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# Effect of Average Happiness for Twitter on the Dow Jones Industrial Average Return Volatility

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# EFFECT OF AVERAGE HAPPINESS FOR TWITTER

# ON THE DOW JONES INDUSTRIAL AVERAGE

# RETURN VOLATILITY

BY

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## ABSTRACT

The stock market is well known for its volatility and many models are proposed to capture the volatility. Volatility is naturally unobservable and the absolute values of the returns work as the realized volatility. The Dow Jones Industrial Average is the study object and the models used are generalized autoregressive conditional heteroskedasticity (GARCH) models with different extensions. The unique extension in this study is to add happiness data into the model and check whether it helps to better capture the volatility and improve the forecasting accuracy. The happiness data is extracted from Twitter and it is an index to show people's happiness level based on their online expressions. The one day lagged happiness data is also used as one extension to the models. The leverage effects and the heavy tails problems are also addressed in this study, EGARCH models and GJR-GARCH models with other error distributions such as student's T distribution are used to deal with these specific problems. The forecasting performance of these models is checked and we find out that the happiness data does help to better capture the volatility. However, the forecasting accuracy of the models with happiness data is not statistically different compared to the models without happiness data. This illustrates that the happiness data does not help to improve the forecasting performance.

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# CHAPTER 1

## INTRODUCTION

A stock of a corporation constitutes the equity stake of its owners. It represents the residual assets of the company after the discharge of all the other senior claims. The intrinsic value of the stock is the present value of future dividends. A stock market is where the price of the stock forms because it is the aggregation of the buyers and sellers. It helps companies to raise money and the smooth function of this activity contributes to the economic growth.

The stock market index is created in order to describe the stock market, and it is the measurement of the value of a portion of the stock market. It is computed using the selected stock values. The stock selected depends on the goal of the index. One example is the Dow Jones Industrial Average Index. The Dow Jones Industrial Average (DJIA) is the most quoted stock market index in the world (Shoven and Sialm, 2000). It was first published on May 26, 1896 by Charles Dow, one of the founders of Dow Jones & Co. It included twelve industrial companies listed on the New York Stock Exchange at the beginning. In 1916, the number of companies in the index increased to twenty and in 1928, the number extended to thirty. The DJIA is calculated as a price-weighted measure of these thirty influential companies in the United States and it remains as a good indicator of the entire economy.

There are four main price indicators each day in the stock market index:

opening price, high price, low price and closing price. In order to calculate the daily return, the closing price is often used reflecting the most up-to-date price. There are two ways to calculate the daily return: discrete return and logarithmic return.

$$\text{Discrete:} \quad R_t = \frac{P_t - P_{t-1}}{P_t}$$

$$\text{Logarithmic:} \quad R_t = \log(P_t/P_{t-1})$$

Where $P_t$ = the closing price at time t;

$P_{t-1}$ = the closing price at time t-1.

The logarithmic of the closing price is the discrete return with continuous compounding (Fama et al., 1969). In this work, the logarithmic return is preferred to use. There are both theoretical and empirical reasons for preferring the logarithmic return (Strong, 1992). Theoretically, the logarithmic return is analytically more tractable when returns are calculated over longer intervals (simply add up the sub-period returns). Empirically, the logarithmic return is more likely to be normally distributed.

Volatility of the stock market return is often perceived as a measure of risk. It is a statistical measure for variation of the return over time. In finance, the volatility is also a core parameter in many models such as the Capital Asset Price Model (Sharpe, 1964).

Volatility is inherently unobservable, and what we know about volatility has been learned either by fitting parametric econometric models, or by studying some indicators of volatilities such as the absolute returns (Andersen et al., 2001). It is often calculated as the standard deviation of the return (Poon and Granger, 2003) denoted by σ.

Many researchers have studied the movement of stock market volatility, and raised the question of why the volatility changes so much over time. Officer (1973) relates changes to the macroeconomic variables. There are also attempts to connect volatility to changes in expected stock returns, including Merton (1980), French et al. (1987). Also, a number of studies have used measure of the variance or “volatility” of speculative asset prices to provide evidence against simple models of market efficiency (Shiller, 1981).

## 1.1 Basic Time Series Concepts

### Stochastic Process

A stochastic process is a sequence of random variables $\{X_t,\ t = 1,2, \ldots\}$ defined at fixed sampling intervals, representing the evolution of random values over time. The index t represents time, and a stochastic process is also known as a random process.

### Time Series

A time series is a sequence of observations on a particular variable, and it can be interpreted as a realization of the stochastic process. Examples of time series are inflation rates, unemployment rates and market shares. The main features of time series include trends, seasonal variations and the observations that are close in time are correlated. So time series models are needed to explain this correlation.

**Autocorrelation**

A correlation of a variable with itself at different times is known as autocorrelation. The number of time steps between the variables is known as the lag. The autocorrelation function, or ACF, express the autocorrelation as a function of the lag k for k = 1,2…. Let $\{x_t, t \in T\}$ be a time series and $\bar{x}$ is the sample mean. The autocorrelation can be estimated by the sample autocorrelation function (ACF), or the empirical ACF. The sample autocorrelation function or correlogram is given by

$$\rho_k = \frac{\sum_{t=1}^{T-k}(x_t - \bar{x})(x_{t+1} - \bar{x})}{\sum_{t=1}^{T}(x_t - \bar{x})^2}$$

**White Noise**

White noise is a stochastic process used for many time series models. A time series $\{w_t\}$ is white noise if $w_1$, $w_2$, $w_3$,…, $w_n$ are independent and identically distributed random variables with mean of zero. This means that the variables have the same variance and the covariance between them is zero.

The stock market returns are expected to be white noise under the efficient market hypothesis. In an efficient market, asset prices adjust instantaneously to reflect new information, which eliminates the possibility to predict future prices using only past prices (Logue and Sweeney, 1977). This implies that the current price of a security “fully reflects” available information (Fama, 1970). Thus, the successive price changes (or returns) are independent and identically distributed, which makes them a white noise. However, there are many reasons which may cause violations to the efficient market hypothesis. The arbitrage risk, for example, is one of them. Under the efficient market hypothesis, any arbitrage opportunities results from mis-pricing will be removed by rational traders' transactions. In the real world, however, arbitrageurs are subject to many constraints, such as transaction fees and holding costs (Pontiff, 2006). Therefore, the price may not fully reflects available information which violates the efficient market hypothesis.

**Autoregressive (AR) Models**

An AR model is a linear combination of p most recent past values of a random variable and the current white noise term.

The series $\{x_t\}$ is an autoregressive process of order p, if

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + w_t$$

where $\{w_t\}$ is white noise and $\alpha i$ are the model parameters with $\alpha_i \neq 0$.

**Moving Average (MA) Models**

A moving Average (MA) Model is also one foundation of other models. A moving average (MA) process of order q is a linear combination of the current white noise and the q most recent past white noise terms.

The series $\{x_t\}$ is a moving average (MA) process of order q, if

$$x_t = w_t + \beta_1 w_{t-1} + \beta_2 w_{t-2} + \cdots + \beta_q w_{q-2}$$

where $\{w_t\}$ is white noise.

**Autoregressive Moving Average (ARMA) Models**

In the time series analysis, Box-Jenkins method (Box and Jenkins, 1970) applies autoregressive moving average (ARMA) models to find more appropriate fit of one time series. The ARMA model is the combination of AR and MA model. Dependence is very common in time series data, and ARMA models could be used to capture this dependence.

The time series $\{x_t\}$ is an autoregressive moving average (ARMA) process of order (p,q), if

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + w_t + \beta_1 w_{t-1} + \beta_2 w_{t-2} + \cdots + \beta_q w_{q-2}$$

Where $\{w_t\}$ is white noise. If $d_{th}$ difference of the $\{x_t\}$ series are an ARMA(p,q) process, then $\{x_t\}$ follows an autoregressive integrated moving average ARIMA(p,d,q).

1.2 Time Series Models for Financial Data

In financial area, the random walk is often used to predict the price of the financial asset. That means we can use normal distribution to simulate the trend of the stock price. It is quite convenient to use this easy model to predict the stock price but the shortcoming is also quite obvious. For example, in Figure 1, the return does change with time but we can find the volatility clustering happens. That means the volatility is higher during one period of time (like in 2009) compared to other periods of time. According to Poon and Granger (2003), there are two ways to forecast the volatility, one is to use the time series data and the other one is to use the option prices. In this study, we will use the time series data and we need to use other models to predict the volatility.

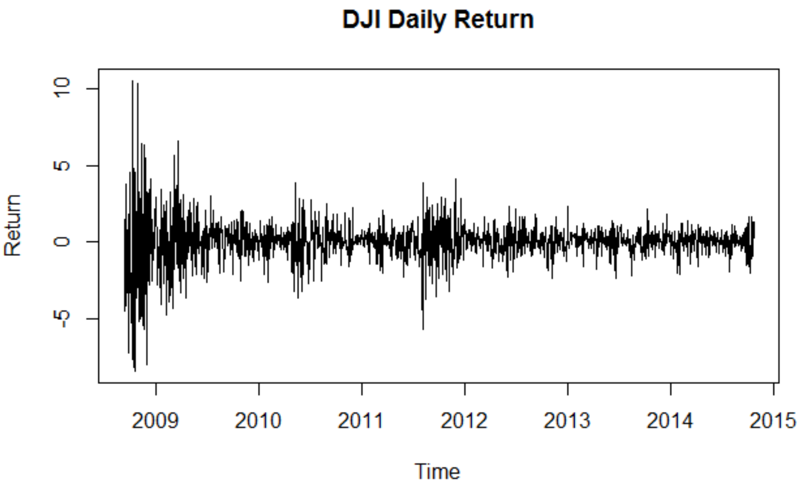


Figure 1. Dow Jones Return from 9/11/2008 to 10/25/2014.

A lot of volatility forecasting models have been investigated in the previous studies, but no consensus has been reached on which model is better than others (Poon and Granger, 2003). Therefore, many researchers try to add other external variable in the model (like the Investor sentiment index) to better fit the volatility. According to (Lee et al., 2002), the shifts in sentiment are negatively correlated with the market volatility. In this research, the data used is from Twitter instead of using proxy like the turnover ratio from the market (Baker and Wurgler, 2006).

Engle (1982) introduced Autoregressive conditional heteroskedasticity (ARCH) to model the volatility changed with time.

Autoregressive conditional heteroskedasticity (ARCH) model of order p

$$r_t = \varepsilon_t h_t$$

Where

$$\mathrm{h}_\mathrm{t}^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \cdots + \alpha_\mathrm{p} r_{p-1}^2$$

Bollerslev (1986) proposed Generalized ARCH model as an extension of the

ARCH model. It has longer memory and more flexible lag structure by adding lagged conditional variance into the model.

Generalized Autoregressive conditional heteroskedasticity (GARCH) model of order (p,q)

$$r_t = \varepsilon_t\, h_t, \quad \varepsilon_t \sim N(0, h_t)$$

Where

$$\mathrm{h}_{\mathrm{t}}^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \cdots + \alpha_{\mathrm{p}} r_{p-1}^2 + \beta_1 \mathrm{h}_{\mathrm{t-1}}^2 + \cdots + \beta_{\mathrm{q}} \mathrm{h}_{\mathrm{t-q}}^2$$

Specifically, the GARCH (1,1) model is often used in finance.

Basic GARCH (1, 1) model:

$$r_t = \varepsilon_t\, h_t, \quad \varepsilon_t \sim N(0, h_t)$$

Where

$$\mathrm{h}_{\mathrm{t}}^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \mathrm{h}_{\mathrm{t-1}}^2$$

In the classic GARCH model, the error is normally distributed $\varepsilon_t = \sigma_t z_t,\ z_t$ is standard normal distribution ~ iid (0,1). The density function of normal distribution is

$$\mathrm{f}(\mathrm{x}, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where μ is the mean of the distribution and σ is the standard deviation.

### 1.3 Research Goal and Thesis Outline

Bollen (2011) proposed that with the development of social media, people’s emotions can be easily measured through their online expressions. In this study, happiness works as the representative of people’s emotions. It is interesting to see what the effect of happiness is on the stock market return. Does the happiness data help to fit the return data? Or it may have a lagged impact on the return volatility. In the term of forecasting, the happiness data may help to forecast the return volatility. That is to check whether there is any improvement of the prediction accuracy when adding the happiness data into the model.

The thesis is organized as follows: Chapter 2 will fit the data with basic GARCH models and the result is shown to decide which model has more appropriate fitting. Also, some basic features of the dataset will be discussed. In Chapter 3, more advanced models are used to deal with asymmetry problems and heavy tails. Chapter 4 is going to present forecasting based on the advanced models selected and compare the estimation power of different models with each other.

CHAPTER 2

# FITTING BASIC GARCH MODELS

## 2.1 Data

Dow Jones Industrial Average (DJIA) Daily Return. The daily closing prices of the DJIA, which are downloaded from Yahoo Finance, are used to calculate the DJIA daily returns. The time range for the DJIA return is from 9/11/2008 to 10/25/2014.

## 2.2 Data Description

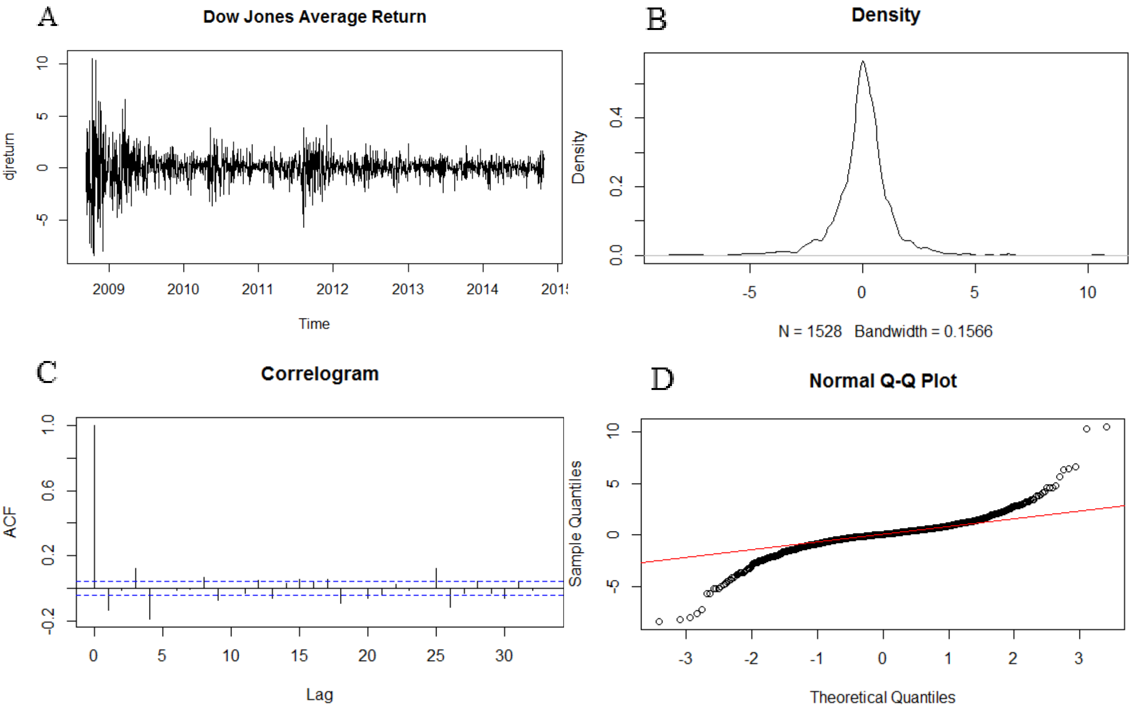


Figure 2. Plot A: time series of the Dow Jones Industrial Average Return; Plot B: the density of the DJIA return; Plot C and D: Correlogram and QQ plot of the DJIA return.

Plot A in Figure 2 shows that the return is a stationary series in mean averaging around zero. However, the volatility is clustered especially during the end of 2009 when the financial crisis still has its influential impact all over the world. This is the reason non-linear models (like GARCH) are needed to fit the data. The correlogram plot implies that autocorrelation exists in this series. The auto-correlation means the correlation of a variable with itself at different times. It is typically modeled with autoregressive moving average model (ARMA). In this study, ARMA component is added to GARCH model and I will check whether it is significant as one extension to GARCH model. The density and the QQ plot indicate this series has heavy tails and potential asymmetric problems. Especially in the QQ plot, the two tails deviated from the red line which represents the normal distribution. So in Chapter 3, more advanced models which deal with these two problems.

2.3 Results from Basic Models

**Benchmark Model**

After checking the significance of the parameters, the preferable model in the GARCH (p,q) for p from 1 to 5 and q from 1 to 2 was GARCH (1,1). So GARCH (1,1) is used as the benchmark model.

Basic GARCH (1, 1) model:

$$r_t = \varepsilon_t \, h_t, \quad \varepsilon_t \sim N(0, h_t)$$

Where

$$\mathrm{h}_{\mathrm{t}}^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \mathrm{h}_{\mathrm{t-1}}^2$$

The maximum likelihood method is used to estimate the parameters in the

GARCH models. In basic GARCH models, the error is normally distributed with mean of $\mu$ and standard deviation of $\sigma$. So the likelihood function is:

$$\mathrm{L}(\theta|x_1, \ldots, x_n) = f(x_1, \ldots, x_n|\theta) = \prod_{i=1}^{n} f(x_i|\theta)$$

Where $\theta$ will be the parameters. In practice, it is more convenient to use the logarithm of the likelihood function which is:

$$\ln\mathrm{L}(\theta|x_1, \ldots, x_n) = \sum_{i=1}^{n} \ln\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right)$$

Table 1. Estimates of GARCH(1,1) model.

| Model | μ | ω | α | β |
|---|---|---|---|---|
| GARCH(1,1) | 0.0691*** | 0.0224*** | 0.1256*** | 0.8564*** |
| | (0.0184) | (0.0053) | (0.0239) | (0.0213) |

The mean is modelled for the GARCH(1,1) model, so μ is the estimated mean. ω is the variance intercept and α is the ARCH(q) parameter and β is the GARCH(q) parameter. They are all significant.

So, the estimated benchmark model is

$$r_t = \varepsilon_t h_t + 0.0691$$

Where

$$\mathrm{h}_\mathrm{t}^2 = 0.0224 + 0.1256 r_{t-1}^2 + 0.8564 \mathrm{h}_{\mathrm{t}-1}^2$$

GARCH(1,1) is frequently used as the benchmark model because it is a relative simple model but with great performance fitting the financial time series data. The plots in figure 3 display the performance of GARCH(1,1) model.

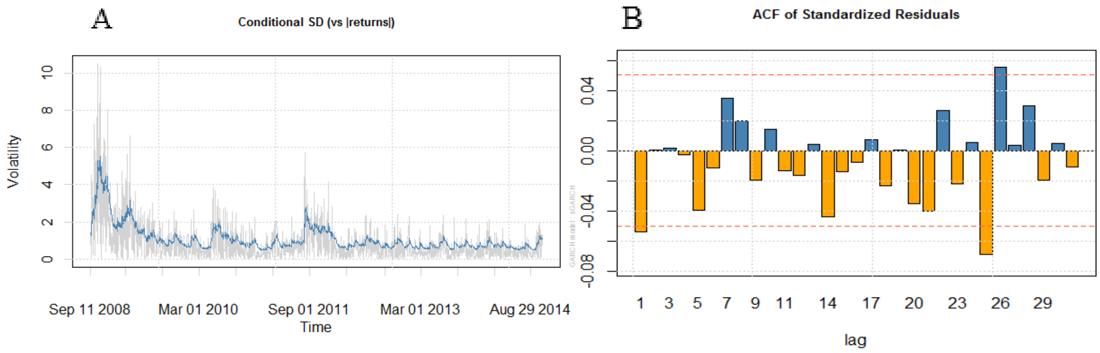


Figure 3. The plot of fitted values (A) and correlogram (B) of the standardized residuals from GARCH(1,1) model.

Plot A in Figure 3 shows the conditional standard deviation (blue line) which indicates the fitted volatility against the absolute value of return (grey line). The absolute value of return is used as one proxy of the volatility. The GARCH(1,1) model captures a lot of the volatility as displayed in the first plot. The first significant value in the correlogram plot is at lag 1, which implies the underlying autocorrelation is not all zero. Hence, a GARCH-ARMA model is fitted next.

**GARCH(1,1)-ARMA(p,q) Models**

As indicated in the correlogram of the standardized residuals, one potential extension is to add ARMA component into the GARCH model. That is to include an ARMA model for the conditional mean of the process. I will specify the mean equation with a low order of ARMA process to capture the autocorrelation of the return.

GARCH (1, 1) - ARMA (p, q) Model:

$$r_t = \alpha + \sum_{1}^{p} \theta_{\mathrm{i}} r_{t-\mathrm{i}} + \varepsilon_t + \sum_{1}^{q} \delta_{\mathrm{q}} \varepsilon_{t-\mathrm{q}},$$

Where

$$\mathrm{h}_{\mathrm{t}}^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \mathrm{h}_{\mathrm{t-1}}^2$$

After checking the AIC and the significance of the parameters, the model chosen is GARCH(1,1)-ARMA(1,1).

Table 2. Estimates of GARCH(1,1) -ARMA(1,1) model.

| Model | μ | ω | α | β | θ | δ |
|---|---|---|---|---|---|---|
| GARCH(1,1)-ARMA(1,1) | 0.0727*** | 0.0222*** | 0.1264*** | 0.8558*** | 0.9167*** | -0.9460*** |
| | (0.0107) | (0.0053) | (0.0241) | (0.0214) | (0.0184) | (0.0148) |

The parameters for the ARMA component is θ for the autoregressive process and δ for the moving average. These two parameters are all significant meaning the ARMA component is helpful to be added into the GARCH model.

The GARCH(1,1)-ARMA(1,1) model is

$$r_t = 0.0727 + 0.9167 r_{t-1} + \varepsilon_t - 0.9460 \varepsilon_{t-1}$$

Where

$$\mathrm{h}_{\mathrm{t}}^2 = 0.0222 + 0.1264 r_{t-1}^2 + 0.8558 \mathrm{h}_{\mathrm{t-1}}^2$$

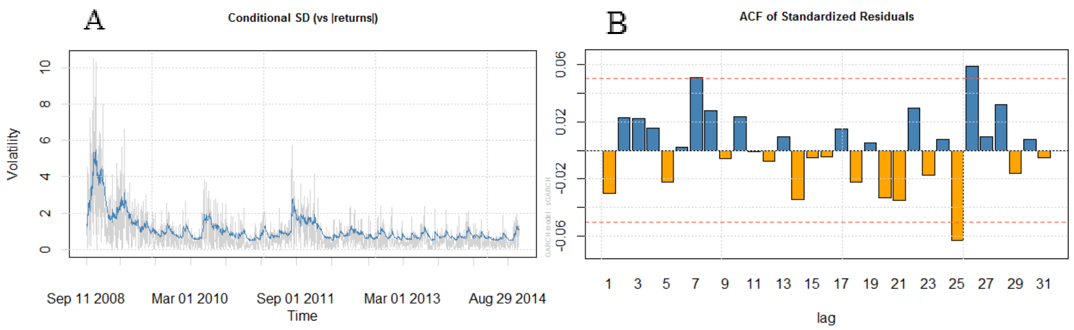


Figure 4. The plot of fitted values (A) and the correlogram (B) for the standardized residuals from GARCH(1,1)-ARMA(1,1) model.

Table 3. AICs for the GARCH(1,1) and GARCH(1,1)-ARMA(1,1) models.

| Model | AIC |
|---|---|
| GARCH (1,1) | 4208.592 |
| GARCH(1,1)-ARMA(1,1) | 4204.822 |

The difference between the fitting of these two models (GARCH and GARCH-ARMA) is quite small based on the plot. The autocorrelation at lag 1 for GARCH-ARMA model is zero indicating that it is reasonable to add ARMA component into the mean function of the GARCH model.

**GARCHX-ARMAX Framework**

The main goal of this study is to check the impact of happiness data on the stock market return volatility. The very intrinsic application is to add happiness data as an external regressor in the GARCH-ARMA model.

The happiness data comes from Hedonometer.org, which is based on people's online expressions on Twitter. To quantify happiness, it merged 5,000 most frequent words from a collection of: Google Books, New York Times articles, Music Lyrics, and Twitter messages, resulting in a composite set of roughly 10,000 unique words.

These words scored on a nine point scale of happiness: (1) sad to (9) happy. Based on (Bollen et al., 2010), Twitter posts are a sensible way to measure the sentiment of people. Hedonometer.org currently measures Twitter's Gardenhose feed, a random sampling of roughly 50 million (10%) of all messages posted to the service. Words in messages written in English are thrown into a large bag and the bag is assigned a happiness score based on the average happiness score of the words contained within. In this study, the time range for happiness data is from 09/11/2008 to 10/25/2014. There are 13 missing values in the happiness dataset, which are the values for dates from 05/14/2009 - 05/19/2009, 08/03/2009 -08/05/2009, 12/18-12/20/2009 and 04/22/2012. The adjustments made here is to use the linear interpolation to create the data for these days. It is fair to use linear interpolation because of the relative small amount of the missing values (13) compared to the number of observations (1527) in the dataset. Another issues about the dataset is the weekend data. The stock market will be closed during weekends, so the happiness data during the weekends are not included in the study.

In the GARCHX-ARMAX model, external regressor can be added into the conditional mean equation or the conditional variance equation or both. So I will check these different combinations of the models and choose some of them to fit more advanced models. The notation for happiness data is $H_t$.

Basic GARCHX(1,1)-ARMAX(1,1) model (with happiness in mean and variance):

$$r_t = \alpha + \theta_1 r_{t-1} + \varepsilon_t + \delta_1 \varepsilon_{t-1} + \gamma H_t$$

Where

$$h_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}^2 + \tau H_t$$

The GARCH-ARMA model with happiness in mean or variance equation is considered as the same scenario as the basic GARCHX-ARMAX model.

Also, if happiness data has a lagged influence on the volatility, we can add lagged value into the GARCH-ARMA model. Then the model will be:

$$r_t = \alpha + \theta_1 r_{t-1} + \varepsilon_t + \delta_1 \varepsilon_{t-1} + \gamma_2 H_{t-1}$$

Where

$$h_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}^2 + \tau_2 H_{t-1}$$

Again, for the models with lagged happiness data, there are 2 other scenarios which are the GARCH-ARMA model lagged happiness in mean or variance function.

Based on the significance of the parameters and the AIC of different models, the GARCH-ARMA model with happiness data in the mean equation and the model with lagged happiness in the mean equation are the preferable models.

Table 4. Estimates of GARCH(1,1)-ARMA(1,1) with happiness data in the mean equation and GARCH(1,1)-ARMA(1,1) with lagged happiness data in the mean function.

| GARCH(1,1) -ARMA(1,1) | μ | ω | α | β | θ | δ | $\gamma$ |
|---|---|---|---|---|---|---|---|
| $H_t$ in mean | -2.6155***<br>(0.1359) | 0.0223***<br>(0.0053) | 0.1261***<br>(0.0243) | 0.8561***<br>(0.0216) | 0.9197***<br>(0.0372) | -0.9494***<br>(0.0289) | 0.4484***<br>(0.0227) |
| lag $H_t$ in mean | -2.0329***<br>(0.5113) | 0.0223***<br>(0.0053) | 0.1253***<br>(0.0241) | 0.8565***<br>(0.0216) | 0.9208***<br>(0.0134) | -0.9497***<br>(0.00964) | 0.3513***<br>(0.0858) |

Note: the word lag is short for lagged, the word sim is short for simultaneous happiness data and $H_t$ short for happiness data in all the tables.

GARCH(1,1)-ARMA(1,1) with happiness data in the mean function:

$$r_t = -2.6155 + 0.9197r_{t-1} + \varepsilon_t - 0.9494\varepsilon_{t-1} + 0.4484H_t$$

Where

$$\mathrm{h}_t^2 = 0.0223 + 0.1261r_{t-1}^2 + 0.8561\mathrm{h}_{t-1}^2$$

GARCH(1,1)-ARMA(1,1) with lagged happiness data in the mean function:

$$r_t = -2.0329 + 0.9208r_{t-1} + \varepsilon_t - 0.9497\varepsilon_{t-1} + 0.3513H_{t-1}$$

Where

$$\mathrm{h}_t^2 = 0.0223 + 0.1253r_{t-1}^2 + 0.8565\mathrm{h}_{t-1}^2$$

The estimated GARCH parameter, β is close to one and the ARCH parameter, α is close to zero. The sum of them is very close to one indicating that the conditional variance is covariance stationary.

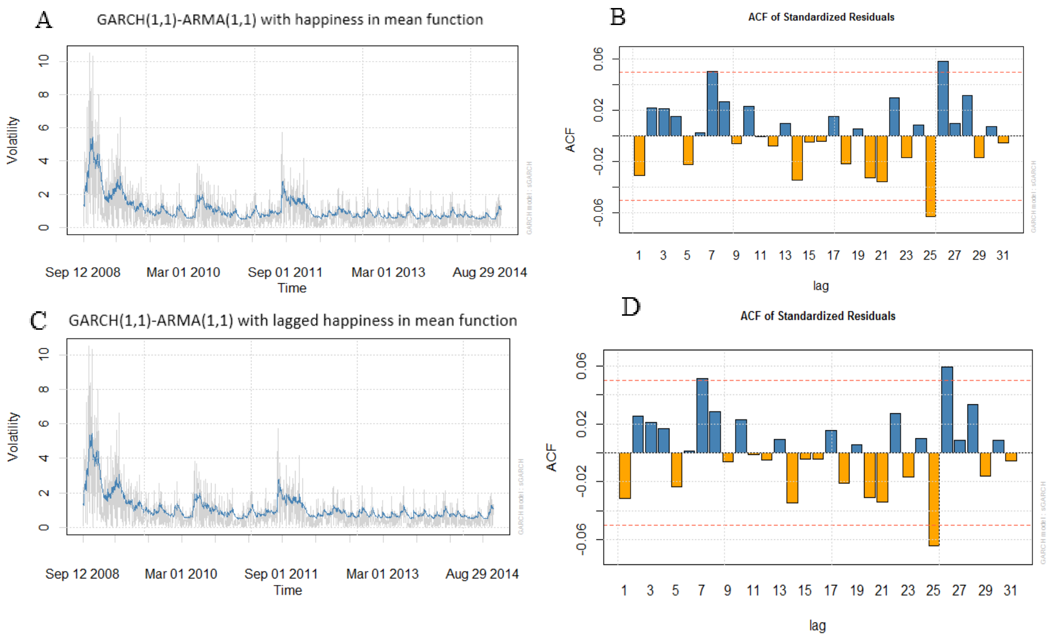


Figure 5. Plot A and B: Plot of fitted values and correlogram for the standardized residuals from GARCH(1,1)-ARMA(1,1) model with $H_t$ in the mean function; Plot C and D: Plot of fitted value and correlogram for the standardized residuals from GARCH(1,1)-ARMA(1,1) model with $H_t$ in the mean function.

These two models are able to capture most of the volatilities based on the two plots of the fitted values. The underlying autocorrelations are all zero; the statistically significant values at lag 25 and 26 are due to sampling variation and are small in magnitude.

Table 5. AICs for the GARCH(1,1) and GARCH(1,1)-ARMA(1,1) models.

| Model | AIC |
|---|---|
| Garch (1,1) | 4208.592 |
| Garch(1,1)-ARMA(1,1) | 4204.822 |
| Garch(1,1)-ARMA(1,1) with $H_t$ in the mean equation | 4204.350 |
| Garch(1,1)-ARMA(1,1) with lag $H_t$ in the mean equation | 4202.915 |

The table 5 provides more information about the fitting of these models. To sum up, the GARCH(1,1)-ARMA(1,1) with lagged happiness in the mean equation is considered as the model with superior fitting results. Therefore, adding happiness in the model does help to capture more features about the DJIA return as the AIC becomes smaller.

Another interesting extension is to keep lagged happiness data and simultaneous happiness data both in the model. That is to add lagged happiness data in the mean equation and the simultaneous happiness in the variance equation or vice versa. The model with lagged happiness in the variance equation and simultaneous one in the mean equation will be

$$r_t = \alpha + \theta_1 r_{t-1} + \varepsilon_t + \delta_1 \varepsilon_{t-1} + \gamma_1 H_t$$

Where

$$h_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}^2 + \tau_2 H_{t-1}$$

The model with lagged happiness in the mean equation and simultaneous one in the variance equation will be

$$r_t = \alpha + \theta_1 r_{t-1} + \varepsilon_t + \delta_1 \varepsilon_{t-1} + \gamma_2 H_{t-1}$$

Where

$$h_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}^2 + \tau_1 H_t$$

Furthermore, simultaneous and lagged happiness can be kept both in the mean equation or both in the variance equation. For both of them in the mean equation, the model will be

$$r_t = \alpha + \theta_1 r_{t-1} + \varepsilon_t + \delta_1 \varepsilon_{t-1} + \gamma H_t + \gamma_2 H_{t-1}$$

Where

$$h_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}^2$$

Both of them in the variance equation, the model will be

$$r_t = \alpha + \theta_1 r_{t-1} + \varepsilon_t + \delta_1 \varepsilon_{t-1}$$

Where

$$h_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}^2 + \tau H_t + \tau_2 H_{t-1}$$

Based on the significance of the parameters and the AIC of different models, the models chosen are:

GARCH(1,1)-ARMA(1,1) with simultaneous happiness data in the mean equation and lagged happiness data in the variance equation

$$r_t = -2.5135 + 0.9762 r_{t-1} + \varepsilon_t - 0.9917 \varepsilon_{t-1} + 0.4293 H_t$$

Where

$$h_t^2 = 0.1249 r_{t-1}^2 + 0.8569 h_{t-1}^2 + 0.0037 H_{t-1}$$

GARCH(1,1)-ARMA(1,1) with lagged and simultaneous happiness data both in the mean equation

$$r_t = -2.5222 + 0.919 r_{t-1} + \varepsilon_t - 0.9492 \varepsilon_{t-1} + 1.2655 H_t - 0.8329 H_{t-1}$$

Where

$$h_t^2 = 0.0223 + 0.1258 r_{t-1}^2 + 0.8559 h_{t-1}^2$$

## 2.4 Basic Conclusions

The table 6 shows the AICs of six basic models. ARMA component does help to capture the autocorrelation of the return as the AIC of GARCH(1,1)-ARMA(1,1) model is smaller than that of GARCH(1,1) model. When the happiness data is added into the model as one external regressor, the AIC becomes smaller. This implies happiness data does help to fit the return volatility.

Table 6. AICs for the models chosen in chapter 2.

| Models | AIC |
|---|---|
| GARCH (1,1) | 4208.592 |
| GARCH (1,1)-ARMA(1,1) | 4204.822 |
| GARCH (1,1)-ARMA(1,1) with sim. $H_t$ in mean | 4204.35 |
| GARCH (1,1)-ARMA(1,1) with lag $H_t$ in mean | 4202.915 |
| GARCH (1,1)-ARMA(1,1) with sim. $H_t$ in mean and lag $H_t$ in variance | 4202.589 |
| GARCH (1,1)-ARMA(1,1) with sim. & lag $H_t$ both in mean | 4201.859 |

CHAPTER 3

FITTING ADVANCED GARCH MODELS

## 3.1 Asymmetric Leverage Effects

Leverage effect refers to the phenomenon that the volatility tends to be negatively correlated with the return (Ait-Sahalia et al., 2013). Specifically, a negative shock will cause a larger increase in volatility than a positive shock. That is to say an unexpected drop in price (bad news) increases volatility more than an unexpected increase in price (good news).

Diagnostic tests introduced by Engle and Ng (1993) including sign bias test, negative sign bias, and positive sign bias. These tests will be used to check whether there is leverage effect in the DJIA returns.

The diagnostic procedure is to test for the significance of $\beta_1$ in the regression:

$$\hat{\epsilon}_t^2 = \beta_0 + \beta_1 \hat{\omega}_{t-1} + \xi_t$$

Let $$S_{t-1} = \begin{cases} 1, & if \quad \epsilon_{t-1} < 0 \\ 0, & if \quad \epsilon_{t-1} \geq 0 \end{cases}$$

Then $$\hat{\omega}_{t-1} = \begin{cases} S_{t-1}, & sign\ bias\ test \\ S_{t-1}\, \epsilon_{t-1} & negative\ sign\ size\ bias\ test \\ (1 - S_{t-1})\epsilon_{t-1} & positive\ sign\ size\ bias\ test \end{cases}$$

The table 3.1 shows the sign bias test result from the model GARCH (1,1)-ARMA(1,1) with lagged happiness in the mean equation. This model is chosen as one representative model out from six basic models from chapter 2.

Table 7. T-value and p-value of diagnostic tests for the estimated GARCH (1,1)-ARMA(1,1) with lag $H_t$ in the mean equation.

| Diagnostic Test | t-value | p-value |
|---|---|---|
| Sign Bias | 2.565 | 0.0104 ** |
| Negative Sign Bias | 1.372 | 0.1702 |
| Positive Sign Bias | 1.755 | 0.0795 * |

The sign bias and positive sign bias are significant. These statistics indicate that the sign and size of the volatility from last period does influence the current volatility. Therefore, there is asymmetric effect in the stock market volatility. The reason is quite obvious: in the basic GARCH model, since only squared residuals $\varepsilon_{t-i}^2$ enter the conditional variance model. It assumes the squared values of the residuals have a symmetric response to shocks. So the asymmetric extensions are often needed.

**EGARCH** model is used to deal with the leverage effect. In GARCH model, we assume that good and bad news have same effects on the volatility. In the real world, however, the volatility usually increased more after bas news compared to the good news. Exponential GARCH (EGARCH) model:

$$r_t = \varepsilon_t h_t$$

Where

$$\log(\mathrm{h}_\mathrm{t}^2) = \alpha_0 + \sum_{i=1}^{p}[\alpha_i r_{t-i} + \gamma_i(|r_{t-i}| - \mathrm{E}(\,|r_{t-i}|))] + \sum_{j=1}^{q}\beta_j \log(\mathrm{h}_{\mathrm{t-j}}^2)$$

The EGARCH(1,1) model will be

$$r_t = \varepsilon_t h_t$$

$$\log(\mathrm{h}_\mathrm{t}^2) = \alpha_0 + \alpha_1 r_{t-1} + \gamma_1(|r_{t-1}| - \mathrm{E}(|r_{t-1}|))] + \beta_1 \log(h_{t-1}^2)$$

In the EGARCH model, the coefficients $\alpha_i$ capture the sign effect. If the leverage effect does happen, $\alpha_i$ should be negative numbers. The parameters $\gamma_i$, on the other hand, capture the size effect. The bigger $\gamma_i$ imply a larger leverage effect, so it should be positive numbers.

Another model for asymmetric GARCH specification is the **GJR-GARCH** model:

$$r_t = \varepsilon_t h_t$$

$$h_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i r_{t-i}^2 + \gamma_i S_{t-i} r_{t-i}^2) + \sum_{j=1}^{q} \beta_j h_{t-j}^2$$

$$S_{t-i} = \begin{cases} 1, & if \ X_{t-i} < 0 \\ 0, & if \ X_{t-i} > 0 \end{cases}$$

The GJR-GARCH(1,1) model will be:

$$r_t = \varepsilon_t h_t$$

$$h_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \omega_1 S_{t-1} r_{t-1}^2 + \beta_1 h_{t-1}^2$$

$$S_{t-1} = \begin{cases} 1, \ if \ r_{t-1} < 0 \\ 0, \ if \ r_{t-1} > 0 \end{cases}$$

In the GJR-GARCH model, if $r_{t-i}$ is positive, the total effects are $\alpha_i r_{t-i}^2$. When $r_{t-1}$ is negative, the total effects are $(\alpha_i + \gamma_i) r_{t-i}^2$. Therefore, leverage effect implies that $\gamma_i$ are positive numbers.

### Comparison Between EGARCH and GJR-GARCH Models

The table 8 displays the AIC of EGARCH and GJR-GARCH model. GJR-GARCH seems to be the right model to choose because it has lower AIC. However, when the signbias is tested for the GJR-GARCH, the result is not as good as EGARCH. The possible reason for this is the exponential functional form of EGARCH. EGARCH model actually creates a ridiculously high variance because of the exponential function.

Table 8. Model selection for EGARCH and GJR-GARCH models.

| Model | AIC (EGARCH) | AIC (GJR-GARCH) |
|---|---|---|
| Basic GARCH | 4149.219 | 4140.023 |
| Sim $H_t$ in mean | 4145.59 | 4138.557 |
| Lag $H_t$ in mean | 4142.426 | 4136.024 |
| Sim $H_t$ in mean and lag $H_t$ in variance | 4135.407 | 4135.522 |
| Sim lag $H_t$ both in mean | 4139.209 | 4132.701 |

The table 9 provides the results from the diagnostic tests for EGARCH and GJR-GARCH models. The diagnostic bias test for the GJR-GARCH model with lagged happiness in mean could work as one representative table for all the GJR-GARCH models. All these six GJR-GARCH models still have the sign bias problem (all the results for signbias test is included in the table A.4 in the appendix).

Table 9. T-value and p-value of diagnostic tests for the estimated GJR-GARCH (1, 1) with lag $H_t$ in mean equation.

| Diagnostic Test | t-value | p-value |
|---|---|---|
| Sign Bias | 2.121 | 0.0340 ** |
| Negative Sign Bias | 2.278 | 0.0227 ** |
| Positive Sign Bias | 1.694 | 0.0903 * |

Table 10. T-value and p-value of diagnostic tests for the estimated EGARCH (1,1) with lag $H_t$ in mean equation.

| Diagnostic Test | t-value | p-value |
|---|---|---|
| Sign Bias | 1.274 | 0.2028 |
| Negative Sign Bias | 1.543 | 0.1228 |
| Positive Sign Bias | 1.366 | 0.1721 |

The test results from the EGARCH model have no significant values which means the leverage effect has been taken care of by the model. Comparing the results from these two tables, although the EGARCH models have bigger AICs, its power to deal with leverage effect is obvious stronger than the GJR-GARCH models.

**EGARCH and GJR-GARCH Models with ARMA Component**

The ARMA component is added to the EGARCH and GJR-GARCH models in order to remove the potential autocorrelations in the residuals. Figure 6 shows the correlogram of the residuals of the EGRACH model with simultaneous happiness in the mean equation and lagged happiness in the variance. The significant value at lag one is the reason to add ARMA component into EGARCH and GJR-GARCH models.

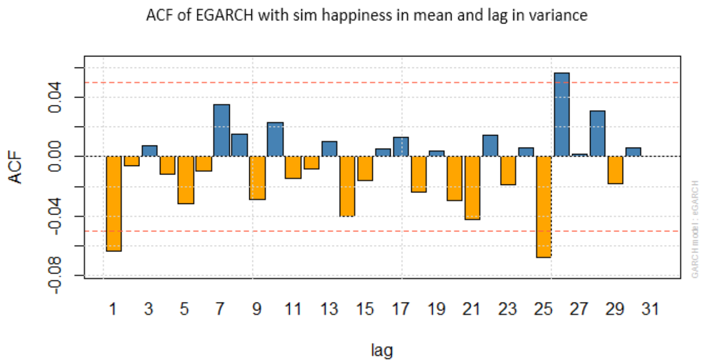


Figure 6. The correlogram of the standard residuals from EGARCH with sim. $H_t$ in the mean equation and lag $H_t$ in the variance equation.

The estimates of the parameters for the EGARCH-ARMA models are in the table A.5 in the appendix. The table indicates that most of the ARMA parameters are not significant. The only model that has significant AR and MA parameters is the EGARCH-ARMA model with simultaneous happiness data in the mean equation and the lagged happiness data in the variance. However, the lagged happiness is not significant anymore when the ARMA component is added into this EGARCH model.

The estimates of the GJR-GARCH-ARMA parameters are displayed in table A.6 in the appendix. The GJR-GARCH-ARMA with simultaneous happiness data in the mean equation and the lagged happiness data in the variance is the only one model that has significant parameters. The signbias test results from GJR-GARCH-ARMA model in table 11 still indicates that GJR-GARCH-ARMA model is not preferable in dealing with the asymmetric problems compared to the EGARCH model.

Table 11. T-value and p-value of diagnostic tests for the estimated GJR-GARCH (1, 1) – ARMA(1,1) with sim $H_t$ in mean and lag $H_t$ in variance.

| Diagnostic Test | t-value | p-value |
|---|---|---|
| Sign Bias | 2.044 | 0.0411 ** |
| Negative Sign Bias | 2.394 | 0.0167 ** |
| Positive Sign Bias | 1.743 | 0.0815 * |

The ARMA component is not significant when added to the EGARCH model and the GJR-GARCH-ARMA model still has limited ability to deal with the asymmetric problem. Hence, the EGARCH models without ARMA component will be the models discussed in the next section.

**Comparison between EGARCH and GARCH Models**

From last section, the selected models are the EGARCH models for the asymmetric problems after testing potential ARMA intensions to the EGARCH and GJR-GARCH models. In this section, the compassion between EGARCH models and GARCH models will be addressed to show the benefice to use the EGARCH models.

Table 12. Models comparison between EGARCH and basic GARCH models.

| Model | AIC(GARCH) | AIC(EGARCH) |
|---|---|---|
| Basic GARCH | 4208.592 | 4149.219 |
| Sim $H_t$ in mean | 4204.35 | 4138.511 |
| Lag $H_t$ in mean | 4202.915 | 4142.426 |
| Sim $H_t$ in mean lag $H_t$ in variance | 4202.589 | 4135.407 |
| Sim lag $H_t$ both in mean | 4201.859 | 4139.209 |

In the table 12, the decreasing AIC from basic GARCH to EGARCH implies that the EGARCH model does help to better fit the return volatility. Among all the asymmetric models, EGARCH model with simultaneous and lagged happiness data both in the mean equationhas the lowest AIC.

Table 13. Estimates of EGARCH(1,1) with simultaneous and lagged happiness in the mean equation.

| EGARCH | μ | ω | α | β | $\gamma$ | $\tau_1$ | $\tau_2$ |
|---|---|---|---|---|---|---|---|
| with sim lag | -2.5221 *** | 0.0006 | -0.1668*** | 0.9752*** | 0.1477** | 1.6555 *** | -1.2306 *** |
| $H_t$ in mean | (0.2195) | (0.0072) | (0.0126) | (0.0007) | (0.0655) | (0.0083) | (0.0468) |

Table 13 provides the estimated parameters for the EGARCH(1,1) model with happiness and all of them are significant.

EGARCH(1,1) with simultaneous and lagged happiness both in the mean equation:

$$r_t = -2.5221 + \varepsilon_t h_t + 1.6555 H_t - 1.2306 H_{t-1}$$

$$\log(h_t^2) = 0.0006 - 0.1668 r_{t-1} + 0.1477(|r_{t-1}| - \mathrm{E}(|r_{t-1}|))] + 0.9752 \log(h_{t-1}^2)$$

The parameter α is less than zero, which means the leverage effect does happen. The news impact curve is used to check the effect of news on conditional heteroskedasticity.

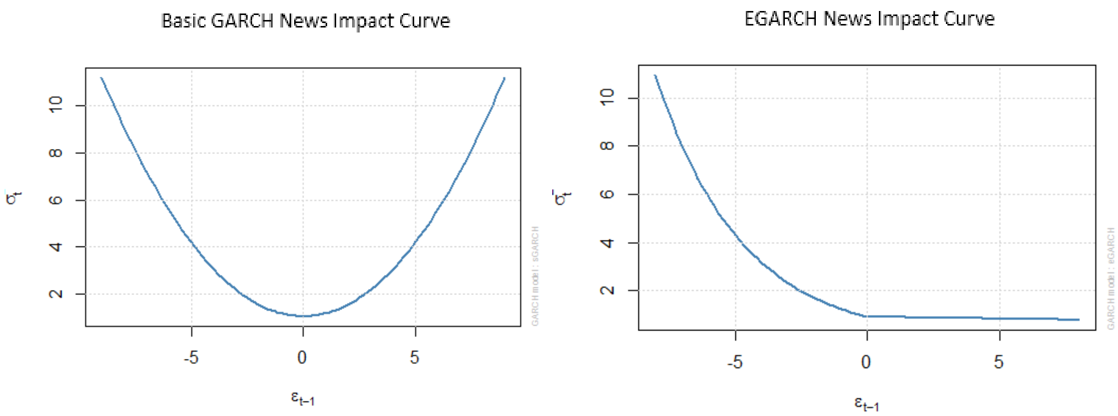


Figure 7. News impact curve for the GARCH model and the EGARCH.

The news impact curve is the functional relationship between conditional variance at time t and the shock at time t-1, assuming all the information before time t-2 is constant. Difference between these two plots is quite clear. The curve in the first plot is symmetric meaning the shock has the same impact on the conditional variance no matter it is positive or negative. The second curve is asymmetric, that is why EGARCH model allows good news and bad news to have different impact on volatility. The leverage effect implies that the bad news tends to increase the volatility more than the good news does. This is why the curve has a steeper slope in the left part.

3.2 Heavy Tails

One of the features of the financial series is the observed excess of kurtosis in the error distribution which also means heavy tails exist in the distribution. The classic GARCH assumes the error is normally distributed, but in reality, this is often not the case. The extensions of models to other distributions with heavier tails are needed. The QQ plot from basic models also shows that heavy tails problem exists. A few more distributions are needed instead of only using normal distribution in the GARCH model to deal with it. The possible distributions are Student's T, the generalized error, and the generalized hyperbolic distributions.

In addition, excess of skewness is another issue with the financial series. There are other distributions with both heavy tails and skewness like skewed student's T distribution, and generalized hyperbolic distribution.

**Student's T Distribution**

The density function is

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\beta\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1 + \frac{(x-\alpha)^2}{\beta\nu}\right)^{-\frac{\nu+1}{2}}$$

where $\alpha, \beta,$ and $\nu$ are the location, scale and the shape parameters and $\Gamma$ is the gamma function which is defined as

$$\Gamma(t) = \int_0^{\infty} x^{t-1} e^{-x}\, dx$$

Table 14. Model selection for EGARCH model with t distribution.

| Models | AIC |
|---|---|
| T with EGARCH | 4100.648 |
| Sim $H_t$ in mean T with EGARCH | 4097.65 |
| Lag $H_t$ in mean T with EGARCH | 4095.676 |
| Sim $H_t$ in mean lag $H_t$ in variance T with EGARCH | 4092.581 |
| Sim lag $H_t$ both in mean T with EGARCH | 4092.332 |

The selected model based on AIC is the EGARCH-T with simultaneous and lagged happiness data in the mean function. The estimated parameters are in the table 15.

Table 15. Estimates for EGARCH-T model with sim and lag $H_t$ in the mean equation.

| Parameters | μ | ω | α | β |
|---|---|---|---|---|
| EGARCH-T with sim and lag $H_t$ in the mean equation | -2.5221 ***<br>(0.0207) | -0.0102 ***<br>(0.0061) | -0.1892 ***<br>(0.0193) | 0.9806 ***<br>(0.0005) |
| | γ | $\nu$ | $H_t$ in mean | lag $H_t$ in mean |
| | 0.1570 ***<br>(0.0197) | 6.5345***<br>(1.1330) | 1.5071 ***<br>(0.0013) | -1.0772 ***<br>(0.0003) |

The estimated GARCH coefficients α and β are significant at 1% level, and the sum of them is less than one implies that the GARCH model is stationary. The estimated degree of freedom of the conditional t-distribution is 6.53 which means that the return is conditionally non-normally distributed. According to Connolly (1989), the estimated degree of freedom may indicate the source of the excess kurtosis in the return. If it is less than 10, both non-normality and conditional heteroskedasticity explain the excess kurtosis, where as if it is bigger than 30, the conditional heteroskedasticity is the only source of heavy tails in the return. Therefore, both non-noamality and conditional heteroskedasticity explain the excess turtosis.

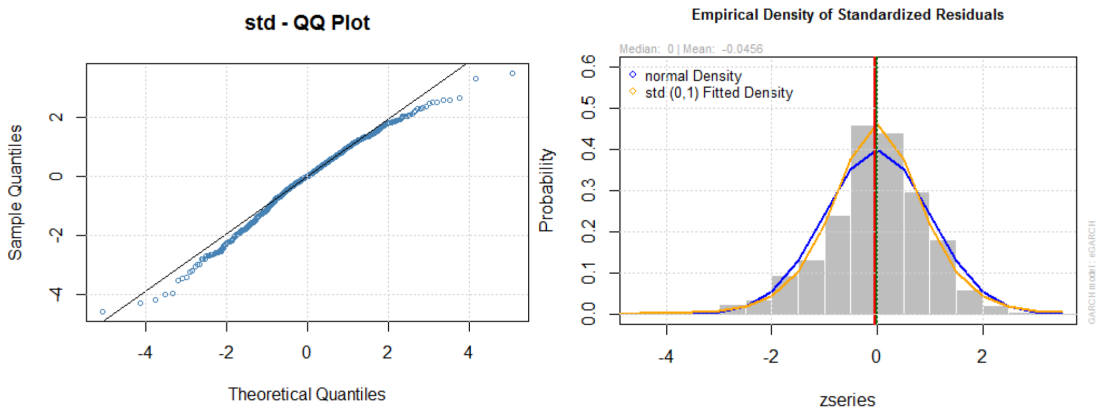


Figure 8. Standard normal QQ plot and density of the standardized residuals from the EGARCH-T model.

The QQ plot helps to check the power of student’s T distribution. Comparing this plot to the one in data description, the heavy tails problem is not an issue anymore. The density of the standardized residual shows that student’s T distribution captures the shapes of the residuals more accurately than the normal distribution.

**Generalized Error Distribution**

Generalized Error Distribution (GED) is defined with parameter $\nu > 0$. If x is GED distributed then

$$f(x) = \frac{\nu \exp(-0.5\left|\frac{x}{\lambda}\right|^{\nu})}{\nu \cdot 2^{\frac{\nu+1}{\nu}}\Gamma(\frac{1}{\nu})}$$

Where

$$\lambda = \sqrt{\frac{2^{-\frac{2}{\nu}}\Gamma(\frac{1}{\nu})}{\Gamma(\frac{3}{\nu})}}$$

Table 16. AICs for the EGARCH-GED models.

| Models | AIC |
|---|---|
| Ged with EGARCH | 4093.532 |
| Sim $H_t$ in mean | 4089.805 |
| Lag $H_t$ in mean | 4088.231 |
| Sim $H_t$ in mean lag $H_t$ in variance | 4083.961 |
| Sim lag $H_t$ both in mean | 4081.873 |

The GARCH-GED model with simultaneous and lagged happiness data in the mean equation should be the model to choose based on the AIC. The parameters estimated are in the table 17.

Table 17. Estimates for EGARCH-GED model with simultaneous and lagged happiness data in the mean function.

| Parameters | μ | ω | α | β |
|---|---|---|---|---|
| EGARCH-GED with sim and lag $H_t$ in mean | -2.5221 *** (0.0141) | -0.0112 *** (0.0049) | -0.1806 *** (0.0126) | 0.9784 *** (0.0005) |
| | γ | $\nu$ | $H_t$ in mean | lag $H_t$ in mean |
| | 0.1556 *** (0.0158) | 1.3309 *** (0.0756) | 1.8740 *** (0.0013) | -1.4447 *** (0.0006) |

In the generalized error distribution, if $\nu$ is between 0 and 2, the distribution will have a fatter tail than normal distribution; if $\nu$ equals to 2, it is the normal distribution. In our case, the estimated $\nu$ is 1.33 which is less than 2. This means the distribution has a fatter tail than normal distribution which is also shown in the plot in figure 9.

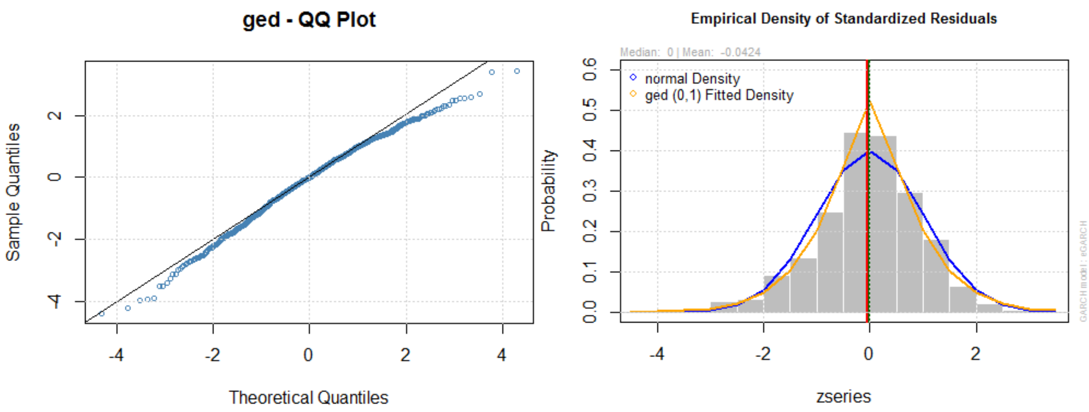


Figure 9. Standard normal QQ plot and density of the standardized residuals from the EGARCH-GED model.

The performance of GED is similar to the student's T distribution. For the return data, GED seems preferable based on AIC, but the difference between them is small. These plots provide the same information as the plots for the student's T distribution, and the other issue is the excess skewness.

**Generalized Hyperbolic Distribution**

The generalized hyperbolic distribution can be parameterized as (Prause, 1999)

$$\mathrm{f}\,(\mathrm{x}) = \frac{(\alpha^2-\beta^2)^{\frac{\lambda}{2}} K_{\lambda-\frac{1}{2}}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)\exp(\beta(x-\mu))}{\sqrt{2\pi}\,\alpha^{\lambda-\frac{1}{2}}\delta^{\lambda}\left(\delta\sqrt{\alpha^2-\beta^2}\right)\left(\sqrt{\delta^2-(x-\mu)^2}\right)^{\frac{1}{2}-\lambda}}$$

In the above expression, $K_j$ is the modified Bessel function of the third kind of order j (Abramowitz and Stegun, 1972) and

$$\delta \geq 0, |\beta| < \alpha \quad \text{if } \lambda > 0$$

$$\delta > 0, |\beta| < \alpha \quad \text{if } \lambda = 0$$

$$\delta > 0, |\beta| \leq \alpha \quad \text{if } \lambda < 0$$

According to Necula, C. (2009), $\alpha$ determines the shape, $\beta$ determines the skewness, $\mu$ is a location parameter, $\delta$ serves for scaling, and λ influences the kurtosis and represents the subclass of the generalized hyperbolic distribution. The first important subclass is when λ =1, the GED will become Hyperbolic Distribution. The second subclass is when λ =-0.5, this distribution is Normal Inverse Gaussian Distribution (NIG).

Table 18. AICs for the EGARCH-GH models.

| Models | AIC |
|---|---|
| GH EGARCH | 4073.774 |
| Sim $H_t$ in mean GH EGARCH | 4070.793 |
| Lag $H_t$ in mean | 4068.621 |
| Sim $H_t$ in mean and lag $H_t$ in variance | 4119.513 |
| Sim lag $H_t$ both in mean | 4066.458 |

The model selected is the EGARCH-GHD with simultaneous happiness in the mean function and lagged happiness in the variance function. Although the last model is the one with lowest AIC, the parameter λ is not significant. The estimated parameters are presented in the table 19.

Table 19. Estimates for EGARCH-GH model

| Parameters | μ | ω | α | β | γ |
|---|---|---|---|---|---|
| EGARCH-GH with sim $H_t$ in mean and lag $H_t$ in variance | -1.8419 *** (0.0100) | -6.1731 *** (0.1650) | -0.2890 *** (0.0467) | 0.9000 *** (0.0482) | 0.2577 *** (0.0635) |
| | skew | Shape | λ | $H_t$ in mean | Lag $H_t$ in variance |
| | -0.2077*** (0.0691) | 1.4103 *** (0.5954) | 0.6914*** (1.1461) | 0.3069 *** (0.0017) | 1.0291 *** (0.0278) |

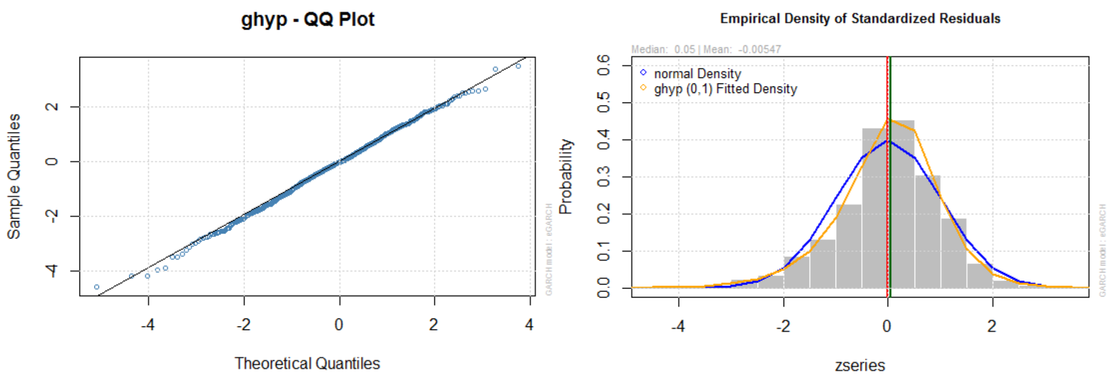


Figure 10. Standard normal QQ plot and density of the standardized residuals from the EGARCH-GH model.

The skewness parameter is -0.2 < 0, which implies the distribution is skewed to the left. The skewness is also shown in the density plot. The GHD deals with the skewness as well as the heavy tails. Also displayed in the QQ plot, the GHD has one almost perfect performance dealing with the heavy tail problem.

**Skewed Student's T Distribution**

Skewed student's T distribution can be defined in many ways. In this study, skewed student's T distribution will be defined as a limiting case of the Generalized Hyperbolic distribution.

Let $\lambda = -\nu/2$ and $\alpha \to |\beta|$ in the generalized hyperbolic distribution, it will become generalized hyperbolic skewed student distribution. This distribution is popularized by Aas and Haff (2006) because of its uniqueness in having one tail with polynomial and one with exponential behavior. The skewness and kurtosis do not exist when $\nu \leq 6$, and $\nu \leq 8$, respectively. The density function is given by

$$F(x) = \frac{2^{\frac{1-\nu}{2}} \delta^{\nu} |\beta|^{\frac{\nu+1}{2}} K_{\frac{\nu+1}{2}}\left(\sqrt{\beta^2(\delta^2+(x-\mu)^2)}\right) \exp(\beta\,(x-\mu\,))}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\pi}\, \left(\sqrt{\delta^2+(x-\mu)^2}\,\right)^{\frac{\nu+1}{2}}}, \quad \beta \neq 0,$$

And

$$F(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi}\,\delta\,\Gamma\left(\frac{\nu}{2}\right)} \left[1 + \frac{(x-u)^2}{\delta^2}\right]^{-(\nu+1)2}, \quad \beta = 0$$

Table 20. AICs for the EGARCH-ST models.

| Models | AIC |
|---|---|
| Skew T EGARCH | 4078.875 |
| Sim $H_t$ in mean | 4076.136 |
| Lag $H_t$ in mean | 4073.824 |
| Sim $H_t$ in mean lag $H_t$ in variance | 4070.91 |
| Sim lag $H_t$ both in mean | 4072.222 |

Based on the AIC, the recommended model is the model with simultaneous happiness in mean and lagged happiness data in variance. The estimated parameters are showed in the table 21.

Table 21. Estimates for the EGARCH-ST model with simultaneous happiness in mean and lagged happiness in variance.

| Parameters | μ | ω | α | β | γ |
|---|---|---|---|---|---|
| EGARCH-ST with sim $H_t$ in mean and lag $H_t$ in variance | -2.5221 ***<br>(0.0086) | -1.2455 ***<br>(0.2263) | -0.2027 ***<br>(0.0219) | 0.9735 ***<br>(0.0007) | 0.1503 ***<br>(0.0273) |
| | skew | Shape | λ | $H_t$ in mean | Lag $H_t$ in variance |
| | 0.8410 ***<br>(0.0293) | 7.8809 ***<br>(1.5689) | 0.6914***<br>(1.1461) | 0.4251 ***<br>(0.0014) | 0.2068 ***<br>(0.0376) |

The skew parameter is the skewness of the distribution. It is 0.84 which is slightly bigger than 0 meaning it is positively skewed. The shape parameter is 7.88 which is almost 8, the existence of kurtosis is uncertain.

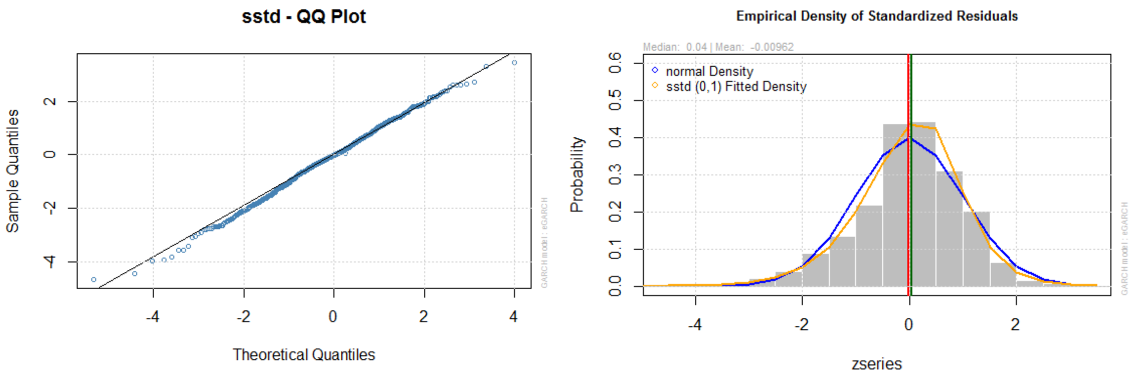


Figure 11. Standard normal QQ plot and density of the standardized residuals from the EGARCH-SSTD model.

These two plots in the figure 11 provide very similar results as the generalized hyperbolic distribution does. Compared with the results from GED, generalized hyperbolic skewed student distribution is not as good as GED. The tail is still a little heavy to the left in the QQ plot and the fitting of the residuals is definitely much better than normal distribution but not as good as generalized hyperbolic distribution.

To sum up, the generalized hyperbolic distribution has the biggest power to deal with the heavy tail problem and excess skewness. The simultaneous happiness data works in the mean equation and the lagged happiness data works in the variance function.

CHAPTER 4

FORECASTING THE RETURN VOLATILITY

## 4.1 Forecasting Methods

**Unconditional Forecasting**

Unconditional forecasting, also named as recursive forecasting. It means a series of data is used to predict the data n times ahead. Mean Squared Forecasting Error (MSE) and Mean Absolute Forecasting Error (MAE) are calculated to measure the error of the forecasting. Assume $y_T$ is the absolute value of the return at time T and $\widehat{y_T}$ is the estimated conditional variance. The mean squared forecasting error and the mean absolute forecasting error are used to test the accuracy of the forecasting.

The Mean Squared Forecasting Error is

$$\text{MSE} = \frac{\sum_{i=0}^{n}(y_{T+i} - \widehat{y_{T+i}})^2}{n}$$

The Mean Absolute Forecasting Error is

$$\text{MAE} = \frac{\sum_{i=0}^{n}|y_{T+i} - \widehat{y_{T+i}}|}{n}$$

**Rolling Forecasting**

Another way to do the forecasting is to use the rolling forecasting method. There is one set of time period in the rolling forecast which will shift each time a new value is collected. The number of rolling is set to be 100 in this research and each time get the 1 step ahead forecasting. The length of dataset is 1528, so the first 1429 data is used as the training data and the last 100 data is used as the testing data. For the rolling forecasting, MSE and MAE are also the ways to get a cumulative measurement of the error over the forecasting range.

4.2 Diebold-Mariano Tests for Predictive Accuracy

The Diebold-Mariano (DM) Test is used to check whether two models are equally good about the forecasting. We assume the actual values are $\{y_t$; t=1,2,…, T$\}$ and the two forecasting values are $\widehat{y_{1t}}$ $\{\widehat{y_{1t}}$; t=1,2,…, T$\}$and $\{\widehat{y_{2t}}$; t=1,2,…, T$\}$. The error is defined as $\varepsilon_i = \widehat{y_{\iota t}} - y_t$, (i=1,2). The loss function is the square or the absolute value of the error as g($\varepsilon_i$)=$\varepsilon_i{}^2$ or g($\varepsilon_i$)= |$\varepsilon_i$|. The Diebold-Mariano (DM) Test is based on the loss differential $d_t$= g($\varepsilon_1$)- g($\varepsilon_2$) and we say that the two forecast are equally good if the differential has zero expected value. So the null hypothesis is $H_0$ : E ($d_t$) =0 versus the alternative hypothesis $H_1$ : E ($d_t$) $\neq$0. In the DM test, one density used is the spectral density: $f_d(0)$ is the spectral density of the loss differential at frequency 0, which means

$$f_d(0) = \frac{1}{2\pi}\left(\sum_{k=-\infty}^{\infty} \gamma_d(k)\right)$$

$\gamma_d(k)$ is the autocovariance of the loss differential at lag k.

The DM test statistics is

$$\text{DM} = \frac{\bar{d}}{\sqrt{\frac{2\pi \hat{f}_d(0)}{T}}}$$

Where $\hat{f}_d(0)$ is a consistant estimate of the $f_d(0)$ defined by

$$\hat{f}_d(0) = \frac{1}{2\pi}\sum_{k=-(T-1)}^{T-1} I\left(\frac{k}{h-1}\right)\hat{\gamma}_d(k)$$

Where

$$\hat{\gamma}_d(k) = \frac{1}{T}\sum_{t=|k|+1}^{T}(\,d_t - \bar{d})\,(d_{t-|k|} - \bar{d})$$

And $$I\left(\frac{k}{h-1}\right) = \begin{cases} 1\ for\ \left|\frac{k}{h-1}\right| \le 1 \\ 0\quad elsewhere \end{cases}$$

Under the null hypothesis, the DM test statistics is N(0,1) distributed. So if the DM statistics falls outside of ($-Z_{\alpha/2}$, $Z_{\alpha/2}$), we will reject the null hypothesis. That is to say the two models have differences in the prediction accuracy.

4.2 Results

**Unconditional Forecasting Results**

The table 22 shows the MSE and MAE from the unconditional forecasting. In this conditional forecasting, 10 data points are estimated using unconditional forecasting and compared it to the absolute value of the returns. The models with generalized hyperbolic distributed errors are more preferable than basic GARCH model based on AIC. However, the model with lower AIC has bigger forecasting errors which indicates lower forecasting accuracy.

Table 22. Unconditional forecasting evaluation of selected models.

| Model | AIC | MSE | MAE |
|---|---|---|---|
| Garch (1,1) | 4208.592 | 0.4101 | 0.4912 |
| Garch(1,1)-ARMA(1,1) | 4204.822 | 0.4168 | 0.4932 |
| Egarch (1,1) | 4149.219 | 0.5030 | 0.5143 |
| Lag $H_t$ in variance sim $H_t$ in mean egarch | 4135.407 | 0.5155 | 0.5208 |
| Lag sim $H_t$ both in mean egarch | 4139.209 | 0.5104 | 0.5161 |
| Egarch(1,1) GH | 4073.774 | 0.5510 | 0.5447 |
| Lag $H_t$ in variance sim $H_t$ in mean egarch GH | 4063.131 | 0.5571 | 0.5492 |
| Lag sim $H_t$ both in mean egarch GH | 4056.864 | 0.5372 | 0.5341 |

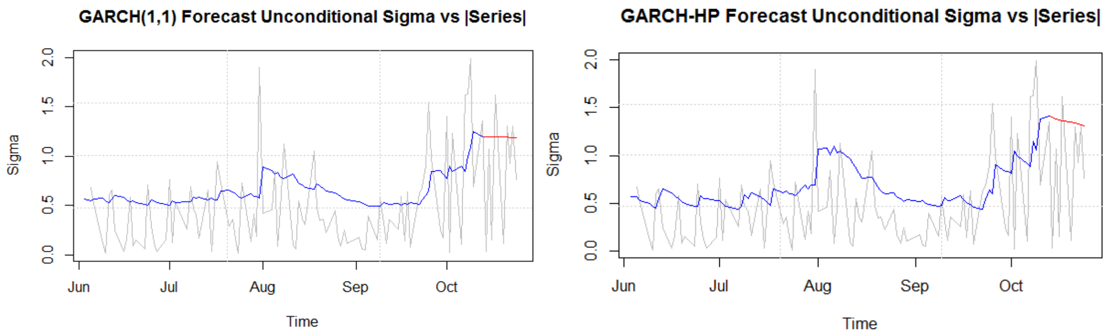


Figure 12. Unconditional forecasting comparison between GARCH and GARCH-GH models.

In the figure 12, the grey line is the absolute value of the returns which works as a proxy of the volatility, the blue line is the fitted conditional variance from the model and the red line represents ten predicted values. The GARCH(1,1) model and the EGARCH-GH (generalized hyperbolic) with lagged happiness in variance and simultaneous one in mean are the two models under comparison. There is clear evidence that the GARCH-GH model captures volatility well which is also indicated by the lower AIC of the model. However, it is not quite clear whether they have the same accuracy or not based only on these two plots. The DM test is shown in the table 23.

Table 23. Diebold-Mariano tests results for unconditional forecasting

| Loss function type | DM statistics | P-value |
|---|---|---|
| Absolute value | -1.2386 | 0.2468 |
| Square value | -2.1005 | 0.06506 |

None of the p-values in table 4.2 is significant, so we cannot reject the null hypothesis which is the two models have the same forecast accuracy. In other words, more preferable fitting models are not statistically different from other models in the forecasting performance.

The sophisticated models are tested about the forecasting power and the result indicates that they actually don't have a better performance compared to GARCH model. This raised the question whether the happiness data helps in the forecasting of basic GARCH models.

Table 24. Unconditional forecasting errors of GARCH models

| Model | MSE | MAE |
|---|---|---|
| Garch (1,1) | 0.4101 | 0.4912 |
| Garch(1,1)-ARMA(1,1) | 0.4168 | 0.4932 |
| GARCH with happiness in mean | 0.4103 | 0.4913 |
| GARCH with lag happiness in mean | 0.4103 | 0.4913 |
| GARCH sim in mean lag happiness in variance | 0.4089 | 0.4908 |
| GARCH lag sim happiness both in mean | 0.4095 | 0.4910 |

The last two models in the table 24 are the two candidates that may prove happiness data improves the forecasting performance of GARCH models. However, as the MSE and MAE are so close, we still need to use DM test to confirm that.

Table 25. D-M test results for GARCH with happiness data

| Model | GARCH sim in mean lag happiness in variance | | GARCH lag sim happiness both in mean | |
|---|---|---|---|---|
| Loss function type | DM statistics | P-value | DM statistics | P-value |
| Absolute value | 0.5910 | 0.5690 | 0.5378 | 0.6037 |
| Square value | 1.8464 | 0.0979 | 1.7565 | 0.1128 |

The DM test results still indicate that the happiness data does not help when added into GARCH models in the forecasting performance.

**Rolling Forecasting Results**

In the unconditional forecasting, only 10 points are estimated. In the rolling forecasting, 100 points are estimated out of sample. The table 26 contains the results from rolling forecasting.

Table 26. Rolling forecasting evaluation of selected models.

| Model | AIC | MSE | MAE |
|---|---|---|---|
| Garch (1,1) | 4208.592 | 0.1787 | 0.3529 |
| Garch(1,1)-ARMA(1,1) | 4204.822 | 0.1798 | 0.3537 |
| Egarch (1,1) | 4149.219 | 0.2081 | 0.3759 |
| Lag $H_t$ in variance sim $H_t$ in mean egarch | 4135.407 | 0.2612 | 0.4185 |
| Lag sim $H_t$ both in mean egarch | 4139.209 | 0.2292 | 0.3917 |
| Egarch(1,1)-GH | 4073.774 | 0.2160 | 0.3790 |
| Lag $H_t$ in variance sim $H_t$ in mean egarch GH | 4063.131 | 0.2369 | 0.3950 |
| Lag sim $H_t$ both in mean egarch GH | 4056.864 | 0.2375 | 0.3951 |

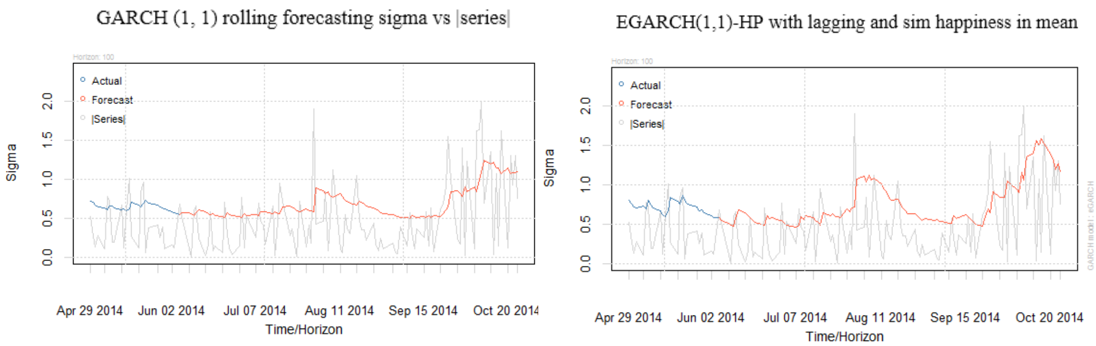


Figure 13. Rolling forecasting comparison between GARCH and GARCH-GH models.

The figure 13 indicates the EGARCH(1,1)-GH with lagged and simultaneous happiness in the mean equation has a better performance of estimation which is contradictory to the MSE and MAE results. Again, DM test is used to find out whether these two models have different estimation accuracy.

Table 27. Diebold-Mariano tests results for rolling forecasting

| Loss function type | DM statistics | P-value |
|---|---|---|
| Absolute value | -1.1225 | 0.2643 |
| Square value | -1.3940 | 0.1664 |

The p-values in table 27 are all bigger than 0.05 which means none of them are significant. Hence, we have no enough evidence to reject the null hypothesis meaning that the two models are not statistically different in the forecasting accuracy.

The table 28 presents the MSE and MAE for the GARCH model with happiness data. For the basic GARCH models with happiness data, there seems no improvement for adding happiness data in the model. When the error distribution changes to student's T distribution, generalized hyperbolic distribution and skewed T distribution in the GARCH model, the errors are dropped significantly. This phenomenon is quite easy to understand as these distributions deal with the skewness and heavy tail problems. However, when happiness is added into these models, although the forecasting is more accurate compared to the GARCH(1,1) model, the MAE and MSE are bigger than the GARCH models with these distributions only. Therefore, there is still no evidence to indicate the happiness data helps to improve the forecasting accuracy.

Table 28. Rolling forecasting MSE and MAE for GARCH models

| Model | MSE | MAE |
|---|---|---|
| Garch (1,1) | 0.1787 | 0.3529 |
| Garch(1,1)-ARMA(1,1) | 0.1798 | 0.3537 |
| GARCH with happiness in mean | 0.1788 | 0.3528 |
| GARCH with lag happiness in mean | 0.2202 | 0.3855 |
| GARCH sim in mean lag happiness in variance | 0.2495 | 0.4097 |
| GARCH lag sim happiness both in mean | 0.2185 | 0.3840 |
| **GARCH with student's T** | | |
| GARCH(1,1) T | 0.1783 | 0.3516 |
| GARCH T happiness in mean | 0.1786 | 0.3518 |
| GARCH T lag happiness in mean | 0.1787 | 0.3518 |
| GARCH T lag in variance and sim in mean | 0.1790 | 0.3522 |
| GARCH T lag sim both in mean | 0.1795 | 0.3529 |
| **GARCH with GH** | | |
| GARCH(1,1) GH | 0.1745 | 0.3488 |
| GARCH GH happiness in mean | 0.1747 | 0.3487 |
| GARCH GH lag happiness in mean | 0.1746 | 0.3486 |
| GARCH GH lag in variance and sim in mean | 0.1750 | 0.3491 |
| GARCH GH lag sim both in mean | 0.1760 | 0.3502 |
| **GARCH with skew T** | | |
| GARCH(1,1) ST | 0.1748 | 0.3489 |
| GARCH ST happiness in mean | 0.1751 | 0.3489 |
| GARCH ST lag happiness in mean | 0.1751 | 0.3488 |
| GARCH ST lag in variance and sim in mean | 0.1754 | 0.3492 |
| GARCH ST lag sim both in mean | 0.1759 | 0.3498 |

# CHAPTER 5

# CONCLUSION

This study has examined the impact of happiness data on the daily DJIA return volatility. The happiness data does help to fit the conditional variance but it does not help to improve the forecasting accuracy of the return volatility.

At the beginning, we find out the GARCH(1,1) model is appropriate to be the benchmark model. The ARMA component then is added to the GARCH(1,1) to remove the autocorrelation within the residuals. The happiness data was included in the GARCH(1,1)-ARMA(1,1) model in order to check the impact of happiness on the stock market return volatility. It turns out that simultaneous and lagged happiness data does help to better fit the return data.

The asymmetric models are raised to deal with potential leverage effects. The asymmetry test results indicate that the EGARCH model is more preferable compared to the GJR-GARCH model. Meanwhile, the ARMA component is not significant in the EGARCH models. Many other distributions such as student's T distribution, the generalized hyperbolic distribution are included as the extensions to the GARCH model. The generalized hyperbolic distribution is the preferable distribution to deal with asymmetric and heavy tails problems. In Chapter 4, forecasting accuracy is tested for representative models and the models with happiness does not have a better prediction accuracy compared to GARCH model.

# APPENDICES

Table A.1. Estimates of models tested in chapter 2.

| Models | GARCH(1,1) | GARCH(1,1) -ARMA(1,1) | GARCH(1,1)-ARMA(1,1) w/ $H_t$ in mean | GARCH(1,1)-ARMA(1,1) w/ lag $H_t$ in mean | GARCH(1,1)-ARMA(1,1) w/sim $H_t$ in mean lag $h_t$ in variance | GARCH(1,1)-ARMA(1,1) w/sim lag $H_t$ both in mean |
|---|---|---|---|---|---|---|
| μ | 0.0691*** | 0.0727*** | -2.6155*** | -2.0329*** | -2.5135*** | -2.5222*** |
| | (0.0184) | (0.0107) | (0.1359) | (0.5113) | (0.0059) | (0.0652) |
| ω | 0.0224*** | 0.0222*** | 0.0223*** | 0.0223*** | 0.0000 | 0.0223*** |
| | (0.0053) | (0.0053) | (0.0053) | (0.0053) | (0.00004) | (0.0053) |
| α | 0.1256*** | 0.1264*** | 0.1261*** | 0.1253*** | 0.1249*** | 0.1258*** |
| | (0.0239) | (0.0241) | (0.0243) | (0.0241) | (0.0237) | (0.0239) |
| β | 0.8564*** | 0.8558*** | 0.8561*** | 0.8565*** | 0.8569*** | 0.8559*** |
| | (0.0213) | (0.0214) | (0.0216) | (0.0216) | (0.0207) | (0.0214) |
| δ | | 0.9167*** | 0.9197*** | 0.9208*** | 0.9762*** | 0.9197*** |
| | | (0.0184) | (0.0372) | (0.0134) | (0.0027) | (0.0097) |
| $\gamma$ | | -0.9460*** | -0.9494*** | -0.9497*** | -0.9917*** | -0.9492*** |
| | | (0.0148) | (0.0289) | (0.00964) | (0.0001) | (0.0013) |
| $H_t$ in mean | | | 0.4484*** | 0.3513*** | 0.4293*** | 1.2655*** |
| | | | (0.0227) | (0.0858) | (0.0008) | (0.0033) |
| $H_t$ in variance | | | | | 0.0037*** | |
| | | | | | (0.0008) | |
| Lag $H_t$ in mean | | | | | | -0.8329*** |
| | | | | | | (0.0073) |

Table A.2. Estimates of EGARCH models in chapter 3.

| EGARCH | GARCH(1,1) | GARCH(1,1) w/ $H_t$ in mean | GARCH(1,1) w/ lag $H_t$ in mean | GARCH(1,1) w/sim$h_t$ in mean lag $H_t$ in variance | GARCH(1,1)- w/sim lag $H_t$ both in mean |
|---|---|---|---|---|---|
| μ | 0.0263* | -2.6155 *** | -2.5221 *** | -2.5221 *** | -2.5221 *** |
| | (0.0162) | (0.0107) | (0.0110) | (0.010) | (0.2195) |
| ω | 0.0017 | 0.0006*** | 0.0007 | -1.6460*** | 0.0006 *** |
| | (0.0055) | (0.0063) | (0.0062) | (0.2455) | (0.0072) |
| α | -0.1570*** | -0.1630 *** | -0.1640 *** | -0.1752 *** | -0.1668 *** |
| | (0.0199) | (0.0203) | (0.0203) | (0.0157) | (0.0126) |
| β | 0.9751*** | 0.9750 *** | 0.9747 *** | 0.9701 *** | 0.9752 *** |
| | (0.0009) | (0.0009) | (0.0009) | (0.0008) | (0.0007) |
| γ | 0.1578*** | 0.1506 *** | 0.1486*** | 0.1492*** | 0.1477*** |
| | (0.0282) | (0.0286) | (0.0282) | (0.0118) | (0.0655) |
| $H_t$ in mean | | | 0.4252*** | 0.4256 *** | 1.6555 *** |
| | | | (0.0018) | (0.0055) | (0.0083) |
| $H_t$ in variance | | | | 0.2742*** | |
| | | | | (0.0408) | |
| Lag $H_t$ in mean | | | | | -1.2306 *** |
| | | | | | (0.0468) |

Table A.3. Estimates of GJR-GARCH models in chapter 3.

| GJR-GARCH | GARCH(1,1) | GARCH(1,1) w/ $H_t$ in mean | GARCH(1,1) w/ lag $H_t$ in mean | Garch(1,1) w/sim $H_t$ in mean lag $H_t$ in variance | Garch(1,1)- w/sim lag $H_t$ both in mean |
|---|---|---|---|---|---|
| μ | 0.0270 | -2.6155 *** | -2.5221 *** | -2.5221 *** | -2.5221 *** |
| | (0.0187) | (0.0243) | (0.0258) | (0.0240) | (0.0177) |
| ω | 0.0216*** | 0.0220 *** | 0.0222 *** | 0.0000 *** | 0.0221 *** |
| | (0.0046) | (0.0046) | (0.0046) | (0.0001) | (0.0046) |
| α | 0.0000 | 0.0000 *** | 0.0000 *** | 0.0000 *** | 0.0000 *** |
| | (0.0237) | (0.0264) | (0.0261) | (0.0264) | (0.0264) |
| β | 0.8776*** | 0.8762 *** | 0.8754 *** | 0.8751 *** | 0.8748 *** |
| | (0.0254) | (0.0268) | (0.0269) | (0.0270) | (0.0269) |
| γ | 0.2027*** | 0.2040 *** | 0.2050 *** | 0.2060 *** | 0.2079 *** |
| | (0.0367) | (0.0372) | (0.0376) | (0.0376) | (0.0375) |
| $H_t$ in mean | | 0.4409*** | 0.4254 *** | 0.4253 *** | 1.6187 *** |
| | | (0.0037) | (0.0040) | (0.0037) | (0.0024) |
| $H_t$ in variance | | | | 0.0037 *** | |
| | | | | (0.0007) | |
| Lag $H_t$ in mean | | | | | -1.1940 *** |
| | | | | | (0.0028) |

Table A.4. Diagnostic tests results for GJR-GARCH models.

| Model(GJR) | GARCH(1,1) | | Sim $H_t$ in mean | | Lag $H_t$ in mean | |
|---|---|---|---|---|---|---|
| Diagnostic Test | t-value | p-value | t-value | p-value | t-value | p-value |
| Sign Bias | 2.2102 | 0.0272** | 2.4746 | 0.0134** | 2.1215 | 0.0340** |
| Negative Sign Bias | 2.3046 | 0.0213** | 2.4140 | 0.0158** | 2.2788 | 0.0228** |
| Positive Sign Bias | 1.6843 | 0.0923* | 1.5452 | 0.1224 | 1.6947 | 0.0903* |
| Model(GJR) | Sim $H_t$ in mean lag $H_t$ in variance | | Sim lag $H_t$ both in mean | | | |
| Diagnostic Test | t-value | p-value | t-value | p-value | | |
| Sign Bias | 2.2372 | 0.0254** | 2.3467 | 0.0190** | | |
| Negative Sign Bias | 2.3258 | 0.0201** | 2.3814 | 0.0173** | | |
| Positive Sign Bias | 1.6439 | 0.1003 | 1.6339 | 0.1024 | | |

Table A.5. Estimates of EGARCH-ARMA models.

| EGARCH-ARMA | GARCH(1,1) | GARCH(1,1) w/ $H_t$ in mean | GARCH(1,1) w/ lag $H_t$ in mean | GARCH(1,1) w/sim $H_t$ in mean lag $H_t$ in variance | GARCH(1,1)- w/sim lag $H_t$ both in mean |
|---|---|---|---|---|---|
| μ | 0.0295 | -2.6155 *** | -2.5221*** | -2.5219 *** | -2.5221 *** |
| | (0.01834) | (0.0111) | (0.0108) | (0.002) | (0.0176) |
| ω | 0.0017 | 0.0005 | 0.0008 | -0.9846 | 0.0007 |
| | (0.0059) | (0.0064) | (0.0063) | (0.6146 ) | (0.0059) |
| α | -0.1504*** | -0.1561*** | -0.1571*** | -0.1838*** | -0.1595 *** |
| | (0.0199) | (0.0200) | (0.0201) | (0.01932 ) | (0.0137) |
| β | 0.9754*** | 0.9753*** | 0.9749 *** | 0.9689 *** | 0.9754 *** |
| | (0.0009) | (0.0009) | (0.0009) | (0.0020 ) | (0.0007) |
| γ | 0.1579 *** | 0.1510 *** | 0.1490*** | 0.1408 *** | 0.1481 *** |
| | (0.0280) | (0.0284) | (0.0281) | (0.0268 ) | (0.0208) |
| ar | -0.0515** | 0.0049 | -0.0172 | 0.9974*** | -0.0071 |
| | (0.0202) | (0.0102) | (0.0111) | (0.0000) | (0.1527) |
| ma | 0.0050 | -0.0485*** | -0.0283 | -1.0000*** | -0.0407 |
| | ( 0.0191) | (0.0138) | (0.0188) | (0.0001) | (0.1409) |
| $H_t$ in mean | | 0.4416 | 0.4257*** | 0.4233*** | 1.6844 *** |
| | | (0.0019) | (0.0018) | (0.0001) | (0.0014) |
| $H_t$ in variance | | | | 0.1634 | |
| | | | | (0.1024 ) | |
| Lag $H_t$ in mean | | | | | -1.2591 *** |
| | | | | | (0.0025 ) |

Table A.6. Estimates of GJR-GARCH-ARMA models.

| GJR-GARCH | GARCH(1,1) | GARCH(1,1) w/ $H_t$ in mean | GARCH(1,1) w/ lag $H_t$ in mean | GARCH(1,1) w/sim $H_t$ in mean lag $h_t$ in variance | GARCH(1,1)- w/sim lag $H_t$ both in mean |
|---|---|---|---|---|---|
| μ | 0.0321 | -2.6155 *** | -2.5221 *** | -1.5279*** | -0.2455*** |
| | (0.0225) | (0.1272) | (0.0321) | (0.0064) | (0.0214) |
| ω | 0.0211 *** | 0.0215 *** | 0.0218 *** | 0.0000 *** | 0.0273 *** |
| | (0.0046) | (0.0046) | (0.0046) | (0.0002) | ( 0.0074) |
| α | 0.0000 | 0.0000 *** | 0.0000 *** | 0.0000 *** | 0.0000 |
| | (0.0273) | (0.0257) | (0.0261) | (0.0639) | (0.3881) |
| β | 0.8801 *** | 0.8788 *** | 0.8779 *** | 0.8598 *** | 0.8593 *** |
| | (0.0256) | ( 0.0265 ) | (0.0267) | (0.0417) | (0.1938) |
| γ | 0.1957 *** | 0.1967 *** | 0.1981 *** | 0.2115 *** | 0.2140 |
| | (0.0431) | (0.0368) | (0.0389) | (0.0522) | (0.2297) |
| ar | 0.1340 | 0.1713 | 0.0999 | 0.9849*** | 0.9868*** |
| | (2.0147) | (0.4092) | (0.8817) | (0.0147) | (0.0414) |
| ma | -0.1858 | -0.2244 | -0.1534 | -0.9958*** | -0.9974*** |
| | (1.9995) | (0.4039) | (0.8748) | (0.0005) | (0.0016) |
| $H_t$ in mean | | 0.4417 *** | 0.4263 *** | 0.2615 *** | 1.4679 *** |
| | | ( 0.0208 ) | (0.0049) | (0.0025) | (0.0485) |
| $H_t$ in variance | | | | 0.0046 *** | |
| | | | | (0.0010) | |
| Lag $H_t$ in mean | | | | | -1.4214 *** |
| | | | | | (0.0523) |

Table A.7. Estimates of EGARCH models with student's T distribution.

| T with EGARCH | GARCH(1,1) | Sim $H_t$ in mean | lag $H_t$ in mean | Sim $H_t$ in mean lag $H_t$ in variance | sim lag $H_t$ both in mean |
|---|---|---|---|---|---|
| μ | 0.0587*** | -2.6155 *** | -2.5221 *** | -2.5221 *** | -2.5221 *** |
| | (0.0165) | (0.0086) | (0.0087) | (0.0085) | (0.0207) |
| ω | -0.0097 | -0.0107 * | -0.0107 * | -1.2614 *** | -0.0102 *** |
| | (0.0059) | (0.0061) | (0.0061) | (0.2687) | (0.0061) |
| α | -0.1810*** | -0.1865 *** | -0.1859 *** | -0.1927 *** | -0.1892 *** |
| | (0.0232) | (0.0234) | (0.0232) | (0.0234) | (0.0193) |
| β | 0.9811 *** | 0.9807 *** | 0.9806 *** | 0.9769 *** | 0.9806 *** |
| | (0.0005) | (0.0005) | (0.0005) | (0.0005) | (0.0005) |
| γ | 0.1635 *** | 0.1584 *** | 0.1561 *** | 0.1562 *** | 0.1570 *** |
| | (0.0273) | (0.0278) | (0.0275) | (0.0292) | (0.0197) |
| $\nu$ | 6.5231*** | 6.5512*** | 6.6142*** | 6.8418*** | 6.5345*** |
| | (1.1047) | (1.1130) | (1.1368) | (1.2122) | (1.1330) |
| $H_t$ in mean | | 0.4463 *** | 0.4307 *** | 0.4304 *** | 1.5071 *** |
| | | (0.0014) | (0.0015) | (0.0014) | (0.0013) |
| $H_t$ in variance | | | | 0.2083 *** | |
| | | | | (0.0447) | |
| Lag $H_t$ in mean | | | | | -1.0772 *** |
| | | | | | (0.0003) |

Table A.8. Estimates of EGARCH models with the generalized error distribution.

| GED with EGARCH | GARCH(1,1) | Sim $H_t$ in mean | lag $H_t$ in mean | Sim $H_t$ in mean lag $H_t$ in variance | sim lag $H_t$ both in mean |
|---|---|---|---|---|---|
| μ | 0.0540*** | -2.6155 *** | -2.5221 *** | -2.5221 *** | -2.5221 *** |
| | (0.0146) | (0.0078) | (0.0080) | (0.0079) | (0.0141) |
| Ω | -0.010* | -0.0109 * | -0.0110 * | -1.3805 *** | -0.0112 *** |
| | (0.0054) | (0.0057) | (0.0057) | (0.2765) | (0.0049) |
| α | -0.1699*** | -0.1762 *** | -0.1759 *** | -0.1849 *** | -0.1806 *** |
| | (0.0210) | (0.0215) | (0.0214) | (0.0223) | (0.0126) |
| Β | 0.9785*** | 0.9781 *** | 0.9781 *** | 0.9741 *** | 0.9784 *** |
| | (0.0005) | (0.0005) | (0.0005) | (0.0010) | (0.0005) |
| $\gamma$ | 0.1634*** | 0.1578 *** | 0.1557 *** | 0.1558 *** | 0.1556 *** |
| | (0.0275) | (0.0282) | (0.0280) | (0.0339) | (0.0158) |
| $\nu$ | 1.3482*** | 1.3460 *** | 1.3527 *** | 1.3605 *** | 1.3309 *** |
| | (0.0748) | (0.0749) | (0.0754) | (0.0764) | (0.0756) |
| $H_t$ in mean | | 0.4453 *** | 0.4301 *** | 0.4296 *** | 1.8740 *** |
| | | (0.0013) | (0.0013) | (0.0013) | (0.0013) |
| $H_t$ in variance | | | | 0.2281 *** | |
| | | | | (0.0460) | |
| Lag $H_t$ in mean | | | | | -1.4447 *** |
| | | | | | (0.0006) |

Table A.9. Estimates of EGARCH models with the generalized hyperbolic distribution.

| GH with EGARCH | GARCH(1,1) | Sim $H_t$ in mean | Lag $H_t$ in mean | Sim $H_t$ in mean lag $H_t$ in variance | Sim lag $H_t$ both in mean |
|---|---|---|---|---|---|
| μ | 0.0236 | -2.6155 *** | -2.5221 *** | -1.8419 *** | -2.5221 *** |
| | (0.0159) | (0.0084) | (0.0117) | (0.0100) | (0.0044) |
| ω | -0.0032* | -0.0038 | -0.0039 | -6.1731 *** | -0.0039 *** |
| | (0.0054) | (0.0061) | (0.0060) | (0.1650) | (0.0090) |
| α | -0.1893*** | -0.1953 *** | -0.1945 *** | -0.2890 *** | -0.1979 *** |
| | (0.0205) | (0.0207) | (0.0205) | (0.0467) | (0.0170) |
| β | 0.9773*** | 0.9766 *** | 0.9766 *** | 0.9000 *** | 0.9765 *** |
| | (0.0005) | (0.0005) | (0.0005) | (0.0482) | (0.0008) |
| γ | 0.1566*** | 0.1506 *** | 0.1489 *** | 0.2577 *** | 0.1503 *** |
| | (0.0240) | (0.0244) | (0.0242) | (0.0635) | (0.0315) |
| Skew | -0.2390*** | -0.2428*** | -0.2443*** | -0.2077*** | -0.2095** |
| | (0.0832) | (0.0868) | (0.0837) | (0.0691) | (0.0855) |
| shape | 2.0863*** | 2.1515 *** | 2.1812 *** | 1.4103 *** | 1.8251* |
| | (0.7429) | (0.7650) | (0.7634) | (0.5954) | (0.9670) |
| λ | 0.4612 | 0.3797 | 0.4095 | 0.6914*** | 0.9642 |
| | (1.4284) | (1.4395) | (0.4095) | (1.1461) | (1.3951) |
| $H_t$ in mean | | 0.4402 *** | 0.4248 *** | 0.3069 *** | 1.4881 *** |
| | | (0.0014) | (0.0020) | (0.0017) | (0.0004) |
| $H_t$ in variance | | | | 1.0291 *** | |
| | | | | (0.0278) | |
| Lag $H_t$ in mean | | | | | -1.0639 *** |
| | | | | | (0.0053) |

Table A.10. Estimates of EGARCH models with the skewed student's T distribution.

| ST with egarch | GARCH(1,1) | Sim $H_t$ in mean | Lag $H_t$ in mean | Sim $H_t$ in mean lag $H_t$ in variance | sim lag $H_t$ both in mean |
|---|---|---|---|---|---|
| μ | 0.0266 | -2.6155 *** | -2.5221 *** | -2.5221 *** | -2.5221 *** |
| | (0.0162) | (0.0088) | (0.4856) | (0.0086) | (0.0149) |
| ω | -0.0031 | -0.0039 | -0.0038 | -1.2455 *** | -0.0037 |
| | (0.0058) | (0.0062) | (0.0061) | (0.2263) | (0.0068) |
| α | -0.1916*** | -0.1970 *** | -0.1964 *** | -0.2027 *** | -0.1993 *** |
| | (0.0219) | (0.0219) | (0.0217) | (0.0219) | (0.0170) |
| β | 0.9779*** | 0.9772 *** | 0.9771 *** | 0.9735 *** | 0.9772 *** |
| | (0.0005) | (0.0005) | (0.0005) | (0.0007) | (0.0008) |
| γ | 0.1585*** | 0.1526 *** | 0.1508 *** | 0.1503 *** | 0.1518 *** |
| | (0.0242) | (0.0247) | (0.0245) | (0.0273) | (0.0174) |
| Skew | 0.8402*** | 0.8411 *** | 0.8391 *** | 0.8410 *** | 0.8472 ** |
| | (0.0294) | (0.0294) | (0.0293) | (0.0293) | (0.0304) |
| shape | 7.4047*** | 7.5101 *** | 7.5948 *** | 7.8809 *** | 7.3961 *** |
| | (1.3868) | (1.4269) | (1.4613) | (1.5689) | (1.4758) |
| $H_t$ in mean | | 0.4408 *** | 0.4252 *** | 0.4251 *** | 1.4052 *** |
| | | (0.0014) | (0.0808) | (0.0014) | (0.0015) |
| $H_t$ in variance | | | | 0.2068 *** | |
| | | | | (0.0376) | |
| Lag $H_t$ in mean | | | | | -0.9804 *** |
| | | | | | (0.0027) |

Table A.11. Unconditional forecasting results comparison

| Absolute value of the return | GARCH(1,1) | Garch(1,1)-ARMA(1,1) | EGARCH(1,1) | EGARCH lag $H_t$ in variance sim $H_t$ in mean | EGARCH lag sim $H_t$ both in mean | EGARCH- GH | EGARCH-GH with lag $H_t$ in variance and sim $H_t$ in mean | EGARCH-GH lag sim $H_t$ both in mean |
|---|---|---|---|---|---|---|---|---|
| 1.357 | 1.2000 | 1.2104 | 1.3458 | 1.3745 | 1.358 | 1.4135 | 1.4307 | 1.4006 |
| 0.036 | 1.1987 | 1.209 | 1.3372 | 1.3649 | 1.3482 | 1.4002 | 1.4167 | 1.3869 |
| 1.0688 | 1.1975 | 1.2077 | 1.3289 | 1.3479 | 1.3388 | 1.3873 | 1.3972 | 1.3738 |
| 0.1519 | 1.1962 | 1.2063 | 1.3208 | 1.3337 | 1.3297 | 1.3749 | 1.3801 | 1.361 |
| 1.6197 | 1.195 | 1.205 | 1.3129 | 1.3211 | 1.3209 | 1.3628 | 1.3645 | 1.3486 |
| 0.1175 | 1.1938 | 1.2037 | 1.3053 | 1.3118 | 1.3123 | 1.3512 | 1.3517 | 1.3367 |
| 1.3033 | 1.1926 | 1.2025 | 1.298 | 1.3085 | 1.3041 | 1.3398 | 1.3436 | 1.3251 |
| 0.9281 | 1.1914 | 1.2012 | 1.2908 | 1.2989 | 1.296 | 1.3289 | 1.3309 | 1.3138 |
| 1.3071 | 1.1902 | 1.2 | 1.2839 | 1.2861 | 1.2882 | 1.3183 | 1.316 | 1.3029 |
| 0.7616 | 1.1891 | 1.1988 | 1.2772 | 1.2732 | 1.2807 | 1.308 | 1.3012 | 1.2924 |

# BIBLIOGRAPHY

Aas, K. and I. H. Haff (2006). "The generalized hyperbolic skew student's t-distribution." Journal of financial econometrics 4(2): 275-309.

Abramowitz, M. and I. A. Stegun (1972). Handbook of mathematical functions, Dover New York.

Ait-Sahalia, Y., et al. (2013). "The leverage effect puzzle: Disentangling sources of bias at high frequency." Journal of financial Economics 109(1): 224-249.

Andersen, T. G., et al. (2001). "The distribution of realized exchange rate volatility." Journal of the American statistical association 96(453): 42-55.

Bollen, J., et al. (2011). "Twitter mood predicts the stock market." Journal of Computational Science 2(1): 1-8.

Bollerslev, T. (1986). "Generalized autoregressive conditional heteroskedasticity." Journal of Econometrics 31(3): 307-327.

Box, G. J., Gwilym (1970). Time series analysis: Forecasting and control. San Francisco, Holden-Day.

Connolly, R. A. (1989). "An examination of the robustness of the weekend effect." Journal of Financial and quantitative Analysis 24(02): 133-169.

Engle, R. F. (1982). "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation." Econometrica: Journal of the Econometric Society: 987-1007.

Engle, R. F. and V. K. Ng (1993). "Measuring and testing the impact of news on volatility." The journal of finance 48(5): 1749-1778.

Fama, E. F., et al. (1969). "The adjustment of stock prices to new information." International economic review 10(1): 1-21.

Fama, E. F. (1970). "Efficient capital markets: A review of theory and empirical work*." The journal of finance 25(2): 383-417.

French, K. R., et al. (1987). "Expected stock returns and volatility." Journal of financial Economics 19(1): 3-29.

Ghalanos, A. (2014). Introduction to the rugarch package.(Version 1.3-1), Technical report.

Glosten, L. R., et al. (1993). "On the relation between the expected value and the volatility of the nominal excess return on stocks." The journal of finance 48(5): 1779-1801.

Lee, W. Y., et al. (2002). "Stock market volatility, excess returns, and the role of investor sentiment." Journal of Banking & Finance 26(12): 2277-2299.

Logue, D. E. and R. J. Sweeney (1977). "'WHITE-NOISE'IN IMPERFECT MARKETS: THE CASE OF THE FRANC/DOLLAR EXCHANGE RATE." The journal of finance 32(3): 761-768.

Merton, R. C. (1980). "On estimating the expected return on the market: An exploratory investigation." Journal of financial Economics 8(4): 323-361.

Necula, C. (2009). "Modeling Heavy Tailed Stock Index Returns Using the Generalized Hyperbolic Distribution." Romanian Journal of Economic Forecasting 10(2): 118-131.

Nelson, D. B. (1991). "Conditional heteroskedasticity in asset returns: A new approach." Econometrica: Journal of the Econometric Society: 347-370.

Officer, R. R. (1973). "The variability of the market factor of the New York Stock Exchange." Journal of Business: 434-453.

Pontiff, J. (2006). "Costly arbitrage and the myth of idiosyncratic risk." Journal of Accounting and Economics 42(1): 35-52.

Poon, S.-H. and C. W. Granger (2003). "Forecasting volatility in financial markets: A review." Journal of economic literature 41(2): 478-539.

Prause, K. (1999). The generalized hyperbolic model: Estimation, financial derivatives, and risk measures, PhD thesis, University of Freiburg.

Sharpe, W. F. (1964). "Capital asset prices: A theory of market equilibrium under conditions of risk*." The journal of finance 19(3): 425-442.

Shiller, R. J. (1981). "The Use of Volatility Measures in Assessing Market Efficiency*." The journal of finance 36(2): 291-304.

Shoven, J. B. and C. Sialm (2000). "The dow jones industrial average: the impact of fixing its flaws." The Journal of Wealth Management 3(3): 9-18.

Strong, N. (1992). "Modelling abnormal returns: a review article." Journal of Business Finance & Accounting 19(4): 533-553.