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INVESTIGATING BEGINNING SECONDARY MATHEMATICS TEACHERS' PERCEPTIONS AND DEVELOPMENT OF THEIR PEDAGOGICAL CONTENT KNOWLEDGE

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BY

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ABSTRACT

Research has shown that there are different types of knowledge possessed by teachers that impact their effectiveness as practitioners. These types of teacher knowledge have been connected to student achievement, teacher retention, teacher efficacy, and teacher quality. Currently, there is a gap in the literature about how secondary mathematics teachers develop their knowledge for teaching, known as Pedagogical Content Knowledge (PCK), during the transition from pre-service to in-service teaching. Understanding how this knowledge develops and individuals' perceptions of their development has implications for teacher preparation programs and school leaders.

The goal of this study was to investigate the development of beginning secondary mathematics teachers' pedagogical content knowledge (PCK) about teaching and learning mathematics over the course of the first year of teaching. Taking into account the concerns about beginning secondary mathematics teachers' preparedness to enter the profession and the gap in the research on PCK development during the transition from teacher pre-service to in-service teaching, I conducted a qualitative study. Data were compiled from multiple sources: a PCK inventory, interviews, classroom observations, and a survey. Each source provided information for understanding how beginning secondary mathematics teachers developed their PCK and their perceptions of their development.

Findings from this study indicated that PCK developed primarily from participants' experiences working with students. The role of reflection and collaboration with others was also found to be instrumental in PCK development. Having opportunities to develop

all aspects of knowledge was not always available for participants in all situations. At times, there were PCK tasks that were beyond to scope of the given experience or teachers were limited in their freedom to exercise their knowledge. This data demonstrated that participants needed opportunities and the agency to act on those opportunities to develop their PCK. My data also suggest the development of knowledge in the different domains of PCK does not happen in isolation. Instead, different domains and types of knowledge develop in parallel.

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My friends, thank you for believing in me and understanding why I have been so busy for the past four years. I am exceptionally lucky to have you all in my corner. Brooke, your friendship has supported me through so much and I'm so thankful for all you do. You constantly reminded me I could do it, even when I felt completely overwhelmed.

Thank you to my family. I would not be the person I am today without you. Through everything, we were one unit who withstood so much. You have supported me in every one of my goals and aspirations, never considering them unreachable or silly. Tom, thank you for all your support, patience, and love. Thank you all for listening to me talk about my research and thoughts, even if you did not understand half of what I was saying as they were half-formed ideas. You helped me persevere through it all.

DEDICATION

To my supportive and selfless family. Mom, Mike, Michelle, Alicia, Stef, and Tom- your love and support mean the world to me and I cannot express my gratitude enough for sticking by my side through everything. You each know exactly what to say and what I need to hear. You keep me laughing and give me the confidence to tackle anything that life brings. I'm proud to be able to call such brilliant and kind people my family.

PREFACE

Throughout this dissertation, different terms are used to indicated different stages in the professional careers of teachers. *Pre-service teachers* and *teacher candidates* are both used when discussing any student enrolled in a teacher preparation program at a college or university. *Student teachers* refer to those enrolled in preparation programs but that are in their final year of study and are in their full-time practicum experience known as student teaching. *In-service teachers* are individuals who are working as teacher in elementary, middle, or high schools. *Beginning teachers* encompasses teachers who are still novices to the profession and have not yet reached tenure. *First-year teachers* are individuals who have just entered the profession and have obtained their first full-time teaching position. *Experienced teachers* are those teachers who have taught for a number of years and are no longer considered novices or beginning teachers. The term *students* is used when discussing children or adolescents in schools.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

WHAT TO BAKE?: INTRODUCTION

> The problem is not the problem. The problem is your attitude about the problem.
>
> Jack Sparrow, *Pirates of the Caribbean*

What makes mathematics “click” for some people and not for others? Is it inherent to the individual or due to outside influences? According to the National Council of Teachers of Mathematics (NCTM, 2000), “students’ understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school” (p. 17). The capacity of teachers to promote student interest and knowledge development in mathematics is an important topic of research within the United States and internationally (Ball, 2000; Kahan, Cooper, & Bethea, 2003; Mitchell, 1993; Trautwein, Lüdtke, Marsh, Köller, & Baumert, 2006).

The level of achievement of United States students in relation to students from other countries is an area of concern for policymakers, administrators, parents, and teachers. International comparisons that exist on mathematical achievement, such as Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA), illustrate that there are differences between educational systems and the associated levels of achievement (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005). One component of interest in the different educational systems is the effects of teachers on student achievement. Additionally, there are international comparisons of teacher preparation and teacher knowledge (Kleickmann et al., 2013; Schmidt, Burroughs, Cogan, & Houang, 2016). One focus of these studies is the different components found in teacher preparation programs and how the different elements contribute to teacher knowledge development.

Over the past three decades, greater attention has been given to mathematics teachers' content knowledge. No Child Left Behind (NCLB) required teachers to be "Highly Qualified" (U.S. Department of Education, 2004, p. 1). Of late, the Every Student Succeeds Act (ESSA) strips down these requirements and now "teachers in schools receiving Title I funds need only fulfill their state's licensing requirements," which typically include passing a content licensing examination (Sawchuk, 2016, p. 14). Validity of these examinations has been established, however, do these standardized tests actually correlate to teacher performance as measured by student achievement? Though teacher preparation has evolved over the decades, a number of the tests have not changed with the times. It has been argued that there is not a clear understanding of whether these tests are an accurate portrayal of teacher knowledge (Angrist, & Guryan, 2008; Goldhaber, 2007; Podgursky, 2005). Nevertheless, the use of these content examinations are widespread and form a potential barrier for teacher candidates entering the profession. It raises the urgency for teacher preparation programs to support teacher candidates in their development of content knowledge for teaching. To do this we need a better understanding of the nature of such knowledge and its development.

In 1986, Shulman began a systematic investigation of how teacher knowledge was defined over the previous century. He was able to examine tests for teachers used in licensing which showed the majority of questions, 90%-95%, pertained to subject matter knowledge while about 5% of the questions were devoted to pedagogical practices. These tests, and accounts found in autobiographies, revealed how important subject matter knowledge was as a prerequisite to teach while “theories and methods of teaching” played “a decidedly secondary role in the qualifications of a teacher” (Shulman, 1986, p. 2). While the pendulum did not swing in the complete opposite direction during the 1980s, Shulman (1986) explains how there was greater emphasis during this decade on assessing teachers’ “capacity to teach” (p. 2). However, basic skills tests became a pre-requisite for many teacher education programs, as they are today. Similarly, there is now a focus on both content knowledge and pedagogical knowledge testing prior to earning licenses. Teachers need to demonstrate their aptitude in the content they will be teaching as well as in pedagogy appropriate to their student populations on standardized testing in order to be granted their teaching certifications.

Statement of the Problem

The importance of quality teacher preparation is well known (Darling-Hammond, 2010; Darling-Hammond & Youngs, 2002; Gansle, Noell, & Burns, 2012; Koedel, Parsons, Podgursky, & Ehlert, 2015). Research has been compiled about the effects of professional preparation and content knowledge on teacher retention (Darling-Hammond, Chung, & Frelow, 2002); student success and achievement (Darling-Hammond, Holtzman, Gatlin, & Heilig, 2005; Gansle et al., 2012; Koedel et al., 2015; National Mathematics Advisory Panel, 2008; Tchoshanov, Lesser, & Salazar, 2008); and teacher quality (Brouwer & Korthagen, 2005). Therefore, teacher educators need to be more informed about what specialized knowledge teachers need in order to better prepare pre-service teachers for their transition into the profession (Cummings, 2010).

The subject matter knowledge for teaching is referred to as pedagogical content knowledge (PCK). This term was first defined by Shulman (1986) and is used to describe specialized knowledge possessed by teachers beyond pure subject matter knowledge. According to Stevens, Harris, Aguirre-Munoz, and Cobbs (2009) this specialized knowledge is comprised of knowledge of: (a) what it means to teach a particular subject, (b) instructional strategies and representations for teaching particular topics, (c) students' understanding and potential misunderstandings of a subject area, and (d) curriculum and curricular materials. Since researchers have found mixed results of the effects of teacher preparation programs on the development of PCK (Goldhaber, Liddle, & Theobald, 2013; Grossman, 1990; Leong, 2013; Saeli, Perrenet, Jochems, & Zwaneveld, 2012; Schmidt et al, 2016), there is a need to investigate the role of preparation programs on teachers' ability to teach mathematics. Similarly, there is a need to understand what experiences and factors influence PCK development and how it develops over the first year(s) of teaching.

Teacher preparation programs vary across the United States and globally. The types of courses required and offered, the variety of placements pre-service teachers engage in, and the organization of the program all influence the development of the various types of teachers' knowledge. Van Driel and Berry (2010) suggest that pre-service teachers possess little to no PCK because they do not yet have teaching experience, however, they can begin to develop PCK in their education programs. Brouwer and Korthagen (2005) looked at the relationship between teacher preparation and teaching competence; they argue that beginning teachers will continue to grow during their first years of teaching. Thus, programs should equip candidates for "entry into the teaching profession"

(Brouwer & Korthagen, 2005, p. 158). Studies examined the influence of preparation type on student achievement (Gansle et al., 2012; Darling-Hammond et al., 2005; Koedel et al., 2015), teacher efficacy (Darling-Hammond et al., 2002; Hoy & Spero, 2005), teacher retention and satisfaction (Andrew, 1990), and other characteristics of teachers and teaching. Together these studies show that not only does the type of preparation matter, but the elements of the programs themselves contribute to the development of PCK. There is an assumption that improving the development of PCK will lead to higher student performance with those teachers. However, there needs to be a greater understanding of how the elements of teacher preparation programs work separately and together to promote PCK development in pre-service and in-service teachers.

After completing teacher preparation programs, teacher candidates enter the profession as beginning in-service teachers. Currently, there is a lack of research about the transition of teachers from pre-service to in-service with a focus on changes in PCK—a gap this study addresses. Studies about this period of transition have been conducted outside of the United States (e.g. Kleickmann et al., 2013- Germany; Mulholland & Wallace, 2003- Australia) which inform this study but are not necessarily comparable to teachers in this country. Similarly, there is existing research regarding PCK development in elementary school teachers (Ball, 1988; Hill, Ball, & Schilling, 2008; Ma, 1999; McAuliffe & Lubben, 2013; Mulholland & Wallance. 2003; Noblet, 2016; Turner & Rowland, 2008) and secondary mathematics teachers within specific subjects (Blasjo, Dalgamoni, & Roberson, 2010; Even, 1993; Saeli et al., 2012). Further investigation needs to occur on how beginning secondary mathematics teachers develop their PCK and what contributes to their PCK development during the first year of teaching.

Purpose of the Study and Research Questions

The purpose of this study is to investigate the development of beginning secondary mathematics teachers' pedagogical content knowledge (PCK) about teaching and learning mathematics over the course of the first year of teaching. Based on the concerns about beginning secondary mathematics teachers' preparedness to enter the classroom and the gap in the research as it concerns PCK development during the transition from teacher pre-service to in-service, I conducted a qualitative study.

With the goal of investigating the development of PCK during the transition from student-teacher to teacher, I needed interact with the same individuals in both settings. In order to gather data on my potential participants' PCK and PCK development during their student teaching year (fall 2016 to spring 2017), I conducted a pilot study. A subset of three of the original nine pilot study participants were recruited for my dissertation research. The participants for this study were first-year teachers who were recent graduates of a teacher preparation program as a secondary education or elementary education majors who earned certification to teach middle and/or high school mathematics and who participated in my pilot study. For this study, secondary education included grades 5-8 for middle school and 9-12 for high school. Through my study, I addressed the following research questions:

1. How does secondary mathematics teachers' PCK change over the first year of teaching?

2. How do secondary mathematics teachers describe the development of their PCK before and during their first year as a teacher?

    2.1 How do beginning secondary mathematics teachers' experiences and views of their development of PCK change from institutional to professional learning of teaching?

3. What experiences and factors influence the development of secondary mathematics teachers' PCK?

    3.1 How does the development of PCK during the student teaching year transfer to their first year of teaching?

    3.2 What experiences and factors do beginning secondary mathematics teachers report supported or hindered the development of their PCK while in their first year of teaching?

To address the research questions, I collected data from multiple data sources: a PCK inventory, interviews, classroom observations, and a survey. Each source provided data for understanding how beginning secondary mathematics teachers developed their PCK and their perceptions of their development.

Significance of the Study

By understanding how PCK develops in beginning mathematics teachers and what factors contribute to PCK, teacher educators can develop a more robust "pedagogy of teacher education" (Feiman-Nemser, 2001; Korthagen, 2010; Loughran, 2006). With this knowledge, beginning teachers would be better prepared for entering the profession since their preparation would provide them with both a start competence and growth or "in-service" competence (Brouwer & Korthagen, 2005, p. 158). Start competence refers to the competence beginning teachers need as they enter the profession which continues to develop into in-service competence over the first years of teaching. In-service competence is the ability for teachers to continue their development as a teacher, and PCK specifically, in a self-sustained and self-directed manner. Researchers estimate between 20 and 50 percent of teachers leave the profession in the first five years with higher percentages associated with high-poverty and high-need areas (Guha, Hyler, & Darling-Hammond, 2017). Thus, another enquiry is whether teachers with a stronger developed PCK are more likely to be retained in the profession than those that have weaker PCK development (Price & Roth, 2011). Investigating practices that enhance and promote strong development of PCK would allow school leaders to develop a better understanding of what knowledge teachers possess at the start of their careers and how that knowledge develops and changes over the first year(s) of teaching. Shulman (1986) defines this period of transition as a person moving from an "expert student to novice teacher" (p. 8). This study will highlight areas of need within the curriculum, programs, and professional development. These changes will hopefully lead to better preparation of future mathematics teachers who will be equipped for the transition into the teaching profession.

This study was designed to contribute to the professional preparation of mathematics teachers and fill the gap in the literature about the transition of teachers from pre-service to in-service with focus on changes in PCK. It will highlight areas of need within the curriculum, programs, and professional development. These changes will hopefully lead to better preparation of future mathematics teachers who will be equipped for the transition into the teaching profession. Understanding this transition may help support and retain teachers in the long-term.

In the following chapter, I synthesize related literature to this study (Chapter 2). This review of the literature focuses on how research on PCK in secondary mathematics teachers has evolved over the past three decades, factors and experiences that influence PCK development, and characteristics of beginning teachers. Next, in Chapter 3, I explain my methodology including my research design, data sources, and analysis methods. In Chapter 4, I introduce my participants and discuss their experiences learning to be teachers. Chapter 5 is my analysis chapter, where I connect my results to the existing literature and explain the findings from my data analysis and answers to my research questions [changes in PCK development, perceptions, and contributing experiences]. Last, Chapter 6 is my discussion, conclusions, and meta-chapter where I propose implications for practice and directions for future research and reflect on my own PCK development as a teacher-educator and researcher through conducting this study.

CHAPTER 2

THE INGREDIENTS: INITIAL REVIEW OF LITERATURE

> The past can hurt, but the way I see it you can either run from it or learn from it.
>
> Rafiki, *Lion King*

In the previous chapter, I described the need for teacher educators and administrators to become more informed about how beginning secondary mathematics teachers develop their knowledge of teaching. To address the research questions, I reviewed literature regarding types and development of teacher knowledge, methods of teacher preparation, and beginning teacher development. This initial review of the literature is split up into three separate yet related sections, the first of which is about teacher knowledge domains and how this field of research has expanded since the 1980s. The second section focuses on what factors are known to influence PCK development. This section is organized around my initial conceptual framework, illustrating foundational experiences and opportunities linked to PCK. The third section synthesizes research on beginning teacher development. Finally, at the end of my literature review, I explain my theoretical framework as it is informed by the literature.

Teacher Knowledge Domains

Japan experienced a "Miracle Growth" period from 1953-1970 which fostered economic growth and increased the nation's presence as a global competitor (Duiker & Spielvogel, 2012). Additionally, Japan became an economic rival of the United States with a rise in automobile and technology exports in the 1980s (Crawford, 1998).

Declinist believed the United States' fall as a global leader was due to "scientific, technological, and educational factors" (Huntington, 1988, p. 76). Further, the report of the National Commission on Excellence in Education (1983), *A Nation at Risk*, indicted America's schools as failing with US students falling behind other nations on international comparisons. As a result, there was a push for increased STEM education and a greater focus on teacher knowledge and preparation.

Over the past decades, research on teachers' knowledge domains has expanded. One particular domain of knowledge, pedagogical content knowledge (PCK), has become a focal point in many disciplines and grade levels. This section is a review of the definitions and models of teachers' knowledge found in the literature with a specific focus on mathematics teachers over the past four decades. The available frameworks illustrate trends in contemporary research, models of interpreting and classifying data, and inform research design. I will discuss models and views of PCK that have developed from the 1980s to the present by looking at the prominent researchers and findings in the different decades. In addition, I will define the terms pedagogical content knowledge as framed for my research study and discuss different methods of improving or prompting PCK development in pre-service mathematics teachers, types of teacher preparation, and how first-year teachers develop.

Shulman (1986) was the first to present and define PCK as *specialized knowledge* possessed by teachers that enables them to effectively promote learning. He proposed that there are three categories of content knowledge that should be distinguished: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. Content knowledge refers to the facts, topics, rules, and "truths" of the domain; this knowledge should parallel the knowledge possessed by a sole content major (Shulman, 1986, p. 6). Pedagogical content knowledge refers to the knowledge of the subject matter necessary for teaching. Within this knowledge, there exists the ability to identify and utilize useful forms of representations, understand what makes learning a specific topic difficult, and discerning possible preconceptions or conceptions possessed about a topic. Lastly, curricular knowledge for a subject area is the knowledge of different programs, materials, and relationship between curriculums. How these knowledge domains are developed, relate to one another, and intersect continues to be a matter of debate and research.

Researchers have applied and re-conceptualized Shulman's original framework toward mathematics teachers (An, Kulm, & Wu 2004; Ball, Thames, & Phelps, 2008; Cummings, 2010; Hauk, Toney, Jackson, Nair, & Tsay, 2014; Lannin et al., 2013; Leong, 2013; Ma, 1999; Marks, 1990). Researchers have included other types of teacher knowledge, renamed domains, added linkages, and contextualized teacher knowledge for specific content areas, levels of teaching, and topics. Through an examination of the models of teachers' knowledge in the literature, ways of thinking about teacher knowledge are explained and illustrated. For example, how researchers describe the different domains of knowledge indicate if they are viewed as static or dynamic. Also, the degree of interconnectedness between the different domains shows the complexity of describing teacher knowledge and its many components.

1990s

With the 1980s' focus on quality and problem solving in mathematics education, there was a rise in constructivist approaches in teaching and research (e.g. Piaget, Vygotsky, Bruner, Gardner, and Goodman). This focus continued into the 1990s and prompted mathematics teachers and teacher educators to consider "the way we think and talk about mathematics learning" (Pejouhy, 1990, p. 6). At the end of the 1980s, the National Council of Teachers of Mathematics (NCTM, 1989) published the Curriculum and Evaluation Standards for School Mathematics. Additionally, NCTM (1991) released their Professional Standards for Teaching Mathematics shortly after. While the first set of standards focuses on the topics and organization of mathematics instruction, the second document focuses on teaching and professional development. Ball (1992) argues that this document was in response to failing reform efforts focused on improving mathematics education of the previous decades. This pivotal publication launched debates about what should be included in mathematics curriculums across the country and who was qualified to make those decisions. The debate of whether teachers are both designers and implementers of curriculums persists to this day. It is unclear where and with whom the responsibilities and expertise for curriculum design are located. These changes in the view of what and how mathematics should be taught again spurred research into teacher knowledge (Ball, 1992; Marks, 1990; Pejouhy, 1990).

Along with the publication of standards, the early 90s ushered in the formation of the Association of Mathematics Teacher Educators (AMTE) in 1991. The goal of the organization was to:

> provide a national forum … to discuss issues of mutual professional concern [and to] share ideas on effective ways of promoting the NCTM [National Council of Teachers of Mathematics] Standards, NCSM [National Council of Supervisors of Mathematics] and MAA [Mathematical Association of America]

recommendations on teaching school mathematics and developing programs to improve the mathematics education of practicing and future teachers. (Spikell, 1992, p. 1)

This organization is focused on improving mathematics teacher education and has contributed to both the conversations about and research in teacher knowledge development. The publication of the NCTM standards and the formation of the ATME steered the research agenda of the 1990s.

Marks (1990) used his study of fifth grade teachers to suggest modifications to how PCK is perceived by teacher educators. He also found there to be three main knowledge categories: knowledge of subject matter, general pedagogy, and pedagogical content knowledge. In addition, Marks (1990) was able to further explain mathematics teachers' PCK by identifying its composition into four specific areas: "subject matter for instructional purposes, students' understanding of the subject matter, media for instruction in the subject matter (i.e., texts and materials), and instructional processes for the subject matter" (p. 4) (see Figure 2.1). While these were separate components, he did find that they were highly interconnected. Additionally, Marks (1990) described that PCK can be primarily rooted in subject-matter knowledge, or pedagogical knowledge, or a mixture of both depending on the particular tasks a teacher is performing. If teachers are relying on their content knowledge, then they are using the process of *interpretation* to use their PCK. Alternatively, *specification* occurs when PCK that is derived from general pedagogical knowledge of teaching and learning experiences. Marks explained this process as "the appropriate instantiation of a broadly applicable idea in a particular context” (p. 8). On the other hand, when content and pedagogy knowledge are both equally necessary, then *synthesis* of the types of knowledge is occurring.

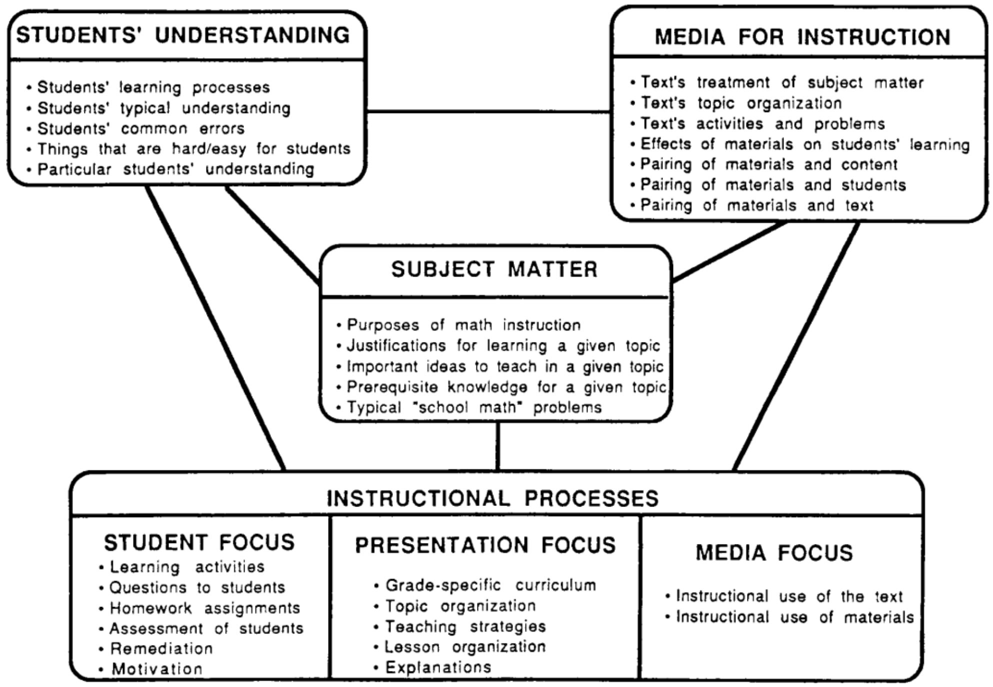


Figure 2.1. Framework of pedagogical content knowledge (Marks, 1990, p. 5).

While this review is focused on mathematics teachers, the research conducted by Grossman (1990) is widely cited by many researchers in different content areas due to the contributions to the understanding of PCK in general. In her study of PCK of beginning English teachers, Grossman (1990) describes four general areas of teacher knowledge: general pedagogical knowledge (PK), subject matter knowledge, PCK, and knowledge of context. General PK refers to the collection of general knowledge, beliefs, and skills related to teaching (examples in Grossman, 1990, p. 6). Similarly, subject matter knowledge encompasses the major facts or concepts central to a subject. Knowledge of context concerns to knowledge of districts, school settings, and specific students and communities. Knowing about a school’s culture or student backgrounds are concrete examples of knowledge of context. Lastly, Grossman expanded Shulman’s definition of

PCK to include knowledge of students' understanding, curriculum, instructional strategies, and purposes for teaching.

In the way Grossman (1990) researched PCK of English teachers, Gess-Newsome (1999) proposed two models of PCK as a result of her research with beginning science teachers: integrative and transformative. The integrative model does not consider PCK as a separate knowledge domain but as being made up of subject matter knowledge, pedagogical knowledge, and contextual knowledge. The transformative model "recognizes the value of a synthesized knowledge base for teaching" (Gess-Newsome, 1999, p. 12). According to Gess-Newsome (1999), PCK is the result of subject matter, pedagogy, and context being transformed and combined to form this new type of knowledge. The differences between the two models are based in how the types of knowledge are learned and taught and in how they are used and applied. For example, the organization of teacher preparation programs where students take separate courses pertaining to subject matter topics and pedagogy and then integrate them in practicum settings utilizes an integrative approach to developing PCK. On the other hand, programs where students engage in classrooms, and thus being immersed in the context, while learning content and pedagogy is a transformative approach. Both the integrative and transformative models have their strengths and weaknesses, which is why viewing PCK in these extreme forms is not the best. Instead, researchers should consider viewing PCK in relation to the other knowledge domains and not necessarily as a stand-alone type of knowledge. Further, when considering the models presented in the literature, having a way to compare and discusses how they are interpreting PCK (integrative or transformative) is useful and helpful for understanding PCK and its development.

2000s

Mathematics teaching in the 1990s was focused on problem solving, which impacted teacher preparation and the knowledge the mathematics teachers needed to possess. The turn of the new millennium continued the push for problem solving skills. Additionally, there was a movement toward finding balance between problem solving and skill work to counteract the focus of the 90s on problem solving side. In 2001, the reauthorization of the Elementary and Secondary Education Act was done under No Child Left Behind (NCLB). As a result, standards-based reforms and standardized testing grew with federal funding being tied to annual test scores. Additionally, a goal of NCLB was to improve teacher quality and increase accountability for state and local school districts. Again, the question of teacher knowledge and preparation were at the forefront of educational legislation.

An, Kulm, and Wu (2004) described the types of knowledge a teacher possesses as a network and specifically focused on a PCK framework that included three components: knowledge of content, knowledge of curriculum, and knowledge of teaching. These categories are "broader than Shulman's original designation" since they encompass both broad knowledge and specific knowledge (An et al., 2004, p. 147). Knowledge of content refers to both general content knowledge as well as knowledge about grade-specific topics. Using and selecting appropriate resources and materials as well as understanding the curriculums are part of knowledge of curriculum. Lastly, knowledge of teaching involves understanding student thinking and designing and delivering instruction. An et al. (2004) posit that while all three domains are important, knowledge of teaching is central and vital to PCK: "although all three parts of pedagogical content knowledge are very important to effective teaching, the core component of pedagogical content knowledge is knowledge of teaching" (p. 147). The researchers explain that the three knowledge domains are connected and interactive and that knowledge of teaching is influenced by knowledge of content and knowledge of curriculum.

Ball, Thames, and Phelps (2008) described a more detailed framework: Mathematical Knowledge for Teaching (MKfT). These researchers further divided the types of knowledge possessed by teachers to help clarify what is meant by PCK, indicating it includes knowledge of content and teaching, knowledge of content and students, and knowledge of content and curriculum. The similarities between the structures of Shulman's theory and the six elements of MKfT theory are shown in Figure 2.2.

Shulman	*MKfT*
Subject matter knowledge	Common content knowledge Specialised content knowledge Horizon content knowledge
Pedagogical content knowledge Curricular knowledge	Knowledge of content and teaching Knowledge of content and students Knowledge of content and curriculum

Figure 2.2. Comparison of areas of teacher knowledge in the Shulman and the MKfT frameworks (McAuliffe & Lubben, 2013, p. 158).

Additionally, Ball and colleagues included a diagram in their publication illustrating how their model maps to Shulman's original conceptualizations (Figure 2.3).

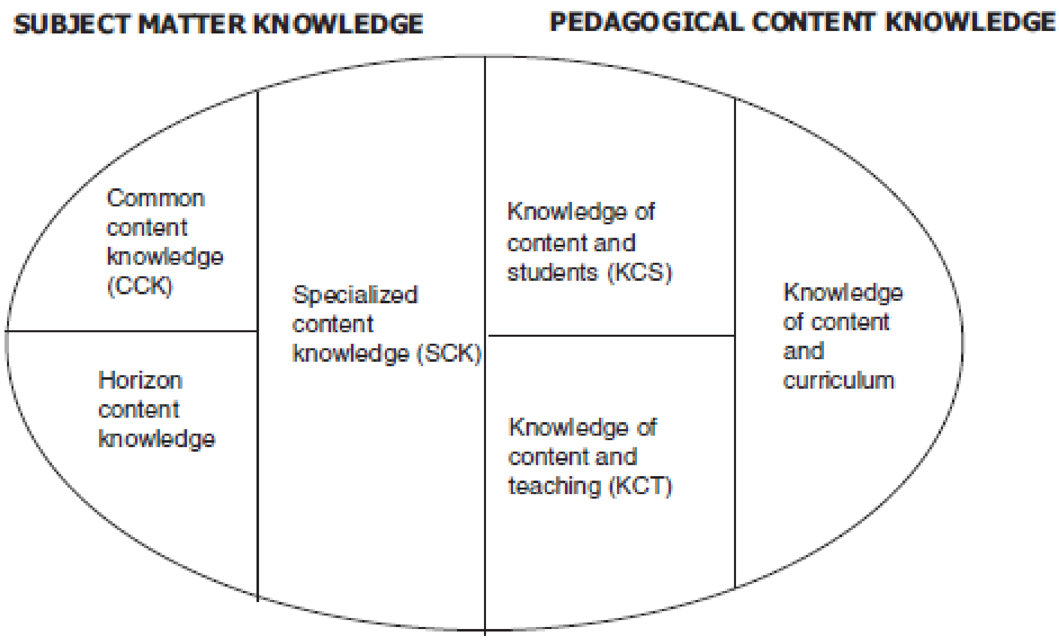


Figure 2.3. Domains of knowledge within the Mathematical Knowledge for Teaching (MKfT) Framework. (Ball, Thames, & Phelp, 2008, p. 403)

The organization of Ball and colleagues' design shows rigid divides between the knowledge domains and how their model maps onto Shulman's original categories of subject matter knowledge and pedagogical content knowledge. Within their description of their model, they explain how they placed Shulman's domain of curricular knowledge with PCK and justify this organizational decision by citing other researcher's organization structure (e.g. Grossman, 1990). The reasoning of the placement and size of the domains in the MKfT framework is unclear.

The original domain of content knowledge was divided into common content knowledge (CCK) and specialized content knowledge (SCK). Common content knowledge is defined as "the mathematical knowledge and skills used in settings other than teaching" or the ability to correctly solve problems (Ball et al., 2008, p. 399). Specialized content knowledge, on the other hand, is the mathematical knowledge uniquely possessed by teachers. Many tasks associated with SCK have to do with recognizing student errors, making connections between topics, constructing explanations, and interpreting methods of solving. In addition, the researchers divided what Shulman referred to as pedagogical content knowledge into knowledge of content and students (KCS) and knowledge of content and teaching (KCT). The ability to interpret student thinking, anticipating what students could perceive as confusing, and identifying areas of common student conceptions and misconceptions are all under the umbrella of KCS; this knowledge "combines knowing about students and knowing about mathematics" (Ball et al., 2008, p. 401). KCT is the merging of the knowledge about teaching and mathematics. For example, the order and sequencing of topics and selection of representations and instructional approaches are tasks associated with KCT. The domain of curricular knowledge from Shulman's model was grouped with PCK in the design by Ball and colleagues. Lastly, horizon content knowledge is the awareness of the interconnectedness of topics across grade levels. The specificity in this model makes it well suited for research and discussions by providing detailed descriptions about tasks associated with the different knowledge domains.

2010s

In 2008, The Great Recession, the worst economic crisis since The Great Depression, affected the United States and countries around the world. Increased unemployment rates, falling house prices, and other consequences caused funding to schools to be cut. As a result, districts had to lay off teachers, cut extracurricular activities, reduce professional development, and limit curricular offerings that were deemed non-essential for graduation (Hull, 2010). Teachers had to make due with fewer resources and more students, in most cases.

One year after the economy took a turn for the worse, the United States Department of Education launched its competitive grant program, Race to the Top (RttT). The goals of this grant focused on college and career readiness, data driven instruction, teacher effectiveness, and improving failing schools (U.S. Department of Education, 2016). The question of teacher quality and preparation was again at the center of legislative decisions and policy-making.

As part of the college and career readiness goal, the US Department of Education asked states to adopt higher standards and assessments. The Secretary of Education at the time stated that there was a “patchwork of 50 [sets of] state standards” that needed to be addressed and states should, instead, adopt common standards (Duncan, 2009). This lead to the Common Core State Standards Initiative; while the federal mandate does not specify which standards states should adopt, “forty-two states, the District of Columbia, four territories, and the Department of Defense Education Activity (DoDEA)” have adopted the CCSS, illustrated in Figure 2.4 below (CCSS, 2018). As a result of the organization and content in the standards, the depth and breadth of teachers’ content knowledge is once more in question.

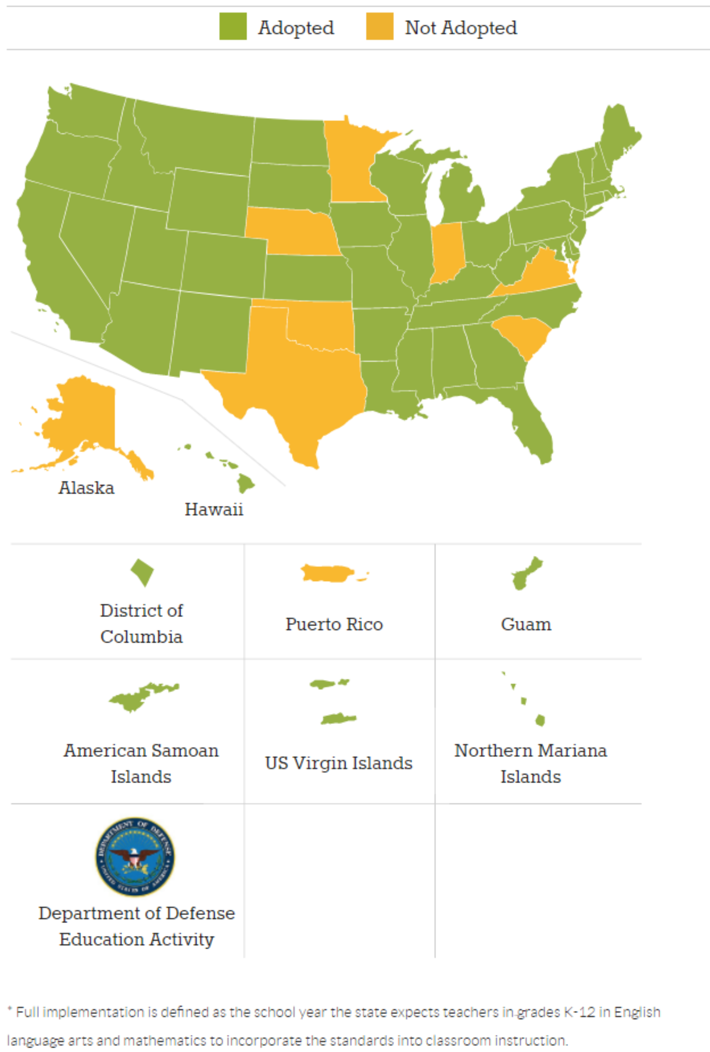


Figure 2.4. States, districts, and territories that have adopted the Common Core State Standards (CCSS, 2018).

In 2015, President Obama signed in the Every Student Succeeds Act (ESSA) which reauthorized the fifty-year-old Elementary and Secondary Education Act into its current form (U.S. Department of Education, 2018). Similar to NCLB, ESSA includes requirements around standardized testing and accountability for student progress. A primary goal of ESSA is for all students to be prepared for college and career. As a result of this act, states should be engaging in “curriculum design, access to materials, and educator development” that meet the needs of all learners (Darling-Hammond et al., 2016, p. 2). Again, teacher quality and knowledge are in question as teachers are central to designing and implementing curriculums and materials.

The Association of Mathematics Teacher Educators (AMTE), as discussed previously, are concerned with the preparation of mathematics teachers. According to AMTE (2017), “those involved in preparing teachers of mathematics must ensure that all their candidates have the knowledge, skills, and dispositions to provide all students access to meaningful experiences with mathematics” (p. xii). In response to this interest, AMTE published the first comprehensive standards for preparing k-12 math teachers. These standards are based on previous research and will stimulate researchers to further research less understood areas (ATME, 2017).

When constructing their framework of PCK of mathematics teachers, Lannin and colleagues (2013) adapted the model by Magnusson, Krajcik, and Borko (1999), initially developed for science teaching, and aligned it to elements of Ball and colleagues’ model. Magnusson et al. (1999) theorized that PCK includes five components: orientation toward science teaching, knowledge and beliefs about science curriculum, knowledge and beliefs about students’ understanding of specific science topics, knowledge and beliefs about assessment in science, and knowledge and beliefs about instructional strategies for teaching science. Lannin et al (2013) did not include orientation towards mathematics teaching in their model but did adapt the other four areas. Knowledge of student understanding within mathematics refers to, for example, “knowledge that students have difficulty developing meaning for mathematical notation” (Lannin et al., 2013, p. 406); this domain was aligned to KCS as defined by Ball et al. (2008). Similarly, KCT correlated to knowledge of instructional strategies for mathematics in these researchers’ model. Knowledge of curriculum from Ball et al. (2008) was more specifically called knowledge of curriculum for mathematics. This model bridged research between science and mathematics and provided yet another conceptualization of teacher knowledge.

Hauk, Toney, Jackson, Nair, and Tsay (2014) considered the duality of the perspectives present in the literature on PCK, stable versus dynamic, when designing their model of PCK. Starting with MKfT framework by Ball et al. (2008), these researchers elaborated on the linkages between the components and added Knowledge of Discourse: "the connections from Knowledge of Discourse to Knowledge of Curriculum *curricular thinking*, to Knowledge of Content and Students (KCS) *anticipatory thinking*, and to Knowledge of Content and Teaching (KCT) *implementation thinking*" (Hauk et al., 2014, p. A26). This additional component indicates the need to understand and consider how mathematics is communicated. Since discourse involves the socially constructed meanings of words and symbols, this added component to PCK illustrates the need for cultural considerations when researching the teaching and learning of mathematics.

Researchers throughout the past three decades have focused their efforts on understanding and describing PCK and its relation to other types of knowledge. Through their efforts it is clear that PCK is a specialized type of knowledge possessed by teachers. However, when considering CK of secondary mathematics teachers, how is it different than mathematicians' CK? Based on Shulman's original definition of CK, this knowledge should not be different between the two groups. Speer and King (2009) discuss that at times there is a blurry boundary between what is specialized and common content knowledge for secondary mathematics teachers. While both mathematicians and mathematics teachers take many content courses for their degrees, it could be argued that teachers need to possess more CK about more topics than mathematicians who specialize in a specific field. This would be an argument that mathematicians have more depth in specific topics of mathematics while teachers need to possess both breadth and a reasonable depth.

The different models of teacher knowledge found in the literature show the growth of research and understanding in this field. Figure 2.5 summarizes the components of PCK frameworks throughout the past three decades; gray indicates inclusion in researcher(s) frameworks while italics explain what the researcher(s) did to modify Shulman's original framework. The last column includes any additional domains that the researcher(s) included in their frameworks that were not in Shulman's framework. While no one model of teacher knowledge encompasses all factors, domains, and contexts, they do illustrate trends and ways of examining PCK and CK. For example, the tasks associated with the different knowledge domains can be operationalized and studied for particular populations. Ball's framework provides the domains of interest for my PCK Inventory since they are specific and have associated tasks.

Researcher(s)	Subject Matter Content Knowledge	Pedagogical Content Knowledge	Curricular Knowledge	Additions to Shulmans original Framework
Shulman (1986, 1987)				
Marks (1990)				+ General Pedagogical Knowledge
Grossman (1990)		Pedagogical Content Knowledge *Merged PCK & Curricular Knowledge*		+ General Pedagogical Knowledge + Knowledge of Context
Gess-Newsome (1999)				+ Pedagogical Knowledge + Contextual Knowledge
An, Kulm, & Wu (2004)	Knowledge of Content *Combined parts of CK with PCK*	Knowledge of Teaching	Knowledge of Curriculum *Combined parts of Curricular Knowledge with PCK*	
Ball, Thames, & Phelps (2008)- MKfT	Common Content Knowledge (CCK); Specialized Content Knowledge (SCK); Horizon Content Knowledge *Sub-divided Content Knowledge*	Knowledge of Content and Teaching (KCT); Knowledge of Content and Students (KCS) *Merged PCK & Curricular Knowledge then Sub-divided*	Knowledge of Content and Curriculum (KCC)	
Lannin et al. (2013)		Knowledge and Beliefs of Instructional Strategies for Mathematics; Knowledge and Beliefs of Student Understanding within Mathematics; Knowledge and Beliefs about Assessment in Mathematics *Sub-divided PCK*	Knowledge and Beliefs about Mathematics Curriculum	
Hauk, Toney, Jackson, Nair, &Tsay (2014)		Knowledge of Content and Teaching (KCT) *Mirrored Ball and colleagues framework*	Knowledge of Content and Students (KCS); Knowledge of Content and Curriculum (KCC)	+ Knowledge of Discourse +anticipatory thinking +curricular thinking +implementation thinking

Figure 2.5. Summary Table of PCK Research in the 1980s, 1990s, 2000s, and 2010s.

Elementary and Secondary Education Act (1965)
Education Consolidation and Improvement Act (1981)
Improving America's Schools Act (1993)
No Child Left Behind (2001)
Race to the Top (2009)
Every Student Succeeds Act (2015)

Transfer of PCK Research from the Elementary School Level to the Secondary School Level

The existing research on PCK development of secondary mathematics teachers during the transition from college to career is limited. Existing research includes international studies (Bukova-Güzel, 2010; Ensor, 2001; Even, 1993; Krauss et al., 2008; Leong, 2013; Lim-Teo, Chua, Cheang, & Yeo, 2007), which are not necessarily generalizable to teachers in the United States, other studies where only certain subjects or topics within subjects are the context under which PCK development is studied (Blasjo, Dalgamoni, & Roberson, 2010; Kinach, 2002), and studies focusing exclusively on either in-service (Cummings, 2010; Goss, Powers, & Hauk, 2013; Speer & King, 2009) or pre-service (Feiman-Nemser & Buchmann, 1986; Kinach, 2002; Kovarik, 2008) teacher populations. No studies where found that addressed the transition from pre-service to in-service with a focus on the development and changes in PCK. Additionally, there is a great deal of existing research about elementary teachers' PCK, which provides a good foundation for research, but is not entirely transferable to the context of secondary mathematics teachers.

Elementary School Level. Mathematics teachers' PCK has been studied historically and recently in the context of elementary education (Ball, 1988; Hill, Ball, & Schilling, 2008; Ma, 1999; McAuliffe & Lubben, 2013; Mulholland & Wallance. 2003; Noblet, 2016; Turner & Rowland, 2008). The focus on elementary school teachers could be due to the common belief that elementary school teachers are not as comfortable as secondary mathematics teachers with the content (Turner & Rowland, 2008) or the disagreement about the depth at which elementary school teachers need to know mathematics (Hill, Schilling, & Ball, 2004). The preparation elementary teachers receive is different than the preparation of secondary mathematics teachers, including the amount of content knowledge they are expect to have unless the candidates choose mathematics as their specialization. This difference alone warrants mentioning since what researchers find in terms of factors influencing elementary teachers' PCK may not be entirely applicable to secondary mathematics teachers, and thus, more research is needed in this area. During the 2011-2012 school year, less than 1% of elementary school teachers identified as having a specialization in mathematics (National Center for Education Statistics, 2013). This is unlike secondary mathematics teachers who do specialize in mathematics.

Speer and King (2009) analyzed the components of PCK used at the elementary level and compared and contrasted those characteristics with secondary and post-secondary teachers. Concerns raised by these researchers include (1) what should and should not be considered common knowledge for secondary mathematics teachers and (2) the work mathematicians and secondary mathematics teachers do is similar in nature, thus making the distinction of specialized content knowledge somewhat more difficult.

Speer and King (2009) state that "further research on teaching and teachers at secondary and post-secondary levels can help strengthen the literature base in this area by identifying aspects of current theory and definitions that are generalizable and others that are in need of refinement” (p. 9).

Secondary and Post-Secondary Level. The existing body of research on PCK at the secondary and post-secondary level tends to be topic-specific (Blasjo et al., 2010; Even, 1993; Saeli et al., 2012). While no individual study providing a comprehensive overview of PCK development of secondary mathematics teachers was found, collectively the current research provides a good basis of understanding PCK. In addition, it is important to note PCK is not entirely transferable between different topics (Sanders, Borko, & Lockard, 1993); therefore, understanding a teacher’s PCK in Algebra may not indicate an understanding of the teacher’s PCK in Geometry. Similarly, different PCK tasks could be more dominate in different topics for different teachers. An example of this would be a teacher has a very strong PCK with anticipating potential areas of confusion or difficulty in fractions but struggles with this same component in area and perimeter.

Further, current studies focus on either in-service or pre-service populations, indicating a lack of research about teachers’ pre-service to in-service transition with focus on changes in PCK – a gap my research will intend to address. Several studies have been conducted outside of the United States and are not necessarily applicable to teachers in this country. Mulholland and Wallace (2003) conducted a 4-year longitudinal study set in Australia about the transition from pre-service to in-service focusing on primary (elementary) teachers’ PCK in regards to science teaching. These researchers concluded "several features of science learning at university allowed the teachers to make tentative crossings into the subculture of science and feel confident about their preparation to teach science" (Mulholland & Wallace, 2003, p. 895). This illustrates that specific structural mechanisms can help foster growth in beginning teachers' PCK development.

Focusing on the development of PCK in pre-service and in-service German secondary mathematics teachers, Kleickmann and colleagues (2013) discussed the influence of pre-service teacher preparation on both content knowledge and PCK development. Findings of their study indicate that both CK and PCK develop over the course of teacher preparation with PCK continuing to develop during the student teaching period and working careers. However, these researchers also stated that in-service teaching "does not seem to contribute to substantial development of CK after initial teacher education" and that it weakly contributes to PCK development after initial teacher education (Kleickmann et al., 2013, p. 100). These researchers attribute this finding to the close tie of PCK to individual's CK and the type of professional development opportunities available to teachers.

Research on the Experiences and Factors that Influence PCK Development

Within the body of literature, factors identified as influential to the development of PCK include: teacher education (Grossman, 1990; Leong, 2013), previous experience with topics (Sanders et al., 1993), how teachers were taught mathematics as K-12 students and the role of previous teachers (Ball & McDiarmid, 1990; Leong, 2013; Lortie, 1975), the socialization of teaching (Brouwer & Korthagen, 2005; Korthagen & Lagerwerf, 1996), and teachers' subject matter knowledge (Ball, 1988; Even 1993). Each of these sources were found to impact PCK development and throughout all of them there were common foci on personal learning, subject matter knowledge, and reflection. For this reason, I have included these three factors as being central core components of PCK development in beginning secondary mathematics teachers. In my initial conceptual framework, which is has a tetrahedral organization, the base is composed of these three factors (Figure 2.6). While these are separate categories, there is overlap between them, as shown with the overlapping Venn diagram.

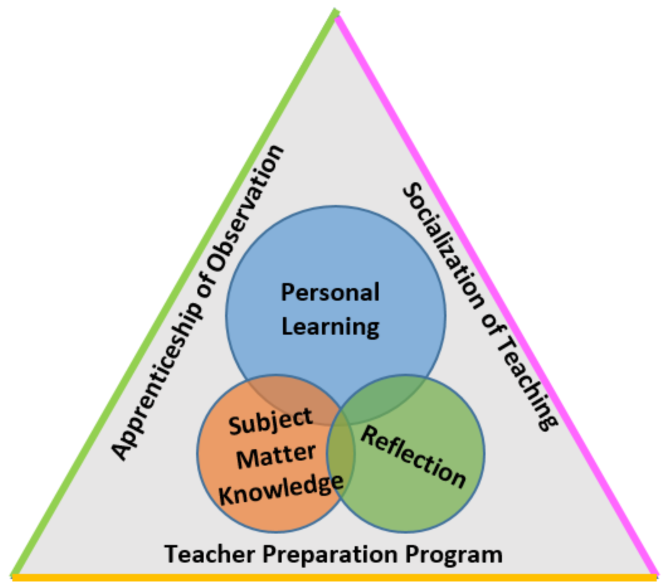


Figure 2.6. View 1 (the bottom) of Initial Conceptual Framework of influential experiences and factors on the development of PCK based on review of literature.

For example, personal learning occurs both when learning subject matter knowledge and other information. Also, reflection occurs in personal learning and in other contexts as well such as making sense of subject matter knowledge. These are not only on the surface of the conceptual framework, but are three-dimensional and extend within the tetrahedron showing their influence throughout PCK development. These three factors are internal to the individual and contribute to a candidate's Gestalt. Additionally, foundational experiences in developing the different components of PCK can be traced to these three central factors. This will be elaborated on in each of the following sections.

In addition to factors internal to teacher candidates, there are also cultural-environmental factors vying for influence over knowledge development. Among these are candidate's teacher preparation program, the socialization of teaching, and the apprenticeship of observation. These are illustrated in Figure 2.6 as surrounding the central core since they shape teacher candidates' perceptions of their knowledge and beliefs. The socialization of teaching refers to how pre-service teachers and in-service teachers are exposed to and internalize the norms, behaviors, and knowledge of teaching as a profession (Maloney, 2013). This process occurs through interaction with professors and practicum teachers, colleagues, and from images of teachers portrayed in the media, movies, and other sources. Similarly, the apprenticeship of observation encompasses how teacher candidates construct images and beliefs about teaching and learning from watching their own teachers. These ideas and the role of teacher preparation programs will be further explored later in this chapter.

Colors on sides of base correspond to the different sides of tetrahedron illustrating different components of PCK (Knowledge of Students, Knowledge of Content, Knowledge of Curriculum). This was done to help orient the viewer of the conceptual framework to how the base is positioned in reference to the sides.

Personal Learning

Beginning as students in K–12 classrooms, individuals start the process of being socialized into the teaching profession, shown as part of the base in Figure 2.6. This continues in their teacher preparation and when they have their own classrooms and experience the reality of schools (Feiman-Nemser, 1983; Lortie, 1975; Yeh, 2017; Zeichner & Gore, 1990). Future teachers experience at least sixteen years of schooling where they are exposed to teaching methods, classroom organizational techniques, assessments, and how teachers and students interact. Cultural scripts and mental models are constructed through these experiences which contribute to the perpetuation of common teaching practices (Stigler & Hiebert, 1998). These experiences are shown as the first layer foundation on each of the three sides of the tetrahedron (Figures 2.7 and 2.8), supporting teachers' knowledge of students, knowledge of curriculum, and knowledge of content. As a result of these experiences, pre-service teacher candidates entering preparation programs with a tacit image (i.e. Gestalt) of what teaching and learning is in their minds (Lortie, 1975).

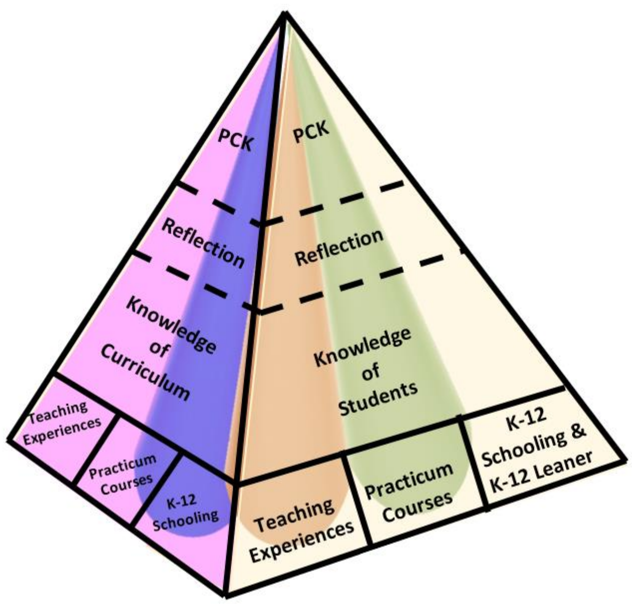


Figure 2.7. View 2 (showing 2 sides) of Conceptual framework of influential experiences and factors on the development of PCK based on review of literature.

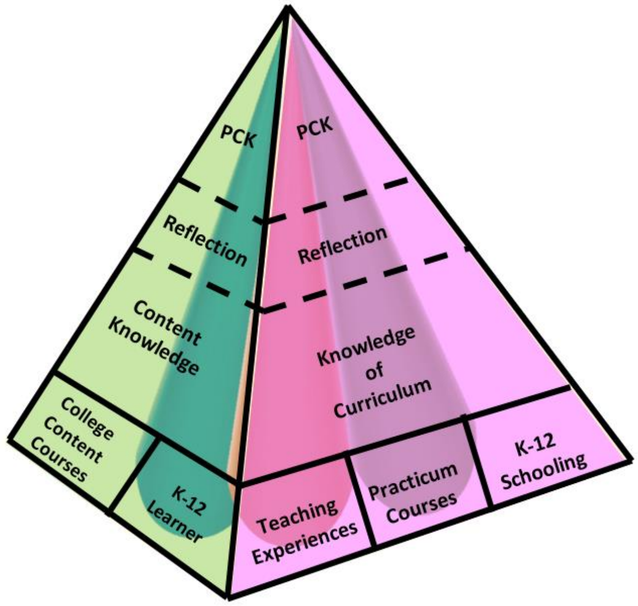


Figure 2.8. View 3 (showing 3rd side) of Conceptual framework of influential experiences and factors on the development of PCK based on review of literature.

Specifically, we can see personal learning occurring as a K-12 Learner, in practicum courses, in college content courses, and in teaching experiences. The difference between K-12 learning and K-12 schooling is the focus of the experiences. The “K-12 Learner” category is focused on individuals’ learning experiences, for example how they studied concepts and how they see themselves as learners. “K-12 Schooling” is focused on how their school was structured, the type of curriculum used, and resources available, for example.

Subject Matter Knowledge

Secondary mathematics teachers are expected to be experts at their content. Through the frameworks discussed earlier in this chapter, we can see how different researchers view subject matter, or content, knowledge as an essential component to PCK development. Teachers need to know what they are teaching before they can consciously decide how to teach it (Ball & McDiarmid, 1989; Gess-Newsome, 1999; Marks, 1990).

Subject matter knowledge is developed throughout a teacher candidate's educational career, starting in K-12 schooling and continuing in college. Schmidt, Burroughs, Cogan, and Houang (2016) analyzed course-taking patterns of pre-service secondary education majors around the world and found trends in what courses were taken: a calculus sequence including linear algebra, probability, and differential equations along with mathematics methods courses. This illustrates that secondary education teacher candidates pursuing certification to teach mathematics take a battery of mathematics courses to deepen their content knowledge. As discussed earlier in this chapter, it could be argued that teachers need to possess a broader subject matter knowledge about more topics than mathematicians who specialize in a specific field. This would be an argument that mathematicians have more depth in specific topics of mathematics while teachers need to possess both breadth and a reasonable depth. Additionally, this could be part of the argument of why mathematics teacher candidates need to take a multitude of content courses beyond merely the topics they will be teaching in the future.

Reflection

Reflection allows for individuals to make sense of experiences, connect new ideas and experiences to prior ones, and revise their thinking about situations based on new experiences. Accordingly, teacher candidates' reflective abilities are an essential skill that needs to be enhanced and practiced (Darling-Hammond, 2006; Korthagen, Loughran, & Russell, 2006; Loughran, Brown, & Doecke, 2001). Since engaging in a reflection process is an essential tenet of Realistic Teacher Education, this idea will be explored in more detail in a later section. It should be noted that the participants in this study learned to reflect systematically using the ALACT model in their preparation program

(Korthagen, 2002). This model consists of five phases: "(1) action, (2) looking back on the action, (3) awareness of essential aspects, (4) creating alternative methods of action, and (5) trial, which itself is a new action and therefore the starting point of a new cycle" (Korthagen, 2002, p. 5). Through reflection, individuals develop *growth competence* where they are able to reflect in-action as oppose to just on-action and think more critically about lesson design and resources. This means they are able to actively reflect in the moment and adjust their actions or thinking while in the moment. Ideally, beginning teachers enter the profession with start competence from their teacher preparation program (Brouwer & Korthagen, 2005). Start competence refers to the competence beginning teachers need as they enter the profession which continues to develop into growth or "in-service" competence over the first years of teaching (Brouwer & Korthagen, 2005, p. 158). Brouwer and Korthagen (2005) explain in-service competence as "an innovative type of competence encompassing teaching behaviors such as stimulating pupil activity during lessons, problem-based learning characterized by authentic contexts and materials, and cooperative learning" (p. 158). Beginning teachers can develop this type of competence by being reflective practitioners.

From involvement in this study, participants were prompted to think about their own PCK development thus stimulating the reflection process. As individuals reflected on their experiences, they developed different components on PCK. They made connections between their own experiences and how their students could experience learning. For instance, as participants reflected on how they learned mathematics and developed their own CK, they considered how they could mediate their students' interactions with mathematics to be positive and successful. Similarly, they considered what lessons, examples, materials, or resources were effective or ineffective with certain students and through this reflection are more capable of planning effective instruction in the future.

Teacher Preparation

The role of teacher preparation on PCK development has been linked to the type of preparation received and connected to elements specifically found within these programs. Not only is a teacher's preparation program at the base of their PCK development, as illustrated in Figure 2.6, it also provides opportunities that support PCK development. For example, teacher preparation programs utilize clinical experiences and practicums as a way for pre-service teacher candidates to learn and develop as teachers. These experiences, labeled "practicum courses," are found in first layer foundation on each of the three sides of the tetrahedron (Figures 2.7 and 2.8), supporting and influencing teachers' knowledge of students, knowledge of curriculum, and knowledge of content. The role of teacher preparation programs is discussed in more detail in the next sections of this chapter.

Teacher preparation programs vary across the United States and globally. The types of courses required and offered, the variety of placements pre-service teachers engage in, and the organization of the program all influence the development of types of teachers' knowledge. Further, teacher preparation and education has been a focus of reform and policy changes over the past few decades: Carnegie Task Force on Teaching as a Profession in the 1980s; the Holmes Group in the late 1980s and early 1990s; National Board for Professional Teaching Standards; and the Obama administration legislations such as the Higher Education Act (Darling-Hammond, 2010; Shulman, 1986;

U.S. Department of Education, 2010; U.S. Department of Education, 2016; Zeichner & Liston, 1990).

Across the United States and worldwide, there are different pathways into teaching. According to Goldhaber, Liddle, and Theobald (2013), there are over 2000 traditional teacher preparation programs in the United States alone. Models of teacher preparation programs differ in their duration, structure, populations they serve, and location or affiliations. Within university-based programs, there exist undergraduate, graduate, and combined undergraduate/graduate models. Additionally, non-university-based routes include (a) substitute teaching; (b) private school teaching; (c) alternative route program (Peace Corps, Teach for America, Teacher Opportunity Corps); (d) no prior experience; or (e) other pathways (Darling-Hammond, Chung, & Frelow, 2002). Alternative teacher certification programs are titled as such since they "provide alternatives to the traditional 4-year undergraduate program path to certification" (Darling-Hammond et al., 2002, p. 287). A variety of pathways into the teaching profession have a spectrum of benefits, including meeting the needs of different candidate populations and potentially reducing the demand for teachers in high need areas quickly. However, the effectiveness of the different pathways on teachers' PCK development still needs further research and analysis.

Existing research on teacher preparation programs varies in methodology, design, location, and focus. Few studies have focused on how different preparation programs impact on PCK development; however, results of studies include references to components of PCK (Andrew, 1990; Darling-Hammond et al., 2002) or use elements of PCK to design frameworks for teacher education (Darling-Hammond, 2006; Feiman-

Nemser, 2001). Studies interested in the connection between teacher preparation and PCK development are from various countries outside of the United States and have called for more research in this area (Kleickmann et al., 2013; Schmidt et al., 2016). In addition, studies examined the influence of teacher preparation type on student achievement (Darling-Hammond, Holtzman, Gatlin, & Heilig, 2005; Gansle, Noell, & Burns, 2012; Koedel, Parsons, Podgursky, & Ehlert, 2015), teacher efficacy (Darling-Hammond et al. 2002; Hoy & Spero, 2005), teacher retention and satisfaction (Andrew, 1990), and other characteristics of teachers and teaching. Together these studies show that not only does the type of preparation matter, but the elements of the programs themselves contribute to the development of PCK.

Research on Program Models and PCK Development

Different types and models of preparations, traditional or non-traditional, influence aspects of teachers' PCK such as teachers' knowledge of curriculum, content, and learners (Andrew, 1990; Darling-Hammond et al., 2002). Andrew (1990) analyzed the effects of a 4-year and 5-year preparation programs at the University of New Hampshire. Though the research article is over ten years old, UNH still has the same programs described in the study (UNH, 2018). While both the 4-year and 5-year programs are at UNH, the designs of the programs differ. For example, the duration of student teaching is essentially doubled in the 5-year program with increased and more frequent supervisor visits. Andrew found that graduates of the 5-year program self-rated higher in areas that could be classified as knowledge domains of PCK as described by Ball and colleagues in the framework for PCK called Mathematical Knowledge for Teaching (MKfT) (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008).

Specifically, Andrew (1990) found areas that would be considered components of Knowledge of Content and Teaching (KCT) and Knowledge of Content and Students (KCS). The difference in graduates' self-evaluation is contributed to "higher entry standards" and the length of the program resulting in "more students with high commitment to teaching" (Andrew, 1990, p. 50). The types of candidates that go on to pursue the 5-year degree could possess different characteristics, resulting in different knowledge development. It is important to consider the structural and candidate differences when analyzing the impact of the preparation programs on teacher knowledge development.

Branching out from a single institution, studies investigated the effects of multiple pathways into teaching on various teacher characteristics and student achievement. The findings from these studies are not solely focused on PCK development. However, I will be focusing on the results found in regards to PCK development. Darling-Hammond and colleagues (2002) investigated the influence of different pathways, such as traditional university-based and non-traditional, non-university based, on New York City teachers' preparation. These researchers stated:

> The contributions made by teacher education programs are most noticeable with respect to the core tasks of teaching, such as the ability to make subject matter knowledge accessible to students, to plan instruction, to meet the needs of diverse learners, and to construct a positive learning environment. (Darling-Hammond et al., 2002, pp. 295-296)

These "core tasks of teaching" are all elements of PCK described in the MKfT framework (Ball et al, 2008; Hill et al, 2008). Darling-Hammond et al. (2002) found that while programs prepared teachers in some ways, no one program prepared professionals sufficiently in all aspects. In another study, Darling-Hammond et al. (2005) looked at the effects of preparation again, but this time with teachers in Houston, Texas. These researchers found that while there was some success for teachers from the Teach for America program and other alternative programs, “students achieved stronger achievement gains in both reading and mathematics when they were taught by standard certified teachers rather than uncertified teachers” (Darling-Hammond, Holtzman, Gatlin, & Heilig, 2005, p. 22).

Brouwer and Korthagen (2005) looked at the relationship between teacher preparation and teaching competence. These researchers acknowledge that beginning teachers will continue to grow during their first years of teaching and programs should equip candidates for “entry into the teaching profession” (Brouwer & Korthagen, 2005, p. 158). While this study was not focused specifically on PCK development, aspects of PCK can be found throughout their analysis. For instance, it is discussed that

> Teachers should be able to go beyond transmitting and having pupils reproduce what is in the standard textbooks (see Bolhuis, 2003). This means that teachers should have a command of the knowledge structures characteristic of the scientific disciplines underlying their school subject as well as the capacity to select, structure, and present learning content in forms learnable by the specific groups of pupils they teach. (Brouwer & Korthagen, 2005, pp. 162-163)

This signifies that teachers need to know how to sequence topics, select appropriate representations and materials, and consider students’ thinking processes, which are all elements of PCK. The activities in the teacher preparation programs of this study became increasingly more complex, building on prior knowledge and experiences. All student teaching, in this program, was completed in triads of student teachers. This was found to foster PCK development since they were able to observe and give feedback to each other and collaborate on lesson construction.

Researchers are aware that analyzing certification type is a "proxy for the real variables of interest that pertain to teachers' knowledge and skills. These include knowledge of the subject matter content to be taught and knowledge of how to teach that content to a wide range of learners" (Darling-Hammond et al., 2005, p. 20). As I have seen during the preparation of my pilot study and dissertation research, measuring teacher knowledge is difficult and imprecise. Thus, I am not surprised that researchers are using certification as a way of determining teacher knowledge. However, teacher knowledge is not merely a degree, piece of paper, or static test score; it is ever changing depending on experiences, resources, challenges, and students.

While alternative pathways help meet the demand for teachers in terms of quantity, their preparedness, retention, and quality tend to suffer (Darling-Hammond et al., 2002). Darling-Hammond, Chung, and Frelow (2002) investigated how different pathways to teaching influenced teachers' preparedness and personal views on five factors: preparedness to promote student learning, teaching critical thinking and social development, using technology, understanding learners, and developing instructional leadership. Teachers who went through a university-based program to earn certification "felt better prepared than noncertified teachers on every factor except preparation to use technology" (Darling-Hammond et al., 2002, p. 288). As the researchers disaggregated the data, it was clear that program differences showed greater variance than within-group variance, meaning the effects of the different programs overshadowed participant differences. Brouwer and Korthagen (2005) argued that beginning teachers will continue to grow during their first years of teaching. The types of experiences candidates have in their preparation program influence how they navigate learning opportunities in their first year and their development in different competencies (Brouwer & Korthagen, 2005). This illuminates the point that teacher preparation programs have a positive impact on teachers' knowledge when entering the workforce.

Focusing on the development of PCK in pre-service and in-service German mathematics teachers, Kleickmann and colleagues (2013) discussed the influence of pre-service preparation on both content knowledge and PCK development. For their study, they used a cross-sectional comparison where data from different groups are collected at the same point in time. These groups were students in years 1 and 3 of their teacher education programs, teacher candidates at the end of student teaching, and experienced teachers. In addition, the researchers looked at pre-service teachers in both academic and non-academic tracks of teacher preparation, meaning teachers are prepared separately depending on if they plan to teach in academic- or nonacademic-track schools. These groups were selected to show how PCK and CK change over the course of teacher preparation and time teaching. Also, the group selection was used to determine if the type of preparation received influence development. The findings of their study indicate that both CK and PCK develop over the course of teacher preparation with PCK continuing to develop during the student teaching period and working careers. However, pre-service teachers in the academic-track initially started the program with different levels of CK and showed higher gains than those in the nonacademic-track. On the other hand, participants in both tracks were similar in their initial PCK development and growth during their time within the programs, but the academic-track teachers showed higher gains during student teaching and then during their in-service work. These finding indicate that in-service teaching "does not seem to contribute to substantial development of CK after initial teacher education" and that it weakly contributes to PCK development after initial teacher education (Kleickmann et al., 2013, p. 100). As a result, the researchers call for targeted professional development to help continue the growth started in teacher preparation.

Referencing the internationally conducted Teacher Education and Development Study—Mathematics (TEDS-M), Schmidt, Burroughs, Cogan, and Houang (2016) discussed how variation in CK and PCK can be attributed to the coursework required by different preparation programs. These researchers found trends in course requirements around the world for what they deemed as "A+ programmes." In creating this international benchmark for mathematics teacher preparation programs, the researchers analyzed the coursework required and completed by the graduates who would be teaching lower secondary grades of the top performing countries on the mathematics content assessment. They found common trends in what courses were taken: a calculus sequence including linear algebra, probability, and differential equations; mathematics methods courses; opportunities in courses for observation, analysis, and reflection on mathematics teaching; and "one school-level mathematics course covering algebra, trigonometry and analytic geometry" (Schmidt et al., 2016, p. 6). As a follow-up study, researchers surveyed US teachers and found there exists a significant relationship between the preparation teachers receive and their perceptions of ability to teach mathematics.

Further, they found that many teachers "do not receive internationally competitive mathematics training before they enter the classroom" and that there is a very clear distinction between the preparation of elementary and secondary teachers (Schmidt et al., 2016, p. 17). Since mathematics builds on previously learned topics within a given domain, it is important that all teachers receive a strong preparation. With this study's focus on traditional teacher preparation programs, it was helpful to see what types of courses were similar across programs and how the increase in requirements—both in mathematics coursework and in methods instruction—influenced CK and PCK. Later, when I discuss the elements of teacher preparation programs, this international comparison of coursework will be useful.

Overall, the literature identified certain elements of teacher preparation programs as influential to PCK development. Most notable elements are GPA, program design, coursework, and the variety of experiences pre-service teacher have. It is of interest that these are all components that preparation programs have some control over. In the previous section I reviewed models of teacher preparation programs and their association with PCK development. In the next section, I will discuss implications of specific program elements in relation to PCK and its development.

Program Elements and Implications on PCK Development

Based on available literature about the impacts of teacher preparation on teacher knowledge development, teacher educators need to consider the types of knowledge candidates will need, the types of experiences they should have, and techniques they should practice when designing preparation programs. A review of how programs use essential elements to embed PCK components in their curriculum will be discussed in this section. I will also be reviewing common trends in these elements and discuss implications on development of teachers' knowledge.

When one considers elements of effective teacher preparation programs, it is clear that having pre-service teachers engage in systematic experiences in schools is essential to their development: "Extensive and intensively supervised clinical work—tightly integrated with course work—that allows candidates to learn from expert practice in schools that serve diverse students" is critically important (Darling-Hammond, 2006, p. 307). The integration of these experiences with education coursework further help to promote growth and development. Korthagen, Loughran, and Russell (2006) state that "teacher preparation needs to focus on how to learn from experience and on how to build professional knowledge" (p. 1025). Thus, pre-service teachers need to have experiences with diverse students and with a variety of topics. Being focused on where students complete clinical experiences and with who is essential to ensure what they see in classrooms aligns to the coursework of the preparation program. However, procuring quality placements can be difficult, which is why it crucial for teacher preparation programs to have a collaborative relationship with area schools. This relationship should be reciprocal, in that the schools get the benefit of having university support, resources, and access to research-based practices while teacher preparation programs have sites to place their students where there are best practices being implemented.

Darling-Hammond (2010) argues that teacher education is the core of the nation and that the future of the Unites States depends on investing in teaching. In addition to having an impact on employment and teaching ability, there are social, political, and economic implications connected to teacher preparation (Darling-Hammond, 2010;

Zeichner, 1999). However, some teachers view their preparation as insufficient for the actual work they do daily. Loughran et al. (2001) state that "it is common to hear them [experienced teachers] speak about teacher education as being a 'waste of time' or something that had to be done to 'get the piece of paper'" (p. 11). These sentiments are a result of how they were prepared and what comes as a "shock to the system" when they are engaged in full-time teaching (Loughran et al., 2001, p. 13). Programs should respond to the concerns held by candidates through self-evaluation and by working with graduates and area schools.

While a teacher preparation cannot prepare teachers for everything they will encounter in their careers, it is important that future teachers be equipped with the abilities of reflection, research, and collaboration (Darling-Hammond, 2006; Korthagen et al., 2006; Loughran et al., 2001). With these skills, teachers will enter the profession with the abilities necessary to locate and use resources, meet the needs of diverse learners, plan effective lessons, and have a support system. For example, Brouwer and Korthagen (2005) state

> The beginning teachers' reflection on their work helped them improve their professional competence in the following ways: making instructions and pupil assignments more precise, clarifying subject matter, activating pupils in more open types of discourse and through a stronger call for individual and group work, and improving their interpersonal relationships with classes and students, most of all by avoiding conflict about rules for behavior. (pp. 209-210)

These skills are difficult to teach in isolation as they are more effective if they are integrated within experiences, "the learning of student teachers is only meaningful and powerful when it is *embedded in the experience* of learning to teach” (Korthagen et al., 2006, p. 1030). Pre-service teachers can take these skills with them into the classroom and continue to grow and learn with each new experience over the course of their careers.

In addition to having integrated clinical experience, it has been discussed in the literature about having a cohesive and developmental progression of coursework for pre-service teachers to go through. Programs consisting of discrete courses that were not part of a cohesive and integrated curriculum were found to be weak in promoting change in practices among new teachers (Zeichner & Gore, 1990). Sequenced coursework in which courses intersect and build off of each other have been found to be highly successful (Darling-Hammond, 2006). In addition, it has been suggested that connecting subject matter learning with pedagogy explicitly through coursework will promote teachers’ knowledge development (Darling-Hammond, 2006; Schmidt et al., 2016).

The development or reform of teacher preparation programs should not be done in isolation; stakeholders from the university and schools should have input into the design. Loughran and colleagues (2001) state, “teachers in schools and Faculties of Education need to continually work together to enhance learning about teaching of our students of teaching” (p. 22). This also illustrates the point that changes should not become fossilized in the institutions, but they should be continually evaluated to determine their effectiveness, how they are meeting the needs of both the pre-service teachers and larger community, and whether the content and instructional methods are still current and up-to-date.

Realistic Teacher Education. Teacher preparation programs around the world are modifying their programs and adopting what Korthagen (2002) terms *Realistic*

Teacher Education (RTE). The core idea of realistic teacher education is that instruction is centered on the experiences and concerns of the individual candidates. Additionally, there is a constant back-and-forth between action and reflection to make sense of what is occurring in those experiences and to learn from it. The ALACT model is primarily utilized as the reflection tool and consists of five phases: "(1) action, (2) looking back on the action, (3) awareness of essential aspects, (4) creating alternative methods of action, and (5) trial, which itself is a new action and therefore the starting point of a new cycle" (Korthagen, 2002, p. 5). Korthagen (2002) includes a model (Figure 2.9) of what the ALACT process looks like.

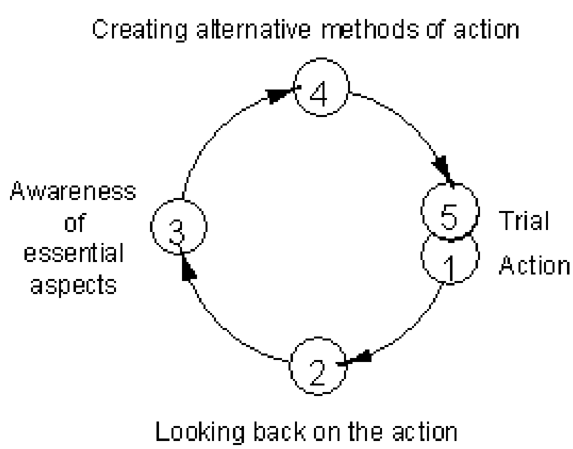


Figure 2.9. The ALACT model describing the ideal process of reflection. (Korthagen, 2002, p. 5).

Programs utilizing a realistic approach to teacher education include certain elements in line with the main tenets of RTE. Once such principle is that there is an integrative nature to the program coursework. For example, all course offerings should not be separated by topic, but rather build in a progression based on the experiences of the candidates. Similarly, practicum experiences should be tightly woven into the pedagogy and theory courses. As such, Korthagen (2002) states “frequent alternation of school teaching days and meetings at the teacher education institute” where the ALACT reflection process is utilized is necessary to the development of teacher knowledge. This also illustrates the need for practicum experiences to occur in appropriate settings, as discussed earlier in this chapter.

Characteristics of Beginning Teachers

Research on the characteristics of first year teachers describe different ways in which these individuals orient themselves to the tasks of teaching. One primary concern of beginning teachers that has persisted throughout many decades pertains to classroom management (Barrett & Davis, 1995; Melnick & Meister, 2008; Veenman, 1984; Wolff, Jarodzka, & Boshuizen, 2017). Melnick and Meister (2008) state "the greatest concern of all the new teachers was their inability to deal with the aberrant behavior and diverse needs of some students" (p. 2007). Additionally, beginning teachers feel pressure in terms of time including the time needed to plan and complete paperwork.

Research has shown that beginning teachers experience an attitude shift when they enter the profession (Brouwer & Korthagen, 2005; Loughran et al., 2001; Veenman, 1984; Wideen, Mayer-Smith, & Moon, 1998). Because of this "shock" to their systems, many teachers "struggle for control and experience feelings of frustration, anger, and bewilderment" (Brouwer & Korthagen, 2005, p. 155). After leaving their preparation programs, beginning teachers feel the isolation of teaching. Mentor programs and collaboration with colleagues can curb these feelings with a positive influence on development and retention. Brouwer and Korthagen (2005) found "the more the beginning teachers experienced collaboration with colleagues as beneficial (*collaboration* *construct*), the more they practiced a variety of teaching activities (*variety construct*)" (p. 186). On the other hand, these researchers found that there are obstacles to teacher development including: use of prescribed textbooks, high number of hours taught per week, and lack of time and collaboration with colleagues. These obstacles can influence how and change the direction of teacher development.

Oosterheert and Vermunt (2002) discuss the ways in which teachers orient themselves to learning and interpreting their experiences. The *inactive/survival orientation* are focused on getting more teaching experiences without necessarily learning from them. In the *closed reproduction orientation*, teachers use their pre-existing knowledge to improve their teaching and are largely focused on overcoming negative teaching experiences. The third orientation discussed, *closed meaning*, is focused on improving their teaching through feedback from others. These teachers are also concerned with negative teaching experiences but actively work to improve their practices. The *open meaning* orientation was the last type identified and, as the name implies, are receptive to learning opportunities. These different orientations dictate how beginning teachers navigate their experiences, what they see as worthy learning opportunities, and their knowledge development.

Defining PCK Operationally

Based on a review of the literature, an operationalized definition of PCK has been adapted from Nardi, Jaworski, and Hegedus (2005). A teacher with a well-developed PCK is able to construct/design an effective and coherent learning trajectory for a given student or group of students based on social, emotional, and cognitive learning needs and background. This can be observed, in part, by assessing to what extent teachers:

- explain students not learning as being placed outside of the teacher's control (locus of control);
- acknowledge a student's difficulty and attempt to analyze this difficulty (reflection in-action and on-action, dealing with unanticipated thinking) (Schön, 1987);
- make connections between mathematics topics (e.g. activating prior knowledge);
- ask probing questions to understand student thinking (reflection in-action);
- demonstrate awareness of common student conceptions, misconceptions, and difficulties (anticipate student thinking and prepare responses); and
- select developmentally appropriate teaching strategies for development level of students and content.

(Adapted from Nardi, Jaworski, & Hegedus, 2005)

Theoretical Framework

The initial frame for this research study is social constructivism (Fosnot, 2013; von Glasersfeld, 1996; Vygotsky, 1978). This perspective acknowledges the role that interest, peers, and community have on development and learning. Pre-service teachers engage in practicums, student teaching, and work with each other, college supervisors, and their cooperating teachers, all of which contribute to the development of their PCK. Similarly, in-service teachers work with colleagues and mentors further influencing their PCK development.

Some of the most powerful (and often overlooked) learning experiences teachers have is from when they were students themselves. The ideas about teaching that teachers develop through these experiences is referred to as the apprenticeship of observation

(Lortie, 1975). These experiences contribute to mental images of teaching, referred to as cultural scripts, that explain patterns of behaviors (Stigler & Hiebert, 1998). One role of teacher education is to help pre-service teachers develop an understanding of, and ways of thinking about, teaching that could differ greatly from their own learning experiences (Darling-Hammond, 1998; Hammerness et al., 2005). Teacher candidates may need to confront these cultural scripts or be presented with alternative scripts to grow and change. Teacher educators may use conceptual change theory as a framework for designing programs, curriculums, and assignments. Conceptual change theory is a theory of learning concerned with how to change pre-existing conceptions or Gestalt (Davis, 2001; Korthagen & Lagerwerf, 1996). This cognitive psychology theory indicates the need for pre-service teachers to have opportunities to test out ideas and develop their different understandings and beliefs. Without chances to be confronted with problems in their thinking, teachers may never feel compelled to grow and continue to learn (Davis, 2001). Teachers can construct their own knowledge in a deliberately constructed learning environment that takes the teacher as a person into account (Korthagen, 2004).

A teacher's experiences, both in teaching and in learning, influence the evolution of the "dynamic and holistic unity of needs, feelings, values, meanings and behavioral inclinations triggered by an immediate situation," referred to as Gestalt (Korthagen & Kessels, 1999, p. 9). Teachers bring their own feelings and experiences to new situations, which orients them on different topics. Specifically, teachers tend to use themselves as the model for the students they will encounter (Grossman, 1990). It is argued that the information and skills necessary to make effective teaching decisions come from the "context of practice" (Hammerness et al., 2005, p. 374). Teachers can develop their ability to anticipate potential areas of confusion, an aspect of PCK, by comparing it to their own learning experiences or by working with groups of students (Hauk, Jackson, & Noblet, 2010; Saeli et al., 2012). However, these experiences are not necessarily transferable to different situations with different groups of students nor may their experiences of learning be the same as those of their students.

Together, these theoretical frameworks will initially inform my perspective of the learning and teaching of beginning secondary mathematics teachers and how they develop their PCK. These theoretical components are also amalgamated in the tenets of Realistic Teacher Education (RTE), including the candidates' concerns being central to coursework and curriculum, the integration and back and forth connections made between theory and practice, and that theories are rooted in the experiences, episteme, of the teacher candidates (Korthagen, 2010; Korthagen & Kessels, 1999; Loughran, 2006). RTE is firmly grounded in social constructivism. Figure 2.10 below illustrate how the different components from these theoretical perspectives come together to form my theoretical framework.

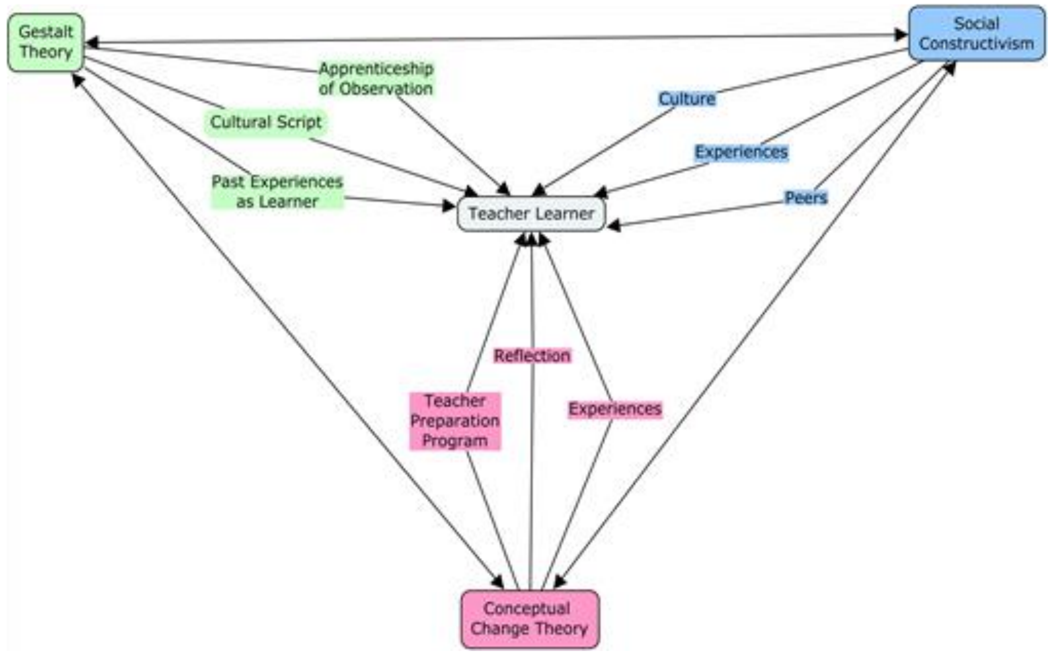


Figure 2.10. Theoretical Framework.

In this chapter I have discussed relevant literature that set the foundation for my study. Specifically, I synthesized studies on the historical development of research on teacher knowledge domains, existing research on PCK development at the elementary and secondary levels, and experiences and factors that influence PCK development. I have also illustrated and explained the theoretical framework that guided this study and the conceptual framework of PCK development. As this was an initial review of literature, additional literature will be integrated in Chapter 5, Analysis, and Chapter 6, Discussion, Conclusions, Implications, and Reflection. In the next chapter, I will explain the methodology used in this study. The participants will be introduced, their experiences with regards to learning and teaching, and their PCK development will be summarized in Chapter 4 and further analyzed in Chapter 5. Lastly, I will discuss the findings and recommendations and reflect on my own PCK development as a teacher educator and as a researcher through conducting this study in Chapter 6.

CHAPTER 3

THE RECIPE: METHODOLOGY

> "In every job that must be done, there is an element of fun. You find the fun and—snap!—the job's a game!"
>
> Mary Poppins, *Mary Poppins*

As I considered *what* I wanted to study, it also became apparent that the *how* would follow. Since I was always a "math person," people expected me to utilize quantitative analytical methods in my research. However, due to the kinds of questions I was asking bourn from my experiences with pre-service teachers, I found myself drawn to approaches where their stories were at the center. Reflecting on my own path to teaching and my experiences in teaching brought me to the question of how teachers develop their knowledge for teaching. How do teachers understand what they are teaching? How can they enact this knowledge in their teaching of mathematics to students with diverse backgrounds, needs, and understandings? Before explaining the research design of this study, I will first introduce myself as a researcher and provide some background knowledge about myself.

Meet the Researcher

I am a white, middle-class woman who is the oldest of five children. I attended public schools and graduated from the same teacher preparation program my participants completed. People in my social and professional circles believed math came easily to me, which was not the case in my experience. I always had to work at my content knowledge and am still learning and deepening this type of knowledge. After graduating from my preparation program, I gained employment at a local high school and was able to work with a wide variety of students. While pursuing my Master's degree in secondary education, I worked as a graduate assistant at my former higher education institution, where I am now a lecturer.

I share some of these details about myself since, in qualitative research, the researcher is the instrument so it is important that I explain my background and potential biases. Through my own experiences, I believe an individual's background, interests, and experiences influence their future interactions, beliefs, and actions. Aside from enjoying teaching and mathematics, I am also an avid baker. As such, I will convey my researcher identity through an analogy with baking.

Baking has a typical set of procedures that you follow, akin to a research methodology. Similarly, you can experiment with your ingredients but there are foundational things that you cannot change, which I equate to theoretical perspectives. For example, to start most recipes you need flour, sugar, eggs, oil, and leavening agents. From there, you can add other ingredients to change the flavor of what you are baking. The processes of baking take practice and time to master. When you bake something for the first time, you follow a recipe closely—measure each ingredient precisely, reread each step, and follow it to the letter. As you develop comfort with a recipe, you do not need to refer back to it for everything and eventually you internalize it. This automaticity with baking is something I equate to learning to be a researcher. While I learned how to conduct qualitative research, I would refer back to my course notes and readings to ensure that I was following procedures accordingly. As I have become more comfortable with qualitative research methods, I am able to recall coding procedures, for example, without having to refer back to my notes. All researchers bring their own theoretical perspectives and background to their research. I believe that people construct knowledge through interactions with others and through exploration; I bring these perspectives to my research.

As discussed earlier, there are certain ingredients that are necessary for baking most things. In addition to those foundational ingredients, what a baker chooses to add is at her discretion. These items are what makes each baked good unique and different. The sweet ingredients added, like chocolate, are what I consider to be the success I've had in teaching and in research. These are learning opportunities that have shown what can go well and things I can use in the future. With most sweet ingredients, there needs to be a balancing addition, usually of salt. While salt is tart and is not usually thought of as an essential ingredient for baking, I think of this as my ineffective lessons or dead ends that are still learning experiences. Sometimes, these "tart" experiences serve as better learning opportunities than the "sweet". As a qualitative researcher, I need to be receptive to the data I receive, even if it conflicts with what I previously thought.

In order to measure and combine the ingredients together, there are tools that a baker needs to utilize. Measuring cups and spoons are useful to make sure you are not distorting your flavors, which are similar to participant and member checking. I am concerned with accurately portraying my participants' experiences and perceptions, which is why I need to use them as my measuring devices. Similarly, my mentors, colleagues, and faculty are soundboards that help ensure my analysis is not overtly swayed by my own experiences and biases. As I developed my own content knowledge and pedagogical content knowledge through working with my students, I consider them my mixer. My students constantly make me reconsider my thinking or delivery approach to my lessons, thus mixing up my ideas. As a researcher, I bring my own knowledge and experiences to my study, which have been thoroughly mixed by my time in classrooms.

The workspace for baking should be clean and organized; you do not want any stray ingredients making their way into your baked goods. The organizational technique can be compared with data management systems and methods in research. The more organized and methodical you are when you are keeping track of your data and findings, the easier it is to see connections between participants, for instance. Pre-portioning your ingredients is a technique that will help your baking go more smoothly. You then combine the pre-portioned ingredients in certain steps—usually your dry ingredients and wet ingredients separately, then together. Similarly, you want to organize your data, coding schemes, and analytical memos and then use them by combining into a coherent analysis and discussion.

In both baking and in research, the individual relies on her intuition based on past experiences. A baker has an intuition about what flavors would work well together based on other baking experiences like a researcher does when entering the field—they bring their own sets of experiences and perspectives on a situation that have to be examined. In baking, the outcomes are not always what you expect and these are sometimes the greatest moments of learning. It could be that you forgot a crucial ingredient or your flavor combination did not work as you predicted. Similarly, in research, you never know quite what you are going to get in terms of data from your participants and it may not always be in line with your preconceptions. These are some of the most valuable learning opportunities available when conducting research. At times, a researcher may need to deviate from her original methodology. She may need to adjust to participants' schedule, modify interview questions, and change coding schemes based on new data, for instance. This is comparable to in baking when sometimes you need to be able to abandon the prescribed recipe in favor of the cake; you adjust the flavorings or proportions of ingredients to have the best possible outcome.

At the start of my doctoral program, while I was thinking about my own development from the perspective of these same questions, I began to work with pre-service teacher candidates. The ways in which these individuals constructed their knowledge, their histories, and what supported or hindered their development not only interested me as their instructor, but also as a researcher. Were there any commonalities and patterns in their development? Are there systemic structures in place that contribute to their development?

Research Questions

All of these wonderings led me to reading a great deal about the types of knowledge teachers develop and the existing research that has been conducted in this field (see Chapter 2). With this literature and the gaps in the literature in mind, I was able to formulate my research questions:

1. How does secondary mathematics teachers' PCK change over the first year of teaching?
2. How do secondary mathematics teachers describe the development of their PCK before and during their first year as a teacher?

2.1 How do beginning secondary mathematics teachers' experiences and views of their development of PCK change from institutional to professional learning of teaching?

3. What experiences and factors influence the development of secondary mathematics teachers' PCK?

    3.1 How does the development of PCK during the student teaching year transfer to their first year of teaching?

    3.2 What experiences and factors do beginning secondary mathematics teachers report supported or hindered the development of their PCK while in their first year of teaching?

Research Design

To address the research questions, a qualitative design was used. This approach was appropriate since it describes the process of an occurrence and captures people's perspectives and experiences through a detailed, thick description situated in the real world (Creswell, 2014; Frankel, Wallen, & Hyun, 2012; Patton, 2015). Due to the lack of existing research about secondary mathematics teachers' PCK development through the transition from "student of teaching" to teacher, an exploratory qualitative approach was suitable; "qualitative methods are especially appropriate for inquiries where no acceptable, valid, and reliable measures exist" (Patton, 2015, p. 229). Qualitative inquiry helps researchers understand the process occurring and to obtain a holistic description of a situation (Frankel et al., 2012). Since the intent of my study was to explore and describe the nature of PCK development in first-year secondary mathematics teachers and to represent participants' voices, perceptions, and experiences, a qualitative approach was fitting.

Participants

This study utilized a convenience, non-random sample of recent graduates of a teacher preparation program who earned certification to teach middle and/or high school mathematics. Any first-year teacher who recently graduated from the target university as a secondary education or elementary education major who earned certification to teach middle and/or high school mathematics and who participated in my pilot study was eligible to participate in the study. My pilot study will be described in more detail in the next paragraph. For this study, secondary education included grades 5-8 for middle school and 9-12 for high school. Undergraduate and graduate elementary education majors were also eligible to participate if they earned a middle level extension to their certification in mathematics.

Since the goal of my study was to investigate the development of PCK during the transition from student to teacher, I needed to interact with the same individuals in both settings. In order to gather data on my potential participants' PCK and PCK development during their student teaching year (fall 2016 to spring 2017), I conducted a pilot study. This eligible cohort consisted of nine students, both males and females, who were of varying ages. Secondary education mathematics students were recruited through their methods class in the fall of 2016, prior to their student teaching semester; since this course is a requirement for their major, the majority of the participants were enrolled. Elementary education majors seeking a middle level extension in mathematics were contacted via email and invited to participate. The participants were also in the mathematics capstone course, of which I am the instructor, and the recruitment was done via the methods course and email to avoid coercion. Only those who chose to participate in the research are part of the data analysis. This pilot study was approved by the IRB. Those who chose to participate, eight of the original nine eligible, were emailed and asked to complete an initial survey, a link through SurveyMonkey, including demographic information and the PCK Inventory Instrument, which served as an initial assessment of their PCK. Participants completed the PCK Inventory Instrument a total of three times (beginning, middle, and end) during the pilot study to track changes in participants' perceived and demonstrated PCK. Two semi-structured interviews were conducted at the beginning and end of the study, after the first and last administration of the PCK Inventory Instrument.

For this study, the beginning teachers of interest were the 2017 graduates of a secondary education mathematics and elementary with middle level extension in mathematics programs who started their first teaching jobs in the fall of 2017 who also participated in my pilot study. These graduates were initially involved in the pilot study (2016-2017 academic year) described above and a subset of three of them were recruited for my dissertation research. In August of 2017, I recruited participants via an email, including the consent letter (Appendix A), prior to their first year of teaching. Within the email and consent letter, participants were informed that they would be asked to complete an initial background survey, PCK Inventory Instrument twice, participate in two interviews and two observations, and submit one closing survey.

When considering who my participants were, I had to consider the type of program they completed as well as background and personal information central to their teacher identity formation. Two of the factors identified in the literature as influential to development of PCK are teacher's content knowledge and previous experiences with the content/topics. One way in which these factors were demonstrated was through the pathway they were accepted into their preparation program. Those that demonstrated a strong academic background through their high school GPA and SAT or ACT scores were directly accepted into the school of education. Traditional undergraduate applicants apply to their programs, usually during their sophomore year of college, and must meet particular GPA requirements overall and in their content area and passing test scores (Praxis I: PPST or Praxis I: CORE, SAT, ACT, or a combination).

Similarly, this university has a National Science Foundation grant which has the goal of recruiting and supporting teachers of science and mathematics in high-need schools. The grant funds the NOYCE scholarship and those awarded these scholarships have their practicum and student teaching placements in high-need districts. Additionally, these teachers are expected to gain employment in a high-need school. This is an important characteristic to consider since participants' experiences in high-need schools may differ from those elsewhere.

Below is Figure 3.1, which provides some background information of each participant, both from the pilot study and those that continued with me to my dissertation research (noted with a star* next to their name). In my pilot study, over half my participants were part of the early acceptance program and three were NOYCE scholars.

Name (Pseudonyms)	Gender	Program	Early Admission Program	NOYCE
Alyssa*	Female	Undergraduate; Elementary Education	Yes	
Ben	Male	Undergraduate; Secondary Education	Yes	Yes
Emma	Female	Undergraduate; Secondary Education	Yes	
Hannah	Female	Graduate; Secondary Education		Yes
Jeff	Male	Undergraduate; Secondary Education		
Kara*	Female	Undergraduate; Secondary Education	Yes	
Lisa	Female	Undergraduate; Secondary Education	Yes	
Molly*	Female	Undergraduate; Secondary Education	Yes	Yes

Figure 3.1. Participant Characteristics.

Of the eight participants in my pilot study, three participants consented to participate in my dissertation research. Of these participants, two were graduates of the secondary education program and one was a graduate of the elementary education program; all three completed the undergraduate programs and earned certification to teach mathematics at the middle and/or high school level. Upon graduation, these three participants all gained employment at the middle level at various schools around the state. All three of these teachers are female and are in their early twenties. Additionally, all of these teachers were accepted early into their preparation program.

Procedures

The recruitment and data collection for this study was modeled after my pilot study illustrated in the timeline below (Figure 3.2). This illustrates how data collection methods were sequenced and paired together.

Research Time Table- Data Collection Procedures											
Month	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
Recruitment	■	■									
PCK Inventory			■					■			
Observation				■	■					■	■
Interview				■	■					■	■
Survey										■	■

Figure 3.2. Timeline of data collection procedures.

Participants completed an initial survey used to gather demographic information (see Appendix B) during the months of August and September of 2017. The results of this survey were used to identify where participants were working and if they were interested in participating in this study. In addition, I was able to determine whether participants' current employment environment was within the context of my study, teaching mathematics in grades 5 through 12. Of the four individuals who responded to the initial survey, three were employed at different middle schools around the state and one was employed as a graduate teaching assistant at a local university. With the demographic survey, participants also completed the PCK Inventory Instrument (see Appendix C) for the first time as licensed teachers. Pilot study participants, which included the three dissertation participants, completed this instrument two to three times as pre-service teachers during the pilot study. The pilot study results provided an assessment of their PCK during their student teaching year and served as a comparison for those who continued into my dissertation research in terms of their PCK development. Further, this helps establish credibility in my study by ensuring those who did not participate in the entire two-year study were not likely to respond differently than those who did participate based on learner characteristics (Fraenkel et al., 2012).

Two observations and two semi-structured interviews were conducted in this study: first observation and first interview in the beginning of the school year and the second observation and interview toward the end of the academic year. Similarly, participants completed the on-line PCK Inventory Instrument twice during the study to investigate changes in their PCK and PCK development. Over the course of the pilot and dissertation studies, the dates for administration of the PCK inventory, interviews, and observations were selected at transition points in both the teacher preparation program experience and work experience. Observations were conducted at each teacher's school at a time of her choosing. Interviews were conducted in person following the observations. Lastly, a survey was administered at the end of the study in which participants were asked to reflect on where they primarily learned various skills and knowledge (see Appendix D).

After completing the first PCK Inventory Instrument in October, participants were contacted to schedule observation and interview dates. The original intention was to conduct these observations and interviews in early November. However, gaining permission from the schools and administration took longer than expected. Additionally, participants were overwhelmed with the end of first quarter so these visits needed to be moved to a more conducive time for my participants. Both of these occurrences initially felt like huge setbacks but I realized the goal of my observations and interviews were to better understand how my participants developed their PCK and what influenced their development. The pressures on them indicated one factor to both of these points. I also realized that I would still be able to visit their classrooms to observe and meet with them during the first half of the year, which was originally why I picked November.

Everything was still new to them, even in late November and early December, so the adjustment to my original timeline did not impact the data I was collecting.

Similarly, the last phase of data collection occurred in May and June of 2018, mirroring what had occurred in the first half of the year. Participants were contacted in the end of April and received a link to the PCK Inventory Instrument. Upon completion, they each received and email to schedule the second observation and interview. With standardized testing and end of the year meetings and events, scheduling visits was again a bit more difficult than I anticipated. Like the first observation and interviews, participants were observed in a class of their choosing and then we met after for thirty to forty-five minutes.

Data Sources

As discussed above, data were collected through interviews, observations, the PCK Inventory Instrument, and a survey. Below is a visual diagram of the main data sources and why each of them were used in my study (Figure 3.3).

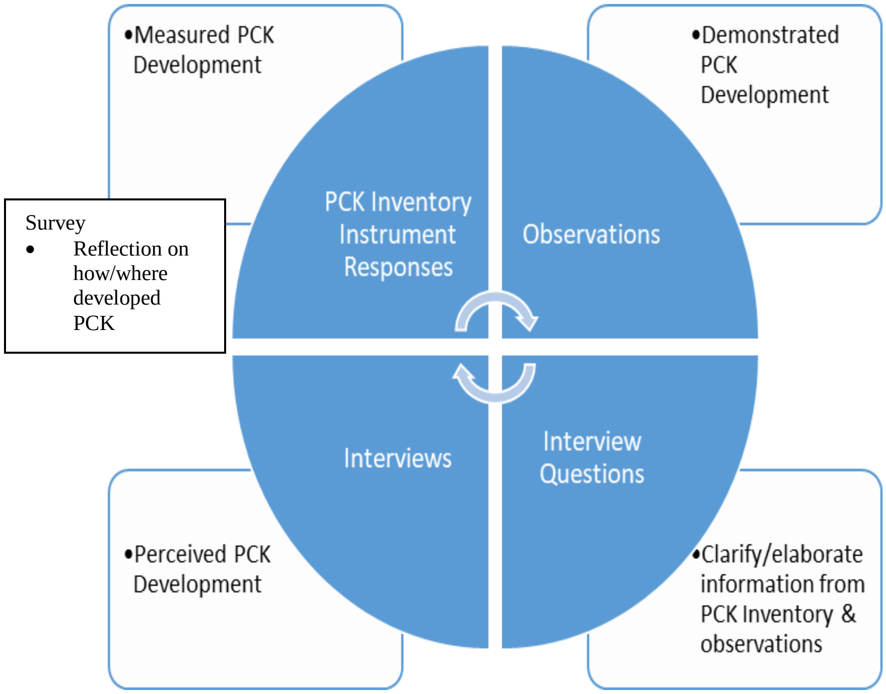


Figure 3.3. Cycle of data collection.

Next, I will elaborate on each of these data sources and connect them more explicitly to my research questions.

PCK Inventory Instrument. Through a review of the literature, no suitable instrument to gather data on PCK for secondary mathematics teachers was located. Instruments that exist in the field are for elementary education teachers (Hill, Schilling, & Ball, 2004) or are quantitative (e.g. Hauk et al., 2010). Further, Orrill et al. (2015) state "there are not many instruments readily available for use by researchers and professional developers, project personnel create their own measures of teacher knowledge, with little uniformity across the developed measures" (p. 12). As a result, by adapting questions from Sultan and Artzt (2011), I developed the PCK Inventory Instrument (see Appendix C). This instrument was used to track participants' PCK development over time. The Learning Mathematics for Teaching (LMT) Project (Ball et al., 2008), an investigation of elementary education teacher PCK development, was also used as a model for constructing this inventory. The LMT utilizes a framework of PCK that describes different domains of knowledge including: knowledge of content and teaching, knowledge of content and students, knowledge of content and curriculum, and content knowledge. Each of the items in the inventory used in this dissertation are associated with specific tasks within each of these domains (see Figure 3.4 below).

Domain	Tasks	Items
Knowledge of Content and Teaching	Design of Instruction	5, 6, 7, 8, 9, 12, 13, 14
	Sequencing of Topics	2, 9, 14
	Selection of Examples	3, 5, 8, 9, 12, 14
	Evaluate Different Representations of Topic	4, 9
	Use of Questioning	3, 6, 7, 12
Knowledge of Content and Students	Anticipate Student Thinking	5, 8, 14
	Anticipate Potential Areas of Confusion or Difficulty	2, 6, 8, 9
	Ways to Motivate Students	2, 4, 14
	Hear and Interpret Students' Thinking	1, 3, 6, 7, 11, 12, 13, 15
Knowledge of Content and Curriculum	Lateral Curriculum Vertical Curriculum	2, 5, 10, 11, 15
	Program/Instructional Materials	5, 13
Content Knowledge		1, 4, 6, 7, 8, 10

Figure 3.4. PCK Inventory Instrument Item Mappings.

After each implementation of the PCK Inventory Instrument, I analyzed the results to ensure that this original alignment stayed true. Participants' responses showed that these questions were collecting data as intended and was eliciting responses about these different domains. Additionally, analysis of their responses demonstrated an additional task and domain to this framework, which will be discussed in later chapters.

The inventory was sent to experts in the field to determine its content and construct validity. Feedback given from the experts was used to condense questions, reword them for clarity, and ensure accessibility to the readers. Participants received a link to access the instrument through SurveyMonkey (https://www.surveymonkey.com) for each administration. The purpose of the PCK inventory was to collect data on participants' PCK, of interest in research question 1 [changes in PCK development]. Similarly, participants' responses to the PCK inventory during the pilot study was included to provide data for research question 1 and 3. Also, the responses on the inventory were compared with interview responses to see continuity or discrepancies in participants' PCK development and their perceptions of their development.

Interviews. Interviews were semi-structured and audiotaped with the permission of the participants. The design and content of the interviews were to elicit descriptions of experiences and beliefs about PCK development. Participants were asked to discuss their experiences in teaching and learning, describe self-perceptions about their abilities and development, and reflect on how they have learned to teach mathematics (Leong, 2013). These interviews provided data regarding research questions 2 [perceptions] and 3 [contributing experiences]. Other interview questions prompted participants to respond to hypothetical situations (Blasjo et al., 2010; Hill et al., 2008). Participants' responses to hypothetical situations, including questions about students' thinking processes, approaches to teaching particular topics, and how they would prepare for student preconceptions or alternate conceptions, illustrated where they were in their PCK development, thus helping provide evidence for an answer to research question 1 [changes in PCK development].

Initial interview questions (Appendix E) were developed through a review of the literature and conducting the pilot study. However, based on participants' responses and ongoing analysis, these tentative interview questions were revised to clarify or elaborate on information from the PCK Inventory Instrument and observations. For this reason, a semi-structured approach was appropriate since follow-up questions were asked in order to probe and have clarifications made to their statements. Interview responses were juxtaposed with their PCK Inventory Instrument responses and observations to illustrate if participants were consistent in the way they discussed and used their PCK as a form of triangulation. Additionally, interview responses from the pilot study were used to determine how participants' PCK and perceptions of their PCK changed over the two years (question 2 and 3).

Observations. During participants' first year of teaching, two observations occurred: once at the beginning and once at the end. When conducting observations, Patton (2015) emphasizes the importance of using factual, detailed, and accurate descriptions of the setting, activities, and participants. Extensive notes were written to capture as much of what was observed as well as my impressions as possible; DeWalt and DeWalt (2011) warn "if you didn't write it in your field notes, then it didn't happen" (p. 157). In the observations, I looked for instances that demonstrated participants' PCK and changes in their development, as operationally defined previously. Additionally, comparing what was observed to interview responses helped construct a better representation of participants' PCK. For example, I looked at how participants structured their lesson that I observed and how this matched to what they discussed in interview questions about their view of instructional practices. Similarly, the types of resources used and answering student questions were visible in my observations and also discussed in the interviews. Comparing what participants did to what they vocalized illustrated in multiple ways their PCK development.

Survey. Participants' perceptions about the factors influencing their PCK development were gathered through a survey that included multiple choice questions (see Appendix D). This survey was developed by Cummings (2010) and has Cronbach's alpha levels that are considered acceptable for each construct (Fraenkel et al., 2012): mathematical knowledge (α=0.94), PCK (α=0.86), pedagogical knowledge (α=0.81), and curricular knowledge (α=0.89). This survey prompted participants to reflect on different experiences and factors, thus providing data to help answer research question 3. Further, answers provided more depth to interview responses and PCK instrument results.

Data Analysis

With a qualitative approach, "data analysis occurs alongside data collection" (Galletta, 2013, p. 119). This means as participants' responses to interview questions, field notes from observations, and the PCK Inventory Instrument were collected, they were also coded and analyzed for initial themes (Auerbach & Silverstein, 2003). In order to do so, the audio recordings of the interviews were transcribed. A sample of a transcript can be found in Appendix F.   As I transcribed the interviews, I was able to listen to pieces repeatedly and begin my initial analysis. Next, both the interviews and the field notes were read in their entirety. After each interview, observation, and PCK Inventory Instrument completion, previously collected data was revisited. Upon each reading, I looked for meaningful sections or units pertaining to participants' experiences and PCK development. These meaningful sections were analyzed to find repeated ideas, which were labeled as codes; Galletta (2013) explains that codes are "ideas [that] represent a core level of meaning" (p. 122). Themes emerge by looking for "patterns across interviews and across other data sources" (Galletta, 2013, p. 125). To help uncover themes, I used MindMup, a mindmapping application that can link to other documents (Appendix G). This allowed me to group condensed text from interviews and observations into codes and themes and facilitated analysis. Lastly, the raw data was revisited and participant checking was done to check interpretations of both the essential meanings and the general structure. In participant checking, participants were sent excerpts of the synthesized data and findings for feedback on whether their experiences, feelings, and thinking were accurately and fully represented (Appendix H).

Responses to the items on the PCK Inventory Instrument were analyzed in two phases. The first phase looked at the mathematical and/or pedagogical correctness and appropriateness of participants' responses to the different questions. The second phase of analysis looked for trends in responses to determine if there were similar aspects of knowledge present among the participants. Participants' results from each administration of the PCK Inventory Instrument were compared to their subsequent or previous results. This illustrated how participants' demonstrated PCK changed over time [research question 1]. In addition, the analysis process described for interviews and observations was utilized to label codes and identify themes within the PCK Inventory Instrument responses.

Survey responses were analyzed through descriptive statistics to determine if trends could be determined in participants' experiences and participant-selected factors. Specifically, participants' identification of which experience(s) they believe were influential to their understanding of how to teach mathematics [research question 3] was of interest. Using means and standard deviations, common experiences and factors identified by participants and which experience(s) the group identified as the most influential was illustrated. This analysis method would normally be considered a form of quantitative data analysis. However, since it was used to describe the situation rather than making inferences with it, it was considered to be another element of qualitative analysis. Inferential statistics are used in quantitative analysis in order to be able to make inferences about a population based on a sample (Fraenkel et al., 2012). On the other hand, descriptive statistics are used to describe the information; in quantitative approaches, descriptive statistics are used to simplify large amounts of data to single measures. However, this study has less than ten participants, so using descriptive statistics to look at the patterns in participants' responses is useful. The data gleaned from the survey was also illustrative of participants' experiences as a K-12 learner, college student, pre-service teacher, and first-year teacher. It showed what participants valued and considered influential in developing their PCK.

Participants' responses during the pilot study interviews and on the PCK Inventory Instrument administrations were included in the data analyses. The process for analyzing each of these data sources was done in the same manner as previously described for each data source. For example, the process for analyzing and coding PCK Inventory Instrument responses from the pilot study were analyzed in two phases as they were for the dissertation study: (1) mathematical and/or pedagogical correctness and; (2) appropriateness types of knowledge present. In addition, the data from the pilot study was revisited during the analysis of new data to ensure appropriateness of coding, themes, and interpretations. This allowed me to look at the development of PCK and participants' perceptions of their development through their student teaching experiences and through their first year of teaching.

Multiple data sources and analysis methods help to give credibility and dependability to the research design (Jones, Torres, & Arminio, 2013). Triangulation occurred since conclusions were "supported by data collected from a number of different instruments" (Fraenkel et al., 2012, p. 458). Patton (2015) argues, "by using a variety of sources and resources, the qualitative inquirer can build on the strengths of each type of data collection while minimizing the weaknesses of any single approach" (p. 390). Also, the use of different data collections produced a more holistic picture of secondary mathematics teachers' PCK development. To help ensure validity, I utilized participant and member checking as well as analytical memos. The analytical memos were used to track my thoughts, feelings, and interpretations while conducting interviews and observations. This will help give assurance that personal biases do not influence the analysis of the interview data and will help distinguish the researcher's feelings and thoughts from those of the participants. In these memos, I was able to start to make sense of my data and begin my initial analysis of the data.

Codes and Themes. An initial coding scheme for analysis was developed during the pilot study where participants' responses to the PCK Inventory Instrument and interview questions were used. After reading the pieces of data, I highlighted chunks of text that seemed meaningful and relevant to the different elements of PCK development. To do so, I first worked through with paper and pencil, writing possible labels in the margins. Opening coding followed this chunking process where key terms were extracted to label the pieces of text. I used my word processor for this part of the process to label the pieces of text using the comment feature as I re-read the data again (Appendix F). To look for trends in my codes, I used a spreadsheet where I could paste in the text, label it by participant and data source, and identify the associated code. I sorted these pieces of text by similarity in the labels and looked for trends and repeating ideas in what the participants said and wrote. Subsequent interview and PCK Inventory Instrument responses shed light on meanings from prior interviews, focused my re-reading of data, and allowed for re-coding and re-conceptualization of ideas.

In total, I identified five major themes relevant to my participants' PCK development and their perceptions of their development: connections, experiences as a student, learning-on-the-go, supports, and job constraints. Participants throughout both the pilot and dissertation studies discussed the overarching topic of connections: making connections to themselves, connections with students, links between and to coursework, connections among topics, etc. Thus, connections emerged as a theme from my participants' experiences. Similarly, the idea of experiences as a student became apparent as a central theme in my study of PCK development; participants discussed their experiences in K-12 education, college general education courses, college mathematics courses, teacher preparation coursework, and practicum courses. Participants discussed items that were outside of their control that either supported or hindered their PCK development, which I labeled as job constraints, including time, control of the curriculum, the evaluation process, construction of assessments, classroom management, etc. In reflecting on their experiences, participants repeatedly discussed different supports during their development of PCK such as their cohort, college experiences, other colleagues, and professional organizations. Lastly, as both pre-service and first-year teachers, participants discussed the role of reflection, effective and ineffective lessons, working with students, their development as a teacher, and their wants and concerns. The topics illustrated learning-on-the go as an integral theme of PCK development. Each of these themes will be explored in more detail in the following chapters.

CHAPTER 4

THE BATTER: FINDINGS

> If you walk in the footsteps of a stranger, you'll learn things you never knew you never know.
>
> Pocahontas, *Pocahontas*

In this chapter, I will introduce my participants and discuss their experiences and their PCK development as they transitioned from a student teacher to a first-year teacher. All three of my participants graduated from the same university, were double majors in mathematics and education, and obtained jobs in middle schools. One participant (Alyssa) graduated from the elementary education program while the other two (Kara and Molly) graduated from the secondary education program. Each obtained employment at different middle schools. The participants' development in the areas of different tasks associated the PCK are discussed in relation to their experiences at different points in their preparation program and first year of teaching. These tasks were originally identified by Ball, Thames, and Phelps (2008) and are listed in Figure 4.1 below. When analyzing what participants discussed during the different interviews, it also became apparent that they were developing knowledge of assessment, an additional domain, and use of mathematical language, an additional task. While both of these could be considered within the different tasks identified above, I believe it is important to considered them separately. Descriptions of participants' development are amalgamated from interviews, observations, and responses on the PCK Inventory Instrument.

Participants' development will be discussed in each of the three original domains and the tasks within each domain (Figure 4.1).

Domain	Tasks
Knowledge of Content and Teaching (KCT)	Design of Instruction
	Sequencing of Topics
	Selection of Examples
	Evaluate Different Representations of Topic
	Use of Questioning
	Use of Mathematical Language*
Knowledge of Content and Students (KCS)	Anticipate Student Thinking
	Anticipate Potential Areas of Confusion or Difficulty
	Ways to Motivate Students
	Hear and Interpret Students' Thinking
Knowledge of Content and Curriculum (KCC)	Lateral Curriculum Vertical Curriculum
	Program/Instructional Materials
Knowledge of Assessment*	Identifying Methods or Strategies of Assessment
	Use of Assessment Data
	Challenges or Difficulties with Assessment
	Selecting Appropriate Topics and Processes to Assess
	Design of Assessment

Figure 4.1. Domains and Tasks of PCK.
*Additions to the Ball et al. (2008) organization.

Kara

Kara is a highly-organized individual who also likes to color-code her notes and assignments. She was the "time-keeper" in many classes, making sure everyone was productive and that they would finish on-time. She is industrious and values collaborations that stimulate learning and discussions. One thing she regards highly is the experiences of others she could learn from. She viewed all her teachers, instructors, and professors as role models for teaching methods and considered their forms of instruction in relation to her forming teaching philosophy. Similarly, she considered the experiences of former students:

> Our advisor brings in a lot of previous student teachers, which I think is really helpful because we ask them a lot of questions about their student teaching experiences and then how they got a job and any advice. So I think that's really helpful just because it's where I'll hopefully be in a year so it gives me a little glimpse into my future. It really puts in into perspective and gives us good advice. Everything they've told us, we all take notes and listen so intently because we really want to know, so they've been really helpful. One of them typed up answers to all our questions and handed it out to us. [Kara- Interview 2- 1: 15-23]

She treasures personal connections with individuals. Since she views these relationships as important, she is mindful of feelings and is supportive in her interactions with others.

Kara came to the university as an out-of-state elementary education major. The traditional application process into the education program was waived since she had an overall strong academic background from high school. She explained that she frequently struggled learning mathematics in middle school and high school, but she viewed this as a way to be relatable for her students:

> I don't think I'm as strong mathematically as kids would think I would be; being a math teacher they think you're an expert at math and I don't think that I am an expert at math. I wasn't always the strongest math person in my middle and high school. I think that almost helps because it didn't always come right away for me. I can see where they're coming from" [Kara- Interview 1- 6: 9-11; Kara- Interview 2- 6- 21-23]

Due to her own struggles with mathematics, she became conscious of and sensitive to other students and their struggles. She also came to value the role of the teacher in fostering a student's mathematical knowledge development.

In her first year of college, Kara focused her coursework on completing her general education requirements. She stated that she "didn't really know what [she] was doing" when picking her classes and "did a little bit of everything" [Kara- Interview 1- 1:

8-10]. When considering the courses she chose to take, she realized that not all of her classes were the best fit for her and her future career:

> I took really random classes like Astronomy, Geology, Theater, which was cool, it was alright. I wasn't really interested in any of them. I'm probably never going to use my knowledge from Astronomy whereas if I had taken Nutrition that may be good background information to have and stuff. I guess I could have looked into it more. I feel like I was trying to take the easy way out and just get my credits done. I think if I had looked at the full list of everything I would have chosen courses that I was more interested in rather than just choosing things that fit in my schedule. [Kara- Interview 1- 1: 11-21]

Different courses could have given her a stronger general foundation to make curricular connections between subject areas. Kara also explained that as soon as she knew she was going to go into the field of education, she critically examined how other people taught during each encounter with them:

> When I decided that I wanted to do education, I was watching how other people teach even in my own classes. I think I've always kind of done it because I've always kind of known that I wanted to be a teacher. For instance, when another teacher does something, I make a mental note if I like that or if I don't like it. I think having experiences with different teachers and being exposed to different teaching styles or different methods and ways they do things is important. I think I say if I like it or not and that's how I'm building my own teaching style. [Kara- Interview 1- 5-6: 17-22 & 1-2]

She viewed all teachers as potential role models of how she could approach teaching and learning and made the conscious decision to reflect on those interactions.

At the time Kara started her education program, elementary education majors were required to have a double major. She chose her double major as mathematics after her first semester freshmen year, which put her behind in the typical mathematics curriculum. While she did well in Calculus I, she struggled in Calculus II though she had taken calculus in high school. In her Calculus II class, the instructor explained to the students that it was one of the hardest mathematics courses students take. She also recalled that the student seated next to her stated he was retaking the course. She believed these priming experiences as well as the structure of the course itself set her up to not be successful. These experiences prompted her to consider switching her major out of mathematics. It was her experiences studying abroad that compelled her to switch from elementary education to secondary education and persevere in her mathematics degree:

> I went abroad for a semester, that's when I decided I was going to switch to secondary because I worked with kids abroad that were ages 12-17 and I liked that age so I decided to switch. When I got back, I had emailed the math education advisor and he said how can I catch up and I had to take like 3 math classes every semester since then. So it was a lot of work but I mean I'm here now and I'm only taking 1 math class so I'm almost done. [Kara- Interview 1- 2: 3-9]

She went on to identify certain advanced mathematics courses that she believed provided her with a good basis for teaching and continuing to learn in her professional life. Specifically, Kara consistently explained the role of the mathematics capstone course and the curriculum course in locating resources and materials, preparing for instruction, and curricular connections.

In reflecting on her experiences in the education coursework, Kara noted the importance of interacting with professionals from the field. She valued the guest speakers that came to classes. Kara stressed the importance of experiences with real students in her practicum settings. She explained that not all of her experiences in her practicums were "the best," but she was still able to learn from those practicums. She provided the example of her third practicum placement in an urban high school where she characterized the teacher as not caring. This setting was difficult for Kara since she wanted to learn how to work with these populations of students and she felt that she could have gotten more out of the experience with a different teacher. On the other hand, she enjoyed and appreciated her student teaching experiences. She explained that both her cooperating teachers for the middle school and high school placements had good classroom management techniques and connections with the students. The styles of instruction were different for each of her cooperating teachers and Kara found that she preferred a mixture:

> I think during student teaching, in my middle school placement, I had a teacher who did a lot of activities like this and showed me a lot of these so that kind of opened my eyes to that teaching style. And then, in my high school placement, I had a teacher who just kind of gave a worksheet, taught on the board, and did that kind of teaching style. So I found that I'm a little bit of both. I like the direct instruction for parts and then I like the activities for parts so I found it through student teaching, I guess. And I was lucky enough to have both of those experiences so I got a feel for each. [Kara- Interview 3- 4-5: 19-21 & 1-5]

While she saw importance in the coursework she had taken in her preparation program, she believed some of the courses were too theoretical. Specifically, she stated she would have much preferred a list of classroom management techniques to use in her class instead of more instructional design methods. Kara explained that she learned a great deal about classroom management from her cooperating teachers, but she "walked into it and it was already setup" [Kara- Interview 3- 15: 5-6]. She went on to give the example that she was unsure of what to do on the first day of school in her new teaching job and how she learned the need to start the year strong: "Next year, I know that the first day of school is super important for setting the tone. I've already thought about that" [Kara- Interview 4- 5: 18-20]. She reflected that her experiences during her first year of teaching will help her start stronger next academic year, both in terms of content and classroom management.

Kara's school had one-to-one Chromebook integration. Many applications and programs were used which she learned about in her methods course and during student teaching. For example, she utilized Desmos when teaching about linear functions and slope; she prompted students to investigate the relationship between slope in an equation and the visual representation on a graph. She explained that she was comfortable using Google Classroom since both her middle school and high school student teaching placements utilized it. These experiences provided her with background knowledge and experiences she was able to use in her first year of teaching.

Throughout Kara's first year of teaching, she experimented with her teaching style, gathered and created resources, and began to further develop her classroom management. She was assigned to an eighth grade team that looped and felt like the "new kid" though the English teacher was new to the team as well [Kara- Interview 3- 2: 2]. While not being assigned a formal mentor, she worked closely with the curriculum coordinator, induction coach, and a teacher who taught eighth grade last year. The former eighth grade teacher gave Kara all of her resources and curriculum binder. At first, Kara used all of the resources and materials as they came, typically using a PowerPoint and direct instruction. However, she soon realized that these methods did not work for her or her students:

> For the first couple of weeks, I used her PowerPoints and I just found it was not how I like to teach. So I look at the PowerPoints, sometimes, actually I barely look at it now, but I kind of just use it as a guideline to go in the direction that I want to go. [Kara- Interview 3- 3: 3-6]

As she has gained experience with teaching and confidence in her own teaching style, she was better able to construct effective lessons. If she did have questions on the curriculum or on the math, she worked with the curriculum coordinator. She created her lesson ideas and activities in collaboration with other teachers and the induction coach. She explained that she created most of the worksheets that she used with her students. Kara explained that the induction coach was her go-to person for all her "silly questions" and a great source of lesson materials and classroom management techniques [Kara- Interview 3- 3: 11-15; Kara- Interview 3- 4: 5-7; Kara- Interview 3- 10 & 14: 13-15 & 14-17]. The induction coach had a budget to purchase lesson materials for the teachers and met with Kara once a week for ninety minutes. She felt that all of these people provide her with a great support network and provided her with opportunities to continue to learn and grow as a teacher.

PCK Development

Kara began the final year of her teacher preparation with certain aspects of her PCK stronger than other areas as these developed from her own experiences learning, coursework, and practicums. She continued this development as a student teacher and as a first-year teacher. She grew rapidly in some areas while remaining constant or wavering in others.

Knowledge of content and teaching. At the start of her pre-student teaching semester, she considered what she would say to students when *designing instruction*. For instance, in response to PCK Inventory Questions 5, she described the directions she would give to students when using a geometric representation to explore squaring a binomial:

> Draw a square. Cut the square into 4 equal squares labeling the sides a and b so that the area of the large square would be (a+b)^2. Then show that if you find all the areas of the smaller squares and take the sum to find the area of the larger square, you get a^2+2ab+b^2, hence (a+b)^2 = a^2+2ab+b^2. [Kara- PCK Inventory 1- Question 5]

However, as she gained more experiences working with students and began her first year of teaching, she moved towards using examples or having students do activities. She initially felt that a teacher should be the one doing the talking and controlling all aspects of student learning. After she student taught, she noticed a change in she presented material to class:

> I know when I was in my practicums, I would be nervous to go up and talk in front of the whole class and it was less conversations and more of me trying to write things on the board and have them hopefully understand it. And now I feel like I can walk around and connect with my students." [Kara- Interview 2- 4: 2-5]

Though she felt she had grown in this area at the completion of her teacher preparation program, Kara felt she was still developing how she designed and delivered instruction. For example, she explained that she had difficulty reflecting in-action about how to modify her instructional design for struggling students:

> Sometimes I'll teaching something and be like "why didn't they understand this?" and I can't figure out how to teach it a different way. I mean, I think it will come naturally as I get more experience and keep working on figuring that out. [Kara- Interview 2- 4: 11-13]

She noted that in her student teaching she used direct instruction more at her high school placement than her middle school placement. Her rationale for this was the amount of time available for a given topic as well as the complexity of the content that needed to be taught. On the other hand, the design of instruction for her middle school student teaching placement utilized more group work such as carousels or a "speed dating activity" [Kara- Interview 2- 9: 14-1]. She explained "getting them [the students] more involved and not just teaching things at them helps them" [Kara- Interview 2- 9: 10-22]. She transferred this knowledge to her work as a first-year teacher in a middle school setting. A former eighth grade teacher at the school gifted Kara her materials including PowerPoints and direct instruction style resources. Kara realized she needed to adapt these resources to be more student-centered and activity-based:

> She gave me this huge binder [shows binder in milk crate at front of room near her desk]. She gave me all her resources and I have them all online

> too. But for the first couple of weeks, I used her PowerPoints and I just found it was not how I like to teach. So I look at the PowerPoints, sometimes, actually I barely look at it now, but I kind of just use it as a guideline to go in the direction that I want to go. […] These kids love activities like this [scavenger hunt with distance formula] and getting up. They can't just sit still; even for ten minutes they just can't sit still. So I try to get them up as much as possible. I gave them a little survey at the beginning of the year and said what do you like and what do you not like and they all love group work so I do a lot of group work. [Kara-Interview 3- 2 & 3: 10-20 & 1-6]

In addition, she was reflective about her instruction at the end of her first year of teaching and had continuously thought about how she would revise her lessons and materials in the future. Through her experiences and reflection on those experiences, she further developed her knowledge of how to design and implement instruction of her students.

Kara had difficulty at first with *sequencing topics* for instruction. She struggled to develop in this area throughout her student teaching and first year of teaching. For example, when asked to select the order she would teach topics in a trigonometry unit on the PCK Inventory (Question 2), she changed the order each time. In addition, the orders she identified did not fully support student development or connections to be fostered between the topics. In one response, she did not plan to teach special right triangles until after reference angles, conterminal angles, and the unit circle [Kara- PCK Inventory 1- Question 2]. Though she did revise the sequence of topics to be more developmentally appropriate, there was still room for improvement. At the end of her first year of teaching, she realized the need to start the unit of trigonometry with special right triangles but chose to teach the unit circle before the definitions of trigonometric functions [Kara- PCK Inventory 5- Question 2]. At the end of her student teaching semester, Kara was reflective about her knowledge in this area and identified it as an area she needed to work on:

> I think I feel less prepared, I don't know if I would say not prepared but I guess less prepared, curriculum-wise again because a lot of time I would have to go up to my cooperating teacher and ask what I should do next or how should I lead into this topic. More finding the sequences for my lessons, again, and building those from there. Once I have a topic, I can go and create a lesson for it and progress a little, but once I finish something I don't know where to go next. If I can get led in a direction, I can kind of do it but figuring that out all on my own will be a little tricky for me. [Kara- Interview 2- 11: 4-10]

When asked how she would decide the sequence of topics in her first year of teaching, she explained she would look at the school's pacing guide and assessments and talk to other teachers in the school. In her first year of teaching, she followed the curriculum map provided by the school and worked closely with the curriculum coordinator. She also used the units provided by the former eighth grade teacher as a guideline for the sequencing of topics. The use of these resources and colleagues provided Kara with a semi-structured sequence of topics. As a result, she had little room to grow in this area during her first year of teaching. However, she considered changes she would make in her future teaching:

> I wish I could reteach this year because I would love to use what I already have and tweak it. I literally make schedules for every unit that I do and I have all of the links and then I make notes about what I would change. [Kara- Interview 4- 9: 18-21]

She recognized the importance of ordering topics so students can make connections and build on prior knowledge; she explained this as having to "figure out what they [the students] need to know before I jump into something else" [Kara- Interview 2- 6: 6].

Kara consistently used *examples* to help her students understand different concepts throughout her last year in her preparation program and as a first year teacher. When asked how she would respond to a student who believed the greatest common divisor is greater than the least common multiple (Question 3), she stated she would show examples in every response on the PCK Inventory. In some cases, she even identified what examples she would use: "think about the definitions of divisor and multiple. Which is bigger? Do an example of the divisors and multiples of 12 and 16, which are bigger? So even the greatest common divisor is smaller than the smallest multiple" [Kara- PCK Inventory 3- Question 3]. She is able to identify examples for different topics and levels of mathematics. For instance, she explained using examples to help a student identify his or her error in solving an absolute value inequality (Question 6), explain the difference between inverse and reciprocal trigonometric functions (Question 12), and work through a student's misconception with rules of logarithms (Question 8). Also, as a first year teacher she began to use counterexamples more to help students identify where their thinking was incorrect. She explained the importance of modeling and using examples to help students understand the problem type more easily: "Modeling a very specific type of question—if they know it's coming up and I model it ahead of time, then they've seen it and they'll understand it a little more. They definitely need it to be seen first" [Kara- Interview 4- 7: 7-9]. Kara began her pre-student teaching semester with a strong knowledge of selecting examples and continued to develop this aspect of PCK through her experiences during student teaching and her first year of teaching.

Unlike the knowledge of examples, Kara was less comfortable with *evaluating different representations of a topic* before starting her first year of teaching. She had experiences in her student teaching placements where she would explain a concept and her students would struggle understanding it. In these moments, she was unable to think about another way of representing the information for her students. After student teaching she realized the importance of real world applications when representing topics.

One example she provided was of a Ferris wheel problem she used in her high school placement where they were using trigonometry to solve certain problems [Kara- Interview 2- 6: 8-10]. The context of the problem allowed them to "picture it" [Kara- Interview 2- 6: 10]. As a first year teacher, Kara explained that she learned new ways of representing material for students from her co-teacher, a special educator:

> She [the special education teacher] has been doing this for a while [19 years] so she knows different strategies to help the kids who maybe don't know the math behind it so she breaks it down to simplify it or to visualize it or any of those things. So I am learning strategies from her, too. [Kara- Interview 3- 16: 4-7]

Through her work with the special educator, she developed alternative ways of representing topics and especially the use of visuals when explaining concepts. In addition, the experiences she had with her students in her first year of teaching showed her the importance of using different representations. For instance, she noted the use of concrete object and visuals to illustrate a problem:

> I try to present it in a different light. With that volume problem, I showed them with physical objects [modeling with her water bottle], "if this was 13 and this was 2, how much is the rest" and they responded with "oh, 11". And I asked them how they got that and they realized they subtracted. [Kara- Interview 4- 5: 4-8]

From her interactions with students and colleagues, she further developed her knowledge and use of different representations.

As with her knowledge of different representations, Kara did not begin her final year of her teacher preparation program with *use of questioning* as a strength. Initially, she did not reference asking questions in her PCK Inventory 1 responses or interviews. However, as she started her student teaching placement she began to consider what questions she could ask students to redirect their thinking. For instance, she explained that she would ask a series of questions to help the student see the error in their thinking

[Kara- PCK Inventory 2- Question 3]. She continued to work on her development in this area as she began her first year of teaching. Again, in her responses to different questions on the PCK Inventory, she repeatedly explained what she would ask students in the different situations. However, many of her questions were leading or she would be giving information with a request for confirmation. For instance, when helping a student understand that there were two possible solutions when solving Question 7 of the PCK Inventory, she stated “think about the different solutions to x^2. There's always a positive and negative solution right? Same for x^4 so before applying the power rule, check out your exponent and think about how many solutions you should have” [Kara- PCK Inventory 4- Question 7]. Though she had difficulty including appropriate questions on the PCK Inventory, she readily used questions in her instruction during her first year of teaching. She explained the importance of answering questions with questions. She noticed many times students would ask her a question without fully thinking about a problem:

> I feel like I try to answer their questions with a question, which sometimes they hate! I don’t know if you could hear me talking to him over here [referring to a student near her desk during the previously observed lesson], he was asking me questions and I was like “well, is that what you would do?” and he was like “I don’t know, I’m just saying that until you say yes.” I had to go like “let’s think about this.” So I think my questioning and answering has been improving but I’m sure there’s still things I can work on too. [Kara- Interview 3- 18-19: 19-23 & 1-2]

As a first-year teacher, she believed this was a skill and knowledge she was still developing. Kara continued to deepen her knowledge of the use of questions during her first year of teaching. In both field observations, I noticed her asking the students questions in order to fully understand their thought processes [Kara- Observations 1 & 2]. She, thus, had the ability to use questions effectively when interacting with real students but could not consider how to use them with hypothetical students on the PCK Inventory. This illustrated how influential working with real students was to her when developing this knowledge.

Kara steadily developed as she gained more classroom experience in the domain of Knowledge of Content and Teaching (KCT), as seen in Figure 4.2.

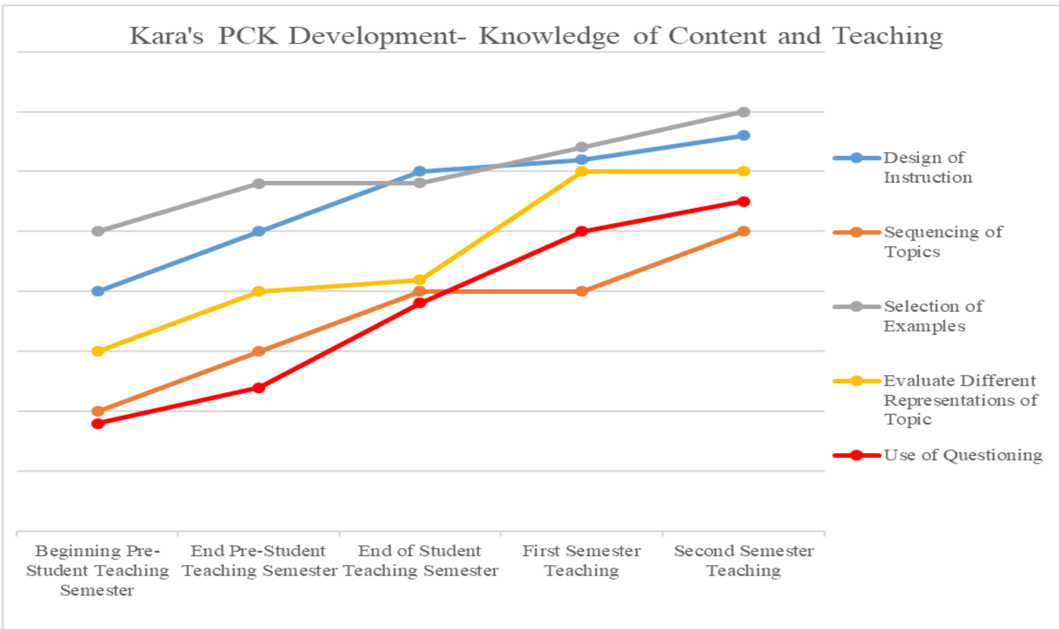


Figure 4.2. Kara's PCK Development in domain of KCT.

Overall, there is an upward trend in all KCT tasks, with some more notable than others, but also there is less spread between the levels of development for each tasks toward the end of the first year. It seems that once people start teaching these tasks come more in focus and they are more able to address them in practice. During the preparation years that is too high a level of complexity and we see that candidates make progress in some areas toward which they have an affinity or a sense of competence, as well as tasks they see connected to being a teacher.

Knowledge of content and students. *Anticipating student thinking* was an aspect of PCK where Kara developed immensely since the start of her teacher preparation program. At first, she had difficulty doing so before she was in a classroom full-time as a student teacher or teacher. This gap in her knowledge was evident in the responses on the PCK Inventory where anticipating how students would approach various topics would have been appropriate but she did not respond in that manner. For example, she was not able to consider how students, in general, would approach multiplying polynomials (Question 5). Instead, she postulated about errors they may have when multiplying polynomials:

> Students may only multiply one of the terms in each polynomial, or forget to go through every term in all polynomials. They also may try to combine what is in the parenthesis before multiplying because that is what they are taught to do when they are working with integers. [Kara- PCK Inventory 1- Question 5]

At the end of student teaching, she reflected that her ability to anticipate her students' thinking was due to her own experiences learning since she struggled and could relate to their experiences: "I wasn't always the strongest math person in my middle and high school. I think that almost helps because it didn't always come right away for me. I can see where they're coming from" [Kara- Interview 2- 6: 21-23]. As she transitioned into her role as a first-year teacher, she considered the importance of knowing her students and their backgrounds in order to anticipate their thinking. She speculated that if she had her students last year (as 7th graders on a looping team) she would have been better able to anticipate their thinking [Kara- Interview 3- 6: 13-20]. Kara also explained that her students would sometimes approach problems or ask questions in a way she was not expecting, which she described as being thrown "curveballs" [Kara- Interview 3- 18: 13-14]. She gave the example of a problem they were working on involving a circle inscribed in a square:

> For instance, yesterday I was showing them "draw this triangle and then you're going to find the legs and then you're going to have to double it at the end to find the full diagonal," we were doing a circle inscribed in a square. And one of the kids just says "well, can't you just find the big lengths of the square and those are your new legs and then that's your diagonal?" I was like "I never even had thought of it like that." So sometimes they think of even new things that I haven't thought of. But they're really good at just sharing their thoughts so that helps me and then I share it with a different class and they all love it. [Kara- Interview 3- 7: 10-18]

However, with experience, she was able to anticipate how her students would approach or think about different topics. She explained that she noticed her students will copy what she does instead of "thinking about what they are actually doing" [Kara- Interview 3- 19 & 20: 14-21 & 1- 17]. Since she was aware of this practice, she designed instruction and assignments were students could not always copy the model exactly. In reflecting on her growth in this area, she realized she was able to anticipate how her students would think or approach a problem or topic since she worked with them: "I'm getting better at anticipating or understanding their thinking. I'm starting to think about doing more different types of problems to work on problem solving skills because I think their problem solving skills are just lacking" [Kara- Interview 4- 4: 18-20]. However, she was worried about next year with having a new group of students. This concern was founded in the value she placed on knowing her students and having prior knowledge about their strengths and weaknesses:

> I came into this team in the middle of a loop so these teachers already knew all of the kids so they could tell me "watch out for this one, he really doesn't like to do this" or "don't put them together." They already kind of knew who to put into which class and I had the backgrounds on all of them and they told me family histories and stuff they're dealing with at home so it was nice to come into it already knowing about all the kids. I guess a worry for me would be to have a whole new group of kids and not know anything about them and having to make those connections and find those things out on my own. [Kara- Interview 4- 10: 13-21].

Even with these concerns, she realized that she will continue to develop her ability to anticipate student thinking with more experience.

Kara began her student teaching year relatively strong in the area of *anticipating potential areas of confusion or difficulty* for students. She was specific about where exactly students would have problems in the different situations presented on the PCK Inventory. For example, she proposed that students would mistake the absolute value symbol as parenthesis when solving [Kara- PCK Inventory 1- Question 6]. As she gained more full-time classroom experiences as a student teacher and then as a first year teacher, she was still specific with her responses and also more detailed. One example of these was in response her to how students would approach multiplying polynomials, as discussed previously on page 95 [Kara- PCK Inventory 1- Question 5]. However, after her student teaching experience she expanded her explanation to also include a different reason why students would make this mistake:

> Students may only multiply the first terms and the last terms, forgetting to distribute all terms to all other terms in both polynomials. They may do this because they are used to only distributing a whole number to a polynomial, or because FOIL states just to do 4 different distributions. [Kara- PCK Inventory 3- Question 5]

She was able to specifically identify a strategy her students would try to use (FOIL) and how it may confuse them in other situations besides a binomial multiplied by another binomial. Similarly, she considered how the representation of a function could lead students to confuse equations and functions (PCK Inventory Question 9). She believed that because, to students, equations and functions "look the same" which leads to the confusion [Kara- PCK Inventory 2- Question 9]. She was very considerate of how students may have difficulty with a topic or problem. This aptitude stems from her own experiences with difficulties learning mathematics [Kara- Interview 1- 2: 3-9]. She also considered her experiences working with students to guide the identification of potential areas of confusion or difficulty. Within her school day, she is able to use her experiences in one class to help anticipate how her other classes may struggle with a given problem for topics:

> It's nice that I teach four classes in a row of the same content so if one class had that misconception, I'll get it in the second class. I can say "I know you might think this but…" and try to present it in a different light. [Kara- Interview 4- 5: 2-4]

Similarly, she explained that next year she can use her experiences from her first year of teaching to help students navigate potential areas of confusion:

> Just knowing some of the questions that they'll have or some of the misconceptions they'll have going into doing a project or something. Like the activity they did today, knowing that they might have the misconception or research "exterior angles" instead of "alternate exterior angles." Just knowing that for next year, I can tell them "this has happened in the past so we're going to research *every* word I tell you." Being able to use what I've learned and make those changes. [Kara- Interview 4- 10: 3-9]

Kara also explained that she initially had some difficulties anticipating areas of difficulty for her students in her first year of teaching. She attributed this to not fully understanding what prior knowledge and experiences her students had since she did not have them the previous year. In addition, her students began the year with differences in their prior knowledge since their pervious seventh grade teacher did "personalized learning" [Kara- Interview 3- 6: 14]. To illustrate, she gave the example of transformations and geometry:

> The math teacher last year did a lot of personalized learning and I'm not exactly sure what they did but it seems like they all got a packet and they had to work at their own pace through it. So they're all at different spots, especially for the first unit—it was transformations on the graph so their geometry knowledge was a little bit of everywhere. So, getting them on a level playfield and just getting them caught up or trying to make sure they're not bored out of their minds. Finding the balance between all those things I think has been difficult, but I've found my way. [Kara- Interview 3- 6: 13-20]

After she was able to ascertain what and how her students learned in the preceding grades, she was better able to anticipate where they would encounter difficulties or areas of confusion. While she started strong in this area of PCK, she continued to develop through her work with students.

Kara consistently believed in the use of real world examples and applications as *ways of motivating students* throughout her pre-student teaching and student teaching semesters and first year of teaching. This can be seen in her explanations of how she would motivate students when teaching solving equations involving radical expressions (PCK Inventory Question 14). She repeatedly explained the use of real world applications when teaching these concepts:

> Give word problems of real life, interesting, relevant examples that will engage students into wanting to know the answers. [Kara- PCK Inventory 1- Question 14]
>
> Using geometry, like the sides of a square when using square roots, in order to find the missing value. Tie this into real life scenarios. [Kara- PCK Inventory 4- Question 14]
>
> Relating this topic to geometry and using real-world geometry problems involving area would help students understand different parts, such as why in D there is no solution, or why -7 in C could not be a solution since we would relate this to distance. [Kara- PCK Inventory 5- Question 14]

She also believed allowing students to collaborate and work together was an effective method of motivation. Letting students have a voice in problem solving and hearing other students' methods of solving could encourage them to continue to persevere in a difficulty situation or try alternative methods of solving. She specifically identified this as a method in PCK Inventory 1 Question 4 where she explained she would "have students work in pairs or groups to see other ways of thinking" in order to motivate them to explore multiple methods to calculate the area of the given triangle. She also explained the importance of giving students "hope" because if "they think they can't do math and then they don't want to try" [Kara- Interview 2- 7: 2-4]. This realization came after her high school student teaching experience. She explained one way to give students hope and to motivate them to persist was by making connections with material or topics they are confident in:

> I definitely try to break it down into things they may be strong at or comparatively. For example, we were just doing long division with polynomials so I was just doing long division with regular numbers, so they would be like "okay, yeah. I know how to do this" and have them be more confident in it, and then scaffolding them into something a little trickier so they would at least feel confident at the beginning. I didn't just dive into something brand new. I think that helps boost their confidence and that's really what they need because they can do it, but sometimes they just believe they can't. [Kara- Interview 2- 7: 8-14]

She took this knowledge into her first year of teaching. Additionally, she was receptive to how the students responded to different activities and for their preferences in lesson design. At the beginning of the year, she gave her students a survey to learn how they preferred to learn. Since the majority responded that they preferred working in groups, she tried to utilize that method of instruction most [Kara- Interview 3- 2: 16-18]. She also noticed how they reacted to the structure of activities. She learned that these students were not motivated by time constraints:

> They definitely don't like being timed. I noticed that. I did a scoot activity so they had two minutes at each desk with their group and it stressed them out so much so I've learned not to time them. That's why with this one, they can kind of go at their own pace. I found that works a lot better. Originally I was wondering if I should time them so they know they need to get to work right away and start it but they really do just start their work right away. So I've laid off of the timers since I know that stresses them out. [Kara- Interview 3- 9: 7-13]

Another way Kara noticed she could motivate student was by giving direct feedback or positive reinforcement. While she wanted students to explore concepts on their own, sometimes she noticed they needed confirmation that what they were attempting was valid:

> Worksheets sometimes work when I give them feedback instead of just saying "ok, do this and then see how you do." I give them direct feedback on it and some of them like that so I've been doing more of that. But, for the most part, yeah just activities, moving around. [Kara- Interview 3- 9: 13-16]

Similarly, she started to integrate technology more as her first year of teaching progressed. Her school was one-to-one with Chromebooks but she did not use them often in the first half of the year. She explained she was not ready to use them much as she began the year since she was adjusting and "not ready for it yet" [Kara- Interview 4- 7: 22]. When they started the unit on linear functions, she integrated the use of Desmos into her instruction:

> They did really like using Desmos; they liked playing around with it. The first time I ever let them go on it was a bit crazy because they were all zooming out as far as they could or in as far as they could go. Once they got the hang of it, they liked it. We were talking about which functions were linear or non-linear and they liked being able to see. They were like "wow this is cool! I can really see it!" So that was cool. [Kara- Interview 4- 8: 4-9]

She reflected that students explored more while using the technology. As with real world examples, she explained the role of exploration as a motivator throughout the two years. For example, she stated she would use activities to allow students to delve into the concept of area:

> Give students both labeled and unlabeled shapes and ask them to find area. Also, using graph paper to find area will show another method. Cutting out shapes and having students measure the side lengths of the shape will also motivate students to try another method to calculate area. [Kara- PCK Inventory 2- Question 4]

From her own experiences learning and teaching, she was able to learn more about ways of motivating students.

Like her knowledge of anticipating potential areas of difficulty, Kara began the final year of her teacher preparation program with some knowledge of how to *hear and interpret students' thinking*. One way she exhibited this knowledge was in her responses on the PCK Inventory where she identified students' thought processes. She also, in most cases, explained why they most likely thought in that particular way. For example, she was able to read the student work provided for Question 12 and interpret why the student used the reciprocal trigonometric function instead of the inverse, misreading the exponent. Similarly, she was able to read a student's statement about the properties of a rectangle and determine the accuracy of those statements [Kara-PCK Inventory 1-5-Question 15]. Her development of this knowledge continued in her student teaching semester and through her first year as a classroom teacher. The responses she provided on the PCK Inventory became more detailed and evaluative of the students' thought processes. One instance of this was in her response about interpreting the reasoning and ideas the student might have used when solving the area of the given trapezoid (PCK Inventory Question 1). In her first response, she explained:

> The student used the numbers they were given to substitute into the formula and solve for the area. They knew base 1 and base 2 were opposite sides so they used the given numbers as bases and the other number as a height. [Kara- PCK Inventory 1- Question 1]

At the start of her first year of teaching, the detail in her response grew when she started to identify *why* the student approached the problem in the manner they did:

> The way the trapezoid is presented, it looks as though the bottom number (20) would be a base, and therefore 29 would be the other base. Since 18 is the length going from bottom to top, this looks like the height so the student substituted those numbers into the formula and solved. [Kara-PCK Inventory 4- Question 1]

Not only did she recognize that the student interpreted the values incorrectly, she provided an explanation of what led to this students thinking—the orientation of the trapezoid. She also discussed her abilities in this area when she provided the example of a student presenting an alternative method of solving the problem involving the square inscribed in the circle, discussed on page 96 [Kara- Interview 3- 7: 10-16]. She was able to evaluate the student's idea of a method a determine that it was appropriate and practical. Another example was when she was also able to interpret her students' thought process and determine why they presented on the wrong topic for their angle properties presentation [Kara- Observation 2; Kara- Interview 4- 10: 6-7]. This illustrated her ability to consider students' claims on the spot and evaluate their correctness and accuracy.

Kara developed her Knowledge of Content and Students (KCS) steadily over the end of her preparation program and during her first year teaching (Figure 4.3).

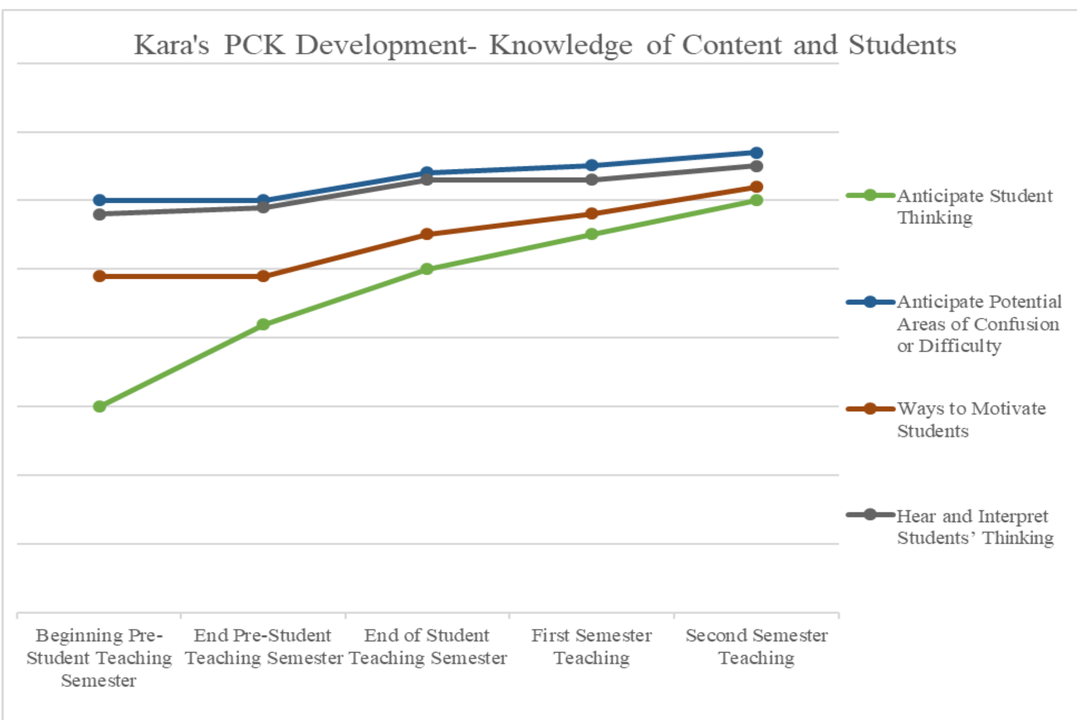


Figure 4.3. Kara's PCK Development in domain of KCS.

Again, there is an upward trend in all KCS tasks with a noticeable narrowing of the spread between the levels of development for each tasks toward the end of the first year. It seems that the tasks in this domain are highly integrated and as an individual begins teaching, they tend to develop in these tasks simultaneously.

Knowledge of content and curriculum. One area of PCK which Kara made considerable growth in was in her *curricular knowledge*. At the start of her pre-student teaching semester, she did not fully understand connections between many topics. This knowledge seemed to depend on how advanced the topics were that she needed to consider. For example, she was able to identify the connection between a topic in an Algebra curriculum, parallel and perpendicular lines, with a topic in Geometry, properties of quadrilaterals [Kara- PCK Inventory 1- Question 15]. She was also able to draw connections between two topics in Geometry, congruence and similarity with transformations. However, she was not able to explain the connection between these topics fully: "to show that shapes are congruent or similar you can use transformations to manipulate the given shape to look like the other one" [Kara- PCK Inventory 1- Question 10]. She did not explain the relationship between these topics in the other direction—how transformations construct either congruent or similar figures. Similarly, she had difficulty when recognizing appropriate connections between topics in trigonometry [Kara- PCK Inventory 1- Question 2]. Before she started her student teaching placements full-time, she reflected that she felt more knowledgeable about the middle school curriculum since she's "seen more" when compared to high school [Kara- Interview 1- 5: 1-6]. From her student teaching experiences, Kara realized the need to make connections between topics and prior knowledge to help students make connections and have a “smooth flow” [Kara- Interview 2- 6: 4]. She gave the example of teaching roots of polynomials without reviewing or teaching factoring [Kara- Interview 2- 6: 1-9]. Though she knew this connection within the Algebra curriculum, she learned the impact of not making it explicit for her students. As part of her teacher preparation program, Kara took a technology and curriculum course during her pre-student teaching semester. She recalled completing a curriculum report as an assignment that helped her understand the structure of a school’s curriculum. Coupling the experiences in the course with her experiences in her student teaching placements, she developed her knowledge slightly. She recognized this as a gap and wished she has more practice working within a pre-existing curriculum in her preparation program:

> I guess I feel that we need to connect more to a curriculum, to trying to plan lessons around curriculum because we do a lot of lessons where we’re just given a topic and making a lesson off of that but tying it in with maybe something that might actually be in a real-life scenario. [Kara- Interview 2- 2: 15-18]

At the start of her first year of teaching, her responses on the PCK Inventory illustrated her growth in this area. For instance, she connected the teaching of equations with radical expressions to side lengths of a square [Kara- PCK Inventory 3, 4, & 5- Question 14]. This was a connection between topics that she did not make until after having full-time experience in a classroom. As a first year teacher, Kara relied on the curriculum map provided by the school and the units she received from a previous eighth grade teacher. At first, she was anxious about keeping up with the pace of the curriculum map but she realized it was more important to consider the learning needs and connections to prior knowledge. She explained this as “figuring out what my kids need and then taking it from there and just going day-by-day while also trying to stick to the pacing of it” [Kara- Interview 3- 6: 9-11]. Through experiences with her students and by working within different curriculums, she was able to be more knowledgeable about vertical and lateral curriculum connections with the topics she taught. There was still room for growth in this area as she still struggled slightly when considering topics outside of her current scope.

Like her growth in curricular knowledge, Kara also made noticeable strides in her knowledge of *program and instructional materials* as she progressed into and through her first year of teaching. When she was first asked to consider materials to teach different topics, she was not consistent in demonstrating this knowledge and was, in general, not specific in her responses. As a pre-student teacher, she explained she would use "physical objects to move or rotate around" when connecting geometric transformations to congruence and similarity [Kara- PCK Inventory 1- Question 10]. At the same time, she explained the use of visuals and cutting paper when teaching the area model [Kara- PCK Inventory 1- Question 5]. Similarly, she indicated the use of real dice or an online simulator when teaching probability [Kara- PCK Inventory 1- Question 13]. As a student teacher, she began to develop more knowledge about materials to use with both middle school and high school students. In addition to the "concrete objects" she identified again for Question 10, she also included grid paper and GeoGebra [Kara- PCK Inventory 2- Question 10]. Her answer to this question was further developed when she noted the use of patty paper [Kara- PCK Inventory 3- Question 10]. It was during student teaching that Kara learned about different manipulatives for teaching fractions. During her first year of teaching, she was able to utilize different materials that she created or received from colleagues. She stated that she worked closely with the induction coach who helped locate and procure resource. Some materials she learned about in the last year of her preparation program that she transferring to her first year of teaching were digital resources. Since her school used Chromebooks, like her student teaching placements, she was able to integrate these materials in some of her lessons, such as Desmos to explore slope and GoogleClassroom where students worked in interactive activities and to review or practice [Kara- Interview 3- 8 & 9: 12-22 & 1-5]. She also explained the use of videos as an instructional material including one on the Pythagorean theorem and different Math By Fives [Kara- Interview 3- 9: 15-17]. In addition to the digital materials, she described how she used different manipulatives when teaching concepts:

> In one of my co-taught classes we did, we took out the chips and we showed them "here, you have three, I'm subtracting a negative, how can I do that? So we had to add the chips" and we show them and they're like "Oh yeah, I remember doing that." […] When we just started learning perfect cubes and perfect squares, we brought out the tiles and brought out the cubes and played with them. Some of the kids were like "I feel like I'm in Kindergarten" but some of the kids loved it. [Kara- Interview 3- 17 & 18: 1-4 & 1-4]

She used her experiences with her students to provide more detail in her response to PCK Inventory Question 10:

> Physically cut out shapes for students to move around (rotate, reflect, translate). Also, I recently did an activity using rubber bands to attach to your pencil and stretch it out to create a similar shape (doing a dilation). [Kara- PCK Inventory 4- Question 10].

Her knowledge of program and instructional materials developed as exhibited by her awareness and use of a variety of materials in her lessons.

Alongside the development of knowledge about curriculums and materials, Kara developed her knowledge of *assessment*. At her high school student teaching placement, she remarked about the format and content of the common assessments and tasks:

> One thing my school does that a really, really, really don't like is these unit assessments after a really long span of time. I know I just gave one in geometry and it covered so many topics so they had forgotten a lot of them from before. I would just rather do little increments of smaller assessments or something like that so they don't get lost in the whole bunch and then show how they build on it. One thing I do like is they do these tasks at the end that ties everything in together in these real world situations. I like that. [Kara- Interview 2- 7: 19-24]

Along with the scope being too large of these assessments, she explained that the multiple-choice format did not support students' motivation or development of deep content knowledge:

> In my geometry class they were also doing this assessment that was five multiple-choice questions so a lot of them got 20s on it because if you only get one right… those are harder, I feel like. And then that discourages them and they feel like "well I suck at math so I can't do it." [Kara- Interview 2- 8: 11-14]

When considering the environment where she was going to work in after completing her preparation program, she was aware most schools had curriculum guides and assessments she would most likely have to use. This turned out to be the case for Kara—the school she gained employment at for her first year of teaching had common assessments [Kara- Interview 3- 4: 12]. In addition, Kara recalled completing an assignment that mirrored Student Learning Objectives (SLOs) which prepared her for using student data for SLOs in her evaluation:

> We did these two assignments and I remember hating them and we had to take data in our middle school placement and we had to collect data and we had to write this big paper about it and I was like "this is so dumb" and I hated it at the time, but it is just like a Professional Development Plan (PLP) and just like a Student Learning Objective (SLO) and now I'm so thankful I did it because I feel like it prepared me. [Kara- Interview 3- 12: 16-20]

The use of data or formative assessments to drive instruction was not a point Kara discussed; she did not explain *how* she used different types of assessments to determine if and how her students were learning. While she did not discuss this explicitly, she seemed to observe students in her classes as they worked on activities [Kara- Observation 1]. Similarly, she explained the role in giving direct feedback to students to help them, typically on worksheets [Kara- Interview 3- 8: 13-16]. Another form of assessment Kara used during her first year of teaching was projects and presentations. For example, in the last classroom observation, groups of students were presenting about different angle relationships resulting from two parallel lines are intersected by a transversal. Students were given a packet to help them organize their thinking along with the rubric of how they would be assessed that they had to turn in when they presented. The use of this performance assessment along with her awareness of different assessment types illustrated a growth in her knowledge about and uses of assessments.

As with the other domains of PCK, Kara developed her Knowledge of Content and Curriculum (KCC) throughout her student teaching year and first year of teaching (Figure 4.4).

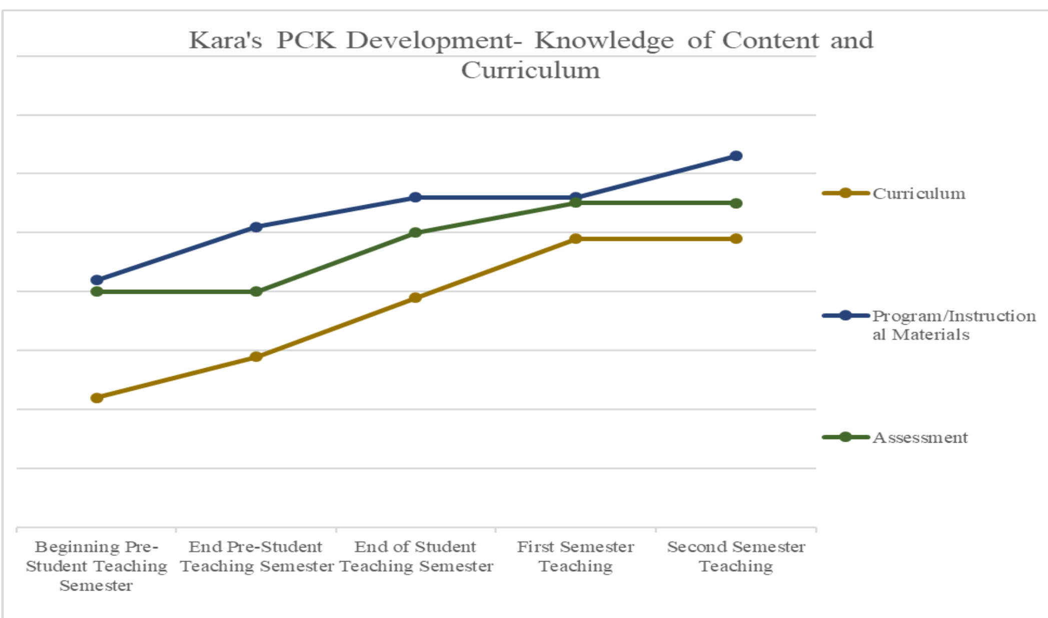


Figure 4.4. Kara's PCK Development in domain of KCC.

For summaries of this domain, I chose to include assessment within this knowledge domain since curriculum and assessments are closely related and can be conflated in schools. It is important to note that there were both instances of growth and then plateaus in her development of this domain. Growth in knowledge of curriculum and program and instructional materials can be linked with assignments and experiences in her preparation program.

Summary of Kara's PCK Development

A summary of Kara's development can be seen visually in the diagram below (Figure 4.5). Each line represents a different aspect of PCK and the horizontal axis illustrates the different transition points.

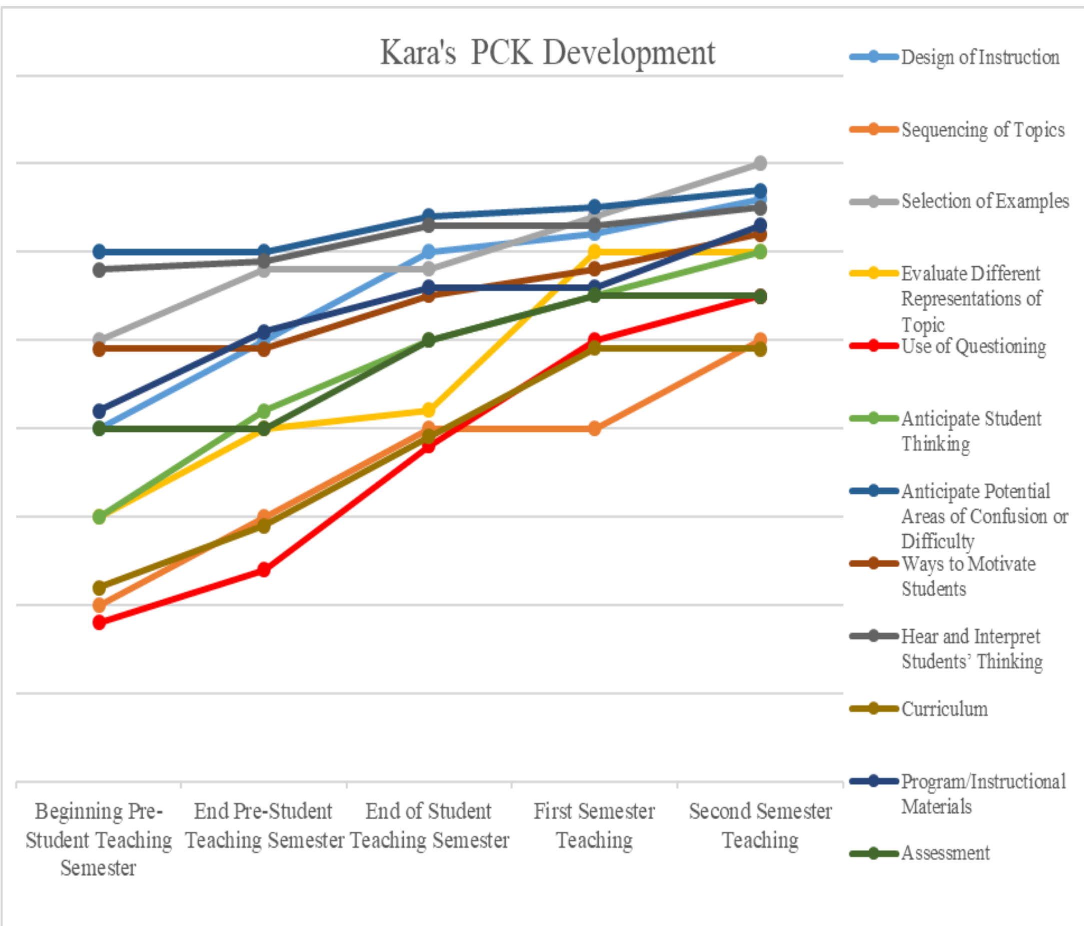


Figure 4.5. Summary of Kara's PCK Development.

Kara developed her knowledge of designing instruction in a consistent, positive way, changing from mainly using direct instruction to using a variety of instructional approaches. On the other hand, she did not have much opportunity to further develop her knowledge of how to sequence topics and relied on the scope and sequence she received. From experiences in student teaching and in her own classroom, she developed in her use of examples and counterexamples. She grew in evaluating and using different representations of topics during her first year of teaching through her work with her co-teacher. From working with real students, she developed her knowledge of questioning a great deal. Similarly, her ability to anticipate student thinking initially started out as inconsistent but developed immensely through her work with students as a student teacher and first-year teacher. She also was better able to anticipate potential areas of difficulty or confusion through her experiences working with students. Kara was reflective about her experiences with different students and was able to grow in the ways she motivated her students, including collaborative learning, inquiry activities, and types of examples. In regards to hearing and interpreting student thinking, she began the final year of her preparation program with some ability and then developed greatly from her work with students. One area she struggled with initially was in her curricular knowledge. However, through her coursework and classroom experiences, she developed her knowledge of both lateral and vertical curriculums for the most part. Alongside her curricular knowledge, she developed her knowledge of program and instructional materials from coursework and experiences in classrooms. Lastly, her knowledge of assessments grew from the experiences she gained in student teaching and during her first year of teaching. She became more conscious of the different types and uses of assessment present in schools.

Molly

Molly is a self-described “people-person” [Molly- Interview 1- 9: 11]. She considers herself to be “weird” in that she can be eccentric and think in an out-of-the-box way [Molly- Interview 2- 6: 9-12]. She tends to be self-critical and hard on herself since she was always pushing herself to be better. She is organized and always tries to make connections, be it with people or what she was learning. She has a love of mathematics and repeatedly reminded me that she even uses the Pythagorean Theorem to get from place to place.

Molly came to the university as an out-of-state student, pursuing her double major in secondary education and mathematics from the start. Like the other participants in this study, she had the traditional application process waived based on her strong academic background from high school. As an aspiring teacher, Molly viewed all interactions with instructors as a way to learn teaching methods, classroom management, and lesson design. She speculated that

> A lot of the math professors just have the math piece. They know what they’re talking about and the rest of us [students] are just kind of like “I’m going to copy this over and look up a video about it later.” [Molly- Interview 1- 2: 8-11]

This assertion showed that she believed many math professors in higher education are experts at mathematics but not necessarily at teaching. She described the instructional methods used in most mathematics courses involving lecturing, passively taking notes, and being “bored” [Molly- Interview 1- 2: 16]. In contrast, Molly described her education courses as discussion-based and centered around students’ personal beliefs and development. How Molly had envisioned learning to be a teacher was very different than what she had experienced in her preparation program. She attributed this preconception to her past experiences learning mathematics since she was unaware that there were other instructional methods besides direct instruction. When she learned about these practices during her preparation program, she embraced them. She felt empowered by being able to discover concepts on her own through these student-centered methods and said she would never want to take that feeling away from students by just telling them the answers. [Molly- Interview 3- 16: 13-17]. Being in charge of her own learning gave her a sense of autonomy she believed was lacking in the traditional lecture structure of other mathematics courses.

Molly was a recipient of a grant for which she committed to teach in high-need areas in return. Since she was part of this program, her field practicums and student teaching placements were all in high-need, urban and urban-ring districts. She continually discussed her experiences in her practicums and student teaching placements. She was able to extract pieces from her different experiences and combine them into her own teaching style. She also made the connection between her coursework and what she was able to observe in her placements:

> I've had some really good practicums; I think one of the cool things that I've seen in my practicums, not that this is cool but I've seen teachers do things that I know not to do. And I know that because of my classes and I see things because of the opinions that I have and I can say wow I would never do that in a classroom or I would never talk to a kid like that because I'm learning outside the practicum. But that being said, I've had some really, really good practicum teachers, one in particular, who I work with…he's the best. One thing I really like about him is that he plans his day kind of around me being there so he makes sure that I'm not just going to be sitting there and watching him talk to the kids. He'll have me take a group in the hall or he asked me to design something to present to the students and having the opportunity has been helping me feel more

> confident for my student teaching next semester. [Molly-Interview 1- 3: 6-18]

The last teacher Molly referred to was one of the teachers for a practicum experience as well as her cooperating teacher for her high school student teaching placement. She spent three semesters with this teacher and is still in constant communication with him during her first year of teaching. He was a great resource for her and someone who continues to push her to develop both her content knowledge and pedagogical content knowledge. As she reflected on the types of experiences she had in her practicums, she realized that she had a wide range of experiences in almost every grade-level for middle and high-school. The breadth of these experiences provided her with a good basis of how to work with a variety of learners and a look into the vertical mathematics curriculum.

In addition to her practicum experiences, Molly started substitute teaching in her hometown after her sophomore year of college. She noticed that as she progressed through her preparation program and gained experiences in teaching, there was a transformation in her identity:

> I've kind of seen the transformation in myself how I started substitute teaching at my high school my sophomore year in college. I wouldn't say anything because I'd be all nervous or whatever. Then this past summer after I finished up for the year I was in there and I chased a kid down the hall when he left the room. I've seen that transition from me being so timid to me knowing when I need to kind of intervene. [Molly- Interview 1- 11: 5-12]

This transformation continued during her student teaching experiences and her first year of teaching. She gained confidence in her teaching abilities and content knowledge. She was still developing her classroom management style and confidence in herself as "*the* teacher" [Molly- Interview 1- 10: 15]. When Molly considered her future as a teacher, she expressed some concern and anxiety about finding the right fit for her:

Concerned
first 2 years
everyone always says they're going to be hard.

Nervous
figure out where I belong
how my philosophy fits into the philosophy of the school?
I know I'll figure it out.
I know it'll be fine.

Nervous
stigma
first 2 years
building resources
stressful.

Once I get my feet wet
find out where I am
I'm going to be really happy.

When I shared this poem with her halfway through her first year of teaching, she expressed that there were similarities between her feelings then, at the point of transition from student teacher to teacher, and how she felt during her first year of teaching. She was still feeling overwhelmed and was questioning whether she made the right choice of job environment. Much of what she was questioning revolved around classroom management; she was confident in her mathematics knowledge and designing effective instruction. She recognized that she has very high expectations for herself and understood why it was sometimes unreasonable for her to be at a "fifth-year" teacher's level when she was in her first year of teaching. She explained "I'm not where I want to be because I just want to be better and I think I get a little better every day but I think I'm where I'm supposed to be" in terms of her development as a teacher [Molly- Interview 3-1: 12-14]. These trepidations persisted throughout her first year of teaching. She was still committed to working with students, but she was unsure if the district she was currently in was the best place for her.

Molly described feeling alone and a sense of isolation in her current job environment [Molly-Interview 4- 1]. She went from having a very close cohort during her preparation program to being alone in her classroom with little contact to other teachers. She recognized that she could have sought people out and made herself more visible in the teachers' room but she chose to avoid these common meeting areas due to high incidences of gossip and complaining that occurred. She preferred to only talk to select teachers and school professionals instead of participating in negative conversations. While there were only a few people at her current school she interacted with, she was still in contact with her former cooperating teachers and academic advisor.

Molly explained that she did not receive her curricular scope and sequence until a month after school had started. Also, she did not utilize the textbook and few resources she received from the school but instead used them for guidance on topics. Molly used the scope and sequence as a guide for the order of topics and adhered pretty closely to it. However, she did vocalize when she believed the order of topics should be switched. For instance, the explicit teaching of the Pythagorean Theorem was not supposed to occur until the fourth quarter but she pointed out that it would make more sense to teach this topic alongside rational and irrational numbers. Many of the resources and supports she utilized during her first year of teaching came from her university experiences: professors, cooperating teachers, coursework, practicums, etc. Her preferred method of instruction, inquiry, did not always align with the common assessments required by the school. This meant she had to reteach some of the material in ways students would be expected to use on these assessments. As she became more comfortable with the curriculum, the need to "re-teach" did not happen as often. On the other hand, Molly was conscious and aware of how her students were progressing through the material and was willing to spend longer on a given topic if she felt they needed more time:

> I was ready to move on and I looked at their quizzes and I saw that they didn't get the distributive property at all. I do a lot of exit slips that help me, formative assessments. Looking at the quizzes helped me say "ok, we're going to take another day." So that stuff is really kind of how I plan my own scope. [Molly- Interview 3- 5: 15-19]

The willingness to be flexible with her schedule allowed Molly and her students to have deeper explorations about concepts and to ensure that students were not left behind.

Though Molly did not feel connected to many of the teachers within her school, she was a part of a mathematics teaching network initiative in her district. The Algebra I teachers from the high school met with middle school teachers to discuss strategies focused on problem solving. The teachers were asked to implement tasks with their students and utilize a guided reflection sheet to help foster problem solving skills. She believes that, while the ideas where good in theory, there were gaps in students' prior knowledge and experiences that hinder their ability to work on many of the tasks. While she did not say this to the group since she was still working on "finding her voice", Molly stated that the elementary teachers should also be included in curricular discussions since everything in mathematics builds on each other [Molly- Interview 4- 4: 9]. One piece of enjoyment she found at these meetings was talking about mathematics and mathematics teaching with other teachers. She reflected that she does not "get to talk math a lot" after leaving college and did value the opportunities she had do so with other teachers [Molly- Interview 4- 4: 19]. Additionally, she was able to share her experiences teaching mathematics using the three-act math model and provided the other teachers with resources.

As discussed in chapter 2, reflection is an important component of PCK development. Molly is a deeply reflective individual who believes she learns best through consciously reflecting on her experiences. She explained that "falling on your face" is sometimes the most valuable learning experiences you can have both as a pre-service and in-service teacher since you can reflect on what was effective or ineffective after having experienced it first-hand [Molly- Interview 4- 7: 6-10]. Similarly, she speculated that by looking back on how she learned mathematics or from her other experiences teaching (in her practicums or student teaching), she was better able to adapt her instruction to her current group of learners or recognize when they were struggling. With all of the pressures of being a first-year teacher including the evaluation process, learning a new curriculum, and managing a classroom, Molly felt she did not have much time to reflect. She explained that she was looking forward to summer when she would have time to reflect on her first year of teaching and "make a plan" for next year.

> If I had more reflection time and more downtime, because even when I go home I'm grading and planning. I work fourteen hours a day! If I had more time to just sit and think about it, I would be a much better teacher. I'm really ready and excited for that opportunity and hoping it will bring some of that joy back into this job since it's been a really draining year. [Molly- Interview 4- 12: 12-17]

Thus, not only is reflecting part of the sense-making process, but a rejuvenating practice that helps sustain Molly in her teaching.

PCK Development

Molly developed aspects of her PCK during her K-12 schooling, college mathematics courses, teacher preparation program, and her first year of teaching. Her development of the different components of PCK varied and occurred at different paces.

Knowledge of content and teaching. Her knowledge of how to *design instruction* deepened during her teacher preparation program and she was able to apply this knowledge in her first year of teaching. This was exhibited in her responses on the PCK Inventory where she frequently responded with what she would tell students or how she would use direct instruction when she began her pre-student teaching semester. For example, when asked how she would help the student struggling with solving equations involving absolute values, she stated "what I tell students to do is to make it into two separate inequalities instead of keeping it all together with the X term in the middle" [Molly- PCK Inventory 1- Question 6]. At this point in her development, she repeatedly explained the use of modeling for students. This seemed illustrate the conflict she was experiencing with the beliefs she was developing about teaching and how to design effective instruction:

> I can think of at least 2 professors that I've had that have followed a lot of the same things that we talk about in how we should be presenting math and a lot of the inquiry stuff and not just sitting there and taking notes and not absorbing anything, which is something that I have experienced a lot. I think back to Calc I, specifically, and Calc III, and Abstract Algebra [laughs] the list continues. And it just being straight lecturing, direct instruction and getting lost a lot and not feeling like I can ask questions. […] I just hate lectures. I really hate it. I know there's a place for it but I think that most students respond so much better when they have to do the work. And even if they're kind of slacking a little bit, they're still doing the work and they're getting more out of it than just copying things into a notebook. I mean, I know it's important but minimal of that. [Molly- Interview 1- 1 & 2: 21-23 & 1-4; Molly- Interview 1- 8: 8-19]

She reached a resolution with her internal struggle after her student teaching experiences. Her responses on the PCK Inventory began to include more activities, group discussion, and use of manipulatives. When asked to describe how she would help students visually interpret multiplying polynomials, she explained the use of algebra tiles in detail:

> Using algebra tiles is a really good tool to show the relationships in multiplying polynomials, particularly in giving an example with a variable in it. Lining up the terms in algebra tiles perpendicularly and then creating the appropriate term you would get from multiplying (which involves you using a squared term tile as well) will help to show the array that is created and where each term comes from in the distributed answer. [Molly- PCK Inventory 3- Question 5]

This development continued during her first year of teaching. Molly recognized that she had strong content knowledge and utilized a variety of instructional strategies that were less “traditional”; she explained that some of the other mathematics teachers at her school were not as strong in their content knowledge and other teachers received many complaints from students and parents since the instructional methods they used were “traditional” and “leaves a lot of kids behind” [Molly- Interview 4- 1: 15-23]. She explained that she tried to utilize inquiry lessons as much as possible but realized that sometimes she did need to provide further explanation in the form of a lecture or discussion: “I prefer to start with inquiry and then explain more in depth with another lesson and then give them practice. I do a lot of application” [Molly- Interview 3- 6: 1-2]. She reflected that she was initially nervous to use inquiry with her students since they had low confidence in their math abilities, but her students impressed her with their perseverance and growth:

> I’ve been trying really hard to stick true to the mathematics and methods I learned in college. Sometimes it goes really well and sometimes it doesn’t. I’ve been trying really hard to do inquiry lessons when I can. Sometimes I get really nervous because I’m finding the kids have really low confidence in math and I go in and I plan something inquiry and I can just see them saying “I don’t get it” or “what is this? I don’t get it.” And every single time it goes above my expectations. They wow me! And I tell them how impressed I am because I think it’s important for them to hear that. I did a three-act math the other day for the first time with a

> group and I wanted to cry it went so well. I've taken stuff from what we did in methods class on the Pythagorean Theorem. I didn't tell them what it was; I printed out the square tiles that our advisor used with us and they got it! I was like "you just figured out the Pythagorean Theorem!" They thought they were so smart! And it was awesome. I'm trying really hard and sometimes it's not easy and I cut myself some slack because sometimes I have to stand up there [points to her board] and tell them things. But, yeah, that's what I'm trying to stick with. [Molly- Interview 3- 2 & 3: 12-21 & 1-6]

In addition to their confidence in mathematics, she noted that her eighth-grade students seemed resistant to inquiry activities. She attributed this to how they previously learned mathematics and that they typically just want the method or formula. From her own experiences both in learning mathematics and in teaching, she realized the need to help students understand the underlying structures of mathematics or "the why" [Molly- Interview 4- 8: 11]. Her knowledge of designing instruction greatly developed from her preparation program and time in the classroom.

At the start of her preparation program, Molly had some knowledge of how to select an appropriate *sequence of topics*. For example, she was able to identify a reasonable order of topics when teaching a unit in trigonometry, though there was room for improvement [Molly- PCK Inventory 1- Question 2]. She continued to develop this aspect of her PCK somewhat during her last semester of coursework and student teaching experience. This was exhibited in how she modified the order of topics she identified in her PCK Inventory responses for Question 2. She was able to indicate a more effective and coherent order of topics to teach for the unit of trigonometry and then remained consistent in this order during more of her responses. With the scope and sequence provided to her from the school, she had little opportunity to fully control the order of topics she taught during her first year of teaching. However, she was able to further develop her knowledge of how to sequence topics. Her growth in this area was also illustrated when she described changes she made in the prescribed curriculum as a first-year teacher: “We’re supposed to do the Pythagorean Theorem until fourth quarter but we thought it was better to teach it after we did rational and irrational numbers” [Molly-Interview 3- 5: 11-13]. She realized the connections between these topics and proposed and implemented the change in the sequence of these topics. Though she did not have many chances to apply this knowledge, she was conscious of what prior knowledge her students needed in order to be successful with different topics. This awareness illustrated her view on the importance of sequencing of topics.

Molly was consistently strong in her knowledge of *selecting examples*. She repeatedly cited the use of examples in each implementation of the PCK Inventory on a variety of questions. For example, at the start of her final year in her preparation program, she explained the use of an example to help students understand the difference between the greatest common divisor and least common multiple:

> I would ask the student to write out all of the factors and all of the multiples of some number and compare the two. It will be obvious that their conjecture is incorrect, and I will ask them to define a divisor and a multiple, and remind them to look at phrases and topics as a whole. [Molly- PCK Inventory 1- Question 3]

Though she did not specify which numbers she would use, this illustrated her view on how an example would help a student consider their misconception. She responded in the same way at the start and end of her student teaching semester and at the start and end of her first year of teaching. Similarly, to help a student understand the error in his or her thinking about the quotient of logarithms, she indicated an example would be helpful (PCK Inventory- Question 8). Again, she maintained the use of an example throughout her student teaching semester and first year of teaching. In addition, she identified a specific example she could use to help them identify their misconception: “I would try to show the student the difference between the two, maybe doing an example of log(1/2) versus log(1)/log(2)" [Molly- PCK Inventory 2- Question 8]. Molly realized the important of selecting appropriate and relevant examples as a pre-student teacher. She specifically stated the need show students "how math is real and in the real world" [Molly- Interview 1- 8: 2]. Her knowledge of how to use real world examples and applications to illustrate mathematics concepts transferred from her pre-service experiences to her work as a first-year teacher. In addition, she began to utilize counterexamples in her explanations to help students understand their mistakes. For instance, when helping a student understand how he misinterpreted the exponent when solving an equation with an inverse trigonometric function, she explained she would use a counterexample:

> This student is applying rules of algebra into trigonometry. They are thinking that the -1 requires them to rewrite with positive exponents, but in reality it is the same thing as arctan. I would ask the student to explain to me what that -1 stands for and how we can write an equivalent statement with the proper rule. I would have them solve correctly and show them how their answers are the same, but the way they got there was not correct. I would also give another example where they would not get the same answer and ask them to solve both ways to see the difference. [Molly- PCK Inventory 5- Question 12]

Overall, her knowledge of selecting examples began and remained strong, while developing slightly as a first-year teacher to include the use of counterexamples.

At the start of her pre-student teaching semester, Molly was anxious about her knowledge of *different representations for topics*. Her main concern was choosing representation of the content that are accurate to the mathematics without resorting to tricks or pseudo-math:

> This semester we're learning so much about how we were taught things wrong and how the way that teachers say things, the way we've heard teachers say things for years is wrong, and what I think I'm nervous about

> is knowing how to say things right and having all of these thoughts in my head at once and knowing how to make words come out [laughs]. [Molly- Interview 1- 5: 12-16]

Even with these concerns, she was able to use her content knowledge to consider multiple representations of different content. For example, she was able to consider multiple methods of how to calculate the area of a triangle on a Geoboard (PCK Inventory Question 2). She was conscious of the need to consider and select appropriate representations throughout her student teaching experiences. In reflecting on these experiences, she noted a growth in her knowledge in this area:

> I think, it's funny because it was so frustrating at first with all of these things we were taught wrong and you know FOIL, the f-word. It was so frustrating at first because I was like how am I going to a) remember that this is all wrong and b) know how to teach it in the right way? And I found myself correcting kids like my advisor would correct us for things like that so…and I don't even think about the way that they were taught to me anymore. [Molly- Interview 2- 7 & 8: 18-23 & 1]

Her knowledge continued to develop throughout her first year of teaching. She continued to try to "stay true" to the mathematics she was taught in her coursework and avoid using tricks like "keep-change-flip" and "FOIL" [Molly- Interview 3- 2: 1-6]. For instance, she frequently used visuals to represent abstract concepts, such as the Pythagorean Theorem. She worked on choosing representations that would help students make connections between topics and to the real world. Through her coursework and experiences working with students she continued to develop her knowledge of evaluating and utilizing different representations.

Unlike the initial development of her knowledge about selecting appropriate examples, Molly did not begin the final year of her preparation program with a strong understanding of how to *use questioning* appropriately. This was evident in her responses on the first PCK Inventory where including questions to help students would have been appropriate but she did not respond in such a way. For instance, when asked what feedback she would give to a student who incorrectly calculated the area of a trapezoid, she included information aligned with telling instead of using questions: “I would remind them of the definition of a base and ask that they reconsider their labels” [Molly- PCK Inventory 1- Question 1]. After both her last semester of coursework and her student teaching semester, she developed this aspect of her PCK greatly. She began to consider how to use questions with her students to help prompt their thinking and engage them in discussions. This growth was illustrated in the changes in her response on the PCK Inventory. Instead of stating what she would tell the student, she included questions she could ask him to help prompt his thinking: "Good job remembering formulas. What are your bases? How might you go about finding the missing side? Think about the Pythagorean Theorem" [Molly- PCK Inventory 2- Question 1]. She continued to develop her use of questions as a first-year teacher. She explained how she viewed answering students’ questions and the methods she chose to use:

> I usually like to answer questions with questions. I think I’ve always kind of done that because then then they have that “soooooh” moment. I don’t like to just tell them. I don’t like to just give them the answer because that’s not them…. whenever someone is like “how do I do this” and I tell them I feel bad after because it takes away their thinking. And they ask a lot. Usually I’ll circulate a lot while some of these teachers just sit at their desks all day. I look for commonalities in student questions and then I’ll bring it to the front. And I like to have students explain their work. I like to use student work as models. I don’t like to just give answers; I like to have them figure it out. It’s a lot of redirecting and scaffolding. [Molly- Interview 3- 16: 13-22]

Similarly, the types of questions she asked students tended to be ones that promoted in-depth thinking or to have them consider another aspect to a problem. For instance, in the second field observation she was conducting a lesson using a three-act math design. During this lesson she used questions to help students consider lines of inquiry and information they would need. She did so *after* students had generated their own questions and tried to devise a plan and arrive at a solution. Her reasoning for providing the scaffolding questions was she realized many students were struggling and chose to help them struggle productively while still giving them the freedom aligned with the three-act math style. As she gained experience with students, she was able to develop her knowledge of the use of questions and the role of different types of questions.

Molly's development in her Knowledge of Content and Teaching (KCT) developed positively during her time as a student teacher and as a first-year teacher (Figure 4.6).

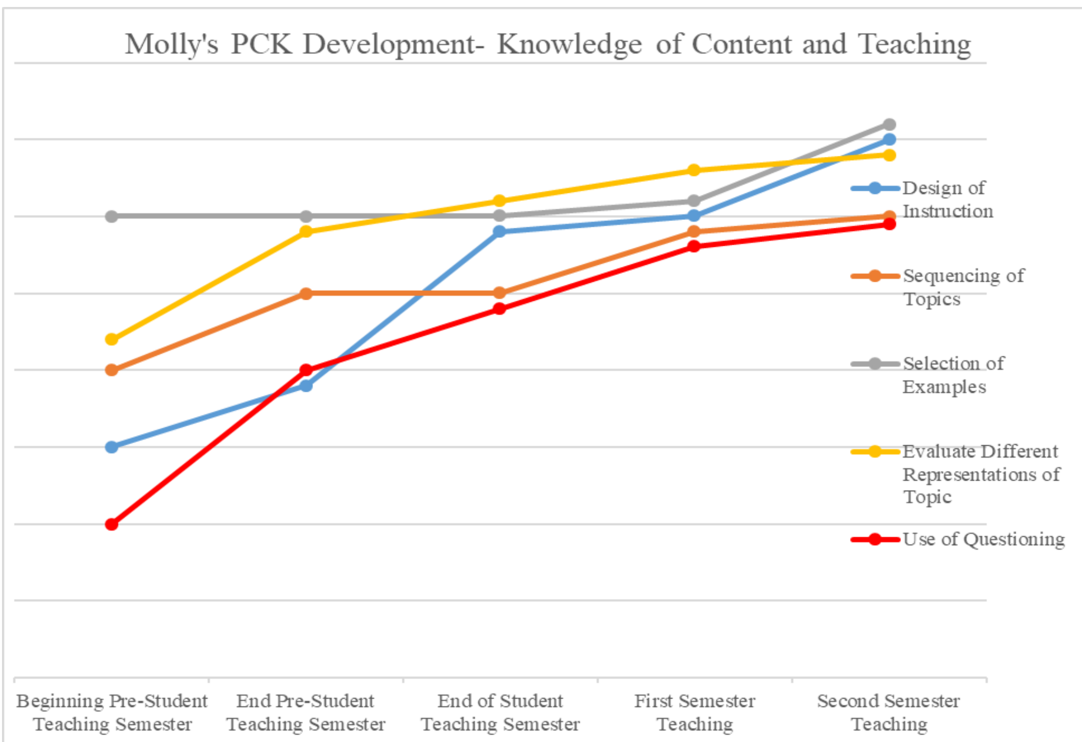


Figure 4.6. Molly's PCK Development in domain of KCT.

Like Kara, there is an upward trend in all KCT tasks and there is less spread between the levels of development for each tasks toward the end of the first year. This further supports the idea that these tasks develop most through experiences in a classroom full-time. However, it also illustrates that development does not occur consistently at the same pace for each of the tasks and it depends on the complexity of the tasks. Molly experienced periods of both rapid and slower development in her KCT.

Knowledge of content and students. Unlike Kara, Molly began the last year of her preparation program with slightly more knowledge about how to *anticipate student thinking*. The reason she was more developed in this area was due to her prolonged placement with the same students during her high school student teaching placement. Other student teachers first interact with their students during the pre-student teaching semester right before their student teaching semester. Instead, Molly had a practicum in the second semester of her junior year, which led into her pre-student teaching and student teaching semesters. This extended interaction with the same students afforded her more time to learn how to anticipate their thinking. She believed she was able to anticipate their thinking better:

> I know about what makes students understand things more, what students need me to relate things to. So knowing them personally but also knowing them and their learning preferences. And knowing how they're going to learn best. So I think that's been cool for me at my high school placement because I've had them for longer. [Molly- Interview 1- 9 & 10: 21-22 & 1-6]

She recognized this area as one that she needed to improve on during her student teaching experiences. She was consciously considering her knowledge and was making connection between her practice and the coursework she completed during her pre-student teaching semester: "what I've been working on a lot in my student teaching, is anticipating student thinking and I think a lot of that comes from what we did in the math capstone class" [Molly- Interview 2- 8: 6-8]. By being reflective about her experiences, she was able to continue to develop this knowledge. In addition, she considered how she teaches in relation to how she was taught and realized she needed to discern how she would think about a topic from how her students would: “sometimes I have to stop and think ‘is a student going to think about this the way that I am?’” [Molly- Interview 2- 8: 14-17]. Similarly, she was able to anticipate the thinking of students in her classes during her first year of teaching to some extent. She was not confident in her students’ prior knowledge and experiences, which led her to be uncertain about how they would approach different topics. With more experience and by interacting with her students, she felt she had become better at anticipating their thinking. However, for students she was unfamiliar with or those she did not have a rapport with, she was unable to anticipate their thinking. For example, she considered areas of difficulty rather than how they would approach a problem in general. Even after her first year of teaching, she was still unable to consider how hypothetical students would be thinking. This illustrates that her knowledge of anticipating student thinking is still developing and is contingent on working with real students.

From the start of her pre-student teaching semester, Molly was able to *consider potential areas of confusion or difficulty* when asked to anticipate how students would think about topics. She demonstrated this knowledge by explicitly identifying elements of a problem that she believed could be difficult. For example, she explained the way in which students learn trigonometry could influence what difficulties they have with the content:

> I think that trigonometry can be very confusing without the proper introduction. I anticipate many students struggling with memorization of the unit circle and essentially what it even means. Many students are just asked to memorize with no proper explanation of what it all means, and without a proper basis of understanding, more confusion will be created as trigonometry builds a lot on itself. [Molly- PCK Inventory 1- Question 2]

Similarly, she stated that students would confuse functions and equations since they have "same essential structure" [Molly- PCK Inventory 1- Question 9]. The responses she provided on the PCK Inventory after the last semester of coursework but before she student taught remained relatively consistent with how she responded at the start of that semester. However, her responses after student teaching became more specific and detailed. She also referenced what she had witnessed students do in different situations to give context to her response. This indicated that her knowledge of anticipating potential areas of difficulty continued to develop through her student teaching experiences. For example, from her experiences she was able to include terminology and concepts students would be confused with when solving equations with radical expressions:

> Students will absolutely become confused about when things are unions and when they are intersections. Students tend to also have a tough time remembering and understanding when they change the sign around (multiplying/dividing by negative numbers). I have also seen students struggle with understanding how to manipulate these equations because they are used to an equal sign being there, not an inequality. [Molly- PCK Inventory 3- Question 14]

At the start of her first year of teaching, she reflected that though her students lacked confidence in mathematics, she should not underestimate their abilities. She started the year believing many students would struggle with certain topics or her teaching style. But instead, students persevered and tried to learn the concepts to the best of their ability; she stated "every single time it goes above my expectations" [Molly- Interview 3- 2: 17]. Through her experiences as a first-year teacher, she was able to further anticipate areas of difficulty or confusion for the hypothetical students on the PCK Inventory. One example of this is in her description of how students would approach squaring a binomial:

> The first thing that students may struggle with is how we can combine terms using multiplication. My students often get confused about how we only combine like terms with addition and subtraction and think that it is the same for multiplication and division. In addition, students may forget to distribute both terms in parentheses. They are often used to using the distributive property with one term to be distributed, so throwing in another term can be a strange concept to them. [Molly- PCK Inventory 5- Question 5]

Therefore, through working with students, she was able to continue to develop her knowledge and ability of anticipating potential areas of difficulty.

Among the aspects of PCK that developed throughout student teaching and while being a classroom teacher, *ways of motivating students* progressively transformed for Molly. At the start of her last of year of her preparation program, she viewed making connections as a main method of motivation for students. In addition, she believed mnemonics or tricks would be helpful for students to remember different concepts:

> There are a lot of ways like mnemonic devices to help with remembering each rule and topic in trigonometry. These "fun" tips can help students memorize things for a long time. However, I think what is even more important is developing connections between new material and past. Teaching trigonometry is really teaching about triangles, which a lot of students learn about early on in their schooling. Introducing the connection first and the origin of what sine, cosine, etc. is will help students to build knowledge instead of just starting from scratch. [Molly- PCK Inventory 1- Question 2]

After completing more coursework on instructional design and learning more about the content found in middle school and high school curriculums, she began to integrate technology and alternative instructional designs into her responses about motivating students. For instance, she explained the use of GeoGebra and investigations when having students calculate the area of a given triangle: "there are some really cool tools on Geogebra that can be implemented through technology. I think also showing the relationships between the different area formulas and having students investigate them can motivate their understanding of area" [Molly- PCK Inventory 2- Question 4]. The developmental trend she began at the start of her student teaching semester continued as she began her middle school and high school placements. She still identified the use of inquiry lesson designs as a method of motivating students for different topics. Also, she still believed the importance of connecting new information to prior knowledge which she began to realize she may need to review or re-teach for some students. By explicitly connecting previously learned material to new information, she believed students would be more confident and motivated to learn. This belief persisted throughout her first year as a teacher:

> My advisor made a good point when I went to talk to his seminar class. Sometimes it is better to review or reteach or teach the skill that they're lacking because it will make the rest of it come easier. [Molly- Interview 4- 9 & 10: 21-22 & 1]

As a first year teacher, she also explained how it was her "mission" to boost her students' confidence in their math abilities [Molly- Interview 3- 8: 6]. If students had more confidence, then they would be more motivated to learn new material and try new things. In addition to the methods of motivating students she believed in entering her first year of teaching, she also began to explicitly identify hands-on activities as another way. For example, she identified making connections and inquiry activities in general as a way of motivating students during a lesson on trigonometry in her response on the PCK Inventory before being a first-year teacher. However, after having her own classroom, she specifically identified lessons ideas she could utilize:

> The more you can make trigonometry hands-on and less memorizing the better. Showing students how to find the cosines and sine for each angle on the unit circle using paper plates and special right triangles is a cool activity, among others. [Molly- PCK Inventory 5- Question 2]

The activity she was referring to involving paper plates was something she learned in her mathematics capstone course during her pre-student teaching semester. After having experiences in classrooms, she was able to reflect on lesson ideas and strategies she learned in her coursework and connect them.

Molly began the final year of her preparation program with some of the knowledge necessary to *hear and interpret student thinking*. In her first responses on the PCK Inventory, she hypothesized as to why students answered in the way they did and attempted to ascertain what they meant by their answers. For instance, when explaining the student's work in Question 7, she stated "the student is only accounting for the positive case when this is not the only one. They are looking to making both sides of the equation match rather than solving for x" [Molly- PCK Inventory 1- Question 7]. She was able to look at the student work and interpret their thinking. Similarly, she demonstrated this knowledge when she explained that the student was incorrectly applying the distributive property instead of using trigonometric theorems [Molly- PCK Inventory 1- Question 11]. As with methods of motivating students, Molly's development of this aspect of PCK really enhanced as she gained full-time classroom experience. After student teaching, she began to include possible root causes of why students on the PCK Inventory responded in the different ways. One example of this is in her responses to Question 1 on the PCK Inventory. At the start of her pre-student teaching and student teaching semesters, she explained that the student incorrectly substituted in the values given into the area formula for a trapezoid and that the student was confused about the orientation of the shape. However, in her response after student teaching, she tried to explain what why the student was confused or mistaken by interpreting their work. She concluded:

> The student has successfully shown that they know the area formula for a trapezoid. However, they have substituted in their values incorrectly, showing that they do not have a deep understanding of the meaning of the formula. I think this student may have just been focusing on the memorization of the formula rather than understanding the meaning of it. They were most likely disoriented by the fact that the trapezoid was rotated, and thus assumed 18 would be the height rather than one of the bases. [Molly- PCK Inventory 3- Question 1]

By interpreting the student's work, she concluded that the student had a superficial understanding of the formula instead of a conceptual one. She also explained that she needed to deduce what a student did not understand when he or she says "I just don't get it" during a lesson [Molly- Interview 2- 8: 16-19]. To do so, she needed to have one-on-one interactions with them to be able to conclude how they are thinking about a problem or topic, what was confusing or problematic for them, and how to help them further. She continued to develop this knowledge during her first year of teaching. As she interacted with more students on a daily basis, she began to notice trends in how they were thinking about a topic. During lessons and activities, she would circulate the classroom and listen to students at work. When she noticed commonalities in their thought process, regardless of being correct or incorrect, she would have the whole class discuss and analyze the different ideas and methods [Molly- Interview 3- 16: 17-22]. In addition to hearing students talk about their work and thought processes, she was able to interpret their thinking through their written work. She used the work they submitted as a way to determine if they understood the material, if they were thinking about the concepts correctly, or if there were misconceptions which she needed to addresses. Through her interactions with students, she further developed her knowledge of hearing and interpreting student thinking.

As we can see from Figure 4.7 below, Molly had different starting points at the beginning of her final year in her preparation program for the tasks associated with Knowledge of Content and Students (KCS).

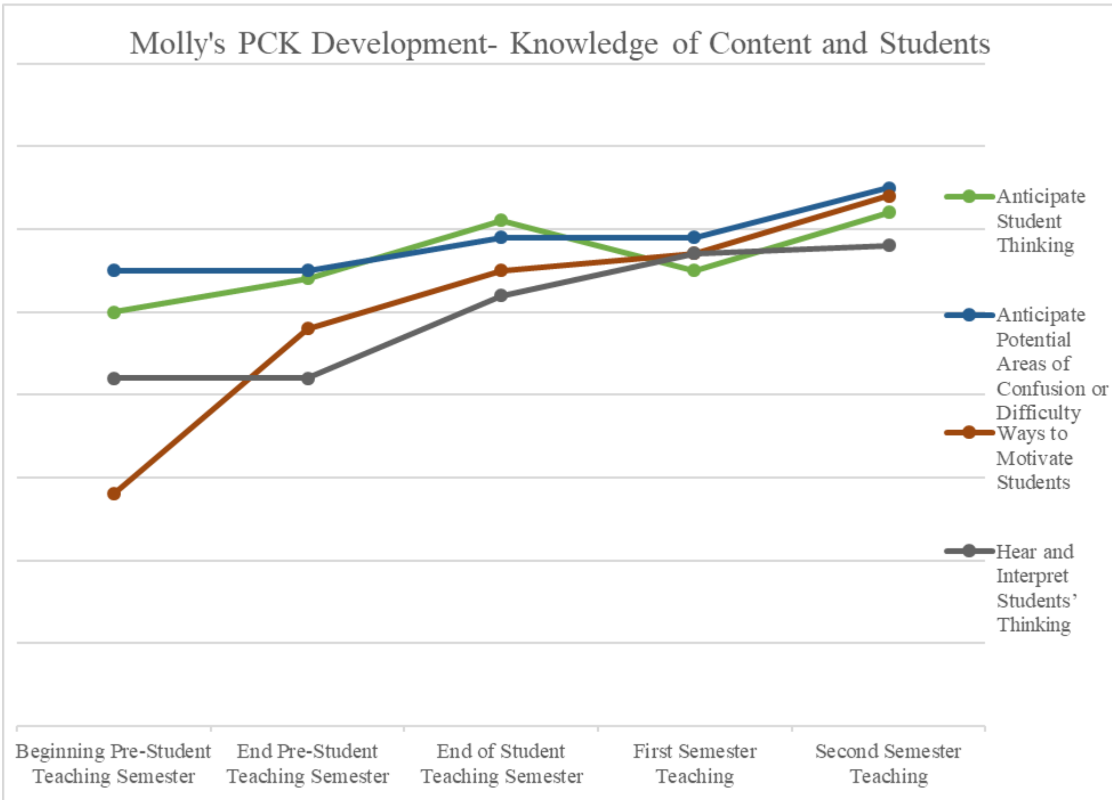


Figure 4.7. Molly's PCK Development in domain of KCS.

However, by the start of her first year of teaching, she had developed in these areas so they were close in terms of her development. This meant some grew dramatically while other stayed relatively constant. The only task that showed some regression in her knowledge was in anticipating student thinking. The reasoning behind this change in knowledge was due to her comfort and time spent with her students; when she started with a new group of students, she needed to learn about them more in order to anticipate their thinking effectively.

Knowledge of content and curriculum. *Knowledge of curriculum*, both vertical and lateral, developed slowly and not as continuously as the other areas of PCK. Molly was apprehensive at the start of her student teaching semester about her lack of knowledge about the topics in different curriculums: "I think coming into senior year, I was like wow, what is in the curriculum for Algebra I? I have no idea" [Molly- Interview 1- 9: 1-6]. She explained that through her practicums, she had "seen" most of the grade levels from middle school to high school [Molly- Interview 1- 4: 4-7]. However, while she had a "wide spectrum" of experiences, they did not necessarily contribute much to her curricular knowledge [Molly- Interview 1- 4: 7]. This was apparent in her initial difficulty when answering questions on the PCK Inventory about the sequencing of topics, as discussed earlier. She did reflect that her coursework contributed to her understanding of connections between topics, subjects, and grades. In conjunction with her coursework, she believed her student teaching experiences helped develop her knowledge of curriculums further. For instance, she was not given a curriculum at her middle school placement, she used resources from coursework and from online to construct an appropriate curriculum [Molly- Interview 2- 2: 10-14]. At her high school placement, she was given more structure and had to work within a certain curriculum. These experiences together helped provide her with more knowledge about how to structure a curriculum and what connections should be made amongst topics and to prior or future knowledge. Molly explained that she believed her comfort and knowledge of curriculum would improve as she gained experiences teaching different topics [Molly- Interview 2- 7: 9-14]. Her notion became somewhat true as she began her first year of teaching. As a first-year teacher, she expected she would receive her curriculum in the form of a scope and sequence before the start of the school year. However, she did not receive her curriculum until a month into school. This caused some difficulty in her development of curricular knowledge since she did not have time to review it prior to implementing it. She also expressed concern over her lack of knowledge about the vertical curriculum: “sometimes I just forget that they don’t know things. That’s one thing that has been hard for me, I’ll say “did you learn this last year?” and I kind of have to look back at the other standards to see if they covered it” [Molly- Interview 3- 6: 13-21]. She realized she needed to become more comfortable with the curriculums in other grades besides her own in order to connect her lessons to prior knowledge. Within her lateral curriculum, she was developing her knowledge through her experiences as a first-year teacher. She could see connections between different topics that spanned throughout the school year. For example, as discussed previously, she recognized the need to teach the Pythagorean Theorem earlier in the year when they taught rational and irrational numbers. Because of this change in her sequencing of topics, she was able to help students transfer this knowledge to different topics. For instance, students recognized the use of the Pythagorean Theorem when investigating the distance formula:

> We were doing the Pythagorean Theorem, like the distance between points and it’s pretty easy. They haven’t really done a lot with it so I was having them count but I asked them to find the distance between two points like this [points to example on worksheet where it’s not a vertical or horizontal line] and one student said “I know how to do it! You just count down and you count over and that’s the length.” And they were like “No it’s not but that’s a right triangle! We can use the Pythagorean Theorem!” And they figured it out all on their own and I almost cried it was so beautiful. [Molly- Interview 3- 7: 15-22]

Later in the year, students were calculating the perimeter and area of different two-dimensional and three-dimensional figures. Most students recognized the need to use the Pythagorean Theorem to find different measurements needed for the formulas [Molly-

Observation 2]. She was able to recognize the Pythagorean Theorem as an important construct in her curriculum which her students would need during the whole academic year. She also reflected that there was a great deal of content in the different grade level curriculums and that is was hard to teach everything in depth: “I think the nature of the curriculum is, especially the seventh grade is, it’s hard to cover everything. I didn’t cover everything. I tried really hard to but that really ruins things in the long run” [Molly- Interview 4- 10: 10-12]. Through her experiences, she learned what topics in her curriculum she needed to focus on more and which ones she could rearrange or combine to be more efficient in the future. Her knowledge in this area of PCK was relatively consistent but did increase in general.

Molly grew in her knowledge of *program and instructional materials* from seeing them to actually using them in her own classroom. During her pre-student teaching and student teaching semesters, she witnessed the use of many different resources both concrete and digital. She explained that she “gathered so many resources” during her teacher preparation program [Molly- Interview 1- 12: 3]. In her practicum experiences and in her coursework she saw the use of many “cool resources” such as 3X Math [Molly- Interview 1- 7: 22]. She was able to reference these materials in her responses on the PCK Inventory, such as describing materials she could use to teach geometric transformations: “the coordinate plane is obviously huge in this type of instruction, and technology like Geogebra is helpful in exploring these topics. You could also just use your old fashioned graph paper and construction paper to show congruence” [Molly- PCK Inventory 2- Question 10]. Similarly, she explained the use of Algebra Tiles when teaching about squaring a binomial, a material she learned about in her methods course:

> Using algebra tiles is a really good tool to show the relationships in multiplying polynomials, particularly in giving an example with a variable in it. Lining up the terms in algebra tiles perpendicularly and then creating the appropriate term you would get from multiplying (which involves you using a squared term tile as well) will help to show the array that is created and where each term comes from in the distributed answer. [Molly- PCK Inventory 3- Question 5]

Before beginning her student teaching experiences, she expressed anxiety about finding appropriate instructional materials as a first-year teacher. She explained that she would be working towards gathering resources and building her collection: "I think I'm just nervous because there's this stigma attached to the first 2 years of teaching and building resources" [Molly- Interview 1- 14: 6-7]. As a student teacher, she created the majority of her materials for her middle school placement and observed her high school cooperating teacher construct the majority of his own resources. These experiences informed how she viewed materials and resources as a first-year teacher. The school she was hired at provided her with some materials, such as an online textbook which she did not utilize often. She explained that her work with her high school cooperating teacher showed her how valuable it could be to create your own resources, but also time-consuming:

> I think working with my high school cooperating teacher was really helpful because he makes a lot of his own stuff and I think that was kind of a blessing and a curse because now I'm trying to reinvent the wheel. [Molly- Interview 4- 5: 13-16]

Being a grant recipient entitled her to receive some classroom resources as a beginning teacher; she was unaware of this fact until another grant recipient informed her. Molly chose to use the money to purchase calculators and other manipulatives she needed for her instruction. When asked what materials and resources she used in her instruction during her first-year of teaching, she explained she used illustrative mathematics, 3-Act

Math, teachers-pay-teachers, Desmos, learnzillion, blendspace and edpuzzles [Molly- Interview 3- 13: 6-18]. As discussed earlier, she utilized the three-act math instructional model frequently, an instructional resource she learned about during her preparation program and high school student teaching experience. She also stated that she frequently referred to her advisor's Wikispace to utilize resources she learned about in her methods and seminar courses and to see if there were any new resources. During her preparation program, she had to join the local association of mathematics teachers and attended one of their yearly conferences. She continued her membership as a first-year teacher and explained the role of professional organizations in the materials she utilized in her classroom:

> Kara and I went to a state math association meeting and it was **awesome**! It was so good! And I've been doing some of the stuff with them [her students] and not only am I finding that it fits with the common assessments but its expanding them and it's focusing on the math practices and that's all from my preparation program. [Molly- Interview 3- 12: 5-9]

In reflecting on her growth in this area of PCK, she noted that the majority of her knowledge developed during the final year of her preparation program. One assignment she noted that was particularly influential in her development was the resource evaluation project she completed in her math capstone course:

> One project, and I've told other people about this, that we did in the math capstone class was looking at the resources and determining whether or not it's effective because I do it every day! Sometimes I'll realize that I skimmed through something too quickly and its crap [laughs]…it's not as good as I thought that it was. That was huge! And those are things that I many not have even thought about before. [Molly- Interview 4- 5 & 6: 18-22 & 1]

Thus, her knowledge of instructional materials developed primarily during the last year of her preparation program. But, she was able to implement and utilize many of the resources during her first year of teaching, also contributing to her knowledge development.

At the start of her first year of teaching, Molly discussed *assessments* often. From her preparation program, she recalled learning about performance assessments and their uses [Molly- Interview 1- 12: 17]. She recognized that performance assessments were only one type of assessment she could utilize and that sometimes she would not have control over the assessments she would be administering. She expressed nervousness about time and the pressures associated with standardized testing and deadlines. For the first month, as discussed earlier, she did not have her curriculum or access to the school's common assessments. As a result, she needed to re-teach some concepts in a different way from what she had original done:

> I started teaching them ways that I knew how to do things but we have common assessments that I didn't get until a month in so I had to re-teach some things the way that they should expect on the common assessments. And I hate teaching to the test so what I try to do is find ways to implement the questions without taking away from the way that I think they need to understand it. [Molly- Interview 3- 5 & 6: 18-22 & 1]

One quality of the common assessments that she identified as being good was there were some applications associated with the questions: "the good thing about the common assessments is that it's not a lot of this, 'solve,' it's more 'here's a story. How would you use it?'" [Molly- Interview 3- 6: 2-4]. In addition to the common assessments mandated by the school, she explained that she frequently utilized formative assessments. She recognized the use of assessments as a way to determine if her students understood the material and whether she needed to spend more time on different concepts. One example was from the quizzes she gave her students on the distributive property:

> Today, for example, I was ready to move on and I looked at their quizzes and I saw that they didn't get the distributive property at all. I do a lot of

> exit slips that help me, formative assessments. Looking at the quizzes helped me say "ok, we're going to take another day." So that stuff is really kind of how I plan my own scope. [Molly- Interview 3- 5: 15-19]

Her knowledge of different assessments and how to use the types of assessments developed as she transitioned from a pre-service teacher to a first-year teacher. She had to make decisions about the types of assessments she would use as well as how to conduct classroom instruction around already formulated assessments.

Molly's development in the domain of Knowledge of Content and Curriculum (KCC) showed interesting trends including spikes and instances of little to no noticeable development (see Figure 4.8 below).

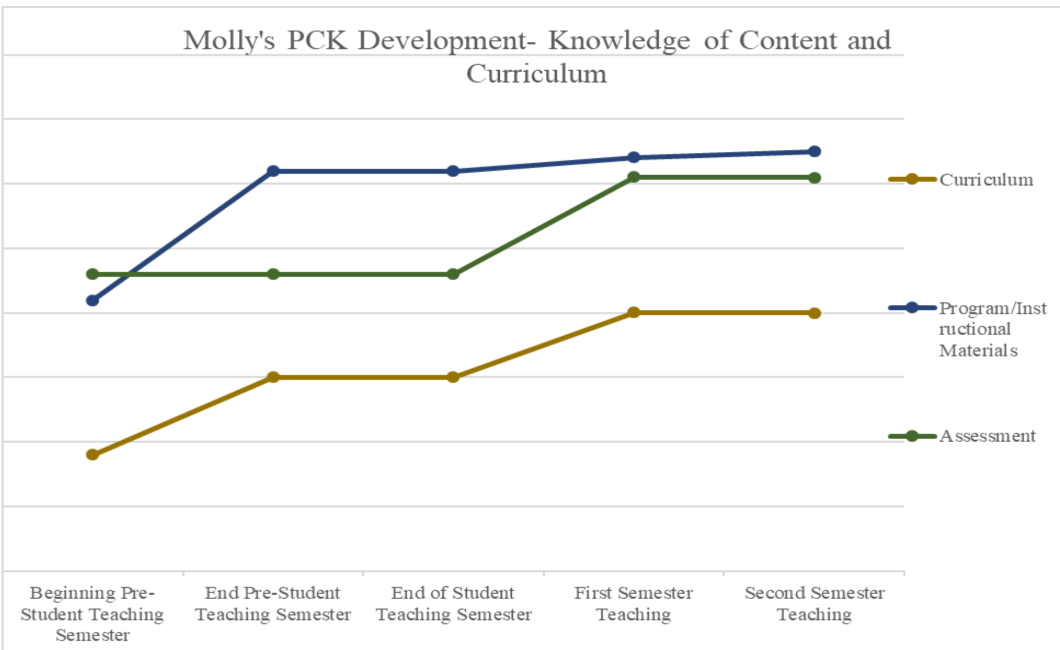


Figure 4.8. Molly's PCK Development in domain of KCC.

We can also see a discrepancy between her knowledge of other tasks and her knowledge of curriculum. It also seems that she develops her knowledge of curriculum when she begins in a new environment and then remains relatively constant without much growth after that. She did explain that she would become more knowledgeable about the curriculum of her school and grades as she gained more experience in the classroom as a teacher. It is also visible that she did not rely on assessments to inform instruction as much during her preparation program and really grew in her knowledge of assessments as a first-year teacher.

Summary of Molly's PCK Development

A summary of Molly's growth and plateaus in her PCK development can be seen visually in the diagram below (Figure 4.9).

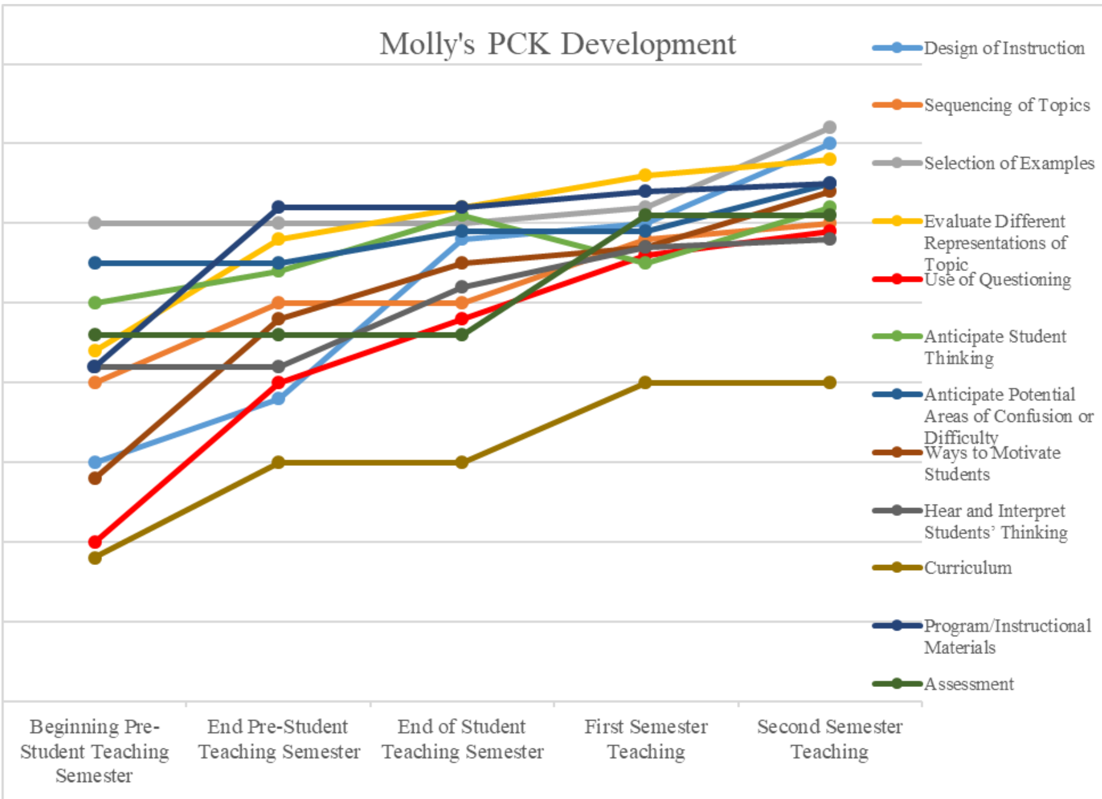


Figure 4.9. Summary of Molly's PCK Development.

Molly grew drastically in her knowledge of designing instruction during the final year of her preparation program. She was able to transfer this knowledge effectively into her first year of teaching. She demonstrated some knowledge development in designing instruction and selecting the order of topics by explaining the importance of connecting prior knowledge. However, since she was given a scope and sequence during her first year of teaching, she did not have much opportunity to further develop her knowledge of how to sequence topics.   She was consistently strong in her knowledge of how to select examples. In addition, she grew in this area by including counterexamples during her first year of teaching. She grew in evaluating and using different representations of topics through her coursework and classroom experiences.   She developed her knowledge of questioning a great deal from working with real students in her student teaching placements and first year of teaching. In the same way, her ability to anticipate student thinking developed through her experiences with students and developing a connection with them. Similarly, she grew in her ability to anticipate potential areas of difficulty or confusion through her experiences working with students. She also progressively developed her knowledge of motivating students by considering instructional design and the role of students' confidence in their mathematical abilities on motivation. Molly was able to hear and interpret student thinking initially but did develop in this area through her work with her students as a student teacher and as a first-year teacher. One area of PCK that she struggled developing was her curricular knowledge. She was initially not strong in this knowledge and grew slightly through her coursework and during her first year of teaching. On the other hand, she developed her knowledge of program and instructional materials during her pre-student teaching and student teaching semesters. Her knowledge also developed when she implemented the materials and resources in her classroom as a first-year teacher. Lastly, her knowledge of assessments developed significantly as began her year as a first-year teacher.

Alyssa

Alyssa is a social individual who values productive interactions with peers and colleagues. She is conscientious of what other people think and was attune to the feelings of her peers during her preparation program. For example, she empathized with her classmates' frustrations with the Praxis tests but understood why the assessments were a requirement. She enjoyed collaborating on assignments and group projects and sees the importance in learning from everyone she comes into contact with. Other's perceptions of her abilities are important to her, especially in group discussions in class. She was reflective about their feedback and thoughts and used them to inform her future practices. One example is when a classmate would present an alternative method of solving a problem, she would ask probing questions in order to fully understand their method and learn from them. As a highly organized individual, she color-codes her notes and materials and keeps a detailed planner. She enjoys following a schedule and knowing what is coming up next. For instance, she would highlight all due dates found in course syllabi at the start of the semester. She would adhere closely to the requirements of assignments and valued feedback from her instructors. When she did not received feedback, she was left with a feeling of uncertainty since she did not know where she stood and felt uncomfortable gauging her own learning. She gave the example of one her courses where she did not get any feedback during the semester:

> She wouldn't hand things back and you would get a course grade but you wouldn't know why you got the course grade because you didn't get any of your work back and so it was just like she just kind of awarded grades however she thought, which I didn't like. […] You didn't get any feedback so you didn't know if you were doing it right. So I think that was really difficult, I did not like that. Yeah, it was frustrating. [Alyssa-Interview 1- 5 & 6: 18-21 & 4-8]

She strived to continue to learn and make connections between her courses and experiences. She is able to do so by being reflective about her experiences and contemplating how they would influence her future.

Entering college, Alyssa had a strong academic background. She attended a local private school for her K-12 education and took many Advanced Placement courses which gave her college credit. The traditional application process to get into the education program was waived due to her SAT scores and academic record in high school. Her experiences learning mathematics were not always easy, but she was able to overcome obstacles and valued the experiences she had with certain teachers:

> I did not do well in math in middle school. I didn't. That's why I had to take Algebra I again and not Geometry as a freshman in high school which is ok, I'm ok with that. I ended up loving my Algebra I teacher; he was awesome. He was very realistic about the problems you're going to see, "these are the ones that I want you to do and that are going to be important," not just do 50 problems. They were very strategic, which I liked and I think that's important to do, strategic problem selection. You have to give skill practice and problems for exploration, thus the need to being selective in the problems you assign. [Alyssa- Interview 1- 15: 2-10]

Alyssa also reflected on the connection she felt with her Algebra II teacher and how that colored her experiences with learning mathematics. She felt her teacher cared about her success, so she believed more in her abilities. The experiences with her Algebra I and Algebra II teachers illustrated to her the role of the teacher in effecting the learning experience for his or her students. Her previous experiences with mathematics contributed to her understanding in her college-level mathematics courses. For instance, she had taken pre-calculus in high school which allowed her to view pre-calculus at the college-level as "fun" or "easy." In college, as she progressed through the calculus sequence and other higher-level mathematics courses, she explained the importance of the teacher:

> Calculus was fun. I liked it because I was able to understand it with the teacher. He gave good notes and we were able to work through all the problems and we had a nice basis for going into Calc II. Calc II, not so fun [laughs]. Integrals kicked my butt but it's ok because I can kind of do them now. So that's ok.

> I really didn't like the teacher for Calc II. He just didn't present it in a way where you knew it. He just assumed you were going to get it from one thing and then he expected you to teach yourself a lot which I think is understandable since it's college but at the same time with those kinds of concepts, especially if you're doing 3-D or double integrals it's very hard to set it up or to evaluate it. So I didn't appreciate his style.
>
> I would do the homework and stuff, it wasn't like I didn't do it, but I feel it was him. I love Professor Jones. He is my favorite person because he has a degree in education and he understands how kids learn and I think that's something you rarely find in our math department, which I think is important. I mean you're teaching a higher level, so ok, and you're here to work on your work, I guess, and you teach on the side, kind of. It's that kind of thing. But he really gets it and wants us to understand it because he's so passionate about it and you can just see that in the way that he teaches and yeah, how he presents the information. He really cares that you're going to understand it so he was one of my favorites, my favorite teacher that I've had with math. [Alyssa- Interview 1- 2+3: 18-23 & 1-12]

Further, she reflected on the design of the mathematics program at the university and expressed concern that it is not necessarily the most appropriate for education majors. Instead, the degree program is geared towards "pure mathematics" and does not foster the needed knowledge of future educators. Alyssa saw the value in courses that provided her with background knowledge, techniques, or tools she would need when working with her future students.

> The proof classes where we learned how to write proofs and number theory will be helpful when I'm teaching. The other ones you're just kind of learning that topic really. Other than that, it's not really teaching applicable. You have to know it because it's why you can do all of the things you do in classes but you don't see that until you get past that point when you're in the math capstone course and it's after the fact and you're like "I have to relearn all of this math" because you're not learning it as you go which is hard. [Alyssa- Interview 1- 2+3: 18-23 & 1-12]

Alyssa continued to reflect on the mathematics she learned in college. She began to see how these experiences developed her content knowledge and pedagogical content knowledge. Specifically, she identified the importance of Abstract Algebra, the math capstone course, and the math methods course for elementary education majors. She stated that she realized the need for taking these courses as she was teaching her own students:

> Thinking about groups, like Abstract Algebra, like the commutative property-- what has the commutative property? All of that. Ok, I get all of this now doing it, teaching them in this context kind of thing. I can do that with addition and multiplication but I can't do that with subtraction. So that's been really cool actually seeing that, like oh yeah, that applies. That's why I had to take that class. I had to be able to explain things to kids about why we can do this and prove it to them. [Alyssa- Interview 3-7: 3-9]

Being faced with student questions and designing lessons prompted her to reflect on the structures of mathematics, which she learned in her high-level mathematics courses.

Alyssa took a variety of methods and practicum courses as an elementary education major seeking an extension in middle level. She noted that the education coursework tended to focus on theory and she found true value in her practicum experiences, "I think a lot of the time we talk a lot of theory and that's great, theories great, but I think it's hard to see how apply it in the classroom and which can be hard" [Alyssa- Interview 1- 3: 20-22]. Through her practicum settings, she worked with diverse students in a variety of settings. Below is a found poem summarizing her different placements, including the grade, types of students, and what she valued most in the experiences.

Practicums and Placements
First- urban second grade
Very influenced from Latin America
All spoke Spanish
Teacher- amazing, made it fun,
Control over her classroom but they were free to kind of learn.
Next- 6th grade in an elementary school
Switched classes
Math teacher- great, good classroom management,
everyone knew what they were supposed to be doing.
Engage New York- don't know how I feel, like it but at the same time I don't.
Then- 7th grade, inclusion model and honors students

All awesome.
Excited to learn the math even though it was hard,
Kids sometimes don't like math.
Finally- 3rd grade for student teaching
Math, I'm actually able to see what we talked about in my math method course
They use the manipulatives and they make arrays,
They're actually doing it.
[Alyssa- Interview 1- 6+8: 13-23 & 1-23 & 1-17]

She went on to explain that she understood visiting a classroom is only a "snapshot" of the day but that all of the experiences contributed to how she viewed teaching and learning:

> Every placement that I've had I've always wanted to go and I've loved being there working with them. It's a snapshot but watching my practicum teachers teach really influenced how I think I'm going to teach in the future. Taking bits and pieces that you like or you think is effective is what I think I'm going to do. [Alyssa- Interview 1- 9+10: 20-23 & 1-8]

Participating in a variety practicum experiences is thus extremely important as it gave Alyssa time to work with different populations and see different "snapshots" of days. As she transitioned to a first-year teacher, Alyssa contemplated the value of working with diverse students during all of her practicums but especially in her student teaching placement. During her final semester, when she student taught, she worked with a large population of students with special needs. The value of these experiences became apparent to her in the population she is currently working with as a full-time teacher at her school, which has similar demographic groups:

> I think that I got a lot of it from student teaching because I had so many kids who were special needs or we were trying to get qualified for special ed. I think that that's a struggle. I think that working with English Learners (ELs) is a struggle. Not everyone wants to do that. [Alyssa- Interview 3- 12: 19-23]

She reflected that her "elementary school practicums and student teaching experience somewhat prepared her for working with special needs populations where her middle school experiences did nothing to further that knowledge" [Alyssa- Follow-up Email- 7-

24-18]. She also felt none of her prior experiences fully prepared her for working with ELs.

After graduating from her teacher preparation program, Alyssa gained employment as a fifth-grade teacher at a local middle school. This public charter school is part of a network that serves students of four districts around the state. In this model, she had a co-teacher and followed a block schedule with 100-minute meeting periods. Alyssa explained that this design gave her someone to "bounce ideas off of," plan with, and collaborate with on classroom management. The amount of time she was able to spend with each group of students made her feel that she did not need to rush through material or lessons. The school provided a curriculum written by the STEM director that dictated how many days teachers should spend on different topics.

> We have a curriculum we have to follow. It's all in our unit plan. For instance, the first bullet points that were in the lesson today, they give us that. That's their notes that the students have to take and then we do fill-ins so that they're actively doing everything. And then we come up with the Do Nows and the modeling and applications (MAP) questions. The school gives us suggested ones but we kind of see and adjust how we see fit. Last week we put in a lesson where they learned how to convert fractions into decimals and decimals into fractions because that wasn't a lesson and the student were supposed to use that skill to add them. We asked "how are we supposed to do that all in one day?" So we change things based on what we think. For the most part, they give us the units and we kind of follow through them. [Alyssa- Interview 3- 1: 12-22]

Alyssa explained that while it is very prescriptive, teachers do have flex days and can adjust or modify how they see fit. However, she also clarified that the common assessments ask for certain skills to be highlighted or methods to be used when solving problems which meant she needed to be sure to include those techniques in her daily instruction.

As part of the structure of the school, Alyssa was part of a fifth grade team, which met on Mondays for common planning time. At these meetings, teachers discussed students and their upcoming lessons. She noted that there was a lack of communication between the elementary teachers and the fifth grade team which left them with little prior knowledge about their students. One example she provided was of a selectively mute student; until a classmate spoke for this student on the first day of school, Alyssa was unaware of the student's condition. As the year went on, she noticed the disconnect of prior knowledge and experiences of her students to her own expectations in terms of content knowledge and language usage. For example, the way in which students talk about division lacked proper terminology. The teachers struggled with correcting their language and re-teaching concepts while not confusing the learners. She also explained that all the teachers on the fifth grade team had access to each other's curriculums and lesson plans online. With her experiences as a pre-service elementary education major, she was able to make curricular connections between different subject areas. Alyssa pulled topics or passages from ELA, history, and science into her mathematics lessons to help students make connections and situate mathematics in real world contexts.

Additionally, the school had content team meetings where all mathematics teachers came together on Wednesdays. At these content team meetings, the teachers would do "deep dives into [their] grade books and decide if [they] have all the data [they] need and what that means" [Alyssa- Interview 3- 11: 4-6]. There is an intense focus on data collection and data points which Alyssa recounted in different instances during her first year of teaching. She also explained that the STEM director did "unit launches" at the content team meetings where teachers explored the connections between the topics in the upcoming unit, resources, and common student pre-conceptions. Through these meetings, working with her co-teacher, and from her own interactions with her students, she felt better able to plan for instruction, anticipate student needs, and locate resources to utilize in her lessons.

PCK Development

During her final year of college, Alyssa continued her development of PCK through her coursework and practicum experiences while student teaching. She grew significantly in some areas while others stayed relatively plateaued. A similar trend happened in her first year of teaching—some tasks associated the PCK development changed while others did not.

Knowledge of content and teaching. When it came to *designing instruction*, Alyssa consistently used direct instruction or modeling as the primary method during both her student teaching year and first year of teaching. When considering how she was taught mathematics, she explained that these were the primary ways *she* learned mathematics. She did recall using manipulatives on some occasions during her K-12 schooling, however these were not the main method of instructional delivery:

> Umm, to be honest, I don't remember using manipulatives much in my own learning. […] I remember using in 2nd grade the unit squares and building with 10s. […] I think I did in 5th grade. I'm pretty sure, I remember being in the classroom for doing math, because we would switch and using things on our desks but I can't really remember. [Interview 1- 14 & 15: 16-18 & 16-18]

In her own instruction, she did utilize direct instruction and modeling the most, though she did explain the importance of allowing students to work in groups and explore some concepts on their own. For example, before her first year of teaching she explained that she would have students make lists or tree diagrams to solve the problem of rolling two dice on the PCK Inventory Question 13 and determine the probability of getting two 1s. However, after she started her first year teaching, she changed her answer to have students roll actual dice to see how their thinking was incorrect and to help them determine the probability.

Alyssa explained that the age group she is working with, fifth graders, likes consistency and competition in their instructional routine. The structure of her lessons typically included a warmup, guided notes, independent practice, and then applications. The class format was prescribed by the school and she stated that she was receiving pressure from the administrators to not to deviate from it:

> A lot of what my co-teacher and I have decided, and there's push back on this from the higher-up people, is to spend a lot of time on our warmups. They are typically awake and alert during the first part of the class so then we do our warmups and its either concept review but we do application problems, like word problems, and we work on analyzing them and doing that during the first part. That's why we don't get to the designated application problem time, because we kind of move it to the beginning. Explaining that to higher-ups is difficult. The warmups typically connect to each other because then they'll do an assessment on it so we have a data point on old skills. We also do assessments in the application time. The last two days, their application time has been writing expressions and then today their assessment was on it. We've been doing it that way but there is push back on how we're setting it up, strategically like that. They're telling us what we should be doing, like having them work more in groups. [Alyssa- Interview 4- 6: 7-20]

This illustrates that though she would like to have had more flexibility in the structure of her classes, she found it difficult to do so with administrative pressure. Thus, this limited how much development she could gain in the area of design of instruction.

The effect of having a prescribed curriculum also extended to Alyssa's ability to select appropriate *sequences of topics*. She had little control over the sequencing of topics in her first year of teaching which resulted in only minor changes in this aspect of her PCK development. Though there was rigidity in the scope and sequence of the curriculum, there were some flexibility which she did utilize. For example, she realized that the current sequence of the curriculum separated order of operations and expressions. Her co-teacher and herself decided to change this arrangement:

> They wanted us to be able to teach writing expressions and written expressions before they even knew what GEMS (Grouping, Exponents, Multiplication or Division, and Subtraction or Addition) was and what parenthesis's function was. And we were like "why? Why would we do that?" You know what I mean? If they don't know what the symbols mean when they're solving, then how are they going to know where to put the when you're doing it? [Alyssa- Interview 3- 5: 16-21]

As Alyssa gained experience in her first year of teaching, she was able to sequence topics on the PCK Inventory into a more developmentally appropriate order. However, she admitted that she was not comfortable with the mathematical concepts and did research before answering Question 2 on the PCK Inventory:

> Each time I get to this question, I have to look up different unit plans to determine the best possible order for these topics. I look at different unit plans and different places have different orders, so I need to use what I know and determine if I agree with their order. [Alyssa- PCK Inventory 5- Question 2]

This shows that in order to appropriately determine a sequence of topics, Alyssa needed to be confident in her content knowledge and be familiar with the concepts themselves.

Throughout her pre-service program and first year of teaching, Alyssa valued the *use of examples* when working with a wide range of students. She worked with students from second grade to seventh grade with a wide range of abilities and needs, including English Learners. Though she knew the importance of selecting appropriate examples to use with her students, she was inconsistent with when she utilized them. This was illustrated in her responses to PCK Inventory Question 3 where she had to explain how she would respond to a student confusing the greatest common divisor as being greater than the least common multiple. At the beginning of the pre-student teaching semester, she explained that she would review the definitions of the concepts and provide an illustrative example. However, at the end of the pre-student teaching semester she stated she would remind the students to consider the whole terms and then tell them the definitions; she would no longer use an example. This was again her response at the start of her first year of teaching. At the end of her first year of teaching, she went back to using an example to illustrate why the student's thinking was incorrect and was very specific in the example she chose to use. This inconsistency in her ability select appropriate examples was also evident in the responses on the PCK Inventory where the use of examples would have been appropriate but she did not respond in that manner. When I considered the topics of the questions, it seems that she was only able to decide to use examples and give examples with content she was comfortable with. For example, she explained that she was not confident in her content knowledge for trigonometry and while the use of examples would have been appropriate in PCK Inventory Question 12, she did not respond in that way ever. Further, when asked how she would teach mathematics if she had free reign at the beginning of her first year of teaching, Alyssa reflected that she should utilize examples more: "definitely learning math in more real world examples is something that I wish that we could do more and that is something that I would probably do" [Alyssa- Interview 3- 13: 17-19]. This was a change from how she previously responded, which included using a problem-posing model where students would need to investigate problems on their own and the use of manipulatives.

As with selecting examples, Alyssa's ability to *evaluate different representations of topics* was inconsistent and seemed to be dependent on her comfort with the topics. For example, when responding to the question on the PCK Inventory about a student's error in solving a compound inequality (Question 6), she only explained using a visual representation alongside a verbal explanation at the beginning of her pre-student teaching semester. In all later responses to this question, she included only what she would say to the student. She did focus on the role of opposites in her last response which occurred at the end of her first year of teaching. This representation connected to how she described integers to her students when they had to order them on a number line [Alyssa- Interview 4]. Again, in situations on the PCK Inventory when she could have considered other representation of topics, she did not. For instance, she never identified the use of graphs to explain differences between various logarithmic functions (PCK Inventory Question 8). With more experience as a classroom teacher, Alyssa began to realize the power of visual representations to help her students understand what was occurring in the problem:

> We've also been having them draw a picture, which maybe we haven't been focusing on as much. You can tell when they don't know what is happening in a problem, they just put whatever they see on the page as their answer. Then we have them draw or we draw a picture for them and ask them what's going on in the problem and they can see it better. Sometimes getting them to draw the picture is difficult. [Alyssa- Interview 4- 2: 9-14]

Similarly, she was able to consider different mnemonic representations for remembering the order of operations. While she learned PEMDAS in her own K-12 experiences, she chose the representation of GEMS (Grouping, Exponents, Multiplication or Division, and Subtraction or Addition) as what she would teach her students. She chose this mnemonic since she believed it was clearer that parenthesis are not the only grouping symbol and it help promote remembering to read the problem from left to right [Alyssa- Interview 3- 5: 15-23]. I wondered if Alyssa's slight development in this area was due to the rigid curriculum with the prefabricated guided notes and independent practices. She does explain that together with her co-teacher she evaluates the content of the information and what to keep or change:

> We have our own units that we follow and our math is pretty strict on day-by-day, so our days are planned out with different objectives and then they're typically aligned to EngageNY content. My co-teacher and I, we tend to go into the EngageNY, look at it, and take the independent practice and guided notes from there or make our own, but then decide whether or not we like everything. [Alyssa- Interview 4- 3: 8-13]

It seemed that they typically took most of the material at face value and did not change many of the representations provided, especially since it aligned with the assessments given by the school. Since she has seen firsthand the power of different representations with her students, particularly visual representations, she has enhanced her PCK slightly in this area.

One task of PCK that Alyssa continuously developed over student teaching and her first year of teaching was the *use of questioning*. At the start of her pre-student teaching semester, she would primarily "tell" students what they should be doing instead of prompting them with questions to reconsider their thinking [example: Alyssa- PCK Inventory 1- Question 1]. This was interesting since she described in detail the question posing method that she learned in her science methods course:

> You pose a question on the board, and then you have them highlight the important words that you're going to need and then you do your experiment. I really liked that model because it got them thinking about when you pose the question on the board and they really get to explore. [Alyssa- Interview 1- 8 & 9: 22-23 & 12-14]

While she did not necessarily transfer this idea from her science methods to her mathematics instruction initially, she developed in this area during her first year of teaching. One of her concerns when she was asking prompting questions to her students was that she was leading them to the answer instead of allowing them to explore their own thinking.

> I'm working on making my questions critical thinking questions. I try to ask why questions but I'm also quick to give hints. I don't want to but

here's a push kind of. I think that's something I'm still struggling with. [Alyssa- Interview 3- 9: 12-15]

Being reflective about the types of questions she was asking and the experience of working with students led her to develop in her use of questioning.

In the domain of Knowledge of Content and Teaching (KCT), Alyssa experienced spurts of development and then times of little growth. In the visual below (Figure 4.10), the dotted lines indicate where data was extrapolated from other interviews and evidence since she was unable to complete the PCK Inventory or participate in an interview at the end of her student teaching experiences.

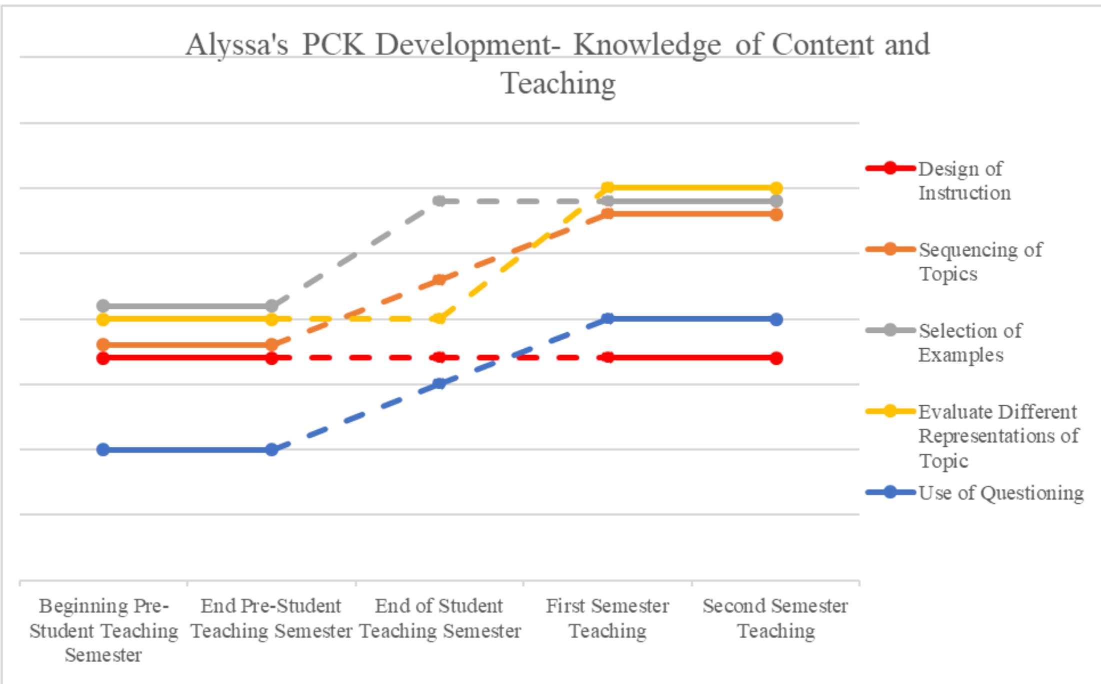


Figure 4.10. Alyssa's PCK Development in domain of KCT.

In all the tasks associated with this domain, we can see that little development occurred during her pre-student teaching semester. Tasks Alyssa felt confident in did develop during her student teaching semester, such as use of questions and selecting examples. She also developed when she began her first year of teaching, but then fell into a routine that did not stimulate any further growth. It seemed that occasions did not arise to prompt her development in designing instruction.

Knowledge of content and students. One area of PCK that Alyssa had difficulty in developing was *anticipating student thinking*. It seemed that if she was comfortable with the content, she was more confident and able to consider how her students would approach different problems. For example, she explains that she enjoys teaching area and volume so when Question 2 on the PCK Inventory asked her to consider multiple ways in which students may solve for the area of a given triangle, she was able to consistently consider ways in which students could approach the problem. Even during her first year of teach, she felt that she was "not ready for the questions that they have" [Alyssa- Interview 3- 9: 10]. She explained that she did not know much about her students prior to starting her first year of teaching so she had difficulty anticipating their prior knowledge and experiences and how they would approach problems. However, after 2 months, she felt she was better able to do so though she still had room for improvement. At the end of her first year of teaching, she was concerned that she was inaccurate in her evaluation of her students' prior knowledge: "maybe I assume where my students are higher than they should be" [Alyssa- Interview 4- 10: 10]. She also speculated that she will be better able to anticipate student thinking having "already done it" once, meaning she gained experiences working with her students that prompted development in this area of PCK [Alyssa- Interview 4- 10: 7-8].

When asked to consider how her students would think about a problem or topic, Alyssa *considered the difficulties or areas of confusion* they might encounter. She answered in this way on PCK Inventory questions where is specifically asked about potential pitfalls and even when it was asking to anticipate their thinking in general. She relied on experiences with students or her own experiences learning for her ability to anticipate these areas. In areas where she had considerable experiences teaching or had her own difficulties, she was able to consider what parts of a concept would be difficult or confusing for the student. These answers were very specific with what problems students would have. For example, Question 5 of the PCK Inventory asked her to consider how students would approach squaring a binomial. She consistently responded that students would incorrectly distribute the exponent to the terms in the binomial: "Students will most likely only square a and b and they will forget to do 2ab. They believe that you can just distribute the exponent to the values, similar to the distributive property of multiplication" [Alyssa- PCK Inventory 4- Question 5]. Similarly, she was able to consider issues that might arise when students encounter solving equations with absolute values (PCK Inventory- Question 6). These are two topics where she had experiences working with students. However, in topics that were more difficult, such as trigonometry, she listed almost every topic as a potential area of confusion (PCK Inventory- Question 2). Also, she consistently did not answer how students would confuse functions and equations (PCK Inventory- Question 9). However, when she reflected on what areas her students from her first year struggled with, she was able to identify ordering integers, dividing decimals, and finding common factors were difficult for them [Alyssa- Interview 4]. She will be able to use this knowledge in the future when working with these topics again. This illustrates that in order for Alyssa to be able to anticipate potential areas of difficulty or confusion, she needed to be comfortable the topics and it was best if she had experiences with students to develop in this area of PCK.

Alyssa considered *ways of motivating* her students during her student teaching experiences and first year of teaching. In both environments, she considered how real world examples and applications can stimulate students to engage with the material. With this as her starting points, she also began to realize the role of lesson design and the organization of curriculum on student motivation. This awareness developed at the end of her first year of teaching. For example, when responding to PCK Inventory Question 14, she explained that she could motivate students by sequencing topics and examples to build to more complicated understanding [Alyssa- PCK Inventory 5]. She also consistently identified peer collaboration and cooperative learning as a method of motivating students. For example, she explained that she might have students discuss the method they chose to use when calculating the area of a given triangle: “we might have students turn and talk with their group members who solve the problem differently or using a different method. Challenge them to prove their way is correct by solving in another way” [Alyssa- PCK Inventory 5- Question 4]. Similarly, Alyssa identifies the role of games and competition on student motivation throughout student teaching and first year of teaching. She viewed this method of motivating students as useful for different topics and ages. For instance, she explained the use of an online games for exploring methods of calculating area (a middle school topic) and a bingo game when teaching solving equations involving radicals (a high school topic). Since she was aware that her middle school students were competitive, she was able to design games as part of her instruction:

> They are competitive. They are. We would have a block in our schedule (100 minutes) that’s set aside for working with applications where they have to do real life stuff and during those blocks, sometimes we don’t always get to them because we focus on the skill since they have to be

> able to do it. So we have days set aside, which we write into the curriculum, and they're doing word problems and analysis for those days and we create games where all the groups have to work on a problem and the group that picked it, if they get the answer right, they get to take x's from another team and the goal is to have the most x's by the end. Each group starts off with 10 x's and they can either take 2 from one team or split it up and take 1 from two different teams. But the other team only loses an x if they got the question wrong. That means you have to be strategic and a lot of them are super competitive so they yell out wrong answers to throw other teams off. They love that. [Alyssa- Interview 4- 3 & 4: 18-23 & 1-9]

She was able to motivate the students to work through problem and apply the skills they learned in class by creating a game.

Alyssa began her pre-student teaching semester with being able to *hear and interpret students' thinking* in a limited manner. She was able to recognize how most students arrived at the various conclusion on the PCK Inventory. For example, she determined that the student incorrectly used the distributive property when expanding $\sin(A + B)$ [Alyssa- PCK Inventory 1- Question 11]. However, in other instances she was unable to fully explain what a student's thought process was when arriving at a solution. One example of this was when she responded to the question about solving an equation with logarithms. She stated "honestly, I am not sure" when asked about the error in the student's solution [Alyssa- PCK Inventory 1- Question 7]. Since she was not confident with her content knowledge on logarithmic functions, she could not interpret completely how this student was thinking. As she transitioned into her first year of teaching, she became more descriptive in her explanations and better at understanding students' thinking. Both on the inventory and in person, she was able to ascertain why students thought in the manner they did and whether their thinking was accurate and appropriate. For instance, on Question 1 she explained in detail why the student made the error when solving for the area of the given trapezoid:

> This student assumed that bases means the sides of the shape that are on the top and bottom. Because there is not a side length listed on segment DC, this child assumed that the segments AD and BC were the bases. [Alyssa- PCK Inventory 4- Question 1]

However, in earlier responses to the same question she stated that the student's error involved forgetting the bases of a trapezoid must be parallel. When she considered her own student's thinking, she provided the example of a student who was having difficulty with subtraction.

> You have a subtraction problem, say 9 – 6 ["nine minus six"]. He wasn't understanding when I was saying it like. He was like "okay?" Instead I asked him "can you take 6 from 9?" [meaning she re-phrased it for him] and he was like "yeah!", I said "great! What is it?" He didn't understand when it's top to bottom. [Alyssa- Interview 3- 6: 10-15]

She asked him a series of questions to understand that is was the way the question was posed that was confusing for the student. Alyssa also explained that she saw her students' thinking through their work or writing; "writing is difficult for them but it's good to see how they're thinking about them" [Alyssa- Interview 3- 4: 10-11]. By examining their work, she could see how they are approaching a problem and whether they are truly understanding the concepts:

> Looking at their misconceptions through their writing and if they're kind of close or almost there, but they can't completely verbalize it yet but they're thinking along the same path is nice. […] You can tell when they don't know what is happening in a problem, they just put whatever they see on the page as their answer. [Alyssa- Interview 4- 7-12]

She also gave the example of a student being stuck in the procedure without really understanding what they were doing:

> Like today, when I'm looking at what we were working on in RtI, I'm seeing that she can't see that the least common multiple is going to be when they multiply the numbers together. She couldn't see that because she's stuck in the method [using a t-chart]. [Alyssa- Interview 4- 5: 15-17]

As she has gained more experience with students, her ability to hear and interpret students' thinking developed further.

Alyssa developed her Knowledge of Content and Students (KCS) at different points in her preparation program or as a first year teacher (Figure 4.11).

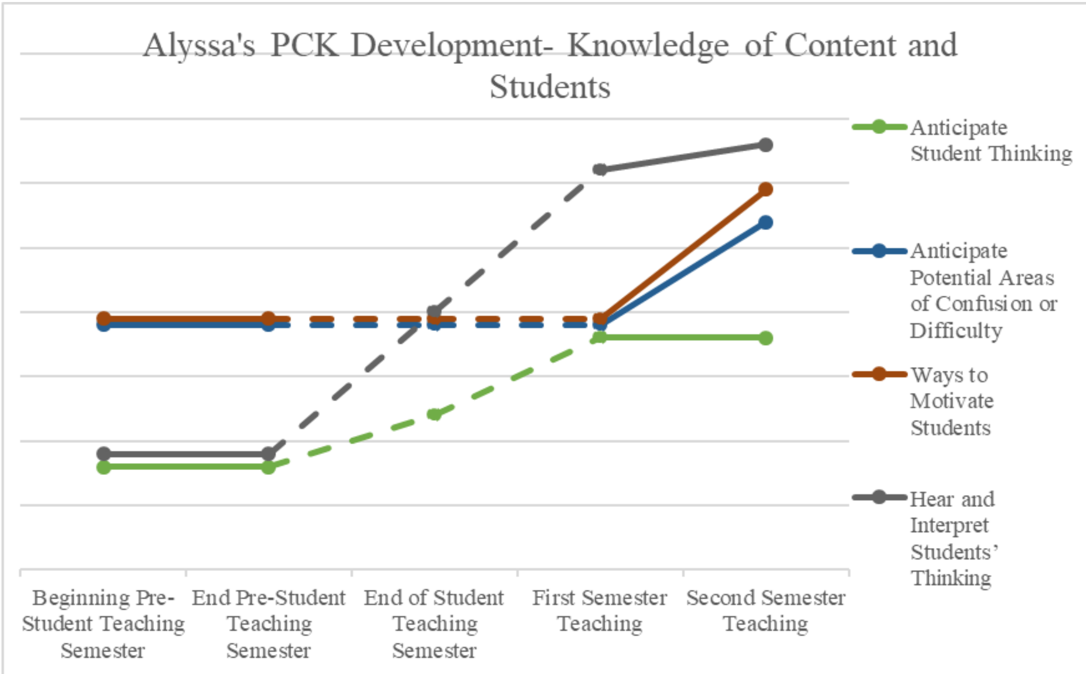


Figure 4.11. Alyssa's PCK Development in domain of KCS.

Again, she did not really grow much as a pre-student teacher. Her work as a student teacher led her to develop in some tasks of this domain, anticipating student thinking and hearing and interpreting student thinking. The development she started in these areas as a student teacher continued into her first year of teaching. The fact that it was these two tasks that develop while the other stayed constant could indicate a link between them. For instance, if you are unable to understand what a student is saying or doing, then how can you anticipate his or her future thinking. It was during her first year of teaching when Alyssa really developed her knowledge in this domain after working with her own students.

Knowledge of content and curriculum. Knowledge of different *curricular* structures and the content of curriculums began developing from Alyssa's teacher preparation coursework. As an elementary education major, she took methods courses in each of the content areas. She specifically recalled learning about linear and spiral curriculums in her social studies methods course:

> In a linear curriculum, you have a very stepped program; everyone learns the same thing every year. Whereas in a spiral curriculum, there are interwoven topics that are throughout the whole thing. You learn about these topics every year and you will build on that your prior knowledge. You're not just learning something completely different which is nice. [Alyssa- Interview 1- 5: 4-9]

Her ability to connect topics within and across grades was limited when she was in her final year of her preparation program. She could identify connections among certain topics but had difficulty with others. For instance, she linked multiplying whole numbers to multiplying polynomials using the area model [Alyssa- PCK Inventory 1- Question 5], taking a topic learned in elementary school and relating it to an Algebra I topic. However, she struggled initially when connecting the ideas of similarity and congruence to geometric transformations (PCK Inventory- Question 10). For instance, she did not identify all types of transformations in either response during her pre-student teaching and student teaching semesters, forgetting about dilations first and translations in both. After beginning her first year of teaching, she was able to connect these two geometric concepts with detail and accuracy. Though she had knowledge of different curriculums, she was anxious for having to either design her own curriculum or implement a curriculum she was not familiar with:

> There's so many different [math] programs and they [methods courses] can't teach you how to do a program because everyone uses something different. Like we had Engage New York in my 6th grade placement but in my 7th grade placement they had a really great math curriculum

> coordinator who kind of found everything, which, that scares me—having to find lessons for every single topic. [Alyssa- Interview 1- 10: 17-21]

Alyssa also reflected on the importance of connecting new information with what was previously taught and teaching something in a way that will help students in the future. She felt that since she was a mathematics major and had experiences in middle school, that she has an advantage over other elementary school majors who struggle with their content knowledge:

> Learning how to take the appropriate steps in the beginning is hard for an elementary education person because you don't really know what they learn in high school. For me, I do because I'm in the math capstone course and I've taken these classes and I'm teaching middle school so I have a good idea but for so many other students, they struggle with math. [Alyssa- Interview 1- 12: 1-5]

With a stronger content knowledge background, she entered her first year of teaching being able to identify gaps in the prescribed curriculum and in students' prior knowledge and experiences. One example, as discussed previously, was in the separation of the order of operations and writing expressions. Another example of an instance in the curriculum where Alyssa felt she should modify it was the connections between decimals and fractions. She explained her co-teacher and herself felt the need to add in a lesson on "how to convert fractions into decimals and decimals into fractions because that wasn't a lesson" as the curriculum expected students to perform addition using fractions only [Alyssa- Interview 3- 1: 18-19]. She expressed frustrations with gaps in her students' prior knowledge since she was aware of the content of that curriculum and had planned to build off of it: "I can also see where they're lacking from last year and I ask myself how they don't have this skill or knowledge already. That's really difficult. We curse their names sometimes, their old teachers. Why is it like that?" [Alyssa- Interview 4- 7: 5-12]. This shows Alyssa has an awareness of not only the lateral fifth-grade curriculum but of the vertical curriculum of her school. This is further exhibited when she identified the next few units during our interview at the start of the school year including subtracting, multiplying, and dividing decimals and geometry concepts. In addition, she was also able to discuss topics in the sixth-grade curriculum such as dividing fractions, integers, area, and volume. As she gained experience in her workplace, she was able to continue to develop her PCK in the area of curriculums.

Connected to the knowledge of curriculum, is the knowledge of *program and instructional materials*. Alyssa initially developed this knowledge during her own K-12 learning, undergraduate methods courses, and practicum experiences. For example, she explained the use of different types of manipulatives as allowing students to letting students explore concepts and come to their own understanding [Alyssa- Interview 1- 13: 14-23].   She noted that she learned the importance of using manipulatives in her different methods courses. When responding to questions on the PCK Inventory as a student teacher, she identified concrete resources, such as graphs, manipulatives, and number lines [Alyssa- PCK Inventory 1- Questions 6 & 10]. During her first year of teaching, she began to identify digital resources as well as the concrete ones, such as online videos or games [Alyssa- PCK Inventory 4 & 5- Questions 6 & 10]. As a first-year teacher, she identified different resources she used in her classroom. However, many of the resources she referenced were used either in her Response to Intervention (RTI) class or for assessment purposes:

> They [the school] gives us illustrative mathematics, links to problems specifically aligned to standards that we can use for application practice but it's not enough for everyday so we kind of create a lot of stuff. They also put in the different EngageNY lessons into the units, so we can model ours after and change it. I use a lot of their word problems and tweak them and put them in for application practice or independent practice so

> that they [the students] can get exposed to them. PARCC released questions, we use those a lot, or released problems from the state's assessment system, we use those. A lot of illustrative mathematics. We're not one-to-one so I use the computers with my RtI kids and we do extra math where they're working on fact fluency: addition, subtraction, multiplication, and just basic facts. And then we go on TenMarks where I assign work on there. And then some of my kids are working on Khan Academy because they're missing a lot of content knowledge and I work with them while they're doing that. [Alyssa- Interview 3- 14: 1-19]

She was able to take the variety of resources her school provided, evaluate them, and combine or modify them to be relevant for her students. As with the area of curricular knowledge, her knowledge of resources and materials developed as she gained experience working with her own students.

Alyssa encountered many types of *assessments* being a student, pre-service, and first-year teacher. She grappled with role of standardized testing as undergraduate student when witnessing her classmates struggle with the licensure exams:

> They can't pass the Praxis test or they're still trying to take it and, which is really hard and you're like, ok how are they going to teach it? I think, which is bad to say, it is because I feel like I'm looking down on them and I don't want to, you know what I mean? But at the same time, you're like you have to pass these exams because it's important, and while it's a standardized exam and they have their drawbacks, but at the same time…and it shouldn't define how you teach because you could be a wonderful teacher and fail your Praxis but at the same time, it's like, it's knowledge. Knowing content knowledge is not necessarily essential to be a good teacher, but I feel like you should be able to pass it and you should…yeah…yeah, I guess so, yeah. Which, people don't like. I've heard mixed opinions about that from others don't feel that way but for some things I feel like you have to know it to be able to teach it so why wouldn't you know it. [Alyssa- Interview 1- 11: 5-21]

She was struggling to explain how the assessment was used to measure content knowledge was important yet it could be preventing some from becoming teachers. It seemed she felt guilty, at first, that she had passed her tests while her classmates did not. However, towards the end of her thinking, she began to feel that the test was an indicator of whether an individual had the knowledge necessary to enter the field of teaching. She also recognized how standardized assessments guide schools and teachers' decision where to focus:

> I think in a lot of ways science and social studies are kind of off shoots of math and reading. I think the emphasis is because math and reading are on the tests. They're on the standardized tests that teachers have to have the students take, the PARCC or the whatever they're going to, because aren't they changing it? It's on the PARCC, it was on the NECAP, it was on the CAT test that you have to take for the catholic schools. It's not that they're not important but in a lot of ways your reading skills and your writing skills really factor into your science and social studies skills. While science may be more interesting or social studies may be more interesting for you to read about, it's just that you need that foundation in reading to be able to do it. [Alyssa- Interview 1-21: 1-12]

This internal battle between the pros and cons of assessments continued into her student teaching semester and first year of teaching. In her first year of teaching, Alyssa was constantly thinking about assessment since the school was focused on the collection of data. For example, she explained that utilize some of their weekly faculty meeting time to ensure they have the data they need; "we have Wednesday as our math content team meeting where we do deep dives into our grade books and make sure we have all the data we need and what that means" [Alyssa- Interview 1- 11: 14-16]. Also, when Alyssa explained why they chose to include more writing in their classes, she stated:

> Writing in math is very important for their district assessments and for the PARCC and the things they have to take—they have to explain why things work and why they are able to do the things they that they can do. [Alyssa- Interview 4- 1: 19-21]

Again, the district and state assessments were responsible for instructional decisions. Though there were concerns with the use of some assessments, she was interested to see her students' results on the different trimester assessments. She was "excited but nervous" to see their "growth or lack thereof" [Alyssa- Interview 4- 10: 17]. Overall, Alyssa was aware of the important role assessment has in learning and teaching and how teachers use assessments to inform their instruction.

Alyssa began stronger overall in the domain of Knowledge of Content and Curriculum (KCC) when compared to the other domains. However, she experienced little growth overall in this area, as seen in Figure 4.12 below.

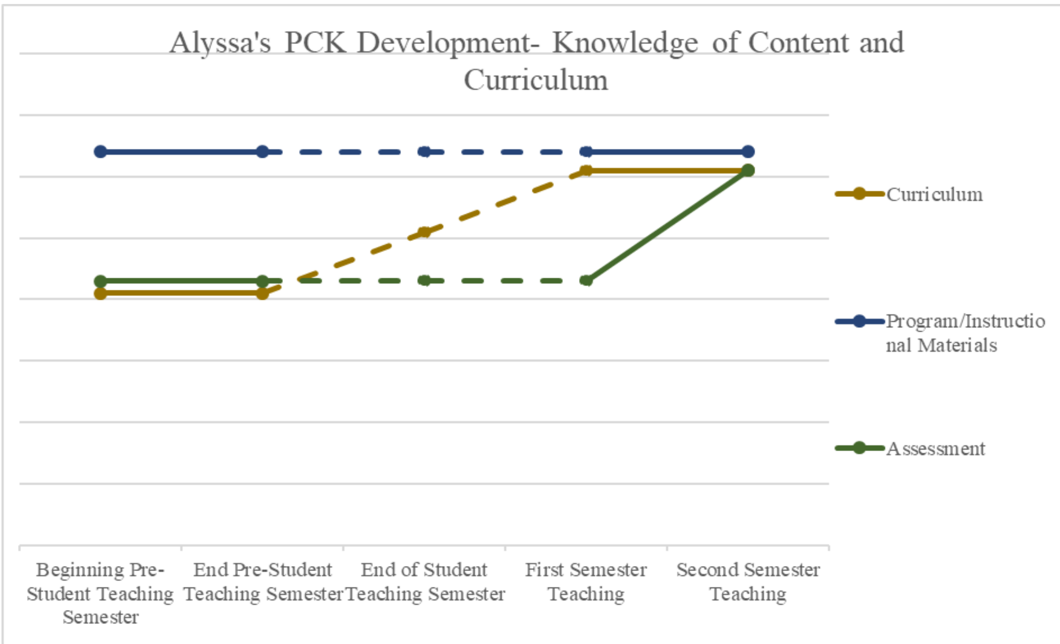


Figure 4.12. Alyssa's PCK Development in domain of KCC.

Her knowledge of curriculum increased as she had to implement prescribed curriculums in her student teaching placements and at the start of her first year of teaching. Once she became comfortable with the curriculum, she did not grow significantly in this area. Similarly, her knowledge of assessments did develop further when she became a first year teacher due to her school's structure and focus on data.

Summary of Alyssa's PCK Development

From the visual of Alyssa's PCK Development below (Figure 4.13), we can see trends in her growth. Again, the dotted lines indicate where data was extrapolated from other interviews and evidence since she was unable to complete the PCK Inventory or participate in an interview at the end of her student teaching experiences.

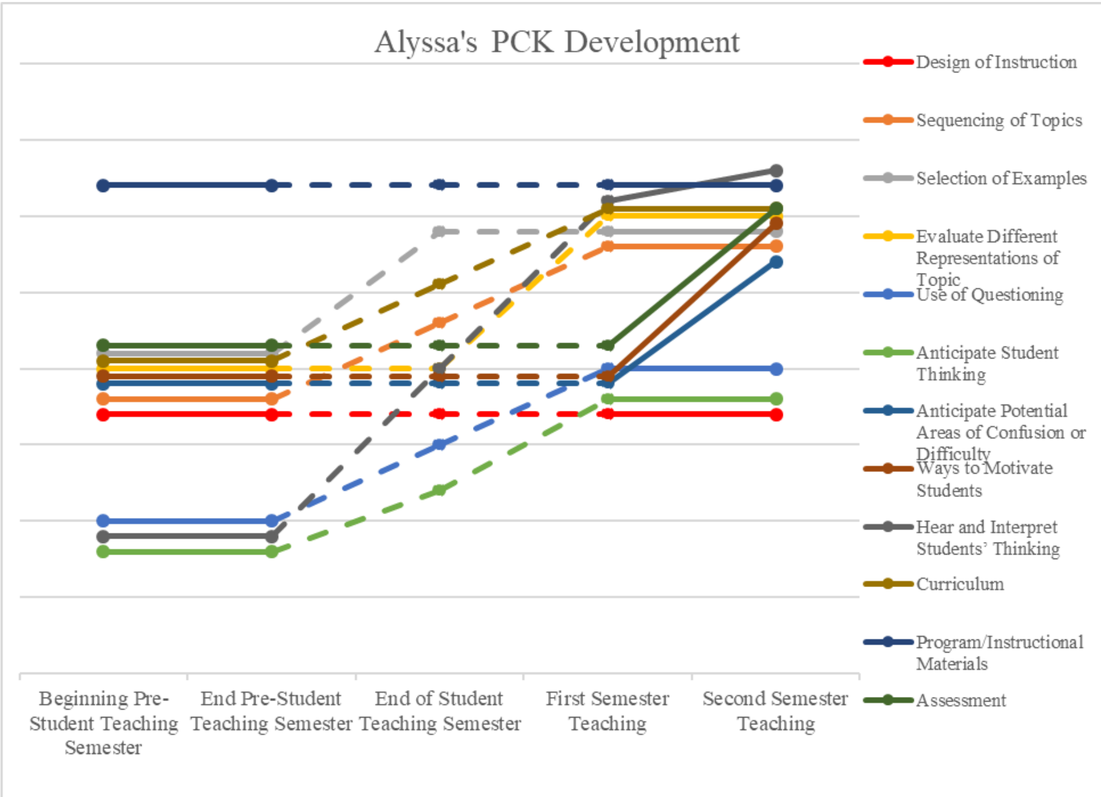


Figure 4.13. Summary of Alyssa's PCK Development.

Alyssa remained relatively stagnant in her knowledge of designing instruction, mainly utilizing direct instruction. Similarly, she did not have much opportunity to further develop her knowledge of how to sequence topics, though she did makes some gains in this area. From experiences in her own classroom, she developed an appreciation for using examples and began to select more real-world ones to utilize in her lessons. Along the same lines, she grew in evaluating and using different representations of topics during her first year of teaching. An area in which she developed great deal in was in her use of questioning. She did so by transferring knowledge from her preparation program and by being reflective about how her use of questioning functioned with her students. On the other hand, she had great difficulty in anticipating student thinking and only really began to develop this aspect of PCK after months of working with her own students as a first-year teacher. In the same way, she had difficulty anticipating potential areas of confusion or difficulty for learners. She could only do so if she had experiences with the content from learning it herself or through teaching it. She did develop her knowledge of how to motivate students and began to consider how instructional design and examples could promote motivation. When it came to hearing and interpreting student thinking, Alyssa developed immensely from her student teaching experiences and throughout her first year of teaching. By working within a pre-construct curriculum as a first-year teacher, she developed her knowledge of both lateral and vertical curriculums. Linked with her curricular knowledge, she developed her awareness and use of different instructional materials. As a first-year teacher, she received a great deal of resources from her school. Lastly, her knowledge of assessments grew from the experiences she gained during her first year of teaching.

Summary

It appears that Kara, Molly, and Alyssa all developed throughout the course of this study, to different extents. Their programs, student teaching experiences, and first years of teaching all contributed to how they grew as teachers. It is also important to consider their experiences before the start of this study, as learners in K-12 classrooms themselves. As discussed in Chapter 2, the socialization of teaching instilled an image of teaching and learning that these participants held to be true for a long time. Through their preparation program and from experiences in classrooms, they began to modify this Gestalt. They were able to transfer the knowledge they developed during their education program years into their first year of teaching.

In this chapter, the experiences of Kara, Molly, and Alyssa were explicated by discussing their development in the different domains of PCK and the tasks within each of these domains. This was culled from their responses to questions on the PCK Inventory and interview questions and from the classroom observations. In the next chapter I will discuss what contributed or hindered their PCK development in more detail. The different themes used to explore these supports will be organized using the Onion Model described by Korthagen (2004). In addition, more relevant literature will be integrated into this discussion.

CHAPTER 5

TIME IN THE OVEN: ANALYSIS

There's a lot more to ogres than people think. […] Ogres are like onions! […] Layers. Onions have layers. Ogres have layers... You get it? We both have layers.
Oh, you both have LAYERS. Oh. You know, not everybody likes onions. CAKE! Everybody loves cake! Cakes have layers!
Shrek & Donkey, *Shrek*

Layers

The different themes identified and used to explore what influenced participants' PCK development will be organized using the model of levels of change, or Onion Model (see Figure 5.1 below), described by Korthagen (2004).

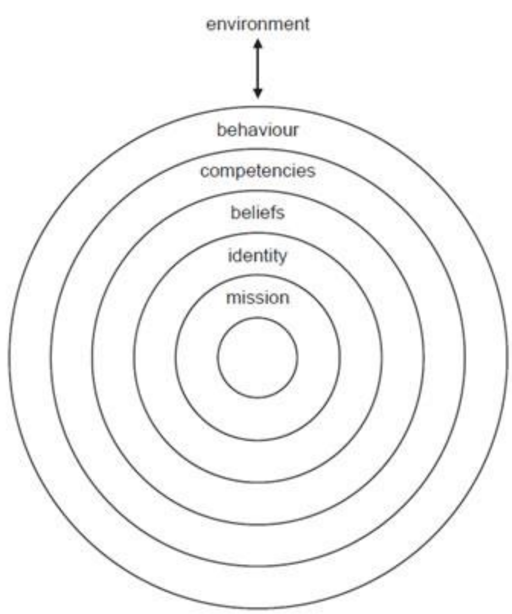


Figure 5.1. The onion: a model of levels of personal and professional change (Korthagen, 2004, p. 80).

Each layer indicates a level in which an individual can experience change, with outer-most layers being visually observed by others and inner layers progressively intrinsic to the person. There is two-way influencing occurring, from the outer layers inward and the inner layers outward. For example, the behaviors an individual can enact are influenced by the milieu and, conversely, how the individual behaves can change the climate of the environment. For instance, students' behaviors in a class can prompt a reaction from the teacher, showing the environment to behavior relationship. In the other direction, a teacher's behaviors can shape the environment, such as if she establishes the setting as a space for exploration and investigation by positively reacting to questions being asked. The immediate environment and behaviors are usually the focus of student teachers: "they often focus on problems in their classes, and the question how to deal with these problems" (Korthagen, 2004, p. 80). The next layer discusses competencies of the individual followed by beliefs. An understanding of what is meant by competency comes from Self-Determination Theory: "*Competence* relates to a person's ability to comply to a range of externally agreed standards, whereas *competency* refers to personal attributes that a person draws upon as part of their work activities" (Dainty, Cheng, & Moore, 2004, p. 878). Korthagen stresses that a teacher's beliefs influence her competencies and must be investigated since beliefs about teaching and learning are highly integrated with teaching practices. The next level is an individual's (professional) identity, which includes self-concept and perceptions and awareness about oneself. One central idea that will be explored in more detail later in this chapter, is how teacher identity is formed and what contributes or influences it. At the core of the model is the mission of the individual. The mission describes the inner force that propels an individual in his or her life and work. Korthagen (2004) explained that the mission could be considered spiritual or religious and described as "deeply felt, personal values that the person regards as inextricably bound up with his or her existence" (p. 85). Discrepancies between the levels can occur if the influencing forces do not match and can cause tension and problems for the teacher and for others in the environment. Exploring the different levels in more detail will provide a framework to discuss what supported or hindered participants' PCK development since the different experiences, interactions, environments, and people all *influenced* their development as a teacher on different levels. Each section of this chapter will utilize a layer or two to analyze participants' experiences and development. There are interactions between the layers and these dynamics explain growths or plateaus in development.

Environments and Behaviors: Wants, Needs, and Concerns

This section will discuss the environments (the classes, people, and schools) where Kara, Molly, and Alyssa learned to be teachers in more detail. Many of the elements in the different settings contributed to PCK development and identifying their beliefs and missions. Both as pre-service and first-year teachers, participants discussed the role of reflection, implementing effective and ineffective lessons, working with students, their development as a teacher, and their wants and concerns. Throughout many of the reported and observed experiences, thoughts, and feelings by the participants, the integral theme of Learning-On-The-Go and the central role it plays in PCK development emerged. In this chapter I will describe this in detail and will lift it out in the next chapter as a central focus of discussion. While these were identified in the previous chapter and described to a large extent, this discussion will be more analytical and focused on how the different settings and elements of the settings contributed or hindered their PCK development. Participants' behaviors will also be analyzed more critically and used to understand their wants, needs, and concerns as developing teachers.

Practicums

In both the elementary and secondary preparation programs, there was a variety of practicum experiences. As pre-service teachers, Kara, Molly, and Alyssa spent time in classrooms ranging in age, location, ability, language used, and other distinguishing characters. The hope was to give them experiences in a multitude of settings so they would be prepared for any future job environment. Kara recalled being in a practicum course as a second-semester freshmen during her preparation program. She stated, "I really like how quick you get experience" which helped solidify her passion for becoming a teacher [Kara- Interview 1- 3: 27-21]. Molly explained that as a grant recipient, all of her practicum experiences were in high-need areas and that some attributes of her placements surprised her. She recalled reading a great deal about the needs of students from underserved populations and could see these characteristics in the classes she observed. When I asked about their experiences in the different practicum settings, it was interesting to note that they all discussed what the students and the teachers were doing. They occasionally discussed what they did in the classroom, but much of the description of the environment was spent on explaining the teachers' role, student behaviors, and topics being taught. For example, Kara described one of her practicum placement teachers as being a "great teacher" and when asked to explain why he was a great, she stated:

> He has a great connection with the kids. They're always just joking around together and stuff so I think they listen to him from that. Which is

> hard because it's kind of a diverse high school so I think it could be difficult and I think he does a really good job working with that. I just feel they listen to him when he talks and asks them questions and I think that has to do a lot with him. He can get them to be engaged in an upper-level math class where not a lot of kids want to be there. [Kara- Interview 1- 4: 16-22]

In a similar way, Alyssa described her first practicum setting and identified the teacher as being in control of the class:

> It was in an urban elementary school and that population is very influenced from Latin America and they all speak Spanish. I was in a higher class so they spoke more English as oppose to the lower classes where they couldn't understand a conversation that we were having, even if it was more basic words. I loved them, they were the cutest things in the world. I loved the class and the teacher, she was amazing. She made it fun. She kind of let them be free but she had control over her classroom but at the same time they were free to kind of learn. [Alyssa- Interview 1- 6: 16-23]

This illustrated the importance placed on what learning and teaching should look like. They were focused on the classroom setting and visible behaviors of the teacher and students. These practicum courses served as a bridge to connect theoretical concepts discussed in education coursework to the practice of teaching (Smith & Lev-Ari, 2005). Molly explained this connection when she explained how she was able to critique lessons and actions she observed:

> I think one of the cool things that I've seen in my practicums, not that this is cool but I've seen teachers do things that I know not to do. And I know that because of my classes and I see things because of the opinions that I have and I can say wow I would never do that in a classroom or I would never talk to a kid like that because I'm learning outside the practicum. [Kara- Interview 1- 3: 7-11]

All three participants identified effective and ineffective lessons they witnessed in their practicum settings. This indicated that having explicit connections between education coursework and practical coursework promoted the development of some areas of PCK. What participants learned in their pedagogy courses influenced their behaviors in their practicums which then altered that setting and prompted discussions back in those courses. The core principle of realistic teacher education is instruction centered on the experiences and concerns of each candidate. Additionally, there is a constant back-and-forth or cyclic relationship between action and reflection to make sense of what is occurring in those experiences and to learn from it. In terms of the Onion Model, this demonstrated the interconnectedness of the environment with behaviors. In addition, these participants' beliefs about teaching and learning started to change as they participated in environments different than what they experienced as students. They were confronted with situations that either confirmed their beliefs or caused them to reevaluate. Through a connection between practicum and education courses, they were supported in these tense situations. Thus, the settings in which teacher candidates engaged with students and deepened their pedagogy by being integrated with education coursework. The education courses were most productive when the boundaries between the environments were blurred and the practicum settings were not stand-alone environments.

The behaviors of the three participants in their different practicum settings depended on where in their preparation program the experience occurred. During the first clinical experiences, all three participants were placed in high-need, urban districts. Participants mainly observed during these classroom visits, so their behavior was a bit disconnected from the classroom interactions. As they progressed further into their preparation program, they took on a more active role in their practicum placements. Their role in these classrooms could be characterized as being "participant observers" since they were learning about teaching by both watching practicing teachers and by working with small groups of students or teaching parts of lessons. Practicums presented teacher candidates with learning environments that may have differed from their own, which prompted them to reconsider their beliefs about teaching and learning. Although these experiences gave them some time working with students, Molly explained that she wished she had done more, such as teaching full lessons. She was unsure whether more experience would have impacted the quality of her instruction, but she did think it would have increased her confidence [Molly- Interview 2- 5: 17-22]. Her lack of confidence demonstrated that she was still forming her identity as a teacher, which caused her some disorientation in her beliefs and reservedness in her evaluation of her competencies. Again, the interaction between the layers of the Onion Model showed that while the inner layers were in flux, the behaviors of the individual demonstrated the unease they felt. For instance, Kara's behaviors in the practicum settings was a bit more timid than as a first-year teacher since she was still learning her place. She was still in the process of forming her teacher identity, understanding her own beliefs about teaching and learning, and solidifying her mission as a teacher. Kara and Alyssa echoed Molly's characterization of practicums as being a surface-level experience at teaching. Kara stated, "some of the practicums before student teaching gave me less experience than student teaching since I was just observing and seeing how they [other teachers] did things and you don't see as much background of it" [Kara- Interview 2- 2: 21-24]. Alyssa explained that going to practicums was seeing a "snapshot" of the day and of what teaching entailed [Alyssa- Interview 1- 5: 21]. However, the practicum courses did promote development in some areas of the PCK. This could be attributed to the fact that the practicum courses were integrated with education courses they were taking in their preparation programs

(Darling-Hammond, 2006). They would also write journal reflections on what they saw or did in the different classrooms, thus one of the behaviors was to be reflective about their experiences. Being reflective required participants to consider their environment, including their students' behaviors, and their own actions and juxtapose them with their own beliefs about learning, self-evaluate their competencies, and consider their identity development. Also, reflective practices establish conditions for teacher candidates to be able to develop different competencies. The act of reflecting helped them to make sense of their experiences (Eisner, 2002; Korthagen, Loughran, & Russell, 2006) and further developed some areas of their PCK, such as designing lessons, sequencing topics, and locating and using instructional materials. At times, through reflection, participants also considered the driving force that compelled them towards a certain action, demonstrating their inner mission.

> I'm really just excited to be a teacher and I'm really excited for those relationships I have with students and I know all teachers say that "if you can make a difference in one student's life, that's all you need," and I mean I guess it's really true. I'm really looking forward to those students who can take a lot out of my class. I'm looking forward to those who don't because it will be a good lesson for me too, as corny as it sounds. I just watched a video that my roommate tagged me in on Facebook, it was students- they wrote letters to their teachers about how much of an impact they had made. I feel like one thing that I never saw from my teachers ever was how hard they were on themselves when a student doesn't get something, you take it personally, and I get that now because I get how it works. And so, to hear from students who really took a lot out of what I taught them or even just the relationship I have with them, I'm really looking forward to that. Even if it doesn't happen a lot. [Molly- Interview 1- 13: 8-20]

Molly realized that her main purpose as a teacher was to make a difference in the lives of her students and to be there to support them in any way she could. She did not want her students to feel abandoned or alone as they grow up and felt that by teaching she could impact their lives. Kara also expressed excitement about being able to help students develop and grow: “I’m mostly excited for just working with students and seeing how I can make a difference and help them grow” [Kara- Interview 1- 10: 20-21]. Alyssa recognized that being at teacher was not always focused on content or academics, but also in helping students develop socially and morally.

One characteristic of the teacher preparation program for secondary education, which Kara and Molly completed, was that they had designed the sequence of courses to be developmentally stimulating. Neither Kara nor Molly identified this sequence as a vertical progression with connections made between practicum experiences. They both recognized the horizontal connections between the theory and pedagogy courses they were taking concurrently with the practicum courses. In a similar way, Alyssa noted connections between her methods courses and her practicum experiences, but not necessarily a vertical developmental progression between practicum experiences. However, through retrospective reflection on their preparation program and practicums, they were able to identify areas of development in their PCK from their experiences in those environments. For example, Kara explained her growth in designing instruction and use of questioning with students when she compared her behaviors in different practicum settings [Kara- Interview 2- 4: 2-8]. Fieman-Nemser (2001) explained the importance of having a cohesive, interconnected curriculum with a series of integrated learning opportunities:

> Through a careful sequence of multiple placements […] programs make it possible for teacher candidates to see and practice the kind of teaching they are learning about in their courses as they move from observation to limited

participation to full responsibility with appropriate modeling and supervision. (p. 1024).

In realistic teacher education, this design is referred to as the gradual increase of complexity in the tasks teacher candidates encounter over time. Brouwer and Korthagen (2005) argue “the benefit of the gradual increase in complexity resides in the fact that it creates opportunities for students to come to grips with the teacher role and its many demands” (p. 192). Carefully sequenced and integrated coursework provides opportunities for teacher candidates to be confronted with their beliefs, develop their competencies, grow in their identity development, and come to a better understanding of their mission. Participants’ different competencies were targeted by the coursework in the various practicum environments where prompted their development. They also worked with students who came from a multitude of backgrounds, many of which were very different from their own. This allowed them to re-examine their own beliefs and develop a sense of why there were pursuing this profession. Since participants did not always see the coursework as a developmental progression, this could indicate that the programs need to make connections between the practicum experiences more explicit to help promote the developmental progression of PCK.

Student Teaching

For all three participants, their student teaching experiences provided launch points for many aspects of their PCK development. This is congruent with Kleickmann and colleagues (2013) identification of student teaching experiences as influential in the development of CK and PCK for pre-service teachers. Though Kara, Molly, and Alyssa were placed at different schools and grade levels, they each described their students and classes which provided images of these environments. Brouwer and Korthagen (2005) found collaborating with peers in student teaching as nurturing to the PCK development. Kara was placed at a middle school and high school with another student teacher. This meant she could collaborate with her peer as well as her cooperating teachers on lessons and materials. Molly also explained that she would share resources with another student teacher since they were teaching the same grade at the middle level. This illustrated that the environments where they student taught were collaborative and social at the peer level which supported development of certain areas of PCK. On the other hand, Alyssa did not discuss collaborating in her student teaching placements and, as explained in the previous chapter, her development in some aspects of PCK remained stagnant during this time. While this can not be entirely attributed to the lack of peer collaboration, it is noteworthy since working with peers during student teaching has been found to be supportive in stressful situations (Murray-Harvey et al., 2000) and promote the "development of practice" (Korthagen et al., 2006, p. 1027). One environmental element, peer discussion, influenced changes in the inner levels of an individual's development. Working with colleagues is an important skill for practicing teachers and peer collaboration contributes to developing that competency. Peers also offer other perspectives as teacher candidates continue to modify their beliefs and form their professional identities. They also speak and think about teaching in similar ways that are very understandable to each other, while sometimes the language of teaching of a cooperating teacher or a university supervisor is less penetrable. Molly also explained that it was important that they were able to discuss their experiences in student teaching with their peers in their seminar course:

> We all had so much talk about with our own experiences. Which is good, because I can go home and talk to my roommates and tell them what's going on but they don't really get it. They smile and nod but it was really good to kind of compare my experiences to what my peers were going through. [Molly- Interview 2- 6: 4-7]

Korthagen, Loughran, and Russell (2006) identified discussions with peers as a method of developing PCK: "They learn not so much by being taught by their teacher educators, but by structured reflection on their experiences and discussions with peers. In this way the student teachers begin to create their own professional knowledge" (p. 1029). As discussed previously, reflection plays a mediating role between the interactions of the layers in the Onion Model. Through interacting with peers, individuals are exposed to other opinions, viewpoints, and beliefs. By reflecting on these experiences and comparing their beliefs to their peers, teacher candidates recognize their own set of beliefs, further construct their identity, and works towards understanding their personal missions. Student teachers appreciate collaborative learning (Hauge & Wittek, 2003) and seek feedback from their peers (Smith & Lev-Ari, 2005). Soini, Pietarinen, Toom, and Pyhältö (2015) explain, "the quality of peer relations is a key regulator for student teachers' sense of professional agency from the very beginning of teacher studies" (p. 651). Thus, it was not only the immediate environments of their student teaching placements that influenced PCK development, but also environments of their associated education coursework that contributed to the growth in many areas.

When participants were asked to describe their student teaching experiences, they explained that there was a clear structure to most of the classes they took over as student teacher. For instance, Kara and Molly noted there were already establish classroom management systems present at their placements and it sometimes felt awkward to change it. Kara stated: "When I student taught my middle school placement had awesome classroom management so I walked into it and it was already setup for me and I just went from there" [Kara- Interview 3- 15: 4-6]. Similarly, Molly viewed her placements as established and did not want to disrupt the environment with her behavior: "I also think that a part of it has been that I'm in someone else's classroom and I don't want to step on anyone's toes or be someone that I'm not supposed to be" [Molly- Interview 2- 6 & 7: 22-23 & 1]. However, they felt supported in their placements and attributed much of their PCK development to the work they did with their cooperating teachers and with the students at their placements. They felt welcomed into their student teaching placements and felt comfortable in the environment to try some of their ideas about teaching and learning in a limited way. They did not want to deviate too much from the norms of the classroom. This behavior is consistent with the socialization of teaching where pre-service teachers learn what behaviors are appropriate and expected through interactions with mentor teachers (Maloney, 2013). For these reasons, finding the right placement for each teacher candidate is so paramount and not always an easy undertaking. Preparation programs and teacher educators need to secure placements that both support and challenge candidates.

Students' and their behaviors are considered part of the environment in the Onion Model since they are external to the individual (Korthagen, 2004). The demographics of the students as well as the behaviors of the students are of interest since the interactions the participants had with the students in their placements influenced different aspects of PCK development. Molly, as explained earlier, was a grant recipient and all of her practicums and student teaching placements were in high-need, urban and urban-ring districts. She worked with a diverse population of students during her student teaching placements, which prompted her to develop how she designed instruction, what materials she would use, and how she could motivate her students. Her environments prompted her to consider her competencies, especially working with English language learners, and she actively worked to improve them through her behaviors in these settings. She noticed her own growth in this area and explained she needed to expand her knowledge of instructional methods in order to both interest students and to teach them in the best way possible. For example, she explained how students responded to different forms of instruction she tried in her student teaching placement:

> Kids don't want to listen; they don't want to learn when they're just being talked at. I don't want to just talk at them. And even when I tried to engage them when I'm doing direct instruction, they're kind of like "meh." Whereas they get excited about inquiry and excited about independent and group work compared to me telling them what to do. [Molly- Interview 1- 11: 16-21]

She was able to notice how students behaved in response to different instructional methods, which supported her PCK development. Similarly, Kara was reflective about the behaviors of students in various lessons which also developed aspects of her PCK. In one instance, she explained how students engaged in the different activities:

> We did a lot of carousel activities and students loved it; they loved getting out of their seats and being anonymous and writing things down but also getting the chance to be the one who writes things down. A lot of group work and different activities like that. They loved that. And even sometimes, in my high school, if I had them stand up and go to a group. We did a speed dating activity and they loved just getting up. I think those are helpful ways because we did the "experience a high school student day" and you just sit for the whole day. I've realized that getting them up and getting them more involved and not just teaching things at them helps them. [Kara- Interview 2- 9: 11-18]

Kim and colleagues (2018) explain, "teaching behaviors are acquired and maintained as a result of reinforcement and stopped by the absence of reinforcement and/or punishment" (p. 134). Among the reinforcements and punishments identified by these researchers are students being on-task or off-task and their success or lack-there-of. Since students had a positive reaction to the activities, Kara viewed them as effective and developed in the areas of her PCK including designing instruction and motivating students. Alyssa's description of the students she worked with during student teaching indicated that they may have hindered her development in some aspects of her PCK: "I had a really crazy group of third graders [in student teaching]. Lots of behaviors, like which ones doing what" [Alyssa- Interview 3- 8: 18-20]. She also explained that many of her students were either special needs or they were working to have them identified as special needs, which is another example of student characteristics that can be confounding for beginning teachers and make them question their competencies. Working with these students prompted her development in hearing and interpreting student thinking, selecting examples, and anticipating their thinking. On the other hand, she stayed relatively stagnant in other areas of her PCK, such as design of instruction and evaluating different representations of topics. This could be due to the consistency required in her placement and the lack of freedom she experienced in constructing her own lessons.

The classroom and resources are also important components of the environment to consider in regards to participants' PCK development. Participants' beliefs about learning and teaching were either confirmed or challenged by the structures of the classrooms. When they implemented lesson plans aligned to their beliefs and viewed them as effective, their beliefs were confirmed. On the other hand, if they deemed the lesson as ineffective based on how they felt or what they witnessed in their students' behaviors, then their beliefs were challenged. If the student teachers were presented with resources unfamiliar to them, their experiences utilizing those materials contributed to their competencies. Both Kara and Molly were in placements where technology was integrated into the classrooms. Molly explained she used technology in her middle school placement: "When I was at my middle school for my student teaching, blended learning was my team's whole thing" [Molly- Interview 3- 9: 3 & 4]. This supported her development of certain areas of PCK and she transferred her knowledge of this type of instructional design and resources to her first year of teaching. In addition, it contributed to her beliefs about technology's role in instructional design as well as her competencies for using different programs and design structures. Similarly, Kara used Google Classroom in both her middle school and high school student teaching placements. She also described other digital resources she used in her placements, such as math-by-fives videos. While Alyssa did not discuss the types of technology used in her student teaching placements, she did describe the use of manipulatives and different representations of a multiplication including the array method. Kara also noted the use of manipulatives in her middle level student teaching placement during the teaching of fractions. The exposure and use of different materials and resources supported participants' knowledge development in design of instruction, evaluating different representations of topics, and program and instructional materials. As with technology and resources, the classroom arrangement was also important to consider. Molly was the only one who discussed how her classrooms were arranged:

> I moved all my students into groups in the middle school where they were sitting in rows before. And my cooperating teacher actually ended up keeping it that way after I left which made me really happy and he said he's actually been doing a lot more group work with them. [Molly- Interview 2- 10 & 11: 21-23 & 1]

While this alludes to Molly's beliefs about learning, which will be discussed later in this chapter, it also showed how her behavior influenced the learning environment. And it also shows how the environment influenced the design of instruction she used—since her students were moved into groups, she utilized cooperative learning more.

As student teachers, Kara, Molly, and Alyssa began to behave differently in the classroom. They began to take some ownership of their classes, referring to the students as "my students" or "my kids" instead of "the students." This change in language indicated they started to view the students as part of their responsibility which could have prompted some of the development in their PCK. It also illustrated development in their identity as they began to view themselves as teachers. Similarly, they began to have confidence in their competencies since their identity as a teacher became clearer. It seems that once people start teaching and interacting with "their" students, these tasks start to become real and applicable and they are more prone to address them in practice. Some of the tasks were either too complex for them to address in their student teaching placements or they were out of the scope of the placements. For instance, student teachers really do not have control over the sequence of topics or the curriculum for their classes—for the most part, they have to go with whatever is established by their cooperating teacher. Kara expressed how out of the norm student teaching can be when compared to beginning the year as a teacher: "it's hard to just jump in from mid-January on a random Monday where I just appear in the class and I just take over" [Kara-Interview 2- 11: 22-23]. The lack of flexibility in some aspects of the student teaching placements limits the amount of development possible for student teachers. Student teachers also tend to work towards tasks they see as central to teaching (Kennedy, 1997; Mulholland & Wallace, 2003), such as managing behaviors, designing instruction, and hearing and interpreting student thinking. These behaviors are physical manifestations of their beliefs about the role of a teacher. Also, their professional identities are still in formation and they tend to use references of teachers from their past as models. Among the competencies they view as central to a teacher's role are those they are visible, like constructing lesson plans. Through more experience in the classroom, they begin to be confronted with situations where their beliefs do not hold which prompts their development. Smith and Lev-Ari (2005) found that while theory courses provided student teachers with a basis for working with certain populations, such as special needs students or those with different backgrounds, "the more tacit components of knowledge of teaching, such as handling spontaneous problems, decision making, developing a professional vision, class management, are best acquired during the practicum when student teachers are engaged in active learning, learning by doing" (p. 298). Kara and Molly's survey responses were consistent with these researchers' findings; they identified their student teaching experiences as where they learned tasks for daily classroom practice but their pedagogy courses for their broader, theoretical knowledge development. Molly also verbally explained this Learning-On-The-Go feeling: "Some of this stuff you're only going to *really* learn from experience and you're not going to get that authentic experience in a student teaching situation" [Molly- Interview 3- 15: 3-15]. The experiences in participants' student teaching placements were directly influenced by the environments they were in coupled with their behaviors in those placements. Through their experiences, they developed in the different domains of PCK, some domains more so than others.

First Year Teaching

Upon graduating from their preparation programs, Kara, Molly, and Alyssa all obtained employment at different middle schools, as discussed in the previous chapter. While some of the conditions present in their work environments were discussed with the findings, a more detailed look at elements of their first year of teaching atmospheres will be explored in this section. The school structure, classroom setup, students and their behaviors, and relationships with colleagues and school leaders are among the characteristics of the environments that will be further explored in this section.

In terms of school type, I have already explained that all three participants gained employment at the middle level. However, there were some differences in the grade levels and consequently the ages of the students that each teacher worked with. Alyssa worked with fifth grade students, a population that is typically considered elementary age but was included at the middle level in this particular district. Molly taught students in both seventh and eighth grade while Kara taught four classes of eighth grade students. All three first-year teachers were concerned about their classroom management, which reveals their focus on the immediate learning environment. Their apprehensions about classroom management indicated that they wanted to be seen as competent professionals but were still in the process of forming their professional identities. It also demonstrated that they believed a teacher should be in control of their classes and without that control, learning would not happen. When Alyssa reflected on her experiences as a first-year teacher, she provided a description of her work environment through identifying some characteristics of her students:

> I just think that this year has been really stressful with kids and all the different needs that we have and all the different behaviors that we have. We have nine kids with IEPS and fourteen 504s and a bunch of different

> stuff we're trying to balance with everything that's happening. [Alyssa-Interview 4- 9: 21-24]

She also explained that the makeup of her students included a large number of special needs and English learners; all of her classes were co-taught with a special educator. Similarly, Molly worked in a high-need district with a diverse population of students, and, looking back on her first year of teaching, she stated "the group of kids that I had was really, really tough" [Molly- Interview 4- 1: 7 &8]. She described behaviors of her students that also demonstrated her focus on the environment and actions of her students. Through interviews and in classroom observations, it was apparent that Alyssa and Molly both experienced many disruptive behaviors from their students that occurred on an almost daily basis. Kara did describe some behaviors she experienced in her classes but it mostly consisted of students being talkative. As discussed earlier, beginning teachers tend to focus on the immediate environment and behaviors in that environment (Korthagen, 2004), which results in little energy focused on further developing in other areas of PCK. The effort on the outer layers on the Onion Model again demonstrated that they were still constructing their professional identity and understanding their mission which left them feeling uncertain and wanting to control their environments.

In addition to having the youngest students, Alyssa also worked in the most structured environment. There were school norms, curricular requirements, common assessments, and other structures in place that she had to work within. She attended weekly fifth grade common planning and math content meetings twice a week as well as PDs that included analyzing resources and unit launches. The prescriptive curriculum with specific requirements for what and how a topic was to be taught limited her ability to develop in many aspects of her PCK. Kara and Molly also received curriculums in the form of scope and sequences but had a bit more flexibility in its implementation when compared to Alyssa. There were instances where they felt locked into teaching a topic in a certain way due to the common assessment but did have more freedom overall in their instructional design. All three participants' experiences indicated the need for articulation between grade levels. Though they had experiences from their practicums or student teaching in other grade levels, they were still uncertain about the content taught in previous or subsequent grades. This hindered development since they could not expand their knowledge of vertical curriculum or learning trajectories. This struggle made them confront their competencies and realize there was a gap that needed to be addressed. These teachers appeared to want to act from a layer of identity and beliefs, while the system forced a focus on environment and competencies.

A discussion on the participants' first-year of teaching environments would be remiss if it did not include the classrooms where they spent the majority of their time. All three teachers had their own classrooms and did not need to travel to another room to teach any of their classes. They could organize and decorate the space how they saw fit, with a few requirements from the different schools. Among these requirements was the need to post the standards in their classrooms. Alyssa also had to have the school's motto visible. Student work was showcased in their classrooms as well as motivational quotes and sayings. Some of these decorations revealed their underlying beliefs about learning. For example, Molly's belief about learning through mistakes was evident in her bulletin board decorations—she had a wall dedicated to how mistakes can inform the learning process. The arrangement of the classroom also illustrated some of their beliefs about learning and the role of the teacher, both of which impacted their behavior in the classroom and the PCK development. All of them had their own desks to the side of their classrooms. This forced them to not be stationary at their own desks and to engage with the students in their own spaces. The interactions with students supported their development in many tasks of PCK, such as use of questioning, hearing and interpreting student thinking, and evaluating different representations of topics. Alyssa and Molly arranged the student desks into groups of four or five, which increased their use of cooperative learning. Kara's students sat predominantly in rows, but she did explain that they were usually up and walking around the room in groups while engaging in activities. These beginning teachers spent countless hours considering how they would arrange their room and were excited to have their own space. Molly believed in the importance of letting students collaborate and construct their own knowledge through experiences, like inquiry activities, which is why she arranged her classroom into groups. She specifically planned her bulletin boards to reflect her philosophy of learning and teaching, such as learning through mistakes. Alyssa believed in supporting students' knowledge development by being able to work with them in small groups and address their individual needs. She arranged her classroom so students were semi-homogenously grouped and they could differentiate support as needed. All three participants saw their shared mission as making a difference in the lives of their students, so by having their classroom arranged in the way they did, they could circulate and build relationships with each student. They recognized the importance the environment for their students learning, but did not realize how it would impact their own PCK development.

In addition to the arrangement of the classroom, the resources and technology present also impacted the environment and thus their PCK development. Alyssa, Molly, and Kara all had computers provided by the school linked to projectors, which they used in their instructional delivery. Kara and Molly were at schools that were one-to-one with each student having a Chromebook. Alyssa also had a cart of Chromebooks in her classroom that she utilized in her Response to Intervention (RTI) class. The access to the technology and the initiatives by the schools to integrate its use into daily instruction influenced their PCK development. They had to consider effective and appropriate uses for technology while designing instruction, selecting examples, motivating students, anticipating their thinking, and connections to the curriculum. In some of their practicum and student teaching settings, they used similar types of technology and resources, which contributed to their beliefs and competencies. For example, Kara and Molly both used videos in their student teaching placements and in their first years of teaching that provided students' with real-world contexts for mathematics. Since they believed in real-world applications for students as being crucial to student learning, these videos linked their environment, beliefs, and competencies. When there was alignment between the teachers' beliefs and the environment, in this case use of technology, and they felt strong in their competencies, they were able to focus their attention towards their personal missions.

One environmental component that did support their development in tasks of their PCK, specifically in the domain of KCC, was the collaboration with other professionals in the schools. While none of them were assigned a formal mentor, each participant identified as least one other professional they felt supported by during their first year of teaching. This is in line with Marable and Raimondi (2007), who found that "in the absence of mentors, peers were identified overwhelming as the primary source of support" (p. 35). For Alyssa, that person was her co-teacher; they collaborated daily on lessons, resources, student support, classroom management, and assessments. While she also identified the STEM director as a resource during her first year of teaching, she saw her in an administrative role since she would do observations of Alyssa's teaching. She also described some push-back from administrators about how she was designing instruction and structuring her class time. This could have hindered development of her PCK in certain tasks since she was not able to alter the structures of the environment. Kara explained how her relationship with the other eighth grade math teacher and the former eighth grade math teacher provided her with resources and lesson plans, ideas about student thinking, and ways of motivating students. She also stated she felt supported by the other members of her team for learning about student backgrounds, which helped her with anticipating their thinking and potential pitfalls, and the administrative tasks required as a classroom teacher, such as entering grades. There was also an induction program in the district where each new hire met with the induction coach once a week and then once a month all the new-hires came together for an additional meeting. Kara also identified the induction coach as a person she could go to with "silly questions" and who provided her with resources and activities [Kara-Interview 3- 3: 13]. Overall, she felt incredibly supported by a variety of individuals in her school: "I have a ton of people. Everyone has been overly welcoming" [Kara-Interview 3- 4: 1].

Like Alyssa and Kara, Molly identified other professionals as being her main source of support. She explained how her work with a second-year math teacher provided her with emotional support and gave her instructional ideas. However, her experience overall was more in line with how Feiman-Nemser (2003) describe the view of new teachers by mentors: "mentors often offer help only if the new teacher asks; they don't think of new teachers as learners and themselves as their teachers" (p. 28). Molly described a feeling of isolation during her first year of teaching: "I get support when I ask for it but I don't feel like a lot of people will go out of their way, but again that's not their job. I've felt kind of alone this year in terms of everything" [Molly- Interview 4- 2: 6-8]. Feeling supported during their first year of teaching by other professionals coincides with how student teachers value and need to be able to work with their peers. A supportive environment, then, can contribute to PCK development for first-year teachers. Colleagues also influenced identity formation and understanding one's mission. For example, Molly's feeling of isolation and her choice of separating herself from many of her colleagues illustrated discrepancies between her mission and that of her colleagues. This caused her to question whether her mission was appropriate and possible in her work environment.

Research on induction methods often demonstrate that programs are designed to help teachers fit into an already established system, essentially enculturing them into the profession (Feiman-Nemser, 2003). Before these three teachers entered the profession, they explained that they wanted to fit into the environment where they gained employment. This indicated their awareness of how influential the climate of the school could be on their future work and development. Molly expressed nervousness during her student teaching semester about the possibility of a mismatch between her and the school: "I'm just nervous to figure out where I belong and how my philosophy fits into the philosophy of the school" [Molly- Interview 2- 14: 4-5]. Again, Molly demonstrated the need to work in an environment where a teacher's mission aligns to that of her environment. She explained the importance of "interviewing the school" to ensure there was synergy between herself as a teacher and the environment where she would be working [Molly- Interview 2- 3: 10-13]. She illustrated that they did not want to just find a job, but that they wanted to find a place where their ideas about teaching and learning were welcomed. The importance of finding suitable environments for work demonstrated that they wanted to be able to apply what they learned in their preparation programs. Though they were able to transfer most of their knowledge, they faced obstacles from the environment such as students' resistance and required common assessments.

Competencies and Beliefs: CK, PCK, and Language

This section will move from a focus on influences external to the individual to types of forces internal to a person, namely competencies and beliefs. Different states also identify different competencies they want teachers to exhibit, sometimes in the form of standards. For example, Minnesota has 10 standards with 120 associated competencies aligned to the national InTASC standards. The University of South Dakota adapted these standards and competencies and produced six main competencies. There are countless lists of competencies that teachers "should" possess, but no list is complete. What is more important is the teachers' beliefs about their own competencies and abilities to perform duties as a classroom teacher. Among the integral competencies are knowledge of one's subject and tasks associated with PCK.

The structure of the preparation program contributed to participants' knowledge of pedagogy. Among the competencies connected to this domain where how to write objectives, construct lesson plans, identify appropriate accommodations and modification for students from different populations, and select relevant standards aligned to lessons. Through their coursework, they continued to formulate their personal beliefs about teaching and learning which began developing during their K-12 learning experiences. Participants referred to their philosophies of teaching, which they wrote and revised in different courses throughout their preparation program. This philosophy summarized their beliefs about the role of the teacher, the processes of learning, effective and ineffective teaching methods, and other views they held. As they transitioned into their first year of teaching, many of their beliefs were tested. Beginning teachers tend to focus on influences external to themselves and are reluctant to attributed difficult experiences to their own lack of competencies or errors in their beliefs: "new teachers may find some comfort in ascribing their difficulties to traits in pupils or parents or blaming the administration" (Feiman-Nemser, 2003, p. 27). Initially discussed in the previous section, the environment of the schools and classrooms during teachers' first years of teaching either supported their beliefs about teaching and learning, caused them to reconsider or modify them, or made them reassess their choice of workplace. However, when there was alignment between the different layers described in the Onion Model, these teachers could focus on their personal missions and continue to develop their identities.

Throughout their student teaching year and first year of teaching, Alyssa, Kara, and Molly reflected on their content knowledge and how it impacted their abilities to construct effective lessons, anticipate student thinking, utilize resources, and identify links between topics. The more knowledgeable the individual was about a given topic, the more able she was to apply different tasks of PCK. For example, Kara exhibited confidence in her knowledge of logarithms and was able to use that content knowledge to effectively interpret a student's thought process [Kara- PCK Inventory- Question 7]. On the other hand, Alyssa was not comfortable working with logarithms and thus had difficulty in this task of PCK for this particular topic [Alyssa- PCK Inventory- Question 7]. Similarly, Molly explained that having a strong content knowledge base allowed her to relearn concepts that she may have forgotten more easily:

> Having that depth of knowledge has helped me in my teaching. More specifically, not only does it help me remember all the things and understand them more deeply, but when I have to reteach myself something I can. I have such a deep level of understanding of mathematics concepts and I can connect all of them, it comes back to me. And I think as I progress in my career I won't forget things as much. Some of the topics I really haven't thought about in a long time, but it doesn't matter because I understand math so well that it just comes right back, which is really good. [Molly- Interview 2- 4 & 5: 21-23 & 1-7].

Initially, participants began with predominately separate knowledge domains of their subject matter and of pedagogy. They knew the content for themselves and were exposed to theories and practices of teachers with little formal or effective integration of the two. Through experiences in classrooms with students, their knowledge of content and of pedagogy began to merge together into their PCK. They also noted that the coursework they had in the semester before their student teaching contributed to the development of their competencies. As discussed earlier, the settings where they learned how to teach directly influenced their competencies and PCK development.

As participants gained experience in classrooms and began working with a variety of students, they also began to realize that it was not sufficient to have amassed content knowledge but the way in which they understood that content. Initially, they possessed surface knowledge or what Skemp (1976) would call "instrumental understanding" (p. 20). They were able to recall facts without fully understanding the reasoning behind those concepts. This produced difficulty in their PCK when they needed to anticipate student thinking, hear and interpret their thinking, and evaluate different representations of topics since they did not possess a deep understanding of the concepts. Through coursework and work in classrooms, they developed “relational understanding” or knowing “what to do and why” (Skemp, 1976, p. 20). For example, Molly explained that her content knowledge was strong enough so she could explain the why behind different concepts if she needed to or if students asked. Their growth in this area allowed them to become more effective in tasks of PCK and altered their behavior in the different settings. It also contributed to their identity development since they began to view themselves more as teachers and less like students.

Models theorizing the relationship between content knowledge and pedagogical content knowledge were discussed in Chapter 2. Kara and Molly both explained that the courses they took in their pre-student teaching semester helped them integrate their knowledge of pedagogy with their content knowledge. They felt those courses provided the basis of their knowledge for teaching rather than education courses focusing on other aspects of teaching, such as assessment, working with students with special needs, and so on. Kara stated, “I can learn how to teach math rather than just in general” [Kara- Interview 2- 2: 10 & 11]. Similarly, Molly stated, “I think the math capstone course and the math methods course have been the most influential. I think all of the courses I’ve taken, I’ve taken bits and pieces from. But I felt most prepared because of those two courses in the fall” [Molly- Interview 2- 8: 8-10]. These experiences are reminiscent of an integrative view of PCK: students take separate subject matter and pedagogy courses and then integrate them in practicum settings to developing PCK (Gess-Newsome, 1999).

The development of the different competencies and types of knowledge was also dynamic (An, Kulm, & Wu, 2004) where growth in one area could mean growth in another area. For example, as participants gained experiences in the different environments and developed their content knowledge, they also were better able to anticipate student thinking.

In responses on PCK Inventory, discussions during interviews, and classroom observations, participants began to use more precise and accurate language over time. The growth in their use of mathematical language illustrated their development in their content knowledge as well as them becoming more conscious of the role of language in developing mathematics knowledge with their students. From her first year of teaching, Alyssa gave one example of trying to help a student understand subtraction but who was having difficulty with the terminology. She also explained she was trying to connect new concepts to things they learned in the past but realized they had not learned the appropriate terms:

> It's been a battle, the teachers versus what the students have learned last year. Mostly in the vocabulary that they're using. We noticed that they would be saying borrow a lot and we try to get away from that. […] We're trying to get them to understand and say "it's groups of" when we're doing division and connecting it to subtraction but they may call it take away. So having that battle and how much do you want to have that battle with them and possibly confuse them but teach them the right vocabulary. Because then they'll revert back to their old ways. So I tried doing it today, you may have noticed. I try doing it every day but yeah, that has been difficult. Learning how to teach what to teach. [Alyssa- Interview 3-2-3: 18-23 & 1-4]

Though Alyssa realized the need for proper language, she also felt unsure about whether re-teaching concepts with the correct terminology would be more of a confusion rather than a benefit to her students. In my field observations with her, I noticed her waver between using appropriate language and more common or imprecise language. Some of the times it was her regressing to how she was taught while other times she was trying to align her language to the language on the pre-constructed, mandated, common assessments. This demonstrated how influential the environment was on developing different competencies and how powerful the initial learning experiences from K-12 are to teachers (Lortie, 1975). Molly explained that she tried to stay true to the mathematics and not teach any tricks or pseudo-math to her students. She would model appropriate language to her students and rephrase their statements to help them connect concepts to terminology. She also felt strong in her content knowledge, which directly influenced her language competency, beliefs about teaching and learning, and behavior in the classroom. Similarly, Kara would generally model appropriate language and rephrase students' responses. She also explained she was comfortable with her content knowledge and this helped her develop her use of mathematical language. Teachers with a more developed content knowledge were more likely to use appropriate language throughout different topics and teaching experiences. As their content knowledge deepened, their language usage developed. However, when their environment disrupted their beliefs or they were not confident in themselves, their behaviors changed and they responded impulsively and not always from their beliefs. Through classroom experiences and reflection these impulsive responses tended to become more in line with their beliefs since their competencies developed by working with students.

Teacher Identity Development

Throughout this study, one of the most pronounced areas of development for Alyssa, Kara, and Molly was in their professional identities. Korthagen (2004) explains "professional identity often takes on the form of a Gestalt: an unconscious body of needs, images, feelings, values, role models, previous experiences and behavioral tendencies, which together create a sense of identity" (p. 85). At different points during their student teaching semester, they began to take ownership of the classes and students, referring to them as "my students." They also expressed anxiousness yet excitement at the prospect of being "*the* teacher" [Molly- Interview 1- 10: 15]. One of the symbols demonstrating status as teachers was having their own classrooms. As student teachers, they each expressed excitement about establishing their own class norms and having their own students. Kara stated, "I'm excited to start day one and get my classroom how I want it to be and have my own classroom where I can be in charge of everything" [Kara- Interview 2- 12: 1-2]. Similarly, Molly explained she was looking forward to her first year of teaching:

> I'm ready for a job. I just want to teach. I don't want the summer to happen. I know that's bad. [laughs] I think I'm really anxious about what's to come but I'm really excited. I think I've gotten a taste of it and I'm just ready for it and to make it my own because there were some limitations in student teaching. I think that's just a universal thing; like you're in someone else's classroom and I think I'm really excited to make my own classroom and not need to be supervised all the time. [Molly- Interview 2- 15: 8-14]

Their outlook on their future careers demonstrated a change in their professional identities. Though self-concept is general resistant to change, it is connected to "status": "overall conception of one's own place or position in relation to all the elements in one's world, including oneself" (Korthagen, 2004, p. 84). Through their success in their student teaching classrooms and first year of teaching, they began to realize they possessed the status of a teacher. For Molly, when she did not always experience success, her status was threatened which made her question her professional identity. When these experiences are considered through the lens of conceptual change theory, we can see them as opportunities for pre-service teachers or first-year teachers to be confronted with their beliefs. This impacts their self-concept and views of teaching, and thus could alter their professional identity. The importance of providing experiences with purposeful pedagogy by teacher preparation programs is again evident. Similarly, their collaboration with peers during their preparation program and with colleagues in their first year of teaching impacted the formation of their identities (Watzlawick, Beavin, & Jackson, 1967). They were able to discuss their beliefs with others which also promoted critical reflection on their experiences.

Teacher's professional identity development begins with individuals comparing themselves to the images of teaching and learning they developed from their own learning experiences (Korthagen, 2004). These role models serve as powerful totems in the construction of individuals' beliefs, how they evaluate their competencies, how they behave in classrooms, and what they pay attention to in their environment. Each participant experienced their own transition from student-of-teachers to student-teachers to identifying themselves as teachers. They started by emulating their cooperating teachers and then developed their own style of teaching. Kara explained this process as she reflected on how she developed her own teaching style:

> I think during student teaching, in my middle school placement, I had a teacher who did a lot of activities like this and showed me a lot of these so that kind of opened my eyes to that teaching style. And then, in my high school placement, I had a teacher who just kind of gave a worksheet, taught on the board, and did that kind of teaching style. So I found that I'm a little bit of both. I like the direct instruction for parts and then I like the activities for parts and I found it through student teaching, I guess. And I was lucky enough to have both of those experiences so I got a feel for each. [Kara- Interview 3- 4 & 5: 19-21 & 1-5]

She realized she did not fit perfectly in the model of either of her cooperating teachers but took what she learned in those settings and merged them into her own identity. Molly was constantly comparing herself to her high school cooperating teacher but realized she was not at his level of experience yet. She also realized that his work environment differed from hers so she needed to develop her own identity beyond emulating his behaviors. Molly expressed that she was still "finding her voice" as a teacher, which indicated she was still developing her professional identity and coming to terms with her competencies and beliefs [Molly- Interview 4- 4: 9]. Participants' identities were not completely formed after completing their preparation programs, began their first year of teaching, or even at the end of their first year. The connection between identity development and PCK development will be discussed more in the next chapter.

Missions

Kara, Molly, and Alyssa all gained employment at different middle schools. When asked why they chose to work in a middle school setting, they each explained that they liked working with this particular age group. Kara described that she felt more secure in identity as a teacher and authority figure in a middle school as opposed to a high school. Alyssa explained that she enjoyed working with students in early middle school since they are at a transition point, going from "little elementary schoolers" to becoming "little adults" [Alyssa- Interview 4- 6: 6-11]. She stated that this is the time where students are able to start building on their basic skills and begin exploring more complicated concepts in mathematics. Molly felt her personality fit best at a middle school and that she could relate to that age group. Since this age is a period of rapid growth and research shows that many children move away from STEM areas during middle school (Carlone, Scott, & Lowder, 2014; Simpkins, Davis-Kean, & Eccles, 2006), having positive role models is a way to counteract this phenomenon (Else-Quest, Linn, &

Hyde, 2010; Simpkins, Davis-Kean, & Eccles, 2006). Else-Quest, Linn, and Hyde (2010) found that girls and boys perform at the same proficiency in the classroom when there is representation from positive female role-models. These researchers found that these role models provided students with the encouragement and the educational tools necessary to succeed. Kara, Molly, and Alyssa all identified making connections with students as important to them, thus making them role models for their students.

When describing the force or calling that leads many into the field of mathematics education, Korthagen (2004) states that is usually the love of mathematics that draws people:

> It is not uncommon for our own mathematics student teachers to be enthusiastic about their subject; in fact they often find their main inspiration in mathematics, and—at least at the beginning of their professional preparation—much less in their relationship with students at school. (Korthagen, 2004, p. 88)

However, that did not seem to be the case for the participants in this study. While they expressed that they enjoyed doing mathematics, they all explained that they wanted to make a difference in the lives of learners. Learning mathematics did not always come easy for these teachers, be it in their elementary, secondary, or college education. They saw the impact that teachers and classmates could have on the learning environment. Thus their shared mission was to change perceptions of mathematics for the next generation. Molly recognized she may not make a difference in every student's life, but did hope to impact some students in a positive way:

> I always tell people I love the math part, for sure, love it. I'm a nerd. Like I said, I use the Pythagorean Theorem to get from place to place. But I'm really excited for those relationships I'll have with students and I know all teachers say that "if you can make a difference in one student's life, that's all you need," and I mean I guess it's really true. I'm really looking forward to those students who can take a lot out of my class. I'm looking forward to those who don't because it will be a good lesson for me too, as

> corny as it sounds. I just watched a video that my roommate tagged me in on Facebook, it was students- they wrote letters to their teachers about how much of an impact they had made. I feel like one thing that I never saw from my teachers ever was how hard they were on themselves when a student didn't get something. You take it personally and I get that now because I get how it works. And so, to hear from students who really took a lot out of what I taught them or even just the relationship I have with them, I'm really looking forward to that. Even if it doesn't happen a lot. [Molly- Interview 1- 13: 6-20]

She hoped to show students why she loved mathematics and how you can apply your knowledge of mathematics to real world situations. She repeatedly gave the example of how she used the Pythagorean Theorem when navigating around places, to find the shortest distance. She wanted to make mathematics real, interesting, and relatable for students. Similarly, Kara explained that she too looked to make a difference in students' lives. She also theorized that she might want to be pursue a graduate degree in teaching or become an administrator in the future to be able to help more student:

> I feel like eventually I will go to grad school, maybe for teaching and administration so kind of do want to get more involved within the school…maybe, I don't know exactly what yet. But I'm excited to work my way up and work my way around the school and get different positions, work with different students. But yeah, mostly just working with students and seeing how I can make a difference and help them. [Kara- Interview 1- 10: 15-22].

For Alyssa, she chose to work with younger-aged students than the others which helped her realize the importance of helping student have a solid foundation in mathematics for future development. She also realized the importance of supporting personal, social, and moral development in her students:

> I am looking forward to making those connections with kids. I think that's, as much as teaching it, but I really am looking forward to and having them find a connection with learning and helping them find what they like and to become confident in themselves which, you know, because in teaching you do both. You teach social skills and you also teach content and you teach kids to know themselves. You teach them

> about themselves or how to get to know themselves. [Alyssa- Interview 1-23: 15-20]

Each of their personal missions propelled them to make connections with their students and gain employment is an environment where they felt they could make a difference.

Mission is the inner core that directs an individual's motivation, behavior, and beliefs. As pre-service teachers, Alyssa, Kara, and Molly were developing their understanding of their missions. Through their experiences in practicum courses and through work with students, peers, and mentors they were able to further uncover what their calling about teaching was. They were able to explore their beliefs about teaching and learning with institutional supports which helped them determine what aligned to their current understandings of their missions. Experiences in a variety of settings forced them to confront some of their own preconceptions and become aware of why they held certain beliefs. Tensions between influences, the layers, helped bring clarity about their selves as teachers in terms of their beliefs and missions. This awareness also helped them recognize their missions. Being cognizant of their beliefs and missions focused their attention on competencies they deemed relevant to the identity of a teacher. Among these competencies were improving their content knowledge and tasks of PCK which they felt were less developed than others. The participants developed their PCK through working with students and being reflective about these experiences. Reflection helps individuals make sense of their environments and contextualizes beliefs, which contributes to identity and solidifies their missions. Therefore, it was the interaction between the layers that contributed to the development of PCK.

For example, Molly experienced some instances of tension between how she wanted to teach mathematics and students' receptiveness to her methods. However, she believed that everyone could learn and recognized one element of her mission was to boost her students' confidence in their mathematical abilities [Molly- Interview 3-8: 3-7]. This helped her realize why some students acted disruptive in class or why they were resistant to learning mathematics in a different way, they lacked confidence in mathematics and she needed to help build it up. One way she did so was by focusing on providing feedback to students and encouraging them to share their thinking with each other. Another example of a participant acting from their awareness of their missions was in Alyssa's work towards helping student develop as a whole. She recognized that teaching mathematics was only one part of her job, or mission as a teacher, and that she also had to support students in their personal development. Like Molly, she also believed part of her mission was to help "them find what they like and to become confident in themselves" [Alyssa- Interview 1- 23: 15-23]. She structured her lessons to be interdisciplinary and foster collaboration between students to promote development and learning.

Connecting the Onion Model to the PCK Framework

As I analyzed my data using the onion model, there were key elements from each layer that either supported or hindered various tasks of PCK. In order to better understand the levels of influence and what supported or hindered participants' PCK development, I constructed the following figures (5.2, 5.3, and 5.4). Within each layer, the different experiences, factors, or other type of knowledge are identified. If that element was found to be supportive of my participants' PCK development, it is denoted in green with a "+" sign. On the other hand, if the element was found to be a potential hindrance to their development, it is denoted in red with a "-" sign. Last, elements that could have been supportive or a hindrance are in orange with a "+/-" sign. For example, past experiences as a student could support development of PCK or it could have hindered development, as discussed by Lortie (1975). Elements that dominated participants' experiences were bolded in the tables for emphasis. The bar where the element is written spans the task or tasks it influenced in that particular domain. In addition to the domains and the associated tasks in my original conceptualization of PCK which was adapted from Ball, Thames, and Phelps (2008), I have added an additional task to the domain of Knowledge of Content and Teaching (KCT), Use of Mathematical Language[1], and an additional domain, Knowledge of Assessment[2] (KA). These additions developed from my findings and my participants' experiences and will be discussed in more detail in the next chapter.

From representations of the findings in the figures, it is apparent that there were some experiences or factors that supported or hindered participants' PCK development throughout all of the domains. For example, experiences with students, reflection, beliefs about teaching and learning, and having an understanding of one's mission spanned all the tasks of PCK. This could explain why, as participants had increased time with students in their student teaching semester and in their first year of teaching, there was noticeable growth in multiple tasks of PCK. In addition, since some experiences and factors influence many tasks of PCK at once, it also helps explain why some tasks seemed to develop in parallel to each other. Some hindrances to participants' PCK development were found to also have influence throughout different tasks and domains. For example, if participants had a lack of content knowledge about a particular topic or subject, this limited their PCK development in all domains. Thus, this hindrance, not only effected their knowledge about designing instruction, for example, but also their ability to anticipate student thinking or to select appropriate instructional materials.

<table>
<tr><td colspan="2"></td><td colspan="6">Knowledge of Content and Teaching (KCT)</td></tr>
<tr><td rowspan="34">Onion Model Layers</td><td></td><td>Sequencing of Topics</td><td>Design of Instruction</td><td>Evaluate Different Representations</td><td>Selection of Examples</td><td>Use of Questioning</td><td>Use of Mathematical Language[1]</td></tr>
<tr><td rowspan="7">Environment</td><td colspan="6">**(+) Integration of practicum experiences with education coursework**</td></tr>
<tr><td colspan="6">**(+) Experiences with students**</td></tr>
<tr><td colspan="6">**(+) Peer or colleague support**</td></tr>
<tr><td colspan="6">(+/-) Previous teachers/ Own Learning Experiences as Student</td></tr>
<tr><td colspan="3">**(-) Lack of flexibility**</td><td colspan="3"></td></tr>
<tr><td colspan="6">(-) Unfamiliarity with needs of certain population(s)</td></tr>
<tr><td colspan="6">(-) Socialization of Teaching (Enculturation)- Pressure from colleagues or administration</td></tr>
<tr><td rowspan="5">Behaviors</td><td colspan="6">(+) Being Reflective</td></tr>
<tr><td colspan="6">(+) Collaboration with Others</td></tr>
<tr><td colspan="6">**(+) Experiences with students (learning by doing)**</td></tr>
<tr><td colspan="6">(-) Not trying new things (repeating past behaviors only)</td></tr>
<tr><td colspan="4"></td><td colspan="2">(-) Being reactive</td></tr>
<tr><td rowspan="6">Competencies</td><td colspan="6">**(+) Being Reflective**</td></tr>
<tr><td colspan="6">(+) Collaboration with Others</td></tr>
<tr><td colspan="6">(+) Relational Understanding of Content Knowledge</td></tr>
<tr><td colspan="6">(+) Knowledge of Pedagogy</td></tr>
<tr><td colspan="6">(-) Instrumental Understanding of Content Knowledge</td></tr>
<tr><td colspan="6">(-) Lack of Content Knowledge</td></tr>
<tr><td rowspan="5">Beliefs</td><td></td><td colspan="4">(+) Beliefs about learning</td><td></td></tr>
<tr><td></td><td colspan="4">(+) Beliefs about role of teacher</td><td></td></tr>
<tr><td colspan="4"></td><td colspan="2">(+) Beliefs about role of language</td></tr>
<tr><td colspan="6">(+) Alignment between individual and school philosophies</td></tr>
<tr><td colspan="6">(-) Mismatch between individual and school philosophies</td></tr>
<tr><td rowspan="7">Identity</td><td colspan="6">**(+) Forming professional identity (identifying self-as-teacher instead of mimicking previous teachers or CTs)**</td></tr>
<tr><td colspan="6">**(+) Taking ownership of students**</td></tr>
<tr><td colspan="6">(+) Alignment between individual and school philosophies</td></tr>
<tr><td colspan="6">(+) Confidence in one's abilities</td></tr>
<tr><td colspan="6">(-) Lack of confidence (e.g. in one's ability or in CK)</td></tr>
<tr><td colspan="6">(-) Mismatch between individual and school philosophies</td></tr>
<tr><td colspan="6">(-) Ill-formed identity (identity development is still in flux); finding place or voice</td></tr>
<tr><td rowspan="3">Mission</td><td colspan="6">**(+) Alignment between individual and school philosophies**</td></tr>
<tr><td colspan="6">(+) Understanding of one's purpose (mission/driving force)</td></tr>
<tr><td colspan="6">**(-) Mismatch between individual and school philosophies**</td></tr>
</table>

Figure 5.2. Connecting Layers of Onion Model to PCK Domain KCT.

		Knowledge of Content and Students (KCS)			
Onion Model Layers		Anticipating Student Thinking	Anticipating Potential Areas of Confusion or Difficulty	Ways of Motivating Students	Hear and Interpret Students' Thinking
	Environment	**(+) Integration of practicum experiences with education coursework**			
		(+) Experiences with students			
		(+) Peer or colleague support			
		(+) Articulation between grades			
		(+/-) Own Learning Experiences as Student			
		(-) Unfamiliarity with needs of certain population(s)			
		(-) Disruptive behaviors by students			
	Behaviors	(+) Being Reflective			
		(+) Collaboration with Others			
		(+) Experiences with students (learning by doing)			
		(-) Not engaging with students (e.g. during practicum experiences)			
	Competencies	**(+) Being Reflective**			
		(+) Collaboration with Others			
		(+) Relational Understanding of Content Knowledge			
		(+) Knowledge of different student populations' needs (PK)			
		(+) Curricular Knowledge (knowledge about previous or future grades' content, methods, and experiences)			
		(-) Unfamiliarity with knowledge and experiences from previous or future grades			
		(-) Unfamiliarity with needs of certain population(s)			
		(-) Lack of Content Knowledge			
	Beliefs	**(+) Beliefs about learning**			
		(+) Alignment between individual and school philosophies			
		(+) Beliefs about teaching and role of teacher			
		(-) Mismatch between individual and school philosophies			
	Identity	**(+) Forming professional identity (identifying self-as-teacher instead of mimicking previous teachers or CTs)**			
		(+) Taking ownership of students			
		(+) Alignment between individual and school philosophies			
		(-) Mismatch between individual and school philosophies			
		(-) Ill-formed identity (identity development is still in flux); finding place or voice			
	Mission	(+) Alignment between individual and school philosophies			
		(+) Understanding of one's purpose (mission/driving force)			
		(-) Mismatch between individual and school philosophies			

Figure 5.3. Connecting Layers of Onion Model to PCK Domain KCS.

<table>
<tr><td colspan="2"></td><td colspan="2">Knowledge of Content and Curriculum (KCC)</td><td colspan="5">Knowledge of Assessment[2] (KA)</td></tr>
<tr><td></td><td></td><td>Vertical and Lateral Curriculum</td><td>Program and Instructional Materials</td><td>Selecting Appropriate Topics and Processes to Assess</td><td>Challenges or Difficulties with Assessment</td><td>Identifying Methods or Strategies of Assessment</td><td>Use of Assessment Data</td><td>Design of Assessments</td></tr>
<tr><td rowspan="35">Onion Model Layers</td><td rowspan="6">Environment</td><td colspan="7">(+) Integration of practicum experiences with education coursework</td></tr>
<tr><td colspan="7">(+) Experiences with students</td></tr>
<tr><td colspan="7">(+) Peer or colleague support</td></tr>
<tr><td colspan="6">(+) Articulation between grades</td><td></td></tr>
<tr><td colspan="7">(-) Lack of experience</td></tr>
<tr><td colspan="7">(-) Lack of control or freedom</td></tr>
<tr><td rowspan="4">Behaviors</td><td colspan="7">(+) Being Reflective</td></tr>
<tr><td colspan="7">(+) Collaboration with Others</td></tr>
<tr><td colspan="7">(+) Experiences with students (learning by doing)</td></tr>
<tr><td colspan="7">(-) Not engaging with colleagues (e.g. not speaking up)</td></tr>
<tr><td rowspan="9">Competencies</td><td colspan="7">(+) Being Reflective</td></tr>
<tr><td colspan="7">(+) Collaboration with Others</td></tr>
<tr><td colspan="2">(+) Pedagogical Knowledge</td><td colspan="2"></td><td colspan="3">(+) Pedagogical Knowledge</td></tr>
<tr><td colspan="3">(+) Relational Understanding of Content Knowledge</td><td colspan="4"></td></tr>
<tr><td colspan="7">(+) Knowledge of different student populations’ needs (PK)</td></tr>
<tr><td colspan="7">(+) Curricular Knowledge (knowledge about previous or future grades’ content, methods, and experiences)</td></tr>
<tr><td colspan="7">(-) Unfamiliarity with knowledge and experiences from previous or future grades</td></tr>
<tr><td colspan="7">(-) Unfamiliarity with needs of certain population(s)</td></tr>
<tr><td colspan="3">(-) Lack of Content Knowledge</td><td colspan="4"></td></tr>
<tr><td rowspan="4">Beliefs</td><td colspan="7">(+) Beliefs about learning</td></tr>
<tr><td colspan="7">(+) Alignment between individual and school philosophies</td></tr>
<tr><td colspan="7">(+) Beliefs about teaching and role of teacher</td></tr>
<tr><td colspan="7">(-) Mismatch between individual and school philosophies</td></tr>
<tr><td rowspan="7">Identity</td><td colspan="7">(+) Forming professional identity (identifying self-as-teacher instead of mimicking previous teachers or CTs)</td></tr>
<tr><td colspan="7">(+) Taking ownership of students</td></tr>
<tr><td colspan="7">(+) Alignment between individual and school philosophies</td></tr>
<tr><td colspan="7">(+) Confidence in one’s abilities</td></tr>
<tr><td colspan="7">(-) Lack of confidence (e.g. in one’s ability or in CK)</td></tr>
<tr><td colspan="7">(-) Mismatch between individual and school philosophies</td></tr>
<tr><td colspan="7">(-) Ill-formed identity (identity development is still in flux); finding place or voice</td></tr>
<tr><td rowspan="3">Mission</td><td colspan="7">(+) Alignment between individual and school philosophies</td></tr>
<tr><td colspan="7">(+) Understanding of one’s purpose (mission/driving force)</td></tr>
<tr><td colspan="7">(-) Mismatch between individual and school philosophies</td></tr>
</table>

Figure 5.4. Connecting Layers of Onion Model to PCK Domain KCC & KA.

It is not a surprise that having a strong content knowledge base was found to be a support while having a weak or unstable base to be a hindrance as each of the domains involves the connection between “knowing of mathematics” to other ways of knowing (Ball et al., 2008, p. 401). Similarly, when participants perceived a lack of control in their experiences, they were unable to further develop their PCK in multiple domains. For instance, when they were unable to extensively modify their prescribed curriculums, they had difficulty developing their knowledge of sequencing topics, vertical and lateral curriculum connections, and evaluating or using different representations of topics.

Also, some of the experiences and factors were repeated in the different layers of the onion model, demonstrating their influence at the different levels of an individual. One example of a factor that influenced an individual on multiple layers is when there was a match between the individual’s and school’s philosophies. This factor influenced individual’s beliefs, identity, and mission. Having experiences and factors that influenced participants on multiple layers could have helped produce synergy between the layers, thus promoting or hindering development more substantially than at a single layer. Some experiences and factors spanned multiple domains and were influential on multiple layers, which is an important insight for teacher educators and school leaders. Understanding the influence of these experiences and factors could help further promote development in teacher candidates or beginning teachers, target professional development or program enhancement, or identify learning gaps that could be addressed to ease the transition from pre-service to in-service environments. In the next chapter, I will discuss further conclusions and implications from my study and contextualize my study’s findings in the current literature.

CHAPTER 6

THE BAKED GOODS: DISCUSSION, CONCLUSIONS, IMPLICATIONS, AND REFLECTION

Venture outside your comfort zones. The rewards are worth it.
Rapunzel, *Tangled*

The purpose of this study was to contribute to and broaden the existing research concerning the development of beginning teachers' pedagogical content knowledge (PCK) and their perceptions of that development as they transition from their preparation programs to their first year of teaching. Studying teachers' knowledge is not a new topic of interest for educational researchers, teacher educators, and policy makers. It is important to understand where and how teachers develop their knowledge and the role teacher preparation programs and job environments play in that development. Korthagen (2017) explains the need to understand how teachers learn in order to more accurately explain the connection between theory and teaching practices. Research has shown the impact of preparation programs on teacher retention, student achievement, and teacher quality, among others (Brouwer & Korthagen, 2005; Darling-Hammond, 2010; Darling-Hammond et al., 2002; Darling-Hammond et al., 2005; Darling-Hammond & Youngs, 2002; Gansle et al., 2012; Koedel et al., 2015; National Mathematics Advisory Panel, 2008; Tchoshanov et al., 2008). However, there have been mixed results of the effects of teacher preparation programs on the development of PCK (Goldhaber et al., 2013; Grossman, 1990; Leong, 2013; Saeli et al., 2012; Schmidt et al., 2016). There is also a lack of research conducted in the United States about the transition from pre-service to in-service settings with a focus on PCK development. To explore how PCK develops throughout student teaching and during the first year of teaching, I investigated the following research questions:

1. How does secondary mathematics teachers' PCK change over the first year of teaching?
2. How do secondary mathematics teachers describe the development of their PCK before and during their first year as a teacher?
    2.1 How do beginning secondary mathematics teachers' experiences and views of their development of PCK change from institutional to professional learning of teaching?
3. What experiences and factors influence the development of secondary mathematics teachers' PCK?
    3.1 How does the development of PCK during the student teaching year transfer to their first year of teaching?
    3.2 What experiences and factors do beginning secondary mathematics teachers report supported or hindered the development of their PCK while in their first year of teaching?

In this chapter, I will discuss my findings and connect them to my research questions. Many of my findings demonstrated commonalities in the ways my participants developed their PCK. In particular, participants' experiences with students were main sources of PCK since many of the tasks associated with PCK involved student thinking. My data also suggest that these three participating teachers possess a domain of PCK which was not explicitly represented in my original conceptualization of PCK adapted from Ball, Thames, and Phelps (2008): Knowledge of Assessment (KA). In this chapter I will revisit my theoretical and conceptual frameworks and explain how my study contributed to changes in them. I also discuss implications of my research as well as directions for future research. Last, I will reflect on my own development both as a researcher and as a teacher educator by conducting this study.

The Development of Secondary Mathematics Teachers' PCK

It became evident that the main source of participants' PCK development was their experiences in classrooms with real students (research question 3). Similarly, Van Driel and Berry (2010) posit that teaching experiences are fundamental for developing PCK from their meta-analysis of the literature on pre-service teachers' PCK. Veteran teachers, from their experiences in their classrooms, possess more developed PCK including "ways of organizing content for learning, a store of specific explanations, awareness of common errors and misconceptions, and an understanding of the learning characteristics of the students in their classes" (Livingston & Borko, 1990, p. 384). In general, these participants grew in different domains and tasks of PCK by engaging with students, implementing their ideas about teaching and learning, and using students' behaviors and success or lack thereof as indicators of effective or ineffective knowledge. Learning-On-The-Go contributed to participants' development and helped connect theory to practice as they continued to develop their own professional identities. Noblet (2016) describes PCK in the developing stages as "potential PCK" (p. 317). She also explains that the different domains of PCK develop individually and in conjunction with other domains. For example, the domain of Knowledge of Content and Students (KCS) develops through experiences with students and also by building on teachers' content knowledge. Data from my participants also indicated this to be the case. Specific tasks seemed to develop parallel to each other, like designing instruction and selecting program and instructional materials.

Below are visual representations showing the same information presented in Chapter 4 but in an alternative format (Figures 6.1, 6.2, and 6.3).

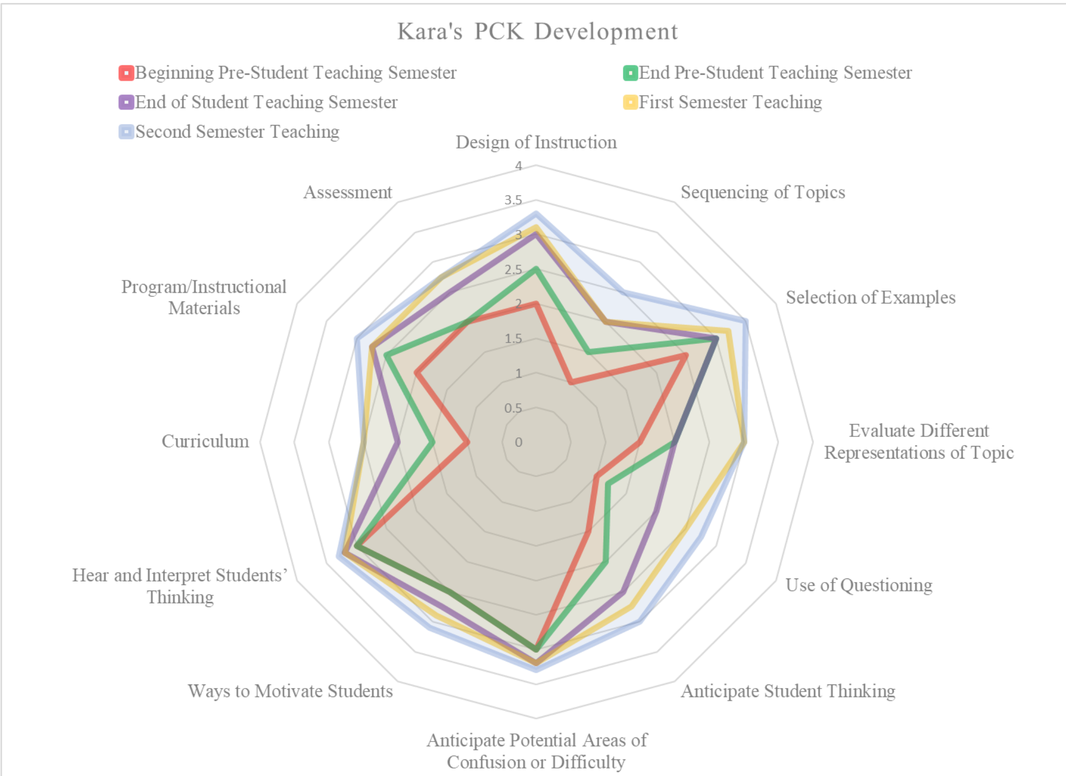


Figure 6.1. Summary of Kara's PCK Development using a Radar Plot.

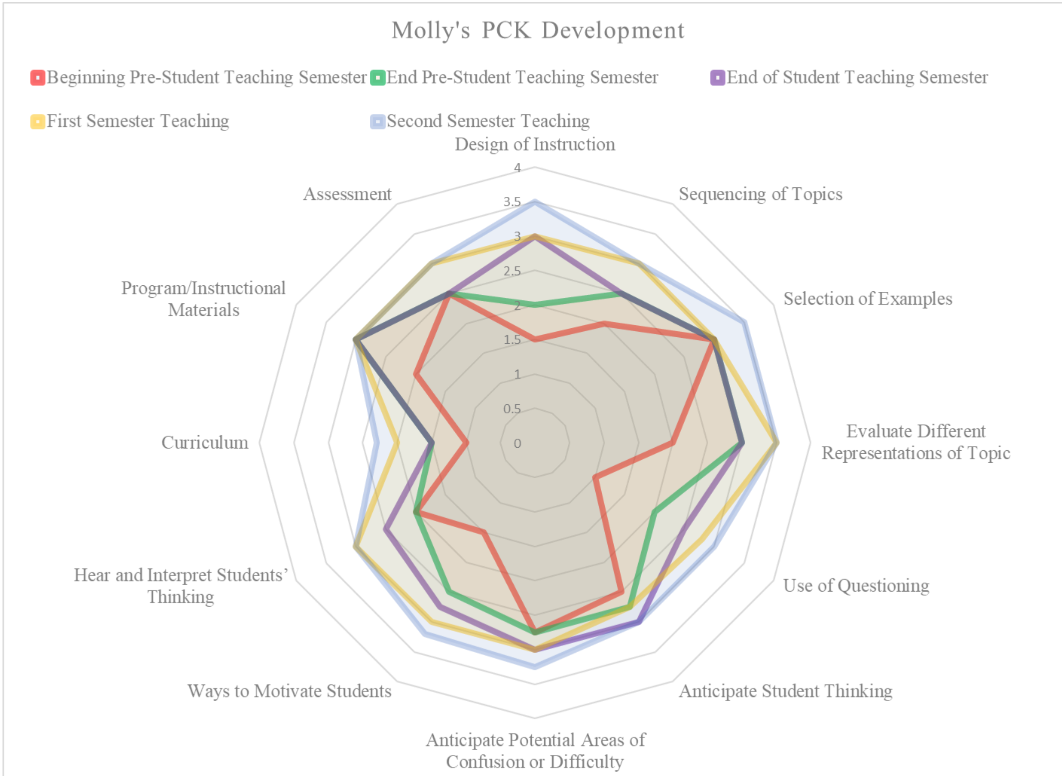


Figure 6.2. Summary of Molly's PCK Development using a Radar Plot.

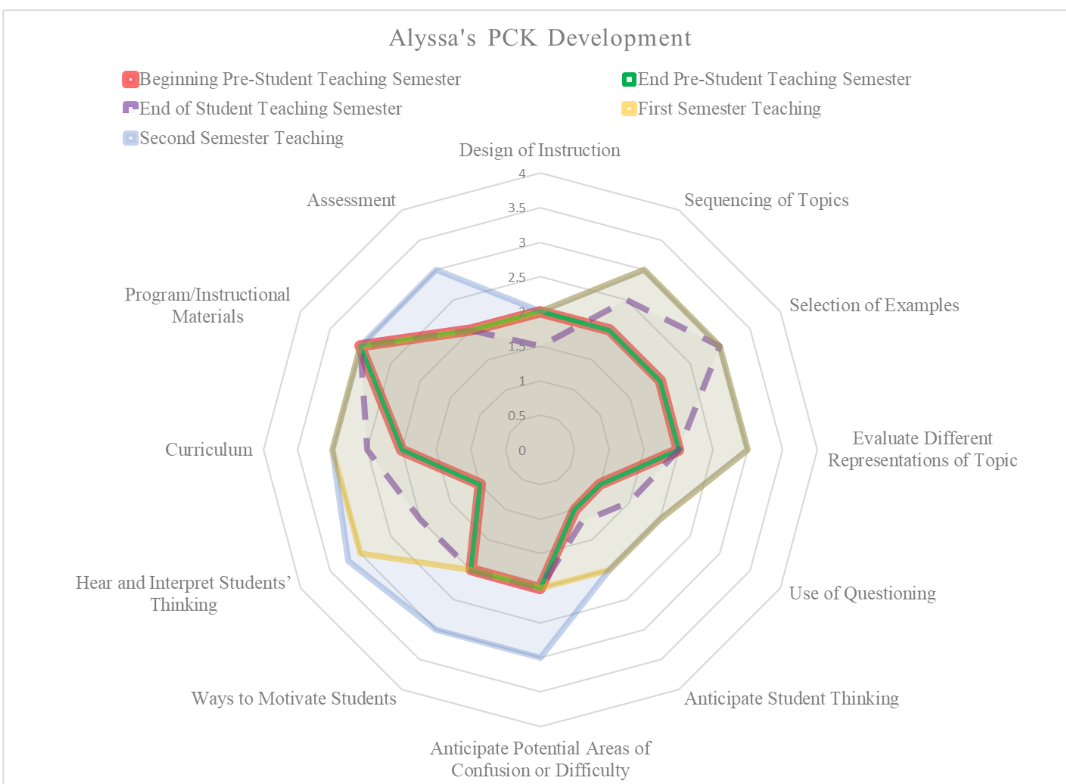


Figure 6.3. Summary of Alyssa's PCK Development using a Radar Plot.

Each ring in the figures above shows participants' PCK development at different time periods throughout the study, from the beginning of their last year in their preparation program to the end of their first year of teaching. Each spoke corresponds to a different task of PCK. For each of the participants there were some periods of rapid development in the different tasks, as indicated by the expansion in the ring, while other remained relatively stable, shown with points or rings coinciding (research question 1). For Alyssa (Figure 6.3), the dotted lines indicate where data was extrapolated from other interviews and evidence since she was unable to complete the PCK Inventory or participate in an interview at the end of her student teaching experiences. The periods of growth correlate to increased classroom time and having more responsibilities in the daily processes of a classroom. For example, in the figure illustrating Kara's development (Figure 6.1), there is space between the rings after a period of time where she has increased classroom responsibilities. One noticeable instance of this growth in all three figures was between the end of their student teaching semester and the start of their first semester teaching. In tasks where participants had little control or freedom, like sequencing of topics, there was minimal growth, as shown by the overlaying of the lines or points. Tasks that were less developed are illustrated as "dents" in the figures above (Figures 6.1, 6.2, and 6.3). During student teaching, some of these tasks were out of the scope of the experiences. Similarly, participants began their first year of teaching with a less developed knowledge in some domains of PCK which results in less confidence and insecurity in their beliefs and identity (research question 1). They felt uncomfortable disrupting the norms of their schools which resulted in few changes and stagnation in their development. For these reasons, it is increasingly important that teacher candidates be provided with opportunities to explore these tasks before entering the profession. Similarly, administrators need to be confident in their teachers' knowledge and support their critiques about curriculum, instructional design and materials, and other tasks of PCK. Participants had experiences that were similar to what most teachers experience when working with administrators, including identifying administrative support as important during their first years of teaching (Marabel & Raimondi, 2007). Without administrative support or with fear of repercussions, teachers could become unmotivated or unwilling to apply their knowledge or continue their development, like in Alyssa's case. They also may experience confusion about expectations in their roles and difficulty navigating the politics that reverberate through in schools (Marabel & Raimondi, 2007).

As alluded to earlier, development in the different domains of PCK was supported by participants' content knowledge (research question 3). Participants were able to engage in tasks of PCK more effectively when they had a deeper understanding of the subject matter, as discussed in Chapters 4 and 5. As they developed in the different tasks of PCK, the rings in the figures above became smoother with less protuberances and dents (Figures 6.1, 6.2, and 6.3). However, having a strong knowledge base in mathematics is not the only type of knowledge necessary to be a teacher; Monk (1994) claims that "a good grasp of one's subject area is a necessary but not sufficient condition for effective teaching" (p. 142). The findings of this study indicate that teacher preparation programs should further integrate the use of specialized mathematics courses. When looking at the figures above, participants' development in different tasks of PCK through participating in the specialized mathematics courses can be seen in the differences between the rings representing the start and end of their pre-student teaching semester (Figures 6.1, 6.2, 6.3). While their development cannot be entirely attributed to these courses, participants expressed positive gains in both their CK and PCK development from their experiences in these courses and referred to this coursework during their first year of teaching. They felt these specialized courses should be offered earlier and throughout the preparation program since they were integral to their knowledge development (research question 2). Molly proposed a curriculum modification where students would take specialized mathematics courses alongside their other mathematics coursework, allowing for more connections to be made and to go more in depth [Molly- Interview 1- 15: 3-13]. The explicit integration of content knowledge with PCK made tasks of teaching more real and fostered reflection and deeper thinking about the subjects and topics they could be teaching. It is a commonly held belief that teachers' content knowledge development becomes relatively dormant after graduating from their preparation programs unless teacher actively works to continue developing. Kleickmann et al. (2013) summarize this occurrence, stating "the inservice phase does not seem to contribute to substantial further development of CK after initial teacher education" (p. 11). Instead, teachers become "really good" at their grade's content but become less confident or comfortable with other areas. This became evident in participants' PCK Inventory responses, where questions were developed and aligned to the different tasks of PCK and covered a wide range of topics and grade levels. This instrument was designed in this way since it was uncertain where and at what grade level participants would obtain employment as first-year teachers. There were questions on the PCK inventory with topics geared towards high school content but all three participants became middle school teachers. As a result, there were some questions about topics which they had not taught or worked with in many years. In some cases, this resulted in difficulties with performing the different tasks aligned in those particular questions. For example, Molly had some difficulty anticipating student thinking for situations where she did not have recent experiences (Figure 6.2). Research suggests that when teachers are less familiar with topics, or less confident, they tend to rely on learning theories or general PK instead of PCK (Noblet, 2016; Van Driel & Berry, 2010). However, for questions structured around topics they had recent experiences with, they were more able and comfortable in the different PCK tasks. Also, when participants experienced communication between grade levels, they were more comfortable with lesson designs and topics (research question 3). For example, knowing what prior knowledge and experiences students had in previous grades helped them prepare for potential areas of difficulty. Similarly, when they were aware of topics or requirements in future grades, they felt better about their own curricular decisions. For Kara, she did not show much development in her curricular knowledge after beginning her first year of teaching, as shown by the overlapping of points in Figure 6.1. One reason for this is because she taught only eighth grade and worked to understand and implement that curriculum. On the other hand, Molly taught both seventh and eighth grade which supported her knowledge development in curriculums and students' prior knowledge and experiences (Figure 6.2). Findings from this study demonstrated that beginning teachers need to be engaged in communication with other grade levels and have access to those curriculums. This would better support development of specific tasks of PCK like sequencing of topics, designing instruction, selecting representations, and their overall curricular knowledge. It also illustrates how vital having experiences in different grade levels is while enrolled in a preparation program.

New PCK Domain: Knowledge of Assessment (KA)

In addition to the domains originally described by Ball and colleagues (e.g. Ball, Thames, & Phelps, 2008), my data indicated an additional domain: Knowledge of Assessment. Including this domain as part of other domains and tasks within their framework detracts from the importance and influence assessments have on the daily work teachers do. It has been argued that knowledge of assessment is an important component of PCK for science teachers, which includes: knowledge of which concepts and methods of learning can and should be assessed and knowledge of specific instrument, approaches, or activities (Magnusson, Krajcik, & Borko, 1999; Novak, 1993; Park & Oliver, 2008; Tamir, 1988). Similarly, Lannin and colleagues (2013) adapted the framework developed by Magnusson et al. (1999) for science teaching for use with mathematics teaching and identified knowledge of assessment as a missing component of the model by Ball and colleagues. Participants in this study recognized the central role of assessments in both students' experiences and their own. Through practicums and education coursework, they noted use of different types of assessments for a variety of purposes, such as formative and summative assessments (research questions 2 and 3). They discussed the importance of constructing assessments and having alignment between the assessment, curriculum, and instructional design. As first-year teachers, they experienced using common assessments and collecting data on student performance, sometimes tied to their teacher evaluations. While they expressed confidence in their abilities to construct effective lessons and engage students, they did express some feelings of restrictions by the common assessments. From these findings, I propose the following tasks be associated with knowledge of assessment for mathematics teaching include: identifying methods or strategies of assessment; use of assessment data; challenges or difficulties with assessment; selecting appropriate topics and processes to assess; and design of assessment.

As with the other domains, knowledge of assessment (KA) did not develop in isolation. It is intimately tied to all three original domains: knowledge of content and curriculum (KCC), knowledge of content and teaching (KCT), and knowledge of content and students (KCS). For example, if teachers can anticipate potential areas of difficulty or confusion (a task of KCS), they can design assessments in such a way that either avoid or highlight those areas. Similarly, teachers can design instruction, select examples, and evaluate different representations of topics (all tasks of KCT) that are appropriately connected to assessments. This can be seen in the figures above as the developmental rings become more rounded as the tasks develop in tandem (Figures 6.1, 6.2, and 6.3). Teachers also need to have a well-developed knowledge of assessments to ensure they are not "teaching to the test," a concern expressed by some of my participants as they began their first year of teaching. These participating teachers explained how they utilized assessment to determine if their students were understanding concepts. If they felt students were still confused or not proficient enough, they would readdress topics. This showed the connection between developing their knowledge of assessment, hearing and interpreting student thinking, and anticipating potential areas of confusion. They also discussed how the use of common assessments impacted their decision making when designing instruction, sequencing topics, and selecting representations and examples. In some cases, they wanted to teach concepts in a certain way that deviated from the methods expected on the common assessments. These situations caused them to consider their knowledge of assessments alongside other domains and tasks of PCK to provide effective learning experiences for their students.

The Role of Others in PCK Development

For participants in my study, the role of peers and colleagues was almost as important as working with students to developing different tasks of PCK (research question 3). Brouwer and Korthagen (2005) explain that teacher preparation programs should equip students with both start competence and growth or "in-service" competence (p. 158). Start competence refers to the competence beginning teachers' need as they enter the profession which continues to develop into in-service competence over the first years of teaching. In-service competence is the ability for teachers to continue their development as a teacher, and PCK specifically, in a self-sustained and self-directed manner. Data from this study indicate that one way in which pre-service teachers developed these competences is by collaborating and discussing with their peers and through reflecting on their experiences. Participants were able to discuss their experiences with their peers during their methods and seminar courses which promoted reflection and development of PCK (Brouwer & Korthagen, 2005; Korthagen, Loughran, & Russell, 2006). In addition, they shared resources with each other, which promoted development in their knowledge of designing instruction, selecting different representations, and identifying program and instructional materials, and developed different competencies needed to enter the field. Thus, pre-service teachers utilize their peers as supports in their development and also as critical mirrors through which they examine their own development and beliefs. Teacher educators need to be aware of the impact peers have on PCK development and support meaningful and productive conversations between cohorts (Korthagen et al., 2006; Soini et al., 2015).

In a similar way, beginning teachers utilize their colleagues and other professionals during their first year of teaching as sources to support their transition to the profession (research question 3). Being novices, they compare themselves to more experienced teachers and tend to model some of their behaviors after them. Colleagues play an influential role in molding beginning teachers' professional identity, beliefs, and practice by providing explanation or advice (Feiman-Nemser, 2003). Since none of the participants had formal mentors during their first year of teaching, they explained how important colleagues were to their development. This is consistent with findings from Marable and Raimondi (2007), who explained peers as the main source of support when there were not formal mentors assigned. My participants identified colleagues or other school professionals, like induction coaches or curriculum coordinators, as resources that facilitated their instructional design, aided in selecting program materials, promoted development in their curricular knowledge, and enhanced their knowledge of their students. However, not all interactions with colleagues enhanced the participants' PCK development since some of their beliefs or practices deviated from the norms of the schools.

Many schools have induction programs to help new teachers become acclimated to the new environment and facilitate in the adjustment to the practice of teaching. Though many programs have good intentions, the designs of some promote trying to "fit [new] teachers into the existing system" (Feiman-Nemser, 2003, p. 26). At times, this can be viewed as enculturation into a community with new teachers receiving explicit instruction about specific methods, concepts, skills, and procedures that are valued by the school (Putnam & Borko, 2000). This caused some tensions for participants as they had to renegotiate aspects of their professional identities (Eteläpelto, Vähäsantanen, & Hökkä, 2015). For example, departments had detailed curriculums in the form of scope and sequences that participants had to adhere to fairly closely (research question 3). Since participants were still developing their professional identities, some of their beliefs about how the curriculum should be arranged were left unsaid. While they became knowledgeable about the prescribed curriculum, they did not have my opportunities to fully explore their curricular knowledge or different sequences of topics. They did not feel confident in their position so they did not speak out often about changes they felt would enhance student learning. Participants possessed competencies but they did not yet have agency and perceived status or competence to enact some of their beliefs (research questions 1 and 3). Lack of confidence also directly impacted their PCK development as self-confidence is considered a precursor for PCK development (Van Driel & Berry, 2010). As a result, participants were unable to further develop tasks of their PCK since they did not want to deviate too far from their colleagues' practices. Induction programs, like teacher preparation programs, would be more effective by supporting teachers from where they begin instead of trying to fit everyone into the same model. Induction practices could be more effective if there was congruence with designs of pre-service training, such as the Realistic Teacher Education approach. Considering the tenets of Realistic Teacher Education (Korthagen, Kessels, Koster, Lagerwerf, & Wubbels, 2001) and reframing them for an induction program would include:

- Start from the concrete practical problems and concerns experienced by the teachers in real contexts (e.g. their classrooms);
- Programming and professional development starts with the Gestalts of the teachers for continued professional learning;
- Promote systematic reflection on their own and their students' wanting, feeling, thinking, and acting, on the role of context, and on the relationships between those aspects;
- Builds on the personal interactions between educators and school leaders or mentors and among teachers;
- Strong integration between theory and practice.

Participants in this study sought out supports outside of their induction programs as they were not central to the existing structures. Kara was the only one who described interactions with other beginning colleagues as being facilitated by the induction program in monthly district-wide meetings. Molly and Alyssa both sought out other colleagues and professionals to interact with. At times, the induction process was disconnected from the needs of these three beginning teachers. For example, at the beginning of the year, they needed support establishing classroom norms but this was not necessarily part of their induction program agenda. Induction programs would be more effective by beginning with the concerns of the individual and enhancing their already formed skills and knowledge.

Identity and PCK Development

As discussed previously, teachers' identity formation is an ongoing process that begins when they are students in K-12 schools and continues throughout their careers.

Participants in this study experienced noticeable development in their professional identities as they gained experiences in classrooms and began to take ownership of their students (research question 2). Their identities were shaped from interactions with peers and colleagues, working with students, conducting effective and ineffective lessons, and reflecting on their experiences. In tandem with their identity development, participants also developed in the different domains of PCK. While it was difficult to tease out which development influenced the other, it seemed that as teachers became more knowledgeable in the different tasks of PCK, their professional identity became a bit more established. For example, as participants developed their knowledge about designing instruction, they could change their view on the role of teachers thus impacting their identity. Molly is the embodiment of this change occurring: as she began to learn about different methods of instruction, in particular the use of inquiry, she began to change how she viewed teaching and learning. This altered her beliefs, identity, and helped her understand her mission as a teacher. I expect that their identities will continue to be altered as a result of their different experiences throughout their entire teaching careers. After a while, their identities will become relatively well-developed and if they remain reflective and receptive to their experiences, then they will continue to grow and develop. It has also been argued that identity development is an on-going process: "a process of interpreting oneself as a certain kind of person and being recognized as such in a given context" (Beijaard, Meijer, & Verloop, 2004, p. 108).

One way in which participants' identity development and PCK development was evident was in the language they used. Participants began to use precise mathematical language when explaining concepts to students. While this demonstrates a growth in their content knowledge, it also showed growth in their knowledge of content and teaching (KCT). Ball and colleagues describe the domain KCT as combining "knowing about teaching and knowing about mathematics" and involve tasks that require "an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning" (p. 401). For this reason, I propose "use of mathematical language" as an additional task within this domain. Well-developed abilities in this task include being able to provide clear, precise, and complete communication with others, which is also one of the Common Core State Standards for Mathematical Practice: attending to precision. This includes providing using the language of mathematics verbally, in writing, and symbolically when providing directions and discussing content. Participants in this study explained the role of language in developing mathematics knowledge with their students. They demonstrated the importance of accurate mathematical language by rephrasing student responses to facilitate connection to prior knowledge or to future lessons. Teachers model the use of mathematical language alongside teaching content, again illustrating the connection between content and teaching. Participants explained how the use of proper language was stressed by their advisor which made them watchful of their own language and their students' language (research question 3). Thus, teacher educators played a profound role in helping these teacher participants develop in this task of PCK.

When teachers begin their work as first-year teachers, they run the risk of regressing back to less precise language, as evident by my participants' experiences. They have to align their language to that of other teachers while navigating how to integrate proper language. It is recommended that departments promote the use of accurate and precise language and support teachers to continue to develop this task of PCK though common assessments, common planning times, and professional development meetings.

Role of Reflection in Development of PCK

Many times participants did not realize they were learning or developing in the different tasks of PCK and only by reflecting on their past experiences did they become cognizant of their growth (research questions 2 and 3). Participants who received instruction during their preparation program about methods of reflection utilized this habit of mind even after graduating. Being reflective in-action and on-action enhanced their abilities to anticipate student thinking and potential areas of difficulty, design instruction, hear and interpret student thinking, motivate students, and select examples. This is in line with findings from Korthagen (2017) who explains that teacher learning occurs as the teacher experiences different occurrences and through reflection on those experiences: “Although a lot of teacher behaviour and learning seem to take place unconsciously, in-depth reflection is an important instrument in establishing fruitful connections between practice, theory and person” (p. 398). However, participants had difficulty making progress in some tasks of PCK when they were strongly focused on classroom management. As discussed in the previous chapter, beginning teachers already tend to focus extensively on the environment (Korthagen, 2004), which leaves little time for deliberately reflecting, daily lesson preparation, and self-care. This can lead to teacher burn-out since they begin to question their own knowledge and competencies and become exhausted and frustrated with their lack of progress (Marable & Raimondi, 2007). Feiman-Nemser (2003) warns of the risks if effective induction programs are not present for beginning teachers and if they do not feel supported as they enter the profession: “If we leave beginning teachers to sink or swim on their own, they may become overwhelmed and leave the field. Alternatively, they may stay, clinging to practices and attitudes that help them survive but do not serve the education needs of students” (p. 25). This is an issue of district leadership and vision about how people learn teaching and something teacher preparation can do little about.

Participants discussed and demonstrated the role of reflection in their PCK development and the formation of their professional identity. Through reflection, individuals make sense of new or different experiences, connect new ideas and experiences to prior ones, and revise and develop their thinking. It has been stated that teacher candidates’ reflective abilities are an essential skill that needs to be nurtured and practiced (Darling-Hammond, 2006; Korthagen, Loughran, & Russell, 2006; Loughran, Brown, & Doecke, 2001). During their preparation program, participants explained how reflection was integrated into their practicum experiences and methods course. They continued to utilized the ALACT process of reflection (see Korthagen, 2002) during their first year, though they explained they sometimes had difficulty in setting aside time to do so. The pace of schools does not allow for active reflection by teachers. Participants explained that they were spending countless hours outside of the classroom grading, preparing, writing reports, and evaluating materials, resulting in little time or energy left to spend reflecting. Beginning teachers particularly are focused on daily instruction, classroom management, and other job requirements which leaves them with little time to consciously reflect on their teaching. As seen from research on teacher preparation programs, discussions with peers is a power method of fostering reflection and developing PCK (Hauge & Wittek, 2003; Korthagen, Loughran, & Russell, 2006; Smith & Lev-Ari, 2005). As a result, it is recommended that schools support systematic reflection by all teacher, but especially beginning teachers since this is a time of rapid development in many tasks of PCK. Supporting reflection could take many forms in schools, such as a reduced teaching load to allow for time during the day to reflect or use time in common planning times (CPTs) for teachers to discuss their teaching and be reflective together.

Revisiting and Revising Conceptual Framework

In my original conceptualization of PCK development consisting of elements I culled from the literature, I utilized a tetrahedral organization with the base composed of three factors: personal learning, subject matter knowledge, and reflection. The findings of my study further supported these three characteristics as central to PCK development (research question 3). In addition, candidates' beliefs were influential throughout all the levels of development and across the different domains of PCK (research question 3), which is why it has been added to the composition of the base (Figure 6.4).

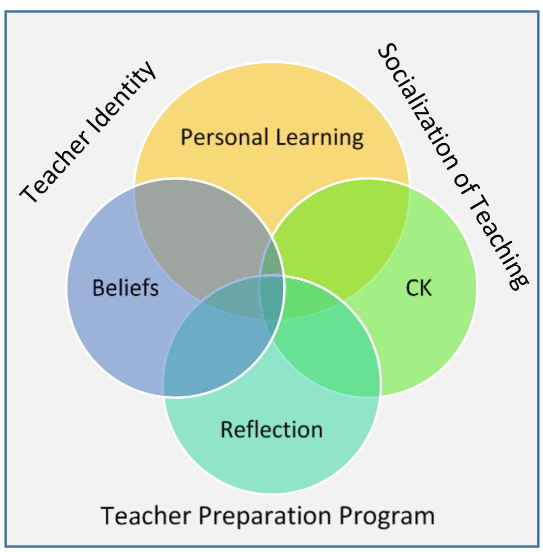


Figure 6.4. View 1 (the bottom) of Revised Conceptual Framework of influential experiences and factors on the development of PCK.

Further, the factors external to teacher candidates have been re-evaluated in light of finding from my study. Participants' experiences demonstrated the role of teacher preparation programs and the socialization of teaching as forces trying to influence their knowledge development (research question 3). However, the apprenticeship of observation was one way in which teachers were socialized into the profession, which is why it is now included under that umbrella instead of on its own. In addition to these forces vying for influence was the role of participants' development of their professional identity. For example, as participants developed their teaching identity, they also had agency to utilized the characteristics at the core of their development, illustrated in the Venn diagram in the center of the base in Figure 6.4.

Another modification to my original conceptualization was in how I viewed the structure of PCK. At the pinnacle of my original organization was the general category of PCK being supported by different types of knowledge as indicated by the different faces. The original design showed PCK as a combination of Knowledge of Students, Knowledge of Curriculum, and Content Knowledge. This view lacked detail and glossed over the relationship between the types of knowledge teachers possess. My participants' experiences demonstrated that PCK is comprised of four different yet related domains that come together to form an individual's PCK: Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), Knowledge of Content and Curriculum (KCC), and Knowledge of Assessment (KA). The development of these domains were supported by different experiences and factors, which are illustrated as the foundational blocks of each face (Figures 6.5 and 6.6).

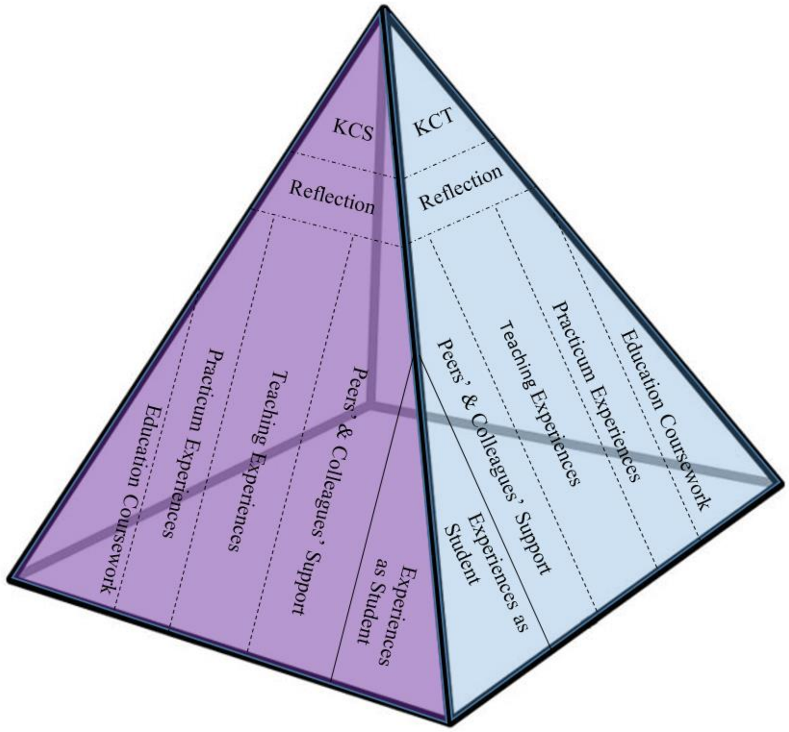


Figure 6.5. View 2 (showing 2 sides) of Revised Conceptual framework of influential experiences and factors on the development of PCK.

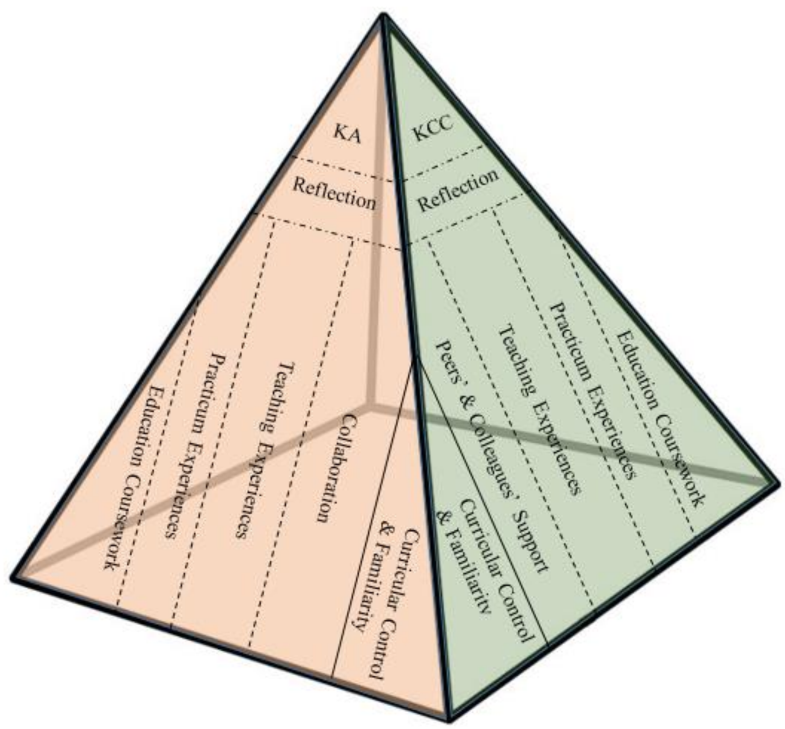


Figure 6.6. View 3 (showing other 2 sides) of Revised Conceptual framework of influential experiences and factors on the development of PCK.

Again, these “building blocks” were originally chosen from my initial review of the literature and were refined based on findings from my study. I was able to be more specific in what supported the development in the different domains from my analysis using the Onion Model (Korthagen, 2004) discussed in my previous chapter. For instance, instead of including just practicum courses as a foundational experience, I also noted the interplay between the education courses and practicum courses by including both as a base block with a dotted line as the interface. This illustrates that both are important experiences and learning opportunities separately and taken together when developing PCK (research question 3). Similarly, participants’ PCK development noticeably developed as they gained teaching experiences and more so when they had their own classrooms. Thus, teaching experiences, like in my original conceptualization, were integral of all domains of PCK development (research question 3). However, my participants’ experiences also demonstrated the influential role of peers and colleagues during those teaching experiences on development. Again, the relationship between those factors was indicated through use of a dotted line. Experiences as students encompasses both when participants were students in K-12 schools and as college students. I chose to categorize these experiences together since participants used themselves as models when thinking about their students based in their experiences as learners throughout their education. In addition, they viewed their teachers, instructors, and professors as archetypes for teaching which contributed to their initial knowledge in many domains of PCK.

Recommendations

Having a more descriptive and accurate representation of how PCK develops can help teacher educators and school leaders enact more targeted supports and necessary changes. In addition, participants' experiences can provide insights for other pre-service and in-service teachers in the form of "words of advice" for the next generation of teachers.

Recommendations for Teacher Candidates

Through interviews, responses on the PCK Inventory, and classroom observations, participants offered their own experiences as examples for up-and-coming pre-service and in-service teachers. From their experiences and words, I've constructed a found poem with the recommendations they have for others:

Find your people,
Seek them out and find your supports.
Try,
Be open to new experiences,
Put yourself out there.
Make mistakes,
That's where learning happens.
Learn from the good, bad, and in between.
Be critical,
Don't take everything at face value.
Grow,
Remember you're still learning.
It will get better.
Reflect.

When these participants considered their own paths to becoming a teacher, they noticed beacons of supports and what helped them develop. These pieces of advice are shown in the poem above. Participants in this study were reflective but also critical of their knowledge abilities and held themselves to high standards. However, they also recognized that they were still beginning in their careers and their learning and development did not end after completing their preparation program. All of the participants acknowledged the need to find people who shared in their beliefs and supported them in a variety of ways. As a result, they would suggest pre-service and in-service teachers construct a network of people they can rely on and who will help them reflect on their experiences. While everyone's experiences are different, these recommendations can help provide others with ideas and practical actions at the start of their careers. In summary, these pieces of advice seem to have three main messages: build collegial relationships, reflect on your practice, and persevere.

Recommendations for Teacher Preparation Programs

Recommendation 1: develop systematic approaches to provide pre-service teachers with opportunities to develop different tasks of PCK. One consideration for teacher preparation programs is enhancing the back-and-forth between theory and practice facilitated by connections between education coursework and practicum experiences (Darling-Hammond, 2006; Darling-Hammond, 2010; Korthagen, 2002; Smith & Lev-Ari, 2005; Zeichner & Gore, 1990). Participants' development in all domains of PCK were supported by the content they learned in their education courses, the experiences they had working with real students during their practicums, and through unpacking those experiences with their peers and instructors in their courses. In addition to connecting practicum experiences with other coursework, PCK developed when participants had occasions to explore the different tasks before entering the profession. While some tasks were beyond the scope of many practicum experiences including student teaching, it is a role of teacher educators to facilitate such opportunities. For example, it is difficult for candidates to have control over the curriculums in their placements but they can explore different curriculums in their coursework to develop an understanding of how topics and subjects fit together. Finding occasions for these experiences is difficult in an already packed schedule of courses and program requirements, but candidates benefit from practicing applying their knowledge in safe and supported environments. Programs can enhance their effectiveness by developing systematic approaches to provide candidates with these opportunities.

Recommendation 2: provide pre-service candidates with experiences in a variety of settings. Further examining participants' experiences in their practicums and student teaching placements indicated the importance of having experiences in a variety of settings. Findings from this study indicated that when teacher candidates and beginning teachers were unfamiliar with the needs of certain populations, their PCK development was hindered. As a result, practicum experiences would be most effective if candidates worked with a variety of populations, such as English learners and students with special needs. Also, participants expressed more confidence and displayed more developed knowledge as a result of working with different age groups of students. They were able to make connections between topics taught at different grade levels and grew in their knowledge of different tasks of PCK by having had practicum experiences in a variety of grades. They were also more able to anticipate student thinking by being aware of how they were taught and behaved in previous classes. Since self-confidence is considered a precursor for PCK development (Van Driel & Berry, 2010), building candidates' confidence is an important implication for teacher preparation programs. Participants also described a better understanding of their mission and which populations they felt most drawn to after having experiences in a variety of settings. For example, Molly described feeling most "like a teacher" while in her middle school placements and realized her personality fit best with working with that age group [Molly- Interview 4- 14 & 15: 16-21 & 1-9]. Thus, it is recommended that preparation programs provided candidates with experiences in a variety of settings (Darling-Hammond, 2006).

Recommendation 3: utilize specialized mathematics courses specifically focused on developing PCK. As discussed earlier, the domains of PCK seemed to develop in parallel to each other and to CK. One inference from this finding is that as candidates deepen their content knowledge by taking high-level mathematics courses, their PCK development can be supported as well if given opportunities to do so. Along with the integration of practicum experiences with other education coursework, participants stated the specialized mathematics courses they took were central to the development of their PCK. Molly proposed having courses similar to the mathematics capstone course throughout the preparation program to begin that knowledge development earlier. Similarly, Alyssa felt the current mathematics program was not designed for those pursuing a career in education and felt there was a gap between her mathematics coursework and what she needed to know as a teacher. Kara explained that the specialized courses were more helpful and specific to the preparation of mathematics teachers than the other mathematics and education coursework. Participants felt the specialized mathematics courses allowed them to explore the tasks of PCK while also making connections between their content knowledge and the knowledge they needed to be effective teachers. Programs could consider integrating specialized mathematics courses or increasing the number of courses to facilitate PCK development.

Recommendation 4: provide opportunities for pre-service teachers to discuss their experiences with peers. Participants in this study repeatedly talked about the role of discussions with peers, instructors, and mentors in relation to how they developed their PCK. Teacher educators need to support meaningful and productive conversation among students that foster reflection and growth (Hauge & Wittek, 2003; Korthagen, Loughran, & Russell, 2006; Smith & Lev-Ari, 2005). Courses can include opportunities for candidates to share their experiences or beliefs with peers.

Recommendation 5: integrate modeling and instruction about clear, precise, and complete language usage. In addition to supporting discussions in general, participants explained the role of their instructors and advisor in stressing the importance of precise, clear, and complete language through modeling and explicit instruction. Through their experiences in classrooms, participants realized the role of language when working with students and greatly appreciated and valued this part of their preparation program. This was not an element of mathematics teacher preparation discussed widely in the literature so it is an important finding from this study.

Recommendation 6: provide pre-service teachers with methods and opportunities for reflection. One finding from this study that is congruent with the existing literature is the role of reflection in teacher development. Participants demonstrated how reflection impacted their abilities to make sense of their environments, reevaluate their beliefs about teaching and learning, and develop their PCK. They were able to be reflective since they learned methods to do so in their teacher preparation coursework. In addition, their advisor facilitated meaningful discussions during their methods and student teaching seminar that prompted them to be reflective about their experiences, their development, and their beliefs. Participants' experiences demonstrated that it would be beneficial if teacher preparation programs equip candidates with tools and strategies to be reflective throughout their careers.

Recommendations for Administrator and School Leaders

Recommendation 1: facilitate communication between grades and/or access to other grades' curriculums. Findings from this study also highlighted areas of need that impact the experiences of beginning teachers during their first year of teaching. Among these needs is for school leaders to facilitate communication between teachers of different grade levels. For example, Molly was part of a professional development opportunity where middle school teachers met with high school teachers from the same district. This experience provided her with the opportunity to further develop her PCK by understanding the expectations and needs her students will encounter in upper grades. Similarly, she taught both seventh and eighth grade so she had a better understanding of what prior knowledge and experiences students may have had. On the other hand, Alyssa did not communicate with her students' previous teachers so she was unable initially to effectively anticipate their thinking. However, she was engaged in departmental meetings that reviewed the curriculums across grade levels, which helped her make connections between her content and future grades' content. Even if scheduling or facilitating meetings between grade-level teachers is difficult, beginning teachers could be given access to other grades' curriculums. Though many beginning teachers are focused on their own scope and sequence and planning their own daily lessons, it would be a great resource they could access if they were questioning students' prior knowledge or skills or what they would need in the future. It would better help them see how their content fit into the overall learning progressions in their district and determine whether their sequence of topics, representations, and examples are appropriate.

Recommendation 2: involve beginning teachers as full partners in a community of practice. Along with promoting communication between grades and the sharing of curriculums, participants' PCK development was supported through discussions with colleagues and by having opportunities to apply their knowledge. However, at times they were faced with instances of inflexibility which limited their development in some tasks. They also felt uncomfortable going against some of the norms and established curriculums or assessments since they were still developing their professional identities. While being provided with scope and sequences and pre-constructed assessments helped participants with their daily planning as first-year teachers, they needed chances to implement their own ideas and beliefs. For example, participants had thoughts about the curriculums they received but did not necessarily enact some of the revisions they wanted to. At times, they wanted to see how their first year went and make necessary changes in the following year. However, other times, they felt uncomfortable making suggestions. Beginning teachers need to feel supported in their efforts and that their voices are important to school leaders in order to further develop their PCK.

Recommendation 3: provide professional development opportunities aligned to the needs of the teachers. Methods of supporting practicing teacher development typically takes the form of professional development (PD) sessions. Participants' experiences with forms of PDs during their first year of teaching varied depending on the schools in which they were employed. For example, Molly participated in PDs that did not necessarily align to her needs: “We had a PD day where we all met at the high school and talked. The high school curriculum coordinator has been somewhat of resource, she’s not a math teacher, but she has ideas” [Molly- Interview 3: 13: 1-2]. Similarly, she participated in a series of PDs focused on developing students’ problem solving abilities. She felt the principle of the PD was great in theory but their plan of implementation was ineffective. Instead, she valued the PD opportunities she sought out for herself such as attending meetings of the local professional organization. She was able to attend sessions that met her self-identified needs and provided her with more knowledge or additional strategies she could implement in her classroom.

Alyssa viewed the weekly PDs she attended as contributing to her PCK development. One example she provided was learning more about integrating writing into her mathematics lesson, which in turn helped her to see her students’ thinking more explicitly. She also said that the PDs in the form of common planning times (CPTs) where opportunities for her to collaborate with her colleagues in unit launches to “analyze what some of the pitfalls or misconceptions students fall into and how [they] would manage that and teach it the correct way” [Alyssa- Interview 4- 4: 6-7]. These meetings helped her develop in several tasks of PCK. Findings from this study indicate that professional development opportunities for beginning teachers would be most effective if they were rooted in the needs of those individuals and demonstrated a clear focus and connection to classroom applications. This finding is in line with other research on effective professional development practices, which include elements of being long-term and coherent PD programs that engage teacher in active learning with a connection to practice (Darling-Hammond et al., 2009; Garet, Porter, Desimone, Birman, & Yoon, 2001; Kleickmann et al, 2013).

Recommendation 4: support systematic reflection by teachers. Topics of professional development for beginning teachers can vary depending on the needs of those individuals. However, participants in this study expressed the need to have time to reflect and findings from this study demonstrated the power of reflecting alone and with others. Some PD opportunities could focus on fostering reflective practices in teachers and provide them with strategies and spaces to think about their own teaching practices and development. With the pace of schools and the amount of demands placed on first-year teachers, they feel there is little time left to reflect. Reflection has been shown to help individuals make sense of their experiences, develop their professional identity, and further develop PCK (Korthagen et al., 2006; Korthagen & Evelein, 2016). Thus, it is an important habit to maintain as teacher candidates transition into the profession.

Recommendation 5: promote the consistent use of clear, precise, and complete mathematics language by teachers. Participants in this study explained the role of having models for accurate and precise language usage during their preparation programs. However, when they began their work as first-year teachers they noticed the language usage was not consistent between teachers or grade levels. As a result, they had to navigate between using clear, precise, and appropriate terminology but aligning their communication to that which students previously learned. This caused some difficulty in their ability to apply their knowledge and further develop their PCK. School and department leaders could promote the consistent use of accurate and precise language by their teachers. For example, departments already utilizing common assessments can ensure the language on those assessments is mathematically precise and clear. Schools can help promote and further develop teachers' use of mathematical language by including this topic in department meetings, common planning times, and professional development opportunities.

Recommendation for Teacher Preparation Programs and School Leaders

Recommendation 1: maintain and enhance partnership between schools and teacher preparation programs. As with communication between grade levels, the partnership between schools and teacher preparation programs should be further explored and integrated. Stronger communication between school partners and teacher preparation programs would help teacher educators become more aware of what schools expect beginning teacher to know and be able to do. Similarly, school leaders would be able to have reasonable expectations of their beginning teachers and develop an understanding of the learning needs and supports necessary as they transition into the profession. One form this partnership takes on for different preparation programs is "professional development schools" where school and university educators collaborate to improve teaching and learning (Berry & Loughran, 2002; Darling-Hammond, 1998; Darling-Hammond, 2010; Feiman-Nemser, 2001). However, this structure is not necessarily available or possible for every preparation program, which is why teacher educators and schools need to consider the use of wrap-around supports for beginning teachers. An example of this is through designing induction programs. Feiman-Nemser (2001) state:

> Building an induction program that extends and enriches initial preparation and addresses the realities of specific teaching contexts would provide a forum for school and university educators to think together about the learning needs of

> teachers and K-12 students. It would also provide a basis for designing more powerful and coherent forms of ongoing professional development. (p. 1038)

In addition to designing induction programs, schools can utilize local teacher educators as a resource for conducting professional development opportunities. Building, maintaining, and supporting partnerships between schools and preparation programs takes time and effort but is valuable in the development of teachers and for student learning (Berry & Loughran, 2002; Feiman-Nemser 2001).

Reflecting on My Own Development: PCK, Teacher Educator, and Researcher

When I started teaching high school, I had my own ideas of what would work in a classroom based on how I was taught and from my coursework in my teacher preparation program. Many of those ideas fell flat when it came time to working with my students for many reasons. Reflecting back on those ineffective lessons showed me that while I was trying to consider the needs of my students, I was still trying to teach them as I was taught. In the same way, I experienced a learning curve when I transitioned from a classroom teacher to working with pre-service teachers. I realized that I was again comparing these students to myself but the self that had been a classroom teacher already. I needed to remember what it was like and what I was feeling right before going into my student teaching semester. From those lessons that went awry, from the ones that went well, and from working with many different types of students, I learned the importance of many elements of a concept I now know referred to as Pedagogical Content Knowledge.

While I worked with teacher candidates are improving their PCK, I was also improving my “teacher educator PCK”. Like my participants, as I had more experiences with pre-service teachers, I further developed my knowledge in many tasks of PCK. For example, I became better at anticipating their thinking and possible misconceptions. As a result, I was able to design instruction, select better examples and representations, and re-design my curriculum to meet their needs. In addition, conducting this study further helped me understand the interconnectedness of knowledge development, the needs of pre-service and beginning teachers, and how these individuals develop their knowledge. I also developed my understanding of teacher candidates' prior knowledge and experiences and ways to support their development. The experiences of participants in this study were not unique to themselves, meaning they exemplified experiences of other pre-service and beginning teachers. Analyzing their experiences gave me anchor points and insight into other candidates' experiences and development.

In addition to improving my "teacher educator PCK" by conducting this study, I also developed in my identity as a researcher. Through my coursework in my Ph.D. program, I learned about research design and methodologies but it was not until I had to put together my own study, recruit and interact with my participants, analyze my data, and write up my findings that I understood how all the pieces work together. For example, using interview and PCK Inventory responses together with observational data provided me with a more complete picture of my participants' experiences. Reflecting back on the two years during which I conducted this study, I noticed a growth in my abilities as a researcher. In the beginning I was not confident in my interviewing skills, so I may have missed opportunities for follow-up or probing questions. But as I gained more experiences interviewing and treated them as planned conversations, I did not miss as many chances. Initially, I struggled with finding methods of organizing and analyzing my data, of which there were pages and pages of transcripts and text. I realize now, the process of finding a way to condense and examine my data was part of the process for me to make sense of it all. I also had to be flexible and receptive to new data that did not fit with any of my previous data.

In addition, I developed an appreciation for qualitative study that many find surprising given my mathematics background. I believe hearing and telling my participants stories give depth to the data that is sometimes lost when just considering numbers. While my study had a small number of participants, these participants are representative of others like them. There are many Karas, Mollys, and Alyssas that experience the same things as my participants. Understanding their development and what supported or hindered it sheds light on how others also develop. While this study is not generalizable to other situations, the descriptions of my participants and their experiences can facilitate transferability as others recognize their own students in Kara, Molly, and Alyssa.

Conducting this study also showed me there is a great deal more we need to understand about how PCK develops and the transition from pre-service to in-service for beginning teachers. I hope to continue to investigate how beginning teachers develop in the different domains of PCK. Also, more research needs to be conducted on how PCK development differs between pre-service teachers, beginning teachers, and experienced teachers. Participants in my study all gained employment at middle schools, which have different structures in place (e.g. teaming) than high schools. This indicates that further study of how PCK develops during the transition needs to occur and explore whether there is a difference for those that work in middle schools or high schools during their first years of teaching. Lastly, as my participants all start their second year of teaching and build on their experiences and the knowledge they developed, I wondered how experienced teachers maintain or further develop their PCK. Research and supports generally target beginning teachers, but are experienced teachers supported in their development? This is also an area that needs further research since development and growth do not necessarily cease over a teacher's career. The results of this study highlight ways in which teacher candidates and beginning teachers develop their knowledge for teaching and their perceptions of their development. It also highlighted supports or hindrances to PCK development that could be addressed by teacher preparation programs, school leaders, or both.

APPENDICES

APPENDIX A: RECRUITING EMAIL AND LETTER OF CONSENT

Dear Recent Education Graduate,

You are receiving this email because you have been identified as a recent graduate with certification to teach either middle school or high school mathematics or both. Starting in the fall, we will be conducting a study on how beginning mathematics teachers develop their mathematical knowledge for teaching. You are invited to participate at the start of the 2017 school year; this email is informative so you have time to consider your participation before signing and returning the attached consent letter.

Description of the project:

The purpose of this study is to investigate the development of beginning secondary mathematics teachers' pedagogical content knowledge (PCK) about teaching and learning mathematics over the course of the first year of teaching. Findings from this research may be used to enact changes that will help prepare future teachers of mathematics. If you decide to take part in this study, you will complete a PCK inventory twice over the course of the year (October 2017 & April 2018), participate in two interviews and two observations (November 2017, April/May 2018), and complete one summary survey (May 2018).

What will be done:

If you decide to take part in this study here is what will happen:

You will complete a PCK inventory a total of 2 times throughout the course of your first year teaching (2017-2018). Each administration of the inventory should take you approximately 30-45 minutes to complete. In addition, you will be asked to participate in two interviews, which will be audio-recorded during the year and each should last about 45 minutes. The observations will take place in a class and day and time of your choosing. The survey should take you about 20-30 minutes. In total, you will be asked for 6 hours of time for the inventory, survey completion and interviews. Your name will not be identified in any way in the presentation of the research. All of your responses will be held in confidence, and a pseudonym will be assigned. All data will be stored in a locked filing cabinet in the researcher's private, locked office in Chafee Hall, or on a password protected computer.

Risks or discomfort:

There are no anticipated risks involved in participating in this study. Your name and other identifiers will not be used in any way in the presentation of the research and all of your identifying data will be held in confidence. It is not anticipated that you will experience any negative effects as a result of this study and <u>*participation, non-participation, or withdrawal from the study will not affect your employment or your academic standing in any way*</u>.

Benefits of this study:

Participating in this study will provide you with more insight into how you learn and develop as a teacher. In addition, you will be providing valuable information that may facilitate program changes that will better support and prepare future mathematics teachers for the transition to being a working professional.

Confidentiality:

Your participation in this study is confidential. None of the information will identify you by name or otherwise. All records will be saved in a password protected file and pseudonyms will be assigned. If you have any questions or concerns, please do not hesitate to contact me.

Thank you,
Nicole Hersey
ndhtennis@uri.edu
(401)874-4165

THE UNIVERSITY OF RHODE ISLAND
ALAN SHAWN FEINSTEIN COLLEGE OF EDUCATION AND PROFESSIONAL STUDIES
THINK BIG WE DO℠
SCHOOL OF EDUCATION
705 Chafee Hall, 142 Flagg Road, Kingston, RI 02881 USA

Investigating the Development of PCK in Beginning Secondary Mathematics Teachers

CONSENT FORM FOR RESEARCH

You are invited to take part in a research project described below. The researcher will explain the project to you in detail. You should feel free to ask questions. If you have more questions later, Dr. Cornelis de Groot (faculty supervisor: degrootc@uri.edu, (401)874-4149) or Nicole Hersey (doctoral researcher: ndhtennis@uri.edu, (401)874-4165), the people mainly responsible for this study, will discuss them with you. You must be at least 18 years old to be in this research project.

Description of the project:

The purpose of this study is to investigate the development of beginning secondary mathematics teachers' pedagogical content knowledge (PCK) about teaching and learning mathematics over the course of the first year of teaching. Findings from this research may be used to enact changes that will help prepare future teachers of mathematics. If you decide to take part in this study, you will complete a PCK inventory twice over the course of the school year (October 2017 & April 2018), participate in two interviews and two observations (November 2017, April/May 2018), and complete one summary survey (May 2018). In order to be eligible to participate in this study you must be teaching mathematics in either a middle school or high school setting, substitute teaching, or pursuing a graduate degree.

What will be done:

If you decide to take part in this study here is what will happen:

You will complete a PCK inventory a total of 2 times throughout the course of your first year teaching (2017-2018). Each administration of the inventory should take you approximately 30-45 minutes to complete. In addition, you will be asked to participate in two interviews during the year and each should last about 45 minutes at a site and time of your choosing. The observations will take place in a class of your choosing. The survey should take you about 20-30 minutes. In total, you will be asked for about 6 hours of time for the inventory and survey completion and interviews. Your name will not be identified in any way in the presentation of the research, all of your responses will be held in confidence, and a pseudonym will be assigned. All data will be stored in a locked filing cabinet in the researcher's private, locked office in Chafee Hall, or on a password protected computer.

Risks or discomfort:

There are no anticipated risks involved in participating in this study. Your name and other identifiers will not be used in any way in the presentation of the research and all of your responses will be held in confidence. It is not anticipated that you will experience any negative effects as a result of this study and <u>*participation, non-participation, or withdrawal from the study will not affect your employment or your academic standing in any way*</u>.

Benefits of this study:

Participating in this study will provide you with more insight into how you learn and develop as a teacher. In addition, you will be providing valuable information that may facilitate program changes that will better support and prepare future mathematics teachers for the transition to being a working professional.

Confidentiality:

Your participation in this study is confidential. None of the information will identify you by name or otherwise. All records will be saved in a password-protected file and pseudonyms will be assigned.

Decision to stop at any time:

The decision to take part in this study is entirely voluntary. You do not have to participate. If you decide to take part in the study, you may stop at any time. Whatever you decide will in no way penalize you or affect your grades. If you wish to stop, simply inform Dr. Cornelis de Groot, (401)874-4149, or Nicole Hersey, (401) 874-4165, of your decision. Upon your decision to stop participating in the study, all data gathered will be destroyed.

Rights and Complaints:

If you are not satisfied with the way this study is performed, you may discuss your complaints with Dr. Cornelis de Groot (401) 874-4149, or Nicole Hersey, (401)874-4165, anonymously, if you choose. In addition, if you have questions about your rights as a research participant, you may contact the office of the Vice President for Research and Economic Development, 70 Lower College Road, Suite 2, University of Rhode Island, Kingston, Rhode Island, telephone: (401) 874-4328.

You have read the Consent Form. Your questions have been answered. Your signature on this form means that you understand the information and you agree to participate in this study.

______________________________ ______________________________

Signature of Participant Signature of Researcher

______________________________ ______________________________

Typed/printed Name Typed/printed name

______________________________ ______________________________

Date Date

Your signature below means that you understand and agree to being audio recorded during the interviews.

______________________________ ______________________________

Signature of Participant Printed Name

Please sign both consent forms, keeping one for yourself

APPENDIX B: INITIAL DEMOGRAPHIC SURVEY

Thank you for participating in this research study. Please provide some background information. Your participation in this study is confidential. None of the information will identify you by name or otherwise. All records will be saved in a password protected file and pseudonyms will be assigned.

Background Information

1. Name ____
2. Age _______
3. Gender ________________________
4. High School Attended

   ____
5. Year of Graduation from High School ____________
6. Program Completed
   - Secondary Education & Mathematics
   - Elementary Education with Middle Level Extension in Mathematics
   - Other
7. Type of Program Completed
   - Undergraduate
   - Graduate
8. Please indicate if either apply to you:
   - Teacher Education Scholar
   - NOYCE Scholar

Where are you employed? ____

APPENDIX C: PCK INVENTORY INSTRUMENT

Item Mappings

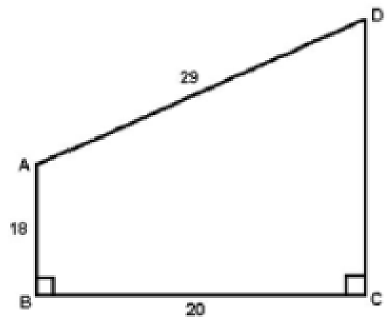
Domain	Tasks	Items
Knowledge of Content and Teaching	Design of Instruction	5, 6, 7, 8, 9, 12, 13, 14
	Sequencing of Topics	2, 9, 14
	Selection of Examples	3, 5, 8, 9, 12, 14
	Evaluate Different Representations of Topic	4, 9
	Use of Questioning	3, 6, 7, 12
Knowledge of Content and Students	Anticipate Student Thinking	5, 8, 14
	Anticipate Potential Areas of Confusion or Difficulty	2, 6, 8, 9
	Ways to Motivate Students	2, 4, 14
	Hear and Interpret Students' Thinking	1, 3, 6, 7, 11, 12, 13, 15
Knowledge of Content and Curriculum	Lateral Curriculum Vertical Curriculum	2, 5, 10, 11, 15
	Program/Instructional Materials	5, 13
Content Knowledge		1, 4, 6, 7, 8, 10

PCK Inventory Instrument Items

PCK Inventory Instrument Fall 2017-Spring 2018

Question 1

Consider the following calculation for the area of the trapezoid pictured.



$$A = \frac{h(b_1 + b_2)}{2}$$
$$A = \frac{18(20+29)}{2}$$
$$A = 441 \text{ sq. units}$$

* Describe the work shown by the student.

* Interpret the reasoning and ideas the student might have used in his/her problem.

* What feedback would you give the student?

* Describe how you would solve the problem to achieve the correct answer.

PCK Inventory Instrument Fall 2017-Spring 2018

Question 2

* Please indicate the order in which you would teach the topics listed below by numbering 1 (first) through 7 (last). Please indicate your rationale for the order you select.

- Special Right Triangles
- Unit Circle
- Graphs of Trigonometric Functions
- Reference Angles
- Relationship between Degree and Radian Measures of Angles
- Definitions of Trigonometric Functions
- Coterminal Angles

* Use this space to indicate your rational for the order you selected above.

* As you are planning a unit on trigonometry, what type of areas of confusion or misconception do you anticipate students having within this topic?

* What strategies or activities might you use to motivate students for this topic?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 3

Your student says the greatest common divisor of two positive integers is always greater than the least common multiple.

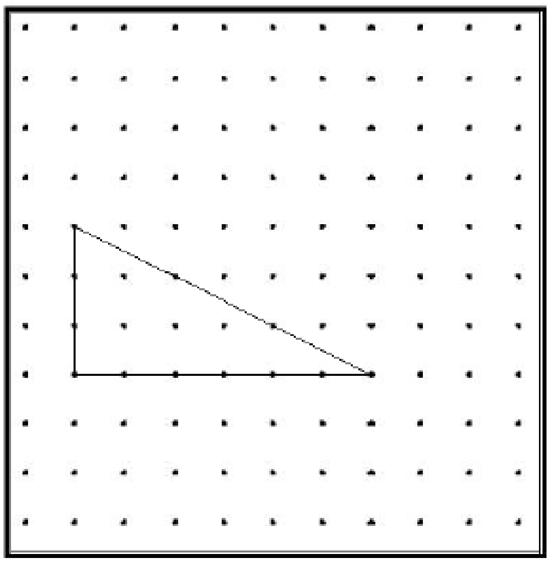
* Why do you think the student is making this conjecture?

* How do you respond?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 4

Your students have learned that there are multiple strategies for solving a problem and some methods come easier to them than others. Given the problem of finding the area of the triangle below,



do the following:

* List at least three different methods they could use to calculate the area.

Method 1:

Method 2:

Method 3:

* Which of the methods might be helpful for your more visual learners? Explain.

* What strategies or activities might you use to motivate students to explore multiple methods to calculate area?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 5

You have decided that it would be helpful for all your students to see a geometric representation of the algebraic relationship that for positive numbers *a* and *b*, $(a+b)^2 = a^2 + 2ab + b^2$.

* How will you accomplish this?

* When teaching multiplying with polynomials, what is or are the most likely way(s) students will approach this? What is the basis for this thinking?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 6

* When teaching equations with absolute values, what type of areas of confusion or misconception do you anticipate students having within this topic?

One of your students has solved the following absolute value inequality as given below and has concluded that there is no solution since, x > 5 and x < -1, and there are no such numbers that satisfy both inequalities.

$$|4-2x|<6$$
$$-6<4-2x<6$$
$$-10<-2x<2$$
$$5<x<-1$$

* What strategy would you use to help this student gain insight in his/her thinking?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 7

A student makes the following series of statements:

"Given $\log x^4 = 4\log 3$, it follows that $4\log x = 4\log 3$, hence $x = 3$."

Since we know that there is another solution, *x* = -3 :

* What is the error in the student's solution and how would you solve it correctly?

* How would you help your student understand what happened?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 8

Students often say that:

$$\frac{\log_c a}{\log_c b} = \log_c a - \log_c b$$

* What rule are they confusing it with?

* How would you convince a student that the identified misconception is, in fact, a misconception?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 9

* How do students often confuse functions and equations? How would you help them learn the difference?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 10

* How are geometric transformations connected with the concepts of congruence and similarity?

* What materials would you use in your instruction?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 11

Your students make the frequent error of saying that:

$\sin(A+B) = \sin A + \sin B$

* What prior experiences or knowledge might lead them to believe this?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 12

One of your students is asked to solve the equation $y = \tan^{-1} x$ for x in terms of y. He solves for x as follows:

$$y = \tan^{-1} x$$
$$y = \frac{1}{\tan} x$$
$$\tan y = x$$

* He checks the answer in the back of the book and sees he is right. How can you help him understand what is wrong with his work?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 13

Your student makes the following computations.

If the probability of getting a 1 on a roll of one die is 1/6, then the probability of getting two 1's on one roll of a pair of dice is 2/12, since each die can fall 6 ways for a total 12 ways and 2 of them are successes.

* Explain what is wrong with it and identify an activity that can help the student see his or her mistake.

PCK Inventory Instrument Fall 2017-Spring 2018

Question 14

You are teaching your Algebra I students how to solve equations with radical expressions. Below are examples of radical equations.

A

$$\sqrt{4x-3} = x$$
$$4x-3 = x^2$$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 3 \quad x = 1$$

B

$$\sqrt{x+3} = 5$$
$$x+3 = 25$$
$$x = 22$$

C

$$\sqrt{18-x} = x+2$$
$$18 - x = x^2 + 4x + 4$$
$$x^2 + 5x - 14 = 0$$
$$(x+7)(x-2) = 0$$
$$\cancel{x = -7} \quad x = 2$$

D

$$\sqrt{2x} = -8$$
no solution

E

$$\sqrt{2x-1} = 1$$
$$2x - 1 = 1$$
$$2x = 2$$
$$x = 1$$

* Please indicate the order in which you would use them and explain why.

- A
- B
- C
- D
- E

* What strategies or activities might you use to motivate students for this topic?

* What is or are the most likely way(s) students will approach solving radical expressions? What is the basis for this thinking?

PCK Inventory Instrument Fall 2017-Spring 2018

Question 15

A student is presented with the following statement.

“A rectangle is a parallelogram with at least one right angle.”

The student responds as follows: “Not true, it must have four right angles.”

* What might be the student's reasoning that prompts this reply?

* What prior knowledge might be useful to activate to help convince the student of the correctness of the statement?

APPENDIX D: TEACHER KNOWLEDGE SURVEY

Please indicate where you PRIMARILY learned each of the knowledge or skills items by indicating the letter of the experience next to the numbered items.

a) In my college *mathematics* classes
b) in my college general *education or licensure* classes
c) in my college *mathematics method or pedagogy* classes
d) during my *student teaching* experience
e) from my own *personal experiences* (e.g., as a student or tutor)
f) during my initial *teaching* experience
g) other; please specify

1. Evaluate the usefulness and appropriateness of mathematics curriculum materials for your students.
2. Help students become self-motivated and self-directed.
3. Use effective verbal and non-verbal communication strategies to guide student learning and behavior.
4. Use a variety of assessments (e.g., observations, portfolios, tests, performance tasks, anecdotal records) to determine student strengths and needs.
5. Maintain discipline and an orderly, purposeful learning environment.
6. Modify instruction, practice, dialog, and assessment for learners who require special education accommodations.
7. Modify curriculum to meet the need of English language learners.
8. Identify and address special learning needs or difficulties.
9. Address the needs of students who receive special education services.
10. Develop and select mathematics curriculum.
11. Use Internet and software for instruction.
12. Use the standards and objects of the Common Core State Standards in selecting curriculum to use for instruction.
13. Use the state's core curriculum and performance standards to plan instruction.
14. Teach mathematical representations, i.e., write variable expressions or

equations.

15. Teach connections among mathematical ideas, i.e., identify relationships between algebra and geometry.
16. Take into account students' prior understandings about mathematics when planning curriculum and instruction.
17. Use standardized mathematics assessments to guide your decision about what skills, concepts, and processes to teach.
18. Help students move from concrete understandings of mathematics to abstract understandings, i.e., teach student how to draw pictures of problem situations and then use the picture to write a mathematical expression or equation for the problem.
19. Help students use prior mathematical understandings to build new understandings, i.e., help student connect adding simple fractions to adding algebraic fractions.
20. Help students use comprehension strategies in mathematics to understand problems and make predictions.
21. Analyze student mathematical work to determine what the student understands or does not understand about mathematical concepts.
22. Explain the algorithm of "invert and multiply" for dividing fractions to students both pictorially and numerically.
23. Use problem or tasked based curriculum to develop mathematical understanding.
24. Explain simplification rules such as why $\sqrt{(x+y)^2} = |x+y|$ but that $\sqrt{x^2+y^2} \neq (x+y)$ in a manner that is accessible to secondary students.
25. Explain mathematics symbols in a manner that helps students understand their mathematical meaning, i.e., helping students understand the difference between 2x, x^2, and 2^x.
26. Explain why multiplying two negative numbers renders a positive product.
27. Explain the algorithm for an integral using area.
28. Explain the relationship between area models for multiplication, the standard algorithm for multiplication of multi-digit numbers and the distributive

property.

29. Explain why multiplication involving two fractions renders a product smaller than both factors.
30. Prove the quadratic equation.
31. Explain the difference between polynomial and exponential functions.
32. Explain graphing transformation rules (why does f(x-h)+k translate the graph of f(x) k-units vertically and h-units horizontally).
33. Explain why one would want to convert rectangular coordinates to polar coordinates or polar coordinates to rectangular coordinates.
34. Prove fundamental trigonometric identities ($1+\tan^2 x=\sec^2 x$).

APPENDIX E: INITIAL INTERVIEW QUESTIONS

Interview 1: October 2017 during first semester teaching

1. Tell me about your experiences in teaching.
2. What are some of the experiences that were influential in developing your understanding of how to teach mathematics?
3. What are some obstacles or difficulties you experienced in developing your understanding of how to teach mathematics?
4. How does the way you think mathematically compare to the ways the students you have worked with so far think?
5. How would you teach mathematics if you had free reign?
6. In your experiences what instructional methods work best for middle and high school students?
7. At this moment in your teaching career what do you feel ready/prepared for? What about what you do not feel ready/prepared for?
8. What do you believe of what you learned from both your mathematics and education courses will be most useful to you in future experience?
9. Is there anything you are concerned about in work this year?
10. Is there anything you are concerned about in your future work?
11. Is there anything you feel missing in your mathematics and education coursework you completed, including practicum experiences?
12. Are you learning new ways of thinking about mathematics from your colleagues?
13. Thank you for all that valuable information, is there anything else you'd like to add before we end?

Interview 2: April 2018 during second semester teaching

1. Tell me about your experiences in teaching in your first year.
2. What are some of the experiences that were influential in developing your understanding of how to teach mathematics?
3. What are some obstacles or difficulties you experienced in developing your understanding of how to teach mathematics?
4. How does the way you think mathematically compare to the ways the students you have worked with so far think?
5. What would you have changed about this year in terms of your teaching?
6. In your experiences what instructional methods work best for middle and high school students?
7. At this moment in your teaching career what do you feel ready/prepared for? What about what you do not feel ready/prepared for?
8. As your first year of teaching comes to a close, what are you most looking forward to in the future?
9. Is there anything you are concerned about in your future work?
10. Are you learning new ways of thinking about mathematics from your colleagues?

Thank you for all that valuable information, is there anything else you'd like to add before we end?

APPENDIX F: SAMPLE TRANSCRIPT AND CODING

In this appendix is a sample of Interview 1 with Kara: page 5 line 14 through page 9 line 23. The line numbers of this excerpt do not match to the transcript due to changes in formatting.

N: What are some of the experiences that you have had that were influential in developing your understanding of how to teach?

K: In understanding how I'm going to teach?

N: Yes, how you're going to teach.

K: Hmmm…I don't know. I think….just as I've decided that I wanted to do education and even my classes at college, watching how other people teach and I think I've always kind of done it because I've always kind of known that I wanted to be a teacher. Like when another teacher does something, I make a mental note if I like that or if I don't like it so I think experience with different teachers and being exposed to different teaching styles or different methods or just ways they do things. I think I say if I like it or not and that's how I'm building how I'll have my own classroom.

Commented [NH1]: Experiences- Personal Learning

Watching teachers- Seeing

N: How does the way you think mathematically compare to the ways the students think mathematically?

K: I mean math always came kind of easy to me but also not…I don't think I'm as strong mathematically as kids would think I would be, being a math teacher they think you're an expert at math and I don't think that I am an expert at math. I think I do…I can be relatable at that and tell them that I struggled with this too or things like that. But it definitely has come more easy to me whereas I know some of my friends can't understand it so I think there's a difference there obviously but I think I'll understand, like I understand that because I've had friends that are like that and I've tried to help them through. I think the pace that they get it but I think with persistence…I believe that everyone can understand math, you just need someone there to help you.

Commented [NH2]: Experience- Personal Learning

Commented [NH3]: CK- not confident

Commented [NH4]: Connection- with students

Commented [NH5]: Experience- working with "students"

Commented [NH6]: Make a difference

Kara-Interview1- Page #

N: How would you teach math if you had free reign?

K: Ummmm….well I think I'd go a lot slower and not introduce topics so fast because I feel like that's where a lot of kids get lost because the second they don't know something it just builds on it and they're just lost for the rest of the year. I think that if everything was a lot slower and I could just individually make sure that every student really does understand it and then move. I mean that could take forever but, I mean, if I had free reign I would definitely do it a lot slower and build on topics very slow just to make sure everyone has a great understanding of the basics before they move on. I also feel like in my high school class, some student don't just know very basic skills so then they get confused when they're doing something. It just makes everything else harder. I think if they had more time to really lock in those skills then that would be how I would do it.

Commented [NH7]: Pace

N: In your experiences, what are some instructional methods work for middle school and high school students?

K: Ummm…. For my middle school, my teacher doesn't do a lot of lecturing, she does a lot of activities and I think they learn really well from that. For instance, I just made a dividing decimal game, so it was like a board game and you all did the division and if you got it right you got to move on the board. And as I walked around I could tell students were catching their own mistakes and understanding why they were doing it wrong. I think letting students work on it instead of just telling them information and then having them write notes, I think having them really practice in a fun way instead of doing timed practice or anything. I think having them practice in a fun way is a good skill. And then, my high school classes….I guess they do have to

Commented [NH8]: Instruction- Student-centered
Seeing it
Experience- working with students

do a little more lecturing, I've seen that more. I feel like group work also works well because they can discuss how they're doing and, then the same thing, find out what they're doing wrong and figure out how to fix it with each other.

Commented [NH9]: Instruction- Lecturing

Seeing it

Commented [NH10]: Instruction- Student-centered

N: At this moment in your teacher preparation program what do you feel ready or prepared for?

K: What do I feel ready or prepared for? Ummm….I feel….ready for…to have my own students and have that connection with that. I feel like I'm more of a personable person where I can get along, like I love going to my practicum and just talking to the students and stuff and working with them. So I like the personal connections and I feel like I'm ready to have…so I just talked to a girl who just graduated last year and she said it's so nice having your own classroom because she kind of sets the tone and everything and has those connections with her students and they're like *her* students, it's not like you're working with someone else's. I'm very ready and excited for that aspect of it and just to have it all my way, and kind of run it and organizational, and very like, I don't know. I like to have my own way so I'm excited for that and for my classroom to set up everything and have my own classroom really.

Commented [NH11]: Connections- with students

Commented [NH12]: Instruction- Implementing own ideas

N: So what don't you feel ready or prepared for?

K: Umm…I feel….I guess I'd say I feel less prepared for maybe the content of it, especially in a high school classroom. I feel like sometimes, when I'm watching my cooperating teacher right now, I mean so far what he's done I'm pretty good at but some of the other classes I've been in…I don't know if I, I mean I know it but sometimes it just takes me a little more time to go over it. So, I guess, but at the same time, I feel like if I do…I mean if I'm going to teach a lesson I'm going to go into it prepared, like I'm going to take that time for myself but I feel like, I

Commented [NH13]: CK- unsure of

guess, just being ready for being up in front of a class and maybe having them ask me a question and not know the answer or not really know what I'm doing or just messing up and having to look at the board and being like hold on, where did I go wrong. But even in my practicum right now, my teacher messed up and he like looked at me and was like "what did I do wrong?" and I couldn't figure it out either so I do know that teachers still make mistakes too, I just think, I guess, content-wise like really nailing down the lessons and what needs to be learned by them exactly. So I guess the content-ish.

N: So what do you believe of what you learned from your math and education courses will be most useful to you in future experiences?

K: Hmm…most useful…ummm….I want to say classroom management but I guess it's because I'm in my classroom management class right now [laughs] so it's most fresh in my mind. But, I feel like learning a lot of those techniques and skills, especially if I want to be in a middle school classroom where they're still kind of getting adjusted and growing up and figuring all that out. I think those are really important skills. At the same time, I feel like practicing in my methods class, like writing inquiry lesson plans and practicing with the common core skills, I feel like those are good skills to have too so I can work on building my content knowledge and figuring all that out.

Commented [NH14]: Instruction- Whole Group

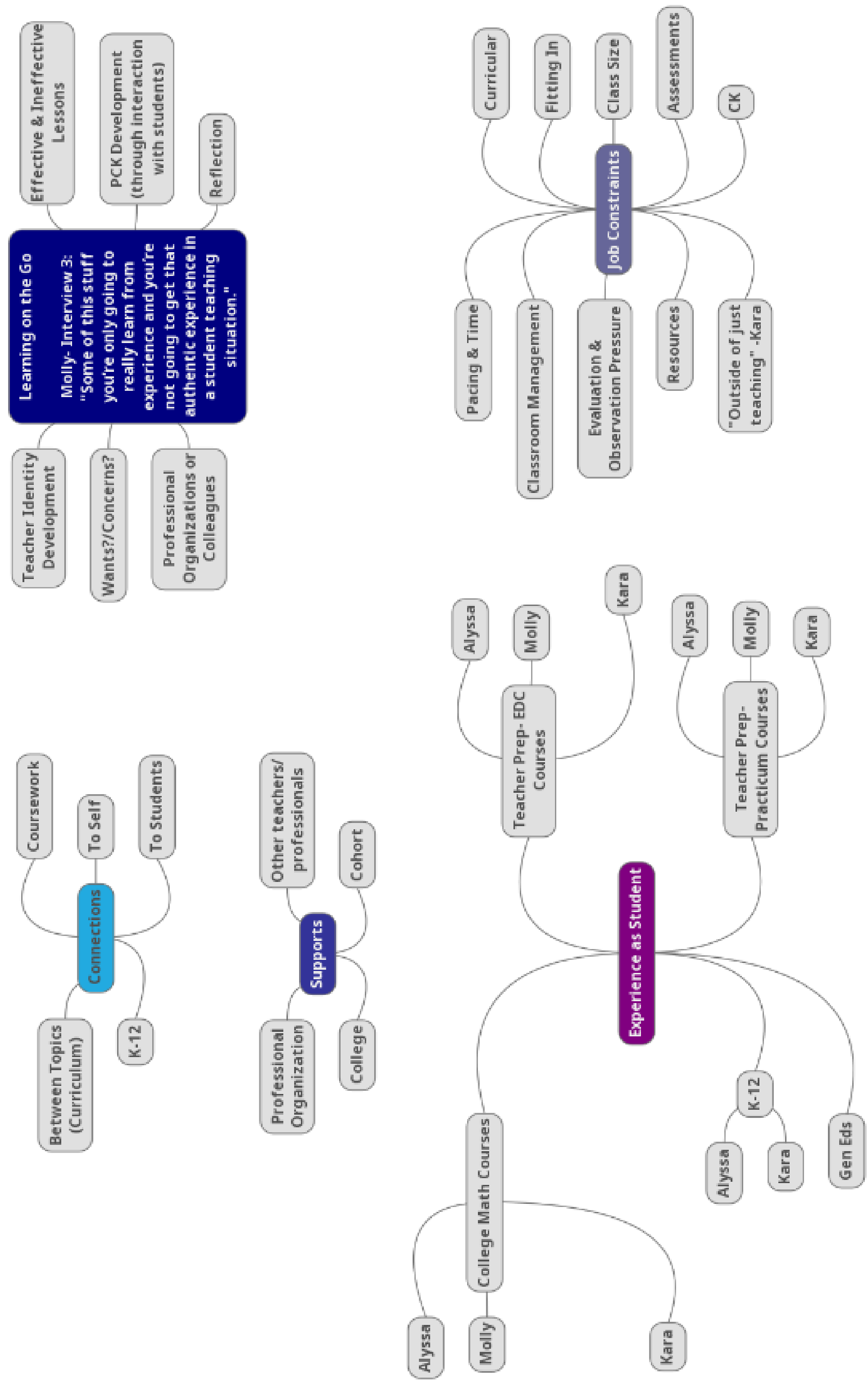
Commented [NH15]: CK- Unsure of

Commented [NH16]: Classroom Management- coursework

Commented [NH17]: Instruction- lesson planning

CK- building

APPENDIX G: SAMPLE OF MINDMUP ANALYSIS



APPENDIX H: SAMPLE PARTICIPANT CHECK

Email correspondence with Molly

Thursday, March 29, 2018

Hi Molly,
I was reanalyzing my transcripts from our interviews and I came across a passage from our first interview together at the start of your senior year. I condensed it into a poem of sorts and wanted to know your thoughts. I hope everything is going well and I can't wait to visit you at your school again!

Concerned
first 2 years
everyone always says they're going to be hard.

Nervous
figure out where I belong
how my philosophy fits into the philosophy of the school.
I know I'll figure it out.
I know it'll be fine.

Nervous
stigma
first 2 years
building resources
stressful.

Once I get my feet wet
find out where I am
I'm going to be really happy.

Let me know what you think!

Friday, March 30, 2018

Hi Nicole,
This is awesome, and means so much to me that you did this! It's very interesting to see all those main points of things I used to feel and compare them to how I'm feeling now, a lot of similarities! This beautifully sums it all up! Thank you so much for sharing with me. Can't wait for your second visit! March has been ROUGH but I still have a pulse so that's really all I can ask for! Messages like this bring my head back to where it needs to me :).

Email correspondence with Alyssa

Tuesday, July 24, 2018

Hi Alyssa,
I just had a quick follow-up question to something you said during our most recent interview. What populations do you think your practicum experiences prepared you to with?

Thank you!

Tuesday July, 24, 2018

Hi Nicole,
My elementary experiences somewhat prepared me for special education but middle school did nothing to further that knowledge. It did not prepare me to work with English learners (ELs) either.

Let me know if you have any other questions!

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