240	995	995	999	999	996	993	997	981	992	996
12	.6	480	997	997	999	<u>999</u>	998	996	999	989	996	998
12	.4	80	952	922	943	775	791	750	769	903	940	986
12	.4	160	991	981	987	996	991	973	978	954	970	995
12	.4	240	994	981	986	998	994	975	980	958	972	995
12	.4	480	996	990	994	999	996	987	991	974	986	996
6	.8	80	980	982	<u>999</u>	988	974	969	987	928	979	983
6	.8	160	989	989	<u>999</u>	999	990	988	999	958	989	990
6	.8	240	993	994	<u>999</u>	<u>999</u>	994	993	999	975	993	994
6	.8	480	997	997	<u>999</u>	<u>999</u>	997	996	<u>999</u>	986	996	997
6	.6	80	966	974	989	878	882	853	875	928	969	986
6	.6	160	985	987	998	997	987	982	993	947	981	989
6	.6	240	987	987	999	999	988	985	996	951	984	989
6	.6	480	993	992	999	999	993	991	998	970	990	993
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	868	878	893	935	910	904	920	891	939	983
6	.4	240	925	918	937	957	953	934	953	902	947	983
6	.4	480	983	977	988	998	986	973	984	935	965	990
4	.8	80	968	974	998	981	962	956	982	903	970	977
4	.8	160	982	982	<u>999</u>	998	983	980	998	930	981	982
4	.8	240	985	985	<u>999</u>	999	986	984	999	942	984	985
4	.8	480	993	993	<u>999</u>	<u>999</u>	993	993	999	972	992	993
4	.6	80	887	906	934	856	836	832	863	864	941	972
4	.6	160	958	964	985	993	966	963	984	905	963	979
4	.6	240	975	977	996	998	979	975	994	919	970	982
4	.6	480	987	988	999	999	989	986	997	952	984	989
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	860	881	907	985	879	889	915	835	908	973
4	.4	480	903	910	931	975	922	923	943	876	936	978

Note. The decimal point is omitted before correlations.

Table C-2

Mean Correlations among Convergent Components for
Principal Component, Image Component, and Factor Scores:
Promax Position, p=36

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	996	997	999	999	996	995	998	986	995	997
12	.8	160	998	998	999	999	998	997	999	992	997	998
12	.8	240	998	998	999	999	999	998	999	994	998	999
12	.8	480	999	999	999	999	999	999	999	997	999	999
12	.6	80	996	995	997	998	996	989	991	982	989	998
12	.6	160	997	996	998	999	998	994	996	987	993	998
12	.6	240	998	997	999	999	998	996	998	991	996	998
12	.6	480	999	999	999	999	999	998	999	997	998	999
12	.4	80	950	883	896	988	962	871	886	848	881	987
12	.4	160	995	975	978	998	996	964	967	952	961	997
12	.4	240	997	987	989	999	997	983	985	974	979	998
12	.4	480	998	992	993	999	998	989	991	982	987	998
6	.8	80	989	990	999	998	989	988	998	958	987	990
6	.8	160	995	995	999	999	996	994	999	981	994	996
6	.8	240	996	996	999	999	996	995	999	984	995	996
6	.8	480	998	999	999	999	998	998	999	993	998	998
6	.6	80	983	984	995	993	986	973	984	942	972	989
6	.6	160	990	991	998	999	992	987	994	963	984	993
6	.6	240	993	992	998	999	994	990	996	971	988	994
6	.6	480	996	996	999	999	996	995	998	983	994	997
6	.4	80	774	763	795	937	820	784	812	797	840	981
6	.4	160	917	889	914	986	925	876	900	830	882	978
6	.4	240	971	948	961	998	978	942	956	895	926	988
6	.4	480	990	982	988	999	992	977	984	949	969	993
4	.8	80	979	979	999	998	981	976	997	920	977	979
4	.8	160	991	991	999	999	992	990	998	965	989	992
4	.8	240	994	994	999	999	995	994	999	977	993	995
4	.8	480	996	997	999	999	997	996	999	987	996	997
4	.6	80	933	950	967	961	960	954	972	892	941	982
4	.6	160	974	979	995	997	979	973	989	920	964	984
4	.6	240	985	986	998	999	988	983	995	945	979	988
4	.6	480	991	992	999	999	993	990	997	968	987	993
4	.4	80	819	852	883	949	818	820	848	797	864	973
4	.4	160	871	896	919	961	895	903	926	819	876	978
4	.4	240	879	879	903	955	908	891	915	828	882	978
4	.4	480	970	967	979	999	974	962	974	907	942	987

Note. The decimal point is omitted before correlations.

Table C-3

Mean Correlations among Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=72

p/m	a _{ij}	N	U C	U I	U F1	U F2	U F3
12	.8	80	993	987	997	974	993
12	.8	160	995	994	999	980	995
12	.8	240	997	997	999	990	997
12	.8	480	999	998	999	994	999
12	.6	80	983	965	985	959	982
12	.6	160	992	990	994	978	991
12	.6	240	994	993	995	984	994
12	.6	480	997	997	998	991	997
12	.4	80	942	743	908	867	891
12	.4	160	959	952	952	938	950
12	.4	240	969	967	967	947	958
12	.4	480	985	983	983	975	983
6	.8	80	978	968	988	932	978
6	.8	160	990	989	997	961	990
6	.8	240	994	994	998	977	994
6	.8	480	997	997	999	988	997
6	.6	80	942	838	924	905	933
6	.6	160	973	969	975	943	972
6	.6	240	982	980	985	954	982
6	.6	480	991	990	992	975	991
6	.4	80	---	---	---	---	---
6	.4	160	881	835	785	775	794
6	.4	240	893	861	850	820	844
6	.4	480	938	935	926	909	928
4	.8	80	962	946	971	899	960
4	.8	160	983	982	994	936	983
4	.8	240	987	987	996	948	987
4	.8	480	994	994	998	977	994
4	.6	80	900	797	824	817	853
4	.6	160	941	935	929	887	930
4	.6	240	961	959	960	917	959
4	.6	480	984	983	984	959	984
4	.4	80	---	---	---	---	---
4	.4	160	---	---	---	---	---
4	.4	240	850	839	764	759	786
4	.4	480	878	859	813	799	823

Note. The decimal point is omitted before correlations.

Table C-4

Mean Correlations among Convergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=36

p/m	a _{1j}	N	U C	U I	U F1	U F2	U F3
12	.8	80	997	996	998	988	997
12	.8	160	998	998	999	993	998
12	.8	240	999	998	999	994	999
12	.8	480	999	999	999	998	999
12	.6	80	995	991	992	988	994
12	.6	160	997	995	996	991	997
12	.6	240	998	997	998	993	998
12	.6	480	999	999	999	998	999
12	.4	80	969	955	920	855	868
12	.4	160	977	973	972	953	960
12	.4	240	988	986	985	980	984
12	.4	480	994	993	992	989	993
6	.8	80	989	988	989	969	989
6	.8	160	996	995	998	983	996
6	.8	240	997	996	999	986	997
6	.8	480	999	998	999	994	999
6	.6	80	974	964	963	952	971
6	.6	160	988	986	987	969	987
6	.6	240	992	991	992	978	992
6	.6	480	996	995	996	988	996
6	.4	80	933	867	731	711	746
6	.4	160	934	917	863	825	859
6	.4	240	950	945	927	890	911
6	.4	480	973	971	967	955	969
4	.8	80	981	979	984	935	981
4	.8	160	993	992	996	971	993
4	.8	240	995	995	998	981	995
4	.8	480	998	997	999	989	997
4	.6	80	942	909	893	893	918
4	.6	160	975	971	966	936	971
4	.6	240	986	984	983	959	984
4	.6	480	992	991	990	977	991
4	.4	80	910	873	761	791	820
4	.4	160	900	869	802	804	832
4	.4	240	893	863	813	801	829
4	.4	480	951	948	928	915	936

Note. The decimal point is omitted before correlations.

APPENDIX D

Table D-1

Standard Deviations of the Mean Correlations among the
Convergent Components for Principal Component, Image
Component, and Factor Scores: Promax Position, p=72

p/m	Δ_{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	002	002	001	003	002	002	003	006	001	002
12	.8	160	002	002	001	001	002	002	001	006	002	002
12	.8	240	001	001	001	001	001	001	001	004	001	001
12	.8	480	001	001	001	001	001	001	001	003	001	001
12	.6	80	002	002	001	007	007	009	007	007	003	002
12	.6	160	001	001	001	001	002	002	001	004	002	001
12	.6	240	001	002	001	001	001	002	001	005	002	001
12	.6	480	001	001	001	001	001	001	001	002	001	001
12	.4	80	055	106	074	087	086	095	096	060	021	016
12	.4	160	003	004	004	002	003	006	006	011	007	003
12	.4	240	002	006	006	001	002	006	007	009	007	002
12	.4	480	001	002	001	001	001	002	002	004	002	001
6	.8	80	002	002	001	005	004	006	005	007	002	002
6	.8	160	002	002	001	001	002	002	001	008	002	002
6	.8	240	001	001	001	001	001	001	001	004	001	001
6	.8	480	001	002	001	001	001	001	001	004	001	001
6	.6	80	013	009	008	044	036	042	039	005	003	002
6	.6	160	002	002	001	001	002	003	001	007	002	002
6	.6	240	002	002	001	001	002	002	001	007	003	002
6	.6	480	001	001	001	001	001	001	001	003	001	001
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	047	048	048	039	029	030	027	022	015	004
6	.4	240	060	071	062	050	046	055	046	036	026	005
6	.4	480	007	008	005	001	005	008	005	017	009	004
4	.8	80	006	004	002	012	010	015	012	012	005	003
4	.8	160	001	002	001	001	001	002	001	004	001	002
4	.8	240	002	002	001	001	002	002	001	007	002	002
4	.8	480	001	001	001	001	001	002	001	003	001	001
4	.6	80	021	019	021	052	034	050	048	020	007	004
4	.6	160	013	012	009	002	011	010	008	014	007	003
4	.6	240	004	004	002	001	003	003	002	011	004	004
4	.6	480	002	002	001	001	002	002	001	007	002	002
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	025	021	025	007	024	028	032	034	027	005
4	.4	480	022	026	024	035	021	022	021	011	010	001

Note. The decimal point is omitted from tabled values.

Table D-2

Standard Deviations of the Mean Correlations among the Convergent Components for Principal Component, Image Component, and Factor Scores: Promax Position, p=36

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	002	002	001	001	002	002	001	007	002	002
12	.8	160	002	001	001	001	002	002	001	005	002	002
12	.8	240	001	001	001	001	001	001	001	003	001	001
12	.8	480	001	002	001	001	001	001	001	003	001	001
12	.6	80	002	002	002	001	001	002	003	006	003	002
12	.6	160	001	001	001	001	001	001	001	003	002	001
12	.6	240	001	002	001	001	001	001	001	004	001	001
12	.6	480	001	001	001	001	001	001	001	001	001	001
12	.4	80	079	121	119	005	049	098	092	097	084	009
12	.4	160	002	009	008	001	002	012	011	017	012	002
12	.4	240	002	005	005	001	002	006	006	008	007	002
12	.4	480	002	005	004	001	002	006	004	010	006	002
6	.8	80	004	003	001	001	003	004	001	012	004	003
6	.8	160	002	002	001	001	002	002	001	006	002	002
6	.8	240	001	001	001	001	001	001	000	005	001	002
6	.8	480	001	001	001	001	001	001	001	002	001	001
6	.6	80	005	005	002	002	006	006	002	017	006	004
6	.6	160	003	003	001	001	003	003	002	010	003	003
6	.6	240	001	002	001	001	001	002	002	004	002	002
6	.6	480	002	002	001	001	002	002	001	007	002	002
6	.4	80	102	120	110	036	050	053	048	039	031	006
6	.4	160	058	068	066	016	061	081	079	051	054	006
6	.4	240	028	032	026	002	020	031	025	046	041	005
6	.4	480	002	004	003	001	002	005	004	009	007	002
4	.8	80	004	005	001	001	003	005	001	015	004	005
4	.8	160	002	003	001	001	002	003	001	008	002	003
4	.8	240	001	001	001	001	002	001	001	003	001	002
4	.8	480	001	001	001	001	001	001	001	003	001	001
4	.6	80	037	030	029	061	015	009	008	015	010	004
4	.6	160	010	007	002	001	010	008	003	024	009	006
4	.6	240	004	003	001	001	003	004	002	012	006	003
4	.6	480	002	002	001	001	002	003	002	008	003	003
4	.4	80	054	044	041	035	101	090	090	049	042	005
4	.4	160	040	035	033	070	029	018	017	028	031	005
4	.4	240	040	036	036	076	025	016	017	025	022	004
4	.4	480	020	011	009	001	020	011	009	025	017	004

Note. The decimal point is omitted from tabled values.

Table D-3

Standard Deviations of the Mean Correlations among the
Convergent Components for Scale Scores with Principal
Component, Image Component, and Factor Scores:
Promax Position, p=72

p/m	a _{ij}	N	U C	U I	U F1	U F2	U F3
12	.8	80	002	003	001	007	002
12	.8	160	002	002	001	006	002
12	.8	240	002	001	001	003	001
12	.8	480	001	001	001	003	001
12	.6	80	004	008	004	007	004
12	.6	160	002	002	001	004	002
12	.6	240	001	001	001	004	001
12	.6	480	001	001	001	002	001
12	.4	80	005	086	047	079	053
12	.4	160	008	010	013	013	014
12	.4	240	004	004	004	012	013
12	.4	480	002	002	003	003	002
6	.8	80	001	005	004	006	001
6	.8	160	002	002	001	007	002
6	.8	240	001	001	001	003	001
6	.8	480	001	001	001	003	001
6	.6	80	005	037	017	010	010
6	.6	160	004	004	004	007	003
6	.6	240	002	002	002	005	002
6	.6	480	001	001	001	002	001
6	.4	80	---	---	---	---	---
6	.4	160	011	024	030	037	033
6	.4	240	013	046	048	060	054
6	.4	480	012	012	014	028	022
4	.8	80	003	012	005	011	004
4	.8	160	001	001	001	003	001
4	.8	240	002	002	001	007	002
4	.8	480	001	001	001	002	001
4	.6	80	007	046	027	016	021
4	.6	160	009	009	012	016	011
4	.6	240	005	005	006	010	006
4	.6	480	001	001	002	005	001
4	.4	80	---	---	---	---	---
4	.4	160	---	---	---	---	---
4	.4	240	006	011	016	021	017
4	.4	480	018	036	017	030	029

Note. The decimal point is omitted from tabled values.

Table D-4

Standard Deviations of the Mean Correlations among the
Convergent Components for Scale Scores with Principal
Component, Image Component, and Factor Scores:
Promax Position, p=36

p/m	a _{ij}	N	U C	U I	U F1	U F2	U F3
12	.8	80	002	002	001	007	002
12	.8	160	001	002	001	005	002
12	.8	240	001	001	001	003	001
12	.8	480	001	001	001	002	001
12	.6	80	001	001	002	004	002
12	.6	160	001	001	001	003	001
12	.6	240	001	001	001	003	001
12	.6	480	001	001	001	001	001
12	.4	80	012	015	074	112	109
12	.4	160	003	004	005	016	017
12	.4	240	003	003	003	009	008
12	.4	480	002	002	002	005	002
6	.8	80	005	005	009	007	005
6	.8	160	001	002	001	005	002
6	.8	240	001	001	001	004	001
6	.8	480	001	001	001	002	001
6	.6	80	007	009	007	015	008
6	.6	160	002	002	002	008	002
6	.6	240	001	001	001	004	001
6	.6	480	002	002	001	005	002
6	.4	80	007	036	090	097	089
6	.4	160	009	027	062	074	078
6	.4	240	011	014	031	038	034
6	.4	480	009	009	008	012	009
4	.8	80	003	004	006	011	004
4	.8	160	001	002	001	007	001
4	.8	240	001	001	001	003	001
4	.8	480	001	001	001	002	001
4	.6	80	011	048	033	029	028
4	.6	160	005	005	006	018	005
4	.6	240	002	002	003	009	002
4	.6	480	002	002	002	006	002
4	.4	80	009	038	053	043	039
4	.4	160	006	056	023	028	025
4	.4	240	021	046	036	040	040
4	.4	480	009	009	021	019	017

Note. The decimal point is omitted from tabled values.

APPENDIX E

Table E-1

Mean Correlations among Divergent Components for
Principal Component, Image Component, and Factor Scores:
Promax Position, p=72

p/m	a _{1j}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	042	043	002	002	042	042	002	000	042	043
12	.8	160	036	037	001	000	036	037	001	000	036	037
12	.8	240	025	026	000	000	025	026	000	000	025	026
12	.8	480	019	020	000	000	019	020	000	000	019	020
12	.6	80	049	051	004	009	051	046	009	000	049	051
12	.6	160	036	036	002	001	036	036	002	000	036	036
12	.6	240	033	033	001	001	033	033	001	000	033	033
12	.6	480	026	025	001	000	026	025	001	000	026	025
12	.4	80	060	056	040	106	114	101	102	000	048	049
12	.4	160	037	038	009	006	037	037	009	000	037	038
12	.4	240	035	036	005	003	035	035	005	000	035	037
12	.4	480	032	033	002	002	032	033	002	000	032	033
6	.8	80	044	044	002	005	044	042	005	000	044	044
6	.8	160	034	035	001	001	034	035	001	000	034	035
6	.8	240	026	026	001	000	026	026	001	000	026	026
6	.8	480	019	020	000	000	019	020	000	000	019	020
6	.6	80	044	042	017	046	064	054	048	000	040	040
6	.6	160	036	036	005	004	036	036	004	000	036	036
6	.6	240	034	035	003	002	034	035	002	000	034	035
6	.6	480	028	029	002	001	028	029	001	000	028	029
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	084	078	070	049	072	069	058	000	043	043
6	.4	240	060	058	042	029	050	048	030	000	041	043
6	.4	480	036	035	011	004	035	035	009	000	033	034
4	.8	80	043	042	008	010	044	041	010	000	042	042
4	.8	160	035	037	002	001	035	037	001	000	035	037
4	.8	240	032	034	001	001	032	034	001	000	032	034
4	.8	480	022	023	001	000	022	023	000	000	022	023
4	.6	80	065	061	048	056	073	066	057	000	046	045
4	.6	160	042	042	019	009	040	040	015	000	037	038
4	.6	240	037	038	009	004	036	037	007	000	036	037
4	.6	480	029	029	003	002	029	029	002	000	029	029
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	072	067	058	022	068	063	051	000	044	044
4	.4	480	058	055	043	020	053	051	036	000	039	039

Note. The decimal point is omitted before correlations.

Table E-2

Mean Correlations among Divergent Components for
Principal Component, Image Component, and Factor Scores:
Promax Position, p=36

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	047	047	002	001	047	047	002	000	047	047
12	.8	160	038	037	001	000	038	037	001	000	038	037
12	.8	240	034	034	000	000	034	034	000	000	034	034
12	.8	480	021	021	000	000	021	021	000	000	021	021
12	.6	80	040	042	002	002	040	041	003	000	041	042
12	.6	160	039	039	001	001	039	038	001	000	039	039
12	.6	240	034	035	001	001	034	035	001	000	034	035
12	.6	480	018	018	000	000	018	017	000	000	018	018
12	.4	80	119	105	066	029	105	090	054	000	084	096
12	.4	160	041	041	007	005	041	039	006	000	041	043
12	.4	240	038	037	003	003	038	037	003	000	037	037
12	.4	480	036	035	002	001	036	035	001	000	036	035
6	.8	80	050	050	002	002	050	050	002	000	050	050
6	.8	160	035	036	001	001	035	036	001	000	035	036
6	.8	240	031	032	001	000	031	032	001	000	031	032
6	.8	480	021	022	000	000	021	022	000	000	021	022
6	.6	80	050	053	010	008	050	051	009	000	049	053
6	.6	160	043	043	004	002	043	043	003	000	043	043
6	.6	240	037	039	002	002	037	039	002	000	037	039
6	.6	480	029	029	002	001	029	029	001	000	029	029
6	.4	80	129	140	126	072	129	123	118	000	057	068
6	.4	160	077	073	052	021	074	075	047	000	061	074
6	.4	240	052	053	027	007	050	050	023	000	052	055
6	.4	480	042	043	007	003	042	042	006	000	041	043
4	.8	80	053	057	004	003	053	057	003	000	053	057
4	.8	160	035	037	002	001	035	037	001	000	035	037
4	.8	240	029	030	001	001	029	029	001	000	029	030
4	.8	480	023	023	001	000	023	023	000	000	023	023
4	.6	80	065	063	040	029	056	056	029	000	050	052
4	.6	160	049	050	012	006	048	048	009	000	047	049
4	.6	240	041	043	005	003	041	042	004	000	040	043
4	.6	480	032	032	003	001	032	032	002	000	032	032
4	.4	80	109	107	095	055	111	105	097	000	060	065
4	.4	160	090	086	072	038	083	077	060	000	057	059
4	.4	240	086	085	070	040	075	074	055	000	053	058
4	.4	480	049	049	021	004	048	048	018	000	042	044

Note. The decimal point is omitted before correlations.

Table E-3

Mean Correlations among Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=72

p/m	a _{ij}	N	U C	U I	U F1	U F2	U F3
12	.8	80	039	039	080	008	039
12	.8	160	034	034	069	002	034
12	.8	240	024	024	048	002	024
12	.8	480	018	018	037	002	018
12	.6	80	048	050	094	019	048
12	.6	160	032	032	067	009	032
12	.6	240	029	029	062	007	029
12	.6	480	021	021	047	006	021
12	.4	80	048	105	089	067	066
12	.4	160	036	038	068	035	041
12	.4	240	029	029	060	023	031
12	.4	480	022	022	052	016	023
6	.8	80	044	044	085	014	044
6	.8	160	031	031	064	005	031
6	.8	240	024	024	050	004	024
6	.8	480	017	017	035	003	017
6	.6	80	052	069	080	048	055
6	.6	160	034	035	068	019	035
6	.6	240	028	029	061	013	029
6	.6	480	020	020	048	010	020
6	.4	80	---	---	---	---	---
6	.4	160	054	067	090	079	080
6	.4	240	046	054	073	059	060
6	.4	480	029	030	055	032	034
4	.8	80	046	048	084	024	047
4	.8	160	032	032	065	008	032
4	.8	240	028	028	060	006	028
4	.8	480	018	018	041	005	018
4	.6	80	062	075	091	071	075
4	.6	160	041	042	071	036	045
4	.6	240	031	032	064	023	034
4	.6	480	019	020	047	013	020
4	.4	80	---	---	---	---	---
4	.4	160	---	---	---	---	---
4	.4	240	051	054	078	071	072
4	.4	480	040	045	066	058	057

Note. The decimal point is omitted before correlations.

Table E-4

Mean Correlations among Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=36

p/m	a _{1j}	N	U C	U I	U F1	U F2	U F3
12	.8	80	043	043	091	007	043
12	.8	160	036	036	074	002	036
12	.8	240	032	032	067	002	032
12	.8	480	019	019	040	002	019
12	.6	80	037	037	071	017	037
12	.6	160	032	032	071	009	032
12	.6	240	029	029	062	007	029
12	.6	480	014	014	032	004	014
12	.4	80	039	050	126	094	082
12	.4	160	032	035	070	034	037
12	.4	240	028	028	065	016	027
12	.4	480	021	022	057	018	022
6	.8	80	045	045	087	020	045
6	.8	160	032	032	066	005	032
6	.8	240	027	027	058	005	027
6	.8	480	018	018	039	003	018
6	.6	80	045	047	082	045	047
6	.6	160	034	034	076	014	035
6	.6	240	026	027	063	015	027
6	.6	480	020	020	049	010	020
6	.4	80	057	097	128	133	127
6	.4	160	047	054	101	070	082
6	.4	240	040	042	080	049	060
6	.4	480	027	028	066	028	031
4	.8	80	047	047	093	019	047
4	.8	160	029	029	063	008	030
4	.8	240	024	024	053	006	024
4	.8	480	018	018	041	005	018
4	.6	80	058	066	093	072	069
4	.6	160	036	037	080	028	041
4	.6	240	028	028	067	021	029
4	.6	480	019	019	050	015	020
4	.4	80	071	085	121	119	114
4	.4	160	061	072	103	096	092
4	.4	240	055	068	094	092	085
4	.4	480	031	032	070	044	043

Note. The decimal point is omitted before correlations.

APPENDIX F

Table F-1

Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Promax Position. p=72

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	006	006	001	000	006	006	001	000	006	006
12	.8	160	005	006	000	000	005	006	000	000	005	006
12	.8	240	005	005	000	000	005	005	000	000	005	005
12	.8	480	005	005	000	000	005	005	000	000	005	005
12	.6	80	005	006	001	004	006	004	003	000	005	006
12	.6	160	006	005	001	000	006	005	001	000	006	005
12	.6	240	005	005	000	000	005	005	000	000	005	005
12	.6	480	004	003	000	000	004	003	000	000	004	003
12	.4	80	029	023	035	031	028	025	026	000	017	015
12	.4	160	008	008	004	002	008	008	001	000	008	008
12	.4	240	007	007	002	001	007	008	001	000	007	007
12	.4	480	004	006	001	000	004	006	000	000	004	006
6	.8	80	003	003	001	001	003	003	001	000	003	003
6	.8	160	004	003	000	000	004	003	000	000	004	003
6	.8	240	002	002	000	000	002	002	000	000	002	002
6	.8	480	002	003	000	000	002	003	000	000	002	003
6	.6	80	003	003	004	010	008	008	011	000	002	002
6	.6	160	003	003	001	000	003	003	001	000	003	003
6	.6	240	003	003	001	000	003	003	000	000	003	003
6	.6	480	001	002	000	000	001	002	000	000	001	002
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	017	012	017	018	013	014	016	000	003	003
6	.4	240	018	018	026	024	017	013	023	000	005	006
6	.4	480	006	006	005	001	005	005	005	000	005	005
4	.8	80	003	003	003	003	003	003	003	000	003	003
4	.8	160	001	002	000	000	001	002	000	000	001	002
4	.8	240	002	002	000	000	002	002	000	000	002	002
4	.8	480	001	001	000	000	001	001	000	000	001	001
4	.6	80	008	005	008	012	006	005	011	000	008	004
4	.6	160	005	004	006	002	004	004	005	000	003	003
4	.6	240	003	003	002	001	003	003	002	000	003	003
4	.6	480	002	002	000	000	002	002	000	000	002	002
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	003	003	007	006	004	004	009	000	007	005
4	.4	480	006	006	007	014	005	004	006	000	002	002

Note. The decimal point is omitted from tabled values.

Table F-2

Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Promax Position. p=36

p/m	a _{1j}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F2	F2 F3
12	.8	80	016	013	001	000	016	013	001	000	016	013
12	.8	160	016	015	001	000	016	015	001	000	016	015
12	.8	240	010	010	000	000	010	010	000	000	010	010
12	.8	480	013	013	000	000	013	013	000	000	013	013
12	.6	80	013	014	002	000	013	014	002	000	012	014
12	.6	160	009	009	001	000	009	009	001	000	009	009
12	.6	240	014	015	001	000	014	015	001	000	015	015
12	.6	480	007	007	000	000	007	007	000	000	007	007
12	.4	80	094	068	111	028	068	046	083	000	032	037
12	.4	160	018	019	003	003	018	019	002	000	018	021
12	.4	240	018	018	001	001	019	018	002	000	019	019
12	.4	480	017	015	001	001	016	015	001	000	017	016
6	.8	80	009	007	001	001	008	007	001	000	009	007
6	.8	160	005	006	000	000	005	006	000	000	005	006
6	.8	240	004	005	000	000	004	005	000	000	004	005
6	.8	480	003	003	000	000	003	003	000	000	003	003
6	.6	80	010	010	003	001	010	010	003	000	010	010
6	.6	160	009	008	001	001	009	008	001	000	009	008
6	.6	240	002	004	001	000	003	004	001	000	003	004
6	.6	480	007	006	000	000	007	006	000	000	007	006
6	.4	80	030	038	034	036	025	029	027	000	007	012
6	.4	160	029	018	025	018	019	019	025	000	010	012
6	.4	240	007	011	016	004	005	008	014	000	006	007
6	.4	480	003	003	002	001	003	003	002	000	002	003
4	.8	80	006	009	001	000	001	009	000	000	006	009
4	.8	160	004	006	000	000	004	006	000	000	004	006
4	.8	240	002	002	000	000	002	002	000	000	002	002
4	.8	480	003	003	000	000	003	003	000	000	003	003
4	.6	80	017	017	019	024	008	008	010	000	006	005
4	.6	160	009	008	003	001	009	009	004	000	008	009
4	.6	240	005	005	002	001	004	005	001	000	004	005
4	.6	480	004	005	001	000	004	005	000	000	004	005
4	.4	80	017	017	022	027	035	034	041	000	005	005
4	.4	160	022	016	018	037	013	012	009	000	009	008
4	.4	240	022	021	028	042	017	017	023	000	008	006
4	.4	480	011	012	012	001	011	012	012	000	007	007

Note. The decimal point is omitted from tabled values.

Table F-3

Standard Deviations of the Mean Correlations among the Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores: Promax Position, p=72

p/m	a _{ij}	N	U C	U I	U F1	U F2	U F3
12	.8	80	007	007	013	001	007
12	.8	160	005	005	009	000	005
12	.8	240	004	004	008	000	004
12	.8	480	004	004	008	000	004
12	.6	80	005	004	009	005	005
12	.6	160	005	005	013	001	005
12	.6	240	003	003	009	001	003
12	.6	480	003	003	007	001	003
12	.4	80	003	028	023	024	022
12	.4	160	004	006	008	006	007
12	.4	240	004	004	007	004	005
12	.4	480	004	004	008	002	003
6	.8	80	002	002	006	003	002
6	.8	160	003	003	007	001	003
6	.8	240	002	002	004	000	002
6	.8	480	002	002	004	001	002
6	.6	80	004	004	005	006	006
6	.6	160	003	003	005	002	003
6	.6	240	003	003	006	001	003
6	.6	480	001	001	002	001	001
6	.4	80	---	---	---	---	---
6	.4	160	002	010	010	009	009
6	.4	240	003	009	008	012	012
6	.4	480	005	005	009	006	008
4	.8	80	003	004	006	002	004
4	.8	160	001	001	003	000	001
4	.8	240	002	002	004	000	002
4	.8	480	001	001	002	000	001
4	.6	80	004	005	005	005	005
4	.6	160	003	003	005	004	003
4	.6	240	003	003	005	002	003
4	.6	480	001	001	003	001	001
4	.4	80	---	---	---	---	---
4	.4	160	---	---	---	---	---
4	.4	240	003	004	005	005	005
4	.4	480	004	009	004	003	004

Note. The decimal point is omitted from tabled values.

Table F-4

Standard Deviations of the Mean correlations among the Divergent Components for Scale Scores with Principal Component, Image Component, and Factor Scores:
Promax Position, p=36

p/m	a _{ij}	N	U C	U I	U F1	U F2	U F3
12	.8	80	014	014	030	009	014
12	.8	160	015	015	031	001	015
12	.8	240	009	009	018	001	009
12	.8	480	012	012	025	001	012
12	.6	80	011	011	026	011	010
12	.6	160	008	008	017	001	008
12	.6	240	012	012	026	003	012
12	.6	480	005	005	013	001	005
12	.4	80	012	016	075	069	086
12	.4	160	010	011	025	013	013
12	.4	240	014	014	032	005	013
12	.4	480	009	009	026	010	009
6	.8	80	007	007	011	011	007
6	.8	160	005	005	010	001	005
6	.8	240	003	003	006	001	003
6	.8	480	003	003	006	001	003
6	.6	80	008	008	020	012	009
6	.6	160	006	006	016	003	006
6	.6	240	002	002	004	002	002
6	.6	480	005	005	013	002	005
6	.4	80	005	030	023	028	026
6	.4	160	007	011	028	018	020
6	.4	240	010	011	014	016	022
6	.4	480	007	007	008	004	006
4	.8	80	005	005	011	005	005
4	.8	160	003	003	005	002	003
4	.8	240	002	002	003	001	002
4	.8	480	002	002	005	001	002
4	.6	80	008	013	011	015	012
4	.6	160	008	008	015	003	007
4	.6	240	004	004	008	002	004
4	.6	480	002	002	006	002	002
4	.4	80	007	019	015	018	015
4	.4	160	006	023	011	008	009
4	.4	240	010	023	014	020	018
4	.4	480	008	008	011	012	013

Note. The decimal point is omitted from tabled values.

APPENDIX G

Table G-1

Mean Correlations among Convergent Components for
Principal Component, Image Component, and Factor Scores:
Procrustes Position, p=72

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	999	999	999	994	994	994	994	999	999	999
12	.8	160	999	999	999	999	999	999	999	999	999	999
12	.8	240	999	999	999	999	999	999	999	999	999	999
12	.8	480	999	999	999	999	999	999	999	999	999	999
12	.6	80	998	998	998	981	981	977	977	992	992	999
12	.6	160	999	999	999	999	999	996	996	996	996	999
12	.6	240	999	999	999	999	999	997	997	996	996	999
12	.6	480	999	999	999	999	999	999	999	999	999	999
12	.4	80	957	949	950	828	852	820	822	947	947	999
12	.4	160	996	987	988	996	996	979	979	974	975	999
12	.4	240	998	986	986	998	998	980	980	976	976	999
12	.4	480	999	994	994	999	999	991	991	989	989	999
6	.8	80	997	999	999	989	990	988	988	996	996	999
6	.8	160	999	999	999	999	999	999	999	999	999	999
6	.8	240	999	999	999	999	999	999	999	999	999	999
6	.8	480	999	999	999	999	999	999	999	999	999	999
6	.6	80	979	991	991	897	905	895	897	981	981	999
6	.6	160	996	998	998	997	998	994	994	991	991	999
6	.6	240	998	999	999	999	999	996	996	994	994	999
6	.6	480	999	999	999	999	999	998	998	996	996	999
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	936	949	949	969	962	961	962	951	952	999
6	.4	240	963	970	970	993	974	966	966	956	957	999
6	.4	480	993	989	989	999	996	984	985	973	974	999
4	.8	80	988	999	999	982	981	983	983	990	990	999
4	.8	160	998	999	999	998	999	998	998	997	997	999
4	.8	240	999	999	999	999	999	999	999	998	998	999
4	.8	480	999	999	999	999	999	999	999	999	999	999
4	.6	80	944	979	980	922	904	921	922	964	965	999
4	.6	160	977	992	992	994	986	989	990	979	979	999
4	.6	240	991	997	997	998	996	994	994	986	986	999
4	.6	480	998	999	999	999	999	997	997	994	995	999
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	920	963	964	993	933	961	961	920	922	999
4	.4	480	956	974	974	997	967	974	974	950	951	999

Note. The decimal point is omitted before correlations.

Table G-2

Mean Correlations among Convergent Components for
Principal Component, Image Component, and Factor Scores:
Procrustes Position, p=36

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	999	999	999	999	999	998	998	998	998	999
12	.8	160	999	999	999	999	999	999	999	999	999	999
12	.8	240	999	999	999	999	999	999	999	999	999	999
12	.8	480	999	999	999	999	999	999	999	999	999	999
12	.6	80	998	997	997	998	998	991	991	991	991	999
12	.6	160	999	998	998	999	999	996	996	995	995	999
12	.6	240	999	999	999	999	999	998	998	997	997	999
12	.6	480	999	999	999	999	999	999	999	999	999	999
12	.4	80	980	916	919	990	983	896	900	881	884	998
12	.4	160	997	978	978	998	998	967	967	962	963	999
12	.4	240	999	989	989	999	999	985	985	981	981	999
12	.4	480	999	993	993	999	999	991	991	989	989	999
6	.8	80	998	999	999	998	999	998	998	997	997	999
6	.8	160	999	999	999	999	999	999	999	999	999	999
6	.8	240	999	999	999	999	999	999	999	999	999	999
6	.8	480	999	999	999	999	999	999	999	999	999	999
6	.6	80	992	996	996	993	995	984	984	981	981	999
6	.6	160	997	998	998	999	999	994	994	990	990	999
6	.6	240	998	998	998	999	999	996	996	994	994	999
6	.6	480	999	999	999	999	999	998	998	997	997	999
6	.4	80	845	879	883	971	884	876	882	832	838	997
6	.4	160	934	918	926	988	943	900	908	877	888	995
6	.4	240	978	959	964	998	985	951	958	924	930	997
6	.4	480	997	988	989	999	998	984	985	975	976	999
4	.8	80	996	999	999	998	998	997	997	994	994	999
4	.8	160	998	999	999	999	999	998	999	997	997	999
4	.8	240	999	999	999	999	999	999	999	998	998	999
4	.8	480	999	999	999	999	999	999	999	999	999	999
4	.6	80	962	988	989	986	979	978	979	951	954	999
4	.6	160	987	996	996	997	992	989	989	975	976	999
4	.6	240	996	998	998	999	998	995	995	989	989	999
4	.6	480	998	999	999	999	999	997	997	993	993	999
4	.4	80	876	946	947	979	894	923	925	866	870	998
4	.4	160	912	958	959	994	931	954	955	879	882	999
4	.4	240	916	944	945	977	938	942	943	881	884	999
4	.4	480	982	982	983	999	986	977	978	948	949	999

Note. The decimal point is omitted before correlations.

APPENDIX H

Table H-1

Standard Deviations of the Mean Correlations among the Convergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, p=72

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	001	001	001	003	003	003	003	001	001	001
12	.8	160	001	001	001	001	001	001	001	001	001	001
12	.8	240	001	001	001	001	001	001	001	001	001	001
12	.8	480	001	001	001	001	001	001	001	001	001	001
12	.6	80	001	001	001	007	006	007	007	002	003	001
12	.6	160	001	001	001	001	002	002	001	001	001	001
12	.6	240	001	001	001	001	001	001	001	001	001	001
12	.6	480	001	001	001	001	001	001	001	001	001	001
12	.4	80	062	065	065	061	059	063	063	017	017	001
12	.4	160	003	004	004	001	001	006	006	008	008	001
12	.4	240	001	006	006	001	001	007	007	008	008	001
12	.4	480	001	001	001	001	001	002	002	002	002	001
6	.8	80	001	001	001	005	005	005	005	001	001	001
6	.8	160	001	001	001	001	001	001	001	001	001	001
6	.8	240	001	001	001	001	001	001	001	001	001	001
6	.8	480	001	001	001	001	001	001	001	001	001	001
6	.6	80	014	008	008	033	030	026	026	003	003	001
6	.6	160	001	001	000	001	001	001	001	001	001	001
6	.6	240	001	001	001	001	001	001	001	001	002	001
6	.6	480	001	001	001	001	001	001	001	001	001	001
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	026	033	033	022	011	010	010	016	015	001
6	.4	240	042	033	033	003	034	028	028	027	027	001
6	.4	480	004	004	003	001	003	004	004	008	007	001
4	.8	80	006	001	001	011	010	011	011	004	004	001
4	.8	160	001	001	001	001	001	001	001	001	001	001
4	.8	240	001	001	001	001	001	001	001	001	001	001
4	.8	480	001	001	001	001	001	001	001	001	001	001
4	.6	80	013	010	010	026	025	028	027	007	006	001
4	.6	160	010	005	005	001	008	004	004	006	006	001
4	.6	240	002	001	001	001	002	002	001	005	005	001
4	.6	480	001	001	001	001	001	001	001	001	001	001
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	028	012	011	002	023	014	014	035	034	001
4	.4	480	020	015	015	002	017	013	013	013	013	001

Note. The decimal point is omitted from tabled values.

Table H-2

Standard Deviations of the Mean Correlations among the
Convergent Components for Principal Component, Image
Component, and Factor Scores: Procrustes Position, p=36

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	001	001	001	001	001	001	001	001	001	001
12	.8	160	001	001	001	001	001	001	001	001	001	001
12	.8	240	001	001	001	001	001	001	001	001	001	001
12	.8	480	001	001	001	001	001	001	001	001	001	001
12	.6	80	001	001	002	001	001	003	002	003	003	001
12	.6	160	000	001	001	001	001	001	001	001	001	001
12	.6	240	001	001	001	001	001	001	001	001	001	001
12	.6	480	001	001	001	001	001	001	001	001	001	001
12	.4	80	028	074	069	004	019	068	062	092	086	003
12	.4	160	001	008	008	001	001	011	011	012	012	001
12	.4	240	001	005	005	001	001	007	006	007	007	001
12	.4	480	001	004	004	001	001	004	004	005	005	001
6	.8	80	001	001	001	001	001	001	001	001	001	001
6	.8	160	001	001	001	000	001	001	001	001	001	001
6	.8	240	001	001	001	001	001	001	001	001	001	001
6	.8	480	001	001	001	001	001	001	001	001	001	001
6	.6	80	002	002	002	002	003	002	002	003	003	001
6	.6	160	001	001	001	001	001	002	002	002	002	001
6	.6	240	001	001	001	001	001	001	001	002	002	001
6	.6	480	001	001	001	001	001	001	001	002	002	001
6	.4	80	066	071	072	016	048	043	042	031	031	002
6	.4	160	053	069	061	012	059	087	079	071	058	006
6	.4	240	031	030	024	002	023	032	023	050	045	004
6	.4	480	001	004	003	001	001	004	004	007	007	001
4	.8	80	001	001	001	001	001	001	001	002	002	001
4	.8	160	002	001	001	001	001	001	001	002	002	001
4	.8	240	001	001	001	001	001	001	001	001	001	001
4	.8	480	001	001	001	001	001	001	001	001	001	001
4	.6	80	017	007	007	011	009	008	007	012	011	001
4	.6	160	009	002	002	001	009	003	003	008	008	001
4	.6	240	003	001	002	001	001	002	002	005	005	001
4	.6	480	001	001	001	001	001	002	002	003	003	001
4	.4	80	044	013	013	008	048	031	029	047	045	001
4	.4	160	035	013	013	002	030	008	008	040	040	001
4	.4	240	019	013	013	039	009	011	011	029	027	001
4	.4	480	019	005	004	001	018	005	004	023	022	001

Note. The decimal point is omitted from tabled values.

APPENDIX I

Table I-1

Mean Correlations among Divergent Components for
Principal Component, Image Component, and Factor Scores:
Procrustes Position, p=72

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	001	001	000	001	002	001	001	000	001	001
12	.8	160	001	001	000	000	000	000	000	000	001	001
12	.8	240	000	000	000	000	000	000	000	000	000	000
12	.8	480	000	000	000	000	000	000	000	000	000	000
12	.6	80	001	002	001	003	006	004	004	000	002	002
12	.6	160	001	001	000	000	000	001	001	000	001	001
12	.6	240	001	001	000	000	000	000	001	000	001	001
12	.6	480	001	001	000	000	000	000	000	000	001	001
12	.4	80	006	009	009	034	038	037	040	000	007	007
12	.4	160	001	002	002	001	002	002	003	000	003	003
12	.4	240	001	002	003	000	001	001	003	000	004	004
12	.4	480	001	001	001	000	001	001	001	000	001	001
6	.8	80	001	001	000	002	003	002	002	000	001	001
6	.8	160	001	001	000	000	000	000	000	000	001	001
6	.8	240	001	001	000	000	000	000	000	000	001	001
6	.8	480	000	000	000	000	000	000	000	000	000	000
6	.6	80	004	004	003	019	027	024	026	000	004	004
6	.6	160	002	002	001	000	001	001	001	000	002	002
6	.6	240	001	002	001	000	001	001	001	000	002	002
6	.6	480	001	001	000	000	000	000	001	000	001	001
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	009	008	008	005	006	007	008	000	006	006
6	.4	240	005	007	006	001	004	005	006	000	005	006
6	.4	480	002	003	002	000	002	002	002	000	004	004
4	.8	80	002	002	001	004	004	004	003	000	002	002
4	.8	160	001	001	000	000	001	001	000	000	001	001
4	.8	240	001	001	000	000	000	000	000	000	001	001
4	.8	480	001	001	000	000	000	000	000	000	001	001
4	.6	80	008	007	004	014	024	020	020	000	005	005
4	.6	160	005	004	002	001	003	003	002	000	004	004
4	.6	240	002	003	001	000	002	002	001	000	003	003
4	.6	480	001	001	000	000	001	001	001	000	002	002
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	009	009	007	001	008	009	007	000	007	007
4	.4	480	007	007	005	001	006	005	005	000	005	005

Note. The decimal point is omitted before correlations.

Table I-2

Mean Correlations among Divergent Components for
Principal Component, Image Component, and Factor Scores:
Procrustes Position, p=36

p/m	a _{1j}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	001	001	000	000	001	001	000	000	001	001
12	.8	160	001	001	000	000	000	000	000	000	001	001
12	.8	240	000	000	000	000	000	000	000	000	000	000
12	.8	480	000	000	000	000	000	000	000	000	000	000
12	.6	80	001	001	001	001	001	001	002	000	002	002
12	.6	160	001	001	001	000	001	001	001	000	002	002
12	.6	240	001	001	000	000	000	000	001	000	001	001
12	.6	480	001	001	000	000	000	000	000	000	001	001
12	.4	80	004	010	018	003	005	008	020	000	023	029
12	.4	160	002	003	004	000	002	003	005	000	006	006
12	.4	240	001	001	002	000	001	001	002	000	002	002
12	.4	480	001	001	002	000	000	001	002	000	002	002
6	.8	80	002	002	000	000	001	001	001	000	002	002
6	.8	160	001	001	000	000	000	000	000	000	001	001
6	.8	240	001	001	000	000	000	000	000	000	001	001
6	.8	480	001	001	000	000	000	000	000	000	001	001
6	.6	80	003	004	002	001	003	003	004	000	004	004
6	.6	160	002	002	001	000	001	001	001	000	002	002
6	.6	240	002	002	000	000	001	001	001	000	002	002
6	.6	480	001	001	000	000	000	000	000	000	001	001
6	.4	80	021	031	032	006	023	030	035	000	021	027
6	.4	160	014	021	028	002	011	015	027	000	023	028
6	.4	240	004	010	015	001	003	009	017	000	017	021
6	.4	480	003	004	003	000	001	002	003	000	006	006
4	.8	80	002	002	000	000	002	001	001	000	002	002
4	.8	160	001	002	000	000	001	001	000	000	002	002
4	.8	240	001	001	000	000	000	000	000	000	001	001
4	.8	480	001	001	000	000	000	000	000	000	001	001
4	.6	80	008	010	006	003	006	007	007	000	009	010
4	.6	160	004	005	002	001	003	003	003	000	006	006
4	.6	240	003	003	001	000	001	001	001	000	003	003
4	.6	480	002	002	000	000	001	001	001	000	002	002
4	.4	80	016	021	016	004	020	022	020	000	013	015
4	.4	160	014	016	013	001	012	014	012	000	012	014
4	.4	240	015	017	013	003	011	013	013	000	013	015
4	.4	480	005	007	006	000	004	006	007	000	009	009

Note. The decimal point is omitted before correlations.

APPENDIX J

Table J-1

Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, p=72

p/m	a _{ij}	N	C F1	C F2	C F3	C I	I F1	I F2	I F3	F1 F2	F1 F3	F2 F3
12	.8	80	000	000	000	000	001	000	000	000	000	000
12	.8	160	000	000	000	000	000	000	000	000	000	000
12	.8	240	000	000	000	000	000	000	000	000	000	000
12	.8	480	000	000	000	000	000	000	000	000	000	000
12	.6	80	000	001	000	001	003	001	002	000	001	001
12	.6	160	000	000	000	000	000	000	000	000	001	000
12	.6	240	000	000	000	000	000	000	000	000	000	000
12	.6	480	000	000	000	000	000	000	000	000	000	000
12	.4	80	007	007	007	012	009	012	010	000	002	002
12	.4	160	000	001	001	000	001	001	001	000	001	002
12	.4	240	000	001	002	000	000	000	002	000	002	002
12	.4	480	000	000	000	000	000	000	001	000	001	001
6	.8	80	000	000	000	001	001	001	001	000	000	000
6	.8	160	000	000	000	000	000	000	000	000	000	000
6	.8	240	000	000	000	000	000	000	000	000	000	000
6	.8	480	000	000	000	000	000	000	000	000	000	000
6	.6	80	002	001	002	006	014	013	013	000	001	001
6	.6	160	000	000	000	000	000	000	000	000	000	000
6	.6	240	000	000	000	000	000	000	000	000	000	000
6	.6	480	000	000	000	000	000	000	000	000	000	000
6	.4	80	---	---	---	---	---	---	---	---	---	---
6	.4	160	003	002	003	003	002	002	002	000	002	003
6	.4	240	004	005	006	001	002	004	004	000	002	003
6	.4	480	001	001	001	000	000	001	001	000	001	001
4	.8	80	001	001	000	003	002	002	002	000	000	001
4	.8	160	000	000	000	000	000	000	000	000	000	000
4	.8	240	000	000	000	000	000	000	000	000	000	000
4	.8	480	000	000	000	000	000	000	000	000	000	000
4	.6	80	002	001	001	006	008	009	009	000	001	001
4	.6	160	002	002	001	000	001	001	001	000	001	001
4	.6	240	000	001	000	000	000	000	000	000	001	001
4	.6	480	000	000	000	000	000	000	000	000	000	000
4	.4	80	---	---	---	---	---	---	---	---	---	---
4	.4	160	---	---	---	---	---	---	---	---	---	---
4	.4	240	002	002	002	000	002	002	002	000	002	003
4	.4	480	002	002	002	000	002	002	002	000	000	001

Note. The decimal point is omitted from tabled values.

Table J-2

Standard Deviations of the Mean Correlations among the Divergent Components for Principal Component, Image Component, and Factor Scores: Procrustes Position, p=36

<u>p/m</u>	<u>a_{1j}</u>	<u>N</u>	<u>C</u> <u>F1</u>	<u>C</u> <u>F2</u>	<u>C</u> <u>F3</u>	<u>C</u> <u>I</u>	<u>I</u> <u>F1</u>	<u>I</u> <u>F2</u>	<u>I</u> <u>F3</u>	<u>F1</u> <u>F2</u>	<u>F1</u> <u>F3</u>	<u>F2</u> <u>F3</u>
12	.8	80	001	001	000	000	000	000	000	000	001	001
12	.8	160	000	000	000	000	000	000	000	000	000	000
12	.8	240	000	000	000	000	000	000	000	000	000	000
12	.8	480	000	000	000	000	000	000	000	000	000	000
12	.6	80	001	001	000	000	000	000	001	000	001	001
12	.6	160	001	000	000	000	000	000	001	000	000	000
12	.6	240	000	001	000	000	000	000	000	000	001	001
12	.6	480	000	000	000	000	000	000	000	000	000	000
12	.4	80	005	009	015	001	005	005	016	000	019	030
12	.4	160	001	001	001	000	001	002	002	000	003	003
12	.4	240	001	001	001	000	000	001	001	000	001	001
12	.4	480	000	001	001	000	000	000	001	000	002	002
6	.8	80	000	000	000	000	000	000	000	000	000	000
6	.8	160	000	000	000	000	000	000	000	000	000	000
6	.8	240	000	000	000	000	000	000	000	000	000	000
6	.8	480	000	000	000	000	000	000	000	000	000	000
6	.6	80	001	001	001	000	001	001	000	000	001	001
6	.6	160	001	001	000	000	001	000	000	000	001	001
6	.6	240	001	001	000	000	000	000	000	000	001	001
6	.6	480	000	000	000	000	000	000	000	000	000	000
6	.4	80	009	013	011	003	013	012	014	000	004	006
6	.4	160	019	024	030	001	015	019	027	000	018	025
6	.4	240	002	009	013	001	002	008	014	000	012	016
6	.4	480	001	002	001	000	000	001	001	000	003	003
4	.8	80	000	000	000	000	000	000	000	000	000	000
4	.8	160	000	000	000	000	000	000	000	000	000	000
4	.8	240	000	000	000	000	000	000	000	000	000	000
4	.8	480	000	000	000	000	000	000	000	000	000	000
4	.6	80	003	004	003	001	002	002	003	000	004	004
4	.6	160	001	001	001	000	001	001	001	000	001	002
4	.6	240	001	000	000	000	000	000	000	000	001	001
4	.6	480	000	001	000	000	000	000	000	000	001	001
4	.4	80	004	006	004	001	006	008	007	000	003	004
4	.4	160	002	004	004	000	002	003	003	000	002	003
4	.4	240	004	005	005	004	002	002	003	000	002	003
4	.4	480	002	004	003	000	003	004	003	000	003	003

Note. The decimal point is omitted from tabled values.