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High Speed Crack Propagation and Branching Under Uniaxial and Biaxial

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HIGH SPEED CRACK PROPAGATION

AND BRANCHING UNDER

UNIAXIAL AND BIAXIAL

LOADING

BY

S. ANAND

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

APPROVED:

Thesis Committee

Major Professor

TEAM OF THE GRADUATE SCHOOL

UNIVERSITY OF RHODE ISLAND

UNIVERSITY OF RHODE ISLAND

1983

ABSTRACT

A photoelastic investigation of high speed crack propagation and branching in a brittle polymer material under dynamic loading was carried out for both uniaxial and biaxial loading conditions. Crack type specimens were loaded in a specially designed loading fixture where loads perpendicular and parallel to the crack could be controlled independently.

The photoelastic data obtained were analyzed to get the stress intensity factor K_I , the crack tip velocity v , the crack velocity $\dot{a}$ and other non-steady state stress field coefficients including the stress acting parallel to the crack, σ_{xx} .

It was observed that K_I showed an increasing trend as the crack propagated through the specimens. Results showed parallel to the crack gave rise to higher stress intensity factors, compared to the compressive case for the same crack tip velocity and initial crack loads. It was also found that velocity of crack propagation increased with increasing branching angle. While the crack was propagating, the stress intensity factor was measured at different points along the crack. The stress intensity factor, which was high at the crack tip, decreased as the crack propagated.

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It was concluded in this study that any crack branching criterion has to take into account the nature of the remote stress parallel to the crack and that the stress intensity factor is affected by non-steady state stress field coefficients.

UNIVERSITY OF RHODE ISLAND

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ABSTRACT

A photoelastic investigation of high speed crack propagation and branching in a brittle polyester material called Homalite 100 was conducted for both uniaxial and biaxial loading conditions. Cross type specimens were loaded in a specially designed loading fixture where loads perpendicular and parallel to the crack could be controlled independently.

The photoelastic data obtained were analyzed to get the stress intensity factor K , the crack tip position a , the crack velocity $\dot{a}$ and other non-singular stress field coefficients including the stress acting parallel to the crack, σ_{ox} .

It was observed that K showed an increasing trend as the crack propagated through the specimens. Tensile stress parallel to the crack gave rise to higher stress intensity factors, compared to the compressive case for the same crack tip positions and initial normal loads. It was also found that tensile parallel remote stress enhanced branching and branching angle. While branching angles of 22° to 29° were encountered in uniaxial normal loading and biaxial tension-compression loading, angles as high as 45° and 73° were obtained in the tension-tension case.

It was concluded in this study that any crack branching criterion has to take into account the nature of the remote stress parallel to the crack and that crack branching is strongly influenced by non-singular stress field coefficients.

ACKNOWLEDGEMENTS

I would like to express my thanks and appreciation to my advisor, Dr. Shukla, for his guidance throughout this investigation. The help of the instrument shop in the project is also acknowledged. I would also like to add that this project was funded by the National Science Foundation under Grant No. MEA-8203277. The assistance of my colleague, K. Kumar, in carrying out the experiments, is also appreciated.

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LIST OF SYMBOLS

$\dot{a}$	crack velocity
B_O'	normalized remote parallel stress
c_1	dilatational wave velocity
c_2	shear wave velocity
c_R	Rayleigh wave velocity
f_σ	material fringe value
G	shear modulus
λ	modulus associated with dilatation
K	stress intensity factor
K_{br}	branching stress intensity factor
ρ	density of material
σ_{ox}	remote parallel stress
$\sigma_x, \sigma_y, \tau_{xy}$	stress components
τ_m	maximum in-plane shear stress
h	thickness of model
N	fringe order
N^*	No. of data points
$ \Delta n $	average fringe order error
a^*	crack tip position from edge of model
$A_0, A_1, A_2, B_0, B_1, B_2$	series terms

1. INTRODUCTION

This study is concerned with high speed crack propagation and branching in a brittle polyester model material called Homalite 100, under uniaxial and biaxial loading conditions.

High speed crack propagation and branching are of interest in several fields like mining and aircraft industries and large structures like nuclear reactors, pipelines, ships, etc. In the mining industry, one wants to optimize fragmentation and thus reduce the cost of mining operations by controlled fracturing. On the other hand, in structures like ships, bridges, pressure vessels etc., one wants to know if the crack will arrest after initiation or will go through the structure.

Dynamic fracture has always been one of the most important and difficult problems in mechanics. The existence of singularity at the crack tip and the inertia effects make it extremely difficult to obtain a solution by either analytical or numerical techniques.

Researchers [1,2,3,4] have been attempting to characterize dynamic fracture in terms of a relation between the stress intensity factor, K and crack velocity $\dot{a}$ for different materials. However, till now, no study has been conducted to investigate the influence of non singular stress field coefficients on the $\dot{a}$ - K behavior.

In the experimental studies of Fracture Mechanics, photoelasticity is a widely used technique to determine the stress intensity factor K at the crack tip. Though Post [5] and Wells and Post [6] applied photo-

elasticity to study the static and dynamic aspects of fracture and Irwin [7] developed a simple method to determine K from isochromatic fringe loops, the photoelastic community largely ignored fracture mechanics, in its formative period. Dixon [8] and Gerberich [9] were exceptions as they applied birefringent coatings to study strain fields near the crack tip in metallic plates.

In the past decade, major research programs in fracture mechanics have been started by C.W. Smith at Virginia Polytechnic Institute, A.S. Kobayashi at the University of Washington and J.W. Dally at the University of Maryland. C.W. Smith and his associates [10] have applied the stress-freezing technique to determine the stress intensity factor in a number of three dimensional crack problems. A.S. Kobayashi and his associates have used both dynamic photoelasticity and finite element methods to determine the dynamic state of stress at the tip of a propagating crack. Dally and his associates [2,11] have also employed dynamic photoelasticity in their studies of propagating cracks.

In photoelastic crack tip stress analysis, it is not advisable to use data from near the crack tip for practical reasons like fringe clarity, light scattering from the caustic at the crack tip, an unknown degree of plane strain constraint and so on [12]. In order to perform meaningful analysis from a larger and more desirable area in the stress field, additional non-singular terms have to be included. In this research, the equations derived by Irwin [13] were used and the series coefficients, including the stress intensity factor, were obtained using a multipoint over-deterministic technique developed by Sanford and Dally [14]. A total of six stress field coefficients were included for the analysis and a computer program was written in BASIC language

to obtain the values of the coefficients.

The results showed that the stress intensity factor, K and the far-field stress parallel to the crack, σ_{ox} , varied systematically as the crack propagated across the specimen and higher order terms showed an oscillatory behavior. The value of the stress intensity factor at branching seemed to vary slightly depending on the initial normal load applied. The branching angle depended on the nature of the far-field stress.

2. REVIEW OF RELATED RESEARCH ON HIGH SPEED

CRACK PROPAGATION AND BRANCHING

Dynamic crack propagation and branching is best discussed in terms of crack propagation velocity $\dot{a}$ and the stress intensity factor, K . The Fracture Mechanics groups at the University of Maryland [1,2,3], University of Washington [4] and other institutions have made important contributions to the characterization of dynamic crack propagation by means of the relationship between the crack velocity and the stress intensity factor.

Irwin, Dally and others [1] performed a series of fracture tests with various types of specimens fabricated from Homalite 100 and observed that the $\dot{a}$ versus K curve had three distinct regions, the stem, the slope range and the plateau, as shown in Fig. 1. In the stem region, the crack velocity is independent of K . Small changes of K cause considerable changes in the crack velocity, up to velocities of about 200 m/s. The slope range is the transition region covering crack velocities from 200 to 381 m/s. For higher velocities, a large increase in K is needed even for small increases in $\dot{a}$. This is the plateau region. The highest velocity of crack propagation recorded in these experiments was 432 m/s. Rossmanith and Irwin [3] suggested that the $\dot{a}$ vs K relationship, as obtained from experiments with test specimens, depends in the high velocity region on the type of test specimen used. Irwin et al [1] concluded that the value K_{Im} , which is the stress intensity factor below which the crack cannot propagate, can be treated

as a material property. Though it has been shown theoretically [16] that the maximum crack velocity which can be achieved is $\dot{a} = c_R$, the Rayleigh wave velocity, this value is not attained for most of the materials in practice because branching occurs at lower velocities and the energy driving the crack is divided. Dally and his associates [2] observed that in the case of Homalite 100, branching occurs with the SEN specimen when $\dot{a}/c_2 = 0.32$ to 0.35 . The velocity at branching is often called the terminal velocity $\dot{a}_T$ and it is generally believed that $\dot{a}_T$ is the maximum which can be achieved for a given material. However super-velocities of the order of 726 m/s were obtained by explosive testing wherein branching was suppressed to an extent by stress waves [2].

Coming to the phenomenon of crack branching, numerous researchers have performed prebranching and post branching analyses but their results rarely agree. Pre branching analysis looks for criteria for crack branching while post branching analysis deals primarily with the direction and the propagation of the branched crack.

Yoffe [16] was the first to study the dynamic stress field around a moving crack tip and found that the circumferential stress $\sigma_{\theta\theta}$ reaches a maximum at an angle of 60° for $\dot{a}/c_2 = 0.6$ which is different from its original direction of $\theta = 0$ for lower crack velocities. It was also concluded that the branching angle was $2 \times 60^\circ = 120^\circ$. Craggs [17] derived a critical crack velocity of $\dot{a}/c_1 = 0.4$ for a propagating semi-infinite crack. However, in experiments, velocities close to the critical velocity were never attained and in phenomena like stress corrosion cracking, branching occurred at very low velocities. So some other parameter, rather than just the velocity, should be connected with branching. Clark and Irwin [18] suggested that the dynamic stress

intensity factor should reach a critical value, K_b (or the strain energy release rate should reach a critical value, G_b) for branching to occur at terminal velocity. Kalthoff [19] found that there is a characteristic forking angle in which case the branches neither repel nor attract each other. The corresponding branching angle was about 30° .

Kobayashi et al [4] observed that the branching stress intensity factor for Homalite 100 was approximately three times the fracture toughness. K values of the order of 1.54 and 1.98 $\text{MPa}\sqrt{\text{m}}$ were encountered for 3/8" and 1/8" thick sheets at branching. These values were equal to the maximum K observed in fractured plates without branches. The results indicated that the dynamic stress intensity factor was a necessary but not sufficient condition for branching. The combination of excessively large strain energy release rate, shown by the large static stress intensity factor, available at the time when the running crack tip is subjected to a maximum dynamic stress intensity factor, could be a plausible cause for the crack to branch.

Dally et al [2] obtained crack branching at branching stress intensity factor, K_{Ib} ranging from 3.3 to 3.8 times K_{Im} , the arrest toughness when the crack moved at terminal velocity in Homalite 100. Research by Dally, Kobayashi and others pointed to the fact that K_{Ib} is not a material property, contrary to results obtained by Congleton, Anthony [20,21] and Doll [22]. Kirchner [23] found that K_{Ib} and Young's Modulus had a strong correlation and developed a strain intensity criterion for crack branching in ceramics. Ramulu et al [15] have developed a branching criterion which contends that a necessary and sufficient condition for branching is a crack branching stress intensity factor, K_{Ib} , accompanied by a minimum characteristic distance

$r_0 = r_c$. H.P. Rossmanith and G.R. Irwin [3] suggested that non-singular stress field coefficients may influence $\dot{a}$ -K behavior in the terminal velocity region. Till now, no research has been conducted on high velocity crack propagation and branching by systematically varying the non-singular coefficients. This is the objective of this project.

Crack propagation studies need a high speed recording device to take pictures of the transient crack tip stress patterns in dynamic photoelastic stress analysis. One of the best frequently used recording systems is the multiple Spark Camera, originally designed by Cook and Seaborn [12]. A camera similar to this was designed and built at MIT and is known as the Photoelastic Laboratory. This chapter will briefly discuss the method of operation of all the cameras of the Spark type camera.

Many transparent polycrystalline materials that are subjected to stress when free of stress become optically anisotropic and display characteristic similar to crystals when they are stressed. These characteristics persist while loads on the material are maintained but disappear when the loads are removed. This behavior is called temporary double refraction and the method of photoelasticity is based on this physical behavior of transparent polycrystalline materials [26].

For experimentation, the model is fabricated from a polycrystalline transparent, birefringent material. When circularly polarized light passes through the stressed model and then through another circular polarizer, an optical interference occurs due to stress-induced birefringence in the model. This optical interference produces a series of light and dark bands which are termed isochromatic fringe patterns. The stress optic law, which relates the stress state of the model to

3. THE MULTIPLE SPARK CAMERA AND ITS APPLICATION TO DYNAMIC PHOTOELASTICITY

Crack propagation studies need a high speed recording system to take pictures of the transient crack tip stress patterns in dynamic photoelastic stress analysis. One of the most frequently used recording systems is the Multiple Spark Camera, originally designed by Cranz and Schardin [25]. A camera similar to this was designed and built at URI and is housed in the Photomechanics Laboratory. This chapter will briefly discuss the method of photoelasticity and the components of the Spark gap Camera.

Many transparent noncrystalline materials that are optically isotropic when free of stress become optically anisotropic and display characteristics similar to crystals when they are stressed. These characteristics persist while loads on the material are maintained but disappear when the loads are removed. This behavior is called temporary double refraction and the method of photoelasticity is based on this physical behavior of transparent noncrystalline materials [26].

For experimentation, the model is fabricated from a polymeric, transparent, birefringent material. When circularly polarized light passes through the stressed model and then through another circular polarizer, an optical interference occurs due to stress-induced birefringence in the model. This optical interference produces a series of light and dark bands which are termed isochromatic fringe patterns. The stress optic law, which relates the stress state of the model to

the order of the associated interference pattern is given by

$$2\tau_m = \sigma_1 - \sigma_2 = Nf_\sigma/h,$$

where σ_1 and σ_2 are the in-plane principal stresses, τ_m the maximum in-plane shear stress, N the fringe order, f_σ the material fringe value and h the thickness of the model. In dynamic photoelasticity, the fringes move at high velocities and so a high speed recording system like the Multiple Spark Camera is used for purposes of stress analysis.

3.1 Description of the Camera

The camera consists of three main subdivisions, viz., the spark gap circuit, the optical arrangement and the control circuit.

3.1.1 The Spark Circuit - The Multiple Mach spark circuit is shown in Fig. 2. In the camera that was used in this project, there are twenty spark gaps (SG), each of them connected to L-C circuits in series. TSG is the trigger spark gap. In operation the condensers are charged to about 15 KV and the circuit L_1C_1 closed by applying a trigger pulse to the spark gap TSG. The firing sequence is initiated at a pre-determined time after the crack initiation by applying a 30 KV pulse to the trigger gap. When the trigger gap is fired, the capacitor C_1 discharges to below the ground potential. When the voltage on C_1 becomes sufficiently negative, a spark occurs at gap SG_1 and capacitor C_1^* discharges, producing a short, intense flash of light.

The timing between the first and second sparks depends on the inductance L_2 in the $C_1^* L_2 C_2$ loop. When the gap SG_1 fires, the voltage on C_2 decays with time and the gap SG_2 fires when the voltage on C_2 is sufficiently negative. Likewise, all the twenty gaps fire. The light from the spark gaps is led out of the camera by fibre optics.

3.2 Optical Arrangement - The optical setup is shown in Fig. 3. Two circular polarizers are kept on each side of the specimen. As an example, light from the spark SG1 passes through a field lens, the first polarizer, the specimen, the second polarizer and the second field lens onto the camera lens L_1 . In a similar manner, light from spark Gaps SG2, SG3, SG4 etc. is focused on the corresponding camera lenses L_2 , L_3 , L_4 etc. The spark gaps, the field lenses and the camera lenses are so placed that the light from one particular spark falls on one particular camera lens so that the image from one camera lens is due to one spark only. So twenty pictures of the propagating crack are obtained at twenty different spots on the same film. Kodak Wratten No. 8 filters and Kodak Grauvre Positive 4135 film are used, the combination of film and filter yielding blue light of wavelength of $4920\text{\AA}$.

3.1.3 Control Circuit - The control circuit is used to initiate the firing sequence at a required delay after the dynamic event is started. A schematic of the circuit is shown in Fig. 4 [27]. When the conductive paint on the specimen is broken by the moving crack, a 20V pulse is emitted which initiates a Digital Delay Generator (Model 103CR of California Avionics Labs Inc.) and a Nicolet Oscilloscope (Model 206).

The light from the sparks is picked up by a high frequency response photocell and its output is recorded on the oscilloscope as an Intensity Time trace. The timings of the peaks on this trace represent the time at which the picture of the crack was taken. The delay generator holds the pulse from the broken conductive paint line for a predetermined interval of time. Once the pulse is passed to the Trigger Spark Gap after the required delay, the voltage at the trigger gap is stepped

up to about 30 KV, after which the firing occurs in the sequence as described before.

4. CRACK TIP STRESS FIELD ANALYSIS

4.1 Derivation of the Dynamic Equations

The analysis shall make use of Irwin's crack tip stress function (28) and shall assume plane strain ($\epsilon_z = 0$). The leading edge of the crack will be taken as the positive x-axis and the crack tip as the origin of the coordinate system, as shown in Fig. 5.

For crack propagation in the x-direction and assuming that the stress field does not change with time with respect to the moving coordinate system, the following equations apply:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (4.1)$$

$$\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial x^2} \quad (4.2)$$

where u and v are the displacements in the x and y directions and c the crack velocity.

Equilibrium conditions in the x and y directions lead to the following results:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2} = \rho c^2 \frac{\partial^2 u}{\partial x^2} \quad (4.3)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} = \rho c^2 \frac{\partial^2 v}{\partial x^2} \quad (4.4)$$

where σ_x , σ_y and τ_{xy} are the stress components and ρ the density of the body.

4. CRACK TIP STRESS FIELD ANALYSIS

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$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} = \rho c^2 \frac{\partial^2 v}{\partial x^2} \quad (4.4)$$

where σ_x , σ_y and τ_{xy} are the stress components and ρ the density of the body.

Using Hooke's law in the form

$$\sigma_x = \lambda \Delta + 2G\epsilon_x \quad (4.5)$$

$$\sigma_y = \lambda \Delta + 2G\epsilon_y \quad (4.6)$$

$$\sigma_{xy} = G\gamma_{xy} \quad (4.7)$$

where G is the shear modulus and λ the modulus associated with dilatation.

We can rewrite the equations (4.3) and (4.4) in terms of dilatation Δ and rotation ω , where

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad (4.8)$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (4.9)$$

Equations (4.3) and (4.4) then take the following form,

$$(\lambda+2G) \frac{\partial \Delta}{\partial x} - G \frac{\partial \omega}{\partial y} = \rho c^2 \frac{\partial^2 u}{\partial x^2} \quad (4.10)$$

$$G \frac{\partial \omega}{\partial x} + (\lambda+2G) \frac{\partial \Delta}{\partial y} = \rho c^2 \frac{\partial^2 v}{\partial x^2} \quad (4.11)$$

Noting that

$$\frac{\partial}{\partial x} \left(\frac{\partial^2 u}{\partial x^2} \right) + \frac{\partial}{\partial y} \left(\frac{\partial^2 v}{\partial x^2} \right) = \frac{\partial^2 \Delta}{\partial x^2} \quad (4.12)$$

$$\text{and } \frac{\partial}{\partial x} \left(\frac{\partial^2 v}{\partial x^2} \right) - \frac{\partial}{\partial y} \left(\frac{\partial^2 u}{\partial x^2} \right) = \frac{\partial^2 \omega}{\partial x^2} \quad (4.13)$$

we can derive the equations

$$(\lambda+2G) \nabla^2 (\Delta) = \rho c^2 \frac{\partial^2 \Delta}{\partial x^2} \quad (4.14)$$

$$G\nabla^2\omega = \rho c^2 \frac{\partial^2\omega}{\partial x^2} \quad (4.15)$$

Equations (4.14) and (4.15) can be simplified in appearance in the following ways. In equation (4.14), we substitute $y_1 = \lambda_1 y$ where

$$\lambda_1^2 = 1 - \left(\frac{c}{c_1}\right)^2 = 1 - \frac{\rho c^2}{\lambda + 2G} \quad (4.16)$$

In equation (4.15), we substitute $y_2 = \lambda_2 y$, where

$$\lambda_2^2 = 1 - \left(\frac{c}{c_2}\right)^2 = 1 - \frac{\rho c^2}{G} \quad (4.17)$$

The resulting pair of equations is

$$\frac{\partial^2\Delta}{\partial x^2} + \frac{\partial^2\Delta}{\partial y_1^2} = 0 \quad (4.18)$$

$$\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y_2^2} = 0 \quad (4.19)$$

Clearly Δ is a harmonic function of x and $\lambda_1 y$ while ω is a harmonic function of x and $\lambda_2 y$.

It can be shown there is no loss of generality for the proposed problem [13] if we assume

$$\Delta = (1 - \lambda_1^2) A \operatorname{Re} Z_1 \quad (4.20)$$

$$\omega = (1 - \lambda_2^2) B \operatorname{Im} Z_2 \quad (4.21)$$

where $Z_1 = f'(z_1)$, $Z_2 = f'(z_2)$, $z_1 = x + iy_1$ and $z_2 = x + iy_2$.

In addition it is helpful to recognize that the displacements can be divided into non rotational (u_1, v_1) and nondilatational (u_2, v_2) components. Thus $u = u_1 + u_2$, $v = v_1 + v_2$ where

$$\Delta = \frac{\partial u_1}{\partial x} + \lambda_1 \frac{\partial v_1}{\partial y_1} = (1 - \lambda_1^2) A \operatorname{Re} Z_1 \quad (4.22)$$

$$\frac{\partial v_1}{\partial x} - \lambda_1 \frac{\partial u_1}{\partial y_1} = 0 \quad (4.23)$$

Also

$$\omega = \frac{\partial v_2}{\partial x} - \lambda_2 \frac{\partial u_2}{\partial y_2} = (1 - \lambda_2^2) B \operatorname{Im} Z_2 \quad (4.24)$$

$$\frac{\partial u_2}{\partial x} + \lambda_2 \frac{\partial v_2}{\partial y_2} = 0 \quad (4.25)$$

Comparing equations 4.22 and 4.23, we get

$$\left. \begin{aligned} u_1 &= A \operatorname{Re} [f(z_1)] \\ v_1 &= -\lambda_1 A \operatorname{Im} [f(z_1)] \end{aligned} \right\} \quad (4.26)$$

Similarly from equations 4.24 and 4.25,

$$\left. \begin{aligned} u_2 &= -\lambda_2 B \operatorname{Re} [f(z_2)] \\ v_2 &= B \operatorname{Im} [f(z_2)] \end{aligned} \right\} \quad (4.27)$$

Since the shear stress on the crack faces is zero, $\tau_{xy} = 0$ on $y = 0$ for $x < 0$. Along this line $f'(z_1) = f'(z_2) = f'(x)$. One finds that on $y = 0$

$$\frac{1}{G} \tau_{xy} = \gamma_{xy} = [-2\lambda_1 A + (1 + \lambda_2^2) B] \operatorname{Im} [f'(x)] \quad (4.28)$$

Since $\operatorname{Im} [f'(x)]$ on $y = 0$ is not zero along the entire x axis, we must assume

$$B = \frac{2\lambda_1}{1 + \lambda_2^2} \cdot A \quad (4.29)$$

In order to determine A, consider next that

$$f'(z) = \sum_{n=0}^{\infty} A_n z^{n-\frac{1}{2}} \quad (4.30)$$

Since $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ are proportional to $\text{Re}[f'(x)]$ along $y = 0$, equation (4.30) ensures σ_x and $\sigma_y = 0$ when x is negative. When x is positive, σ_y is not zero.

'A' can be determined from the defining equation for the Stress Intensity Factor, K. That is, for x very small and positive and $y = 0$

$$\sigma_y = \frac{K}{\sqrt{2\pi x}} = \lambda \Delta + 2G \frac{\partial v}{\partial y} \quad (4.31)$$

From previous expressions

$$\begin{aligned} \lambda \Delta &= \lambda(1-\lambda_1^2) A A_0 / \sqrt{x} \\ 2G \frac{\partial v_1}{\partial y} &= 2G \lambda_1 \frac{\partial v_1}{\partial y_1} = -2G \lambda_1^2 A A_0 / \sqrt{x} \\ 2G \frac{\partial v_2}{\partial y} &= 2G \lambda_2 \frac{\partial v_2}{\partial y_2} = 2G \frac{2\lambda_1 \lambda_2}{1+\lambda_2^2} A A_0 / \sqrt{x} \end{aligned}$$

When the above relations are inserted into equation 4.30, it is convenient to replace λ using the equation

$$(\lambda+2G) = \frac{(1-\lambda_2^2)}{(1-\lambda_1^2)} G \quad (4.32)$$

It is also convenient to assume $K = A_0 \sqrt{2\pi}$. This can be done because all of the constant coefficients in equation (4.30) must be adjusted to fit the boundary conditions.

$$\text{Thus } A = \frac{1}{G} \frac{1}{\left[\frac{4\lambda_1 \lambda_2}{1+\lambda_2^2} - (1+\lambda_2^2) \right]} \quad (4.33)$$

The expressions for σ_x , σ_y and τ_{xy} are derived as follows:

$$\begin{aligned}\sigma_x &= \lambda \Delta + 2G \frac{\partial u}{\partial x} \\ &= \lambda(1-\lambda_1^2)A \operatorname{Re} Z_1 \\ &\quad + 2G(A \operatorname{Re} Z_1 - \lambda_2 B \operatorname{Re} Z_2)\end{aligned}\quad (4.34)$$

We know from eqn. (4.29) that

$$B = \frac{2\lambda_1}{1+\lambda_2} A$$

and from eqn. (4.32) that

$$\lambda = G \left[\frac{1-\lambda_2^2}{1-\lambda_1^2} - 2 \right]$$

Substituting these in eqn. (4.34)

$$\begin{aligned}\sigma_x &= G(2\lambda_1^2 - \lambda_2^2 - 1)A \operatorname{Re} Z_1 \\ &\quad + 2G \left[A \operatorname{Re} Z_1 - \lambda_2 \frac{4\lambda_1}{1+\lambda_2} A \operatorname{Re} Z_2 \right] \\ &= AG \left[(1+2\lambda_1^2 - \lambda_2^2) \operatorname{Re} Z_1 - \frac{4\lambda_1 \lambda_2}{1+\lambda_2} \operatorname{Re} Z_2 \right]\end{aligned}\quad (4.35)$$

Similarly we can obtain

$$\sigma_y = AG \left[-(1+\lambda_2^2) \operatorname{Re} Z_1 + \frac{4\lambda_1 \lambda_2}{1+\lambda_2} \operatorname{Re} Z_2 \right] \quad (4.36)$$

$$\text{and } \tau_{xy} = AG \cdot 2\lambda_1 [\operatorname{Im} Z_2 - \operatorname{Im} Z_1] \quad (4.37)$$

$$\text{where } AG = \frac{1}{\left[\frac{4\lambda_1 \lambda_2}{1+\lambda_2} - (1+\lambda_2^2) \right]} \quad \text{from eqn. (4.33)}$$

$$\text{and } \frac{1-\lambda_1^2}{1-\lambda_2^2} = c_2^2/c_1^2$$

A second choice for the function Z can be made in the form

$$Y = \sum_{m=0} B_m z^m \quad (4.38)$$

One can observe that on $y = 0$ and for any value of x , Y_1 and Y_2 have zero imaginary parts. Thus $\tau_{xy} = 0$ on $y = 0$. In order to have $\sigma_y = 0$ on the line segment occupied by the crack, it is necessary to require

$$B = \frac{1+\lambda_2^2}{2\lambda_2} \cdot A \quad (4.39)$$

The resulting stress equations are

$$\sigma_x = AG[(1+2\lambda_1^2-\lambda_2^2)\text{Re}Y_1 - (1+\lambda_2^2)\text{Re}Y_2] \quad (4.40)$$

$$\sigma_y = AG(1+\lambda_2^2)(\text{Re}Y_2 - \text{Re}Y_1) \quad (4.41)$$

$$\tau_{xy} = AG[(1+\lambda_2^2)^2 \frac{\text{Im}Y_2}{2\lambda_2} - 2\lambda_1 \text{Im}Y_1] \quad (4.42)$$

AG in equations (4.40)-(4.42) can be obtained by using the boundary condition $\sigma_x = \sigma_{ox} = 2B_o$ as $x \rightarrow 0$ on the positive x -axis. So from equation (4.40), we get σ_x as $x \rightarrow 0 = AG[(1+2\lambda_1^2-\lambda_2^2)\text{Re}Y_1 - (1+\lambda_2^2)\text{Re}Y_2]$ where $\text{Re}Y_1 = \text{Re}Y_2 = B_o$ from equation (4.38) i.e. $\sigma_{ox} = 2AG[\lambda_1^2 - \lambda_2^2]B_o$. Assuming that σ_x as r and $\theta \rightarrow 0$ is a constant value of $\sigma_{ox} = 2B_o$, we get

$$AG = \frac{1}{[\lambda_1^2 - \lambda_2^2]}$$

Since the above two series functions satisfy the boundary conditions of the problem, the sum of the two should also satisfy the boundary conditions.

$$\begin{aligned} \text{So } \sigma_x &= \frac{1}{[\Omega - (1 + \lambda_2^2)]} \cdot [(1 + 2\lambda_1^2 - \lambda_2^2) \text{Re}Z_1 - \Omega \text{Re}Z_2] \\ &+ \frac{1}{(\lambda_1^2 - \lambda_2^2)} [(1 + 2\lambda_1^2 - \lambda_2^2) \text{Re}Y_1 - (1 + \lambda_2^2) \text{Re}Y_2] \end{aligned} \quad (4.43)$$

$$\begin{aligned} \sigma_y &= \frac{1}{[\Omega - (1 + \lambda_2^2)]} [-(1 + \lambda_2^2) \text{Re}Z_1 + \Omega \text{Re}Z_2] \\ &+ \frac{(1 + \lambda_2^2)}{(\lambda_1^2 - \lambda_2^2)} (\text{Re}Y_2 - \text{Re}Y_1) \end{aligned} \quad (4.44)$$

$$\begin{aligned} \tau_{xy} &= \frac{1}{[\Omega - (1 + \lambda_2^2)]} \cdot 2\lambda_1 (\text{Im}Z_2 - \text{Im}Z_1) \\ &+ \frac{1}{(\lambda_1^2 - \lambda_2^2)} \left[\frac{(1 + \lambda_2^2)^2}{2\lambda_2} \text{Im}Y_2 - 2\lambda_1 \text{Im}Y_1 \right] \end{aligned} \quad (4.45)$$

where $\Omega = 4\lambda_1\lambda_2/(1 + \lambda_2^2)$

From equations (4.43) and (4.44), we obtain

$$\begin{aligned} \frac{\sigma_y - \sigma_x}{2} &= \frac{1}{[\Omega - (1 + \lambda_2^2)]} [\Omega \text{Re}Z_2 - (1 + \lambda_1^2) \text{Re}Z_1] \\ &+ \frac{1}{(\lambda_1^2 - \lambda_2^2)} [(1 + \lambda_2^2) \text{Re}Y_2 - (1 + \lambda_1^2) \text{Re}Y_1] \end{aligned} \quad (4.46)$$

The first three terms in each of the series Z and Y shall be included in the analysis. So Z_1 and Z_2 can be expressed as follows:

$$Z_1 = A_0 z_1^{-\frac{1}{2}} + A_1 z_1^{\frac{1}{2}} + A_2 z_1^{\frac{3}{2}} \quad \text{and} \quad (4.47)$$

$$Z_2 = A_0 z_2^{-\frac{1}{2}} + A_1 z_2^{\frac{1}{2}} + A_2 z_2^{\frac{3}{2}} \quad (4.48)$$

Y_1 and Y_2 can be expressed as

$$Y_1 = B_0 + B_1 z_1 + B_2 z_1^2 \quad (4.49)$$

$$Y_2 = B_0 + B_1 z_2 + B_2 z_2^2 \quad (4.50)$$

where $A_0 \sqrt{2\pi} = K$, the stress intensity factor and remote stress $\sigma_{ox} = 2B_0$.

$$z_1 = \rho_1 e^{i\phi_1} \quad \text{and} \quad z_2 = \rho_2 e^{i\phi_2} \quad (4.51)$$

where ρ_1 , ρ_2 , ϕ_1 and ϕ_2 are shown in Fig. (6). Substituting for z_1 and z_2 in equations (4.47) to (4.50) and separating the real and imaginary parts, we get

$$\text{Re}Z_1 = A_0 \rho_1^{-\frac{1}{2}} \cos \frac{\phi_1}{2} + A_1 \sqrt{\rho_1} \cos \frac{\phi_1}{2} + A_2 \rho_1^{\frac{3}{2}} \cos \left(\frac{3\phi_1}{2} \right) \quad (4.52)$$

$$\text{Im}Z_1 = -A_0 \rho_1^{-\frac{1}{2}} \sin \frac{\phi_1}{2} + A_1 \sqrt{\rho_1} \sin \frac{\phi_1}{2} + A_2 \rho_1^{\frac{3}{2}} \sin \frac{3\phi_1}{2} \quad (4.53)$$

Replacing z_1 by z_2 in equations (4.52) and (4.53)

$$\text{Re}Z_2 = A_0 \rho_2^{-\frac{1}{2}} \cos \left(\frac{\phi_2}{2} \right) + A_1 \sqrt{\rho_2} \cos \frac{\phi_2}{2} + A_2 \rho_2^{\frac{3}{2}} \cos \left(\frac{3\phi_2}{2} \right) \quad (4.54)$$

and

$$\text{Im}Z_2 = -A_0 \rho_2^{-\frac{1}{2}} \sin \frac{\phi_2}{2} + A_1 \sqrt{\rho_2} \sin \frac{\phi_2}{2} + A_2 \rho_2^{\frac{3}{2}} \sin \left(\frac{3\phi_2}{2} \right) \quad (4.55)$$

Similarly,

$$\text{Re}Y_1 = B_0 + B_1 \rho_1 \cos \phi_1 + B_2 \rho_1^2 \cos 2\phi_1 \quad (4.56)$$

$$\text{Im}Y_1 = B_1 \rho_1 \sin \phi_1 + B_2 \rho_1^2 \sin 2\phi_1 \quad (4.57)$$

$$\text{Re}Y_2 = B_0 + B_1 \rho_2 \cos \phi_2 + B_2 \rho_2^2 \cos 2\phi_2 \quad (4.58)$$

$$\text{Im}Y_2 = B_1 \rho_2 \sin \phi_2 + B_2 \rho_2^2 \sin 2\phi_2 \quad (4.59)$$

From Fig. 6, it is evident that

$$\phi_1 = \tan^{-1}[\lambda_1 \tan \theta] \quad (4.60)$$

and

$$\phi_2 = \tan^{-1}[\lambda_2 \tan \theta] \quad (4.61)$$

$$\rho_1 = r[1 - (1 - \lambda_1^2) \sin^2 \theta]^{\frac{1}{2}} \quad (4.62)$$

$$\rho_2 = r[1 - (1 - \lambda_2^2) \sin^2 \theta]^{\frac{1}{2}} \quad (4.63)$$

where $\lambda_1 = [1 - (\frac{c}{c_1})^2]^{\frac{1}{2}}$ and $\lambda_2 = [1 - (\frac{c}{c_2})^2]^{\frac{1}{2}}$

Substituting the expressions (4.52) to (4.59) in equations (4.45) and (4.46) and making use of eqns. (4.60) to (4.63), we can obtain expressions for $\frac{\sigma_y - \sigma_x}{2}$ and τ_{xy} in terms of the series constants and the polar coordinates r and θ .

2 Application of Photoelasticity to the Dynamic Equations:

The stress optic law, which relates the optical properties of the material to its stress state, is given by

$$2\tau_m = Nf_\sigma/h \quad (4.64)$$

where τ_m is the maximum in plane shear stress, f_σ the material fringe value, h the thickness of the model and N the order of the fringe in consideration.

It is known that

$$(2\tau_m)^2 = (\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2.$$

$$\text{i.e. } \tau_m^2 = \left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2 \quad (4.65)$$

Combining eqns. (4.64) and (4.65), we get

$$(Nf_\sigma/2h)^2 = \left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2 \quad (4.66)$$

The expressions for $\frac{\sigma_y - \sigma_x}{2}$ and τ_{xy} (4.46) and (4.45) can be substituted into eqn. (4.66) to obtain an equation connecting the series constants, the fringe order, the material fringe value and the polar coordinates r and θ . The function $(\frac{Nf_\sigma}{2h})^2 - (\frac{\sigma_y - \sigma_x}{2})^2 - \tau_{xy}^2$ is denoted by G_K and $\frac{\sigma_y - \sigma_x}{2}$ and τ_{xy} are denoted by D and T respectively.

$$\text{i.e. } G_K = (Nf_\sigma/2h)^2 - D^2 - T^2, \text{ at the } K^{\text{th}} \text{ data point.} \quad (4.67)$$

A combination of least squares and Newton-Raphson techniques is applied to this function as follows.

4.3 The Overdeterministic Method:

The approach used is that suggested by Sanford and Dally [14]. The series constants have to be determined to make $G_K = 0$. Though equation (4.67) can be solved in closed form, the algebra becomes quite involved and a simpler numerical method based on the Newton-Raphson technique is employed.

In the overdeterministic method, the function G_K is evaluated at a large number of data points in the stress field. If initial estimates are given for the series constants in eqn. (4.67), $G_K \neq 0$, since the initial estimates will usually be in error. To correct the estimates, a series of iterative equations based on a Taylor series expansion of G_K are written as

$$(G_K)_{i+1} = (G_K)_i + \left[\frac{\partial G_K}{\partial A_0}\right] \Delta A_0 + \left[\frac{\partial G_K}{\partial A_1}\right] \Delta A_1 + \dots \quad (4.68)$$

where the subscript i refers to the i^{th} iteration step and ΔA_0 , ΔA_1 , etc. are corrections for the previous estimates of A_0 , A_1 etc.

The corrections are determined so that $(G_K)_{i+1} = 0$ and thus eqn. (4.68) gives

$$\left[\frac{\partial G_K}{\partial A_0}\right] \Delta A_0 + \left[\frac{\partial G_K}{\partial A_1}\right] \Delta A_1 + \dots = - (G_K)_i \quad (4.69)$$

The method of least squares involves the determination of the series coefficients so that eqn. (4.67) is fitted to a large number of data points in the isochromatic field.

Eqn. (4.69) in the matrix form gives

$$- \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_N \end{bmatrix} = \begin{bmatrix} \Delta A_0 \\ \Delta A_1 \\ \Delta A_2 \\ \Delta B_0 \\ \Delta B_1 \\ \Delta B_2 \end{bmatrix} \times \begin{bmatrix} \frac{\partial G_1}{\partial A_0} & \frac{\partial G_1}{\partial A_1} & \dots & \frac{\partial G_1}{\partial B_2} \\ \frac{\partial G_2}{\partial A_0} & \frac{\partial G_2}{\partial A_1} & \dots & \frac{\partial G_2}{\partial B_2} \\ \vdots & \vdots & \dots & \vdots \\ \frac{\partial G_N}{\partial A_0} & \frac{\partial G_N}{\partial A_1} & \dots & \frac{\partial G_N}{\partial B_2} \end{bmatrix}$$

where N is the total number of data points considered. The above equation can be put in the form

$$G = [a][\Delta A] \quad (4.70)$$

$$\text{where } G = \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_N \end{bmatrix} \quad \Delta A = \begin{bmatrix} \Delta A_0 \\ \Delta A_1 \\ \vdots \\ \Delta B_2 \end{bmatrix}$$

$$\text{and } [a] = - \begin{bmatrix} \frac{\partial G_1}{\partial A_0} & \frac{\partial G_1}{\partial A_1} & \cdots & \frac{\partial G_1}{\partial B_2} \\ \frac{\partial G_2}{\partial A_0} & \frac{\partial G_2}{\partial A_1} & \cdots & \frac{\partial G_2}{\partial B_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial G_N}{\partial A_0} & \frac{\partial G_N}{\partial A_1} & \cdots & \frac{\partial G_N}{\partial B_2} \end{bmatrix}$$

Premultiplying by a^T (Transpose of a),

$$[a]^T G = [a]^T [a] [\Delta A] \text{ or}$$

$$[d] = [c] [\Delta A] \quad \text{where } [d] = [a]^T G$$

$$\text{and } [c] = [a]^T [a]$$

The correction factors are given by

$$[\Delta A] = [c]^{-1} [d] \quad (4.71)$$

The iterative procedure is employed till the series constants are determined to obtain a close fit of the function G to the N data points.

The differentials of G with respect to the series coefficients are obtained as follows:

Differentiating eqn. (4.67) with respect to A_0 gives

$$\frac{\partial G_K}{\partial A_0} = 2D \frac{\partial D}{\partial A_0} + 2T \frac{\partial T}{\partial A_0} \quad (4.72)$$

$$\frac{\partial D}{\partial A_0} = \frac{\partial}{\partial A_0} \left[\frac{\sigma_y - \sigma_x}{2} \right] = \frac{1}{[\Omega - (1 + \lambda_2^2)]} [\Omega_2 \rho_2^{-\frac{1}{2}} \cos \frac{\phi_2}{2} - (1 + \lambda_1^2) \rho_1^{-\frac{1}{2}} \cos \frac{\phi_1}{2}] \quad (4.73)$$

$$\frac{\partial T}{\partial A_0} = \frac{2\lambda_1}{(\Omega - 1 - \lambda_2^2)} (-\rho_2^{-\frac{1}{2}} \sin \frac{\phi_2}{2} + \rho_1^{-\frac{1}{2}} \sin \frac{\phi_1}{2}) \quad (4.74)$$

Substituting (4.73) and (4.74) into (4.72), we can get the expression for $\partial G_K / \partial A_0$. Likewise, the partial differentials of G with respect to the other coefficients can be obtained and substituted in eqn. (4.71) to obtain the correction factors for the series constants.

The whole iterative process has been set up in a computer program, written in BASIC. A sizeable number of data points are taken in the isochromatic field and their polar coordinates are fed into the computer with the help of a Calcomp 9000 digitizer. The HP9845A microprocessor is used for computing the series constants iteration after iteration. A listing of the program has been attached in the appendix.

4.4 Error Analysis:

The number of coefficients necessary for an adequate representation of the stress field over the data acquisition region can be estimated by examining, as a function of the number of coefficients, the value of the average fringe order error, $|\overline{\Delta n}|$, which is defined in this study as

$$|\Delta n| = \frac{1}{N^*} \sum_{K=1}^{K=N^*} |n_i - n_c|_K \quad (4.75)$$

where n_i is the specified (input) fringe order for a data point, n_c the fringe order calculated from the computed set of coefficients and N^* the total number of data points used [12]. Typically, errors of one tenth of a fringe order or less should indicate that the constants are accurate [12].

The error analysis has also been included in the computer program in the appendix.

5. EXPERIMENTAL PROCEDURE AND RESULTS

Cross type models were used in this study. The geometry of the specimen is shown in Fig. 7. The specimen length and width were made fairly large to insure that the boundaries were far away from the crack tip as the crack propagated. The starter crack is made with a band saw and the crack tip blunted with a fine file. The model is mounted on the loading frame, shown in Fig. 8, which was so designed that it could be used for both uniaxial and biaxial loading. Loading was applied using ENERPAC hydraulic cylinders and the loads recorded by in-line PCB Model 200A quartz transducers (of Piezotronics, Inc.), used in conjunction with a 484B line power unit. The transducers had built-in ICP (Integrated Circuit Piezoelectric) amplifiers. The cracks were initiated at fairly high loads by means of a solenoid actuated cutting tool so that the K_Q value (stress intensity factor associated with load level Q) lies in the plateau region of the $\dot{a}$ - K curve and the crack velocity was close to terminal velocity. Pictures of the propagating crack were taken by the High Speed Recording System and magnified prints were made out of the negatives for analysis. A fairly large number of data points, say 60, were taken on each picture and their locations with respect to the crack tip and crack line were fed into the main analysis program through a Calcomp 9000 digitizer and the series constants obtained.

In Experiment 1, a uniaxial normal stress of 233 psi (1.61 MPa) was applied. The variation of crack tip position with respect to time is shown in Fig. 9. A constant crack velocity of 383 m/s was obtained.

The stress intensity factor, as shown in Fig. 10 and Table 1, showed an increasing trend, values ranging from $1 \text{ MPa}\sqrt{\text{m}}$ to $1.55 \text{ MPa}\sqrt{\text{m}}$. The remote stress σ_{ox} varied as in Fig. 11 and Table 1, with values oscillating about a mean of -2 MPa .

In Experiment 2, a higher uniaxial normal stress of 558 psi (3.85 MPa) was applied. A crack velocity of 400 m/s was obtained, as shown in Fig. 12. The stress intensity factor showed a sharply increasing trend, as in Fig. 13 and Table 2, with values ranging from $1.1 \text{ MPa}\sqrt{\text{m}}$ to $1.84 \text{ MPa}\sqrt{\text{m}}$. The crack started the first of a series of multiple attempts to branch when K was $1.84 \text{ MPa}\sqrt{\text{m}}$. Thenceforth, the fracture surface was extremely rough with tiny incomplete branches coming out of the crack and the crack finally branched close to the boundary. Because of the belated branching, pictures of actual branching were not recorded. The branching angle was slightly different on the two surfaces, being 24° on one side and 22° on the other. σ_{ox} was fairly negative through most of the time because of the forward bend of the loops and it reached up to a maximum of -3.3 MPa , as shown in Fig. 14 and Table 2.

To study the influence of far-field stresses parallel to the crack, a series of biaxial tension-compression and tension-tension experiments were conducted.

Typical isochromatics for tension-compression loading are shown in Fig. 15. It is seen that the fringe loops exhibit a strong forward tilt for this kind of loading, which indicates the nature of the remote parallel stress.

In Experiment 3, a tensile normal stress of 443 psi [3.05 MPa] and a compressive parallel remote stress of 197 psi [1.36 MPa] were applied. The dynamic K was not enough at any stage to produce branching. A con-

stant crack velocity of 368 m/s was obtained, as shown in Fig. 16. Again, K showed an increasing trend as in Fig. 17 and Table 3, with values going up to $1.81 \text{ MPa}\sqrt{\text{m}}$ and σ_{ox} was oscillating about a mild negative mean of -2.1 MPa , as in Fig. 18 and Table 3, with values ranging from -3.4 MPa to 0.4 MPa . The compressive parallel stress was not high enough to produce a sizeable increase in σ_{ox} . However, the isochromatic fringe loops showed a forward tilt, indicating the nature of the remote parallel stress.

In Experiment 4, a tensile normal stress of 500 psi [3.45 MPa] and a compressive parallel remote stress of 180 psi [1.24 MPa] were applied. Because of the higher normal stress, the crack was initiated at a higher K_Q than Experiment 3. Unfortunately the crack did not branch but there was a conspicuous roughness of the fracture surface in the final stages of crack propagation, showing that the crack would have branched at a slightly higher initial K_Q . The K value associated with the roughness of fracture surface was fairly high ($1.77 \text{ MPa}\sqrt{\text{m}}$) as was observed by researchers at the University of Maryland [3] and the University of Washington [15]. Again, K showed a sharply increasing trend with values ranging from $0.98 \text{ MPa}\sqrt{\text{m}}$ to $1.77 \text{ MPa}\sqrt{\text{m}}$, as in Fig. 20 and Table 4 and σ_{ox} oscillated about a mild negative mean of about -1.3 MPa , as in Fig. 21 and Table 4. The crack velocity was 400 m/s in this experiment and the velocity plot is shown in Fig. 19.

In Experiment 5, the tensile normal stress was increased to 667 psi [4.6 MPa] and the compressive remote parallel stress was 174 psi [1.2 MPa]. The crack branched at a K_{br} of $2.01 \text{ MPa}\sqrt{\text{m}}$ and K showed an increasing trend as the crack moved across the model, as in Fig. 23 and Table 5. Because of a higher initial K_Q , the K values encountered in

this test were fairly high, ranging from 1.7 to 2.01 MPa $\sqrt{m}$. The crack velocity was a constant 385 m/s, as in Fig. 22. σ_{ox} values were oscillating about a negative mean of -3.7 MPa as in Fig. 24 and Table 5. The branching angle was 29°.

In Experiment 6, a tensile normal stress of 667 psi [4.6 MPa] and a much higher compressive remote parallel stress of 340 psi [2.34 MPa] were resorted to. For comparison purposes, the tensile normal stress was kept the same as in Experiment 5. It was observed that the K values for the same crack tip positions were lower in this experiment than Experiment 5. σ_{ox} reached high negative values up to -9.8 MPa because of a much higher compressive parallel stress, applied in this experiment. The crack velocity plot and the variation of K and σ_{ox} with crack tip position are shown in Figures 25, 26 and 27 respectively. K and σ_{ox} values are also listed in Table 6. Crack branching occurred at a K_{br} of 1.91 MPa $\sqrt{m}$.

It seems that remote compressive stress parallel to the crack tends to lower the stress intensity factor and hence suppresses branching. In Experiment 5, the crack made an attempt to branch - in fact, it almost branched at a crack jump distance of 7.2 cm, though actual branching occurred at a crack jump distance of 11.8 cm. In Experiment 6, the crack travelled 9.8 cm before it branched. This evidence adds to the fact that compressive remote parallel stress suppresses branching. This was also suggested by Dally et al [2] and Ravichandar [29]. It was also observed in experiments 5 and 6 that the fracture surface roughness was very high either when an attempt was made to branch or when actual branching occurred. Again, the branching angle in Experiment 6 was 24.5° as compared to 29° obtained in Experiment 5, indicating that compressive

parallel remote stress affects the branching angle, though not considerably. The case of compressive σ_{ox} has to be studied more by going in for very high compressive parallel loads.

Next, a series of biaxial tension-tension experiments were carried out. Typical isochromatics for this case are shown in Fig. 28. The fringes show a strong backward tilt and exhibit a behavior typical of a DCB specimen. The lean of the fringe patterns again signifies the nature of the remote parallel stress applied.

In Experiment 7, a tensile normal stress of 689 psi [4.75 MPa] and a tensile parallel stress of 240 psi [1.65 MPa] were applied. Even though the normal stress in this experiment was only slightly higher than that in Experiment 6, the K values in this experiment were much higher than the previous experiment for the same crack tip positions. For example at $a^*/w = .38$, K was $1.18 \text{ MPa}\sqrt{\text{m}}$ in Experiment 6 compared to $1.82 \text{ MPa}\sqrt{\text{m}}$ in Experiment 7. In fact, the stress intensity factor was so high that the crack branched very early. K varied with crack tip position as in Fig. 30 and Table 7, values starting from $1.81 \text{ MPa}\sqrt{\text{m}}$. K_{br} was close to $2 \text{ MPa}\sqrt{\text{m}}$. A crack velocity of 375 m/s was obtained as in Fig. 29. The fracture area before branching was 4.8 cm^2 and the branching angle was 30° . These observations point to the fact that tensile remote parallel stress enhances branching, which is in accordance with observations made in explosive testing by Dally et al. [2] and Rossmanith and Shukla [30]. σ_{ox} values were negative, because the tensile parallel stress was not enough to bend the isochromatic fringes backward. σ_{ox} variation is shown in Fig. 31 and Table 7.

In Experiment 8, a tensile normal stress of 422 psi [2.91 MPa] and a tensile parallel stress of 400 psi [2.76 MPa] were applied. Though

the normal stress applied in this test was considerably lower than Experiment 6, the K values for the same crack tip positions in this experiment were higher than Experiment 6, again showing that tensile remote parallel stress enhances the K value, which in turn, enhances branching. In this experiment, branching was not obtained because of an inadequate K_Q at initiation. A constant crack velocity of 377 m/s was obtained as in Fig. 32. K and σ_{ox} values varied as in Table 8 and Figures 33 and 34, with K ranging from 1.27 to 1.68 $\text{MPa}\sqrt{\text{m}}$ and σ_{ox} ranging from -2 to 2 MPa with a mean of -.1 MPa. σ_{ox} values were less negative than the uniaxial case because of the applied parallel tensile stress but again, the parallel stress was not enough to bend the fringe loops backward sufficiently and hence high positive values were not obtained.

In Experiment 9, a tensile normal stress of 538 psi [3.71 MPa] and a very high tensile parallel stress of 924 psi [6.4 MPa] were applied. A crack velocity of 400 m/s was obtained from Fig. 35. K values were fairly high compared to experiments with compressive σ_{ox} and the uniaxial case for the same crack tip locations and comparable initial K_Q 's and showed an increasing trend as shown in Table 9 and Fig. 36. The crack branched at a K_{br} value of 1.91 $\text{MPa}\sqrt{\text{m}}$ and σ_{ox} was hovering around high positive values (2.8 to 6 MPa) as in Fig. 37 and Table 9, because of the considerable backward tilt of the fringe loops due to the high parallel stress applied. The branching angle was 73° , which was much higher than the compressive σ_{ox} tests. One more interesting comparison can be made between Experiment 9 (normal stress 538 psi and parallel tensile stress 924 psi) and Experiment 2 (uniaxial normal stress 553 psi). In spite of the fact that the applied normal stress

was less in Experiment 9, the K values for the same crack tip locations were much higher in that experiment than Experiment 2, as shown in Tables 2 and 9. As an example, for $a^*/w = .39$, K was $1.56 \text{ MPa}\sqrt{\text{m}}$ in Experiment 9, compared to $1.28 \text{ MPa}\sqrt{\text{m}}$ in Experiment 2. The crack jump distance prior to branching was 16.2 cm (fracture area was 17.82 cm^2) in Experiment 2 compared to 10.7 cm (fracture area was 10.7 cm^2) in Experiment 9. The branching angle was 23° in Experiment 2, compared to 73° in Experiment 9. All these observations point to the fact that tensile parallel remote stress enhances K and hence branching. It also increases the branching angle considerably.

In Experiment 10, a tensile normal stress of 594 psi (4.10 MPa) and a tensile parallel stress of 614 psi (4.23 MPa) were applied. A crack velocity of 380 m/s was obtained and the stress intensity factor at branching was $2 \text{ MPa}\sqrt{\text{m}}$. The branching angle was 45° . From this experiment, one can conclude that the ratio of the applied normal stress to the applied parallel tensile stress has a marked influence on the branching angle. For the same normal stress, the branching angle increases with the tensile parallel stress. Since branching occurred very early in this experiment, only a few pre-branching pictures were recorded and hence K and σ_{ox} variation prior to branching was not plotted.

Stress Intensity Factors at branching, K_{br} , branching angles and pre-branching fracture areas for the various experiments are listed in Table 10. K_{br} seems to vary very slightly with different magnitudes of applied stresses and probably depends on initial K_Q . More experimentation has to be performed to confirm this. The variation of branching angle and the pre-branching fracture area with respect to

the ratio of the applied normal stress to the applied parallel stress (σ_y/σ_x) is shown in Figures 38 and 39 respectively, just to highlight the facts that branching occurs earlier in the case of positive σ_{ox} , even with lower normal stresses, compared to the uniaxial and the tensile-compressive cases and that branching angles are much higher in the tensile-tensile case.

The isochromatics for uniaxial, tensile-compressive and tensile-tensile cases are compared in Fig. 40 to explicitly show the slight forward tilt of the loops in the uniaxial case and the considerable backward and forward tilt in the tensile-tensile and tensile-compressive cases respectively.

The crack velocities and K-values, encountered in various experiments are plotted in Fig. 41. The points form a fairly narrow band in the high velocity region, with velocities ranging from 370 m/s to 400 m/s. The remote parallel stress does not seem to have any major or well-defined influence on the crack velocity.

Photographs of the tested specimens for the experiment 2 (uniaxial normal stress of 558 psi), experiment 5 (normal tensile stress of 667 psi and parallel compressive stress of 174 psi), experiment 9 (normal tensile stress of 538 psi and parallel tensile stress of 924 psi) and experiment 10 (normal tensile stress of 594 psi and parallel tensile stress of 614 psi) are given in Figures 42, 43, 44 and 45 respectively to show the branching angles obtained.

6. CONCLUSION

A photoelastic investigation of high speed crack propagation and branching was performed for cross-type specimens fabricated from Homalite 100 under uniaxial and biaxial loading conditions. Special consideration was given to the variation of the singular and non singular stress field coefficients as the crack propagated through the model. The effect of remote parallel stress σ_{ox} on crack propagation and branching was studied by applying very high tensile and compressive stresses parallel to the crack. The stress intensity factor showed an increasing trend as the crack propagated in the models. The value of K increased till crack branching was achieved. The branching stress intensity factor K_{br} was found to vary slightly with K_Q at initiation and the nature of the remote parallel stress. Branching angles in tensile σ_{ox} experiments were as high as 73° , compared to values between 22° and 29° encountered in compressive σ_{ox} and uniaxial cases, which point to the fact that tensile remote parallel stress increases the branching angle considerably. It was also observed that higher compressive σ_{ox} lowers the branching angle, though not sizably, for the same normal stress applied. This was evident from the branching angle of 24.5° obtained in Expt. 6 (normal stress 667 psi and parallel stress 340 psi) as compared to 29° obtained in Expt. 5 (normal stress 667 psi and parallel stress 174 psi). More experimentation has to be done to confirm the effect of compressive σ_{ox} .

Again, the pre-branching fracture area was higher in the compressive

σ_{ox} cases than that in all the tensile-tensile experiments, signifying that tensile σ_{ox} tends to enhance branching and compressive σ_{ox} tends to suppress it. Also, tensile σ_{ox} gives rise to higher stress intensity factors in comparison to the compressive case for the same crack tip positions and comparable K_Q values, which in turn, favors branching. A high roughness of fracture surface was associated with branching or attempts to branch.

Ramulu et al [15] have formulated a criterion for crack branching which says that a necessary and sufficient condition for branching is a crack branching stress intensity factor K_{Ib} , accompanied by a minimum characteristic distance $r_o = r_c$, which is a material property. They have also used equations for r_o and crack curving angle θ_c which are as follows:

$$r_o = \frac{1}{128\pi} \left[\left(\frac{K_I}{\sigma_{ox}} \right) f(c, c_1, c_2) \right]^2 \quad (6.1)$$

$$\theta_c = \cos^{-1} \left\{ f \left[\left(\frac{\sigma_{ox}}{K_I} \right)^2, r_o \right] \right\} \quad (6.2)$$

In this work, r_o was calculated for the various branching situations and the values, which ranged from 1.1 to 8.1 mm, were found to differ considerably from the r_c value of 1.3 mm, obtained by Ramulu and co-workers. The r_o values are shown in Table 11. Also, the branching angle was found to be dependent on the sign of σ_{ox} , i.e. the nature of the remote parallel stress, in contradiction to the equation (6.2), used by Ramulu et al, which seems to signify that the angle is independent of the sign of σ_{ox} .

This study concludes that high velocity crack propagation and crack branching are considerably influenced by non-singular stress field co-

efficients. More investigation has to be carried out to formulate an empirical branching criterion in terms of K , the crack velocity $\dot{a}$ and higher order stress field coefficients.

In this juncture, it has to be mentioned that the branching angles obtained were the macroscopic angles measured just at the crack tip. The branches tend to attract or repel each other, as was observed by Kalthoff [19], which makes it difficult to measure the exact macroscopic branching angle.

It was also observed that the results obtained by the crack tip stress analysis procedure, outlined in Chapter 4, are sensitive to all the input parameters including the radius, theta and the fringe order of the data points. For some unknown reason, there was a convergence problem in the six parameter model when the remote parallel stress was compressive. This was not the case when it was tensile or when uniaxial normal loads were applied.

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FIG. 1. Typical $\dot{a}$ - K Curve for Homalite 100.

FIG. 1. Typical $\dot{a} - K$ Curve for Homalite 100.

FIG. 2. Pulsing Circuit of the Camera.

R_{L1}	LOAD RESISTOR $3M\Omega$ 24 Watts
R_{D1}	DUMP RESISTOR $1M\Omega$, 16 Watts
$C_1 - C_{10}$	CAPACITORS $0.05\mu F$, 20 KVDC
$C_1^* - C_{10}^*$	
$L_1 - L_{10}$	INDUCTORS
$SG_1 - SG_{10}$	SPARK GAPS
R_M	METER RESISTOR $400M\Omega$
M	METER 0-50 μA
TSG	TRIGGER SPARK GAP
$R - R_{10}$	BLEED RESISTORS $2M\Omega$

FIG. 2. Pulsing Circuit of the Camera.

FIG. 3. Optical Arrangement of the Camera.

FIG. 4. Schematic Diagram for the Synchronization Circuits.

FIG. 5. Co-ordinate System for Crack Tip Stress Analysis.

FIG. 6. Relationship of Radii of Complex Variables of the Stress Functions.

FIG. 7. Specimen Geometry for Biaxial Tension-Tension Loading.

FIG. 8. Loading Frame for Experimentation.

FIG. 9. Crack Extension vs. Time Plot for Experiment 1.

FIG. 10. Stress Intensity Factor as a Function of Normalized Crack Tip Position for Experiment 1.

FIG. 11. Normalized Remote Stress as a Function of Normalized Crack Tip Position for Experiment 1.

FIG. 12. Crack Extension vs. Time Plot for Experiment 2.

FIG. 13. Stress Intensity Factor as a Function of Normalized Crack Tip Position for Experiment 2.

FIG. 14. Normalized Remote Stress as a Function of Normalized Crack Tip Position for Experiment 2.

FIG. 15. Typical Isochromatics for a Biaxial Tension-Compression Experiment.

FIG. 16. Crack Extension vs. Time Plot for Experiment 3.

FIG. 17. Stress Intensity Factor as a Function of Normalized Crack Tip Position for Experiment 3.

FIG. 18. Normalized Remote Stress as a Function of Normalized Crack Tip Position for Experiment 3.

FIG. 19. Crack Extension vs. Time Plot for Experiment 4.

FIG. 20. Stress Intensity Factor as a Function of Normalized Crack Tip Position for Experiment 4.

FIG. 21. Normalized Remote Stress as a Function of Normalized Crack Tip Position for Experiment 4.

FIG. 22. Crack Extension vs. Time Plot for Experiment 5.

FIG. 23. Stress Intensity Factor as a Function of Normalized Crack Tip Position for Experiment 5.

FIG. 24. Normalized Remote Stress as a Function of Normalized Crack Tip Position for Experiment 5.

FIG. 25. Crack Extension vs. Time Plot for Experiment 6.

FIG. 26. Stress Intensity Factor as a Function of Normalized Crack Tip Position for Experiment 6.

FIG. 27. Normalized Remote Stress as a Function of Normalized Crack Tip Position for Experiment 6.

FIG. 28. Typical Isochromatics for Biaxial Tension-Tension Experiment.

FIG. 29. Crack Extension vs. Time Plot for Experiment 7.

FIG. 30. Stress Intensity Factor as a Function of Normalized Crack Tip Position for Experiment 7.

FIG. 31. Normalized Remote Stress as a Function of Normalized Crack Tip Position for Experiment 7.

FIG. 32. Crack Extension vs. Time Plot for Experiment 8.

FIG. 33. Stress Intensity Factor as a Function of Normalized Crack Tip Position for Experiment 8.

FIG. 34. Normalized Remote Stress as a Function of Normalized Crack Tip Position for Experiment 8.

FIG. 35. Crack Extension vs. Time Plot for Experiment 9.

FIG. 36. Stress Intensity Factor as a Function of Normalized Crack Tip Position for Experiment 9.

FIG. 37. Normalized Remote Stress as a Function of Normalized Crack Tip Position for Experiment 9.

FIG. 38. σ_y/σ_x as a Function of Branching Angle.

FIG. 39. Pre-Branching Fracture Area as a Function of σ_y/σ_x .

FIG. 40. Comparison of Isochromatic Fringe Patterns for Uniaxial, Tensile-Compressive and Tensile-Tensile Loading.

FIG. 41. $\dot{a} - K$ Plot in the High Velocity Region.

FIG. 42 Post-mortem photograph of specimen - Expt. 2.

FIG. 43 Post-mortem photograph of specimen - Expt. 5.

FIG. 44 Post-mortem photograph of specimen - Expt. 9.

FIG. 45. Post-mortem photograph of specimen - Expt. 10.

TABLE 1
OUTPUT VALUES FOR EXPT. NO. 1
($\dot{a} = 383 \text{ m/s}$)

FRAME NO.	TIME (μs)	a^*/w	K $\text{MPa } \sqrt{\text{m}}$	A_1 $\text{MPa}/\sqrt{\text{m}}$	A_2 $\text{MPa}/\text{m}^{1.5}$	σ_{ox} MPa	B_1 MPa/m	B_2 MPa/m^2	$B'_O = \frac{\sigma_{ox}\sqrt{w}}{K}$
1	13.0	0.44	1.06	-2.8	-71.9	-2.3	4.8	7.9	-1.17
3	30.0	0.46	1.47	-37.6	43.6	1.4	20.6	-683.0	0.52
4	39.5	0.47	1.15	0.9	109.6	-2.2	-8.8	-477.0	-1.03
5	49.0	0.48	1.09	-29.5	-653.0	-1.8	58.4	545.0	-0.89
6	57.5	0.49	1.24	8.2	339.5	-2.1	-28.7	-732.0	-0.92
7	67.5	0.51	1.00	-28.7	-977.0	-2.6	78.1	1322.0	-1.41
8	77.5	0.52	1.28	-34.6	-518.0	-0.7	55.4	456.0	-0.30
9	86.5	0.53	1.29	-14.4	-52.2	-1.3	9.6	-4.9	-0.55
10	95.0	0.54	1.11	-18.2	-541.7	-2.6	39.1	606.0	-1.27
11	139.5	0.60	1.40	-51.5	-996.0	-0.5	99.9	1155.0	-0.19
12	151.5	0.62	1.30	-0.84	29.1	-2.7	-8.6	-112.0	-1.12
14	171.5	0.64	1.05	-21.9	-878.0	-3.8	61.6	1191.0	-1.96
15	181.0	0.56	1.55	-59.4	-963.0	0.4	104.0	1074.0	0.14
16	190.5	0.67	1.11	-21.1	-1156.0	-4.0	82.7	1857.0	-1.95
17	203.5	0.68	.99	-22.9	-1469.0	-4.8	97.0	2459.0	-2.62

TABLE 2
OUTPUT VALUES FOR EXPT. NO. 2
($\dot{a} = 400 \text{ m/s}$)

FRAME NO.	TIME (μs)	a^*/w	K $\text{MPa } \sqrt{\text{m}}$	A_1 $\text{MPa}/\sqrt{\text{m}}$	A_2 $\text{MPa}/\text{m}^{1.5}$	σ_{ox} MPa	B_1 MPa/m	B_2 MPa/m^2	$R'_O = \frac{\sigma_{ox}\sqrt{w}}{K}$
1	15.0	0.33	1.12	-17.5	-253	-1.6	14.4	33	-0.77
2	26.0	0.34	1.09	-10.7	-430	-2.4	18.6	616	-1.19
3	37.5	0.35	1.13	-10.3	-557	-2.6	28.5	884	-1.24
4	44.5	0.37	1.21	18.3	27	-2.6	-19.2	469	-1.16
5	53.0	0.38	1.37	-45.0	-930	-0.3	83.4	1194	-0.12
6	63.5	0.39	1.28	0.2	-99	-2.4	-3.0	172	-1.01
8	83.5	0.42	1.37	6.8	87	-2.2	-11.4	-126	-0.87
9	93.5	0.43	1.65	-39.5	-329	1.0	50.0	405	0.33
10	103.0	0.45	1.63	-42.0	-472	0.6	59.0	509	0.20
11	139.5	0.50	1.77	-49.0	-648	0.4	78.0	560	0.12
12	145.0	0.51	1.80	-30.0	-278	-0.4	35.0	71	-0.12
14	167.5	0.54	1.89	-38.0	-438	-0.1	54.0	343	-0.03
15	174.0	0.55	1.67	-22.0	-706	-2.9	53.0	966	-0.94
16	183.0	0.56	1.73	-10.8	-333	-2.9	23.0	389	-0.91
17	193.5	0.58	1.80	-13.4	-365	-3.0	23.0	374	-0.90
19	223.5	0.62	1.84	-21.0	-478	-3.3	41.0	444	-0.97

TABLE 3
OUTPUT VALUES FOR EXPT. NO. 3
($\dot{a} = 368 \text{ m/s}$)

FRAME NO.	TIME (μs)	a^*/w	K $\text{MPa } \sqrt{\text{m}}$	A_1 $\text{MPa}/\sqrt{\text{m}}$	A_2 $\text{MPa}/\text{m}^{1.5}$	σ_{ox} MPa	k_1 MPa/m	k_2 MPa/m^2	$B_0' = \frac{\sigma_{ox}\sqrt{w}}{K}$
1	13.5	0.39	0.90	8.6	250	-2.6	-27.0	-74	-0.78
3	30.5	0.40	0.96	-2.7	-285	-2.4	11.0	577	-0.68
4	40.5	0.42	1.00	-21.2	-400	-1.8	31.0	377	-0.46
6	57.5	0.44	0.96	17.7	-507	-3.4	20.0	1558	-0.97
7	66.5	0.45	1.12	4.6	-331	-2.2	11.0	1039	-0.55
9	85.0	0.48	1.05	13.2	-281	-3.2	10.0	769	-0.82
12	152.5	0.56	1.40	-11.6	60	-1.4	-4.0	-176	-0.26
15	179.0	0.59	1.81	-13.2	460	0.4	-40.0	-602	0.07

TABLE 4
OUTPUT VALUES FOR EXPT. NO. 4
($\dot{a} = 400 \text{ m/s}$)

FRAME NO.	TIME (μs)	a^*/w	K $\text{MPa } \sqrt{\text{m}}$	A_1 $\text{MPa}/\sqrt{\text{m}}$	A_2 $\text{MPa}/\text{m}^{1.5}$	σ_{ox} MPa	B_1 MPa/m	B_2 MPa/m^2	$B'_0 = \frac{\sigma_{ox}\sqrt{w}}{K}$
1	31.0	0.37	0.98	-25	-706	-2.2	47	983	-1.21
2	39.0	0.38	1.18	-4	330	-1.4	0	-862	-0.64
4	57.5	0.41	1.12	-15	-376	-2.0	28	593	-0.97
5	66.0	0.42	1.10	-19	-519	-2.2	39	742	-1.10
6	75.5	0.43	1.20	-16	-42	-1.6	9	-77	-0.72
7	84.0	0.44	1.06	-20	-599	-3.0	40	786	-1.53
9	103.5	0.47	1.34	-11	-206	-1.8	5	649	-0.73
10	113.5	0.48	1.05	-86	-2523	-2.2	146	2727	-1.13
11	142.5	0.51	1.03	-27	-2066	-5.0	94	4486	-2.62
12	153.0	0.53	1.39	-32	-720	-2.4	34	1057	-0.93
14	171.5	0.55	1.36	-35	-1169	-2.8	81	1908	-1.11
15	180.5	0.57	1.57	-36	-935	-1.8	63	1572	-0.62
16	190.5	0.58	2.46	-75	434	5.2	13	316	1.14
17	201.5	0.60	2.47	-73	-758	4.4	92	1941	0.96
19	226.0	0.63	1.77	-64	-1229	-0.4	92	1589	-0.12

TABLE 5
OUTPUT VALUES FOR EXPT. NO. 5
($\dot{a} = 385 \text{ m/s}$)

FRAME NO.	TIME (μs)	a^*/w	K $\text{MPa } \sqrt{\text{m}}$	A_1 $\text{MPa}/\sqrt{\text{m}}$	A_2 $\text{MPa}/\text{m}^{1.5}$	σ_{ox} MPa	B_1 MPa/m	B_2 MPa/m^2	$B'_0 = \frac{\sigma_{ox}\sqrt{w}}{K}$
1	33.5	0.44	1.86	-74.4	-1193	-0.6	117	1155	-0.17
2	41.5	0.45	1.76	-7.1	443	-3.8	-38	-1516	-1.17
3	50.0	0.46	1.71	-18.8	-330	-4.2	22	165	-1.33
4	60.0	0.48	1.38	40.5	-2028	-8.6	102	6617	-3.37
5	69.0	0.49	1.85	-93.4	-2715	-2.4	173	3869	-0.70
7	87.5	0.51	1.98	-22.2	-240	-3.4	19	11	-0.93
8	98.0	0.53	2.01	-100.9	-3696	-2.8	266	5333	-0.75

TABLE 6
 OUTPUT VALUES FOR EXPT. NO. 6
 ($\dot{a} = 372 \text{ m/s}$)

FRAME NO.	TIME (μs)	a^*/w	K MPa $\sqrt{\text{m}}$	A_1 MPa/ $\sqrt{\text{m}}$	σ_{ox} MPa	B_1 MPa/m	$k_o = \frac{\sigma_{ox}\sqrt{w}}{K}$
1	34.5	0.38	1.18	9.0	-6.8	-17	-3.11
2	43.0	0.39	1.22	6.7	-6.6	-20	-2.92
3	51.0	0.40	1.29	8.2	-6.5	-16	-2.72
4	61.0	0.42	1.36	7.3	-6.3	-15	-2.50
7	88.5	0.45	1.54	1.4	-5.4	-18	-1.90
8	97.5	0.46	1.58	2.0	-5.5	-17	-1.88
9	106.5	0.47	1.62	6.0	-6.4	-25	-2.14
10	116.0	0.48	1.58	4.0	-6.2	-15	-2.12
11	120.5	0.49	1.69	1.6	-5.4	-22	-1.73
12	131.0	0.50	1.72	-1.6	-5.4	-15	-1.70
14	148.0	0.53	1.66	4.8	-7.4	-20	-2.41
15	156.5	0.54	1.65	7.8	-8.4	-15	-2.75
16	166.0	0.55	1.76	8.9	-8.4	-26	-2.58
17	176.0	0.57	1.71	6.5	-8.4	-23	-2.66
18	185.5	0.58	1.91	6.4	-7.4	-30	-2.09
19	197.5	0.59	1.75	14.0	-9.8	-23	-3.03

TABLE 7
 OUTPUT VALUES FOR EXPT. NO. 7
 ($\dot{a} = 375 \text{ m/s}$)

FRAME NO.	TIME (μs)	a^*/w	K $\text{MPa } \sqrt{\text{m}}$	A_1 $\text{MPa}/\sqrt{\text{m}}$	A_2 $\text{MPa}/\text{m}^{1.5}$	σ_{ox} MPa	B_1 MPa/m	B_2 MPa/m^2	$B'_0 = \frac{\sigma_{ox}\sqrt{w}}{K}$
1	32.5	0.37	1.81	46	-610	-6.0	8	1568	-1.79
2	40.0	0.38	1.82	-27	-1195	-4.4	86	960	-1.31
3	49.5	0.39	1.98	48	397	-5.4	-58	-696	-1.47
4	60.0	0.41	2.21	27	171	-3.4	-34	134	-0.83
5	70.0	0.42	2.34	27.4	30.1	-0.2	-102	79.1	-0.50
6	80.0	0.43	2.50	28.0	34.2	0.2	-121	69.5	-0.34
10	117.5	0.46	2.90	27.2	34.8	0.2	-114	111.0	-0.20
11	162.0	0.52	3.45	26.2	32.8	0.8	-94	148.0	-0.29
12	174.0	0.55	3.62	26.7	33.4	2.0	-122	149.0	-0.40
13	181.0	0.56	3.70	26.5	33.8	0.2	-127	141.0	-0.46
14	181.0	0.56	3.70	26.5	33.8	0.2	-127	141.0	-0.46
15	187.0	0.57	3.82	26.2	34.0	0.8	-128	147.0	-0.55
17	218.0	0.59	4.00	25.5	34.4	0.2	-121	178.0	-0.57
18	228.0	0.61	4.22	24.1	32.1	0.4	-142	183.0	-0.73
19	243.0	0.62	4.38	23.7	30.1	0.2	-146	186.0	-0.76

TABLE 8
OUTPUT VALUES FOR EXPT. NO. 8
($\dot{a} = 377 \text{ m/s}$)

FRAME NO.	TIME (μs)	a^*/w	K $\text{MPa } \sqrt{\text{m}}$	A_1 $\text{MPa}/\sqrt{\text{m}}$	A_2 $\text{MPa}/\text{m}^{1.5}$	σ_{ox} MPa	B_1 MPa/m	B_2 MPa/m^2	$B'_0 = \frac{\sigma_{ox}\sqrt{w}}{K}$
1	35.5	0.36	1.27	50.9	102	-2.0	-84	94	-0.85
2	46.0	0.37	1.35	45.1	237	-1.2	-78	704	-0.48
3	52.0	0.38	1.46	24.8	926	0	-101	-1933	0
4	62.5	0.39	1.49	-31.2	-1346	1.0	104	2807	0.36
5	71.5	0.40	1.50	18.7	1146	0.6	-87	-2277	0.22
6	79.5	0.42	1.35	52.4	943	-0.8	-108	-541	-0.32
7	89.0	0.43	1.44	28.7	867	0	-81	-1396	0
8	99.5	0.44	1.58	-14.0	642	2.0	-21	-1663	0.68
10	117.5	0.46	1.50	23.3	849	0.2	-74	-1710	0.07
11	162.0	0.52	1.60	26.7	359	0.6	-46	310	0.20
12	173.5	0.54	1.65	20.7	544	1.0	-51	-483	0.33
13	181.0	0.55	1.50	44.0	219	-1.2	-59	142	-0.43
14	191.5	0.56	1.55	39.1	248	-0.6	-60	262	-0.21
15	199.5	0.57	1.57	36.2	250	-0.6	-58	147	-0.21
17	219.0	0.59	1.65	29.5	594	-0.2	-71	-695	-0.07
18	228.5	0.61	1.63	34.1	437	-0.4	-62	-523	-0.13
19	241.5	0.62	1.68	13.1	481	0.3	-34	-1167	0.08

TABLE 9
OUTPUT VALUES FOR EXPT. NO. 9
($\dot{a} = 400$ m/s)

FRAME NO.	TIME (μ s)	a^*/w	K MPa $\sqrt{m}$	A_1 MPa/ $\sqrt{m}$	A_2 MPa/m ^{1.5}	σ_{ox} MPa	B_1 MPa/m	B_2 MPa/m ²	$k_o' = \frac{\sigma_{ox}\sqrt{w}}{K}$
1	31.0	0.35	1.52	5.1	-401	5.0	31.1	1874	1.78
3	50.0	0.38	1.60	-8.4	77	6.1	1.0	151	2.06
4	58.0	0.39	1.56	-2.5	-216	5.6	10.7	690	1.94
5	67.0	0.40	1.50	0.6	26	5.8	9.3	724	2.09
6	76.0	0.41	1.64	-35.0	1167	7.6	1.4	-2958	2.51
7	85.0	0.42	1.57	3.2	-252	5.6	13.5	1248	1.93
8	95.5	0.44	1.55	2.8	-325	5.6	18.0	1167	1.95
9	103.5	0.45	1.53	5.3	-387	5.4	30.5	1737	1.91
10	113.0	0.46	1.54	-0.6	-275	5.8	18.6	897	2.04
11	162.5	0.53	1.71	-3.8	87	5.8	-5.7	-1195	1.83
12	173.0	0.54	1.83	-5.1	-237	5.6	-1.8	342	1.65
13	181.5	0.55	1.73	4.4	-910	4.6	47.6	2265	1.44
14	191.0	0.57	1.74	11.5	-143	4.4	-7.6	683	1.37
16	209.5	0.59	1.82	9.5	486	3.8	-48.4	-1000	1.13
17	220.0	0.60	1.82	21.2	-3	2.8	-32.8	1032	0.83
18	231.5	0.62	1.91	13.3	-379	3.4	-6.5	1851	0.96

TABLE 10

K_{br} , Branching Angles and Pre-branching Fracture Areas for all the Experiments

EXPT. NO.	σ_y (MPa)	σ_x (MPa)	Pre-branching fracture area (cm) ²	Branching Angle (degrees)	K_{br} (MPa $\sqrt{m}$)
#2	3.85	0	17.8	23	1.84 (branching attempt)
#5	4.6	-1.2	15.0	29	2.01
#6	4.6	-2.34	12.4	24.5	1.91
#7	4.75	1.65	4.8	30	2.2
#9	3.71	6.4	10.8	73	1.91
#10	4.51	4.66	3.5	45°	2

TABLE 11

Summary of Calculated r_o Values

EXPT. NO.	K_{br} (MPA $\sqrt{m}$)	σ_{ox} (MPA)	r_o (MM)
5	2.01	-2.8	8.1
6	1.91	-7.4	1.1
7	2.21	-3.4	6.8
9	1.91	3.4	4.8

APPENDIX

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10  DISP "HELLO, PLEASE SEE CRT OR PRINTER FOR INSTRUCTIONS"
20  WAIT 3000
30  DISP "THIS PROGRAM USES DYN.6 PARA-MODEL : ALL OUTPUT IN S.I.UNITS"
40  WAIT 3000
50  DISP "DATA TO BE FED THRU DIGITIZER INCLUDING FR.ORDER"
60  WAIT 3000
70  DISP "IF THE VALUES CONVERGE BEFORE ASSIGNED ITRNS. STOP PROGRAM"
80  WAIT 3000
90  DISP " NOW YOU MAY START"
100 WAIT 2000
110 PRINT "FEED EXPT & FRAME NO."
120 INPUT O2,O3
130 PRINTER IS 0
140 PRINT USING "K";"EXPT",O2," ", "FRAME",O3
150 PRINT "-----"
160 PRINTER IS 16
170 PRINT "FEED IN INITIAL A0 ETC.-ALL IN S.I. UNITS"
180 INPUT A0,A1,A2,B0,B1,B2
190 PRINT "INITIAL A0 ETC.=" ,A0,A1,A2,B0,B1,B2
200 PRINT "FEED IN C,C1,C2-ALL IN M/SEC"
210 INPUT C,C1,C2
220 PRINT "C=",C,"C1=",C1,"C2=",C2
230 PRINT "FEED IN F(SIGMA) IN MPa-M/FRINGE & THICKNESS(H) IN METER"
240 INPUT F,H
250 PRINT "FSIGMA=",F,"THICKNESS=",H
260 PRINT "FEED IN MINIMUM FRINGE ORDER TAKEN,N100"
270 INPUT N100
280 PRINT "N-MINIMUM=",N100
290 DISP "DO YOU WANT INSTRUCTNS. FOR FEEDING FR.ORDERS ON THERMAL PRINTER?"
300 WAIT 3000
310 DISP "IF YES, FEED I=1; IF NO, FEED I=0"
320 WAIT 3000
330 INPUT "FEED I",I
340 IF I=0 THEN PRINTER IS 16
350 IF I=1 THEN PRINTER IS 0
360 PRINT USING "K";"FR.ORDER", " ", "BUTTON ORDER"
370 FOR J=0 TO 6
380 PRINT USING 390;N100+.5*J,J
390 IMAGE 2X,D.D,10X,D
400 NEXT J
410 PRINTER IS 16
420 DIM R(60),O(60),N(60),G(60),A(60,6),U(6,60),Q(6,6),W(6,6),Y(6,6),P(6),X(6
),X1(60),Y1(60),C(60),N1(60),N2(60),D(60),T(60)
430 PRINT "FEED TOTAL NO. OF DATA PTS."
440 INPUT N3
450 PRINT "INPUT ACTUAL DISTANCE BETWEEN 2 ADJACENT MARKS(D) IN INCHES"
460 INPUT D
470 PRINT "TO FIND SCALE FACTOR, INPUT CO-ORDINATES OF MARKS(X1,Y1,X2,Y2)"
480 ENTER 701;S5,X1,Y1
490 ENTER 701;S5,X2,Y2
500 PRINT X1,Y1,X2,Y2
510 R1=SQR((Y2-Y1)^2+(X2-X1)^2)
520 M0=D/R1
530 PRINT "SCALE FACTOR=",M0
540 PRINT "TO FIND INCLINATION OF CRACK LINE,FEED CO-ORDINATES OF 2 PTS.ON CR
ACK LINE(X3,Y3,X4,Y4)"
550 ENTER 701;S5,X3,Y3
560 ENTER 701;S5,X4,Y4
570 Y7=Y4-Y3
580 X7=X4-X3
590 O1=ATN(Y7/X7)
600 IF (Y7<=0) AND (X7>=0) THEN O1=01
610 IF (Y7>=0) AND (X7<=0) THEN O1=01+PI
620 IF (Y7<0) AND (X7<0) THEN O1=01-PI
630 IF (Y7<0) AND (X7>0) THEN O1=01

540 A3=01+180/PI
550 PRINT "INCLINATION OF CRACK LINE(IN DEG.)=",A3
560 PRINT "FEED CRACK TIP LOCATION (X5,Y5)"
570 ENTER 701;S5,X5,Y5
580 PRINT "CRACK TIP CO-ORDINATES ARE AS FOLLOWS;-"
590 PRINT X5,Y5
600 PRINTER IS 0
700 PRINT USING "X,K,14X,K,4X,K,6X,K";"PT.NO.", "RADIUS IN INCHES", "THETA IN D
EG.", "FR.ORDER"
710 DISP "INPUT DATA FOR RADIUS(R), THETA(O),FR.ORDER(N)"
720 WAIT 100
730 FOR I=1 TO N3
740 ENTER 701;C(I),X1(I),Y1(I)
750 X6=X1(I)-X5
760 Y6=Y1(I)-Y5

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770 R(I)=M0*SQR(X6^2+Y6^2)
780 O2=ATN(Y6/X6)
790 IF (Y6>0) AND (X6>0) THEN O2=O2
800 IF (Y6>0) AND (X6<0) THEN O2=O2+PI
810 IF (Y6<0) AND (X6<0) THEN O2=O2-PI
820 IF (Y6<0) AND (X6>0) THEN O2=O2
830 A4=O2*180/PI
840 O(I)=A4-A3
850 N(I)=N100+C(I)/2
860 PRINTER IS 0
860 PRINT I,R(I),O(I),N(I)
861 PRINTER IS 16
870 R(I)=R(I)*.0254
880 O(I)=O(I)*PI/180
890 NEXT I
900 INPUT "ITRNS.=?",J
910 REAL L1,L2,L12,L22,I21,I22,I31,I32
920 L12=1-(C/C1)^2
930 L22=1-(C/C2)^2
940 L1=SQR(L12)
950 L2=SQR(L22)
960 FOR L=1 TO J
970 FOR I=1 TO N3
980 S1=SIN(O(I))
990 S2=COS(O(I))
1000 S3=TAN(O(I))
1010 X1=R(I)*S2
1020 Y1=R(I)*L1*S1
1030 X2=X1
1040 Y2=R(I)*L2*S1
1050 F1=ATN(L1*S3)
1060 F2=ATN(L2*S3)
1070 IF (X1>0) AND (Y1>0) THEN F1=F1
1080 IF (X1>0) AND (Y1<0) THEN F1=F1
1090 IF (X1<0) AND (Y1>0) THEN F1=3.14159+F1
1100 IF (X1<0) AND (Y1<0) THEN F1=-3.14159+F1
1110 IF (X1>0) AND (Y2>0) THEN F2=F2
1120 IF (X1>0) AND (Y2<0) THEN F2=F2
1130 IF (X1<0) AND (Y2>0) THEN F2=3.14159+F2
1140 IF (X1<0) AND (Y2<0) THEN F2=-3.14159+F2
1150 S12=S1^2
1160 S22=S2^2
1170 R01=R(I)*SQR(1-(1-L12)*S12)
1180 R02=R(I)*SQR(1-(1-L22)*S12)
1190 S01=SIN(F1/2)
1200 S02=COS(F1/2)
1210 S03=COS(3*F1/2)
1220 S04=COS(F2/2)
1230 S05=COS(3*F2/2)
1240 S06=COS(F1)
1250 S07=COS(2*F1)
1260 S08=COS(F2)
1270 S09=COS(2*F2)
1280 S010=SIN(3*F1/2)
1290 S011=SIN(F2/2)
1300 S012=SIN(F1)
1310 R21=R01^1.5+A1*R01^1.5+S02+A2*R01^1.5+S03
1320 R22=R01^1.5+S04/R02^1.5+A1*R02^1.5+S04+A2*R02^1.5+S05
1330 R31=B0+B1*R01+S06+B2*R01^2+S07
1340 R32=B0+B1*R02+S06+B2*R02^2+S07
1350 I21=-R01*S01/R01^1.5+A1*R01^1.5+S01+A2*R01^1.5+S010
1360 I22=-R01*S01/R02^1.5+A1*R02^1.5+S011+A2*R02^1.5+SIN(3*F2/2)
1370 I31=B1*R01+S012+B2*R01^2+SIN(2*F1)
1380 I32=B1*R02+SIN(F2)+B2*R02^2+SIN(2*F2)
1390 W=4*L1*L2/(1+L22)
1400 D(I)=(W*R22-(1+L12)*R21)/(W-(1+L22))+((1+L22)*R32-(1+L12)*R31)/(L12-L22)
1410 T(I)=2*L1*(I22-I21)/(W-1-L22)+((1+L22)/(2*I32)-(2*L1*I31)/(L12-L22)
1420 G(I)=D(I)^2+T(I)^2-(N(I)*F/(2*H))^2
1430 D1=(W*S04/R02^1.5-(1+L12)*S02/R01^1.5)/(W-1-L22)
1440 D2=(W*S04/R02^1.5-(1+L12)*R01^1.5*S02)/(W-1-L22)
1450 D3=(W*S05/R02^1.5-(1+L12)*S03/R01^1.5)/(W-1-L22)
1460 D4=-1
1470 D5=((1+L22)*R02*S08-(1+L12)*R01*S06)/(L12-L22)
1480 D6=((1+L22)*R02^2*S09-(1+L12)*R01^2*S07)/(L12-L22)
1490 T1=(S01/R01^1.5-S011/R02^1.5)*2*L1/(W-1-L22)
1500 T2=2*L1*(S011/R02^1.5-S01/R01^1.5)/(W-1-L22)
1510 T3=2*L1*(R02^1.5*SIN(1.5*F2)-R01^1.5*S010)/(W-(1+L22))
1520 T4=0
1530 T5=((1+L22)^2*R02*SIN(F2)/(2*L2)-2*L1*R01*S012)/(L12-L22)
1540 T6=((1+L22)^2*R02^2*SIN(2*F2)/(2*L2)-2*L1*R01^2*SIN(2*F1))/(L12-L22)
1550 A(I,1)=2*D(I)*D1+2*T(I)*T1
1560 A(I,2)=2*D(I)*D2+2*T(I)*T2
1570 A(I,3)=2*D(I)*D3+2*T(I)*T3
1580 A(I,4)=2*D(I)*D4+2*T(I)*T4
1590 A(I,5)=2*D(I)*D5+2*T(I)*T5
1600 A(I,6)=2*D(I)*D6+2*T(I)*T6
1610 NEXT I

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1620 MAT U=TRN(A)
1630 MAT P=U*G
1640 MAT Q=U*A
1650 MAT W=INV(Q)
1660 MAT Y=(-1)*W
1670 MAT X=Y*P
1680 A0=A0+X(1)
1690 A1=A1+X(2)
1700 A2=A2+X(3)
1710 B0=B0+X(4)
1720 B1=B1+X(5)
1730 B2=B2+X(6)
1731 IF L>J-4 THEN 1733
1732 GOTO 1740
1733 PRINTER IS 0
1740 PRINT "ITERATION NO.=",L
1750 PRINT "-----"
1760 PRINT "INTERMEDIATE A0 ETC.=",A0,A1,A2,B0,B1,B2
1770 MAT G=ZER(N3)
1780 FOR I1=1 TO N3
1790 N1(I1)=SQR(D(I1)^2+T(I1)^2-G(I1))*2*H/F
1800 N2(I1)=ABS(N(I1)-N1(I1))
1810 NEXT I1
1820 FOR I2=2 TO N3
1830 N2(I2)=N2(I2)+N2(I2-1)
1840 NEXT I2
1850 N60=N2(N3)/N3
1851 PRINT "AVERAGE FR. ORDER ERROR=",N60
1852 PRINTER IS 16
1860 NEXT L
1870 K1=A0+SQR(2*PI)
1871 PRINTER IS 0
1880 PRINT "FINAL VALUES OF THE SIX PARAMETERS ARE:--"
1890 PRINT "-----"
1900 PRINT USING "K:": K1=" ", "K1," ", "MPA-SQR(METER)"
1910 PRINT USING "A:": A1=" ", "A1," ", "MPA/SQR(METER)"
1920 PRINT USING "K:": A2=" ", "A2," ", "MPA/M^1.5"
1930 PRINT USING "K:": B0=" ", "B0," ", "MPA"
1940 PRINT USING "K:": B1=" ", "B1," ", "MPA/M"
1950 PRINT USING "K:": B2=" ", "B2," ", "MPA/M^2"
1951 PRINTER IS 16
1960 DISP "IF THE PARAMETERS HAVEN'T CONVERGED DON'T USE THE VALUES"
1970 WAIT 3000
1980 END

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