Design of an All-Optical Long-Distance Solition-Based Optical Fiber Communication System Using Flouride Fibers

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DESIGN OF AN ALL-OPTICAL LONG-DISTANCE
SOLITON-BASED OPTICAL FIBER COMMUNICATION SYSTEM
USING FLUORIDE FIBERS

BY
HATEM A.H. ABDELKADER

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
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ABSTRACT

The stability of optical solitons amplified periodically by stimulated Raman scattering process in heavy fluoride fibers, is numerically studied for a range of parameters using computer simulation. An optimum single-mode fluoride fiber was used to design an all-optical soliton based optical fiber communication system. The value of the chromatic dispersion is 0.6 psec/nm/km with acceptable manufacturer tolerances. The length-bandwidth product is four times that of a soliton-based optical communication system using present day silica fibers. In this design we numerically justify why we neglect the higher order terms in the Taylor series expansion of the propagation constant around the pulse central frequency.

The single channel bandwidth-length product for bit error rate (BER) less than $10^{-9}$ is nearly 120,000 GHz.km. Typical amplification periods are in the range of 200-300 km; average soliton and pump power are in the range of milliwatts and hence well within the capability of semiconductor lasers.
ACKNOWLEDGEMENTS

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I also thank Professor A. Lindgren for his generous support throughout my graduate education. Also I would like to thank S.Bastein for the use of the computer programs he developed for fluoride fiber design.

Finally, I wish to express my appreciation to my father, mother, sister, brother and friends for their continuous support and encouragement.
This thesis is prepared in accordance with the "MANUSCRIPT THESIS PLAN" as described in the Guidelines for Thesis Preparation, Graduate School, University of Rhode Island, 1988. Section one contains the manuscript which is prepared in the format for publication in professional journals.

Section two contains one appendix. This appendix is the mathematical formulation and derivation of the problem which was solved. A complete list of the references cited is also included.
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1 Introduction

In present-day optical fiber communication systems (OFCS) operating at many Gbits/s, the bit rate between regenerators is limited by the chromatic dispersion characteristics of the fiber material and its design. For a single mode fiber, which has intrinsically higher bit rate, the chromatic dispersion is the deterministic factor in influencing the rate of pulse spread, hence the channel capacity.

A natural choice to minimize the pulse spread is to carefully tune the laser wavelength to the wavelength at which the chromatic dispersion passes through zero. Nevertheless still pulse spreading occurs due to the higher order effects in the dispersion, also thus far it has been proven difficult to control the wavelength match between the laser diode output and the zero chromatic dispersion sufficiently well for this scheme to achieve its full potential.

Also, optical signals are detected and electronically repeated every 20-100
km before continuing along the next fiber span, electronic repeaters can limit bit rate of transmission to a few Gbit/s, which means that only a tiny fraction of the tremendous information carrying capacity of the single mode optical fibers is used. Thus the only sensible way to overcome electronic rise time limitations is by ensuring that the signal remains strictly optical in nature throughout the system.

Hasegawa suggested [1] a high bit rate all-optical communication system could be created through the combined use of loss compensating Raman gain [2] and solitons [3], [4]. Numerical studies [5] showed that solitons can indeed be made to propagate over long fiber spans with little change in shape, when Raman gain is used to compensate for energy loss. Experimental demonstration of soliton propagation over short fiber lengths was reported [6] using Raman gain to exactly compensate fiber intrinsic loss.

Fig1 shows a schematic of the system configuration. CW Raman pump light is injected periodically into the fiber through wavelength dependent directional couplers spaced L(km) apart. The pump wavelength is determined from the signal wavelength, which is normally chosen to coincide with the wavelength of minimum predicted loss, and the frequency shift that corresponds to the peak of the broad band Raman gain. For fluoride fibers the minimum loss occurs at $\lambda = 2.55 \mu m$ and the peak Raman gain shift occurs around $580 \, cm^{-1}$ [7], hence the pump
wavelength will be 2.22 µm.

2 Soliton theory

The single mode optical fiber has small core diameter (about 10 µm). Since light can propagate over long distances, it is possible to observe nonlinear phenomena in fibers at power levels much smaller than that required for bulk media. Nonlinear effects in single mode fibers have many important applications. Among these are the generation of new frequencies through the Raman process, the compression of optical pulses and the all-optical soliton based optical fiber communication system.

A light propagating through a single mode fiber of length $L$ and refractive index $n$ will acquire a phase shift $\phi = \frac{2\pi}{\lambda} n L$. If the refractive index is intensity dependent, $n(I) = n_0 + n_2 I(r,t)$, $n_2$ is the nonlinear index coefficient, $r$ is the radial distance, $t$ is time and $I$ is the input intensity, and if the input beam has an intensity modulation, the transmitted light will exhibit a temporally varying phase given by [8]

$$\Delta \Phi = \frac{2\pi L}{\lambda} \delta n$$  (1)
\[ \delta n = \frac{1}{2} n_2 E^2 \]

\[ E^2 = 10^7 \left( \frac{8\pi p(t)}{ncA_{eff}} \right) \]

Where \( n_2 \) is in esu units, \( E \) is the peak field amplitude in cgs units, \( p(t) \) is the transmitted power in watts, \( \delta n \) is the change in the refractive index \( n \), \( c \) is the velocity of light and \( A_{eff} \) is the core effective area of the fiber [see Appendix A]

Thus self phase modulation (SPM), can be regarded as a conversion of amplitude modulation to phase modulation through the non-linear refractive index. The instantaneous frequency shift at any point in the pulse is given by the time derivative of the phase modulation \( \omega - \omega_0 = \frac{\partial}{\partial t} (\Delta \phi) \). According to the rate of change of the input light with respect to time, the instantaneous frequency shift could be either positive or negative, which means that some points in the pulse will be accelerated while others will be retarded with respect to the pulse center. This leads to frequency broadening (chirp) and consequently pulse spreading, which limits the bit rate of the system.

Group velocity chromatic dispersion causes pulse broadening, in particular, a gaussian pulse of duration 5 psec will double its width after propagating a distance
of about 800 m if the value of the dispersion parameter is 16 psec/nm/km at wavelength of 1.5 µm. The group velocity dispersion \((\partial v_g/\partial \lambda)\) may be positive (normal) or negative (anomalous), depending the fiber composition, waveguide geometry and light wavelength.

The presence of group velocity dispersion regardless of its sign, will always lead to broadening of the transform limited pulse. If the pulse was chirped, however, then pulse compression can occur if the dispersion has the correct sign to reverse the chirp. The frequencies in the leading half of the pulse are lowered, while those in the trailing half are raised. When the dispersion is negative, the group velocity increases for increasing frequency, thus the trailing half of the pulse containing the higher frequencies is advanced, while the leading half containing the lowered frequencies is retarded. The pulse then tends to collapse on itself. Thus the combined effects of non linearity and negative group velocity dispersion achieves pulse narrowing. It also becomes possible to have the propagation of the so called "solitons", pulses that either does not change shape or have shapes that change periodically with propagation along the fiber.

Although the above argument can explain simple narrowing, it is not sufficient to account for pulse shaping effects and soliton behavior. To correctly predict the latter, one must solve the non-linear Schrödinger equation, which governs the evo-
3 Optimum fluoride fiber design

Because of their ultra low loss in the 2-5 μm region, multicomponent heavy metal fluoride glass fibers appear to be a serious contender as the transmission medium of future long-haul telecommunication networks. It remains to be seen whether this loss can be achieved in practical fibers. The best loss achieved at 2.55 μm is approximately 1 dB/km whereas the predicted minimum is 0.03 dB/km.

As will be indicated, for the sake of pulse stability, it is desirable to have values of the soliton characteristic period \((Z_0)\) to be a good fraction of the amplification period length \((L)\). Since the pulse width scales as \((Z_0D)^{1/2}\), it is necessary to reduce the fiber dispersion parameter \(D\) to the smallest possible value so as to increase the bit rate of the system. This represents the first criterion in the fiber design.

As we have explained earlier, soliton propagation in fibers is only possible in the region of negative group velocity dispersion \((\partial v_g/\partial \lambda \leq 0)\). This is the second criterion.
Using the programs developed in [9] to optimize fluoride fiber performance in the 1.7 - 5 μm for a ZBLAN composition, with a simple step refractive index profile and the proper values for the refractive index difference (Δ) and core radius (a); we were able to tailor the shape of the fiber dispersion parameter (D) as a function of wavelength (λ) and obtained the best possible fiber design for a soliton-based system. In that design:

1- D is minimized to the lowest possible value allowing for tolerances in the manufacturing techniques.

2- The position of the zero crossing of the D vs. λ curve occurs at λ = 2.48 μm. This means at λ = 2.55 μm, which corresponds to the signal wavelength, the group velocity dispersion is negative. This is an essential condition to reverse the pulse chirp produced by self phase modulation, hence pulse compression and soliton propagation is possible. On the other hand, at λ = 2.22 μm, which corresponds to the pump wavelength, the group velocity dispersion is positive, and hence solitons cannot be produced at this pump wavelength.

3- The slope of the D vs. λ curve at λ - 2.55 μm is small. This means that the third order term in the Taylor series expansion of the propagation constant k around the pulse or soliton frequency, and consequently the other higher order
terms, can be neglected. We numerically verified, using the calculated values of the second and third derivatives of the propagation constant, that the ratio of the third order to the second order term in the Taylor series expansion of the propagation constant around the pulse central frequency is less than $10^{-3}$.

It is important to note that:

1- The values of the first, second, and third derivatives of the propagation constants were not obtained by numerical differentiation of the propagation constant, which is calculated by solving the scalar wave equation and matching the boundary conditions of the tangential fields. We obtained these by the method developed by Sharma [10], which transforms the scalar wave equation into three first order differential equations that are solved using the fourth order Runge-Kutta algorithm [9]. This is more accurate than calculating the derivatives of the propagation constant by numerical differentiation.

2- It is well known that the fractional changes in $D$ resulting from variation in $(a)$ or $(\Delta)$ during fiber manufacture, undoubtedly becomes large in the limit of small $D$, however, sensitivity analysis was done on this design. With $a = 3 \, \mu m$ and $\Delta = 0.95 \%$, it was found out that variation in $(a)$ $\pm 1\%$ will change $D$ by $\pm 0.3$ psec/nm/km, while a change in $(\Delta)$ $\pm 10\%$ can lead to change in $\pm D = 0.15$
Fig 2 shows the transverse field distribution of that specific fiber design. Fig 3 shows the fiber dispersion parameter D versus λ, the value of D at 2.55 µm is 0.6 psec/nm/km and the crossover point occurs at λ = 2.48 µm.

4 Raman gain and pulse energy

Optical solitons can propagate free of distortion in a fiber with group dispersion, however the pulse width increases in the course of propagation because of loss [3] in the soliton power due to fiber attenuation.

If the Raman amplification is distributed such that it exactly compensates for the fiber loss, the soliton can propagate free of distortion practically for an unlimited distance. Although the Raman gain cannot be kept constant because of the fiber loss and pump depletion, the amplification process is still adiabatic, hence allowing the soliton to keep its characteristic property as a soliton (width × amplitude) = constant.
In contrast, if the amplification is made locally, only the amplitude is increased (without reducing the width) and the soliton radiates away excess energy in the form of dispersive waves \cite{11}. In this case it has been found necessary to amplify the soliton again before the dispersive waves escapes from the soliton. This means that the amplifier spacing is determined by the distance controlled by the group velocity dispersion.

Since the Raman gain is non uniform with each period L, the corresponding effective gain(loss) coefficient and pulse energy variation must be established first before any further calculations of the soliton properties.

Consider soliton pulses at wavelength $\lambda_s$ propagating down a fiber span of length L, while subject to the simultaneous effects of fiber loss and bidirectional CW Raman pumping at wavelength $\lambda_p$. The loss(gain) is given by

$$\frac{dE}{E} = (-\alpha_s + \alpha_g)dz = \alpha_{eff}dz \quad (2)$$

$\alpha_s$ is the fiber loss coefficient (km$^{-1}$) at the soliton wavelength, $\alpha_{eff}$ is the effective fiber loss coefficient and $\alpha_g$ is given by:
\[ \alpha_g = R(e^{-\alpha_p z} + e^{-\alpha_p(l-z)}) \]  

\[ R = g \frac{P_p}{A_{eff}} \]  

\( \alpha_p \) is the fiber loss coefficient \((km^{-1})\) at the pump wavelength, 

\( R \) is the Raman gain factor \((km^{-1})\), 

\( g \) is the Raman gain coefficient \((cm/W)\), 

\( P_p \) is the Raman pump power \((mW)\), 

and \( A_{eff} \) is the effective area [see Appendix A] of the fiber core \((\mu m^2)\).

The Raman gain coefficient \((g)\), for fluoride fibers, is between \(1 - 3 \times 10^{-11}\) \(cm/W\) around 1.0 \(\mu m\) [12], since \(g\) is inversely proportional to \(\lambda\), then \(g\) at \(\lambda_p = 2.22 \mu m\) will be approximately \(0.9 \times 10^{-11}\) \(cm/W\).

If we choose \(\alpha_g\) such that the pulse energy, at the end of one amplification period \(L\), is equal to the pulse input energy \(E_0\), then \(\alpha_g\) becomes

\[ \alpha_g = \frac{\alpha_s \alpha_p L}{2 \sinh(\alpha_p \frac{L}{2})} \cosh[\alpha_p(z - \frac{L}{2})] \]
Using (2) and (5) and integrating we obtain

\[ \ln\frac{E}{E_0} = \alpha_s L \left[ \frac{\sinh(\alpha_s (z - L/2)) + \sinh(\alpha_p L/2)}{2 \sinh(\alpha_p L/2)} - \frac{z}{L} \right] \]  

(6)

Fig4 and Fig5 show the quantities \( \alpha_s, \alpha_g, \alpha_{eff} \) and E all graphed as a function of \( Z \) for a period of 250 km and for assumed loss coefficients \( \alpha_s = 0.03 \) dB/km (0.007 \( km^{-1} \)) and \( \alpha_p = 0.06 \) dB/km (0.014 \( km^{-1} \)). These loss coefficients have not yet been obtained due to manufacturing difficulties. However the theoretically predicted values are in good agreement with the above mentioned figures. Fig6 shows the normalized pulse energy as a function of distance along one amplification period span, L is the parameter and takes the values of 200, 250 and 300 km. Note that fluctuations increases with increasing L.

5 Soliton propagation in a lossless medium

The envelope \( A(z,t) \) of the electric field \( E(z,t) = A(z,t)\exp(i(kz - \omega t)) \) in an optical fiber satisfies the following dimensionless non linear Schrödinger (NLS) equation [see Appendix A].
The first term on the right hand side describes the effect of dispersion while the second term describes the effect of non linearity. In the absence of these two terms the left hand side alone would describe the distortionless propagation of the pulse envelope, thus one can imagine that when non linearity balances dispersion similar distortionless propagation might occur.

If a pulse shape of the form:

\[ A(\xi = 0, s) = N \text{sech}(s) \]  

is inserted in eqn.(7), then it will propagate as a pure soliton for integer \( N \) (which we refer to as the order of the soliton). Explicit expressions for \( A(\xi, s) \) have been obtained for \( N = 1, 2 \) [13],[14] and these are

\[ A_1(\xi, s) = e^{-i\frac{\xi}{2}} \text{sech}s \]

\[ A_2(\xi, s) = \frac{4e^{-i\frac{\xi}{2}}[\cosh(3s) + 3e^{-4i}\cosh(s)]}{\cosh(4s) + 4\cosh(2s) + 3\cos(4\xi)} \]
We see that $|A_1|^2 = \text{sech}^2 s$ is independent of $\xi$, however, this is not true for $|A_2|^2$ which develops periodical structure in $\xi$ with period $\pi/2$. This, in fact, is the same for all higher order solitons ($N \geq 2$).

We can also generate a continuous set of solutions from these by using the invariance of the NLS equation under the following transformations:

\begin{equation}
\begin{align*}
A &= \beta^{-1} \tilde{A}, \\
\xi &= \beta^2 \tilde{\xi}, \\
s &= \beta \tilde{s}
\end{align*}
\end{equation}

i.e. $A(\xi = 0, s) = N \beta \text{sech}(\beta s)$ is also a soliton.
6 Non-linear Schrödinger equation and numerical solution

For systems with finite losses, or for higher order solitons \( N \geq 2 \), there is no analytical solution for the NLS equation. Numerical methods have to be used and loss has to be treated as a perturbation. The dimensionless NLS equation including the loss term takes the following form

\[
-i \frac{\partial A}{\partial \xi} = \frac{1}{2} \frac{\partial^2 A}{\partial s^2} + |A|^2 A - i \Gamma A
\]

where \( \Gamma \) is the amplitude gain(loss) coefficient, \( \xi \) and \( s \) are the dimensionless versions of real world quantities \( Z \) and \( t \) respectively.

To make \( \Gamma \) consistent with the definition of \( \alpha_{eff} \), as in eqn. (2), the sign of the first and last term in (12) should be the same, such that positive \( \Gamma \) applies to gain and negative \( \Gamma \) applies to loss; the quantities \( \xi \) and \( \Gamma \) are related to their real world counter parts [see Appendix A] by the following

\[
\xi = \frac{Z}{Z_c}
\]
The factor 2 in eqn. (14) converts $\Gamma$ into an energy coefficient. Unit propagation of dimensionless length ($\xi = 1$) corresponds to the real world quantity $Z_c$, "the soliton characteristic length". For the fundamental soliton it is given [see Appendix A] by the following

$$Z_c = 0.322 \frac{2\pi c \tau^2}{\lambda^2 D}$$

(15)

where $\tau$ is the full width at half intensity maximum (FWHM) of the pulse and $D$ (psec/nm/km) is the fiber dispersion parameter.

To avoid the appearance of the factor $\pi$ in some formulae we shall more often use the closely related quantity $Z_o$, known as the soliton period, and given by

$$Z_o = \frac{\pi}{2} Z_c$$

(16)

Fig. 7 shows $D$ vs. $Z_o$ in the region of interest for different values of $\tau$. The best measure of transmission fidelity is the pulse area $S$, defined as the integral of the absolute value of the amplitude envelope with respect to time, but here for convenience divided by $\pi$. In the appropriate dimensionless units, $S=1$ for a fundamental
soliton and deviation of $S$ from unity indicates the generation of a non-soliton component of the pulse.

Let $\Lambda$ be the dimensionless amplification period, then from eqn.(13)

$$\Lambda = \frac{L}{Z_c}$$  \hspace{1cm} (17)

To obtain pulse behavior in the fiber with gain (loss), we have solved eqn.(12) numerically on a DG/Unix computer. Each solution corresponds to a set of values for the parameters $L$, $Z_0$ (or $Z_c$), $\alpha_s$ and $\alpha_p$. Once these quantities have been chosen, $\Gamma(\xi)$ was computed using eqns. (14),(2) and (5). With the $R$ (Raman gain factor) adjusted such that the pulse energy is conserved at the end of each amplification period, $\Gamma(\xi)$ was then inserted in the the NLS equation (12) which is then ready to be solved numerically.

The NLS equation is a parabolic, initial value, boundary value problem in one space variable. It is a well posed problem if the initial condition, $A(\xi = 0, s)$ and the two boundary conditions, are known. Note that the initial condition is related to the space variable and the boundary conditions are in time variables.
The initial condition is \( A(\xi = 0, s) = \text{sech}(s) \), which is the fundamental soliton, and the boundary conditions are periodic, i.e. the envelope function and its first derivative are continuous at the boundaries.

The pulse envelope amplitude wings are truncated at \(|s| \geq 10\) where it reaches a negligibly small value of approximately \(9 \times 10^{-5}\), i.e., the boundaries are at \( s = \pm 10\). With the boundary defined, initial and boundary conditions determined, the solution proceeded over a span of \( \xi \) corresponding to one or more amplification periods \( \Lambda \).

The NLS equation was solved numerically, in the complex plane using the FRANKEL-DUFORT 3-level finite difference scheme \([15]\). The first level only was obtained using a 2-level explicit scheme, the second level was obtained using the first level and the initial condition, the third level was obtained using the second and the first levels and so on. Each space level consisted of 2000 points, spaced apart by 0.01 (step in dimensionless time units), propagation in space was done by increments of \(10^{-4}\) (step in dimensionless space units) to increase the computational efficiency. One space level (2000 points) takes about 0.1 seconds of CPU time.

Reference \([15]\) shows that this scheme is stable for any value of \( r = k/h^2 \) since the absolute value of the eigenvalues of the associated block tridiagonal matrix
are less than or equal to one. In our case $r$ is taken to be one, since for large values of $r$ the solution becomes less accurate.

The scheme stability and convergence were tested numerically by changing the increments in both time and space continuously and comparing the results until no significant change occurred in the solution.

Errors were incurred from truncation of the input pulse's wings $[A(\xi, s) = 0 \text{ for } |s| \geq 10]$. Also the local truncation error of this scheme is proportional to $h^2$ and $k^2$ in time and space respectively and from approximating the derivative of the solution at $s = -10$ and $s = +10$ by a forward and a backward difference scheme respectively. However, contributions from all these error sources are negligible because the value of the function (soliton envelope) is already very small at $|s| \geq 10$. Also the value of $k$ and $h$ were originally chosen to be small (0.01 and 0.0001 respectively) and hence $k^2$ and $h^2$ are even smaller.

Before proceeding with the calculations, the following checks on self consistency and accuracy were done:

1- With $\Gamma(\xi)$ temporarily set to zero and for $\lambda = \frac{\pi}{2} (L = Z_o)$, the program was run for an input function of $2 \text{ sech } (s)$ which is the second order soliton. Since the
pulse width changes by more than a factor of 4 at midpoint \( \xi = \pi/4 \), the \( N = 2 \) soliton constitutes a severe test on the program's ability to handle efficiently a rapidly varying pulse. The numerical solution matched the analytically known expression eqn. (10) at the midpoint. Figures 8 to 16 show some numerically obtained solutions for the NLS equation for \( N = 2,3 \) at different distances of propagation.

At each eighth of a span, the computation was halted and a subroutine was called to calculate the following quantities, from the dimensionless amplitude \( A(s) \): the area \( S \), intensity profile \( I(s) \), pulse peak intensity and the energy \( E \) of the pulse calculated by integrating \( I(s) \) with respect to \( s \). \( S \) and \( E \) were obtained numerically by integrating the absolute value of \( A(s) \) and \( [A(s)]^2 \) respectively with respect to \( s \), where \( A(s) \) is complex. Integration was done by calling a subroutine that interpolates the points using cubic splines then; calling another subroutine that does integration using gaussian quadratures. Another check on the validity and consistency of the numerical computation is:

2- The pulse energy computed as above from the solution itself was always in excellent agreement with pulse energy given by eqn. (6). This agreement is an independent and sensitive test on the solution. Table (1) shows this fact for \( L = 250 \) km, \( \alpha_s = 0.007 \) km\(^{-1} \) and \( \alpha_p = 0.014 \) km\(^{-1} \).
7 Soliton power, Raman pump power and pump depletion

7.1 Soliton power

In real world quantities, the fundamental soliton peak power $P_1$ is given by [see Appendix A]

$$P_1 = \frac{A_{\text{eff}} \lambda}{4n_2 Z_0} \quad (18)$$

Fig 17 shows the peak power of the fundamental soliton vs. $Z_0$ for various values of $A_{\text{eff}}$.

7.2 Pump power and pump depletion

In ordinary single mode fibers, the relative pump and signal polarizations change rapidly with propagation. This makes the average gain $\alpha_p$ just half of its value given by eqn. (3). Also in a bidirectional pumped system, the quantity $P_p$ represents the sum of the contributions from both ends of the fiber span; thus modifying eqn. (3) we obtain
\[ P_p = \frac{2A_{\text{eff}}}{g} \frac{\alpha_g(L,0)}{1 + \exp(-\alpha_p L)} \]  

(19)

where \( \alpha_g(L,0) \) is the value of \( \alpha_g \) required at either ends of each amplification period \( L \). Fig 18 shows the CW pump power for unity gain over the amplification period \( L \) in a fiber with \( A_{\text{eff}} = 75 \mu m^2 \), \( \alpha_s \) and \( \alpha_p \) as were given earlier.

There is an additional attenuation (depletion) of the pump power brought about by the amplification process. That is, one pump photon is lost for every signal photon acquired through the Raman amplification process. From that basic equation one can calculate the additional loss coefficient \( \Delta \alpha_p \) from

\[ \Delta \alpha_p = g \frac{\lambda_s \, P_a}{\lambda_p \, A_{\text{eff}}} \]  

(20)

where \( P_a \) is the average signal power and is given by

\[ P_a = \frac{1.13 \, \tau}{2 \, T} P_1 \]  

(21)

where \( T \) is the time separation between soliton pulses; the factor 1/2 accounts for the average occupation of pulse slots in a typical data stream. It is required that this pump depletion be small, such that the Raman pump power will not be signif-
icantly dependent on the signal power. As an illustrative example, for $\tau = 16$ psec, which corresponds to $Z_0 = 62.5$ km, $T/\tau = 10$, $A_{eff} = 75 \mu m^2$, $P_1 = 30$ mw, the additional attenuation at the pump wavelength is $2.4 \times 10^{-3} \text{ km}^{-1}$, which is only 17% of the fiber intrinsic attenuation at $\lambda_P$.

8 Optimum system design

For the sake of soliton stability, it is desirable to use large values of $Z_0$ such that $L/Z_0 \ll 8$, [16]. This fact will be proven numerically by showing that the change ($\delta S$) in the pulse area $S$, for different values of $Z_0$, is small for large $Z_0$ and peaks when $L = 8Z_0$. In this domain (of large $Z_0$) the soliton itself is not preserved everywhere, but nevertheless recovered along with the pulse energy at the end of the amplification period. As we will also show, the soliton peak power is reduced significantly over that required for small $Z_0$. Note that increasing $Z_0$ will increase $\tau$ and hence decreasing the bit rate for certain pulse spacing $(T)$. The steps of the system design are in the next section.
8.1 Determination of the amplification period length (L)

The maximum amplification period length is determined by the desire to keep the pulse energy fluctuations within certain limits (± 20 %) and by the fiber loss coefficients at the soliton signal and pump wavelength. Fig 19 shows pulse energy fluctuations vs. L; note how these fluctuations increase rapidly when L increases.

8.2 Determination of the soliton period $Z_o$

This step serves as a way to determine $\tau$, since for given values of $\lambda_s$ and D, $\tau$ can be directly calculated if $Z_o$ is determined, see eqn. (15). Fig 20 shows the transmission fidelity $\delta S$ (the change in pulse area after one amplification period) for different values of $Z_o$ corresponding to $L = 250$ km produced by the numerical solution of the NLS equation. Note that the position of the peak (maximum distortion) occurs at $Z_o = \frac{L}{8}$. This can be explained if we consider the soliton phase term $i\xi/2$, see eqn. (9); this phase term undergoes a change of $2\pi$ when $\delta z = 8Z_o$, this point represents a peak for $\delta S$ and corresponds to a resonance between the perturbation period L and the soliton phase.

As discussed earlier, for the sake of stability, one would then pick the largest possible value of $L/Z_o$ (smallest possible $Z_o$) on the left side of the resonance peak.
and consistent with a certain maximum allowable value for \( \delta S \). Once \( Z_0 \) is determined, \( \tau \) can be calculated. As an example, for \( L = 250 \) km and \( L/Z_0 = 4 \), \( Z_0 = 62.5 \) km gives \( \tau \) approximately equal to 16 psec for \( \lambda_s = 2.55 \) \( \mu m \) and fiber dispersion parameter \( D = 0.6 \) psec/nm/km. Figures 21 to 25 show, for this specific example, the soliton amplitude envelope, energy envelope, the soliton area \( S \), intensity peak and the soliton energy \( E \) as a function of propagation distance along one amplification period span produced by numerical solution of the NLS equation.

### 8.3 Determination of the period (T) between adjacent pulse slots

There are two major limiting factors that impose an upper bound on the bandwidth of a soliton-based optical communication system. These are soliton interaction and the random walk of the coherently amplified solitons caused by spontaneous Raman emission.

#### 8.3.1 Soliton interaction

In the NLS equation, the term \( |A|^2 \) corresponds to the potential function in the classical quantum mechanical form of the Schrödinger equation. This potential
causes both attractive and repulsive forces among soliton pairs. Starting from the general 2-soliton solution, it has been shown [17] that solitons in fibers exert forces on their neighbors, which decrease exponentially with increasing the spacing between them, and depend sinusoidally on their relative phases. These forces account for the displacement suffered by solitons during collisions, and their effects must be taken into account in system design. Equation (18) in [17], if transformed into real world quantities [18] gives

\[
\sigma_{ou} = \sigma_{in} + \ln f \left( \frac{Z}{Z_o} \exp \left( -0.88 \frac{\sigma_{in}}{\tau} \right) \right)
\]  

(22)

where \(\sigma_{in}\) and \(\sigma_{ou}\) are the initial and final pulse separation respectively, \(f\) is the cosine function for the attractive case and hyperbolic cosine function for the repulsive case. Fig 26 shows \(\sigma_{in}\) vs. \(\sigma_{ou}\) for the specific example we gave earlier when \(\sigma_{in}\) changes from 0 to 200 psec.

The interaction between solitons can lead to a significant reduction in the bandwidth. However, it can be neglected when the separation (T) between pulses is \(\geq 10\ \tau\). For large values of \(Z_o\) (which is our case), T can be as small as \(5\tau\). We shall soon see that it is desirable to have a bit of extra spacing between adjacent pulses to allow for other effects which we will discuss in the next section; therefore let us conservatively set \(T/\tau = 10\).
8.3.2 Random walk of coherently amplified solitons

Periodic Raman amplification is needed to maintain the energy of the solitons. Coherent amplification is always accompanied by the generation of spontaneous emission noise. Because the system is non-linear, its behavior is not simply additive; some of the noise field will be incorporated into the soliton. The most troublesome resulting effect is a random shift in the soliton carrier frequency with a corresponding change in its velocity. This in turn, through dispersion, leads to random pulse arrival time. This random walk effect causes timing errors and puts an upper limit on the length-bandwidth product of the system.

In reference [19] Gordon and Haus have calculated the mean square inverse velocity shift ($\delta \Omega$) due to a coherent amplifier of gain $G$ (similar results have been obtained using the inverse scattering theory [13] and [14]).

For a system of overall length $L$, consisting of $n$ amplifiers, the overall shift in arrival time is calculated by summing up the contribution of each individual amplifier. Assuming gaussian distribution of the change in pulse arrival time, probability of error less than $10^{-9}$, and with a detection window $W$ at the receiver to allow for the finite pulse width, and the detector response time, such that $W/T = 1/3$. Then the product of the length of the system $L$ and the bit rate $R$ ( $R$ is taken as the
inverse of $T$ is given by:

\[
(RL)^3 = 0.1372\frac{\tau W^2 R^3 A_{\text{eff}}}{h\Gamma_s n_2 D}
\] (23)

In the above example, with $R = 1/T$, $T = 10\tau$, $\tau = 16$ psec, (i.e. $R = 6.25$ GHz), $A_{\text{eff}} = 75 \mu m^2$, $\Gamma_s = \alpha_s = 0.007\ km^{-1}$, $n_2 = 2.4 \times 10^{-16}\ cm^2/W$, $D = 0.6\ psec/nm/km$ and $h$ is Plank's constant, the length bandwidth product $RL$ (also called the Gordon-Haus limit) = 119,624 GHz.km. Such a system, with 6.25 GHz information rate, could traverse nearly 19,139 km before the effect of the random walk sets in.

To complete the picture, we note that this system would use solitons of peak power $P_s = 30\ mW$ and therefore containing $3.7 \times 10^6$ photons. Because the system has an overall gain of unity, the mean number of noise photons per mode at the receiver will be \[19\] $\alpha_s L = 134$. This is small compared to the number in a soliton and hence should cause no problem for the detection process.
9 Stability with repeated amplification

The system is said to be stable if the pulse distortion $\delta S$ does not grow monotonically with repeated amplification, but instead is always contained within certain limits, no matter how many amplification periods are involved.

The program was run for 20 consecutive amplification periods, which corresponds to 5000 km, for different values of $Z_0$. The pulse shape obtained at the end of each amplification period become the input to the next and so on (as a closed loop). The pulse envelope and area were examined at the end of each fifth period. The change in the pulse area was never more than 4 times the change in the initial pulse area at the end of the first amplification period span.

10 Design examples of a soliton-based OFCS

TABLE (2) lists the parameters of interest for various design examples based on the error curve similar to Fig 20 for amplification periods of length $L = 200, 250$ and $300$ km and for some selected values of $Z_0$; $D = 0.6$ psec/nm/km and $T/\tau = 10$ were assumed throughout.
11 Conclusions

Several design examples have been shown. It is obvious that the soliton based OFCS using Fluoride fibers has a promising future. Recent experimental verification [20] showed by recirculating a 55 psec soliton pulses (\(\lambda_s = 1600 \text{ nm}\)) many times around a closed 42 km loop, with loss exactly compensated by Raman gain at \(\lambda_p = 1497 \text{ nm}\), faithful pulse transmission in the form of solitons has been demonstrated without electronic regeneration over distances in excess of 4000 km. Much work remains to be done before a soliton based communication system could be realized in practice, for example developing a system for pump laser diodes and the wavelength selective coupling. Nevertheless this experimental demonstration revealed no fundamental barrier to its realization.

An important fact to be stressed, however, is that solitons are not just one stable solution of the NLS equation, but the only stable solution. Thus for example, it is not feasible to base an all-optical system on pulses propagating at the wavelength of zero dispersion. That is, for pulses of usefully large energies, the combination of non-linear effects and higher order dispersion will produce severe pulse broadening and distortion.

The primary advantage of the soliton based system lies in the tremendous infor-
mation handling capacity, however, it has many other advantages; some of fundamental nature, others of a matter of engineering convenience and economics. Several very important advantages stem from the all optical nature of the system and are as follows:

1- Simple CW laser pump diodes at the ends of each amplification period would replace the high performance signal laser diodes required at each repeater of a conventional system; the same number of high speed photodetectors would also be eliminated.

2- The high speed electronics of conventional repeaters and the severe bottleneck they impose on the bit rate are eliminated. The only electronics required along the transmission line would be for simple servo control of the Raman gain.

3- The limitations on system performance imposed by the poor frequency stability and excess pulse bandwidth of conventional laser diodes would be eliminated. It would be economically feasible to use more complex, high performance lasers in place of diode lasers. In particular a mode-locked laser synchronously pumped, with a laser diode or diode array, can be used.

4- Multiplexing and demultiplexing would be carried out only at each end of the soliton based system. Not only would this greatly reduce overall complexity and expense, but also the multiplexing hardware.
Another set of advantages arises from the fact that the pulse energies are always maintained at a high level in the soliton based system.

1- Since each pulse always contains $10^6$ or more photons, the question of error caused by poor photon statistics, a constant threat to conventional systems would never arise in the soliton based system. 2- The large signal levels may allow reduced detector sensitivity to be traded for increased speed of response.
<table>
<thead>
<tr>
<th>Z(km)</th>
<th>E(calculated)</th>
<th>E (theoretical)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.999999995909732</td>
<td>1.12290077865737</td>
<td>-4.09026799931056E-09</td>
</tr>
<tr>
<td>31.25</td>
<td>1.12304405914872</td>
<td>1.1351291538555</td>
<td>1.43280491349484E-04</td>
</tr>
<tr>
<td>62.5</td>
<td>1.13542195144042</td>
<td>1.08019605749539</td>
<td>2.92797584918513E-04</td>
</tr>
<tr>
<td>93.75</td>
<td>1.08061350555553</td>
<td>1.08019605749539</td>
<td>4.17448055130886E-04</td>
</tr>
<tr>
<td>125.0</td>
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<td>1.08019605749539</td>
<td>5.0309223656893E-04</td>
</tr>
<tr>
<td>156.25</td>
<td>0.926317803915605</td>
<td>0.92576571895785</td>
<td>5.61232019819543E-04</td>
</tr>
<tr>
<td>187.5</td>
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<td>0.880956579174923</td>
<td>6.1677220222682E-04</td>
</tr>
<tr>
<td>218.75</td>
<td>0.891258376350067</td>
<td>0.890551516700377</td>
<td>7.06859649689611E-04</td>
</tr>
<tr>
<td>250.0</td>
<td>1.00090571490442</td>
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<td>9.02330986262134E-04</td>
</tr>
</tbody>
</table>

Comparison between the normalized soliton energy obtained by solving the NLSE and the theoretical values obtained using (6) in the text at different values of propagation along one amplification period L of length 250 km when L/Zo is 6, and using the same fiber loss coefficients mentioned earlier.
**TABLE (2)**

**SOME DESIGN EXAMPLES OF A SOLITON-BASED LONG DISTANCE HIGH BIT RATE OPTICAL FIBER COMMUNICATION SYSTEM USING FLUORIDE FIBERS**

<table>
<thead>
<tr>
<th>Design number</th>
<th>Amplification period length L(km)</th>
<th>Soliton period length Z₀(km)</th>
<th>Pulse FWHM τ(psec)</th>
<th>Soliton peak power ( P₁(mw) )</th>
<th>Raman pump power ( Pₚ(mw) )</th>
<th>System bit rate ( R(\text{GHz}) )</th>
<th>System length ( Z(\text{km}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>40</td>
<td>12.8</td>
<td>50</td>
<td>16</td>
<td>7.8</td>
<td>15,300</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>50</td>
<td>14</td>
<td>40</td>
<td>20</td>
<td>7</td>
<td>17,100</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>125</td>
<td>23</td>
<td>16</td>
<td>20</td>
<td>4.4</td>
<td>27,200</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>100</td>
<td>20</td>
<td>20</td>
<td>25</td>
<td>5</td>
<td>23,900</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>300</td>
<td>35</td>
<td>6.6</td>
<td>20</td>
<td>2.8</td>
<td>27,200</td>
</tr>
</tbody>
</table>
Figure 1: Segment of the all-optical soliton based system. Loss is compensated by Raman amplification using bidirectional wavelength dependent directional couplers.
Figure 2: Normalized transverse Electric field distribution of the fluoride fiber design to be used in the all-optical soliton based system.
Figure 3: Dispersion parameter $D$ (psec/nm/km) vs. wavelength for the fluoride fiber design presented.
Figure 4: Fiber loss, Raman gain and the effective loss/gain coefficients over one amplification period of length L km.
Figure 5: Normalized pulse energy over one amplification period of 250 km length. The curve is a plot of (6).
Figure 6: Normalized pulse energy for three different amplification periods of 200, 250 and 300 km length.
Figure 7: Soliton period $Z_o$ vs the dispersion parameter $D$ for different values of FWHM pulse durations.
Figure 8: Amplitude envelope of N=2 soliton at each 1/10 of propagation distance along one half of amplification period obtained by numerical solution of NLS equation.
Figure 9: Intensity envelope of N=2 soliton at each $1/10$ of propagation distance along one half of amplification period obtained by numerical solution of NLS equation.
Figure 10: Initial intensity envelope for N=3 soliton.
Figure 11: Intensity envelope of N=3 soliton after a propagation distance of 0.1 $Z_0$ obtained by numerical solution of NLS equation.
Figure 12: Intensity envelope of $N=3$ soliton after a propagation distance of $0.2 \ Z_0$ obtained by numerical solution of NLS equation.
Figure 13: Intensity envelope of $N=3$ soliton after a propagation distance of $0.25 Z_0$ obtained by numerical solution of NLS equation.
Figure 14: Intensity envelope of $N=3$ soliton after a propagation distance of $0.35 Z_0$ obtained by numerical solution of NLS equation.
Figure 15: Intensity envelope of N=3 soliton after a propagation distance of 0.40 $Z_0$ obtained by numerical solution of NLS equation.
Figure 16: Intensity envelope of $N=3$ soliton after a propagation distance of $0.5 \, Z_0$, obtained by numerical solution of NLS equation.
Figure 17: Peak power of fundamental soliton as a function of the fiber core effective area vs the soliton period.
Figure 18: Bidirectional Raman CW pump as a function of amplification period length L.
Figure 19: Maximum pulse energy fluctuations vs amplification period length L.
Figure 20: Percent change in pulse area $S$ as a function of the ratio $L/Z_0$ for $L = 250$ km at the end of one amplification period.
Figure 21: Amplitude envelope for fundamental soliton at each 1/8 of propagation distance along one L span for $L/Z_0 = 4$. 
Figure 22: Intensity envelope for fundamental soliton at each $1/8$ of propagation distance along one $L$ span for $L/Z_0 = 4$.
Figure 22: Intensity envelope for fundamental soliton at each 1/8 of propagation distance along one L span for $L/Z_o = 4$. 
Figure 23: Change in fundamental soliton pulse area at each 1/8 of propagation distance along one amplification period L, obtained from the numerical solution of the NLS equation.
Figure 24: Change in fundamental soliton peak intensity at each 1/8 of propagation distance along one amplification period $L$, obtained from the numerical solution of the NLS equation.
Figure 25: Change in fundamental soliton pulse energy, each 1/8 of propagation distance along one amplification period $L$, obtained from the numerical solution of the NLS equation.
Figure 26: Intersoliton attractive and repulsive forces for $\sigma_i n = 16$ psec and $Z/Z_0 = 4$. 
APPENDIX A

Derivation Of The Non Linear Partial Differential Equation, That Governs The Propagation Of Solitons In Optical Fibers, And Basic Properties Of Optical Solitons.

The propagation of transverse electromagnetic waves in an optical fiber transmission line, can be described by the scalar dimensionless function $A_1(z,t) = A_0 \exp(ikt - i\omega t)$, where $A_1(z,t)$ is proportional to the complex field amplitude, such that the power $P$ in the optical fiber is given by

$$P = P_c |A_1|^2$$

where $P_c$ is the proportionality constant in watts

For frequencies near some central frequency $\omega_0$, the non linear Schrödinger partial differential equation results from the following dispersion relation, (let $\Omega = (\omega - \omega_0)$)
Figure 26: Intersoliton attractive and repulsive forces for $\sigma_{in} = 16$ psec and $z/Z_0 = 4$

\[ k = k_0 + k'_\Omega + \frac{1}{2} k''\Omega^2 + k_2 P \quad (3) \]

The above equation is the Taylor series expansion of $k(\omega, P)$ in the neighborhood of $(\omega_0, 0)$. The higher order terms (e.g., $k'''\Omega^3$, $k'_2\Omega P$, etc.) can be neglected or sufficiently treated as perturbation, the term $k_2 P$ represents the non linear effect (self phase modulation), resulting from the intensity dependent refractive index of the fiber.

If we remove the central frequency from $A_1$ by defining

\[ A(z, t) = A_1(z, t) \text{exp}(i\omega_0 t - ik_0 z) \quad (4) \]

then the wave equation necessary and sufficient to reproduce (3) exactly is

\[ -i \frac{\partial A}{\partial z} = ik' \frac{\partial A}{\partial t} - \frac{1}{2} k'' \frac{\partial^2 A}{\partial t^2} + k_2 P_c |A|^2 A \quad (5) \]

This can be verified by direct substitution of eqn. (4) into eqn. (5). However, eqn. (5) is not easy to deal with in its present form. To eliminate the first t derivative in (5), we transform it to a retarded time frame by using the substitution
where $t_c$ is an arbitrary time scale, that allows a pulse in the dimensionless retarded time domain ($s$), to correspond to any pulse of FWHM duration $\tau$ in the real world time domain ($t$), and $Z_c$, is the proportionality constant, that gives the correspondence between the dimensionless space domain in $\xi$ coordinates, and the real world distance ($z$). In other words, unit propagation length in the dimensionless domain, corresponds to $Z_c$(km) in real world quantities. $Z_c$ is called the soliton characteristic length.

If we chose unit values of time and distance such that $k = -1$ and $k_2 = 1$ when measured in those units, and the power unit already mentioned, then $t_c$, $Z_c$ and $P_c$ satisfy the relations

$$- \frac{t_c^2}{Z_c} = k''$$

$$(Z_cP_c)^{-1} = k_2$$

the resulting equation is the non linear Schrödenegr equation

$$s = \frac{t - k'z}{t_c}$$

$$\xi = \frac{Z}{Z_c}$$
Small values of gain or loss can be accommodated by adding a term \((-\alpha/2)A\) to the right hand side of the NLS equation, where \(\alpha\) is the energy gain per unit length. Correspondingly, eqn. (8) can be modified, in the case of loss, by adding the term \(-i\Gamma A\), where \(\Gamma = \alpha Z_c/2\) is the dimensionless amplitude gain coefficient.

\[ Z_c \text{ can be expressed in terms of the fiber dispersion parameter } D \text{ rather than } k''; \text{ the expression for the reciprocal group velocity is} \]

\[ v_g^{-1} = \frac{\partial k}{\partial \omega} = k' + k''(\omega - \omega_0) \]  

(9)

The dispersion parameter \(D\) used widely to describe fibers, it is the wavelength derivative of \(v_g^{-1}\), hence is related to \(k''\) by

\[ D = -\frac{2\pi c}{\lambda^2} k'' \]  

(10)

For the fundamental soliton \((N=1)\), the value of the dimensionless time \(s\) at which the function \(sech^2 s\) falls to one half of its maximum value FWHM, is \(s = 0.881374\), therefore, \(\tau/\tau_o = 0.881374\), or \(\tau_o^2 = 0.3218248 \tau^2\). Using this together with eqn. (10) and eqn. (7) we obtain
To calculate the value of the non-linear refractive index coefficient $n_2$, the refractive index of fibers, can be expressed as

$$n_2 = n_0 + \frac{1}{2} \hat{n}_2 |E|^2$$  

(12)

For ZBLAN fluoride fibers, $n_0 = 1.4837$ at $\lambda = 2.55 \mu m$, and the intensity dependent part $\hat{n}_2 = 0.85 \times 10^{-13} cm^2/statvolt^2$ in cgs units [21]. If we translate this to the form

$$n = n_0 + n_2 I$$  

(13)

where $n_2$ is the nonlinear refractive index coefficient, and I is the intensity in $W/cm^2$.

In cgs units, $I = (8\pi)^{-1}cn|E|^2$, so that

$$n_2 = 10^7(4\pi/n_0c)\hat{n}_2 = 2.4 \times 10^{-16} \text{ cm}^2/W$$  

(14)

To derive an expression for the effective area $A_{eff}$ and the intensity dependent
propagation constant $k_2$:

The wave vector of in a single mode fiber can be expressed in the form $k = \omega n_{eff}/c$, where $n_{eff}$ is the effective index of the fiber. The linear part of $n_{eff}$ varies depending on frequency, in a range between the indexes of the cladding and the core. The nonlinear part is independent on frequency. Its perturbing effect on the effective index of the fiber, can be sufficiently evaluated, as an average of $n_2 I$ over the fiber cross section weighted by the intensity distribution $I$. If we thus write $n_{eff} = n_{eff0} + n_2 P/A_{eff}$, where $P = \int I dA$ is the power, we find the effective area given by

$$A_{eff} = \frac{P^2}{\int I^2 dA} \quad (15)$$

For the fiber design presented, the core radius $a = 3 \ \mu m$ and the effective area $A_{eff} = 75 \ \mu m^2$.

If we substitute $n_{eff} = \frac{n_2 P}{A_{eff}}$ in the propagation constant $k = \frac{\omega n_{eff}}{c}$, then the intensity dependent $k_2$ can be written as
Finally, the fundamental soliton peak power $P_1$, can be calculated using eqn. (7), eqn. (16) and $Z_0 = \frac{\pi}{2} Z_e$ which yields eqn. (18) in the text.
References


[15] Numerical solutions of partial differential equation:


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