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An Experimental Investigation of the Nearfield Pressures of Ultrasonic Transducers and Arrays

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AN EXPERIMENTAL INVESTIGATION OF THE NEARFIELD
PRESSURES OF ULTRASONIC TRANSDUCERS
AND ARRAYS

BY
GEORGE ANTHONY FISHER

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS OF THE DEGREE OF
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IN
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MASTER OF SCIENCE THESIS

OF

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ABSTRACT

This thesis describes the procedures and results of research conducted on pulsed ultrasonic piezoceramic transducers and arrays which operate in the megahertz range. These transducers and arrays are typical of the types employed currently in biomedical diagnostic systems. Two areas were addressed: The verification of the Impulse Response Approach as a method to predict the spatial and temporal characteristics of the acoustic field pressures of ultrasonic transducers of general shapes and excitations, and the investigation of the element interaction and spatial resolution of field pressures of arrays of piezoceramic elements.

An automated computer controlled high frequency acoustic measuring system was constructed. Several high frequency transducers were built to obtain acoustic nearfield data with this system. The data were subsequently processed using computer programs which were created to obtain impulse responses via the deconvolution of the far field on axis and nearfield time dependent pressures. The acoustic fields of an array driven under focussed and unfocussed conditions were also extensively mapped.

The experimentally derived impulse responses were in good agreement with the theoretical responses and thus provided important new verification of the Impulse Response Approach. The results of the experiment also indicated that extraneous modes of vibration of piezoceramic discs

were, in this case, insignificant. A multi-element transducer array was constructed and operated in such a manner as to focus the acoustic energy, confining it to a narrow region as opposed to a continuous beam. The element to element interaction effects caused by electrical and acoustic coupling were found to be of significant importance in the single driven element case. The effects of interaction were not great enough to cause a perturbation from the expected field pressure pattern when the array was driven for focussing.

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I. INTRODUCTION

This thesis describes procedures and results of research carried out at the University of Rhode Island, under a grant from the National Institute of Health, GM 23104. The objectives of this study are: The investigation of the nearfield characteristics of certain ultrasonic transducers and arrays which operate in the megahertz range, and the verification of the impulse response approach to evaluate the spatial and temporal characteristics of the acoustic field which results from the pulsed wide bandwidth excitation of ultrasonic transducers and arrays. Ultimately the implementation of this knowledge will be used to design and construct an ultrasonic biomedical diagnostic system which operates in real time, with a higher order of resolution than previously attained.

Reports generated by the National Science Foundation have summarized the present state of the art in the use of medical ultrasonic devices for diagnostic purposes. These reports have also made explicit recommendations concerning the type of research which is required in order to advance this area. [See references (1) through (5)], In particular, the need for research on evaluating the

operating characteristics and limitations of pulsed ultrasonic systems which utilize two-dimensional transducer arrays, is indicated. It is expected that, as the state of the art advances, new uses for these ultrasonic diagnostic systems will be found.

It was intended that the work performed under this grant produce significant advances in the area of pulsed acoustic radiators and arrays. To this end, the project proceeded in three distinct stages. First a collection of equipment was assembled and constructed to form a Measurement System capable of making the necessary ultrasonic field measurements in the megahertz range. Secondly, computer programs to process this data were written and evaluated. Finally, measurements were made on several single element transducers, and also multi-element arrays. These measurements were then used to verify the Impulse Response Method of evaluating the characteristics of pulsed acoustic fields from single transducers and arrays.

An experimental verification of the impulse response approach to evaluate the spatial and temporal characteristics of pulsed acoustic fields from ultrasonic transducers and arrays had not been previously performed. A few investigators have measured time dependent pressures of pulsed baffled pistons, comparing them to those predicted by theory; however, various difficulties in obtaining measurements have appeared. These experimental

difficulties, as explained below, have provided stumbling blocks to verifying any analytical method of predicting acoustic fields for pulsed excitations of transducers and arrays.

In a recent experimental investigation of the transient acoustic field which results from an impulsive electrical excitation of a piezoceramic cylinder, Carome (6) used a fairly large piezoceramic element, in the form of a cylinder two or three centimeters in diameter, 2.54 or 5.08 centimeters thick. The motivation for this large length was based on the observation that compressional waves propagate from both the front and the rear faces of the element when the transducer is electrically pulsed. The wave of interest is that propagating from the front of the disc into the medium. However, a short time later the wave from the rear face has travelled through the element and arrives at the front face, making the face velocity hard to predict. A large length was used to increase the time separation between the front and rear waves, hopefully to the point where they were non-overlapping. Also, a large diameter was chosen to create a similar separation of edge pulses in the pressure time history.

Several other researchers [(7) and (8)] have mounted elements on materials chosen to match the acoustic impedance of the element, thereby minimizing the back face pulses. Also, backing materials having a high loss factor, such as mixtures of epoxy and Tungsten, have often been

employed, yielding transducers of significantly wider bandwidth. Kossoff (7) extensively documented the electrical and mechanical properties of piezoceramic transducers, for cases with and without the use of lossy backing materials. The properties of the lossy backing materials were also described in detail as functions of the relative concentration and size of the Tungsten particles. Redwood (8) treated the use of such backings as an approximate resistive load in his electrical analog model of pulsed transducers.

Since the dynamics of the probe become critically important in any wideband acoustic experiment, Weight and Hayman, (9) , constructed their own probes for these reasons, and used them to investigate the field of fairly small (5-8 mm.) diameter pistons , which were also backed with loaded epoxy. Using a Schlieren visualization system, the pulses travelling from the face and the edges of the projector were clearly seen. These results are in general agreement with those predicted via the use of the impulse response approach, (11) .

In studies comparing experimental results to theoretical predictions of acoustic transient fields, both the dynamics of the projector and probe are important. In particular it should be noted that the behavior of the radiating face of the piston is critically important. Either assumptions must be made that the face is moving with a uniform velocity distribution, or this velocity

distribution must be measured. In regard to the probe it is noted that its directivity and dynamic response can greatly affect the wideband measurements.

There is considerable conjecture presently as to the importance of vibrations other than the simple thickness expansion of the elements which results in the uniform velocity distribution on the radiating face. Elasticity theory predicts that radial modes of vibration are possible, and actually probable. The excitation of these modes can of course be expected due to the wideband electrical excitation and the non-uniform loading presented to the faces of the element by the fluid and the backing material. However their importance relative to the thickness mode is presently undefined.

This study presents experimental procedures which are used to give new verification to the Impulse Response Approach. The acoustic field which results from the pulsed excitation of several transducers is investigated, and the results are shown to shed some light on the accuracy of the uniform velocity distribution. Finally, a multi-element array of small piezoceramic transducers is constructed and analysed, and the resultant acoustic pressure fields for pulsed excitation are presented. The subject of interaction between different elements of the array is addressed, as are the differences between nearfield beam focussing and far field beam forming.

II MEASUREMENT SYSTEM

As put forth in the grant proposal, the work was to start with an investigation of the pulsed ultrasonic field of single element high frequency piezoceramic transducers. A verification was sought of the Impulse Response Approach as a method of acoustic field prediction, which is elegantly simple in its mathematics, yet able to predict time dependent pressures not only in the far or near field but at all points with equal ease. The research was to continue with a study of multi-element electronically steerable phased arrays.

In the early stages of planning of this research, it was foreseen that a prerequisite to performing the types of measurements needed was the construction of a complete measurement system. Early experiments made obvious these needs by their absence.

In view of the high frequencies involved, and the precise physical relationships required to obtain the beam patterns and field behaviors which were sought, two areas of equipment were lacking. These were: First ; a high speed analog to digital converter to be used to digitize pressure time histories; and second, a precision positioning system to maintain and control the physical relationships desired between the projecting transducer(s) and field pressure measuring probe.

The above requirements should not be taken to indicate

that the research team was completely devoid of resources ; indeed, quite a collection of test equipment which was partially or completely suited to acoustic research had already been amassed at the University of Rhode Island Department of Ocean Engineering's South Lab. This included a Data General NOVA 1200 minicomputer which was richly endowed with peripherals, including a 9-track magnetic tape drive, a dot matrix plotter, a dual floppy disk, and a MODEM data link to the Academic Computer Center . [see figure (1)] . Also available were a large tank with movable carriages for acoustic test work; several low frequency oscilloscopes; and the existing technical services and personnel of the department.

It was therefore decided to expand upon the existing data processing capabilities of the NOVA, at least as far as was practical. This expansion took the form of the construction of a high speed analog to digital converter, configured to the NOVA as a dedicated I/O device; and the construction of a microprocessor-based computer with capability to interface to the NOVA. Where processing exceeded the capabilities of the NOVA, the data would be transferred to the Academic Computer Center system for processing on a large mainframe computer.

While it was fortunate that such equipment existed to be expanded upon for data acquisition and processing needs, the team was less fortunate as to the positioning system. Although the large scale acoustic tank was available, it

Figure 1. Data Acquisition System

was felt that, in view of the small distances and high precision necessary, and due to the high frequency nature of the work, the system would prove too large and unwieldy. As it later turned out, certain measurements had to be carried out in the large tank. However, all measurements were made using a precision positioning system designed and built as part of this research.

The positioning system, in its final configuration, was a fully automatic device consisting of a computer controlled carriage positioning system, as shown in figure (2). The position of a small staging was placed under complete computer control of the NOVA via the use of small D.C. motors, which were controlled by a slave microcomputer, and shaft position and direction encoders. This made possible extensive field mappings containing several thousand points.

With the addition of these specialized devices, the NOVA was transformed into a system capable of performing automated acoustic measurements and storing the resultant data for later processing on the Academic Computer Center's System.

A discussion of the above mentioned additions to the NOVA will now be described in greater detail in the following subsections.

ANALOG to DIGITAL CONVERTER

For some of the beampattern measurements needed, a

Figure 2. Positioning Carriage

peak detector and relatively slow analog to digital converter (referred to as A/D henceforth for convenience) would have sufficed. However, for the Impulse Response work, accurate pressure versus time waveforms were desired. The frequency range of interest, in view of the biomedical orientation of this work, was between 1 to 10 MHz. To accurately reproduce waveforms as opposed to just spectra, a very high sampling rate of at least several tens of megahertz and preferably 100 MHz. was indicated.

The present state of technology does not offer A/D converters of this speed at reasonable cost, so a repetitive sampling approach was adopted. Borrowing a technique from sampling oscilloscopes, a slower A/D was utilized to make conversions on triggered analog signals. In theory the analog event is repetitive, and hence the sampling times are shifted slightly relative to the successive events. These samples are then rearranged into correct order to yield a time sampled record with a sampling rate corresponding to the frequency associated with the small time shift, [see figure (3)]. To control this shift, a reliable jitter-free timer with binary input and resolution of 10 nanoseconds was purchased as a unit from Evans Associates, and is referred to as the Interleave Delay Timer. Using readily available logic devices, a reliable time shift of 10 nanoseconds was obtainable, yielding an effective sampling rate of 100 MHz, which proved to be more than sufficient.

Continuous
Periodic
Signal

Real Time
Samples

Reconstructed
Event

Figure 3. Analog to Digital Converter
Timing Diagram

As far as the analog to digital conversion of analog waveforms was concerned, the only remaining need was a fast Sample-and-Hold to accurately freeze the waveforms for digitizing. A considerable amount of digital logic was required to control the timing of signals to the converter module itself and the Sample-and-Hold.

A / D conversion at high rates was half the battle; what remained was the handling of digital data input at this high speed. As previously mentioned, the A/D was to be configured as a dedicated I/O device for the NOVA. As a result, the analog to digital conversion circuitry and associated timing devices were constructed on a commercially available I/O board, designed for custom device implementation.

This type of A/D converter, because of its sampling scheme, produces a trigger pulse from the computer when it is ready to sample. This differs from slower A/D converters, which are themselves triggered by the external event. Because of this, some provision had to be made for inserting a propagation delay, which is separate from the interleave delay, between the trigger pulse and the start of repetitive sampling. This accounts for varying times-of-flight of pulses propagating from projector to receiving probe. A 16-bit binary programmable counter was installed with the A/D, Sample-and-Hold, and Interleave Timer modules, as well as other control logic, on the wire-wrap section of the I/O board.

The A/D converter chosen was a Datel G3BC 8-bit unit with a conversion time of 800 nanoseconds. Although sampling could be as fast as 1.25 MHz. in real time, the fastest continuous rate of entering data into the NOVA's memory is one 16-bit word every 1.2 microseconds. This is not a programmed I/O transfer; rather this speed is available only with the use of direct memory access.

The NOVA CPU interrogates every I/O device after each program instruction. This interrogation is to determine if any devices are requesting interrupt service or direct memory access (termed " Data Channel " by Data General, manufacturer of the NOVA) . This interrogation takes the form of two digital signals, INTP and DCHP , that pass 'daisy chain ' fashion from the CPU, sequentially through the I/O devices, then back to the CPU. If these signals return to the CPU, no device has made a request. If any device does make a request, it inhibits the signal corresponding to the type of request from passing through it. The first device along the chain to stop the signal is the one which will receive service. This means that there is a different priority for each device, depending upon its electrical location on the bus. The high speed A/D was installed to have the highest priority in the machine, so no other device would be able to disrupt its operation.

Because the CPU must see the interrogation signals return to it (no requests being made) before it continues with the next program instruction, it is possible for the

I/O devices to inhibit normal program execution continually.

This interruption of program flow is the technique by which timing between the A/D converter and the CPU is accomplished. The program controlling the converter sends certain control words to the device, programming its behavior. Then a start pulse is sent, which triggers the A/D to go into operation. Immediately the converter makes a Data Channel request, and continues to do so until it has finished its assigned task. Refer to figure (4) for a block diagram of the system.

The pre-programming of the high speed A/D consists of writing four data words to internal registers. One of these words is written to the Address Counter Register, which is read by the CPU each time a Data Channel transfer takes place; it contains the address of memory where the current data is to be stored. Another register to be written is the Word Zero Detect Counter. This counter is programmed with the negative of the number of words to be stored in memory, and is incremented with each valid Data Channel transfer, as is the previous counter. In this manner, consecutive transfers are stored in successive memory locations. When the Word Zero Detect Counter has been incremented up to one more than its maximum count, (i.e. up to zero) it sends signals to the rest of the device to relinquish control to the CPU. A third data word from the CPU sets up the Interleave Delay Timer, which

Figure 4. Analog to Digital Converter
Block Diagram

programs the correct number of sample delay time increments between the start pulse and the trigger output. The fourth data word is directed to the propagation delay counter. This number represents the integer number of tenths of microseconds of delay between the start pulse and the start of repetitive sampling.

Following the setup of these registers, the CPU sends a start pulse to the A/D converter. This triggers three events. This start pulse is synchronized to the 10 MHz. clock of the Interleave Delay Timer, which is used as a master clock for the whole A/D. The synchronized start pulse enables the 10 MHz. clock into the Propagation Delay Counter. It also triggers the Interleave Delay Timer, which sends a trigger pulse after its programmed delay to the outside world. The third function of the start pulse is to set up the control logic which in turn presets a byte packer and picks up the Data Channel continuously.

The A/D converter holds control at this point via the Data Channel, but during the propagation delay, because no data samples are being taken, the Address and Word Zero Detect counters are not incremented. Although the Data Channel is running at full speed, it merely stores irrelevant data in the first word of the memory block to receive the data. When the propagation delay has elapsed, the 10 MHz. clock is divided by twelve to give an actual sampling rate of 833 KHz., or one sample every 1.2 microseconds. These sampling commands are sent to the

Sample-and-Hold, a Datel model SHM-UH. This unit has an acquisition time of 35 nanoseconds and a 200 picosecond aperture time. The sample commands to the A/D module are delayed by 80 nanoseconds to allow the Sample-and-Hold time to acquire the analog signal.

When the conversion is complete, the A/D module sends a signal to load the data appearing on its outputs into one side of a 16-bit register. Which side is determined by a byte multiplexer in such a way that the odd numbered samples are stored in the left eight bits, while the even numbered samples are sent to the right eight bits. A set of latches and gates determine when both sides of the 16-bit register contain new data, and at this time they increment the Address and Word counters, which results in the data being stored in memory via the Data Channel.

This sampling continues until the Word Zero Detect Counter signals the control logic to drop the Data Channel. Other logic in the I/O board is set so that when the device is done, an interrupt is generated to restart program execution. The resident program must then re-order the new data with data from other pings in such a way that the analog waveform is reconstructed. Because the actual sampling rate is once every 1.2 microseconds, while the effective sampling rate is 100 MHz, a slight calculation shows that 120 identical analog pings must be sampled to yield one reconstructed waveform.

POSITIONING SYSTEM

The precision positioning carriage system constructed for this work was a two-axis unit of suitable size to be used with a common aquarium. The area of coverage was 41 by 80 centimeters. A main carriage which travels on vee shaped rails itself supported a smaller transverse carriage whose direction of motion was at right angles to that of the main carriage. Both were moved and held in position by metric leadscrews, chosen such that one turn resulted in a translation of 1.5 mm., as seen in figure (2). This odd amount was desired to match an economical type of shaft encoder available having an output of 150 digital pulses per revolution, thereby resulting in an output of one digital pulse per each .01 mm. of translation.

Direction indicators were constructed and installed on each shaft, so that cumulative position data could be had after movements in both directions. The leadscrews were operable by hand cranks, and also chain driven by small D.C. motors. These control and feedback signals were interfaced to a Southwest Technical Products' 6800 microcomputer. The SWTPC 6800 was in turn interfaced to the NOVA. A short program was written for the SWTPC 6800 to control the carriage according to commands received from the NOVA.

The shaft encoders were interfaced to the SWTPC 6800 in such a way that an interrupt was generated for each pulse from either encoder. Under program control, a

destination count received from the NOVA is compared to a current location counter maintained in memory of the SWTPC 6800. If a difference exists, the corresponding drive motor is turned on. The direction of rotation is determined by the sign of the difference between the destination counter and the current position counter. As the motor runs, pulses from the shaft encoders generate interrupts to the main program, which is waiting in a comparison step. At each interrupt, the 6800's CPU examines the input interface to determine which encoder generated the interrupt, and which direction is indicated for its shaft. Based on this information, one count is added or subtracted from the current position counter. Note that interrupts from either shaft are processed, so the shafts could move simultaneously without interference.

Following the correction to the current position counter, the interrupt routine ends with a return to the comparison step of the main program. As the interrupt service routine is transparent to the counter comparison routine, when the current location counter matches the destination counter, the motor is shut off. The motors have internal locking brakes which engage quickly when the motors stop, resulting in negligible coast which has been determined to be 3 counts or .03mm. Also, because interrupts are allowed even after the motors are shut off, this coast does not accumulate. The exact accuracy and precision of the carriage was not measured as it was

obviously sufficient for our work. It has been estimated to be better than $1/4$ mm.

ANALOG SIGNAL SOURCE and CONDITIONING

The remainder of the system includes some sort of waveform generator to drive the transducer under investigation, and the necessary amplifiers to tailor the received signal to the dynamic range of the A/D. Because the focus of the study was on transient phenomena, the ideal input waveform would have been an impulse which would have provided an infinitely wide bandwidth. Such is not realizable in actual practice, and the compromise made was a pulse of very short duration as noted below.

Two pulsers were used, both manufactured by Metrotek Inc. They were essentially identical on the output side, differing only in that one was a more elaborate unit with extra controls for pulse amplitude, damping, width, and rep rate. The first was a model P-105A module implemented with a 300 volt power supply of our own construction. The second was designed to fit as a plug-in for a Tektronix power rack model TM 506. The model number of the second pulser was MP 215. While these units were not identical they both produced pulses of up to 300 volt magnitude with a rise time of approximately 10 nanoseconds. Pulse width was influenced greatly by the type of transducer, the length of interconnecting cable, and the amount of damping or termination employed. Generally the pulse was narrower

than 200 nanoseconds, with several oscillations of 30 to 50 nanosecond period superimposed on the pulse. While not the ultimate in bandwidth, this proved to be more than sufficient excitation.

RECEIVE AMPLIFIERS and ULTRASONIC PROBES

Two different receive amplifiers were used for this study. The first was a solid state device of our own construction, which was modified to provide greater gain and bandwidth. Later, a commercial amplifier, also a plug-in design from Metrotek, was purchased. Having many more conveniences, this proved to be a valuable addition to our instrumentation. This amp, a model MR101, initially provided a bandwidth having a high pass characteristic with adjustable cutoff frequency. When the unit was received, the lowest frequency to pass through the unit was 500 KHz.; however, this was lowered to 100 KHz. to provide better low frequency response. The input impedance of the commercial unit was surprisingly low, approximately 3000 ohms, considering it was designed expressly for ultrasonic work. Because of this, the final set up used a combination of the two amplifiers. The first stage of the 'home-brew' unit was used with as short a cable as possible between it and the receiving probe, to present a high input impedance (approximately 10 Megohms) to the probe. The output of this first stage then drove the lower impedance input of

the commercial unit, which was extremely convenient in view of its db attenuator pad.

The description of the measurement system would not be complete without a thorough discussion of the receiving probe which were used in the system. Here again, the objective was wide bandwidth. Also desirable was a probe which was as close to omni-directional as possible. These two constraints result in the conclusion that the size of the element used must be very small compared to a wavelength of sound in water at the highest frequency contemplated. This constraint on size results in a very low receiving voltage response. These considerations prompted the use of probes having element diameters of less than 1 mm.

Two commercial probes were used successfully. The first probe to be tested was actually manufactured as a blast transducer, available at surprisingly low cost. This transducer is called a "Pinducer" by its manufacturer, Valpey - Fisher. The construction is a 1 mm. diameter disc of PZT (Lead Zirconate Titante), mounted in the end of a tube of similar inner diameter. The back electrode lead is brought out through the tube, which is used as the other connection via gold plating to the front face. Despite its intended use, the Pinducer has an extremely good receiving voltage response. The frequency response starts at less than 200 KHz. and peaks at about 2 MHz., starting to drop sharply at 4 MHz. Our Pinducers performed

excellently and gave us good results.

At one point in the research, which will be detailed in the later sections, it was thought that the directionality of the Pinducers at higher frequencies was having too great an effect to tolerate. For this reason, a very high frequency small size probe was borrowed from Battelle Northwest. This is termed a "Microprobe", because of its extremely small physical size. The construction is identical to the Pinducers, apart from the reduction in size. Again, a small disc element is inserted in the end of a tube, the tube resembling a hypodermic needle. A very short (6") cable is attached, to prevent capacitive division and frequency limiting. This probe exhibited much higher frequency behavior, having its peak at five to six MHz., and a low frequency cutoff of greater than 500 KHz.. The receiving voltage response was much lower, and this unit was much more affected by input impedance than the Pinducers. Also, the directivity was much less noticeable. This probe gave us good results when sufficient signal level was available, but the Pinducers had the edge in terms of gain and low frequency response.

Some probes were constructed as part of the research, and dimensions were chosen as a compromise between the Pinducer and the Microprobe. Two methods of construction were used. One attempt was to solder a small chip of PZT about 250 microns thick to the end of a brass rod, then file the assembly carefully until the chip was circular,

with a diameter of about 600 microns. The other connection consisted of a thin wire soldered to the front electrode. The second method consisted of attaching a similar chip to the center conductor of miniature coaxial cable with conductive epoxy. The chip was then filed to size as before, and the remaining electrode was connected to the shield of the cable with a piece of fine wire and conductive epoxy. The conductive epoxy was used to avoid heating the element in soldering, which could result in depolarization due to exceeding the Curie temperature. These probes seemed quite variable from unit to unit because of differences in construction.

In conclusion to this section describing the measurement system, it can be seen that, at a nominal investment of funds, and a sizeable investment of time, an excellent automated high frequency acoustic measurement system was constructed. Hopefully, as more research continues, the system will be expanded and tuned. Possible additions would include the following: A third axis on the carriage would be extremely convenient, this taking the form of a vertical displacement capability, and also perhaps a rotational capability on the same axis. This would allow the generation of beam patterns with out fear of the directivity of the receiving probe becoming a source of error. Also, three dimensional beam patterns could be generated, which would provide a better measure of transducer directivity. Another addition would be a larger

aquarium. The present tank in use allows a maximum distance of about 80 centimeters between projector and probe, less if either has appreciable length. In some cases, this is not in the far field of the projectors. While the carriage itself is only 1 meter in length, a larger (longer) tank would allow measurements in the far field, even if under some restriction of movement.

III THEORY of the IMPULSE RESPONSE APPROACH

The acoustic pressures which are radiated into a fluid from a circular piston are well known for points in the far field and for harmonic piston velocities. Until recently, the pressure response in the near field and for non-harmonic excitations had not been adequately described. The exact integral expression (1) for the field of a circular piston radiator having a uniform velocity across its face can be expressed as follows (10):

$$p(x,t) = \text{Re} \left[\tilde{p}(x) e^{j\omega t} \right], \quad \tilde{p}(x) = \frac{j\rho c k}{2\pi} U_0 \int_0^a \sigma d\sigma \int_0^{2\pi} \frac{e^{-jk r'}}{r'} d\gamma \quad (1. a)$$

where

$$r' = (r^2 + \sigma^2 - 2r\sigma \sin\theta \cos\gamma)^{1/2} \quad (1. b)$$

In the above expressions and those that follow, ρc is the specific acoustic impedance of the fluid, k is the wave number, U_0 is the peak velocity of the piston, σ is the radius from the center of the piston to the differential element, $d\sigma$; γ is the angle between this radius and a reference axis, and θ is the angle between the axis of the transducer and the position vector $\vec{R}$, whose length is r . The geometrical variables of interest are shown for clarification in figure (5).

The usual far field approximation applied to (1. b),

Figure 5. Geometry of General Integral Equation

and valid if $r \gg a > \sigma$ is:

$$r' = r - \sigma \sin \theta \cos \gamma \quad (2)$$

After substituting equation (2) into (1) and performing the integration, the closed form expression for the far field harmonic pressure is:

$$p(r, \theta) = \frac{j \rho c k a^2 U_0}{2r} e^{-jkr} \left[\frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right] \quad (3)$$

where $J_1(x)$ is a Bessel function of the first kind and order one.

The point at which the approximation loses validity is associated with the separation of near field and far field regions. This is usually taken to be the zone where a regular beam of sound pressure starts to form. Zemanek (10) evaluated equation (1. a) without approximation using computer techniques, for regions from the face of the transducer out into the near field far field transition area, where the pressure magnitude for an harmonic excitation exhibits a highly undulatory behavior. These results can be carried over to excitations consisting of many cycles of a sine wave with some success; however, if the duration of the excitation becomes less than the travel time of sound across the face of the piston, the

expressions lose validity.

The next development in the description of the pressure field of pulsed piston radiators was a time domain approach using a transformation of coordinates applied to the Green's function integral solution for the pressure time history of a pulsed radiator, see Stepanishen, (11). This resulted in the following convolution integral for the time dependent pressures in the field:

$$p(x, t) = \rho \frac{\partial}{\partial t} \int_{\tau=0}^t h(x, t - \tau) \cdot v(\tau) d\tau \quad (4)$$

where $p(x, t)$ is the time dependent pressure at the spatial point x , ρ is the density of the medium, $h(x, t)$ is the impulse response of the piston to the spatial point, and $v(t)$ is the velocity time history of the piston face. Equation (4) can be written as:

$$p(x, t) = \rho h(x, t) * a(t) \quad (5)$$

where $*$ denotes convolution.

The impulse response, $h(x, t)$, for a planar radiator of any shape, was also given in the same reference as follows:

$$h(x, t) = \frac{L(x, ct)}{2\pi t \sin(x, ct)} \quad (6)$$

Here $L(x, ct)$ is the length of the arc formed by the intersection of the radiating piston and a sphere centered

at the point of interest, x . The radius of the sphere is $R=ct$. $\Theta(x,ct)$ is the angle between the radius of the sphere extending to the arc and the perpendicular of the plane passing through the center of the sphere. [see figure (6)]. Of particular importance is the fact that the above expressions are valid for any uniform velocity input and for any point in the half space above the piston.

The evaluation of equation (5) for a circular piston of radius a is simplified if a substitution is made for t , $\tau=ct/a$, and for x , $x'=x/a$, and if x is resolved into two coordinates of cylindrical geometry. [see figure (7)]. Depending on the field point of interest, the resulting function is described in either three or four sections. If the point is inside the right cylinder formed by the projection of the edge of the piston into the half space above, as characterized by the relation $a/r > 1$, the impulse response can be expressed as follows:

(7)

$$h(x', \tau) = \begin{cases} 0 & \tau < \frac{z}{a} \\ 1 & \frac{z}{a} < \tau \leq R_1 \\ \frac{1}{\pi} \cos^{-1} \left[\frac{\tau^2 - (z/a)^2 + (r/a)^2 - 1}{2r/a(\tau^2 - (z/a)^2)^{1/2}} \right] & R_1 \leq \tau \leq R_2 \\ 0 & R_2 \leq \tau \end{cases}$$

Figure 6. Geometry of Impulse Response Function

Figure 7. Geometry of Coordinate Transformation

If $r/a > 1$, the impulse response can be expressed as follows:

$$h(x, \tau) = \begin{cases} 0 & \tau \leq R_1 \\ \frac{1}{\pi} \cos^{-1} \left[\frac{\tau^2 - (z/a)^2 + (r/a)^2 - 1}{2r/a(\tau^2 - (z/a)^2)^{1/2}} \right] & R_1 \leq \tau \leq R_2 \\ 0 & R_2 \leq \tau \end{cases} \quad (8)$$

where

$$R_1 = \left[\left(\frac{z}{a} \right)^2 + \left(\left(\frac{r}{a} \right) - 1 \right)^2 \right]^{1/2} \quad R_2 = \left[\left(\frac{z}{a} \right)^2 + \left(\left(\frac{r}{a} \right) + 1 \right)^2 \right]^{1/2}$$

Also note that equation (7) reduces to the following expression when applied to the far field on axis case:

$$h(R_0/a, 0, \tau) = \frac{\pi}{2\pi R_0} \delta(\tau - R_0/a) \quad (9)$$

Here δ is the Dirac delta function. Thus it follows from equations (5) and (9) that the far field on axis pressure is simply proportional to the piston acceleration weighted by the piston area and range.

Some theoretical impulse responses are presented in figures (8) through (11). The figures show that there are three basic shapes for the impulse response of a circular piston which are dependent on the location of the field point. The on axis impulse response is a rectangle, whose amplitude is unity and time duration is equal to the

difference in time of propagation of sound from the center and from the edges of the piston. This pulse duration is thus a maximum for a point at the surface of the piston and monotonically decreases as axial range of the field point increases, [see figure (8)]. At points lying inside the projection of the circle of the transducer, the impulse response starts off with a step discontinuity as indicated in figure (9). The response is then constant until the arc depicted in figure (6) has arrived at the edge at its closest point. From this time on, the impulse response decreases monotonically, until it goes to zero at the time the arc has arrived at the opposite edge. Figure (9) shows this. The third family of impulse responses is the case where the measuring point is outside the cylinder of projection. Here, the curve takes on the shape of a non-symmetrical hemisphere, whose amplitude decreases with increasing angle from the axis as indicated in figure (10). The degree of non-symmetry is dictated by the radial distance of the point from the center of the piston. As the radial distance increases, the impulse response becomes more symmetric as indicated by a comparison of the results in figures (10) and (11).

Figure 8. Theoretical Impulse Response: On Axis

Figure 9. Theoretical Impulse Response: Inside Disc

Figure 10. Theoretical Impulse Response: Outside Disc

Figure 11. Theoretical Impulse Response: Far Field Outside Disc

IV MATHEMATICAL TECHNIQUES for DECONVOLUTION

The key difference between our research and that mentioned previously is that this experiment attempts to start at the end and work back. That is, rather than attempt to compare resultant pressures with theoretical predictions, we wish to work backwards from experimental pressures to yield the impulse response. Once the impulse response for a particular geometry and spatial point is known the pressure can then be evaluated for any arbitrary excitation without having to perform additional experiments. More importantly once the impulse response method is verified, the method can be used to design wide bandwidth arrays with specified spatial and temporal properties.

An examination of equation (5) shows that if the face acceleration and the pressure time history at a field point were known, and if a method of deconvolution was available, it would be possible to deconvolve the pressure history by the input acceleration, yielding a time domain impulse response. There are two difficulties associated with this method: First, some method of measuring piston velocity or acceleration is required. Second, a method of deconvolution is necessary.

In view of the first requirement, consider again equation (9):

$$h(R_0/a, 0, \tau) = \frac{A}{2\pi R_0} \delta(\tau - R_0/a) \quad (9)$$

As mentioned before, this implies that the far field on axis pressure is proportional to piston acceleration. Therefore, information about the face velocity is available from far field on axis pressure measurements.

Consider the black box or system model of the typical experimental setup in figure (12). As can be seen from the model, the signal passing through the typical measurement system is not solely a function of face velocity and impulse response. Other factors which ultimately affect the received voltage are electrical input, transducer dynamics, receiving probe dynamics, and dynamics associated with the received signal processing. The beauty of this new experimental approach is that if the on axis far field pressure and the nearfield pressure at the point of interest are taken with the same system, as outlined above, when the deconvolution is performed, any effects which are due to common factors will be cancelled out. The deconvolution gives information only about the difference of the two measurements.

The experimental impulse responses were developed using an approach based on the preceeding observations. Experimental pressure histories were taken at two points, an on axis field point at large range, and at another field point where the impulse response was desired. These two measurements were then processed using computer techniques

Figure 12. Box Model of Typical Measurement System

to generate an experimental impulse response.

The preceding sections indicate that the time-domain Impulse Response can be experimentally derived from field pressure time histories if a method of deconvolution is available. To this end, considerable time was spent investigating various approaches to deconvolution. Those not chosen will be outlined in the order in which they were investigated, and their reasons for failure will be discussed in the following subsections. The method which was finally developed to perform deconvolution is then presented in a separate subsection in light of its importance.

EVALUATION CRITERIA and PROCEDURES

Each deconvolution method which was investigated was first tested with arbitrary boxcar, i.e., rectangular pulse, time histories. Two boxcar functions were analytically convolved to obtain a function which was subsequently deconvolved using the following programs. If these indicated success, the program was run using simulated acoustic data generated by another program. The modelling program, PULSECHO, was written by a previous student involved in this research, Glenn Andrew. Its purpose was to produce theoretical pressure time histories at various field points, based on the theoretical Impulse Response. [See equations (7) and (8).] One of several simple electrical inputs could be specified, as could the presence or absence of transducer dynamics.

The program proceeded stepwise, as follows: First, the theoretical Impulse Response was calculated from the expressions cited above. Next this was transformed to the frequency domain, where convolution can be performed as a simple multiplication. This frequency representation was next multiplied by the transform of the chosen electrical input, and the transform of the transducer dynamics, if included. At this point an inverse Fourier Transform was calculated, yielding the pressure time history for the point specified in the calculation of the impulse response.

As originally written, the program would treat this signal as if it had been reflected by a perfect point reflector. Then the aforementioned process would continue in reverse to yield the electrical output of the transducer. Plots were generated after each stage in order as follows: Electrical input, transmit velocity, transmit pressure, receive pressure, receive force response, and receive voltage response.

This program was modified for our purposes as follows: Only the transmitting section was retained. This portion was executed twice, once for the long range on axis pressure history, and again, calculating the theoretical pressure response for a nearfield point of interest. In addition to plots, permanent disk data sets were made of the field pressure time histories. These records were accessed by the deconvolution programs under test.

In the initial phases of testing, no transducer

dynamics were included. This meant that the piston face velocity was identical to the electrical input. Under these conditions, the simulated pressure waveform was simple enough that the theoretical impulse response was apparent in the data. This allowed conclusions to be drawn as to what was happening in the deconvolution program relatively easily.

If the deconvolution process under study showed promise, transducer dynamics would be introduced into the input data. At this point, the input data bore a closer resemblance to real pressure time histories, but the theoretical impulse response was no longer visible in the data. This was considered the hardest artificial test of a deconvolution program: Could it perform in the presence of other transfer functions applied to the two input signals? This was absolutely necessary in view of the experimental setup, which would include transfer functions for the receiving probe and receive amplifier.

SPECIFIC TECHNIQUES

Many different approaches to perform deconvolution were tried, working both in the time domain and in the frequency domain. The time domain expression for the convolution of the spatially dependent impulse response and the acceleration of the piston may be expressed as follows:

$$p(x, t) = \rho c \int_0^t h(x, t-\alpha) \cdot a(\alpha) d\alpha \quad (10)$$

after normalizing the pressure $p(x,t)$ by the specific acoustic impedance of the fluid, the normalized pressure can then be expressed as follows:

$$p(x,t) = h(x,t) * a(t) \quad (11)$$

where the asterisk $*$ denotes the convolution operation and τ has been replaced by t . The convolution integral of equation (11) is reduced to a much simpler expression if the Fourier Transform is taken of both sides, noting that the spatial dependence has been dropped from this point on for clarity:

$$P(\omega) = H(\omega) \cdot A(\omega) \quad (12)$$

Deconvolution, as we wish to perform it, is the processing of two time functions which are the input and output of a linear system, to yield the time domain impulse response of the system. The most obvious approach is to start with equation (12) and divide both sides by $A(\omega)$, yielding:

$$H(\omega) = \frac{P(\omega)}{A(\omega)} \quad (13)$$

Then an inverse transform should result in the impulse response, $h(\tau)$.

The most obvious flaw here is that if $A(w)$ ever goes to zero, $H(w)$ is undefined. Intuition is satisfied in this case, as it is seen that for this frequency w' , no information about the linear system can be determined if no input is supplied. Theoretically, as long as $A(w)$ does not go to zero, equation (13) followed by an inverse Fourier transform should still yield a correct impulse response. In practice, any noise present in $P(w)$ is exaggerated greatly as $A(w)$ goes to zero.

An approach which was based on extending the above technique was implemented on the Academic Computer Center's system, and studied at length with little success. The approach consisted of the controlled averaging in the frequency domain of the quotients of output and input data frequency values. Part of the reasoning behind using such a technique was that although any given signal input would probably display nulls in their spectra, by using several different inputs and processing in parallel, information needed at a null of one signal would be supplied by another signal not having a coincident null.

The division in the frequency domain was performed for each discrete frequency value of input and output. The quotients were averaged with those from other input and output pairs of the same frequency. If any input contained a null at this frequency, its quotient was omitted from the

average. An example of the results is presented as figure (13). This should resemble a rectangular pulse, but does not. This approach was finally abandoned in favor of other more promising techniques. The cause of the distortion was not determined exactly; however, it is a likely probability that the cause was truncation error resulting in cumulative phase errors incurred while the data was transformed back and forth.

The second method which was considered proceeded as follows: The numerator and denominator of equation (13) were multiplied by the conjugate of $A(w)$:

$$H(\omega) = \frac{P(\omega) \cdot A^*(\omega)}{A(\omega) \cdot A^*(\omega)} \quad (14)$$

This technique is recommended in Bendat and Piersol, reference (12). This formulation could be handled in matrix algebra form, noting that $A(w)$ multiplied by its conjugate is simply the frequency transform of the autocorrelation of $a(t)$. Likewise, the multiplication of $P(w)$ and the conjugate of $A(w)$ is the frequency transform of the cross correlation of $a(t)$ and $p(t)$. After inverse transforming both sides of the frequency domain expressions, the resulting integral equation in the time domain can be converted into the following matrix equation:

$$[a][h] = [R] \quad (15)$$

Figure 13. Test Result of Frequency Division Technique

Here R is a cross correlation vector, h is the impulse response vector, and a is the autocorrelation matrix which result from the discretizing process.

Note however that due to the physical size of the transducers involved and the extremely high sampling rate utilized, the autocorrelation matrix a is of the size of a few thousand elements on a side. To determine h from a and R , the inverse of a must be found. This presents a problem of programming, as first, such an amount of storage is difficult to find and extremely expensive. Secondly, as the size of an array increases beyond 20 or 30 elements on a side, if the matrix is not well conditioned, the inversion routine runs into numerical error problems. The above difficulties were sufficient to abandon the frequency domain approach to deconvolution completely.

The next approach sought was a direct 'number crunching' type which was based on equation (10) written in time sampled form:

$$p_J = \Delta t \sum_{k=1}^J a_{J-k+1} h_k \quad J = 1, N \quad (16)$$

where Δt is the time sampling interval. Equation (16) can be used to develop the following recursion relationship for the sampled values of the impulse response:

$$h_J = \frac{p_J / \Delta t - \sum_{k=1}^{J-1} a_{J-k+1} h_k}{a_1} \quad J = 1, N \quad (17)$$

A computer program which was based on equation (17) was developed to evaluate the h_j 's. The program started with p_1 and a_1 to get h_1 . Then p_1 , p_2 , a_1 , a_2 , and h_1 were used to determine h_2 . This series proceeded, with each iteration using the information previously generated and two more data points. This was the first technique to yield correct results from simple geometric waveforms and also PULSECHO generated data with no dynamics. Although the method was suitable for certain types of functions which exhibited initial discontinuities, this method is unstable for other classes of functions. This is readily apparent since any error in a data point affects all following points in a geometrically increasing manner. When tested with PULSECHO data which included transducer dynamics, these instabilities were intolerable. Often the program would not be able to finish execution due to overflow caused by wildly propagating truncation errors. A sample of this is presented as figure (14).

The fourth method examined was a gradient search. In this type of analysis, a trial estimate of the impulse response $h(t)$ is convolved with the input data. The result of this is compared with the output data, and a root mean square error is calculated. The gradient of the error function is calculated, weighted by an amount determined to minimize the error, and is then added to the previous trial approximation. This approach was implemented in a very small number of samples (32) to determine if a larger

Figure 14. Test Result of Time Domain Approach

sample size version should be tested. This method did show a tendency to converge; however, the rate of convergence was very slow, and each iteration required at least three full convolutions, and sometimes more, to arrive at the weighting factor needed to minimize the r.m.s. error.

It was during this evaluation that the final deconvolution program was developed. The program was based on work contained in reference (13) which described a technique for determining the inverse filter given the impulse response was known. Ultimately the objective of the authors was to take the output of their system, convolve it with the impulse response of their inverse filter, yielding an approximation to the actual input to their system, which was unknown. The paper further outlined certain restrictions on the impulse response of their system to guarantee convergence of their iterative technique. This information proved to be the necessary clue to deconvolution as we wished to perform it.

DESCRIPTION of IMPLEMENTED METHOD of DECONVOLUTION

The technique which finally proved successful is based on an iterative approach. The reference mentioned above outlined a procedure for determining the unknown input of a linear system, given the impulse response and the output. Conditions necessary for convergence of the iterations are discussed. Our approach was to perform a substitution on their method. The technique iterates, each time comparing

the convolution of the latest estimate of $h(t)$ with $a(t)$ to the output of the system, $p(t)$, and adds this difference to the estimate:

$$h_1(t) = h_0(t) + [p(t) - h_0(t) * a(t)] \quad (18)$$

or in general:

$$h_n(t) = h_{n-1}(t) + [p(t) - h_{n-1}(t) * a(t)] \quad (19)$$

The corrections added with each iteration can be written as an infinite series:

$$h_n(t) = h_0(t) + \epsilon_1(t) + \epsilon_2(t) + \dots + \epsilon_n(t) + \dots \quad (20)$$

here

$$\epsilon_1(t) = p(t) - h_0(t) * a(t) \quad (21.a)$$

$$\epsilon_2(t) = p(t) - [h_0(t) + \epsilon_1(t)] * a(t) \quad (21.b)$$

$$\epsilon_3(t) = p(t) - [h_0(t) + \epsilon_1(t) + \epsilon_2(t)] * a(t) \quad (21.c)$$

and

(22)

$$\epsilon_n(t) = p(t) - [h_0(t) + \epsilon_1(t) + \epsilon_2(t) + \dots + \epsilon_{n-1}(t)] * a(t)$$

This can be transformed to the frequency domain as follows:

$$e_n(\omega) = [P(\omega) + H_0(\omega) \cdot A(\omega)] [1 - A(\omega)]^{n-1} \quad (23)$$

This means that the series of (20) can be written as:

$$H(\omega) = H_0 + a + ar + ar^2 + \dots = \frac{a}{(1-r)} \quad |r| < 1 \quad (24)$$

where

$$a = P(\omega) - H_0(\omega) \cdot A(\omega), \quad r = 1 - A(\omega)$$

Therefore the series of (20) converges in the limit as follows:

$$\lim_{n \rightarrow \infty} H_n(\omega) = \frac{P(\omega)}{1 - [1 - A(\omega)]} \quad |1 - A(\omega)| < 1 \quad (25)$$

The above condition for convergence is too strict to deal with, as it implies that $A(\omega)$ contains no phase inversions. In view of this, the above process is modified by first processing both input and output signals in such a way to allow a less stringent convergence criteria. Here, both input and output are convolved with the input signal, which has been inverted in time:

$$g(t) = a(t) + a(-t) \quad (26)$$

$$x(t) = p(t) + a(-t) \quad (27)$$

The method above is again applied, with the result that the series converges if:

$$|1 - G(\omega)| < 1 \quad (28)$$

but

$$G(\omega) = |A(\omega)|^2 \quad (29)$$

therefore the series converges if

$$|A(\omega)|^2 < 2$$

which is satisfied by proper scaling of the input signal.

The above analysis has been formulated to allow any starting estimate at all. Note that as implemented on the computer, the technique is valid only for impulse responses which are finite, within the time window of the program, and causal, meaning that $h(t)$ is non-zero only for $t > 0$. The general convergence condition makes no restrictions on the first estimate, so any function may be used without fear of biasing the results. Obviously many more iterations are needed for the same degree of accuracy in

the result if the first guess is considerably different from the solution. In view of this, when applied to real data, the initial estimate used was the theoretical impulse response computed for the spatial point of interest.

This method was implemented in full size on the Academic Computer Center system. The operation is broken down into two separate programs. Data is first processed in a program called NORM, listed in Appendix A. As implied by its name, this program normalizes both the input and the output time histories so that in terms of actual values, their discrete frequency transforms are always less than 2. It is not necessary for convergence to scale the output time history; this is merely done to keep all the signals at a relatively constant level. Next NORM convolves the input and output signals with the time inverse of the input. Lastly, NORM calculates the theoretical impulse response, and creates plots of $g(t)$, $x(t)$, and the theoretical $h(t)$. This data is then stored on permanent disk records. NORM does not take an extremely great amount of computer time, but because an input time history is 1024 samples long, and the output history is twice that, considerable memory is used. Intermediate data was stored often throughout the processing to provide backup if the program ran over its allotted time or if the system crashed.

Next after the data sets have been normalized and pre-processed, a program is run which does the iterations. Several versions of this exist, differing only by the

number of iterations performed. Each iteration involves only one convolution, which is done by the FFT method. One factor of this convolution is always $g(t)$, which instead of being stored in the time domain and transformed every time, is stored as $G(w)$, eliminating the need to transform it every iteration. By this saving, the time for each iteration has been reduced to approximately six seconds of CPU time.

The method was tested as described in the preceeding section, and performed well. With no dynamics present in the simulated data, the deconvolved impulse response tracked almost perfectly, the only deviation being caused by discontinuities in the original trial fit. In testing this program, the first guess impulse response was a simple box, whereas the solution impulse response was known to be a sort of nonuniform semicircle containing discontinuities only at its endpoints. This was done to cause the deconvolution program to require several iterations, to force it to 'work'. The steep rising and falling edges of the box function caused small spikes to appear at the corresponding points of the solution impulse response. A typical output from the program run on this data is shown in figure (15).

Next the technique was tested with simulated data which included transducer dynamics. The program successfully deconvolved the correct impulse response, and the only difference from the previous test was that the

Figure 15. Test Result of Iterative Technique: No Dynamics

solution had a small amount of high frequency noise present. This was relatively insignificant in terms of the ratio of signal to noise. It has been surmised that the noise is caused by truncation error in the computer. Although this is small for any given iteration, the noise probably accumulates when many iterations are performed. In the above two tests, the solution was found after less than 100 iterations, often after less than 20. An output from the program as tested with dynamics included is given in figure (16).

In further study with the technique using experimental data for each of the three cases of the impulse response, it was found that the rate of convergence and ultimate accuracy was significantly affected by the value of an arbitrary scale factor applied to the initial trial $h(t)$ in the first iteration. In view of this, each set of experimental data was run through the first iteration several times to determine the best scale factor. This scale factor was adjusted to a value as close as possible to the value where a phase inversion of the first correction occurred at the time corresponding to the first excursion of the impulse response. This significantly improved the rate of convergence and also the ultimate accuracy. The mathematical analysis of the technique shows that convergence is independent of the initial estimate h_0 ; however, in actual practice this affects the rate of convergence. The technique generates a few artifacts of

Figure 16. Test Result of Iterative Technique: Dynamics Included

processing with many iterations. These artifacts have a fundamental frequency where there is little energy in the input signal, and for this reason, these artifacts tend not to affect the reconvolution check. The technique was tested with experimental data such that the initial estimate was wildly in error; in this situation the technique failed to converge in several thousand iterations. This failure was clearly indicated in the reconvolution check. Therefore, the reconvolution check was used to verify all experimental results.

The next step was to test the program on real data. Here one may inquire how the degree of accuracy of a given solution was tested. To this end, a small routine was written to reconvolve the impulse response determined by the iterative technique with the input pressure time history. The result was plotted along with the output time history. If the solution was correct, the two curves would track well together. A typical example of these two waveforms is shown in figure (17). Note that the curves are extremely similar until the midpoint, where they cease to track exactly due to the fact that the input data has been truncated at this point to fit in the time window.

Figure 17. Convolution Check of Iterative Technique

EXPERIMENTAL PROCEDURES and RESULTS for SINGLE TRANSDUCERS

This section contains a detailed description of the types of transducers which were analysed in the test setup. Initial investigations of the acoustic fields of these transducers are described, and as the initial results did not compare with theoretical predictions, subsequent experiments were conducted to discover the cause of the discrepancies. An important observation concerning the experimental procedure was noted and additional experiments were then conducted that yielded results which were in agreement with the theoretical predictions.

TRANSDUCER CONSTRUCTION

Several projectors, which differed as to their construction, were built for this project. However, in each case the goals behind construction techniques were the same, i.e. to develop a wide bandwidth piezoelectric transducer which exhibited a uniform velocity across its radiating face. All the transducers were constructed from piezoelectric discs and all were backed with a medium chosen specifically for its acoustic impedance or loss.

Several transducers were constructed with a one inch (2.54 mm.) Lead Metaniobate element. Lead Metaniobate was chosen since it has greater internal loss than other materials. The greater loss factor means that the transducer would tend to be more damped and would ring for

a shorter period of time. This was advantageous for two reasons: First the bandwidth was increased, giving us more information in the frequency domain concerning the impulse response. Second, the pressure time history tended to be shorter, meaning more of the significant portion would fit in the heavily sampled time window of NORM.

Several other researchers [(7), (8)], have found that the use of a backing material fabricated out of epoxy and Tungsten powder causes a significant increase in bandwidth, due to the resistive loading on the back face. It is interesting to note that Tungsten has the highest density of any elemental metal. Some prototype transducers were constructed as a test of construction techniques. At first, the amount of Tungsten used was about one part in six by weight. This was kept in suspension in the epoxy during the curing time by slowly rotating the sample. Transducers backed with this compound did not show a considerable difference from those backed on other materials. On comparing construction techniques with a commercial manufacturer, it was found that the amount of Tungsten should be so great that the epoxy should have the consistency of cookie batter before curing, obviating the necessity of spinning to keep the particles in solution. Other investigators have used the centrifuge technique with higher concentrations to promote a non-uniform distribution of Tungsten particles.

The first transducer which was constructed was a Lead

Metaniobate disc of 25.4 mm. diameter, mounted on the end of an Aluminum bar approximately 10 centimeters in length and 7.5 centimeters in diameter. Aluminum was chosen as the best commonly available material to closely match the acoustic impedance of the element. The disc element was 1.0 mm. thick, and was recessed into the end of the Aluminum so that the face of the assembly was as planar as possible.

The second transducer constructed was also a Lead Metaniobate disc, this time mounted on a backing plug molded of Araldite epoxy which was mixed with a large proportion of Tungsten powder. The element was baffled with a layer of pure Araldite formed around the edge of the element. After the epoxy had set, the baffle was filed flat.

The third transducer investigated was a crystal of Lithium Sulphate. This was also bonded to a Tungsten and Araldite plug as before, and an epoxy baffle molded on. The diameter of the Lithium Sulphate was only 15 mm. Lithium Sulphate was chosen at the suggestion of a commercial biomedical transducer manufacturer, who had an intuitive feeling that it would be less susceptible to vibration in the radial mode because it was a single crystal as opposed to an amorphous ceramic. The thickness of the Lithium Sulphate crystal was only .45 mm., resulting in the resonant frequency of between 5 and 10 MHz when mounted on the plug as noted above.

MEASUREMENTS and RESULTS

The time dependent pressures which were radiated from these transducers under shock excitation were measured in the test setup. Programs were written for the NOVA to take several pressure time histories with the high speed A/D converter, average these together, remove any mean value, plot the resulting ensemble average, and store the data on 9-track tape. Another program was run on the NOVA to read the time histories from the tape and send them via the telephone link, to the Academic Computer Center system. Two pressure time histories were obtained for each deconvolution: One pressure time history was measured at an on axis far field point and the other pressure time history was measured at the field point of interest.

Three general areas were concentrated on in the field; on axis at close range (Z less than two radii), off axis but still inside the cylindrical projection of the transducer at close range, and at angles of from 30 to 45 degrees from the normal to the face of the transducer at ranges from Z equal to two to ten radii. Due to the size of the aquarium and the length of the transducers, the maximum range used for the on axis case was 35 centimeters. This was repeated for the first two transducers described above and for several field points.

The results of the deconvolution to obtain the experimental impulse response were at this time disappointing. Although subjected to sometimes as many as

1400 iterations, the computed impulse response curves were very noisy and high frequency in nature, and did not track at all with the theoretical impulse responses. Figure (18) is a typical example. Many suggestions were made as to the cause of this failure. These included excessive radial mode vibrations, insufficient bandwidth throughout the measurement system, and directivity of the receiving probes. The most crucial question at this point was whether the problem was in the transducers themselves or in the measurement system. The deconvolution programs were not suspected, because the impulse responses, when reconvolved with on axis data, tracked extremely well with the off axis pressures recorded.

At this point, an experiment was devised to determine the location of the problem. Since the pressure field which emanates from the end of an open pipe or through a circular aperture in an infinite rigid baffle, assuming a plane wave incident, tracks closely with the field of an ideal piston radiator, this field could be used to test the experimental approach. The only significant difference occurs at low frequencies, as characterized by $kd < 10$, where k is the wave number and d is the characteristic diameter.

In view of this information, it was thought that measurements of the acoustic transmission through an aperture in a baffle would indicate the source of disagreement. Measurements of this type have been done in

Figure 18. Experimental Impulse Response: Insufficient Range

air with good results, ref (14). A thin steel sheet was perforated with a one inch diameter aperture, and covered on the back with a pressure release material, a foam rubber layer only a few cells thick. An objective of the construction was to keep the whole assembly as thin as possible, to minimize effects at the edges of the opening. This baffle was insonified in the test setup with a large (6 centimeter diameter) 200 KHz. transducer. Measurements were taken at similar off axis field points and also right at the aperture. Because the pressure measurement in the aperture resulted in data which was proportional to particle velocity and not acceleration, the expected impulse response would be the derivative of what was expected in the case of the piston radiators.

The experimental results were in fair agreement with the theoretical impulse responses, See figure (19). This prompted another set of experiments, this time taking the input measurement on the axis of the aperture, in it's far field. This should have yielded results identical to those expected for the piston radiators. Basic to this experiment is the requirement that the acoustic wave at the aperture be planar. However, the aquarium used was not large enough to allow placing the baffle in the far field of the signal source, and also allow the receiving probe to be at sufficient distance from the aperture. To get around this, the entire setup was moved down to the large test tank mentioned before. This time, to get a shorter

Figure 19. Experimental Impulse Response: Steel and Foam Baffle

acoustic pulse, the Lead Metaniobate transducer with the Tungsten backing was used as a source.

And here was Divine Inspiration or the accident of science: The pressure field of the source transducer, as measured at a distance of more than two meters, was significantly different from what had been seen a few moments ago in the aquarium at a distance of 35 centimeters. This was what had been interfering with the previous results. The data taken on axis at a distance of approximately 30 radii was not linearly proportional to the piston acceleration at all; rather it was the result of the convolution of the piston acceleration and a narrow boxcar (which is the corresponding impulse response, but not an impulse!). This meant that the input data had been subjected to a $\sin x / x$ filtering in the frequency domain. The result of this was that the experimentally computed impulse responses had been subjected to the inverse of this filter, i.e., a filter which contains poles at frequencies where the argument $\sin x$ goes to zero.

With this stunning information, all the following attempts were carried out in the large tank, using the positioning system for the receiving probe and another support for the transducer under test, at a distance of greater than one and one half meters for the on axis measurements. The results from this setup bore an excellent resemblance to the theory as described in the following paragraphs.

For the three transducers tested, measurements were made at points to yield the three types of impulse response outlined in a previous section. The best agreement with the theoretical impulse responses was found for the case where the point was outside the cylinder of projection, which is also the impulse response having the smallest bandwidth. Examples of this are given as figure (20) for the Aluminum bar transducer, figure (21) for the Tungsten backed Lead Metaniobate transducer, and figure (22) for the Lithium Sulphate on Tungsten. A reconvolution check of the experimental result for this geometry is presented here in figure (23).

Additional agreement between experiment and theory was obtained with those cases not on axis, but inside the cylinder of projection. Here the falling edge tracks well, but the sharp leading edge is obscured as indicated in figure (24) for the Tungsten backed Lead Metaniobate, and figure (25) for the Lithium Sulphate. The corresponding reconvolution check for this family of impulse responses is shown in figure (26).

The least satisfying agreement was obtained in the on axis cases. See figure (27), applying to the Aluminum bar transducer, figure (28) for the Lead Metaniobate on Tungsten, and figure (29) for the Lithium Sulphate. Again, the reconvolution check is given in figure (30).

Note that results obtained from different transducers agree, an indication that excessive radial modes were not

Figure 20. Experimental Impulse Response: Lead Metaniobate
on Aluminum Bar, Outside Disc
 $R = 20. \text{ mm.}$ $Z = 60. \text{ mm.}$

Figure 21. Experimental Impulse Response: Lead Metaniobate
on Tungsten Backing, Outside Disc
R=40. mm. Z= 140. mm.

Figure 22. Experimental Impulse Response: Lithium Sulphate
on Tungsten Backing, Outside Disc
R=50. mm. Z=130. mm.

Figure 23. Reconvolution Check: Outside Disc

Figure 24. Experimental Impulse Response: Lead Metaniobate
on Tungsten, Inside Disc
R=10. mm. Z=12. mm.

Figure 25. Experimental Impulse Response: Lithium Sulphate
on Tungsten Backing, Inside Disc
R=4. mm. Z=16. mm.

Figure 26. Reconvolution Check: Inside Disc

Figure 27. Experimental Impulse Response: Lead Metaniobate
on Aluminum Bar, On Axis
Z=12. mm.

Figure 28. Experimental Impulse Response: Lead Metaniobate
 on Tungsten Backing, On Axis
 $Z=12.$ mm.

Figure 29. Experimental Impulse Response: Lithium Sulphate
on Tungsten Backing, On Axis
 $Z=18. \text{ mm.}$

Figure 30. Reconvolution Check: On Axis

present, because if so, the perturbations caused in the impulse responses would more likely differ than coincide.

Tests made at the same field point on the same transducer with different probes agree well, supporting the hypothesis of the experiment that it is affected little by the receiving probe used. This also suggests that the bandwidth of the probes was not a factor contributing to the smearing of edges. Measurements made with the Pinducer aimed at the center of the piston, when compared with tests made with the Pinducer aimed away from the transducer at an angle of about 60 degrees, show that it's directionality effects were also insignificant.

Another comparison made was intended to discover if the epoxy baffles were meeting the rigid boundary condition intended, or if they were supporting wave propagation internally. In this case two sets of measurements were made with identical test setups, the only difference being that before the second run was made, the epoxy baffle was milled off, and the transducer was mounted in the steel plate mentioned before. The transducer was the Lead Metaniobate element on a Tungsten backing. Here a slight difference was noticeable, in that with the steel baffle, the negative excursion before and after the endpoints of the impulse response was less pronounced. However the difference is slight; too slight to draw any solid conclusions. Other factors could explain this deviation, such as the fact that any radial mode would certainly be

affected by the loading presented to the edge of the crystal, which changes with the presence or absence of the epoxy baffle.

EXPERIMENTAL PROCEDURES and RESULTS for ARRAYS

INTRODUCTION

With the work of the single element transducers finished, investigation turned toward the area of multi-element arrays. Arrays of several elements arranged in some simple geometric configuration are of particular interest for biomedical systems in view of the fact that under the proper driving conditions, they offer significant gains in terms of directivity and resolution. Also, the region of maximum acoustic intensity can be moved electronically.

Work with acoustic transducer arrays has been in existence for more than thirty years. Most of this information concerns SONAR system transducers, where the physical size is large, and the object of interest is in the far field of the array. Under these conditions, the direction of maximum response is controlled by proper shading of the amplitude and phase of the driving signals. Expressions have been generated to optimize the directivity of such an array, for certain boundary conditions such as number or maximum allowable level of side lobe structure. These expressions result, for a line array, in a constant phase difference in driving signal from element to element, assuming a constant element to element spacing. Recently,

work has been done in the area of high frequency arrays; however, this was based on a CW excitation, not pulsed operation. [See reference (15).]

The above information becomes of little importance in the biomedical oriented implementation, as the object of interest in the acoustic field is often no longer in the far field of the array. Also because of the use of transient excitation to the individual transducers, the term phase gives way to the term delay. Beamforming can now be thought of as 'spotforming', because if the point of interest is close to the array in terms of its size, it is possible to concentrate the acoustic pressure to a spot instead of simply confining it to a range of angles as a beam. It was the investigation of this phenomena that was of particular interest .

In contrast to the operation of a single transducer, anomalies often appear in the operation of an array of densely packed projectors whose dimensions are small relative to the acoustic wavelength of interest. [See references (16) through (19).] The acoustic radiation impedance experienced by each element is then no longer simply the self radiation impedance of the isolated element, but becomes a sum of the self radiation impedance and mutual radiation impedances which result from the effects of the acoustic pressures caused by the remaining elements. The degree of interaction is influenced principally by the size of each element in comparison to

the wavelength of interest. This interaction problem is of great potential importance, since the increased radiation impedance seen by an element of an array can cause the phase of the radiated signal to vary from that of an uncoupled element. The resultant phase perturbations can thus significantly affect the beamforming process.

ARRAY TRANSDUCER CONSTRUCTION

Using the knowledge of construction techniques gained from the single element work, a line array of 13 small rectangular elements was constructed as follows: A rectangular element of Lead Metaniobate 25.4 mm. long by 9 mm. wide was epoxied to a plug backing of Tungsten loaded Araldite. The thickness of this element was such that it had a fundamental resonance at 1.0 MHz. After the epoxy was cured, the element was sawed almost all the way through with a diamond saw. The depth of the cut was such that the back electrode remained intact. The cuts were spaced 2.0 mm apart, meaning that there were 13 small elements each about 1.5 mm. wide and 9 mm. long, spaced 2 mm. from center to center. A baffle was formed as before, by pouring epoxy until the surface was flush with the element faces. Hair fine wires were attached from one end of the elements to a ribbon cable. The assembly was then waterproofed and tested.

To investigate the possibilities of beamsteering, a multiple pulser system was needed which would allow a

precise delay between a common trigger pulse and each element's firing. This was implemented with TTL logic devices, which were the only suitable choice in view of the speed involved. The resolution of incremental steering is dependent on the smallest possible delay from element to element. In view of this, the driver for the array consisted of two four-bit binary counters which were preset to the desired number of counts by the trigger pulse, then proceeded to count down from the preload value to zero. When the count was complete, the counters generated a carry out pulse which triggered an SCR pulser circuit, and also reset the counters so no more than one pulse would be generated. The circuit for the SCR pulser is very similar to the output section of the Metrotek pulsers. Because the counters are rated at a maximum count frequency of 35 MHz., the smallest delay, one count, was 28.5 nanoseconds. Being 8-bit binary counters, the maximum delay that could be achieved was 255 counts, or 7.3 microseconds. A circuit diagram of one counter section and its associated pulser is shown in figure (31). Each element of the array was fed by one of these for the focussed cases.

MEASUREMENT PROCEDURE

The objective of this section of the research was to map the field of the array under certain driving conditions, to determine its behavior. Therefore the NOVA was programmed to command the SWTPC 6800 to drive the probe to a point, take a pressure time history, compute the peak

Figure 31. Digital Delay Pulser

to peak value of the pressure, and store this data point. The NOVA then signalled the SWTPC 6800 to drive to an adjacent point, and take another peak pressure measurement. This sampling was continued, resulting in a matrix of data points 100 points long by 51 points wide, corresponding to an area of the same size in millimeters. This data was then sent to the Academic Computer Center via the telephone link.

The peak pressure data was then processed by a package called " SYMVU ", which was adapted from a program of the same name from Harvard University. [See reference (29).] This program produces a unique display of the three dimensional data in a two dimensional plot.

MEASUREMENTS and RESULTS

The array was first tested without the phasing network, to gain some background data about the behavior of the individual elements. The center element was driven with the Metrotek pulser at a low level. As the center element was driven, the pressure time history was observed on the oscilloscope as the adjacent elements were electrically shorted intermittently, to see any effects of interaction. One of the adjacent elements did show a considerable effect, which was much more pronounced than the effect of the other adjacent element. The effect of the second element was small, consisting of not more than a net change of ten percent of the instantaneous pressure at any point along the waveform, which was in agreement with

the analytical study.

Next the remaining elements were left open circuited, and the pressure field of the center element in the array was mapped as described above. The SYMVU plot of this data is shown figure (32). In this case, note that the pattern is not omni-directional; rather there are lower level side lobes which were not accounted for in terms of the size of element relative to a wavelength, both of which were 1.5 mm. A beam like pattern is clearly seen which is similar to the patterns from single element transducers whose dimensions are the same as the overall width of the array. The existence of these side lobes was surprising, and it was theorized that they may have been caused by the coupling of acoustic or electrical energy from the driven element to the other undriven elements.

To investigate the possibility of element to element interaction, another measurement was made of the array with only the center element being excited. In this case, the other elements were shorted to ground at the far end of the cable, to provide a clamping effect. The plot of this data is presented as figure (33). Note that the beam like pattern has been affected by the clamping, and a much more omni-directional field is seen.

As a final effort to characterize the interaction problem, another field map was made, this time firing the end element of the array, with the remaining elements clamped. the idea here was that due to the non-symmetry of

Figure 32.

FIELDMAP PLOT -- CENTER ELEMENT, OTHER ELEMENTS OPEN

AZIMUTH = 51.30 ALTITUDE = 35.00

♦WIDTH = 2.00 ♦HEIGHT = 1.00

♦ = BEFORE FORESHORTENING / FRI. - MAY 5, 1978 /

Figure 33.

-IELMAP PLOT -- CENTER ELEMENT: OTHER ELEMENTS CLAMPED

AZIMUTH = 51.30 ALTITUDE = 35.00

♦WIDTH = 2.00 ♦HEIGHT = 1.00

♦ = BEFORE FORESHORTENING / FRI. - MAY 5, 1978 /

the undriven elements about the driven one, any interaction would show up as a corresponding non-symmetry in the field. This data is presented in figure (34). The firing element is on the right hand side of the far end of the rectangular area, firing towards the viewer. The remaining unexcited elements extend to the left. In this case, the direction of greatest response appears unchanged, however, note that on the side towards the center of the array, the side lobes are fewer and more distinct, indicative of some interaction. It must be noted that for the geometry and frequencies involved, no side lobe structure should be seen for a single element.

The next test was to drive all the elements in parallel, and the field was again mapped. Theory predicts that the field of several elements in parallel is the product of the directivity function for the large array geometry and the directivity function of each individual element. Here, if no interaction was present, the field of the array being fired "broadside" should appear as a composite of 13 elements, weighted by the directivity function of one large rectangular element. The plot of this data is presented in figure (35). Here the field takes on the appearance of the field from a large circular or rectangular transducer, in that the width of its main lobe is approximately the same as the transducer width. However, it is interesting to note the several small side lobes, which do not appear in the field of a larger pulsed

Figure 34.

IELDMAP PLOT -- END ELEMENT , OTHER ELEMENTS CLAMPED

AZIMUTH = 10.30 ALTITUDE = 35.00

♦WIDTH = 2.00 ♦HEIGHT = 1.00

♦ = BEFORE FORESHORTENING / WED. - MAY 10 , 1978 /

Figure 35.

-IELDMAP PLOT -- ALL ELEMENTS IN PHASE

AZIMUTH = 51.30 ALTITUDE = 35.00

♦WIDTH = 2.00 ♦HEIGHT = 1.00

♦ = BEFORE FORESHORTENING / MON. - APR. 17, 1978 /

radiator.

With background data logged for the array operating in the broadside mode, investigation continued with fieldmaps of the array driven such as to focus the acoustic pressure to a point. This is caused by controlling the driving signals so that the pulses from all of the elements coincide in time at one point in the field. Figure (36) shows the pressure time history of the array taken at a point where the acoustic pulses do not coincide. Figure (37) shows the pressure time history at a focal point. The delays necessary to focus the array were calculated on the Academic Computer Center system via the use of a simple algorithm which calculated the distance and propagation delay from each element to the point of interest. The difference in propagation time between each element and the farthest element to the point was computed, and converted to an integer number of clock pulses of a certain frequency. A diagram of the geometry involved is given as figure (38)

A few electrical problems were discovered when the array was first run by the digital driver. In all our tests, the electrical signal feeding the transducer(s) was introduced into the receive system by stray coupling. In single element cases this did not cause concern, as the electrical signal is short, and does not overlap the acoustic signal, due to the propagation delay. With the array running under control of the digital delay

Figure 36. Pressure Time History; Not at Focal Point

Figure 87. Pressure Time History: Focal Point

Figure 38. Geometry for Delay Calculation

controller, the electrical input pulses were now spread out over more than 2 microseconds. This meant that the electrical pulses overlapped the acoustic signals at close range. To prevent this from affecting the data, the acoustic signals were passed through a Metrotek Gate unit model MG 701, which eliminated all signals for a short interval after the first element was fired. This would tend to reduce data taken at very close range, but did not affect the majority of the measurements. This problem would not be significant in a biomedical application, as the point of interest would not usually be this close to the transducer.

The field was mapped for two cases. In the first case, the driving logic was programmed to focus the array at a point on the center line of the array, at a distance of 20 mm. In the second case, the array was focussed at a point 10 mm. to one side of the center line, at a distance of 30 mm. from the face of the array.

The field of the array focussed at the point $R = 0$. mm. and $Z = 20$. mm. is shown in figure (39). Here the peak pressure is indeed focussed at approximately the correct point. The main spot of interest is highly insonified, being greater than any other characteristic by at least six to ten db. The side lobe structure of this case is surprisingly depressed, in view of the number of radiating elements. Note that there are no aliases, or points other than the main point of interest where the acoustic pulses

Figure 39.

FIELDMAP PLOT -- ARRAY FOCUSSED AT R =0. Z =20. MM.

AZIMUTH = 51.30 ALTITUDE = 35.00

♦WIDTH = 2.00 ♦HEIGHT = 1.00

♦ = BEFORE FORESHORTENING / MON. - APR. 17, 1978 /

add to form a second peak.

The second focussed field plot is listed as figure (40). Here the array was driven according to the delays generated by the computer routine so as to focus the acoustic energy at the spot $R = 10. \text{ mm.}$ and $Z = 30. \text{ mm.}$ Here the extraneous side lobe energy is found in two areas, on either side of the main pressure feature. The side structure on the 'near' side of the main feature, that is, in front of the center of the array, is concentrated in close to the face. The side structure on the 'far' side of the main feature is at considerably greater range than the main feature itself.

The above study of arrays provides a small insight into the behavior of high frequency pulsed acoustic arrays. Work must be continued to more fully characterize the acoustic field under pulsed conditions and to fully realize the usefulness of these arrays. Some areas pertaining to this that could prove informative would be as follows: Experimentally determining the impulse response of the entire array, under various driving conditions, would help to indicate further the extent of element to element interaction. Also, more precisely controlled construction techniques, employing smaller elements, would improve the spatial resolution of the system. Making the elements smaller, i.e., a half wavelength or less, would decrease the directionality of the individual elements, thereby lessening the pressure variation as the array is focussed

Figure 40.

-IELDMAP PLOT -- ARRAY FOCUSSED AT R =10. Z =30. MM.

AZIMUTH = 51.30 ALTITUDE = 22.00

*WIDTH = 2.00 *HEIGHT = 1.00

* = BEFORE FORESHORTENING / MON. - APR. 17 ; 1978 /

toward greater angles from the axis.

CONCLUSIONS

This section outlines the results and states the conclusions drawn from the present research. The key accomplishments, results, and conclusions of each aspect of this research are summarized as follows:

An automated measurement system designed for high frequency acoustic research was constructed. This system included a high speed Analog to Digital converter with an effective sampling rate of 100 MHz., and a motorized computer controlled precise positioning system. Commercially obtained acoustic probes were evaluated and compared to probes constructed in the course of this research.

A mathematical technique for performing deconvolution on the input and output of a linear system was implemented and evaluated on the Academic Computer System. This procedure was found to be useful in the verification of the Impulse Response Approach to evaluate the spatial and temporal characteristics of the pulsed acoustic field which results from the broadband excitation of medical ultrasonic transducers and arrays.

Three piezoelectric transducers similar in construction to those presently in biomedical applications were constructed. The use of Tungsten loaded epoxy as a backing material was found to be helpful in increasing transducer bandwidth.

These transducers were investigated in the measuring system, and their corresponding impulse responses were arrived at via digital signal processing techniques. Several measurements were made for each transducer, to provide data for the computer deconvolution routine. For each of the transducers tested, and each of the three types of impulse response, the results of the experimental work were in good agreement with the theoretical predictions, thereby strengthening the validity of the use of the theoretical approach for unconfirmed analytical simulation.

The impulse responses obtained from the broadband acoustic fields which were generated by these transducers provided important new experimental verification to the Impulse Response Approach for describing the pressure fields of pulsed piston radiators. The agreement of theoretical and experimental results implied that vibration of the elements in modes other than thickness expansion was not significantly troublesome. Identical results obtained from different acoustic receiving systems showed that the experimental approach and assumptions were valid.

A thirteen element array of similar materials chosen for wide bandwidth was constructed. A digital delay driving system with high voltage high speed pulsers was built to drive the array. The background characteristics of the array were measured. The array was successfully driven in such a way as to concentrate the acoustic pressure to a very small region. Element to element

interaction effects were examined, and found to be insignificant as far as pressure focussing was concerned.

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10 //DECONVLU JOB (LDG100,384,1), TONY FISHER GSO
20 /*ROUTE PRINT REMOTE2
30 // EXEC FORTHCLG,REGION=384K,PARM.FORT='OPT=02',LIB1=SSP,LIB2=TFLC
40 //FORT.SYSIN DD *
50 C
60 C*****
70 C
80 C      THIS PROGRAM PERFORMS THE INITIAL STEPS OF DECONVOLUTION
90 C      VIA A METHOD OF SUCCESSIVE APPROXIMATION. THIS SECTION
100 C      INITIALIZES THE DATA SETS, PLOTTING INTERMEDIATE STEPS.
110 C
120 C      THE RECURSIVE FORMULA IS AS FOLLOWS:
130 C
140 C      
$$H_N(T) = H_{N-1}(T) + (X(T) - G(T) \# H_{N-1}(T))$$

150 C
160 C
170 C      WHERE:
180 C
190 C      # STANDS FOR CONVOLUTION
200 C
210 C       $X(T) = P(T) \# A(-T)$ 
220 C
230 C      AND
240 C
250 C       $G(T) = A(T) \# A(-T)$ 
260 C
270 C*****
280 C      INITIALIZATION SECTION
290 C*****
300      DIMENSION ACC(1024),M(3),S(1024),INV(1024),X(1024)
310      DIMENSION PRESS(2048),H(1024)
320      COMPLEX ARRAY1(4096),ARRAY2(4096)
330      COMMON M,INV,S,IFERR
340      M(1)=12
350      M(2)=0
360      M(3)=0
370      DO 25 I=1,4096
380 25  ARRAY1(I)=(0.0,0.0)
390      CALL HARM(ARRAY1,M,INV,S,0,IFERR)
400 C*****
410 C      NORMALIZE A(T) SUCH THAT  $|A(W)|^2 < 2.0$ 
420 C*****
430      READ(8)(ACC(I),I=1,1024)
440      DO 12 I=1,4096
450      ARRAY1(I)=(0.0,0.0)
460      IF(I.LT.1025)ARRAY1(I)=CMPLX(ACC(I),0.0)
470 12  CONTINUE
480      CALL NORM(ARRAY1)
490      DO 17 I=1,1024
500      ACC(I)=REAL(ARRAY1(I))
510 17  CONTINUE
520 C*****
530 C      CONVOLVE A(T) WITH A(-T) TO GIVE G(T)
540 C*****
550      WRITE(6,1)ACC
560 1    FORMAT(10(2X,E10.4))

```



```

570      DO 100 I=1,4096
580      IF(I.GT.1024)GO TO 50
590      ARRAY1(I)=CMPLX(ACC(I),0.0)
600      ARRAY2(I)=CMPLX(ACC(1025-I),0.0)
610      GO TO 100
620  50  ARRAY1(I)=(0.0,0.0)
630      ARRAY2(I)=(0.0,0.0)
640  100 CONTINUE
650      CALL CONVOL(ARRAY1,ARRAY2)
660 C*****
670 C   PLOT G(T)
680 C*****
690      CALL PLTOUT(ARRAY2,'G(T)',4)
700 C*****
710 C   SPLIT G(T) INTO G1(T) AND G2(T)
720 C*****
730      DO 200 I=1,4096
740      IF(I.LT.1025)ARRAY1(I)=CMPLX(REAL(ARRAY2(I)),0.0)
750      IF(I.LT.1025)ARRAY2(I)=CMPLX(REAL(ARRAY2(I+1024)),0.0)
760      IF(I.GE.1025)ARRAY1(I)=(0.0,0.0)
770      IF(I.GE.1025)ARRAY2(I)=(0.0,0.0)
780  200 CONTINUE
790 C*****
800 C   TRANSFORM G 'S TO FREQUENCY DOMAIN
810 C*****
820      CALL HARM(ARRAY1,M,INV,S,2,IFERR)
830      CALL HARM(ARRAY2,M,INV,S,2,IFERR)
840      WRITE(10)(ARRAY1(I),I=1,4096)
850      WRITE(11)(ARRAY2(I),I=1,4096)
860 C*****
870 C   NORMALIZE PRESS
880 C*****
890      READ(9)PRESS
900      DO 237 IU=1,4096
910      ARRAY1(IU)=(0.0,0.0)
920      IF(IU.LT.2049)ARRAY1(IU)=CMPLX(PRESS(IU),0.)
930  237 CONTINUE
940      CALL NORM(ARRAY1)
950      DO 242 IO=1,2048
960  242 PRESS(IO)=REAL(ARRAY1(IO))
970      WRITE(6,1)PRESS
980 C*****
990 C   CONVOLVE A(-T) WITH P(T) TO GIVE X(T)
1000 C   FIRST HALF:
1010 C*****
1020      DO 300 I=1,4096
1030      IF(I.LT.1025)ARRAY1(I)=CMPLX(PRESS(I),0.0)
1040      IF(I.LT.1025)ARRAY2(I)=CMPLX(ACC(1025-I),0.0)
1050      IF(I.GT.1024)ARRAY1(I)=(0.0,0.0)
1060      IF(I.GT.1024)ARRAY2(I)=(0.0,0.0)
1070  300 CONTINUE
1080      CALL CONVOL(ARRAY1,ARRAY2)
1090      DO 400 I=1,1024
1100      X(I)=REAL(ARRAY2(I+1024))

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1100      X(I)=REAL(ARRAY2(I+1024))
1110      400 CONTINUE
1120 C*****
1130 C   SECOND HALF:
1140 C*****
1150      DO 500 I=1,4096
1160          IF(I.LT.1025)ARRAY1(I)=CMPLX(PRESS(I+1024),0.0)
1170          IF(I.LT.1025)ARRAY2(I)=CMPLX(ACC(1025-I),0.0)
1180          IF(I.GT.1024)ARRAY1(I)=(0.0,0.0)
1190          IF(I.GT.1024)ARRAY2(I)=(0.0,0.0)
1200      500 CONTINUE
1210      CALL CONVOL(ARRAY1,ARRAY2)
1220      DO 600 I=1,1024
1230          X(I)=X(I)+REAL(ARRAY2(I))
1240      600 CONTINUE
1250      WRITE(12)X
1260      CALL RESET
1270      CALL ORIGIN(0.,5.,0)
1280      DO 700 I=1,4096
1290          IF(I.LT.1025)ARRAY1(I)=CMPLX(X(I),0.0)
1300          IF(I.GT.1024)ARRAY1(I)=(0.0,0.0)
1310      700 CONTINUE
1320 C*****
1330 C   PLOT X(T)
1340 C*****
1350      CALL PLTOUT(ARRAY1,'X(T)',4)
1360 C*****
1370 C   COMPUTE THEORETICAL H(T)
1380 C*****
1390      R=20.0E-3
1400      Z=60.0E-3
1410      A=12.5E-3
1420      C=1500.
1430      TAU=C/A*2.0E-8
1440      R1=((Z/A)**2.+ABS((R/A)-1)**2.）**.5
1450      R2=((Z/A)**2.+ABS((R/A)+1)**2.）**.5
1460      WRITE(6,2)R1,R2
1470      2  FORMAT(3X,' R1 = 'E20.8,' R2 = 'E20.8)
1480      TAU1=R1
1490      IF(R.LT.1.0E-4)R=1.0E-4
1500      IF(A/R.GT.1.)GO TO 800
1510      DO 900 I=1,1024
1520          H(I)=0.
1530          TIME=TAU1+TAU*I
1540          IF(TIME.LE.R2.AND.TIME.GE.R1)
1550              $H(I)=(1./3.14159265)*ARCOS(((TIME)**2.-(Z/A)**2.+(R/A)**2
1560              $,-1.)/(2*(R/A)*((TIME)**2.-(Z/A)**2.）**.5))
1570      900 CONTINUE
1580      GO TO 1000
1590      800 TAU1=Z/A+4*TAU
1600      DO 210 LI=1,1024

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1610      H(LI)=0.
1620      TIME=TAU1+TAU*LI
1630      IF(TIME.GE.Z/A.AND.TIME.LE.R1)H(LI)=1.
1640      IF(TIME.GT.R1.AND.TIME.LE.R2)
1650      $TERM=((TIME)**2.-(Z/A)**2.+(R/A)**2.-
1660      $1.)/(2.*(R/A)*((TIME)**2.-(Z/A)**2.))**.5))
1670      IF(TERM.GT.1.)TERM=1.
1680      IF(TIME.GT.R1.AND.TIME.LE.R2)H(LI)=(1./3.14159265)*ARCOS
1690      $(TERM)
1700      210 CONTINUE
1710      1000 L=0
1720      CALL FINDEX(ACC,PRESS,INDEX)
1730      IF(INDEX.LT.1)GO TO 7777
1740      DO 977 I=1,1024
1750      IF(1025-I-INDEX.LT.1)GO TO 877
1760      H(1025-I)=H(1025-I-INDEX)
1770      GO TO 977
1780      877 H(1025-I)=0.
1790      977 CONTINUE
1800      7777 CONTINUE
1810      WRITE(13)H,L
1820      CALL RESET
1830      CALL ORIGIN(6.,-5.,0)
1840      DO 750 I=1,4096
1850      IF(I.LT.1025)ARRAY1(I)=CMPLX(H(I),0.0)
1860      IF(I.GT.1024)ARRAY1(I)=(0.0,0.0)
1870      C*****
1880      C   PLOT H(T)
1890      C*****
1900      750 CONTINUE
1910      CALL PLTOUT(ARRAY1,'H(T)',4)
1920      CALL RESETN
1930      STOP
1940      END
1950      C*****
1960      C   SUBROUTINE TO PERFORM CONVOLUTION VIA FFT
1970      C*****
1980      SUBROUTINE CONVOL(Y1,Y2)
1990      COMPLEX Y1(4096),Y2(4096)
2000      COMMON M(3),INV(1024),S(1024),IFERR
2010      CALL HARM(Y1,M,INV,S,2,IFERR)
2020      CALL HARM(Y2,M,INV,S,2,IFERR)
2030      DO 100 I=1,4096
2040      Y2(I)=Y1(I)*Y2(I)
2050      100 CONTINUE
2060      CALL HARM(Y2,M,INV,S,-2,IFERR)
2070      RETURN
2080      END
2090      C*****
2100      C   SUBROUTINE TO PLOT COMPLEX TIME HISTORIES
2110      C*****

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2120      SUBROUTINE PLTOUT(A,B,C)
2130      COMPLEX A(4096)
2140      DIMENSION X(2048),Y(2048)
2150      YMAX=0.
2160      YMIN=0.
2170      DO 100 I=1,2048
2180      Y(I)=REAL(A(I))
2190      X(I)=FLOAT(I)
2200      IF(YMAX.LT.Y(I))YMAX=Y(I)
2210      IF(YMIN.GT.Y(I))YMIN=Y(I)
2220      100 CONTINUE
2230      IF(ABS(YMAX).GT.ABS(YMIN))FNORM=ABS(YMAX)
2240      IF(ABS(YMIN).GT.ABS(YMAX))FNORM=ABS(YMIN)
2250      DO 200 I=1,2048
2260      Y(I)=Y(I)/FNORM
2270      200 CONTINUE
2280      YMIN=-1.2
2290      YMAX=1.2
2300      CALL SETUPN(0.,2048.,YMIN,YMAX,5.0,3.0,'TIME',
2310      $B,4,C,5,5,1)
2320      CALL NPLLOT(X,Y,2048,1,0,P)
2330      RETURN
2340      END
2350 C*****
2360 C  SUBROUTINE TO NORMALIZE DATA FOR CONVERGENCE CONDITION
2370 C*****
2380      SUBROUTINE NORM(ARRAY)
2390      COMMON M(3),INV(1024),S(1024),IFERR
2400      COMPLEX ARRAY(4096)
2410      CALL HARM(ARRAY,M,INV,S,2,IFERR)
2420      FMAX=0.
2430      DO 100 I=1,4096
2440      FMAG=ARRAY(I)*CONJG(ARRAY(I))
2450      IF(FMAX.LT.FMAG)FMAX=FMAG
2460      100 CONTINUE
2470      FMAX=SQRT(FMAX)
2480      DO 200 I=1,4096
2490      200 ARRAY(I)=(ARRAY(I)/FMAX)*SQRT(1.9999)
2500      CALL HARM(ARRAY,M,INV,S,-2,IFERR)

2510      WRITE(6,1)FMAX
2520      1 FORMAT(2X,'IN NORM,  FMAX = ',F30.8)
2530      RETURN
2540      END
2550 C*****
2560 C  SUBROUTINE TO FIND STARTING POINT OF H (T)
2570 C*****
2580      SUBROUTINE FINDEX(ACC,PRESS,INDEX)
2590      DIMENSION ACC(1024),PRESS(2048)
2600      AMAX=0.

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2610      PMAX=0.
2620      DO 100 I=1,2048
2630      IF(I.LT.1025.AND.AMAX.LT.ABS(ACC(I)))AMAX=ABS(ACC(I))
2640      IF(PMAX.LT.ABS(PRESS(I)))PMAX=ABS(PRESS(I))
2650 100 CONTINUE
2660      IANDEX=0
2670      IPNDEX=0
2680      AMAX=AMAX/20.
2690      PMAX=PMAX/20.
2700      DO 200 I=1,1024
2710      IF(IANDEX.NE.0) GO TO 250
2720      IF(ABS(ACC(I)).GT.AMAX)IANDEX=I
2730 250 IF(IPNDEX.NE.0) GO TO 200
2740      IF(ABS(PRESS(I)).GT.PMAX)IPNDEX=I
2750 200 CONTINUE
2760      INDEX=IPNDEX-IANDEX
2770      WRITE(6,1)INDEX,IANDEX,IPNDEX
2780 1  FORMAT(3X,'IN FINDEX : INDEX = ',I8,' ACC DELAY = ',I8,'
ESS
2790      $ DELAY = ',I8)
2800      RETURN
2810      END
2820/*
2830//LKED.SYSLIB DD DSN=SYS1.FORTLIB,DISP=SHR
2840//          DD DSN=URI.QDN1.TPLOTLIB,DISP=SHR
2850//          DD DSN=UCC.LC01.SSPLIB,DISP=SHR
2860//GO.FT99F001 DD DSN=URI.LDG1.PLOTFIL1,DISP=OLD
2870//GO.FT08F001 DD DSN=URI.LDG1.EXP.ACC25,DISP=SHR
2880//GO.FT09F001 DD DSN=URI.LDG1.EXP.PRESS25A,DISP=SHR
2890//GO.FT10F001 DD UNIT=DISK,DSN=URI.LDG1.G1.PRESS1,DISP=OLD
2900//GO.FT11F001 DD UNIT=DISK,DSN=URI.LDG1.G2.PRESS1,DISP=OLD
2910//GO.FT12F001 DD UNIT=DISK,DSN=URI.LDG1.XX.PRESS1,DISP=OLD
2920//GO.FT13F001 DD UNIT=DISK,DSN=URI.LDG1.H.PRESS1,DISP=OLD
2930/*
2940//

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10//DECONITR JOB (LDG100,384,1), ' TONY FISHER GSO '
20*ROUTE PRINT REMOTE2
30// EXEC FORTHCLG,REGION=384K,LIB1=TPLOT,LIB2=SSP
40//FORT.SYSIN DD *
50 C
60 C
70 C*****
80 C THIS PROGRAM PERFORMS 5 ITERATIONS OF DECONVOLUTION
90 C VIA SUCCESSIVE APPROXIMATION
100 C*****
110 C
120     DIMENSION HLAST(1024),HNEXT(1024),X(1024),CHANGE(1024)
130     COMMON M(3),INV(1024),S(1024),IFERR
140     COMPLEX G(4096),ARRAY(4096),HFREQ(4096)
150     EQUIVALENCE(CHANGE(1024),INV(1024)),(X(1024),S(1024))
160     DO 1000 III =1,5
170 C*****
180 C  INITIALIZATION SECTION
190 C*****
200     READ(8)HLAST,ITER
210     WRITE(9)HLAST,ITER
220     WRITE(6,5)ITER
230     5  FORMAT(2X,'ITER = ',I10)
240     M(1)=12
250     M(2)=0
260     M(3)=0
270     DO 50 I=1,4096
280     G(I)=(0.0,0.0)
290     50  CONTINUE
300     CALL HARM(G,M,INV,S,0,IFERR)
310 C*****
320 C  CONVOLVE G(T) WITH LAST H(T) IN TWO PARTS
330 C  FIRST HALF;
340 C*****
350     READ(10) G
360     DO 100 I=1,4096
370     IF(I.LT.1025)HFREQ(I)=CMPLX(HLAST(I),0.0)
380     IF(I.GT.1024)HFREQ(I)=(0.0,0.0)
390     100 CONTINUE
400     909 FORMAT(2X,'CHECKPOINT')
410     CALL HARM(HFREQ,M,INV,S,2,IFERR)
420     DO 200 I=1,4096
430     ARRAY(I)=HFREQ(I)*G(I)
440     200 CONTINUE
450     CALL HARM(ARRAY,M,INV,S,-2,IFERR)
460 C*****
470 C  SECOND HALF;
480 C*****
490     READ(11) G
500     DO 300 I=1,4096
510     IF(I.LT.1025)HNEXT(I)=REAL(ARRAY(I+1024))
520     ARRAY(I)=HFREQ(I)*G(I)
530     300 CONTINUE
540     WRITE(6,909)
550     CALL HARM(ARRAY,M,INV,S,-2,IFERR)

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560      READ(12) X
570      DUMMY=REAL (ARRAY(500))
580      WRITE(6,77)X(500),HNEXT(500),DUMMY
590      77 FORMAT(2X,'X(T)=' ,E10.4,'H1='E10.4,'H2=' ,E10.4)
600      DO 400 I=1,1024
610 C*****
620 C   COMPUTE LATEST CORRECTION
630 C*****
640      CHANGE(I)=X(I)-HNEXT(I)-REAL (ARRAY(I))
650      HNEXT(I)=HLAST(I)+CHANGE(I)
660      400 CONTINUE
670      ITER=ITER+1
680      WRITE(6,5)ITER
690 C*****
700 C   SAV LATEST ESTIMATE, PLOT RESULTS
710 C*****
720      REWIND 8
730      WRITE(8)HNEXT,ITER
740      YMAX=0
750      YMIN=0
760      DO 500 I=1,1024
770      IF(YMAX.LT.HNEXT(I))YMAX=HNEXT(I)
780      IF(YMIN.GT.HNEXT(I))YMIN=HNEXT(I)
790      500 CONTINUE
800      IF(ABS(YMAX).GT.ABS(YMIN))FNORM=ABS(YMAX)
810      IF(ABS(YMIN).GE.ABS(YMAX))FNORM=ABS(YMIN)
820      DO 600 I=1,1024
830      HNEXT(I)=HNEXT(I)/FNORM
840      X(I)=FLOAT(I)
850      600 CONTINUE
860      WRITE(6,10)FNORM
870      10 FORMAT(2X,'MAXIMUM MAGNITUDE OF HNEXT = ' ,E20.8)
880      REWIND 99
890      CALL SETUPN(0.,1024.,-1.2,1.2,5.0,3.0,'TAU',
900      $' H (T), ITERATION # ',3,19,5,5,1)
910      CALL NPLT(X,HNEXT,1024,1,0,P)
920      CALL SLEW(0.,0.)
930      CALL RESET
940      CALL NUMPLT(-.25,3.3,90.,.125,FLOAT(ITER),0)
950      CALL ORIGIN(0.,5.,0)
960      YMAX=0.
970      YMIN=0.
980      DO 700 I=1,1024
990      IF(YMAX.LT.CHANGE(I))YMAX=CHANGE(I)
1000     IF(YMIN.GT.CHANGE(I))YMIN=CHANGE(I)
1010     700 CONTINUE
1020     FNORM=YMAX
1030     IF(YMAX.LT.ABS(YMIN))FNORM=YMIN
1040     WRITE(6,110)FNORM
1050     FNORM=ABS(FNORM)
1060     110 FORMAT(2X,'MAXIMUM MAGNITUDE OF CORRECTION = ' ,E20.8)
1070     DO 800 I=1,1024
1080     800 CHANGE(I)=CHANGE(I)/FNORM
1090     CALL SETUPN(0.,1024.,-1.2,1.2,5.0,3.0,'TAU',
1100     $' CORRECTION',3,10,5,5,1)

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```
1110      CALL NPLOT(X,CHANGE,1024,1,0,P)
1120      CALL ORIGIN(6.,-5.2,0)
1130      CALL RESETN
1140      REWIND 8
1150      REWIND 9
1160      REWIND 10
1170      REWIND 11
1180      REWIND 12
1190 1000 CONTINUE
1200      STOP
1210      END
1220/*
1230//LKED.SYSLIB DD DSN=SYS1.FORTLIB,DISP=SHR
1240//           DD DSN=URI.QDN1.TPLOTLIB,DISP=SHR
1250//           DD DSN=UCC.LC01.SSPLIB,DISP=SHR
1260//GO.FT99F001 DD UNIT=SYSDA,DSN=URI.LDG1.PLOTFIL1,DISP=OLD
1270//GO.FT08F001 DD UNIT=DISK,DSN=URI.LDG1.HNEXT1,DISP=OLD
1280//GO.FT09F001 DD UNIT=DISK,DSN=URI.LDG1.HLAST1,DISP=OLD
1290//GO.FT10F001 DD UNIT=DISK,DSN=URI.LDG1.G1.PRESS1,DISP=SHR
1300//GO.FT11F001 DD UNIT=DISK,DSN=URI.LDG1.G2.PRESS1,DISP=SHR
1310//GO.FT12F001 DD UNIT=DISK,DSN=URI.LDG1.XX.PRESS1,DISP=SHR
1320/*
1330//
```