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Sales Forecasting Using Exponential Smoothing

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SALES FORECASTING USING EXPONENTIAL SMOOTHING

BY

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ABSTRACT

Sales forecasting affects almost every area of activity in industry. The importance of a sales forecast can never be underestimated. The choice of the right forecasting technique is essential for a company to operate efficiently.

The purpose of this thesis is to show that exponential smoothing as a sales forecasting device and as a device to predict demand for production and inventory control purposes is more accurate, more efficient and less time consuming in its application than other conventional forecasting techniques.

Exponential smoothing is a simple procedure for calculating a weighted moving average; the greatest weight is assigned to the most recent data of actual or predicted sales. This paper discusses the effectiveness of simple methods of exponential smoothing with regard to accuracy, computational simplicity, and flexibility in order to adjust the prediction to the rate of response of the forecasting system. It is not necessary to select and work with complicated economic indicators, etc.

A number of commonly used forecasting devices are presented and an analysis of their strengths and weaknesses are discussed. Among the statistical forecasting

tools constructively criticized are: correlation analysis (simple, multiple, linear and non linear), time series analysis, and moving averages (simple and weighted). Actual problem solutions and valid arguments are presented to prove that exponential smoothing is highly advantageous. Practical applications of exponential smoothing show real life cases where the experiences of a number of companies indicate exponential smoothing to be extremely beneficial.

The academic discussions as well as practical applications of the technique in operation indicate exponential smoothing to be a most successful method. It requires less data than any type of forecasting method while remaining highly flexible because a modified forecast can be made by simply changing the smoothing constant. When used in conjunction with data processing equipment, exponential smoothing makes it possible to forecast demand accurately on a weekly basis. It is easily adapted to high speed electronic computers so that expected demand as well as detection of and correction for trends can be measured as a routine matter. It makes it possible to measure current distribution of forecast errors item by item. Therefore, exponential smoothing is particularly well suited for item forecasts which may be needed for determining re-order points, materials planning, economic order quantities in materials management and scheduling in production control.

ACKNOWLEDGEMENTS

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I

INTRODUCTION

I. ROLE OF SALES FORECASTING

A sales forecast may be defined as an attempt to determine the volume of sales which can reasonably be expected by a company at some future time.[1] It is usually a statistical projection modified by the judgement, knowledge, opinion, and intuition of the individuals who know the product under study. The accuracy, frequency, and details of forecasts, as well as the periods to which they apply, will quite probably vary, depending upon the type of industry, the type of company, and the type of management problems which are encountered. There is no single pattern. In the business world today, management is making more and more use of forecasts in many areas of their businesses. As one author puts it, "Forecasts are indispensable in planning. All decisions involve planning."[2] However, many forecasters say that it is

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[1] Robert J. Paulis, "Sales and Manufacturing Forecasts," (Kingston, R.I.: University of Rhode Island, 1967), p. 1.

[2] Elmer C. Bratt, Business Forecasting (New York: McGraw Hill Book Company, 1958), p. 1.

almost impossible to reach any definite, concrete statement of future conditions; that the best any man can do is to make his assumptions very carefully and make a complete and detailed study of them. In such a way his forecasts will then become more plausible, reliable, or useful. One may very well ask at this point, "Why do businessmen forecast?" In economic environments, business conditions tend to fluctuate. Due to the uncertainty, the investor attempts to reduce the possibility of a loss. This is best done by attempting to predict what the future has in store for the business.

In recent years more and more attention has been given to business forecasting. The businessman sees the need for forecasting in two broad areas of planning: the needs resulting from short-term changes in demand and the area of those needs arising from long-term changes in demand. In its inventory planning and the ability to fill orders, businessmen need to know how much to produce. Almost every company needs a long range forecast. That is, a non-routine report made expressly to suit the special needs of a company's management. Long range forecasts are most useful for planning of production facilities and raw materials needed for production. They are consequently expressed in terms of normal and peak levels, rather than actual levels.[3] It is expensive to have too much or too little capacity. With limited production capacity, some orders will not be filled and the per unit cost is increased due to high operational costs. In order to have a production capacity which will satisfy most of the demand in prosperity, yet not leave the businessman bankrupt in less prosperous times, the businessman must have the knowledge of the long term demand for his product. Corning Glass Works has a five year sales forecast. Its purpose is to provide a guide for long range sales and manufacturing plans. It has provided that company with a means to plan future requirements for manpower, manufacturing facilities, and research and development projects, so that an orderly approach to meet these requirements may be developed.[4] Some other areas in which long-range planning is necessary are capital investment and the procurement of plant or equipment. In deciding whether or not to undertake long-term borrowing, the company must be aware of future

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[3]B.E.Estes, "What Management Expects of Forecasting," Sales Forecasting, Special Report No. XVI of American Management Association, (New York: American Management Association, Inc., 1956), p. 17.

[4]Richard L. Pately, "Preparation and Coordination of Forecasts at Corning Glass Works," Sales Forecasting, Special Report No. XVI, of American Management Associa-

conditions. If the forecasts show that demand for the product will fall off during the next five or ten years, the executives and board of directors would probably agree that it would not be advisable to float a long-term loan. If on the other hand, future demand for sales of the product is on the upsurge, the officers of the company would probably agree that, in view of the optimistic future outlook, this would be a good time to make a capital investment and take out a long-term loan if necessary. Whether a sales forecast looks 5, 10, or 15 years ahead will depend on the individual company, the nature of its product, the relative cost of its equipment as well as its economic life, and the rate of obsolescence of its products. Some companies, particularly in those industries which depend directly on natural resources for their raw materials, may need extremely long range projections. Forecasts of this nature are used in making decisions regarding prospecting activities, rate of development of mineral and other resources and location of major new facilities. Other less important areas where the long-range demand for the product must be taken into consideration are industries such as the alcohol and

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tion, (New York: American Management Association, Inc., 1956), p. 115.

tobacco industries where the process of aging plays an important role.

In addition to long-term forecasts, there is a continuous need for forecasting "Near-future" developments.[5] As contrasted with long-term forecasting, short-term forecasts indicate actual future levels rather than an average expectation. Private business uses short-term forecasting in many areas. Among them are anticipation of future sales and the setting of sales quotas, inventory management, pricing and wage policies, budget making, production planning, and employment policies.

In arriving at an effective inventory policy, the company must gear its production schedules to anticipated sales rather than to actual sales. If a forecast could come close to predicting and understanding the prospects in the near future, it would be possible for the company to avoid some of the rapid changes which often occur in production planning, scheduling and inventory control. The effect of future demand for the product can also be seen in the area of employment. If future sales are going to be good, the company will hire more employees in order to meet the demand by increasing production and inventory. However, if the outlook for the near future

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[5]Bratt, op. cit., p. 4.

is poor, the company may be forced to impose a cut-back in employment, with a resulting cut-back in production and inventory.

The future level of sales is also important to know when setting prices. If a company saw that in the future its costs would rise, it certainly wouldn't want to lower its prices. And conversely, if the near-future markets are not expected to be strong, the company would most likely avoid raising its prices.

Short-term forecasting is necessary also in the area of budget making. If the budget in production and sales for the coming year is geared to future sales levels rather than to actual sales levels, it will be much more practical. If a budget is geared to current sales, it may need periodic adjustment if and when sales levels change in the future. By gearing the budget to future sales right from the start, the adjustments which may be costly and time consuming will appear unnecessary.

The government as well as some private agencies use short and long-term forecasts to aid and influence private business. The government makes extensive studies of the entire economy and in turn makes the results of this research and these studies available to private businessmen. Gross national product, a weighted index of all types of economic activities made by the Joint

Economic Committee of Congress, is the most widely used federal government forecasting aid.[6] Many companies feel gross national product is a good starting point for developing industry and/or company sales forecasts. A variety of other federal agencies such as the Treasury Department, Commerce Department, Federal Power Commission, Bureau of Mines, Bureau of Labor Statistics, and Federal Reserve Board forecast data useful to the businessman. Economic indicators are either leading, concurrent or lagging with sales. The break-down of some indicators as listed by Business Cycle Developments of January, 1968, a publication of the U. S. Department of Commerce, is as follows:

Leading Indicators

1. New incorporations
2. Business failures
3. Common stock - industrial
4. New orders for manufacturers' durable goods

Concurrent Indicators

1. Gross national product - quarterly

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[6] E. Jerome McCarthy, Basic Marketing (Homewood: Richard D. Irwin, Incorporated, 1964), p. 107.

2. Corporate profits - quarterly

3. Unemployment

Lagging Indicators

1. Personal income

2. Retail sales

3. Bank rates on business loans

If a series or index leads with substantial regularity it would be of great value to a company to predict sales. For forecasting then the leading indicator is of primary value but examples of all sets are presented here for the sake of completeness. Without such information business would be forecasting on conditions prevailing in their own markets; however, with the aid of forecasting studies made by the government and others, the private businessman may avoid some of the errors and pitfalls which would occur if he made the forecasts without considering the broad economic outlook.

Thus, we see that any company desirous of forecasting may analyze and consider first the general, overall economic outlook, and then the outlook for its own particular industry. Once this has been accomplished, the executive is more able to see how expected changes in the economy and in the industry will affect his own company. This knowledge of the factors affecting the economy, the industry as a whole, and his company in particular, will enable the executive to appraise and evaluate the outlook for his company.

As mentioned earlier, sales forecasting can be done objectively or subjectively. Objective sales forecasts are those techniques which rely on the accumulation and interpretation of quantitative data. Generally, subjective sales forecasting methods rely on the judgement and experience of the salesmen, sales manager, and high-level management.

In the spring of 1965, Robert Reichard surveyed 300 companies (with a 42 per cent response) with regard to their sales forecasting approach. He found that no two companies use the same approach to forecast sales. He did find, however, that 90 per cent of the firms responding to the inquiry used a combination of objective and subjective methods. There was a 2-1 preference to use an objective approach first and then to smooth out the rough edges by subjective or judgemental modification.[7] Although he concluded that more weight will be given to objective techniques of sales forecasting in the next

[7]Robert S. Reichard, "What's New in Sales Forecasting," <u>Management Review</u>, LIV (September, 1965), pp. 35-37.

five years, subjective forecasting techniques are and will be an integral part of the forecasting process.

II. SALES FORECASTING AS A MANUFACTURING AID

The sales forecast represents the starting point in the manufacturing cycle. From a forecast of sales a manufacturing plan is derived which authorizes production and determines inventory of raw materials, spare parts, labor force and machines. Information may be derived to establish relationships between demands for different products so that a sound balance of production runs can be worked out in terms of the quantities required of various products. The efficiency of the plant as a whole is affected since the forecast has great bearing on the utilization of equipment and manpower.

In this thesis the significance of sales forecasts shall be related primarily to production and inventory control. The importance of an accurate sales forecast can be easily understood. An accurate forecast enables management to make sound and timely decisions concerning what to produce and when.

In preparing a manufacturing forecast production and inventory control managers must consider the following factors:

1. The demand for the items to be sold.

2. Existing inventory levels.
3. Required inventory levels at the end of the forecast period.
4. Decline in sales of old products.
5. New product allowances.
6. Scrap allowances.
7. Adjustment to backlog levels.[8]

The type of manufacturing exerts a great influence on the importance and feasibility of the sales forecast. In continuous manufacturing concerns, such as companies making automobiles, refrigerators, and television sets, the sales forecast actually becomes the production program month by month. The company produces directly on the basis of forecasted sales. The continuity of flow in mass production depends on the proper allocation of plant capacity, on the adequate flow of raw materials, or in-process inventory and on employment (suffering from absenteeism, turnover). The continuity of the flow of production will in turn determine the cost of production and the company's profit. An improper appraisal of market conditions can lead either to under or over-production. If a forecast is underestimated by the manufacturer, his

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[8]Robert J. Paulis, "Industrial Forecasting," (Kingston, R.I.: University of Rhode Island, 1967), p. 1.

products may reach the market too late and in the meantime competition would make inroads in his market. No large-scale producer can afford to be caught short of supply, because it would mean loss of sales for him. "Never run out of stock," is the most important slogan in manufacturing and sales. By the same token no continuous manufacturer can afford to produce in excess of market demand because of high inventory carrying costs and the surplus would become either obsolete or must be sold at cut-rate prices.

In intermittent manufacturing the forecast has less direct bearing on immediate production. The long-term forecast is taken, but it does not become the exact production program. Production is performed when orders are received so that the production program cannot be set up in advance of orders that have not yet been received for the product. Economic fluctuations have great impact on this type of manufacturing.

III. COMPUTERS IN FORECASTING

More and more, companies are using computerized forecasts as a bench mark for assessing conventional forecasting methods, as well as for updating the original forecast with the latest sales data for fresh insights into the short-term future.

Good statistical projections with the aid of an electronic computer have a number of advantages:[9]

1. Speed (ten to fifteen thousand projections within a day).
2. Stability and optimum response to underlying facts that cannot be achieved by judgement.

Then this stability is reflected in stability of the manufacturing schedule and in the manufacturing costs which is the most important yardstick of production efficiency.

3. Cost of doing the statistical projections with modern computers is a negligible fraction of the earlier costs of manual studies.
4. Accuracy: projections are mostly more accurate than forecasts based on judgement or experience.
5. Error estimate can be computed only when the forecast is prepared in a formalized, consistent manner. This estimate is important in doing much of a company's planning, particularly that of safety stock. An analysis of forecasting error will reveal how good the forecast is.

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[9]Robert J. Paulis, "Industrial Forecasting," <u>op. cit</u>., pp. 2-3.

A McGraw Hill survey in the spring of 1965 covering 800 big companies, found that one-third of them are running their business forecasts on electronic computers. This was far from the 74 per cent use of the machines for inventory control, 58 per cent for production planning, and 47 per cent for scientific and engineering applications.[10] As new models improve the computer's capabilities an increased use will build up savings so that business will make greater use of computerized forecasts than in the past.

Impressive as their contributions to sales forecasting may be, computers have not reduced forecasting to strictly a machine function. The human factors are still very critical catalysts, in particular judgement, intuition and experience.

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[10] Thayer C. Taylor, "The Computer in Marketing," Sales Management, XCVI, (January 7, 1966), pp. 52-53.

II

SALES FORECASTING TECHNIQUES

In order to make forecasts, it is necessary to have some mechanical methods whereby the forecaster can come to some reliable conclusions on the basis of his assumptions. Hence, it is necessary to investigate the methods both quantitative or objective, qualitative or subjective; and to evaluate these methods.

I. SUBJECTIVE SALES FORECASTING TECHNIQUES

Subjective sales forecasting techniques generally utilize a collective opinion or judgemental approach. Although there may be refinements or variations of this method, fundamentally the process is as follows: Salesmen make estimates of future sales in their territories by projecting the needs of their customers into the future; these estimates are submitted to sales managers who may review and modify the estimates before forwarding the forecast to territorial or regional sales managers for further appraisal. The estimates are then sent to the company's head office where they are assembled and studied by a committee. The committee is often made up of the sales manager, marketing manager, production manager, chief engineer, treasurer, and economist.

The advantage of this method is its simplicity. It requires little or no technical skill while making use of experienced personnel who have their finger on the pulse of the market. Salesmen, however, are often poor estimators, being either too optimistic or too pessimistic.[1] In addition, salesmen cannot accurately forecast sales for more than one year. A company consequently finds itself at a disadvantage when facing such problems as acquisition of new equipment and plant expansion - typically long-range projects. Group judgement also suffers from a lack of consistency in the standards of evaluation.[2]

II. STATISTICAL SALES FORECASTING METHODS

"By far the most generally relied upon approach to sales forecasting is the application of various statistical methods."[3] There are many statistical tools

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[1]Bratt, _op_. _cit_., p. 239.

[2]Warren K. Schoonmaker, "What You Should Know About Sales Forecasting," _Industrial_ _Marketing_, IIL (October, 1963), p. 110.

[3]Harold Koontz and Cyril O'Donnel, _Principles of Management_ (New York: McGraw Hill Book Company, 1964), p. 127.

used to calculate estimated sales: correlation analysis (simple and multiple), method of least squares, time series analysis, moving averages, and exponential smoothing to name a few.

Correlation analysis.

> The purpose of correlation and regression analysis is to arrive at a mathematical equation, called an estimating equation, prediction equation or regression equation, which best discloses the nature of the relationship between a dependent variable and one or more independent variables.[4]

In sales forecasting correlation analysis can measure the relationship between sales and some independent variables - often economic indicators.

To find the relationship between sales and the economic index in Table I, page 18, the method of least squares is used. The method of least squares serves to construct a line of best fit in the correct position. A line of best fit or regression line is defined as a line where the deviations between the actual figures and the calculated figures equal zero.[5]

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[4] Milton H. Spencer, Colin G. Clark, and Peter W. Hoguet, Business and Economic Forecasting (Homewood: Richard D. Irwin, Inc., 1961), p. 44.

[5] Edward E. Lewis, Methods of Statistical Analysis in Economics and Business (Boston: Houghton Mifflin Company, 1953), p. 389.

TABLE I

SALES VS. ECONOMIC INDEXES

Year	Sales	Economic Index
1	10.0	100
2	13.1	113
3	15.1	136
4	15.8	144
5	13.5	126
6	14.7	129
7	16.5	157
8	18.0	160
9	18.5	174
10	21.2	191
11	23.0	214
12	20.1	180
13	23.3	215
14	24.6	224
15	24.4	218

Source: Raymond R. Mayer, Production Management (New York: McGraw Hill Book Company, 1959), p. 242.

The general expression for the equation of any regression line is

$$Y = a + bX$$

and for the computation of the values of a and b

$$\Sigma Y = n.a + b\Sigma X$$

$$\Sigma XY = a\Sigma X + b\Sigma X^2$$

where:

n = number of paired observations given

a = Y intercept of the line of best fit (initial value of dependent variable)

b = slope of the line of best fit

Y = value of the dependent variable, which is the variable whose magnitude is to be predicted (sales)

X = value of the independent variable, which is the variable in terms of which the magnitude of another variable is to be predicted.[6]

After substituting the data from Table II, page 20, into the earlier mentioned formulas, one gets

$$271.8 = 15a + 2{,}481b$$

$$48{,}001.2 = 2{,}481a + 433{,}645b$$

Solving for a and b, one finds that a is equal to .1 and

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[6]Raymond R. Mayer, Production Management (New York: McGraw Hill Book Company, 1959), p. 238.

TABLE II

SALES VS. ECONOMIC INDEXES;
LEAST SQUARES CALCULATIONS

Year	Sales (Y)	Economic Index (X)	XY	X^2
1	10.0	100	1,000	10,000
2	13.1	113	1,480.3	12,769
3	15.1	136	2,053.6	18,496
4	15.8	144	2,752.2	20,736
5	13.5	126	1,701.0	15,876
6	14.7	129	1,896.3	16,641
7	16.5	157	2,590.5	24,649
8	18.0	160	2,880.0	25,600
9	18.5	174	3,219.0	30,276
10	21.2	191	4,049.2	36,481
11	23.0	214	4,922.0	45,796
12	20.1	180	3,618.0	32,400
13	23.3	215	5,009.5	46,225
14	24.6	224	5,510.4	50,176
15	24.4	218	5,319.2	47,524
	271.8	2,481	48,001.2	433,645

Source: Original calculations based on Raymond R. Mayer, Production Management (New York: McGraw Hill Book Company, 1959), p. 242.

b is equal to .109. Thus, the line of best fit is

$$Y = 0.1 + .109X$$

If the index was predicted to be 200 for the year, the regression equation would yield the following forecast:

$$0.1 + 0.109\,(200) = 22$$

This means that about 22 units will be sold in the next year. Plotting this information on a graph, a clearer picture of the relationship between the sales economic index and sales results. See Figure 1, page 22.

Because there is very little scatter, it is obvious that a high degree of correlation exists. To measure the goodness of the relationship, a coefficient of correlation is computed:

$$r = \text{coefficient of correlation} = \sqrt{1 - \frac{\Sigma(Y_a - Y_c)^2}{\Sigma(Y_a - \bar{Y})^2}}$$

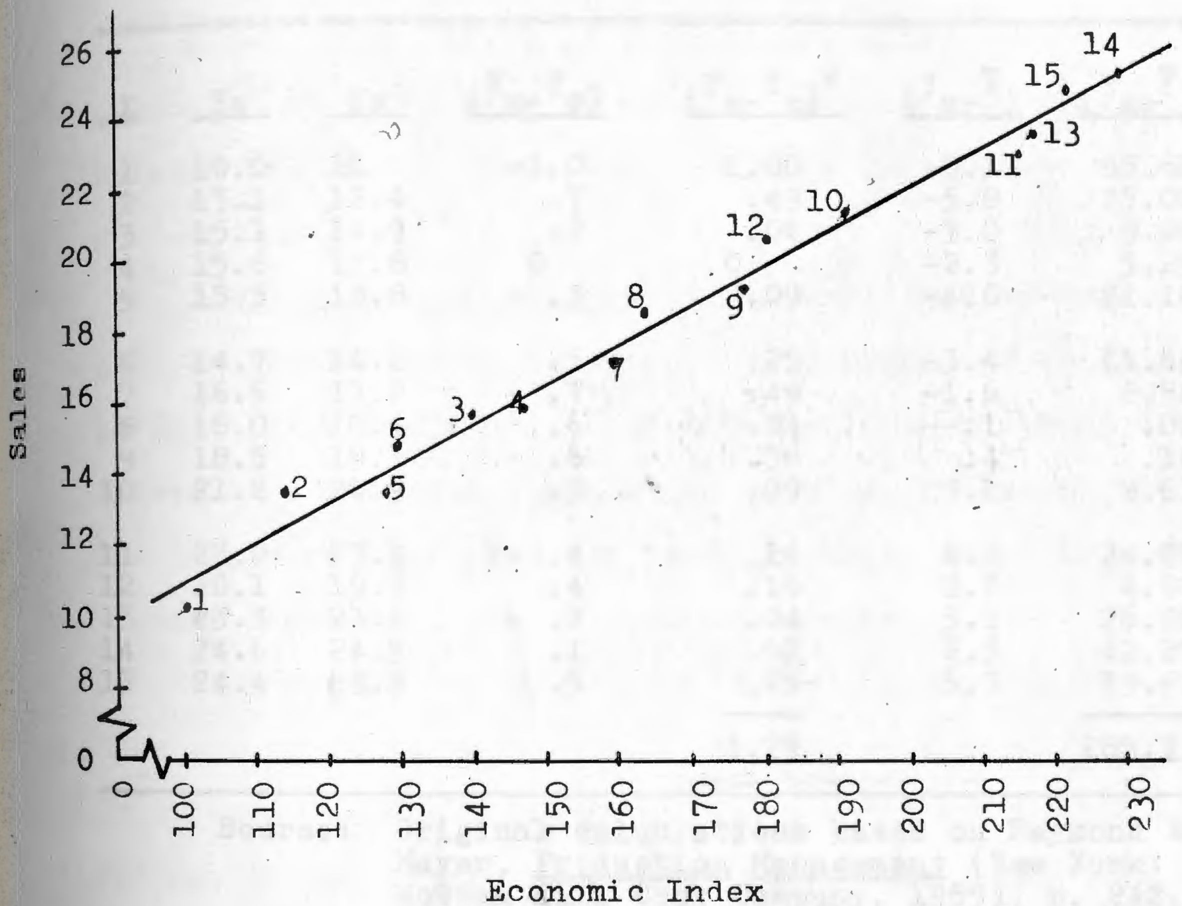
where:

Ya = original value of the dependent variable for which the symbol Y was used earlier

Yc = corresponding calculated value found from the least squares equation

$\bar{Y}$ = average of given values of the dependent variable.

Hence, one may set up a table of Coefficient of Correlation Calculations which is shown on page 23, Table III.



Source: Raymond R. Mayer, Production Management (New York: Mc Graw Hill Book Company, 1959), p.243.

FIGURE I

LINE OF BEST FIT

TABLE III

COEFFICIENT OF CORRELATION
CALCULATIONS

X	Ya	Yc	$(Y_a - Y_c)$	$(Y_a - Y_c)^2$	$(Y_a - \bar{Y})$	$(Y_a - \bar{Y})^2$
1	10.0	11	-1.0	1.00	-8.1	65.60
2	13.1	12.4	.7	.49	-5.0	25.00
3	15.1	14.9	.2	.04	-3.0	9.00
4	15.8	15.8	0	0	-2.3	5.29
5	13.5	13.8	- .3	.09	-4.6	21.16
6	14.7	14.2	.5	.25	-3.4	11.56
7	16.5	17.2	- .7	.49	-1.6	2.56
8	18.0	17.4	.6	.36	- .1	.01
9	18.5	19.1	- .6	.36	.4	.16
10	21.2	20.9	.3	.09	3.1	9.61
11	23.0	23.4	- .4	.16	4.9	24.01
12	20.1	19.7	.4	.16	2.0	4.00
13	23.3	23.5	- .2	.04	5.1	26.01
14	24.6	24.5	.1	.01	6.5	42.25
15	24.4	23.9	.5	.25	6.3	39.69
				3.79		285.91

Source: Original calculations based on Raymond R. Mayer, Production Management (New York: McGraw Hill Cook Company, 1959), p. 242.

After making the appropriate substitutions:

$$r = \sqrt{1 - \frac{3.79}{285.91}} = 0.99$$

A very high degree of correlation exists since a generally accepted rule of thumb interprets any figure between 0.90 and 1.00 as having a very high degree of correlation.[7]

<u>Multiple Correlation</u>. Often more than one variable is used to estimate sales. This technique is known as multiple correlation. The mathematical procedure involved in a multiple correlation problem is similar to the procedure utilizing the least squares technique previously described, but it is more complex.

Naturally, in many cases the addition of more than one variable improves the relationship considerably. In a recent cement industry multiple correlation application, it was found that demand for cement could best be explained by four variables: residential construction, non-residential construction, highway construction, and all other types of construction.[8] Splitting up the ex-

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[7] Mayer, <u>op</u>. <u>cit</u>., p. 241.

[8] <u>Construction Review</u>, United States Department of Commerce, (Washington: Government Printing Office), January, 1964, p. 12.

plaining or independent variable is only one of many uses of multiple correlation. It is possible that two or more completely different variables may insure a good relationship.

Table IV, on page 26, illustrates hypothetical data for a four variable correlation problem.

The equation or line of regression found by applying the principle of least squares is:

$$X_{c1.234} = -0.7694 + 1.0173X_2 + 0.1969X_3 + 0.0483X_4$$

where:

$X_{c1.234}$ = calculated value of X, based on the economic indexes of X_2, X_3 and X_4.

Table V, on page 27, shows the calculated values of X, for each of the ten observations as well as the deviations of actual from calculated values and the deviations squared.

The coefficient of multiple correlation is computed in the same manner as the coefficient of correlation for simple correlation.

Since there are ten observations the square of the multiple scatter is $\frac{5.8237}{10}$ or .58237.

The square of the standard deviations of X_1 is found to be 5.81000, so

$$r = \sqrt{1 - \frac{0.58237}{5.81000}} = .9486$$

TABLE IV

SALES WITH THREE
ECONOMIC INDICATORS

Observation	Sales X_1	Economic Indexes X_2	X_3	X_4
1	7	6	8	10
2	6	5	10	11
3	3	1	5	13
4	4	4	6	14
5	8	7	5	13
6	1	1	3	11
7	9	6	9	10
8	7	5	8	16
9	5	4	5	12
10	3	2	8	10

Source: Edward E. Lewis, Methods of Statistical Analysis in Economics and Business (Boston: Houghton Mifflin Company, 1953), p. 603.

TABLE V

COEFFICIENT OF
CORRELATION CALCULATIONS

X_1	$X_{c1.234}$	$x_{1.234}$ (deviations)	$x^2_{1.234}$ (deviations squared)
7.0000	7.3923	- .3923	.1539
6.0000	6.8171	- .8171	.6677
3.0000	1.8600	1.1400	1.2996
4.0000	5.1570	-1.1570	1.3386
8.0000	7.9636	.0364	.0013
1.0000	1.3696	- .3696	.1366
9.0000	7.5892	1.4108	1.9904
7.0000	6.6646	.3354	.1125
5.0000	4.8635	.1365	.0186
3.0000	3.3232	- .3232	.1045
			5.8237

Source: Edward E. Lewis, Methods of Statistical Analysis in Economics and Business (Boston: Houghton Mifflin Company, 1953), p. 604.

Nonlinear correlations. The correlation relationship need not be straight. Correlation problems can be worked out using exponential, parabolic, or any other type of curve that can be explained mathematically. "Many firms report their best correlation results with a combination of several variables, all related in some kind to nonlinear relationship."[9]

Analysis of the correlation techniques. Correlation depends upon the existence of an economic indicator or group of indicators. In Reichard's survey many firms complained of difficulty in finding a leading or predicting indicator. (For obvious reasons an economic indicator should lead sales. How else could sales be predicted?) It has been suggested that an extrapolation of the indicator be made when the indicator does not lead sales. This has proven to be an unreliable method of facilitating sales forecasting. There is little use in developing relationships of this type because the series of data may be so difficult to forecast that the reliability and validity of the basic forecast are not improved. Some leading indicators serve as a useful guide to predict the course of the economy, however, they are not always consistent in their tendency to lead; do not

[9] Reichard, op. cit., p. 165.

always predict a turning point in the economy; and merely indicate the direction of the change and disclose nothing about the magnitude of the change.[10]

Another problem which presents itself to a forecaster is making sure that the relationship between the variables is rational - one of cause and effect. Also, the lead-lag relationship of sales to economic indicators normally prevents a forecast being made for a period of more than one year. The disadvantage of using the correlation technique is further compounded when monthly or weekly sales forecasts are desired, for there are few indexes which are other than annual reflections.

The illustration presented earlier in this paper was an extremely simple example of the correlation process. In practice the calculations and data handled by the forecaster are time consuming and difficult to adapt to computerized operations.

Correlation analysis does have an advantage. It provides management with a good starting point for making a final forecast.[11]

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[10] Spencer, Clark and Hoguet, _op_. _cit_., p. 11.

[11] Mayer, _op_. _cit_., p. 247.

Time series analysis

Time series analysis is an analysis of past sales data in order to determine the nature of an existing trend. The trend is then extrapolated and used as a basis for a forecast. The method of trend projection assumes that the recent rate of change of the variable will continue in the future. These trends (like correlation trends) may be curvilinear, but for simplicity, only linear trends are described here. See Table VI, page 31.

Utilizing the least squares simultaneous equations to forecast sales for the first quarter of the fifth year (17^{th} quarter), the trend value will be:

$$Y = 1.775 + 0.1956(17) = 5.10$$

Figure 2, page 32, which is a graphic representation of Table VI and the trend line reveals that most of the points represented by the trend line do not fall on that line. In order to predict sales, the forecast must be adjusted for these variations.

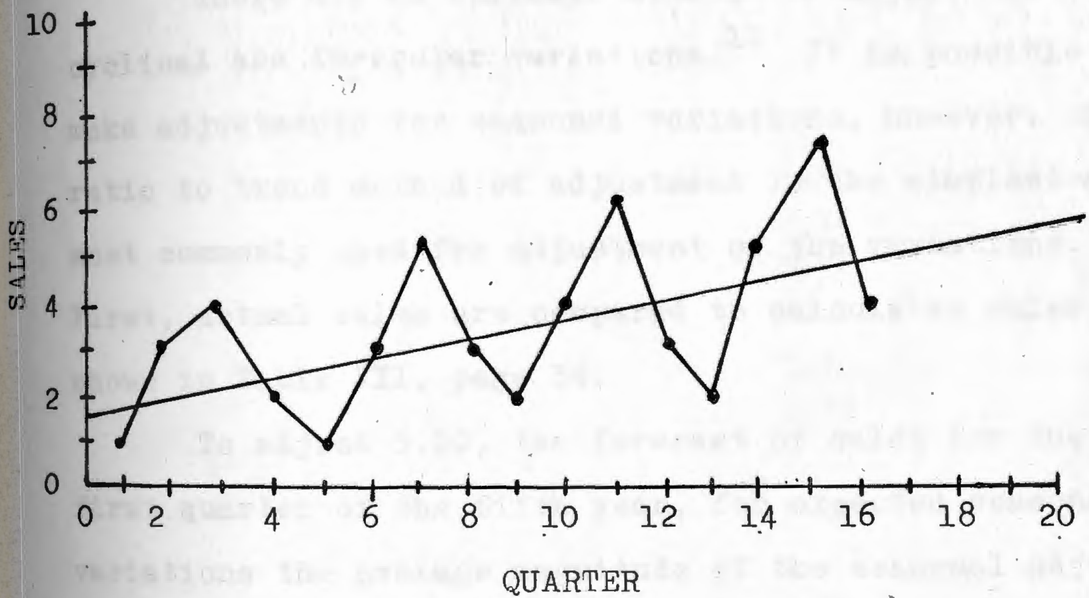
Forecasters generally divide the factors influencing sales into four basic categories: trend, cyclical, seasonal, and irregular. Trend refers to the general movement of sales over a period of several years. The cyclical influence is a temporary wave, usually two to seven years in magnitude, caused by economic fluctuations

TABLE VI

LEAST SQUARES CALCULATIONS

Quarter (X)	Sales (Y)	XY	X^2
1	1	1	1
2	3	6	4
3	4	12	9
4	2	8	16
5	1	5	25
6	3	18	36
7	5	35	49
8	3	24	64
9	2	18	81
10	4	40	100
11	6	66	121
12	3	36	144
13	2	26	169
14	5	70	196
15	7	105	225
16	4	64	256
136	55	534	1,496

Source: Raymond R. Mayer, Production Management (New York: McGraw Hill Book Company, 1959), p. 238.



Source: Raymond R. Mayer, Production Management (New York: Mc Graw Hill Book Company, 1959), p. 248.

FIGURE 2

TREND LINE

or by something inherent in the particular industry or product analyzed. Seasonal variations are those influences in calendar years which are caused by weather or some custom manifestation. Finally, irregular variations are all variations which cannot be explained by other influences (strikes, wars, floods, hurricanes, etc.).

There are no reliable methods to adjust for cyclical and irregular variations.[12] It is possible to make adjustments for seasonal variations, however. The ratio to trend method of adjustment is the simplest and most commonly used for adjustment of the variations. First, actual sales are compared to calculated sales as shown in Table VII, page 34.

To adjust 5.10, the forecast of sales for the first quarter of the fifth year, for expected seasonal variations the average magnitude of the seasonal adjustment for the first quarter of each of the past four years is computed working with values from Table VII.

	Actual sales as a per cent of calculated				
Quarter	Year 1	Year 2	Year 3	Year 4	Average %
1	51	36	56	46	47

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[12]Mayer, op. cit., p. 250.

TABLE VII

COMPARISON OF ACTUAL TO CALCULATED SALES

Quarter (X)	Actual Sales (Y)	Calculated Sales (Y)	Ya/Yc %
1	1	1.97	51
2	3	2.17	138
3	4	2.36	170
4	2	2.56	78
5	1	2.75	36
6	3	2.95	102
7	5	3.14	159
8	3	3.34	90
9	2	3.54	56
10	4	3.73	107
11	6	3.93	152
12	3	4.12	73
13	2	4.32	46
14	5	4.51	111
15	7	4.71	149
16	4	4.90	82

Source: Raymond R. Mayer, Production Management (New York: McGraw Hill Book Company, 1959), p. 251.

The trend value of sales is then multiplied by the adjustment factor to give the forecast:[13]

Trend value of sales for the 17^{th} quarter equals 5.10 so the forecast with a seasonal adjustment factor of 47 per cent is:

$$(5.10)\ (.47) = 2.4 \text{ units.}$$

In the same manner one could forecast sales with seasonal adjustments for quarters 18, 19 and 20 of the fifth year. However, it is obvious that a seasonal adjustment for trend would not be necessary if one were using a time series based on annual data.

Analysis of the time series technique. When compared to correlation analysis, the advantage of employing a time series forecasting procedure is clear - there is no reliance on outside variables such as the economic index. In addition time series analysis permits forecasting by months or even weeks if a company so desires.

Moving averages

"When an inventory control system includes an objective method of forecasting demand item by item, the method is usually some form of moving average."[14]

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[13]Mayer, op. cit., pp. 251-254.

[14]Robert G. Brown, "Less Risk in Inventory Estimates," New Decision Making Tools for Managers, Edward C.

Table VIII shows the demand for bedroom suites experienced by a small furniture manufacturer from January to June.

TABLE VIII

DEMAND FOR BEDROOM SUITES

MONTH	NUMBER OF BEDROOM SUITES SOLD
January	65
February	93
March	85
April	105
May	71
June	115
Total	534

Source: Leonard J. Garrett and Milton Silver, Production Management Analysis (New York: Harcourt, Brace & World, Inc., 1966), p. 291.

To predict sales of bedroom suites for the next month, July, a simple moving average would be used:

$$\text{Moving average} = \frac{\text{Sum of demand values}}{\text{Number of demand values in sum}} =$$

$$\frac{534}{6} = 89 \text{ suites per month.}$$

Bursk and John F. Chapman, editors. (New York: Harcourt, Brace and World, Inc., 1966), p. 291.

The major flaw of this method is obvious. Eighty-nine units understate demand. The cause of this understatement is the smoothing factor built into the simple moving average which forces the average to lag. Adjustments can be applied to make this technique more responsive to the trend of sales by weighting the data of individual months so that most recent ones receive the highest weights.

To illustrate this approach, let us examine the data shown in Table IX, page 38, which represents the number of dining room suites sold for each month of a seven month period.

Utilizing a version of least squares the slope of the line is equal to:[15]

$$\text{Slope} = \frac{\text{sum of weighted demand}}{\text{sum of squared weights}} = \frac{194}{28} = 6.9$$

To calculate the expected sales in any month the procedure would be as follows:[16]

$$\text{Expected demand} = \frac{\text{sum of demand}}{\text{number of months}} + \text{Slope (number of months from the base period)}$$

[15]Leonard J. Garrett and Milton Silver, Production Management Analysis (New York: Harcourt, Brace & World, Inc., 1966), p. 293

[16]Ibid.

TABLE IX

TREND ADJUSTMENT CALCULATIONS

Month	Number of Dining-Room Suites Sold	Weighting Factor	Weighted Demand	Square of Weight Factor
January	20	-3	-60	9
February	24	-2	-48	4
March	30	-1	-30	1
April	34	0	0	0
May	45	1	45	1
June	52	2	104	4
July	61	3	183	9
	266		194	28

Source: Leonard J. Garrett and Milton Silver, Production Management Analysis (New York: Harcourt, Brace and World, Inc., 1966), p. 293.

where:

base period = A

Sales for August, which is four months from April would be:

$$\frac{266}{7} + (6.9)\ (4) = 65.6$$

A simple correlation between the weighted average and the actual demand is presented graphically in Figure 3 on page 40.

If management intends to plan and control its inventory correctly it should have an indication of the variations that can be expected for any forecast in addition to the forecast of average demand.

Look again at the demand for bedroom suites. The moving average was 89 units. To find out the best estimate of probable error of the forecast, compute the standard deviation of the forecast.[17]

$$Sx = \sqrt{\frac{\Sigma(x - \bar{x})}{N-1}}$$

where:

x = Individual monthly demands

$\bar{x}$ = Average demand

N = Number of periods included

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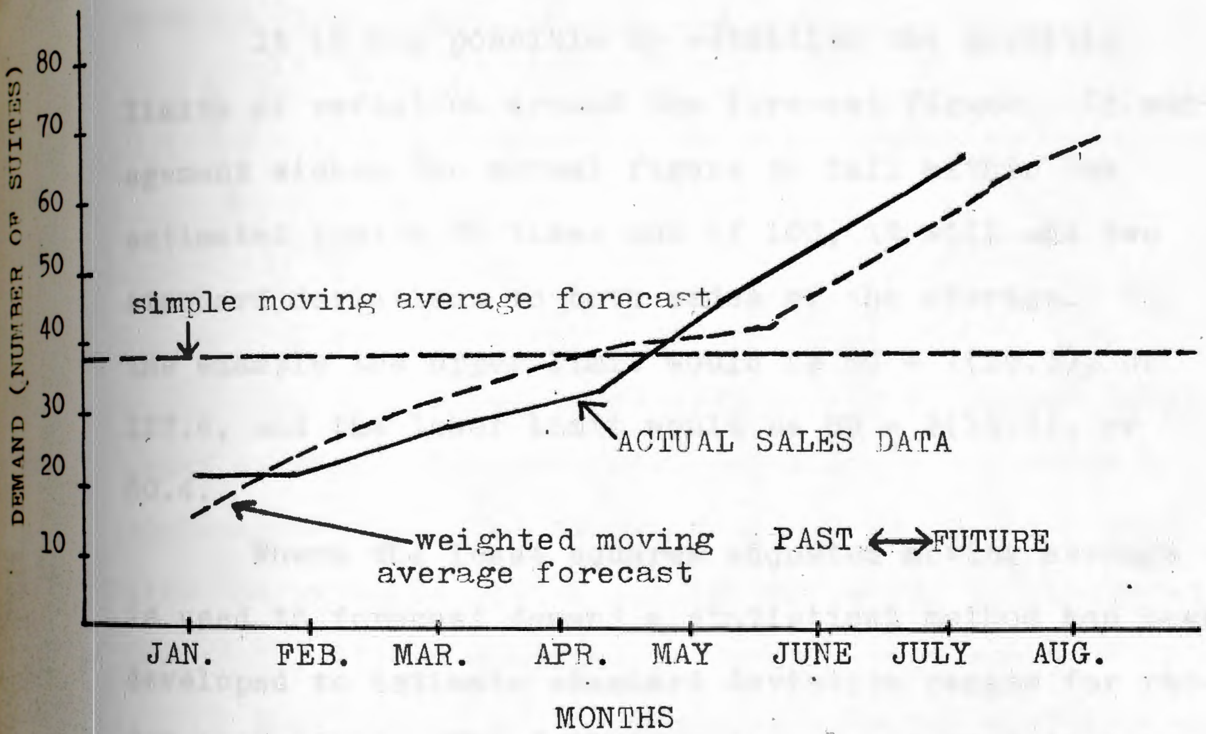
[17] Ibid., p. 301.



Source: Leonard J. Garrett and Milton Silver, Production Management Analysis, (New York: Harcourt, Brace and World, Inc., 1966), p. 294.

FIGURE 3

WEIGHTED AVERAGE VS. ACTUAL DEMAND

so

$$Sx = \sqrt{\frac{(65-89)^2+(93-89)^2+(85-89)^2 + \ldots + (115-89)^2}{6-1}}$$

or

Sx = 19.3 units.

It is now possible to establish the probable limits of variation around the forecast figure. If management wishes the actual figure to fall within the estimated limits 95 times out of 100, it will add two standard deviations to both sides of the average. In the example the upper limit would be 89 + 2(19.3), or 127.6, and the lower limit would be 89 - 2(19.3), or 50.4.

Where the least squares adjusted moving average is used to forecast demand a statistical method has been developed to estimate standard deviation ranges for random variations. The following formula is used to measure variation that exists around the trend line:[18]

$$S_x t = \sqrt{\frac{\Sigma(X - X_t)}{N-2}}$$

where:

$S_x t$ = standard error of estimate

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[18]*Ibid.*, p. 302.

X = demand for period

X_t = estimated demand for a period from the trend line

N = number of demand period

Referring to Table IX the standard error of estimate is equal to:

$$S_xt = \sqrt{\frac{(20 - 17.3)^2 + (24 - 24.2)^2 + (30 - 31.3)^2 + \ldots + (61 - 58.7)^2}{7 - 2}}$$

$$= \sqrt{\frac{29.88}{5}}$$

$$= 2.44$$

The S_xt figure is similar to the S_x figure calculated earlier. However, its limits are drawn about a regression line. The amount 2.44 can be used to establish the probable limits for the variation in estimated demand for dining room suites for the next month.[19]

Analysis of moving averages. Moving averages can smooth out abrupt fluctuations in a demand pattern. The stability of its response to change can be adjusted by altering the number of periods included in the average. Moving averages are simple, straightforward, and can be easily programmed for punched-card machines in electronic computers. However, a record of sales must be kept for

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[19] *Ibid.*, p. 303.

all past periods which can mean long files. In addition, another major problem is the difficulty of changing the rate of response.

III

EXPONENTIAL SMOOTHING

Exponential smoothing is a weighted sum of all past demand with the heaviest weight being placed on the most recent information. It is designed to overcome the limitations of moving averages. It does away with the necessity of keeping extensive records of past data. It has the ability to sense a change in the demand picture as well as the ability to track this changing pattern of demand. The exponential smoothing technique can be extended to account for long-term trends and seasonal effects. Exponential smoothing can also allow for uncertainty in management estimates. With data processing equipment an exponential smoothing program can update forecasts regularly for thousands of items and provide information to control inventories far better than could be accomplished manually.[1]

In a sales forecasting application a forecast of sales for the next period is predicted by using a weighted average of sales in the current period, and the forecast

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[1] Robert G. Brown, "Less Risk in Inventory Estimates," op. cit., p. 279.

of sales made during the previous period.[2]

The fundamental concept of exponential smoothing or as it is sometimes called exponential weighting is quite simple. The new estimate of average demand (called a forecast) is equal to the old estimate adjusted by a fraction of the difference between the old estimate and the actual demand. The fraction of this difference used to modify the previous estimate, is called a smoothing constant and is referred to as K or the Greek letter α, (alpha).

The value of alpha is always between zero and one and this value determines how much of the past demand figure has any significant effect on the estimate of the average.[3]

If a small smoothing constant is chosen, for example, 0.01 the response to changes in demand will be slow and gradual. (0.01 is based on the average of 199 past observations.)

where:

N = number of observations = 199

$$\alpha = \frac{2}{N + 1} = \frac{2}{200} = 0.01$$

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[2]Peter R. Winters, "Forecasting Sales by Exponentially Weighted Moving Averages," Management Science, VI (April, 1960), p. 235.

[3]Robert G. Brown, "Less Risk in Inventory Estimates,"

If fewer past data were included in the calculation of α the response to changes in the demand that occur would be faster.

Any smoothing constant is a compromise between being too sluggish and too erratic. Generally 0.1 is regarded as a compromise between a stable system that is slow to recognize changes in demand and a very sensitive system that fluctuates greatly with demand.

The appropriate value of the smoothing constant can best be determined by trial on a sample of actual past demand. Here is an example, using the data in Table X, page 47.

If the management thinks that some factors are going to affect demand resulting either in increasing demand or decreasing demand that is not reflected in the past history the value of the smoothing constant can be increased temporarily for a short period to account for the predicted change.

The formula for exponential smoothing is as follows:

New average = old estimate - α (actual usage - old estimate)

For example, if a forecast of demand of 80 units

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op cit., pp. 283-284.

TABLE X

ORDERS AND PREDICTIONS WITH DIFFERENT SMOOTHING CONSTANTS

Year	Month	Orders (000)	Forecasts (000)			
			$\alpha=0.8$	$\alpha=0.6$	$\alpha=0.4$	$\alpha=0.2$
1963	January	30.6	24.0	24.0	24.0	24.0
	February	30.0	29.3	28.0	26.6	25.3
	March	44.6	29.9	29.2	27.9	26.2
	April	30.2	41.7	38.4	34.6	29.9
	May	41.2	32.5	33.5	32.8	30.0
	June	15.0	39.5	38.1	36.2	32.2
	July	36.7	20.0	24.2	27.7	28.8
	August	20.8	33.4	31.7	31.3	30.4
	September	38.1	23.3	25.2	27.1	28.5
	October	29.8	35.1	32.9	31.5	30.4
	November	40.5	30.9	31.0	30.8	30.3
	December	36.8	38.6	36.7	34.7	32.3
1964	January	27.8	37.2	36.8	35.5	33.2
	February	30.5	29.7	31.4	32.4	32.1
	March	40.7	30.3	30.9	31.6	31.8
	April	38.8	38.6	36.8	35.2	33.6
	May	34.8	38.7	38.0	36.6	34.6
	June	35.0	35.6	36.1	35.9	34.6
	July	38.0	35.1	35.4	35.5	34.7

Source: John F. Magee and David Boodman, Production Planning and Inventory Control (second edition; New York: McGraw Hill Book Company, 1967), p. 111.

was made for the week while actual usage was 90 units, using a smoothing constant of 0.1 the new average is equal to:

$80 + 0.1\ (90 - 80) = 81$ units.

The formula for exponential smoothing can be rearranged to simplify the calculation as follows:

New average $= \alpha$(actual usage) + $(1 - \alpha)$ old average

$0.1\ (90) + 0.9\ (80) = 81$ units.

Simple exponential smoothing performs very efficiently when a forecastor deals with fairly stable items. Most practitioners have found that simple smoothing gives satisfactory results and is particularly well suited for short range forecasts, for order point calculations in inventory management and for scheduling in production control.[4] Simple exponential smoothing will detect trends quite readily, however, this average will lag behind a demand that follows a systematic trend.

The aforementioned average of 81 units will lag behind a demand that follows a systematic trend. Adjusted exponential smoothing can estimate the magnitude of a trend, thus a correction can be made to eliminate the lag. To measure the change in trend the formula is:

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[4] G. W. Plossl and O. W. Wright, Production and Inventory Control (Englewood Cliffs: Prentice Hall, Inc., 1967), p. 41.

Current trend = new average - old average

In the example illustrated earlier the current trend is equal to 81 less 80 or 1.

Just as exponential smoothing measures the average demand so can it measure the average trend:

$$\text{New trend} = \alpha(\text{current trend}) + (1 - \alpha)\ \text{old trend}$$

Assuming the averages have been increasing by two units per period (old trend) the new trend equals 0.1(1) + (1 - 0.1)2 = 1.9 units.

The correction for lag due to trend can be expressed as:[5]

$$\text{Expected demand} = \text{new average} + \frac{1-\alpha}{\alpha}(\text{new trend})$$

or in our example: $81 + \frac{(1-0.1)}{0.1}(1.9) = 98.1$ units

Comparing the actual demand figures and the new average figures we see the forecast lags behind demand. However, when the process was adjusted to account for lag due to trend characteristics by the method described previously the lag was nearly eliminated. A suitable example of a trend correction can be seen comparing the demand to the next period forecast in Table XI, page 50.

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[5]Dean S. Ammer, Materials Management (Homewood: Richard D. Irwin Inc., 1962), pp. 187-188.

TABLE XI

FORECASTS WITH
TREND ADJUSTMENTS

($\alpha = 0.6$)

Month	Demand	New Average	Current Trend	New Trend	Next Period Forecast
Jan.	20	20	0	0	20
Feb.	24	22.4	2.4	1.44	23.3
March	30	27.0	4.6	3.34	29.0
April	34	31.2	4.2	3.86	33.5
May	45	39.5	8.3	6.52	43.4
June	52	47.0	7.5	7.11	51.3
July	61	55.4	8.4	7.88	60.1

Source: Leonard J. Garrett and Milton Silver, <u>Production Management Analysis</u> (New York: Harcourt, Brace & World, Inc., 1966) p. 300.

The formulas needed for adjusted exponential smoothing require only the previous values to be stored. Hence, data processing is simple, but the results involve some mathematical errors. The elimination of these errors will be discussed later in the chapter.

<u>Second order smoothing</u>. An alternative to account for trends would be to use a more complicated formula called second order exponential smoothing. Second order smoothing can track any trend in demand without making

corrections. The calculations of the trend uses two values of the average, hence, the name second order smoothing is used. A decision on whether or not to use second order smoothing is determined by taking real data on a few items and simulating the results.

Combining the average and the trend second order smoothing can track a steadily rising demand. The equation is as follows:

$$\text{New forecast} = \alpha(\text{new demand}) + 2(1-\alpha)(\text{average computed last month}) - (1-\alpha)(\text{average computed previous month})$$

> Although this system does automatically track a trend without correction, it does have a very bad tendency to oscillate whenever there is a sudden change in demand.[6]

The tendency to oscillate is shown in Figure 4, a graphic presentation of a random sequence of demand with trend.

The first order system smooths out peaks and valleys. The second order system with an α of 0.1 introduces even more instability by exaggerating peaks and valleys. There is still a tendency to oscillate when the smoothing constant is increased to 0.5 but the oscil-

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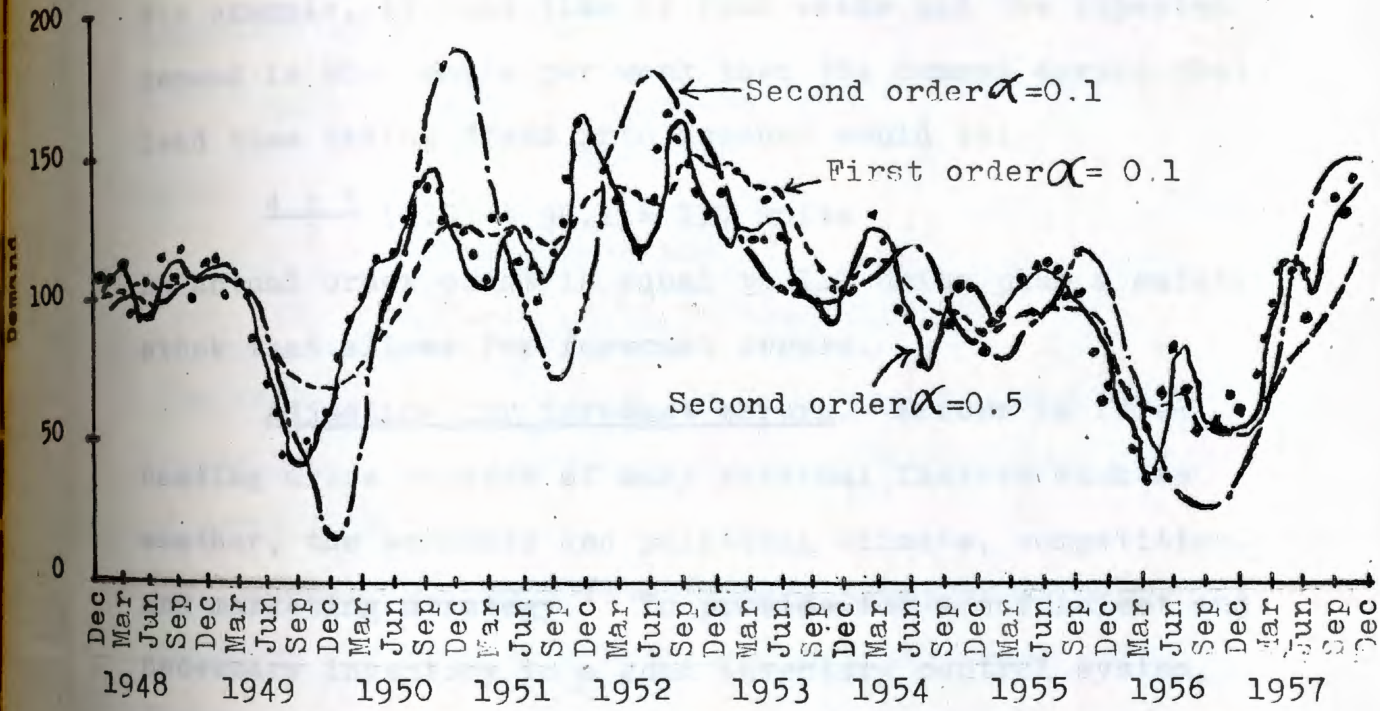
[6]Robert G. Brown, Statistical Forecasting for Inventory Control (New York: McGraw Hill Book Company, 1959), p. 66.



Source: Robert G. Brown, <u>Statitistical Forecasting for Inventory Control</u> (New York: Mc Graw Hill Book Company, 1959), p. 66.

FIGURE 4

SMOOTHED ESTIMATES
OF
CURRENT EXPECTED VALUE

lation has been reduced.

Lead Time. The estimate of demand projected through a lead time of T weeks would be:

$$\text{Demand expected in lead time} = \frac{\text{Lead time (lead time + 1) (new trend)}}{2} + \text{Expected demand.}$$

For example, if lead time is four weeks and the expected demand is 98.1 units per week then the demand during that lead time taking trend into account would be:

$$\frac{4 \times 5}{2}(1.9) + 98.1 = 117 \text{ units}$$

An actual order point is equal to 117 units plus a safety stock that allows for forecast errors.

Adjusting for forecast errors. Errors in forecasting arise because of many external factors such as weather, the economic and political climate, competition, and marketing strategy.[7] To provide for a sufficient and necessary inventory in a good inventory control system, the variability of demand is measured. This measure of demand variability is a mean absolute deviation (MAD). The mean absolute deviation is equal to the alpha smoothing constant times the current deviation (sometimes called the absolute error) plus ($1 - \alpha$) times the pre-

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[7]Robert G. Brown, "Less Risk in Inventory Estimates," op. cit., p. 283.

vious MAD.

For the normally distributed demand, the average value of the absolute difference is proportional to the standard deviation of the forecast error. (The average or MAD is equal to approximately .8 times its standard deviation). The new mean absolute deviation can then be expressed in the formula:

$$MAD = \alpha \left| D_{cu} \right| + (1 - \alpha)\ MAD_p$$ [8]

where:

D_{cu} = current deviation

MAD_p = old mean absolute deviation.

(The verticle lines around the current deviation value in the preceeding formula mean: Use the absolute value of current deviation.)

Table XII, page 55, presents an exponential smoothing forecast with mean absolute deviation calculations.

The complete forecast, for example, February, 1963 is for expected demand of 25.3 units plus a mean absolute deviation of 5.6 units.

If one wanted to establish the probable limits of variation around the average forecast figure, one could

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[8] Ibid., p. 293.

TABLE XII

EXPONENTIAL SMOOTHING FORECAST
WITH MEAN ABSOLUTE DEVIATION

Year	Month	Orders (000)	Forecast (α =0.2)	Absolute error, actual minus forecast	Mean absolute deviation (initial MAD = 5)
1963	January	30.6	24.0	6.6	5.3
	February	30.0	25.3	4.7	5.6
	March	44.6	26.2	18.4	5.4
	April	30.2	29.9	0.3	8.0
	May	41.2	30.0	11.2	6.5
	June	15.0	32.2	17.2	7.4
	July	36.7	28.8	7.9	9.4
	August	20.8	30.4	9.6	9.1
	September	38.1	28.5	9.6	9.2
	October	29.8	30.4	0.6	9.3
	November	40.5	30.3	10.2	7.6
	December	36.8	32.3	4.5	8.1
1964	January	27.8	33.2	5.4	7.4
	February	30.5	32.1	1.7	7.0
	March	40.7	31.8	8.9	5.9
	April	38.8	33.6	5.1	6.5
	May	34.8	34.6	0.2	6.2
	June	35.0	34.6	0.4	5.0
	July	38.0	34.7	3.3	4.1
	August	40.2	35.4	4.8	3.9
	September	29.2	36.4	7.2	4.1
	October	51.1	35.0	16.1	4.7
	November	39.3	38.2	1.1	7.0
	December	35.9	38.4	2.5	5.8
1965	January	32.5	37.9	5.4	5.1
	February	45.7	36.8	8.9	5.2
	March	35.4	38.6	3.2	5.9
	April	39.8	38.0	1.8	5.4
	May	56.5	38.4	18.1	4.7
	June	55.0	42.0	13.0	7.4
	July	46.5	44.6	1.9	8.5
	August	44.2	45.0	0.8	7.2

Source: John F. Magee and David Boodman, _Production Planning and Inventory Control_ (second edition, New York: McGraw Hill Book Company, 1967), p. 113.

proceed using the following data.

Actual demand	=	105
Expected demand	=	98.1
Forecast error	=	6.9
Alpha	=	0.1
Old MAD	=	4

then:

New mean absolute deviation = 0.1 (105-98.1) + (1 - 0.1) 4 = 4.29

Since the standard deviation is equal to 1.25 times the mean absolute deviation a safety stock for the preceding observation would be equal to 4.29 times 1.25 (2) or 11 units for 95.44 occurrences out of 100. Since the lead time expected demand was 117 units (see page 53) a safety stock of 11 units should be added to allow for noise in the forecast. The actual order point then is equal to 128 units.

Knowing the general statistical properties of a normal distribution and having calculated the standard deviation or mean absolute deviation of forecast error the calculation of an order point is straightforward. The service level or safety factor is obviously related to the number of standard deviations provided as a reserve or safety stock. The knowledge of a normal distribution tells us that adding two standard deviations to the anticipated demand during lead time gives 95.44 per cent service and adding three standard deviations during lead time increases the service level to 99.73 per cent. Service level means the percentage of replenishment periods during which demand should not exceed the order point quantity. An order point is designed to cover the demand during the lead time so that a replenishment order can be placed in time for a product or part to be delivered before all stock is withdrawn.[9]

Accumulation of error. The error of the forecast is needed in order to make certain decisions concerning the maximum demand likely to be experienced during the period over which the forecast is made.[10] This suggestion was described when discussing how to adjust for forecast error.

If this error is accumulated (called Sum Delta) it can be of interest to management in the following ways. When Sum Delta is not zero it can provide an indication of bias in forecasting. Positive numbers indicate an underestimate, negative numbers indicate an over-estimate. Management can establish an exception reporting technique using this information. As long as demand occurs in a random fashion the cumulative error

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[9] Plossl and Wright, op cit., p. 107.

[10] Magee and Boodman, op cit., p. 111.

should be small. The logical choice of a control limit for such exception reporting is to establish the bounds of this cumulative error as a multiple of MADs. A forecast evaluation program can contain the capability to examine the result of such exception reporting requirements.[11]

Management may set a tracking control system at, for instance, five times MAD. An out-of-control column could print an asterisk when the tracking signal exceeds the predetermined management control value.

Exponential smoothing for a product group forecast. A product group forecast utilizing exponential smoothing can be used when a judgement forecast is made based on anticipated changes in market penetration, competitive reactions, public acceptance of a product, etc.[12] Naturally, such a product group forecast will not be exactly the same as the sum of the exponential smoothing forecasts for the individual items. For example, if a marketing department decided to lower the price on a line of products expecting demand to increase as a result of this price policy the increase in demand would

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[11] Burroughs Inventory Control System (Detroit, Michigan: Burroughs Corporation, 1966), pp. 3-4.

[12] Plossl and Wright, op. cit., p. 39.

not be shown in an exponential smoothing forecast because the forecast is based on only past data.

Table XIII shows how the exponential smoothing forecasts for each of ten items in a product group may be revised as a result of a new product group forecast of 20,000 units received from the marketing department.

TABLE XIII

REVISED EXPONENTIAL SMOOTHING FORECAST

Item	Exponential smoothing forecast	New prorated item forecast
1	1,500	1,875
2	2,500	3,125
3	500	625
4	3,500	4,375
5	1,000	1,250
6	4,000	5,000
7	2,000	2,500
8	200	250
9	300	375
10	4,500	5,625
TOTAL	20,000	25,000

The individual forecasts totaled only 20,000 units and the difference is then pro-rated across ten products, each of them being adjusted in proportion to its share of the total so that the sum of the forecasts for the individual products equals 25,000. The new pro-rated forecast for each item may now be used as a basis for forecasting for the next period. Using a computer, this type of forecast adjustment is very readily made, even for numerous items.

Seasonal patterns. A few products like Christmas cards or antifreeze follow a seasonal pattern. A forecast system designed under application of exponential smoothing can adjust for and detect seasonal patterns. Two principles, however, are necessary for this method to be implemented:

1. The reason for the seasonal pattern must be known.
2. The variation resulting from the seasonal pattern must be less than the variation caused by unpredictable fluctuations.[13]

A solution to a sales forecasting problem where a seasonal pattern is involved rests upon the selection of a base series (some historical data) and comparing this base series with current demand so as to compute a demand ratio. The most common base series is the actual demand for the product during the corresponding period

[13] Robert G. Brown, Statistical Forecasting For Inventory Control, op. cit., p. 129.

last year. An average of the demand for the three months surrounding the corresponding month in the previous year may be used as a base series if the history of a product reveals that peak demand shifts back and forth by a month or two from year to year. This base series (called average of the surrounding quarter) is found quite often where the demand for a product depends on an unpredictable factor like weather. For example, antifreeze may have a peak demand in any one of two or three winter months depending on when cold weather arrives. Sometimes when a number of stock keeping units are related their total demand may be used as a base series.

As the first step each month a demand ratio must be computed:

$$\text{Demand ratio} = \frac{\text{Demand in the current month}}{\text{Value of the base series for the current month.}}$$

Table XIV, page 62, illustrates the procedure to find expected demand for the seasonal. Our example also assumes that a definite trend exists and consequently adjusted exponential smoothing is employed.

The first four columns of Table XIV are self explanatory. Column five is equal to (1- α) times the average for the previous month, plus α times the demand

TABLE XIV

EXPECTED DEMAND FOR A SEASONAL

Data Source: Air passengers carried (in thousands). Base Series: Surrounding quarter last year. Smoothing constant = 0.1. $1 - \alpha = 0.9$; $\frac{1-\alpha}{\alpha} = 9.0$

	1 Date	2 Demand	3 Base Series	4 Ratio	5 Average Ratio	6 Change	7 Trend	8 Expected Ratio	9 Expected Demand
1950	Jan	115			(1.050)		0	1.050	
	Feb	126	120.7	1.043	1.049	-0.001	-0.0001	1.048	126.5
	Mar	141	126.3	1.116	1.056	0.007	0.0006	1.061	134.0
	Apr	135	127.3	1.060	1.056	0.000	0.0005	1.061	135.1
	May	125	128.3	0.974	1.048	-0.008	-0.0004	1.044	133.9
	Jun	149	134.7	1.106	1.054	0.006	0.0002	1.056	142.2
	Jul	170	143.7	1.183	1.067	0.013	0.0015	1.081	155.3
	Aug	170	144.0	1.180	1.078	0.011	0.0025	1.101	158.5
	Sep	158	134.3	1.176	1.088	0.010	0.0033	1.118	150.1
	Oct	133	119.7	1.111	1.090	0.002	0.0032	1.119	133.9

TABLE XIV (continued)

	1 Date	2 Demand	3 Base Series	4 Ratio	5 Average Ratio	6 Change	7 Trend	8 Expected Ratio	9 Expected Demand
	Nov	114	113.7	1.002	1.081	-0.009	0.0020	1.099	125.0
	Dec	140	112.3	1.246	1.098	0.017	0.0035	1.130	126.9
1951	Jan	145	119.7	1.211	1.109	0.011	0.0043	1.148	137.4

Source: Robert G. Brown, Statistical Forecasting for Inventory Control (New York: McGraw Hill Book Company, 1959), p. 134, citing CAA Statistical Handbook of Aviation, 1954-1958.

ratio for the current month. (The initial value for column five is chosen by inspection of the first four values in column four.) Column six, change, is the average ratio last month subtracted from the value this month (column five). Column seven (trend values) are computed in the same manner.[14]

Computations for trends were described earlier in this chapter, i.e. $(1-\alpha)$ times old trend plus α times the change for the current month. Initial value is arbitrarily determined as zero. Expected ratio equals column five plus $\frac{1-\alpha}{\alpha}$ (column seven). And finally, expected demand equals column eight times column three.[15]

Balancing holding costs and acquisition costs by using exponential smoothing. There are two approaches for balancing holding costs and acquisition costs. The "fixed order quantity-variable order interval" approach and the "fixed order interval-variable quantity" approach. Of the two the former is most frequently used and is specified by: (1) the lead time, (T), elapsing between placing an order and receiving stock; (2) the order size, (Q); (3) the safety stock, (U); and, (4) the expected demand rate, $(\overline{M})$. In inventory management the order point is specified to indicate when the economic

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[14] Ibid., p. 135.

[15] Ibid.

order quantity, (Q), should be ordered. The order point is equal to the expected demand during the lead time plus the safety stock. The safety stock is usually estimated to be twice the square root of lead time usage. The formulas are:

$$Q = \sqrt{\frac{24\bar{M}S}{KC}}$$

$$U = 2\sqrt{\bar{M}T}$$

$$O.P. = \bar{M}T + U$$

where:

Q = Order quantity in units

$\bar{M}$ = Current month demand in units

S = Acquisition cost

K = Holding cost per dollar of inventory per year

C = Unit cost of item

T = Lead time in months

U = Safety Stock

$O.P.$ = Order point in units

The second system, the "fixed order interval-variable order quantity" approach is specified by: (1) lead time, (T); (2) the period between reviews, (P); (3) the variable order quantity, (Q); (4) the safety stock, (U); and, (5) the demand rate, ($\bar{M}$). Here a forecast is made of the demand for a standard review period plus the delivery lead time. At each review an order is placed to bring total inventory "on hand" and "on order" up to the forecast for lead time plus one review period, plus the standard safety stock. Here safety stock is estimated to be twice the sum of the lead time usage plus the usage during the review period. Under this system the formulas are:

$$P = \text{length of review period (in months)} = \sqrt{\frac{24S}{K\bar{M}C}}$$

$$U = 2.3$$

$$T = P\bar{M}$$

$$Q = P\bar{M} + T\bar{M} + U$$

Each system has certain advantages. The fixed order interval system makes it possible to do clerical work on a regular basis when a large number of items must be reviewed. The fixed order quantity system is used to review many low dollar value items in a large inventory on a continuous basis.[16]

<u>Production scheduling with exponential smoothing</u>. In order to balance the cost of change of production and the change in inventory level a system has been designed which accomplishes the following:

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[16] Robert J. Paulis, "Inventory Control," (Kingston, R.I.: University of Rhode Island, 1967), pp. 4-6.

(1) Reviews sales each period while arriving automatically at an individual item forecast.

(2) Provides for smooth ordering or production while maintaining the most economical inventory levels, even in seasonal items.

(3) Reviews a production schedule periodically and compensates for changes in sales.

(4) Balances the two sets of conflicting costs discussed previously so as to minimize cost operations. The formula is:

$$Q = \overline{M}P + R(Id - Ia)$$

where:

Q = Economic order quantity

$\overline{M}$ = Current month forecast of sales

P = Period between schedules (months)

Id = Desired inventory at order points

Ia = Actual inventory on hand and on order

R = A production response factor between zero and one which is proportional to the ratio of inventory change costs to production change costs.[17]

R is taken to be .05 to .2 depending on estimated costs. It is used to spread out inventory and production

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[17]*Ibid.*, pp. 4-5.

change requirements over several schedules. Here I_d includes lead time usage and safety stock, where $\overline{M}$ equals current month forecast, T equals lead time in months and U equals safety stock, so that I_d equals MT + U.

Substituting for I_d:

$$Q = \overline{M}(P + RT) + R(U - I_a)$$

Assigning a value of .2 for α then $\overline{M}$ equals $.2M + .8\overline{M}_2$ and if R equals .2 then we get:

$$Q = (.2M + .8\overline{M}_2)\ (P + .2T) + .2(U - I_a)$$

M = latest actual month's sale

$\overline{M}_2$ = last month's forecast

Utilizing the formula presented earlier for P $\left(P = \sqrt{\frac{24S}{K\overline{M}C}}\right)$ where:

S = Acquisition ordering cost = $50,00

C = Unit cost = $10.00

K = Holding cost = $0.24

$\overline{M}$ = Demand per month = 500 units

Then $P = \sqrt{\frac{24 \times 50}{.24 \times 500 \times 100}} = \sqrt{\frac{1,200}{1,200}}$ = one month.

Our formula now reads:

$$Q = (.2M + .8\overline{M}_2)\ (1.1) + (.2U - .2\ I_a)$$

Table XV, page 69, illustrates how this formula responds to a typical series of sales data. The safety

TABLE XV

EXAMPLE OF PRODUCTION PLANNING

$$Q = (.2M + .8\overline{M}_2)(1.1) + (.2U - .2I_a)$$

Period	Actual Demand M	Current Month Forecast $\overline{M}$	Safety Stock U	Inventory on Hand I_a	Order Quantity Q
1	525				
2	500				
3	225				
4	550				
5	500				
6	300				
7	550				
8	625				
9	600				
10	625				
11	510				
12	490	500	400	1500	328
13	280	456	400	1550	233
14	550	475	400	1271	365
15	525	485	400	1096	403
16	375	463	400	1115	347
17	590	489	400	890	451
18	565	505	350	765	475
19	690	543	350	547	570
20	625	559	350	479	600
21	490	545	350	578	542
22	550	546	350	582	566
23	325	502	350	812	422
24	580	518	410	692	527

stock in this table is determined as the difference between the highest and lowest sales for the last twelve months. This corresponds roughly to a shortage probability of one per cent.

U is determined by using the formula of the statistical small sample theory (t table).

Here we have a simple formula for planning production on a typical item in terms of current demand, demand forecast, safety stock and inventory on hand and on order. It involves only addition, subtraction and multiplication. It can be solved using a computer, slide rule, or calculator. All costs have been balanced. It is efficient, stable, and responds faithfully to all sales trends while keeping production stable and inventories at or near their optimum level.[18]

Figure 5 is a graphic presentation of a material control cycle. It pictures the importance and interrelationship of sales forecasts, production scheduling and materials control. As materials pass through four stages of cycle; raw materials inventory, in-process inventory, finished stock inventory and finally warehouse inventory its time table and size or number is affected by sales forecasts and planning, and actual sales. The amount of

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[18]*Ibid*., p. 5.

goods to be ordered, produced and stocked depends on sales and sales forecasting information correlation with the key control points illustrated.

Double exponential smoothing. Single exponential smoothing considered a process where the model was constant, and there was but one coefficient. However, where a process is changing steadily at a certain rate the rate should be included in a model. Double exponential smoothing, by the addition of a singular linear coefficient extends single exponential smoothing to account for the rate of change. Double exponential smoothing can be defined in the same way as a double moving average. The formulas for double exponential smoothing are:

New single smoothed average = α (new demand) + $(1 - \alpha)$ old single smoothed average

New double smoothed average = α(new single smoothed average) + $(1 - \alpha)$ old double smoothed average

and the formulas for the coefficients are:

Estimated coefficient at time T = $\hat{a}_t$= 2(new single smoothed average) - new double smoothed average

Estimated trend at time T = $\hat{b}_t = \frac{\alpha}{1 - \alpha}$ (new single smoothed average) - new double smoothed average

forecast for sales:[19]

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[19]Robert G. Brown, Smoothing, Forecasting and Pre-

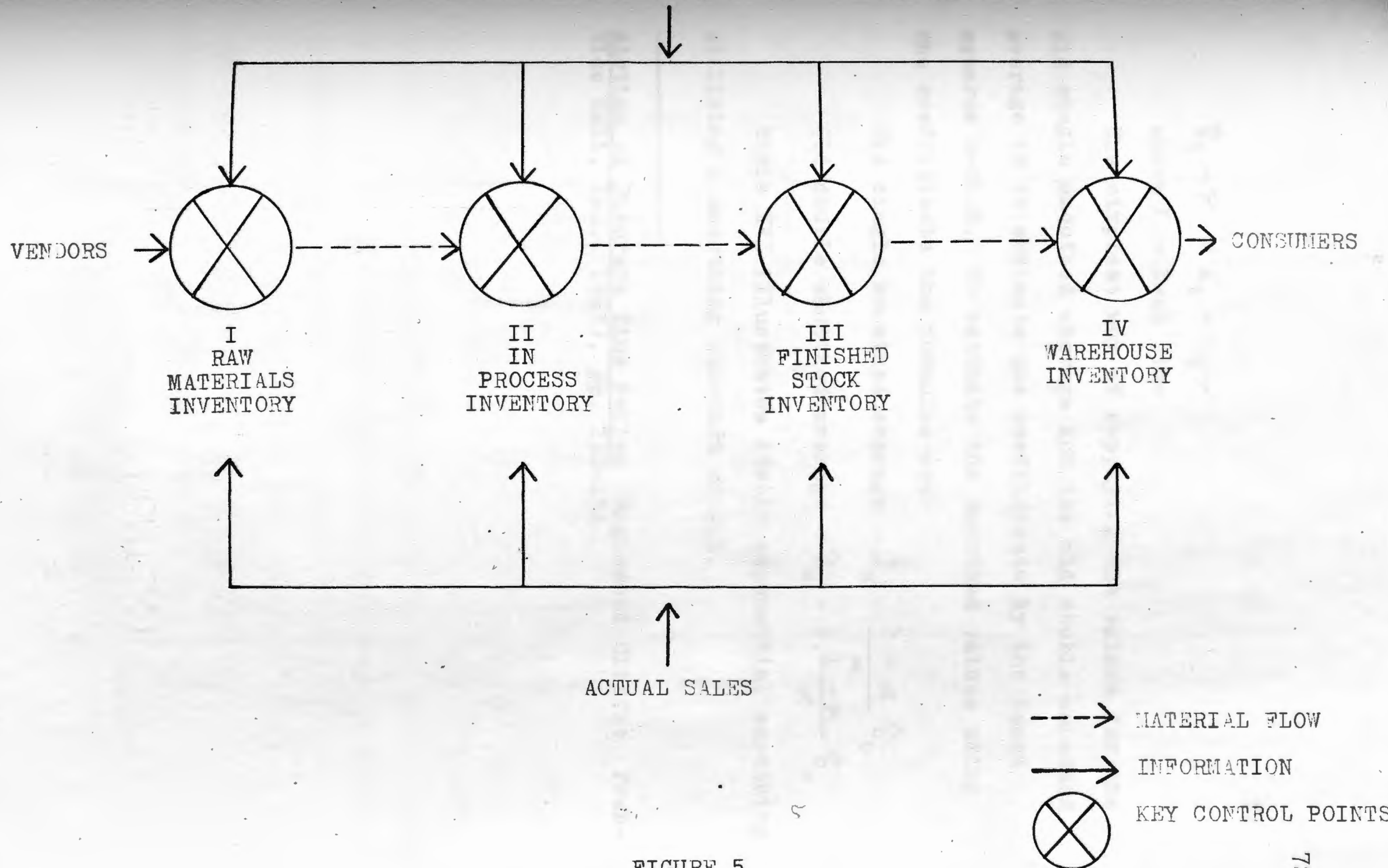

FIGURE 5

THE MATERIALS CONTROL CYCLE
SALES FORECASTS AND PLANNING

$$\hat{x}_{t+\tau} = a_t + b_t \tau$$

where τ = lead time

The simplest way of supplying the values for the old single smoothed average and the old double smoothed average is to estimate the coefficients by the least squares method. To estimate the smoothed values using the coefficients the formulas are:

$$\text{Old single smoothed average} = \hat{a}_0 - \frac{1-\alpha}{\alpha}\hat{b}_0$$

$$\text{Old double smoothed average} = \hat{a}_0 - 2.\frac{1-\alpha}{\alpha}\hat{b}_0$$

Table XVI illustrates double exponential smoothing utilizing a smoothing constant of 0.1.

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diction of Discrete Time Series (Englewood Cliffs: Prentice Hall, Inc., 1963), pp. 132-134.

TABLE XVI

DOUBLE EXPONENTIAL SMOOTHING

Date t	Sales Data x_t	Smoothed Data $S_t(x)$	Double Smoothed Data $S_t^{[2]}(x)$	Coefficients $\hat{a}_t$	Coefficients $\hat{b}_t$	Lead Time τ	Forecast $\hat{a}_t + \hat{b}_t \tau$
13		196.6	196.6	196.6	0	2	196.6
14	189	195.8	196.5	195.1	-0.08	2	194.94
15	244	200.6	196.9	204.3	0.41	2	205.11
16	209	201.4	197.4	205.4	0.44	2	206.28
17	207	202.0	197.9	206.1	0.44	2	207.00
18	211	202.9	198.4	207.4	0.50	2	208.40

Source: Robert G. Brown, _Smoothing, Forecasting and Prediction of Discrete Time Series_ (Englewood Cliffs: Prentice Hall, Inc., 1963), p. 131.

IV

EXPONENTIAL SMOOTHING IN USE

Burroughs Corporation

The Burroughs Corporation, a producer of a wide variety of business machines and data processing equipment, has experienced rapid growth. Burroughs' 1966 sales reached $489,652,523.00, more than double the 1955 figure of $217,805,044. As a service to their computer customers, Burroughs offers an Inventory Control System which utilizes exponential smoothing as a forecasting tool.

In discussing the characteristics of demand the Burroughs Corporation emphasizes that change is a significant characteristic of demand.[1] When selecting a forecast technique this criterion must be considered. Another criterion for a forecasting technique is the accuracy of the forecast. Since accuracy and response to change are opposing choices the management must decide on a system that produces the best mix of these objectives. The accuracy of exponential smoothing can be measured quantitatively.

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[1] Burroughs Inventory Control System (Detroit, Mich.: Burroughs Corporation), Ch. 3, p. 4.
1966,

As stated earlier, of considerable importance in the use of this dynamic forecasting technique is the determination of the smoothing constant which should be used to best cope with the particular circumstances of a given item's requirement. The Burroughs Inventory Control System examines the use of various values for the smoothing constant by presenting a series of programs in their publications. In order to begin the demonstration of response characteristics Burroughs assumes that it has inaccurate information concerning the mean of variance demand characteristics. Table XVII on page 83, describes the results of this analysis.

While the estimate of demand improves steadily in comparison to actual demand there is still substantial error from the actual demand track, even after sixty-five revisions of the forecast. Using the same set of circumstances, including the initial error of the estimate the alpha value of the smoothing constant is increased to .20 in Table XVIII on page 84. By observation, we see that the forecast tended to the actual demand pattern by the end of twelve months. But there was some instability in the continuing forecast of requirements.

Table XIX on page 85 illustrates the results when a demand history has upward trend characteristics. When analyzing the values of these characteristics with a time series approach and plotting the values on the chart it appears obvious that exponential smoothing is constantly lagging the upward trend.

In order to correct for lag due to trend as well as account for forecast errors six additional elements of information are presented for the first time on the right hand side of Table XX, page 86.

The column following forecast error in Table XX represents averages. Since the objective is to produce a forecast for the next demand period, we must add a new trend to the average plus the trend correction. So this column represents the forecast minus the trend adjustment. It will vary from expected demand only when the trend adjustment is used. Following the average is the trend value (computed in the same manner as described in the previous chapter). Sometimes a smoothing constant for the trend is designated by the Greek letter β, (beta). To increase the flexibility of the technique, beta is assigned a different value than the value of alpha. Occasionally the noise magnitude is different with respect to the trend than for the expected value.[2] To simplify the presentation Burroughs allocates the same value of 0.1 to both alpha and beta. The mean absolute

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[2]*Ibid.*, Ch. 3, p. 22.

deviation is next and the steps in the computational procedure are exactly the same as described earlier. Sum Delta is the cumulative error of the estimate. Burroughs has established an exception reporting technique using the Sum Delta information as a multiple of MADs. The next column is the tracking column. It expresses Sum Delta in MADs. The column headed "OC" is the "out of control column," it shows an asterisk when the tracking signal exceeds the predetermined tracking signal, in our illustration, the value of 5.

In Table XX the Sum Delta reflects a value of zero at forecast interval number 16. Here another element of flexibility in this management tool has been added. Since in setting up this particular analysis Burroughs planned to consider the first fifteen stages as a transient period, the length of time allowed for the forecasting technique to home in on actual demand, the error during this period is ignored. The accuracy of this technique is evaluated only after this point. It is generally true that the number of out-of-control signals increases as the accuracy of the forecasting technique increases. As a company's forecasting ability increases the unexplained variations generally decrease. Thus as the ability to forecast item requirements increases, the yardstick which triggers this management exception report shrinks.

A further modification of the computational procedure involved in exponential smoothing can be made to evaluate the existence of a seasonal demand characteristics. Table XXI on page 87 demonstrates the effectiveness of exponential smoothing with a seasonal modification.

The seasonal adjustment factors for each period were determined by a comparison of the demand for the current month to the value of the base series for the current month. A look at the graphic presentation portion of Table XXI reveals that the forecast has been brought in phase with the demand pattern. This advantage was achieved with a small smoothing constant meaning less tracking of noise and improved accuracy.

Table XXII on page 88 presents the analysis of a combination demand pattern. A demand pattern has been structured to contain an obvious seasonal pattern, superimposed on an upward trend. The analysis is made using the exponential smoothing technique with both trend and seasonal modifications. The improved accuracy and response in forecasting the demand for this item with these characteristics would be impossible without application of seasonal indicates.

A computer program developed by Burroughs can analyze the demand characteristics of an item in order that the forecasting techniques of exponential smoothing (single order, trend or second order, average with seasonal, or trend with seasonal) may be chosen which will effectively solve this management problem. Management should also consider the impact of using a range of smoothing constants. The <u>Burroughs Inventory Control System</u> includes a forecast evaluation model which facilitates making a systematic analysis of the effect of varying the options to the basic exponential smoothing techniques previously discussed and the effect of a range of smoothing constants.

The analysis results in Table XXIII on page 89 pertain to the forecast analysis conducted on the item presented in Table XXII. The smoothing constant values to be identified are listed from .10 to .40 in stages of .10 each. Beta values are also stepped up to .40 by .10. Now the model evaluates all of these smoothing constant values for the four (single order, trend or second order, average with seasonal, or trend with seasonal) forecasting techniques. The seasonal ratios to be used are identified. Two rows of information are presented; first, the seasonal ratio and second, the number of business days of demand that the particular ratio is applicable to, so that the seasonal ratios can be tailored to the business habits of an organization. The tracking signal is set at 5.00. The forecast interval is specified as 20 working days - the demands are aggregated in 20 day increments and the forecast of future requirements reassessed at these intervals. The transient period is set at 15.

The summary report, Table XXIV on page 90 representing the model of Table XXIII is of more immediate value to an inventory control manager in reviewing the results of a comprehensive forecast system method evaluation. A review of the summary report indicates that the forecast using trend and seasonal adjustment, with an alpha value of .20 and a beta value of .40 produced an average error of 150 units. It is easy to compare this result with other techniques evaluated. The maximum unit error that occurred throughout the simulation gives an additional element of information of value to management.

There are instances where the technique producing the lowest average unit error does not produce the lowest maximum error. There are situations where management may choose to be protected against wide swings in demand, at the expense of average error. Three columns of information are presented in the summary which deal with measures of the variability of demand in terms of

MAD. The summary report also indicates the number of times we would have received an out-of-control report or exception notice, had we been using an exception technique. The final column of the summary report reflects the final Sum Delta for each analysis, as discussed previously.

Uniroyal, Inc.

Uniroyal is a large, diversified corporation that manufactures everything from tires to sporting goods. World-wide sales of $1,320,794,000 in 1966 were 7.8 per cent higher than in 1965.

Mr. William S. Gere, Jr., Manager of Corporate Systems Development at Uniroyal, Inc., reports that his company uses short range sales forecasts to control production costs. The sales forecast is the first step in Uniroyal's inventory management program and there are two areas where exponential smoothing is applied - canvas footwear and tires. Each month short range computerized forecasts provide an item by item projection of monthly sales. In the footwear area over 1,000 items are being forecasted by style, size, color, and gender. In forecasts for tires the demand for over 1,500 items is being predicted. The forecast is used by plant managers to prepare production schedules for the next six

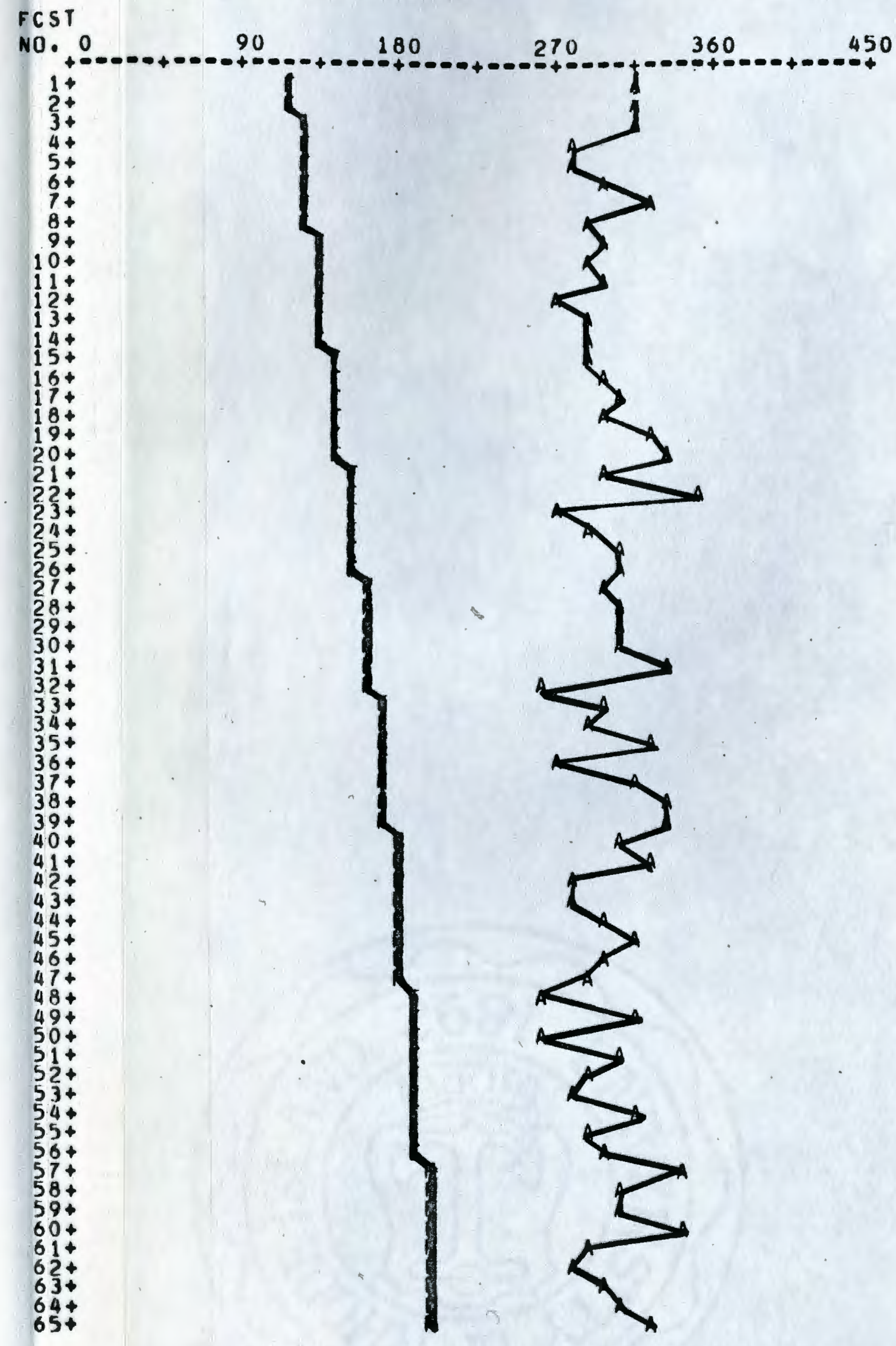
TABLE XVII

AVERAGE OR SINGLE SMOOTHING WITH ALPHA = 0.10

ITEM IDENTIFICATION --- LOW VARIABILITY DEMAND

A = ACTUAL, F = FORECAST

ACTUAL DEMAND	EXPECT DEMAND	FCST ERROR
313	118	+195
317	120	+197
319	122	+197
276	124	+152
275	125	+150
295	127	+168
325	128	+197
286	130	+156
293	132	+161
288	133	+155
294	135	+159
274	136	+138
289	138	+151
285	139	+146
286	141	+145
293	142	+151
308	143	+165
294	145	+149
328	146	+182
329	148	+181
295	150	+145
354	151	+203
273	153	+120
289	155	+134
310	156	+154
304	157	+147
297	159	+138
310	160	+150
310	162	+148
306	163	+143
329	164	+165
264	166	+98
300	167	+133
284	168	+116
320	169	+151
270	171	+99
311	172	+139
333	173	+160
331	175	+156
309	176	+133
325	177	+148
280	179	+101
277	180	+97
298	181	+117
314	182	+132
296	183	+113
289	184	+105
262	185	+77
313	186	+127
258	187	+71
307	188	+119
286	189	+97
276	190	+86
315	191	+124
291	192	+99
297	193	+104
343	194	+149
306	195	+111
308	196	+112
342	197	+145
292	199	+93
281	200	+81
297	200	+97
307	201	+106
328	202	+126



Source: Burroughs Inventory Control System (Detroit; Michigan: Burroughs Corporation, 1966), Ch. 3. p. 4.

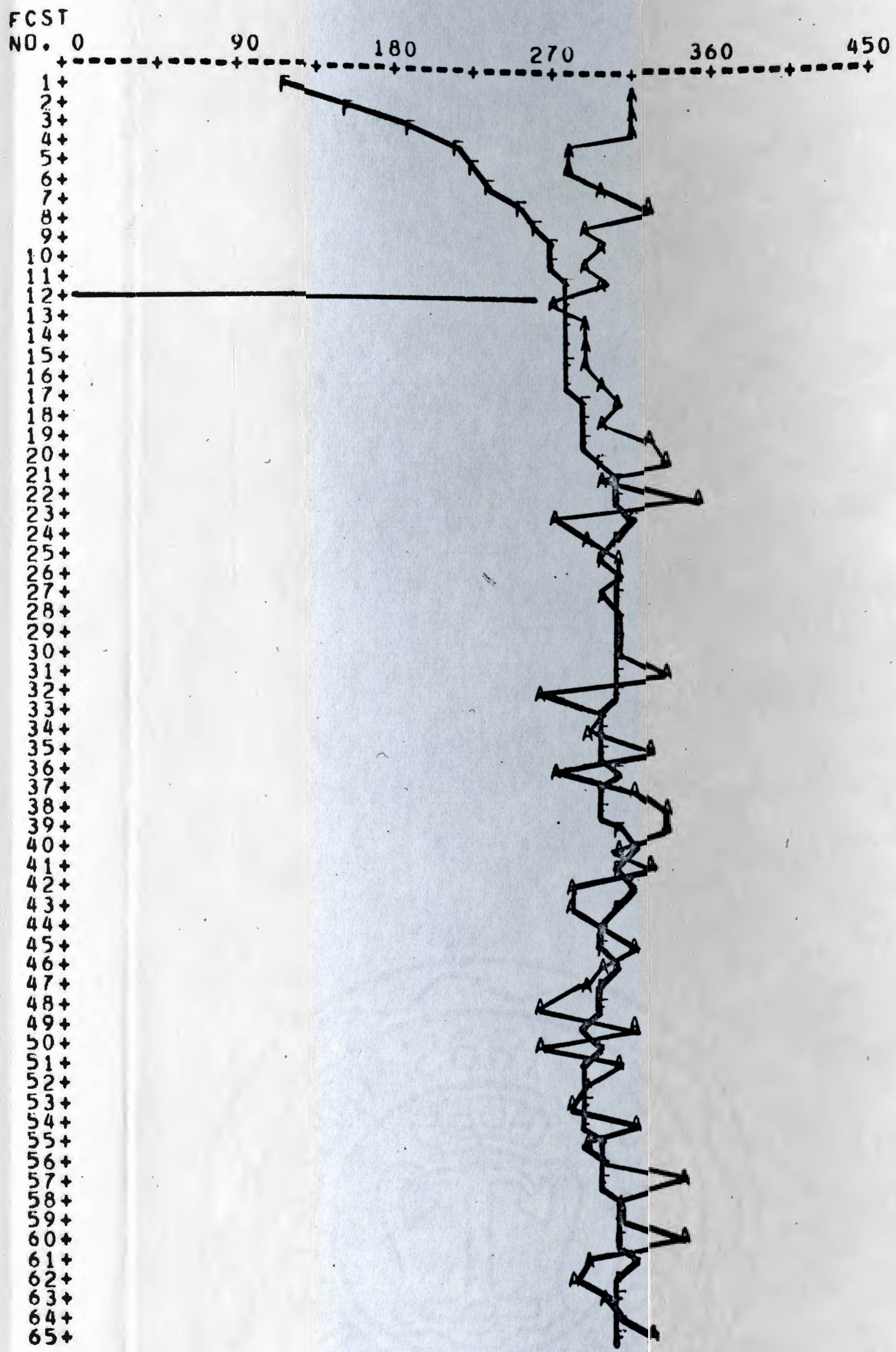
TABLE XVIII

AVERAGE OR SINGLE SMOOTHING WITH ALPHA = 0.20

ITEM IDENTIFICATION --- LOW VARIABILITY DEMAND

ACTUAL DEMAND	EXPECT DEMAND	FCST ERROR
313	118	+195
317	157	+160
319	189	+130
276	215	+61
275	227	+48
295	237	+58
325	248	+77
286	264	+22
293	268	+25
288	273	+15
294	276	+18
274	280	-6
289	279	+10
285	281	+4
286	281	+5
293	282	+11
308	285	+23
294	289	+5
328	290	+38
329	298	+31
295	304	-9
354	302	+52
273	313	-40
289	305	-16
310	301	+9
304	303	+1
297	303	-6
310	302	+8
310	304	+6
306	305	+1
329	305	+24
264	310	-46
300	301	-1
284	301	-17
320	297	+23
270	302	-32
311	295	+16
333	299	+34
331	305	+26
309	311	-2
325	310	+15
280	313	-33
277	307	-30
298	301	-3
314	300	+14
296	303	-7
289	301	-12
262	299	-37
313	292	+21
258	296	-38
307	288	+19
286	292	-6
276	291	-15
315	288	+27
291	293	-2
297	293	+4
343	294	+49
306	303	+3
308	304	+4
342	305	+37
292	312	-20
281	308	-27
297	303	-6
307	302	+5
328	303	+25

A = ACTUAL, F = FORECAST



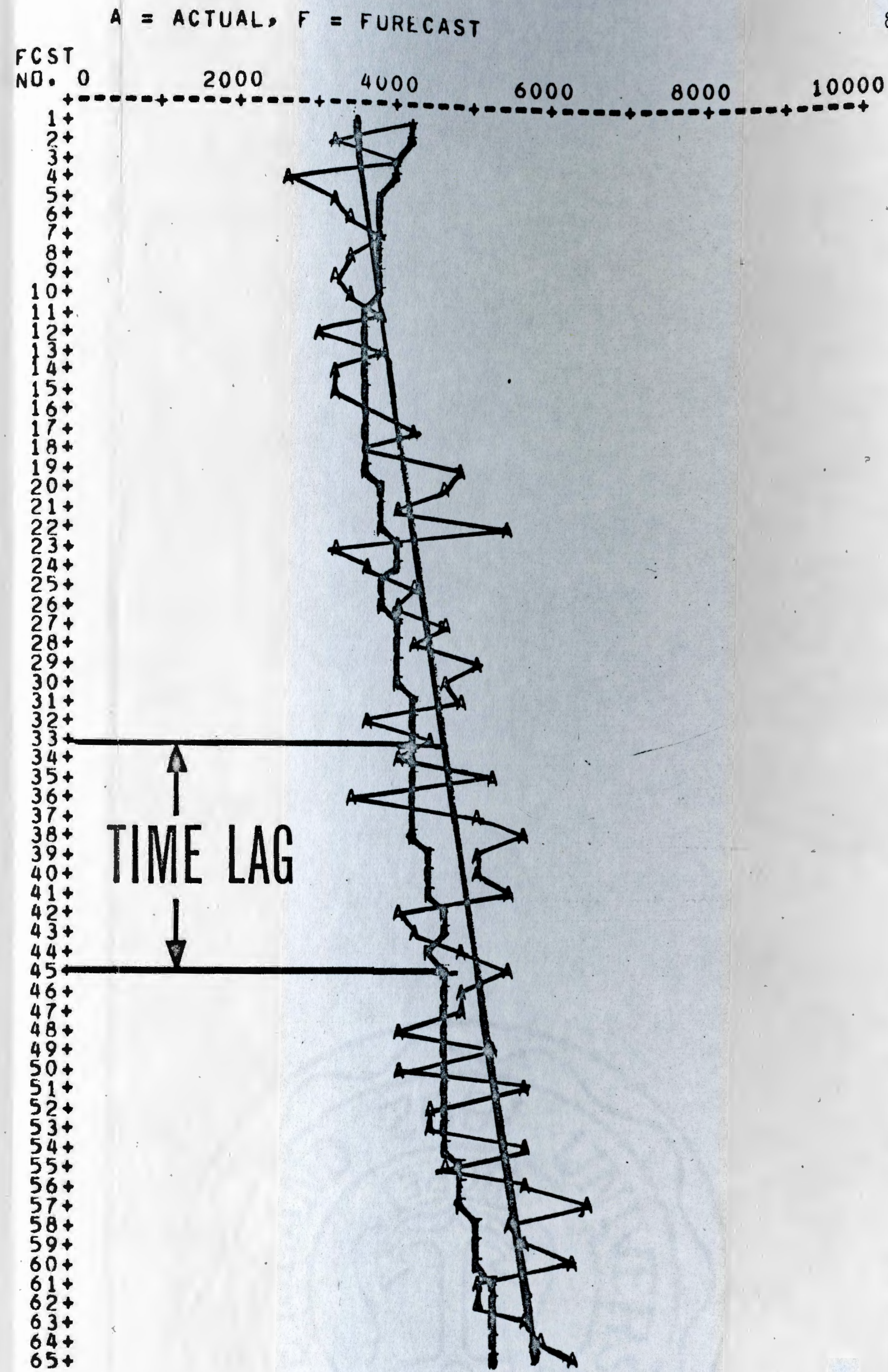
Source: Burroughs Inventory Control System (Detroit; Michigan: Burroughs Corporation, 1966), Ch. 3, p. 6.

TABLE XIX

AVERAGE OR SINGLE SMOOTHING WITH ALPHA = 0.10, UPWARD TRENDING DEMAND

ITEM IDENTIFICATION --- UPWARD TRENDING DEMAND

ACTUAL DEMAND	EXPECT DEMAND	FCST ERROR
4128	4128	+0
3281	4128	-847
3936	4043	-107
2589	4033	-1444
3238	3888	-650
3365	3823	-458
3841	3777	+64
3477	3784	-307
3265	3753	-488
3333	3704	-371
3704	3667	+37
2994	3671	-677
3757	3603	+154
3207	3619	-412
3249	3577	-328
3689	3545	+144
4122	3559	+563
3520	3615	-95
4714	3606	+1108
4529	3717	+812
4019	3798	+221
5316	3820	+1496
3291	3970	-679
3577	3902	-325
4160	3869	+291
3964	3898	+66
4557	3905	+652
4191	3970	+221
4910	3992	+918
4585	4084	+501
4775	4134	+641
3526	4198	-672
4449	4131	+318
3984	4163	-179
5114	4145	+969
3441	4242	-801
4901	4162	+739
5685	4236	+1449
5065	4380	+685
4938	4449	+489
5396	4498	+898
3996	4588	-592
4120	4528	-408
4793	4488	+305
5319	4518	+801
4891	4598	+293
4893	4628	+265
3951	4654	-703
5286	4584	+702
3925	4654	-729
5563	4581	+982
4331	4679	-348
4354	4644	-290
5598	4615	+983
4687	4714	-27
5520	4711	+809
6464	4792	+1672
5364	4959	+405
5642	5000	+642
6272	5064	+1208
5047	5185	-138
5043	5171	-128
5645	5158	+487
5801	5207	+594
6156	5266	+890



Source: Burroughs Inventory Control System (Detroit; Michigan: Burroughs Corporation, 1966), Ch. 3, p. 12.

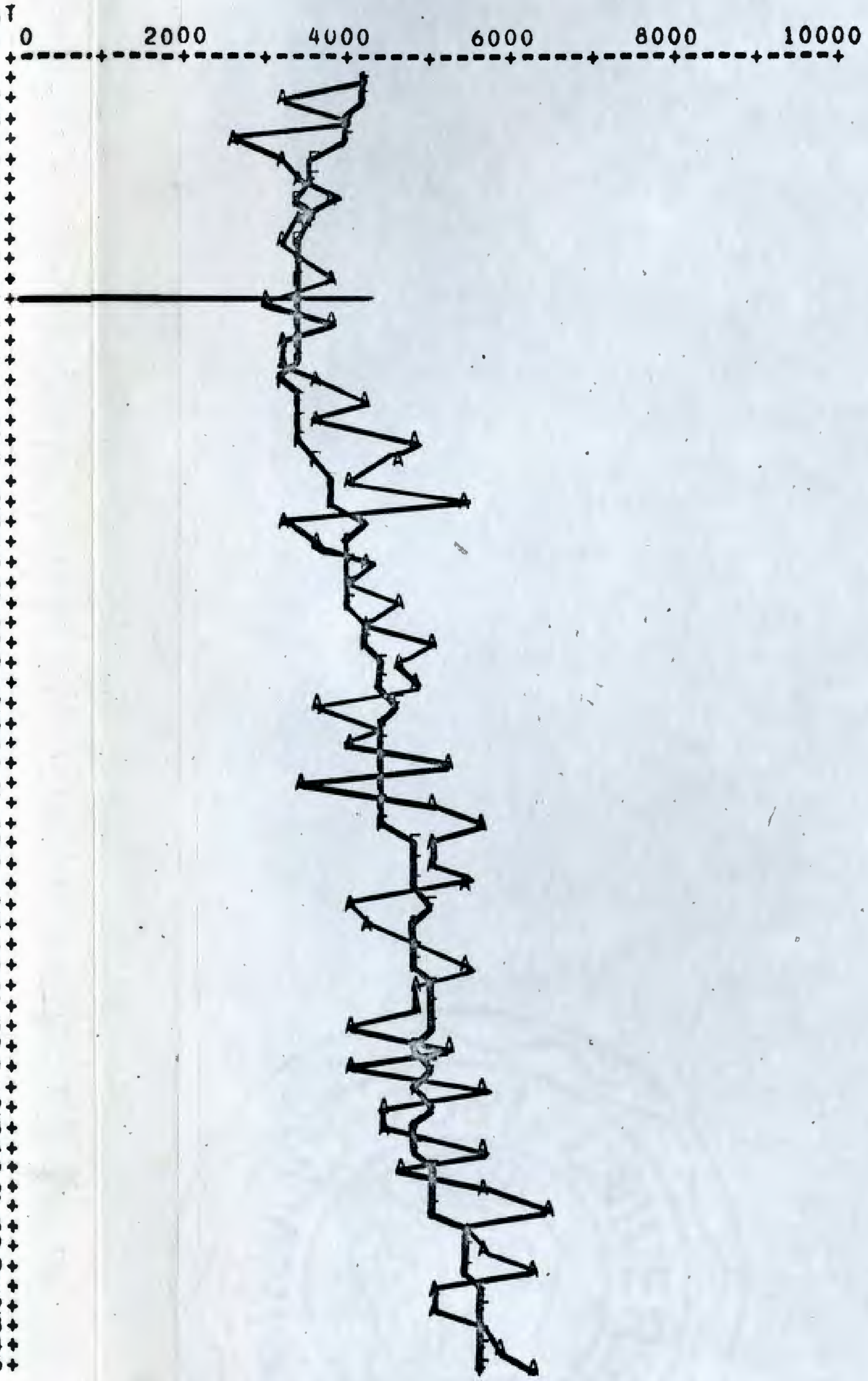
TABLE XX

TREND OR SECOND ORDER SMOOTHING WITH ALPHA = 0.10, BETA = 0.10

ITEM IDENTIFICATION --- UPWARD TRENDING DEMAND

A = ACTUAL, F = FORECAST

ACTUAL DEMAND	EXPECT DEMAND	FCST ERROR	EXP AVERAGE	SMOOTHED TREND	SMOOTHED MAD	SUM DELTA	TRK SIG	O C	FCST NO.
4128	4128	+0	4128.0	+0.00	0.00	+0	0.0	T	1
3281	4128	-847	4128.0	+0.00	0.00	+0	0.0	T	2
3936	3959	-23	4043.3	-8.47	84.70	-847	10.0	T	3
2589	3946	-1357	4032.6	-8.70	78.53	-870	11.0	T	4
3238	3666	-428	3888.2	-22.27	206.38	-2227	10.7	T	5
3365	3558	-193	3823.2	-26.55	228.54	-2655	11.6	T	6
3841	3493	+348	3777.4	-28.48	224.99	-2848	12.6	T	7
3477	3534	-57	3783.8	-25.00	237.29	-2500	10.5	T	8
3265	3497	-232	3753.1	-25.57	219.26	-2557	11.6	T	9
3333	3425	-92	3704.3	-27.89	220.53	-2789	12.6	T	10
3704	3379	+325	3667.2	-28.81	207.68	-2881	13.8	T	11
2994	3415	-421	3670.9	-25.56	219.41	-2556	11.6	T	12
3757	3306	+451	3603.2	-29.77	239.57	-2977	12.4	T	13
3207	3366	-159	3618.6	-25.26	260.71	-2526	9.6	T	14
3249	3309	-60	3577.4	-26.85	250.54	-2685	10.7	T	15
3689	3270	+419	3544.6	-27.45	231.49	+0	0.0		16
4122	3326	+796	3559.0	-23.26	250.24	+419	1.6		17
3520	3462	+58	3615.3	-15.30	304.82	+1215	3.9		18
4714	3459	+1255	3605.8	-14.72	280.14	+1273	4.5		19
4529	3695	+834	3716.6	-2.17	377.63	+2528	6.6	*	20
4019	3860	+159	3797.8	+6.17	423.27	+3362	7.9	*	21
5316	3898	+1418	3819.9	+7.77	396.84	+3521	8.8	*	22
3291	4189	-898	3969.5	+21.95	498.96	+4939	9.8	*	23
3577	4031	-454	3901.7	+12.97	538.86	+4041	7.4	*	24
4160	3954	+206	3869.2	+8.43	530.37	+3587	6.7	*	25
3964	4003	-39	3898.3	+10.50	497.93	+3793	7.6	*	26
4557	4006	+551	3904.9	+10.11	452.04	+3754	8.3	*	27
4191	4126	+65	3970.1	+15.62	461.94	+4305	9.3	*	28
4910	4155	+755	3992.2	+16.27	422.25	+4370	10.3	*	29
4585	4322	+263	4084.0	+23.82	455.53	+5125	11.2	*	30
4775	4399	+376	4134.1	+26.45	436.28	+5388	12.3	*	31
3526	4500	-974	4198.2	+30.21	430.25	+5764	13.3	*	32
4449	4336	+113	4131.0	+20.47	484.63	+4790	9.8	*	33
3984	4379	-395	4162.8	+21.60	447.47	+4903	10.9	*	34
5114	4321	+793	4144.9	+17.65	442.22	+4508	10.1	*	35
3441	4498	-1057	4241.8	+25.58	477.30	+5301	11.1	*	36
4901	4312	+589	4161.7	+15.01	535.27	+4244	7.9	*	37
5685	4445	+1240	4235.6	+20.90	540.64	+4833	8.9	*	38
5065	4714	+351	4380.5	+33.30	610.58	+6073	9.9	*	39
4938	4817	+121	4449.0	+36.82	584.62	+6424	10.9	*	40
5396	4878	+518	4497.9	+38.03	538.26	+6545	12.1	*	41
3996	5020	-1024	4587.7	+43.21	536.23	+7063	13.1	*	42
4120	4858	-738	4528.5	+32.97	585.01	+6039	10.3	*	43
4793	4744	+49	4487.7	+25.59	600.31	+5301	8.8	*	44
5319	4779	+540	4518.2	+26.08	545.18	+5350	9.8	*	45
4891	4913	-22	4598.3	+31.48	544.66	+5890	10.8	*	46
4893	4940	-47	4627.6	+31.26	492.39	+5868	11.9	*	47
3951	4962	-1011	4654.1	+30.79	447.85	+5821	12.9	*	48
5286	4791	+495	4583.8	+20.68	504.17	+4810	9.5	*	49
3925	4910	-985	4654.0	+25.63	503.25	+5305	10.5	*	50
5563	4739	+824	4581.1	+15.78	551.43	+4320	7.8	*	51
4331	4920	-589	4679.3	+24.02	578.69	+5144	8.8	*	52
4354	4826	-472	4644.5	+18.14	579.72	+4555	7.8	*	53
5598	4750	+848	4615.5	+13.42	568.95	+4083	7.1	*	54
4687	4933	-246	4713.8	+21.90	596.86	+4931	8.2	*	55
5520	4906	+614	4711.1	+19.44	561.77	+4685	8.3	*	56
6464	5048	+1416	4792.0	+25.59	566.99	+5299	9.3	*	57
5364	5357	+7	4959.2	+39.75	651.89	+6715	10.3	*	58
5642	5398	+244	4999.7	+39.82	587.40	+6722	11.4	*	59
6272	5487	+785	5063.9	+42.26	553.06	+6966	12.5	*	60
5047	5686	-639	5184.7	+50.12	576.25	+7751	13.4	*	61
5043	5608	-565	5170.9	+43.73	582.53	+7112	12.2	*	62
5645	5539	+106	5158.1	+38.08	580.78	+6547	11.2	*	63
5801	5598	+203	5206.8	+39.14	533.30	+6653	12.4	*	64
6156	5678	+478	5266.2	+41.17	500.27	+6856	13.7	*	65



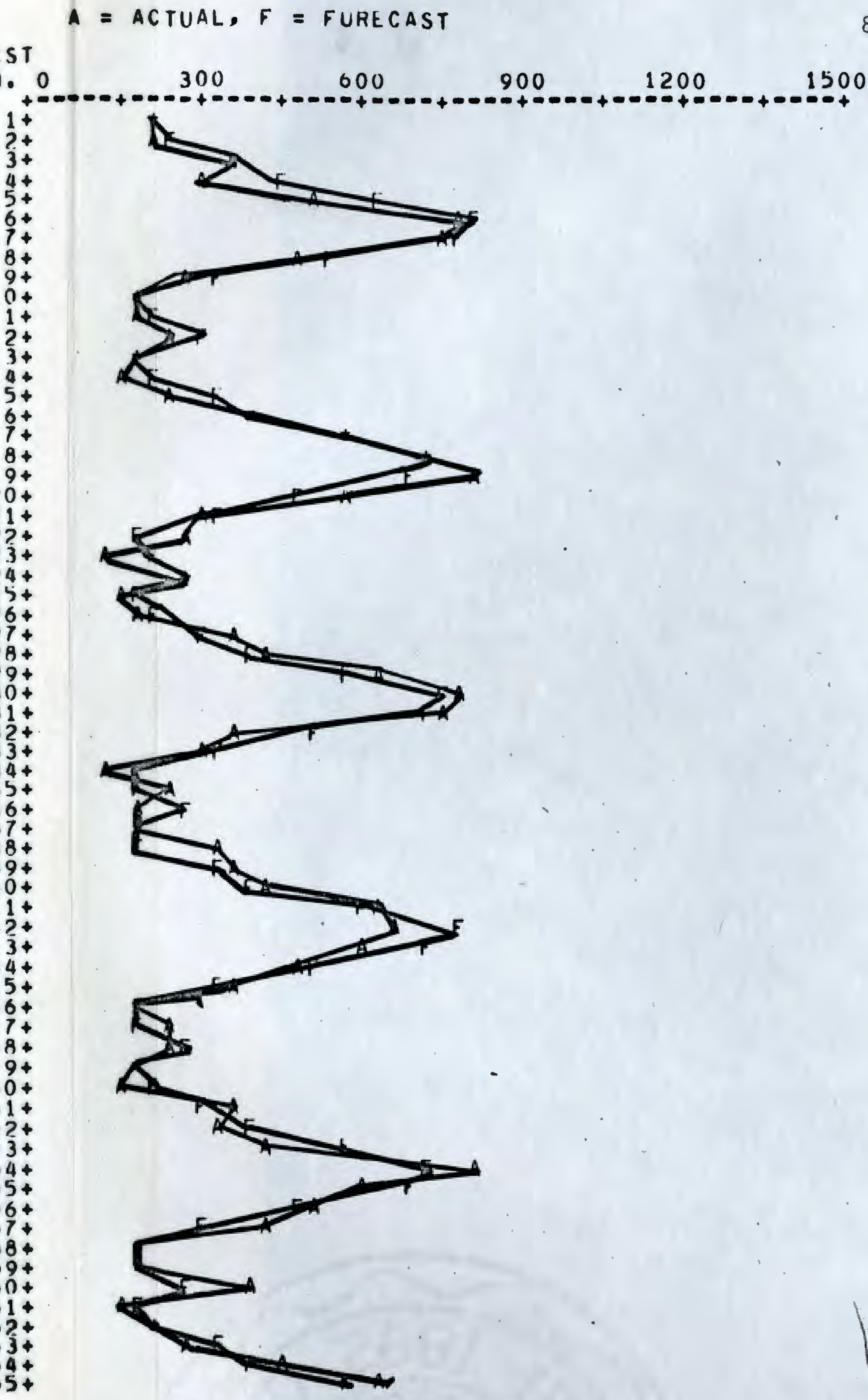
Source: Burroughs Inventory Control System (Detroit; Michigan: Burroughs

TABLE XXI

AVFRAGE WITH SEASONAL WITH ALPHA = 0.10

ITEM IDENTIFICATION --- LAWN FURNITURE

ACTUAL DEMAND	EXPECT DEMAND	FCST ERROR	EXP AVERAGE	SMOOTHED TREND	SMOOTHED MAD	SUM DELTA	TRK SIG	U C	FCST NO.
212	212	+0	451.0	+0.00	0.00	+0	0.0	T	1+
200	239	-39	451.0	+0.00	0.00	+0	0.0	T	2+
353	373	-20	443.6	+0.00	3.90	-39	10.0	T	3+
308	454	-146	441.2	+0.00	5.51	-59	10.7	T	4+
511	640	-129	426.9	+0.00	19.56	-205	10.4	T	5+
770	815	-45	418.2	+0.00	30.50	-334	10.9	T	6+
738	769	-31	415.8	+0.00	31.95	-379	11.8	T	7+
473	538	-65	414.1	+0.00	31.85	-410	12.8	T	8+
261	344	-83	409.0	+0.00	35.17	-475	13.5	T	9+
182	192	-10	399.1	+0.00	39.95	-558	13.9	T	10+
190	199	-9	397.1	+0.00	36.95	-568	15.3	T	11+
236	285	-49	395.3	+0.00	34.15	-577	16.8	T	12+
184	183	+1	388.5	+0.00	35.64	-626	17.5	T	13+
152	206	-54	388.7	+0.00	32.18	-625	19.4	T	14+
231	318	-87	378.5	+0.00	34.36	-679	19.7	T	15+
398	379	+19	368.1	+0.00	39.62	+0	0.0		16+
582	555	+27	369.9	+0.00	37.56	+19	0.5		17+
716	725	-9	371.7	+0.00	36.50	+46	1.2		18+
796	687	+109	371.2	+0.00	33.75	+37	1.0		19+
558	490	+68	377.1	+0.00	41.28	+146	3.5		20+
300	321	-21	382.3	+0.00	43.95	+214	4.8		21+
279	182	+97	379.7	+0.00	41.65	+193	4.6		22+
128	200	-72	399.8	+0.00	47.19	+290	6.1	*	23+
265	277	-12	385.4	+0.00	49.67	+218	4.3		24+
136	180	-44	383.6	+0.00	45.90	+206	4.4		25+
183	198	-15	374.1	+0.00	45.71	+162	3.5		26+
345	312	+33	371.2	+0.00	42.64	+147	3.4		27+
418	386	+32	375.1	+0.00	41.68	+180	4.3		28+
634	567	+67	378.1	+0.00	40.71	+212	5.2	*	29+
787	746	+41	382.5	+0.00	43.34	+279	6.4	*	30+
751	712	+39	384.6	+0.00	43.11	+320	7.4	*	31+
371	503	-132	386.7	+0.00	42.70	+359	8.4	*	32+
300	316	-16	376.5	+0.00	51.63	+227	4.3		33+
110	180	-70	374.5	+0.00	48.07	+211	4.3		34+
239	180	+59	359.9	+0.00	50.26	+141	2.8		35+
186	268	-82	371.7	+0.00	51.13	+200	3.9		36+
191	169	+22	360.3	+0.00	54.22	+118	2.1		37+
324	193	+131	364.9	+0.00	51.00	+140	2.7		38+
345	327	+18	389.5	+0.00	59.00	+271	4.5		39+
434	403	+31	391.6	+0.00	54.90	+289	5.2	*	40+
628	592	+36	394.5	+0.00	52.51	+320	6.0	*	41+
654	774	-120	396.9	+0.00	50.86	+356	6.9	*	42+
592	723	-131	390.7	+0.00	57.77	+236	4.0		43+
471	499	-28	383.6	+0.00	65.09	+105	1.6		44+
358	320	+38	381.4	+0.00	61.38	+77	1.2		45+
166	185	-19	385.8	+0.00	59.04	+115	1.9		46+
234	191	+43	381.8	+0.00	55.04	+96	1.7		47+
234	281	-47	390.4	+0.00	53.84	+139	2.5		48+
189	180	+9	383.8	+0.00	53.16	+92	1.7		49+
142	204	-62	385.6	+0.00	48.74	+101	2.0		50+
352	314	+38	373.8	+0.00	50.07	+39	0.7		51+
341	390	-49	378.3	+0.00	48.86	+77	1.5		52+
432	560	-128	373.5	+0.00	48.87	+28	0.5		53+
811	712	+99	364.9	+0.00	56.78	-100	1.7		54+
612	684	-72	369.9	+0.00	61.00	-1	0.0		55+
510	476	+34	365.9	+0.00	62.10	-73	1.1		56+
426	310	+116	368.5	+0.00	59.29	-39	0.6		57+
179	184	-5	382.3	+0.00	64.96	+77	1.1		58+
184	191	-7	381.3	+0.00	58.96	+72	1.2		59+
381	274	+107	379.9	+0.00	53.76	+65	1.2		60+
144	186	-42	394.8	+0.00	59.08	+172	2.9		61+
206	205	+1	385.9	+0.00	57.37	+130	2.2		62+
276	324	-48	386.1	+0.00	51.73	+131	2.5		63+
447	392	+55	380.3	+0.00	51.36	+83	1.6		64+
623	578	+45	385.6	+0.00	51.72	+138	2.6		65+



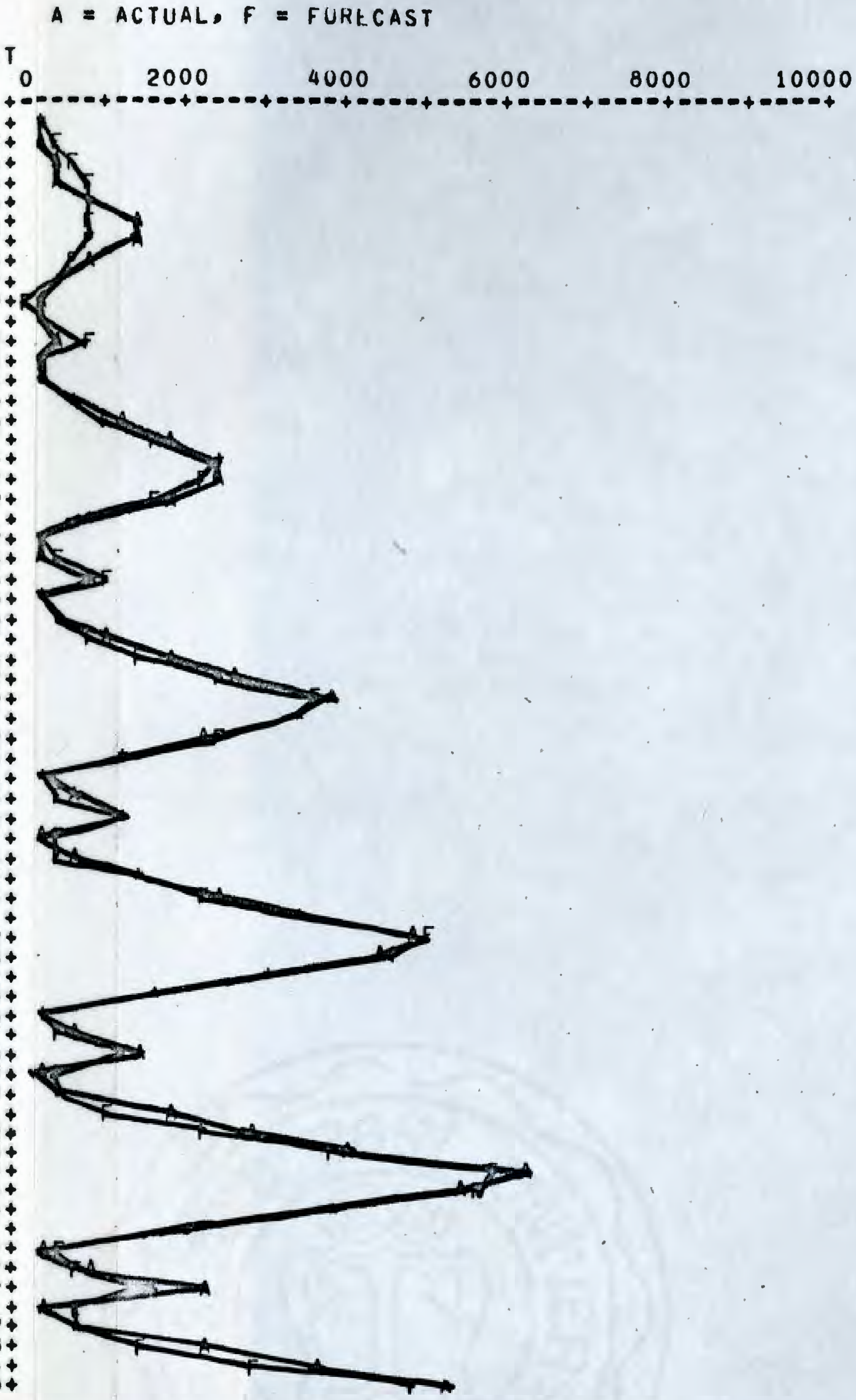
Source: Burroughs Inventory Control System (Detroit; Michigan: Burrou[ghs] Corporation, 1966), Ch. 3, p. 17.

TABLE XXII

TREND WITH SEASONAL WITH ALPHA = 0.20, BETA = 0.40

ITEM IDENTIFICATION --- GOOD LAWN FURNITURE

ACTUAL DEMAND	EXPECT DEMAND	FCST ERROR	EXP AVERAGE	SMOOTHED TREND	SMOOTHED MAD	SUM DELTA	TRK SIG	O C	FCST NO.
212	212	+0	1177.7	+0.00	0.00	+0	0.0	T	1
208	353	-145	1177.7	+0.00	0.00	+0	0.0	T	2
423	683	-260	1080.8	-38.75	29.00	-145	5.0	T	3
455	749	-294	974.5	-65.77	75.20	-405	5.3	T	4
773	723	+50	858.0	-86.05	118.96	-699	5.8	T	5
1314	866	+448	777.9	-83.69	105.17	-649	6.1	T	6
1313	856	+457	731.4	-68.83	173.74	-201	1.1	T	7
853	642	+211	703.9	-52.28	230.39	+256	1.1	T	8
467	358	+109	680.8	-40.62	226.51	+467	2.0	T	9
182	68	+114	669.2	-29.03	203.01	+576	2.8	T	10
262	276	-14	815.4	+41.05	185.21	+690	3.7	T	11
487	712	-225	846.4	+37.02	150.97	+676	4.4	T	12
184	157	+27	818.3	+10.96	165.78	+451	2.7	T	13
230	292	-62	859.1	+22.89	138.02	+478	3.4	T	14
600	672	-72	840.6	+6.33	122.82	+416	3.3	T	15
1117	954	+163	828.3	-1.11	112.66	+0	0.0		16
1729	1531	+198	855.2	+10.10	122.73	+163	1.3		17
2452	2377	+75	888.8	+19.48	137.78	+361	2.6		18
2459	2264	+195	914.5	+21.98	125.22	+436	3.4		19
1703	1594	+109	954.1	+29.04	139.18	+631	4.5		20
851	880	-29	998.2	+35.05	133.14	+740	5.5	*	21
279	154	+125	1025.5	+31.94	112.31	+711	6.3	*	22
266	484	-218	1249.6	+108.81	114.85	+836	7.2	*	23
872	978	-106	1196.7	+44.13	135.48	+618	4.5		24
136	246	-110	1210.1	+31.84	129.58	+512	3.9		25
355	310	+45	1119.2	-17.26	125.66	+402	3.1		26
1065	852	+213	1132.0	-5.23	109.53	+447	4.0		27
1736	1470	+266	1182.2	+16.95	130.22	+660	5.0		28
2580	2403	+177	1245.1	+35.31	157.38	+926	5.8	*	29
3734	3663	+71	1301.4	+43.71	161.30	+1103	6.8	*	30
3495	3495	+0	1351.0	+46.06	143.24	+1174	8.1	*	31
2201	2360	-159	1397.1	+46.07	114.59	+1174	10.2	*	32
1212	1206	+6	1421.3	+37.31	123.47	+1015	8.2	*	33
110	215	-105	1460.2	+37.96	99.98	+1021	10.2	*	34
521	326	+195	1337.4	-26.35	100.98	+916	9.0	*	35
1143	1113	+30	1455.8	+31.57	119.78	+1111	9.2	*	36
191	301	-110	1495.9	+35.00	101.82	+1141	11.2	*	37
621	402	+219	1408.9	-13.78	103.46	+1031	9.9	*	38
1450	1358	+92	1541.1	+44.62	126.57	+1250	9.8	*	39
2391	2181	+210	1609.5	+54.13	119.66	+1342	11.2	*	40
3462	3452	+10	1699.8	+68.61	137.73	+1552	11.2	*	41
4802	5097	-295	1769.5	+69.06	112.18	+1562	13.9	*	42
4392	4664	-272	1814.1	+59.28	148.74	+1267	8.5	*	43
3066	3039	+27	1848.7	+49.42	173.39	+995	5.7	*	44
1619	1617	+2	1901.8	+50.91	144.11	+1022	7.0	*	45
166	287	-121	1953.2	+51.09	115.69	+1024	8.8	*	46
551	459	+92	1817.9	-23.45	116.75	+903	7.7	*	47
1456	1298	+158	1862.5	+3.75	111.80	+995	8.8	*	48
189	364	-175	1912.0	+22.06	121.04	+1153	9.5	*	49
447	438	+9	1739.6	-55.72	131.83	+978	7.4	*	50
1808	1095	+713	1689.7	-53.40	107.26	+987	9.2	*	51
2844	2232	+612	1821.4	+20.62	228.41	+1700	7.4	*	52
4098	3822	+276	1947.5	+62.80	305.13	+2312	7.5	*	53
6157	5838	+319	2043.0	+75.86	299.30	+2588	8.6	*	54
5499	5696	-197	2145.3	+86.45	303.24	+2907	9.5	*	55
3818	3785	+33	2213.9	+79.30	281.99	+2710	9.6	*	56
2080	2027	+53	2297.7	+81.12	232.19	+2743	11.8	*	57
179	367	-188	2392.8	+86.72	196.35	+2796	14.2	*	58
700	552	+148	2189.6	-29.24	194.68	+2608	13.3	*	59
2127	1617	+510	2270.2	+14.69	185.34	+2756	14.8	*	60
144	504	-360	2432.7	+73.81	250.27	+3266	13.0	*	61
637	502	+135	2106.2	-86.33	272.22	+2906	10.6	*	62
2102	1430	+672	2109.6	-50.43	244.78	+3041	12.4	*	63
3582	2703	+879	2233.6	+19.36	330.22	+3713	11.2	*	64
5114	4739	+375	2404.5	+79.96	439.98	+4592	10.4	*	65



Source: Burroughs Inventory Control System (Detroit: Michigan: Burroughs

TABLE XXIII

FORECASTING METHOD ANALYSIS - BURROUGHS CORP., MANAGEMENT SCIENCE DEPT.

ITEM IDENTIFICATION --- GOOD LAWN FURNITURE

THE FOLLOWING METHODS WILL BE USED:

AVERAGE OR SINGLE SMOOTHING
TREND OR SECOND ORDER SMOOTHING
AVERAGE WITH SEASONAL
TREND WITH SEASONAL

THE FOLLOWING PARAMETERS WILL BE USED:

ALPHA FROM .10 TO .40 BY STEPS OF .10
BETA FROM .10 TO .40 BY STEPS OF .10
FORECAST INTERVAL 20
TRACKING SIGNAL LIMIT 5.0
LENGTH OF TRANSIENT 15
BUSINESS DAYS PER YEAR 240

THE FOLLOWING OPTIONS WILL BE USED:

DETAIL REPORT
INITIALIZATION

THE FOLLOWING SEASONALS WILL BE USED:

PER VAL	1 DAT	PER VAL	2 DAT	PER VAL	3 DAT	PER VAL	4 DAT	PER VAL	5 DAT	PER VAL	6 DAT	PER VAL	7 DAT	PER VAL	8 DAT	PER VAL	9 DAT	PER VAL	10 DAT	PER VAL	11 DAT	PER VAL	12 DAT
0.18	20	0.30	40	0.77	60	1.16	80	1.69	100	2.41	120	2.21	140	1.45	160	0.75	180	0.13	200	0.27	220	0.69	240

Source: Burroughs Inventory Control System (Detroit; Michigan: Burroughs Corporation, 1966), Ch. 3, p. 19.

TABLE XXIV

SUMMARY REPORT

ITEM IDENTIFICATION --- GOOD LAWN FURNITURE

FCST METH	USE SEAS	S/C ALPHA	S/C BETA	UNIT-ERROR AVG	UNIT-ERROR MAX	PERCENT-ERROR AVG	PERCENT-ERROR MAX	MEAN-ABS-DEV AVG	MEAN-ABS-DEV MAX	NO. O-C	END-OF-RUN VALUES EXPECT	AVERAGE	TREND	MAD	SUMDELTA
AVRG	NO	.10		1158	+4843	114.8%	900.0%	895.41	2128.04	54	2745	2745.4	+0.00	2087.52	+23063
AVRG	NO	.20		1155	+4565	122.6%	814.8%	1005.09	2502.61	27	2659	2659.2	+0.00	2397.11	+11070
AVRG	NO	.30		1124	+4226	121.0%	825.1%	1017.28	2669.12	12	2265	2264.9	+0.00	2522.98	+6141
AVRG	NO	.40		1093	+3824	110.9%	814.0%	1014.72	2923.20	8	1791	1791.2	+0.00	2456.32	+3432
TRND	NO	.10	.10	1197	+4485	146.5%	941.8%	921.86	2152.37	14	2977	2745.5	+23.13	2210.63	+1674
TRND	NO	.10	.20	1210	+4287	118.1%	795.8%	926.89	2234.77	8	2572	2745.5	-17.32	2268.49	-929
TRND	NO	.10	.30	1179	+3919	105.6%	747.6%	898.49	2253.25	5	2025	2745.5	-72.06	2243.62	-2326
TRND	NO	.10	.40	1126	+3484	116.6%	950.8%	860.44	2182.78	5	1473	2745.5	-127.22	2124.20	-3116
TRND	NO	.20	.10	1210	+4286	118.1%	795.8%	1046.98	2620.94	2	2573	2659.0	-17.31	2611.15	-928
TRND	NO	.20	.20	1211	+3988	111.0%	802.4%	1044.50	2769.85	2	2073	2659.0	-117.16	2669.68	-2702
TRND	NO	.20	.30	1177	+3551	109.7%	974.3%	1017.11	2766.62	1	1477	2659.0	-236.32	2561.70	-3719
TRND	NO	.20	.40	1112	+3083	120.1%	921.2%	966.82	2651.70	1	924	2659.0	-347.02	2343.36	-4211
TRND	NO	.30	.10	1179	+3919	105.6%	747.6%	1063.18	2997.96	0	2025	2265.1	-72.07	2745.67	-2326
TRND	NO	.30	.20	1177	+3551	109.7%	974.3%	1063.32	3112.79	0	1477	2265.1	-236.33	2701.55	-3720
TRND	NO	.30	.30	1135	+3082	122.7%	963.7%	1030.94	3060.91	1	882	2265.1	-414.85	2484.94	-4455
TRND	NO	.30	.40	1077	+2615	103.7%	781.6%	987.29	2922.45	1	370	2265.1	-568.43	2189.11	-4699
TRND	NO	.40	.10	1126	+3484	116.6%	950.8%	1044.28	3212.67	0	1474	1791.4	-127.19	2566.40	-3114
TRND	NO	.40	.20	1112	+3083	120.1%	921.2%	1036.77	3269.83	0	924	1791.4	-347.00	2405.90	-4212
TRND	NO	.40	.30	1077	+2615	103.7%	781.6%	1012.68	3184.27	1	370	1791.4	-568.41	2101.76	-4697
TRND	NO	.40	.40	1045	+2214	101.3%	981.7%	989.18	3018.21	0	70	1791.4	-744.69	1881.73	-4720
AVRG	YES	.10		387	+1661	22.9%	175.7%	322.52	827.00	51	1703	2468.8	+0.00	657.59	+25544
AVRG	YES	.20		286	+1234	21.6%	204.2%	262.33	746.86	51	1773	2569.9	+0.00	441.80	+17782
AVRG	YES	.30		234	+991	21.1%	217.4%	223.60	689.11	51	1753	2540.3	+0.00	287.11	+13839
AVRG	YES	.40		199	+935	21.0%	227.1%	194.01	641.78	50	1716	2487.3	+0.00	197.14	+11163
TRND	YES	.10	.10	229	+949	21.0%	223.6%	194.71	451.64	53	1896	2469.3	+27.88	354.29	+13300
TRND	YES	.10	.20	189	+880	20.5%	231.9%	158.12	401.32	53	1843	2469.3	+20.15	310.56	+10023
TRND	YES	.10	.30	171	+902	20.7%	238.2%	144.63	366.18	53	1778	2469.3	+10.68	277.96	+7984
TRND	YES	.10	.40	155	+879	21.0%	244.4%	134.50	341.42	52	1720	2469.3	+2.41	245.72	+6365
TRND	YES	.20	.10	189	+880	20.5%	231.9%	175.22	504.86	52	1843	2570.1	+20.16	275.37	+10026
TRND	YES	.20	.20	170	+922	20.7%	238.9%	158.60	474.20	51	1779	2570.1	+1.69	248.99	+7652

TABLE XXIV (Continued)

SUMMARY REPORT

ITEM IDENTIFICATION --- GOOD LAWN FURNITURE

	FCST METH	USE SEAS	ALPHA	S/C BETA	UNIT-ERROR AVG	UNIT-ERROR MAX	PERCENT-ERROR AVG	PERCENT-ERROR MAX	MEAN-ABS-DEV AVG	MEAN-ABS-DEV MAX	NO. O-C	END-OF-RUN VALUES EXPECT	AVERAGE	TREND	MAD	SUMDELTA
	TRND	YES	.20	.30	156	+923	21.1%	243.8%	147.79	456.63	49	1712	2570.1	-17.76	222.21	+5951
1ST	TRND	YES	.20	.40	150	+879	21.6%	250.0%	142.23	439.98	46	1659	2570.1	-33.08	224.00	+4535
	TRND	YES	.30	.10	171	+902	20.7%	238.2%	164.37	539.17	51	1778	2540.5	+10.70	202.07	+7985
	TRND	YES	.30	.20	156	+923	21.1%	243.8%	151.25	527.50	49	1712	2540.5	-17.75	188.40	+5951
2ND	TRND	YES	.30	.30	152	+900	21.7%	249.3%	147.23	534.91	44	1650	2540.5	-44.83	202.69	+4402
	TRND	YES	.30	.40	155	+835	22.5%	256.3%	148.73	534.88	35	1606	2540.5	-63.70	236.53	+3115
	TRND	YES	.40	.10	155	+879	21.0%	244.4%	151.71	575.23	51	1720	2487.4	+2.43	167.36	+6363
1ST	TRND	YES	.40	.20	150	+879	21.6%	250.0%	146.38	601.73	43	1659	2487.4	-33.06	185.66	+4536
	TRND	YES	.40	.30	155	+835	22.5%	256.3%	150.22	606.86	37	1606	2487.4	-63.70	223.38	+3116
	TRND	YES	.40	.40	158	+793	23.3%	264.6%	152.01	590.73	27	1576	2487.4	-81.35	252.74	+1977

Source: Burroughs Inventory Control System (Detroit; Michigan: Burroughs Corporation, 1966), Ch. 3, pp. 24-25.

months. Balancing the forecast against goods on hand, the plant manager can schedule future production to avoid the danger of over- or under-stocking when orders start flowing in. Although exponential smoothing has been applied only to these two areas of high volume sales Uniroyal has plans to use this techniques for other products later. Exponential smoothing was introduced at Uniroyal in 1961 because the manual forecasting techniques then in use proved to be clumsy, time consuming and inaccurate. Prior to the introduction of exponential smoothing Uniroyal's sales department relied upon methods based on judgement applied to the previous year's sales. Ordinarily data for two years (monthly) are included in the estimate of the moving average. Using the IBM 360 computer the smoothing constant, delta factor and R factor (all mentioned in Chapter III) are calculated easily and quickly. In forecasting demand for tires Uniroyal has found it necessary to change the value of the smoothing constant, (increase) when error in the forecast exceeds predetermined limits. Like most companies, Uniroyal has found single smoothing to be quite satisfactory. Trends and seasonal effects are included in the smoothing technique. In the footwear division it has been found advantageous to make adjustment for seasonal effects during the fall when, "back to school" business increases the demand for these canvas products. To give such a period its proper attention and prevent its weighted index from providing a false picture, each month's sales are smoothed out by seasonal factors. A base series of two or more years is computed when accounting for the seasonal effects. Forecast error is noted in all smoothing calculations and the error is calculated monthly for each item in order to determine the safety stocks required.

Briefly summarizing Uniroyal's implementation of exponential smoothing, the newest sales data is each month fed into the computer. Then weighted seasonal and trend factors for each product are calculated. Thus, each monthly forecast has the value of being based on the latest known facts. In turn, this revises the monthly forecast so that the reports on sales predicted for April in March differs only slightly from what was indicated in the February print-out.

What has exponential smoothing meant to Uniroyal?

1. Reduction in paper work.
2. More efficient handling of estimates in the sales department.
3. Better production scheduling, hence, improved customer service.

Uniroyal has achieved a unique way of forecasting sales for new products. In the footwear division, Uniroyal has found that by matching the new product against a forecast of a product it most closely resembles an accurate forecast for the new product can be achieved. The same numerical factors are used until enough sales data on the new product has been amassed to allow the computer to calculate pertinent, weighted, seasonal and trend averages. At the same time, the sales department is making its own forecasts, though not in the detail provided by inventory control's prediction. If the two projections vary sharply Gere's people sit down with sales people and find out the reason for the departure.[3]

Honeywell, Inc.

Honeywell, Inc. makes thousands of different electronic, electrical and pneumatic types and models of automatic control instruments. 1966 sales, $914,384,094 are five times greater than sales in 1952 of $165,710,384.

Honeywell's Heating and Air Conditioning Division finds itself in an expanding market. It manufactures over 7,000 relatively low-priced items ranging from cooling and air conditioning controls to swimming pool controls. Honeywell introduced exponential smoothing in

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[3]Letter from William S. Gere of Uniroyal, Inc. dated October 17, 1967.

1964 to achieve a sales forecast improvement. Using data processing equipment the sales forecasting department has found that all 7,000 products can be easily forecasted with a large reduction of clerical work. They have tried multiple exponential smoothing techniques but dropped it rather soon when double and/or triple exponential smoothing proved of little additional value while complicating the system. The smoothing constant is calculated (rather than estimated) on the basis of selected observations and history and demand rates for their products. If necessary, they may change the smoothing constant for any of their 7,000 products to respond to trends. They include the past three months of actual demand in their initial estimate of the moving average. A forecast error is also calculated.[4]

<u>Eli Lilly Company</u>

The Eli Lilly Company is a large diversified drug corporation (1966 sales of $366 million). In the United States they have about 950 prescription drug products in over 2,000 package sizes. Exponential smoothing was introduced at the Lilly Company in January, 1967 after experimentation before introduction revealed that exponential smoothing yielded better forecasts and faster re-

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[4]Letter from James A. Seabloom of Honeywell, Inc.,

covery for inventory control purposes than other conventional methods of forecasting. Prior to the exponential smoothing utilization Lilly relied on graphic forecasts entered into a computer program which generated monthly production requirements on the basis of predicted sales inventory levels, withdrawals, and other production constraints, such as uniform workload, optimum lot size, etc.

Lilly uses a 12 month moving average in order to deseasonalize the data. The company does not calculate the smoothing constant. Rather, it is available on file for each time series. Although the smoothing constant is not much adaptive it is changed when the model detects trouble and makes a shift. The smoothing factor is usually 0.1 but may be changed within the range of .06 to .24. How long the smoothing factor remains constant depends on the model and the behavior of the particular time series involved. First order adjusted smoothing is used with characteristics described by Robert G. Brown. John R. Virts, Staff Economist for Eli Lilly and Company, reports adjusted smoothing is used because when the computer program was written it was the latest thing in common usage. For seasonal adjustments

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dated August 8, 1967.

the company uses a base series of a twelve component section. The elements of the vector are updated by simple exponential smoothing every month (only one component is altered by the new information from sales). The vector is then so normalized that its total components amount to twelve. Multiplicative models are used so these factors are modified according to forecasts. A 2.5 mean average deviation accounts for forecast errors.[5]

The older computerized inventory control system saved many hours of clerical work and made possible reasonably good inventory control. The recent shift to exponential smoothing actually consumes more computer time than the old system. However, the shift to an exponential smoothing technique reduced the clerical staff at Eli Lilly and Company by two women and has permitted a considerable reduction of inventories.[6]

Champion Spark Plug Company

The Champion Spark Plug Company manufactures spark plugs for internal combustion, diesel and turbine

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[5] Letter from John R. Virts of Eli Lilly and Company dated October 31, 1967.

[6] *Ibid*.

engines. The number of products marketed is over 300. In 1966 net sales were over 153 million dollars. This figure is $20,000,000 more than 1965 sales. In the past inventories were determined by taking moving averages on historical data. Sales were predicted through the use of correlation of historical data with external indicators of business activity such as Gross National Product, etc. Internal statistical data is also relied upon to create a more comprehensive picture of future sales predictions.

Simple exponential smoothing was introduced in 1965 to test out its advantages in controlling spark plug inventories for better customer service. Although the technique has been utilized for over two years the Champion Spark Plug Company feels that no graduation to more complicated exponential smoothing techniques is required. Consequently, simple smoothing with a seasonal adjustment remains in use. A base series of 12 months is used and an estimate of the smoothing constant has proven to be adequate. The smoothing technique is used for all products with its greatest benefit having occurred in a reduction of computer running time.[7]

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[7]Letter from E. H. Reifeis of Champion Spark Plug Company dated November 20, 1967.

Brown & Sharpe Company

Brown & Sharpe, a leader in the machine tool industry showed sales of over $80,000,000 in 1966. Located primarily in North Kingstown, Rhode Island, a modern production facility should continue to assist in establishing Brown & Sharpe's solid position.

Under certain circumstances exponential smoothing has proven to be an inefficient forecasting technique as asserted by Brown & Sharpe. The following make the adoption of exponential smoothing unwise:

A. Where there are only a few items to forecast and the cost of these items is relatively high.

B. Where there is a great deal of noise in the forecast because of the nature of the industry.

C. Where the forecasting process is performed infrequently.

D. Where lead time is great.

E. Where cyclical variations are prevalent and the reasons for these variations cannot be definitely ascertained.

Cyclical variations are prevalent in the machine tool industry. Although Brown and Sharpe Company is making progress in determining why these variations occur in their business the reasons for the variations have not definitely been ascertained. Brown and Sharpe's Machine Tool Division manufactures a relatively low volume of individual machine tools. The products are highly priced. The lead time for delivery of a large machine tool is now from 12 to 18 months. Forecasting is done by the marketing department on a dollar basis and judgement is utilized by production control personnel to determine what the requirements for a particular machine tool will be each year. These forecasts are reevaluated each quarter, but extensive lead time does not affect inventory or production. The company has experimented with exponential smoothing with little success. Using historical figures for the year 1963, Brown and Sharpe tried to predict figures for 1964. Comparing the results to actual figures showed that calculated figures constantly lagged behind actual demand. The smoothing constant was changed, the forecast errors were analyzed but to no avail. It was found impossible to eliminate noise in the forecast. Methods based on judgement which were inaccurate within the range of 10 to 15 per cent proved to be closer to actual figures than exponential smoothing. The two primary reasons for not using exponential smoothing as stated by Mr. J. A. Newton, Manager, Operations Manufacture and Control were; the time to make the product is too long and the volume of production is comparatively low.[8]

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[8] Statement by J. A. Newton of Brown & Sharpe Company, personal interview on October 15, 1967.

V

CONCLUSION

This study reports various methods used in sales and production forecasting. However, the main purpose of this paper is to present evidence that the exponential smoothing method is one of the best.

When choosing a forecasting system the most important criterion is accuracy. Let us begin with examination of a trend analysis approach to forecasting. Time series analysis treats sales data as though it were produced by one stable casual phenomenon. Inherent in this technique is the allotment of equal weight to each piece of data. The technique is questionable in its validity - not questionable in terms of the vigor of its mathematical computation, but questionable in the light of the total span of history to the job of forecasting the immediate future. For recent history is more relevant as a base of computation, than the earlier points of time. A time series technique cannot track a changing demand pattern. Hence, it is difficult for management to minimize adverse consequences of uncertainty, or to capitalize upon the opportunities created by unforeseen possibilities. This inability to change with a changing character of demand is a major weakness in the use of this statistical forecasting technique.

The best that can be said for correlation analysis is that the use of economic indicators is a supplement to other forecasting devices. Lead-lag relationships are sought between a firm's sales and some outside indicators. Correlation is of little use for predicting demand on a monthly or weekly basis as required in production and inventory control.

No series leads with consistency. The turning points shown by indicators are to a considerable degree accidental. Unpredictable events such as weather or labor strikes, may distort the position of peaks and troughs in the life of an indicator thus having little forecasting value for a company.

Subjective forecasting techniques sometimes are surprisingly accurate, however, they do not provide consistent reliability and their value is, like correlation analysis, a supplement to other devices.

We must choose a forecasting technique which will deal effectively with the possibility of change in the character of demand. A system that will track a changing pattern of demand and change its interpretation of this history can be established by giving different weights to various segments of past experience in estimating what the future will be. The concept of a moving average or more specifically a weighted moving average would seem to be the answer. It can allot more weight to the latest periods of demand history than to demand that occurred long ago. It is possible to discount in favor of the most recent experience. As mentioned earlier some form of a moving average is in common usage as a forecasting tool in industry today. The disadvantage of employing a moving average as a forecasting tool is primarily the requirement of numerous data which necessitates long files.

Exponential smoothing has solved that problem. A prediction can be made in any period based on current sales data for the item, but in such a way that only one number (the most recent estimate for the current period) must be retained to be combined with latest incoming sales information.

This system has the characteristics of a good prediction technique. The forecast calculation is fast and easy because only a limited amount of information should be maintained. It makes compensations for demand pictures with a marked trend and/or seasonal patterns. The extension of the basic exponential smoothing system to take these two factors into account is not difficult. Although more information is required with this slightly more complex model the accuracy of prediction is increased for most items.

In summary, exponential smoothing has several advantages over more conventional methods. It provides accurate forecasts while requiring less information. And it responds more rapidly to sudden shifts in the demand picture.

It is visible from the paper, that a system need not be complex to be accurate. The field research performed has confirmed this hypothesis. Most of the companies queried revealed that their use of exponential smoothing in its simple or adjusted form proved successful. Indeed, a simple system of forecasting demand can be viewed as a necessity, for the cost of a complex system would not justify its results. One must remember that forecasts are often made for thousands of products. It must be done on a routine basis so that prediction can be obtained quickly and at a low cost in terms of computing time and information storage. Exponential smoothing has these characteristics.

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