1996

Nominalistic Elements in the Work of Stanislaw Lesniewski

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NOMINALISTIC ELEMENTS IN THE WORK OF
STANISLAW LESNIEWSKI
BY
CHRISTOPHER FLEMING

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS
IN
PHILOSOPHY

35964833

UNIVERSITY OF RHODE ISLAND

1996

Abstract

Stanislaw Lesniewski (1886-1939) is called a nominalist, even though his published works contain no developed philosophical doctrine. Yet, in order to understand and interpret his logical systems, we must understand his nominalism. This thesis will investigate, in detail, the philosophical origins of the "nominalistic" elements of Lesniewski's logical systems and will offer a characterization of his nominalism.

This thesis will provide a brief historical sketch of Lesniewski's career as a logician and of the times in which his logical systems were developed. A definition of nominalism will be developed within the context of the realist/nominalist debate over the existence of universals and a realists notion of universals will be given as a background against which Lesniewski's philosophical beliefs can be measured. The philosophical origins of Lesniewski's nominalism will be explored and will provide the basis for an examination of the nominalistic elements of his logical systems and the basis for a characterization of his nominalism.

Lesniewski's nominalism avoids traditional classification and can only be examined indirectly through an analysis of his logical systems and through his attitude towards Russellian classes. In the final analysis, it is best to say that Lesniewski was a philosopher who created consistent logical systems in which to "talk" about objects.

Acknowledgments

I would like to thank Professor James G. Kowalski for his time and patience in answering my many questions about logic, for introducing me to Lesniewski's logical systems and for his assistance in the completion of this thesis. I would like to thank Professor John Peterson for taking the time to read this thesis toward the end of a busy semester and for his many suggestions for revision. I would like to thank Professor Albert Silverstein for his helpful comments on the sections of the thesis concerning Aristotle.

I would especially like to thank my wife Nancy and my daughter Ana who took on the burden of our business in order to allow me the time to research and write this thesis. Without Nancy's patience, understanding and support, this thesis would not have been possible.

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Introduction:

Stanislaw Lesniewski (1886-1939) taught logic at Warsaw University between the First and Second World Wars. He developed three logical systems - Mereology, Ontology and Protothetic - which he believed would provide a more sound basis for mathematics than the other logical systems in use at that time, especially the logical system developed by Whitehead and Russell in *Principia Mathematica*.

Lesniewski is called a nominalist, even though his published works contain no developed philosophical doctrine. Yet, in order to understand and interpret his logical systems, we must understand his nominalism. This thesis will investigate, in detail, the "nominalistic" elements of Lesniewski's logical systems and will offer a more precise characterization of his nominalism.

In order to investigate the origin of Lesniewski's nominalism and to more precisely characterize his philosophical tendencies, this thesis will provide a brief historical sketch of Lesniewski's career as a logician and of the times in which his logical systems were developed. A definition of nominalism will be developed within the context of the realist/nominalist debate over the existence of universals and a realists notion of universals will be given as a background against which Lesniewski's philosophical beliefs can be measured. The philosophical origins of Lesniewski's nominalism will be explored and will provide the basis for an examination of the nominalistic elements of his logical systems and the basis for a characterization of his nominalism.

Chapter I - Background

1) A Brief Historical Sketch¹

Stanislaw Lesniewski was born on March 30, 1886. Although the history of his university training is not complete, it is known that he spent at least some of his university days in Munich where he attended lectures by Hans Cornelius. Lesniewski came to Lvov University in 1910 to study logic and completed his doctoral work under the direction of Kazimierz Twardowski in 1912. Lesniewski characterized his student days in the following way.

Steeped in the influence of John Stuart Mill in which I mainly grew up, and 'conditioned' by the problems of 'universal-grammar' and of logico-semantics in the style of Edward Husserl and by the exponents of the so-called Austrian School, I ineffectually attacked the foundations of 'logistic' from this point of view.²

Lesniewski was first introduced to modern symbolic logic in 1910 when he read Jan Lukasiewicz's "On The Principle Of Contradiction In Aristotle." Lesniewski focused on Russell's Paradox involving the "class of all classes not subordinated to themselves" for 11 years (1911 through 1922). During his "initial period" (1911 through 1914), Lesniewski published seven papers which he later repudiated as immature works. He lived in Moscow during the First World War (1915 through 1918). In 1916, Lesniewski's published "Foundations of the General Theory of Sets" which can clearly be taken as a "bridge between Lesniewski's early and later writings." Lesniewski's mature period extended from 1916 until his death in 1939. Lesniewski's most important published work, "On the Foundations of Mathematics," was

¹ The facts in this brief summary of Lesniewski's philosophical career and his intellectual development have been extracted mainly from two sources: Jan Wolenski's book, Logic And Philosophy In the Lvov-Warsaw School, and the Introduction to Stanislaw Lesniewski Collected Works.

² Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Szrednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 181.

published from 1927 through 1931 as a series of journal articles. Lesniewski published little in his lifetime. All of his personal papers were destroyed near the end of the Second World War. The list of lecture topics Lesniewski delivered at Warsaw University from 1919 through 1939 contained on pages xii and xiii of "Stanislaw Lesniewski Collected Works" are a most valuable source of information because Lesniewski presented the details of his logical discoveries in his lectures. The list of his lectures, therefore, is a helpful outline of the development of his logical systems.

2) Lesniewski's Logical Systems

Lesniewski developed three logical systems during his mature period: Protothetic, Ontology, and Mereology. Lesniewski was not concerned with formal semantics³; his work, therefore, was mainly focused on the syntactic development of his logical systems. Because Lesniewski developed a highly unique symbolism for his logical systems and, formally, he favored the axiomatic development of his systems, his published works are difficult to access. In his lectures, however, Lesniewski adopted the more common Peano/Russell notation and developed his systems using the method of natural deduction. Even though there has been considerable work by others in presenting the formal features of Lesniewski's logical systems, his logical systems are still not as accessible as the logic of Whitehead and Russell. In order to understand Lesniewski's logical systems, it is necessary not only to read the literature, but also to actually work with his systems, even if one already has a familiarity with the logic of Whitehead and Russell.

The primitive terms of Protothetic, Ontology, and Mereology are "=", "ε", and "part" ("cz" in Polish), respectively, as well as the quantifier. The axiom(s)

³ Frederick V. Rickey, ("Interpretations of Lesniewski's Ontology", *Dialectica*, Vol. 39, No. 3 (1985), pp. 182 through 184.), gives an account of what he believes Lesniewski's informal semantics were.

of each logical system "defines" the primitive term of that system by its use in the axiom(s). The quantifier, however, is not "defined" in the same way; its "definition" will be the subject of much discussion in the Chapter IV below.

Protothetic is very similar to a quantified propositional calculus; it is based on a single axiom which "defines" equivalence.

Ontology is similar to a quantified predicate calculus, except that $[A] a(A)$ in Whitehead and Russell's system would be written $[Aa] (A \varepsilon a)$ in Ontology (in my examples, I have used the same shaped variables in order to make the comparison more obvious) and, unlike $[A] a(A)$, where "A" and "a" refer to two different kinds of things, i.e. objects and their properties respectively, "A" and "a" in Lesniewski's system refer to only one kind of thing, namely, objects. Ontology is based on the single axiom:

$$[Aa]:A\varepsilon a.\equiv.[B] B\varepsilon A:.[CD] C\varepsilon A.D\varepsilon A.\supset.C\varepsilon D:.[E] E\varepsilon A.\supset.E\varepsilon a$$

A helpful English translation of " $A\varepsilon a$ " might read "A is the a" or "A is one of the a's," but " $A\varepsilon a$ " for Lesniewski is most correctly "defined" by the use of " ε " in the Axiom of Ontology.

Mereology is a deductive system Lesniewski designed to treat parts and wholes. Logically, Mereology is built on Ontology and Ontology is built on Protothetic. The order in which Lesniewski constructed these systems, however, is just the reverse. As we will see, Lesniewski constructed Mereology in order to "define" class as something no different from its members. Mereology is not based on a single axiom. It will be sufficient in the introduction, however, to present the first axiom of Mereology along with an English translation of that axiom. The first axiom of Mereology is:

$$[P,Q]:P\varepsilon cz(Q).\supset.Q\varepsilon N(cz(P)).$$

This axiom might most easily be understood if translated into the English sentence: "If P is one of the parts of Q, then Q is not one of the parts of P." If

applied to botany, the first axiom of Ontology might be translated by the following English sentence: "If a cell is one of the parts of a leaf, then a leaf is not one of the parts of a cell." Lesniewski would agree to these translations only in so far as they are helpful in understanding the first axiom of Mereology.

Each logical system "grows," theses are derived, according to directives, i.e. very precise rules of transformation/inference, which are directly related to the axiom of each logical system through terminological explanations which develop the "vocabulary" of the directives through direct reference to the axioms of the logical system. Lesniewski's terminological explanations and his directives are not proper parts of his logical systems. The shapes of parentheses surrounding the variables of all Lesniewski's logical systems "mark" the semantic category of expressions which can be substituted for those variables.

Lesniewski's logical systems are carefully constructed to avoid the possibility of contradiction and to avoid the lack of precision he detected in other logical systems, e.g. the logical systems of Whitehead and Russell, Chwistek, Fraenkel, Zermelo and Frege. A good example of Lesniewski's own precision is his extended treatment of the use of the turnstile in Whitehead and Russell's *Principia Mathematica*.⁴

3) The Polish School Of Logic

The Polish School of Logic played a significant role in the development of logic between the First and the Second World Wars. In their written works there is the energy and enthusiasm of people engaged in an important intellectual mission. They supported one another; they shared a common enthusiasm for discovery; and, most importantly, they did not accept the

⁴ Stanislaw Lesniewski, "On the Foundation of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1991, pp. 181 - 196.

limits of any philosophical doctrine. The tone of the Polish School was mainly a result of the influence of Kazimierz Twardowski who was the father of modern Polish philosophy. Lesniewski studied with Twardowski and, it seems, was imbued with the high standards of precision which Twardowski instilled in all of his students.

Twardowski also represented a certain moral attitude towards philosophy, which he tried to convey to his disciples. It was his belief that philosophy is the true school of the spirit because it creates important moral ideals and defends them. This fact must be very strongly emphasized because the ideological unity of the Lvov-Warsaw School consisted among other things in a very serious treatment of philosophical enquiries and the teaching of philosophy, in treating philosophy and its propagation as an intellectual and moral mission.⁵

It is easy to detect in Lesniewski's writing the tone of intellectual mission Twardowski's students shared. Twardowski succeeded in inspiring students with a variety of interests because he did not impose any definite philosophical doctrine on them.

Now Twardowski - and later his successors - did not impose upon his disciples any definite philosophical doctrine. His primary goal was to teach them above all a clear and critical way of thinking. It seems that his philosophical minimalism was an intended element of the total conception of building philosophy in Poland.⁶

Lesniewski lived in an exciting time for the logicians of the Polish School, a time when they believed that it was possible to develop with logic a "general theory of existence"⁷ or to discover through logic "unshakable and eternal truths."⁸ The excitement found at the Polish School was not the result of a

⁵ Jan Wolenski, Logic And Philosophy In The Lvov-Warsaw School, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1989, p. 5.

⁶ Ibid., p. 5.

⁷ Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 374.

⁸ Jan Lukasiewicz, "In Defence Of Logistic", Jan Lukasiewicz Selected Works, Ed. L. Borkowski, Amsterdam: North-Holland Publishing Company, 1970, p. 249.

belief that answers to the questions of philosophy are beyond the reach of man, but was partially the result of a hope that logic might provide the basis for important discoveries with sweeping implications for human knowledge. Even though Lesniewski almost completely concentrated his efforts on the development of his logical systems in order to provide a consistent logical foundation for mathematics and avoided traditional problems of philosophy, during his mature period he was still capable of writing:

...I used the name 'ontology'¹⁹ to characterize the theory I was developing, without offense to my 'linguistic instincts' because I was formulating in that theory a certain kind of 'general principles of existence'.⁹

⁹ Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 374.

Chapter II - Towards An Understanding Of Nominalism

Introduction

This Chapter is intended to build a very general background against which Lesniewski's nominalism can be analyzed.

Eberle, who characterizes Lesniewski as a contemporary nominalist, lists eight features of contemporary nominalism.

Although it is difficult to discover views common to all contemporary nominalists, the following appear to be widespread: (1) criticism of the notion of a class, in as much as a class differs from the individual whole composed of its elements, (2) a refusal to postulate the existence of an infinity of objects, (3) an aversion to treat predicates as expressions which designate non-individuals, (4) objections to the use of such entities as concepts, meanings, senses, and propositions in the theory of meaning, (5) preference for a syntax where expressions are construed as non-repeatable inscriptions, (6) efforts to reconstruct or reinterpret portions of mathematics in such a fashion that reference to numbers or classes is replaced by reference to concrete objects, individual inscriptions, or wholes which are determined by their least parts, (7) advocacy of parsimony with regard to the number of distinct categories of entities to which a theory makes reference, even at the expense of greater complexity in the construction of definitions and proofs, (8) a tendency to identify individuals, if possible, with phenomenal data or with observable macroscopic things or events. Concrete things are preferred to abstract items, actual entities to possible ones, occurrent qualities to dispositional ones, and observables to theoretical constructs.¹⁰

The characteristics Eberle chose to highlight are, coincidentally, those features of Lesniewski's logical systems which are most often cited as proof of his, Lesniewski's, nominalistic inclinations. Yet, although these characterizations accurately "define" those nominalistic features, they neither describe the origins of Lesniewski's nominalism nor do they define the scope of Lesniewski's philosophical nominalism.

¹⁰ Rolf A. Eberle, Nominalistic Systems, Dordrecht-Holland: D. Reidel Publishing Company, 1970, p. 8.

It is necessary, therefore, to develop a rough understanding of nominalism within the context of the realist/conceptualist/nominalist debate over the existence of universals. Aristotle's Theory of Universals will be summarized and will be contrasted with Lesniewski's own limited notion of universals. And, finally, several points in the nominalist/realist contention over the existence of universals will be used to form a limited critical framework against which Lesniewski's nominalistic philosophy can be judged.

1) A Definition Of Nominalism

Nominalism is sometimes defined as "the doctrine that whatever exists is a particular, and nothing but a particular."¹¹ Nominalism can also be defined negatively, if "... by nominalism we mean the view that universals do not exist."¹² These definitions do not help to explain what the nominalist takes to be a particular, a universal or what he means for something to exist. But, in a general way, one can understand from this that the nominalist wants to claim that the world is made of only one kind of thing, namely, particulars, that we can know these things, and that there are no other things, especially things called universals.

Lesniewski, as we will see in the next Chapter, Chapter III, never expressed his nominalism as a developed philosophical doctrine. As a matter of fact, once Lesniewski believed that he could construct logical systems that might provide a consistent foundation for mathematics free of the possibility of contradiction, he lost all interest in the traditional problems of philosophy. He constructed logical systems that only allow consistent "talk" about objects and he believed that he had "proven" that a certain, limited, kind of universal did

¹¹ D. M. Armstrong, Nominalism and Realism, Cambridge, England: Cambridge University Press, 1978, p. 138.

¹² Tadeusz Kotarbinski, Gnosiology. The Scientific Approach to the Theory of Knowledge, Trans. Olgierd Wojtasiewicz, Oxford, New York: Pergamon Press, 1966, p. 430.

not exist, but he did not engage in the debate over the existence of universals. For that reason, it is necessary to provide a background against which we might capture Lesniewski's philosophical beliefs and from which a clearer sense of Lesniewski's nominalism can be defined.

The realist/conceptualist/nominalist debate over the existence of universals and an example of a realistic notion of universals will provide such a background, since nominalism makes no sense outside of that debate and since that debate makes no sense apart from what it means for a universal to exist. The background I will provide makes no claim to completeness, it is purposely constructed to highlight Lesniewski's philosophical beliefs.

It is, perhaps, best to begin our effort to provide a background for an understanding of nominalism from the common origins of realism, conceptualism and nominalism . These philosophical doctrines find their roots in the beliefs of common sense realism. A common sense realist believes that things like houses, trees, and people, things which we sense, i.e. particular things or natural objects, actually exist in the world. He believes that we can use ordinary language to talk about the things that exist, to talk about our knowledge of what exists, and to talk about the correspondence between the two. The realist, the conceptualist, and the nominalist, then, attempt to give accounts of the same "world." They differ in that the realist emphasizes the correspondence between objects which actually exist in the world and our inner knowledge of those objects; the conceptualist emphasizes our inner knowledge of objects; and the nominalist emphasizes the outer world of objects.

It has been suggested that the roots of nominalism lie in the Medieval controversy over the "existence" of universals.

Medieval Realists...held that universals are real entities subsisting independent of mind and sense. *Medieval Conceptualists*...maintained that universals are real but mind-

dependent entities whose existence in the understanding may, however, be due to similarities among objects. *Medieval Nominalists*...advocated that only particular things exist. Thus, members of a species were thought to have nothing in common, except possibly the name of the species which refers to each individual of the species.¹³

Since, as we will find, Lesniewski rejects epistemology and he did not concern himself with the inner existence of objects, it is safe to eliminate further consideration of conceptualism. Even if Lesniewski believed that universals exist in mind, we will see that he rejects the realist belief that universals somehow exist in the world. For that reason, I will focus my considerations below on the realist/nominalist debate over the existence of universals.

It is clear that in order to understand nominalism, we need to know what universals are for a realist in order to understand why a nominalist might reject them.

2) Aristotle vs. Lesniewski

Aristotle's account of natural objects¹⁴ includes a description of universals. My discussion of Aristotle's Theory of Universals, therefore, will be limited to a discussion of how Aristotle might answer the following two questions:

What is a natural object?

How do we know a natural object?

Although Aristotle believed that philosophy begins and ends with an account of the common sense world - "In general ... philosophy seeks the cause of perceptible things..."¹⁵ -, he also believed that a philosopher must extend

¹³ Rolf A. Eberle, *Nominalistic Systems*, Dordrecht, Holland: D. Reidel Publishing Company, 1970, p. 4.

¹⁴ This section contains an abbreviated interpretation of Aristotle's treatment of universals and particulars. Appendix A contains a more complete treatment.

¹⁵ Aristotle, *The Complete Works Of Aristotle*, 2 Volumes, Ed. Jonathan Barnes, Princeton: Princeton University Press, 1985, p. 1568.

beyond what is merely sensed in order to provide a true account of natural objects.

... while ... thinkers were inquiring into the truth of that which is, they thought that which is was identical with the sensible world; in this, however, there is largely present the nature of the indeterminate ... and, therefore, while they speak plausibly, they do not say what is true.¹⁶

Although it is true that Aristotle uses the word "substance" in different ways, I believe that for Aristotle a natural object, a particular thing, is a combination of two "substances", that is a combination of matter and form, neither of which exist separately. He also believed that we "know" these "substances" in two distinct ways. Matter causes sensation; form causes knowledge. Form, for Aristotle, is "what is truly universal in a particular." Matter is that persistent substance which, along with form, combine to form a particular thing.

Although matter and form are what we sense and what we know directly and although we can "see" evidence of the existence of these "substances" in the sensible world - that there can be both a bronze sphere (form) and a bronze circle (form), i.e. two different forms made of identical matter, and that there can be a bronze and a wooden sphere, i.e. two identical forms made of different matter - matter and form are, in a sense, posited as substances necessary to give a more complete account of change. Although I will not treat Aristotle's account of motion, Aristotle believed that motion caused matter to "flow" from one form to another.

"The form is a kind of power in matter - a duct as it were."¹⁷

It is almost as if motion were a general force, moving matter from one form into another.

¹⁶ Ibid., p. 1594.

¹⁷ Ibid., p. 527.

What can be a bit confusing about Aristotle's account of universals, is, that while form is 'truly what is universal in a particular' and form is a "substance" we know directly in a different way from matter, the word "universal" is most frequently used to refer to the accumulation of experiences of the same form in the mind. In any case, when one knows form, 'what is universal in a particular,' one actually knows a universal. The more we experience the same form, however, the more aware of the universal we become. This accumulation is neither a different sort of knowledge nor is it knowledge of a different sort of thing than knowledge of 'what is universal in a particular.'

For Aristotle, if we reject the existence of universals, we reject not only a thing that exists differently than matter, but also the immediate object of our knowledge.

Returning to the questions raised above:

What is a natural object?

How do we know a natural object?

Aristotle finds it necessary to answer both questions simultaneously. He seems to feel that any discussion about "what is" necessarily requires a discussion of "how we know what is." He developed his theory of universals, in part, as an epistemological counterpart to his ontological belief that particulars, natural objects, actually exist in the world. If he had not, he would have been confronted by two problems: what there is, is not, simply because we know it AND what we know must be a different sort of thing than what is, otherwise existence, in a sense, would be contingent on our knowledge. In final analysis, universals are required in order to say how we know what is, since what is seems to be always changing. A particular thing, a natural object, is both form and matter. Our knowledge of form, then, provides us our knowledge

of what is similar in particulars and gives us the means to group natural objects according to their similarities. General names, then, actually denote universal substance, i.e. form.

How does Lesniewski's notion of universal fair against Aristotle's? In his earlier works Lesniewski believed that his definition of "general object" was sufficiently broad to encompass several kinds of universals.

'General objects', according to the various authors dealing with them, possess a single characteristic property and that irrespective of particular forms assumed by those objects in the different systems in which they exist either as e.g.: 'notions' in the sense of ancient and medieval 'realism', or Locke's 'general ideas', or Twardowski's 'objects of general representations', or Husserl's 'ideal' objects existing 'outside time'.¹⁸

Even though Lesniewski later eliminates his references to other philosophers, he retains the same definition of "general object," i.e. "general objects" are objects which "possess a single characteristic property ... that the object which is allegedly 'general' with respect to a group of 'individual' objects can possess only those properties which are common to all corresponding 'individual' objects."¹⁹

Even though Lesniewski's notion of "general objects" and his "proofs" against the existence of universals will be more thoroughly discussed in Chapter III, it is important to note here that he did not comment on other notions of universals which might have provided justification for his own restricted notion of what he defined as universals, i.e. "general objects". Neither his definition of "general objects" nor his "proofs" against the existence of "general objects", thrust Lesniewski into the realist/nominalist debate over the existence of universals.

¹⁸ Stanislaw Lesniewski, "The Critique of the Logical Principle of the Excluded Middle", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Szrednicki, D.I. Barnett, Trans. S. J. Surma and J. Wojcik, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 51.

¹⁹ Ibid., p. 51.

Aristotle's Theory of Universals was meant to provide an epistemological and ontological basis for a discussion of change in natural objects.

Lesniewski's logical systems are said to be ontologically neutral.

So his [Lesniewski's logical - my note] systems are logically pure and metaphysically neutral in that they do not *logically* presuppose or imply what is the population of the universe or the denotation of expressions...²⁰

As true as this characterization might be, Lesniewski does mention the existence of natural objects in his published works. It is obvious that Lesniewski would not deny the existence of quite ordinary things. One must realize, however, that Lesniewski occupied himself almost completely with the construction of a language, his logical systems, which might provide a consistent language for science and a consistent basis for mathematics; he avoided metaphysical discussion and rejected the usefulness of epistemology.

3) Realism vs. Nominalism

My intent is not to argue for or against the nominalist or realist point of view, but, in this section, to provide a critical framework in which to highlight the differences between nominalistic and realistic view of the world and, therefore, to highlight the fact that certain features of Lesniewski's logical systems are nominalistic.

D. M. Armstrong presents a critique of nominalism in his book "Nominalism and Realism." He presents the nominalist/realist controversy as follows:

Nominalists deny that there is any genuine or objective identity in things which are not identical. Realists, on the other hand, hold that the apparent situation is the real situation. There genuinely is, or can be, something identical in things which are

²⁰ Eugene C. Luschei, The Logical Systems Of Lesniewski, Amsterdam: North-Holland Publishing Company, 1962, p. 108.

not identical. Besides particulars, there are universals.²¹

For Armstrong an account of our knowledge of natural objects necessarily includes the existence of universals. The fact that we can simultaneously know things as individuals and as identical in certain respects requires the existence of something in addition to the particular. Because of that, Armstrong notes, even though "...all Nominalists agree that all things that exist are only particulars,"²² at some point in their account of natural objects, in their effort to reduce the "apparent identity in things which are not identical," they inevitably must appeal to properties, that is to universals.

All that the Nominalist can hope to do is to give a reductive analysis or account of what it is for something to have a property or to be of a certain kind or sort: a reductive analysis or account of types.²³

And,

If, then, in the course of an attempted Nominalist analysis it should happen that covert appeal is made to the notion of property, kind or type, the analysis has failed to achieve its purpose.²⁴

In his account of different kinds of nominalism (predicate nominalism, concept nominalism, class nominalism, mereological nominalism, resemblance nominalism, Ostrich or Cloak-and-dagger Nominalism), he shows that each attempt fails to explain away similarities in things because it must, in the end, either appeal to properties, that is universals, or, in the case of Ostrich or Cloak-and-dagger Nominalism, simply ignore the "apparent identity in things which are not identical."

In his general critique of nominalism, Armstrong concludes:

Ordinary thought and discourse recognizes identity both of particulars and property, sort, or kind. Indeed, without the

²¹ D. M. Armstrong, Nominalism and Realism, Cambridge, England: Cambridge University Press, 1978, p. 12.

²² Ibid., p. 12.

²³ Ibid., p. 19.

²⁴ Ibid., p. 19.

distinction between sameness of thing and sameness of property or kind, thought and discourse would be impossible.²⁵

For Armstrong, then, in order to be successful, a nominalist must be able to explain the sameness of things without appealing to properties.

D. F. Pears critiques both realism and nominalism. He believes that both nominalism and realism fail for the same reason.

Their (realism/nominalism- my note) goal is the unattainable completely satisfactory explanation of naming.²⁶

He criticizes realism in the following way:

But ultimately there must be some exit from the maze of words, and, wherever this exit is made, it will be impossible to give an informative reason except by pointing. For the only other way of giving an informative reason is to give a new word, and this would prevent the exit from the maze of words from being made at this place²⁷. Still at the place where the exit is made it is always possible to give a detailed reason like 'We are able to call things red because they are red', which is too obviously circular even to look informative. Or alternatively it is possible to say 'We are able to call things ϕ because they are ϕ ', and this is a general reason which is almost as obviously circular and uninformative. What philosophers who propose the existence of universals do is to propose a general reason which looks informative because it shifts to another level, but unfortunately is not.²⁷

He offers a similar criticism of nominalism:

The type of nominalism which says that a name is applied to a number of things which are similar immediately falls into the same circularity.²⁸

Because this

...similarity can be specified only by a backward reference to the name.²⁹

²⁵ Ibid., p. 19.

²⁶ D. F. Pears, "Universals", Universals And Particulars: Readings In Ontology, Editor Michael Loux, Garden City, New York: Anchor Books, 1970, p. 48.

²⁷ Ibid., p. 38.

²⁸ Ibid., p. 41.

²⁹ Ibid., p. 42.

Although Pears' article is critical of both nominalism and realism, at one point he criticizes nominalism in a way that seems to support the realistic assertion that universals serve an important function.

The type of nominalism which suggests that things which are called by one name have only their name in common represent the extreme of artificiality.²¹ It suggests that there are never any ways of telling even approximately whether a word is used in one sense or two senses [i.e. that there is a reason one can call two things by the same name - my note].³⁰

In another place, where he is critical of both nominalism and realism, he actually seems to support the nominalist's assertion that only particulars exist, only the natural objects of common sense realism, by claiming that naming is justified by pointing to a particular thing or a collection of particular things.

If a word is explained ostensively, then however difficult this process may be it really is explained ostensively. It is no good trying to combine the concreteness of ostensive definition with the clarity of verbal definition. Verbal definitions have such an easy task just because ostensive definitions have such a difficult task.³¹

In the following Chapters, it will become clear that Lesniewski constructed his logical systems in such a way as to avoid Pears' particular criticism of nominalistic languages. Lesniewski does not have to resort to properties to be able to "talk" about collections. In Lesniewski's logical systems, naming is analogous to pointing to a particular thing or a collection of things.

4) Remarks

In the next Chapter, Chapter III, I will explore Lesniewski's nominalism and also discuss his notion of universals. Both Armstrong's and Pears' critique should provide a loose background against which Lesniewski's own notions can

³⁰ Ibid., p. 47.

³¹ Ibid., p. 48.

be measured. If Armstrong is correct, Lesniewski can not be a successful nominalist if he appeals to properties in his account of natural objects or if properties are necessary for the success of his logical systems. Pears' critique is pertinent because Lesniewski, I believe, will be able to overcome his criticism of nominalist language in a way Pears has not considered. At this point, however, we must begin to investigate the origins of Lesniewski's rejection of universals.

Chapter III- Lesniewski's Nominalism

1) The Origins Of Lesniewski's Nominalism

The first evidence of Lesniewski's nominalism, his belief that universals do not exist, can be traced to the time when he first became interested in modern symbolic logic (1910) and, more specifically, when (1911) he first became familiar with Russell's Paradox ("class of all classes not members of (subordinate to) themselves"). Underlying his earlier and his transitional works was an intense, long-term, study (1911-1922) of Russell's Paradox and similar antinomies.

While publishing in turn the above works [His earlier works; seven papers written during the period (1911-1914) - my note], I occupied myself zealously with the 'antinomies'. From the time when in the year 1911 I began an acquaintance with them by meeting with the 'antinomies' of Russell related to the 'class [klasy] of classes not elements of themselves', and problems concerning the antinomies were the most demanding subject of my deliberations for over eleven years.³²

2) Russell's Paradox and Lesniewski's "Solution"

In *Principia Mathematica*³³ Whitehead and Russell determined that, if a class were considered the same sort of thing as one of its members, a paradox would result. The paradox, "Russell's Paradox" results when we consider whether or not "K," "a class of classes not subordinated to themselves," is subordinated to itself. Lesniewski presents the Paradox quite clearly; I follow his presentation below.

In order to understand the way in which the Russell's Paradox arises, one must make two distinctions. First, a distinction must be made between a "full class" and an "empty class." A full class, the class of all men, for example,

³² Stanislaw Lesniewski, "On The Foundations Of Mathematics, Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 199.

³³ Alfred North Whitehead and Bertrand Russell, Principia Mathematica, Vol. 1, Cambridge, England: Cambridge University Press, 1950, pp. 24, 37, and 60 - 65.

contains some individuals; an empty class, the class of all square circles, for example, contains no individuals. Secondly, one must make a distinction between classes that are and classes that are not subordinated to themselves. A class not subordinated to itself, the "class of men" for example, is not a man, while a class subordinated to itself, the class of names, for example, is a name or, to use Lesniewski's example, "the class of full classes" is also a full class and, therefore, it is subordinated to itself.

With these distinctions made, we are then prepared to ask our question: Is "K," "a class of classes not subordinated to themselves," subordinated to itself or not?

If we accept that class K is subordinated to itself, then because each class subordinated to the class K is not subordinated to itself, we reach the conclusion that class K is not subordinated to itself. Therefore a contradiction arises, for from this, that class K is subordinated to itself, it results that K is not subordinated to itself.

If we wish to avoid this contradiction, we must accept that class K is not subordinated to itself. However, if it is not subordinated to itself, then it belongs to class K, consequently it is subordinated to itself. And so here also there arises a contradiction, since from the fact that class K is not subordinated to itself, it results that it is subordinated to itself. Whichever way we turn we encounter a contradiction.³⁴

In short, Russell's Paradox results when a class is simultaneously treated as the same sort of thing as its members.

... the appearance of contradiction is produced by the presence of some word which has systematic ambiguity of type, such as ... *class* ...³⁵

Whitehead and Russell believed that a class is not one of its members.³⁶ They were not prepared to claim that a class actually exists.³⁷ Instead,

³⁴ Stanislaw Lesniewski, "On The Foundations Of Mathematics, Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 201.

³⁵ Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, Vol. 1, Cambridge, England: Cambridge University Press, 1950, p. 64.

³⁶ *Ibid.*, p. 24.

³⁷ *Ibid.*, p. 72.

Whitehead and Russell introduce class into their logical system as an "incomplete symbol":

The symbols for classes ... are incomplete symbols: their uses are defined, but they themselves are not assumed to mean anything at all.³⁸

... classes, so far as we introduce them, are merely symbolic or linguistic conveniences, not genuine objects as their members are if they are individuals.³⁹

Although Whitehead and Russell did not believe that a class really exists, they admit that they can neither prove that it exists nor prove that it does not exist. Their own belief that classes do not exist is based on arguments from "... the ancient problem of the One and the Many*."⁴⁰

Briefly, the arguments reduce to the following: If there is such an object as a class, it must be in some sense *one* object. Yet it is only of classes that *many* can be predicated. Hence, if we admit classes as objects, we must suppose that the same object can be both one and many, which seems impossible.⁴¹

In the quotation below Lesniewski quoted and criticized Whitehead and Russell's reason for believing that classes do not exist. Figure 1, below, is referenced in his comments.

Figure 1.

Despite my sincerest wish, I am unable to treat seriously the thesis which proclaims that, "if we admit classes as objects, we must suppose that the same object can be both one and many which seems impossible", as I feel in it some gross misunderstanding: even *posito* that the expression 'many' causes no doubts on the theme 'at least how many?', I can find no sense in saying about some object that it is 'many' even though, by assuming that the meaning of the expression 'many' is not uncertain with respect to quantity, that it means e.g., the same as 'at least two', I fully understand e.g., the utterance that 'many objects'

³⁸ Ibid., p. 71.

³⁹ Ibid., p. 72.

⁴⁰ Ibid., p. 72.

⁴¹ Ibid., p. 72, fn. *.

exist in the world, or even the utterance that segment AB of Fig. 1 has 'many' parts. Seeing no sense in the thesis quoted, I cannot regard it as an even slightly 'cogent' argument for anything at all in the world.⁴²

Because Whitehead and Russell can give no reason to reject or accept their notion of class, Lesniewski simply dismissed that notion.

Scenting in the 'classes' of Whitehead and Russell and in the 'extensions of concepts' of Frege, the aroma of mythical specimens from a rich gallery of 'invented' objects, I am unable to rid myself of an inclination to sympathize 'on credit' with the authors' doubts as to whether objects which are such 'classes', do exist in the world.⁴³

Even though Whitehead and Russell did not resolve the issue of the existence of classes, they embedded their notion of class in their logical system and proposed a Theory of Types to rid that system of "Russell's Paradox".⁴⁴ Lesniewski was dissatisfied with Whitehead and Russell's notion of class because it left open the possibility that a class existed as a different sort of thing than its members.

Lesniewski, on the other hand, developed Ontology and Mereology in such a way that Russell's Paradox could not appear in his logical systems because he "defined" a class as an object which, if it existed, exists as a collection of objects of the same type.

3) Lesniewski's View Of Classes

Lesniewski realized that we can talk about "class" in two ways and, in order to eliminate the possibility of "Russell's Paradox" from his logical systems, he realized that he needed to include both ways of talking about "class" in the construction of his logical systems. In the distributive sense,

⁴² Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 225 and 226.

⁴³ Ibid., pp. 224 and 225.

⁴⁴ Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, Vol. 1, Cambridge, England: Cambridge University Press, 1950, p. 60.

"class" can be used in phrases like " a is a member of class P ." In the collective sense, "class" can also be used in phrases like " P is a class of a 's."

In Ontology, Lesniewski was able to eliminate the distributive use of "class" by reducing " a is a member of class P " to " a is P " or to " a is one of the P 's," since the Axiom of Ontology specifies that both a and P are the same sort of thing.

...expressions of the form ' N is a member of the class cs ', for instance, may be regarded simply as long-winded ways of saying ' N is a c ' or ' N is one of the cs '.⁴⁵

...a Lesniewskian sentence of the form ' $a \in b$ ' is true just in case the term ' a ' is (semantically) singular, and the individual it denotes is one of the one or more individuals denoted by ' b '. I suggest that the ordinary language expression coming closest to expressing this is ' a is one of b ': we think of examples such as 'Romeo is one of the Montagues', 'John Williams is one of Sky', and so on.⁴⁶

Through the Axiom of Ontology Lesniewski was able to specify that both " a " and " P " must be the same kinds of things. Since "class" is removed from Ontology, the possibility of contradiction from the use of "class" in Ontology is eliminated.

In the axioms and definitions of Mereology, Lesniewski was able to specify that a "collective" class is a collection of things and nothing more. If we ask what a class is in Mereology, it is a collection and that is all. In order to demonstrate this, we must examine the first two axioms and the first two definitions of Mereology.

AI. $[P, Q]: P \in cz(Q) \supset Q \in N(cz(P))$.⁴⁷

⁴⁵ Peter M. Simons, "On Understanding Lesniewski", History And Philosophy Of Logic, Volume 3, 1982, p. 188.

⁴⁶ Ibid., p. 184.

⁴⁷ Srzednicki, Jan T.J. and Stachniak, Zbigniew, Ed., Lesniewski's Lecture Notes in Logic, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1988, p. 60.

AII. $[P, Q, R]: P \in cz(Q). Q \in cz(R). \supset P \in cz(R).$ ⁴⁸

D.I. $[P, Q]: P \in el(Q). \equiv P = Q. \vee P \in cz(Q)$ ⁴⁹

D.II. $[P, a]: P \in Kl(a). \equiv P \in obj.: [Q]: Q \in a. \supset Q \in el(P):.$
 $[Q]: Q \in el(P). \supset [\exists R, S]. R \in el(Q). S \in a. R \in el(S).$ ⁵⁰

Before a detailed look at Lesniewski's "collective" class, "Kl(a)" in D. II above, I would like to make three observations. First, the most basic element of Mereology is something which is designated a part, i.e. "cz", of something else. So, whoever would use Mereology to talk about the world, must first specify what is to be taken as a part. From this it must be recognized that, if anything can be "built" in Mereology, it must be built of parts. Second, a class and its members are defined as the same sort of thing and, ultimately, they must be specified in terms of things, i.e. part(s), which are in the world. Thirdly, "Kl" is not an object in D.II.. "Kl" specifies how objects "a" are to be collected.

Let us now look closely at D. II and determine exactly what "P is a class of a's", i.e. " $P \in Kl(a)$ ", means. " $P \in Kl(a)$ " is "defined" by the three conjuncts which follow the equivalence sign. " $P \in obj.$ ", the first conjunct, specifies that "P" in " $P \in Kl(a)$ " must be an object. The second conjunct, " $[Q]: Q \in a. \supset Q \in el(P)$," determines the relationship between the object "P" and the object(s) "a" in " $P \in Kl(a)$ " and can be translated into the English sentence: "If any thing is an 'a,' then that thing is 'P' or is one of the parts of 'P'." The third, the last conjunct, $[Q]: Q \in el(P). \supset [\exists R, S]. R \in el(Q). S \in a. R \in el(S)$, specifies that, "if any Q is one of the elements of P" or, another way of saying the same thing, "if any Q is P or is a part of P", "then" three conditions must be met. First, "there is some thing R which is Q or a part of Q." Second, "there is some thing S which is one of the a's." And, lastly, "the thing which is Q or part of Q, i.e. R, is the same

⁴⁸ Ibid., p. 60.

⁴⁹ Ibid., p. 60.

⁵⁰ Ibid., p. 60.

thing as the thing S (which is one of the a's) or is a part of that thing S (which is one of the a's)." In short, " $P\epsilon Kl(a)$ " could be read "P is the collection of a's" or, as Lesniewski suggests, " $P\epsilon Kl(a)$ " could also be read "P is the '... heap of objects a consisting of every a.'⁵¹"

Lesniewski believed in a quite concrete notion of class and, therefore, could reject a Theory of Types.

Expressions of the type 'class of objects a' are, on the basis of my mereology, names denoting definite and quite ordinary objects. These expressions naturally have nothing in common either with any mythology of 'classes', considered as objects of some 'higher type' or 'higher order', or with a use of the word 'class' in which the latter is not the name of any object(s), but rather a surrogate *facon de parler* of some entirely different syntactical type, as for example the system of Whitehead and Russell. The totality of theorems of my system of the foundations of mathematics, which in practice can be handled as theoretical correlates of this or that thesis of these authors' 'theory of classes', forms a proper part of my ontology.⁵²

In addition to his "definition" of class in Mereology, Lesniewski offered his objections to certain kinds of classes which were excluded from that logical system. Lesniewski did not accept the existence of empty classes which, if accepted, would necessarily have to exist apart from their members because an empty class has no members.

Being of the opinion that, if an object is the class of some a (e.g., people, points, square circles), then it actually consists of a, I always rejected, in accordance with thesis (1) on page 186, the existence of theoretical monstrosities like the class of square circles, understanding only too well that nothing can consist of something which does not even exist. There has never been a time in my life in which I would not have been in complete agreement with the lapidary remark of Frege *a propos* the

⁵¹ Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 225.

⁵² Stanislaw Lesniewski, "Introductory Remarks to the Continuation of My Article: 'Grundzuge eines neuen Systems der Grundlagen der Mathematik'", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. W. Teichmann and S. McCall, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 709 - 710.

theory of classes of Ernest Schroeder: "If" ... "a class consists of objects, is an aggregate, a collective unity of them, then it must vanish when the objects vanish. If we burn all the trees in a forest, we thereby burn the forest. Thus there can be no empty class."⁵³

Lesniewski did not believe that a class of just one object existed. A natural object, for example, may be considered a single thing, and, therefore, "is a class, whose only element is just that object," but Lesniewski insisted that even that object might contain elements (refer to Figure 1. repeated below).

Figure 1.

... I firmly reject ... the view according to which (E*) every object is a class, whose only element is just that object, because, considering e.g., segment AB of Fig. 1 and taking into account the circumstance that, from the point of view of my conception of classes (see thesis (17, Chapter II), segment AC is an element of segment AB which is in accordance with thesis (13) the class of the segments which are segment AC or segment CB, I am, of course, entitled to assert that although one and only one object is segment AB of Fig. 1, and although such being the case, in accordance with thesis (3) segment AB is also the class of segments AB, that segment is by no means the class whose only element is just that segment AB.⁵⁴

At the same time that Lesniewski eliminated the ambiguous notion of class from his logical systems, I believe that he saw a similarity between the ambiguous notion of class and the notion of a "general" object, objects which Lesniewski believed were universals and which he thought were ambiguous as well. Lesniewski, therefore, believed that he could eliminate the ambiguous notion of universals just as he had eliminated the ambiguous notion of class. If

⁵³ Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 214 and 215.

⁵⁴ Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 216.

his notion of class as a collection of objects and nothing more, eliminated "Russell's Paradox" and Whitehead and Russell's Theory of types, he believed that "general" objects, universals, could be analogously eliminated by showing that universals could not exist apart from the individuals whose common property they were supposed to possess and, therefore, that they did not exist.

4) Lesniewski's Two "Proofs" That "General" Objects Do Not Exist.

Lesniewski offered two "proofs" against the existence of "general" objects. His first "proof"⁵⁵ appeared in 1913 in an article titled "The Critique of the Logical Principle of Excluded Middle"⁵⁶ and is the only part of any earlier work not later repudiate. Tadeusz Kotarbinski includes a version of Lesniewski's first "proof," along with similar proofs of his own and others, in his book titled "Gnosiology" which was published in 1929.⁵⁷

Lesniewski's second "proof"⁵⁸ appeared during his "mature" period in 1927 in the footnote⁵⁹ of an article titled "On Russell's 'Antinomy' Concerning 'The Class of Classes Which Are Not Elements of Themselves'".⁶⁰ This article was the second of a series of papers collectively titled "On the Foundations of Mathematics." These papers were published between 1927 and 1931. Eugene Luschei includes a version of Lesniewski's second proof in his book "The Logical

⁵⁵ The wording of Lesniewski's First Proof is included as Appendix B.

⁵⁶ Stanislaw Lesniewski, "The Critique of the Logical Principle of Excluded Middle", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. S. J. Surma and J. Wojcik, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 51 through 53.

⁵⁷ Tadeusz Kotarbinski, Gnosiology, The Scientific Approach to the Theory of Knowledge, Trans. Olgierd Wojtasiewicz, Ed. G. Bidwell and C. Pinder, Oxford and New York: Pergamon Press, 1966, p. 36, (please see footnote 5).

⁵⁸ The wording of Lesniewski's Second Proof is included as Appendix C.

⁵⁹ Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 198. The proof, interestingly enough, appears where Lesniewski actually repudiates all that he wrote during his 'initial period'.

⁶⁰ *Ibid.*, pp. 198 and 199, fn. 6.

Systems Of Lesniewski" published in 1962.⁶¹ Kotarbinski also gives a summary of "the essential point" of both "proofs"⁶² and hints that Lesniewski believed that both "proofs" were essentially the same.⁶³

There are similarities between both "proofs." In both "proofs" "general" objects are defined in the same way: they "possess a single characteristic property ... that the object which is allegedly 'general' with respect to a group of 'individual' objects can possess only those properties which are common to all corresponding 'individual' objects."⁶⁴ Both "proofs" have the same form; they are both *reductio ad absurdum* "proofs."

There are also differences between the first and the second "proof." In the second "proof":

1. Lesniewski withdrew his comparison between his notion of a "general" object and different authors, e.g. Locke's, Twardowski's and Husserl's, notion of "universal" objects, that is, he eliminated his desire to "ascribe this or that opinion on the question of 'general object' to the authors mentioned" in his first "proof."

I regard my treatment as the result of careful formulation of theoretical tendencies involved, more or less explicitly, in the argumentation of opponents of the different kinds of 'universals' in various phases of their 'disputes' about them. If one takes the position that this assertion is a banal one, I would cite in defense the circumstance that exponents of 'philosophy' defend,

⁶¹ Eugene C. Luschei, The Logical Systems Of Lesniewski, North-Holland Publishing Company, 1962, footnote 21, pp. 308 through 310.

⁶² Tadeusz Kotarbinski, Gnosiology, The Scientific Approach to the Theory of Knowledge, Trans. Olgierd Wojtasiewicz, Ed. G. Bidwell and C. Pinder, Oxford and New York: Pergamon Press, 1966, p. 430. "An essential point in this borrowed proof was the assumption that each object of a given set of objects has a specific property of its own, and hence a universal can by definition neither have that property nor its negation, which runs contrary to the law of excluded middle, and indirectly contrary to the law of contradiction."

⁶³ Ibid., p. 48, fn. 5.

⁶⁴ Stanislaw Lesniewski, "The Critique of the Logical Principle of Excluded Middle", Trans. S. J. Surma and J. Wojcik, Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Kluwer Academic Publishers, 1991, p. 51.

regrettably often, positions at variance with banal assertions.⁶⁵

2. In referring back to his first "proof," Lesniewski states that he no longer believes in "features" (properties) or "relations."

At the time I wrote that passage I believed that there are in existence in this world so called features and so called relations, as two special kinds of objects, and I felt no scruples about using the expressions 'feature' and 'relations'. It is a long time since I believed in the existence of objects which are features, or in the existence of objects which are relations and now nothing induces me to believe in the existence of such objects (see: Tadeusz Kotarbinski, [1921], pp. 7-11)...⁶⁶

3. Lesniewski did not provide examples of a "general" object in his second "proof."

Even in his first "proof," Lesniewski did not enter directly into the nominalist/realist debate over the existence of universals; he, evidently, considered his "proof" sufficient evidence that things such as universals did not exist.

'General objects', according to the various authors dealing with them, possess a single characteristic property and that irrespective of particular forms assumed by those objects in the different systems in which they exist either as e.g.: 'notions' in the sense of ancient and medieval 'realism', or Locke's 'general ideas', or Twardowski's 'objects of general representations', or Husserl's 'ideal' objects existing 'outside time'.⁶⁷

Lesniewski believed that "general" objects encompassed what several philosophers believed were universals. Lesniewski seems to believe that the universals of realism are 'notions.' Is he confusing conceptualism with realism? None the less he rejects all such things!

⁶⁵ Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 199.

⁶⁶ *Ibid.*, p. 198.

⁶⁷ Stanislaw Lesniewski, "The Critique of the Logical Principle of Excluded Middle", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. S. J. Surma and J. Wojcik, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 51.

The fact that Lesniewski did not repudiate his first "proof" and actually provides a second, mature, "proof" indicates that the nominalistic motivation for the creation of his logical systems had not decreased, even though he withdrew criticism of various specific notions of "general" objects (universals). If Kotarbinski is correct, Lesniewski considered his second "proof" to be essentially the same as his first and, therefore, an analysis of the first "proof" is sufficient for the purposes of this paper.

In both "proofs" Lesniewski argued that "general" objects (which "possess a single characteristic property ... that the object which is allegedly 'general' with respect to a group of 'individual' objects can possess only those properties which are common to all corresponding 'individual' objects."⁶⁸) does not exist. His first "proof" seems to work, if we believe that an object's existence is contingent on having properties, i.e. that a thing must have a property to be an object. By definition a "general" object can have only a certain property. To paraphrase Kotarbinski, if there is no property for a "general" object to possess, then a "general" object can not exist. And that is the reasoning behind Lesniewski's "proof" which I outline below:

Lesniewski assumed that a "general" object, P_k , exists as defined above.

1) P_k exists

a) 1. Every individual object, P'_k , has a single property, e.g.

C_k , that makes it unique.

2. C_k is not common.

3. P_k , therefore, does not possess C_k .

b) 1. The individual object, P'_k , possessing C_k , can not

possess not C_k . (Reason: P'_k can possess C_k or not C_k , but not both)

⁶⁸ Ibid., p. 51.

2. "Not Ck" is not common. ("Not Ck" is not possessed at least by P'k)

3. Pk, therefore, does not possess not Ck.

4. Pk, therefore, possesses Ck.

2) Pk does not exist. (The assumption that a "general" object exists leads to a contradiction, i.e. Pk possesses and does not possess Ck. Therefore, by *reductio ad absurdum* we are entitled to negate our assumption.)

Lesniewski's "proof" seems to work if we believe two things. First, we must believe that a "general" object possesses properties in the same way as individual objects, that is, an object's existence must be contingent on the fact that it possesses properties. Second, we must believe that there is a single property that "makes" each individual individual. Also, I believe that Guido Kng is correct in his observation that Lesniewski's "proof" fails because he, Lesniewski, assumes that a 'general' object and an 'individual' object are the same type of thing, that 'general' objects are to be "...regarded to be exactly like concrete objects."⁶⁹ The problem with Lesniewski's "proof" is that, if the existence of an object is contingent on possessing a unique property, then a "general" object, by definition, could never exist because its "existence" is contingent on possessing a common property. Lesniewski's "proof" plays on the fact that "existence" and "possessing properties" are used equivocally and, therefore, it fails. Unlike the success Lesniewski experienced in eliminating the ambiguous notion of class from his logical systems, his "proof" that "general objects," universals, do not exist fails because it uses "existence" and "possessing properties" ambiguously.

⁶⁹ Guido Kng, Ontology And The Logistic Analysis Of Language, Dordrecht - Holland: D. Reidel Publishing Company, 1967, p. 104.

Additionally, Lesniewski's proof is not a fair representation of the kind of thing a realist would call a universal. In the most general sense, if P_k were taken to be a "general" object in respect to "all individuals being individual," that is, C_k is the property of being an "individual," then no realist would claim that P_k is an "individual." Just the reverse; the fact that P_k is not an "individual" object, could provide proof for a realist that P_k is a different sort of thing, namely a "general" object or a universal.

In "Is All Truth Only True Eternally Or Is It Also True Without A Beginning" Lesniewski writes about objects, properties, and the relation of inherence.⁷⁰ In the case of an "individual" object, Lesniewski believes that: objects possess properties; properties inhere in objects; and, so, therefore, properties relate to objects differently than objects relate to properties. Why does Lesniewski deny that there might be a difference between the way in which properties inhere in "individual" objects and the way properties inhere in "general" objects? It seems that Lesniewski simply insists that only "individual" objects possess properties and properties, properly speaking, can only inhere in "individual" objects.

Lesniewski's second "proof," I believe, merely shows that things defined similarly to "general" objects cannot exist in his systems. And, from Lesniewski's second "proof," it is clear that he no longer believed that he was providing a "proof" against any specific notion of a universal. Lesniewski's second "proof" demonstrates his continued belief that nominalistic logical systems could best provide a sound basis for mathematics, but his comments are evidence that he had abandoned discussion of traditional problems of

⁷⁰ Stanislaw Lesniewski, "Is All Truth Only True Eternally or Is It Also True Without Beginning", *Stanislaw Lesniewski Collected Works*, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. S. J. Surma and J. Wojcik, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 89.

philosophy to include any part in the realist/nominalist debate over the existence of universals.

5) Lesniewski's Shift In Philosophical Focus

During his "mature" period (1916 - 1939), Lesniewski repudiated his earlier, more philosophical works - except, of course, his first "proof" against the existence of universals -; he abandoned work on the traditional problems of philosophy; and he focused his efforts on the development of his logical systems. This "shift" in focus can be seen as early as 1916 when Lesniewski published the "Foundations of the General Theory of Sets,"⁷¹ a paper which forms "a bridge between Lesniewski's early and later writings."⁷² In that paper Lesniewski formally announced the new direction of his thinking.

I wish to add a few words as a preventative measure against possible critical objections from the 'philosophical' camp: that is - in my system the expressions are treated as a hypothetical-deductive system, from which it follows that, properly speaking, I assert only that those propositions which I call 'theorems' result from the propositions which I call 'axioms'. The psychic 'sources' of my axioms are my intuitions, which simply means, that I believe in the truth of my axioms, but I am unable to say why I believe, since I am not acquainted with the theory of causality. My axioms do not have a logical 'source', which simply means that these axioms do not have proofs within my system, just as in general no axioms, in the nature of things, have proofs in that system for which they are axioms. I am quite unable to answer the question, what is the 'objective value' of my axioms, nor any other similar questions, which concern the exponents of the so-called theory of knowledge - because I admit sadly and to my clear disadvantage, that despite my most sincere wishes, I am still unable to understand even one of the problems which occur in the just mentioned respectable 'science'.⁷³

⁷¹ Stanislaw Lesniewski, "Foundations of the General Theory of Sets", Stanislaw Lesniewski Collected Works, (Vol 1.), Ed. Stanislaw J. Surma, Jan T. Srzednicki, and D. I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 131 and 132.

⁷² Stanislaw Lesniewski, Stanislaw Lesniewski Collected Works, (Vol 1.), Ed. Stanislaw J. Surma, Jan T. Srzednicki, and D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. ix.

⁷³ Stanislaw Lesniewski, "Foundations of the General Theory of Sets", Stanislaw Lesniewski Collected Works, (Vol 1.), Ed. Stanislaw J. Surma, Jan T. Srzednicki, and D. I.

By abandoning work on the traditional problems of philosophy, Lesniewski never constructed a complete philosophical system. He distanced himself from the "philosophical camp" - and focused almost completely on the development of his logical systems. Sobocinski, a student of Lesniewski, recalls that Lesniewski even avoided "philosophical 'asides'" in published works.

There is an interesting contrast on this score between the two great figures on the Warsaw School of Logic, Lukasiewicz and Lesniewski. The latter was also a philosopher by training; he too moved away from philosophy and avoided even philosophical "asides" in his published work.⁷⁴

Lesniewski's philosophical tendencies, his formalism, his intuitionism, his nominalism, his pragmatism, his common-sense, his penchant for precision, all provided a loose background against which Lesniewski built his logical systems. From Lesniewski's statement above, it appears that he not only abandoned the traditional problems of philosophy, but he also found the disciplines of philosophy, i.e. ontology and epistemology, irrelevant as well.

There is no evidence that his attitude toward philosophy or towards logic changed after 1916. Lesniewski did, however, continue to believe that "general" objects, universals, do not exist.

6) Remarks

After Lesniewski's "shift" in thinking, he abandoned work on the traditional problems of philosophy. Yet, I believe, he maintained his belief that objects exist in the world and universals do not. His belief in this regard can hardly be called a philosophical doctrine. In Lesniewski's mature works there is no discussion of traditional philosophical issues; his nominalistic inclinations can only be inferred from certain metalogical statements about the

Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 130 and 131.

⁷⁴ Jan Wolenski, Logic And Philosophy In The Lvov-Warsaw School, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1989, p. 85.

construction of his logical systems, from the fact that features of his logical systems can be interpreted nominalistically, and from the few comments others have made about his philosophical beliefs.

Chapter IV - Nominalistic Features Of Lesniewski's Logical Systems.

Many authors⁷⁵ have noted that Lesniewski's nominalism is evident in the features of his logical systems. By considering together the various aspects of Lesniewski's logical system, i.e. terminological explanations, directives, the axioms and definitions of Protothetic, Ontology and Mereology, and the actual theses of Protothetic, Ontology and Mereology which Lesniewski developed, Lesniewski's nominalistic tendencies are strongly evident. In these features we can see how Lesniewski's commitment to things is expressed in his logical systems.

Logical systems can be said to be nominalistic in two ways. 1. They can be nominalistic in the way they are constructed. 2. They can be nominalistic in the way they are used.

Lesniewski constructed his logical systems as if they were objects. There are three aspects of the construction of Lesniewski's logical systems which are relevant. I will examine Lesniewski's notion of Equiformity, his Terminological Explanations, and his Directives in the first section of this chapter.

Logical systems are nominalistic in the way they are used, if their variables can be replaced by the expressions of some actual language, actual things, without the requirement that those expression denote and if, as will be discussed below in the examination of Lesniewski's quantifiers, the author chooses to interpret his quantifiers substitutionally. I believe that Lesniewski's notion of quantification meets those requirements.

1) Are Lesniewski's Logical Systems Nominalistic In Construction?

⁷⁵ Examples of authors who discuss the nominalistic features of Lesniewski's logical systems are: Rolf Eberle in "Nominalistic Systems", John T. Kearns in "The Contribution of Lesniewski", Guido Küng in "Ontology And The Logistic Analysis Of Language", Eugene C. Lushei in "The Logical Systems Of Lesniewski" and Peter M. Simons in "On Understanding Lesniewski". Complete citations are given in the Bibliography.

A) Equiformity and Terminological Explanations

Before Lesniewski introduced his directives, i.e. the rules of inference, for any of his logical systems, he introduced the axiom(s) of that system and, through the terminological explanations, built up sufficient metalanguage to be able to precisely define exactly how theses and definitions were to be introduced, that is, just how his logical systems were to be constructed. Lesniewski did not consider either the terminological explanations or the directives parts of his logical systems.

...directives [and, therefore, Terminological Explanations - my note] do not themselves belong to the system of Protothetic [Ontology, Mereology - my note] which they affect...⁷⁶

They are parts of his overall logical enterprise, however, and, as such, add considerably to the nominalistic tone of his logical systems.

Consideration of the axiom of Protothetic and several of the terminological explanations which apply to that axiom will be sufficient to provide an understanding of the way in which terminological explanations are nominalistic. Lesniewski chose Lukasiewicz's Axiom (L) not only as a basis for protothetic but also as the object used in the development of his terminological explanations for Protothetic. That Axiom is:

$$CCC\alpha C\beta\alpha CCCN\gamma C\delta N\epsilon CC\gamma C\delta\zeta CC\epsilon\delta C\epsilon\zeta\eta C\theta\eta^{77}$$

Two examples of terminological explanations will be sufficient to show that terminological explanations are independent statements about Axiom (L) and the way Axiom (L) is to be regarded. It should be noted that these

⁷⁶ Stanislaw Lesniewski, "Fundamentals Of A New System Of The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. M. P. O'Neil, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 468.

⁷⁷ Stanislaw Lesniewski, "On Definitions In The So-Called Theory Of Deduction", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. E. C. Luschei, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 631.

terminological explanations preserve the notion of collection which Lesniewski developed in Mereology. My example of a terminological explanation will be taken from Lesniewski's development of Protothetic.

Terminological explanation I. I say of object A that it is (the) complex of (the) α ⁷⁸ if and only if the following conditions are fulfilled:

- (1) A is an expression;
- (2) if any object is a word that belongs to A, then it belongs to a certain α ;
- (3) if any object B is α , any object C is α , and some word that belongs to B belongs to C, then B is the same object as C;
- (4) if any object is α , then it is an expression that belongs to A.⁷⁹

In the first terminological explanation, Lesniewski is developing a way to talk about Axiom (L) in terms of a complex of (the) α , i.e. as a collection of words.

In the second terminological explanations, Lesniewski points to a word in Axiom (L) in his explanation of what it is for one object to be the negate of another.

Terminological explanation II. I say of object A that it is (the) negate of B if and only if the following conditions are fulfilled:

- (1) A is an expression;
- (2) B is the complex of objects that are either A or the first of the words that belong to B;
- (3) B is not a word;
- (4) the first of the words that belong to B is an expression equiform to the 11th word of Axiom (L).⁷⁹

In this way, terminological explanation after terminological explanation, Lesniewski develops an entire vocabulary surrounding Axiom (L) in advance of introducing his directives.

Before proceeding to formulate the directives of this system [protothetic - my note] of the theory of deduction based on Axiom (L), I give the following series of terminological explanations of

⁷⁸ Ibid., p. 631.

⁷⁹ Ibid., p. 633.

the technical expressions peculiar to these directives.⁸⁰

Terminological Explanation II contains the word "equiform." Lesniewski refers to "an expression equiform to the 11th word of Axiom (L)." Lesniewski talks about equiformity in the following ways:

Every expression is equiform to itself.⁸¹

Two expressions equiform to each other written in two different places are never the same expression. (Not taking these facts into consideration could lead the reader to a completely wrong interpretation of my terminological explanations ...⁸²

When Lesniewski talks about equiformity, he is really talking about "seeing" similarities. It is true that Lesniewski wishes to emphasize the fact that each expression is an individual, even though it is the same shape as another expression. And, in order to do that, he has to appeal to their similar shape, a property. This appeal to similarity of shape, however, is not a fatal flaw in the construction of Lesniewski's logical systems. Terminological explanations refer to actual objects which exist, i.e. expressions, and, in order to understanding thoes similarities, one does not need to appeal to a similarity of shape.

B. Directives

Having provided a way to talk about Axiom (L), Lesniewski, then, introduced the directives for substitution, detachment and definition and for the use of quantifiers. Lesniewski believes that his directives save his logical systems (Protothetic, Ontology and Mereology) from the possibility of contradiction:

The directives represented above of the system Protothetic, together with the directives of Ontology and Mereology which I

⁸⁰ Ibid., p. 631.

⁸¹ Stanislaw Lesniewski, "Fundamentals Of A New System Of The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. M. P. O'Neil, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 470.

⁸² Ibid. p. 471.

shall deal with in further sections of this article, I consider, in all due modesty, to be but one of the numerous attempts to formalize mathematics which have been undertaken on a larger scale by various investigators since the time of Frege, and which are represented today under the 'metamathematical' banner (as it is best known) of David Hilbert. But even among those works whose purpose is to construct a foundation for mathematics I do not know of a single one which actually stipulates, in a way that causes no doubts about its interpretation, a combination of directives sufficient for the derivation of all the theses effectively admitted into its system, and which at the same time would not lead to a contradiction in one way or another not foreseen by its author.⁸³

Once specified, the directives allow the rigorous construction of each of Lesniewski's logical systems. Lesniewski viewed each "growing" logical system as a definite thing, albeit a growing thing.

As theses of this system in addition to Axiom (L) I count only those 'definitions' and 'theorems' effectively added to the system, not various other expressions that might be added according to its directives. So the extent of the expression 'thesis of this system' is by no means univocally determined in advance, but rather is conceived as 'growing' by stages. Axiom (L) is the only expression already a thesis of this system.⁸⁴

Lesniewski would never, it appears, talk about possible theses of his logical systems. The exactness of his terminological explanations and directives assured him that he was able to define precisely the growth of his systems. In understanding the development of Lesniewski's logical systems as objects, I cannot imagine a system of logic more precise or more nominalistic in construction. They are constructed by appealing only to objects.

2) Are Lesniewski's Logical Systems (Quantifiers) Used Nominalistically?

⁸³ Ibid., p. 488, fn. 86.

⁸⁴ Stanislaw Lesniewski, "On Definitions in the So-Called Theory of Deduction", Stanislaw Lesniewski Collected Works, (Vol. II.), Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. E. C. Luschei, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 635, fn. 15.

I will show that Lesniewski intended that actual expressions of any language of the appropriate semantic category could be substituted for the variables of his logical systems and, because of that, his notion of quantification is substitutional and, therefore, nominalistic.

W. V. Quine believed that Lesniewski's quantifiers are to be interpreted substitutionally and, therefore, are evidence of his nominalism.

Ruth Marcus construes quantification substitutionally, and so, less explicitly, did Lesniewski: she for reasons having to do with modal logic, he for reasons of nominalism.⁸⁵

We can explain universal quantification as true when true under all substitutions; and correspondingly for existential. Such is the course that has been favored by Lesniewski and by Ruth Marcus.⁸⁶

Quine's view of Lesniewskian quantification is based on conversations he had with Lesniewski himself.

I cannot locate an adequate statement of Stanislaw Lesniewski's philosophy of quantification in his writings; I have it from his conversations. E. C. Luschei, in *The Logical Systems of Lesniewski* (Amsterdam: North-Holland, 1962), pp. 108f, confirms my attribution but still cites no passage.⁸⁷

Lesniewski's logical systems are ontologically neutral; his "quantifiers assert only the the existence of expressions."

Once constants of a specific semantical category are introduced into the system, then quantified variables of this semantical category may also be used, since Lesniewski's quantifiers assert only the existence of expressions.⁸⁸

It may, therefore, seem contradictory to say that Lesniewski's notion of quantification provides evidence of his nominalism and, at the same time, to

⁸⁵ W. V. Quine, *The Roots Of Reference*, La Salle, Illinois: Open Court, 1973, p. 99.

⁸⁶ Willard Van Orman Quine, *Ontological Relativity and Other Essays*, New York: Columbia University Press, 1969, p. 63.

⁸⁷ *Ibid.*, p. 63, footnote 15.

⁸⁸ Küng, Guido, *Ontology And The Logistic Analysis Of Language*, Dordrecht - Holland: D. Reidel Publishing Company, 1967, pp. 123 and 124.

say that Lesniewski's logical systems are ontologically neutral. There is, however, no contradiction.

According to Quine, a nominalist, such as Lesniewski, chose substitutional quantification to avoid reference to "...indenumerable and indefinite universes ... [which - my word]...are what, in the end, give point to objectual quantification and ontology."⁸⁹ The fact that Lesniewski's logical systems are ontologically neutral, that his quantifiers assert only the existence of expressions, then, does not contradict the Quine's assertion that Lesniewski chose substitutional quantification because of his nominalism.

Luschei agrees with Quine's interpretation of Lesniewski's quantifiers.

Quantification in language_L is an ontologically innocent, strictly logical device for making generalizations, whose variables stand for (i.e., in place of) arbitrary substituents, or substitutable expressions of the same semantic category.⁹⁰

Quine describes two common "attitudes" toward quantifiers:

- 1) substitutionally conceived, the "variable is strictly a place holder for the constants that can be substituted for it. Such variables do not purport to refer to objects as values. The constants that may be substituted for them need not be names at all; they may belong to any grammatical category."⁹¹
- 2) objectually (or referentially) conceived, "the variable refers to objects of some sort as its values; and these need not even be objects each of which is separately specifiable by name or description."⁹²

He also notes:

There are not really different interpretations of the quantifiers. There are instead different views of variables - how one understands a quantifier depends on the way he regards the quantified variable.⁹³

⁸⁹ W. V. Quine, *Ontological Relativity and Other Essays*, New York and London: Columbia University Press, 1969, p. 107.

⁹⁰ Eugene C. Luschei, *The Logical Systems Of Lesniewski*, Amsterdam: North-Holland Publishing Company, 1962, p. 110.

⁹¹ Quine, W. V., *The Roots Of Reference*, La Salle, Illinois: Open Court, 1973, p. 98.

⁹² Ibid., p. 98.

⁹³ John T. Kearns, "Two Views Of Variables", *Notre Dame Journal of Formal Logic*, Volume X, Number 2, April 1969, p. 167.

Quine criticized the substitutional interpretation of quantifiers. Since he believed that Lesniewski's notion of quantification was substitutional, his criticism extended to Lesniewski's logical systems. Küng and Canty attempted to defend Lesniewski from such criticism by writing an article titled "Substitutional quantification and Lesniewskian quantifiers" in which they asserted that Lesniewski's notion of quantification was neither substitutional nor referential, but rather something else.

The controversy over the interpretation of Lesniewski's quantifiers is not yet resolved in the literature. I believe, however, that one must make a distinction between an interpretation of Lesniewski's quantifiers and Lesniewski's own thoughts on quantification. In this thesis my emphasis is on attempting to determine how Lesniewski viewed quantifiers in an effort to understand his nominalism.

I will divide my treatment of Lesniewskian quantification into two parts:

In Part I, I will answer three questions about Lesniewski's notion of quantification, in order to give our discussion a straightforward orientation:

- 1) What are quantifiers in Lesniewski's logical systems?
- 2) Are quantifiers "primitive" and in what sense are quantifiers "primitive" in Lesniewski's logical systems?
- 3) What do quantifiers do in Lesniewski's logical systems?

In Part II, I will attempt to construct a consistent account of quantification from the hints he has given us in his published works.

PART I

- 1) What are Lesniewski's quantifiers?

Lesniewski's Terminological Explanation VII, namely,

T.E. VIII [A]: A is qntf.=.

lingr (A) is vrb1.

Uingr (A) is vrb 2:

$[\exists B]. B \text{ is int}(A) \therefore$

$[B] : B \text{ is int}(A) \supset B \text{ is trm} \therefore$

$[B, C] : B \text{ is int}(A). C \text{ is int}(A). B \text{ is cnf}(C) \supset B \text{ is Id}(C)^{94}$

"describes" "quantifier" in Lesniewski's logical systems. A quantifier, according to this T.E., is a set of brackets, "[]", filled with one or more terms, e.g. "[x]", "[x y]", etc.

There are, however, a series of quotations from various works of Lesniewski that seem to indicate that he thought that quantifiers [one must remember that Lesniewski officially used only the universal quantifier] were just the brackets and not the terms contained within them.

...all or only some variables contained in the quantifier into the quantifiers in front of the left ...⁹⁵

...universal quantifier containing variable function-signs...⁹⁶

...quantifiers containing propositional variables...⁹⁷

...universal quantifiers containing propositional variables...⁹⁸

...the variables occurring in the quantifier...⁹⁹ - which Lesniewski precedes by an example similar to $[\alpha\beta \dots \kappa\lambda \dots] (f(\alpha\beta \dots \kappa\lambda \dots))$.

But there are also places where Lesniewski supports the "definition" of quantifier we found in Terminological Explanation VII, for example:

...P and Q are any meaningful propositions in Protothetic containing no variables dependent upon the quantifier.¹⁰⁰

And I have to agree with Luschei when he writes:

⁹⁴ Stanislaw Lesniewski, "Fundamentals of a New System of the Foundations of Mathematics", Stanislaw Lesniewski Collected Works, Editors, Stanislaw J. Surma, Jan T. Srzednicki, and D.I. Barnett, Trans. M. P. O'Neil, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 472 and 473.

⁹⁵ Ibid., p. 442.

⁹⁶ Ibid., p. 445.

⁹⁷ Ibid., p. 448.

⁹⁸ Ibid., p. 457.

⁹⁹ Stanislaw Lesniewski, "Introductory Remarks to the Continuation of My Article: 'Grundzuge eines neuen Systems der Grundlagen der Mathematik'", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. W. Teichmann and S. McCall, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 664 and 665.

¹⁰⁰ Ibid., p. 463.

"...A quantifier enclosing no term to bind variables would be useless..."¹⁰¹

I believe that we have given enough evidence to show what quantifiers are in Lesniewski's systems; we have developed enough of an understanding to be able to "point" to them, even though there might be the lingering suspicion that, strictly speaking, the quantifier does, somehow, seem to be just a pair of brackets

2) Are quantifiers primitive and in what way primitive?

Lesniewski thought that quantifiers were primitive.

The problem for which I present the solution is the following: is it possible to construct a system of logistic recognizing the equivalence sign as the only primitive term (in addition, of course, to the quantifiers (3)?¹⁴ 102

The primitive nature of the universal quantifier, Lesniewski "officially" used only universal quantifiers, seems different than the way in which " ϵ ," " $=$," and "part" (cz) are primitive terms for Lesniewski. " ϵ ," " $=$," and "part" (cz) are "defined," so to speak, in the way they are used in the axiom in which each first appears. The Primitive nature of universal quantifiers seems to demand, however, that we "see" some connection between an inscription within the quantifier bracket and the inscription(s) in other parts of an axiom or thesis which are equiform with that inscription in the quantifier. But Lesniewski does not clarify precisely what he means when he writes that quantifiers are primitive.

By calling quantifiers primitive, Lesniewski avoids discussion of the relationship between the quantifier and the variously shaped parentheses

¹⁰¹ Eugene C. Luschei, The Logical Systems Of Lesniewski, Amsterdam: North-Holland Publishing Company, 1962, pp. 181 and 182.

¹⁰² Stanislaw Lesniewski, "Fundamentals of a New System of the Foundations of Mathematics", Stanislaw Lesniewski Collected Works, Eds., Stanislaw J. Surma, Jan T. Srzednicki, and D.I. Barnett, Trans. M. P. O'Neil, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 419. - a quotation from Tarski's doctoral thesis, a thesis he supervised.

which surround variables. I will discuss this relationship more fully in my treatment of Lesniewski's mature notion of quantification (PART II below), but it appears as if the quantifier bracket and the variable bracket are to be seen as pointing to the same kinds of things, expressions, expressions whose "type," semantic category, is determined by the shape of the parentheses surrounding the variable.

3) What do quantifiers do?

Our third question - what do quantifiers do in Lesniewski's logical systems? - requires two answers. "Internally", quantifiers (more correctly, the variable(s) within the quantifier) "point to" or "govern" the inscription(s), the term(s), equiform with it (them) in other parts of a thesis.

Externally, quantifiers point to the actual expressions of some language of the same semantic category as indicated by the shape(s) of parentheses surrounding the variables which it governs.

The "internal" relationships between the quantifier and the variable(s) within a logical thesis is made explicit by Lesniewski's Terminological Explanations and Directives.

The directives allow no possibility of introducing into the system any kind of quantifier other than the previously mentioned universal quantifier governing any number of variables.¹⁰³

But Lesniewski's failure to carefully define the "external" relationship between the quantifier and things outside his logical systems is the source of a controversy over interpreting his quantifiers.

I believe that we can find evidence in both Lesniewski's "initial" and mature works to support my contention that Lesniewski's quantifiers range

¹⁰³ Stanislaw Lesniewski, "Introductory Remarks to the Continuation of My Article: "Grundzüge eines neuen Systems der Grundlagen der Mathematik", Stanislaw Lesniewski Collected Works, Editors, Stanislaw J. Surma, Jan T. Srzednicki, and D.I. Barnett, Trans. W. Teichmann, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 664.

over the expressions of a language which are *a priori* arranged into semantic categories, that is, his quantifiers can be interpreted substitutionally.

For hints, let us look at Lesniewski own works first.

PART II

1) Quantification In Lesniewski's "Initial Period"

Our scant knowledge of Lesniewski's notion of quantification begins with "A Class of Classes not Subordinated to Themselves"¹⁰⁴ written in his "initial period."

In the early paper [25], where the theorems are not yet given in symbolic notation, Lesniewski expressed the universal quantifier by saying "prsy kazdem znaczeniu wyrazu 'a'" ("for every meaning of the expression 'a'") and the particular quantifier by saying "przy pewnym znaczeniu wyrazu 'a'" ("for some meaning of the expression 'a'"); i.e. he explicitly referred to the meaning of the expression.¹⁰⁵

Lesniewski's uses the word "meaning" in this article and, therefore, may mislead us in believing that "meaning" is to be used in the conventional sense. I take the conventional sense of meaning to be semantic, that is the simultaneous acknowledgment of an inscription being a word which denotes or connotes or which does both. Lesniewski, during his "initial" period, believed that words both denote and connote.

There is good evidence, that even during his "initial" period, Lesniewski did not use "meaning" in the ordinary sense. I believe that it is best to think of "meaning" more syntactically, that is as any "meaningful expression," an inscription with meaning, but in the sense of an inscription being an expression in any language.

¹⁰⁴ Stanislaw Lesniewski, "Is the Class of Classes not Subordinated to Themselves, Subordinated to Itself?", Stanislaw Lesniewski Collected Works, Editors, Stanislaw J. Surma, Jan T. Srzednicki, and D.I. Barnett, Trans. S. J. Surma and J. Wojcik, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 115-128.

¹⁰⁵ Guido Kng, "The Meaning of the Quantifiers in the Logic of Lesniewski", Studia Logica, April 1977, p. 319, fn. 9.

A few examples will make Lesniewski's use of "meaning" more clear.

I shall call any object P an object subordinated to class K if given some meaning of the word 'a' the following two conditions are fulfilled: (1) K is a class of (objects) a;² (2) P is a. Examples: (A) any human being C is subordinated to the class of human beings, given that the word 'a' is employed to mean 'human being', both the above mentioned conditions are fulfilled, and namely: (1) the class of human beings is the class (of objects) a, (2) a human being C is a...¹⁰⁶

If this were our only example, we might think that "given some meaning of the word 'a'" is being used both syntactically and semantically, i.e. that we must simultaneously choose an expression to be substituted for 'a,' consider the denotation and/or connotation of that expression, and, then, determine the truth or falsity of the new expression with the denotation and/or connotation of the substituend[um] in mind. But "meaning" is not being used to indicate the denotation and/or connotation of the substituend[um]. "Meaning" announces the purely syntactic operation of substituting an expression for 'a,' that any expression is to be substituted for 'a,' which is a variable in this case.

Lesniewski assumes that expressions of a language already have a "meaning."

Our observation is supported more clearly by a second example where Lesniewski introduces the semantic notion of interpretation.

I shall call any object P' an object not subordinated to class K, is with no interpretation of the meaning of the word 'a' the following two conditions will be fulfilled together: (1) K is a class (of objects) a; (2) P' is a (that is if given every possible meaning of the word 'a' at least one of these conditions fails to be fulfilled).¹⁰⁷

The word "meaning" is still used, and still used syntactically as it was in our first example, but Lesniewski's introduction of the word "interpretation" adds the semantic dimension to his definition that one usually expects from the

¹⁰⁶ Stanislaw Lesniewski, "Is the Class of Classes not Subordinated to Themselves, Subordinated to Itself?", *Stanislaw Lesniewski Collected Works*, Editors, Stanislaw J. Surma, Jan T. Srzednicki, and D.I. Barnett, Trans. S. J. Surma and J. Wojcik, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 116.

¹⁰⁷ Ibid., p. 117.

word "meaning" alone. Lesniewski plainly intends that "meaning" be taken as an equivalent to "substitution (of an expression of any language)." I believe that Lesniewski uses "meaning" in order to assure that possible substituendum are expressions of some language; "substitution" by itself would not add the necessary connotation Lesniewski desired. That is the reason, it seems to me, that he does not use the word "substitution."

"Meaning," therefore, restricts the domain of quantification to "meaningful" expressions of some language and not, as the use of "substitution" might imply, any inscription whatsoever. Given this restriction, "meaningful" need not be taken semantically. An expression of some language need not be interpreted prior to substitution. By restricting the domain of quantification, however, Lesniewski wants guarantee that what is substituted for 'a' is an expression of some language.

Additionally, Lesniewski's parenthetical explanation in the last quotation above supports our interpretation that "meaning" is to be taken syntactically. By the use of "meaning" in that parenthetical explanation, Lesniewski is simply instructing us to substitute every possible expression for the word 'a'; "interpretation" clearly is to be taken semantically, that is, after substituting an expression for 'a' we are then to "read" the entire expression with the denotation and/or connotation of the substituend[um] in mind.

In a like manner, Lesniewski uses "meaningful" syntactically. In "...all analogous meaningful theses can be proved..."¹⁰⁸, the expression "meaningful theses" is being used syntactically, i.e. as an alternative expression for "well-formed formula."

¹⁰⁸ Stanislaw Lesniewski, "Fundamentals of a New System of the Foundations of Mathematics", Stanislaw Lesniewski Collected Works, Eds., Stanislaw J. Surma, Jan T. Srzednicki, and D.I. Barnett, Trans. M. P. O'Neil, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 450.

And he uses "meaningful" syntactically in "...P and Q are any meaningful propositions in Protothetic..."¹⁰⁹

I could provide further support from Lesniewski's earlier works that "meaning" is to be taken syntactically, that Lesniewski's notion of "quantification" is substitutional over a domain of expressions of some language, but I believe that one more example will be sufficient. Lesniewski writes:

Substituting for the word 'a' its new meaning, that is the expression 'a non-universe object', we obtain ...¹¹⁰

The reading of this quote, and many others like it in the same article, makes sense only if read syntactically. The "new meaning" of 'a' in this quotation is the expression 'a non-universe object.' The use of the word "substituting" along with "meaning" adds significantly to my interpretation. "New meaning" is merely an indication that Lesniewski is "substituting" another expression for the old "meaning," the old expression.

Lesniewski also emphasizes the straight forward syntactic exchange of expressions in his use of "meaning" through his use of single quotes. Single quotes "mark" the variable "space" into which one expression is to be substituted for another, i.e. one 'meaning' substituted for another 'meaning.' At this point in the article, interpretation is not relevant. Lesniewski's treatment of possible substituendum is isolated from any notion of denotation and/or connotation of the expressions which are substituted.

Over and over in this article, Lesniewski uses the word "meaning" to announce the syntactic operation of substituting expressions, that is to alert

¹⁰⁹ Ibid., p. 463.

¹¹⁰ Stanislaw Lesniewski, "Is the Class of Classes not Subordinated to Themselves, Subordinated to Itself?", Stanislaw Lesniewski Collected Works, Editors, Stanislaw J. Surma, Jan T. Srzednicki, and D.I. Barnett, Trans. S. J. Surma and J. Wojcik, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 122

us to the fact that some expression can be substituted for another. When we are to pay attention to the denotation or connotation of a substituend(um), he either uses the word "interpretation" or some another word like "compare." He consistently maintains the distinction between these two operations.

2) Lesniewski's Mature Notion of Quantification.

I will show that Lesniewski's ideas about quantification in his mature period are essentially unchanged from his "initial" period. His quantifiers continue to range over the expressions of any language; he did not concern himself with the "meaning" of the substituend(um) as long as it had "meaning", i.e. was an expression of some language. After Lesniewski adopted Husserl's notion of semantic categories and incorporated that notion into his own ideas about quantification, Lesniewski lost interest in the actual "meaning" of an expression; the fact that language includes only those inscriptions with "meaning" and the fact that semantic categories *a priori* order those expressions into groups to which reference can be in a logical system, released Lesniewski from concern about semantics. He increasingly focused his attention on the syntactical development of his logical systems.

Lesniewski, it should be mentioned, used conventional truth tables to provide a formal semantics for Protothetic. Peter M. Simons provides an interesting formal semantics for Lesniewski's Ontology.¹¹¹ I assume that Simons' ideas could be extended to Lesniewski's Mereology, but Mereology, according to some commentators,¹¹² is not, strictly speaking, a logical system.

¹¹¹ Peter M. Simons, "A Semantics for Ontology", *Dialectica*, Vol. 39, No. 3 (1985), pp. 193 - 216.

¹¹² John T. Kearns, "The Contribution Of Lesniewski", *Notre Dame Journal of Formal Logic*, Volume VIII, Nos. 1&2, (April, 1967), p. 65. "...only Protothetic and Ontology are, properly speaking, logical systems. The constants introduced by Mereology are terms used for relating objects in the world; they do not affect the grammatical structure of language (in contrast, the fundamental terms of a logical system are terms that bring with them a new grammatical-syntactical structure)."

[Mereology is not considered a logical system because it is built on axioms and definitions which contain non logical constants, e.g. "cz" and "Kl."]

Lesniewski's first mention of quantification in his mature period is, oddly enough, the place where he refers to his use of "quantifiers" in his "initial" period.

(at the time I did not yet know how to operate with quantifiers,¹⁴ and requiring in the colloquial language which I was using, some equivalents of expressions of the type $(\exists a) \cdot f(a)$, $(\exists X, a) \cdot f(X, a)$, etc. familiar in 'symbolic' language,¹⁵ I used corresponding expression of the type 'for some meanings of expressions 'X' and 'a f(X, a)' and the like, treating these complicated expressions in the same way in practice, mutatis mutandis, as one would treat the corresponding expressions of the types $(\exists a) \cdot f(a)$, $(\exists X, a)$, etc.¹¹³

This quotation is interesting, not only because it reflects Lesniewski's mature thoughts on quantification, but also because, in footnote 14 (quoted immediately below), he references and adopts aspects of Peirce's notion of quantifiers.

See: C. S. Peirce, On the Algebra of Logic, 'A contribution to the Philosophy of Notation', American Journal of Mathematics, VII, 1885, p. 197.¹¹⁴

Pierce indexed quantifiers and variables in order to indicate the universe of discourse which those quantifiers and variables referenced. Lesniewski constructed his logical systems in a way to avoid indexing either his quantifiers or his variables. A more complete treatment of this footnote to Pierce, though important because it supports my contention that Lesniewski viewed quantification substitutionally, would interrupt the flow of this section and can be, therefore, found in Appendix D.

¹¹³ Stanislaw Lesniewski, "On The Foundations Of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 203.

¹¹⁴ Ibid., p. 203.

There is nothing in the quotation above which would lead us to believe that Lesniewski's "mature" notion of quantification has changed from that of his "initial period." As a matter of fact, by making reference to an earlier work and by not challenging the views on quantification he held there, Lesniewski, himself, supports my assertion that his notion of quantification has not changed. And, if Lesniewski had changed his notion of quantification, this would have been the most appropriate place to describe that change. Instead, Lesniewski provides a symbolically equivalent way of expressing a notion of quantification which he was only able to express colloquially during his "initial" period.

In Lesniewski's published works, I have found nothing to indicate that Lesniewski changed his mind about quantifiers. I have found nothing in the works of his students to indicate that Lesniewski thought about quantifiers in a way other than I have presented above. There is no evidence that Lesniewski presented an interpretation of quantification different from what we find in his published works, especially when we take into account Peirce's influence.

It was Lesniewski's habit to pay particular attention to the use of symbols in logical systems. We need only recall Lesniewski's treatment of Whitehead and Russell's use of the assertion sign in "*Principia Mathematica*" to remind ourselves of the weight Lesniewski placed on such detail. Lesniewski advised "careful readers" of "*Principia*" to ignore the assertion sign because any attempt to interpret the sign creates so many alternative interpretations that the issue of interpretation cannot be clearly decided.

The variety of interpretations

... illustrates very well the doubts of a semantic nature...
... which can repel an interest in 'logistic' for a considerable
number of scientific workers who are not satisfied just by the

delights of writing signs and the transformation of formulas, and who - in contrast to the devotees of empty mathematics (and such occur) - wish to understand the meaning of the transformed formulas and to know 'of what' and 'what', respectively, 'of what' and 'what' it is wished to assert by means of those formulas.¹¹⁵

It is difficult to believe that Lesniewski would have introduced an obscure or unique notion of quantification into his logical systems without carefully preparing a written justification, without having given lectures on quantification, without some evidence of that unique notion appearing in the works of his students.

From Lesniewski's point of view, then, quantifiers refer to the expressions of actual languages and, therefore, the expressions of these actual languages are the things which may be substituted for the variables of his logical systems.

3) Remarks

The discussion of certain features of Lesniewski's logical systems in this Chapter, along with the brief discussion of some of the axioms and definitions of Ontology and Mereology found in Chapters II and III above, it is clear that Lesniewski did construct his logical systems in order to avoid "talk" about things other than quite ordinary objects. In that respect, Lesniewski constructed his logical systems to accommodate his nominalistic tendencies. That he did not develop a philosophical doctrine to support the construction of his logical systems does not diminish his nominalistic motivation.

Direct knowledge of Lesniewski's nominalism is limited to those issues discussed in the preceding Chapters. There is, however, additional, but indirect evidence of his nominalism which will be discussed in the next chapter.

¹¹⁵ Ibid., pp. 194 and 195.

Chapter V - Additional Considerations.

Introduction:

As we move toward a characterization of Lesniewski's nominalism, several additional issues should be addressed.

I believe that it is necessary to address the fact that Lesniewski never characterized himself a nominalist. I will examine Lesniewski's formalism and his intuitionism in the first section of this Chapter. These considerations are speculative but necessary.

I also believe that no characterization of Lesniewski is complete without a mention of Tadeusz Kotarbinski, a close friend of Lesniewski's who can provide several important insights into Lesniewski's mature thinking on several topics pertinent to Lesniewski's nominalism. I will consider those topics in the second section of this Chapter.

In the last section of this Chapter, I will consider several recent characterizations of Lesniewski's nominalism which appear to be exaggerations.

1) Lesniewski's Formalism and Intuitionism

Lesniewski's beliefs about what exists in the world and the way in which he constructed his logical systems are nominalistic, but he never called himself a nominalist. He characterizes himself quite differently?

I see no contradiction, therefore, in saying that I advocate a rather radical 'formalism' in the construction of my system even though I am an obdurate 'intuitionist'.¹¹⁶

W. V. Quine, I believe, provides a reason that Lesniewski might call himself a formalist and rather than a nominalist.

¹¹⁶ Stanislaw Lesniewski, "Fundamentals of a New System of the Foundations of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. M. P. O'Neil, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 487.

The three main medieval points of view regarding universals are designated by historians as *realism*, *conceptualism*, and *nominalism*. Essentially these same three doctrines reappear in twentieth-century surveys of the philosophy of mathematics under the new names *logicism*, *intuitionism*, and *formalism*.¹¹⁷

Quine believes that formalists "...like the nominalists of old, object to admitting abstract entities at all, even in the restrained sense of mind-made entities."¹¹⁸ Lesniewski's philosophical inclinations fit Quine's description quite well.

The other half of the characterization - the fact that Lesniewski also calls himself an intuitionist - is not as easy to deal with. For Quine the modern intuitionist shares a belief with his medieval counterpart, the conceptualist, that universals exist in the mind. Lesniewski does not believe that universals exist in the world and he never tells us whether or not he believes they exist in the mind. Lesniewski's own characterization of his intuition is, most likely, a better guide to what he means by intuition.

...in my system the expressions are treated as a hypothetical-deductive system, from which it follows that, properly speaking, I assert only that those propositions which I call 'theorems' result from the propositions which I call 'axioms'. The psychic 'sources' of my axioms are my intuitions, which simply means, that I believe in the truth of my axioms, but I am unable to say why I believe, since I am not acquainted with the theory of causality.¹¹⁹

Although Lesniewski wrote this in 1916, there is no reason to believe that his attitude changed. Accompanying his "shift" in focus was a shift in attitude. He wrote nothing in his "mature" period to defend his intuition; he simply developed his logical systems.

¹¹⁷ Willard Van Orman Quine, "On What There Is", Universals And Particulars: Reading In Ontology, Editor Michael J. Loux, Garden City, New York: Doubleday & Company, 1970, p. 28.

¹¹⁸ Ibid., p. 29.

¹¹⁹ Stanislaw Lesniewski, "Foundations of the General Theory of Sets.", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 130 and 131.

Although Jan Lukasiewicz may not have had Lesniewski in mind, he makes an important observation about logicians of the Polish School Of Logic.

While in practice they adopted the nominalistic standpoint, the logicians, as far as I see, have not yet discussed nominalism thoroughly enough as a philosophical doctrine.¹²⁰

And, he also observed, that:

...while we use nominalistic terminology, we are not true nominalists but incline toward some unanalyzed conceptualism or even idealism.¹²¹

2) Evidence of Lesniewski's Nominalism in the Works of Tadeusz Kotarbinski.

Tadeusz Kotarbinski offers three valuable insights into Lesniewski's thinking during his "mature" period. First, Kotarbinski confirms the fact that Lesniewski abandoned the notion of connotation. Secondly, Kotarbinski provides evidence about Lesniewski's thoughts on epistemology. Thirdly, Kotarbinski accepted Lesniewski's "proofs" as proof that universals did not exist. I believe that one must understand the close relationship which existed between these two men in order to appreciate the importance of Kotarbinski's comments on Lesniewski's thinking.

Kotarbinski and Lesniewski had remarkably similar academic careers and, interestingly enough, were even born in the same year, 1886. They both received their Ph. D. in 1912 from Lvov University; they both studied with Kasimir Twardowski at Lvov University; they both spent the war years (1915-1918) in Moscow; they were both given teaching post at Warsaw University after the war - Kotarbinski in 1918, Lesniewski in 1919; they both began to publish works that defined the direction of their mature philosophy in the late

¹²⁰ Jan Lukasiewicz, Jan Lukasiewicz Selected Works, Editor L. Borkowski, North-Holland Publishing Co., 1970, p. 223.

¹²¹ Jan Lukasiewicz, "Logistic And Philosophy", Jan Lukasiewicz Selected Works, Amsterdam: North-Holland Publishing Company, 1970, p. 224.

1920's - that is when Kotarbinski published "Gnosiology" and Lesniewski began publication of "On The Foundations Of Mathematics;" they both taught at Warsaw University until the beginning of the Second World War. Their association, unfortunately, ended with Lesniewski's early death in 1939.

Their respect was mutual. In the preface to the first edition of "Gnosiology," Kotarbinski acknowledges his intellectual debt to Lesniewski as follows:

But certainly I have learned most from Professor Stanislaw Lesniewski. I explicitly indicate this in the various places in the text, but those indications pertain only to the most important points. But I admit that all my reflections have been imbued with the influence of that extraordinary intellect of whose priceless gifts favourable fortune has enabled me to partake for many years in almost daily contact. I am decidedly a pupil of Professor Lesniewski whom I have the pleasure here of thanking cordially and respectfully for all he has ever taught me.¹²²

Lesniewski returns the compliment in "On The Foundations Of Mathematics" indirectly by quoting lengthy sections from "Gnosiology," sections which he offers as accurate statements of his own thinking, and directly as follows:

On the part of one of those authors, Dr. Tadeusz Kotarbinski, Professor of Philosophy in the University of Warsaw, my dear friend and colleague from student years - my ontology has met with a systematic and favourable scientific support from the time of its birth. Judging favourably the theory which I had concocted, Kotarbinski introduced its elements into his lectures at the university for a number of years, and also expressed himself decidedly in its favour in his new work entitled 'Elements of the Theory of Knowledge, Formal Logic and Methodology' as is clear from the section which, full of pride, I introduce shortly. - From the remote period of our common 'philosophical' past when each one of us, pursuing our various aims, was straying along blind alleys in semantics and theories of 'truth', whether in a pilgrimage to the unpromising land of 'free activity' or in

¹²² Tadeusz Kotarbinski, Gnosiology, The Scientific Approach to the Theory of Knowledge, Trans. Olgierd Wojtasiewicz, Ed. G. Bidwell and C. Pinder, Oxford and New York: Pergamon Press, 1966, p. xii.

disorganized flight from the formidable spectre of Epimenides' - The Liar and other fearsome creatures belching contradictions, I became accustomed to check my various ideas and theoretical projects in scientific discussions with Tadeusz Kotarbinski: I availed myself on various occasions of his subtle analytical help; I constantly referred to his sharp insights during the establishment of various assumptions in the different deductive theories which I was constructing; I listened to his relevant and fair critical observations and felt concerned whenever I deviated too much from his theoretical conceptions of my own views. The most sincere joy fills my heart in the light of the fact that, as far as my ontology is concerned, I have established right to claim that in Tadeusz Kotarbinski I have a learned ally.¹²³

A) Lesniewski abandoned the notion of connotation.

Kotarbinski's reference to Lesniewski's thoughts on connotation are straightforward.

While concretism owed its birth to the renunciation of the idea that properties inhere in things, Lesniewski freed himself from the concept of connotation in the interpretation of the truth of a singular sentence with the subject - copula - subjective complement structure. Under Mill's influence, he used to state that such a sentence is true if the object denoted by the subject has the property connoted by the subjective complement. Reconsideration of the problem led him, however, to a formula free from "connotation" and hence also from "property". That formula states that a singular sentence with the subject - copula - subjective complement structure (of the type "A is B") is true, if the object denoted by the subject is the object denoted by the subjective complement.¹²⁴

B) Lesniewski believed in "immanent images."

Lesniewski does not discuss the "existence of immanent images"¹²⁵ in his published works, therefore, Kotarbinski provides us an important insight to Lesniewski's mature epistemology. When Kotarbinski discusses "immanent images," he writes:

¹²³ Stanislaw Lesniewski, "On the Foundations of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 371-373.

¹²⁴ Tadeusz Kotarbinski, Gnosiology. The Scientific Approach to the Theory of Knowledge, Trans. Olgierd Wojtasiewicz, Ed. G. Bidwell and C. Pinder, Oxford and New York: Pergamon Press, 1966, p. 430.

¹²⁵ *Ibid.*, p. 430.

Now reism (for Kotarbinski, reism is the belief that objects are as we perceive them in the world. - my note.) dared to storm the stronghold of that doctrine [the doctrine that "immanent images" exist - my note], without availing itself of Lesniewski's assistance, since that assistance was in this matter refused.¹²⁶

What are immanent images?

Lesniewski, evidently, believed that for us to determine the truth or falsity of the existence of something in the world: first, something in the world must exist; simultaneously, if we look at that something we have an image of it, an immanent image, and, therefore, that that immanent image exists and; then, there is a correspondence between the existent thing in the world and our immanent image of it. Knowledge of the world, therefore, is contingent on our having images of things in the world and our being able to compare the immanent image with a particular thing in the world. This comparison or realization that we have an image of something in the world seems to take place simultaneously with "seeing" a thing in the world. For the radical realist, Kotarbinski, immanent images do not exist, so the existence or non-existence of a thing in the world is not contingent on our image of them.¹²⁷

What does this have to do with Lesniewski's nominalism? Would it be consistent for a nominalist to believe in immanent images? From Kotarbinski's discussion it is difficult to say. Immanent images do not seem like universals or a type of abstract entity. It is possible, of course, but not at all certain, that Lesniewski believed that immanent images were things, but we do not know. The importance of Kotarbinski's reference to Lesniewski's belief in immanent images is that we are given a glimpse of Lesniewski's epistemology, however slight, which Lesniewski, himself, does not offer in his published works.

¹²⁶ Ibid. p. 431.

¹²⁷ Ibid., p. 85.

C) Lesniewski Proved That Universals Do Not Exist

Kotarbinski, at the time of his life when he wrote "Gnosiology", was an enthusiastic extreme nominalist, though later in his life he viewed nominalism as just another interesting hypothesis. As we have learned from our brief sketch of Twardowski's influence on the Polish School, there was a great tolerance, even enthusiastic support, for a wide divergence of philosophical beliefs amongst the philosophers of the Polish School. There is no reason to believe that Lesniewski embraced Kotarbinski's beliefs, any more than we can burden Kotarbinski with Lesniewski's beliefs, however, Kotarbinski did believe that Lesniewski proved that universals did not exist.

...universals, are the first to fall victim to eliminating analysis as carried out by nominalism; it might be said it was only concerning them that nominalism succeeded in convincingly proving their non-existence by a *reductio ad absurdum*.¹²⁸

Now, Lesniewski provided proofs, i.e. his first "proof" and his second "proof" against the existence of "general" objects. It is important to realize that Kotarbinski did not call Lesniewski a nominalist in "Gnosiology".

3) Recent Characterizations of Lesniewski's Nominalism.

Most authors who discuss Lesniewski's nominalism mention the nominalistic construction and interpretation of his logical systems and the fact that he offered "proofs" against the existence of "general" objects. More recent characterizations, however, make it appear as if Lesniewski actually developed a philosophical doctrine to support his nominalistic tendencies.

Peter Simons in his 1985 article "A Semantics for Ontology" writes:

...Lesniewski was a staunch nominalist. Not only did he not believe in the existence of abstract entities, but, more than any other logician, he went out of his way to avoid doing logic *as if* there were such entities.¹²⁹

¹²⁸ Ibid., p. 55.

¹²⁹ Peter M. Simons, "A Semantics for Ontology", *Dialectica*, Vol. 39, No. 3 (1985), p. 195.

And Jan Wolenski in his 1989 book "Logic And Philosophy In The Lvov-Warsaw School" writes:

Lesniewski was a declared nominalist.¹³⁰

These characterizations seem to imply that Lesniewski developed a nominalistic philosophy. The fact that Lesniewski withdrew his criticism of specific notions of universals by the time he developed his second "proof" and the fact that his logical systems are ontologically neutral, seem to contradict such strong characterizations. Lesniewski's philosophical tendencies support a limited sort of nominalism.

¹³⁰ Jan Wolenski, *Logic And Philosophy In The Lvov-Warsaw School*, Kluwer Academic Publishers, 1989, p. 194.

Summary and Conclusion:

If a nominalist believes that particular things exist, then Lesniewski's beliefs do support that aspect of nominalism. In Lesniewski's published works and in accounts of his philosophical attitudes, one can say he sympathized with those who believed that the world was populated by quite ordinary objects and nothing more. It seems clear from his belief in immanent images and his belief that language can be used to denote things, that he also believed that there is some sort of a correspondence between objects in the world and the language we use to talk about the world and, therefore, he believed that there is some sort of a correspondence between mind and the world. From this, we can infer that Lesniewski believed that we can know things which exist in the world, even though he rejected epistemology. He did not, however, develop a philosophical doctrine to support these beliefs. As a matter of fact, Lesniewski abandoned all discussion of the traditional issues of philosophy.

If a nominalist believes that only particular things exist, then Lesniewski's "proofs" against the existence of "general objects," things of a different sort than individual objects, can be considered qualified support of that aspect of nominalism. Lesniewski, however, did not enter the realist/nominalist debate over the existence of universals, so his "proofs," even if they were valid, are "proofs" against the existence of one kind of universal.

Lesniewski's nominalism, then, is a highly qualified sort. Beyond the philosophical inclinations mentioned, his nominalism must be inferred from the way in which he constructed his logical systems and in the ways in which those systems are interpreted.

Lesniewski constructed his logical systems very precisely in order to avoid the possibility of contradiction and, in doing so, he treated the theses of his systems like particular things and he believed that his quantifiers actually

"pointed" to the expressions of actually existing languages. Although his logical systems are said to be ontologically neutral, that is hardly support for a nominalist belief that particular things actually exist in the world, it seems that he believed that his notion of quantification allowed him to avoid reference to things other than particular things, if, in fact, particular things exist. The kind of nominalism we can infer from the construction and interpretation of his logical systems, then, is as highly qualified as the nominalism which his philosophical inclinations support.

In the final analysis, it is best to say that Lesniewski was a philosopher who created formal language with which one could consistently "talk" about objects. A more complete understanding of his philosophy is available only to those who would use and expand his logical systems.

Appendix A.

Aristotle's Theory Of Universals Applied To Natural Objects.

For Aristotle a natural object is a particular and a "substance" which is a composite of two different "substances", i.e. matter (substratum) and form, neither of which exists separately. So, matter and form both actually exist in a natural object (in a particular) as two different kinds of "substance" and not as aspects of the particular. Aristotle, therefore, rejected a kind of nominalism (fn. 329-330), i.e. what exists is only matter, and he also rejected a kind of universalism (fn. 329-330), i.e. what exists is only form. Actually, Aristotle's realism might be viewed as an effort to unite nominalism and universalism, since he believed that a natural object is a combination of the 'particulars' (individuals) of nominalism and the 'formed' (universals) of universalism.

There are three kinds of substance - the matter, which is a 'this' by being perceived (for all things that are characterized by contact and not by organic unity are matter and substratum); the nature, a 'this' and a state that it moves towards; and again, thirdly, the particular substance which is composed of these two, e.g. Socrates or Callias.¹³¹

Aristotle also believed that we know matter and form - and, therefore, the composite, the natural object or particular - in two different ways. Form is known by the thinking part of the soul.

The thinking part of the soul must therefore be, while impassible, capable of receiving the form of an object; that is, must be potentially identical in character with its object without being the object. Thought must be related to what is thinkable, as sense is to what is sensible.¹³²

Matter is "known" through sensation.

Since it seems that there is nothing outside and separate in existence from sensible spacial magnitudes, the objects of thought are in sensible forms, viz. both the abstract objects and

¹³¹ Aristotle, The Complete Works Of Aristotle, 2 Volumes, ed. Jonathan Barnes, Princeton: Princeton University Press, 1985, p. 1690.

¹³² *Ibid.*, p. 682.

all the states and affections of sensible things. Hence no one can learn or understand anything in absence of sense, and when the mind is actively aware of anything it is necessarily aware of it along with an image; for images are like sensuous contents except in that they contain no matter.¹³³

The relationship between what is known, i.e. matter and form, and the way we know these things, sensation and thinking, must be carefully considered because Aristotle, at times, seems to blur the distinction between the outer, i.e. what is known, and the inner, i.e. how we know, it appears as if what is universal, that is, form, is really in the mind and not a substance in the natural object or particular.

... the objects that excite the sensory powers to activity, the seen, the heard, etc., are outside. The ground of this difference is that what actual sensation apprehends is individuals, while what knowledge apprehends is universals, and these are in a sense within the soul itself. That is why a man can think when he wants to but his sensation does not depend upon himself - a sensible object must be there. A similar statement must be made about our knowledge of what is sensible - on the same ground, viz. that the sensible objects are individual and external.¹³⁴

And when Aristotle considers particulars and universals from a linguistic point of view, the universal seems to be a general term existing only in the mind. This appears to confirm Aristotle's observation that "... universals ... are in a sense within the soul itself."

Now of actual things some are universal, other particulars (I call universal that which is by its nature predicated of a number of things, and particular that which is not; man, for instance, is a universal, Callias a particular).¹³⁵

Yet Aristotle clearly writes that:

... no universal exists apart from the individuals.¹³⁶

What Aristotle fails to make absolutely explicit is the difference between 'what is universal in a particular', i.e. form, that is something which we know directly

¹³³ Ibid., p. 686.

¹³⁴ Ibid., p. 664.

¹³⁵ Ibid., p. 27.

¹³⁶ Ibid., p. 1643.

in thinking, and our knowledge of form as it exists in our mind (in the soul). Knowing form in a particular produces knowledge of "what is universal in that particular", to be sure, but, when we come to know the identical form in many particulars, our knowledge is of the same universal is now a mixture of the immediate knowledge of a natural thing with that specific form and our knowledge (memory) of past experiences of that form.

Psychology was in its infancy in Aristotle's time, however, he did acknowledge the role of memory on producing an experience:

... from memory experience is produced in men; for many memories of the same thing produce finally the capacity for a single experience.¹³⁷

Aristotle believed that knowledge of a universal is prior to "knowledge" of particulars; his order of knowledge is quite advanced.

The natural way of doing this is to start from the things which are more knowable and clear to us and proceed towards those which are clearer and more knowable by nature; for the same things are not knowable relatively to us and knowable without qualification. So we must follow this method and advance from what is more obscure by nature, but clearer to us, towards what is more clear and more knowable by nature.

"Now what is plain and clear at first is rather confused masses, the elements and principles of which become known to us later by analysis. Thus we must advance from universals to particulars; for it is a whole that is more knowable to sense-perception, and a universal is a kind of whole, comprehending many things within it, like parts. Much the same thing happens in the relation of the name to the formula. A name, e.g. 'circle', means vaguely a sort of whole: its definition analyses this into particulars. Similarly a child begins by calling all men father, and all women mother, but later on distinguishes each of them."¹³⁸

His observations of how we come to know natural objects is very sophisticated.

The way we first learn about the world is not the same as the way we learn about the world once habits of observation are set and we begin to "see" particular things and we come to appreciate more how are knowledge of

¹³⁷ *Ibid.*, p. 1582.

¹³⁸ *Ibid.*, p. 315.

universals contributes to our knowledge of universals. A child may call all spherical things "ball," until he learns to distinguish between one spherical thing and another.

Conclusion:

Aristotle believed that natural objects were composite of two different kinds of things. This belief is spread throughout his written works and he discusses particulars and universals from many points of view, so that the language he uses varies, sometimes slightly, sometimes more radically, in the way he treats these two primitive things. Sometimes, as for example in his treatment of "substance", he reaches conclusions indirectly almost deductively; matter and form almost seem to be hypotheses.

And, in general, if only the sensible exists, there would be nothing if animate things were not; for there would be no faculty of sense [i.e. faculty of sense in an animate object - my note]. The view that neither the objects of sensation nor the sensations would exist [i.e. if there were no sensible things - my note] is doubtless true (for they are affections of the perceiver), but that the substrata which cause the sensation should not exist even apart from sensation is impossible. For sensation is surely not the sensation of itself, but there is something beyond the sensation, which must be prior to the sensation; for that which moves is prior in nature to that which is moved,...¹³⁹

The underlying nature can be known by analogy. For as the bronze is to the statue, the wood to the bed, or the matter and the formless before receiving form to anything which has form, so is the underlying nature to substance, i.e. the 'this' or existent.¹⁴⁰

The matter comes to be and ceases to be in one sense, while in another it does not. As that which contains the privation, it ceases to be in its own nature; for what ceases to be - the privation - is contained within it. But as potentiality it does not cease to be in its own nature, but is necessarily outside the sphere of becoming and ceasing to be. For if it came to be, something must have existed as a primary substratum from which it should come and which should persist in it; but this is its own very nature, so that it will be before coming to be. (For my definition of matter is just this - the primary substratum of each

¹³⁹ Ibid., p. 1596.

¹⁴⁰ Ibid., p. 326.

thing, from which it comes to be, and which persists in the result, not accidentally.)¹⁴¹

He also gives examples which make it clear that he derives the existence of matter and from directly from evidence provided through the observation of natural objects. I summarize: That a sphere (form) and a circle (form) can both be bronze demonstrates for Aristotle that different forms can be made of the same kind of matter and that a sphere can be both bronze (matter) and wood (matter) demonstrates that one form can be present in different matter; these facts provide evidence for Aristotle that natural objects are a combination of both form and matter.

It is clear that Aristotle's "Theory of Universals" is an hypothesis meant to give an account of the perceived stability and the instability of natural objects. For the purposes of this thesis, it has been sufficient to detail the way in which Aristotle regarded natural objects. In our further discussions of nominalism, it will be useful to have an exact account of what Aristotle believed to exist and how he believed we come to know these existents.

¹⁴¹ Ibid., p. 328.

Appendix B.

Lesniewski's First Proof:

"In order to prove the thesis that no object is 'general', I shall argue by *reductio ad absurdum*. I assume that there is an object P_k which is 'general' and it corresponds to 'individual' objects $P'_1, P'_2, P'_3, \dots, P'_n$; for every 'individual' object' P'_k , one can always find certain property C_k which is not common to all 'individual' objects $P'_1, P'_2, P'_3, \dots, P'_n$. On the basis of the above the 'general' object' P'_k [misprint in text; should read P_k - my note.] does not possess the property C_k (I). The 'individual' object P'_k possessing the property C_k does not possess the property of not possessing the property C_k . It is so because if it possessed the property of not possessing the property C_k , i.e. if it was not possessed of the property C_k , then it would be contradictory since it would be an object both possessing and not possessing the property C_k . The property of not possessing the property C_k is not common to all 'individual' objects $P'_1, P'_2, P'_3, \dots, P'_n$ because any 'individual' object P'_k possesses the property C_k . Consequently the 'general' object P'_k [misprint in text; should read P_k - my note.] cannot possess the property of not possessing the property C_k , therefore it is not possessed of the property C_k , *ergo* it possesses the property C_k (II). From the comparison of the theses (I) and (II) it follows that the assumption that any object P_k is 'general' leads to contradiction because it implies that this object both possesses the property C_k (I) and does not possess it (II). We conclude that the assumption that any object is 'general' is false. I believe that the above reasoning demonstrates that no object is a universal or general object."¹⁴²

¹⁴² Stanislaw Lesniewski, "The Critique of the Logical Principle of the Excluded Middel", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Szrednicki, D.I. Barnett, Trans. S. J. Surma and J. Wojcik, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1991, pp. 51 and 52.

Appendix C.

Lesniewski's Second Proof

"... to all those who, by reason of the meaning they give to expressions of the type 'general object with respect to objects a', are inclined to state the proposition 'if X is a general object with respect to objects a, X is b, and Y is a, then Y is b', I wish to state here that this proposition entails the proposition 'if there exist at least two different a, then a general object with respect to objects a does not exist', in accordance with the following schema:

(1) if X is a general object with respect to objects a, X is b, also Y is a, then Y is b. (assumption) from (1) it results, that

(2) if X is a general object with respect to objects a, X is different from Z, and Z is a, then Z is different from Z,

and

(3) if X is a general object with respect to objects a, X is identical with Z, and Y is a, then Y is identical with Z;

from (2) it follows, that

(4) if X is a general object with respect to objects a, and Z is a, then X is identical with Z,

from (4) however, that,

(5) if X is a general object with respect to objects a, Z is a, and Y is a, then (X is a general object with respect to objects a, X is identical with Z, and Y is a);

from (5) and (3) it follows, that,

(6) if X is a general object with respect to objects a, Z is a, and Y is a, then Y is identical with Z,

from (6) however, that if there exist at least two different a , then a general object with respect to objects a , does not exist."¹⁴³

¹⁴³ Stanislaw Lesniewski, "On the Foundations of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. D. I. Barnett, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 198 and 199.

Appendix D.

Peirce Footnote

On page 53 above, I make reference to a footnote, footnote 14, on page 203 of "Stanislaw Lesniewski Collected Works" which directs our attention to Peirce.

See: C. S. Peirce, On the Algebra of Logic, 'A contribution to the Philosophy of Notation', American Journal of Mathematics, VII, 1885, p. 197.¹⁴⁴

This footnote is mentioned in almost every article which discusses Lesniewskian quantification. Commentators, however, generally quote from an article by Tarski in which the same reference is made to the same Peirce article as Lesniewski makes above, but Tarski's reference is believed to have included an actual quotation from the Peirce article and not just a reference to page 197 of the article. Commentators usually mention the fact that Lesniewski, in a later work¹⁴⁵, actually quotes the entire Tarski reference, including what is taken to be an actual quotation from the Peirce article. Tarski's reference, which includes what is believed to be an actual quotation for Peirce, is found in footnote 3 of his article, which reads as follows:

³ In the sense of Peirce (see Peirce, C. S. (58a), p. 197) who gives this name to the symbols ' Π ' (universal quantifier) and ' Σ ' (particular or existential quantifier) representing abbreviations of the expressions: 'for every signification of the terms ...' and 'for some signification of the terms ...'¹⁴⁶

For the sake of completeness, I provide the sentence in the body of Tarski's article to which footnote above is attached:

¹⁴⁴ Ibid., p. 203.

¹⁴⁵ Stanislaw Lesniewski, "Fundamentals of a New System of the Foundations of Mathematics", Stanislaw Lesniewski Collected Works, Eds. Stanislaw J. Surma, Jan T. Srzednicki, D.I. Barnett, Trans. M. P. O'Neil, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, pp. 419 and 420.

¹⁴⁶ Alfred Tarski, Logic, Semantics, Meta-Mathematics, Hackett Publishing Company, 1983, p. 1, fn. 3.

The problem of which I here offer a solution is the following: *is it possible to construct a system of logic in which the sign of equivalence is the only primitive sign* (in addition of course to the quantifiers³)¹⁴⁷

This sentence is a statement of the problem Tarski addressed in his doctoral thesis, a thesis written under the direction of Lesniewski - Tarski was the only doctoral student Lesniewski ever supervised. Although it is obvious from the quote, it is important to mention that Tarski considered quantifiers to be primitive signs.

Let us return to the Lesniewski's and Tarski's reference to Peirce.

I have not found a commentator who mentions that Tarski actually misquotes Peirce. Nowhere on page 197, as a matter of fact nowhere in Peirce's article, do we find the phrases "'for every signification of the terms...'" and "'for some signification of the terms...'" . As a matter of fact, the only possible section on page 197 of Peirce's article to which Tarski (and Lesniewski) could reasonably be referencing, is the "4th" "step" of Peirce's instructions for manipulating indices, namely, instructions for "...making the indices refer to the same collections of objects..." .

On pages 194 and 195 of Peirce's article, there are phrases similar to the ones Tarski quotes:

Thus $\sum x_i$ means that x is true of some one of the individuals denoted by i or $\sum x_i = x_i + x_j + x_k + \text{etc.}$ In the same way, $\prod x_i$ means that x is true of all these individuals, or $\prod x_i = x_i x_j x_k, \text{etc.}$ ¹⁴⁸

But there are no phrases closer than these to the ones Tarski attributes to Peirce.

I believe that both Lesniewski and Tarski refer to Peirce, not so much for the phrases which Peirce uses to translate the symbolic logic tokens for

¹⁴⁷ Ibid., p. 1.

¹⁴⁸ Peirce, C. S., "On the Algebra of Logic: A Contribution To The Philosophy Of Notation", American Journal Of Mathematics, Vol. VII, 1885, p. 197.

universal and particular quantifiers into ordinary language, but because they both actually make use of Peirce's instructions for "...making the indices refer to the same collections of objects..." in their own works on logic.

In order to see the relevance of the Peirce reference to Lesniewski's notion of quantification, I have used Peirce's notation to notate Lesniewski's Axiom of Ontology. Lesniewski's Axiom of Ontology using Peirce's notation would look something like this:

$$\begin{aligned} [[\prod x_{en} \prod x'_{en}] : x_{en} \varepsilon x'_{en} \therefore \equiv \therefore \neg [[\prod y_{en}] \neg (y_{en} \varepsilon x_{en})] : \\ [[\prod y_{en} \prod z_{en}] y_{en} \varepsilon x_{en} \cdot z_{en} \varepsilon x_{en} \cdot \supset \cdot y_{en} \varepsilon z_{en} : \\ [[\prod y_{en}] y_{en} \varepsilon x_{en} \cdot \supset \cdot y_{en} \varepsilon x'_{en} \end{aligned}$$

Peirce used indices to make explicite the collection of things outside his system to which his quantifiers "pointed". In my example, each variable is indexed, actually double-indexed ("indices of indices"¹⁴⁹ for Peirce). If we assume that Lesniewski quantified over expressions of any language which are *a priori* ordered into semantic categories, then first index, "e" of my example symbolizes expressions of any language, the widest range of things which can be substituted for the variable. The second index, "n", symbolizes the semantic category of the expression (nouns) which can be substituted for the particular variables of this axiom. By specifying the semantic category the range of quantification is further restricted to noun expressions; for Lesniewski, the functor "e" required two expressions of the semantic category of nouns.

It is important to note that, for Peirce, variables were always to be replaced by tokens.¹⁵⁰ Peirce's use of the word "token" is very close to Lesniewski's use of the word "expression" (another reason for choosing "e" as the first index in the Peirce notated Axiom of Ontology above). I assume that neither Lesniewski nor Tarski restricted their reading of Peirce to page 197 of

¹⁴⁹ Ibid., p. 199.

¹⁵⁰ Ibid., p. 199.

the above quoted article. I assume that they read and understood the entire article and, perhaps other articles of Peirce as well.

What we realize from this analysis is that Lesniewski could and did drop the first index by simply restricting his universe of discourse to expressions. Lesniewski was able to avoid using the second index and , therefore, the use of indices altogether by using variously shaped parenthesis to mark the semantic categories of the variables of his logical systems.

...I adopted from my 'theory of types',...,a variability in parentheses that depends upon semantical categories of the particular expressions involved.¹⁵¹

The phrases Tarski seems to quote from Peirce's article are not direct quotations at all. When we realize that the Peirce footnote appears, in Tarski's original works, in either Polish or French, then we understand that those phrases, "For every signification of the terms" and "for some signification of the terms" could just as well be translated into english as "for every meaning of the terms" and "for some meaning of the terms", respectively. These phrases represent precisely Lesniewski's notion of quantification during his "initial" period. By quoting Peirce, therefore, Tarski simply wishes to express the concurrence of Lesniewski's and Peirce's notions of quantification.

If we do not take Peirce's indices as an alternate way to describe both Lesniewski's and Tarski's range of quantification, then the fact that both Lesniewski and Tarski quoted Peirce makes no sense. If we cannot index Lesniewski's variables to expressions and semantic categories, again, quoting Peirce makes no sense.

¹⁵¹ Stanislaw Lesniewski, "Fundamentals of a New System of the Foundations of Mathematics", Stanislaw Lesniewski Collected Works, Editors, Stanislaw J. Surma, Jan T. Srzednicki, and D.I. Barnett, Trans. M. P. O'Neil, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1992, p. 453.

After reading Peirce's article, Lesniewski, it seems, not only understood that variables needed to be indexed to a specified range of expressions, but also, if he choose a fixed universe of discourse for all the variables of his logical systems, then no index was required. Dropping the index made his own systems less cumbersome. Lesniewski's use of variously shaped parenthesis to enclose variables that could only be replaced by expressions of a certain semantic category seems to be a more elegant and a more "iconical" way of showing a correspondence between the semantic categories of any language and the semantic categories of Lesniewski's logical systems. Lesniewski's methods of "indexing" variables is syntactically superior because his variables are, then, not burdened with any other duty than as place holders and as primitive expressions of the particular semantic category indicated by the shape of the parentheses surrounding them. Indices, it seems to me, are not as simple syntactically as Lesniewski's parentheses; indices seem to confuse the notion of simple substitution.

That Peirce had a significant influence on the development of Lesniewski's logical systems is further supported in Lesniewski's mature works by a continuous reference to words, terms, expressions, and, in general, to linguistic entities. Lesniewski's mature writing makes absolutely no sense, unless the syntactic development of symbols in his logical systems somehow finds a correspondence to the expressions of natural language.

Certain other similarities to Lesniewski's logical systems appear in Peirce's article and, although they are not pertinent to our discussion, I mention these similarities because they are interesting; they show, I believe, that Peirce's influence on Lesniewski's intellectual development was greater than a simple reference might indicate.

Peirce treated terms taken in the distributive sense separately from terms treated in a collective sense. Though it might seem that "externally" Peirce's quantifiers "pointed to" objects in the world, we see in the quote below, that that opinion is a bit too simplistic.

The index [quantifier - my note] asserts nothing; it only says "There!" It takes hold of our eyes, as it were, and forcibly directs them to a particular object, and there it stops. Demonstrative and relative pronouns are nearly pure indices, because they denote things without describing them; so are the letters on a geometrical diagram, and the subscript numbers which in algebra distinguish one value from another without saying what those values are.¹⁵²

Peirce attempts to describe the "primitive" nature of quantifiers, not by saying that they attempt to provide an absolute correspondence between things in the world and the variables of his logical systems, but rather that whatever quantifiers do, they do it "without saying what those values are".

¹⁵² Peirce, C. S., "On the Algebra of Logic: A Contribution To The Philosophy Of Notation", American Journal Of Mathematics, Vol. VII, 1885, p. 181.

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