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The Construction of a Mathematical Manual to Serve Secondary Schools with Particular Reference to Those of North Kingstown

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THE CONSTRUCTION OF A MATHEMATICAL MANUAL TO SERVE
SECONDARY SCHOOLS WITH PARTICULAR
REFERENCE TO THOSE OF
NORTH KINGSTOWN

BY
 LEROY EDWIN ALGREN

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
 REQUIREMENTS FOR THE DEGREE OF
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MASTER OF SCIENCE THESIS

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ABSTRACT

This is not a course of study, and it is not intended as a directive which tells teachers what they should teach in their day by day routine in the classroom. It is designed as a resource for teachers and others who wish to develop mathematics materials and to work effectively for and with their pupils. It must be acceptable in both theory and practice if it is to be used in the secondary schools of North Kingstown.

Much thought has been given to the problems of teachers and the types of materials needed to solve some of these problems. They seem to be more in the field of how to teach than what to teach. Teachers seem to be faced with such continuing problems as providing for a wide range of differences in pupils within a group, handling large groups, overcoming the handicap of inadequate materials, and applying their knowledge of the learning process to the education of children.

This guide to the development of curriculum materials in arithmetic and mathematics provides a flexible framework of content and procedures. The nature and meaning of arithmetic and mathematics as an essential part of everyday life are stressed in many ways within the reach of each pupil and within each section of the manual. There are included many suggestions as to the inter-relationship of the skills, abilities, and competencies of arithmetic and mathematics with other subjects of

the curriculum.

As teachers use this manual to give direction and provide suggestions in the building of arithmetic and mathematics curricular materials to meet the needs of pupils, they will find certain areas of emphasis projected for careful consideration. The skills, abilities, and competencies of arithmetic and more advanced mathematics are necessary to the total development of pupils. The place of drill in learning, the use of the textbook in teaching, the principles for evaluation, and many other aspects of teaching have been included. It may well be said that this manual is designed in organization and content to aid teachers in the preparation of curricular materials that will help pupils in the daily use of skills, abilities, and competencies in the field of both arithmetic and mathematics in general.

No time limits have been specified for teaching of various phases of the subject, thus allowing for greater flexibility on the part of the teachers. The intent here has been to guide the teacher into the requirements of the various courses. Such standard courses as General Mathematics, First-Course Algebra, Plane and Solid Geometry, Second-Course Algebra, and Trigonometry have been included in this study and will be found in detail within this manual.

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CHAPTER I

INTRODUCTION

This is a mathematics manual designed for teachers of secondary grades with particular reference to the schools of the town of North Kingstown. The need for such a manual or guide has grown out of the study of mathematics, particularly that of recent years. Revision and scope have been so greatly extended that, as a result, many teachers find themselves in a quandary when trying to determine what topics will be adequate at any given point in the mathematics sequence.

The purpose of this manual is three-fold: first, to be of use in a helpful and guiding sense to inexperienced, experienced, and training teachers, both in North Kingstown and elsewhere. Such a manual can be of much value to all. There are oftentimes questions in the minds of new teachers as to proven methods, as well as to acceptable suggestions. It is hoped that both may be found in the body of this manual under the desired subject heading. Also, there are questions as to what material should be covered and to what extent, and whether or not a specific topic should be taken up at a certain grade level. These are sincere questions in the minds of new teachers.

Experienced teachers can find valuable use here. New suggestions for various units might alter and brighten a

routine which has become overused and, thus, border on boredom. The varied and ample supply of suggestions in each topic under the various subject headings can be of much value to such teachers. The manual is so constructed that any part of a topic can be singled out, as well as its accompanying suggestions. Training teachers will doubtless find the manual an indispensable aid with its valuable suggestions as well as subject content coverage. The answers to many of their questions can be found to their own satisfaction.

For adequate teaching of mathematics there needs to be a definite plan. This study was concerned with the development of such a sequence of courses integrated around mathematics to be presented in usable manual form. It must not only be acceptable in theory but also in practice. There is need for building some type of manual that will incorporate the best methods of mathematics teaching so as to give experienced, as well as inexperienced, teachers assistance when needed. In conclusion, the need for such a manual has grown out of the study and increased scope of mathematics in more recent years.

The material included in this manual has grown out of a careful study of various reports which have been made on the subject of mathematics revision. The organization of the material has resulted from twenty-one years of experience on the part of the writer in teaching mathematics in secondary schools to all levels of students. Furthermore, the suggestions as to methods and aids result largely in the same manner.

From one's experience as a teacher of mathematics, there can be seen a need for such a manual which will serve the above-mentioned groups and do specifically the following three things:

1. Organize carefully subject matter at the proper grade level;
2. Offer helpful suggestions for clear, easy, and acceptable presentation to classes;
3. Integrate subject matter with other subjects wherever and whenever possible.

With this in mind, the following manual has emerged. This has been the very purpose of the entire study and contribution.

Prior to this study, no organized syllabus in mathematics has been in evidence in the secondary schools of North Kingstown. It is anticipated that the initiation of such a program at this level, and at this time, will present no problem not usually associated with such a task but rather, as has been stated, serve as a manual for teachers as they feel the need for such a reference.

This manual has emerged from a philosophy which has guided the very preparation of the curriculum itself. A statement of this philosophy serves as an introduction to the manual as it will be used in the secondary schools of North Kingstown. The outline form was chosen purposely as it facilitates the extraction of any material a teacher may wish to use or the

addition of further material.

While the very purpose in writing this work was to serve primarily the teachers in the aforementioned town, this should not be misconstrued with the belief that it cannot serve other teachers. Mathematics teachers in any school system can, without a doubt, find valuable aids and suggestions for their own peace of mind in studying this manual.

CHAPTER II

CRITICAL REVIEW OF LITERATURE

Some research in mathematics tends to focus on word counts, objective tests, and other elements which are subject to quantitative handling. However, the current trend is to place less faith in these statistical studies and more in judgments of value. Carr, Wesley, and Murra¹ conclude that current research is concerned with: (a) identification and measurement of intangible outcomes, (b) certain aspects of social learning and the development of conception through both direct and non-direct experience, (c) efficacy of equipment, particularly visual aids, and (d) development of a great diversity of courses of study, units, and curricula. This study falls into the latter category and its contents are certainly the results of judgments of value.

Research never determines objectives. However, it does make analyses of social trends and purposes, syntheses of opinions, and classifications of educational purposes. In this study, the objective of meeting the needs of children has been sought. Inclusion of certain items in the guide were placed there through personal judgment. However, many articles

¹E. R. Carr, H. B. Wesley, and W. F. Murra, Social Studies, Encyclopedia of Educational Research, ed. Walter S. Monroe (1950), p.1215

of contemporary literature on this subject or in this field have been studied and have had their influence.

Articles on guidance, teaching methods, and supervision have been examined as an aid in determining a philosophy and the objectives for organizing the manual. From such reading one concludes that an awareness of the needs of children because of the unsettled times in which they live is common to nearly every article regardless of the field with which they are primarily concerned.

According to Cox, Duff, and McNamara in Basic Principles of Guidance¹, the fifth basic principle of guidance reads:

The need for guidance is particularly acute today because of:

- a) Increased complexity of our social organization
- b) Rapidity of changes in our social organization
- c) The changing character of sanctions as determined by
 - 1.) The home
 - 2.) The community
 - 3.) The church
- d) The industrial situation
- e) The economic situation
- f) The demands of life in a modern democracy

In the foreword of Curriculum Planning² by Krug, the Educational Policies Commission is quoted as stating:

¹P. W. L. Cox, J. C. Duff, and N. McNamara, Basic Principles of Guidance (New York: Prentice-Hall, Inc. 1949), p. 75

²E. A. Krug, Curriculum Planning, (New York: Harper and Brothers, 1950), p. ix.

culum program in a local school system, are considered of sufficient importance to list here:

1. Start the problems with an emphasis on real problems of teaching and school life.
2. Don't dawdle over philosophy or objectives.
3. Place stress on materials and activities used in classroom teaching. As soon as possible, the curriculum discussions should get down to cases in terms of what teachers can do in the classroom.
4. Don't gibe at subject matter.
5. Emphasize needed changes and improvements in content of instruction.
6. Avoid sweeping demands for structural reorganization.
7. Recognize and build on individual differences in teachers.
8. Utilize as much as possible diversified tools of instruction and learning activities.
9. Recognize and provide for emotional needs involved in group process.
10. Recognize the time problem and try to make provision for it.¹

Of the experience method, Featherstone says:

In order to avoid the artificiality, formalism, and aimlessness that often come from trying to teach skills, appreciations, or knowledges separately or by direct and special methods, it is necessary to teach all these skills within the matrix of the total situation in which they operate; i.e., to teach them in the context of situations, problems, and activities that have an obvious and convincing resemblance to the patterns of actual daily living, emphasizing those elements of knowledge, skill, and attitude which are clearly necessary for dealing effectively with the situation at hand.²

Thus, Featherstone's statement has exercised consider-

¹Ibid., pp.218-236

²W. S. Featherstone, A Functional Curriculum for Youth, (New York: American Book Company, 1950), p.230

able influence in the decision to outline the subject matter and objectives by subject rather than to construct a series of suggested experience units.

Many appropriate textbooks and courses of study were reviewed to determine current practice in this field. Noticeable were one or more seeming weaknesses in their compilation. This fault has been given due attention in this manual. Oftentimes, the very size is considered a handicap when it interferes with the easy location of material which is to be used in instruction.

A weakness common to most manuals is the lack of concrete aids and suggestions for teachers. Such aids and suggestions are essential particularly for new and inexperienced teachers. However, their use must be so conceived by teachers as not to limit free selection of materials, free adaptations of suggestions, or the inclusion of other materials which might be pertinent to the topic under study. In this sense a manual can be of real value to imaginative and creative teaching. As a result of the use of the manual it is hoped that additional aids and suggestions can be incorporated in its body.

Another weakness common to most manuals of this type is the lack of suggestions to the teacher that the child needs to be assisted in finding his particular role or function in the class. Instruction seems incomplete when it does not assist the child in finding what his contribution may be to the success of home, school, or community groups. In his search

for security, the child wants to know what he can do now, not in the future, to become an important member of his group, at the same time contributing to its success.

The improvement of teaching in mathematics, as in other subjects, depends to a large degree on the extent to which conclusions from research find their way into the classroom. Vital questions to the teacher of mathematics today might be: What does research say about the effective ways of teaching and in what areas of mathematics education has considerable research been carried on? On the secondary school level, readings in mathematics teaching seem to emphasize considerably the need for students to understand. Much investigation is going on at present in regards to the extent to which physical aids and methods are able to contribute to attaining that goal. Experiments with introducing "modern" mathematics concepts to high school students reflect the desire of frontier thinkers in mathematics education to restudy the entire mathematics program.¹ Mathematics today as a science or discipline is an entirely different subject than it was a generation ago. New fields have been added: mathematical logic, mathematical statistics, topology, information theory, and others. Applications of mathematics have been extended into many new fields. Psychology, economics and even sociology are now using mathematical concepts and techniques

¹Modernizing the Mathematics Curriculum, Commission on Mathematics of the College Entrance Board, (New York. 1958)

in much the same way that physicists and engineers used mathematics at the turn of the century.

Devices themselves seldom teach, but in the hands of skillful teachers they can serve as aids to instruction. Investigation has concluded that the chalkboard is the most frequently used aid in mathematics teaching. However there is increasing emphasis placed upon the use of models, filmstrips, and the like. Lecture-recitation still leads as the most popular method of teaching.¹

There is some considerable activity of late in the study of the content of high school mathematics courses. This varies from a mere listing of some practical problems for a given course to a controlled experiment in developing a mathematical sequence of topics in which mathematics teachers have themselves become involved. Such experiments have failed to produce any definitive results as to what should be included, although there is great activity in what the schools should do in the mathematical sequence. For implementation to take place, research will need to determine what specific changes can be put into effect with the present teachers. There are problems with reference to what each school system can do, such as how much, which "modern" mathematics can present teachers incorporate in their classes, and how much change can be made in the content of high school courses and still give

¹United States Department of Health, Education and Welfare, Analysis of Research in the Teaching of Mathematics 1956 and 1957 (Washington: U.S. Government Printing Office, 1958).

security not only to the teachers but to the parents. These and similar questions remain unanswered.

The Commission on Mathematics indicates that a combination of scores on an aptitude test and an achievement test, when considered with courses taken in mathematics previously, are a good predictor of success in high school mathematics. However, while this may be so, there has not been much to show what motivates students to pursue courses in advanced mathematics.¹ There are indications that often secondary school mathematics teachers have been a dominant force in causing high school pupils to carry on such a study. But there is no evidence to show why these teachers were outstanding in their ability to influence pupils.

The Mathematics Commission emphasizes that mathematics should be taught as though it were a language.² Numerals are only the names one uses for numbers. In addition, equations are merely statements about numbers that are true, if the correct names are inserted.

New topics, many of which are not taught except to advanced college students, have been added to the current development of mathematics. Intuitive geometry has made vast strides to find a place in the curriculum, as well as many of the previously mentioned advanced topics. New methods have

¹Ibid. p. 27.

²Ibid. p. 31.

been substituted for old. Consolidation has taken place and mathematics has been re-evaluated. Curricular revision is necessary because mathematics is a different subject today than it was a generation ago, its applications are vastly more extensive, and its essential nature is now considered to be entirely different. Thus, it had been a growing concern of mathematics examiners that the standard curriculum in the secondary schools was outmoded and outdated. As a result the Commission on Mathematics of the College Entrance Board was appointed in 1955 to study the situation and make recommendations for a more up-to-date and functional mathematics.

As has already been stated, mathematics should be taught creatively in terms of structure and pattern rather than through tricks and "gimmicks." On the premise that it does little good to devise a program so strange and unfamiliar that teachers are unable or unwilling to teach it, it is recommended that a modification of, rather than an outright substitution for, the existing curricula be the aim.

What is needed is a careful revision in point of view. Algebra must still be taught, but teachers of mathematics agree that it is more important to teach the student to understand the fundamental ideas and concepts of algebra and their application, than to train the student in a certain number of basic skills, such as the use of parentheses, factoring, solution of linear equations and operations with fractions.

A process of elimination and consolidation in determining what should be taught in mathematics must be evolved,

since topics of greater importance than the classic ones have been developed and are suitable for presentation to high school students.

Just to plan a new curriculum is not enough. It should be recognized that the new curriculum could rapidly become as out-dated as a present one and that mathematics must be taught as a vital force in our lives in which the mathematician must not be afraid to change the rules as new developments make them obsolete.

CHAPTER III

THE MANUAL FOR SECONDARY SCHOOL TEACHERS OF MATHEMATICS

INTRODUCTION

There are many ways in which the mathematics taught to pupils in grades 7 through 12 may be organized and presented. This manual is not intended as an instrument by which these varied practices may be reduced to one standard practice. Nevertheless, it has become apparent that a program of mathematics for the secondary school level, which may be considered sound and feasible, should be studied and developed for a given school system in which the plan is to operate. In the light of situations as they exist in the secondary schools of North Kingstown, a program of mathematics has been developed and presented in this manual. Its presentation is in terms of content that can be reasonably expected to be taught in the various levels such as:

1. All teachers of grades 7 and 8 would turn substantially to the same body of content from which to teach mathematics in these grades. This content would be what is usually called general mathematics. It would certainly include a redevelopment and extension of the concepts of arithmetic taught in the earlier grades. It would also include those aspects of geometry and algebra which are simple enough to be comprehended by pupils at this age level and which are applicable in the lives of most people.
2. In grade 9 a dual offering was assumed. Some pupils would be guided into another year of general mathematics. In the main, these would be pupils who are not likely to need extensive study

of mathematics in their future plans. Other pupils would be guided into first-year algebra. These would be the pupils who anticipate further study of mathematics because they have an aptitude for and an interest in it, and because it is necessary for their plans.

3. Most of the pupils who take general mathematics in grade 9 would take only one more year of mathematics in high school. This would be a course called Senior General Mathematics. It was assumed that such pupils would usually take this course in grade 11 or 12 when their maturity would make the topics studied of first-hand importance to them. The mathematics (mainly arithmetic) of buying, budgeting, saving, investing, and the like will be studied by those pupils with an enthusiasm that comes from an awareness of the immediate relationship of such topics to their daily living.
4. Many of the pupils who successfully complete first-year algebra in grade 9 would continue with plane geometry in grade 10, second-year algebra in grade 11, and the solid-geometry-trigonometry course in grade 12. Advanced students would complete their second course in algebra in the first semester of grade 11 thus allowing time during that year for trigonometry and thus leaving open their twelfth year for solid geometry during the first semester and the more advanced work in analytic geometry and the calculus during the latter half of this year.

Algebra would be treated as a study of mathematical structure, rather than merely the development of manipulative skill in one particular mathematical system. The study of inequalities as well as equations, and of expressions involving the concept of absolute value should be added to the syllabus. Provision should be made for experience in deductive reasoning in algebra as well as in geometry.

The introduction of this point of view will require that teachers familiarize themselves with certain concepts not

ordinarily included in their college training, in particular the notions of set, statements, variable, relations and functions, as these are formulated in modern mathematics.

Geometry should build a sound understanding of the nature of deductive reasoning, a more adequate knowledge of geometric facts, and an experience in mathematical creativity on a level appropriate for each student. Subject matter of Euclidean geometry may include exercises sufficiently easy for almost any student to have the experience of discovering something for himself, but also problems sufficiently complex to challenge the brightest minds.

Existing courses do not always meet these objectives; instead they include distressingly long lists of theorems to be proved, exclusive use of synthetic methods, and rigidly formalized proofs. Meaningless memorization by rote is often substituted by less able students for any attempt at creative understanding. In this manual there will be found a greatly curtailed list of required theorems, as well as every opportunity to develop spatial concepts in addition to the concepts of plane geometry.

In the past the emphasis in trigonometry has been on the computational aspects of the subject, particularly the solution of triangles. Today, the problems of trigonometry involve vectors and their components rather than the solution of triangles. The "trigonometry of angles" presently emphasized can be reduced to a much smaller compass than is now devoted to it. Therefore, certain topics can be completely

discarded; certain topics can be included in the second year algebra course; and the remainder to constitute the trigonometry program.

It is not recommended here that the inclusion of a course in calculus be a part of the normal high school curriculum. The average student cannot be adequately prepared for such a course in three years, and anything less than a full course will ordinarily be time wasted, since it will not fit into any typical college program. A course in calculus deals with ideas that are mathematically quite sophisticated, and mathematical maturity is absolutely essential. There is no value in a course in calculus that merely sets forth rules for calculation and formulas for solving certain types of problems without adequate attention to conceptual difficulties. However, it is not the desire here to close the door on the study of calculus for as the program moves along there could conceivably be a place for the course. Even then high schools should offer calculus only to able students who have found it possible to complete a full four-year program in three years, and then should be sure that it is taught on a college level with college standards.

A STATEMENT OF PHILOSOPHY AND OBJECTIVES

At present, teaching methods generally employed in the secondary schools of North Kingstown are tradition-centered. However, they have progressed to a transitional stage because some changes in methodology have been made by individual teachers. These changes fortunately are often concerned with newer and better ways of presenting subject matter material; others are innovations for handling some specific aspect of mathematics by experienced teachers.

Certainly the sincerity and ability of the present faculty are not to be questioned. However, as newer demands are made upon the teaching of mathematics there remains constant need for reevaluation to keep mathematics meaningful. If there is a fault to be found with present methodology, it may be found primarily because it is subject-centered. Teachers who use this subject-matter method of approach generally recognize this inadequacy. Often they are heard commenting that children today are subject to too many distractions and upsetting influences and are, therefore, unable to cope with them. Also, there is danger in stagnation. This will inevitably occur in any school system where active study and experimentation are not both encouraged and practiced.

The transitional stage between traditional teaching and more modern concepts of teaching certainly cannot progress

until a majority of the teaching staff has a true appreciation of the meaning of child-centered teaching based on a child's interests, abilities, needs, and experiences. When this appreciation has been acquired, methods change rapidly until the desired goal or level in teaching is attained.

Teachers are fully cognizant of the fact that they must concern themselves with the "whole" child which includes his special interests, needs, and experiences while they teach the subject material. The difference in concept which is desired is that, while teaching the "whole" child, teachers must be positive that the child is acquiring the necessary basic skills and subject matter. It is then a question of emphasis, which in turn may alter to a very high degree the teaching methods employed. The child is taught the desired skills, attitudes, appreciations, and understandings by his own experiences in a well-planned, meaningful situation.

Certainly a requirement for bringing about a change in teaching methods is the awareness of the teachers that present methods are inadequate. Changes in curriculum content and method become effective as the result of cooperative effort on the part of the entire staff. This effort, in turn, must be stimulated and coordinated to produce concrete results.

Because of the many changes in the world of the child, brought on by profound social, political, and economic trends, there is an increasing need for alertness on the part of teachers in recognizing opportunities for guidance.¹ There is

¹P. W. L. Cox, J. C. Duff, and N. McNamara, Basic Principles of Guidance (New York: Prentice-Hall, Inc., 1949), p.15.

need for close personal guidance on the part of teachers. This need is often brought about by the immediate unsettled environment of the child, caused by broken homes, working mothers, and, in some cases, fathers in the Navy or in some other branch of the service.

Better than fifty percent of the secondary school population of North Kingstown are transient. These children are from families of civilian or service personnel connected with the Quonset Naval Air Station. Through necessity, they often follow the father from one assignment to another. Close friends and familiar places are abandoned on short notice. As can easily be seen, these children need special attention; yet they, in return, have many cultural contributions to offer because of their background of travel. School is becoming increasingly important to all our students. It compensates for the many threats to their security and the many causes for emotional instability. Thus, the school becomes a second home.

Teachers and parents are becoming increasingly aware of the pressure of political factions in our society which wish to influence our thinking. Consequently, the need becomes greater for opportunities to practice analytical or critical thinking in our schools. Our youth need to learn to examine controversial issues with intelligence and forethought, and to reserve opinion and judgment until evidence has been carefully weighed.¹ Our schools must provide opportunities for our youth

¹Association for Supervision and Curriculum Development, Leadership Through Supervision, Yearbook (Washington, D. C., 1946) pp. 1-16.

to exercise their judgment. They need to be guided through learning situations in such a way that it becomes habitual for them to make up their minds and decide issues, basing their judgments on the merits of the case or the truths involved rather than to make their decisions because of special group pressures.

Thus, it can be seen that much more is required of the school than was formerly. Tradition cannot be the sole basis for making decisions on what is important in the school program. And this does not mean discarding the basic tools of reading, writing, and arithmetic. Rather, it means developing a high skill in using these tools, and, at the same time, taking responsibility for new levels of understanding and behavior.

It would be folly to assume that teachers could accept these responsibilities and still keep all the old content of school experience. The increasing complexity of demands on the schools forces many decisions. A redistribution of both time and effort then is necessary.

The necessary shift away from subject-centered teaching cannot be abrupt. The shift in emphasis is slow, since both teachers and pupils must understand, accept, and have a desire for, as well as contribute to, the change. It is the intent that this manual may assist teachers who use it, in planning experiences for their students which will prove more adequate than mere subject-matter teaching. It is hoped that it may assist in the development of learning situations which

will provide more opportunities for guidance, and chances to learn desired skills, attitudes, and appreciations.

Skills are taught most effectively when students find that they have a need for them during the work of the unit. When such a need arises, teachers use the opportunity to teach the skills at once. Basic skills must be maintained, whether or not they are related to a unit; thus, one keeps his teaching traditional to some extent by scheduling periods of study in the required basic skills.

This manual will contain ideas and suggestions for experience units with recommendations for their use. No teacher is limited by the material in the manual; she may use it as she sees fit by extracting or altering some material or by using her own ideas. However, the desired skills or competencies should remain the goal, no matter whose ideas are used.

Granted that a plan may prescribe the focal points around which the activities of a group are to be developed during a designated period of time and it may indicate the competencies which such activities should advance, there are many equally acceptable alternatives for developing the activities and attaining the specified competencies. Featherstone says:

If you do not like any of these suggestions and can think of others that are just as good, or better, for achieving the designated purpose, you are free to develop your own plan. You should, however, undertake to make a suitable record of what you do so that your plan can be added to the collection of suggested ways of working. Other people can profit from your experience as well as you can profit from theirs.¹

¹William B. Featherstone, A Functional Curriculum for Youth (New York: American Book Company, 1950), pp.215-217

School subjects are a hindrance to creative education only when subjects are looked upon as ends in themselves, and learning is regarded as a process of passively absorbing subject matter. Subjects are capable of being taught in a highly creative and functional way. Certainly, teaching a subject need not be a mere matter of expounding what has already been learned nor does learning need be a mere matter of absorption. It is possible to use the experience method, in the teaching of highly organized subjects.

The problem of determining what curriculum materials to assign to each year has not been solved by research.¹ The experience of North Kingstown teachers, not unlike others elsewhere, have given them many varying opinions concerning the grade placement of the curricular material. There should be provision for the orderly sequence of the experience units presented and assignment of content material to a year level in terms of the level of abilities, interests, and experience of the pupils.

In this manual developed for the secondary schools of North Kingstown, units and materials presented are suggestions only and teachers are free to use them or any part of them, or to develop their own units which may fulfill the desired objectives. These units do not need to be used necessarily at the grade level suggested. It is possible to repeat a unit at a later level if it provides new experiences and higher levels of accomplishment.

¹Ibid., p.222

SUGGESTIONS FOR TEACHING EFFECTIVELY

It is well for teachers to refresh themselves on some of the salient points in teaching. The answer, in the final analysis, is obvious; teach the child in a way that he learns best. This requires an understanding of the way in which growth, development, and learning occur.

Some of the most important factors to be kept in mind regarding the nature of learning are:

1. An individual learns best when he is guided by his own purposes or goals. Purpose determines what he learns and the degree to which he learns.
2. Children grow physically, mentally, and socially at different rates. While there are many growth curves which apply generally to children at a given age, many children deviate from these curves in various ways.
3. Learning is a continuous process. It begins in the home before the child enters school and continues both in and out of school throughout his adult life.
4. Behavior is caused and learned. Individuals react to the various forces of their environment in ways that have been learned as a result of the influences of the culture in which they live.
5. A child grows through experiences which provide both security and challenging adventure. A balance in such experiences is needed. Many times all new experiences become overwhelming to the child and cause frustration. On the other hand, the absence of challenge discourages exploration, retards development, and prolongs dependence.

6. Each individual is unique. Each child needs conditions and surroundings which will provide for the fullest development of his potentialities in ways which are suitable for his stage of growth and development.
7. We learn what we live. Learning is real only when it is acquired through actual living or through close relationship to former experiences already lived and learned.
8. Individuals learn many things at the same time. The good school provides the kinds of learning situations in which knowledge, skill, and attitudes may be learned simultaneously.
9. Children learn many things through example. The pattern of living predominant in the school is reflected in the attitude and behavior of the child.

It should be remembered that no one of these factors in learning operates separately; they are interrelated in a total instructional program. Learning will take place most effectively when these factors are characteristic of the development of a program for children at any level in the educational system. The test of their applicability often occurs when teachers analyze their classroom practices.

Since every child needs new experiences and at the same time the feeling of security that comes from working with ideas, people, and things that are familiar to him, the teacher in thinking over the development of any particular child should ask herself: Am I providing a balance between new experiences and the use of old experiences for this child? Does this child face enough new problems to keep him stimulated? On the other hand, does he have opportunities to use his previous experiences often enough to build relationships

and learn from them? Has the child opportunities to grow and develop in all areas of social living within the limits of his ability? Should he have more opportunities to think for himself and to question statements of others? Are the experiences that the child is having in school making a contribution to his development as a person? Are there situations existing through which he can develop those attitudes, understandings, and practices involved in the production and distribution of goods and services, communication, transportation of goods and people, the expression of aesthetic and religious impulses, the conservation of life and property and an adequate recreational life? If the school has helped the child to have meaningful experiences that are appropriate for him in all these areas, its program is contributing to balance in his life.

Every child needs to participate in group activities and at the same time he needs opportunities to work by himself. Some pupils work best in group situations, others do better when working alone, but all children need experiences in both situations. The teacher should analyze the activities of any individual and those of the class as a whole to see whether she is providing for balanced living in this respect.

All children need the recognition that comes from having their contributions to the group accepted as valuable. Every member of the group needs to be recognized for what he is or can contribute and each member of the group needs to be able to recognize others. Therefore, if each pupil's life is to be balanced there must be times when he recognizes others

and is recognized himself. When the teacher is reviewing the experiences of any child in her group, she should ask herself whether recognition is constantly given to a few or whether each child is having opportunities to lead and to make his maximum contribution to the group. A child needs to acquire, within the limits of his ability, the skills which are involved in living a satisfactory life in an increasingly complex society. These should be developed when children reach a level of maturity at which they would normally have use for them. In a well-balanced program not all children are expected to develop the same skills nor to develop any one skill to the same degree of proficiency. The teacher who is re-examining her efforts to develop skills may ask herself: Is the child developing skills which he can use and which have meaning for him? Am I providing for different degrees of academic achievement? Am I challenging the child to the limit of his capacity? Am I fitting the program to each child or am I fitting the child into a ready-made program?

Children learn in many different ways--by observing, constructing, examining, experimenting, talking, listening, collecting, seeing pictures, reading, and problem solving. To determine whether the teacher's program is balanced with respect to the kind of learning situations that are provided for the children, she can ask herself: Am I giving each child an opportunity to learn by using different media or are his classroom activities confined to one or a few of the possible avenues for learning? Am I using a variety of instructional materials?

These are the principal ways in which the school program can be viewed in order to see whether it helps pupils to develop well-rounded personalities. This type of analysis should be made for the individual child. Teachers can use the same procedure when analyzing the program as a whole. In making this analysis it is essential that the teacher never lose sight of these facts: (1) that living can be balanced or unbalanced in many different respects; (2) that what constitutes balanced living is different for each individual; and (3) that the task of developing a balanced program is less difficult when teachers take children into their confidence and use them in planning their school program.

GENERAL MATHEMATICS: GRADES 7, 8, AND 9

INTRODUCTION

It is expected that pupils who enter grades 7, 8, and 9 will come with varying degrees of ability and understanding of the basic arithmetic principles and skills. These and body of content may be regarded as equally appropriate for all groups of pupils in these grades. Hence, the following topics are suggested only as a breakdown of the scope of mathematics for grades 7 through 9.

THE MANUAL

- I. Maintaining and deepening the understanding of the fundamental processes.
- II. Percentage.
- III. Direct measurement.
- IV. Word and practical problems.
- V. Graphs and tables.
- VI. Informal geometry and the formulae.
- VII. Preview of algebra.
- VIII. Individual measurement.
- IX. Mathematics of the school.
- X. Mathematics of the community.
- XI. Travel and communication.
- XII. Government income and expenditures.

No specific classification of these topics by grade is suggested in this manual but rather it is recommended that all topics be partially developed in each of the grades listed. The amount of time to be devoted to each topic in any one of these grades will depend upon two things: the need and readiness of the pupils.

It should be observed that general mathematics in grade 9 is not regarded as a course in general first-year algebra. It is the intent here that both courses be offered in grade 9 with some pupils taking algebra and others taking general mathematics.

Mathematical Needs of Pupils

The mathematical needs of all pupils are, for practical purposes, set forth in the Guidance Report of the Commission on

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- I. Maintaining and deepening the understanding of the fundamental processes.
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- III. Direct measurement.
- IV. Home and personal problems.
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The mathematical needs of all pupils are, for practical purposes, set forth in the Guidance Report of the Commission on

Post-War Plans of the National Council of Teachers of Mathematics.
These "functional competencies" are listed below.

1. Computation. Can you add, subtract, multiply, and divide effectively with whole numbers, common fractions, and decimals?
2. Per cents. Can you use per cents understandingly and accurately?
3. Ratio. Do you have a clear understanding of ratio?
4. Estimating. Before you perform a computation, do you estimate the result for the purpose of checking your answer?
5. Rounding numbers. Do you know the meaning of significant numbers? Can you round numbers accurately?
6. Tables. Can you find correct values in tables; e.g. interest and income tax?
7. Graphs. Can you read ordinary graphs: bar, line and circle graphs? the graph of a formula?
8. Statistics. Do you know the main guides that one should follow in collecting and interpreting data; can you use averages (mean, median, and mode); can you draw and interpret a graph?
9. The nature of measurement. Do you know the meaning of a measurement, or a standard unit, or the largest permissible error, of tolerance, and of the statement that "a measurement is an approximation"?
10. Use of measuring devices. Can you use certain measuring devices, such as an ordinary ruler, other rulers (graduated to thirty-seconds, to tenths of an inch, and to millimeters), protractor, graph paper, tape, caliper micrometer, and thermometer?
11. Square root. Can you find the square root of a number by table or by division?
12. Angles. Can you estimate, read, and construct an angle?
13. Geometric concepts. Do you have an understanding of point, line, angle, parallel lines, perpendicular lines, triangles (right, scalene, isosceles, and equilateral), parallelogram, (including square and rectangle), trapezoid, circle, regular polygon, prism, cylinder, cone, and sphere?

14. The 3-4-5 relation. Can you use the Pythagorean relationship in a right triangle?
15. Constructions. Can you with ruler and compass construct a circle, a square, and a rectangle, transfer a line segment and an angle, bisect a line segment and an angle, copy a triangle, divide a line segment into more than two parts, draw a tangent to a circle, and draw a geometric figure to scale?
16. Drawings. Can you read and interpret reasonably well, maps, floor plans, mechanical drawings, and blueprints? Can you find the distance between two points on a map?
17. Vectors. Do you understand the meaning of a vector, and can you find the resultant of two forces?
18. Metric system. Do you know how to use the most important metric units (meter, centimeter, millimeter, kilometer, gram, kilogram)?
19. Conversion. In measuring length, area, volume, weight, time, temperature, angle, and speed, can you shift from one commonly used standard unit to another widely used standard unit: e.g., do you know the relationship between yard and foot, inch and centimeter, etc.?
20. Algebraic symbolism. Can you use letters to represent numbers; i.e., do you understand the symbolism of algebra--do you know the meaning of exponent and coefficient?
21. Formulas. Do you know the meaning of a formula--can you, for example, write an arithmetic rule as a formula, and can you substitute given values in order to find the value for a required unknown?
22. Signed numbers. Do you understand signed numbers and can you use them?
23. Using the axioms. Do you understand what you are doing when you use the axioms to change the form of a formula or when you find the value of an unknown in a simple equation?
24. Practical formulas. Do you know from memory certain widely used formulas relating to areas, volumes, and interests, and to distance, rate and time?
25. Similar triangles and proportion. Do you understand the meaning of similar triangles, and do you know

how to use the fact that in similar triangles the ratios of corresponding sides are equal? Can you manage a proportion?

26. Trigonometry. Do you know the meaning of tangent, sine, cosine? Can you develop their meanings by means of a scale drawing?
27. First steps in business arithmetic. Are you mathematically conditioned for satisfactory adjustment to a first job in business; e.g., have you a start in understanding the keeping of a simple account, making change, and the arithmetic that illustrates the most common problems of communications and everyday affairs?
28. Stretching the dollar. Do you have a basis for dealing intelligently with the main problems of the consumer; e.g., the cost of borrowing money, insurance to secure adequate protection against the numerous hazards of life, the wise management of money, and buying with a given income so as to get good values as regards both quantity and quality?
29. 'Proceeding from hypothesis to conclusion.' Can you analyze a statement in a newspaper and determine what is assumed and whether the suggested conclusions really follow from the given facts or assumptions?¹

TOPIC I--MAINTAINING AND DEEPENING THE UNDERSTANDING OF THE FUNDAMENTAL PROCESSES

<u>Content</u>	<u>Teaching suggestions</u>
A. Meaning, origin, and use of numbers <ol style="list-style-type: none"> 1. Roman 2. Arabic 	Arouse interest through use of the origin of numbers, by having students bring in newspaper clippings and have the pupils read the numbers.
B. Place value	Emphasize importance of place value as well as development of zero. This may be done by solving problems on an abacus and on an adding machine. Students might make an abacus. Then contrast the old methods with the new improved methods.
C. Origin and use of zero	
D. Rounding numbers	
	Contrast Roman and Arabic systems of numbers, pointing out the difficulties

¹"Guidance Report of the Commission on Post-War Plans."
The Mathematics Teacher, Vol. LX, November, 1947, pp. 318-19.

E. Meaning of the fundamental processes, whole numbers, fractions (common and decimals), and denominate numbers

1. Adding whole numbers, fractions, and denominate numbers
2. Subtracting whole numbers, fractions, and denominate numbers
3. Multiplying whole numbers, fractions, and denominate numbers
4. Dividing whole numbers, fractions, and denominate numbers

met in performing the fundamental operations with the use of Roman numbers.

Place value should certainly receive attention at this level. It should not merely be a review with excessive drill but rather, the emphasis should be on the redevelopment and extension of meanings.

Stress importance of decimals in every day affairs as well as the meaning of the decimal.

Fractions continue to trouble many pupils here. The difficulty usually arises from lack of understanding of the nature of fractions and operations with them. Charts and other learning aids, made and used by pupils, are helpful in reteaching meanings. It is important that time be found for this redevelopment.

At this time the reasons for many of the rules can be easily shown. When pupils rely upon rules for placing the decimal point in quotients and products, often absurd answers result. Hence, pupils should be taught to think about the reasonableness of their answers. This will be done if they learn to place the decimal by estimate before they use the rules, e.g.,

$$.6 \overline{) 6.36}$$

Here pupil should estimate before any work starts that the quotient will be something more than 6 because he is dividing by less than 1. This might avoid such incorrect answers as 1.6 or even 16.

Money matters and measurements should furnish many practical problems in the uses of decimals.

Evaluation:

Have the pupils gained a clearer insight into the nature of the Hindu-Arabic number system, and the four operations with whole numbers?

Have the pupils increased their speed and accuracy in

performing the fundamental operations of integers and fractions?

Do the pupils have a thorough understanding of the decimal number system since many of the concepts and operations in this unit depend upon this understanding? (Use of very small and very large numbers will give some indication of this mastery).

Do pupils exhibit a high degree of skill in locating the decimal point in products and quotients not only by using the rules but by the application of common sense?

Are the pupils able to use the four fundamental processes with denominate numbers?

Are pupils able to change fractions, from common to decimal form and vice versa?

TOPIC II--PERCENTAGE

<u>Content</u>	<u>Teaching suggestions</u>
A. Extension of meaning of per cent	Have a bulletin board display of illustrations showing uses of per cent.
B. Changing fractions and decimals to per cents and vice versa	Emphasize that the % sign is a symbol for expressing fractions with denominators of 100.
C. Finding the per cent of a number	Present figures of sales for a paper route and compute the commission with the class.
D. Finding what per cent one number is of another	Discuss bargain sales and buying on discount.
E. Finding the whole number when a part of it is known	Find per cent of profit earned on a hot-dog sale. Estimate percentage of time devoted to various activities at home.
F. Rounding off per cents	Make a bar graph to show percentage of qualified voters in a community who cast ballots; use per cents in sports: finding per cents and interpreting batting averages, team standings, etc.
G. Short cuts in per cent	
H. Applications of per cent	Find per cent of tax money allotted locally for transportation, instruction, maintenance, and other items in the local school program. Find what per cent of the total cost of education is paid by state funds and what per cent

is paid from local taxes.

Develop the formula $p = br$ and solve problems using it.

Evaluation:

The pupil should know:

That the per cent sign is only a substitute for the denominator 100; have an understanding of percentage through knowledge of common and decimal fractions; have a working knowledge of per cent in solving problems of simple interest, commission, trade discount, per cent of increase and decrease, and what per cent one number is of another; appreciate the uses of per cent in everyday affairs such as advertising, discount in buying, borrowing or lending money, interpreting newspaper articles using per cent, etc.; and be able to find the whole number when a part of it is known.

TOPIC III--DIRECT MEASUREMENT

<u>Content</u>	<u>Teaching suggestions</u>
A. Origin, need, and meaning of measurement	Have various pupils report either in writing or orally about the history of measurement.
1. Common units of measurement	Create a display of as many measuring devices as the pupils can assemble.
B. Metric system-- meaning, use, relation to the English system	In order to motivate interest have the class list the measurements necessary in cooking, in sewing, in taking a trip, etc.
C. English system-- denominate measures, linear, liquid, dry, weight, money, time, temperature, etc.	Discuss the Bureau of Standards and its value as a public safeguard. Interview a mechanic, a carpenter, an industrial arts teacher etc., on the value of accuracy in measurement.
D. Nature and computation with approximate numbers.	Measure your classroom and various articles in it and then examine, discuss, and use a variety of measuring devices to determine the degree of accuracy.
E. Precision in measurement	Read the gas, light, and water meters at your home at different intervals; get the rates on such utilities, and compute the bill.

Interview a merchant to see how the public is protected against inaccurate weights when buying.

Make a list of all units of measure involved in operating your home.

Make estimates of heights and weights of a number of your friends, then check by actual measurements. Retest yourself with different people.

Compare the English and metric systems of measurements and solve practical problems with both, at the same time stressing approximation.

Show advantages of the metric system over our system of weights and measures.

Have ample practice in changing one unit of measure to another unit.

Give pupils practice in estimating and measuring lengths to show that all measurements are approximations.

Evaluation:

Do your pupils measure with any degree of accuracy?

Do your pupils have a reasonably clear understanding of the relationship of the metric system to the English?

Do your pupils show a workable knowledge of the vocabulary in this unit?

Are the pupils familiar with all the units of measures such as linear, liquid, weight, time, money, etc.?

Are the pupils able to read light and gas meters, and to solve such practical problems as finding the amount of covering needed for a floor?

TOPIC IV--HOME AND PERSONAL PROBLEMS

Content

Teaching suggestions

- A. Bank services
 - 1. Savings accounts

Practice using the bank forms.

Discuss types of bank accounts.

2. Checking accounts
3. Loans
4. Safety deposit box

B. Borrowing money

1. Banks
2. Individuals
3. Loan agencies
4. Insurance policies

C. Budgeting

1. Time
2. Money

D. Buying

1. Installment buying
2. Mail order buying
3. Cash purchases
4. Charge accounts

E. Investments

1. Government bonds
2. Savings
3. Real estate
4. Stocks and bonds
5. Loan associations
6. Home projects

F. Insurance

1. Life
 - a. Ordinary life
 - b. Limited payment
 - c. Endowment
 - d. Term
2. Health and accident
3. Hospitalization
4. Property
 - a. Fire
 - b. Personal

Follow the travels of a check.

Write checks and complete the stubs.

Find the answers to such questions as: What rate of interest does the bank pay on savings accounts? What rate did it pay 10 years ago? What use does the bank make of the money deposited in savings accounts? How does one open a savings account?

Set up a class bank and conduct the ordinary bank transactions.

Discuss advantages and disadvantages of borrowing money from the various agencies.

Assign such problems as: Which is more economical, to buy an automobile through a finance company or to borrow the money from a bank? On what kinds of insurance policies may money be borrowed?

Compare actual interest rates of small companies with bank rates.

Make a personal and a household budget for time and money.

Plan a monthly or yearly family budget and make a graph to show wise distribution of income. Either a bar or a circle graph may be used.

Have class keep records of personal income and expenses and make a graph of this material.

Discuss advantages and disadvantages of the various kinds of buying.

Solve typical problems associated with buying food, constructing an addition to the home, etc.

Discuss the ways of investing one's savings and bring out any possible advantages and disadvantages. The following list may suggest ways: savings and loan associations, savings banks, real estate, postal savings, U. S. Government Bonds, and shares of stock.

property
c. Automobile

- G. Shelter
1. Buying
2. Renting
3. Up-keep

Discuss such problems as the differences between investment and speculation; elements of risk in all investments; differences between mortgages, stocks, and bonds; and the varied types of investments.

Study tables which show redemption values of United States Savings Bonds of all denominations.

Study various kinds of insurance recognizing advantages and disadvantages of each.

Discuss benefits of life insurance to the individual insured and to his family not only as a protection but as a means of savings.

List yearly expenses involved in owning or renting a home.

Evaluation:

Does the pupil know:

The services offered by banks; how to use the ordinary forms used in banking procedures; institutions which lend money and the routine by which it may be borrowed; the various rates of interest and how to compute them; the differences between interest-bearing and non-interest-bearing notes; the value of a personal budget; what per cent of the income should be set aside for each item in the budget; how to set up a sensible budget; how to keep an expense account; the advantages and disadvantages of the various kinds of buying; how to compute the rate of interest paid for goods purchased on the installment plan; the value of consumer credit; how to figure the cost of owning a home, an automobile, etc.; how to compare the cost of renting and owning a home; the value of systematic savings or wise investment; how and where to make investments and the risks involved in investment; types of insurance--life, fire, automobile, health, etc.; and, among many other important factors, the cautions to be observed when purchasing insurance.

TOPIC V--GRAPHS AND TABLES

Content

Teaching suggestions

- A. Reading graphs found in newspapers

Have students bring to class as many kinds of graphs and tables as can be found in newspapers and magazines.

- B. Making graphs
1. Rounding numbers
 2. Selecting scales
 3. Selecting appropriate kind of graphs

Make bar, circle, and line graphs which use statistical data that have meaning for the class, such as: number of pupils in different grades, expenditures of income, variations of temperatures, growth of school populations, accident and death rates, and pupil drop-out by grades.

Have pupils interpret completed graphs.

- C. Bar graph

Know what a table is and be able to make one from known facts.

- D. Circle graph

Stress importance of rounding numbers.

- E. Line graph

- F. Pictograph

Check completed graphs for neatness and accuracy.

- G. Making and using tables

When teaching circular graphs drill on finding the number of degrees in fractional and decimal parts of a circle.

Many kinds of tables are used today, among which are: interest, cost, mileage charts, and tables showing the amount of antifreeze to use in a radiator. Pupils should collect and learn to read a variety of such tables.

Evaluation:

The pupil should:

Be able to read and interpret the tables which he uses; be able to construct, read, and interpret pictograph, bar, line, and circle graphs; be able to round off numbers to use in the construction of graphs and tables; be able to name and draw each type of graph that the teacher has demonstrated; be able to make graphs neat, interesting, well-spaced, easily read, and adapted to the material at hand; be able to criticize a graph which misrepresents the facts.

TOPIC VI--INFORMAL GEOMETRY AND THE FORMULA

<u>Content</u>	<u>Teaching suggestions</u>
A. Geometric concepts	
1. Meaning and measurement of angles	Have class bring in pictures of geometric figures which we see daily in order to reestablish an understanding of geometric figures.

2. Recognition of kinds of:
 - a. lines
 - b. angles
 - c. plane and solid figures

Develop skill and accuracy in the use of drawing instruments such as the compass, protractor, rule, etc.

Practice measuring many angles with the protractor.

Name the angles formed by the hands of the clock.

B. Basic constructions

1. Bisecting an angle
2. An angle equal to a given angle
3. Perpendicular to a line from a point outside the line
4. Perpendicular bisector

Develop the meaning of volume by placing blocks of uniform dimension into a rectangular box in which they fit.

Draw diagrams of a football field, tennis court, baseball field, etc.

Allow time for making original designs involving the use of circles and regular polygons.

C. Construction and properties of geometric figures

1. Perpendicular and parallel lines
2. Angles
3. Triangles
4. Squares, Rectangles, Parallelograms, hexagons, etc.
5. Circles

Construct a square inch and a square foot.

Develop formulas experimentally to help with understanding, such as,

- a. To find the area of a rectangle, cut a number of one-inch squares from paper and with these form as many rectangles as possible;
- b. To find the area of a given triangle, cut another triangle of equal size to form a parallelogram.

D. Development and application of geometric formulas

1. Perimeters of plane figures
2. Areas of plane figures
3. Surface area and volume of solid figures
4. Angle and triangle relationships
 - a. Sum of the

Measure and compute perimeter and area of the floor, blackboard, basket-ball court, etc.

Measure a bin and estimate the number of tons, for example, that it will hold and check by computation.

Find number of cubic feet of air per pupil in the classroom.

Have pupils measure the circumference and diameter of various wheels, lamp shades, cans, etc. and divide to find

- angles of a triangle the value of π .
- b. Congruent triangles Bring out relationship between the cylinder and cone, using one of each of equal base and height.
 - c. Similar triangles
- Introduce meaning of complimentary and supplementary angles, giving practice in their construction.
- Discuss meaning of pairs of equal angles when parallel lines are cut by a transversal.
- Construct similar triangles. Cut them out and compare them.
- Run an experiment showing that in similar triangles the corresponding angles are equal and the corresponding sides have the same ratio.
- Use figures cut from paper to show the meaning of congruence.
- Use protractor to measure angles of various sizes and kinds of triangles to discover that the sum of the angles equals 180 degrees.
- In like manner, have pupils discover that the sum of the angles of a quadrilateral equal 360 degrees.
- Encourage pupils to create original designs.

Evaluation:

The pupil should:

Have a working knowledge of the vocabulary of the topic; be his own critic of his own accuracy in measuring; be able to make with great accuracy standard constructions; have an awareness of geometric forms in the world about him; recognize the commonly used geometric figures; know the relationships of one, two, and three dimensions; recognize the varied application of formulas and be able to apply a formula to a situation; demonstrate understanding of congruence, similarity, symmetry, size, and nature of angles, etc.

TOPIC VII--REVIEW TO ALGEBRA

<u>Content</u>	<u>Teaching suggestions</u>
A. Algebra as general number	Elementary algebra is basically the mathematics of general number. Therefore, it is a poor comparison to teach that arithmetic uses the ten digits and algebra uses letters.
B. Language of algebra	
C. Directed numbers <ol style="list-style-type: none"> 1. Nature 2. Computation 	To help develop the "unknown" number concept have pupils erase the letter and replace it by its numerical value.
D. Computation with simple literal numbers	Translate verbal expressions into general number form such as:
E. Equations	<ol style="list-style-type: none"> a. If pencils cost 5¢ each, discuss the method of finding the cost of various numbers of pencils. b. Percentage equals base times rate. c. Circumference of a circle equals pi times the diameter.

Introduce the four ways of showing number relationship--graph, table, rule, and formula.

In order to illustrate positive and negative numbers and to help develop rules for the fundamental operations use:

- a. Temperatures above and below zero, number scales, spending and saving, locations above and below sea level, profit and loss, overweight and underweight.

Emphasize zero as a position to help in understanding directed numbers.

Develop the concept of balance in equations by comparing with scales used in a candy store.

Also the meaning of the word equation may be developed from its base word EQUAL.

Present equation through its close relationship to the formula and show that the equation is an expression of equality involving an unknown quantity.

Constantly remind pupils that the letters actually represent numbers. Translate verbal statements as follow into equations using X to represent the unknown number:

- a. Some number divided by six equals two.
- b. The sum of a number and four is eleven.
- c. A certain number decreased by nine is one.

When solving equations, use the following principles, better known as axioms:

- a. Adding the same number to both sides
- b. Subtracting the same number from both sides
- c. Multiplying both sides by the same number.
- d. Dividing both sides by the same number.

Transposing should not be taught since it is a mechanical device and lacks meaning. In solving an equation, it is always best to use the four basic axioms listed above.

This may take more time but the results in pupil understanding will be worth the effort.

Avoid such expressions here as "cross-multiplying".

It is important also to have the pupil check his equations.

This topic is not intended to prepare pupils for further study of algebra. Rather it is intended to take from alge-

bra those basic ideas which should be a part of any person's general education. Most of the pupils who take general mathematics in these grades will take only one more mathematics course in high school--senior general mathematics.

Evaluation:

Can the pupil think more clearly and organize his material better when solving word problems?

Does the pupil use his knowledge of equations to solve other word problems?

Can the pupil use formulas with greater ease and efficiency?

Does the pupil have a clear understanding of the meaning and importance of positive and negative numbers?

Has the pupil increased his interest in mathematics?

Can the pupil add, subtract, multiply, and divide signed numbers accurately?

Does the pupil thoroughly understand that the letters used in algebra actually represent numbers?

TOPIC VIII--INDIRECT MEASUREMENT

<u>Content</u>	<u>Teaching suggestions</u>
A. Scale drawing	Make all constructions with compass and straightedge.
B. Similar triangles	Measure height of the school flagpole, a tree, a church spire, etc., by using similar triangles, and scale drawing.
C. Ratio	Show results on paper, making all drawings to scale.
D. Proportion	Solve problems in ratio and proportion.
E. Properties of right triangle	Give ample problems in measuring distances indirectly.
F. Square root (by table and division)	Make a scale drawing of a baseball diamond. Then, how far is it from home plate to second base?
	Draw to scale on squared paper.

Use scales on maps and blueprints.

Decide upon a trip and figure the distance covered from a map.

Illustrate the rule of Pythagoras by constructing the square on each side of a right triangle.

Individual pupil and class projects in outdoor measurements recommended.

Evaluation:

Does the pupil have the ability to find the ratio of two items?

Does the pupil understand that a proportion is the equality of two ratios? Can he make and solve a proportion?

Can the pupil make, read, and interpret scale drawings?

Does the pupil recognize similar and congruent triangles and their parts?

Can the pupil use similar and congruent triangles to make indirect measurements?

TOPIC IX--MATHEMATICS OF THE SCHOOL

<u>Content</u>	<u>Teaching suggestions</u>
A. Operational cost	Have a group of pupils secure information concerning the cost of operating the school. Use this information to work up averages, graphs, and per cents.
1. Transportation	
2. Fuel	
3. Electricity	Find the per cent of attendance for the class and the school.
4. Janitorial service	
5. Cafeteria	Compare the attendance records by graphs.
6. Building	
7. Repairs	Find out the cost per pupil and use the figures to compute the cost of failures.
8. Care of grounds	
9. Water	Compute the per cent of failures.
B. Attendance	
1. Average daily attendance	Make ample use of statistics and graphs here.
2. Drop-outs	Set up tables showing the standing of

- C. Cost of failures athletic teams.
- D. Extra-curricular activities
 1. Athletics Lay off athletic fields and make scale drawings of same.
 2. Clubs Measure the floor space and air space of the classroom in terms of square and cubic units-per pupil and total.
 3. Class parties, etc.
- E. Physical properties of the school plant
 1. Square and cubic foot space per pupil Learn to read thermometers and be able to check the temperature in the classroom occasionally. Show this information graphically.
 2. Height, length, etc., of buildings and grounds Measure the treads and risers of stairways in relation to safety regulations.
 3. Healthful conditions of heat, light, and safety

Evaluation:

Are the pupils able to compute the average daily attendance each month for their class?

Are the pupils able to keep accurate accounts of their school expenses by the month and year?

Are the pupils aware of statistics in the cafeteria, athletic department, in solving problems involving sales, cost of goods sold, cost of lunches served, margin or gross profit, operating expenses, net profit?

TOPIC X--MATHEMATICS AND THE COMMUNITY

<u>Content</u>	<u>Teaching suggestions</u>
A. Supporting the community 1. Local and state taxation a. Where money	<p>This topic should be easily coordinated with social studies since its background is the community.</p> <p>Have the pupils find ways the community serves its people and how these services are supported through taxation.</p>

- comes from
- b. How it is spent
- B. Utilities
- C. Occupations
1. Trades-carpenter, plumber, machinist, mason, beauty parlor operator, sales clerk, etc.
 2. Professions-law, teaching, ministry, medicine, nursing, etc.
 3. Occupations typical to the local community-agriculture, fishing, manufacturing, etc.
- Make problems and graphs based upon above information.
- Make use of special community projects such as construction of public buildings, water systems, parks, roads, etc.
- After teaching reading of meters, have each student read meters at his home for a period and figure the cost of the utility.
- Have either individuals or groups select an occupation of personal or local interest; by investigation find out and list the mathematical knowledge needed; illustrate by the use of problems from real life.
- From newspapers, magazines, or other sources collect examples of what would be suitable for a bulletin board display on the subject, "Mathematics in Leisure Time Activities".

Evaluation:

Are the pupils able to use community resources as a means for problem making and problem solving?

Have the pupils shown an interest in studying occupations in the local community and are they able to interpret and use information found?

Have the pupils shown further improvement in their use of the fundamentals of arithmetic?

TOPIC XI--TRAVEL AND COMMUNICATION

<u>Content</u>	<u>Teaching suggestions</u>
A. Travel	
1. History of travel where mathematics is used	Trace the development of travel from early days to the present. Compare the time and expense of trips.
2. Ways of travel	What advantages and disadvantages of each method of travel would you

- a. Private automobile consider before you made a trip?
- b. bus What factors would determine the type of transportation?
- c. Train
- d. Plane Compute the approximate cost of a trip by plane, train, and bus.
- e. Boat
- 3. Services for travelers
 - a. Time tables Collect timetables, maps, and other information from railway offices, bus terminals, etc.
 - b. Maps
 - c. Rates
 - d. Reservations Make maps of time zones in the United States.
 - e. Methods of carrying money Find answers to such typical questions as:

B. Communications

- 1. History of communications where mathematics is directly used
- 2. Means of communication
 - a. Telephone What time is it in each of the time zones when it is 8 o'clock in your home?
 - b. Telegrams If you need \$1000 for a trip, how would you carry this amount safely? How much extra would it cost?
 - c. Mail services What types of reservations can you make when you travel by train, plane, or bus? How are these reservations made? What is the cost of such reservations?
 - d. Newspaper Collect information on communication such as the cost of long distance telephone calls to various places from your home. Show this information graphically.
 - e. Radio
 - f. Television List all ways of communication available to you, comparing them as to speed, cost, efficiency, convenience, reliability, and extent of use.

Collect information from the U. S. Post Office and make tables of rates. Then develop formulas for first and second class mail.

What is the relative cost of sending each kind of telegraph message?

Discover opportunities for employment offered boys and girls of your age by the press, radio, telephone, and tele-

graph companies. What preparation in mathematics is needed?

Evaluation:

The pupil should:

Be able to use a scale of miles in map reading to find distances between towns; compare time and cost of air travel with those of train, bus, or boat; know the advantages and disadvantages of each means of travel; be able to figure the approximate cost of trips by car, train, bus, or plane; be able to read and interpret time tables; be familiar with all the ways of communication available in the community; know the services rendered by the post office; know how we can be sure that a package or letter reaches the person to whom it is sent.

TOPIC XII--GOVERNMENT INCOME AND EXPENSES

<u>Content</u>	<u>Teaching suggestions</u>
A. Taxation	The teacher might introduce this topic by such a question as follows:
1. Direct and indirect	
a. Distinction made	Do your parents or the community in which you live buy the food and pay the rent for your family?
b. Examples	
2. Local and state taxes	What things in your community are not bought directly by your family? How are such goods and services paid for?
a. Real and personal property	
b. State income	What are taxes? Explain the difference between direct and indirect taxes and give examples of each.
c. Highway taxes	
d. Fees, fines	Draw out the reasons for taxes by a group discussion.
e. Utility taxes	
f. Dog, poll, vehicle, etc.	Study community to find ways it best serves its people.
B. Government expenses	Make use of local community projects such as public buildings, water system, parks, roads, etc.
1. Local and state governments	Learn how assessments are made by studying the real estate in your community.
2. Police and	

fire depart-
ments	Study types of licenses issued in your
community.
3. Health and
sanitation	Study the amount of tax money spent for
schools, roads, etc.
4. Public wel-
fare	
5. Highways and
bridges	Study spendings for public welfare.
6. Library and
recreation	Compute the amount of gasoline tax paid
by an individual on a long trip.
7. Principal and
interest on
debts	Learn the direct tax on gasoline in
Rhode Island and the amount of Federal
tax. Study and fill out tax forms.
8. Public works	
9. Public educa-
tion	What power did the Sixteenth Amendment
of the Constitution give Congress?
How long ago did it become a law?
C. Federal (National)
taxes	
1. Where money
comes from	Bring in tax receipts for class to
study.
a. Income
taxes	Help pupils develop a clear understand-
ing of where the tax money comes from.
b. Internal
revenue	Include in this such indirect taxes as:
c. Social
Security,
retire-
ment, and
unemploy-
ment funds	gasoline, amusement, tobacco, inherit-
ance, licenses, sales tax, etc.
d. Customs	Find out how Rhode Island spends her
tax money for administration (govern-
ment), education, highways, parks,
public welfare, conservation, etc.
e. Miscella-
neous re-
ceipts	Have pupils make graphs showing the
sources and distribution of the tax
dollar in Rhode Island.
2. Federal expend-
itures	
a. National
defense	Compare money spent for education,
highways, and recreation in Rhode Island
with money spent for same in other
states.
b. Pensions
and bene-
fits for
veterans	
c. Relief
and recov-
ery ex-
penditures	
d. Interest on
federal
debt	
e. Insurance
for veter-
ans	
f. Agriculture	

- g. Department
Social Security and
retirement
- h. Retired
workers

Evaluation:

Do pupils have a better understanding of the fundamental purposes of taxation?

Do pupils demonstrate a working knowledge of the vocabulary used in the unit?

Do pupils know what agencies levy taxes and the uses made of tax money? Group or individual reports might be used.

Do pupils understand the difference between direct and indirect taxes?

Have pupils become conscious of the three taxing agencies: local, state, and federal?

Are pupils able to compute simple tax problems? For instance, rate when the amount of tax and assessed valuation are known, or amount of tax when assessed valuation and rate are known.

Changes in the growth of pupils during the entire unit such as interest, work habits, evidences of thinking, cooperation, etc. should be noted.

FIRST-COURSE ALGEBRA

INTRODUCTION

It is herewith recommended that students who plan more work in mathematics be encouraged to take First-Course Algebra. This further study of mathematics may be necessary for future vocational or professional aims or it may be studied for the pleasure to be derived from the study of mathematics.

Certainly the skills, ideas, and principles of algebra need to be taught carefully. Skill alone in the mechanical manipulation of algebraic symbols is a seriously inadequate outcome of the teaching of such an important branch of mathematics. Skill without understanding is so likely to produce the temporary learning of the subject which so many teachers complain of and which plagues a pupil who enters college and studies mathematics at this level.

In this manual, First-Course Algebra will be divided into nine topics:

- I. Introduction to algebra.
- II. The equation.
- III. Signed numbers.
- IV. Fundamental operations in algebra.
- V. Equations of the first degree in two unknowns.
- VI. Special products and factoring.
- VII. Fractions and fractional equations.
- VIII. Finding square roots and using radicals.
- IX. Quadratic equations,

TOPIC I-INTRODUCTION TO ALGEBRA

<u>Content</u>	<u>Teaching suggestions</u>
A. Why study algebra?	List and discuss professions for which algebra is needed. Help students find out the mathematics requirements for his proposed vocation or for the college
B. How to study algebra?	

- C. Origin of algebra
- D. The nature of general number
- E. The nature and use of the formula
- F. Terms, coefficient, and power
- G. Combining like and unlike terms
- H. Making formulas from verbal statements
- I. Evaluation of and substitution in a formula

Supplementary Work

- A. Interpreting the meaning of more complex literal expressions
- B. Expressing more complicated verbal data algebraically
- C. Preparing booklets or reports on history of our modern symbols or on the works of great men who contributed to the study of algebra

he intends to enter.

Study the origin of algebra and point out the historical development of the subject. Find out what men made contributions to algebra.

Present algebra as a language and tool of mathematics.

Pretest to estimate the present ability of the class with reference to:

- a. The ability to interpret simple algebraic expressions.
- b. The ability to express simple verbal statements symbolically.
- c. The ability to combine literal numbers.
- d. The ability to interpret and evaluate simple formulas.

Then, the class should be ready for grouping for practice. Each group should proceed at its own rate, receiving help when needed.

All pupils should complete the work listed in content, except that marked supplementary which is to be assigned as enrichment material.

Briefly review the use of symbols to indicate sum, difference, product, and quotient.

Give some drill in combining terms and the substitution of values in formulas; take the rest of the time in making formulas from verbal statements, tables, and graphs.

The idea that algebra is the study of general number needs to be thoroughly taught. This means that many examples of universal number relations will be derived by pupils and expressed in algebraic language. Tables which pupils build and from which they discover a relationship serve this purpose well.

Here the formula is used as a topic to provide a ready transition from arithmetic to algebra. Any uninteresting review is to be avoided here. Instead, the formula is used as a familiar topic through which the general number idea of algebra is taught.

Pupils should be given drill material in terms of their needs—either in groups or individually through the use of the blackboard, dictated exercises, or mimeographed sheets.

Allow enough drill on each new concept to develop accuracy and efficiency in its use and application. However, drill should be used only after the process is understood.

Demonstrations and explanations made by the teacher should, in so far as possible, be exercises within the experiences of the pupils.

Rules are of value only when they summarize a process already learned and understood. These rules should be developed by the class in class discussions whenever possible.

Often pupils begin the study of algebra with arithmetic deficiencies. Research has indicated that it is futile for a teacher to try to correct these at the beginning of his algebra course before getting into the content of the subject. This can much better be done incidental to the study of the new topics of algebra.

Evaluation:

Check list for PUPIL of things to be remembered:

Algebraic numbers, formulas, factor, general numbers, like terms, coefficient, algebraic expression, unlike terms, and power.

Important skills:

Student should be able to identify like terms; add like terms; subtract like terms; multiply and divide algebraic numbers; translate rules into formu-

las; and translate verbal statements into algebraic language.

Observation check list for teachers:

- a. Does pupil understand how and why letters are used to represent general numbers?
- b. Does pupil know the meaning of words used?
- c. Can pupil make and apply simple formulas?
- d. Can pupil collect similar terms?
- e. Can pupil evaluate simple formulas and algebraic expressions, etc.?

TOPIC II--THE EQUATION

<u>Content</u>	<u>Teaching suggestions</u>
A. Nature of equations	Give a brief test at the beginning of the unit to measure the present ability of the pupils with reference to:
B. Nature of a solution to an equation	<ol style="list-style-type: none"> a. Knowledge of terms used in working with equations.
C. Reviewing the solution of equations by the four axioms	<ol style="list-style-type: none"> b. Ability to solve linear equations of the simplest form.
D. Translating verbal problems into equations and solving	<ol style="list-style-type: none"> c. Ability to select pertinent data from a stated problem and form an equation.
E. Checking equations	A set of platform balances will help to develop an understanding of equations. Be sure that the student sees that there is still a balance when the same amount is added or subtracted from both sides or when each side is multiplied or divided by the same number.
<u>Supplementary work</u>	
A. Solving more complex literal equations	Emphasize orderly arrangement and sequential steps in solving an equation.
B. Solving more advanced problems	<p>Go over the four axioms for solving an equation(in the beginning it is a good idea to have the students write out the axiom used in solving an equation).</p> <p>Give ample drill--always keeping before the class the idea of an equation being a balance.</p> <p>Short cuts are apt to be mechanical de-</p>

The equation cannot receive too much emphasis. Algebra has even been defined by some as the science of the equation. The majority of the units studied in algebra receive an application in the solution of equations. Therefore, it is of prime importance that the student receive a firm foundation in the basic principles of the equation. Thus, the teacher should regularly present the pupils with problems and exercises which require them to make use of the basic ideas of the equation.

TOPIC III--SIGNED NUMBERS

<u>Content</u>	<u>Teaching suggestions</u>
A. Meaning of signed numbers	It is important that the teacher find out the extent to which her students have acquired an understanding of signed numbers and operations with them in the previous course. She could spend much time in useless repetition. Should the teacher decide that this unit should be taught in this class the following suggestions are offered: A thorough understanding of signed numbers must be established. A number scale seems basic to this understanding. Once the scale is understood, pupils can place on it temperature readings, altitudes above and below sea level, profits and losses, and the like. All of this teaching should <u>precede</u> any attention to the four operations. The teacher must make clear the distinction between the use of + and - to indicate addition and subtraction and to indicate signed numbers, e.g., the two minus signs below have different meanings: $3 - (-7) = ?$ When teaching the four operations with signed numbers a fine opportunity presents itself for developing an understanding of the process before the rule is introduced. For example, the pupils may use the number scale to discover how to add signed numbers. Then they may express it in their own words, comparing it with the rule found in the text. Thus it becomes their rule for adding signed numbers.
B. Adding signed numbers	
C. Subtracting signed numbers	
D. Multiplying signed numbers	
E. Dividing signed numbers	

Evaluation:

Do the pupils have an understanding of the nature of signed numbers? Do they have an ability to add, subtract, multiply, and divide signed numbers on the number scale and by rule?

Teachers should not wait for a complete mastery of the objectives of this unit before going on to the next; there are many uses of signed numbers throughout the first course in algebra, consequently there will be many opportunities for re-teaching if it is needed.

TOPIC IV--FUNDAMENTAL OPERATIONS IN ALGEBRA

<u>Content</u>	<u>Teaching suggestions</u>
A. Four operations with simple literal numbers	Allow for plenty of practice in fundamental operations with monomials, including fractional and decimal coefficients to both increase skill in handling them and to increase the understanding of the meaning of each operation.
1. Adding, subtracting, multiplying, and dividing monomials and polynomials	Show that exponents indicate a short form of multiplication much as multiplication itself indicates a short form of addition.
B. Use of fundamental processes in solving	
1. Equations	Help students discover for themselves the laws of exponents in multiplication and division, allowing for ample practice in the application of these laws.
2. Verbal problems	
a. Number relation	Teach the solution of equations as the tools necessary for solving verbal problems using, in so far as possible, those which have real life meaning and which will interest pupils to develop the power to reason.
b. Perimeter	
c. Motion	
d. Age	
C. Symbols of grouping	
1. Historical background for use of grouping symbols	Develop the skill needed in performing the four fundamental operations with monomials and polynomials.
2. Purpose of symbols of grouping	Have sufficient practice with symbols to insure mastery.
3. Placing terms within symbols	Compare arithmetic multiplication and division with that of polynomials.
4. Uses of symbols of grouping	Use some division exercises that have

D. Exponents

remainders as well as those in which terms need rearranging.

Show that parentheses and other symbols of grouping are used to indicate a unity of the quantity enclosed. The removal or insertion is possible only by some law of operation as addition, subtraction, or multiplication.

Removal of parentheses preceded by a positive or negative sign may be taught as addition or subtraction of a quantity and as multiplication. For example, $-(4y - 7)$ could mean subtract giving $-4y + 7$. Or it might mean (-1) times $(4y - 7)$, which also gives $-4y + 7$.

At this time complicated exercises involving more than two sets of symbols of grouping might be avoided. Seldom does a pupil find use for anything more than parentheses and brackets.

Evaluation:

The pupil should:

- a. Be able to add, subtract, multiply, and divide monomials and polynomials;
- b. Be able to prove the laws of exponents in multiplication and division;
- c. Understand the meaning of parentheses and be able to insert or remove them from an algebraic expression or equation.

TOPIC V--EQUATIONS OF THE FIRST DEGREE IN TWO UNKNOWNNS

<u>Content</u>	<u>Teaching suggestions</u>
A. Nature of linear	In so far as possible, use problems within the reach of pupils that will illustrate equations with two unknowns, such as:
B. Meaning of a solution of a linear equation in two unknowns	
C. Solution of simultaneous linear equations in two unknowns	
	a. How many candy bars at 5¢ and 7¢ each can be bought with \$1.20?
	b. There were 1200 persons watching a ball game. The adults paid 65¢ for an admission ticket and the children paid 10¢. if

1. Graphing
2. Elimination by addition or subtraction
3. Substitution

the total receipts amounted to \$516.00, how many adults and how many children saw the game?

Pupils will need practice in plotting points and graphing single equations before graphing systems of equations.

D. Terms needed:

1. Horizontal axis
2. Vertical axis
3. Origin
4. Abscissa
5. Ordinate
6. Coordinate
7. Point
8. Variable
9. Simultaneous equations
10. Independent equations
11. Dependent equations

Explain carefully and illustrate the construction of a graph of an equation.

Make tables of values used in drawing graphs. Stress the function concept by emphasizing that one of the variables depends on the other to which values have been assigned.

Show that the graph of an equation of the first degree having two variables is a straight line, explaining that a linear equation having two variables has an indefinite number of solutions.

Allow for sufficient practice for graphical solution of linear equations.

E. Verbal-problem solving

1. Mixture
2. Number-relations
3. Investment
4. Others

Have pupils plot two linear equations on the same set of axes.

Emphasize that if a point lies on the graph of two different lines, its coordinates must satisfy each equation, and the coordinates of this point are the roots of the system of equations.

Prove by substituting values in the equations.

Allow for some equations to be graphed that have no common solution or all common solutions and thus bring in independent and dependent equations.

Discuss the need for algebraic solutions of simultaneous equations because the answers then are exact and require less time than the graphic solution.

Start verbal problems involving two unknowns.

Evaluation:

Observation check list:

- a. Is the pupil familiar with terms used?
- b. Does the pupil understand and appreciate the value of two variables in the solution of problems?
- c. Does the student understand what is meant by the common solution of a set of equations from both the algebraic and geometric points of view?
- d. Can pupils graph reasonably fast and accurately?

Can the student solve sets of equations:

- a. By graphing?
- b. By elimination, by addition, or subtraction?
- c. By substitution?

Is the student able to reason out a verbal problem and solve it?

TOPIC VI--SPECIAL PRODUCTS AND FACTORING

<u>Content</u>	<u>Teaching suggestions</u>
A. Terms used: 1. Monomial 2. Binomial 3. Trinomial 4. Square 5. Cube 6. Prime factor	If special products and factoring are developed together, pupils will more easily grasp the meaning of such forms as: $a^2 - b^2 = (a - b)(a + b)$
B. Finding the square of a monomial and binomial by inspection	Determine the present ability of the pupils with reference to: a. Knowledge of meaning of factoring; b. Recognition of similarity between division and factoring; c. Knowledge of terms used.
C. Factoring by removing common factor	Discover special ways of obtaining products mentally: a. Polynomial by monomial; b. Binomial by binomial; c. Product of sum and difference; d. Square of binomial.
D. Finding the product of two binomials of the form $(ax + by)(cx + dy)$	Discuss the meaning of the word <u>factor</u> and define what is done when an expression is factored. This can be easily brought out in arithmetic; the factors
E. Finding product of sum and difference of two terms	
F. Factoring perfect square trinomials	

of type $ax^2 + bx + c$

- G. Finding principal factors of algebraic expressions

of 24 are: 1 and 24 ; 4 and 6; 3 and 8. Here prime factors can well be introduced.

Lead pupils into the understanding that factoring grows out of multiplication.

Present simple applications as:

- Removing common monomial factor.
- Solving the trinomial form:
 $ax^2 + bx + c$.
- Difference of two perfect squares.
- Perfect square trinomial.

Drill through illustrations and applications to specific examples involving both literal and numerical terms. Include nonfactorable expressions as well.

Use quadratic equations only as an example of factoring.

Use ample verbal problems.

Evaluation:

Does the pupil:

- Understand the meaning of the word factor?
Does he indicate this understanding in factoring rather than a meaningless, mechanical operation devoid of meaning?
- Find special products mentally with reasonable speed and accuracy?
- Have the ability to select the type to which certain problems in factoring belong? Then can he factor the expression readily?
- Understand why each factor is set equal to zero in a quadratic equation?

TOPIC VII--FRACTIONS AND FRACTIONAL EQUATIONS

Content

Teaching suggestions

- Explanation of terms to be used in the topic of fractions
- Reduction of

Here is an excellent opportunity to enlarge the general number idea of algebra. For example, pupils should be taught how algebraic fractions are the generalized equivalents of the arithmetical fractions with which they are

fractions to lowest terms

- C. Finding a common denominator of several fractions
- D. Multiplying and dividing fractions
- E. Adding and subtracting fractions
- F. Solving fractional equations
- G. Analyzing problems that involve fractions and forming and solving equations

Supplementary work

- A. Adding and subtracting complex fractions
- B. Multiplying and dividing complex fractions

already familiar. Also, operations with algebraic fractions follow the laws as arithmetical fractions. For example:

$$\frac{1}{2} + \frac{1}{4} = \frac{2 + 1}{(2)(4)}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{a + b}{ab}$$

Begin with simple arithmetical fractions and proceed in gradual steps to literal fractions.

Test to measure the present ability of the pupils with references to the following:

- a. A knowledge of terms such as equivalent, numerator, denominator, etc.;
- b. ability to reduce fractions to lowest terms;
- c. ability to find a common denominator;
- d. ability to add, subtract, multiply, and divide fractions from arithmetic;
- e. ability to solve simple fractional equations.

Review meaning of fractions by using some idea within the everyday experience of the pupils. Example: if you spend 60¢ you spend 60/100 of a dollar which is 6/10 or 3/5.

Make sure that terms used are definitely understood. Operations with fractions should receive special attention and thorough treatment.

Practice finding a common denominator.

Pupils should understand the basic principle of fractions, namely, that the value of a fraction is not changed if both the denominator and numerator are multiplied or divided by the same number.

When solving fractional equations, pupils should see that they apply the four axioms to reduce a fractional equation to its

equivalent without fractions.

Consider multiplication and division first because of their similarity in treatment to reduction to lowest terms. However, attention should be called to the following:

Factors, not terms, are divided out. Use divide rather than cancel.

When all the factors in both numerator and denominator have been divided out, the result is 1, not zero.

Have one or two pupils demonstrate processes on the board, giving help only when needed.

Addition and subtraction of fractions will necessitate considerable emphasis.

Exercises in addition and subtraction of fractions may be checked by numerical substitution, however do not use numbers that will make any denominator zero.

Review finding a common denominator with arithmetic fractions in demonstration of addition and subtraction of fractions.

Have a drill in these fundamental processes after the process has been understood.

Pupils should understand that in fractions three signs are present, either expressed or implied: the sign of the numerator, of the denominator, and of the fraction itself. Practice changing signs to assure clear understanding of this process.

In reducing mixed expressions to improper fractions, the rule of arithmetic may be applied or the exercise may be considered one of addition or subtraction.

With complex fractions, two methods should be pointed out:

Reduce numerator and denominator to simple

fractions and divide, or

Multiply both numerator and denominator by a common multiple of all the denominators.

Present simple fractional equations.

Demonstrate process of changing a fractional equation into one of a simpler form. Help pupils discover the principle involved. Provide drill in solving and checking simple fractional equations.

Include solution of verbal problems involving fractional equations.

Evaluation:

Check list for pupil of things to be remembered:

- The numerator tells how many.
- The denominator tells what kind.
- The line separating the numerator and the denominator means divide.
- Algebraic fraction; signs of a fraction; common denominator; like and unlike fractions.

Important skills to develop are:

- Making equivalent fractions; multiplying fractions; adding like and unlike fractions; subtracting like and unlike fractions; dividing fractions; changing signs of a fraction; solving fractional equations.

Observation checklist for teachers:

- a. Are the pupils familiar with the common terms used?
- b. Can pupils add and subtract, multiply and divide fractions with reasonable speed and accuracy?
- c. Do pupils understand how by changing signs the fraction is simplified?
- d. Are pupils able to select and organize data in preparing to solve fractional equations?
- e. Do pupils understand how to reduce fractions to lowest terms?
- f. Do pupils understand that, in adding or subtracting fractions, they must have a common denominator, but to multiply or divide they do not require a common denominator?

TOPIC VIII--FINDING SQUARE ROOT AND USING RADICALS

<u>Content</u>	<u>Teaching suggestions</u>
A. Computing arithmetical square roots	Test to measure the present ability of pupils with reference to:
B. Finding square root by table	a. Ability to recognize squares of numbers from 1 to 15.
C. Becoming familiar with common terms used in working with square root such as:	b. Ability to find square roots of numbers.
1. Radical	c. Understanding of the terms: square and square root.
2. Square	
3. Factor	Emphasis should come in the understanding of square roots, radicals, and radical equations. Complicated problems here are seldom used.
4. Radicand	
D. Simplifying radicals	Introduce the unit by calling attention to the scientist's uses of squares and square roots as in the formula for the distance for falling bodies,
E. Adding, subtracting, multiplying, and dividing radicals	$s = \frac{1}{2}gt^2$
F. Rationalizing a binomial surd in the denominator	Show that both time and energy in computation can be saved by the use of radicals. It should also be shown that this unit is a foundation for further study in both mathematics and science.
G. Solving equations containing radicals	Discuss what is meant by "radical expression."
<u>Supplementary work</u>	Discover any historical facts about the radical sign.
A. Extracting square root of numbers by use of the slide rule	The following should receive attention:
B. More complex problems of types above	a. Any positive number has two square roots. The positive one is called the principal root.
1. Multiplication or division of radicals of different order	b. The coefficient of the square root of x or of 6 is 1.
2. Square roots of binomial surds	c. The radical should be treated as a term in algebra.
	Bring out the reason for rationalizing a fraction with radicals in the denominator--facility in evaluating the fraction.

Introduce the squaring of both sides of an equation as Axiom Five in the solution of equations. Still, the pupil should recognize this as the equivalent of the multiplication axiom. He should be cautioned to check all roots as extraneous roots may be introduced.

A clear distinction should be made between square and square root as pupils often are confused on the two.

The following skills and concepts should be emphasized in this unit by demonstrations, explanations, and drills:

- a. Understanding of terms used.
- b. Addition, subtraction, multiplication, and division of radicals.
- c. Square roots by computations and tables.
- d. Solution of equations involving radicals.

Pupils should make a list of fields of studies and vocations that require a knowledge of the material of this unit.

After the process is understood, provide ample drill.

Evaluation:

Check list for pupil of things to be remembered:

Base, power, radical, radicand, square root, and square.

Important skills to develop are:

Finding the square root of a number; simplifying a radical; solving equations involving radicals.

Observation checklist for teachers:

- a. Can pupils find squares and square roots with reasonable speed and accuracy?
- b. Do pupils understand the meaning of the concepts and terms used?
- c. Are pupils acquainted to some extent with the history and origin of squares and square roots?
- d. Are pupils familiar with the fundamental principles governing the simplification of radicals?

TOPIC ~~E~~--QUADRATIC EQUATIONS

Content

- A. Nature of quadratic equations
- B. Incomplete quadratics
 - 1. Solved by square root method
 - 2. Solved by graphing
- C. Complete quadratics
 - 1. Solved graphically
 - 2. Solved by factoring
 - 3. Solved by completing square

Teaching suggestions

Introduce the study of quadratic equations by graphing a quadratic equation that has no real root; one that has equal real roots; and one that has unequal real roots. Examples are as follows:

a. $x^2 + 4x + 5 = 0$ no real roots.

b. $x^2 + 4x + 4 = 0$ equal real roots.

c. $x^2 + 5x + 6 = 0$ unequal real roots.

Thus show what it means for a quadratic equation to have real roots and why any particular quadratic may have not any, one, or two real roots.

Factor some quadratics as was done in previous units and then present one that cannot be factored. Now apply the basic axiom for the solution of equations and lead to the solution by the method of completing the square.

Use such familiar formulas for the square method as:

$$A = (p1) r^2$$

and problems as:

Find the side of a square whose area is 64 square inches.

The derivation and use of the quadratic formula is to be left to Second-Course Algebra.

PLANE GEOMETRY

INTRODUCTION

Logical proof is the crux of a course in plane geometry. This means that the ability to state and prove independently a theorem of geometry is a primary purpose of this course and implies an understanding of the relationship of assumptions, definitions, hypotheses, and conclusions to proofs. No doubt of equal importance is the ability to apply the method of proof used in geometry to non-mathematical situations. Then too there are these other purposes: an awareness and appreciation of geometry in art, nature, industry, and architecture; skill in the use of instruments for making geometrical measurements and constructions; and knowledge of the basic facts and relationships of geometry.

It is to be left with each teacher and her group to decide how many theorems will be formally proved. In general, she will select enough to develop a thorough understanding of the nature of a mathematical proof as well as the ability to make an original proof. Constructions, corollaries, and applications will be used extensively, though formal proofs will not be required of all of them.

Teachers should find valuable the laboratory method of teaching that employs many models and other learning aids. Through the use of models, charts, etc., pupils may be led to discover geometric relationships which they can state as theorems to be proved in words which they understand. In this way reality may be introduced to plane geometry for pupils, particularly the slower ones. In addition, pupils will be stimulated in this manner to develop proofs which differ from those found in the textbook.

The teacher of plane geometry must remember that her pupils have most likely learned some facts and relationships of geometry in earlier courses in general mathematics. However, in these earlier courses conclusions were reached inductively through experimentation and measurement. Now in plane geometry, pupils must learn the difference between a conclusion established inductively and one established deductively through a formal mathematical proof. This transition

from what has been called the "intuitive geometry" of general mathematics to the demonstrative geometry course of the tenth grade is, therefore, of primary importance.

This course in plane geometry has been planned around eleven topics:

- I. Introduction
- II. Informal Geometry
- III. Formal Geometry
- IV. Parallel Lines
- V. Circles
- VI. Polygons
- VII. Inequalities
- VIII. Locus
- IX. Ratio and Proportion
- X. Similar Polygons
- XI. Measurement of Geometric Figures

TOPIC I--INTRODUCTION

<u>Content</u>	<u>Teaching suggestions</u>
A. The origin and early use of geometry	Pupil should read about Thales, Pythagoras, Plato, Euclid, and others. After a general discussion, have essays written or reports given on the life and work of important mathematicians.
1. Contributions made by great mathematicians and nations	
2. Egyptian rope-stretchers	Allow pupils an opportunity to list and bring to class geometric figures observed on way to school in order to point out the elements of symmetry.
B. Geometry found in our surroundings	
1. Nature	Have pupils find out in what ways geometry directly or indirectly enters into the pursuits of various occupations.
2. Science	
3. Industry	
4. Art	Have pupils construct original designs as used in wall paper, fabrics, or linoleum.
5. Architecture	
	Have on display a number of optical

C. Need for, and nature of logical thinking

1. Habits of correct thinking
2. Assumptions and definitions

illusions to show the danger of basing conclusions on appearances.

Have pupils make an analysis of their thinking by rating themselves --sometimes, always, or never on such questions as:

- a. Do you exaggerate?
- b. Do you question the validity of statements made in the newspapers or on the radio?
Can you detect faulty reasoning in yourself and others?
- c. Do you jump to conclusions?
- d. Are you willing to accept criticism when it is valid?

Then, select a few traits for which improvement is necessary.

Give pupils several statements, such as, "It always rains when the sun does not shine. The sun is not shining" to illustrate the fact that, if a single exception can be found, the conclusion is false.

Have pupils make similar statements and see what conclusions they can draw from each.

Encourage pupils to look for errors in reasoning in advertisements and in other sources familiar in their daily experiences.

Draw from pupils reactions as to how to avoid faulty reasoning.

Discuss with pupils the meaning of definitions, how definitions are made, the necessity of leaving certain terms undefined, the use of definitions in proving simple geometric exercises, and the importance of definitions in non-mathematical situations.

Point out that a good definition is

reversible and places the object in the smallest known class and gives the characteristics which makes it different from the other objects in the same class.

Develop inductively an understanding of axioms and postulates as they are needed.

TOPIC II--INFORMAL GEOMETRY

<u>Content</u>	<u>Teaching suggestions</u>
A. Basic constructions	Check to see how many pupils understand how to do the basic constructions as taught in general mathematics.
1. Bisect an angle	
2. Bisect a line segment	Allow for further attention to these constructions if this check reveals the necessity.
3. An angle equal to a given angle	
4. A perpendicular to a line from a point on the line.	
5. A perpendicular to a line from a point not on the line	

TOPIC III--BEGINNING FORMAL GEOMETRY

<u>Content</u>	<u>Teaching suggestions</u>
A. Nature of deductive proof	Have pupils draw two intersecting lines and ask if the vertical angles appear equal. Have them check by folding the paper and by measurement.
1. Contrast with conclusions reached by induction or experimentation	Have pupils pick out the "if" - "then" clause in any number of statements so they will see the condition and the conclusion.
2. Relationship of definitions, axioms, and postulates to proof	Help pupils discover the postulates and definitions needed in a formal

B. Proofs of simple theorems such as: If two straight lines intersect, the vertical angles are equal

1. Systematic arrangement of proofs

2. Identifying hypothesis and conclusion in theorems

3. Representing hypothesis in a figure

C. Congruent theorems

1. Definitions and assumptions

2. Theorems

3. Applications

D. Proof of basic constructions

proof that vertical angles are equal. Have pupils write out the formal proof. Point out that measurement and observation alone were not used in the formal proof of the equality of vertical angles.

Give pupils several examples from the text that will require simple proofs; for example, supplements of the same angle are equal.

Develop the meaning of altitude, median, etc., of a triangle by using models.

Have pupils construct triangles when SSS; SAS; or ASA are given. Then construct three triangles equal to them, cut out, and place one on the other to illustrate the three congruent theorems.

Proof of the triangle congruence theorems is hard for beginners to understand. Therefore, it might be advisable to postulate these theorems at this point and proceed with other proof more easily understood. If this is done, it should be clearly indicated what this means for subsequent proofs based on these theorems.

Have pupils prove exercises given in text using these three theorems.

Let pupils suggest congruency used in clothing, industry, and design.

Have pupils prove the basic constructions and explain why the constructions are possible.

TOPIC IV--PARALLEL LINES

Content

Teaching suggestions

A. Definitions

1. Parallel lines

Have each pupil construct a model as suggested at the end of the unit and lead him by a series of questions

2. Transversal

3. Corresponding angles

4. Alternate interior angles

5. Alternate exterior angles

B. Theorems

1. Indirect proof

2. Converse theorems

3. Practical application

4. Theorems related to parallel lines

to discover the theorems based on parallel lines. This procedure should help him understand the if-then relationship for he knows that what is fixed on his model tells him what is given and what results as he moves the wires on his model tells him what he must prove. Have the pupil state his own definitions of angles and the propositions of parallel lines. Have the class polish the definitions and propositions and then have each pupil prove the propositions.

Introduce pupils to the indirect method of proof by making use of non-mathematical situations involving indirect reasoning.

Have them list all the possibilities that may arise and show that all possibilities except one leads to a contradiction of the hypothesis so the remaining possibility is true.

Pupils are to write the converse and inverse of several theorems and decide which are true.

Cite examples of false conclusions which result from accepting the truth of a converse or an inverse statement in both mathematical and non-mathematical situations.

Ex.-The Hoover sweeper runs quietly.

Con.-If a sweeper runs quietly, it is a Hoover.

Inv.-If the sweeper does not run smoothly, it is not a Hoover.

Point out that this type of reasoning is a trick used by the propagandist.

Draw attention to the use of parallel lines used in mechanical drawing, architecture, interior decorating, costume designing, and engineering.

Have pupils construct and cut out isosceles, scalene, right, and equilateral triangles. Cut off two of the angles and place them on either side of the third to show that the sum of the angles of a triangle equals a straight angle.

Now have pupils give a formal proof of this theorem by constructing a line parallel to the base through the vertex and by constructing a line through one of the base vertices parallel to the opposite side.

Help pupils make a table showing the sequence of theorems, definitions, and assumptions on which this theorem depends.

Suggest pupils read a detective novel. Study the logic and list the steps used in solving the case. Eliminate steps that were misleading.

Suggested model referred to at the beginning of the unit:

Cut a piece of cardboard 8" by 12". Notch the centers of four protractors and paste them on this board. Cut 4 pieces of wire from coathangers and sew them on this board to the center of the circles made by the protractors (AB and CD). Use buttons on back to hold wires securely.

TOPIC V--CIRCLES

Content

- A. Definitions-
Assumptions
- B. Theorems
- C. Exercises

Teaching suggestions

Pupils are to review such terms as radius, diameter, chord, etc. and develop an understanding and a definition of a circle by use of a globe. Through the discussion bring out the difference in size

D. Angle measurement

1. Angles formed by tangents, chords, secants, and radii

E. Constructions

F. Applications

of circles formed by longitude and latitude.

Through experimentation let pupils conclude that one and only one circle can be constructed through three points not in a straight line.

Using two circular cardboard figures, with a sector cut out of one, illustrate by placing one upon the other the idea that equal angles (central) have equal arcs.

Help pupils develop a definition of tangency (both internal and external) by rolling a coin along a line and by using rubber bands.

To help pupils review theorems draw on the board figures representing each one studied and ask pupils to state the theorem each drawing represents.

Have pupils solve several exercises involving the use of these theorems.

Have pupils construct designs using the knowledge they have obtained about circles and angle measurement. Ask them to complete a circle when an arc of the circle is given.

Discuss advantages of the circle over the straight line as in the arrangement of seats in churches or amphitheaters.

Point out that civilization moves along on wheels (circles).

A project that could be developed for extra assignment is "Civilization on Wheels."

TOPIC VI--POLYGONS

Content

A. Definitions

Teaching suggestions

Pupils develop definitions for members of the polygon family.

B. Constructions

Construct polygons for a polygon tree.

C. Theorems

1. Angles of polygons

Have pupils derive the theorem informally for the sum of the angles of any polygon, exterior and interior angles.

2. Quadrilaterals, parallelograms, etc.

Definitions of quadrilaterals and theorems on parallelograms may be formulated by the pupil upon discovery of the relationship between the parts.

D. Practical applications

Point out to the pupil the many applications of the parallelogram of forces in aeronautics, physics, etc.

Use a plane table to show the location of points of intersection. Polygons are formed by joining the end points of the intersecting lines.

Pupils are to inscribe a pentagon in a circle by the method used in drafting.

TOPIC VII--INEQUALITIES

ContentTeaching suggestions

A. Axioms

Assist pupils in formulating the fundamental theorems of inequality of triangles, sides, and angles. The inequality of arcs and chords of a circle may be studied now.

B. Theorems

Help pupils develop proof.

C. Applications

Illustrate from arithmetic for the inequality axioms.

Point out the circular reasoning involved in proving the proposition: If two sides of a triangle are unequal, the angles opposite these sides are unequal and the greater angle lies opposite the greater side, by the indirect method in

which each one is made to depend upon the other.

Have pupils cite examples of circular reasoning from non-mathematical situations.

TOPIC VIII--LOCUS

Content

- A. Definitions
- B. Theorems
- C. Examples
- D. Applications

Teaching suggestions

Pupils are to define word locus.

Illustrate the definition by tacking a large piece of paper on the bulletin board on which are placed points and lines. Let pupils, supplied with a circle to roll, a line to move, or a ruler for measuring, carry out the required movements to develop the fundamental loci, using a thumb tack or colored crayon to mark successive positions of the moving point or line.

The result is a number of points which show the locus as a pattern traced by the moving point.

Remind pupils of the fact that when they were constructing graphs of equations they were really constructing loci.

Locate algebraically:

- a. The locus of points equidistant from the points (3,5) and (-1,5);
- b. The locus of points which are 4 units from the line whose equation is $Y = 2$;
- c. The locus of points at a given distance R from the origin (0,0) $X^2 + Y^2 = A^2$

The better pupils might extend this theorem to include the locus of points at a given distance from a

given point whose coordinates are a and b .

Solve any examples given in basal texts.

Call attention of pupils to the illustration of loci in "Treasure Island."

Have pupils locate points in a classroom and on the school grounds by use of a straight edge and string compasses.

Call attention to the fact that star paths are often studied by photographing their positions at various intervals with a fixed camera. Stars near the poles trace circular paths on the photographic plate while stars near the equator trace straight lines.

Locus in three dimensions might be introduced here.

TOPIC IX--RATIO AND PROPORTION

<u>Content</u>	<u>Teaching suggestions</u>
A. Definitions	Teach ratio as a fraction, and proportion as equal fractions.
B. Exercises	
C. Theorems and related constructions	Point the change in value as the numerator increases and denominator remains constant, or the denominator increases and the numerator remains constant.
D. Practical application	Practice solving fractional equations and apply the axioms of algebra.
	Cut a triangle from ordinary notebook paper using a line as the base and the space between lines as the common unit of measure. Have pupils analyze and write out a proof for, "A parallel to one side of a triangle, intersecting the other two

sides, divides the other two sides proportionally."

Emphasize the necessity of a common unit of measure.

Have pupils write the converse of this theorem and prove the examples in the text related to each.

Have pupils measure the classroom and make a scale drawing of same.

Solve several problems such as:

- a. Given a 15 foot board, divide it in two parts which will have the ratio 2 to 3.
- b. Measure the length and width of two American flags of different sizes and determine whether they have the same ratio.
- c. A brakeman pulls with a force of 125 pounds on a brake wheel 20 inches in diameter. The force is communicated to the brake chain by means of an axle $2\frac{1}{2}$ inches in diameter. Find the pull on the brake chain.

Explain the use of ratio in determining the number of representatives in Congress.

TOPIC X--SIMILAR POLYGONS

<u>Content</u>	<u>Teaching suggestions</u>
A. Definitions	Have pupils construct a square 4×4 , a rectangle 8×2 , a square 5×5 , and another square 4×4 . Compare the four figures and point out the difference between similarity, equality, and congruence.
B. Theorems	
C. Practical application	
D. Related theorems	
1. Proportional segments in circles	Have each pupil construct a triangle with one angle measuring 60 degrees and another measuring 80 degrees.

2. Right triangle and mean proportional

Two pupils working together could compare the triangles and write out a proof for, "Two triangles are similar if two angles of one are equal respectively to two angles of the other."

It might be of help to point out that the combination of letters in the numerators usually indicates one triangle and those of the other by the denominators when the pupil is selecting similar triangles from a given proportion.

Another aid in indicating corresponding parts of similar figures is the use of colored chalk.

Use such examples as:

A photograph $3\frac{1}{2}$ by $4\frac{1}{2}$ is to be enlarged so that the width of the enlarged picture will be 9 in. Find the length of the enlarged picture.

Develop the theorem: "If two chords intersect in a circle, the product of the segments of one equals the product of the segments of the other" by measuring the segments and finding the product. Develop by similar method the theorem dealing with secants and tangents.

If chords AB and CD intersect at B and AB = 10 inches, CE = 4 inches, ED = 4 inches, find AE and EB.

Solve any exercises in the text using these theorems. Also use numerical applications of students' choice to illustrate the meaning of these theorems.

It might be necessary to teach or re-teach radicals at this point.

TOPIC XI--MEASUREMENT OF GEOMETRICAL FIGURES

<u>Content</u>	<u>Teaching suggestions</u>
A. History	Have pupils do research on the origin and development of measurement. (The Nineteenth Yearbook of the National Council of Teachers of Mathematics has valuable help for the history and use of surveying instruments). Have pupils construct some of these and use them to lay off a baseball diamond, a football field, or survey the school grounds.
B. Definitions	
C. Theorems	
1. Polygons	Emphasize the fact that area is measured in square units. Also emphasize the difference between area and perimeter.
a. Special triangles	
b. Parallelograms	
c. Rectangles	
d. Rhombus	
e. Square	
f. Regular polygon	
g. Trapezoid	
h. Any polygon	
2. Exercises	Develop an informal proof for the area formulas of the following:
3. Pythagorean theorem	
a. History	a. Parallelogram
b. Proof	b. Triangle
c. Exercises	c. Regular polygon
d. Applications	d. Trapezoid
4. Circles	Have pupils solve the numerical exercises in the text giving special attention to the manipulation of formulas to solve for the missing terms.
a. Definitions and relationship	
b. Theorems and constructions	
c. Exercises and practical application	Study and explain the history and use of the Pythagorean theorem.
D. Numerical trigonometry (optional)	Help pupils develop an understanding of the formal proof of this theorem using the square and encourage the abler pupils to experiment and develop proofs using the semi-circle or similar polygons in place of the square.
1. Definition	
a. Trigonometric ratio	Have pupils solve a number of examples applying the Pythagorean theorem.
2. Application	

3. Construction

- a. A triangle equal to a given polygon.
- b. A square equal to the difference of two squares.
- c. A circle whose area is equal to the sum of two circles.

Have pupils construct geometric figures for the square root of 2, 3, etc. (The diagonal of a square, with the side equal to one inch is the square root of 2. Use the length of this diagonal for one side of a rectangle and one inch for the other and the diagonal of this rectangle is the square root of 3).

By referring to the Seventeenth Yearbook of the National Council of Teachers of Mathematics have pupils point out the use of the Pythagorean theorem in locating foreign objects such as bullets, needles, or glass in a human body when x-rays are made.

Develop the relationship between the circle, the regular inscribed and circumscribed polygons, the apothem, and the radius.

Inscribe a triangle, square, pentagon, hexagon, octagon, in a circle to introduce the basic concept of the theory of limits.

Measure the diameter and circumference of a number of circles and divide each circumference by the diameter of that circle. If you are accurate, you will find that this ratio equals 3.14.

Call attention again to the fact that geometry is a complete and interrelated system by having the pupils list the independent theorems in this unit.

Allow pupils to solve the numerical examples in the text using the special area formulas for triangles, the formulas for sector and segment to give practice in arithmetical and algebraic skills.

On a blackboard graph draw right triangles of different sizes and show the pupils how sine, cosine, and tangent change as the size of the angle changes.

Teach pupils how to use the trigonometric tables.

Have pupils solve numerical exercises in the text using the trigonometric tables.

Have pupils use simple instruments they have constructed to measure heights of objects around the school (trees, flagpole, posts, etc.).

EVALUATION:

Evaluation of any topic should include the following:

- Demonstration of theorem proof;
- Construction problem;
- Objective questions to test vocabulary and facts;
- One or more original problems to show application and to test ability to use what has been learned.

SECOND-COURSE ALGEBRA

INTRODUCTION

Capable pupils should be encouraged to elect a second course in algebra, especially if they wish further training in mathematics or the sciences. The second course in algebra should help the student to develop: (1) an advanced understanding of the number system through the study of irrational and imaginary numbers; (2) an understanding of the use of graphical representation; and (3) an understanding of the function concept through the study of tables, graphs, formulas, and equations. This course should also facilitate computation by giving the pupil the new computational tool of logarithms. It should help him acquire more advanced algebraic skills and the ability to apply them to wider fields as well as an increased understanding of the locus concept.

Often too much time is spent reviewing first-year algebra, consequently it is often difficult to complete the work of the second year. With this in mind, it is suggested that a review covering the fundamentals, factoring, fractions, first-degree equations in both one and two unknowns be completed in approximately six weeks or less.

The new content of the second course should allow immediate follow-up, beginning with the unit on functional relations, and a short period of time be given along with the advanced topics for such reviews as are necessary to retain skills and extend the applications of the underlying principles.

In this way, a better comprehension of the entire course will be achieved. Second-Course Algebra is herewith divided into eleven topics:

- I. First Course review.
- II. Functional relations.
- III. Square roots-surds-radicals-radical equations.
- IV. Quadratic equations in one unknown.
- V. Imaginary numbers-theory of quadratic equations.
- VI. Graphs and solution of quadratic equations in two unknowns.
- VII. Exponents and radicals.

- VIII. Logarithms
- IX. Numerical Trigonometry
- X. Progressions
- XI. Equations of higher degree

TOPIC I--FIRST-COURSE REVIEW

<u>Content</u>	<u>Teaching suggestions</u>
PART I	
Addition, Subtraction, Multiplication, and Division	Allow for practice material to make sure that pupils understand the procedure. Follow this with a diagnostic test to determine individual pupil assignments.
A. Signed numbers	
B. Monomials	Assign to each pupil related practice exercises that correspond to the exercises he was unable to solve in the test.
C. Polynomials	
PART II	A review here will refresh the concepts. If all pupils do not achieve the goals, short reviews during the year will help more than too much time spent here.
Special Products and Factoring	
A. Factoring trinomials of the type: $ax^2 + bx + c$	Since this is a review, pupils will enjoy a new approach in which they see all regular cases, except removing common monomials, as special cases of $ax^2 + bx + c$. For example, the usual separate cases, illustrated with these trinomials: $x^2 + 4x + 4$; $x^2 - 2x - 8$; $3x^2 - 13x - 10$; are all special cases of $ax^2 + bx + c$.
B. Special cases of factoring	
1. By grouping: $ax+ay+bx+by$ $a(x+y)+b(x+y)$	Teach the multiplication of each type along with the factoring processes.
2. Polynomials reducible to difference of two squares: $a^2+2ab+b^2-c^2$ $(a+b)^2 - c^2$	Often the topic of factoring is given too much time in the review at the outset of second-course algebra. The teacher must bear in mind that further opportunity is provided to study factoring throughout the course.
3. Trinomials reducible to difference of two squares: $16a^4 - 12a^2 + 1$ $(4a^2 - 1)^2 - 4a^2$	In complete factoring, remove the simplest factor first and then refactor the general trinomial, difference of two squares, and sum or difference of two cubes, if such expressions remain.
4. Binomials reducible to the	Any more advanced special cases of factoring are recommended as enrichment

difference of
two squares:

$$a^4 + 4$$

$$(a^2 + 2)^2 - 4a^2$$

5. Sum or difference of like powers:
- $$a^n + b^n$$
- $$a^n - b^n$$

PART III

Fractions

- A. Reduction of a fraction to its lowest terms
- B. Multiplication and division of fractions
- C. Sign of fractions
- D. Addition and subtraction of fractions
- E. Complex fractions
- F. Mixed fractions

material for pupils who need less time on review.

A short review of the fundamental ideas of arithmetical fractions should help to increase understanding of algebraic fractions.

Relate the work in algebraic fractions to that of arithmetical fractions.

Require pupils to express the line separating the numerator and denominator of the fraction as "divided by."

Remind students of the three signs of a fraction, either written or implied, i.e., the sign of the numerator, the sign of the denominator, and the sign of the fraction.

Emphasize the fundamental law that only fractions with equal denominators can be combined.

At this level pupils should be encouraged to replace much step-by-step written work with sound mental work. For example, many pupils should be able to eliminate writing down the invert step in division of fractions.

Briefly review the solution of equations with one unknown by use of the four basic axioms, i.e., the same number may be added to or subtracted from both sides of an equation and both sides of the equation may be multiplied or divided by the same number.

Review equations involving parenthesis and fractions.

Graph the first degree equation in two unknowns. Use the graph to point out that such an equation has an infinite number of solutions since there are an infinite number of points on the line. Then try to develop the idea that there must be as many sets of data as there are unknowns. Problems in non-algebraic language should help clarify the presentation. Then proceed with the solution of

PART IV

First-Degree Equations

- A. Equations with one unknown
- B. Equations with two unknowns
- C. Fractional equation
- D. Verbal problems expressing a first-degree equation
- E. Equations with three unknowns (as enrichment material)

first-degree equations in two variables.

Review methods of solution by elimination by addition or subtraction and substitution.

Apply the solution of linear equations by the use of verbal problems drawn from the experiences of the pupils, whenever possible.

The teacher should always bear in mind that the review of this work is to refresh and to extend the meaning of FIRST-COURSE ALGEBRA.

Evaluation:

The pupil should:

Understand the terminology used;
be able to carry out the fundamental operations with signed numbers, monomials, polynomials, and fractions;
solve problems involving parentheses;
solve equations using the basic axioms of addition, subtraction, multiplication, and division;
translate verbal problems into equations and solve them; find special products mentally and factor all types completely; solve systems of equations by graphing, by elimination, by addition, by subtraction, and by substitution.

TOPIC II--FUNCTIONAL RELATIONS

<u>Content</u>	<u>Teaching suggestions</u>
A. The function concept	Develop the concept of functional relations by discussing with the class the idea of one quantity <u>depending upon</u> another quantity. Some examples are:
B. Ways of expressing the functional relations:	
1. The word-rule	a. Upon what will the total cost of a family's grocery order depend?
2. The formula	b. Upon what will the total distance a car travels depend?
3. The table	c. Upon what do the total wages a man earns depend?
4. The graph	d. Upon what does the amount of taxes depend?

C. The meaning and use of the words:

1. Formula
2. Variable
3. Constant
4. Independent variable
5. Dependent variable

Give many other examples from both pupils' and teacher's experience.

Show each of the four ways of expressing the functional relations, i.e., the word-rule, the formula, the table, and the graph.

Show that many functional relationships cannot be expressed in mathematical terms.

D. Graphing by plotting points

Give and require of the class various examples of the use of the words in C. Use the language in class discussions throughout the topic and thereafter.

E. Graphing by the intercept method

Given the word-rules, allow the pupils to write the formulas, prepare the tables, and plot the corresponding graphs.

F. The slope of a line

G. Finding the slope of a line from its equation

Reteach the meaning of axis, origin, quadrants, abscissa, ordinate, co-ordinates.

H. Families of first-degree functions

1. Type
 $y = mx$
2. Type
 $y = mx + b$

Have class plot various points in each of the four quadrants.

I. Graphs of systems of first-degree equations

Develop concept that two points determine a straight line. A third point is located only as a check.

J. Ratio and proportion

Explain meaning of x-intercept and y-intercept.

K. Variation

1. Direct
2. Inverse
3. Joint

Present the intercept method of graphing as the easy way of locating two points through which a straight line can be drawn.

Explain meaning of the slope of a line. It can be defined as the ratio of rise to run.

Define positive and negative slope as follows:

- a. If a line rises to the right, its slope is positive.
- b. If a line rises to the left, its slope is negative.

The following teaching procedure

should prove helpful in teaching the finding of the slope of a line from its equation.

- a. Plot the graph of the equation $3y = 2x$ and determine the slope of this line.
- b. Solve the equation for y .
- c. Compare the slope of the line with the coefficient of x .
- d. Plot the graph of the equation $3y = 2x + 6$ and determine the slope of this line.
- e. Solve the equation for y .
- f. Compare the slope of the line with the coefficient of x .
- g. Compare the constant term with the y -intercept. Hence, m is the slope of the line $y = mx + b$, and b is the y -intercept.

The following procedure should prove helpful in explaining the equation $y = mx$.

Have the class graph the following equations on the same set of axes:

$y = \frac{1}{4}x$	$y = -\frac{1}{4}x$
$y = \frac{1}{2}x$	$y = -\frac{1}{2}x$
$y = 2x$	$y = -2x$
$y = 4x$	$y = -4x$

Be sure that student observes that:

- a. All the graphs pass through the origin.
- b. As the value of m increases, the graph of $y = mx$ appears to rotate in a clockwise direction. The teacher may need to point out that the change in m from $-\frac{1}{4}$ to -4 is a decrease in m .

The following procedure should prove helpful in explaining the equation $y = mx + b$.

Have the class graph the following equations on the same set of axes:

$$y = x + 2$$

$$y = 2x + 2$$

$$y = 3x + 2$$

$$y = -4x + 2$$

$$y = -2x + 2$$

Notice that when m changes and b remains constant, the graphs appear to rotate about the point (0,b).

Have the class graph the following equations on the same set of axes:

$$y = 2x + 1$$

$$y = 2x + 6$$

$$y = 2x + 4$$

$$y = 2x + 8$$

Notice that when m remains constant and b changes, the graphs appear to be translated (moved so as to remain parallel to one another).

After these examples have been presented and observations made by the pupils, help the students to generalize concerning any first-degree equation of the form $y = mx + b$. Make sure the student understands that the coefficient of x alone determines the slope of a line when the equation of the line is in the form $y = mx + b$, with the constant term b determining the location of the line on a graph.

The student should now realize that either of the following sets of data determine a straight line:

- a. two points, or
- b. one point and the slope of the line.

In graphing systems of first-degree equations, it is important that the student be able to recognize dependent, inconsistent, and independent equations. The following procedure will help:

Have the class graph the following

types of pairs of equations:

$2x + 4y = 2$	Dependent equations
$4x + 8y = 4$	
$3x - 2y = 6$	Inconsistent equations
$6x - 4y = 8$	
$4x + 3y = 6$	Independent equations
$2x + y = 4$	

Teach pupils to recognize each of the above types of equations:

- a. If the equations are dependent, one equation can be derived from the other by multiplying by a constant.
- b. If the equations are inconsistent, the ratio of the coefficients of x , and the ratio of the coefficients of y are the same, but the ratio of the constant term is different.
- c. If the equations are independent, the ratios of the coefficients are not the same.

The student should recognize that there is no solution for either dependent or inconsistent equations, but there is a solution for independent equations. The graphical representation of each should make this clear.

The concept of ratio as being a comparison between like quantities can be presented by a brief review of the slope of a line. Many other examples should be given so that the concept of ratio is clear.

Demonstrate to the class that a ratio is a fraction and obeys all the laws of fractions.

The student should understand that a proportion is a statement that two ratios are equal. Thus, a proportion is a fraction's equation.

Similar triangles may be used to illustrate the concept of proportion and

problems in indirect measurement offer practical application.

Both the colon and the fractional forms of proportion should be taught.

The main idea in this study of variation is an understanding of the three types of variation. The student will need much practice in stating verbal problems involving variation into algebraic language.

DIRECT VARIATION

If y varies directly as x , then y equals some constant times x , $y = kx$.
For example:

- a. The distance a train travels at a uniform rate varies directly as the time. (The uniform rate is the constant.)
- b. The amount of property tax, at a fixed rate, paid varies directly as the assessed value. (The tax rate is the constant).
- c. The area of a circle varies directly as the square of the radius. (π is a constant term).

INVERSE VARIATION

If y varies as the inverse of x , then y equals some constant times $1/x$, $y = k 1/x$.
For example:

- a. The volume of a gas in a container varies inversely as the pressure.
- b. The number of revolutions made by a wheel rolling over a given distance varies inversely as the radius.
- c. The resistance a wire offers to an electric current varies inversely as the square of the radius.

JOINT VARIATION

If y varies jointly as x and z , then y equals some constant times xz , $y = kxz$.
For example:

- a. The area of a triangle varies jointly as the base and altitude.
- b. The total cost of a set of books varies jointly as the number of volumes and the cost of each volume.
- c. The volume of a rectangular solid varies jointly as the length, width, and height.

Evaluation:

The pupil should be able to:

Think in terms of functions;
express, interpret, and use functional relationships in the form of words, formulas, tables, and graphs;
demonstrate and understand the concepts, vocabulary, and symbolism of the topic;
compare changes in two related variables;
recognize functional relations in life situations, some of which cannot be expressed mathematically.

TOPIC III--SQUARE ROOTS--SURDS--RADICALS AND RADICAL EQUATIONS

<u>Content</u>	<u>Teaching suggestions</u>
A. Squares and square root	Test to determine what should be included in this unit.
1. From a table	
2. By approximation	Have a brief drill on the use of the table to find both the square and square root of numbers. A clear distinction should be made here as pupils often fail to distinguish the difference.
3. By rule	
B. Surds	
1. Simplification of:	
a. Whole numbers	Pupils should memorize the squares of numbers from 1 to 25 and the cubes from 1 to 12.
b. Decimals	
c. Fractions	
2. Addition and subtraction	The students should be taught to use the square and cube of the sum and the difference of two numbers for

3. Multiplication
and division

C. Radical equations

facility in squaring numbers, e.g.

$$32^2 = (30+2)^2 = 900+120+4 = 1024.$$

At this point teachers should give the algebraic explanation of the common rule for finding square root of 5184. First let t equal the tens in the square root and the u the units. Then

$$\begin{aligned}(t + u)^2 &= t^2 + 2tu + u^2 \\ &= t^2 + u(2t + u) = 5184\end{aligned}$$

Now t^2 must be 4900 and $t = 70$
Then $u(2t + u) = 284$, but $t = 70$,
hence $u(140 + u) = 284$

But 140, the coefficient of u , is contained about 2 times in 284.
Substituting

$$2(140 + 2) = 184.$$

Therefore,

$$\sqrt{5184} = 72.$$

In simplifying surds, it is well to require a systematic plan in the beginning such as:

$$5 \text{ in.} + 3 \text{ in.} - 2 \text{ in.} = 6 \text{ in.}$$

$$5x + 3x - 2x = 6x$$

$$5\sqrt{2} + 3\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$$

Multiplication and division of radicals should be introduced by a short review of products of binomials. Students easily transfer from

$$(5 + x)(3 - x) \text{ to } (5 + \sqrt{2})(3 - \sqrt{2})$$

$$15 - 2x - x^2 \quad 15 - 2\sqrt{2} - 2.$$

This might be a good time to stress the fact that squaring a radical whose index is 2 removes the radical sign.

A review of the product of the sum and difference of two numbers will help the class to understand rationalization of

binomial surd denominators.

Stress the idea that denominators are rationalized only to facilitate the evaluation of the fraction.

Apply the basic axiom that the numerator and denominator of a fraction can be multiplied by the same number. To rationalize the denominator, the numerator will be the conjugate of the denominator.

Begin the study of radical equations with very simple equations. Formulas might be used which involve radicals.

For ease in solution and to avoid introducing extraneous roots, the teacher should stress the idea that one radical expression should be the only term on one side of the equation as $\sqrt{2y} + 4 = 5$ should become $\sqrt{2y} = 1$ before solution.

Evaluation:

Pupils should:

- Be able to find squares and square roots with accuracy; be able to solve problems using the rule of Pythagoras that involves square and square root; know the language related to the content of the topic;
- simplify, add, subtract, multiply, and divide radicals;
- be able to rationalize the denominator of fractions;
- be able to solve linear equations involving radicals.

TOPIC IV--QUADRATIC EQUATIONS IN ONE UNKNOWN

<u>Content</u>	<u>Teaching Suggestions</u>
A. Graphing quadratic equations	Graphical representation of quadratic equations offers the best approach to quadratic equations because here the student can grasp the meaning of the equation and the roots of the equation.
B. Solution of quadratic equations	A quadratic equation should be graphed. Then the teacher should point out that the roots of the quadratic are the values
1. Incomplete quadratics	

- a. Square root method of x where the curve crosses the x axis. Have the students locate the points at which the quadratic equation crosses the x axis and give the roots of the equation.
- b. Factoring method

2. Complete quadratics

- a. Factoring method Plenty of practice should be given in the graphical solution of quadratic equations in which the roots are real, rational, and unequal.
- b. Completing the square method Then present a quadratic in which the roots are real, unequal, and irrational. This should bring out the inadequacy of the graphical solution and lead to the algebraic method of solution.
- c. Formula method

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- d. Graphic method

A suggestion here is that a graphical representation of each type of quadratic equation be made before the algebraic solution is given. The reason for imaginary roots is readily seen when a graph is drawn.

C. Fractional quadratic equations

Stress the fact that there are two roots to a quadratic equation.

D. Radical equations

Have students solve some incomplete quadratics as found in formulas such as the formula for falling bodies, $S = \frac{1}{2}gt^2$.

E. Verbal problems

1. Numerical
2. Consecutive integer
3. Reciprocals
4. Numerator and denominator
5. Geometric
6. Distance
7. Picture frame

Use the method of factoring when the factors are readily seen.

The method of completing the square is important as a background for the derivation of the quadratic formula.

After the derivation the student should memorize the formula.

Stress that the quadratic formula gives two roots to a quadratic equation but that sometimes these roots are equal.

Solve many equations of varied types by the formula.

Have the students check some of all types of roots by substitution.

In radical equations, extraneous roots are sometimes introduced when both

members of an equation are raised to a certain power. Thus, it is necessary that the roots of all radical equations be checked.

Verbal problems often involve quadratic equations. The aim here should be that of developing the ability of translating the problems into equations.

Pupils need much help with reading and analyzing. Teach pupils to write down and organize all relevant data with care.

Evaluation:

The pupils should:

Solve a quadratic of the form $ax^2 + bx + c = 0$ by:

- a. Graphing it;
- b. factoring, if possible;
- c. completing the square;
- d. the formula;
- e. solve verbal problems leading to quadratic equations;
- f. solve fractional and radical equations which lead to quadratic equations;
- g. check roots always and discard inappropriate ones.

TOPIC V--IMAGINARY NUMBERS--THEORY OF QUADRATIC EQUATIONS

<u>Content</u>	<u>Teaching suggestions</u>
A. Imaginary numbers	Show how imaginary numbers come about.
1. Simplification using imaginary unit	Graph the equation $x^2 + 2x + 3 = 0$. Show that the roots are imaginary. Then solve the equation by the quadratic formula to show how imaginary numbers arise.
2. Operations with imaginary numbers	Point out the use of imaginary numbers in electricity in physics.
a. Addition and subtraction	Explain the imaginary unit $\sqrt{-1} = i$ and powers as $-1 = i^2$. Then show how every imaginary number can be expressed as a product of the <u>real</u> number and the imaginary unit:
b. Powers of the imaginary unit	$\sqrt{-36} = \sqrt{-1} \text{ or } 6i$.

- c. multi-
plication
and
division

Here the teacher might explain that two of the three cube roots of 8 involve imaginary numbers.

$$\sqrt[3]{8} = 2, -1 + \sqrt{-3}, \text{ and } -1 - \sqrt{-3}$$

B. Complex numbers

1. Addition and subtraction
2. Multiplication and division
 - a. Conjugate complex numbers
3. Quadratics having imaginary roots
4. Character of roots as determined by
 - a. the discriminant
5. Relation between roots and coefficients
6. Forming equations from given roots
 - a. Numerical and literal roots
 - b. Integral and fractional roots
 - c. Rational and irrational roots
 - d. Complex roots
7. Literal quadratics
8. Equations solved like quadratics

Imaginary numbers are subject to all the normal laws of operation which are used with real numbers and thus may be added or subtracted as radicals are.

Set up the series 1, -1, -1, 1 for powers of i.

When studying complex numbers, use the law that only like terms can be combined.

Use sufficient drill to insure confidence in working with these numbers.

By the use of the quadratic formula, have the students identify the roots of a quadratic equation as

- a. Real or complex
- b. Equal or unequal
- c. Rational or irrational

After such identifications as were suggested above, develop a summary:

If $b^2 - 4ac$ is		The roots are
Positive and a perfect square		Real Rational Unequal
Positive but not a perfect square		Real Irrational Unequal
0		Real Rational Equal
Negative		Complex Irrational Unequal or Imaginary

The teacher should point out that most complex roots are irrational.

Have students determine character of the roots by use of the discriminant and drill.

From the quadratic $ax^2 + bx + c = 0$, show that the sum of the roots is $-\frac{b}{a}$, and the product is $\frac{c}{a}$.

Encourage the class to make up problems. Literal quadratics should be used as enrichment material.

Any equation which can be put in the form: $a(\quad)^2 + b(\quad) + c = 0$ may be solved as a quadratic. Examples are:

a. $x^4 - 13x^2 + 36 = 0$

b. $(5x + 12)^2 - 5(5x + 12) - 6 = 0$

Evaluation:

The pupil should know:

The meaning of imaginary numbers and roots;
the nature of roots by inspecting the discriminant $b^2 - 4ac$; how to form equations when roots are given as well as solve the equations;
how to use and express the imaginary unit;
how to carry out the fundamental operations with imaginary numbers;
how to solve equations of higher degree that have the form of quadratic;
how to check by using relations between roots and coefficients.

TOPIC VI--GRAPHS AND SOLUTION OF QUADRATIC EQUATIONS IN TWO UNKNOWNNS

<u>Content</u>	<u>Teaching suggestions</u>
A. Graphs of circles $x^2 + y^2 = r^2$	Review graphs of first-degree equations. Emphasize that these graphs were straight lines. Explain why second-degree equations are called conics. Use pictures of sections of cones to show the forms of the conics.
B. Graphs of ellipses $b^2x^2 + a^2y^2 = a^2b^2$	

C. Graphs of parabolas THE CIRCLE

$$y = ax^2 \text{ or}$$

$$y = ax^2 + bx + c$$

$$x = by^2 \text{ or}$$

$$x = ay^2 + by + c$$

D. Graph of the hyperbola

$$xy = c$$

E. Quadratic equations with two unknowns

1. One linear and one quadratic
2. Pairs of quadratics
3. Verbal problems

Using a sample equation such as

$$x^2 + y^2 = 49$$

- a. Solve for y and show the range of real values for x .
 - b. Solve for x and show the range of real values for y .
 - c. Make a table of corresponding values of x and y .
 - d. Plot the points and draw a smooth curve through them.
 - e. Go from this quickly to the use of the compass to draw the graphs without making a table of values.
 - f. Generalize concerning the nature of the equation of a circle, i.e.,
- $$x^2 + y^2 = r^2$$
- g. Point out that the circle is the basis of many fundamental geometric constructions and that it is the form of many objects around us.

As a preview to analytical geometry, the circle may be defined as the path a point traces so that it is a given distance from a fixed point. The distance is the radius and the fixed point is the center of the circle.

THE ELLIPSE

Use the same steps as in a through d for circle graphs.

Study the equation of the ellipse, i. e.,

$$b^2x^2 + a^2y^2 = a^2b^2$$

or

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Explain the meaning of x and y intercepts, and major and minor axis and arcs. Show how these graphs may be sketched.

Point out the use of ellipses in orbits of planets and satellites, arches, auditoriums, bridges, designs, etc.

The ellipse may be defined as the locus of a point which moves so that the sum of the distances of this point from two fixed points is a constant. The two fixed points are the foci and the constant is the length of the major axis.

THE PARABOLA

Briefly review parabolas with the y -axis as the axis of symmetry as they were studied in Topic IV.

Set up tables and graph the following parabolas:

$$x = y^2 \qquad x = \frac{1}{4} y^2$$

$$x = \frac{1}{2} y^2 \qquad x = 2y^2$$

$$x = -y^2$$

In so doing, note that:

- a. All curves pass through the origin.
- b. The axis of symmetry is the x -axis.
- c. The graph opens to the right if x is positive and to the left if x is negative.

Point out practical examples of the parabola as light reflectors, arches, etc.

The parabola may be defined as the locus of a point which moves so that the distance from a fixed point is equal to the distance from a fixed line the directrix.

THE HYPERBOLA

When graphing the hyperbola, make a table of corresponding values for x and y . Then plot the points and draw smooth curves through them.

Teach the meaning of asymptotes as well as their use.

Point out the applications in scientific formulas, sound ranging, etc., calling special attention to Boyle's Law ($p v = c$).

Graphical representation of quadratics in two unknowns should be made. This adds meaning to the solution of such equations and gives some insight into analytical geometry.

The algebraic solution of one linear and one quadratic equation should be limited to the substitution method.

Pairs of quadratic equations may be solved by the method of substitution or that of addition and subtraction.

Verbal problems should show the need for ability in solving systems of quadratics and give the pupil practice in choosing the method he considers best to use in each solution.

Evaluation:

The pupil should:

Recognize the following about graphs of equations:

$ax + by = c$ gives a straight line

$ax^2 + bx = c$ gives the parabola

$ax^2 + by^2 = c$ gives the ellipse

$xy = c$ gives the hyperbola

$ax^2 - by^2 = c$ gives the hyperbola

$x^2 + y^2 = c$ gives the circle;

Solve quadratic systems and pair the answers correctly.
Be able to graph the systems of equations.

TOPIC VII--EXPONENTS AND RADICALS

<u>Content</u>	<u>Teaching suggestions</u>
<p>A. Exponents</p> <ol style="list-style-type: none"> 1. The fundamental laws of exponents 2. Fractional exponents 3. Negative exponents 4. The zero exponent 5. Application of exponents 	<p>Review with the class the work they have already had about positive integral exponents and powers of the same base. Explain that in this unit those fundamental laws are to be extended in order that meaning may be given to negative and fractional exponents and zero as an exponent. Explain that to do this, we must choose definitions in such a way that the new exponents will obey the same laws as positive integral exponents do.</p>
<p>B. Radicals</p> <ol style="list-style-type: none"> 1. Simplification of radicals <ol style="list-style-type: none"> a. Removing factors from the radicand b. Simplifying fractional radicands c. Reducing to a lower order 2. Addition and subtraction of radicals 3. Fundamental laws of radicals <ol style="list-style-type: none"> a. Multiplication law b. Division law 4. Rationalization of denominators 	<p>Allow for practice in the use of the fundamental laws of exponents with positive integers.</p> <p>Operations with exponents involving whole numbers should become almost automatic. In order to gain this proficiency oral drill might be gainfully used.</p> <p>Extend the fundamental laws of exponents to include fractional exponents, negative exponents, and the zero exponent.</p> <p>Emphasize the difference between the power of a root and the root of a power. When possible, the power of a root should be used in preference to the root of a power. Show the pupils the reason for this preference.</p> <p>Here is a fine example of the many opportunities in algebra to advance the understanding of the nature of mathematical proof. For example, these steps constitute a good proof of $A^0 = 1$.</p>
<p>C. Radical equations</p>	$A^n \div A^n = A^{n-n} = A^0 \quad (\text{Law of exponents in division}).$ $A^n \div A^n = 1 \quad (\text{Any quantity divided by itself}).$ <p>Therefore $A^0 = 1$ (Quantities equal to the same quantity equal each other).</p>

In physics, chemistry, and astronomy, there is constant occasion to deal with very large or very small numbers. Exponents are often used to express these numbers.

Emphasize the fact that radicals having the same index and the same radicand can be added or subtracted.

Emphasis should be placed on the fundamental laws of radicals. However, a high degree of skill of operation with radicals is rarely needed. Therefore, greater emphasis should be placed on understanding the process. It should be enough if a pupil can work with radicals correctly if not speedily.

Stress the fact that radicals must be of the same index before they can be multiplied or divided.

Care should be taken to check roots. We often introduce roots that do not satisfy the equation as given. Students should be shown how these roots are introduced.

Evaluation:

The pupil should:

- Know the meaning of the terms used.
- Know the fundamental laws of exponents.
- Know the meaning of fractional, negative, and zero exponents.
- Be able to operate with them as with positive integral exponents.
- Be able to simplify radicals by using fundamental laws of exponents.
- Be able to solve equations containing radicals.

TOPIC VIII--LOGARITHMS

Content

Teaching suggestions

A. Basic concept of logarithms	Present an effective over-view of this unit at the very outset. This can be done by using the accompanying miniature system of logarithms to base "2".
B. Exponential and logarithmic forms	

- C. Finding the logarithm, the number, and the base
1. $\log_2 4 = x$
 2. $\log_2 x = 2$
 3. $\log_x 8 = 3$
- D. Finding the characteristic and mantissa
- E. Interpolation
- F. Antilogarithms
- G. Operations with logarithms
1. Logarithm of a product
 2. Logarithm of a quotient
 3. Logarithm of a power
 4. Logarithm of a root
- H. Cologarithms
- I. Applications of logarithms
1. Formulas
 - a. Circle
 - b. Cylinder
 - c. Cone
 - d. Sphere
 - e. Cube
 - f. Rectangular solid
 - g. Area of triangle
 - h. Trapezoids
 2. Science formulas

Have this table on the side board and the following examples on the front board when the class comes in. Have the pupils in each row do one of the problems by arithmetic.

$$1. \begin{array}{r} 128 \times 512 \\ \hline 1024 \end{array}$$

$$2. \begin{array}{r} (512 \times 128)^2 \\ \hline 64 \end{array}$$

$$3. \begin{array}{r} \sqrt{65.536} \\ \hline 256 \end{array}$$

$$4. 1/4(32768)$$

<u>x</u>	<u>2^x</u>
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

When the class has finished the exercises, write the answers from the table beside each example on the front board.

See if some pupils have the insight to tell how the teacher could write down the answers from the table.

Such a procedure can be used to convince the class of the efficiency of logarithms. Explain that this system is built to base 2, but that it would not be convenient for practical use. Show the advantage of use of number 10 as a base. Explain that the logarithm used here is actually an exponent.

Emphasize that logarithms are merely exponents. The logarithm of a number is simply the power to which the base is raised to get the number.

Drill on the changing from the exponential to the logarithmic form. For example, change from the exponential form $2^3 = 8$ to the logarithmic form $\text{Log}_2 8 = 3$.

In order to fix the concept of a logarithm, have pupils state orally the powers of 2 in logarithmic form.

Drill on each of the three steps in exercise C for finding $\underline{x}$. It is very important that students are able to state such answers quickly by inspection.

When beginning the study of the characteristic, develop a table of the powers of 10, with spaces between the lines, from at least the third power to the minus third power, using both the exponential and logarithmic forms. For example:

$$10^3 = 1000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$10^{-1} = .1$$

$$10^{-2} = .01$$

$$10^{-3} = .001$$

Have class understand that a logarithm consists of two parts, a whole number and a decimal. The whole number, or characteristic, is found by inspection. The decimal part, called the mantissa, is taken from tables.

From the table of powers of 10 the class should see that a given number greater than one has a positive logarithm, while a number less than one has a negative logarithm. At this point drill on finding the characteristic of the log of a number.

Explain that for convenience, to avoid negative logarithms, we use a positive logarithm followed by a -10, thus if

$$10^{-1} = .1, \text{ then } \log .1 = -1$$

$$\text{or } \log .1 = 9.0000-10$$

$$\text{if } 10^{-2} = .01, \text{ then } \log .01 = 8.0000-10$$

When the student understands the meaning of characteristic and how to derive it,

he should be taught the use of the table of mantissas. Practice with numbers of three digits should be followed by numbers of two, one, and four digits.

When beginning the study of interpolation, the students should be taught to obtain tabular differences by inspection and to interpolate without the use of proportional parts. After this fundamental method is well understood, four or five place tables giving proportional parts may be explained and used.

Explain that the number corresponding to a given logarithm is called the antilogarithm of the logarithm.

Explain how to find antilogarithms with and without interpolation.

The following suggestions will prove helpful in the study of operations with logarithms:

All the operations with logarithms should be preceded by a brief review of operations with exponents.

"Set up" the problem before looking up any logarithms.

After the class has a good concept of the four operations with logarithms, the teacher should explain cologarithms, but their use is optional.

Before applying logarithms distinguish between:

$$\begin{array}{ll} \log (2a) = & 2 \log a = \\ \therefore 2a = & \therefore \log a = \\ \text{and } a = & \text{and } a = \end{array}$$

After logarithms have become understood, apply their use in mathematics, business, and science.

Evaluation:

The pupil should:

Understand that logarithms are exponents;
 be able to give the logarithms of numbers to bases other than base 10;
 be able to use logarithms to facilitate arithmetic operations of multiplying, dividing, taking roots of numbers and raising numbers to certain powers;
 see and feel the need for logarithms in mathematics without the feeling that logarithms are more time consuming and more difficult than the actual arithmetical computation.

TOPIC IX--NUMERICAL TRIGONOMETRY

<u>Content</u>	<u>Teaching suggestions</u>
A. Trigonometry as an aid in indirect measurement	Call attention to the very large number of measurements our modern life requires and lead pupils when listing some of these to see that two ways of measuring are used: direct and indirect. Then bring out the fact that trigonometry is especially helpful in indirect measurements in:
B. The tangent of an angle	
1. Tangent value	
2. Problems	
C. The sine of an angle	a. Navigation b. Astronomy c. Surveying d. Building, etc.
1. Sine value	
2. Problems	
D. The cosine of an angle	

TANGENT

Review briefly ratio and similarity of triangles to link "the unknown to the unknown".

Have students measure carefully the sides and angles of right triangles and compute the numerical values of ratios representing the tangent.

Show the natural tangent values and then the logarithmic tangent values.

Clarify the angles of elevation and depression.

The teacher should make clear that the ratios of sine, cosine, and tangent

are defined in terms of right triangles. Then he should point out that the trigonometric tables make it possible to obtain the value for any angle in the triangle.

SINE

A drawing is most helpful in problem solving. Follow a plan similar to finding the tangent, making clear that the sine is simply another ratio obtained by studying the relationship of the side opposite the angle to the hypotenuse.

COSINE

The cosine ratio may be derived in the same manner as the tangent and sine--showing both natural and logarithmic values. It is advisable to have pupils learn these three functions--tangent, sine, and cosine.

Much drill should be given the three common angles--thirty, forty-five, and sixty degrees.

Field trips in which these three functions are used such as to calculate heights of flagpoles, buildings, etc., should prove most valuable as teaching aids.

Evaluation:

The pupils should:

- Know how the tangent, sine, and cosine of angles are found;
- memorize the definitions of the three functions--tangent, sine, and cosine;
- understand the vocabulary of the unit;
- be able to apply the principles learned in solving practical problems by use of indirect measurement.

TOPIC X--PROGRESSIONS

Content

- A. Progressions as an interesting and

Teaching suggestions

This unit helps us to see the rhythm, harmony, symmetry, and the pleasing

useful device
of mathematics

- B. Arithmetic progressions
1. The n-th term
 2. Arithmetic means
 3. Sum of n terms
 4. Practical applications

- C. Geometric progressions
1. The n-th term
 2. Geometric means
 3. Sum of n-th terms

- D. Infinite geometric progressions
1. Sum to infinity
 2. Repeating decimals

- E. The binomial theorem

- F. Expanding binomials
1. Writing the r-th term

- G. Applications of the binomial theorem
1. Construction of compound interest tables
 2. Problems in compound interest
 3. The normal distribution curve
 4. Law of Probability

- H. Negative and fractional exponents

quality that mathematics possesses. This has already been observed in the conic sections and in the polygons of geometry and further observation is made in the study of series. There is practical value in the study of series. Installment buying, calculating annuities, insurance statistics, etc., are examples where series of one kind or another are used.

In applying for positions of various types after leaving school, pupils will likely be confronted with tests containing numerous problems based upon progressions. When they work certain practical problems they will find it necessary to deal with sequence of numbers in which each number is related in a definite way to the number which precedes it. Four kinds of series are treated-- arithmetic, geometric, infinite, and binomial.

Now develop with the class illustrations of different types of series, such as:

$$a. \quad 5 + 10 + 15 + 20 + \dots = S$$

$$b. \quad S = 1 + 8 + 27 + 64 + \dots$$

$$c. \quad S = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots$$

$$d. \quad S = x + 2x + 4x + 8x + \dots$$

Some of the pupils may be encouraged to compose some series of their own where the differences are negative, fractional, and decimal, and the ratios are integral, fractional, negative, irrational, etc.

ARITHMETIC PROGRESSIONS

Show the class how to derive the formula:

$$l = a + (n - 1)d$$

Assign practice exercises using the formula.

Have class insert one, two and then five arithmetic means between the numbers 4 and 16, such as

4			10			16
<hr/>						
4		8		12		16
<hr/>						
4	6	8	10	12	14	16
<hr/>						

For the first series of numbers, show the class that

$$M = \frac{A + B}{2}, \text{ where } M \text{ stands for the arithmetic mean and } A \text{ and } B \text{ for the first and last terms respectively.}$$

For other series, point out that we may use the formula

$$l = a + (n - 1)d$$

Develop the formula

$$S = \frac{n}{2} (a + l)$$

Practical applications of arithmetical progressions may be shown by falling bodies, interest and banking, the striking of clocks, potato races, salaries with regular increases, etc.

Progressions offer a rich opportunity to link mathematics with important social economic situations.

The following problem might be cited as an example of the above:

A man is offered a home at \$9,500 on the following terms:

\$1,500 cash, semi-annual payments of \$500, and, at the end of each half year, the interest at the rate of 5% a year on the debt during that half year.

- (a) Prepare a table showing the interest and the total payment due at the end of each of the first three half years.
- (b) What will his payments be at the end of the tenth year?
- (c) What will be the total of his payments when he has completely paid for his home?

GEOMETRIC PROGRESSIONS

Show class how to derive the formula:

$$1 = ar^{n-1}$$

Assign some exercises in which the student will need to resort to the use of logarithms.

Have class insert a geometric mean between 4 and 16. Then show them how the formula $x = \pm\sqrt{ab}$ may be used to find geometric means. Prior to introducing the formula for the sum of N terms, the teacher may tell one of the following stories:

- a. As I was going to St. Ives
I met a man with seven
wives, each wife had seven
sacks, each sack held seven
cats, each cat had seven
kits. Kits, cats, sacks,
man and wives, how many
were going to St. Ives?
- b. To reward a faithful minister
a friend offered to gratify
any wish made by the minister.
Either to sober the king or to
be malicious, the minister
made this seemingly modest
request: One grain of wheat
for the first box on a
checker board, two grains
for the second box, four grains
for a third box and so on,
successively doubling the
amounts until 64 boxes are
exhausted. The king foolishly
scoffed at what seemed to him

a modest request, but, in reality, there was not enough wheat in the whole world to meet the request.

Develop the following formulas:

$$S = \frac{a - ar^n}{1 - r} \quad \text{and} \quad S = \frac{a - rl}{1 - r}$$

INFINITE GEOMETRIC PROGRESSIONS

Derive the formula:

$$S = \frac{a}{1 - r}$$

Show by actual addition of decimals that the sum of 1, 1/2, 1/4, 1/16, etc. approaches 2 as the number of terms indefinitely increases:

$$\begin{array}{r} 1.0000000 \\ .5000000 \\ .2500000 \\ .1250000 \\ .0625000 \\ .0312500 \\ .0156250 \\ .0078125 \\ \hline 1.9921875 \end{array}$$

Have class change a number of fractions to decimals such as:

$$8/9 = .8888... \quad 8/15 = .5333...$$

$$5/11 = .4545... \quad 1/7 = .1427...$$

Have class change repeating decimals back to fractions by use of the formula:

$$S = \frac{a}{1 - r}$$

THE BINOMIAL THEOREM

Have pupils develop, by actual multiplication, the first five powers of $(a + b)$.

Arrange coefficients of the successive expansions of $(a + b)$ in the configuration known as "Pascal's Triangle."

1	$(a + b)^0$
1 1	$(a + b)^1$
1 2 1	$(a + b)^2$
1 3 3 1	$(a + b)^3$
1 4 6 4 1	$(a + b)^4$

The student should note:

- a. That the first and last numbers of each row is unity;
- b. That each coefficient in this arrangement is equal to the sum of the two coefficients which are in the line above and nearest to the right and left of it;
- c. That any coefficient of any term after the first term may also be determined by means of the coefficient of the term just preceding according to the following rule: Multiply the coefficient of the preceding term by the exponent of A in that term and divide the product by the number of that term;
- d. That the first term is A^n ;
- e. That from any term to the next following term, the exponent of A decreases by 1 and the exponent of B increases by 1;
- f. That the sum of the exponents of A and B in each term in the expansion is always N;
- g. That the number of terms in the expansion of $(a + b)^n$ is $n + 1$;
- h. That the coefficients of terms equidistant from the ends are the same;
- i. That when B is negative, the terms are alternately plus and minus.

EXPANDING BINOMIALS

Write the expansion of $(a + b)^n$ where N has successive values as 6, 7, 8, 9, etc.

Substitute a number for either a or b and then expand.

Let a and b take on exponents and finally coefficients.

Write the expansion of $(a + b)^n$, etc.

Writing the R^{th} term:

- a. Write the expansion of $(a - b)^5$
- b. Write the first five terms of $(a - b)^n$.
- c. Write the r-th term of $(a - b)^5$.

Applications of the binomial theorem:

Develop a few such tables giving the compound amounts of \$1 at different rates of interest up to five years.

Show the difficulty of this calculation by arithmetic and the facility of it by the binomial theorem method.

Develop and apply the formula for compound interest,

$$A = p(1 + r)^n$$

Applications of the binomial theorem to the normal distribution curves and probability is recommended as enrichment material.

Explain that the ordinates of the points on the normal distribution curve are the coefficients in the binomial expansions of $(a + b)^n$.

Evaluation:

The pupil should:

Be able to find the law which is used to form different series.

Be able to solve accurately and rapidly problems based on arithmetical progressions.

Be able to solve accurately and rapidly problems based on geometric progressions.

Be able to expand a binomial $(a + x)^n$ when N is a positive integer.

Be able to find " r ", any term in the expansion of a binomial.

Be able to change fractions to decimals.

Be able to change repeating decimals back to fractions.

TOPIC XI--EQUATIONS OF HIGHER DEGREE

<u>Content</u>	<u>Teaching suggestions</u>
A. Language	
1. Rational function	Many second courses in algebra do not include this subject, but capable pupils should have an opportunity to learn to solve equations of higher degree. Review quadratic equations and equations that are quadratic in form.
2. Integral function	
3. Degree	
B. Remainder theorem	Clarify the language and definitions to be used.
C. Synthetic division	
D. The factor theorem	Practice on the use of the remainder theorem to find the remainder and prove it for a few exercises by long division. The "short-cut" should be eagerly accepted by the students. Allow for further drill on the use of the theorem.
E. The cubic function	
	Show that synthetic division is a short form of division which can be used when the divisor is a binomial. A few examples worked by both long and synthetic division should arouse interest in the short form.
	When pupils understand synthetic division and the remainder theorem, it should be easy for them to understand and use the factor theorem.
	When studying the cubic function, the

values of Y may be found by direct substitution, but pupils should be encouraged to use synthetic division to make table of values. The general shape of the graph of a cubic function should be clear to students. This may be a good time for a cumulative review of graphs of various functions, starting with the very simplest and building to this or even a quartic function as a climax.

Evaluation:

The pupil should:

Understand the vocabulary involved;
know and be able to use the remainder and factor theorems; be able to use synthetic division, computing mentally; recognize the shape and be able to graph the cubic and quartic functions.

TRIGONOMETRY

INTRODUCTION

Trigonometry is a mathematics course designed for such purposes and uses in engineering, physics, navigation, astronomy, etc. It is, therefore, not just another mathematics course as far as mathematics is concerned but it offers practical applications for many other courses and professions. Thus, it is intended to be an elective course for those pupils who have these purposes.

Since most colleges offer trigonometry, high schools who feel their mathematics curricula overcrowded or understaffed may afford to eliminate trigonometry from their programs. However, it is suggested that it be dropped only in schools where such a situation exists, because trigonometry is a good final course in high school. It requires extensive use of algebra and consequently provides a final review of algebra through trigonometry.

As in other mathematics courses, it is easy to teach here with much formalism. There is a possibility of many formulas and problems in the solving for angles. Yet, it seems much more important that the student grasp the fundamental meanings of trigonometry rather than a collection of formulas for solving "classroom problems." Extensive use should be made of field trips in which the principles of trigonometry are applied to everyday practical problems.

Trigonometry is divided into the following eight topics:

- I. The Language of Trigonometry
- II. Solution of Right Triangles
- III. Use of Tables and Interpolation--Solving Right Triangles by Logarithms
- IV. Measurement of Angles
- V. General Properties of Trigonometric Functions
- VI. Graphs of Trigonometric Functions
- VII. Identities and Equations
- VIII. Solution of the General Triangle

TOPIC I--THE LANGUAGE OF TRIGONOMETRY

<u>Content</u>	<u>Teaching suggestions</u>
A. Historical sketch	Make use of the library by having the students trace briefly the development of trigonometry.
B. The trigonometric functions	Point out clearly that a function of an angle is a ratio and can be expressed as a decimal fraction.
1. sine of an angle	Have pupils use the complete definitions for the functions.
2. cosine of an angle	After the complete meaning is understood, abbreviations may be used to save time.
3. tangent of an angle	Students should be able to derive the value of the function of each angle of a 30° - 60° - 90° triangle and each angle of a 45° - 45° - 90° triangle.
4. cotangent of an angle	It will become helpful to associate the $\sqrt{2}$ with a 45° angle, and $\sqrt{3}$ with 30° and 60° angles. Then the student should memorize the value of the given functions but he should at all times be able to derive their values.
5. secant of an angle	Study functions of the complement of the acute angle showing the co-functional relations:
6. cosecant of an angle	$\sin A = \cos B = \cos (90^{\circ} - A)$ etc.
C. Functions of special angles	Develop with the class the three Pythagorean relations. Have the students transform these equations into other forms.
1. functions of 0°	Show how the function hexagon can be used to recall important relationships.
2. functions of 30°	
3. functions of 45°	
4. functions of 60°	
5. functions of 90°	
D. Co-functional relations for acute angles	
$f(A) = \text{co-function } (90^{\circ} - A)$	
E. Inverse relations	
1. $\sin A = \frac{1}{\text{cosecant } A}$	

$$2. \text{ cosine } A = \frac{1}{\text{secant } A}$$

$$3. \text{ tangent } A = \frac{1}{\text{cotangent } A}$$

F. Pythagorean relations

$$1. \sin^2 A + \cos^2 A = 1$$

$$2. \sec^2 A - \tan^2 A = 1$$

$$3. \csc^2 A - \cot^2 A = 1$$

- a. Show that the functions at the end of any central diagonal are reciprocals of each other.

$$\text{e.g. } \tan A = \frac{1}{\cot A}$$

- b. Any function is the product of the two functions between which it lies on the hexagon.

$$\text{e.g. } \tan A = \sin A \sec A$$

- c. Any function may be expressed as a fraction in which the numerator is either of the two functions next to it on the hexagon, and the denominator is the one next beyond.

$$\text{e.g. } \tan A = \frac{\sin A}{\cos A}$$

- d. The product of the functions at any three alternate vertices of the hexagon is 1.

$$\text{e.g. } \tan A \cos A \csc A = 1$$

TOPIC II--SOLUTION OF RIGHT TRIANGLE

Content

A. Exact and approximate numbers

B. Solving the right triangle with the natural functions

Teaching suggestions

It is well to review here approximate numbers and extend the concepts involved.

Point out that all measurements are approximate, and that an exact number is obtained by the process of counting.

Define significant digits as follows:

- a. Any non-zero digit is significant.
- b. Any zero used merely as an aid in placing the decimal point is not significant.
- c. All other zeros are significant.

Show what is meant, by precision and accuracy.

Have students round off answers to the degree of precision of the given data.

Emphasize the fact that accuracy of measurement cannot be improved by computation.

Use the natural functions to solve right triangles.

Let the given sides contain not more than three digits, and postpone interpolation by using the next smaller number of degrees and minutes.

TOPIC III--USE OF TABLES AND INTERPOLATION--SOLVING RIGHT TRIANGLES BY LOGARITHMS

<u>Content</u>	<u>Teaching suggestions</u>
A. Squares and square root	Review the use of logarithms here as they have been taught previously. Renew the basic concepts of logarithms stressing the idea that logarithms are exponents.
B. Natural trigonometric functions	
C. Logarithms	Drill pupils on interpolation with each of the tables. Emphasize the fact that interpolations are at best approximations.
D. Logarithms of trigonometric functions	

E. Solution of right triangles using logarithms

Show class that the idea of interpolation is based upon the principle of "proportion."

Show that the table of logarithmic functions is a combination of a table of natural functions and a table of logarithms. The slide rule may be used as a check.

In the solution of right triangles with logarithms have the students derive the logarithmic formula from the natural formula and then proceed to the computation.

Insist that the work be orderly and neatly arranged in columns.

Show the advantage of setting up the "frame work" for the logarithms before turning to the tables.

A wide variety of practical problems arising in surveying, navigation, and field work should be used.

TOPIC IV--MEASUREMENT OF ANGLES

<u>Content</u>	<u>Teaching suggestions</u>
A. Generating an angle	Rotate a line about a fixed point. If the direction of rotation is counter-clockwise, the angle is positive; if the rotation is clockwise, the angle is negative. The first position of the rotating line is called the initial side; the final position is called the terminal side of the angle. Illustrate by the use of the hands of the clock.
1. The positive angle	
2. The negative angle	
B. The measurement of angles	
1. The degree-minute-second system	Review the tables for trigonometric functions; emphasize 30° , 45° , 60° , and 90° angles. Students should be able to sketch these special angles with fair precision.
2. The radian system	

3. The mil

The student should be able to convert seconds to the decimal part of a minute. He should also be able to convert minutes and seconds to the decimal part of a degree. The decimal form is preferred today in such work as land surveying.

Illustration:

$$40^{\circ}33'18'' = 40^{\circ}33.3' = 40.555^{\circ}$$

Pupils should be able to express any angle in any of these three forms.

Define the radian.

The Armed Forces make much use of the mil for measuring angles. The students should learn and convert from mils to degrees and from degrees to a mil.

Define a mil as $1/6400$ of a revolution. Thus, $1600 \text{ mils} = 90^{\circ}$

To give a pupil a general concept of the size of a radian, draw in a circle a central angle of 60° . This will have a chord equal to the radius. Since the arc subtended by one radian must be equal to the radius, imagine the chord to be pliable and fitted to the arc. The student should see that this would have the effect of slightly lessening the angle. The student should conclude, therefore, that one radian is slightly less than 60° .

TOPIC V--GENERAL PROPERTIES OF TRIGONOMETRIC FUNCTIONS

Content

Teaching suggestions

A. Trigonometric functions of the general angle 1. Reduction of the function of any	Show how the functions vary in sign as the angle rotates from 0° to 360°. Show that each function is negative in two quadrants and positive in two quadrants.

general angle to the function of an acute angle

2. Change the sign of the function as the angle rotates from 0° to 360° .

B. Functions of useful special angles

1. First quadrant angles of $30^\circ, 45^\circ$, and 60° .
2. Second quadrant angles of $120^\circ, 135^\circ$, and 150° .
3. Third quadrant angles of $210^\circ, 225^\circ$, and 240° .
4. Fourth quadrant angles of $300^\circ, 315^\circ$, and 330° .

C. Transformation formulas

1. Functions of $90^\circ \pm A$
2. Functions of $180^\circ \pm A$
3. Functions of $270^\circ \pm A$
4. Functions of $360^\circ \pm A$
5. Functions of $n \cdot 90^\circ \pm A$

The student should be able to reduce the function of any angle in terms of the function of an acute angle.

Show that quadrantal or boundary angles are $0^\circ, 90^\circ, 180^\circ, 270^\circ$ and 360° . In the first quadrant the special angles are $30^\circ, 45^\circ$, and 60° .

Teach pupils to apply the tables to angles of any quadrant, or to use the general angle.

Find functions of $90^\circ \pm A, 180^\circ \pm A, 270^\circ \pm A$.

To show the derivation of the formula $\sin(90^\circ + A) = \cos A$, select the angle A about 20° and locate the angle $(90^\circ + A)$; on the terminal sides of these angles, locate points, P and Q , equidistant from the origin O , for convenience.

Draw the ordinates of these points RP and SQ ; place signs on the co-ordinates; apply the definition of the sine of $(90^\circ + A)$ to the figure and, using the principle of equal parts of congruent triangles, show that,

$$\sin(90^\circ + A) = \frac{SQ}{OQ} = \frac{OR}{OP} = \cos A$$

The other functions are derived in like manner. By the use of other such figures the functions of the other angles are converted into functions of A .

In the various texts the teacher may find such summarizing formulas as the following. These will be very helpful to students in remembering the relationships of the functions of the general angle.

$$f(180^\circ \pm A) \text{ or } f(360^\circ \pm A) = f(A) \text{ with the proper sign.}$$

In every case the sign of $f(n90^\circ \pm A)$ is determined by the quadrant in which the terminal line lies. Give many exercises to fix these principles.

After the student understands the method of determining the sign of any function in any quadrant, the following device may prove helpful in quickly determining the sign of any function:

By placing the letters of the word CAST in the four quadrants, beginning with the fourth and going counterclockwise, the letter in the appropriate quadrant represents the function, and its reciprocal function, which is positive in that quadrant.

Observe that:

In Quadrant I, ALL functions are positive;
 In Quadrant II, SINE and its reciprocal, cosecant are positive;
 In Quadrant III, TANGENT and cotangent are positive;
 In Quadrant IV, COSINE and secant are positive;
 All other functions are negative.

Make a distinct figure for each angle with the initial side always on the x-axis.

Mark a point, P, on the terminal side of the angle and select convenient coordinates for P.

When a 30° - 60° - 90° triangle appears, show that convenient coordinates are $\sqrt{3}$, 1, and 2, placed in agreement with the theorem "The greater side lies opposite the greater angle, and conversely." When a 45° - 45° - 90°

triangle appears, 1, 1, and $\sqrt{2}$ are convenient. It may be necessary to review the underlying geometry.

Make figures showing the second quadrant special angles. Be careful in placing the proper signs on the coordinates and explain that the radius is positive always because it is regarded as the principle square root of the sum of the squares of the other two sides of the triangle.

In the same way, show the special angles of the third and fourth quadrants.

Emphasize that the definitions apply to the functions of every angle.

Drill until the pupils are able to give rather quickly a desired function of any of the useful special angles. Encourage pupils to turn to a diagram to figure out a relationship whenever memory leaves a question.

Summary of the method of finding the trigonometric functions of any angle A:

- a. Place A in standard position. Take a point P on the terminal side, and complete the triangle of reference DOP, marking each side with its proper algebraic sign.
- b. From the given value of A determine the value of the related angle DOP, by finding the positive difference between the numerical value of A and the nearest multiple of 180° . This value should always lie between 0° and 90° .
- c. Find the numerical value of any function of A by using the table to determine the numerical value of the corresponding function of the related angle of A.

- d. The algebraic sign of the function of A will then be determined by the definition of the function and the signs of the sides of the triangle of reference. The result will be the signed value of the required functions of A.

TOPIC VI--GRAPHS OF TRIGONOMETRIC FUNCTIONS--INVERSE FUNCTIONS

<u>Content</u>	<u>Teaching suggestions</u>
A. The graphs of the functions	Graphing of the trigonometric function is applicable at any time during the course.
1. with intervals of 30°	This topic gives good opportunity to study the ways in which the change in an angle's size produces corresponding changes in its trigonometric functions.
2. with intervals of 10°	It is well to use the same scale for ordinates and abscissas. The curves may be plotted using points at intervals of 30° . This serves as a drill on the functions of the special angles.
3. periodicity	A second method makes use of points at intervals of 10° , with the aid of a table of natural functions. This locates more points and thus leads to more accurate curves.
4. maximum and minimum values	A third method is purely geometric and makes use of a unit circle and the line representations of the functions. To do this quickly and skillfully, a drawing board and a T-square are helpful.
5. use of graphs of trigonometric functions in wave motion	Use the graphs of the functions to teach the changes of value of the functions, periodicity, and their maximum and minimum values in each of the quadrants.
B. Inverse functions	Use the graph to point out related
1. Terminology	
a. inverse sine x	
b. anti-sine x	
c. arc sine x	
d. $\sin^{-1} x$	
2. Change inverse functions to direct functions	
3. Multiple values of any function	

C. Polar coordinates

and equivalent functions. The sine and cosine curves indicate clearly the usefulness of these functions in the mathematics of wave motion.

The inverse functions may not be advisable for all pupils. The students who will continue the study of mathematics or the physical sciences in college will need a complete understanding of the symbolism at least.

Use familiar illustrations (radian or degrees) to impress firmly the inverse notation with reference to the common direct notation:

For example, the direct function

$$\sin \frac{\pi}{6} = \frac{1}{2} \text{ is written as } \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

in the inverse function.

Point out clearly that while the direct trigonometric functions are single-valued, the inverse functions are multi-valued. The smallest possible value is called the principal value.

Show class how to find as many multiple values of any function as may be desired from the principal value of that function. Such formulas are called general values. Thus, if $\tan k = m$, then $\arctan m = k + n\pi$, when n is an integer, positive, negative, or zero.

Polar coordinates may be studied as enrichment material.

The origin of the peculiar inverse symbolism may create interest and at the same time clarify the important distinction between trigonometric and algebraic functions.

Although exponential notation of algebraic equations and the inverse notation of trigonometric equations

seem similar, the pupil must recognize that they are not mathematically equivalent. For example,

Algebra

If $s \cdot x = k$ then $x = s^{-1}k$;

Again, $s \cdot s^{-1}x = x$.

But while $s^{-1} = \frac{1}{s}$

And while $s = \frac{1}{s^{-1}}$

Trigonometry

If $\sin x = k$ then $x = \sin^{-1}k$;

Also, $\sin(\sin^{-1}x) = x$.

Note that $\sin^{-1}x \neq \frac{1}{\sin x}$

Note that $\sin x \neq \frac{1}{\sin^{-1}x}$

TOPIC VII--IDENTITIES AND EQUATIONS

<u>Content</u>	<u>Teaching suggestions</u>
A. Fundamental identities	Students should have a solid understanding of the fundamental trigonometric relationships before identities are introduced.
1. $\sin^2 A + \cos^2 A = 1$	
2. $\sec^2 A - \tan^2 A = 1$	
3. $\csc^2 A - \cot^2 A = 1$	Establish difference between a trigonometric equation" and a "trigonometric identity."
4. $\tan A = \frac{\sin A}{\cos A}$	When selecting identities, keep in mind that there are three chief values of their derivation or proof:
5. $\cot A = \frac{\cos A}{\sin A}$	a. To attain skill in the algebraic manipulation of the trigonometric formulas;
6. $\tan A = \frac{1}{\cot A}$	

$$7. \sec A = \frac{1}{\cos A}$$

$$8. \csc A = \frac{1}{\sin A}$$

B. Miscellaneous identities

C. Functions of special angle relationships

1. sum or difference of two angles

2. twice an angle

3. half an angle

D. Trigonometric equations

b. to be able to reduce a trigonometric expression from a given form to an identical one which is more convenient;

c. to develop the ability to handle the identities of advanced mathematics.

When both sides have been reduced to the same expression, the identity is proved. There are two accepted ways for accomplishing this:

a. Reduce either side to the identical form of the other.

b. Reduce each side independently to a third identical term.

When solving trigonometric equations it is best to transform every function in terms of one trigonometric function. After the algebraic solution is obtained, a check should be made to find valid angles and discard impossible results.

When study of identities and trigonometric equations has been completed, the student should be able to identify an identity and a trigonometric equation from a group containing both.

The geometric method is preferred by most pupils:

Draw a right triangle with one of its sides unity, according to the function in terms of which the other functions are to be derived. When other functions are expressed in terms of the sine or of the cosine, the hypotenuse, or distance, is unity; in terms of the tangent or of the secant, the adjacent side or the abscissa is unity; in terms of the cotangent or of the cosecant, the ordinate or the opposite is unity. The other sides of the right triangle, or the other co-

ordinates are easily placed in terms of the desired function. By applying the fundamental definitions, the desired functions come directly from the figure.

The algebraic method consists of a series of transformations by means of some of the standard identities. Pupils who are skillful in such transformations in algebra will experience little difficulty in the trigonometric set-up.

TOPIC VIII--SOLUTION OF THE GENERAL TRIANGLE

Content

Teaching suggestions

- A. The law of sines
- B. The law of cosines
- C. Formulas for functions of sum and difference of two angles
 - 1. $\sin (x \pm y)$
 - 2. $\cos (x \pm y)$
 - 3. $\tan (x \pm y)$
 - 4. $\cot (x \pm y)$
- D. The double angle formulas
- E. The half angle formulas
- F. Formulas for the sum and difference of the functions of two angles
 - 1. $\sin x \pm \sin y$
 - 2. $\cos x \pm \cos y$

All formulas in this unit are used to solve the oblique triangle. The teacher should take considerable time in the derivation of the formulas and should provide many practical applications.

The law of sines is used to solve a triangle with: (a) two given angles and any side and (b) two sides and the angle opposite one of them. The teacher should take care to avoid giving values which lead to the ambiguous case in (b).

An additional use for the law of sines comes later. This occurs when two sides and the included angle are given and the third side is to be found by logarithmic solution. The two angles not given are previously computed from the law of tangents.

The law of cosines is easily derived by drawing an altitude to one of the sides. The Pythagorean theorem is then applied. Simple algebraic and trigonometric transformations and reductions give the law of cosines,

3. $\tan x \pm \tan y$

4. $\cot x \pm \cot y$

5. Identities involving the sum and differences of the functions of two angles

6. An identity useful in checking triangles

G. The law of tangents

H. The functions of the half angles of a triangle

1. $\sin \frac{1}{2}A = \frac{(s-b)(s-c)}{bc}$

2. $\cos \frac{1}{2}A = \frac{(s)(s-a)}{bc}$

3. $\tan \frac{1}{2}A = \frac{r}{s-a}$

I. The computation of the angles of a triangle when the three sides are given

J. The area formulas

$$a^2 = b^2 + c^2 - 2bc \cos A$$

As can be seen, the law of cosines is not adapted to logarithms. All the problems to be solved by it, therefore, should be confined to triangles whose given sides are expressed by one or two digits; otherwise, laborious multiplications will be necessary.

The law of cosines is used to solve a triangle with: (a) two given sides and the included angle to find the third side and (b) three given sides to find the angles of the triangle.

Triangles whose sides are one- or two-digit numbers solved by the law of cosines give much satisfaction to students and are thoroughly worthwhile, even though the logarithmic method using the functions of half angles is taught later. This case is easily checked by the relation,

$$A + B + C = 180^\circ$$

For convenience, it is suggested that $x + y$ lie in the first quadrant when deriving the formulas for the function of the sum and difference of two angles. The generalized proof should not necessarily be required of all pupils.

The chief objective in the study of the formulas for the sum and difference of the functions of two angles is to attain skill in the application of standard formulas to transformations and reductions. Thus, to prove

$$\frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$$

Simply convert cotangents into tangents and the left member is easily transformed into the right member. Exercises which merely transform one complication into another are generally valueless.

The formula, $a \sin \frac{1}{2}(B - C) = (b - c) \cos \frac{1}{2}A$, contains all six parts of a triangle. It is easily applied logarithmically and makes an excellent check to any set of computed parts.

The analytic derivation of the law of tangents is generally preferred to the geometric derivations.

When two sides and the included angle of a triangle are given, the law of tangents is usually employed, logarithmically, to find the required angles. It is advisable to avoid negatives by placing the larger parts before the smaller parts. After the two angles are thus computed, the law of sines is used to find the third side.

To find the formula, $\sin \frac{1}{2}A$, the transformed law of cosines,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

is substituted into the formula,

$$2 \sin^2 \frac{1}{2}A = 1 - \cos A$$

Algebraic transformation followed by the substitution, $a + b + c = 2s$, readily produces the formula desired. For the formula for $\cos \frac{1}{2}A$, start with the formula $2 \cos^2 \frac{1}{2}A = 1 + \cos A$. The derivation proceeds similar to that above.

To derive the formula for $\tan \frac{1}{2}A$, divide $\sin \frac{1}{2}A$ by $\cos \frac{1}{2}A$ and $\tan \frac{1}{2}A$ is obtained. The formula is then made homogeneous and takes the form,

$$\tan \frac{1}{2}A = \frac{r}{s - a}$$

when

$$r = \sqrt{\frac{(s - a)(s - b)(s - c)}{s}}$$

The students will perhaps be interested in the geometric meaning of $(s - a)$, $(s - b)$, $(s - c)$, s , and r and their derivation from the triangle with its inscribed circle and the related circle.

The law of cosines gives an easy solution by natural functions when the sides are all one- or two-digit numbers. The convenient logarithmic solution comes from the half angle formulas. The formulas for the tangents of the half angles are easier to remember and are generally preferred.

All of these formulas are derived by using the well known principle that the area of a triangle is equal to half the product of its base and its altitude. The altitude is found easily from the formula,

$$h = b \sin A \text{ or } a \sin B$$

The base, if not given, is quickly computed. Hence, it is not necessary to derive special formula for the area. Hero's formula is well known.

SOLID GEOMETRY

INTRODUCTION

This course in solid geometry introduces the pupil to spatial relations and the geometry of three dimensions by means of experiment and perspective drawing. Models should be used to introduce and help the pupil in analyzing the theorems found in these topics:

- I. Introduction
- II. Lines and Planes in Space
- III. Dihedral and Polyhedral Angles
- IV. Locus
- V. Polyhedrons
- VI. Cylinders and Cones
- VII. The Sphere
- VIII. Enrichment Topics

Emphasis is placed on spatial relations and their importance in science, industry, and art. The derivation and application of the mensuration formulas provide an opportunity to develop real mastery in the arithmetic and algebraic skills used in the solution of practical problems. Supplementary topics are suggested to stimulate the superior pupil to do research in his field of interest.

TOPIC I--INTRODUCTION

<u>Content</u>	<u>Teaching suggestions</u>
A. Relationship of plane and solid geometry	Bring out fact that plane geometry consists of points and lines which lie in the same plane, while in solid geometry the points and lines may lie in different planes.
B. Basic terms: point, line, plane, solid	
C. Constructions and perspective	Also, show that plane geometry is two-dimensional and deals with the

- D. Locus of points, lines, planes
- E. Intersections of: lines with lines, lines with planes, planes with planes
- F. Review of nature of geometric proof
 - 1. Nature of axioms, postulates, theorems
- G. Origin and history
- H. Fields of work using solid geometry

measurement of flat surfaces, while solid geometry deals in the measurement of three-dimensional objects. Plane geometry is the surface on the board; solid geometry is the room itself.

Use a weight on the end of a string to show that a point moving in a fixed direction generates a line; a line moving in a fixed direction generates a plane. Then have the pupils move the various figures of plane geometry in a fixed direction to show that a moving plane generates the solids of space.

Have pupils visualize in space the moving center of some circular object such as an embroidery hoop to demonstrate that a point has position only.

Have some modeling clay in a shallow box and with the use of wire or tooth picks demonstrate the four ways planes are determined.

Cut out a cardboard triangle, locate the center of gravity by drawing the three medians. Balance this triangle on a pencil point or a straight wire. Then have the pupils answer these questions:

- a. Is the pencil perpendicular to the three medians?
- b. Will it be perpendicular to all the lines passing through the point?
- c. When is a line perpendicular to a plane?

Locate pictures on post cards or in magazines which illustrate differences in perspective. Paste them on 7" x 10" cardboards. Project them on a screen with an opaque projector. Ask the following questions:

- a. Are parallel lines actually parallel in the pictures?
- b. How are right angles and circles portrayed?
- c. What appears to happen to parallel lines as they recede from the observer?
- d. How is depth in pictures achieved?
- e. How are vertical lines represented?

Bring out the fact that pictures of buildings are made so that the side of the building appears about one-half the length of front.

Discuss the perspective of various pictures shown on a screen.

Teach pupils to draw cubes and other geometric figures in a horizontal plane. Be sure that they observe these rules: Use vertical lines to represent vertical lines of the figure. Use slanting lines to indicate lines that recede from the observer. Use dotted lines to indicate lines which cannot be seen. Use shorter lines to indicate receding edges or lines of the figure. Use oblique or acute angles for right angles when drawing horizontal planes. Use heavier lines to indicate lines near the observer. Make horizontal line at right angles to the line of vision appear parallel.

Have pupils make drawings of the text-book theorems and originals on the relationship of lines and planes on 7" x 10" cardboard.

Again, select the best of these drawings to project on the screen with an opaque projector during the study of lines and planes. State the theorem on some of these cards

and have the pupil give the hypothesis and conclusion to the lettered figure on the screen and discuss how to prove the theorems. On others give the hypothesis and have the pupils see what conclusion can be drawn.

For enrichment the pupils could study and read blueprints to develop the ability to visualize three-dimensional objects when represented in a plane drawing.

A project might be, to have some of the pupils report on the origin and history of solid geometry along with its early contributions.

Usually pupils who take solid geometry in high school are a group with special vocational interests. Thus, it will help to motivate the study of the subject to show the use of solid geometry in these vocations.

TOPIC II--LINES AND PLANES IN SPACE

<u>Content</u>	<u>Teaching suggestions</u>
A. Perpendicular lines and planes	It is very important that pupils not learn the theorems of solid geometry as independent items. Rather the teacher should help them see relationships among several theorems. These relationships help to add meaning and to aid learning. The outline of content is intended to suggest some headings around which theorems may be grouped to emphasize relationships.
1. Conditions under which lines are perpendicular to planes	
2. Conditions under which planes are perpendicular to lines	
3. Relationship between oblique and perpendicular lines to a plane	After pupils develop the proof for such a theorem as: "If a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of two lines." They

B. Parallel lines and planes

1. Lines parallel to planes
2. Perpendiculars forming parallels

may discuss such applications as how buildings and posts or poles are made upright.

Establish the logical dependencies of the proof of this theorem and other theorems on the theorems of plane geometry.

Have pupils write the formal proof for the plane geometry theorem, "If two points are each equidistant from the ends of a segment, they determine the perpendicular-bisector of the segment."

Call attention to the fact that the same type of logical reasoning is used in proving the solid geometry theorems.

Stress the fact that axioms, postulates, and theorems in plane geometry may be used as the basis of proof in solid geometry.

As pupils progress in the study of theorems on lines and planes, have them make a table showing the analogous theorems of plane geometry thus:

PLANE GEOMETRY

SOLID GEOMETRY

Two straight lines can intersect in only one point.

Two planes can intersect in only one line.

There can be one and only one perpendicular to a line at a point on the line.

Through a given point in a plane there can be drawn one line, and only one, perpendicular to the plane.

After pupils have developed the proof for the theorem, "If two straight lines are cut by three parallel planes, their corresponding segments are proportional," have them solve numerical problems, such

as $\frac{x}{6} = \frac{2}{8}$ to maintain skill in algebra.

Relate the use carpenters, surveyors, and engineers make of the theorems and corollaries of solid geometry.

Have simple examples of the application of these theorems.

Review theorems and exercises using the opaque projector with the drawings the pupils made during the study of perspective.

Stress the importance of clear definitions and good descriptions. Review the elements of a good definition and develop definitions of key words inductively.

Have pupils define oblique, parallel, perpendicular, plumb line, carpenter's level, carpenter's square, analogous, duality, distance, skew lines, coplanar, dimension, collinear, coincide, concurrent, determine, surface, and plane.

Have pupils prepare a list of groups of theorems proving two lines parallel, two planes parallel, a line perpendicular to a plane, a line parallel to a plane, and two line segments equal.

Allow pupils to work in groups to review the proofs and discuss and state examples of practical applications of these principles in their environment.

TOPIC III--DIHEDRAL AND POLYHEDRAL ANGLES

Content

A. Dihedral angles

1. Nature

Teaching suggestions

Models of various types may be used with great profit in teaching solid geometry. A pupil is well along

2. Plane angles of dihedral angles

3. Classes

a. Complimentary, supplementary, right, acute, obtuse, vertical

4. Congruency

5. Related theorems

B. Polyhedral angles

1. Nature

2. Classes

a. Trihedral, tetrahedral, etc., isosceles, convex

3. Relationships among face angles of polyhedral angles

4. Related theorems

toward the proof of a theorem when he has clearly seen the relationships involved through handling or observing a model. Several suggestions will be given in this guide for such models.

Point out examples of dihedral angles in the classroom.

Illustrate and classify the different kinds of angles formed by folding and unfolding a piece of ruled paper. Fold so that the crease of the fold is perpendicular to the lines of the paper.

Use plastic straws strung together with elastic thread or cardboard to develop an informal proof for the theorems, "The sum of the face angles of a polyhedral angle is less than 360 degrees," and "The sum of any two face angles of a trihedral angle is greater than the third."

Explain the nomenclature and terms, such as similar and congruent polyhedral angles, faces, vertical polyhedral angles, etc.

Point out that the edges and faces of polyhedral angles are unlimited in extent.

TOPIC IV--LOCUS

Content

A. Locus of a point under given conditions to form a plane.

B. Locus of lines

C. Locus of planes

D. Projections of lines on planes

E. Related theorems and applications

Teaching suggestions

This might be regarded as an optional topic. It is recommended, however, because it is usually interesting to pupils and is a basic aspect of some of the mathematics which a pupil will encounter if he continues his study of mathematics beyond high school.

The study should be initiated with simple examples of loci in planes and develop visualization to three-dimensional figures, for example,

"The perpendicular bisector of a segment is the locus of points equidistant from the end points." Then, by analogy develop the corresponding solid geometry concept.

Have the pupils make a table similar to the following showing the corresponding loci of plane and solid geometry:

<u>CONDITION</u>	<u>PLANE</u>	<u>SOLID</u>
At a given distance from a fixed point	A circle	A sphere
Equidistant from two points	A line, perpendicular bisector of the line segment joining the two points	A plane perpendicular to the line segment joining the two points at its midpoint
Equidistant from two parallel lines	A line bisecting the plane of the two lines and parallel to them.	A plane perpendicular to and bisecting the plane of the two lines.
Equidistant from two intersecting lines	Two lines bisecting the vertical angles formed by the lines	Two planes bisecting the angles formed by the lines and perpendicular to their plane

Continue the table to show that the point-line relationship in the plane becomes the line-plane relationship in space.

PLANE GEOMETRY

CONDITIONLOCUS

Locus of points
at given dis-
tance from a line.

Two lines equi-
distant from the
given line and
parallel to it.

SOLID GEOMETRY

CONDITIONLOCUS

Locus of points
at given dis-
tance from a
given plane.

Two planes equi-
distant from the
given plane and
parallel to it.

Treat this topic in an informal way and plan to develop visualization by the use of objects and by discussion of locus problems and exercises.

Stress loci in everyday situations through the study of aerial maps, photographs, etc. Have the pupils bring into the classroom such projections.

Develop the meaning of projection by means of wire, string, etc.

Explain the projections of pictures on a screen.

TOPIC V--POLYHEDRONS

ContentTeaching suggestions

A. Parts of polyhedrons

1. Faces
2. Edges
3. Vertices
4. Diagonals

Copy and enlarge the patterns of the five regular polyhedrons to be found in many textbooks and make models from them.

Demonstrate how the solids are generated by a moving line and a geometric figure.

- B. Names by number of faces
- C. Regular polyhedrons
- D. Prisms
- E. Parallelepipeds
- F. Rectangular solids
- G. Surface area of polyhedrons
- H. Volume of polyhedrons
- I. Pyramids
- J. Truncated solids
- K. Similar polyhedrons

Make flattened out versions of the prisms, the cube, and the rectangular solids. Identify the plane geometry figures and review the area formula for each figure.

Use these models to illustrate the meaning of the definitions and to show why there can be only five regular polyhedrons.

Have pupils solve problems using these formulas.

Review the common mensuration formulas on length, area, and volume.

Help pupils develop formulas for the lateral area of the flattened out solids.

Have pupils write out a formal proof for these theorems and solve the examples in the text.

Make models from the flattened out patterns and develop volume formulas on each.

Have pupils solve many problems involving the application of these formulas. These problems should be selected so as to give practice in solving the fractional and literal equations and in simplifying radicals. Pupils may leave their answers in the simplest radical form.

Solve problems applying principles of construction, to engineering, architecture, science, etc.

Teach pupils to analyze complicated figures, take them apart and draw important parts, such as the base, a lateral face, a right section parallel to the base, etc., again in their correct proportions as though they were in the plane of the paper.

Develop the basic formulas by Cavalieri's theorem and the Prismatoid formula.

TOPIC VI--CYLINDERS AND CONES

<u>Content</u>	<u>Teaching suggestions</u>
A. Cylinders	Relate the formulas for the lateral area and volume of the cylinder to the respective formulas of the prism by means of flattened out figures and the three-dimensional models.
1. Classes	
2. Area	
3. Prisms inscribed in and circumscribed about cylinders	Show how the lateral area and volume of a cone may be developed from those of the pyramid.
4. Volume	Carefully present definitions of new terms such as axis, conical surface, cylindrical surface, directrix, generatrix, slant height, and right cylinder.
5. Similar cylinders	
B. Cones	
1. Parts	Have pupils solve numerical problems to show that the lateral area of a cone is the limit which the lateral areas of regular inscribed and circumscribed pyramids approach when the number of their lateral faces is indefinitely increased and the volume of the cone is the limit of the volume of these pyramids.
2. Classes	
3. Properties of sections	
4. Area	
5. Volume	
C. Conic sections (optional)	Make a pattern of a right circular cone which has a slant height of 6 in. and a base radius of 2 in.
	Tabulate the results and stress the fact that in similar figures areas are to each other as the squares of corresponding lines; volumes are to each other as the cube of corresponding lines.
	Use models to identify and explain the conic sections.

Have pupils report on the uses of the conic sections.

Review the equations of the conic sections. Thus:

EQUATIONS	CONIC SECTION
$x^2 + y^2 = r^2$	Circle
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Ellipse
$y = ax^2 + bx + c$	Parabola
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	Hyperbola

TOPIC VII--THE SPHERE

Content

- A. Parts of a sphere
- B. Sections
- C. Circles of a sphere
- D. Polar distance
- E. Tangents of a sphere
- F. Spheres inscribed in and circumscribed about polyhedrons
- G. Related theorems and applications
- H. Area
- I. Volume

Teaching suggestions

Associate the definitions of the properties of a sphere with those of the circle.

Ask why aviators fly the arcs of great circles when flying from New York to Paris. Use a globe to explain the answer.

Place on exhibit in the classroom a globe, a map showing the air routes around the world, a map showing the time belts on the earth's surface, and other illustrations of the application of the geometry of the sphere to the many fields of learning.

Develop the definitions of great circles, lune, zone, etc., and call attention to their use as latitude, meridians, and other measuring devices used in various phases of life such as: science, engineering, architecture, navigation, and aeronautics.

Have pupils read the proofs and understand the logical sequence of the theorems, but emphasize the numerical and algebraic application of them.

Use the globe to explain the polar distance. Have pupils work in groups and prove the theorems.

Stress the fact that the sides and angles of a spherical polygon are equal respectively in degrees to the face angles and dihedral angle formed at the center of the sphere.

Emphasize difference between congruent and symmetrical triangles.

Assist students in developing the formulas for the area of a sphere, the area of a lune, and a zone. Have them solve many problems using same.

Have pupils construct from cardboard a cone and a cylinder with the height and diameter of each equal to the diameter of a ball. Show that the cylinder requires three fillings of the cone with water to fill it. Empty the water, place the ball in the cylinder and show that it now requires only one filling of the cone to fill it. Have the pupils state the conclusions which they draw from this experiment.

In such an experiment, always emphasize that a conclusion has not been logically reached. However, this type of experimentation is excellent as a stimulation of pupil interest and as a means of clarifying understandings of the relationships discovered. Pupils must, however, be taught to see the need for a deductive proof before the conclusion can be held as a universal one.

TOPIC VIII--ENRICHMENT TOPICS

ContentTeaching suggestions

A. Photogrammetry

1. Definition

2. Application

B. Slide rule

C. Sundial

Have pupils report on topographic mapping with aerial photographs, the geometry of a vertical photograph, the use of tilted oblique photographs in making world aeronautical charts, how horizontal positions of objects are determined graphically from aerial photographs when the position of a few objects is known, how the sensation of depth is achieved, the use of photogrammetry in medicine, in traffic accident investigations, and in the reconstruction of historic buildings.

Have class construct a simple slide rule and apply the rules for multiplying and dividing with logarithms.

Have pupils construct a scale to find square roots and square numbers by applying the rules used in the work with logarithms.

Study the mathematics of the sundial and construct one on the school ground.

CHAPTER IV

DISCUSSION

It is quite evident that the outline form used in this thesis is incomplete. The omission or absence of any suggestion does not, however, constitute a deficiency. Proper references and suggestions can be added after they, too, have been in actual classroom use. Because of the many diverse conditions encountered in teaching, each teacher might experiment, and, over a period of several years, develop her own file of aids and references, suitable to her own manner of teaching and appropriate to the particular method she uses or to the unit which she may have chosen.

However, the present manual is a foundation. Teachers may use any part of it as a basis for building their own finished lessons or units. It would grow through the teachers' own initiative. It could conceivably take much time to build a good manual based on the present philosophy and objectives. If one is to be consistent, changes in both philosophy as well as content should be anticipated. It is not beyond reason and expectation, that, through constant revision and in the light of new experiences, this manual will eventually be outmoded and replaced.

To say that a manual, such as this one, is complete

and final would be inconsistent with the policy of encouraging teacher experimentation. The unfinished manual is needed. As a result, a project to motivate the cooperative effort of teachers as a professional group, stimulating professional inquiry and employing professional interests, attitudes, and skills, is introduced. The unfinished manual should be considered then in a natural yet desirable state.

A subject of paramount importance in the North Kingstown schools should be the possibility, in the very near future, of forming a curriculum study committee embracing all twelve grades. Certainly then, if not sooner, all teachers of mathematics in the system would be working to add their individual aids and suggestions to the present manual which might, in turn, serve as a nucleus for such new ideas. Thus, there might be a question in the minds of many teachers concerning the accuracy and usefulness of any suggestions added at this time. It is herewith suggested that these be added by individual teachers according to their own experience and only after their value has been proven. However, in so doing, it is possible yet dangerous to its usefulness since the manual may become too awkward and cumbersome. Thus, it has been the thought in the mind of the writer to add only those references for which previous experience has been made for their certainty.

Another deficiency in the manual, if it can be called such, is the fact that tradition and the opinions of textbooks have determined its content material and the sequence in which

the material is presented. Research should determine any deviation for anything as important as this in the field of education. This has not been done, however, since education is sometimes referred to as an inexact science. It is still largely a matter of hope and faith.

It goes without doubt that this manual will be revised and added to during its first year of operation. With this possibility in mind, the writer has prepared a copy in loose-leaf form. Certainly revisions and additions to such a manual would be greatly facilitated by such a plan. If each teacher, experienced as well as inexperienced, in the mathematics department of North Kingstown's Junior and Senior High Schools were presented such a booklet, it would be sincerely hoped that the teaching standards in said schools would be substantially raised.

While there was no curriculum study group in operation on this work, there are certain suggestions for the same which can be seen by the writer as being most profitable. Failure to recognize these could result unless certain suggestions for informal meetings of such a group are adhered to. Teachers tend to avoid attending formal meetings after school hours for curriculum study. Even if this seems to be a lack of professional attitude, it cannot be considered entirely the fault of the teachers. Truly, the majority of teachers are exhausted at the end of a school day. Special attention should be given to

group dynamics in planning any activity of this type.¹ Many teachers have been discouraged from participating in any such work for two main reasons: first, the meetings have been too long and have been a real imposition upon teachers; and second, many teachers have been turned away from curriculum work by reading many of the heavy and dull volumes written by theorists who themselves have been away from the classroom much too long. Educational studies, together with their philosophic methods of research are often explained in such a vague manner that teachers consider the study of such beyond their capabilities--something to be left to the experts.

Keep group meetings small in number, for then teachers feel more relaxed and free to talk casually. Then, there is a feeling that they are free to say exactly what they think. With such points in mind, meetings with teachers should be very informal and brief. Therefore, for such meetings to be productive, the leader must plan carefully the points he wishes to bring up for discussion. Also, it is important that he be a good conversationalist with the ability to direct the discussion in a general manner to the topics he wishes. Then, and only then, are teachers more apt to say what they really think rather than what they think is expected of them.

After much time and study had been spent on the preliminary outline of the philosophy, objectives, and subject

¹E. A. Krug, Curriculum Planning. (New York: Harper and Brothers, 1950) p.224.

matter material, the writer concluded that the most essential part of any such manual is its statement of philosophy and its objectives. However, working out and evaluating each phase of the work brought out two things: first, it made clear what was expected of teachers in teaching mathematics, and second, the need for definite aids and suggestions of methods to be used. Such suggestions could be considered as starting points, as something to be evaluated and as a basis for experimentation.

Teachers, especially new ones, often feel insecure when they do not know what is expected of them. They often do not have enough confidence in their own conclusions about teaching methods or content material. When teaching, they look for those things which are clearest to them and which are most easily understood by anyone who might have criticized them.

New or inexperienced teachers are sometimes timid about being left alone to teach in any manner they deem proper to bring about the desired objectives. Realizing the importance of their task they like to have assurance that what they do has the approval of fellow teachers and supervisors. Getting something down in black and white, something they can turn to for help, seems to be a symbol of security to them as well as creating the atmosphere for some experimentation of their own.

Thus, it can be plainly seen that the writer feels that there is a definite need for such a manual in the secondary schools of North Kingstown. However, the intent used in the building and composing of this work in mathematics has not been to limit its ideas and suggestions to this one school system,

but rather, to allow it be an incentive to other mathematics teachers who might find new ideas and aids within its pages.

CHAPTER V

RECOMMENDATIONS FOR THE PREPARATION OF A MANUAL

1. Philosophy, objectives, and suggestions are necessary in a mathematics manual. Only after a philosophy for the task has been formed can specific objectives for any part of the curriculum be stated. The most skilled and experienced teachers usually need only an understanding of these objectives and the accompanying philosophy in order to carry out their teaching assignment successfully. Inexperienced teachers or those unfamiliar with the objectives usually feel strongly a need for suggestions which will help them achieve these very objectives. A sense of insecurity or anxiety is often common among younger or newer teachers who are left to teach by themselves with only an occasional visit from a supervisor. Many teachers have stated that a curriculum manual, whatever the field may be, gives them the assurance which they need that they are teaching the requirements. When experience has brought confidence into a new teacher's personality, she seldom needs to consult the manual to any extent. Also, many older, more experienced teachers have often wondered whether or not they were on the right track. Doubtless, a complete guide with teaching suggestions such as are included here would have been of great help. As a result, it is recommended

that a manual should not be merely a short outline of philosophy or objectives. It should contain all the aids and suggestions that teachers think worthy of inclusion in the manual.

2. There is no preferred sequence in which the curriculum material is presented. Today, when the mathematics of our secondary schools is in such a questionable state, it is rather difficult to agree upon the grade level at which most content material should be presented. There are so many variables encountered in each course to say that it would be ready for any previously selected experience unit. In conclusion, teachers should be left free to select the units or topics which they consider best to meet the needs of the class at any particular point in its progress. However, the material selected by the teacher should have the same objectives and be consistent with the accepted philosophy of teaching.

3. Curriculum study, construction, and revision should be continuous. When a manual is considered finished, it ages quickly. To say that it is finished is to encourage complacency and discourage future investigation and study. Only by leaving it open to further inquiry and investigation as well as amendment can it be kept at the highest level of development and remain useful. There is always new material coming forth and this and more promising material should constantly replace older, less useful, out-of-date material, if curriculum development is to keep abreast of social development and the times. True, philosophy, objectives, and content are largely determined

by the opinions of those engaged in curriculum construction. Consequently, it is necessary to recommend frequent investigations and inquiries into the state of opinions because points of view change rapidly.

4. Changes in method and content should be gradual. Conservative teachers often look with suspicion upon any recommendations for a change in method or content which may, in turn, be considered minor. Then, too, they often reject entirely big changes. Such a condition is no doubt brought on by the loss of security felt by the experienced teachers when familiar skills and knowledges are suddenly no longer of use to them. It means a reformation to them; they must start over in much the same manner that a beginning teacher does.

Therefore, the desired level of teaching should be compromised with that presently accepted. There are two answers here: the first is to make all changes gradually and, if possible, as a result of suggestions by teachers themselves; and the second is to promote in the teachers a sense of confidence in their supervisor and principal. It can help prevent any loss of security on their part if they know that purposeful experimentation will meet with praise, and that they will be considered capable teachers regardless of methods used.

5. Constant attention should be given to the orientation of the child. Children are naturally most concerned with their immediate surroundings, environment, and present needs. They need help in discovering the contributions that they themselves can make now to the welfare of the groups of

which they, as individuals, are members. They need and want to achieve recognition as they are now, at their present state of physical and mental development. Youth can make real contributions to the welfare and success of their respective families, classes, or other groups. They are often serious about their delegated objective. Teachers and parents should reciprocate in these attitudes and the role their children or students play in family or school life.

6. Cooperative planning in the preparation of such a manual is necessary. While such activity was not utilized to any great extent in this manual, opinions and suggestions have been sought at various times from members of the department. They have often been reflected in the body of the subject matter. Very little cooperative planning was used due to the pressure of time. However, with the possibility of a curriculum study group being organized at North Kingstown, it is suggested and anticipated by the writer that this present manual serve as a nucleus for future study and expansion of ideas by a larger group. Oftentimes, a manual passed on by a supervisor or administrator to a new teacher is difficult to accept and use. Many teachers are willing to do so, but as a result, their teaching is very mechanical and subject-matter centered. Then, too, some teachers are devoted to method. They lose sight of valid objectives. It is felt that if teachers were to have worked on the manual's preparation they would necessarily consider both objectives and content.

If teachers share in the manual's preparation, they

share the responsibility for its success. They find themselves in a position of defending themselves as well as their philosophy of teaching. Personal contributions make the added work and time spent in its preparation more worthwhile. Good teachers also contribute their personalities which often serve to inspire others, to stimulate critical thinking, and to promote friendly cooperation.

7. Should the teaching of any item of subject matter or the use of any special method of teaching be identified with the attainment of any of the objectives of mathematics, the construction of a manual would be greatly simplified. It would still be necessary to state a philosophy and to determine objectives. However, the attainment of these objectives would become a more exacting science. The acquisition of certain skills and knowledges is already provided for by the inclusion of a certain amount of drill and the proper topics in the curriculum. Certainly necessary are surer, more positive and concrete methods of teaching certain attitudes, appreciations, and skills--both social as well as mathematical.

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