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Nonlinear Finite Element Modeling of Cellular Materials Under Dynamic Loading and Comparison to Experiments

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**NONLINEAR FINITE ELEMENT MODELING OF CELLULAR MATERIALS
UNDER DYNAMIC LOADING AND COMPARISON TO EXPERIMENTS**

BY

COLIN J. MURPHY

**A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE**

IN

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MASTER OF SCIENCE IN MECHANICAL ENGINEERING AND APPLIED
MECHANICS THESIS
OF
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ABSTRACT

This study used an open source three-dimensional Voronoi cell software library to create nonlinear finite element models of open cell metal foams in the 5% to 10% relative density range. Cubic and Body-Centered Cubic (BCC) seed point generation techniques were compared. The impact of random positional perturbations on original seed points was investigated as it relates to material stiffness and yield strength. The models simulated a 10-cell cube of foam material under uniaxial loading at strain rates of around $10^2/\text{s}$ up to about 80% compressive strain. It was shown that the models created with BCC seed points generally had a higher modulus which was less sensitive to perturbations in seed point location. The models were compared to drop-weight experiments on ERG Duocel metal foams of 10, 20, and 40 Pores Per Inch (PPI) which were filmed with a high speed camera. The models showed good agreement with analytical predictions for material properties, but a comparison with experimental data indicated that they lost accuracy in simulating material response after 50% compressive strain. Past this point, cell-wall contact within the foam was a dominant mechanism in the mechanical response, and model predictions did not appear to match well with experimental data.

In a parallel experimental effort ERG foams of 10 PPI and around 8% relative density were subjected to tensile loading at a strain rate of $73/\text{s}$. High speed photography was again used to interpret the data. The Young's modulus and yield strength of these foams were shown to increase by a factor of ten as compared to quasistatic values, indicating significant rate dependence.

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CHAPTER 1

INTRODUCTION

Cellular materials are made up of repeating internal geometry with interconnected faces or edges and are commonly found in everyday life. Examples are seen in natural materials such as wood, cork, and bone; and in manmade materials such as polymer and metal foams, see Figure 1-1. Cells can be entirely sealed from their neighbors (closed cells) or have faces with no material in between (open cells). Periodic cellular materials have repeating cells which are shaped similarly and aligned regularly, while stochastic materials are those with a more random distribution of cellular shapes and sizes.

Figure 1-1: Examples of cellular materials from Sadd (2013).

Metal foams are a subset of cellular materials. They can be developed to have favorable stiffness-to-weight and strength-to-weight ratios which makes them good candidates for structural applications which demand light components. Figures 1-2 and 1-3 show logarithmic plots of Young's modulus as a function of density and compressive strength as a function of density for various materials. High ratios of strength to density are found towards the upper left regions of the plots. Cellular materials, including polymer and metal foams encompass a wide range in the plot; indicating their broad applicability.

Figure 1-2: Compressive stiffness for various materials by Wadley (2014)

Figure 1-3: Compressive strength for various materials by Wadley (2014).

Electrodes, fuel cells, heat exchangers, and filters all make use of the high surface area and controllable porosity available in metal foams. Due to the large strain that these materials undergo during plastic deformation they can limit the maximum force applied during impact (high-rate) loads, while absorbing large amounts of energy. In fact, according to Gibson and Ashby (1997), a foam will always absorb more energy during impact than a solid made of the same material given the same peak stress. This particular quality warrants their use in protective applications from car bumpers to railway cars.

Figure 1-4 shows a typical curve of uniaxial engineering stress as a function of strain for a foam material. The area under the large, flat, plastic yielding plateau represents energy absorption by the material up to around 80% compressive strain. Figure 1-5 shows a similar curve with a comparison to a solid made of the same material.

Figure 1-4: Typical stress-strain curve for foams by Gibson and Ashby (1997).

Figure 1-5: Peak stress comparison of foams to solids by Gibson and Ashby (1997).

With the introduction of these plots, there is some terminology which would be helpful to define. The comprehensive text on the subject by Gibson and Ashby (1997) describes the properties of cellular materials in detail and sets forth definitions that are commonly used throughout the body of research on the subject.

One of the most important parameters in relating cellular material properties is the *relative density* which is the ratio of the cellular material's density to the density of the solid from which it is made:

$$\rho_{rel} = \frac{\rho_f}{\rho_s} \quad (1-1)$$

where $0 < \rho_{rel} < 1$. This gives rise to other relative parameters such as the relative Young's modulus:

$$E_{rel} = \frac{E_f}{E_s} \quad (1-2)$$

And the relative yield strength:

$$\sigma_{yr} = \frac{\sigma_{yf}}{\sigma_{ys}} \quad (1-3)$$

which are also defined as ratios of the bulk cellular material properties to the properties of the base material.

Similarly, since there are different stress and strain measures for materials which undergo finite deformation, it is helpful to state that stress and strain in these plots are generally reported in the literature as the engineering stress:

$$\sigma = \frac{F}{A_0} \quad (1-4)$$

which is a ratio of the applied force to the original area of the foam; and the engineering strain:

$$\varepsilon = \frac{\Delta L}{L_0} \quad (1-5)$$

which is a ratio of the change in length of the foam to the original length. For the remainder of this work, these will simply be referred to solely as *stress* and *strain*, respectively.

In addition to these material parameters, several other properties of these plots stand out. From left to right in Figure 1-4, it is noted that there are three distinct regimes in which these foams behave: an elastic regime, in which stress as a function of strain has a positive, nearly linear slope; a plastic yielding regime, in which stress is mostly flat as strain increases; and a densification regime, in which a steep rise in stress is noted as strain increases. The presence or absence of these regimes as well as their description in relation to material properties will serve as guides in matching material models to real world behavior.

Open-cell metal foams are made up of small metal ligaments. An example of these foams is shown in Figure 1-6. Bubbles form in a molten metal mix which is then cooled and hardened in to a porous media in which the ligaments form the new cell boundaries.

Figure 1-6: A sample of Duocel ® open-cell aluminum foam.

The mathematical description of this process involves a random seeding of points in a medium with subsequent nucleation and growth of bubbles about these points. The boundary of each bubble encloses a volume in which all locations are closer to the bubble's original seed point than to any other. In two dimensions this forms what is known mathematically as a Voronoi honeycomb. In three dimensions it is a Voronoi foam. The terms "Voronoi lattice" and "Voronoi diagram" will also be used interchangeably to describe this arrangement. Okabe et al. (1992) describes the myriad applications of the Voronoi diagram including the description of particles in

granular flow problems, path planning algorithms in robotics, description of atomic arrangements in materials, nearest neighbor search queries for databases, and more.

Figure 1-7: A two-dimensional Voronoi honeycomb created with MATLAB.

As will be shown, much of the work using finite elements to model open-cell foams has involved using two-dimensional models with a mesh that would look like Figure 1-7. However, models in two dimensions do not fully capture the full spatial deformation of the materials. Moreover, it is not feasible to use these models to analyze out-of-plane anisotropy. Thus, a method was sought in order to create three-dimensional Voronoi lattices in order to explore anisotropic phenomena. A recently created software, Voro++, by Rycroft (2009, 2014) has enabled the creation of the aforementioned lattices which can easily be prescribed within certain shapes. This has been a challenge in the field of computational geometry. Figure 1-8 shows a Voronoi

lattice in the shape of a cube. The work in this thesis will use the Voronoi lattices generated by the Voro++ software.

Figure 1-8: A three-dimensional Voronoi lattice created in Voro++ by Rycroft (2014).

LS-DYNA is a Finite Element Modeling (FEM) solver which can predict material behavior under extreme loading conditions. The software is described by the Livermore Software Technology Corporation and Hallquist (2006) as being especially made for dynamic loading applications in which materials plastically deform. It is heavily used in the defense, aerospace and automotive industries for the evaluation of structural response to dynamic loading. The solver offers many different element types which can delineate between material deformation and rigid body motion. It also offers a wide variety of constitutive material models with nonlinear properties. Both the capacity to solve nonlinear problems and the ability to handle large deformations were needed for this work, and thus LS-DYNA was selected for the computer simulation.

With increasing emphasis on efficiency and cost reductions, many impact-absorption applications seek to reduce weight while maintaining or increasing energy absorption during an impact event. Cellular materials, in particular open-cell metal foams, offer advantages in this area and their response to dynamic loading has long been studied. Recent mass-availability of 3D printing and surging interest in advancing this technology have renewed attention to understanding the mechanical response of these metal foams. The prospect of designing and building a material which will increase energy absorption effectiveness in custom applications necessitates in-depth understanding of the mechanisms by which these materials deform.

This thesis has developed a highly adaptable and computationally efficient finite element model which was used to explore the relation between foam parameters and mechanical response in both linear (elastic) and nonlinear (elastic-plastic) load regimes while under high-rate (dynamic) loading. This was accomplished by tying in the three dimensional Voronoi lattices created in Voro++ with the LS-DYNA solver which simulated large deformations and rate effects. In this way cellular porosity, anisotropy, and shape/size distributions of open-cell foam material make-up was controlled, observed, and altered. Drop-weight experiments, in which the open-cell metal foams were dynamically loaded in compression and tension, were also performed and photographed with a high speed camera. Data from the computational material models were then compared with the experiments, previously published

research, and manufacturer's specifications. This allowed for an evaluation of model accuracy as well as for conclusions to be drawn as to how the aforementioned properties affect dynamic material response.

With an understanding of the previous research on cellular materials it is evident that a three-dimensional foam finite element model would add to the body of research. Additionally, the absence of research on the response of open-cell foams to tensile loading leads to interesting questions about such behavior. This work sought a better understanding and exploration of the means by which open-cell foams deform both in compression and tension. This adaptable model could then facilitate targeted material design and creation.

CHAPTER 2

REVIEW OF LITERATURE

The mechanical behavior of metal foams has been well studied. Since initial patents for methods to produce a “sponge metal” by Sosnik (1948) and “metal foam” by Elliot (1951) these materials have been recognized as having unique properties. According to ERG Aerospace Corporation (2014) open-cell metal foams were mostly limited to military use until after the Cold War in the mid-1990s. Thus, the majority of early work on metal foams was focused mainly on closed-cell materials. Since the main definitions and many of the parameters still apply to open-cell materials, this early research is still a good place to begin.

Initial studies by Thornton and Magee (1975) of closed-cell aluminum foams in the 5-18% relative density range at strain rates of $8 \times 10^{-3}/s$ observe that a “greater than linear” increase in yield strength occurs with increase in density. This leads them to conclude that bending stresses within the foams are important mechanisms of plastic collapse. They report an energy-absorbing efficiency parameter which is given by:

$$P = \frac{\int_0^l F dl}{F_{max} L} \quad (2-1)$$

where F is the instantaneous force which is then integrated over the distance of the deformation, F_{max} is the maximum force during the deformation, and L is the total

length of the deformed sample. Further, they concluded from their analysis that a more regular cell structure would improve energy absorption. This work was also reported by Davies and Zhen (1983) along with the description of manufacturing techniques to create various types of foams in four different categories: casting, metallic deposition, powder metallurgy, and sputter deposition. Along with methods of fabrication they discuss the benefits and drawbacks of metallic foams in various applications. The work concluded that impact absorption would be the application in which metal foams have the greatest possibility for use.

The work by Ashby (1983) went in to great detail to describe the elastic, plastic, creep, and fracture properties of cellular solids (mostly polymeric foams, but Thornton and Magee's work is included as well as some ceramic foams). Many of the original figures and results from the experimental/analytical work are seen in the second edition of the textbook, by Gibson and Ashby (1997). Although primarily regarding nonmetallic foams, Ashby's work sets forth a number of different definitions and parameter relations which are used throughout the body of research on the subject. Relative density and relative yield strength are two such parameters.

Gibson and Ashby then modeled an open-cell foam as the cubic array of struts shown in Figure 2-1. Dimensional arguments were used to generalize the cell properties without regard to specific cell geometry. The dimensional arguments combined the cell size, l ; the ligament thickness, t ; with Timoshenko beam theory to find the deflection, δ , as a function of F .

Figure 2-1: Cubic array used by Gibson and Ashby (1997) to model open-cell foams.

This was then used to develop scaling relations for open-cell foams which relate the relative density to the relative stiffness and relative yield strengths shown in Equations 2-2 and 2-3.

$$\frac{E_f}{E_s} \approx C_1 \left(\frac{\rho_f}{\rho_s} \right)^2 \quad (2-2)$$

$$\frac{\sigma_{yf}}{\sigma_{ys}} \approx C_2 \left(\frac{\rho_f}{\rho_s} \right)^{\frac{3}{2}} \quad (2-3)$$

where E_f is the Young's modulus of the foam and E_s is the Young's modulus of the solid from which it is made; σ_{yf} and σ_{ys} are the yield strengths of the foam and of the

solid; and C_1 and C_2 are constants. The equations were then fit to experimental data to and it was found that the constants C_1 and C_2 were 1.0 and 0.3, respectively.

Further work by Gibson (1989) discussed analytically modelling the elastic behavior of these materials with respect to packaging design and sandwiched panels. Properties for many different foams and cellular materials were given with relation to cell geometry and cell wall materials with the intent to provide an engineering design guide for cellular material selection. The models were fairly simplistic. They used bending and yielding stresses based on beam theory combined with periodic unit cell models to estimate and shed insight into the elastic properties of many different cellular materials, with an emphasis on metal foams. Young's modulus, energy absorption, and fracture toughness plots were given as functions of relative density and strain rate.

In the first work to use Voronoi diagrams in two dimensions, Silva, Hayes and Gibson (1995) diverged from previous studies using periodic unit cells and analytical calculations with the intent to better account for microstructure variability. In order to create a finite element model of a foam, they created a software program which would randomly seed a given area with points. Each point was added to the area and if it was more than a given distance away from any other point, it would be accepted. Once no more points could be added to the area without violating this rule, the Voronoi honeycomb in Figure 2-2 was created using the points as origins for nucleation and linear bubble growth.

Figure 2-2: The Voronoi honeycomb generated by Silva Hayes and Gibson (1995).

A linear-elastic Finite Element Analysis (FEA) was performed with each ligament of the two-dimensional honeycomb discretized using a three-node beam element which accounted for bending, shear, and axial displacements. Studies were done with uniaxial loading, biaxial loading, and anisotropy applied to the material by scaling cell-lengths in one direction of the material by a random factor from 1.0-2.0. Relative densities from 0.1-0.3 were analyzed at low strain rates. FEA results for the relative moduli (both shear and Young's) and Poisson's ratio as functions of the relative density were compared to a closed-form solution. The results tended to agree with previous studies, using hexagonal cells, by Kraynik, Reinelt, and Princen (1991) as well as with previous unit cell studies by the work's authors. The two main conclusions from the FEA work were that variations in cell wall arrangement contributed only minimally (4-9%) to changes in elastic constants for the microstructurally variable materials; and that the elastic constants were also similar to regular, periodic honeycombs.

Three-dimensional foam models started appearing with a study by Roberts and Garboczi (2002) which used a three-dimensional finite element model to study the dependence of Young's modulus on relative density. At the time the work was done, many theoretical models were using periodic cellular structures. Their research noted that a gap existed between the understanding of these periodic models and the behavior of real world cellular solids which have more random makeups. They used Voronoi tessellations similar to the present study; models in which layers of the material have a Gaussian density distribution; and nearest neighbor node-bonded foams. They found that Young's modulus was proportional to the density of the foam to the power of n , where n is between 1.3 and 3.0.

Additional three-dimensional models using the finite element method were created by Gan, Chen, and Shen (2005). Their work developed a three-dimensional Voronoi finite element model which was then used to explore the mechanical response of open-cell polymeric foams in the 1 to 10% relative density range. However, the work focused only on the linear elastic response of the foams and thus diverged from the present study. The Voronoi lattices were compared with a more-regular Kelvin formulation proposed by Warren and Kraynik (1997) which was comprised of regular tetrakaidecahedra (14-sided cells in a BCC lattice arrangement). The model by Gan, Chen, and Shen model consisted of analyzing the response of $N \times N \times N$ Voronoi cell unit blocks ($N = 3, 4, 5$, and 6) and then adding them together to create a "super cell" model of ultimate dimensions $3N \times 3N \times 3N$ cells. The calculated relative Young's

moduli results for $N = 3$ and $N = 5$ models differed by less than 10%. They found that the elastic constants predicted by Voronoi and Kelvin foams for low density open-cell foams were in close agreement, but diverged as relative density increased. In addition, they concluded that Young's modulus of Voronoi foams was sensitive to imperfections in the foam lattice, while the plateau stress of the foams was not.

More recent work has focused on using greater cellular variations in two-dimensional finite element models of Voronoi tessellations to further explore microstructural effects. Alkhader and Vural (2006, 2008) used models of this type to link cellular microstructure with material response in both the quasistatic and dynamic load regimes. Their work focused on characterizing what role specimen size, boundary morphology, cellular topology, and microstructural irregularity have on the mechanical response of two-dimensional foams. These studies looked at similar methods of generating nucleation points for the Voronoi tessellation as the aforementioned study by Silva et al. It also used a second method which began with a point grid and then added random perturbations to the points in the grid (this method is ultimately chosen by the current work). Between 2 and 7 Timoshenko beam elements were used to discretize each ligament in the foam models which had relative densities in the range of 15%. In the quasistatic study, a displacement boundary condition was set at the top of the specimen, enforcing a $1/s$ strain rate. The dynamic study by Alkhader and Vural set up the FE model similarly, but with a strain rate of $1000/s$. The models looked at several sample sizes (in numbers of cells), shapes (pure Voronoi,

tetragonal, hexagonal), and perturbations to the nucleation points in the foam. The authors made several conclusions with direct applicability to the work herein. First, they found that the response of the models was most accurately modeled at a size of at least 10 x 10 cells. Next, they concluded that the arrangement of cells around the boundaries had little effect on compressive response, but boundary conditions did have a significant effect. Lastly, with respect to microstructure, they found that a loss of periodicity can contribute to bending-dominated structural response which, in turn, leads to slight decrease in macroscopic stiffness. These conclusions hold true for both the quasistatic and dynamic loading scenarios. A unique conclusion for the dynamic load scenario was that the simulations suggested a strong sensitivity to load rates, with particular emphasis on the rise time of initial loading. They suggest that more study should be tuned to investigating appropriate rise times in dynamic simulations of cellular materials.

The idea of “functionally graded materials” for enhanced properties was explored by Ajdari et al. (2009) in the quasistatic loading case and again by Ajdari et al. (2011) to look at tailored microstructure’s effects on dynamic crushing properties. The 2009 study created a finite element model with a two-dimensional Voronoi honeycomb in a similar way to the previously mentioned studies: using beam elements in SIMULIA (formerly ABAQUS). Between 2 and 6 elements were used for each ligament. Beam materials were linear elastic and elastic-perfectly plastic for the elastic and elastic-plastic load regimes, respectively; with constants which mimicked

properties of aluminum. The authors diverged from previous studies by simulating a material with a relative density that varied spatially from 10% increasing along the loading direction to a peak density of 53% at the surface to which the load was applied. Results were compared to a sample with 10% constant density. The work concluded that the increasing density gradient increased both the material's stiffness and yield stress, but had a more pronounced effect on the former. The 2011 Adjari work used regular, periodic honeycomb models with similar computational inputs and explored dynamic scenarios. Although the exact strain rates imposed in the research were unclear, the research found that a decreasing gradient in the load direction increased the energy absorption in early stages of crushing.

Jones (2011) studied the effect that topology has on the stress intensification around a circular hole. The work used a two-dimensional Voronoi honeycomb to explore these effects and found that reducing the hole size resulted in a reduction of the stress amplification at the corners, which is the opposite effect found in solid materials. Additionally, the research found that by optimizing relative density distribution to align with areas in the material in which high stresses were present a global reduction in material stresses could be achieved. Breault (2012) expanded on this finding by aligning the topology (struts and ligaments) of a cellular material with stress trajectories found in the continuum material. The Breault research found that alignment of the struts according to stress trajectories lowered both maximum and average stresses found in the material.

Although many efforts to model open-cell foams have involved finite element models of semi-randomly generated networks of beam elements, some research has actually tried to exactly match specific samples and then model them. One such example is from Liebscher et al. (2012) who used X-ray computed tomography to scan a material sample. This data was then used to create a finite element model consisting of Timoshenko beams to predict material response to vibrations.

An experimental work by Tan, Reid, and Harrigan (2012) examined how the structural response of cell walls in open-cell metal foams is affected by the anisotropy of cell shape. They further investigated how this alteration of cell shape would affect crush patterns and the mechanical properties while the materials are subjected to dynamic loads. The experiments looked at open-cell aluminum foams of 6106-T6 Aluminum in the 9-10% relative density range with porosities of 10 and 40 pores-per-inch. The foams were compressed at strain rates from 400/s to 4,000/s. The deformation and crush patterns in the materials are shown to be functions of the load rate. These patterns fit into regimes which are delineated by critical impact velocities at which the deformation patterns are observed to change (the border between the “sub-critical” and “super-critical” load regimes is around 2300/s).

The work concludes that in the sub-critical regime, translational and rotational inertia of the cell ligaments contribute to strength enhancement and the plastic collapse strength increases linearly with load rate. However, in the super-critical load regime, the plastic collapse strength is a function of the square of the impact velocity.

They conclude that this change is based on shock propagation effects. This cell ligament motion is termed “microinertia”. The experiments are compared with previously conducted two-dimensional finite element simulations. Notably, the work investigates the effects of the anisotropy of the foam cells. Samples were observed to have cells which were slightly oblong. Strength and plateau stress were shown to decrease when the load was aligned perpendicular to the longer axis of the cells.

Finite element studies which have analyzed three-dimensional Voronoi lattices under dynamic loads are rare, this is most likely due to the complexity in creating a three-dimensional Voronoi lattice as well as the large computational requirements in analyzing it. Very recent work by Gaitanaros, Kyriakides, and Kraynik (2012), and Gaitanaros and Kyriakides (2013), have studied this behavior. Their research closely parallels the work herein in its use of the LS-DYNA numerical solver and a lattice which departs from previously studies in that varying cell size is accounted for. The 2012 work by Gaitanaros, et al. creates a soap froth using a software created by called Surface Evolver. The creator, Brakke (1992) describes it as a software which can model the shape that liquid surfaces take in response to various forces. The froth is reduced to a set of ligaments through an algorithm which minimizes the surface energy of the bubbles in the froth (similarly to how real open-cell foams form). This gives the model the ability to vary cross-section diameter along ligament length. Each ligament is discretized in great detail by 7-9 beam Hughes-Liu (1981) beam elements, depending on the length of the ligament. The Hughes-Liu beam element accounts for

finite deformations, rotations, and shear and has the capability to account for and apply friction to contact between beam elements. The authors looked at the relationship between the model size (in cell dimensions) and convergence. The models were shown to converge at around 1,000 cells (a 10 x 10 x 10-cell cube) which is in general agreement with previous two-dimensional finite element models.

Foams in the 8% density range were first modeled under quasistatic conditions in the Gaitanaros, Kyriakides, and Kraynik's 2012 study and then under dynamic conditions in Gaitanaros and Kyriakides's 2013 study. Strain rates in each study were 0.1/s and 800-8000/s, respectively. Models were shown to exhibit many of the same physical phenomena (crush zones, densification, and large plastic deformation) as real foams.

The current study seeks a more flexible tool with which to model open-cell foams. The forces of nature (surface forces in molten aluminum slurries) currently drive the ability to control the porosity and microstructure of these foams. However, with the advances in additive manufacturing it is easy to imagine a future in which these foams can be designed and then formed in ways which would not be possible with present manufacturing technologies. Cells and ligaments can take more varied shapes; their sizes and anisotropy can be controlled. An open-cell foam could conceivably be designed and built ligament by ligament with a specific purpose in mind. In order to do this, foam models will need to be more easily adaptable and controllable. The present work seeks to build upon this concept.

Existing methods of mass metal foam fabrication have not been able to alter the distribution of the bubbles (voids) across the material, but limited open-cell metal foam structures have already been made and tested. Cheng, et al (2012) used electron beam melting in order to fabricate open-cell titanium foams intended for biomedical applications. The foams were reported to have strengths and elastic moduli close to those of several types of human bone. Within Technologies (2015), a design and additive manufacturing company, sells software to design small volumes of functionally graded materials for very specialized, statically-loaded applications. Figure 2-3 shows some examples which have been created by these methods.

Figure 2-3: Several cellular material types created by Within Technologies (2015)

There have been many studies of open-cell aluminum foams subjected to compressive loads due to the interest in their long load plateau which is useful in energy absorption. Tensile response has been less well studied. Andrews, Sanders, and Gibson (1998) originally looked at both the uniaxial compressive and tensile response of open-cell metal foams. Their work studied the response of ERG Duocel foams of

8% relative density and 10 PPI in the .0002/s strain rate regime. They published the Young's modulus and yield strength of the foams to be .502 GPa and 1.93 MPa, respectively. They compared the response of open-cell foams to an analytical model and found good agreement. In contrast, they found that closed-cell foams were not well described by this model which they attributed to imperfections in the metal which made up the cell walls. Related tensile work was performed by Olurin, Fleck, and Ashby (2000) on closed-cell foams in the 8-15% relative density range at strain rates of .00017/s and found that the foams exhibited strain hardening and failed in the range of 1-3% strain. Scatter in their sample response data lead them to conclusions which generally agreed with those of Andrews et al in that the samples had heterogeneous microstructure and imperfections.

Tensile work moved to studying fracture mechanics. Andrews and Gibson (2001) studied the effects of "crack-like defects" in open-cell metal foams. Their work with ERG Duocel open-cell aluminum foam of 40PPI and relative density 7-8% aimed to explore how these cracks impacted the tensile strength of the foams at strain rates of .0001/s. They found notch strengthening with increasing crack length and that peak stress in the foams increased with increasing notch size. The strengthening effects of the notches diminished as the ratio of specimen size to notch size increased. Their work concluded with equations to predict the effects of notch strengthening.

Work by Amsterdam et al (2008) looked at how anisotropy in open-cell foam cell shapes correlated to material response. Of primary interest was how the struts and

ligaments bend and reorient themselves when foams are subjected to an applied strain. The ERG Duocel open-cell Aluminum foam of 20PPI and relative density 3-13% were Electro-Discharge Machined (EDM) and strained at .0019/s strain rate. They found that strain hardening in the foams was unaffected by cell orientation and that a higher peak strain is obtained in the foams when the long axis of the sample was oriented transverse to the loading axis. They attributed this to an increase in the contribution of strut bending to the foam mechanical response.

CHAPTER 3

FINITE ELEMENT MODELING

3.1 Preliminaries

A nonlinear finite element model of an open-cell foam was created. The model was used to explore the effect of microstructure changes on the compressive elastic-plastic response of the foam at strain rates $\sim 143/\text{s}$. The general method included creating a seed point cloud, using the point cloud to produce a Voronoi lattice, discretizing the lattice with finite beam elements to form the model, and running explicit simulations to measure the properties of the model foam. The method is described pictorially in Figure 3-1. The following sections describe the finite element modeling procedure in detail.

Figure 3-1: General procedure for creating the finite element models

3.2 Seed Point Generation, Pseudo-Randomization, and Voronoi Lattice Computation

Two methods for creating the Voronoi seed points were explored: the “Cubic” method, and the “Body-Centered Cubic” or BCC method. Each of these methods were taken from natural crystallographic atomic packing arrangements. Attempts were made to produce Face-Centered Cubic (FCC) and Hexagonal Close-Packed (HCP) seed points, but each resulted in dense packing which produced too many elements for the element-limited version of LS-DYNA which was obtained.

With each method, perturbations were added to the original seed point location in order to simulate the apparent randomness of bubbles which would form in a metallic mixture. Each seed point represents a bubble which will ultimately become a cell in the foam material model. Both seed point generation methods created an $N \times N \times N$ cell point cloud. Each seed point was then perturbed in the x, y, and z directions by adding a random coordinate addition, f , upon which a limit was set. Both x and y coordinate additions were set to be equal ($f_x = f_y = f_{xy}$):

$$x_f = x_i + f_{xy} \quad (3-1)$$

$$y_f = y_i + f_{xy} \quad (3-2)$$

$$y_f = y_i + f_{xy} \quad (3-3)$$

in which x_i, y_i, z_i are the x,y,z coordinates of the original seed points created in both the cubic and BCC lattices. The value for f was chosen to be from zero to one full cell's width in the cubic seed point generation technique and from 0 to one-half cell's width in the BCC seed point generation technique. Due to the half-cell spacing of the BCC points, they could not be moved one full cell as that would shift the seed points to a new position. Figures 3-2 through 3-5 show seed point clouds for some specific cases.

Figure 3-2 illustrates a cubic seed point cloud with no perturbations to the original seed point locations. Dimensions in the plots are given as cells. The rows, columns, and pages are ordered and regular. This is shown moving left to right in a three-dimensional view and views as an observer looking down the x and y axes.

Figure 3-2: Cubic seed points with randomness parameters $f_{xy}=f_z=0$.

Figure 3-3 is the same point cloud with the random perturbations increased to a maximum of one full cell's width. The seed points become more disordered and it is difficult to find any regularity in the points.

Figure 3-3: Cubic seed points with randomness parameters $f_{xy}=f_z=1$.

Figure 3-4 illustrates a BCC seed point cloud with no perturbations to the original seed point locations. Again, the rows, columns, and pages are ordered and regularly align with the positions in a BCC lattice. A perturbation with a maximum of one-half cell's with was applied to the same seed point cloud which is shown in Figure 3-5.

Figure 3-4: BCC seed points with randomness parameters $f_{xy}=f_z=0$.

Figure 3-5: BCC seed points with randomness parameters $f_{xy}=f_z=0.5$.

Seed point clouds were then input into Voro++ which created the Voronoi lattices. Voro++ outputs coordinates for the vertex locations of the struts which encapsulate each Voronoi cell. Figures 3-6 and 3-7 show the resulting external faces of both the cubic and BCC Voronoi lattices as seen by an observer looking down the x axis. The disorder in seed point locations manifests as a variation in cell size and shape. Lattices with lesser seed point perturbation are shown on the left while those with greater perturbation are shown on the right.

Figure 3-6: X-faces of the Voronoi lattices created with the Cubic seed point generation technique ($f_{xy}=f_z=0$ on the left, $f_{xy}=f_z=1$ on the right).

Figure 3-7: X-faces of the Voronoi lattices created with the BCC seed point generation technique ($f_{xy}=f_z=0$ on the left, $f_{xy}=f_z=.5$ on the right).

In the final model, vertex locations became node points and the struts were modeled by single beam elements. Custom object-oriented software (K_FILE_CREATE) was written in MATLAB in order to translate these coordinates in to a “keyword” file format. The keyword file is an input deck which is read by LS-DYNA and used to create the finite element model. All MATLAB software created for this research can be found in Appendix A. An abbreviated LS-DYNA keyword file can be found in Appendix B.

K_FILE_CREATE read the output of Voro++ and used that information to determine node and element locations. Up until this point, all coordinates were in the units of cells and the model was a 10 cell cube. The coordinates were multiplied by a scale factor in order to convert them to a real length. The models were created to approximate a 10 Pore Per Inch (PPI) open-cell foam. From ERG Duocel product literature, 10 PPI = 7 cells per inch. Therefore, 10 PPI x 1 inch/7cells = 1.4 inches.

Since the models were in units of meters (m), kilograms (kg), seconds (s) this number was multiplied by .0254 to equate the 10 cell cube to a model cube which was .036m on a side. Prior to putting the final coordinates into keyword format, duplicate nodes and elements were rejected as well as elements which were less than 0.1mm in length. This was done to get rid of non-real elements as well as to expedite the finite element simulation since the time step of an explicit analysis is determined by the shortest element length (for beam elements). Relative density of the foam was computed by

$$\rho_{rel} = \sum_{i=1}^N \frac{\pi d^2 L_i}{4V} \quad (3-4)$$

where N is the number of elements, d is the diameter of each element, L is the length of each element and V is the volume of the 10-cell cube. The value for d is chosen to be .4mm which was reported as the strut diameter in ERG product literature.

3.3 Finite Element Model Input Parameters

After creating the Voronoi lattices, and transforming them into foam models they were set up to run in LS-DYNA explicit. The explicit time integration scheme in calculates nodal forces from equations of motion at each time step and then uses those forces to compute element deformation and rigid body motion. The model was developed to simulate an experiment in which a cube of foam material was struck by a rigid impactor. Each strut in the model was be represented by a single Hughes-Liu (1H) beam element. The 1H beam element was chosen due to its ability to compute

compression, tension, moment, and torsion. In addition, the 1H element reported contact between elements and distinguished between element deflection and rigid body motion.

An isotropic elastic perfectly-plastic nonlinear constitutive material model (MAT_003/MAT_PLASTIC_KINEMATIC in LS-DYNA syntax) was chosen for the beam elements with the properties described in Table 3-1:

Table 3-1: Material constants chosen for the finite elements

Material	ρ (kg/m³)	E (GPa)	ν	σ_y (MPa)	ϵ_u
Aluminum 6101-T6	2700	68.9	0.33	193	0.2

where ρ is the density, E is the Young's modulus, ν is the Poisson's ratio, σ_y is the yield stress, and ϵ_u is the ultimate plastic strain. "Erosion" was applied to the elements which means that once the element reached its ultimate plastic strain the nodal forces disappeared, but the element's inertia and contact was kept in the simulation as it progressed in time steps. Aluminum 6101-T6 was listed in ERG product literature as being the base material for Duocel foam. Boundary conditions for the foam were set so that the nodes were unconstrained on all sides. Rigid walls were created on the top and bottom of the foam cube as shown in blue in Figure 3-8.

Figure 3-8: Visual representation of finite element model.

Nodes and elements were unable to pass through these walls. Standard dry Coulomb friction was applied between the nodes and elements of the foam model and the walls. The static and dynamic coefficients, μ_s and μ_d were given by Avallone, Baumeister, and Sadegh (2007) as 1.1 and 1.4, respectively.

The top wall was given a mass and initial velocity at the beginning of the simulation. For the foam models which were created with Cubic seed points the top wall was given a mass of 6.46 kg to match the mass of the impactor in the experiment. The foam models which were created with BCC seed points had higher relative densities and absorbed more energy during the simulated impact. To keep the strain rates somewhat constant the mass was increased to 8.75 kg for the BCC case. An impactor of this mass underwent similar deceleration and average strain rate when

impacting a BCC model as a 6.46 kg mass experienced when impacting a Cubic model.

Force was measured across the top wall. Stress (given in Equation 1-4) was computed by dividing this force by the initial area, lw . The foam model and real foams undergo changes in cross-sectional area during large deformations. However, due to the microstructure of the foam, this area change is not necessarily constant throughout the z-direction of the foam. Finding the exact area for each of these slices would be difficult to determine and it would vary from slice to slice. Further, most of the literature referenced in Chapter 2, as well as ERG product literature report the stress measure as engineering stress, rather than true stress. For these reasons, it was deemed best to simply use the original area of the foam to calculate the stress.

Similarly, strain was reported as engineering strain (given in Equation 1-5). The z displacement of the impacting wall was recorded and compared to the original length of the sample. The equations below referencing the variables given in Figure 3-8 were substituted in to Equations 1-4 and 1-5.

$$A_0 = lw \quad (3-5)$$

$$h = L_0, \Delta z = \Delta h = \Delta L \quad (3-6)$$

For each seed point method, several variations of the randomness parameter were tested and compared. Additionally, a larger model was run which matched the

size of samples tested experimentally. Table 3-2 lists the finite element simulations which were performed. The full-sized model is noted with an asterisk.

Table 3-2: Point generation types and randomness parameters compared in the finite element models.

Point Generation Type	f_{xy}	f_z
Cubic	0.25	0.25
Cubic	0.5	0.5
Cubic	0.75	0.75
Cubic	1	1
Cubic	1	0.5
Cubic	0.5	1
BCC	0.3	0.3
BCC	0.4	0.4
BCC	0.5	0.5
BCC	0	0.5
BCC	0.5	0
Cubic*	1	1

3.4 Finite Element Model Results

For each simulation, stress was plotted as a function of strain. This stress-strain curve was used to find the Young's modulus and the yield stress of the model foam. Stress-strain curves for Cubic simulation and a BCC simulation are shown in Figure 3-9. For ease of comparison, only two plots are shown since the curves are very similar within each seed point generation class.

Figure 3-9: Representative stress-strain curves for simulations of two separate seed point generation methods.

The Young's modulus was found by calculating the slope, $\frac{\Delta\sigma}{\Delta\varepsilon}$, of the linear portion of the curve (on the left side in Figure 3-9) using a linear regression fit. The yield stress of the foam model was found by recording the stress at which the stress-strain curve ceases to be linear. The curves for simulations of both cubic and BCC seed point generation techniques had similar features: a steep, linear rise in stress followed by a drop-off and subsequent plateau during which the models exhibited plastic deformation. At between 60-70% strain, the curves rise in stress which indicates densification. Tabulated data for each model is shown in Table 3-3.

Table 3-3: Point generation types and randomness parameters compared in the finite element model.

Point Generation Type	f_{xy}	f_z	Relative Density (%)	Number of Elements	Young's Modulus (Mpa)	Yield Stress (Mpa)
Cubic	0.25	0.25	7.47	11328	247	1.25
Cubic	0.5	0.5	7.04	11664	254	2.06
Cubic	0.75	0.75	6.45	11825	204	1.98
Cubic	1	1	5.91	12023	157	1.72
Cubic	1	0.5	6.70	11819	180	1.95
Cubic	0.5	1	7.00	11492	191	2.17
BCC	0.3	0.3	10.83	22657	533	5.06
BCC	0.4	0.4	10.93	22754	461	4.34
BCC	0.5	0.5	11.05	23054	375	3.94
BCC	0	0.5	10.55	21669	354	4.27
BCC	0.5	0	11.04	22995	617	4.66

Table 3-3 shows that the Young's modulus varied between 157 MPa and 254 MPa for the cubic seed point generation models. The BCC seed point generation models had higher moduli of between 354 MPa and 617 MPa. Likewise, yield strength and relative densities were higher for the BCC models than for the Cubic Models.

Figures 3-10 through 3-15 show logarithmic plots of the relative Young's moduli and relative yield strengths from the models compared to equations 2-2 and 2-3. Figure 3-10 shows tight groupings for both seed point generation techniques. Each lie below the line of predicted values from equation 2-2. Figure 3-11 shows only the Cubic models which show a general trend down the line of equation 2-2 with increasing magnitude of f_{xy} .

Figure 3-10: Log-log plot of relative Young's moduli for both seed point generation methods compared with equation 2-2.

Figure 3-11: Log-log plot of relative Young's moduli for the cubic seed point generation method compared with equation 2-2.

Figure 3-12: Log-log plot of relative Young's moduli the BCC seed point generation method compared with equation 2-2.

Figure 3-12 shows that changes in the randomness parameter, f_{xy} , for BCC models affects the relative Young's modulus more than it does the relative density as there is greater vertical spread in the data.

Figure 3-13: Log-log plot of relative yield strength for both seed point generation methods compared with equation 2-3.

Figure 3-14: Log-log plot of relative yield strength for the cubic seed point generation method compared with equation 2-3.

Figure 3-15 Log-log plot of relative yield strength for the cubic seed point generation method compared with equation 2-3.

Figure 3-13 shows that the relative yield strengths of both seed point generation methods lie above the line of values predicted by equation 2-3. Both seed point generation methods are tightly grouped with a less clear trend of the effects an alteration in seed point generation randomness has on the material yield strength.

CHAPTER 4

COMPRESSION EXPERIMENTS

4.1 Introduction

Drop weight experiments were performed on open-cell metal foam samples in order to compare their results with those of the finite element modeling. Samples of varying porosities were subjected to loading at a strain rate of 110/s. High speed photography was used to view the samples as they were compressed. The high speed photography was used to visualize crush patterns within the samples and interpret the results.

4.2 Experimental Method

Samples of Duocel open-cell aluminum foam of porosity 10 PPI, 20 PPI, and 40 PPI were purchased from KRReynolds Company, which is a distributor of ERG Duocel foam materials. All samples were listed to have 8% nominal relative density and were made of 6101-T6 Aluminum base material. Each sample was then weighed and measured in order to obtain the true relative density. The measured relative densities are given in Table 4-1. The densities for both 10PPI and 20PPI foams were close to the manufacturer's specification, but the 40PPI foam diverged by ~1.2%.

Table 4-1: Name, porosity, and relative density of each compression experiment sample.

Sample Name	Porosity (PPI)	Relative Density (%)
10 PPI - 1	10	7.95
10 PPI - 2	10	7.95
20 PPI - 1	20	7.82
20 PPI - 2	20	7.82
40 PPI - 1	40	6.40
40 PPI - 2	40	6.40

The samples were originally 3.5” long by 1.5” wide x .5” thick, but were cut by hand with a band saw to 1.75” long, by 1.5” wide x .5” thick. Although careful attention was paid to maintain square edges, it was noted that after cutting some edges of some samples were slightly skewed. Attempts were made to get samples which were thicker, however .5” was the thickest size available.

After the samples were cut, the 1.5” x 1.75” face of each sample which would be facing the high-speed camera was painted with black Por 15 priming paint. Por 15 was chosen due to its rugged adherence to a range of different metals. The paint was carefully dabbed on to the front ligaments of the sample. Outward-facing ligaments down to approximately one cell’s width beneath the surface were covered across the entire surface. It was hoped that the black color would add contrast to the cellular ligaments to assist in visualizing crush zone formation and cellular deformation.

The samples were then placed in an Instron Dynatup 9210 dynamic testing machine and aligned with the center of the impactor. The impactor was a 2” diameter

by 2" long steel bar which was attached to the machine by a threaded bolt. The surface of the impactor which would contact the sample was machined smooth in a lathe and then sanded with a fine-grit sand paper.

A Photron SA1 high-speed camera was then focused on the front face of the sample with an f8 aperture setting which balanced lighting conditions with depth of field. High-power lighting was focused on the sample. It should be noted that due to the intensity of the light, considerable temperature rise was observed in the sample. Although warm to the touch, the sample was still able to be held.

The impactor was raised and then dropped at free-fall in order to crush the samples. Raising the impactor to its greatest height yielded an impact velocity of 4.86 meters per second which is a strain rate of $\sim 109/s$. The mass of the impactor and frame together was 6.36 kilograms. As the impactor made contact with the sample, a load cell was used to measure the force on the impactor as a function of time. Velocity at the time of impact was also measured by the machine. Concurrently, the camera was triggered to take pictures at 10,000 frames per second. These pictures were then compiled into a video.

The Dynatup machine outputs a force signal from its load cell. This force signal is scaled by a ratio of the total mass impacting the specimen to the mass of the crosshead. The scaled result is the force "seen" by the sample during the impact event. The scaled force is then divided by the total mass in order to get acceleration. The acceleration is numerically integrated to get the velocity. The velocity is then

numerically integrated to get the displacement of the impactor as it hits the sample. Both integrations are trapezoidal and the acceleration due to gravity during sample deformation is neglected. The final displacement is used to get the strain.

MATLAB image analysis was used to turn the video from grayscale to binary (enhancing contrast). The threshold of .25 was chosen because it seemed to give the best view of the ligaments.

4.3 Experiment Results and Data Processing

Figures 4-1 and 4-2 show the force-time curves and stress-strain curves obtained from the samples tested in the experiments. Both 10PPI and 20PPI samples have similarly shaped curves which have steep rises on the left-hand side followed by a plateau and a final rise as sample densification occurs.

The stress-strain curves were then used to compute both the Young's modulus and yield stress of the samples. The modulus of each sample was originally computed using a linear regression fit of the first 250 points of the stress-strain curve, which was assumed to be linear. The values obtained in this approach are listed in the second column, "Young's E ", of Table 4-2.

Figure 4-1: Force as function of time for all samples.

Figure 4-2: Stress as function of strain for all samples.

Duocel foam is listed as having a compressive modulus of 100 MPa and yield strength 2.53 MPa. As seen from the table, the originally computed values are notably lower than these manufacturer's specifications. Comparison between the stress-strain data curves and the high speed camera footage reveals that due to the slightly-skewed edges of some of the samples the impactor makes contact with only an edge before the sample "seats" to have both top and bottom faces parallel with the impactor. This process takes about 2 frames or .2 ms. This .2 ms seating process aligns well with visible nonlinearities, or "kinks" in the initial portions of the force-time and stress-strain curves.

Due to this observation, the Young's modulus was recomputed using the linear portion of the stress-strain curve directly after the kinks (see column "Youngs 2" in Table 4-2). The recomputed Young's values matched well with expected values. Figure 4-3 shows the force-time curve of sample 10PPI-1 with the aforementioned kink. The portion used to get the Young's modulus is noted in red. On the right side of Figure 4-3 is a plot of the rise in the stress-strain curve. The linear regression fits (with zero intercept) of the first 250 points and of the red section of the force-time curve are shown in red and green, respectively. It is evident from the right-hand plot that the green line, "Linear Int. 2" matches the slope of the stress-strain after the kink. This slope adjustment is carried out for the remaining samples. Adjusted values are listed in the third column of Table 4-2.

Figure 4-3: Force-time (left) and stress-strain (right) curves from sample 10PPI-1.

Table 4-2: Young's modulus for each sample computed using the two methods.

Sample Name	Young's 1 (MPa)	Young's 2 (MPa)	Yield Stress (MPa)
10 PPI - 1	67.4	110	3.21
10 PPI - 2	23.1	90.6	2.62
20 PPI - 1	45.6	117	3.27
20 PPI - 2	30.2	74.1	2.77
40 PPI - 1	17.8	55.9	1.26
40 PPI - 2	25.0	26.6	1.26

From Table 4-2 it is clear that the elastic moduli of the samples vary significantly and the 40PPI samples are less stiff than the 10PPI and 20PPI samples.

The yield stresses are in line with listed values, but still show variability, with the 40PPI samples both having significantly lower yield stresses.

Figures 4-4 through 4-6 show high speed photography of the compression experiments of three samples. The pictures provide good clues as to several attributes of the stress-strain curve. As mentioned earlier, it was helpful in determining when the sample was fully loaded in order to obtain the adjusted modulus. Some features of the pictures stand out. For instance, sample 10PPI-1 shows what visible crush zones beginning at .9 ms and developing until 2.4ms when another crush zone forms to the lower right of the sample. Red arrows on the pictures mark the positions at which the crush zones form. This phenomenon was noted by Tan, Reid, and Harrigan in their experimental work. They appear to align with the undulations in the stress-strain curve seen in greater magnitude in samples 10PPI-1 and 20PPI-1. For instance, in Figure 4-4 on the upper right, a crush zone appears which aligns with a drop in force in at .9 ms on the force-time plot shown in Figure 4-1. The 40 PPI samples exhibited a much more even distribution of crushing as seen in Figure 4-6 in which a large portion of the front face crushes all at once. As the crushing is more uniform, the force-time plots of both 40 PPI samples are comparatively smooth.

Figure 4-4: Time-series of 10PPI-1 impact. Times are in seconds.

Figure 4-5: Time-series of 20PPI-1 impact.

Figure 4-6: Time-series of 40PPI-2 impact.

CHAPTER 5

TENSION EXPERIMENTS

5.1 Introduction

As previously mentioned, relatively little work has been done to characterize the response of open-cell metal foams to high-rate tensile loading. Tensile experiments with strain rates in the range of $\sim 73/s$ were performed on two open-cell metal foam samples in order to explore this phenomena.

5.2 Experimental Method

Dynamic tension experiments were performed on two pieces of ERG Duocel 10PPI open-cell aluminum foam of 8% nominal relative density. The base material was Aluminum 6101-T6. The overall sample size was 3.5" long by 1.5" wide by .5" thick. The samples were cut into a dogbone shape using a high-speed 4-flute newly sharpened end mill. The gauge section of the dogbone samples was 2.5" long by 1" wide by .5" thick. A schematic of the sample size is shown in Figure 5-1 below.

Figure 5-1: Tensile sample dimensions.

Specialized fixtures were designed and fabricated in order to transmit the dynamic tensile load. Two end caps (represented by gray areas in Figure 5-1) were machined in which the top and bottom ends of the sample were bonded using 3M Scotch-Weld™ DP 100 two-part epoxy. The adhesive was spread around the outside of the sample and injected inside the cells of the section of the sample in the sample holder. The sample freely hangs from the top holder and the bottom holder has “wings” which protrude out in order to provide load transfer. Figure 5-2 shows the sample from Test 1 in the sample holder. Figure 5-3 is a close-up photograph of the bonded end of the sample in the top sample holder which illustrates how the sample was held.

Figure 5-2: Tensile sample in sample holder.

Figure 5-3: Close-up of tensile sample epoxied into sample holder.

A specialized impactor was then made which would fit around the sample to make contact with the wings in the bottom sample holder and also allow for the sample to be viewed by the high speed camera. Figure 5-4 shows several views of the impactor itself and of the arrangement as it hits the sample.

Impactor

Front view of impactor
and sample

Side view of impactor
and sample

Figure 5-4: Photos of the tensile impactor.

The samples were placed in an Instron Dynatup 9210 impact tester and then dynamically tensioned at an impact speed of ~ 4.67 m/s, which equates to a strain rate at impact of ~ 73 /s. The mass of the impactor and frame together was 7.59 kilograms. As the impactor made contact with the sample, a load cell was used to measure the force on the impactor as a function of time. Velocity at the moment of impact was also measured by the machine. The same numerical integration procedure used during the compression experiments (described in Section 4.2) was repeated to obtain the deflection and strain as a function of time, with the exception that the mass of the impactor was altered. Additionally, a Photron SA1 high-speed camera was used to photograph the two tests in order to view the sample microstructure. Test 1 was filmed at 10,000 frames per second (fps). Test 2 was filmed at 20,000 fps.

5.2 Experiment Results and Data Processing

Figure 5-5 shows the output of force as a function of time for both experiments. Both experiments show curious material behavior with a steep nearly linear increase in stress followed by an abrupt drop-off and secondary force increase. This “valley” in the force-time curve is more pronounced for Test 1 than it is for Test 2. Analysis of the high-speed video was used to interpret these results. Time-series of the videos is shown in Figures 5-6 and 5-7.

Figure 5-5: Plot of force as function of time for both tensile experiments.

Figure 5-6: Time-series photos from tensile experiment Test 1.

Figure 5-7: Time-series photos from tensile experiment Test 2.

Both videos show lateral movement of the samples as they are loaded and it appears as if the sample holder and impactor are not perfectly parallel. Due to this observation in the video, the stress-strain for Test 2, shown in Figure 5-8, is taken from about 0.08 ms on the force-time plot. This is assumed to be the point of zero strain as the impactor makes full contact with the sample holder.

As the entire cross-section starts to deflect, a rupture in cells and subsequent crack propagates through the material. Attempts were made to match this cracking to specific points on the force-time plots, but the images are not high enough resolution to exactly match crack features with features of those curves. At the time the crack is easily distinguishable, the force has dropped to zero. What can be seen of the crack propagation process takes only a few frames (on the order of 0.4-1.0 ms). The crack

also appears not to be planar, having jagged edges which vary across the width of the sample. As the crack propagates cell-by-cell through the material, the load is no longer distributed across the entire cross section. The first section of each curve (on the left-hand side of Figure 5-5) is of interest with regards to stiffness and yield criterion. It is used to compute the tensile Young's modulus and yield strength of the foam. The tensile Young's modulus and yield strength are found from the linear portions of the stress-strain curve shown in Figure 5-8.

Figure 5-8: Plot of stress as function of strain for both tensile experiments.

A linear interpolation fit is done on the curves to find the slope from .3% to .6% in Test 1 and from .4%-.7% strain in Test 2. These are interpreted to be the linear

portions of the curves which best describe the stiffness of both samples as they undergo rapid tensile loading. The results are given in Table 5-1.

Table 5-1: Tensile Young's modulus and yield strength.

	Young's Modulus (GPa)	Yield Strength (MPa)
Test 1	5.83	37
Test 2	6.45	27

CHAPTER 6

CONCLUSIONS

6.1 Compression FEM and Experiments

Figures 6-1 and 6-2 show relative Young's moduli and relative yield strengths obtained from the finite element study and experimental results presented in this thesis as compared to manufacturer's specifications, results from previous research, and the analytical predictions for quasistatic loading cases given by Equations 2-2 and 2-3 which are used only as a guide for material properties.

Figure 6-1: Relative Young's modulus as function of relative density comparison of work from this thesis to previously published work.

In Figure 6-1, the Young's moduli of the finite element models are found to be below the analytical prediction line, but straddle the previous experimental results reported by Andrews et al. The experimental data from this work matches the manufacturer's specifications, but lies below the finite element model. The plots of the relative yield strength in Figure 6-2 show somewhat better agreement between the experimental data and the finite element model.

Figure 6-2: Relative Yield strength as function of relative density comparison of work from this thesis to previously published work.

Aside from material constants, material behavior throughout the compressive stress-strain curve is of interest. Figure 6-3 is a plot of stress as a function of strain

from the models created by Gaitanaros, Kyriakides, and Kraynik (2012). The stress-strain data produced from both models and experiments from the present work is shown in Figure 6-4.

Figure 6-3: Stress-strain curve from Gaitanaros, Kyriakides, and Kraynik (2012)

Figure 6-4: Stress-strain curves from experimental and FEM data

Although both BCC and Cubic models created in this work show a rise in stress indicating densification after ~50% strain, however this slope is not as steep as those of the experimental work or the model created by Gaitanaros et al. This indicates issues with how the models interpret contact between ligaments as they undergo densification which implies that at strain values over 50% the models lose accuracy.

6.2 Tension Experiments

Table 6-1 shows the values obtained from the tension experiments from this thesis compared to values obtained from manufacturer's specifications and previously published work. The results from ERG Duocel and Andrews et al. were done under quasistatic strain rates. While those reported in this work were obtained from strain rates ~73/s.

Table 6-1: Tensile data from these experiments compared with those from previous work .

	Young's Modulus (GPa)	Yield Strength (MPa)
Test 1	5.83	37
Test 2	6.45	26
ERG Duocel Documents	.101	1.24
Andrews and Gibson (1999)	.502	1.93

It is clear from the values in Table 6-1 that the 10PPI open-cell metal foams tested in the tension experiments reported here had significantly higher stiffness. This indicates strong rate-dependence in the tensile modulus and yield strength of open-cell

metal foams. This finding has not been reported in any other research of which the author is aware.

Figures 6-5 and 6-6 show tensile stress-strain behavior as reported by Andrews et al. and from the experiments presented herein, respectively. The ERG Duocel stress-strain curve is noted in Figure 6-5 and begins with a linear rise in the stress with respect to strain before reaching a peak shortly after 1% strain. The experimental data presented in this work begins with a nonlinear rise in stress-strain with a slope that is greater by an order of magnitude. Although the maximum stress values also differ by an order of magnitude, the strain at which the sample exhibits maximum stress is roughly equal ($\sim 1.1 - 1.2\%$ in both plots). It is thought that the increase in stress is due to rate effects in the material.

Figure 6-5: Tensile stress-strain curve given by Andrews et al. (1999)

Figure 6-6: Experimental tensile stress-strain curve

6.3 Concluding Remarks and Recommendations for Further Work

In conclusion, this work has created a three-dimensional nonlinear finite element model in which the topology of an open-cell metal foam can be easily altered. The model shows good agreement with analytical predictions and with some previously published data up to around 50% compressive strain. Explicitly generated stress-strain curves from these models match the attributes of real life open-cell foams: a linear loading region, a long plastic collapse plateau, and densification. However, after ~50% compressive strain contact conditions within the foam appear to dominate the mechanical response and the models fail to accurately predict these contact effects. The work by Gaitanaros et al. went into great detail using a penalty method and

defining an interface stiffness between ligaments. This work used a simpler method which decreased computation time, but limited accuracy at high strains. Better estimates of friction coefficients or erosion parameters within the material could be used to increase accuracy. Mechanical constants taken from below the 50% compressive strain value are deemed to be reliable indicators of foam behavior.

Two seed point generation techniques for a three dimensional Voronoi lattice were explored and used to look at the effects of seed point positional perturbations on the elastic-plastic response of the foam. While more work is needed to make firmer conclusions, several can be made from the data presented herein.

First, a BCC seed point generation technique will create foams with higher relative moduli than a cubic seed point generation technique. Second, increasing perturbations in the seed point position will decrease the relative stiffness of the foam. These results agree with the conclusions made by Alkhader and Vural (2008) who made this same conclusion about the loss of periodicity in 2D foams. Finally, the mechanical properties of foams created with BCC seed points have greater sensitivity to seed point perturbations than do their relative densities. The last point is important because it shows that in a BCC foam, alterations in the seed point perturbations can change the mechanical characteristics of the foam without a comparative increase in the relative density of the foam. As the relative density is related to weight and cost, this finding can be used to tailor foams for specific applications.

Results from the compression experiments performed in this work are deemed to be less reliable as there were imperfections in material preparation which affected the results. In addition, the thin sample size available off the shelf produced out-of-plane bending and buckling which may have impacted the results. Still, crush patterns were clearly visible in the high speed photography and the experiments were valuable to compare to finite element models of the same size. These crush patterns were more visible in the lower porosity samples than the higher ones.

The tensile tests produced results which show evidence of strain-rate hardening in open-cell metal foams. While the low compressive plastic yield stress and longer strain plateau of these foams make them suitable energy absorbers, the significantly higher stiffness attributed to dynamic tensile loading indicates that these materials make poor tensile energy absorbers.

The models created in this work were helpful in comparing material stiffness and yield strength of open-cell metal foams, but their usefulness could also be extended to energy absorption of the foams. This work was limited in computing power, but larger models with more elements per strut could easily be studied. Further, functional gradation of the material is possible by altering the seed point locations or strut diameters throughout. In an effort to improve the modeling beyond 50% strain and into densification much more work could be done in understanding the complicated contact conditions within the foam. Varying friction between struts during the crush plateau would provide interesting insight and targeted studies on

specific contact scenarios would help further define contact conditions. These models are adaptable and have already shown use in understanding foam material properties, but also provide the foundation for further research.

APPENDIX A

MATLAB SOFTWARE TO CREATE SEED POINTS, NODES, AND ELEMENTS

```
%-----
% K_FILE_CREATE
clear all;close all; clc
%
% Created by Colin J. Murphy
%
% Description: K_FILE_CREATE is script which runs the object-oriented
% routines Element and Node. These routines read file output from
%Voro++
% 3D voronoi cell library and
% then create "nod_list.k" and "el_list.k" and node and element files
%in
% LS-DYNA keyword format.
% To use, one would use "random_point_gen.m" to
% output a list of seed points. This list is put into Voro++ using
%the
% "import" function found here:
http://math.lbl.gov/voro++/examples/import/
% following the pack_ten_cube example. Voro++ is a C++ library which
%can
% easily be compiled in linux.
%
% Make sure to change the file names and max/min values in the
%"import"
% file to match the ones created in random_point_gen. Use the file
output
% from import (in the example, it is "pack_ten_cube.gnu") as "fname"
in the
% script below
%
% First, GNU_V_reader parses the file to determine the cells and node
% locations. Node.m uses that information to create the node list and
% Element.m uses the Node information to create the element list.
%-----
% Options
%
% directory -> the directory in which Voro++ output file is placed
and
% where element
%
% sf -> scale factor to scale the model from "cells" to meters
%
% fname -> Voro++ output file with voronoi cell locations
%
%
directory = 'C: ';
fname = 'Cube_10x10x10_fx-1_fy-1_fz-1_1.gnu';
sf = .0033;
```

```

plot_cells = 0;
%EPL = 1;
plot_nodes = 0;
write_dir = directory;
E_L_lim = .0001;
ligament_diameter = .0004;
rho_AL = 2700; % kg/m^3
sc = .5;
%
%-----
% Reading GNU file to get node locations
% MNL = Master Node List
% cells = index locations of breaks between cells. This tells
function
% where not to make elements that would cross cells
%
disp('Reading GNU file...')
[MNL,cells] = GNU_V_reader(directory,fname,plot_cells);
%
%-----
% Taking the node points and putting nodes between them in the case
of
% having more than one element per ligament
% MNL = Master Node List
% EPL = Elements Per Ligament. Number of elements between main
vertices
% plot_nodes = option to plot the nodes (not functional as of
10/27/13)
% NNL = New Node List which has all the between nodes
%
disp('Creating Nodes...')
NNL = Node(MNL,cells,sf);
%
disp('Creating Elements...')
NLlength = length(NNL);
ELL = Element(NNL,E_L_lim,NLlength,sc);
%
disp('Creating Node List...')
KFNL = MakeNodeList(NNL);
xmaxmin = [max(KFNL(1:length(KFNL(:,1))-1,2))
min(KFNL(1:length(KFNL(:,1))-1,2))];
ymaxmin = [max(KFNL(1:length(KFNL(:,1))-1,3))
min(KFNL(1:length(KFNL(:,1))-1,3))];
zmaxmin = [max(KFNL(1:length(KFNL(:,1))-1,4))
min(KFNL(1:length(KFNL(:,1))-1,4))];
%
disp('Creating Element List...')
KFEL = ELL.MakeElementList(E_L_lim,NLlength);
%
%-----
% Defining the borders of the lattice
%
zbord = zmaxmin(1) + .0001;

```

```

xbord1 = xmaxmin(1);
xbord2 = xmaxmin(2);
ybord1 = ymaxmin(1);
ybord2 = ymaxmin(2);
%
% shell_nodes are used for the rigidwalls which impact the Lattice
%
shell_nodes = [199999,xbord2,ybord2,zbord,0,0;...
               199998,xbord2,ybord1,zbord,0,0;...
               199997,xbord1,ybord1,zbord,0,0;...
               199996,xbord1,ybord2,zbord,0,0];

%
Element_Lengths = ELL.Compute_Lengths(E_L_lim);
Area = xmaxmin(1)*ymaxmin(1);
Length = zmaxmin(1);
Volume = xmaxmin(1)*ymaxmin(1)*zmaxmin(1);
Surface_Area = sum(Element_Lengths)*pi*ligament_diameter;
Relative_Density =
((sum(Element_Lengths)*(pi*((ligament_diameter^2))/4)...
 *rho_AL)/(rho_AL*Volume))*100;
%
%-----
% Writing 3 files:
%
% nod_list.k is the list of nodes used in the LS-DYNA simulations.
The
% nodes are then used in the element list. The last four nodes are
used to
% define the rigidwall which impacts the sample
%
% el_list.k is the list of elements created from the node list
%
% run_stats.txt has the size of the sample, relative density
computation,
% and volume. It also has a list of the length of every element in
the
% computation if length statistics are desired
%

cd(write_dir);

disp('Writing Node File...')

fid = fopen('nod_list.k','w');
fprintf(fid,'%s\r\n','*keyword');
fprintf(fid,'%s\r\n','*NODE');
fclose(fid);
dlmwrite('nod_list.k',KFNL,'-
append','roffset',0,'newline','pc','precision',6);
dlmwrite('nod_list.k',shell_nodes,'-
append','roffset',0,'newline','pc','precision',6);

```

```

disp('Writing Element File...')

fid = fopen('el_list.k','w');
fprintf(fid,'%s\r\n','*keyword');
fprintf(fid,'%s\r\n','*ELEMENT_BEAM');
fclose(fid);
dlmwrite('el_list.k',KFEL,'-
append','roffset',0,'newline','pc','precision',6);

disp('Writing Stats File...')

fid = fopen('run_stats.txt','w');
fprintf(fid,'%s\r\n',(['Element Limit: ',num2str(E_L_lim)]));
fprintf(fid,'%s\r\n',(['Scale Factor: ',num2str(sf)]));
fprintf(fid,'%s\r\n',(['Area: ',num2str(Area)]));
fprintf(fid,'%s\r\n',(['Ligament Diameter:
',num2str(ligament_diameter)]));
fprintf(fid,'%s\r\n',(['Height: ',num2str(Length)]));
fprintf(fid,'%s\r\n',(['Volume: ',num2str(Volume)]));
fprintf(fid,'%s\r\n',(['Surface Area: ',num2str(Surface_Area)]));
fprintf(fid,'%s\r\n',(['Relative Density:
',num2str(Relative_Density)]));
fclose(fid);
dlmwrite('run_stats.txt',Element_Lengths,'-
append','roffset',0,'newline','pc')

-----

function[MNL,cells] = GNU_V_reader(directory,fname,plot_cells)

% When in standalone mode, uncomment the lines below
% clear all
% clc
% cd('C:\Users\Colin\Desktop\MATLAB NODE MAKER');
% fname = 'random_points_v.txt';
% fname = 'pack_six_cube.txt';
%% Change in to datafile directory
cd(directory);
% Opening file and reading in data
fid = fopen(fname);
% Taking in all of the data to struct C including spaces
C = textscan(fid,'%s','delimiter','\n');
fclose(fid);
%-----
%indexing for lines
dcol = 1;
drow = 1;
spaces = 1;
numcells = 1;
mrow = 1;
%-----

```



```

% Going through each part of the data and separating the numbers
% If a line is empty, it means that it is a new cell
% Since there are two spaces between cells, it has to read after the
second
% space which is why the second if loop is there
%
for i = 1:length(C{1,1})
    lod = char(C{1,1}(i));
    if ~isempty(lod)
        nlod = textscan(lod,'%f %f %f','delimiter',' ');
        X{dcol}(drow) = nlod{1,1};
        Y{dcol}(drow) = nlod{1,2};
        Z{dcol}(drow) = nlod{1,3};
        MNL(mrow,:) = [nlod{1,1} nlod{1,2} nlod{1,3}];
        drow = drow + 1;
        mrow = mrow + 1;
    end
%
% Sorting the spaces. Also will give the index "mrow" in the master
node
% list "MNL" at which there is a new cell being created
% If there is an empty space at the line and spaces is equal to one,
record
% index and add 1 to space. If it is empty and spaces equals 2,
advance
% main indexes and set space back to one
%
    if isempty(lod)&& spaces == 1
        cells(numcells) = mrow;
        spaces = spaces + 1;
        numcells = numcells + 1;
    elseif isempty(lod) && spaces == 2
        spaces = 1;
        dcol = dcol + 1;
        drow = 1;
    end
end

end

clear C
%
%-----
% Plotting the cells from the file in a MATLAB FIGURE
if plot_cells == 1
figure
for i = 1:length(X)
    x = X{i}(:, :)';
    y = Y{i}(:, :)';
    z = Z{i}(:, :)';
    h=line(x,y,z,'linewidth',2,'Color',[0 0 0]);
    clear x y z
end

```

```

end
-----
classdef Node < handle
    properties
        ID
        coordinates
        translation
        rotation
        cell_begin
        duplicate
        first_instance
        Top_Nodes
    end
    methods
        function obj = Node(nodelist,cells,sf)
            if nargin ~=0
                R = size(nodelist,1);
                obj(R) = Node;
                nodelist = nodelist * sf;
                [~,ia,ic] = unique(nodelist,'rows');

                zmin_indxs = find(nodelist(:,3)==0);
                zmax_indxs = find(nodelist(:,3)==max(nodelist(:,3)));

                zindx = 1;
                ztindx = 1;
                celli = 1;
                for i = 1:R
                    obj(i).ID = i;
                    obj(i).coordinates = nodelist(i,1:3);
                    if ia(ic(i)) < i
                        obj(i).duplicate = 'yes';
                        obj(i).first_instance = ia(ic(i));
                    else
                        obj(i).duplicate = 'no';
                        obj(i).first_instance = [];
                    end

                    if i == cells(celli)
                        obj(i).cell_begin = 'yes';
                        celli = celli + 1;
                    else
                        obj(i).cell_begin = 'no';
                    end

                    if i == zmin_indxs(zindx)
                        obj(i).translation = 7;
                        obj(i).rotation = 7;
                        %disp('Constrained node')
                        if zindx < length(zmin_indxs)
                            zindx = zindx + 1;
                        end
                    end
                end
            end
        end
    end
end

```

```

        else
            obj(i).translation = 0;
            obj(i).rotation = 0;
        end

        if i == zmax_indxs(ztindx)

            obj(i).Top_Nodes = 'yes';
            if ztindx < length(zmax_indxs)
                ztindx = ztindx + 1;
            end

        end

    end

end

end
end
function TNL = Make_Top_Node_List(obj)

    tnlidx = 1;

    for i = 1:length(obj)
        if strcmp(obj(i).Top_Nodes,'yes') &&
~strcmp(obj(i).duplicate,'yes')
            TNL(tnlidx) = obj(i).ID;
            tnlidx = tnlidx + 1;
        end
    end

end

end

end

-----

classdef Element
    properties
        ID
        PID = 1;
        N1
        N2
        N3
        RT1 = 0;
        RR1 = 0;
        RT2 = 0;
        RR2 = 0;
        CS = 2;
        Length;
        Center;
        duplicate;
        first_instance;
    end
end

```

```

        Close_Elements;
        Intersect_Reject = 'no';
    end

    methods
        function obj = Element(Node,E_L_lim,NLlength,sc)
            if nargin ~=0
                C = size(Node,2);
                EID = 1;
                %obj(C) = Element;
                for i = 1:C-1
                    if strcmp(Node(i+1).cell_begin,'yes')
                        % Do not create element
                    elseif strcmp(Node(i+1).cell_begin,'no')
                        obj(EID).ID = EID;

                        if strcmp(Node(i).duplicate,'yes')
                            obj(EID).N1 = Node(i).first_instance ;
                        elseif strcmp(Node(i).duplicate,'no')
                            obj(EID).N1 = Node(i).ID;
                        end

                        if strcmp(Node(i+1).duplicate,'yes')
                            obj(EID).N2 = Node(i+1).first_instance ;
                        elseif strcmp(Node(i+1).duplicate,'no')
                            obj(EID).N2 = Node(i+1).ID;
                        else
                            disp('Duplication property not entered');
                        end
                        d = Node(i+1).coordinates - Node(i).coordinates;
                        l = sqrt((d(1)^2)+ (d(2)^2) + (d(3)^2));
                        obj(EID).Length = l;
                        clear d l
                        EID = EID + 1;
                    else
                        disp('Duplication property not entered');
                    end
                end
            end
        %end
    %end

    %***** Added 4/13-----
    %function [KFEL] = MakeElementList(obj,E_L_lim,NLlength,NNL)
    NNL = Node;
    %----Find centers and create center list
    for i = 1:length(obj)

        N1 = obj(i).N1;
        N2 = obj(i).N2;

        obj(i).Center = [ (NNL(N1).coordinates(1) +
        NNL(N2).coordinates(1))/2 ...

```

```

...
        (NNL(N1).coordinates(2) + NNL(N2).coordinates(2))/2
        (NNL(N1).coordinates(3) + NNL(N2).coordinates(3))/2 ];

CTR_LIST(i,:) = obj(i).Center;

end
%
%---- Find duplicates and mark them
%
[~,ia,ic] = unique(CTR_LIST,'rows');

for i = 1:length(obj)

    if ia(ic(i)) < i

        obj(i).duplicate = 'yes';
        obj(i).first_instance = ia(ic(i));

    else

        obj(i).duplicate = 'no';
        obj(i).first_instance = [];

    end

end

%
%---- Create the list based on duplicates and length

end

% ----- Done creating list
end

function obj = set.Close_Elements(obj,elements)
    obj.Close_Elements = elements;
end

function obj = set.Intersect_Reject(obj,status)
    obj.Intersect_Reject = status;
end
%end
%
%-----
%
function KFEL = MakeElementList(obj,E_L_lim,NLlength);
ei = 1;

```

```

        for i = 1:length(obj)

            if obj(i).Length > E_L_lim && ~strcmp(obj(i).duplicate,'yes')
&& strcmp(obj(i).Intersect_Reject,'no')

                KFEL(ei,:) = [obj(i).ID ...
                obj(i).PID...
                obj(i).N1 obj(i).N2 NLength...
                obj(i).RT1 obj(i).RR1 obj(i).RT2 obj(i).RR2...
                obj(i).CS];
                ei = ei + 1;

            end

        end

    end
    end
    %
    %---- Create a list of node lengths
    %
    function Element_Lengths = Compute_Lengths(obj,E_L_lim)
        ei = 1;
        for i = 1:length(obj)
            if obj(i).Length > E_L_lim &&
~strcmp(obj(i).duplicate,'yes')&&
strcmp(obj(i).Intersect_Reject,'no')
                Element_Lengths(ei,1) = obj(i).Length;
                ei = ei + 1;
            end
        end
    end
    %
    %---- Find elements with centers that are closer than
    %sc*ligament_diameter
    %
    %function obj =
Find_Close_Elements(obj,ligament_diameter,E_L_lim,sc);
function [obj IDX IDs] =
Find_Close_Elements(obj,ligament_diameter,E_L_lim,sc);
        j = 1;
        for i = 1:length(obj)

            if obj(i).Length > E_L_lim &&
~strcmp(obj(i).duplicate,'yes')

                C(j,:) = obj(i).Center;
                IDs(j) = obj(i).ID;
                j = j + 1;

            end
        end
    end
end

```

```

[IDX,~] = rangesearch(C,C,sc*ligament_diameter);

%Creates a list of close elements as a propertie of the
Element

for i = 1:length(IDX)

    if length(IDX{i}) > 1
        %ce =
        obj(IDs(i)).Close_Elements =
        IDX{i}(2:length(IDX{i}));
        %obj(IDs(i)).Close_Elements =
        IDX{i}(2:length(IDX{i}));

    end

end

% Finding intersecting elements and marking the shortest
for
% deletion

for i = 1:length(IDX)

    clear int_lengths

    if length(IDX{i}) > 1 &&
~strcmp(obj(i).Intersect_Reject,'yes');

        for j = 2:length(IDX{i})

            if
strcmp(obj(IDX{i}(j)).Intersect_Reject,'no');

                int_lengths(j-1) = obj(IDX{i}(j)).Length;

            else
                int_lengths = 0;

            end

        end

    end

    if obj(i).Length < max(int_lengths)

        obj(i).Intersect_Reject = 'yes';

    end

end
end

```

```

        end
    end

%-----

function El_Loc = Element_Locations(obj,E_L_lim)

    j = 1;
    for i = 1:length(obj)

        if obj(i).Length > E_L_lim &&
~strcmp(obj(i).duplicate,'yes')

            El_Loc(j,:) = obj(i).Center;
            IDs(j) = obj(i).ID;
            j = j + 1;

        end
    end
end

%-----

function [n close_elements] = Return_Close_Elements(obj)

    n = 0;
    k = 1;
    for i = 1:length(obj)

        if ~isempty(obj(i).Close_Elements)
            num_els = length(obj(i).Close_Elements);
            n = n + num_els;

            for j = 1:length(num_els)

                %el_id = obj(i).Close_Elements
                close_elements(k+j-1) = obj(i).Close_Elements(j);

            end

            k = k + num_els;

        end
    end
end
end

```



```

                %%%%%%%%% Added 4/13
            end
        end

        -----
        % random_point_gen
        % creates seed points for input into Voro++ 3D Voronoi cell library
        %
        clc,clear all;clf
        NX = 2;
        NY = 2;
        NZ = 2;
        fx = 0;
        fy = 0;
        fz = 0;
        fname = 'Cube_2x2x2_fx-0_fy-0_fz-0_1';
        %fsize = ([num2str(NX),'x',num2str(NY),'x',num2str(NZ)]);
        fnrandp = ([ 'fx-',num2str(fx),'_', 'fy-',num2str(fy),'_', 'fz-',
            num2str(fz),'.emf' ]);
        %fname = ([ 'Cube_',fsize,'_',fnrandp,'_1' ]);

        [x,y,z]=meshgrid(1:NX,1:NY,1:NZ);

        r1=fx*rand(NY,NX,NZ);
        r2=fy*rand(NY,NX,NZ);
        r3=fz*rand(NY,NX,NZ);

        xr=x+r1
        yr=y+r2
        zr=z+r3

        figure('Units', 'pixels', ...
            'Position', [100 10 2000 500]);
        hold on;

        m = 1;
        for i = 1:length(zr(:,1,1))
            for j = 1:length(yr(1,:,1))
                for k = 1:length(xr(1,1,:))

                    p(m,:) = [m xr(i,j,k) yr(i,j,k) zr(i,j,k)];
                    m = m+1;
                end
            end
        end

        cd('C:\Users\Colin\Desktop\Point Clouds\Simple Cubic')

        dlmwrite(fname,p,'delimiter','\t','newline','pc');
    
```

APPENDIX B

LS-DYNA KEYWORD FILE REDUCED INPUT

```
*keyword
*title
Title of the run
$
$-----
*CONTACT_AUTOMATIC_GENERAL
0,0
1.1,1.4
1,1
*CONTROL_IMPLICIT_GENERAL
0,0.00001
*CONTROL_TERMINATION
.0085
*DATABASE_BINARY_D3PLOT
0.00001
*DATABASE_BINARY_D3THDT
.00001
*DATABASE_HISTORY_NODE
199999
*DATABASE_NODOUT
.00001,1
*DATABASE_GLSTAT
.00001,1
*DATABASE_RWFORC
.00001,1
*DATABASE_RBDOUT
.00001,1
*DATABASE_ELOUT
.0000001
$
$-----
$
*PART
aluminum beam
1,1,1
*SECTION_BEAM
1,1,1,2,1
.0004,.0004,0,0,0,0
*MAT_003
```

```

1,2700,68.9e+9,.33,1.93e+8,0,0
0,0,.2,0
$-----
$ The following "include" files are created by K_FILE_CREATE
*INCLUDE
nod_list.k
*INCLUDE
el_list.k
$-----
*RIGIDWALL_PLANAR_MOVING_FORCES
$  nsid  nsidex  boxid
0,0,0
$  xt    yt    zt    xh    yh    zh    fric
.0182,.0182,.0364,.0182,.0182,0,.61,0
$ sw mass  sw vel
6.46,4.835

```

BIBLIOGRAPHY

Ajdari, A., Canavan, P., Nayeb-Hashemi, H., Warner, G. (2009). Mechanical properties of functionally graded 2-D cellular structures: A finite element simulation. *Materials Science and Engineering A*, 499, 434-439.

Ajdari, A., Nayeb-Hashemi, H., Vaziri, A. (2011). Dynamic crushing and energy absorption of regular, irregular and functionally graded cellular structures. *International Journal of Solids and Structures*, 48, 506-516.

Alkhader, M., Vural, M. (2006). Influence of cellular topology on dynamic response of solid foams. *Proceedings of the ASME International Mechanical Engineering Congress and Exposition*.

Alkhader, M., Vural, M. (2008). Mechanical response of cellular solids: Role of cellular topology and microstructural irregularity. *International Journal of Engineering Science*, 46, 1035-1051.

Amsterdam, E., Hedde van Hoorn, J., De Hosson, J., and Onck, P. (2008). The influence of cell shape anisotropy on the tensile behavior of open cell aluminum foam. *Advanced Engineering Materials*, 10, 9.

Andrews, E., Sanders, W., Gibson, L. (1998). Compressive and tensile behavior of aluminum foams. *Materials Science and Engineering*, A270, 113-124.

Andrews, E., Gibson, L. (2001). The influence of crack-like defects on the tensile strength of an open-cell aluminum foam. *Scripta Materialia*, 44, 1005-1010.

Ashby, M. (1983). The mechanical properties of cellular solids. *Metallurgical Transactions*, 14A, 1755-1769.

Avallone, E., Baumeister T., and Sadegh, A. (2007). Mark's Standard Handbook for Mechanical Engineers. New York, New York. McGraw-Hill, p. 3-21.

Brakke, K.A., (1992). The surface evolver. *Experimental Mathematics* 1, 141-165. Retrieved from <http://www.susqu.edu/facstaff/b/brakke/evolver>.

Breault, S., (2012). Improving load distributions in cellular materials using stress trajectory topologies. *Open Access Master's Theses*. Paper 100. <http://digitalcommons.uri.edu/theses/100>

Cheng, X.Y., Li, S.J., Murr, L.E., Zhang, Z.B., Hao, Y.L, Yang, R., Medina, F., and Wicker, R.B., (2012). Compression deformation behavior of Ti-6Al-4V alloy with cellular structures fabricated by electron beam melting. *Journal of the Mechanical Behavior of Biomedical Materials*, 16, 153-162.

Davies, G., Zhen, S. (1983). Metallic foams: their production, properties and applications. *Journal of Materials Science*, 18, 1899-1911.

Elliot, J. (1951). Method of producing metal foam. United States Patent 2751289.

ERG Aerospace Corporation (2014). *About Us*. Retrieved from <http://www.ergaerospace.com/about-us.htm>

Gaitanaros, S., Kyriakides, S., Kraynik, A. (2012). On the crushing response of random open-cell foams. *International Journal of Solids and Structures*, 49, 2733-2743.

Gaitanaros, S., Kyriakides, S. (2013). Dynamic crushing of aluminum foams: Part II – Analysis. *International Journal of Solids and Structures*, (yet to be published).

Gibson, L. (1989). Modelling the mechanical behavior of cellular materials. *Material Science Engineering*, A110, 1-36.

Gibson, L., Ashby, M. (1997). *Cellular solids: structure and properties – Second edition*. Cambridge, UK: Cambridge University Press.

Hallquist, J. (2006). LS-DYNA theory manual. *Livermore Software Technology Corporation*, Livermore, CA.

Hughes, T.J.R, Liu, W.K., (1981). Nonlinear finite element analysis of shells. Part II: Two-dimensional shells. *Computer Methods in Applied Mechanics and Engineering*, 27, 167-181.

Jones, R. (2011). Mechanical behavior of functionally graded cellular materials around a circular hole. *Dissertations and Master's Theses (Campus Access)*. Paper

AAI1497503. <http://digitalcommons.uri.edu.uri.idm.oclc.org/dissertations/AAI1497503>.

Liebscher, A., Proppe, C., Redenbach, C., Schwarzer, D. (2012). Uncertainty quantification for metal foam structures by means of image analysis. *Probabilistic Engineering Mechanics*, 28, 143-151.

Kraynik, A.M. Reinelt, A., Princen, H.M., (1991). The nonlinear elastic behavior of polydisperse hexagonal foams and concentrated emulsions. *Journal of Rheology*, 35, 1235-1253.

Ochsner, A., Klaus, L., (2003). On the uniaxial compression behavior of regular shaped cellular materials. *Mechanics Research Communications*, 30, 573-579.

Okabe, A., Boots, B., Sugihara, K. (1992). *Spatial tessellations: concepts and applications of Voronoi diagrams*. New York, NY: Wiley & Sons.

Olurin, O., Fleck, N., and Ashby, M. (2000). Deformation and fracture of aluminum foams. *Materials Science and Engineering*, A291, 136-146.

Predictive Engineering (2011). LS-DYNA analysis for structural mechanics

Roberts, A., Garboczi, E. (2002). Elastic properties of model random three-dimensional open-cell solids. *Journal of the Mechanics and Physics of Solids*, 50, 33-55.

Rycroft, C. (2009). *Voro++: A three-dimensional Voronoi cell library in C++*, *Chaos* **19**, 041111.

Rycroft, C. (2014). *Voro++ - A 3D Voronoi cell software library*. Retrieved from <http://math.lbl.gov/voro++/>.

Sadd, M. (2013). "Micromechanical Simulation of Cellular Materials with Varying Topologies." Special Symposium: Performance of Materials and Structures Under Extreme Loading Conditions. URI's Cherry Auditorium, Kingston. 12 October 2013. Lecture.

Silva, M.J, Hayes, W.C. and Gibson, L.J., (1995), "The Effects of Non Periodic Microstructure on the Elastic Properties of Two-Dimensional Honeycombs" *International Journal of Mechanical Science*, 37, 1161-1177.

Sosnick, B. (1948). Process for making foamlike mass of metal. United States Patent 2434775.

Tan, P.J, Reid, S.R, and Harrigan, J.J, (2012). "On the dynamic mechanical properties of open-cell metal foams – A re-assessment of the 'simple-shock theory'". *International Journal of Solids and Structures*, 49, 2744-2753.

Wadley, H. (2014). Ultralight Cellular Materials: Material property chart comparing the Young's modulus and density of engineering materials. *Wadley*

Research Group, University of Virginia. Retrieved from
<http://www.virginia.edu/ms/research/wadley/cellular-materials.html>.

Warren, W.E., Kraynik, A.M., (1997). Linear elastic behavior of a low-density Kelvin foam with open cells. *ASME Journal of Applied Mechanics*, 64, 787-794.

Within Technologies. (2014, January). *Overview and Case study lattice designs*. Retrieved from <http://withinlab.com/case-studies/index10.php>.