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Proportional Integral Observer (PIO) Design for Linear Control Systems

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PROPORTIONAL INTEGRAL OBSERVER (PIO) DESIGN FOR LINEAR CONTROL SYSTEMS

BY

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ABSTRACT

Proportional integral observer (PIO) has the ability to estimate state variables and disturbances in linear control systems. The observer gain can be obtained by traditional pole-placement methods, however, these methods may not provide good robustness bound for observer based regulators and tracking systems. In this thesis, an extended observer model is derived, and shown that PIO can be regarded as this extended observer. A parameterized method of calculating feedback gain and proportional observer gain is modified and applied to gain calculation of PIO using an extended model for observer, with good robustness for PIO based regulators and tracking systems. Examples of PIO and PIO based control systems in both continuous time and discrete time are provided to show the result of this design method.

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CHAPTER 1

Introduction

In state feedback control systems, all state variables are needed for feedback to make the system stable. However, as is pointed out in [1, 2], state variables might not be available for the reason of inaccessibility of some variables, or the limitation on the number of sensors. Therefore, observers are designed to estimate the unmeasured state variables for feedback purpose. Proportional observer (PO) is first introduced by Luenberger [3, 4] and shown to have the ability of estimating state variables in general cases. However, in case of plant with disturbance, the estimated variables and outputs will not match the actual ones.

Proportional integral observer (PIO) was first proposed by S. Beale and B. Shafai to make the observer based controller design less sensitive to parameter variation of the system by adding an integration path to the observer, which provides additional degrees of freedom [5]. Compared with proportional observer, PIO may offer advantages such as: reducing the effect of disturbances on control system performance, more accurate state-variables estimation and improved stability robustness [2, 6]. PIO is used in several applications to estimate unknown state variables, like in battery charge estimation [7] and flight control [8, 9]. However, design methods for calculating the observer gains are not provided in these papers. Besides, some other applications of PIO are raised by Z. Gao et. al. [10, 11] and F. Bakhshande et. al. [12].

In Chapter 2, the stability and disturbance estimation of PIO will be discussed based on the state-space model provided in [2], and it will be shown that the disturbance observer (DO) model provided in [2] is a special case of PIO. Furthermore, the fact that PIO can be regarded as a higher order PO will also be proved in this chapter.

For observer gain design, loop transfer recovery (LTR) method is a commonly used method [6, 13] to minimize the difference between estimated state variables and actual ones, and to provide guaranteed stability robustness for observer-based control systems. The shortcoming of this method is that the observer poles cannot be chosen on demand, and the settling time of observer cannot be directly controlled. In [14], an observer design method is introduced to minimize disturbance by noise on output measurement. However, this paper does not provide a design method with disturbance in plant. Duan et. al. introduced a parameterized design method of observer gain calculation with desired pole locations for both continuous-time and discrete-time proportional integral observers in [15, 16], but the reason for selecting proper parameters for better system behavior, such as norm of observer gain or system robustness for feedback control, is not given in these papers.

R. Vaccaro introduced an optimization approach to pole placement in [17] to design feedback and observer gain for control systems, with desired pole location and good system robustness. In Chapter 3, it will be shown that these methods can be modified to apply on gain calculation for PIO.

PIO can be applied to various types of control systems. PIO based regulator will be discussed in Chapter 4, and PIO based tracking system will be discussed in Chapter 5.

CHAPTER 2

State and Disturbance Estimation with PIO

The state-space model of a n^{th} order, p input, q output plant with l independent disturbance of constant value is described as:

$$\begin{aligned} \dot{x} &= Ax + Bu + Ed \\ \dot{d} &= 0 \end{aligned} \tag{1}$$

In the previous equation, the plant state vector x is $n \times 1$, the plant input vector u is $p \times 1$, and the independent disturbance d is an $l \times 1$ vector. In case of the disturbance model is unknown, the matrix E can be assumed to be identity matrix with the same order of plant. The output y, a $q \times 1$ vector, of the plant is:

$$y = Cx \tag{2}$$

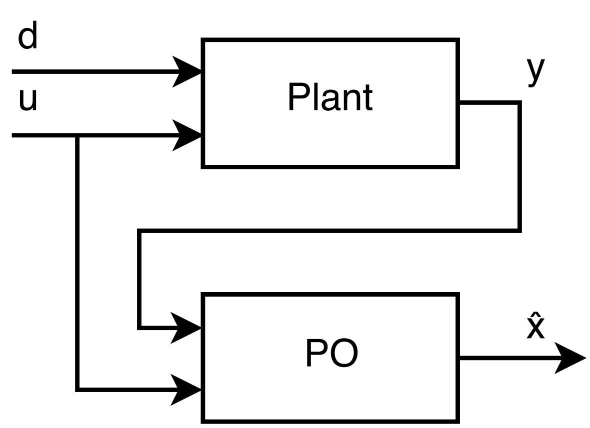

Figure 1. Block diagram of proportional observer

Proportional observers are built to estimate state variables using the plant input and output, as is shown in Figure 1. The state-space model of proportional observer in shown as follows:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \tag{3}$$

in which $\hat{x}$ is the estimated of state variables. Subtract equation (3) from (1), letting $e = x - \hat{x}$, which is the error between actual and estimated variables and disturbances, so that:

$$\dot{e} = \dot{x} - \dot{\hat{x}} = Ax + Ed - (A - LC)\hat{x} - Ly = (A - LC)e + Ed \tag{4}$$

Therefore, if there is no disturbance in plant ($d = 0$), a proportional observer has the ability to estimate the state variables, if $(A - LC)$ is Routh-Hurwitz stable [18, 19], which means all eigenvalues of $(A - LC)$ have negative real parts. However, if d is a non-zero constant, there will be a constant steady-state error between the estimated and actual state variables.

In order to eliminate this error in estimation, disturbance observer (DO) [2] is described as follows:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L_P(y - C\hat{x}) + E\hat{d} \\ \dot{\hat{d}} &= L_I(y - C\hat{x}) \end{aligned} \tag{5}$$

According to the definition [5], the state space mode of proportional integral observer is:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + Ev + G(y - C\hat{x}) \\ \dot{v} &= F(y - C\hat{x}) \end{aligned} \tag{6}$$

Comparing equation (5) and equation (6), it is obvious that disturbance observer can be regarded as proportional integral observer in a special case. The block diagram of proportional integral observer is shown in Figure 2.

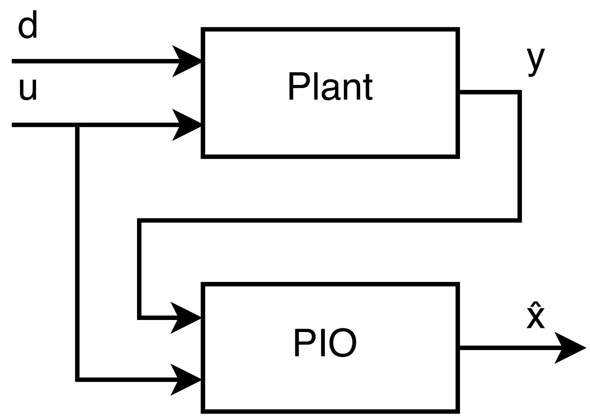

Figure 2. Block diagram of proportional integral observer

Recall the state space model of the plant with disturbance described by equation (1). These two equations can be combined together by defining $z = \begin{bmatrix} x \\ d \end{bmatrix}$, so that:

$$\dot{z} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u = A_Z z + B_Z u \tag{7}$$

The output of the plant is given by the following equation:

$$y = Cx = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} = C_Z z \tag{8}$$

Similarly, the state space model of disturbance observer described by equation (5) can also be rewritten into the following equation by defining $\hat{z} = \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}$:

$$\dot{\hat{z}} = \begin{bmatrix} A - L_P C & E \\ -L_I C & 0 \end{bmatrix} \hat{z} + \begin{bmatrix} L_P \\ L_I \end{bmatrix} y + \begin{bmatrix} B \\ 0 \end{bmatrix} u \tag{9}$$

By forming

$$A_{Z(n+l\times n+l)} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}, B_{Z(n+l\times p)} = \begin{bmatrix} B \\ 0 \end{bmatrix}, C_{Z(q\times n+l)} = \begin{bmatrix} C & 0 \end{bmatrix}, L_{Z(n+l\times q)} = \begin{bmatrix} L_P \\ L_I \end{bmatrix},$$

equation (9) is equivalent to:

$$\begin{aligned} \dot{\hat{z}} &= (\begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} L_P \\ L_I \end{bmatrix} \begin{bmatrix} C & 0 \end{bmatrix})\hat{z} + \begin{bmatrix} L_P \\ L_I \end{bmatrix} y + \begin{bmatrix} B \\ 0 \end{bmatrix} u \\ &= (A_Z - L_Z C_Z)\hat{z} + L_Z y + B_Z u \end{aligned} \tag{10}$$

In this thesis, the state space model of plant with disturbance (A_Z, B_Z, C_Z) described by equation (7) will be called the extended plant model, and the state space model of DO described by equation (10) will be called the extended observer, which is similar to the extended observer described in [20].

Subtract equation (7) by equation (10), letting $e = z - \hat{z}$, which is the error between actual and estimated variables and disturbances, so that:

$$\dot{e}_z = \dot{z} - \dot{\hat{z}} = A_Z z - (A_Z - L_Z C_Z)\hat{z} - L_Z y \tag{11}$$

Noticing that $y = C_Z z$ in equation (8), the equation above can be rewritten as:

$$\dot{e}_z = A_Z z - L_Z C_Z z - (A_Z - L_Z C_Z)\hat{z} = (A_Z - L_Z C_Z)e \tag{12}$$

Thus, as long as $A_Z - L_Z C_Z = \begin{bmatrix} A - L_P C & E \\ -L_I C & 0 \end{bmatrix}$ is Routh-Hurwitz stable, the error between actual and estimated variables will become zero as $t \to \infty$

Recall equation (8) and (10):

$$\begin{aligned} \dot{\hat{z}} &= (A_Z - L_Z C_Z)\hat{z} + L_Z y + B_Z u \\ y &= C_Z z \end{aligned} \tag{13}$$

Comparing the equations above with the state-space model of proportional observer described by equation (2) and (3):

$$\begin{aligned} \dot{\hat{x}} &= (A - LC)\hat{x} + Ly + Bu \\ y &= Cx \end{aligned} \tag{14}$$

It is obvious that the the extended PIO model has the same formation with PO, by changing (A, B, C, L) into (A_Z, B_Z, C_Z, L_Z). Therefore, disturbance observer and proportional integral observer both can be regarded as a higher order proportional observer for the extended model, with $\hat{z} = \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}$. Thus, design methods for proportional observers can be applied to observer gain L calculation for proportional integral observers with this extended observer model.

Examples

In this 3^{rd} order single-input, single-output (SISO) system, the state space model is given by:

$$A = \begin{bmatrix} -4 & 2 & 2 \\ 20 & -20 & -6 \\ -12 & 4 & -4 \end{bmatrix}, B = \begin{bmatrix} 10 \\ 1 \\ 0.1 \end{bmatrix}, C = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 1.414 \\ 1.732 \end{bmatrix}$$

The initial state of the system is $x_0 = \begin{bmatrix} 3.142 & 2.718 & 0.618 \end{bmatrix}^T$, and a constant disturbance $d = 10$ is added to the plant at t = 2 sec.

The observer gain for proportional observer can be obtained by using MATLAB command `>>L=place(A',C',opoles)'`, in which `opoles` are the chosen pole locations for the proportional observer. In this example, `opoles` for proportional observer is chosen to be $\begin{bmatrix} -17.7516 & -14.0572 \pm 13.4111i \end{bmatrix}$. A simulation for using proportional observer to estimate plant state variables is provided by `PO_SISO.m`

The simulation result given by Figure 3 and 4 shows that proportional observer estimates the state variables and output correctly when there is no disturbance in the plant for t < 2 sec. However, there will be a constant steady-state error in estimation after leading a constant disturbance into the plant at t = 2 sec, for both state variables and plant output.

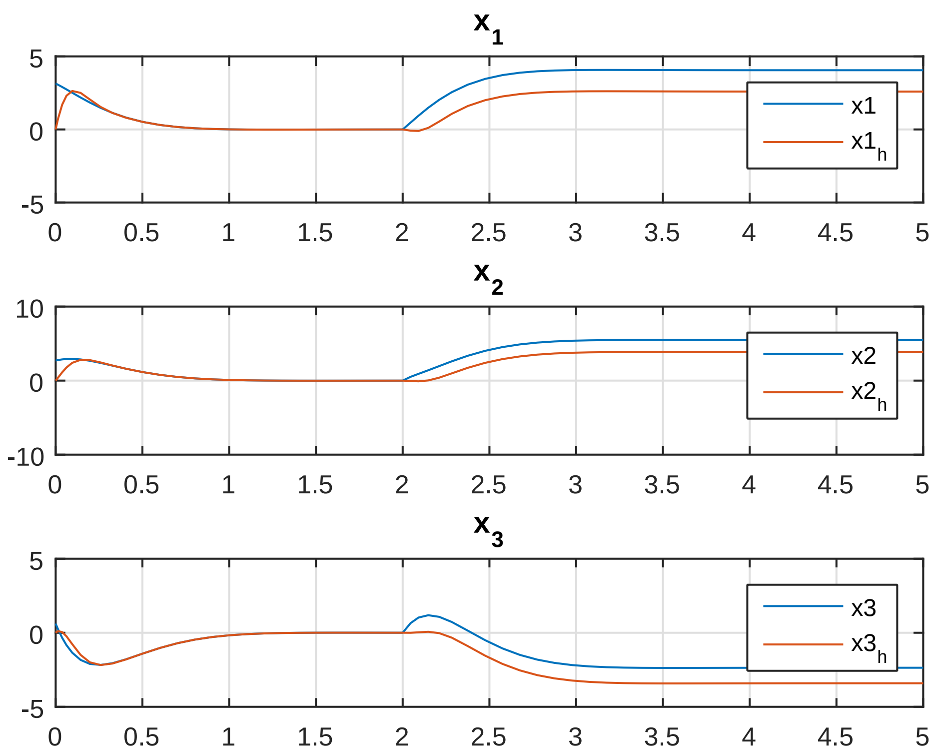

Figure 3. Estimated and actual state variables using PO

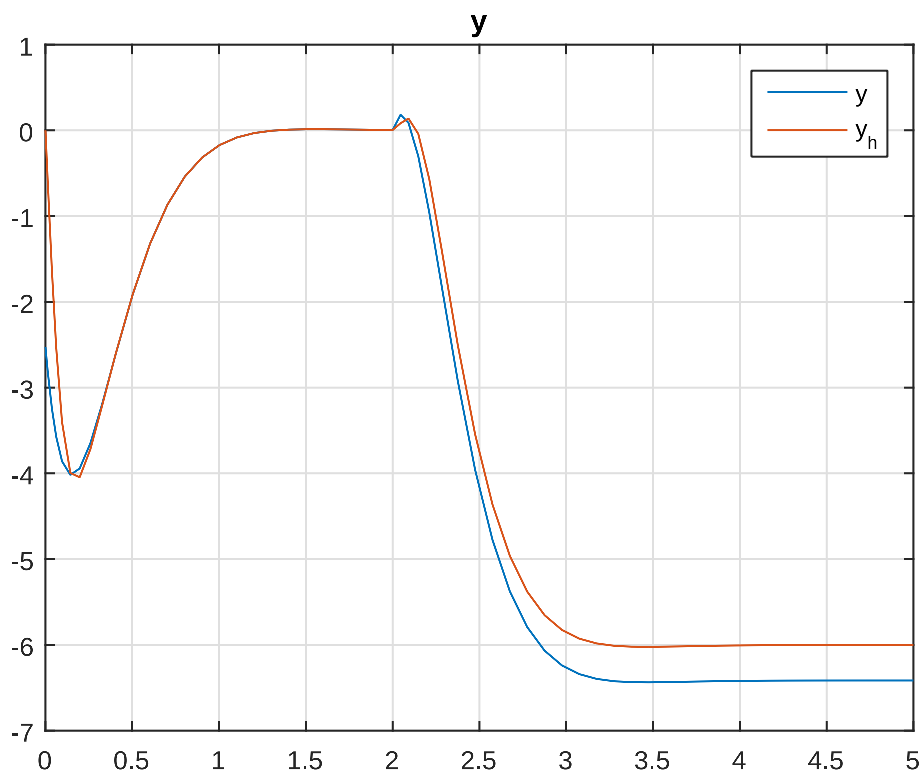

Figure 4. Estimated and actual system output using PO

For proportional integral observers, the observer gain L_Z for the extended model is given by MATLAB command `>>Lz=place(Az',Cz',opoles)'`, in which `opoles` are the chosen pole locations for the extended observer. In this example, `opoles` for proportional integral observer is chosen to be $\begin{bmatrix}-52.6614 & -12.2790 & -10.8783 \pm 5.9727i\end{bmatrix}$. A simulation for using proportional observer to estimate plant state variables is provided by `PIO_SISO.m`

$L_Z = \begin{bmatrix} L_P \\ L_I \end{bmatrix}$ can be separated into two parts to fit equation (5) for the general form of PIO. The proportional observer gain L_P contains the first n rows of L_Z, and the integral observer gain L_I contains the remaining rows of L_Z. The proportional and integral gains calculated using `place` command are:

$$L_P = \begin{bmatrix} -87.4296 \\ -111.3991 \\ -28.7325 \end{bmatrix}, \quad L_I = \begin{bmatrix} -359.3262 \end{bmatrix}.$$

As is shown in Figure 5, 6 and 7, simulation result shows that PIO has the ability to estimate state variables, disturbance and system output correctly within 0.6 sec, with or without the disturbance in plant.

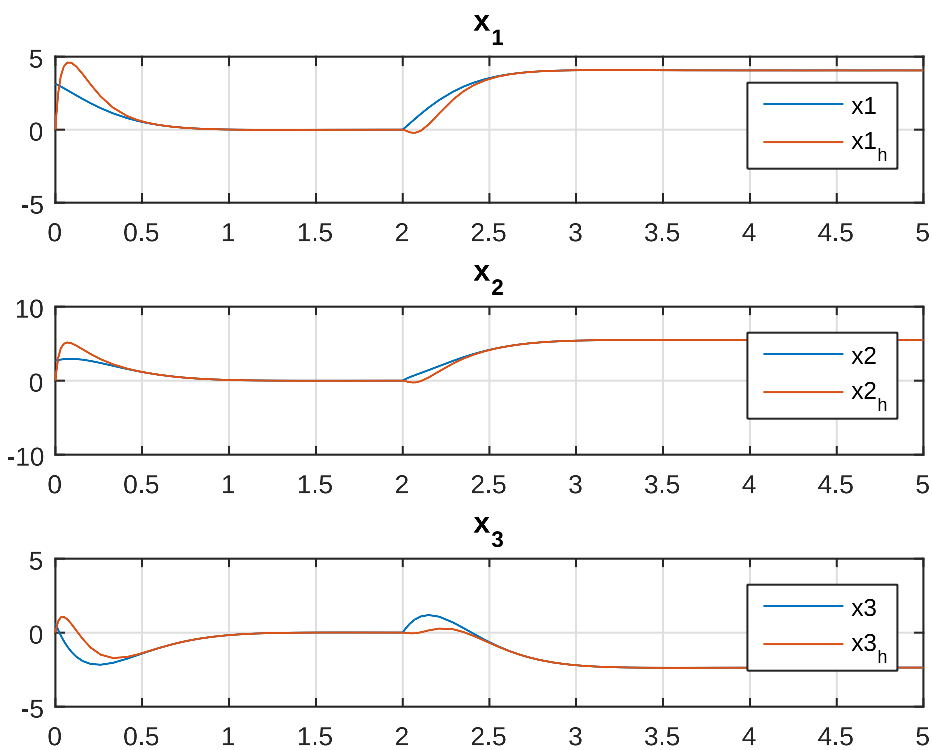

Figure 5. Estimated and actual state variables using PIO

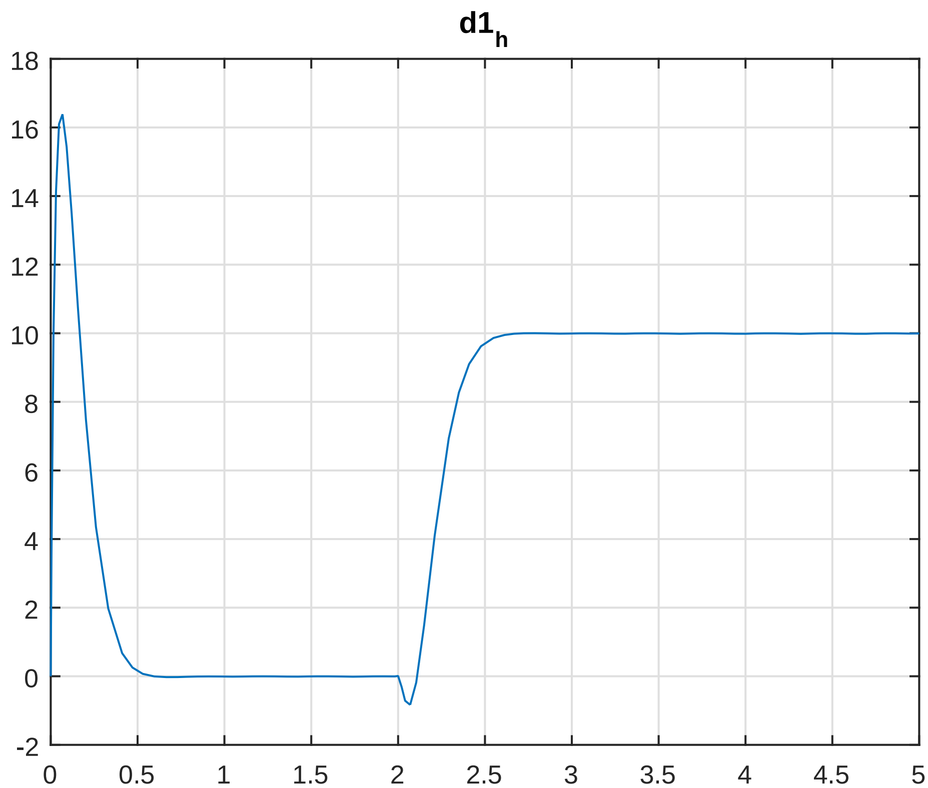

Figure 6. Estimated disturbance using PIO

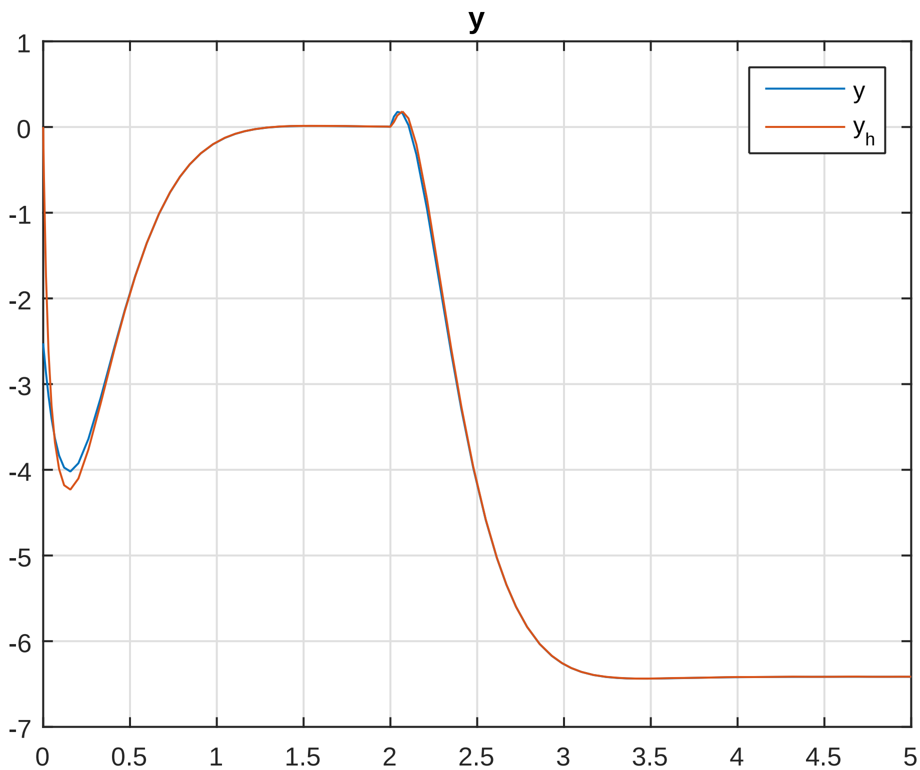

Figure 7. Estimated and actual system output using PIO

The results above are derived in continuous time, and the same process can be applied in discrete time. For a n^{th} order, p input, q output plant with l independent disturbance of constant value, the plant model is described as:

$$\begin{aligned} x[k+1] &= Ax[k] + Bu[k] + Ed[k] \\ d[k+1] &= d[k] \\ y[k] &= Cx[k] \end{aligned} \tag{15}$$

And the PIO model for this discrete time plant is:

$$\begin{aligned} \hat{x}[k+1] &= A\hat{x}[k] + Bu[k] + L_P(y[k] - C\hat{x}[k]) + E\hat{d}[k] \\ \hat{d}[k+1] &= d[k] + L_I(y[k] - C\hat{x}[k]) \end{aligned} \tag{16}$$

Similar to the extended model for plant and observer in continuous time, the extended model for this discrete time plant and observer is given by the following equations by making $z[k] = \begin{bmatrix} x[k] \\ d[k] \end{bmatrix}$:

$$\begin{aligned} z[k+1] &= A_Z z[k] + B_Z u[k] \\ \hat{z}[k+1] &= (A_Z - L_Z C_Z)\hat{z}[k] + L_Z y[k] + B_Z u[k] \\ y[k] &= C_Z z[k] \end{aligned} \tag{17}$$

In which

$$A_{Z(n+l \times n+l)} = \begin{bmatrix} A & E \\ 0 & I \end{bmatrix}, B_{Z(n+l \times p)} = \begin{bmatrix} B \\ 0 \end{bmatrix}, C_{Z(q \times n+l)} = \begin{bmatrix} C & 0 \end{bmatrix}, L_{Z(n+l \times q)} = \begin{bmatrix} L_P \\ L_I \end{bmatrix}$$

The extended model in discrete time described by equation (17) has the exact same form with equation (13) for the continuous time. The only difference between the extended model of continuous time and discrete time is that the A_Z matrix in continuous time model is $\begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}$. Therefore, the design method for observer in continuous time can be directly applied for discrete time systems.

CHAPTER 3

Observer Gain Calculation for MIMO Systems

As is shown in Chapter 2, MATLAB command `place` can be used for observer gain calculation with desired observer pole locations. For single-input, single-output (SISO) systems, `place` command will provide the unique result for feedback gain calculation, while for multiple-input, multiple-output (MIMO) systems, `place` will choose one from an infinite number of results with the eigenvectors of $(A-LC)$ to be most orthogonal. In this chapter, a parameterized method of observer gain calculation will be shown based on the method developed in [17, 21] for feedback gain calculation.

The plant model is assumed to be observable, which means the observer has the ability to estimate all state variables correctly from any initial state in finite time, for observer gain L design. Popov-Belevitch-Hautus (PBH) Test is a commonly used method for checking observability.

Assuming the original plant (A, B, C) is observable, thus, the following equation holds for all eigenvalue λ of matrix A, according to PBH test [22]:

$$rank \begin{bmatrix} \lambda I_n - A \\ C \end{bmatrix} = n \tag{18}$$

Consider the n^{th} order system with q output and l disturbances, as is shown in Chapter 2, the extended model $\dot{z} = A_Z z + B_Z u$ and $y = C_Z z$ is observable if the following equation holds for all eigenvalue λ of matrix A_Z:

$$rank \begin{bmatrix} \lambda I_{n+l} - A_Z \\ C_Z \end{bmatrix} = n + l \tag{19}$$

For $A_Z = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}$ in continuous time, eigenvalues of A_Z are all eigenvalues of A and l zeros. For stable systems in continuous time, matrix A in plant model can not be zero, thus:

Case 1: $\lambda \neq 0$, which are the poles of the original plant:

$$\begin{bmatrix} \lambda I_{n+l} - A_Z \\ C_Z \end{bmatrix} = \begin{bmatrix} \lambda I_n - A & -E \\ 0 & \lambda I_l \\ C & 0 \end{bmatrix} \tag{20}$$

The rank will not be changed by making the following transformation:

$$\begin{bmatrix} I_n & E/\lambda & 0 \\ 0 & I_l & 0 \\ 0 & 0 & I_q \end{bmatrix} \begin{bmatrix} \lambda I_n - A & -E \\ 0 & \lambda I_l \\ C & 0 \end{bmatrix} = \begin{bmatrix} \lambda I_n - A & 0 \\ 0 & \lambda I_l \\ C & 0 \end{bmatrix}$$

which is equivalent to:

$$rank \begin{bmatrix} \lambda I_{n+l} - A_Z \\ C_Z \end{bmatrix} = rank \begin{bmatrix} \lambda I_n - A & 0 \\ 0 & \lambda I_l \\ C & 0 \end{bmatrix} = rank \begin{bmatrix} \lambda I_n - A \\ C \end{bmatrix} + rank \begin{bmatrix} 0 \\ \lambda I_l \\ 0 \end{bmatrix}$$

Noticing that the original plant is assumed to be observable, suggesting that the rank of $\begin{bmatrix} \lambda I_n - A \\ C \end{bmatrix}$ is n as is shown in equation (18), and the rank of $\begin{bmatrix} 0 & \lambda I_l^T & 0 \end{bmatrix}^T$ is l, Thus:

$$rank \begin{bmatrix} \lambda I_{n+l} - A_Z \\ C_Z \end{bmatrix} = n + l \tag{21}$$

Therefore, the poles of original plant are still observable for the extended model.

Case 2: $\lambda = 0$, which are the added poles of extended plant by letting $A_Z = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}$:

$$rank \begin{bmatrix} \lambda I_{n+l} - A_Z \\ C_Z \end{bmatrix} = rank \begin{bmatrix} -A & -E \\ 0 & 0 \\ C & 0 \end{bmatrix} = rank \begin{bmatrix} -A & -E \\ C & 0 \end{bmatrix} \tag{22}$$

Therefore, the added poles to the extended plant are observable, if and only if $rank \begin{bmatrix} -A & -E \\ C & 0 \end{bmatrix} = n + l$ holds.

Noticing that the only condition for $rank \begin{bmatrix} -A & -E \\ C & 0 \end{bmatrix} = n + l$ is that q is greater than or equal to l, which means that the extended system is observable only if the number of measured output is greater than or equal to the number of independent disturbances. In case the disturbance model is not known, E should be selected as I_n, and the system is observable if and only if the number of output is equal to the number of state variables, which is usually invalid for the observer is used when not all state variables are available. The prove process is similar in discrete time, as is shown in [8].

In conclusion, the extended system is observable if the following conditions stands: the original plant is observable, the number of measured output is greater than or equal to the number of independent disturbances, and $rank \begin{bmatrix} -A & -E \\ C & 0 \end{bmatrix} = n + l$ or equivalently, $rank \begin{bmatrix} A & E \\ C & 0 \end{bmatrix} = n + l$.

Now we consider parameterizing of observer gain matrices for systems with more than one measured output. Assuming the extended plant model (A_Z, B_Z, C_Z) is observable, λ_i is one of the eigenvalues of $A_Z - L_Z C_Z$, and u_i is the corresponding eigenvector, which satisfies the following equation:

$$({A_Z}^T - {C_Z}^T {L_Z}^T) u_i = \lambda_i u_i \tag{23}$$

The equation above can be rewritten as:

$$\begin{bmatrix} (\lambda_i I - {A_Z}^T) & {C_Z}^T \end{bmatrix} \begin{bmatrix} u_i \\ {L_Z}^T u_i \end{bmatrix} = 0 \tag{24}$$

Letting $P(\lambda_i) = \begin{bmatrix} (\lambda_i I - {A_Z}^T) & {C_Z}^T \end{bmatrix}$, this $(n+l) \times (n+l+q)$ matrix has rank n for the extended model to be observable, according to the PBH test [22]. Thus $\begin{bmatrix} u_i \\ {L_Z}^T u_i \end{bmatrix}$ would be in the q-dimensional null space of $P(\lambda_i)$. Orthonormal bases $\begin{bmatrix} M_i \\ N_i \end{bmatrix}$ of this null space can be generated by using `null(P)` command in MATLAB.

Multiplying this orthonormal base by a $q \times 1$ parameter α_i, the following equation can be formed:

$$\begin{bmatrix} u_i \\ {L_Z}^T u_i \end{bmatrix} = \begin{bmatrix} M_i \\ N_i \end{bmatrix} \alpha_i = \begin{bmatrix} M_i \alpha_i \\ N_i \alpha_i \end{bmatrix} \stackrel{def}{=} \begin{bmatrix} v_i \\ w_i \end{bmatrix}. \tag{25}$$

Thus, for any selected parameter α_i, there would be a corresponding pair of vectors (v_i, w_i) satisfying ${L_Z}^T w_i = v_i$. Apply this process to all $n + l$ eigenvalues, the following equation will satisfy by forming $V = \begin{bmatrix} v_1 & \dots & v_{n+l} \end{bmatrix}$ and $W = \begin{bmatrix} w_1 & \dots & w_{n+l} \end{bmatrix}$:

$${L_Z}^T V = W \tag{26}$$

By choosing parameters $\alpha_1, \dots, \alpha_{n+l}$ to make matrix V to be nonsingular, the observer gain matrix is:

$$L_Z = (WV^{-1})^T = V^{-T} W^T \tag{27}$$

MATLAB function `obg_reg.m` is provided by R. Vaccaro [17] to calculate proportional observer gain for feedback control system with good stability robustness. A MATLAB function `OBGX.m` has been built by changing the cost function in `obg_reg.m`, making users able to pick out certain parameters $\alpha_1, \dots, \alpha_{n+l}$ and returns corresponding observer gain matrix L_Z for the extended model, with minimized cost function. As is mentioned in Chapter 2, this method can be applied to both continuous time and discrete time systems.

Observer gain for the extended model can be obtained by command `>>Lz=OBGX(Az,Cz,opoles,minoption)`, with the extended plant model (`Az,Cz`) and desired extended observer pole locations (`opoles`). `minoption` is the cost function to be minimized, such as norm of observer gain (`'norm'`) or condition number of $(A_Z - L_Z C_Z)$ (`'cond'`). This MATLAB function can also be used to calculate proportional observer gains, by inputting the original plant model (A, C) and proportional observer pole locations.

Examples

A simulation will be provided using example from [8]. The discrete time state space model of the disturbed plant is

$$A = \begin{bmatrix} 0.9630 & 0.0181 & 0.0187 \\ 0.1808 & 0.8195 & -0.0514 \\ -0.1116 & 0.0344 & 0.9586 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix}, E = \begin{bmatrix} 0.0996 & 0.0213 \\ 0.0050 & 0.1277 \\ 0.1510 & 0.0406 \end{bmatrix}$$

The initial state of the system is $x_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$ and the constant disturbance $d = \begin{bmatrix} 0.3\sin(0.1k) + 0.5\cos(0.03k) \\ 2 + 0.2\cos(0.05k) \end{bmatrix}$. The initial state of observer is zero vector. A simulation for using proportional observer to estimate plant state variables provided by `PIO_MIMO.m` is shown in Figure 8, 9, 10 and 11, with the same pole locations in [8] and cost function to be condition number of $(A_Z - L_Z C_Z)$.

Figure 8 and 9 shows the estimation error on state variables and disturbance will comes to zero in short period. Figure 10 shows the estimated and actual state variables matches very well in long time period. Figure 11 shows that compared to the actual disturbance, there is a delay and error in amplitude for the estimated disturbance in long time period.

The proportional and integral observer gains can be obtained by separating the observer gain L_Z for extended observer with the same process in the previous chapter:

$$L_P = \begin{bmatrix} 2.8404 & -3.5617 \\ 2.6633 & 2.2444 \\ 3.0767 & -2.1287 \end{bmatrix}, \quad L_I = \begin{bmatrix} 18.5780 & 8.6146 \\ 12.9513 & 16.2018 \end{bmatrix}$$

The condition number of $(A_Z - L_Z C_Z)$ is 6.568×10^3

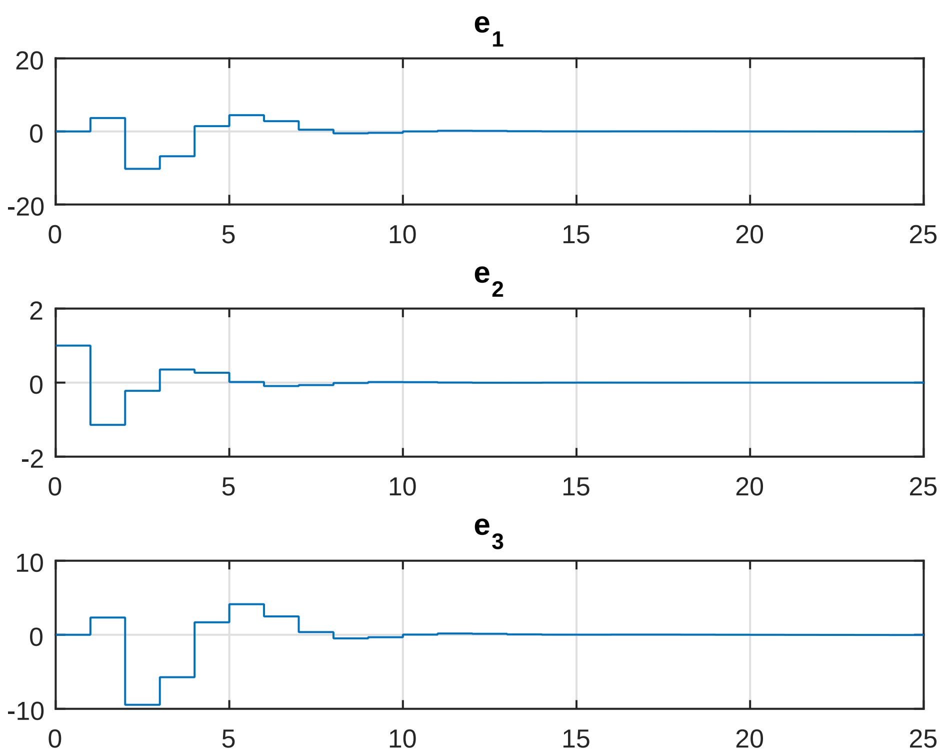

Figure 8. Estimation error on state variables using PIO

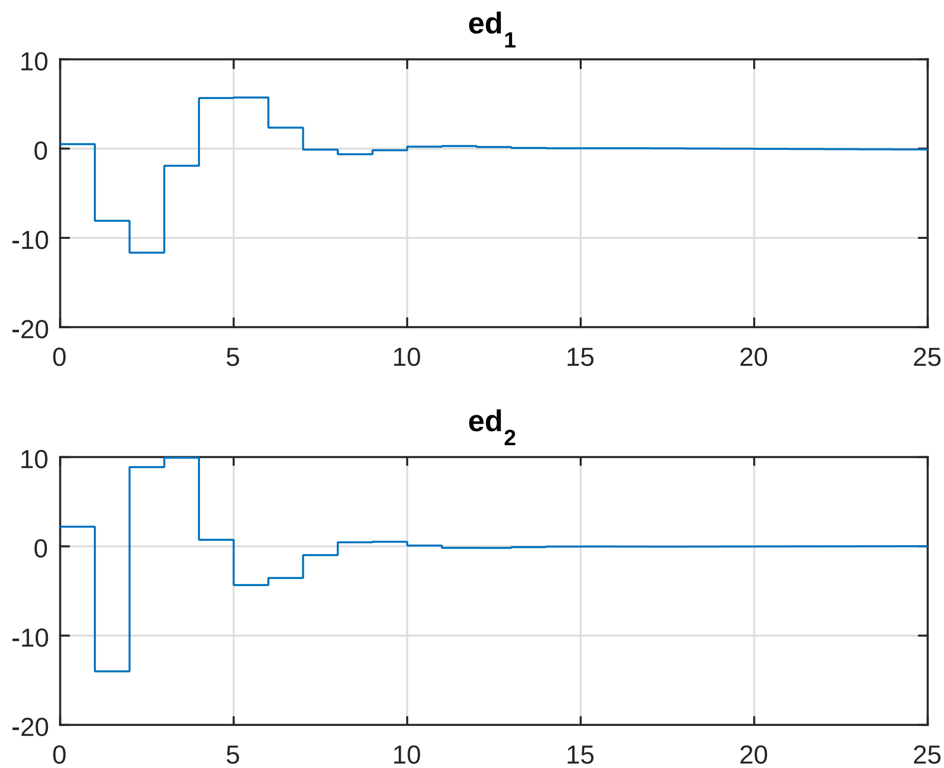

Figure 9. Estimation error on disturbance using PIO

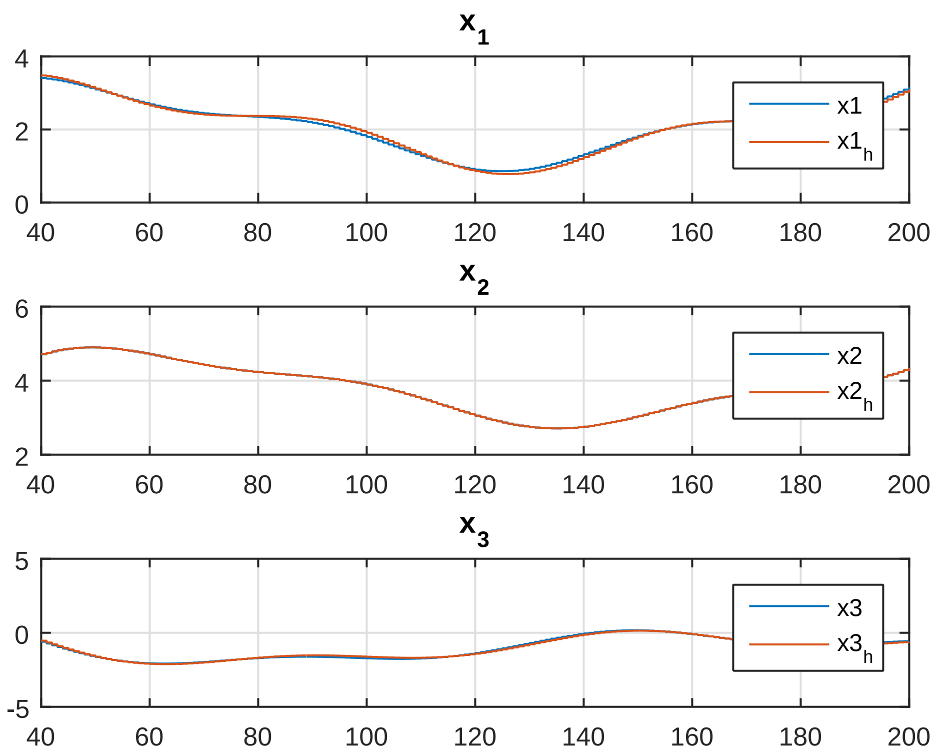

Figure 10. Estimated and actual state variables using PIO

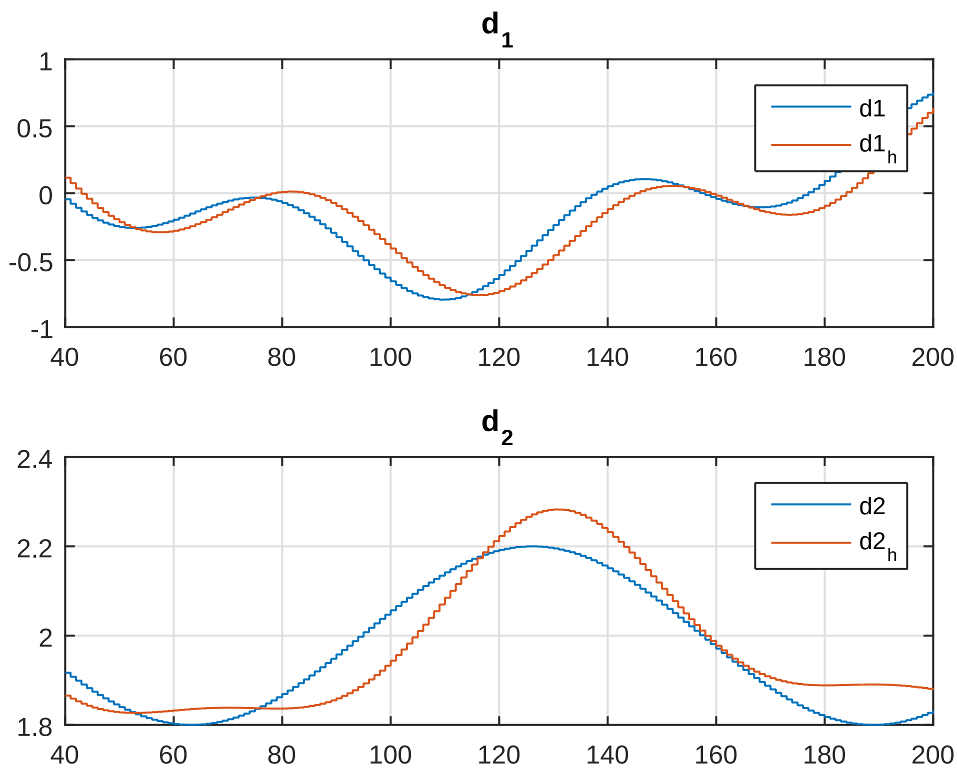

Figure 11. Estimated and actual disturbance using PIO

Compared with the proportional and integral gain in [8],

$$L_P = \begin{bmatrix} 31.7392 & 19.6384 \\ 1.8918 & 1.7307 \\ 29.3767 & 18.9849 \end{bmatrix}, \quad L_I = \begin{bmatrix} 51.1873 & 34.5803 \\ -21.3249 & -10.6399 \end{bmatrix}$$

The condition number of $(A_Z - L_Z C_Z)$ is 3.9957×10^4, which is approximately 6 times higher than the result obtained from `OBGX.m`

Compared to the result in [8], as is shown in Figure 12, 13, 14 and 15, the result using `OBGX.m` shown in Figure 8, 9, 10 and 11 has smaller amplitude on the estimation error in short time period, but not that good result in disturbance estimation in long time period.

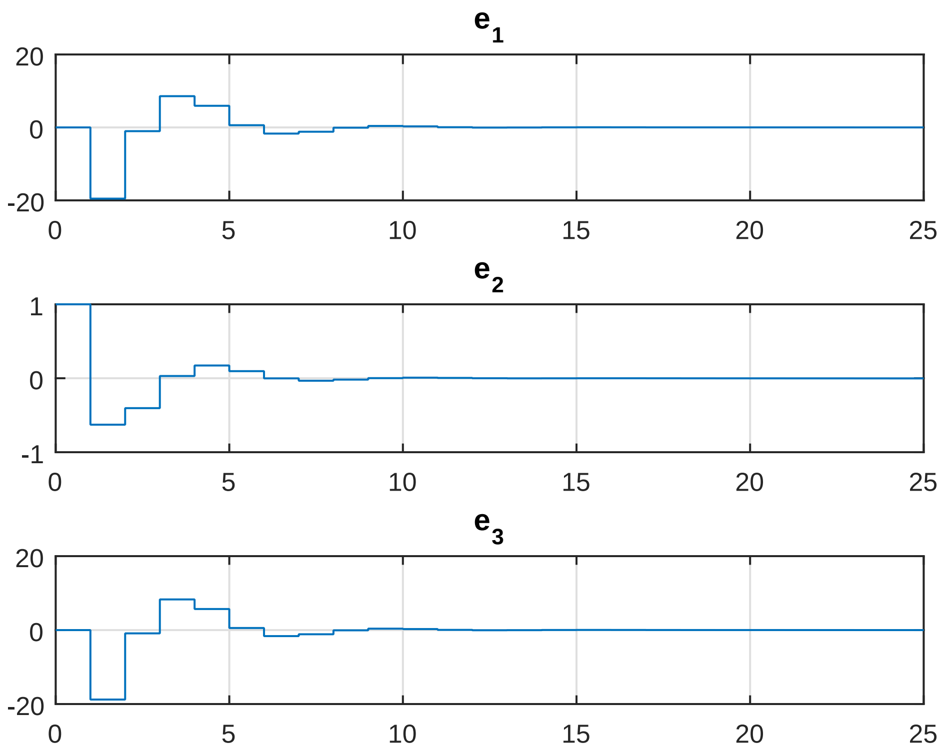

Figure 12. Estimation error on state variables using PIO

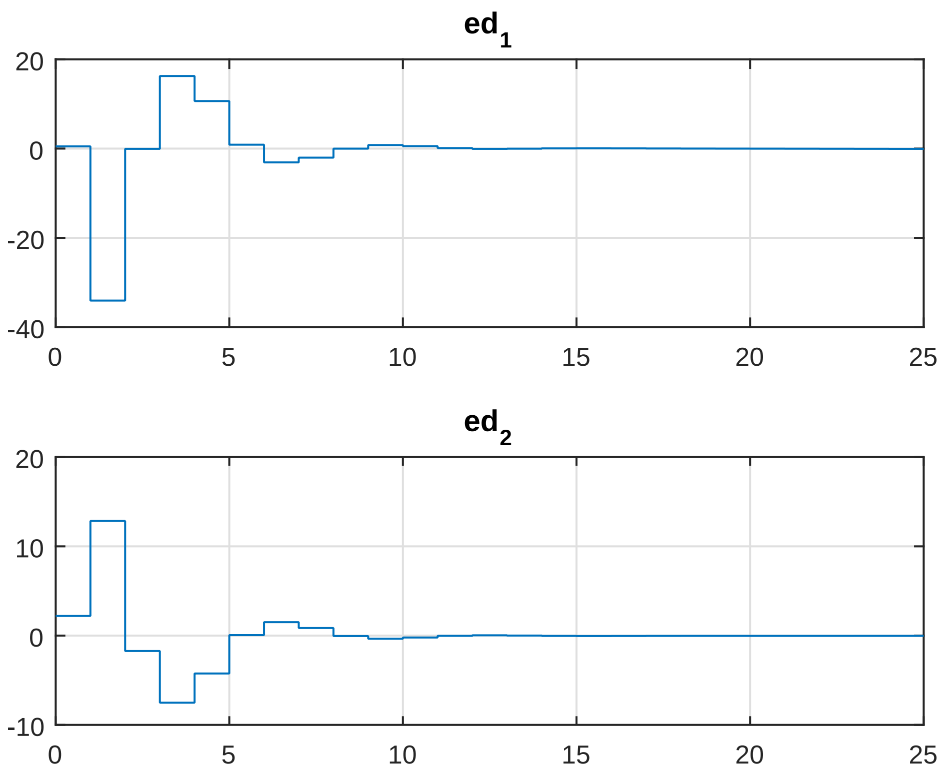

Figure 13. Estimation error on disturbance using PIO

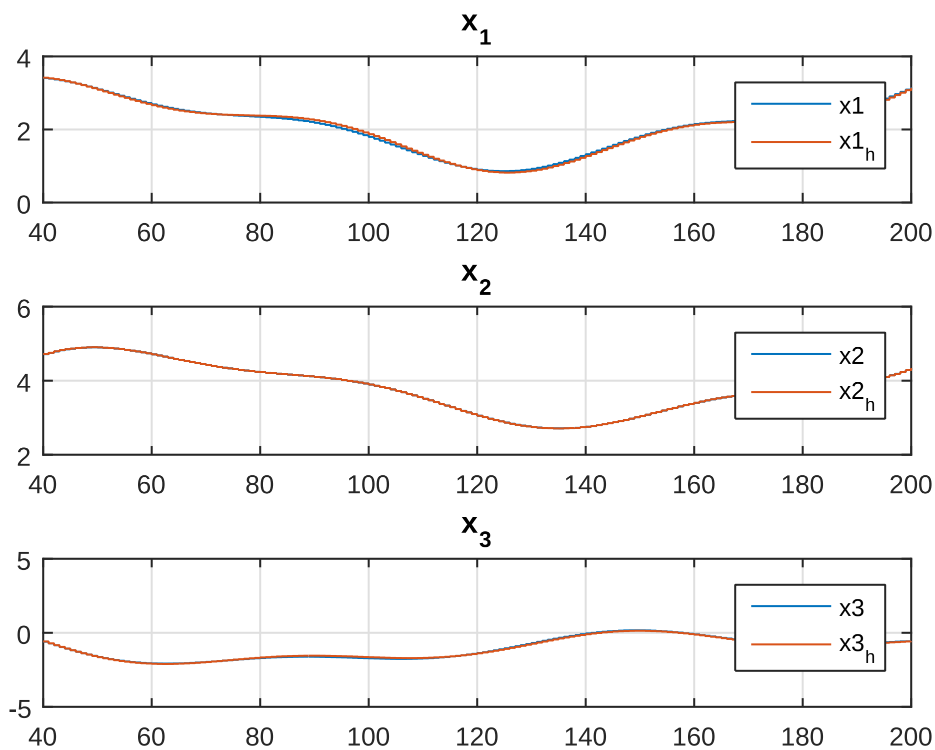

Figure 14. Estimated and actual state variables using PIO

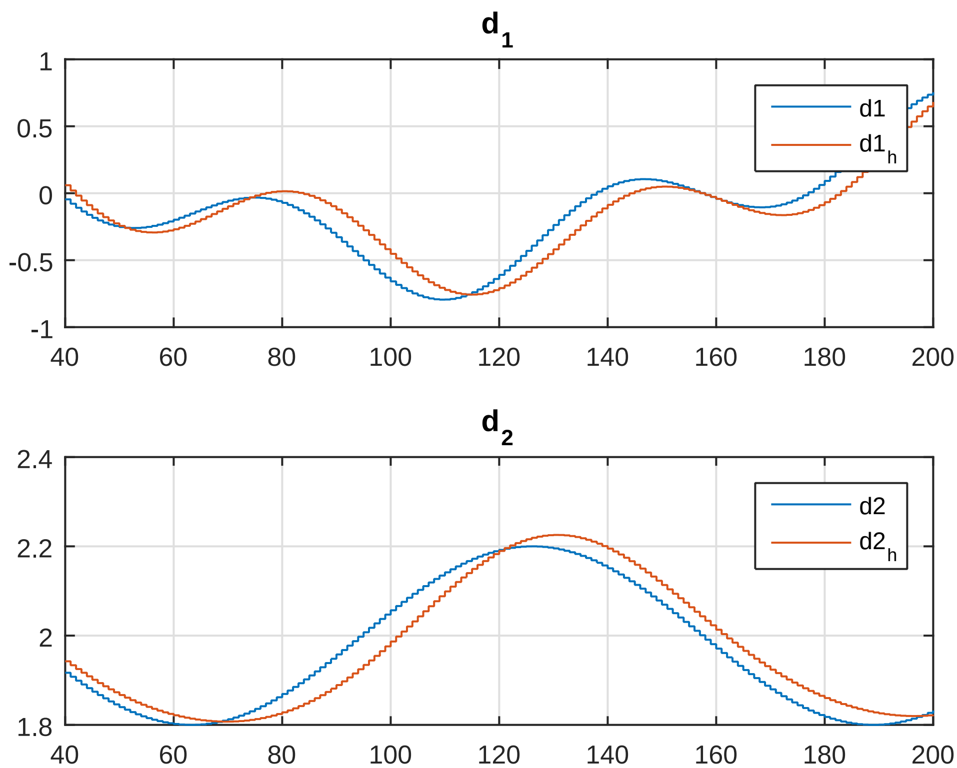

Figure 15. Estimated and actual disturbance using PIO

The reason that the observer can estimate state variables and constant disturbance correctly, while not that accurate on sinusoid disturbance is that this extended model of observer is built for the plant with the assumption that the unknown disturbance is constant. Matrix E can be assumed to be n^{th} order identity matrix if the exact disturbance model is unknown, but the extended observer model will still estimate disturbance with constant value correctly, if the extended system (A_Z, C_Z) is observable. For other types of disturbances, there will always exists an error on disturbance estimation, since the observer model does not fit the disturbance model. In the mean time, the estimation of state variables is still going to be correct, as long as the plant model (A, B, C) is known.

CHAPTER 4

PIO for Regulators

As is shown in Chapter 3, observer gain is not unique for MIMO system, and MATLAB function `OBG.m` was developed to calculate an observer gain to minimize a cost function, such as norm of observer gain or condition number of $(A_Z - L_Z C_Z)$. In this chapter, a similar method of calculating observer gain for PIO based regulators will be provided, by changing the cost function into robustness bound of the feedback control system.

Recall the plant and PIO model described by equation (1), (2) and (5):

$$\begin{aligned} &\dot{x} = Ax + Bu + Ed \\ &\dot{d} = 0 \\ &y = Cx \\ &\dot{\hat{x}} = A\hat{x} + Bu + L_P(y - C\hat{x}) + E\hat{d} \\ &\dot{\hat{d}} = L_I(y - C\hat{x}) \end{aligned} \tag{28}$$

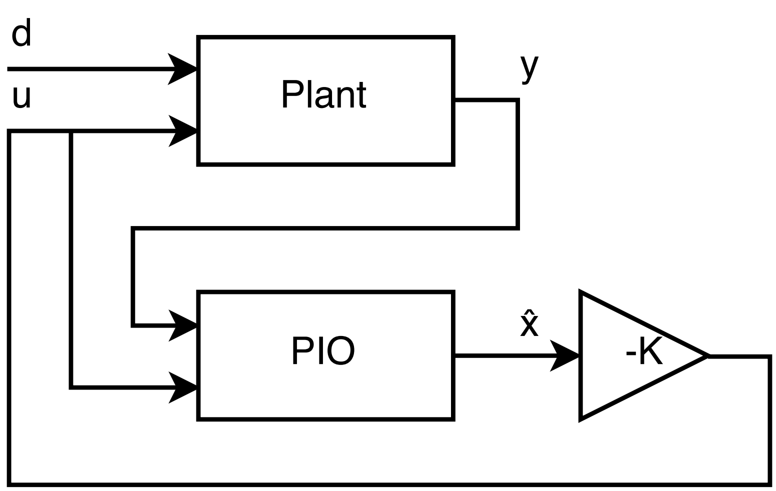

Figure 16. Block diagram of PIO based regulator

As is shown in Figure 16, adding feedback to the system will make the input u to the plant equals to $-K\hat{x}$, and equation (28) can be rewritten into:

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \\ \dot{\hat{d}} \end{bmatrix} = \begin{bmatrix} A & -BK & 0 \\ L_P C & A - BK - L_P C & E \\ L_I C & -L_I C & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \\ \hat{d} \end{bmatrix} + \begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix} d \tag{29}$$

For close loop systems, robustness is an important property of feedback gain and observer gain design. In case the plant is not correctly modeled, the perturbed plant is introduced using small gain theorem as is shown in Figure 17. If the system infinity norm of the unknown plant perturbation is less than a robustness bound δ_1, the perturbed control system is guaranteed to be stable, despite the error in modeling the plant. These robustness bounds are derived using the small-gain theorem [23]. This theorem says that the robustness bound δ_1 is the reciprocal of the system infinity norm of the system from w to v in Figure 17.

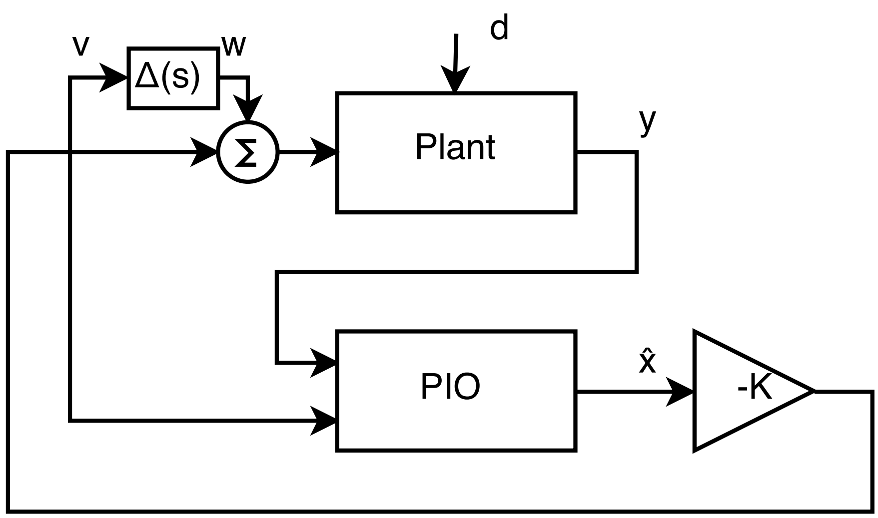

Figure 17. Robustness analysis (δ_1) of PIO based regulator

To calculate robustness bound δ_1 for the PIO based regulator, the model from w to v will be derived by adding a perturbation on plant as is shown in Figure 17. By letting $v = -K\hat{x}$ and $u = w + v = w - K\hat{x}$, equation (28) can be rewritten as:

$$\begin{aligned} \dot{x} &= Ax + B(w - K\hat{x}) + Ed = Ax - BK\hat{x} + Bw + Ed \\ \dot{\hat{x}} &= A\hat{x} - BK\hat{x} + L_P(y - C\hat{x}) + E\hat{d} = L_PCx + (A - BK - L_PC)\hat{x} + E\hat{d} \\ \dot{\hat{d}} &= L_ICx - L_IC\hat{x} \end{aligned} \tag{30}$$

In order to calculate close loop robustness, the disturbance is set to zero, making the equation above equivalent to:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \\ \dot{\hat{d}} \end{bmatrix} &= \begin{bmatrix} A & -BK & 0 \\ L_PC & A - BK - L_PC & E \\ L_IC & -L_IC & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \\ \hat{d} \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} w \\ v &= \begin{bmatrix} 0 & -K & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \\ \hat{d} \end{bmatrix} \end{aligned} \tag{31}$$

By forming

$$A_{Z(n+l \times n+l)} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}, B_{Z(n+l \times p)} = \begin{bmatrix} B \\ 0 \end{bmatrix}, C_{Z(q \times n+l)} = \begin{bmatrix} C & 0 \end{bmatrix}$$

and

$$K_{Z(p \times n+l)} = \begin{bmatrix} K & 0 \end{bmatrix}, L_{Z(n+l \times q)} = \begin{bmatrix} L_P \\ L_I \end{bmatrix},$$

equation (31) can be rewritten into the following equation by letting $\hat{z} = \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}$:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\hat{z}} \end{bmatrix} &= \begin{bmatrix} A & -BK_Z \\ L_ZC & A_Z - B_ZK_Z - L_ZC_Z \end{bmatrix} \begin{bmatrix} x \\ \hat{z} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w \\ v &= \begin{bmatrix} 0 & -K_Z \end{bmatrix} \begin{bmatrix} x \\ \hat{z} \end{bmatrix} \end{aligned} \tag{32}$$

Compared to equation (22) in [17]:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} &= \begin{bmatrix} A & -BK \\ LC & A - BK - LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w \\ v &= \begin{bmatrix} 0 & -K \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \end{aligned}$$

The equation above is equivalent to equation (32) by changing K into K_Z, L into L_Z, and $\hat{x}$ into $\hat{z}$, which suggests that the design method for PIO based regulator would be the same as the PO based regulator, by changing the proportional observer to the extended observer.

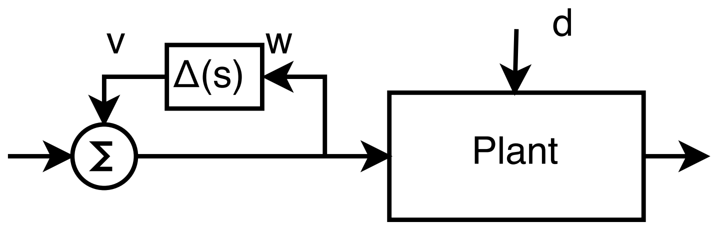

Figure 18. Perturbed plant model using δ_2

If the plant perturbation is of the form as is shown in Figure 18, there is a corresponding robustness bound δ_2 for robustness analysis. Using the similar process, the model from w to v is provided as:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\hat{z}} \end{bmatrix} &= \begin{bmatrix} A & -BK_Z \\ L_Z C & A_Z - B_Z K_Z - L_Z C_Z \end{bmatrix} \begin{bmatrix} x \\ \hat{z} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w \\ v &= \begin{bmatrix} 0 & -K_Z \end{bmatrix} \begin{bmatrix} x \\ \hat{z} \end{bmatrix} + w \end{aligned} \tag{33}$$

The objective of observer design for close loop system is to obtain the maximized robustness bound δ_1 and δ_2. By changing the cost function designed for PO based regulator in `obg_reg.m` provided by R. Vaccaro [17] into the corresponding cost function of robustness bound for PIO based regulator, with A_Z, B_Z, C_Z, K_Z and L_Z, MATLAB function `OBGX_REG.m` is developed to calculate extended observer gain for PIO based feedback control system with good stability robustness. Observer gain for the extended model can be obtained by command `>>Lz=OBGX_REG(Az,Bz,Cz,Kz,l,opoles,T)`, with the extended plant and regulator model (`Az,Bz,Cz,Kz`), number of independent disturbance (`l`), and desired extended observer pole locations (`opoles`). `T` is the sampling interval for discrete systems, and should be set to zero for continuous systems. This MATLAB function can also be used to calculate proportional observer gains, by inputing the original plant and regulator model (`A,B,C,K`) and proportional observer pole locations, with `l` set to be zero.

Examples

A simulation will be provided using the example of cart-pendulum system from [17]. The continuous time state space model of the disturbed plant is

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 23.1 & 0 & 0 & 0.1189 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -25 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 12.525 \\ 0 \\ 2633 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 1.414 \\ 1.732 \\ 2 \end{bmatrix}$$

The initial state of the system is $x_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$, and a constant disturbance $d = 5$ is added to the plant at t = 2 sec. The initial state of observer is set to zero.

The feedback gain given by `>>K=place(A,B,poles)` command with regulator poles equal to `s4/Ts`, in which `s4` is the Bessel poles, and `Ts` is the settling time of the regulator equals to 1 sec.

$$K = \begin{bmatrix} 1.7034 & 0.3610 & -0.0229 & -0.0040 \end{bmatrix}$$

The simulation result given by Figure 19 and 20 shows that proportional observer estimates the state variables and output correctly when there is no disturbance in the plant for t < 2 sec. However, there will be a constant steady-state error in estimation after leading a constant disturbance into the plant at t = 2 sec, for both state variables and plant output.

The robustness bound of the regulator using PO is $\delta_1 = 0.4354$ and $\delta_2 = 0.6158$.

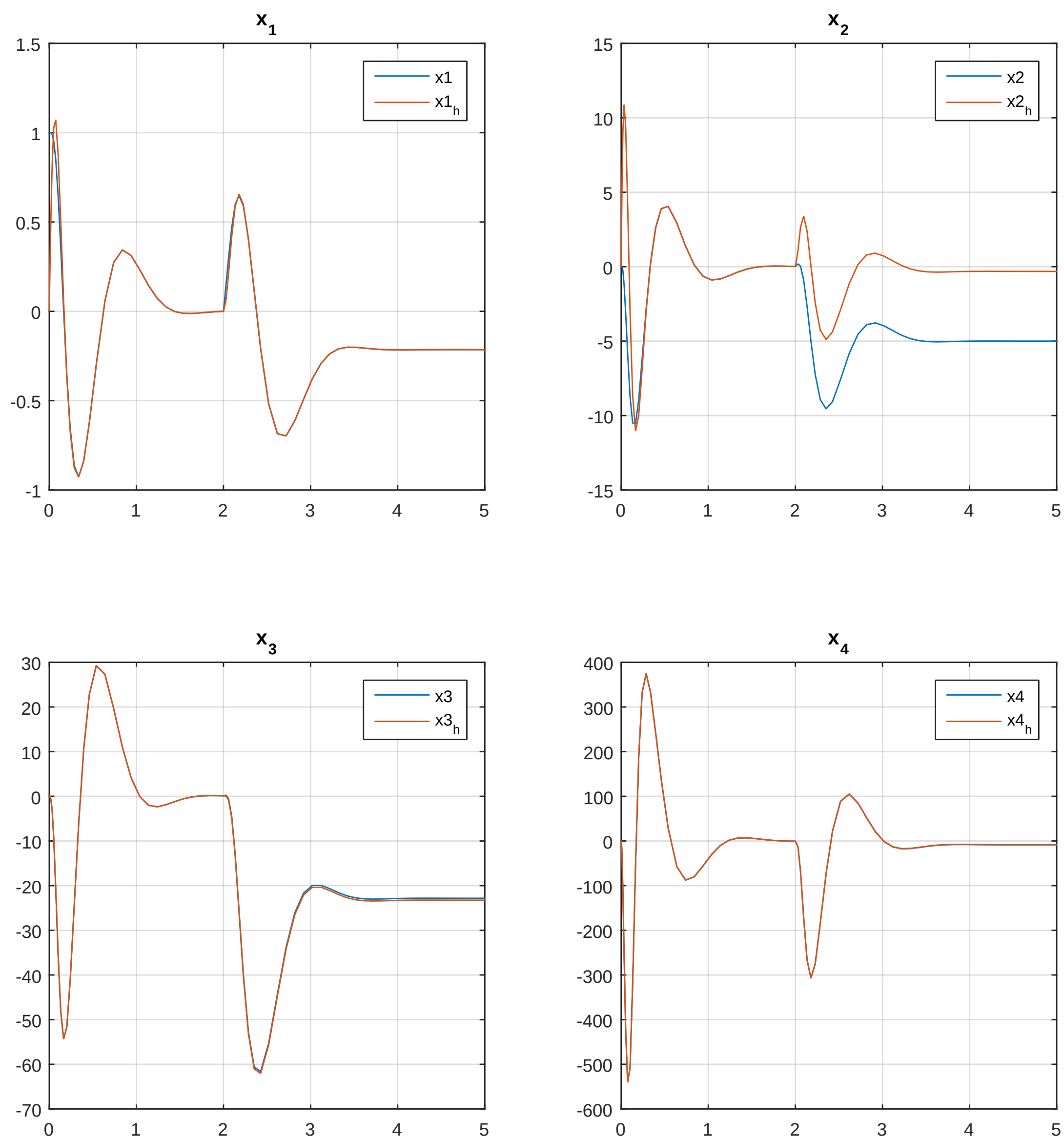

Figure 19. Estimated and actual state variables for PO based regulator

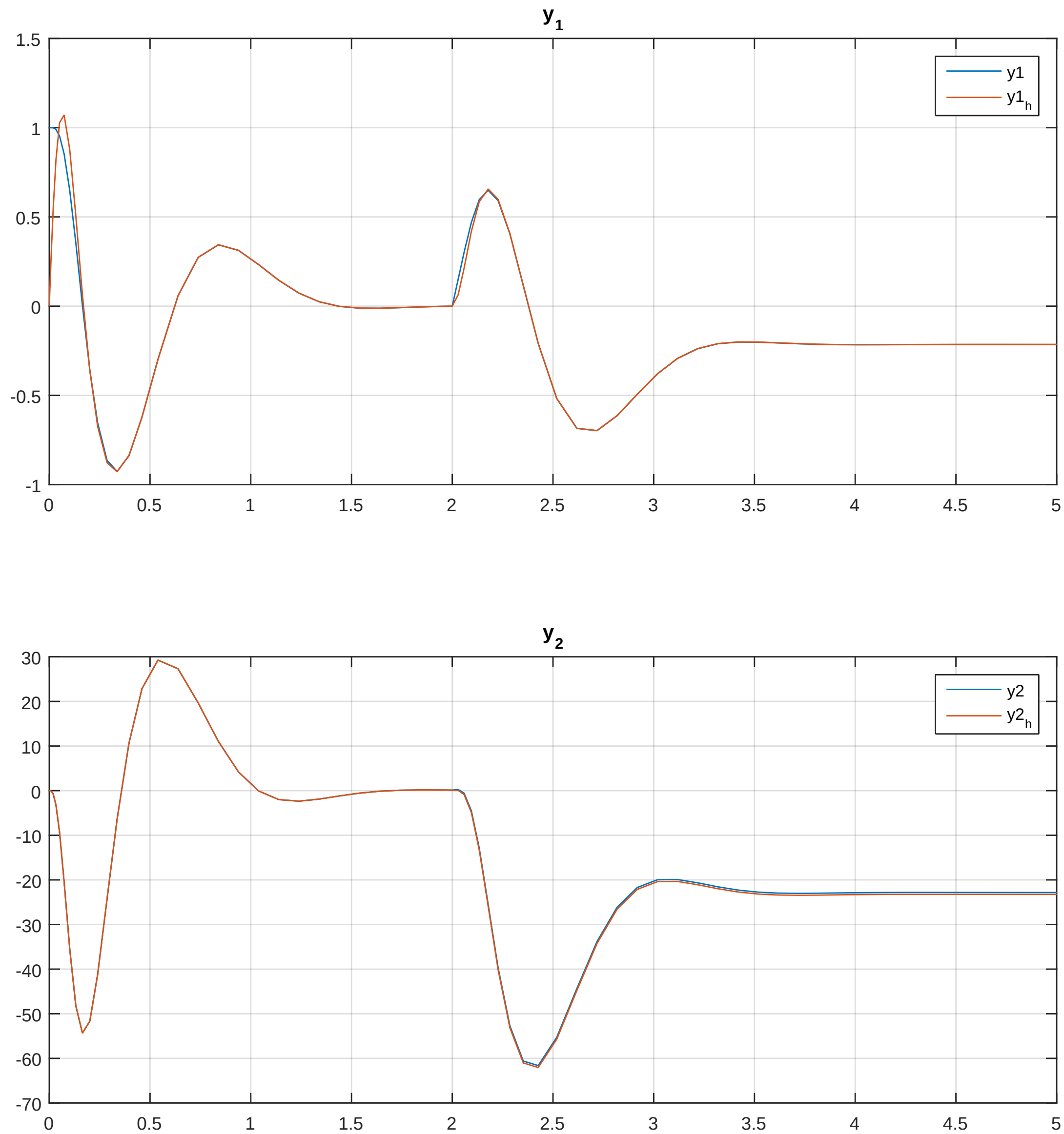

Figure 20. Estimated and actual plant output for PO based regulator

For PIO based regulators, choosing `opoles` for observer to be $\begin{bmatrix} -13.8600 & -16.0622 \pm 20.2891i & -22.1123 \pm 6.6213i \end{bmatrix}$, the observer gain given by '`OBGX_REG`' is:

$$L_P = \begin{bmatrix} 50.7569 & -0.2447 \\ -6385.6711 & 90.0240 \\ 794.5253 & 14.4522 \\ -315.5515 & 61.9187 \end{bmatrix}, L_I = \begin{bmatrix} 7642.0021 & -99.3259 \end{bmatrix}$$

The robustness bound of the regulator without observer is $\delta_1 = 0.3026$ and $\delta_2 = 0.3449$, and by using PIO, the robustness bound is $\delta_1 = 0.4114$ and $\delta_2 = 0.5318$

Compared to the simulation result shown in Figure 19 and 20, simulation result using `PIO_REG.m` shows that the estimated state variables, disturbance and plant output match the actual ones perfectly, with or without the disturbance in plant, as is shown in Figure 21, 22 and 23. As for the robustness bound, the result using PIO is a little bit worse than the result using PO.

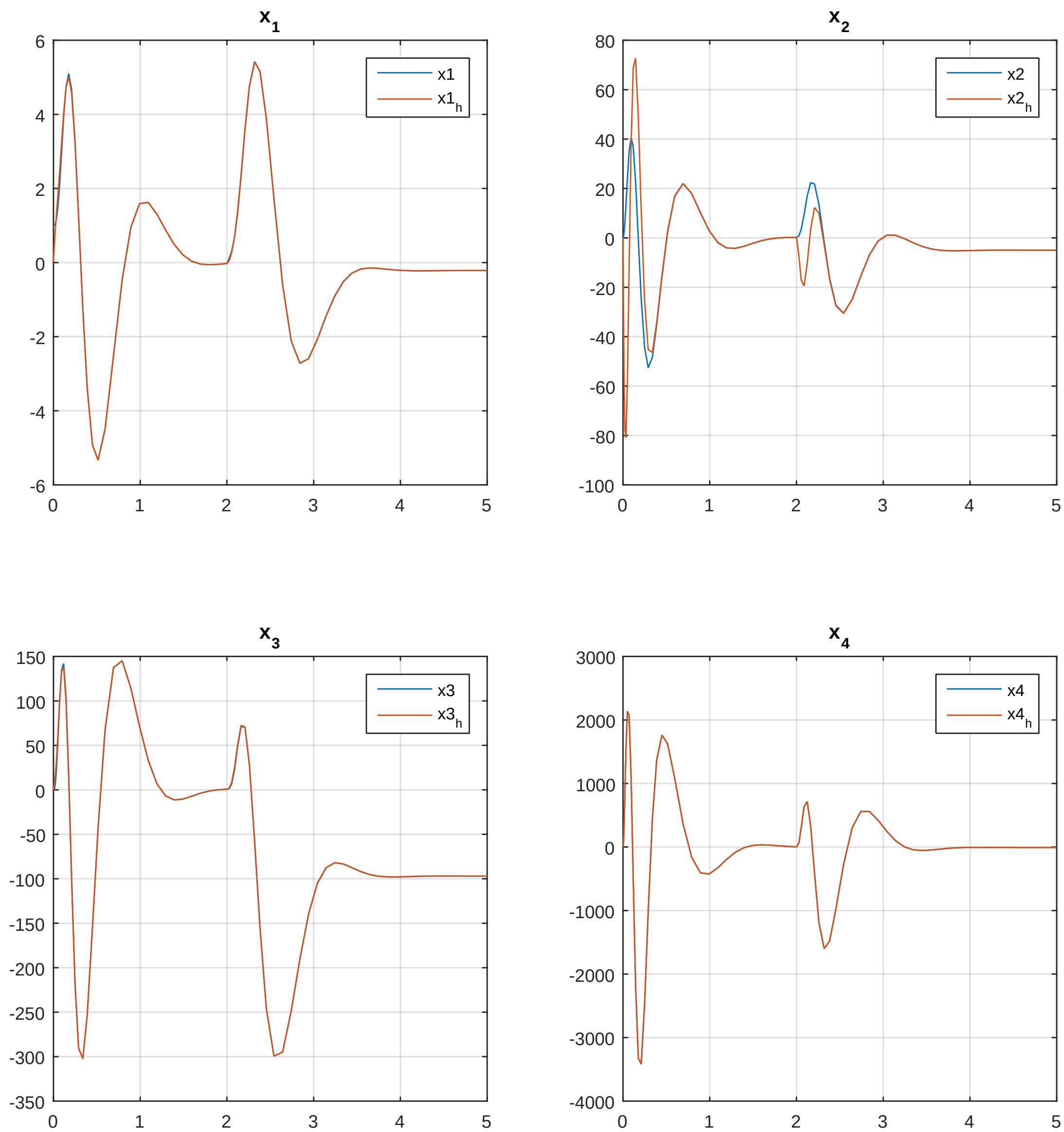

Figure 21. Estimated and actual state variables for PIO based regulator

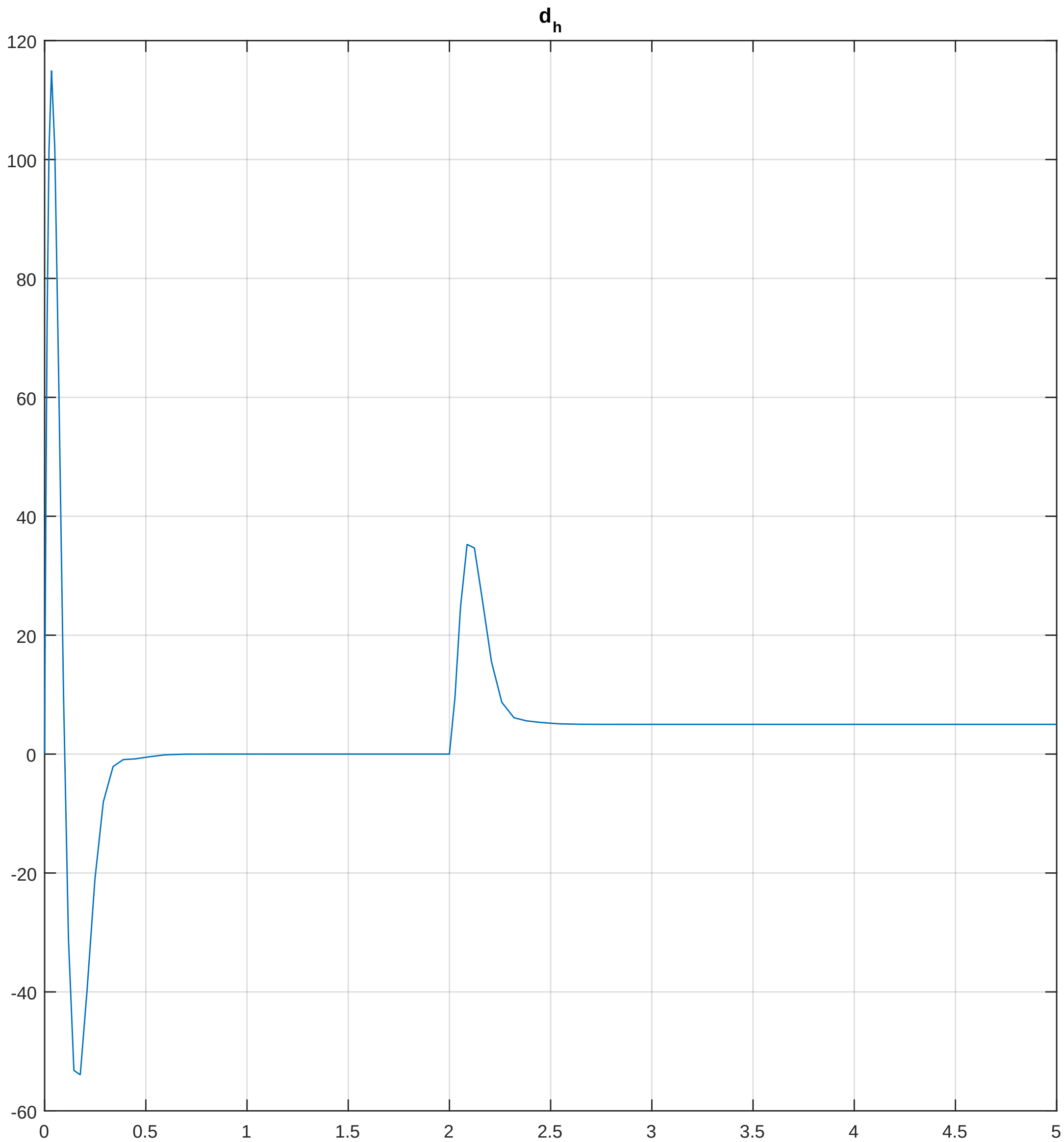

Figure 22. Estimated and actual disturbance for PIO based regulator

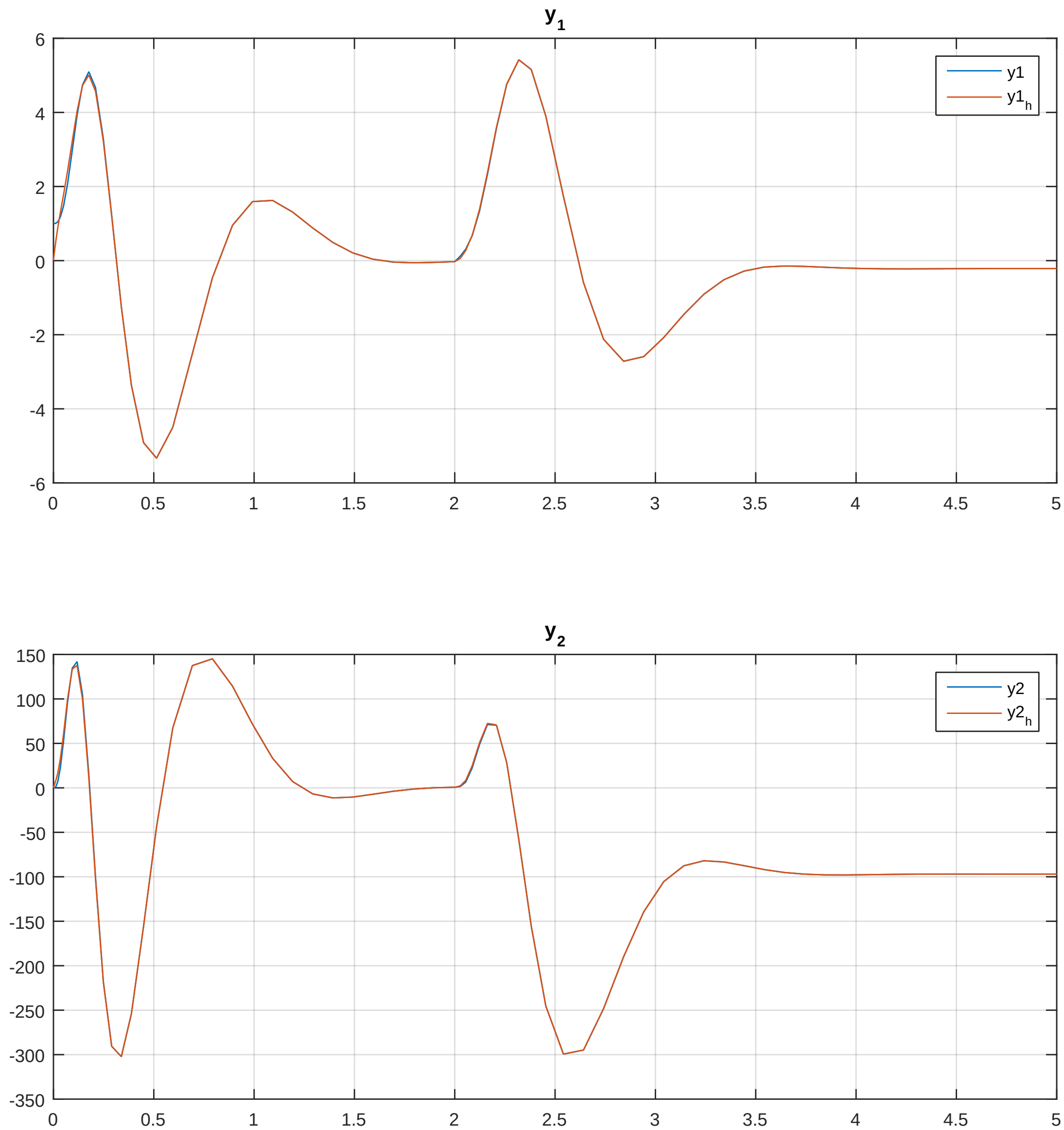

Figure 23. Estimated and actual plant output for PIO based regulator

The simulation above shows that PIO has the ability to estimate both state variables and disturbance correctly, while the feedback gain may not drive state variables and plant output to zero if there are disturbances in the plant. Noticing that the feedback gain $K_{Z(p\times n+l)} = \begin{bmatrix} K & 0 \end{bmatrix}$ is a combination of the feedback gain for the system without observer, and a zero matrix, suggesting that only the estimated state variables are used for feedback control. The state variables can be driven more closely to zero by setting a proper gain $\hat{K}$ for feedback control using estimated disturbances, making $K_Z = \begin{bmatrix} K & \hat{K} \end{bmatrix}$.

For steady-state, $0 = \dot{x} = Ax + Bu + Ed$, the system input u is now $-\begin{bmatrix} K & \hat{K} \end{bmatrix} \hat{z} = -K\hat{x} - \hat{K}\hat{d}$, and estimation of state variables and disturbance using PIO is correct, which means $\hat{x} = x$ and $\hat{d} = d$, thus:

$$0 = Ax - BK\hat{x} - B\hat{K}\hat{d} + Ed = (A - BK)x + (E - B\hat{K})d \tag{34}$$

Therefore, the steady state variables will comes to zero if and only if $B\hat{K} = E$ holds for non-zero disturbances, which suggests that E must be in the column space of B. In case that E is not in the column space of B, MATLAB command `>>B\E` will provide a proper feedback gain $\hat{K}$ for estimated disturbance and make the estimated and actual state variables closer to zero in steady state.

A simulation with $E = B$, which means the disturbance is applied on the plant input, is shown in Figure 24, 25 and 26, and $\hat{K}$ is equal to identity matrix according to the conclusion in previous paragraph. With the additional $\hat{K}$, the state variables and plant output can be driven to zero for steady-state, and the robustness bound of this regulator is $\delta_1 = 0.3951$ and $\delta_2 = 0.5764$

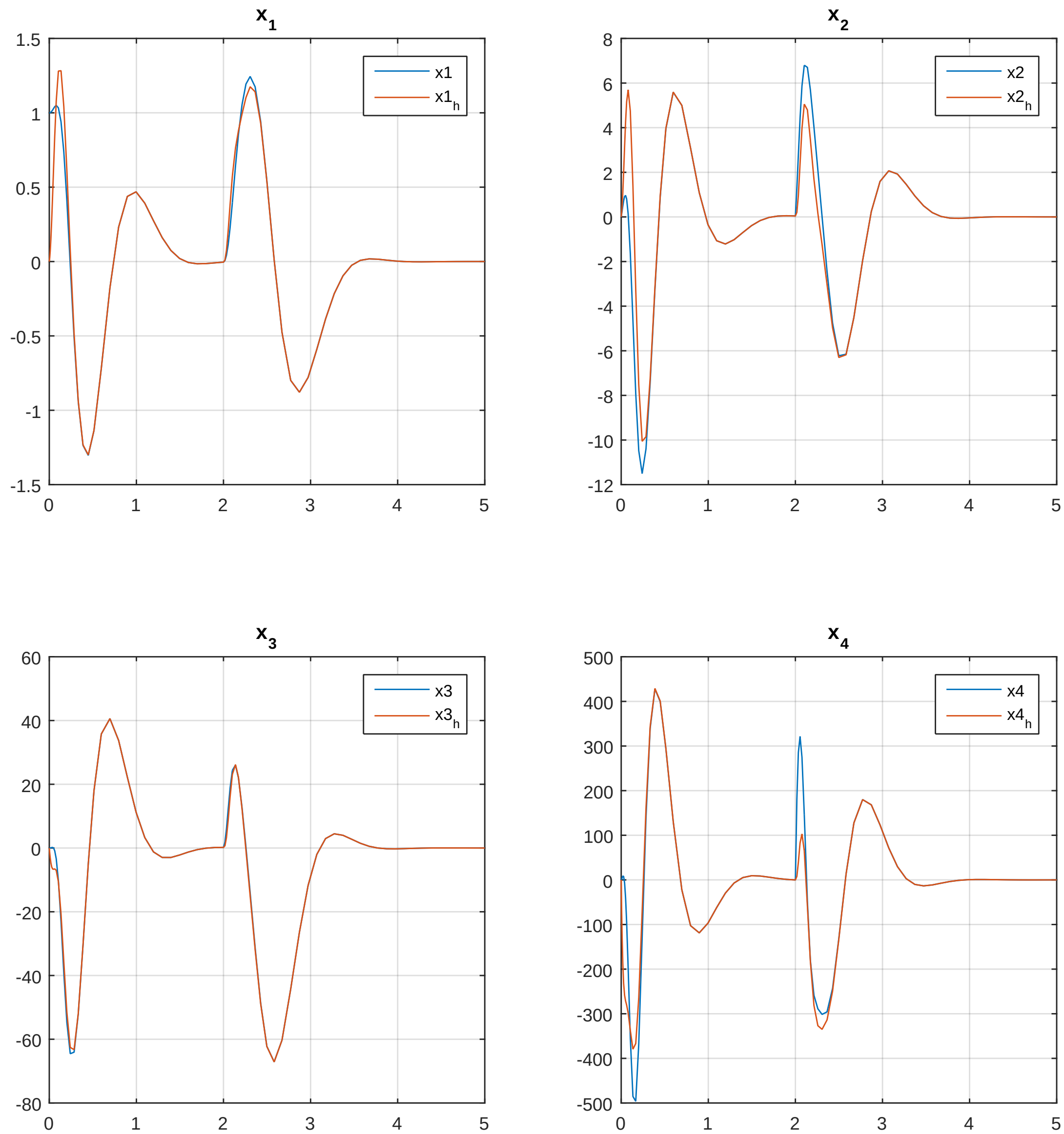

Figure 24. Estimated and actual state variables for PIO based regulator

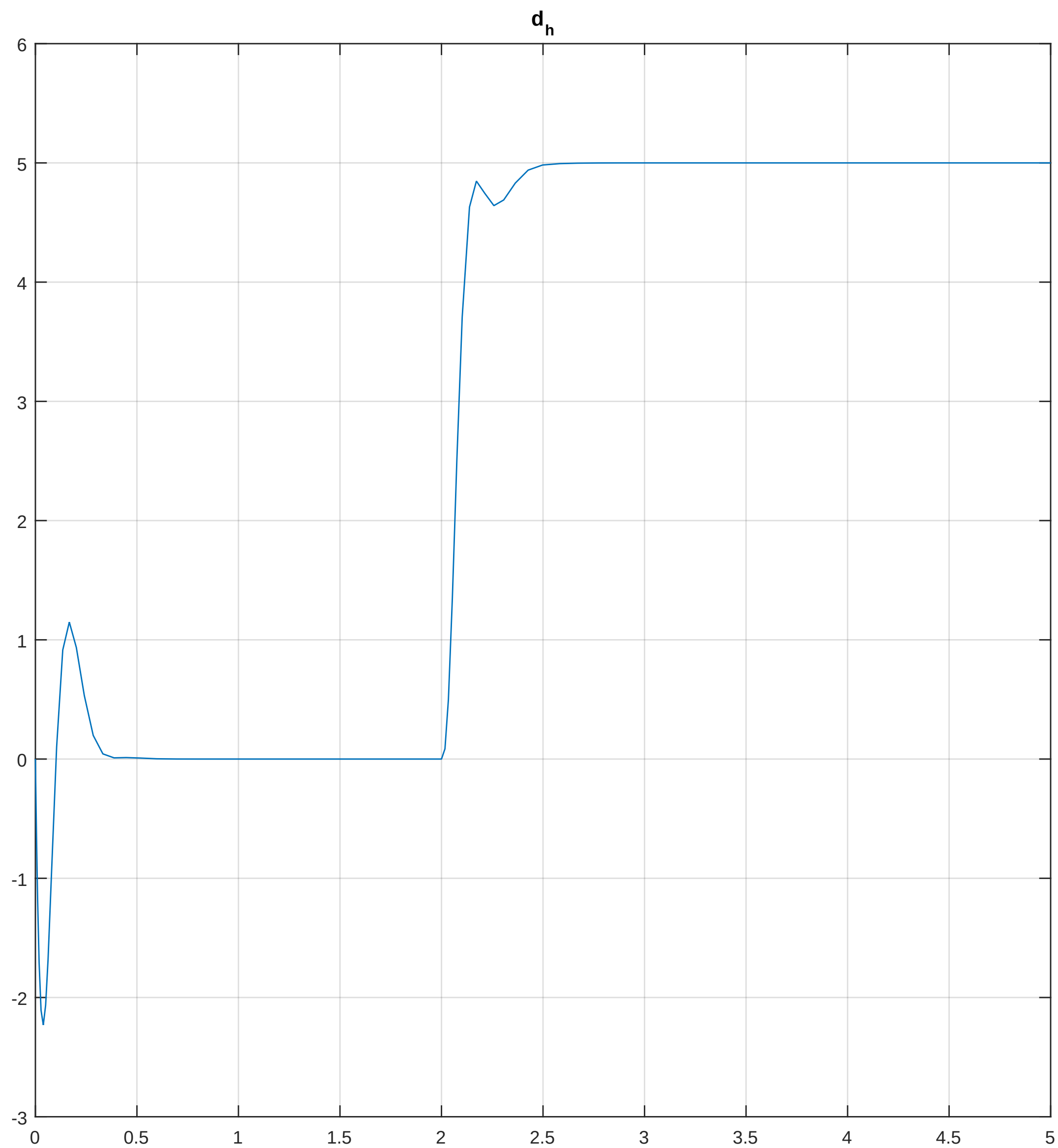

Figure 25. Estimated and actual disturbance for PIO based regulator

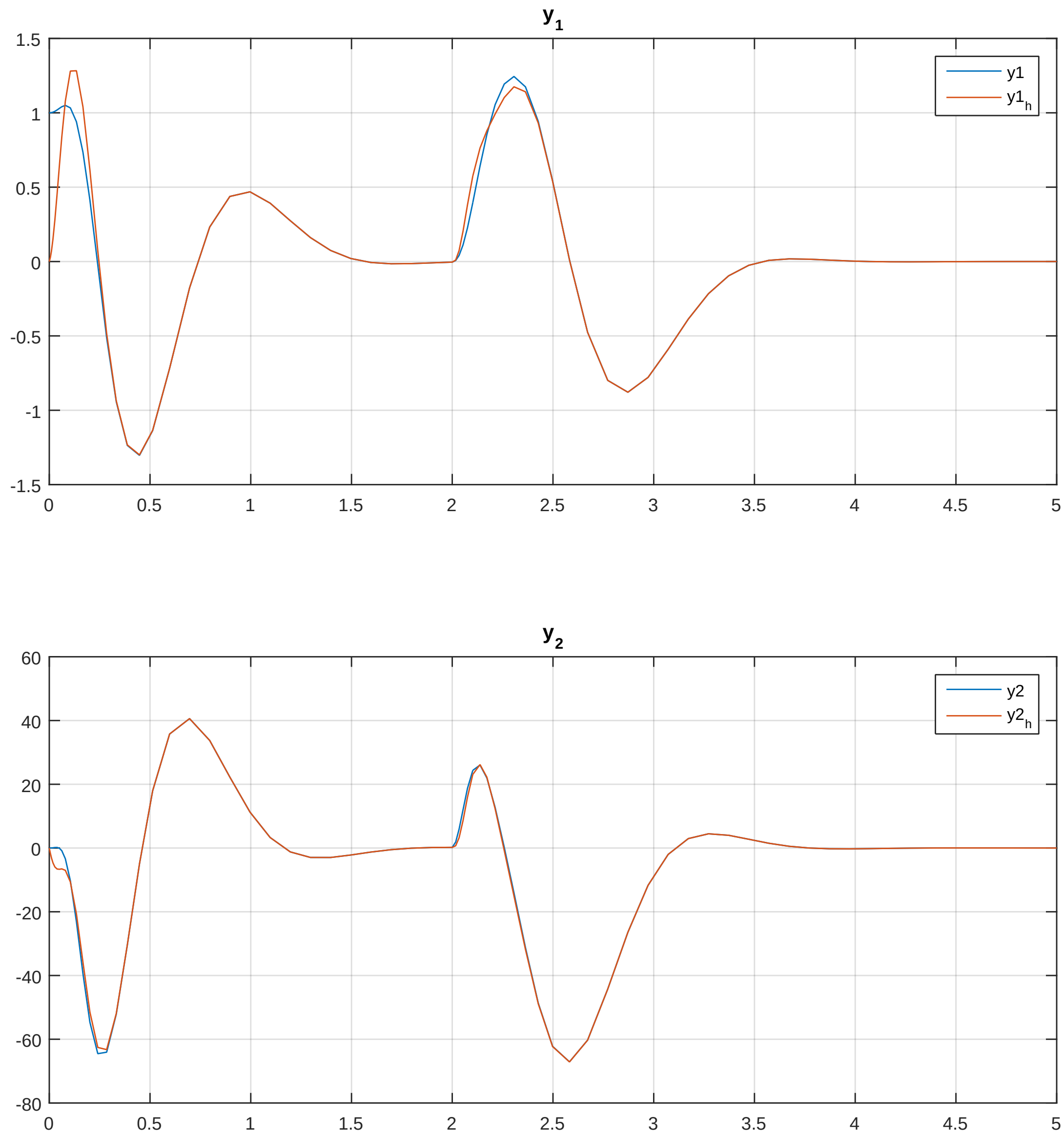

Figure 26. Estimated and actual plant output for PIO based regulator

For PIO based regulators in discrete time, the integral part of PIO, which is the only difference from that of continuous time, is described as $\hat{d}[k+1] = L_I(y[k] - C\hat{x}[k]) + d[k]$. This modification will make equation (31) changed into:

$$\begin{aligned}
\begin{bmatrix} x[k+1] \\ \hat{x}[k+1] \\ \hat{d}[k+1] \end{bmatrix} &= \begin{bmatrix} A & -BK & 0 \\ L_P C & A - BK - L_P C & E \\ L_I C & -L_I C & I \end{bmatrix} \begin{bmatrix} x[k] \\ \hat{x}[k] \\ \hat{d}[k] \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} w \\
v &= \begin{bmatrix} 0 & -K & 0 \end{bmatrix} \begin{bmatrix} x[k] \\ \hat{x}[k] \\ \hat{d}[k] \end{bmatrix}
\end{aligned} \tag{35}$$

The state-space model from w to v is not changed using extended observer, for $A_Z = \begin{bmatrix} A & E \\ 0 & I \end{bmatrix}$ in discrete time is different from $A_Z = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}$ in continuous time:

$$\begin{aligned}
\begin{bmatrix} x[k+1] \\ \hat{z}[k+1] \end{bmatrix} &= \begin{bmatrix} A & -BK_Z \\ L_Z C & A_Z - B_Z K_Z - L_Z C_Z \end{bmatrix} \begin{bmatrix} x[k] \\ \hat{z}[k] \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w \\
v &= \begin{bmatrix} 0 & -K_Z \end{bmatrix} \begin{bmatrix} x[k] \\ \hat{z}[k] \end{bmatrix}
\end{aligned} \tag{36}$$

CHAPTER 5

PIO for Tracking Systems

In this chapter, a design method for PIO based tracking system will be provided with the similar process in the previous chapter.

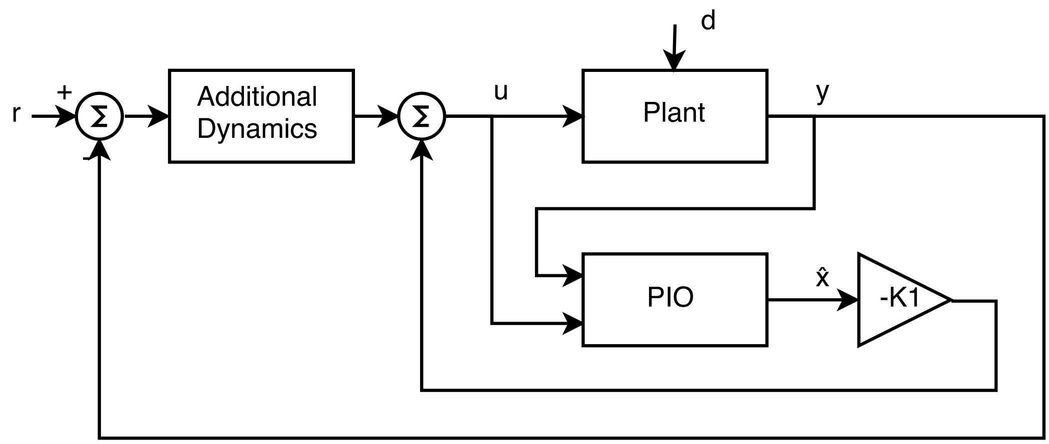

Figure 27. Block diagram of PIO based tracking system

As is shown in Figure 27, in tracking system, the additional dynamics has been added to the regulator to process the difference between system output y and reference input r. The state-space model for additional dynamics is $\dot{x}_a = A_a x_a + B_a(r - y)$, and the output is $K_2 x_a$. K_1 and K_2 can be obtained with the method provided in [17]. A_a is usually set to be zero matrix, and B_a to be identity matrix, which makes the additional dynamics to be an integrator of output error. For steady-state of tracking system, the additional dynamics will make the output equal to the reference input.

The state-space model for the plant and PIO is described as:

$$\begin{aligned}
\dot{x} &= Ax + Bu + Ed \\
\dot{d} &= 0 \\
y &= Cx \\
\dot{\hat{x}} &= A\hat{x} + Bu + L_P(y - C\hat{x}) + E\hat{d} \\
\dot{\hat{d}} &= L_I(y - C\hat{x})
\end{aligned} \tag{37}$$

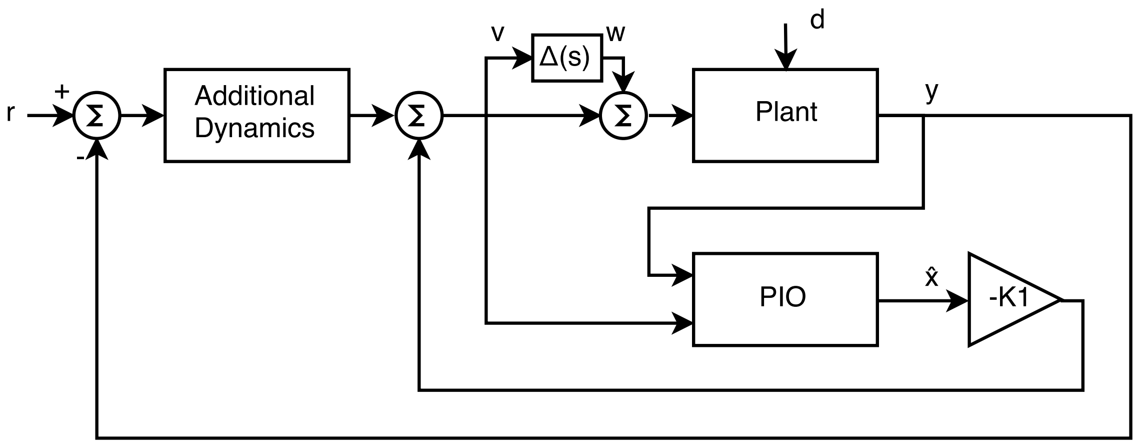

Figure 28. Robustness analysis of PIO based tracking system

To calculate robustness bound for the PIO based tracking system, the model from w to v will be derived by adding a perturbation on plant as is shown in Figure 28. Using the same process as in the previous chapter, the state-space model of PIO based tracking system is shown as the following equation, by letting $v = K_2 x_a - K_1 \hat{x}$ and $u = w + v = w + K_2 x_a - K_1 \hat{x}$:

$$\begin{aligned}
\dot{x} &= Ax + B(K_2 x_a - K_1 \hat{x}) + Bw + Ed = Ax + BK_2 x_a - BK_1 \hat{x} + Bw + Ed \\
\dot{x}_a &= A_a x_a + B(r - Cx) = -B_a Cx + A_a x_a + B_a r \\
\dot{\hat{x}} &= A\hat{x} - BK_1 \hat{x} + L_P C(x - \hat{x}) + E\hat{d} = L_P Cx + BK_2 x_a + (A - BK_1 - L_P C)\hat{x} + E\hat{d} \\
\dot{\hat{d}} &= L_I Cx - L_I C\hat{x}
\end{aligned} \tag{38}$$

In order to calculate closed loop robustness, the reference input and disturbance are set to zero, making the equation above equivalent to:

$$
\begin{aligned}
\begin{bmatrix} \dot{x} \\ \dot{x}_a \\ \dot{\hat{x}} \\ \dot{\hat{d}} \end{bmatrix} &= \begin{bmatrix} A & BK_2 & -BK_1 & 0 \\ -B_aC & A_a & 0 & 0 \\ L_PC & BK_2 & A - BK_1 - L_PC & E \\ L_IC & 0 & -L_IC & 0 \end{bmatrix} \begin{bmatrix} x \\ x_a \\ \hat{x} \\ \hat{d} \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ 0 \end{bmatrix} w \\
v &= \begin{bmatrix} 0 & K_2 & -K_1 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_a \\ \hat{x} \\ \hat{d} \end{bmatrix}
\end{aligned}
\tag{39}
$$

By changing the cost function designed for PO based tracking system in `obg_ts.m` provided by R. Vaccaro [17] into the corresponding cost function of robustness bound for PIO based tracking system, MATLAB function `OBGX_TS.m` is developed to calculate extended observer gain for PIO based tracking system with good stability robustness as is provided in equation 39. Observer gain for the extended model can be obtained by command `>>Lz=OBGX_TS(Az,Bz,Cz,Ad,Bd,K1,K2,poles,T)`, with the extended plant model (`Az,Bz,Cz`), the design model (`Ad,Bd,K1,K2`) as is provided in [17], and the desired extended observer pole locations (`opoles`). `T` is the sampling interval for discrete systems, and should be set to zero for continuous systems. This MATLAB function can also be used to calculate proportional observer gains, by inputing the original plant model (`A,B,C`) and proportional observer pole locations.

Examples

A simulation will be provided using example from [17]. The continuous time state space model of the disturbed plant is

$$
A = \begin{bmatrix} -0.322 & 0.0640 & 0.0364 & -0.9917 & 0.0003 & 0.0008 \\ 0 & 0 & 1 & 0.0037 & 0 & 0 \\ -30.6492 & 0 & -3.6784 & 0.6646 & -0.7333 & 0.1315 \\ 8.5395 & 0 & -0.0254 & -0.4764 & -0.0319 & -0.0620 \\ 0 & 0 & 0 & 0 & -20.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20.2 \end{bmatrix},
$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20.2 & 0 \\ 0 & 20.2 \end{bmatrix}, C = \begin{bmatrix} 0 & 57.2958 & 0 & 0 & 0 & 0 \\ 57.2958 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In this example, matrix E of disturbance model is set equal to B, which means the disturbance is applied on the plant input. The initial state of the system is $x_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & -2 & 2 \end{bmatrix}^T$, and a constant disturbance $d = \begin{bmatrix} -0.2718 & 0.3142 \end{bmatrix}^T$ is added to the plant at t = 3 sec. The initial state of observer is set to zero. The reference input of the plant is set to zero before t = 6 sec, and $r = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ after it.

Feedback gains K_1 and K_2 for tracking system without observer is obtained by using command `>>Kd=rfbg(Ad,Bd,poles,0)` provided in [17], with K_1 equals to the first n columns of K_d, and K_2 equals to the remaining columns of K_d. With the same pole locations in [17], the feedback gain given by MATLAB function `rfbg.m` is:

$$K_1 = \begin{bmatrix} 257.6083 & -86.3554 & -11.6249 & -53.0793 & 0.4393 & 0.0781 \\ 640.5502 & -2.2408 & 3.7307 & -165.0356 & 0.1082 & 0.4609 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -2.8185 & 4.8288 \\ -0.9308 & 30.2860 \end{bmatrix}.$$

For tracking system with PO, as is shown in Figure 29 and 30, the simulation result shows that there would be a constant error in steady-state in both state variables and plant output. The robustness bound for tracking system without observer is $\delta_1 = 0.8089$ and $\delta_2 = 1$, and for the PO based tracking system is $\delta_1 = 0.6192$, and $\delta_2 = 0.5733$.

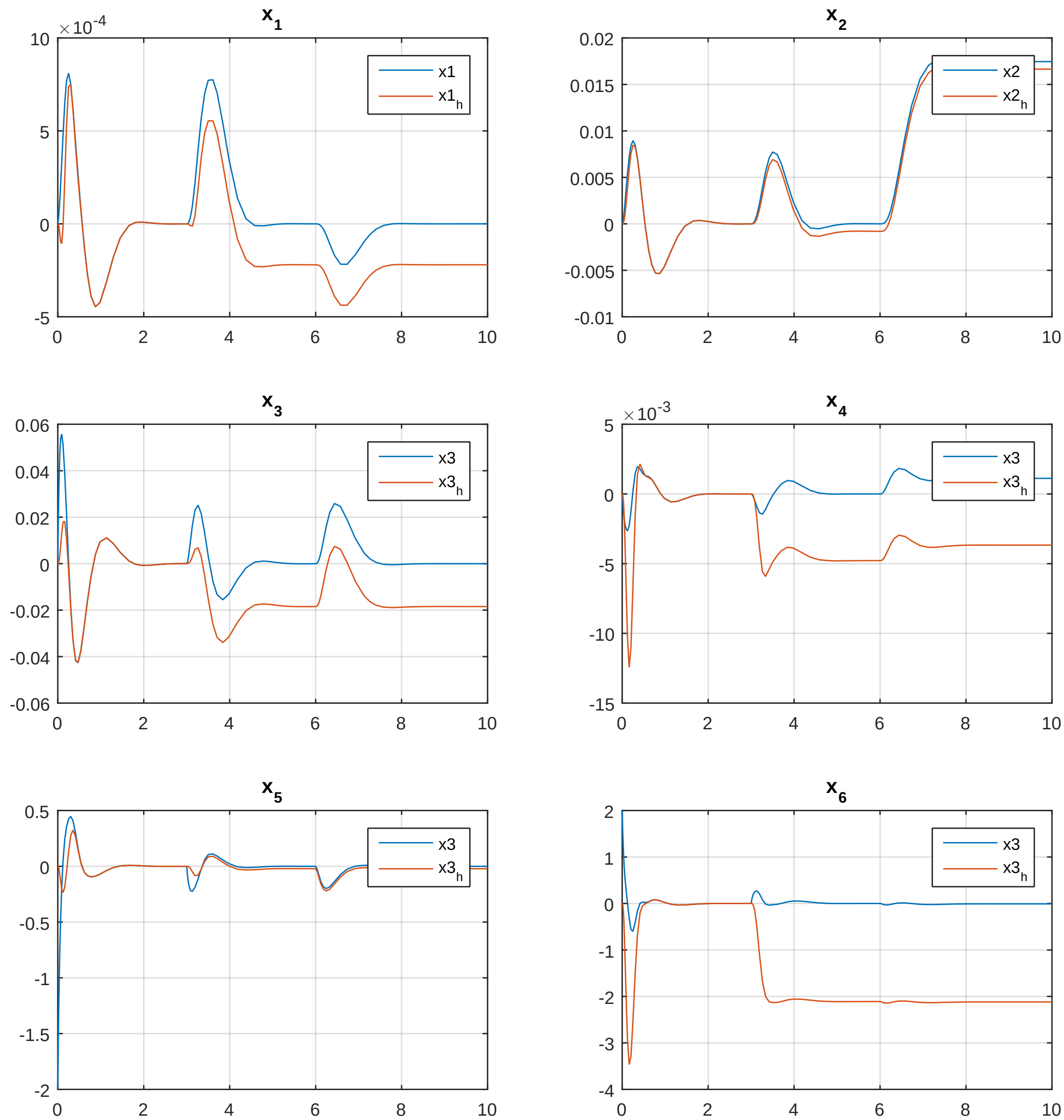

Figure 29. Estimated and actual plant output for PIO based tracking system

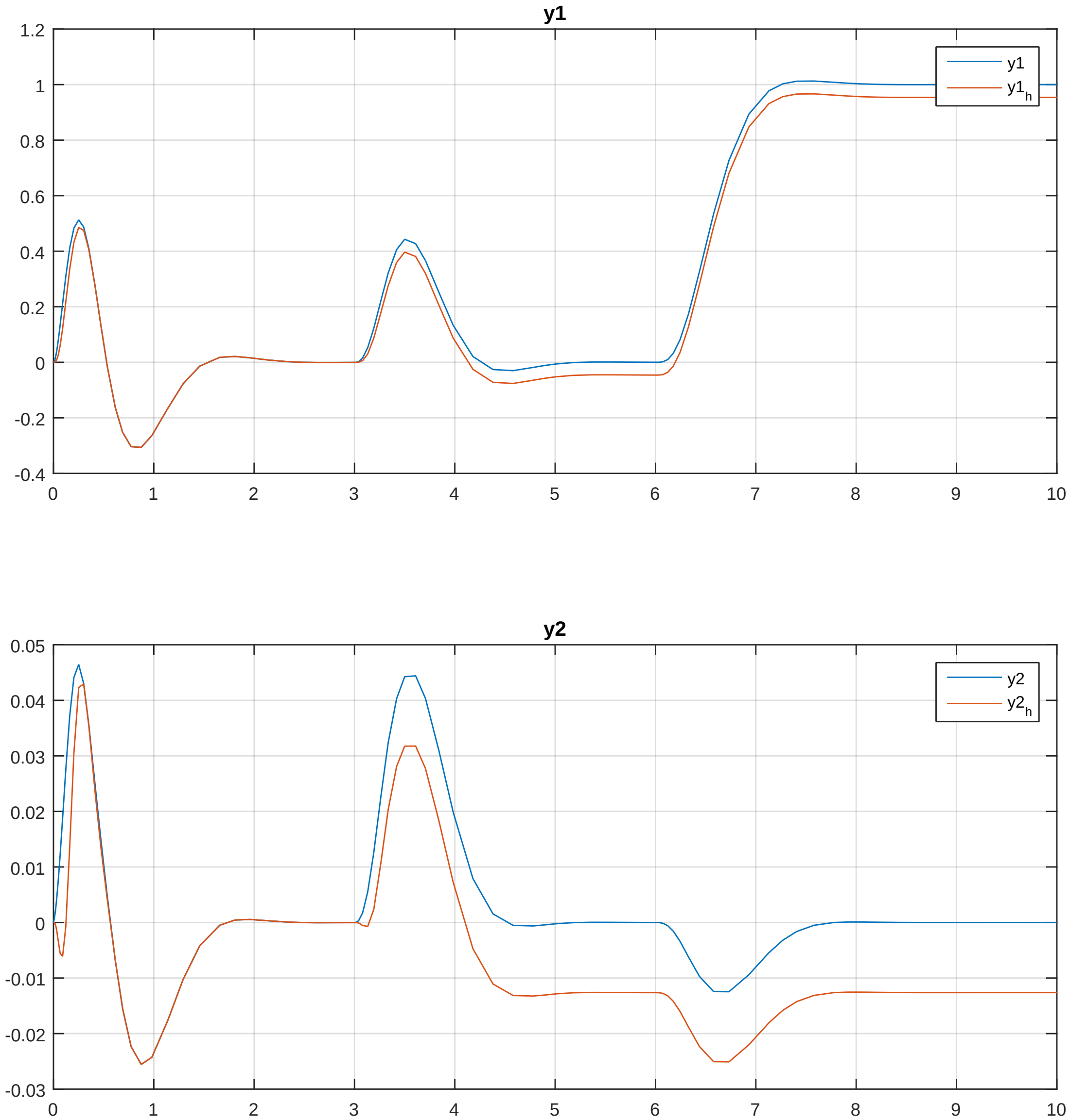

Figure 30. Estimated and actual plant output for PIO based tracking system

For PIO based regulators, by choosing `opoles` for observer to be $\begin{bmatrix}-79.1497 & -12.0223 & -9.5203 \pm 9.0827i & -9.6373 \pm 12.1735i & -13.2674 \pm 3.9728i\end{bmatrix}$, the observer gain given by `‘OBGX_TS’` is:

$$L_P = \begin{bmatrix} -0.1455 & 1.0444 \\ 0.8954 & -1.8155 \\ 9.2596 & -18.4117 \\ -0.0989 & -7.9848 \\ -294.0415 & 990.1249 \\ -665.3529 & 2712.4852 \end{bmatrix}, L_I = \begin{bmatrix} -146.9776 & 524.7721 \\ -104.7635 & 1154.8219 \end{bmatrix}.$$

The robustness bound for PIO based tracking system is $\delta_1 = 0.5410$ and $\delta_2 = 0.4937$

Compared to the simulation result shown in Figure 31 and 33, simulation result using `PIO_TS.m` shows that the estimated state variables, disturbance and plant output match the actual ones perfectly using PIO, with or without the disturbance in plant, as is shown in Figure 31, 32 and 33. As for the robustness bound, the result using PIO is worse than the result using PO.

For regulators, as is shown in Chapter 4, the system outputs may not come to zero for stable system if the disturbance exists. However, tracking system has the ability to drive the output equals to the reference input, discard the existence of disturbance.

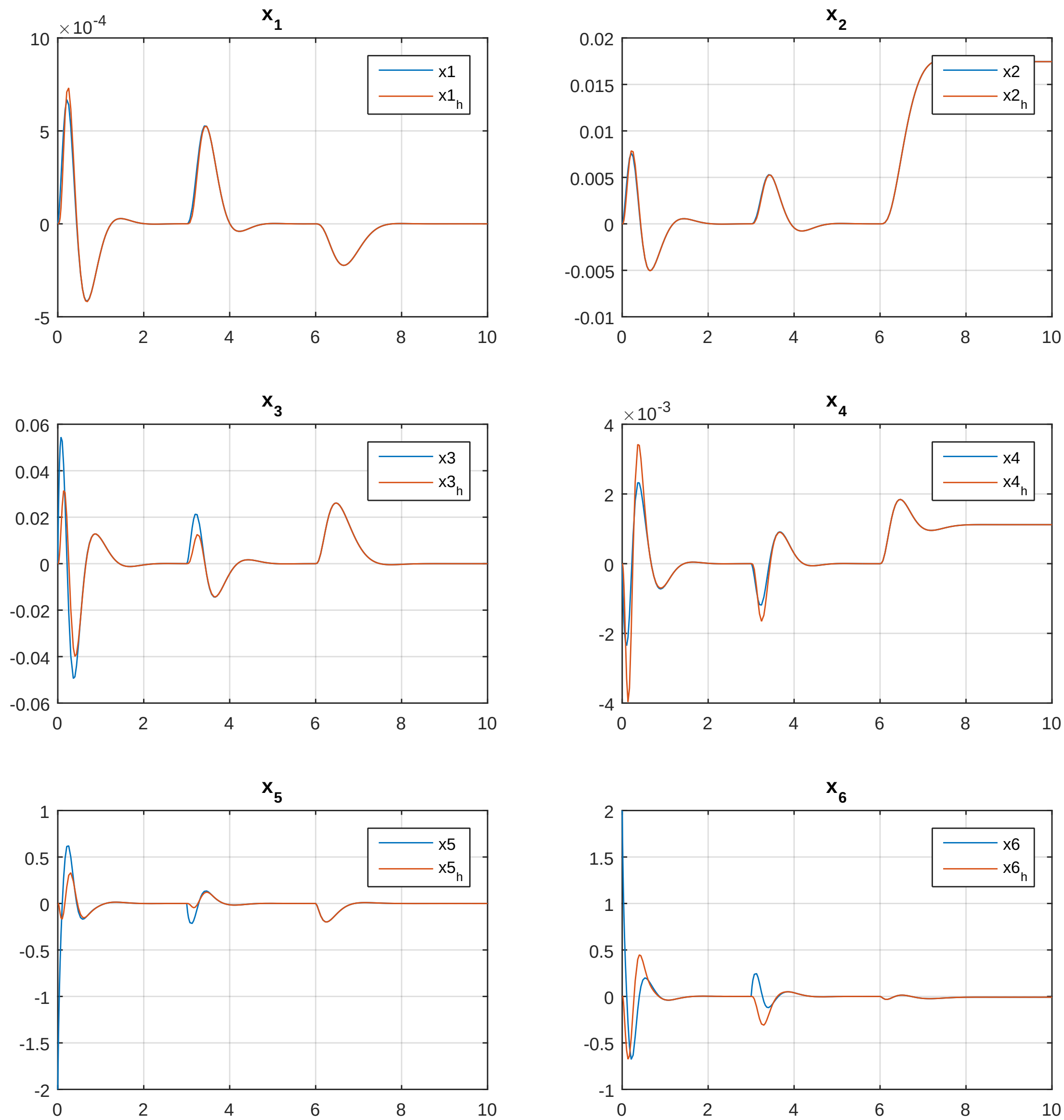

Figure 31. Estimated and actual state variables for PIO based tracking system

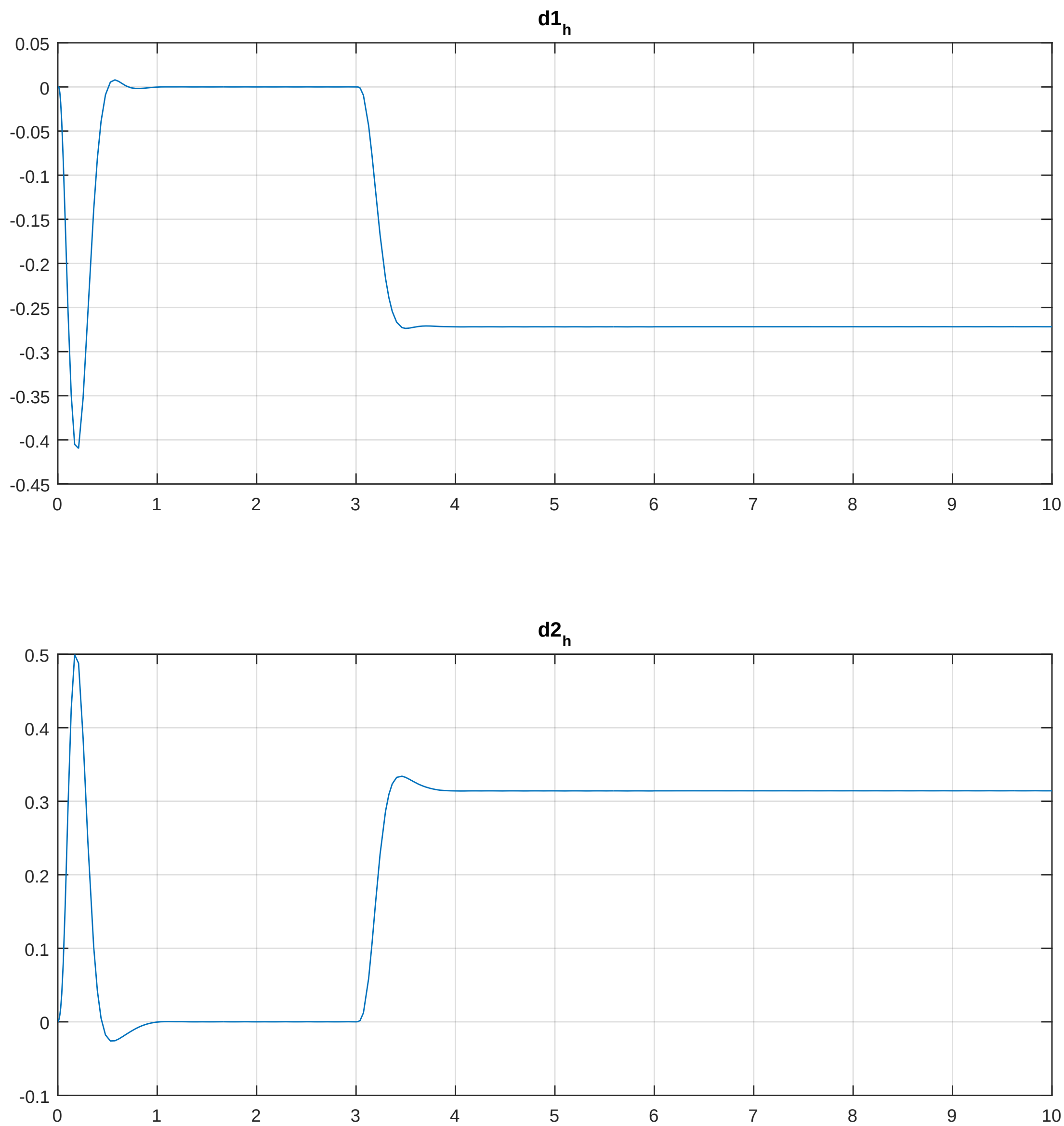

Figure 32. Estimated and actual disturbance for PIO based tracking system

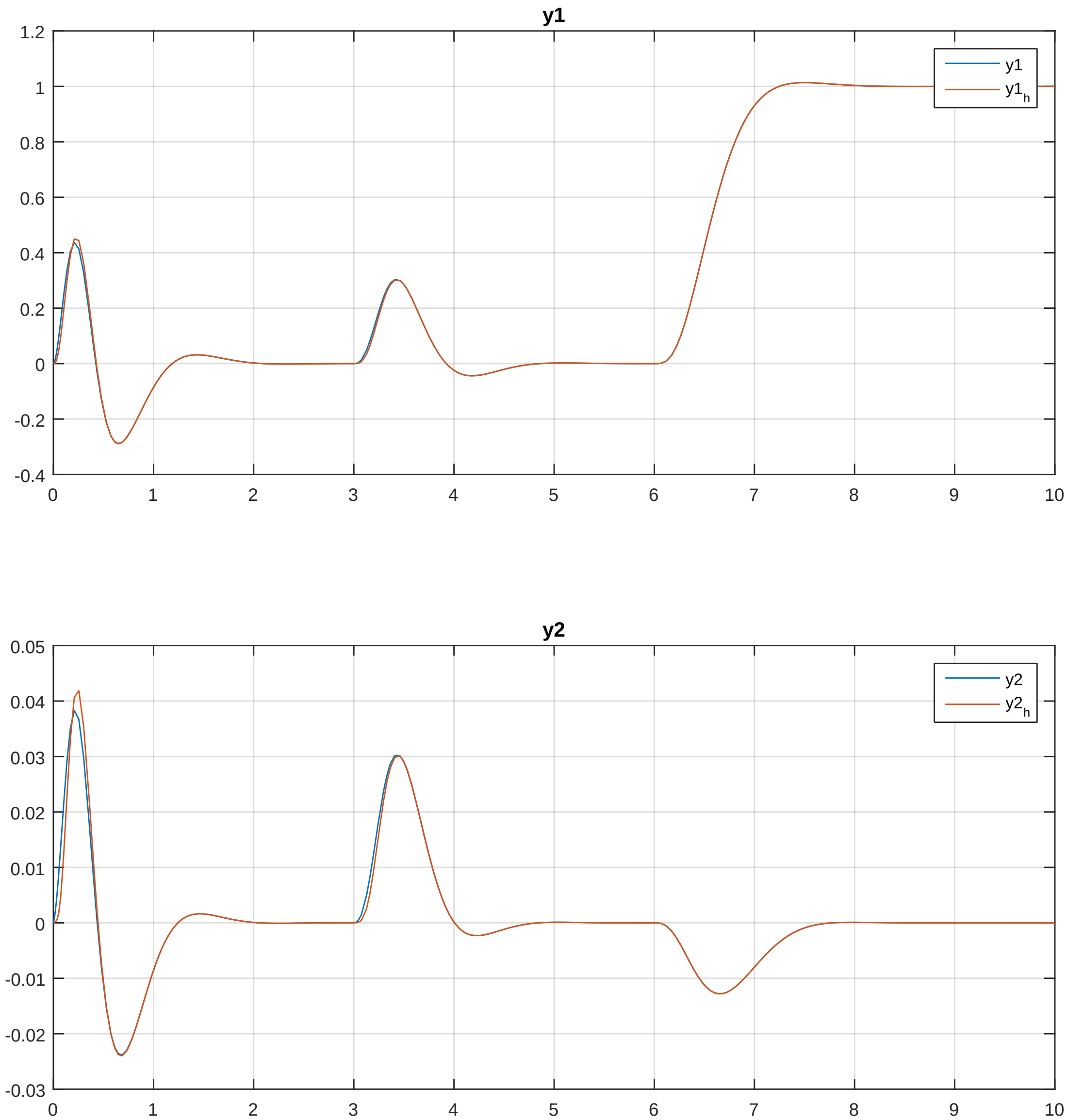

Figure 33. Estimated and actual plant output for PIO based tracking system

CHAPTER 6

Summary and Future Work

6.1 Summary

Observers are systems that estimate the values of unmeasured state variables from input-output measurements. The estimated state variables produced by proportional observers converge to the values of the actual ones if there are no disturbances acting on the plant. In the presence of disturbances, a proportional integral observer is needed to obtain correct estimations.

In Chapter 2, we showed that DO and PIO can be regarded as PO in an extended model, suggesting that design methods for PO can be applied on PIO. Derivation and example have shown that stable PIO is able to estimate state variables and disturbance correctly for a plant with unknown constant disturbance.

In Chapter 3, we showed that for observable systems, a parameterized method for proportional integral observer gain design with desired pole locations is derived based on the design method for feedback gain and proportional observer gain. This method provides observer gain of PIO using extended model with a cost function, such as norm of observer gain or condition number of $(A_Z - L_Z C_Z)$, to optimize system response. An example has shown that by minimizing norm or condition number, the observer will have better transient response, but not that good response for long time period, for the observer model only matches the constant disturbance model.

In Chapter 4, we applied the parameterized method shown in Chapter 3 to observer based regulators, with the cost function set to be closed loop robustness. Simulation result shows that this parameterized method will provide observer gain with good closed loop robustness, while estimating state variables and disturbances correctly. Furthermore, in case of $E = B$, using estimated disturbance for feedback control can drive all state variables to zero in steady-state. In Chapter 5, similar process is applied to an observer based tracking system.

6.2 Future Work

For all works in the thesis, the extended model (A_Z, B_Z, C_Z) is assumed to be observable. However, as is mentioned in Chapter 3, in case the number of unknown disturbance is larger than measured output, the extended system would become unobservable. For observable systems, the observer gain L_Z is unique for certain selected parameters, while there would be infinite number of observer gain matrices for unobservable cases. This problem can be solved by using additional parameters in the gain calculation.

Finally, the design method used in this thesis to calculate observer gain using the feedback gain for an observer-based control system used a feedback gain matrix previously calculated for full-state feedback. A combined method for obtaining observer and feedback gain in the same time could be developed in the future, with cost function set to be robustness bound or other reasonable functions.

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