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THE MISSPECIFICATION OF COMPONENTS AND FACTORS

BY

JOSEPH L. FAVA

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

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ABSTRACT

Factor and component analysis are two similar statistical procedures widely-used to reduce a set of p variables to a smaller set of m variables. This reduced set of m components or factors can be interpreted as an overall pattern structure or used in the derivation of factor and component scores. A commonly occurring and potentially serious problem concerns the misspecification of the number of factors and components (m). Misspecifications can take the form of extracting too many or too few factors or components. A series of simulation studies was undertaken to determine the practical effects of such misspecifications within and between the methods of maximum likelihood factor analysis (MLFA) and principal component analysis (PCA). Computer-simulated data sets, representing baseline factor and component patterns, were generated to represent a wide range of conditions. Item saturation ($a_{ij} = .4, .6 \text{ \& } .8$), sample size ($N = 75, 150, 225 \text{ \& } 450$), and the variable to component and factor ratios ($p:m = 4:1, 6:1 \text{ \& } 12:1$) were systematically varied to create the baseline patterns prior to deliberate misspecifications.

The problem was examined from several perspectives by investigating relationships within MLFA and PCA during both overextraction and underextraction, and by investigating relationships between MLFA and PCA during overextraction, underextraction and for the correct structural patterns. Results indicated an overall degradation in the MLFA and PCA solutions during both overextraction and underextraction. Although degradation within methods occurred during overextraction, little information was lost even at maximal

overextraction for the strongest ($a_{ij} = .8 \text{ \& } .6$) pattern structures during either MLFA or PCA. By contrast, underextraction was a very serious problem with much loss of information occurring at the first underextraction and continuing with each successive underextraction. Greater degradation occurred with MLFA than PCA during underextraction. High similarity between MLFA and PCA solutions occurred for the correct pattern specifications and for the overextracted solutions. Low similarity between MLFA and PCA solutions occurred during underextraction. Item saturation was the major determinant, while sample size and variable to component (factor) ratio were lesser though important determinants, of stable pattern structures during overextraction and for correct solutions, both within and between methods. No condition of interest was found to be a consistent determiner of stable pattern structures during underextraction.

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CHAPTER I

Introduction

A problem of implicit importance to both factor and component analysis as commonly used in social science research involves the selection of the correct number of factors or components to retain. Factor and component analysis, two widely-used and competing methods of data reduction, are most often used to reduce a set of p observed variables to another new set of m variables, such that $m < p$. This reduced set of m factors or components serves to explain and replace the original set of p variables. In fact, accuracy with regard to the specification of m could be viewed as the most important result within a factor or component analysis.

Many different stopping rules have been utilized in the effort to arrive at the solution to the correct number of components to retain, but conflicting decisions are often made (Zwick & Velicer, 1982, 1986) and in a typical factor or component analysis one might end up with a range of possible solutions if one were to employ them all. Given the variability of solutions among the different stopping rules, the lack of consensus as to the best stopping rules, and the unavailability of some of the 'best' stopping rules on major statistical computer packages, it is safe to assume that misspecifications often occur in deciding upon the correct number of components to retain.

A misspecification can take the form of retaining too few or too many factors and a primary example of such misspecification often

occurs in relation to Kaiser's eigenvalue-greater-than-one rule. Kaiser's rule is quite likely to overextract components (Browne, 1968; Zwick & Velicer, 1982, 1986), has received mixed qualitative reviews, often in the same breath, e.g., "... it is sometimes a good practical guide (and occasionally very very bad)" (McDonald, 1965, p. 75), and yet it remains the most widely used stopping rule to date. Other stopping rules, such as the Percent of Variance Accounted For, Cattell's Scree Plot, etc., may also give widely variable and disparate solutions from each other when each is applied to the same problem. These examples serve to illustrate that misspecifications of components in a component analysis or factors in a factor analysis are endemic. The practical consequences of such misspecifications, however, have barely been examined.

A number of researchers have given advice concerning overextraction and underextraction, but this advice appears to be built more upon the 'folklore' attached to the techniques of factor and component analysis, rather than upon any extensive and well-organized empirical evidence. The most often-cited empirical study that dealt with this issue was conducted by Mosier (1939). This small study used a simulated 20 variable data set with a known solution of 4 underlying and orthogonal (although visually complex) factors. An oblique rotation of this data set indicated that rotating one factor too few had a more deleterious effect on the overall factor structure, compared to the overextraction of one or two factors. This simple study by Mosier seems to be the main empirical support for the most common dictum concerning the overextraction and underextraction

of factors that appears in the literature. Some examples of this dictum are "... the researcher should apparently err on the side of too many factors if he errs at all ..." (Rummmel, 1970, p. 365), "one thing is certain on both theoretical, scientific and practical computing grounds — that it pays to extract too many rather than too few factors ..." (Cattell, 1952, p. 302), "... the final conclusions of a factor analysis are likely to be more distorted by underfactoring than overfactoring" (Cattell, 1978, p. 55), "... it is better to err on the side of extracting too many factors rather than too few" (Comrey, 1973, p. 101), and "it appears that extracting too few factors decreases the replicability of those factors, whereas extracting a moderate number with one or two extra factors — and only one or two — does not" (Gorsuch, 1983, pp. 329-330).

There is a logical theoretical thread that supports the idea that overfactoring is less a problem than underfactoring. This concerns the fact that the amount of variance explained by a factor decreases for each succeeding factor extracted (Cattell, 1952, 1978; Comrey, 1973), e.g. the seventh extracted factor in a component or factor analysis will account for less variance than the sixth extracted factor. Thus, if a true solution were a six factor solution, it would be a worse error to only extract five factors than to overextract seven factors, because the information lost by the sixth factor would be greater than the error added in by a seventh factor. Underextraction is also thought to be a more serious problem than overextraction because beyond simply losing some information it is thought that the true factors cannot be accurately portrayed if

underextraction occurs (Cattell, 1958; Comrey, 1973, 1978; Mosier, 1939; Thurstone, 1947). A second theoretical thread supports the advice that it is a less severe problem to overextract factors than to underextract them, and this concerns the idea that upon rotation it is relatively easy to discard extra factors that have been retained as residual or trivial factors without changing the important or substantive factors (Cattell, 1958; Comrey, 1973).

While there is general consensus that suggests underextraction is potentially worse than overextraction, Guertin and Bailey (1970) offer the opinion that "it is the highly individualistic characteristics of the matrices that make it impossible to state as a general rule whether it is better to rotate too many or too few factors" (p. 114). And, as previously noted, while most researchers generally have agreed that merely extracting one or two extra factors would do little harm, Thurstone (1947) offered the opinion that "no matter what method of factoring is used, it is a safe rule to continue the factoring until one is sure that the factoring has gone far enough. Too many factors can do no harm ..." (p. 509). A few of the researchers who disagree with Thurstone's position have stated possible consequences that may ensue if one overextracts liberally. Cattell (1978) notes "... with substantial overextraction 'factor fission' occurs ..." (p. 168), and Gorsuch (1983) states "... that one or two extra factors do little harm but too many may cause a common factor to be missed" (p. 173).

There is also a group of less-cited and peripheral studies which lend support to the notion that the overall factor structure may

change if overextraction occurs. The abstract for a paper presented by Keil and Wrigley (1960) notes that "the initial tendency when another factor is included in the rotation is for existing factors to subdivide ..." (p. 487). Howard and Gordon (1963), while examining the 'number of factors' problem, compared the loadings obtained from an actual 37 variable data set when 5, 6 and 7 factors were extracted. They noted that with the extraction of a seventh factor a specific factor appeared, and also that the loadings of a previously strong sixth factor (as noted when only 6 factors were extracted) degraded noticeably with the introduction of a seventh factor. Dingman, Miller and Eyman (1964) using real data examined the effects of overextraction on a very strong (average loading of about .7), though small (only 9 variables, with 3 variables per factor), orthogonal factor pattern using both varimax and biquartimin rotational solutions. 4, 5, and 6 factor solutions to the data did not cause complete 'factor fission' of the original 3 strong factors, as these factors could still be reasonably observed. There was, however, some gradual deterioration of the original factors and the appearance of some unique factors, as additional factors were extracted. One other interesting finding in this study was that as additional factors were extracted the oblique biquartimin rotation preserved the original factor structure better than the orthogonal varimax rotation. A study by Levonian and Comrey (1966) examined 2 real data sets of 57 and 32 variables, respectively. As more and more factors were extracted from these data sets, a gradual factor division emerged into a near complete change in factor structure, with 'factor

stabilization' occurring as the number of factor extractions became very high, i.e. one third to two thirds the number of variables. And a study by Velicer (1977), which was primarily concerned with comparing principal component, image component, and factor patterns across nine known data sets, noted that two data sets that produced the most discrepant results among the various techniques seemed to be the result of overextraction. When the number of factors for these two data sets was reduced by one in each case, the principal component, image component, and factor patterns all became more congruent with each other.

While there have been numerous opinions proffered about the consequences of the misspecification of the correct number of factors, the hard empirical evidence to support these opinions is, as noted above, meager at best and does not cover a very wide range of possible situations. In fact, no empirical study has examined this problem in depth.

This series of studies was undertaken to examine the consequences of overextraction and underextraction on the true factor solution in a wide variety of situations by means of computer-simulated data. This effort to understand the consequences of a misspecification of the correct number of factors to retain, was also undertaken to seek out possible trends and patterns that might become apparent when misspecifications of factors and components are systematically varied. For example, is there a general degradation from the true solution across all factor patterns if one extracts too many or too few factors? Or do the strongest factor patterns remain

strong and only the weakest factor patterns degrade? Or is there no degradation at all among the true factor patterns?

As there is no strong empirical background to this problem, hypotheses could only be generally stated:

- 1) Misspecification of factors will result in some degradation from the true solution. The degradation may occur less rapidly in strong factor patterns than in weak factor patterns. The rate and extent of degradation is one of the topics of investigation in this study.
- 2) Underextraction of factors will result in greater loss of information than overextraction of factors. The rate and extent of degradation for strong factor patterns versus weak factor patterns is one of the topics of investigation in this study.

CHAPTER II

Method

There are many possible levels of variables (p), factor loadings (a_{ij}), sample size (N), components (m), and variable to component ratios ($p:m$) that could potentially be encountered by the applied researcher utilizing the techniques of factor or component analysis. While the major concern of this study dealt with the effects of the misspecification of m , an attempt was made to approximate situations that might typically confront the applied researcher. Thus, several levels of p , a_{ij} , N , m and $p:m$ were chosen to serve as population pattern benchmarks within which deliberate misspecifications of m were compared. These population pattern matrices, while varied over several levels of factors, are somewhat idealized and do not correspond precisely to data structures in the 'real world'. The resultant computer-simulated data structures used, however, do allow a retention of control that increases precision and objectivity in the analyses of the results.

The number of variables represented in these studies was held to $p = 36$. This value can be considered as representing a small to moderate-sized data set within the context of exploratory data analysis that makes use of the techniques of component and factor analysis (Zwick & Velicer, 1982, 1986). Limitations of available computer time and storage space, as well as the extensive time involved in editing output data sets, nullified the use of a second $p = 72$ condition. Probes done on some data sets utilizing 72 variables

indicated results very similar to $p = 36$ and it was felt that generalizability would not be compromised by only using $p = 36$.

Previous research (Guadagnoli & Velicer, 1988) suggested that the variable to component ratio ($p:m$) was crucial to component stability. Also, studies by Velicer and Fava (1987, 1990), which investigated the effects of variable sampling on principal component analysis, image component analysis, and factor analysis, found that the $p:m$ ratio was important. As strong and weak factor patterns are partially a function of the $p:m$ ratio, and as misspecification of factors may have a differential effect on strong versus weak factor patterns, the $p:m$ ratio was a factor of concern.

The number of components represented in this study was 3, 6, and 9. Components (m) and variables (p) were matched so as to create $p:m$ ratios of 12, 6, and 4. 12 variables per factor can be thought of as representing a very strong factor, 6 variables per factor a moderately strong factor, and 4 variables per factor representing a near minimum for factor replicability. Anderson & Rubin (1956) and Rindskopf (1984) have noted that 3 variables per factor are needed as a minimum for factor replicability, a result confirmed empirically by Velicer and Fava (1987, 1990).

Factor loadings (component saturation) were examined at values of .4, .6 and .8. This represented a very strong loading (.8), a moderate loading (.6), and a borderline limit for an acceptable loading (.4). These loadings were representative of typical values encountered in applied situations. Further, high loadings are important in determining strong factor patterns and, conversely, low

loadings are an important determinant of weak factor patterns. This variable was thus important in helping to create a range of factor patterns from strong to weak, which are of concern to the applied researcher.

Sample size was manipulated under conditions of $N = 75, 150, 225$, and 450 . For most multivariate studies a sample size of 75 is considered to be fairly low and may produce unstable results. Sample sizes in the range of 150 to 225 are thought to produce stable results for many multivariate purposes (Boomsma, 1982; Guadagnoli & Velicer, 1988; Velicer & Fava, 1990; Velicer, Peacock, & Jackson, 1982). And a sample size of 450 is thought to produce very stable results in most situations.

Data Generation

The actual data matrices to be analyzed were constructed in the following manner. First, population matrices were constructed for the different p , m and a_{ij} combinations using a procedure previously employed by Zwick and Velicer (1982, 1986), Velicer, Peacock and Jackson (1982), Guadagnoli and Velicer (1988), and Fava and Velicer (in press). This involved creating a population pattern matrix (A) for each possible p , m , and a_{ij} combination. Table 1 gives an example of an A matrix, when $p = 36$, $m = 6$, and $a_{ij} = .6$. This $p \times m$ matrix, A , was then post-multiplied by its transpose, thus creating a new matrix R^* ($p \times p$), where

$$R^* = AA' . \quad (1)$$

Table 1. Example of Population Pattern Matrix ($p=36$, $m=6$, & $a_{ij}=.6$)

(p)	Components (<u>m</u>)					
	1	2	3	4	5	6
1	.60	.00	.00	.00	.00	.00
2	.60	.00	.00	.00	.00	.00
3	.60	.00	.00	.00	.00	.00
4	.60	.00	.00	.00	.00	.00
5	.60	.00	.00	.00	.00	.00
6	.60	.00	.00	.00	.00	.00
7	.00	.60	.00	.00	.00	.00
8	.00	.60	.00	.00	.00	.00
9	.00	.60	.00	.00	.00	.00
10	.00	.60	.00	.00	.00	.00
11	.00	.60	.00	.00	.00	.00
12	.00	.60	.00	.00	.00	.00
13	.00	.00	.60	.00	.00	.00
14	.00	.00	.60	.00	.00	.00
15	.00	.00	.60	.00	.00	.00
16	.00	.00	.60	.00	.00	.00
17	.00	.00	.60	.00	.00	.00
18	.00	.00	.60	.00	.00	.00
19	.00	.00	.00	.60	.00	.00
20	.00	.00	.00	.60	.00	.00
21	.00	.00	.00	.60	.00	.00
22	.00	.00	.00	.60	.00	.00
23	.00	.00	.00	.60	.00	.00
24	.00	.00	.00	.60	.00	.00
25	.00	.00	.00	.00	.60	.00
26	.00	.00	.00	.00	.60	.00
27	.00	.00	.00	.00	.60	.00
28	.00	.00	.00	.00	.60	.00
29	.00	.00	.00	.00	.60	.00
30	.00	.00	.00	.00	.60	.00
31	.00	.00	.00	.00	.00	.60
32	.00	.00	.00	.00	.00	.60
33	.00	.00	.00	.00	.00	.60
34	.00	.00	.00	.00	.00	.60
35	.00	.00	.00	.00	.00	.60
36	.00	.00	.00	.00	.00	.60

The final population matrix, R , was created by replacing the diagonal elements of R^* with unities. Second, each created population matrix R was input to a computer program by Montanelli (1975). Montanelli's program, which takes sample size (N) into account, adds random error to each created population matrix R , such that R is transformed to an approximate R with

$$\hat{R} = AA' + [e_{ij}] \quad (2)$$

where e_{ij} represents random error added to each cell in the population matrix R . Five sample correlation matrices were generated from each created population matrix for each different sample size, for each p , m , and a_{ij} combination.

The Misspecification of m

The sample correlation matrices created by Montanelli's (1975) program are based on particular values of m . These matrices were analyzed using principal component analysis and factor analysis, and in addition to producing factor pattern solutions based on their true underlying value of m , the value of m was also deliberately misspecified to produce factor and component patterns based on the wrong number of factors and components. These misspecifications of m varied depending on the value of m that was used to generate a particular population matrix.

Values for the overextraction of m are given in Table 2. Various considerations entered into the choices for the values of m displayed in this table. A first consideration involved examining the first few overextracted values in all cases, as it has generally been asserted that a small overextraction, i.e. one or two extra factors, will do little to harm the true factor pattern (Cattell, 1952, 1978; Comrey, 1973; Gorsuch, 1983; Rummel, 1970). Initial overextractions of m were thus incremented by ones in all cases, up to a value of $m+3$, to examine what actually happened in this range where little change was thought to occur. Overextractions of m were then increased by increments of 3 until the $p:m$ ratio became 3. The choice of a $p:m$ value of 3 as a convenient stopping place for the overextraction process is also based on a second consideration, which noted that 3 variables per factor is considered the minimum for factor replicability (Anderson & Rubin, 1956; Rindskopf, 1984), and values of $p:m$ less than 3 would represent dubious factor solutions. And a last consideration relates to the commonly-used Kaiser eigenvalue-greater-than-one rule, which typically retains $m=p/3$ components (Revelle & Rocklin, 1979; Zwick & Velicer, 1982, 1986).

Underextraction of components represented a slightly different problem and attempts to measure its effects were made using values of m as given in Table 2. There was virtually no guidance in the literature to help in the choice of values in examining the underextraction process. As noted earlier, underextraction is thought to result in the loss of information, may make a factor pattern uninterpretable, and in general is thought to be a worse problem than

Table 2. Overextracted, underextracted, and true values of m

p	True value of m	Overextracted values of m	Underextracted values of m
36	3	4, 5, 6, 9, 12	—————
36	6	7, 8, 9, 12	5, 4, 3
36	9	10, 11, 12	8, 7, 6, 3

overextraction. And yet, only in Mosier's (1939) study, where the true number of components was reduced by one, was there an empirical attempt to measure this effect. Within this study it was proposed to study the underextraction process by initially decreasing the value of m by ones, where applicable, down to a value of $m-3$. Underextraction of m was then continued in decrements of 3, until the $p:m$ ratio reached 12:1.

Data Analysis

Factor patterns were compared after overextraction and underextraction had occurred within the context of two different methods of computing scores as used in Fava and Velicer (in press). The sample correlation matrices were first analyzed by principal component analysis (Hotelling, 1933) and maximum likelihood factor analysis (MLFA). The factor analysis employed Brown's (1968) MLFA algorithm. After these analyses the factor and component patterns were rotated to varimax (Kaiser, 1958), promax (Hendrickson & White, 1964), and orthogonal procrustes (Schonemann, 1966) positions, where the principal component and factor scores were computed.

Principal component scores are calculated by the formula

$$F_c = YL_c D_c^{-1} \quad (3)$$

where Y is the $n \times p$ matrix of normalized deviation scores, such that

$$Y'Y = R \quad (4)$$

where L_c is a $p \times m$ matrix of eigen vectors from an eigen decomposition of R , and D_c^2 is a diagonal matrix containing the first m eigen roots of the eigen decomposition of R .

The formula for the factor score that was used in this study is, by contrast, not nearly as straightforward, and the result is essentially a factor score estimate. This is a direct result of the factor indeterminacy problem (Guttman, 1955; Schonemann & Wang, 1972) which means that factor scores cannot be uniquely defined.

There are several methods of computing factor score estimates, as discussed by Harris (1967). The method which was used in this study is

$$F_F = YU^{-2}A(A'U^{-2}RU^{-2}A)^{-1/2} \quad (5)$$

where F_F is an $n \times m$ matrix of factor score estimates, Y is an $n \times p$ matrix of normalized deviation scores following the relationship of Equation 4, R is a $p \times p$ correlation matrix of the variables in the sample, A is a $p \times m$ factor pattern, and U^2 is a $p \times p$ diagonal matrix of unique variances. Technically this factor score estimate is actually a component score since it represents a linear composite of the observed variables.

Harris (1967) notes several widely-used, competing methods of computing factor score estimates. Each of these methods contains some, but not all, of the four properties that are desired in factor scores in the orthogonal factor model (see McDonald & Burr, 1967). These properties include: (1) a high correlation between the

constructed factor score and the true factor score that is being estimated, (2) orthogonality between the constructed factor scores, (3) univocality of constructed factor scores, i.e., each constructed factor score be uncorrelated with each non-corresponding true factor score, and (4) constructed factor scores as conditionally unbiased estimators of their true factor scores.

Fava and Velicer (in press) employed three factor score estimates: (1) the regression estimate (Thomson, 1936; Thurstone, 1935), which is generally popular, and exhibits only property 1; (2) an estimate by Bartlett (1937) that minimizes the sum of squares of the residuals and exhibits properties 1, 3, and 4; and (3) an estimate developed by Anderson and Rubin (1956) which also minimizes the sum of squares of the residuals, but is constrained to satisfy properties 1 and 2. Arguments have persisted over the relative importance of the four properties desired in factor scores, and hence which of the methods of computing factor score estimates is the 'best'. Lacking consensus on a 'best' factor score method, Fava and Velicer (in press) employed all three score methods.

In this study only the Anderson and Rubin (1956) score method was employed. Because of the sheer number of comparisons it was not possible to employ all three factor score estimates in this study. The average correlation among scores was very high at .98 (Fava & Velicer, in press) and this method was chosen because, of the three factor score methods, it correlated the highest with the other two factor score methods and also with the principal component score (Fava & Velicer, in press). It thus represented a good compromise choice

for a factor score method.

Using these two score methods it was possible to compare changes within methods as overextractions and underextractions took place, by correlating the scores based on the correct factor and component patterns with the scores based on the misspecified factor and component patterns. It was also possible to compare changes between the two score methods by correlating the resulting scores of each method with each other after overextraction and underextraction took place.

Rather than calculate the actual scores, the variances and covariances between the estimates were calculated following a method explicated by Harris (1967) and McDonald and Burr (1967). Correlations between factor score estimates were calculated using a basic formula

$$R_{ij} = \text{diag}^{-1/2}(F'_i F_i) (F'_i F_j) \text{diag}^{-1/2}(F'_j F_j) \quad (6)$$

where R_{ij} generically represents a correlation matrix between two different sets of score matrices, F_i represents an $\underline{n} \times \underline{m}$ matrix of true scores, where $\underline{n}$ is the sample size and $\underline{m}$ is the true number of factors or components, and F_j represents a generic score matrix that will be compared to F_i . This formula was used to compare the correct factor and component scores with the misspecified factor and component scores, both within the factor and component score methods after an overextraction or underextraction had taken place, and between the factor and component score methods after an overextraction,

underextraction, and correct specification of factors and components.

The R_{ij} matrix took on 3 basic forms and F_j also took on 3 basic forms. In case 1, the overextraction of factors or components, F_i represents the $n \times m$ true score matrix, F_j represents the $n \times m+q$ misspecified score matrix, where q represents the particular overextraction increment, and R_{ij} is consequently an $m \times m+q$ matrix of score comparisons. In case 2, the underextraction of factors or components, F_i represents the $n \times m$ true score matrix, F_j represents the $n \times m-q$ misspecified score matrix, where q is the particular underextraction decrement, and R_{ij} is consequently an $m \times m-q$ matrix of score comparisons. In case 3, which will only occur in the between scores comparison, F_i represents the $n \times m$ true component score matrix, F_j represents the $n \times m$ true factor score matrix, and R_{ij} is consequently an $m \times m$ matrix of score comparisons. Table 3 provides the within variance-covariance matrices for the correlations that are represented by the above three cases.

Four final summary statistics were also calculated. The first is

$$CRO = \sum_{i=1}^m r_i / m \quad (7)$$

where the r_i represent the m highest correlations within a particular set of score comparisons, and m represents the underlying true number of components for that comparison. This formula was used with all cases involving overextractions, and when m was accurately portrayed

Table 3. Variances and covariances of the component score and the factor score estimate

	F_C	F_F
F'_C	$D_C^{-1} L'_C R L_C D_C^{-1}$ or* I	$D_C^{-1} L'_C R U^{-2} A (A' U^{-2} R U^{-2} A)^{-1/2}$
F'_F		$(A' U^{-2} R U^{-2} A)^{-1/2} (A' U^{-2} R U^{-2} A) (A' U^{-2} R U^{-2} A)^{-1/2}$ or* I

*Note: When $\underline{m} = \underline{m}$ and scores are correlated both $F'_C F_C$ and $F'_F F_F = I$.

in the between scores comparisons.

The second is

$$CRU = \sum_{i=1}^{m-q} r_i / (m-q) \quad (8)$$

where m is as defined in Equation 7, and q represents the number of decrements used to produce an underextraction, and the r_i represent the $m-q$ highest correlations within a particular set of score comparisons. This formula was used with cases involving underextractions.

These first 2 summary statistics measured the average correlation within methods, and between the different methods, for the strongest component and factor scores. This statistic thus measured the convergence within, and between, the principal component scores and the factor score estimates, when they were measuring the strongest factors. Ideally this value would equal 1.0 if the exact same factors or components were recovered after an overextraction or underextraction. Factors and components that degraded when an overextraction or underextraction took place evoked a decrease from the perfect correlational value of 1.0 in this situation.

The third summary statistic is

$$DRO = \sum_{i=1}^{m*} |r_i| / m* \quad (9)$$

where the r_i are the m^* remaining correlations within a particular set of score comparisons after the m highest correlations are removed from that particular score matrix, with m defined as in Equation 7. This formula was used with overextractions, and in the case where $\underline{m}$ was accurately portrayed in the between score comparisons. In the case where $\underline{m}$ is accurately portrayed, m^* equals $m(m-1)$, and in the case involving overextractions m^* equals $(m(m+q)-m)$.

The fourth summary statistic is

$$DRU = \sum_{i=1}^{m^{**}} |r_i| / m^{**} \quad (10)$$

where the r_i represent the m^{**} remaining correlations within, and between, a particular set of score comparisons after the $m-q$ highest correlations are removed, where q represents the number of decrements used to produce an underextraction. This formula was used with cases involving underextractions. In the case of underextraction, m^{**} equals $(m(m-q)-(m-q))$.

The third and fourth summary statistics measured the average correlation within, and between, the different methods when they were measuring the non-matching scores. This statistic measured the divergence among the various scores. Just as true matching factors generated scores that should correlate very highly, there is a corresponding set of relationships among the non-matching factors that produce near-zero correlations. Factors and components that degrade as overextraction and underextraction take place evoke an increase in

divergent correlational values.

A further analysis of the results involved repeated measures analysis of variance of the resulting correlational values. These analyses treated the correlational values within and between the scores as subjects, which were repeatedly measured in a varying number of cases of overextractions and underextractions of $\underline{m}$, as well as the case where $\underline{m}$ was accurately portrayed. In the within method comparisons, type of analysis was also treated as a within factor. The between factors in each ANOVA were sample size ($\underline{N}$) and factor loading ($\underline{a}_{ij}$). Rotational position was added as a between factor of concern for underextraction. The $p:\underline{m}$ ratio, which determined the actual number of overextractions and underextractions in each instance, framed the context for each separate ANOVA. As the overextraction process and the underextraction process appeared to produce qualitatively different results, separate ANOVAs were calculated for the overextracted and underextracted cases.

Fisher's Z statistic was used to transform the correlational values before they were analyzed by the ANOVA method because high correlations, which were predicted for the convergent correlational values, yield a non-normal distribution. In such instances, Fisher's Z statistic is an appropriate transformation to make before comparing values. Similarly, in analyses involving the divergent values, the correlations were first subtracted from 1.0, and then the Fisher's Z statistic was used to transform the resulting values before performing the ANOVA comparisons. In all instances this transformation was applied to individual score correlations within each sample before

mean summary statistics were calculated.

The Fisher's Z values can be calculated by the formula

$$z = 1/2(\log_e(1+r)-\log_e(1-r)). \quad (11)$$

CHAPTER III

Study 1

Overextraction of Components and Factors: Within Method Comparisons

Study 1 was concerned with examining the effects of systematically overextracting components and factors within the methods of principal component analysis and maximum likelihood factor analysis. This study was undertaken because of the dearth of empirical information on the consequences of overextraction. Conventional wisdom suggests overextraction is a 'less' serious problem than underextraction (Cattell, 1952, 1978; Comrey, 1973; Gorsuch, 1983; Mosier, 1939; Rummel, 1970), however no extensive empirical study has been undertaken to examine just how serious 'less' serious is. Advice ranges from the suggestion that one or two extra factors may be extracted without harm (Gorsuch, 1983) to Thurstone's (1947) dictum that "too many factors can do no harm..." (p.509). Cattell (1978) states that 'factor fission' will occur with substantial overextraction. In an effort to sort out such conflicting advice and to provide a strong empirical base for future advice, this study was undertaken.

Method

Factors of interest in the ANOVA designs in this study included 2 between factors (saturation and sample size) and 2 within factors (the number of overextractions and type of analysis). Saturation was examined at three levels: .4 (low), .6 (moderate), and .8 (high).

Sample size was manipulated at four levels: 75 (low), 150, 225 and 450 (high). The number of overextractions is dependent upon the true $p:m$ ratio of the sample matrices. Table 2 notes what values of m will be examined for the appropriate $p:m$ ratio. The type of analysis is represented by principal component analysis and maximum likelihood factor analysis. Dependent measures that will be examined in the various cells of the study design are the CRO and DRO summary statistics.

Repeated measures analysis of variance was chosen as the preferred method of examining the CRO and DRO summary statistics. The Fisher Z transformation of correlational values was chosen to help overcome the non-normal data distributions that can occur with extremely high or extremely low correlational values. Three ANOVAs were planned to examine the CRO values and three ANOVAs were planned to examine the DRO values, one for each true $p:m$ ratio which formed the context for the number of overextractions. These ANOVAs will be used to help describe the pattern of results that are formed by the various cells of the design.

There was extremely high power in the ANOVA designs used to analyze the data in this study. Consequently it was decided to use eta-squared ($SS \text{ effect} / SS \text{ total}$) as the major criterion to determine which factors to interpret as meaningful. Only factors which explained at least 1% of the total variance, in addition to attaining a statistically significant probability level of $p < .0001$, are discussed. Even with these stringent interpretation criteria, some of the significant results are of questionable practical importance.

In each ANOVA the first level of the extraction factor represents the correct factor or component pattern and would theoretically be represented by a 1.0 correlation. Since a Fisher's Z value is not possible for a perfect correlation, it was decided to use the .99999 correlational value to represent this level. As only 11 of 1,440 convergent correlational values within this study exceeded .99999, and none of those 11 exceeded .999995, the .99999 value seemed a reasonable proxy for a 'perfect' correlation. Also, none of the 1,440 divergent correlational values was less than .00001, the divergent equivalent of a .99999 convergent correlation. Furthermore, using Fisher's Z representation for a value greater than .99999 resulted in systematically increasing the effect the extraction factor in the analyses. In an effort to further understand the actual effects of each factor in the ANOVA designs, the analyses were rerun a second time without level one of the extraction factor, which was the level representing the 'perfect' correlation.

Results

Analysis of Convergent Correlations (CRO)

Three initial ANOVAs examined the convergent score correlations for the cases when the true $p:m$ ratio was 12:1 (high), 6:1 (moderate), and 4:1 (low). In each ANOVA very strong main effects were produced by the extraction and saturation factors and a small main effect was found for sample size. A moderate effect was found for the extraction

X saturation interaction and a small effect was found for the extraction X sample size interaction.

Extraction had the strongest impact. The effect for $p:m = 12:1$ was $F(5,240) = 3,708.44$ and eta-squared was 60.1% with mean correlational values ranging from .99999 at baseline to .97+ for 9 overextractions. The effect for $p:m = 6:1$ was $F(4,192) = 6,436.88$ and eta-squared was 57.3% with mean correlational values ranging from .99999 at baseline to .98+ for 6 overextractions. The effect for $p:m = 4:1$ was $F(3,144) = 6,975.47$ and eta-squared was 60.0% with mean correlational values ranging from .99999 at baseline to .99+ for 3 overextractions.

The effects of saturation were also quite strong. The effect for $p:m = 12:1$ was $F(2,48) = 946.06$ and eta-squared was 27.2% with mean correlational values of .999+ at .8 saturation, .99+ at .6 saturation, and .98+ at .4 saturation. The effect for $p:m = 6:1$ was $F(2,48) = 1,327.09$ and eta-squared was 27.9% with mean correlational values of .999+ at .8 saturation, .99+ at .6 saturation, and .99+ at .4 saturation. The effect for $p:m = 4:1$ was $F(2,48) = 1,003.41$ and eta-squared was 24.3% with mean correlational values of .999+ at .8 saturation, .99+ at .6 saturation, and .99+ at .4 saturation.

A small main effect was also found for sample size. The effect for $p:m = 12:1$ was $F(3,48) = 66.23$ and eta-squared was 2.9% with mean correlational values ranging from .99+ at $N = 75$ to .999+ at $N = 450$. The effect for $p:m = 6:1$ was $F(3,48) = 130.25$ and eta-squared was 4.1% with mean correlational values ranging from .99+ at $N = 75$ to .999+ at $N = 450$. The effect for $p:m = 4:1$ was $F(3,48) = 104.32$ and

eta-squared was 3.8% with mean correlational values ranging from .99+ at $N = 75$ to .999+ at $N = 450$.

Moderate effects were found for the extraction x saturation interaction. The effect for $p:m = 12:1$ was $F(10,240) = 172.99$ and eta-squared was 5.6%. Figure 1 displays this interaction. The effect for $p:m = 6:1$ was $F(8,192) = 398.23$ and eta-squared was 7.1%. Figure 2 displays this interaction. The effect for $p:m = 4:1$ was $F(6,144) = 477.87$ and eta-squared was 8.2%. Figure 3 displays this interaction.

A small effect was also found for the extraction x sample size interaction in 2 of the ANOVAs. The effect for $p:m = 6:1$ was $F(12,192) = 43.69$ and eta-squared was 1.2%. Figure 4 displays this interaction. The effect for $p:m = 4:1$ was $F(9,144) = 49.98$ and eta-squared was 1.3%. Figure 5 displays this interaction.

Reanalysis of CRO with baseline case eliminated

These displays suggested that the significant interactions may have been an artifact created by the 1st level of the extraction factor, in which no overextraction has taken place and which is represented by the Fisher's Z value for a .99999 correlation, the value being used as a proxy for a perfect correlation. Consequently the results of these ANOVAs were reanalyzed without level one of the extraction factor.

Given the standards for interpretation previously noted, the ANOVA results of this reanalysis display notable differences. There are now no interaction effects to interpret.

Saturation is now a very strong effect with respect to

Figure 1. Extraction X Saturation Interaction of Within Method
Convergent Score Correlations in the Overextraction Situation:
 $p:m = 12:1$.

Convergent Scores--Overextraction Extraction X Saturation, p:m=12:1

Figure 2. Extraction X Saturation Interaction of Within Method
Convergent Score Correlations in the Overextraction Situation:
 $p:m = 6:1$.

Convergent Scores--Overextraction
Extraction X Saturation, p:m=6:1

Figure 3. Extraction X Saturation Interaction of Within Method
Convergent Score Correlations in the Overextraction Situation:
 $p:m = 4:1$.

Convergent Scores--Overextraction

Extraction X Saturation, p:m=4:1

Figure 4. Extraction X Sample Size Interaction of Within Method
Convergent Score Correlations in the Overextraction Situation:
 $p:m = 6:1$.

Convergent Scores--Overextraction
Extraction X Sample Size, p:m=6:1

Figure 5. Extraction X Sample Size Interaction of Within Method
Convergent Score Correlations in the Overextraction Situation:
 $p:m = 4:1$.

Convergent Scores--Overextraction

Extraction X Sample Size, p:m=4:1

eta-squared. Given the nature of the reanalysis, F values for saturation remain the same but eta-squared changes. For $p:m = 12:1$ eta-squared is now 61.9% with mean correlational values of .999+ at .8 saturation, .99+ at .6 saturation, and .94+ at .4 saturation. For $p:m = 6:1$ eta-squared is now 69.7% with mean correlational values of .999+ at .8 saturation, .99+ at .6 saturation, and .94+ at .4 saturation. For $p:m = 4:1$ eta-squared is now 75.9% with mean correlational values of .999+ at .8 saturation, .99+ at .6 saturation, and .94+ at .4 saturation.

Extraction now varies from a strong to moderate effect especially with regard to eta-squared. Mean correlational values for extraction in the remaining cells of the reanalyzed design remain the same as in the original analysis. The extraction effect for $p:m = 12:1$ was $F(4,192) = 956.02$ and eta-squared was 24.2%. The extraction effect for $p:m = 6:1$ was $F(3,144) = 1,160.13$ and eta-squared was 14.6%. The extraction effect for $p:m = 4:1$ was $F(2,96) = 614.25$ and eta-squared was 6.6%.

Sample size is now shown to have a moderate effect with respect to eta-squared and as was the case with saturation, F values remain the same but eta-squared values change. For $p:m = 12:1$ eta-squared is now 6.5% with mean correlational values ranging from .98+ at $N = 75$ to .99+ at $N = 450$. For $p:m = 6:1$ eta-squared is now 10.3% with mean correlational values ranging from .98+ at $N = 75$ to .99+ at $N = 450$. For $p:m = 4:1$ eta-squared is now 11.8% with mean correlational values ranging from .98+ at $N = 75$ to .99+ at $N = 450$.

A very weak but interpretable main effect for type of analysis

was also found for $p:m = 12:1$, with $F(1,48) = 94.16$ and eta-squared equal to 1.0%. Mean correlational values for factor analysis and principal component analysis were both .99+, with factor analysis higher at the third decimal place.

Analysis of Divergent Correlations (DRO)

Three initial ANOVAs also examined the divergent score correlations for the cases when the true $p:m$ ratio was 12:1 (high), 6:1 (moderate), and 4:1 (low). In each ANOVA an extremely strong main effect was found for the extraction factor, a moderate main effect was found for saturation, and a small main effect was found for sample size. A small effect was also found for the extraction X saturation interaction.

Extraction had an extremely strong effect. The effect for $p:m = 12:1$ was $F(5,240) = 10,134.07$ and eta-squared was 83.3% with mean correlational values ranging from .00001 at baseline to .03- for 9 overextractions. The effect for $p:m = 6:1$ was $F(4,192) = 17,836.74$ and eta-squared was 80.9% with mean correlational values ranging from .00001 at baseline to .02- for 6 overextractions. The effect for $p:m = 4:1$ was $F(3,144) = 27,549.25$ and eta-squared was 81.2% with mean correlational values ranging from .00001 at baseline to .01- for 3 overextractions.

A moderate main effect was found for saturation. The effect for $p:m = 12:1$ was $F(2,48) = 767.19$ and eta-squared was 10.4% with mean correlational values of .01- at .8 saturation, .01- at .6 saturation, and .02- at .4 saturation. The effect for $p:m = 6:1$ was $F(2,48) =$

1,049.29 and eta-squared was 11.8% with mean correlational values of .001- at .8 saturation, .01- at .6 saturation, and .01- at .4 saturation. The effect for $p:m = 4:1$ was $F(2,48) = 1,182.83$ and eta-squared was 11.1% with mean correlational values of .001- at .8 saturation, .01- at .6 saturation, and .01- at .4 saturation.

A small main effect was found for sample size. The effect for $p:m = 12:1$ was $F(3,48) = 91.69$ and eta-squared was 1.9% with mean correlational values of .01- for all sample sizes. Differing only in the third decimal place, the value was highest for $N=75$ and decreased as sample size increased. The effect for $p:m = 6:1$ was $F(3,48) = 141.20$ and eta-squared was 2.4% with mean correlational values of .01- for all sample sizes. Differing only in the third decimal place, the value was highest for $N=75$ and decreased as sample size increased. The effect for $p:m = 4:1$ was $F(3,48) = 151.41$ and eta-squared was 2.1% with mean correlational values ranging from .01- at $N=75$ to .001- at $N=450$.

A small effect was found for the extraction X saturation interaction. The effect for $p:m = 12:1$ was $F(10,240) = 129.72$ and eta-squared was 2.1%. Figure 6 displays this interaction. The effect for $p:m = 6:1$ was $F(8,192) = 333.41$ and eta-squared was 3.0%. Figure 7 displays this interaction. The effect for $p:m = 4:1$ was $F(6,144) = 636.68$ and eta-squared was 3.8%. Figure 8 displays this interaction.

Reanalysis of DRO with baseline case eliminated

As with the previous interactions these displays suggested that the significant interactions may have been an artifact created by the

Figure 6. Extraction X Saturation Interaction of Within Method
Divergent Score Correlations in the Overextraction Situation:
 $p:m = 12:1$.

Divergent Scores--Overextraction

Extraction X Saturation, p:m=12:1

Figure 7. Extraction X Saturation Interaction of Within Method
Divergent Score Correlations in the Overextraction Situation:
 $p:m = 6:1$.

Divergent Scores--Overextraction

Extraction X Saturation, p:m=6:1

Figure 8. Extraction X Saturation Interaction of Within Method
Divergent Score Correlations in the Overextraction Situation:
 $p:m = 4:1$.

Divergent Scores--Overextraction

Extraction X Saturation, p:m=4:1

1st level of the extraction factor and these ANOVAs were consequently reanalyzed without level one of the extraction factor. Results for this reanalysis of divergent correlations parallel those for the convergent correlations. There are now no interaction effects to interpret.

Saturation is now a very strong effect with respect to eta-squared. Eta-squared for saturation in the $p:m = 12:1$ situation is now 63.1% with mean correlational values of .01- at .8 saturation, .02- at .6 saturation, and .05- at .4 saturation. Eta-squared for saturation in the $p:m = 6:1$ situation is now 68.8% with mean correlational values of .01- at .8 saturation, .01- at .6 saturation, and .04- at .4 saturation. Eta-squared for saturation in the $p:m = 4:1$ situation is now 74.9% with mean correlational values of .01- at .8 saturation, .01- at .6 saturation, and .04- at .4 saturation.

Extraction now displayed a moderate effect with respect to eta-squared. The effect for $p:m = 12:1$ was $F(4,192) = 455.88$ and eta-squared was 15.7%. The effect for $p:m = 6:1$ was $F(3,144) = 755.83$ and eta-squared was 11.0%. The effect for $p:m = 4:1$ was $F(2,96) = 734.77$ and eta-squared was 5.1%. Mean correlational values for cell values of extraction do not change in the reanalysis, with the exception of baseline cell values which are removed in the reanalysis.

Sample size is now shown to have a moderate effect with respect to eta-squared. Eta-squared for sample size in the $p:m = 12:1$ situation is now 11.3% with mean correlational values ranging from .03- at $N=75$ to .01- at $N=450$. Eta-squared for sample size in the $p:m = 6:1$ situation is now 13.9% with mean correlational values

ranging from .02- at N=75 to .01- at N=450. Eta-squared for sample size in the $p:m = 4:1$ situation is now 14.4% with mean correlational values ranging from .02- at N=75 to .01- at N=450.

A small but interpretable main effect for type of analysis was also found for $p:m = 12:1$, with $F(1,48) = 201.31$ and eta-squared equal to 2.2%. The mean correlational value for factor analysis was .01- versus .02- for principal component analysis.

Full ANOVA tables for all of the preceding analyses are presented in appendices A through D. Tables presenting and comparing mean correlational values of significant main effects between the original analyses and the reanalyses, for both CRO and DRO values, are contained in appendices O and P.

Discussion

In this study of overextraction within the methods of principal component analysis and maximum likelihood factor analysis the most important finding demonstrates the robustness of strong and clear factor patterns to misspecification by overextraction. Although there were extremely strong statistical effects the practical effects of overextraction seem rather insignificant. Referring back to Figures 1, 2 and 3 one can see that despite the very strong saturation effect, it is not until beyond the 3rd overextraction that the convergent score correlations fall below .90, and this only occurs in the .4 saturation condition. This seems to indicate very strong reproducibility for the original factor patterns marked by the significant loadings on each factor. Parallel findings for the

divergent score correlations imply that variables do not tend to get complex as overextraction occurs. Rather the average divergent score correlations, which ideally would be 0.0 indicating no loading on another factor, did not rise very much even at maximum overextraction. Examining Figures 6, 7 and 8 one notes that despite the very strong statistical effect for saturation, it is only after the 3rd overextraction that the average divergent score correlational value rises above .05, and this only occurs in the .4 saturation condition.

Sample size also had a statistically strong effect on overextraction although only moderate in terms of eta-squared. The rather insignificant practical effect for sample size with regard to CRO values is seen upon examination of Figures 4 and 5. If DRO values were plotted, a similar pattern would result. Sample size findings parallel those for saturation, although they portray less deterioration due to sample size with respect to overextraction. In no instance does sample size drive the average convergent correlation below .94 nor the average divergent correlation above .05.

Conventional wisdom had suggested that overextraction would not change the important or substantive factors (Cattell, 1958; Comrey, 1973). Some, such as Gorsuch (1983) even suggested specific instructions that stated one should not overextract more than one or two factors or replicability would suffer. Gorsuch's recommendation does not appear to fit the case with rather strong and clear factor patterns, rather in this regard Thurstone (1947) was closer to the mark when he flatly stated that "too many factors can do no harm ..." (p. 509). Even this is not entirely accurate, however, as worrisome

deterioration in low loading, low sample size situations, especially in combination, does occur beyond 3 overextractions. This is especially troublesome when one notes that the Kaiser eigenvalue-greater-than-one rule, the most commonly available and used guideline for deciding upon the proper number of components and factors, typically extracts $m = p/3$ components and factors (Revelle & Rocklin, 1979; Zwick & Velicer, 1982, 1986). This equals the maximum overextraction conditions in this study, e.g. the overextraction by 9 condition, which produced the most serious problems.

Given the results of this study some better rules based on empirical observation of score correlations would suggest that (1) strong and clear factor patterns will not suffer with even a very high number of overextractions and (2) clear factor patterns for low loading, low sample situations have robustness when a few, although no more than 3, overextractions occur.

It cannot be plainly commented upon as to the effects of overextraction on more complex factor patterns, i.e. those with mixed loadings (high and low) or with complex variables, but wariness should probably increase as the strength of the factor pattern decreases. It should also be kept in mind that besides using ideal patterns, overextraction in this study was not carried to extremes, i.e. where the $p:m$ ratio would be less than 3:1. In practice, however, this may occur given some liberal rules for deciding the correct number of factors and researchers are cautioned to examine their results carefully if extraction carries their $p:m$ ratio below a 3:1 value.

It is worth noting a caution about the use of scores in this

study. Although a .9 correlation may be quite good for scores, using scores to decide that a factor pattern remains virtually unchanged may mask a possible numeric degradation of factor pattern loadings. Some attempts were made to visually compare factor and component pattern loadings at different overextractions, but because of the sheer number of patterns and the time involved only a sampling could be examined. With the very strongest patterns, however, at very high correlations, i.e. .99, the patterns did appear to remain quite visually obvious and it was relatively easy to pick them out. Further exhaustive investigations may need to be undertaken to compare across more levels of factor patterns in order to make more definitive claims on the merits of using scores to make statements about patterns, but there are some obvious advantages: (1) one can not simply subtract patterns which have a different number of components or factors to obtain a summary statistic such as g (Velicer & Fava, 1987, 1990) which measures an average (squared) difference between comparable loadings, but one can come up with useful summary statistics (CRO, CRU, DRO, DRU) that can be used to compare scores from patterns that have unequal numbers of components or factors; (2) further complications of rotational indeterminacy make it very difficult to pick out the exact components and factors of interest should one succeed in eliminating the 'extra' factors in an overextraction; (3) underextraction and comparison of patterns is even more problematic in that there are less dimensions available to compare with the original patterns; and (4) while visual comparisons provide some clues as to the underlying processes, with large simulation studies this is somewhat impractical

in terms of time, and there are further possible complications of experimenter objectivity that may be involved in visual pattern comparisons.

CHAPTER IV

Study 2

Underextraction of Components and Factors: Within Method Comparisons

Study 2 was concerned with the effects of systematically underextracting components and factors within the methods of principal component analysis and maximum likelihood factor analysis and forms a counterpart to Study 1. Hard empirical evidence on the effects of underextraction is meager and this study was undertaken to provide more factual and practical information concerning this situation. Conventional wisdom had suggested that underextraction is a 'more' serious problem than overextraction (Cattell, 1952, 1978; Comrey, 1973; Gorsuch, 1983; Mosier, 1939; Rummel, 1970), however, no guidelines based on strong empirical evidence exist to tell us just how much 'more' serious a problem underextraction is reputed to be. In contrast to the case of overextraction, where Gorsuch (1983) suggested one might overextract one or two factors safely, and Thurstone (1947) stated there were no limits on the number of factors one could safely overextract without destroying factor pattern integrity, no specific 'safe' number of underextractions has been suggested. Rather, it is generally felt that the true factors and components simply cannot be accurately portrayed if underextraction occurs (Cattell, 1958; Comrey, 1973, 1978; Mosier, 1939; Thurstone, 1947), although Guertin and Bailey (1970) did offer the minority opinion that whether it was better to rotate too many or too few factors depended on the characteristics of the individual matrices.

This study sought to provide some extensive empirical basis for confirming or disconfirming past advice, and to provide a basis for future advice on this issue.

Method

The factors of interest in the ANOVA designs in this study included 3 between factors (saturation, sample size, and rotational position) and 2 within factors (the number of underextractions and type of analysis). Saturation was examined at three levels: .4 (low), .6 (moderate), and .8 (high). Sample size was manipulated at four levels: 75 (low), 150, 225 and 450 (high). Rotational position was examined at varimax (orthogonal) and promax (oblique). The number of underextractions is dependent upon the true $p:m$ ratio of the sample matrices. Table 2 notes what values of m will be examined for the appropriate $p:m$ ratio. The type of analysis is represented by principal component analysis and maximum likelihood factor analysis. Dependent measures that were examined in the various cells of the study design are the CRU and DRU summary statistics.

Repeated measures analysis of variance was chosen as the preferred method of examining the CRU and DRU summary statistics. The Fisher Z transformation of correlational values was chosen to help overcome the non-normal data distributions that can occur with extremely high or extremely low correlational values. Two ANOVAs were planned to examine the CRU values and two ANOVAs were planned to examine the DRU values, one for each true $p:m$ ratio which formed the context for the number of underextractions. These ANOVAs will be used

to help describe the pattern of results that are formed by the various cells of the design.

As with Study 1, there was extremely high power in the ANOVA design of this study. The criteria used in Study 1 to label a factor interpretable for discussion are employed in Study 2 also, i.e. eta-squared for a factor had to equal at least 1% and a factor had to be statistically significant at a probability level of $p < .0001$.

In each ANOVA it was decided to use the .99999 correlational value as a proxy for a perfect correlation, just as it was in Study 1. This choice of a value to represent a 1.0 correlation for convergent values or a 0.0 value for divergent values was more problematic in Study 2, as only 11 of 1680 convergent values were greater than .99 and only 8 of 1680 divergent values were less than .01, and no value was greater than .995 or less than .005. This differential between the 'perfect' correlational value and the actual underextracted correlational values in Fisher Z notation created an overwhelming effect for the extraction factor. Again, as with Study 1, in an effort to understand the actual effects of each factor in the ANOVA design, the analyses were rerun a second time without level one of the extraction factor, which was the proxy value for a perfect correlation.

Results

Analysis of Convergent Correlations (CRU)

Two initial ANOVAs examined the convergent score correlations for the cases when the true $p:m$ ratio was 6:1 (moderate) and 4:1 (low).

In each of these ANOVAs overwhelming main effects were produced by the extraction factor. The effect for $p:m = 6:1$ was $F(3,288) = 25,647.54$ and eta-squared was 98.5% with mean correlational values ranging from .99999 at baseline to .71+ at 3 underextractions. The effect for $p:m = 4:1$ was $F(4,384) = 44,102.33$ and eta-squared was 98.9% with mean correlational values ranging from .99999 at baseline to .61+ at 6 underextractions. No other factor was interpretable for the convergent score correlations.

Reanalysis of CRU with baseline case eliminated

In an effort to examine whether the effects of the extraction factor had been artificially inflated by the inclusion of the level one proxy value for a perfect correlation, the ANOVA analyses were rerun without the level one extraction condition.

The extraction factor still remains very strong in this reanalysis though it is lessened somewhat. The main effect of extraction for $p:m = 6:1$ was $F(2,192) = 768.38$ and eta-squared was 61.9% with mean correlational values ranging from .94+ at 1 underextraction to .71+ at 3 underextractions. The main effect of extraction for $p:m = 4:1$ was $F(3,288) = 2,624.25$ and eta-squared was 82.3% with mean correlational values ranging from .96+ at 1 underextraction to .61+ at 6 underextractions.

There was also a moderate to small effect for type of analysis. This main effect for $p:m = 6:1$ was $F(1,96) = 288.10$ and eta-squared was 11.9% with the mean correlational value of principal component analysis equal to .90+ versus .81+ for maximum likelihood factor

analysis. The main effect for $p:m = 4:1$ was $F(1,96) = 184.70$ and eta-squared was 3.0% with the mean correlational value of principal component analysis equal to .89+ versus .84+ for maximum likelihood factor analysis.

There was a very small type X sample size interaction effect for $p:m = 6:1$ with $F(3,96) = 10.03$ and eta-squared was 1.2%. Figure 9 displays this interaction.

There was a very small saturation X sample size interaction for $p:m = 4:1$ with $F(6,96) = 7.86$ and eta-squared was 1.4%. Figure 10 displays this interaction.

Analysis of Divergent Correlations (DRU)

Two initial ANOVAs also examined the divergent score correlations for the cases when the true $p:m$ ratio was 6:1 (moderate) and 4:1 (low). Paralleling the ANOVA results for the convergent correlations, there were overwhelming main effects for the extraction factor. The effect for $p:m = 6:1$ was $F(3,288) = 24,420.83$ and eta-squared was 99.0% with mean correlational values ranging from .00001 at baseline to .13- at 3 underextractions. The effect for $p:m = 4:1$ was $F(4,384) = 45,226.75$ and eta-squared was 99.3% with mean correlational values ranging from .00001 at baseline to .12- at 6 underextractions. No other factor was interpretable in this analysis of the divergent score correlations.

Reanalysis of DRU with baseline case eliminated

In an effort to examine whether the effects of the extraction

Figure 9. Type of Analysis X Sample Size Interaction of Within Method
Convergent Score Correlations in the Underextraction Situation:
Reanalysis, $p:m = 6:1$.

Convergent Scores--Underextraction
Sample Size X Type of Analysis, p:m=6:1

Figure 10. Saturation X Sample Size Interaction of Within Method
Convergent Score Correlations in the Underextraction Situation:
Reanalysis, $p:m = 4:1$.

Convergent Scores--Underextraction

Saturation X Sample Size, p:m=4:1

factor had been arbitrarily raised by the inclusion of the level one proxy value for a perfect correlation, the ANOVAs were rerun without the level one extraction condition. In these reanalyses the results of the ANOVAs partially paralleled the reanalyses of the convergent score correlations.

The extraction factor still remains quite strong in this reanalysis although its effects are lessened somewhat. The effect for $p:m = 6:1$ was $F(2,192) = 272.16$ and eta-squared was 48.1% with mean correlational values ranging from .04- at 1 underextraction to .13- at 3 underextractions. The effect for $p:m = 4:1$ was $F(3,288) = 1,034.05$ and eta-squared was 72.9% with mean correlational values ranging from .02- at 1 underextraction to .12- at 6 underextractions.

In the reanalysis there was a small effect for type of analysis for $p:m = 6:1$ only, with $F(1,96) = 111.02$ and eta-squared was 4.3% with the mean correlational value of principal component analysis equal to .07- versus .10- for maximum likelihood factor analysis.

Also in the reanalysis there was a very small extraction X type of analysis interaction effect for $p:m = 6:1$, with $F(2,192) = 20.51$ and eta-squared was 1.4%. A graphical display of this interaction is presented in Figure 11.

Full ANOVA tables for all of the preceding analyses are presented in appendices E through H. Tables presenting and comparing F, Eta-squared, and mean correlational values of the extraction main effects between the original analyses and the reanalyses, for both CRU and DRU values, are presented in appendices Q and R.

Figure 11. Extraction X Type of Analysis Interaction of Within Method
Divergent Score Correlations in the Underextraction Situation:
Reanalysis, $p:m = 6:1$.

Divergent Scores--Underextraction
Extraction X Type of Analysis, p:m=6:1

Discussion

In this study of the effects of underextraction within the methods of principal component analysis and factor analysis there was an extremely strong effect for extraction in both the original analyses and the reanalyses. Referring to Appendix Q, it is apparent from the correlational values that the convergent scores begin to rapidly depart from their original configural patterns during the 2nd underextraction. It is at this point that convergent score correlational values start to drop below .90. This suggests that the original factor patterns are losing their reproducibility very quickly when underextraction occurs.

The effect of underextraction on the divergent score correlations is nearly as dramatic. Referring to Appendix R, it is seen that these divergent correlational values begin to increase quite rapidly, especially for the $p:m = 6:1$ analyses. This suggests that the underlying orthogonality of the factor pattern is being compromised, which also would affect the reproducibility of the factor pattern.

The only other consistent effect that occurred in the underextraction reanalyses was for the type of analysis. Although deterioration occurs rapidly within both principal component analysis and maximum likelihood factor analysis, the deterioration is worse within factor analysis.

Three very small interaction effects also were interpretable in the reanalyses, but these effects were neither consistent across the various analyses, nor were they of any real practical significance.

The overall extent of distortion of the factor pattern caused by underextraction when both the convergent score and divergent score correlations are simultaneously taken into account is difficult to gauge. We know that with each underextraction we continue to lose, at a minimum, one complete factor. After only one underextraction, however, the remaining factor scores do continue to correlate at a reasonable rate. Beyond one underextraction serious deterioration occurs. Further, where there are more factors to begin with, i.e. in the $p:m = 4:1$ analysis, there are a greater percentage of factors remaining after each underextraction and the overall factor pattern appears to deteriorate at a slower rate. However, when examined with regard to proportional rates of underextraction, i.e. 3 underextractions in the $p:m = 4:1$ analyses versus 2 underextractions in the $p:m = 6:1$ analyses, thus allowing for $2/3$ of the original number of factors in each of the analyses, it becomes clear that convergent and divergent score correlational values are more nearly identical for each $p:m$ situation, and deterioration thus appears to be taking place at an approximately similar pace. This indicates that beyond the simple number of underextractions, it is the number of true components and factors that is a major determinant of how seriously an underextraction will effect the true component or factor pattern. When the true number of components or factors is small, the loss can quickly become substantial. And further, given the effect of type of analysis it is a more serious deterioration and loss of information that will occur under maximum likelihood factor analysis as opposed to principal component analysis when underextraction occurs.

Some attempts were made to visually compare the factor and component patterns of loadings after underextraction occurred. As was the case with Study 1, because of the sheer number of patterns and the time needed for conscientious visual comparisons, only a sampling of results could be examined in this manner. Some merging of factors and components was observed but no clear patterns emerged as to this process. In the low initial saturation condition (.4) especially, the increase in loadings of variables on other factors suggested some tendency toward the creation of variable complexity by the underextraction process. Further detailed investigations are needed, however, to fully study the process of change in pattern loadings as underextraction occurs.

In general, the results of this study fit in with the conventional wisdom concerning underextraction that is found in the literature (Cattell, 1952, 1978; Comrey, 1973; Gorsuch, 1983; Mosier, 1939; Rummel, 1970). This study goes beyond what was available in terms of advice, however, in suggesting the extent to which underextraction can be a problem and when it is likely to be more serious.

Some cautions about generalizability are applicable to this study. These cautions include the use of only ideal factor patterns in this study and the inability to speak clearly about the effects on more complex factor patterns, and those factor patterns that include complex variables. Given the rapid and continual decline that did take place with the ideal factor patterns in this study, however, it is likely that factor patterns of poor quality (e.g. those with

complex variables) will degenerate more rapidly than ideal patterns. Also, underextraction was not carried to the extremes where all variables would load on a single factor. There is no indication, however, that the degeneration of the factor pattern had reached a plateau and it appears likely that maximal information would be lost with extreme underextraction.

CHAPTER V

Study 3

Overextraction of Components and Factors: Between Methods Comparisons

Study 3 examined the differences between the methods of principal component analysis and maximum likelihood factor analysis as factors and components were systematically overextracted. This study was undertaken for several reasons. First, it would extend the results of Fava and Velicer (in press), which had examined the relationships among component and factor scores in their 'true' configural patterns, into the overextracted direction. Second, if the results of Study 1 suggested that scores within methods did not change substantially with overextraction, it would be important to find out if between method score relationships also did not change substantially. Third, if the results of Study 1 suggested that scores did change substantially within methods as overextraction occurred, it would be important to find out if between method score relationships which were high at baseline would remain high with overextraction or would diverge. And fourth, if there was differential reproduction of scores within methods, it would be very important to examine how the between method score correlations behaved.

Method

The factors of interest in the ANOVA designs in this study included 2 between factors (saturation and sample size) and 1 within factor (the number of overextractions). Saturation was examined at

three levels: .4 (low), .6 (moderate), and .8 (high). Sample size was manipulated at four levels: 75 (low), 150, 225 and 450 (high). The number of overextractions is dependent upon the true $p:m$ ratio of the sample matrices. Table 2 notes what values of m will be examined for the appropriate $p:m$ ratio. Dependent measures that will be examined in the various cells of the study design are the CRO and DRO summary statistics.

Repeated measures analysis of variance was chosen as the preferred method of examining the CRO and DRO summary statistics. The Fisher Z transformation of correlational values was chosen to help overcome the non-normal data distributions that can occur with extremely high or extremely low correlational values. Three ANOVAs were planned to examine the CRO values and three ANOVAs were planned to examine the DRO values, one for each true $p:m$ ratio which formed the context for the number of overextractions. These ANOVAs will be used to help describe the pattern of results that are formed by the various cells of the design.

As in Studies 1 and 2, there was extremely high power in the ANOVA designs of this study, and the criteria used in these earlier studies were also employed in this study in deciding upon which factors to interpret for discussion. These criteria state that eta-squared for a factor had to equal at least 1% and a factor had to be statistically significant at a probability level of $p < .0001$. In total, 900 convergent score correlations and 900 divergent score correlations were analyzed in the ANOVA designs of this study.

Results

Analysis of Convergent Correlations (CRO)

Three ANOVAs examined the convergent score correlations between methods for the cases when the true $p:m$ ratio was 12:1 (high), 6:1 (moderate), and 4:1 (low). In each of these analyses there was a very strong main effect for saturation, a strong to small main effect for extraction, and a small to moderate main effect for sample size. There was also a small effect for the extraction X saturation interaction for each ANOVA, and a single small extraction X sample size interaction effect for the $p:m = 12:1$ situation.

The effect of saturation for $p:m = 12:1$ was $F(2,48) = 1,468.14$ and eta-squared was 66.8% with mean correlational values of .999+ at .8 saturation, .99+ at .6 saturation, and .95+ at .4 saturation. The effect of saturation for $p:m = 6:1$ was $F(2,48) = 759.51$ and eta-squared was 77.3% with mean correlational values of .999+ at .8 saturation, .99+ at .6 saturation, and .94+ at .4 saturation. The effect of saturation for $p:m = 4:1$ was $F(2,48) = 1,050.62$ and eta-squared was 83.9% with mean correlational values of .999+ at .8 saturation, .99+ at .6 saturation, and .92+ at .4 saturation.

The effect of extraction for $p:m = 12:1$ was $F(5,240) = 418.62$ and eta-squared was 21.9% with mean correlational values ranging from .99+ for no overextraction to .97+ for 9 overextractions. The effect of extraction for $p:m = 6:1$ was $F(4,192) = 239.61$ and eta-squared was 8.3% with mean correlational values ranging from .99+ for no overextraction to .98+ for 6 overextractions. The effect of extraction for $p:m = 4:1$ was $F(3,144) = 77.72$ and eta-squared was 2.5%

with mean correlational values of .99+ at all levels of extraction, but differing in the third decimal place with the highest value for no overextraction and the lowest for 3 overextractions.

The main effect of sample size for $p:m = 12:1$ was $F(3,48) = 56.29$ and eta-squared was 3.8% with mean correlational values of .99+ at all levels of sample size, but differing in the third decimal place with the highest value at $N=450$ and the lowest at $N=75$. The main effect of sample size for $p:m = 6:1$ was $F(3,48) = 46.04$ and eta-squared was 7.0% with mean correlational values ranging from .98+ at $N=75$ to .99+ at $N=450$. The main effect of sample size for $p:m = 4:1$ was $F(3,48) = 66.90$ and eta-squared was 8.0% with mean correlational values ranging from .98+ at $N=75$ to .99+ at $N=450$.

The extraction X saturation interaction effect for $p:m = 12:1$ was $F(10,240) = 16.80$ and eta-squared was 1.8%. Figure 12 displays this interaction. The extraction X saturation interaction effect for $p:m = 6:1$ was $F(8,192) = 25.87$ and eta-squared was 1.8%. Figure 13 displays this interaction. The extraction X saturation interaction effect for $p:m = 4:1$ was $F(6,144) = 19.34$ and eta-squared was 1.2%. Figure 14 displays this interaction.

And there was a single small extraction X sample size interaction effect for the convergent score correlations in the $p:m = 12:1$ analysis with $F(15,240) = 6.77$ and eta-squared was 1.1%. Figure 15 displays this interaction.

Figure 12. Extraction X Saturation Interaction of Between Method
Convergent Score Correlations in the Overextraction Situation:
 $p:m = 12:1$.

Convergent Scores--Overextraction

Extraction X Saturation, p:m=12:1

Between Methods Comparison

Figure 13. Extraction X Saturation Interaction of Between Method
Convergent Score Correlations in the Overextraction Situation:
 $p:m = 6:1$.

Convergent Scores--Overextraction

Extraction X Saturation, p:m=6:1

Between Methods Comparison

Figure 14. Extraction X Saturation Interaction of Between Method
Convergent Score Correlations in the Overextraction Situation:
 $p:m = 4:1$.

Convergent Scores--Overextraction

Extraction X Saturation, p:m=4:1

Between Methods Comparison

Figure 15. Extraction X Sample Size Interaction of Between Method
Convergent Score Correlations in the Overextraction Situation:
 $p:m = 12:1$.

Convergent Scores--Overextraction

Extraction X Sample Size, p:m=12:1

Between Methods Comparison

Analysis of Divergent Correlations (DRO)

Three ANOVAs also examined the divergent score correlations between methods for the cases when the true $p:m$ ratio was 12:1 (high), 6:1 (moderate), and 4:1 (low). There was a moderate to very strong main effect in each of these analyses for saturation, a strong to moderate main effect for extraction, and a small to moderate main effect for sample size. There was also a small extraction X saturation interaction effect and a very small extraction X sample size interaction effect in each ANOVA.

The saturation main effect for $p:m = 12:1$ was $F(2,48) = 557.23$ and eta-squared was 15.5% with mean correlational values of .01- at .8 saturation, .02- at .6 saturation, and .05- at .4 saturation. The saturation main effect for $p:m = 6:1$ was $F(2,48) = 702.22$ and eta-squared was 39.5% with mean correlational values of .01- at .8 saturation, .01- at .6 saturation, and .04- at .4 saturation. The saturation main effect for $p:m = 4:1$ was $F(2,48) = 1,010.25$ and eta-squared was 63.1% with mean correlational values of .001- at .8 saturation, .01- at .6 saturation, and .04- at .4 saturation.

The extraction main effect for $p:m = 12:1$ was $F(5,240) = 1,332.34$ and eta-squared was 71.9% with mean correlational values ranging from .001- for no overextraction to .07- for 9 overextractions. The extraction main effect for $p:m = 6:1$ was $F(4,192) = 1,235.27$ and eta-squared was 39.3% with mean correlational values ranging from .001- for no overextraction to .04- for 6 overextractions. The extraction main effect for $p:m = 4:1$ was $F(3,144) = 425.46$ and eta-squared was 13.5% with mean correlational

values ranging from .01- for no overextraction to .02- for 3 overextractions.

The sample size main effect for $p:m = 12:1$ was $F(3,48) = 48.51$ and eta-squared was 2.0% with mean correlational values ranging from .03- at $N=75$ to .01- at $N=450$. The sample size main effect for $p:m = 6:1$ was $F(3,48) = 99.54$ and eta-squared was 8.4% with mean correlational values ranging from .02- at $N=75$ to .01- at $N=450$. The sample size main effect for $p:m = 4:1$ was $F(3,48) = 141.89$ and eta-squared was 13.3% with mean correlational values ranging from .02- at $N=75$ to .01- at $N=450$.

The extraction X saturation interaction effect for $p:m = 12:1$ was $F(10,240) = 43.73$ and eta-squared was 4.7%. Figure 16 displays this interaction. The extraction X saturation interaction effect for $p:m = 6:1$ was $F(8,192) = 91.93$ and eta-squared was 5.8%. Figure 17 displays this interaction. The extraction X saturation interaction effect for $p:m = 4:1$ was $F(6,144) = 57.95$ and eta-squared was 3.7%. Figure 18 displays this interaction.

The extraction X sample size interaction effect for $p:m = 12:1$ was $F(15,240) = 11.35$ and eta-squared was 1.8%. Figure 19 displays this interaction. The extraction X sample size interaction effect for $p:m = 6:1$ was $F(12,192) = 26.53$ and eta-squared was 2.5%. Figure 20 displays this interaction. The extraction X sample size interaction effect for $p:m = 4:1$ was $F(9,144) = 19.82$ and eta-squared was 1.9%. Figure 21 displays this interaction.

Full ANOVA tables for all of the preceding analyses are presented in Appendices I and J. Tables presenting and comparing mean

Figure 16. Extraction X Saturation Interaction of Between Method
Divergent Score Correlations in the Overextraction Situation:
 $p:m = 12:1.$

Divergent Scores--Overextraction

Extraction X Saturation, p:m=12:1
Between Methods Comparison

Figure 17. Extraction X Saturation Interaction of Between Method
Divergent Score Correlations in the Overextraction Situation:
 $p:m = 6:1$.

Divergent Scores--Overextraction

Extraction X Saturation, p:m=6:1

Between Methods Comparison

Figure 18. Extraction X Saturation Interaction of Between Method

Divergent Score Correlations in the Overextraction Situation:

$\underline{p:m} = 4:1.$

Divergent Scores--Overextraction

Extraction X Saturation, p:m=4:1

Between Methods Comparison

Figure 19. Extraction X Sample Size Interaction of Between Method
Divergent Score Correlations in the Overextraction Situation:
 $p:m = 12:1$.

Divergent Scores--Overextraction

Extraction X Sample Size, p:m=12:1

Between Methods Comparison

Figure 20. Extraction X Sample Size Interaction of Between Method

Divergent Score Correlations in the Overextraction Situation:

$p:m = 6:1$.

Divergent Scores--Overextraction
Extraction X Sample Size, p:m=6:1
Between Methods Comparison

Figure 21. Extraction X Sample Size Interaction of Between Method
Divergent Score Correlations in the Overextraction Situation:
 $p:m = 4:1$.

Divergent Scores--Overextraction
Extraction X Sample Size, p:m=4:1
Between Methods Comparison

correlational values of significant main effects for both CRO and DRO values are contained in Appendices S and T.

Discussion

The results of this study argue for the similarity of component and factor scores during overextraction, even as maximal overextraction takes place. A previous study (Fava & Velicer, in press) had shown that for all practical purposes component and factor scores were nearly identical when they were based on an accurate representation of their respective factor patterns. This study extends those results to cases when component and factor scores are based on inaccurate, though idealized, factor patterns in the overextracted direction. Given the results of Study 1 it would have been surprising to find extreme divergence among the two score methods in light of the Fava and Velicer (in press) results. Further, as there is a strong body of evidence that there is little difference between factor and component analysis, and as Study 1 showed that even under maximal overextraction both methods appear to reproduce their respective patterns reasonably well via the proxy of scores, and given the Fava and Velicer (in press) results for similarity among score methods, the results of Study 3 also support the similarity of strong and ideal factor and component patterns under maximal overextraction.

Another interesting result suggested by Study 3 is displayed in the graphs pictured in Figures 12 through 21. Although those interactions are all significant, there is no dramatic loss of information suggested in these displays. However, it does seem that

there may be a sort of asymptotic effect in evidence to explain the cause of the interaction. The lower loadings and lower sample sizes reproduce less well than the higher loadings and sample sizes before overextraction takes place, but as overextraction continues the higher loading and sample size conditions degrade more rapidly than the lower, and further they begin to approach the values of the lower loading and sample size conditions. This suggests that under idealized conditions there may be some bounds as to how far component and factor scores can diverge during overextraction.

There are some cautions about the generalizability of these results. These include the use of ideal (equal loading, equal number of variables per factor, and no item complexity or uniqueness) factor and component patterns as the basis for the baseline sample matrices that were used in the overextraction situations. More complex factor patterns that include unequal loadings, unequal numbers of variables per factor or component, and complex and unique items were not examined. These results also report only on the varimax rotational position although the results were extremely similar for the promax rotational position, values being very slightly worse for the promax condition. Nevertheless, these results do form a strong extension of previous results and a firm foundation for further examination of related score issues.

CHAPTER VI

Study 4

Underextraction of Components and Factors: Between Methods Comparison

Study 4 examined the differences between the methods of principal component analysis and maximum likelihood factor analysis as factors and components were systematically underextracted. This study was undertaken for several reasons. First, it would extend the results of Fava and Velicer (in press), which had examined the relationships among component and factor scores in their 'true' configural patterns, into the underextracted direction. Second, if the results of Study 2 suggested that scores within methods did not change substantially with underextraction, it would be important to find out if between method score relationships also did not change substantially. Third, if the results of Study 2 suggested that scores did change substantially within methods as underextraction occurred, it would be important to find out if between method score relationships which were high at baseline would remain high with underextraction or would diverge. And fourth, if there was differential reproduction of scores within methods, it would be very important to examine how the between method score correlations behaved.

Method

The factors of interest in the ANOVA designs in this study included 3 between factors (saturation, sample size and rotation) and

1 within factor (the number of underextractions). Saturation was examined at three levels: .4 (low), .6 (moderate), and .8 (high). Sample size was manipulated at four levels: 75 (low), 150, 225 and 450 (high). Rotational position was examined at varimax (orthogonal) and promax (oblique). The number of underextractions is dependent upon the true $p:m$ ratio of the sample matrices. Table 2 notes what values of m will be examined for the appropriate $p:m$ ratio. Dependent measures that were examined in the various cells of the study design are the CRU and DRU summary statistics.

Repeated measures analysis of variance was chosen as the preferred method of examining the CRU and DRU summary statistics. The Fisher Z transformation of correlational values was chosen to help overcome the non-normal data distributions that can occur with extremely high or extremely low correlational values. Two ANOVAs were planned to examine the CRU values and two ANOVAs were planned to examine the DRU values, one for each true $p:m$ ratio which formed the context for the number of underextractions. These ANOVAs will be used to help describe the pattern of results that are formed by the various cells of the design.

As in Studies 1, 2 and 3, there was extremely high power in the ANOVA designs of this study, and the criteria used in these earlier studies to determine an interpretable factor of interest were also employed in this study, i.e. eta-squared for a factor had to equal at least 1% along with a probability value $<.0001$ for that factor. In total 1080 convergent score correlations and 1080 divergent score correlations were analyzed in the ANOVA designs of this study.

Results

Analysis of Convergent Correlations (CRU)

Two ANOVAs examined the convergent score correlations between methods for the cases when the true $p:m$ ratio was 6:1 (moderate) and 4:1 (low). In each of these analyses there was a very strong main effect for extraction, and a small main effect for saturation. There was a strong extraction X saturation interaction and a small extraction X sample size interaction for each ANOVA. There was also a small saturation X sample size interaction and a small extraction X saturation X sample size interaction for the $p:m = 4:1$ situation.

The extraction main effect for $p:m = 6:1$ was $F(3,288) = 1,171.42$ and eta-squared was 60.8% with mean correlational values ranging from .99+ at no underextraction to .75+ at 3 underextractions. The extraction main effect for $p:m = 4:1$ was $F(4,384) = 834.66$ and eta-squared was 52.3% with mean correlational values ranging from .99+ at no underextraction to .69+ at 6 underextractions.

The effect of saturation for $p:m = 6:1$ was $F(2,96) = 70.95$ and eta-squared was 4.9% with mean correlational values of .97+ at .8 saturation, .96+ at .6 saturation, and .91+ .4 saturation. The effect of saturation for $p:m = 4:1$ was $F(2,96) = 129.17$ and eta-squared was 8.1% with mean correlational values of .97+ at .8 saturation, .95+ at .6 saturation, and .87+ at .4 saturation.

The extraction X saturation interaction effect for the convergent score correlations for $p:m = 6:1$ was $F(6,288) = 205.85$ and eta-squared was 21.4%. Figure 22 displays this interaction. The extraction X saturation interaction effect for $p:m = 4:1$ was $F(8,384)$

Figure 22. Extraction X Saturation Interaction of Between Method

Convergent Score Correlations in the Underextraction Situation:

$p:m = 6:1.$

Convergent Scores--Underextraction

Extraction X Saturation, p:m=6:1

Between Methods Comparison

Figure 23. Extraction X Saturation Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$.

Convergent Scores--Underextraction

Extraction X Saturation, p:m=4:1

Between Methods Comparison

= 187.33 and eta-squared was 23.5%. Figure 23 displays this interaction.

The extraction X sample size interaction effect for the convergent score correlations for $p:m = 6:1$ was $F(9,288) = 16.86$ and eta-squared was 2.6%. Figure 24 displays this interaction. The extraction X sample size interaction effect for $p:m = 4:1$ was $F(12,384) = 20.30$ and eta-squared was 3.8%. Figure 25 displays this interaction.

The saturation X sample size interaction effect for the convergent score correlations for the $p:m = 4:1$ situation was $F(6,96) = 7.55$ and eta-squared was 1.4%. A graphical analysis of this interaction is displayed in Figure 26.

The extraction X saturation X sample size interaction for the convergent score correlations for the $p:m = 4:1$ situation was $F(24,384) = 4.44$ and eta-squared was 1.7%. Graphical analyses examine this complex interaction in two ways. Extraction X sample size at different levels of saturation are displayed in Figures 27, 28 and 29. Extraction X saturation at different levels of sample size are displayed in Figures 30 to 33.

Analysis of Divergent Correlations (DRU)

Two ANOVAs also examined the divergent score correlations between methods for the cases when the true $p:m$ ratio was 6:1 (moderate) and 4:1 (low). In each of these analyses there was a very strong main effect for extraction, a small effect for saturation, and a small main effect for sample size. There was also a very small main

Figure 24. Extraction X Sample Size Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 6:1$.

Convergent Scores--Underextraction

Extraction X Sample Size, $p:m=6:1$

Between Methods Comparison

Figure 25. Extraction X Sample Size Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$.

Convergent Scores--Underextraction
 Extraction X Sample Size, p:m=4:1
 Between Methods Comparison

Figure 26. Saturation X Sample Size Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$.

Convergent Scores--Underextraction

Saturation X Sample Size, p:m=4:1

Between Methods Comparison

Figure 27. Extraction X Sample Size Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $a_{ij} = .8$.

Convergent Scores--Underextraction

Extraction X Sample Size, $p:m=4:1$ at .8

Between Methods Comparison

Figure 28. Extraction X Sample Size Interaction of Between Method
 Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $a_{ij} = .6$.

Convergent Scores--Underextraction

Extraction X Sample Size, p:m=4:1 at .6

Between Methods Comparison

Figure 29. Extraction X Sample Size Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $a_{ij} = .4$.

Convergent Scores--Underextraction
 Extraction X Sample Size, p:m=4:1 at .4
 Between Methods Comparison

Figure 30. Extraction X Saturation Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $N = 450$.

Convergent Scores--Underextraction

Extraction X Saturation, p:m=4:1, N=450

Between Methods Comparison

Figure 31. Extraction X Saturation Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $N = 225$.

Convergent Scores--Underextraction

Extraction X Saturation, p:m=4:1, N=225

Between Methods Comparison

Figure 32. Extraction X Saturation Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $N = 150$.

Convergent Scores--Underextraction

Extraction X Saturation, p:m=4:1, N=150

Between Methods Comparison

Figure 33. Extraction X Saturation Interaction of Between Method
Convergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $N = 75$.

Convergent Scores--Underextraction

Extraction X Saturation, p:m=4:1, N=75

Between Methods Comparison

effect for rotation in the $p:m = 4:1$ situation. In each analysis there was a strong extraction X saturation interaction effect, and a small extraction X rotation interaction effect. There was also a small extraction X saturation X sample size interaction in the $p:m = 4:1$ situation.

The effect of extraction for $p:m = 6:1$ was $F(3,288) = 664.30$ and eta-squared was 57.2% with mean correlational values ranging from .01- at no underextraction to .09- at 3 underextractions. The effect of extraction for $p:m = 4:1$ was $F(4,384) = 648.59$ and eta-squared was 50.1% with mean correlational values ranging from .01- at no underextraction to .10- at 6 underextractions.

The main effect for saturation for the divergent score correlations for $p:m = 6:1$ was $F(2,96) = 33.06$ and eta-squared was 1.9% with mean correlational values of .02- at .8 saturation, .02- at .6 saturation, and .04- at .4 saturation. The saturation main effect for $p:m = 4:1$ was $F(2,96) = 105.86$ and eta-squared was 5.0% with mean correlational values of .03- at .8 saturation, .03- at .6 saturation, and .05- at .4 saturation.

The main effect for sample size for the divergent score correlations for $p:m = 6:1$ was $F(3,96) = 12.96$ and eta-squared was 1.1% with mean correlational values ranging from .03- at $N=75$ to .02- at $N=450$. The main effect for sample size for $p:m = 4:1$ was $F(3,96) = 22.82$ and eta-squared was 1.6% with mean correlational values ranging from .04- at $N=75$ to .03- at $N=450$.

The main effect in the $p:m = 4:1$ situation for rotation was $F(1,96) = 43.37$ and eta-squared was 1.0%. In this underextraction

situation a varimax rotation slightly improves the divergent score correlations over a promax rotation with mean values of .03- vs. .04-.

The extraction X saturation interaction effect for the divergent score correlations for $p:m = 6:1$ was $F(6,288) = 113.10$ and eta-squared was 19.5%. Figure 34 displays this interaction. The extraction X saturation interaction effect for $p:m = 4:1$ was $F(8,384) = 141.98$ and eta-squared was 21.9%. Figure 35 displays this interaction.

The extraction X sample size interaction effect for the divergent score correlations for $p:m = 6:1$ was $F(9,288) = 17.42$ and eta-squared was 4.5%. Figure 36 displays this interaction. The extraction X sample size interaction effect for $p:m = 4:1$ was $F(12,384) = 22.22$ and eta-squared was 5.1%. Figure 37 displays this interaction.

The extraction X rotation interaction effect for the divergent score correlations for $p:m = 6:1$ was $F(3,288) = 15.99$ and eta-squared was 1.4%. Figure 38 displays this interaction. The extraction X rotation interaction effect for $p:m = 4:1$ was $F(4,384) = 17.93$ and eta-squared was 1.4%. Figure 39 displays this interaction.

And lastly, there was a small extraction X saturation X sample size interaction for the divergent score correlations for the $p:m = 4:1$ situation with $F(24,384) = 4.85$ and eta-squared equal to 2.2%. Graphical analyses examine this complex interaction in two ways. Extraction X sample size at different levels of saturation are displayed in Figures 40, 41 and 42. Extraction X saturation at different levels of sample size are displayed in Figures 43 to 46.

Full ANOVA tables for all of the preceding analyses are

Figure 34. Extraction X Saturation Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 6:1$.

Divergent Scores--Underextraction

Extraction X Saturation, p:m=6:1

Between Methods Comparison

Figure 35. Extraction X Saturation Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$.

Divergent Scores--Underextraction

Extraction X Saturation, p:m=4:1

Between Methods Comparison

Figure 36. Extraction X Sample Size Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 6:1$.

Divergent Scores--Underextraction

Extraction X Sample Size, p:m=6:1

Between Methods Comparison

Figure 37. Extraction X Sample Size Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$.

Divergent Scores--Underextraction

Extraction X Sample Size, $p:m=4:1$

Between Methods Comparison

Figure 38. Extraction X Rotation Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 6:1$.

Divergent Scores--Underextraction

Extraction X Rotation, p:m=6:1

Between Methods Comparison

Figure 39. Extraction X Rotation Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$.

Divergent Scores--Underextraction

Extraction X Rotation, p:m=4:1

Between Methods Comparison

Figure 40. Extraction X Sample Size Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $a_{ij} = .8$.

Divergent Scores--Underextraction
Extraction X Sample Size, $p:m=4:1$ at .8
Between Methods Comparison

Figure 41. Extraction X Sample Size Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $a_{ij} = .6$.

Divergent Scores--Underextraction
Extraction X Sample Size, p:m=4:1 at .6
Between Methods Comparison

Figure 42. Extraction X Sample Size Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $a_{ij} = .4$.

Divergent Scores--Underextraction
Extraction X Sample Size, p:m=4:1 at .4
Between Methods Comparison

Figure 43. Extraction X Saturation Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $N = 450$.

Divergent Scores--Underextraction
Extraction X Saturation, p:m=4:1, N=450
Between Methods Comparison

Divergent Scores--Underextraction

Extraction X Saturation, p:m=4:1, N=225

Between Methods Comparison

Figure 44. Extraction X Saturation Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $N = 225$.

Divergent Scores--Underextraction
Extraction X Saturation, p:m=4:1, N=225
Between Methods Comparison

Figure 45. Extraction X Saturation Interaction of Between Method
 Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $N = 150$.

Divergent Scores--Underextraction

Extraction X Saturation, p:m=4:1, N=150

Between Methods Comparison

Figure 46. Extraction X Saturation Interaction of Between Method
Divergent Score Correlations in the Underextraction Situation:
 $p:m = 4:1$ at $N = 75$.

Divergent Scores--Underextraction
Extraction X Saturation, p:m=4:1, N=75
Between Methods Comparison

presented in Appendices K and L. Tables presenting and comparing mean correlational values of significant main effects for both CRU and DRU values are contained in Appendices U and V.

Discussion

This study of the effects of underextraction between the methods of principal component analysis and maximum likelihood factor analysis produced many statistically significant effects, but only a few had clear practical significance. Perhaps the most striking result, which contrasts with the results of Study 3, is that the principal component score and the factor score tend to rapidly become different as underextraction takes place (see Tables U-1 and V-1 in the appendices). While the effect seems less pronounced in the $p:m = 4:1$ situation, if one examines the results with respect to proportional rates of extraction, i.e. 3 underextractions in the $p:m = 4:1$ analyses versus 2 underextractions in the $p:m = 6:1$ analyses, thus allowing for 2/3 of the original number of factors in each of the analyses, it becomes clear that the component and factor convergent and divergent scores depart from each other at approximately the same pace for each $p:m$ situation. The results of Study 2 suggested similar results for the ability of each method to reproduce its own score matrix, i.e. after the 1st interaction there was rapid, though proportional, deterioration across $p:m$ conditions. Study 2 also suggested principal component scores were slightly better at reproducing their score matrix than were factor scores. Given the departure of component and factor scores from each other during underextraction conditions in

this study and given the results of Study 2 which suggest factor scores reproduce their original score matrix less well during underextraction than component scores, it appears that maximal information will be lost if factor analysis and factor scores are used during the underextraction of factors.

Another very interesting result concerns the series of two way interactions involving extraction X saturation and extraction X sample size displayed in figures 22 to 25 and 34 to 37, and the three way interaction involving extraction X saturation X sample size for the $p:m = 4:1$ situation and displayed in Figures 27 to 33 and 40 to 46. These interaction effects are primarily driven by the difference of the first level of extraction (i.e., no underextraction) with the other levels of extraction. At the first level of extraction both saturation and sample size operate to distinctly separate cell values, with higher levels of saturation and sample size resulting in both higher convergent score correlations and lower divergent score correlations. Starting with the first underextraction, however, cell values across all conditions get closer as a steady overall worsening of cell values occurs with greater underextraction.

Taken in combination with Study 2, the results of Study 4 argue against the use of factor scores when underextraction of factors is a strong possibility. The growing divergence of score correlations between principal component analysis and maximum likelihood factor analysis as underextraction gets more extreme as demonstrated in this study, is probably partly the result of the inability of factor analysis to reproduce its original score matrix as well as component

analysis as demonstrated in Study 2. And while neither maximum likelihood factor analysis nor principal component analysis does a very good job as underextraction takes place, since the original information deteriorates quickly with each extra underextraction, principal component analysis appears to retain a greater proportion of the original true information.

Some cautions pertaining to generalizability mentioned in the previous studies also apply to Study 4. Primarily this involves the use of idealized factor patterns as the basis of comparing score correlations. Given the rapid divergence of score methods using idealized factor patterns under conditions of underextraction, however, it is likely that this divergence would be at least as great with more complex factor and component patterns, as well as those patterns that might include complex or unique items. Despite these cautions the results do form a strong extension of previous results (Fava & Velicer, in press) and a firm foundation for further examination of related score issues.

CHAPTER VII

Study 5

Between Methods Comparison of Component and Factor Scores at Baseline

Study 5 is a partial replication of Fava and Velicer (in press) and examined the differences between the score methods of principal component analysis and maximum likelihood factor analysis in the true component and factor situation. This study examined the baseline condition used in Studies 3 and 4. Analyses used in this study were based on Fisher Z transformations of correlations as opposed to the actual correlational value. This is a more 'correct' approach to handling the non-normal data distribution that can occur with extremely high or extremely low correlational values. Comparable results would substantiate and enhance the results of Fava and Velicer (in press).

Method

The factors of interest in the ANOVA designs used in this study included 3 between factors ($p:m$ ratio, saturation, and sample size). $p:m$ ratio was examined at three levels: 12:1 (high ratio), 6:1 (moderate ratio), and 4:1 (low ratio). Saturation was examined at three levels: .4 (low), .6 (moderate), and .8 (high). Sample size was manipulated at four levels: 75 (low), 150, 225 and 450 (high). Dependent measures that were examined in the various cells of the study design are the CRO and DRO summary statistics.

Analysis of variance was chosen as the preferred method of

examining the CRO and DRO summary statistics. The Fisher Z transformation of correlational values was chosen to help overcome the non-normal data distributions that can occur with extremely high or extremely low correlational values. One ANOVA was planned to examine the CRO values and one ANOVA was planned to examine the DRO values. These ANOVAs were used to help describe the pattern of results that were formed by the various cells of the design.

As in the previous 4 studies, there was extremely high power in the ANOVA designs of this study. Criteria used in the earlier studies to determine an interpretable factor for discussion were also employed in Study 5. These criteria were that eta-squared for a factor had to equal at least 1%, and in addition a factor had to be statistically significant at a probability level of $p < .0001$. In total, 180 convergent score correlations and 180 divergent score correlations were analyzed in the ANOVA designs of this study.

Results

Analysis of Convergent Correlations (CRO)

One ANOVA examined the convergent score correlations between methods. In this analysis there was a significant main effect for each factor.

There was an extremely strong main effect for saturation with $F(2,144) = 2,434.32$ and eta-squared was 85.7% with mean correlational values of .9999+ at .8 saturation, .99+ at .6 saturation, and .96+ at .4 saturation.

There was a moderate main effect for sample size with $F(3,144) =$

154.87 and eta-squared was 8.2% with mean correlational values ranging from .99+ at $N=75$ to .999+ at $N=450$.

And there was a small main effect for $p:m$ ratio with $F(2,144) = 49.74$ and eta-squared was 1.8% with mean correlational values of .99+ at each $p:m$ level. Differences occurred in the third decimal place with the highest value occurring in the $p:m = 12:1$ situation and the lowest value occurring in the $p:m = 4:1$ situation.

Analysis of Divergent Correlations (DRO)

One ANOVA also examined the divergent score correlations between methods. In this analysis there was a significant main effect for each factor, as well as one small saturation X sample size interaction effect.

There was a very strong main effect for saturation with $F(2,144) = 1054.70$ and eta-squared was 62.8% with mean correlational values of .0001- at .8 saturation, .001- at .6 saturation, and .02- at .4 saturation.

There was a moderate main effect for sample size with $F(3,144) = 189.42$ and eta-squared was 16.9% with mean correlational values ranging from .01- at $N=75$ to .001- at $N=450$.

There was a moderate main effect for $p:m$ ratio with $F(2,144) = 209.95$ and eta-squared was 12.5% with mean correlational values of .001- at $p:m = 12:1$, .001- at $p:m = 6:1$, and .01- at $p:m = 4:1$.

There was also a small saturation X sample size interaction with $F(6,144) = 10.56$ and eta-squared was 1.9%. A graphical analysis of this interaction is displayed in Figure 47.

Figure 47. Saturation X Sample Size Interaction of Between Method
Divergent Score Correlations in the True Component and Factor
Situation.

Divergent Scores--Correct Pattern Saturation X Sample Size

Full ANOVA tables for the preceding analyses are presented in Appendices M and N. Tables presenting and comparing mean correlational values of significant main effects for both CRO and DRO values are contained in Appendices W and X.

Discussion

Study 5 provides alternative confirmation for the major results of Fava and Velicer (in press). While Fava and Velicer (in press) was wider in scope as it compared two types of component scores, three types of factor scores, and a scale score within a $p = 72$ and $p = 36$ design, the results were essentially generalizable to a $p = 36$, two score comparison as used in Study 5, as all of the six score types correlated highly and there was essentially no difference between the $p = 72$ and $p = 36$ conditions. Study 5, however, used a more preferred method of comparing correlations as it employed the Fisher Z transformation.

Given the above caveats the results of Study 5 mirror the results of Fava and Velicer (in press). The major finding is the very strong similarity between the principal component score and the factor analysis score over a wide range of factor levels, when using idealized factor patterns as the basis for scores. Saturation is the major determinant of score correlation differences, but at least with regard to idealized factor and component patterns, the differences are not practically significant. The other significant effects in this study are also noted for their lack of practical differences despite being statistically significant. And finally, where there is

departure in the score methods it tends to occur in cells with low saturation and low sample size, situations which are not preferred by the applied researcher.

There are several cautions worth noting with regard to the results of this study. This study used idealized factor and component patterns as the basis of comparing score correlations and did not examine more complex factor and component patterns, as well as those patterns that might include complex or unique items. Nevertheless, these results do provide strong confirmation of the major results of Fava and Velicer (in press).

CHAPTER VIII

Summary Discussion

At one level this series of studies supported the general hypotheses laid out in the initial introduction. At another level there have been several specific results that provide a basis for more detailed guidelines and recommendations that should help researchers who are concerned that they may extract too many or too few factors or components using maximum likelihood factor analysis or principal component analysis.

In accordance with the general hypotheses, there was an implied overall degradation of the true factor and component solutions as judged by the worsening of score correlations when overextraction and underextraction occurred. Further, this degradation was much more severe when underextraction occurred.

As noted earlier, the degradation in the factor and component patterns may be worse than what is actually indicated by the score correlations in that pattern loadings could numerically decrease or degrade, but the basic configural information might possibly remain and preserve the basic score relationships. While visual examination of some samples suggested patterns remain clear for the highest loading samples during overextraction, such visual inspection was less clear for lower loading and lower sample size combinations. The sheer number of samples, as well as computer resource problems, limited the completeness of such a visual evaluation at this time. Despite this caution perhaps the greatest surprise of this series of studies

occurred in Study 1, where it appeared that principal component analysis and maximum likelihood factor analysis are much more robust with regard to overextraction than previously thought by some researchers (Gorsuch, 1983), and as noted in the previous discussion to Study 1. Specific recommendations based on Study 1 are also addressed in that discussion. In general, the advice that erring on the side of overextraction would not change important or substantial factors (Cattell, 1958; Comrey, 1973; Thurstone, 1947) can be adhered to when employing strong and clear factor patterns. Patterns of a more complex nature are potentially more troublesome and should be examined carefully and systematically when trying to establish the 'correct' configural pattern of item loadings.

The most serious problem was encountered in Study 2 which examined underextraction. While this was expected, the degree of the problem had not been previously noted in any empirical manner. The results of Study 2 suggest that even the strongest factor and component patterns appear to suffer greatly after more than one underextraction, as judged by score correlations, with maximum likelihood factor analysis less likely to reproduce its true score matrix. More specific implications of these findings appear in the discussion to Study 2.

With regard to comparisons of score correlations between principal component analysis and maximum likelihood factor analysis, there was very strong similarity when representing the true component and factor patterns, and also in the overextraction situation. Perhaps the second most surprising result, however, occurred with the

relatively lower score correlations found between maximum likelihood factor analysis and principal component analysis in the underextraction situations examined in Study 4. The results of Study 4, when examined in relation to the results of Study 2, where principal component analysis performed better than maximum likelihood factor analysis, suggest that principal component analysis would be a safer choice if underextraction of factors is feared. And further, given the strong similarity between the methods in the overextraction and true situations, it may be a wise choice to simply use principal component analysis in most situations.

The overall results of Studies 1 to 5 suggest that extraction was a strong factor in determining score matrix reproducibility. In addition, saturation generally proved itself to be an equally strong factor, and to a lesser extent sample size and variable-to-component (factor) ratio were consistent factors of importance in score matrix reproducibility. This was not altogether surprising as a number of other monte carlo type studies in the area of factor and component analysis (Fava & Velicer, in press; Guadagnoli & Velicer, 1988; Velicer & Fava, 1987, 1990) have consistently shown saturation and, to a lesser extent, sample size and variable-to-component (factor) ratio to be important in determining strong (and weak) factor and component patterns. Based on the present series of studies it can be safely suggested that level of extraction also plays a key role in determining strong (and weak) factor patterns.

Some cautions in interpreting the results of this series of studies have previously been noted in the specific discussions

attached to each individual study. Chief among these cautions concerns the use of idealized factor and component structures as the basis of the true pattern situation in each of these studies. It was not possible to examine more complex patterns at this time due to the already wide scope of the current series of studies, and due to the very large time and computer resources that were given over to these studies. These current results can be viewed as a first, but essential, step in examining the issues of overextraction and underextraction of factors and components. Future studies might employ 'better' methods of comparing patterns of loadings and could profitably utilize the more complex types of patterns that were not examined in these current studies to provide other invaluable empirical information regarding these issues. For example, while further degradation of scores and pattern structures might be expected using more complex patterns, it is not clear what the degree of degradation might be. Also it is not clear whether one type of complex pattern would lead to different results from another. Such questions are worthy of study and their elucidation would further help to refine our knowledge of the processes of component and factor analysis.

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Appendix A

Table A-1. ANOVA summary table comparing convergent score correlations within methods during overextraction: Original analysis, p:m = 12:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	535.97	267.98	946.06**	27.2%
N	3	56.28	18.76	66.23**	2.9%
SN	6	1.43	.24	.84	.1%
ERROR	48	13.60	.28		
Within					
EXTRAC	5	1185.70	237.14	3708.44**	60.1%
ES	10	110.62	11.06	172.99**	5.6%
EN	15	13.28	.89	13.84**	.7%
ESN	30	2.32	.08	1.21	.1%
ERROR	240	15.35	.06		
TYPE	1	8.96	8.96	94.16**	.5%
TS	2	.98	.49	5.14*	
TN	3	3.58	1.19	12.55**	.2%
TSN	6	2.18	.36	3.83*	.1%
ERROR	48	4.57	.10		
ET	5	2.53	.51	14.37**	.1%
ETS	10	5.13	.51	14.57**	.3%
ETN	15	1.40	.09	2.66*	.1%
ETSN	30	1.23	.04	1.17	.1%
ERROR	240	8.46	.04		
Total	719	1973.57			

*p<.01

**p<.0001

Table A-2. ANOVA summary table comparing convergent score correlations within methods during overextraction: Original analysis, p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	473.62	236.81	1327.09***	27.9%
N	3	69.73	23.24	130.25***	4.1%
SN	6	3.41	.57	3.19	.2%
ERROR	48	8.56	.18		
Within					
EXTRAC	4	974.22	243.56	6436.88***	57.3%
ES	8	120.55	15.07	398.23***	7.1%
EN	12	19.84	1.65	43.69***	1.2%
ESN	24	2.03	.08	2.23*	.1%
ERROR	192	7.26	.04		
TYPE	1	2.52	2.52	34.87***	.1%
TS	2	2.72	1.36	18.76***	.2%
TN	3	1.02	.34	4.69*	.1%
TSN	6	.36	.06	.83	_____
ERROR	48	3.48	.07		
ET	4	.70	.17	6.42**	_____
ETS	8	3.50	.44	16.04***	.2%
ETN	12	.64	.05	1.95	_____
ETSN	24	.64	.03	.97	_____
ERROR	192	5.23	.03		
Total	599	1700.02			

*p<.01

**p<.001

***p<.0001

Table A-3. ANOVA summary table comparing convergent score correlations within methods during overextraction: Original analysis, p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	339.39	169.70	1003.41***	24.3%
N	3	52.93	17.64	104.32***	3.8%
SN	6	5.20	.87	5.13**	.4%
ERROR	48	8.12	.17		
Within					
EXTRAC	3	836.90	278.97	6975.47***	60.0%
ES	6	114.67	19.11	477.87***	8.2%
EN	9	17.99	2.00	49.98***	1.3%
ESN	18	1.98	.11	2.75**	.1%
ERROR	144	5.76	.04		
TYPE	1	1.54	1.54	43.32***	.1%
TS	2	2.20	1.10	30.99***	.2%
TN	3	.52	.17	4.88*	-----
TSN	6	.27	.04	1.26	-----
ERROR	48	1.70	.04		
ET	3	.61	.20	11.68***	-----
ETS	6	1.18	.20	11.24***	.1%
ETN	9	.36	.04	2.27	-----
ETSN	18	.31	.02	.99	-----
ERROR	144	2.51	.02		
Total	479	1394.14			

*p<.01

**p<.001

***p<.0001

Appendix B

Table B-1. ANOVA summary table comparing divergent score correlations within methods during overextraction: Original analysis, p:m = 12:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	161.77	80.88	767.19***	10.4%
N	3	29.00	9.67	91.69***	1.9%
SN	6	.54	.09	.86	—
ERROR	48	5.06	.11		
Within					
EXTRAC	5	1289.52	257.90	10134.07***	83.3%
ES	10	33.01	3.30	129.72***	2.1%
EN	15	6.63	.44	17.36***	.4%
ESN	30	1.67	.06	2.18**	.1%
ERROR	240	6.11	.03		
TYPE	1	5.73	5.73	201.31***	.4%
TS	2	.21	.11	3.71	—
TN	3	1.25	.42	14.66***	.1%
TSN	6	.67	.11	3.90*	—
ERROR	48	1.37	.03		
ET	5	1.21	.24	15.68***	.1%
ETS	10	.43	.04	2.79*	—
ETN	15	.40	.03	1.74	—
ETSN	30	.60	.02	1.29	—
ERROR	240	3.70	.02		
Total	719	1548.86			

*p<.01

**p<.001

***p<.0001

Table B-2. ANOVA summary table comparing divergent score correlations within methods during overextraction: Original analysis, p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	161.73	80.86	1049.29*	11.8%
N	3	32.65	10.88	141.20*	2.4%
SN	6	1.14	.19	2.47	.1%
ERROR	48	3.70	.08		
Within					
EXTRAC	4	1106.22	276.55	17836.74*	80.9%
ES	8	41.35	5.17	333.41*	3.0%
EN	12	8.95	.75	48.10*	.7%
ESN	24	.62	.03	1.67	-----
ERROR	192	2.98	.02		
TYPE	1	2.30	2.30	99.61*	.2%
TS	2	.64	.32	13.94*	-----
TN	3	.74	.25	10.64*	.1%
TSN	6	.35	.06	2.50	-----
ERROR	48	1.11	.02		
ET	4	.75	.19	21.80*	.1%
ETS	8	.34	.04	4.89*	-----
ETN	12	.39	.03	3.76*	-----
ETSN	24	.26	.01	1.27	-----
ERROR	192	1.66	.01		
Total	599	1367.88			

*p<.0001

Table B-3. ANOVA summary table comparing divergent score correlations within methods during overextraction: Original analysis, p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	130.93	65.47	1182.83**	11.1%
N	3	25.14	8.38	151.41**	2.1%
SN	6	2.27	.38	6.84**	.2%
ERROR	48	2.66	.06		
Within					
EXTRAC	3	956.09	318.70	27549.25**	81.2%
ES	6	44.19	7.37	636.68**	3.8%
EN	9	8.57	.95	82.36**	.7%
ESN	18	.81	.05	3.90**	.1%
ERROR	144	1.67	.01		
TYPE	1	1.69	1.69	127.38**	.1%
TS	2	.31	.16	11.75*	—
TN	3	.36	.12	9.10*	—
TSN	6	.04	.01	.47	—
ERROR	48	.64	.01		
ET	3	.59	.20	33.70**	.1%
ETS	6	.15	.02	4.19*	—
ETN	9	.19	.02	3.55*	—
ETSN	18	.08	.00	.80	—
ERROR	144	.84	.01		
Total	479	1177.22			

*p<.001

**p<.0001

Appendix C

Table C-1. ANOVA summary table comparing convergent score correlations within methods during overextraction: Reanalysis. p:m = 12:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	643.16	321.58	946.06**	61.9%
N	3	67.53	22.51	66.23**	6.5%
SN	6	1.71	.29	.84	.2%
ERROR	48	16.32	.34		
Within					
EXTRAC	4	251.51	62.88	956.02**	24.2%
ES	8	3.43	.43	6.52**	.3%
EN	12	2.02	.17	2.56*	.2%
ESN	24	2.03	.08	1.29	.2%
ERROR	192	12.63	.07		
TYPE	1	10.75	10.75	94.16**	1.0%
TS	2	1.17	.59	5.14*	.1%
TN	3	4.30	1.43	12.55**	.4%
TSN	6	2.62	.44	3.83*	.3%
ERROR	48	.64	.01		
ET	4	.74	.18	4.70*	.1%
ETS	8	4.94	.62	15.71**	.5%
ETN	12	.69	.06	1.46	.1%
ETSN	24	.80	.03	.85	.1%
ERROR	192	7.54	.04		
Total	599	1039.38			

*p<.01

**p<.0001

Table C-2. ANOVA summary table comparing convergent score correlations within methods during overextraction: Reanalysis, p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	592.02	296.01	1327.09**	69.7%
N	3	87.16	29.05	130.25**	10.3%
SN	6	4.26	.71	3.19	.5%
ERROR	48	10.71	.22		
Within					
EXTRAC	3	123.83	41.28	1160.13**	14.6%
ES	6	2.14	.36	10.03**	.3%
EN	9	2.41	.27	7.52**	.3%
ESN	18	1.17	.07	1.83	.1%
ERROR	144	5.12	.04		
TYPE	1	3.16	3.16	34.87**	.4%
TS	2	3.40	1.70	18.76**	.4%
TN	3	1.27	.42	4.69*	.1%
TSN	6	.45	.07	.83	.1%
ERRCR	48	4.34	.09		
ET	3	.07	.02	.75	—
ETS	6	2.82	.47	15.49**	.3%
ETN	9	.38	.04	1.41	—
ETSN	18	.55	.03	1.00	.1%
ERROR	144	4.36	.03		
Total	479	849.63			

*p<.01

**p<.0001

Table C-3. ANOVA summary table comparing convergent score correlations within methods during overextraction: Reanalysis, p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	452.53	226.26	1003.41***	75.9%
N	3	70.57	23.52	104.32***	11.8%
SN	6	6.94	1.16	5.13**	1.2%
ERROR	48	10.82	.23		
Within					
EXTRAC	2	39.07	19.53	614.25***	6.6%
ES	4	1.54	.38	12.07***	.3%
EN	6	.35	.06	1.83	.1%
ESN	12	.25	.02	.65	_____
ERROR	96	3.05	.03		
TYPE	1	2.05	2.05	43.32***	.3%
TS	2	2.93	1.47	30.99***	.5%
TN	3	.69	.23	4.88*	.1%
TSN	6	.36	.06	1.26	.1%
ERROR	48	2.27	.05		
ET	2	.09	.05	2.44	_____
ETS	4	.44	.11	5.47**	.1%
ETN	6	.18	.03	1.51	_____
ETSN	12	.22	.02	.91	_____
ERROR	96	1.95	.02		
Total	359	596.31			

*p<.01

**p<.001

***p<.0001

Appendix D

Table D-1. ANOVA summary table comparing divergent score correlations within methods during overextraction: Reanalysis, p:m = 12:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	194.12	97.06	767.19***	63.1%
N	3	34.80	11.60	91.69***	11.3%
SN	6	.65	.11	.86	.2%
ERROR	48	6.07	.13		
Within					
EXTRAC	4	48.40	12.10	455.88***	15.7%
ES	8	.66	.08	3.11*	.2%
EN	12	.83	.07	2.60*	.3%
ESN	24	1.56	.06	2.45**	.5%
ERROR	192	5.10	.03		
TYPE	1	6.88	6.88	201.31***	2.2%
TS	2	.25	.13	3.71	.1%
TN	3	1.50	.50	14.66***	.5%
TSN	6	.80	.13	3.90*	.3%
ERROR	48	1.64	.03		
ET	4	.06	.02	.86	_____
ETS	8	.39	.05	2.71*	.1%
ETN	12	.15	.01	.70	_____
ETSN	24	.46	.02	1.08	.2%
ERROR	192	3.42	.02		
Total	599	307.74			

*p<.01

**p<.001

***p<.0001

Table D-2. ANOVA summary table comparing divergent score correlations within methods during overextraction: Reanalysis, p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	202.16	101.08	1049.29***	68.8%
N	3	40.81	13.60	141.20***	13.9%
SN	6	1.43	.24	2.47	.5%
ERROR	48	4.62	.10		
Within					
EXTRAC	3	32.31	10.77	755.83***	11.0%
ES	6	.92	.15	10.79***	.3%
EN	9	.79	.09	6.14***	.3%
ESN	18	.34	.02	1.31	.1%
ERROR	144	2.05	.01		
TYPE	1	2.88	2.88	99.61***	.9%
TS	2	.81	.40	13.94***	.3%
TN	3	.92	.31	10.64***	.3%
TSN	6	.43	.07	2.50	.1%
ERROR	48	1.39	.03		
ET	3	.18	.06	6.18**	.1%
ETS	6	.18	.03	3.07*	.1%
ETN	9	.21	.02	2.38	.1%
ETSN	18	.18	.01	1.02	.1%
ERROR	144	1.38	.01		
Total	479	293.98			

*p<.01

**p<.001

***p<.0001

Table D-3. ANOVA summary table comparing divergent score correlations within methods during overextraction: Reanalysis, p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	174.57	87.29	1182.83***	74.9%
N	3	33.52	11.17	151.41***	14.4%
SN	6	3.03	.50	6.84***	1.3%
ERROR	48	3.54	.07		
Within					
EXTRAC	2	11.94	5.97	734.77***	5.1%
ES	4	.55	.14	16.85***	.2%
EN	6	.19	.03	3.98***	.1%
ESN	12	.05	.00	.56	—
ERROR	96	.78	.01		
TYPE	1	2.25	2.25	127.38***	.9%
TS	2	.41	.21	11.75**	.2%
TN	3	.48	.16	9.10**	.2%
TSN	6	.05	.01	.47	—
ERROR	48	.85	.02		
ET	2	.03	.02	2.28	—
ETS	4	.04	.01	1.66	—
ETN	6	.07	.01	1.69	—
ETSN	12	.07	.01	.91	—
ERROR	96	.63	.01		
Total	359	233.07			

*p<.01

**p<.001

***p<.0001

Appendix E

Table E-1. ANOVA summary table comparing convergent score correlations within methods during underextraction: Original analysis, p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	.23	.11	1.48	_____
N	3	.39	.13	1.68	_____
ROT	1	.00	.00	.02	_____
SN	6	.35	.06	.75	_____
SR	2	.02	.01	.14	_____
NR	3	.00	.00	.01	_____
SNR	6	.01	.00	.02	_____
ERROR	96	7.39	.08		
Within					
EXTRAC	3	4234.65	1411.55	25647.54**	98.5%
ES	6	1.19	.20	3.60*	_____
EN	9	.39	.04	.79	_____
ER	3	.00	.00	.01	_____
ESN	18	1.62	.09	1.63	_____
ESR	6	.01	.00	.03	_____
ENR	9	.00	.00	.01	_____
ESNR	18	.01	.00	.01	_____
ERROR	288	15.85	.05		
TYPE	1	15.45	15.45	288.10**	.4%
TS	2	.46	.23	4.29	_____
TN	3	1.61	.54	10.03**	_____
TR	1	.01	.01	.15	_____
TSN	6	.31	.05	.97	_____
TSR	2	.00	.00	.02	_____
TNR	3	.00	.00	.01	_____
TSNR	6	.00	.00	.01	_____
ERROR	96	5.15	.05		
ET	3	6.38	2.13	92.97**	.1%
ETS	6	.78	.13	5.66**	_____
ETN	9	1.19	.13	5.76**	_____
ETR	3	.00	.00	.04	_____
ETSN	18	.63	.04	1.54	_____
ETSR	6	.00	.00	.01	_____
ETNR	9	.00	.00	.01	_____
ETSNR	18	.01	.00	.02	_____
ERROR	288	6.59	.02		
Total	959	4300.67			

*p<.01

**p<.0001

Table E-2. ANOVA summary table comparing convergent score correlations within methods during underextraction: Original analysis, p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	1.67	.84	12.94***	_____
N	3	.75	.25	3.88	_____
ROT	1	.03	.03	.50	_____
SN	6	3.05	.51	7.86***	_____
SR	2	.00	.00	.04	_____
NR	3	.00	.00	.02	_____
SNR	6	.03	.00	.07	_____
ERROR	96	6.20	.06		
Within					
EXTRAC	4	4578.18	1144.54	44102.33***	98.9%
ES	8	1.83	.23	8.82***	_____
EN	12	.70	.06	2.24*	_____
ER	4	.03	.01	.33	_____
ESN	24	2.24	.09	3.59***	_____
ESR	8	.00	.00	.01	_____
ENR	12	.01	.00	.03	_____
ESNR	24	.03	.00	.04	_____
ERROR	384	9.97	.03		
TYPE	1	6.74	6.74	184.70***	.1%
TS	2	.19	.10	2.64	_____
TN	3	.44	.15	3.99	_____
TR	1	.00	.00	.10	_____
TSN	6	.67	.11	3.05*	_____
TSR	2	.01	.00	.12	_____
TNR	3	.00	.00	.01	_____
TSNR	6	.01	.00	.03	_____
ERROR	96	3.50	.04		
ET	4	2.47	.62	38.34***	.1%
ETS	8	.55	.07	4.27**	_____
ETN	12	.51	.04	2.63*	_____
ETR	4	.00	.00	.04	_____
ETSN	24	1.60	.07	4.14***	_____
ETSR	8	.00	.00	.03	_____
ETNR	12	.01	.00	.03	_____
ETSNR	24	.01	.00	.02	_____
ERROR	384	6.18	.02		
Total	1199	4627.60			

*p<.01

**p<.001

***p<.0001

Appendix F

Table F-1. ANOVA summary table comparing divergent score correlations within methods during underextraction: Original analysis, p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	.31	.15	2.28	_____
N	3	.25	.08	1.23	_____
ROT	1	.01	.01	.09	_____
SN	6	.26	.04	.63	_____
SR	2	.02	.01	.17	_____
NR	3	.02	.01	.09	_____
SNR	6	.02	.00	.04	_____
ERROR	96	6.52	.07		
Within					
EXTRAC	3	3793.09	1264.36	24420.83***	99.0%
ES	6	.69	.12	2.23	_____
EN	9	.28	.03	.61	_____
ER	3	.01	.00	.07	_____
ESN	18	1.11	.06	1.19	_____
ESR	6	.02	.00	.05	_____
ENR	9	.03	.00	.06	_____
ESNR	18	.04	.00	.05	_____
ERROR	288	14.91	.05		
TYPE	1	2.43	2.43	111.02***	.1%
TS	2	.44	.22	10.13**	_____
TN	3	.24	.08	3.66**	_____
TR	1	.00	.00	.01	_____
TSN	6	.30	.05	2.26	_____
TSR	2	.00	.00	.05	_____
TNR	3	.01	.00	.12	_____
TSNR	6	.01	.00	.09	_____
ERROR	96	2.10	.02		
ET	3	1.85	.62	31.90***	_____
ETS	6	.46	.08	3.95**	_____
ETN	9	.50	.06	2.86**	_____
ETR	3	.00	.00	.01	_____
ETSN	18	.48	.03	1.39	_____
ETSR	6	.03	.01	.29	_____
ETNR	9	.05	.01	.30	_____
ETSNR	18	.04	.00	.11	_____
ERROR	288	5.57	.02		
Total	959	3832.12			

*p<.01

**p<.001

***p<.0001

Table F-2. ANOVA summary table comparing divergent score correlations within methods during underextraction: Original analysis, p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	1.05	.52	9.86**	_____
N	3	.19	.06	1.22	_____
ROT	1	.04	.04	.81	_____
SN	6	1.14	.19	3.59*	_____
SR	2	.01	.01	.13	_____
NR	3	.00	.00	.01	_____
SNR	6	.03	.01	.11	_____
ERROR	96	5.10	.05		
Within					
EXTRAC	4	3808.76	952.19	45226.75***	99.3%
ES	8	1.02	.13	6.06***	_____
EN	12	.68	.06	2.71*	_____
ER	4	.03	.01	.40	_____
ESN	24	1.00	.04	1.99	_____
ESR	8	.01	.00	.06	_____
ENR	12	.01	.00	.06	_____
ESNR	24	.05	.00	.10	_____
ERROR	384	8.08	.02		
TYPE	1	.39	.39	21.66***	_____
TS	2	.22	.11	6.15*	_____
TN	3	.09	.03	1.58	_____
TR	1	.01	.01	.27	_____
TSN	6	.18	.03	1.67	_____
TSR	2	.02	.01	.55	_____
TNR	3	.00	.00	.02	_____
TSNR	6	.04	.01	.40	_____
ERROR	96	1.75	.02		
ET	4	.48	.12	11.48***	_____
ETS	8	.50	.06	5.95***	_____
ETN	12	.44	.04	3.49**	_____
ETR	4	.00	.00	.11	_____
ETSN	24	.56	.02	2.25**	_____
ETSR	8	.01	.00	.13	_____
ETNR	12	.01	.00	.09	_____
ETSNR	24	.04	.00	.14	_____
ERROR	384	4.00	.01		
Total	1199	3835.98			

*p<.01

**p<.001

***p<.0001

Appendix G

Table G-1. ANOVA summary table comparing convergent score correlations within methods during underextraction: Reanalysis, p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	.30	.15	1.48	.2%
N	3	.52	.17	1.68	.3%
ROT	1	.00	.00	.02	-----
SN	6	.46	.08	.75	.3%
SR	2	.03	.01	.14	-----
NR	3	.00	.00	.01	-----
SNR	6	.01	.00	.02	-----
ERROR	96	9.85	.10		
Within					
EXTRAC	2	107.16	53.58	768.38***	61.9%
ES	4	1.11	.28	3.69*	.6%
EN	6	.26	.04	.62	.2%
ER	2	.00	.00	.00	-----
ESN	12	1.50	.13	1.80	.9%
ESR	4	.00	.00	.01	-----
ENR	6	.00	.00	.01	-----
ESNR	12	.00	.00	.01	-----
ERROR	192	13.39	.07		
TYPE	1	20.59	20.59	288.10***	11.9%
TS	2	.61	.31	4.29	.4%
TN	3	2.15	.72	10.03***	1.2%
TR	1	.01	.01	.15	-----
TSN	6	.42	.07	.97	.2%
TSR	2	.00	.00	.02	-----
TNR	3	.00	.00	.01	-----
TSNR	6	.00	.00	.01	-----
ERROR	96	6.86	.07		
ET	2	1.23	.62	24.26***	.7%
ETS	4	.62	.16	6.14**	.4%
ETN	6	.65	.11	4.25**	.4%
ETR	2	.00	.00	.00	-----
ETSN	12	.53	.04	1.74	.3%
ETSR	4	.00	.00	.01	-----
ETNR	6	.00	.00	.01	-----
ETSNR	12	.01	.00	.02	-----
ERROR	192	4.87	.03		
Total	719	173.18			

*p<.01

**p<.001

***p<.0001

Table G-2. ANOVA summary table comparing convergent score correlations within methods during underextraction: Reanalysis, p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	2.09	1.04	12.94***	.7%
N	3	.94	.31	3.88	.3%
ROT	1	.04	.04	.50	-----
SN	6	3.81	.63	7.86***	1.4%
SR	2	.01	.00	.04	-----
NR	3	.00	.00	.02	-----
SNR	6	.04	.01	.07	-----
ERROR	96	7.75	.08		
Within					
EXTRAC	3	230.07	76.69	2624.25***	82.3%
ES	6	1.41	.24	8.06***	.5%
EN	9	.51	.06	1.93	.2%
ER	3	.03	.01	.30	-----
ESN	18	1.48	.08	2.81**	.5%
ESR	6	.00	.00	.01	-----
ENR	9	.01	.00	.03	-----
ESNR	18	.02	.00	.03	-----
ERROR	288	8.42	.03		
TYPE	1	8.42	8.42	184.70***	3.0%
TS	2	.24	.12	2.64	.1%
TN	3	.55	.18	3.99	.2%
TR	1	.00	.00	.10	-----
TSN	6	.83	.14	3.05*	.3%
TSR	2	.01	.01	.12	-----
TNR	3	.00	.00	.01	-----
TSNR	6	.01	.00	.03	-----
ERROR	96	4.38	.05		
ET	3	.78	.26	14.17***	.3%
ETS	6	.50	.08	4.54**	.2%
ETN	9	.40	.04	2.40	.1%
ETR	3	.00	.00	.03	-----
ETSN	18	1.43	.08	4.32***	.5%
ETSR	6	.00	.00	.02	-----
ETNR	9	.01	.00	.03	-----
ETSNR	18	.01	.00	.02	-----
ERROR	288	5.30	.02		
Total	959	279.49			

*p<.01

**p<.001

***p<.0001

Appendix H

Table H-1. ANOVA summary table comparing divergent score correlations within methods during underextraction: Reanalysis, p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	.41	.21	2.28	.5%
N	3	.33	.11	1.23	.4%
ROT	1	.01	.01	.09	_____
SN	6	.34	.06	.63	.5%
SR	2	.03	.02	.17	_____
NR	3	.02	.01	.09	_____
SNR	6	.02	.00	.04	_____
ERROR	96	8.70	.09		
Within					
EXTRAC	2	36.11	18.05	272.16**	48.1%
ES	4	.59	.15	2.22	.8%
EN	6	.20	.03	.50	.3%
ER	2	.01	.00	.07	_____
ESN	12	1.02	.09	1.28	1.4%
ESR	4	.01	.00	.03	_____
ENR	6	.02	.00	.05	_____
ESNR	12	.04	.00	.05	.1%
ERROR	192	12.74	.07		
TYPE	1	3.25	3.25	111.02**	4.3%
TS	2	.59	.30	10.13*	.8%
TN	3	.32	.11	3.66	.4%
TR	1	.00	.00	.01	_____
TSN	6	.40	.07	2.26	.5%
TSR	2	.00	.00	.05	_____
TNR	3	.01	.00	.12	_____
TSNR	6	.02	.00	.09	_____
ERROR	96	2.81	.03		
ET	2	1.04	.52	20.51**	1.4%
ETS	4	.31	.08	3.06	.4%
ETN	6	.42	.07	2.74	.6%
ETR	2	.00	.00	.01	_____
ETSN	12	.38	.03	1.26	.5%
ETSR	4	.03	.01	.32	_____
ETNR	6	.05	.01	.32	.1%
ETSNR	12	.03	.00	.11	_____
ERROR	192	4.87	.02		
Total	719	75.14			

*p<.001

**p<.0001

Table H-2. ANOVA summary table comparing divergent score correlations within methods during underextraction: Reanalysis. p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	1.31	.65	9.86**	1.3%
N	3	.24	.08	1.22	.2%
ROT	1	.05	.05	.81	.1%
SN	6	1.43	.24	3.59*	1.4%
SR	2	.02	.01	.13	-----
NR	3	.00	.00	.01	-----
SNR	6	.04	.01	.11	-----
ERROR	96	6.38	.07		
Within					
EXTRAC	3	73.35	24.45	1034.05***	72.9%
ES	6	.76	.13	5.35***	.8%
EN	9	.64	.07	2.98*	.6%
ER	3	.02	.01	.32	-----
ESN	18	.72	.04	1.69	.7%
ESR	6	.01	.00	.05	-----
ENR	9	.01	.00	.07	-----
ESNR	18	.04	.00	.10	-----
ERROR	288	6.81	.02		
TYPE	1	.49	.49	21.66***	.5%
TS	2	.28	.14	6.15*	.3%
TN	3	.11	.04	1.58	.1%
TR	1	.01	.01	.27	-----
TSN	6	.23	.04	1.67	.2%
TSR	2	.03	.01	.55	-----
TNR	3	.00	.00	.02	-----
TSNR	6	.06	.01	.40	.1%
ERROR	96	2.19	.02		
ET	3	.38	.13	10.23***	.4%
ETS	6	.44	.07	5.93***	.4%
ETN	9	.41	.05	3.72**	.4%
ETR	3	.00	.00	.09	-----
ETSN	18	.52	.03	2.32*	.5%
ETSR	6	.01	.00	.07	-----
ETNR	9	.01	.00	.09	-----
ETSNR	18	.02	.00	.11	-----
ERROR	288	3.56	.01		
Total	959	100.57			

*p<.01

**p<.001

***p<.0001

Appendix I

Table I-1. ANOVA summary table comparing convergent score correlations between methods during overextraction: p:m = 12:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	307.52	153.76	1468.14**	66.8%
N	3	17.69	5.90	56.29**	3.8%
SN	6	1.78	.30	2.84	.4%
ERROR	48	5.03	.10		
Within					
EXTRAC	5	100.59	20.11	418.62**	21.9%
ES	10	8.07	.81	16.80**	1.8%
EN	15	4.88	.33	6.77**	1.1%
ESN	30	3.23	.11	2.24*	.7%
ERROR	240	11.53	.05		
Total	359	460.32			

*p<.001

**p<.0001

Table I-2. ANOVA summary table comparing convergent score correlations between methods during overextraction: p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	319.29	159.64	759.51**	77.3%
N	3	29.03	9.68	46.04**	7.0%
SN	6	1.60	.27	1.27	.4%
ERROR	48	10.09	.21		
Within					
EXTRAC	4	34.36	8.59	239.61**	8.3%
ES	8	7.42	.93	25.87**	1.8%
EN	12	2.01	.17	4.68**	.5%
ESN	24	2.25	.09	2.61*	.5%
ERROR	192	6.88	.04		
Total	299	412.93			

*p<.001

**p<.0001

Table I-3. ANOVA summary table comparing convergent score correlations between methods during overextraction: p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	319.21	159.61	1050.62*	83.9%
N	3	30.49	10.16	66.90*	8.0%
SN	6	2.01	.33	2.20	.5%
ERROR	48	7.29	.15		
Within					
EXTRAC	3	9.53	3.18	77.72*	2.5%
ES	6	4.74	.79	19.34*	1.2%
EN	9	.83	.09	2.25	.2%
ESN	18	.66	.04	.89	.2%
ERROR	144	5.89	.04		
Total	239	380.66			

*p<.0001

Appendix J

Table J-1. ANOVA summary table comparing divergent score correlations between methods during overextraction: p:m = 12:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	71.71	35.85	557.23**	15.5%
N	3	9.36	3.12	48.51**	2.0%
SN	6	.26	.04	.67	.1%
ERROR	48	3.09	.06		
Within					
EXTRAC	5	333.42	66.68	1332.34**	71.9%
ES	10	21.89	2.19	43.73**	4.7%
EN	15	8.52	.57	11.35**	1.8%
ESN	30	3.61	.12	2.41*	.8%
ERROR	240	12.01	.05		
Total	359	463.87			

*p<.001

**p<.0001

Table J-2. ANOVA summary table comparing divergent score correlations between methods during overextraction: p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	113.23	56.62	702.22**	39.5%
N	3	24.08	8.03	99.54**	8.4%
SN	6	1.76	.29	3.63*	.6%
ERROR	48	3.87	.08		
Within					
EXTRAC	4	112.63	28.16	1235.27**	39.3%
ES	8	16.76	2.10	91.93**	5.8%
EN	12	7.26	.60	26.53**	2.5%
ESN	24	2.62	.11	4.79**	.9%
ERROR	192	4.38	.02		
Total	299	286.58			

*p<.01

**p<.0001

Table J-3. ANOVA summary table comparing divergent score correlations between methods during overextraction: p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	132.39	66.20	1010.25***	63.1%
N	3	27.89	9.30	141.89***	13.3%
SN	6	2.19	.36	5.57**	1.0%
ERROR	48	3.15	.07		
Within					
EXTRAC	3	28.38	9.46	425.46***	13.5%
ES	6	7.73	1.29	57.95***	3.7%
EN	9	3.97	.44	19.82***	1.9%
ESN	18	.85	.05	2.13*	-----
ERROR	144	3.20	.02		
Total	239	209.75			

*p<.01

**p<.001

***p<.0001

Appendix K

Table K-1. ANOVA summary table comparing convergent score correlations between methods during underextraction: p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	34.77	17.39	70.95**	4.9%
N	3	1.18	.39	1.61	.2%
ROT	1	.02	.02	.08	_____
SN	6	6.53	1.09	4.44*	.9%
SR	2	.00	.00	.00	_____
NR	3	.00	.00	.00	_____
SNR	6	.00	.00	.00	_____
ERROR	96	23.53	.25		
Within					
EXTRAC	3	428.27	142.76	1171.42**	60.8%
ES	6	150.52	25.09	205.85**	21.4%
EN	9	18.49	2.05	16.86**	2.6%
ER	3	.01	.00	.02	_____
ESN	18	5.77	.32	2.63*	.8%
ESR	6	.01	.00	.01	_____
ENR	9	.01	.00	.00	_____
ESNR	18	.01	.00	.01	_____
ERROR	288	35.10	.12		
Total	479	704.22			

*p<.001

**p<.0001

Table K-2. ANOVA summary table comparing convergent score correlations between methods during underextraction: p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	59.71	29.86	129.17*	8.1%
N	3	.65	.22	.94	.1%
ROT	1	.19	.19	.83	-----
SN	6	10.48	1.75	7.55*	1.4%
SR	2	.00	.00	.01	-----
NR	3	.03	.01	.04	-----
SNR	6	.04	.01	.03	-----
ERROR	96	22.19	.23		
Within					
EXTRAC	4	385.34	96.33	834.66*	52.3%
ES	8	172.97	21.62	187.33*	23.5%
EN	12	28.11	2.34	20.30*	3.8%
ER	4	.09	.02	.20	-----
ESN	24	12.29	.51	4.44*	1.7%
ESR	8	.02	.00	.02	-----
ENR	12	.04	.00	.03	-----
ESNR	24	.02	.00	.01	-----
ERROR	384	44.32	.12		
Total	599	736.51			

*p<.0001

Appendix L

Table L-1. ANOVA summary table comparing divergent score correlations between methods during underextraction: p:m = 6:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	9.53	4.77	33.06***	1.9%
N	3	5.61	1.87	12.96***	1.1%
ROT	1	3.82	3.82	26.51***	.8%
SN	6	3.64	.61	4.21**	.7%
SR	2	.75	.37	2.59	.1%
NR	3	.12	.04	.27	—
SNR	6	.22	.04	.26	—
ERROR	96	13.84	.14		
Within					
EXTRAC	3	291.24	97.08	664.30***	57.2%
ES	6	99.17	16.53	113.10***	19.5%
EN	9	22.91	2.55	17.42***	4.5%
ER	3	7.01	2.34	15.99***	1.4%
ESN	18	6.20	.34	2.36*	1.2%
ESR	6	1.44	.24	1.64	.3%
ENR	9	.50	.06	.38	.1%
ESNR	18	.75	.04	.29	.1%
ERROR	288	42.09	.15		
Total	479	508.84			

*p<.01

**p<.001

***p<.0001

Table L-2. ANOVA summary table comparing divergent score correlations between methods during underextraction: p:m = 4:1

SOURCE	df	SS	MS	F	Eta-Squared
Between					
SAT	2	19.56	9.78	105.86*	5.0%
N	3	6.33	2.11	22.82*	1.6%
ROT	1	4.01	4.01	43.37*	1.0%
SN	6	3.68	.61	6.64*	.9%
SR	2	.40	.20	2.17	.1%
NR	3	.12	.04	.43	—
SNR	6	.19	.03	.35	—
ERROR	96	8.87	.09		
Within					
EXTRAC	4	195.09	48.77	648.59*	50.1%
ES	8	85.41	10.68	141.98*	21.9%
EN	12	20.05	1.67	22.22*	5.1%
ER	4	5.39	1.35	17.93*	1.4%
ESN	24	8.75	.36	4.85*	2.2%
ESR	8	1.47	.18	2.44	.4%
ENR	12	.72	.06	.80	.2%
ESNR	24	.52	.02	.29	.1%
ERROR	384	28.88	.08		
Total	599	389.45			

*p<.0001

Appendix M

Table M. ANOVA summary table comparing convergent score correlations between methods in the true component and factor situation

SOURCE	df	SS	MS	F	Eta-Squared
P:M	2	5.46	2.73	49.74**	1.8%
SAT	2	267.34	133.67	2434.32**	85.7%
N	3	25.51	8.50	154.87**	8.2%
PS	4	1.95	.49	8.88**	.6%
PN	6	.15	.03	.46	
SN	6	2.01	.33	6.10**	.6%
PSN	12	1.59	.13	2.41*	.5%
ERROR	144	7.91	.05		
Total	179		311.92		

*p<.001

**p<.0001

Appendix N

Table N. ANOVA summary table comparing divergent score correlations between methods in the true component and factor situation

SOURCE	df	SS	MS	F	Eta-Squared
P:M	2	35.38	17.69	209.95**	12.5%
SAT	2	177.75	88.87	1054.70**	62.8%
N	3	47.88	15.96	189.42**	16.9%
PS	4	1.05	.26	3.10	.4%
PN	6	.76	.13	1.50	.3%
SN	6	5.34	.89	10.56**	1.9%
PSN	12	2.67	.22	2.64*	.9%
ERROR	144	12.13	.08		
Total	179		282.96		

*p<.01

**p<.0001

Appendix O

Table O-1. Extraction main effects, when comparing within methods for convergent score correlations in the overextraction situation, for the original analyses and the reanalyses.

True p:m	Levels of Extraction						df	F	Eta Squared
	Same	Over1	Over2	Over3	Over6	Over9			
12:1	.99999	.999+	.99+	.99+	.98+	.97+	(5,240)	3708.44*	60.1%
12:1	————	.999+	.99+	.99+	.98+	.97+	(4,192)	956.02*	24.2%
6:1	.99999	.999+	.99+	.99+	.98+	——	(4,192)	6436.88	57.3%
6:1	————	.999+	.99+	.99+	.98+	——	(3,144)	1160.13*	14.4%
4:1	.99999	.99+	.99+	.99+	——	——	(3,144)	6975.47*	60.0%
4:1	————	.99+	.99+	.99+	——	——	(2,96)	614.25*	6.6%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table O-2. Saturation main effects, when comparing within methods for convergent score correlations in the overextraction situation, for the original analyses and the reanalyses.

True p:m	Levels of Saturation			df	F	Eta Squared
	.8	.6	.4			
12:1**	.999+	.99+	.98+	(2,48)	946.06*	27.2%
12:1	.999+	.99+	.94+	(2,48)	946.06*	61.9%
6:1**	.999+	.99+	.99+	(2,48)	1327.09*	27.9%
6:1	.999+	.99+	.94+	(2,48)	1327.09*	69.7%
4:1**	.999+	.99+	.99+	(2,48)	1003.41*	24.3%
4:1	.999+	.99+	.94+	(2,48)	1003.41*	75.9%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z tranformed values that were used in the ANOVAs.

*p < .0001.

**Includes level one of the extraction factor.

Table O-3. Sample size main effects, when comparing within methods for convergent score correlations in the overextraction situation, for the original analyses and the reanalyses.

True p:m	Levels of Sample Size				df	F	Eta Squared
	450	225	150	75			
12:1**	.999+	.99+	.99+	.99+	(3,48)	66.23*	2.9%
12:1	.99+	.99+	.99+	.98+	(3,48)	66.23*	6.5%
6:1**	.999+	.999+	.99+	.99+	(3,48)	130.25*	4.1%
6:1	.99+	.99+	.99+	.98+	(3,48)	130.25*	10.3%
4:1**	.999+	.999+	.99+	.99+	(3,48)	104.32*	3.8%
4:1	.999+	.999+	.99+	.98+	(3,48)	104.32*	11.8%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

**Includes level one of the extraction factor.

Table O-4. Type main effects, when comparing within methods for convergent score correlations in the overextraction situation, for the original analyses and the reanalyses.

True p:m	Factor Analysis	Component Analysis	df	F	Eta Squared
12:1**	.99+	.99+	(1,48)	94.16*	.5%
12:1	.99+	.99+	(1,48)	94.16*	1.0%
6:1**	.99+	.99+	(1,48)	34.87*	.1%
6:1	.99+	.99+	(1,48)	34.87*	.4%
4:1**	.999+	.999+	(1,48)	43.32*	.1%
4:1	.99+	.99+	(1,48)	43.32*	.3%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

**Includes level one of the extraction factor.

Appendix P

Table P-1. Extraction main effects, when comparing within methods for divergent score correlations in the overextraction situation, for the original analyses and the reanalyses.

True p:m	Levels of Extraction						df	F	Eta Squared
	Same	Over1	Over2	Over3	Over6	Over9			
12:1	.00001	.01-	.01-	.02-	.02-	.03-	(5,240)	10134.07*	83.3%
12:1	————	.01-	.01-	.02-	.02-	.03-	(4,192)	455.88*	15.7%
6:1	.00001	.01-	.01-	.01-	.02-	——	(4,192)	17836.74*	80.9%
6:1	————	.01-	.01-	.01-	.02-	——	(3,144)	755.83*	11.0%
4:1	.00001	.01-	.01-	.01-	——	——	(3,144)	27549.25*	81.2%
4:1	————	.01-	.01-	.01-	——	——	(2,96)	734.77*	5.1%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table P-2. Saturation main effects, when comparing within methods for divergent score correlations in the overextraction situation, for the original analyses and the reanalyses.

True p:m	Levels of Saturation			df	F	Eta Squared
	.8	.6	.4			
12:1**	.01-	.01-	.02-	(2,48)	767.19*	10.4%
12:1	.01-	.02-	.05-	(2,48)	767.19*	63.1%
6:1**	.001-	.01-	.01-	(2,48)	1049.29*	11.8%
6:1	.01-	.01-	.04-	(2,48)	1049.29*	68.8%
4:1**	.001-	.01-	.01-	(2,48)	1182.83*	11.1%
4:1	.01-	.01-	.04-	(2,48)	1182.83*	74.9%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

**Includes level one of the extraction factor.

Table P-3. Sample size main effects, when comparing within methods for divergent score correlations in the overextraction situation, for the original analyses and the reanalyses.

True p:m	Levels of Sample Size				df	F	Eta Squared
	450	225	150	75			
12:1**	.01-	.01-	.01-	.01-	(3,48)	91.69*	1.9%
12:1	.01-	.01-	.02-	.03-	(3,48)	91.69*	11.3%
6:1**	.01-	.01-	.01-	.01-	(3,48)	141.20*	2.4%
6:1	.01-	.01-	.01-	.02-	(3,48)	141.20*	13.9%
4:1**	.001-	.01-	.01-	.01-	(3,48)	151.41*	2.1%
4:1	.01-	.01-	.01-	.02-	(3,48)	151.41*	14.4%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

**Includes level one of the extraction factor.

Table P-4. Type main effects, when comparing within methods for divergent score correlations in the overextraction situation, for the original analyses and the reanalyses.

True p:m	Factor Analysis	Component Analysis	df	F	Eta Squared
12:1**	.01-	.01-	(1,48)	201.31*	.4%
12:1	.01-	.02-	(1,48)	201.31*	2.2%
6:1**	.01-	.01-	(1,48)	99.61*	.2%
6:1	.01-	.01-	(1,48)	99.61*	.9%
4:1**	.01-	.01-	(1,48)	127.38*	.1%
4:1	.01-	.01-	(1,48)	127.38*	.9%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

**Includes level one of the extraction factor.

Appendix Q

Table Q. Extraction main effects, when comparing within methods for convergent score correlations in the underextraction situation, for the original analyses and the reanalyses.

True p:m	Levels of Extraction					df	F	Eta Squared
	Same	Under1	Under2	Under3	Under6			
4:1	.99999	.96+	.89+	.81+	.61+	(4,384)	44102.33*	98.9%
4:1	———	.96+	.89+	.81+	.61+	(3,288)	2624.25*	82.3%
6:1	.99999	.94+	.83+	.71+	———	(3,288)	25647.54*	98.5%
6:1	———	.94+	.83+	.71+	———	(2,192)	768.38*	61.9%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Appendix R

Table R. Extraction main effects, when comparing within methods for divergent score correlations in the underextraction situation, for the original analyses and the reanalyses.

True p:m	Levels of Extraction					df	F	Eta Squared
	Same	Under1	Under2	Under3	Under6			
4:1	.00001	.02-	.05-	.08-	.12-	(4,384)	45226.75*	99.3%
4:1	————	.02-	.05-	.08-	.12-	(3,288)	1034.05*	72.9%
6:1	.00001	.04-	.10-	.13-	——	(3,288)	24420.83*	99.0%
6:1	————	.04-	.10-	.13-	——	(2,192)	272.16*	48.1%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Appendix S

Table S-1. Saturation main effects when comparing between methods for convergent score correlations in the overextraction situation.

True p:m	Levels of Saturation			df	F	Eta Squared
	.8	.6	.4			
12:1	.999+	.99+	.95+	(2,48)	1468.14*	66.8%
6:1	.999+	.99+	.94+	(2,48)	759.51*	77.3%
4:1	.999+	.99+	.92+	(2,48)	1050.62*	83.9%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table S-2. Extraction main effects when comparing between methods for convergent score correlations in the overextraction situation.

True p:m	Levels of Extraction						df	F	Eta Squared
	Same	Over1	Over2	Over3	Over6	Over9			
12:1	.99+	.99+	.99+	.99+	.98+	.97+	(5,240)	418.62*	21.9%
6:1	.99+	.99+	.99+	.99+	.98+	——	(4,192)	239.61*	8.3%
4:1	.99+	.99+	.99+	.99+	——	——	(3,144)	77.72*	2.5%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table S-3. Sample size main effects when comparing between methods for convergent score correlations in the overextraction situation.

True p:m	Levels of Sample Size				df	F	Eta Squared
	450	225	150	75			
12:1	.99+	.99+	.99+	.99+	(3,48)	56.29*	3.8%
6:1	.99+	.99+	.99+	.98+	(3,48)	46.04*	7.0%
4:1	.99+	.99+	.99+	.98+	(3,48)	68.90*	8.0%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Appendix T

Table T-1. Saturation main effects when comparing between methods for divergent score correlations in the overextraction situation.

True R:M	Levels of Saturation			df	F	Eta Squared
	.8	.6	.4			
12:1	.01-	.02-	.05-	(2,48)	557.23*	15.5%
6:1	.01-	.01-	.04-	(2,48)	702.22*	39.5%
4:1	.001-	.01-	.04-	(2,48)	1010.25*	63.1%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table T-2. Extraction main effects when comparing between methods for divergent score correlations in the overextraction situation.

True p:m	Levels of Extraction						df	F	Eta Squared
	Same	Over1	Over2	Over3	Over6	Over9			
12:1	.001-	.01-	.03-	.04-	.06-	.07-	(5,240)	1332.34*	71.9%
6:1	.001-	.01-	.02-	.02-	.04-	—	(4,192)	1235.27*	39.3%
4:1	.01-	.01-	.01-	.02-	—	—	(3,144)	425.46*	13.5%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table T-3. Sample size main effects when comparing between methods for divergent score correlations in the overextraction situation.

True p:m	Levels of Sample Size				df	F	Eta Squared
	450	225	150	75			
12:1	.01-	.02-	.02-	.03-	(3,48)	48.51*	2.0%
6:1	.01-	.01-	.01-	.02-	(3,48)	99.54*	8.4%
4:1	.01-	.01-	.01-	.02-	(3,48)	141.89*	13.3%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Appendix U

Table U-1. Extraction main effects when comparing between methods for convergent score correlations in the underextraction situation.

True p:m	Levels of Extraction					df	F	Eta Squared
	Same	Under1	Under2	Under3	Under6			
6:1	.99+	.94+	.87+	.75+	—	(3,288)	1171.42*	60.8%
4:1	.99+	.95+	.92+	.87+	.69+	(4,384)	834.66*	52.3%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table U-2. Saturation main effects when comparing between methods for convergent score correlations in the underextraction situation.

True p:m	Levels of Saturation			df	F	Eta Squared
	.8	.6	.4			
6:1	.97+	.96+	.91+	(2,96)	70.95*	4.9%
4:1	.97+	.95+	.87+	(2,96)	129.17*	8.1%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Appendix V

Table V-1. Extraction main effects when comparing between methods for divergent score correlations in the underextraction situation.

True p:m	Levels of Extraction					df	F	Eta Squared
	Same	Under1	Under2	Under3	Under6			
6:1	.01-	.04-	.06-	.09-	—	(3,288)	664.30*	57.2%
4:1	.01-	.04-	.05-	.06-	.10-	(4,384)	648.59*	50.1%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table V-2. Saturation main effects when comparing between methods for divergent score correlations in the underextraction situation.

True p:m	Levels of Saturation			df	F	Eta Squared
	.8	.6	.4			
6:1	.02-	.02-	.04-	(2,96)	33.06*	1.9%
4:1	.03-	.03-	.05-	(2,96)	105.86*	5.0%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table V-3. Sample size main effects when comparing between methods for divergent score correlations in the underextraction situation.

True p:m	Levels of Sample Size				df	F	Eta Squared
	450	225	150	.75			
6:1	.02-	.03-	.03-	.03-	(3,96)	12.96*	1.1%
4:1	.03-	.03-	.04-	.04-	(3,96)	22.82*	1.6%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Appendix W

Table W-1. Saturation main effects when comparing between methods for convergent score correlations in the true factor and component situation.

Levels of Saturation			df	F	Eta Squared
.8	.6	.4			
.9999+	.99+	.96+	(2,144)	2434.32*	85.7%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table W-2. Sample size effects when comparing between methods for convergent score correlations in the true factor and component situation.

Levels of Sample Size				df	F	Eta Squared
450	225	150	75			
.999+	.99+	.99+	.99+	(3,144)	154.87*	8.2%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table W-3. p:m ratio main effects when comparing between methods for convergent score correlations in the true factor and component situation.

Level of p:m Ratio			df	F	Eta Squared
12:1	6:1	4:1			
.99+	.99+	.99+	(2,144)	49.74*	1.8%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Appendix X

Table X-1. Saturation main effects when comparing between methods for divergent score correlations in the true factor and component situation.

Levels of Saturation			df	F	Eta Squared
.8	.6	.4			
.0001-	.001-	.02-	(2,144)	1,054.70*	62.8%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table X-2. Sample size main effects when comparing between methods for divergent score correlations in the true factor and component situation.

Levels of Sample Size				df	F	Eta Squared
450	225	150	75			
.001-	.001-	.01-	.01-	(3,144)	189.42*	16.9%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

Table X-3. p:m ratio main effects when comparing between methods for divergent score correlations in the true factor and component situation.

Level of p:m Ratio			df	F	Eta Squared
12:1	6:1	4:1			
.001-	.001-	.01-	(2,144)	209.95*	12.5%

Note: Mean values represented in this table are the equivalent correlational values for the Fisher Z transformed values that were used in the ANOVAs.

*p < .0001.

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