2015

# Forecasting the US Unemployment Rate with Job Openings Index

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# FORECASTING THE US UNEMPLOYMENT RATE

# WITH JOB OPENINGS INDEX

BY

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

IN

STATISTICS

UNIVERSITY OF RHODE ISLAND

2015

MASTER OF SCIENCE THESIS

OF

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2015

# ABSTRACT

Predicting the unemployment rate is one of the most important applications for economists and policymakers (Golan, 2002). In this thesis, the focus is on the seasonally adjusted U.S. national unemployment rate (UR). The goal is to introduce the seasonally adjusted job openings (JOB) for UR forecasting.

In order to forecast UR, firstly, an integrated autoregressive moving average model (ARIMA) is constructed as a benchmark mode. For a better comparison, a well known leading indicator – the seasonally adjusted initial claim for unemployment insurance (IC), released by the U.S. Department of Labor, is also included. By using JOB and IC as external variables, integrated autoregressive moving average with external variable(s) models (ARIMAX) are successfully constructed. Multivariate vector autoregressive models (VAR) are also well constructed for UR, JOB & IC. The Akaike Information Criterion (AIC), the Schwarz-Bayesian Criterion (BIC), and the Hannan-Quinn Criterion (HQ) are applied for models selection.

For out-of-sample analysis, both rolling forecasts and recursive forecasts are considered. The Mean absolute forecast error (MAFE) and the mean square forecast error (MSFE) are calculated, along with the Diebold-Mariano (DM) test, for models comparison. The results show that the JOB related models have much better forecasting power than the benchmark model and the IC related models. This suggests that the JOB index can be used as one of the leading indicators to improve UR forecasting.



**Keywords:** Unemployment Rate, Forecast, Job Openings, ARIMA, ARIMAX, VAR

## ACKNOWLEDGMENTS

Foremost, I would like to express my sincere gratitude to my advisor Professor Gavino Puggioni for his continuous support of my master's study and research, for his patience, motivation, enthusiasm and immense knowledge. His guidance helped me in all the time of research and writing of this thesis.

Besides my advisor, I would like to thank the rest of my thesis committee: Prof. Gonzalez, Prof. Merino for their encouragement, patience and great suggestions. My special thanks go to Prof. Gonzalez, for admitting me into the Statistics Program, and for offering me funding for my study at URI.

My sincere thanks also go to my committee chair Prof. Finizio, for teaching me math class, also for agreeing to be part of my committee.

Last but not the least; I would like to thank my wife, my boys and my parents for their selfless loves and their supports.

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# CHAPTER 1

# INTRODUCTION

Predicting unemployment rate is of great importance to many economic decisions (Floros, 2005). Forecasting the unemployment rate accurately is important because it helps economists to have a better idea of what the future economy holds (Lewis and Brown, 2001). Besides, it is also important for the government in terms of decision and policy making.

Over the last century, the U.S. unemployment rate has attracted a lot of attention. It displays steep increases that end in sharp peaks and alternates with much more gradual and longer declines that end in mild troughs. Such cyclical asymmetries have long been noted and much debated (De Long and Summers 1986; Elwood 1996; Neftci 1984; Rothman 1996; Sichel 1989).

The traditional data-driven, time-series models use only lagged observations of unemployment (Golan and Perloff, 2002). In many applications, unemployment related variables are available, and one would like to make use of the relevant information in forecasting. In predicting the U.S. unemployment rate, there exist data on initial jobless claims and some other indicators (Montgomery et al. 1998).

D'Amuri and Marcucci (2009) found that using a Google job-search index as a leading indicator can also increases the forecast precision of the U.S. unemployment rate.

In this thesis, I focus on the seasonally adjusted U.S. national unemployment rate (UR). The goal is to introduce the seasonally adjusted job openings (JOB) from Bureau of Labor Statistic as a useful indicator for UR forecasting.

The UR data are available from Jan 1948 to Feb 2015. However, JOB data are only available from Dec 2000 to Dec 2014 with two months lag in release date. As the literature observed, the U.S. unemployment rate displays no consistent trend at all, and there are no good reasons why it should have either risen or fallen secularly (Montgomery et al. 1998). In this thesis, I am interested in testing the relevance of the JOB data, and finding the best model for short term forecasting, not modeling the long term dynamics of the series. Therefore, for UR data, I choose the relative time period as it for JOB data (from Feb 2001 to Feb 2015).

In order to forecast UR, firstly, an integrated autoregressive moving average model (ARIMA) is built as a benchmark model for UR. For a better comparison, a well known leading indicator – the seasonally adjusted initial claim for unemployment insurance (IC), released by the U.S. Department of Labor, is also included.

By using the JOB and the IC data, four integrated autoregressive moving average with external variable(s) models (ARIMAX) are constructed. The "prewhitening" procedure is performed in the first step. Augmented Dickey-Fuller (ADF) test is applied to check for stationarity. The Akaike Information Criterion (AIC) and the Schwarz-Bayesian Criterion (BIC) are applied for models selection. The Ljung-Box test is chosen to check for autocorrelation of the residuals along with the autocorrelation function (ACF) plot and the partial ACF plot. The Shapiro-Wilk test is applied for the residuals normality check along with QQ plot and histogram plot.

Moreover, three bivariate and one trivariate vector autoregressive models (VAR) are also constructed for UR, JOB and IC. Engle and Granger's two steps procedures and Phillips-Ouliaris (PO) cointegration test are applied in the first step. Model lag selection is based on the AIC, BIC and Hannan-Quinn (HQ) Criterion. In order to check the models' residuals, multivariate portmanteau test is used to check the autocorrelation. Jarque-Bera (JB) test and Royston's multivariate normality test are applied for normality check along with Chi-Square QQ plot. Autoregressive conditionally heteroskedasticity (ARCH) test is applied to test for deviations from the null hypothesis of time-homogeneous variance. The Granger-causality results are produced and impulse response function (IRF) plots are also provided.

After the models are fit, an out-of-sample forecast comparison is applied. Both recursive forecasts and rolling forecasts are chosen. Mean absolute forecast error (MAFE) and mean square forecast error (MSFE) are calculated, along with Diebold-Mariano (DM) test for the model comparison.

Comparing to the benchmark model, JOB related ARIMAX and VAR models have much better forecast accuracy with smaller MAFE and MSFE in all forecasting.

Comparing to IC related models, JOB related bivariate VAR model also has better performance in all steps ahead forecasting with smaller MSFE.

Finally, the trivariate VAR model, which includes both the JOB and the IC index, has the best forecasting performance with lowest MAFE and MSFE. Compared to Benchmark model, the DM test results show the trivariate VAR model is significantly better in 2 and 3 steps-ahead forecasting.

In conclusion, the forecasting comparison results demonstrate that JOB index can be used as one of the useful indicators to improve UR forecasting. When combining the JOB and the IC index together, the ARIMAX and VAR models' performance are further improved.

This thesis is organized as follows. Chapter 1 shows the primary goal of this research and how this thesis is organized. Chapter 2 describes the UR, JOB and IC data and the literature reviews of ARIMA, ARIMAX and VAR time series models. Chapter 3 introduces the modeling methodology, including the time series models and related tests. In Chapter 4, one benchmark ARIMA model and four ARIMAX models with JOB and IC as external variable(s) are well built. In Chapter 5, four bivariate and one trivariate VAR models are successfully constructed. Chapter 6 summarizes all the selected models and shows the out-of-sample comparison results based on MAFE, MSFE and DM test. Chapter 7 presents the conclusions of this research.

# CHAPTER 2

# DATA AND LITERATURE REVIEW

## 2.1 The U.S. Unemployment Rate

The unemployment rate is a measure of the prevalence of unemployment and it is calculated as a percentage by dividing the number of unemployed individuals by all individuals currently in the labor force.

Typically, macroeconomic data are seasonally-adjusted. Seasonal adjustment is a statistical technique that attempts to measure and remove the influences of predictable seasonal patterns to reveal how employment and unemployment change from month to month. Over the course of a year, the size of the labor force, the levels of employment and unemployment, and other measures of labor market activity undergo fluctuations due to seasonal events including changes in weather, harvests, major holidays, and school schedules. Because these seasonal events follow a more or less regular pattern each year, their influence on statistical trends can be eliminated by seasonally adjusting the statistics from month to month. These seasonal adjustments make it easier to observe the cyclical, underlying trend, and other non-seasonal movements in the series (www.bls.gov, 2014).

The seasonally adjusted U.S. national unemployment rate (UR) is released monthly by the Bureau of Labor Statistics (BLS, www.bls.gov). The current seasonal adjustment methodology at BLS is called "X-12 ARIMA", more details of this method can be found at http://www.bls.gov/cpi/cpisahoma.htm. The UR index for month t refer to individuals who do not have a job, but are available for work, in the week including the 12th day of month t and who have looked for a job in the prior 4 weeks ending with the reference week. For the federal level, it is available from Jan 1948 up to the most recent, can be downloaded from www.bls.gov/bls/unemployment.htm.

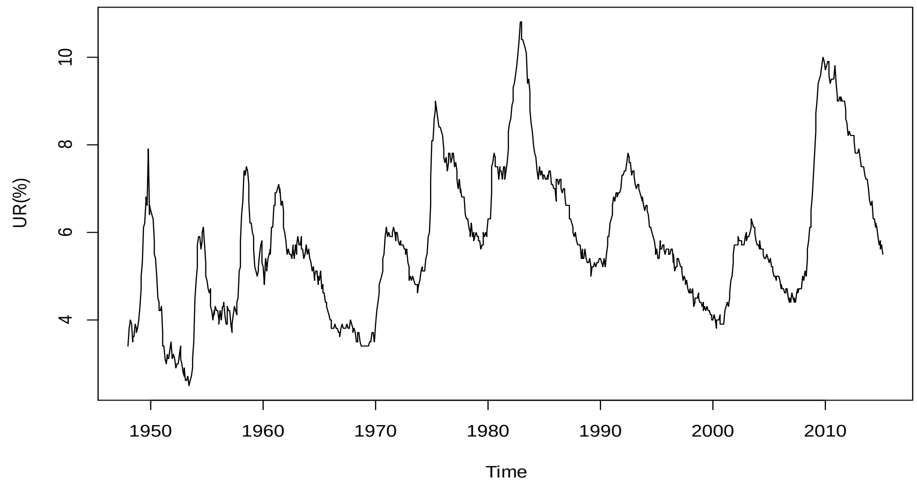


**Figure 2.1** The seasonally adjusted U.S. national unemployment rate (UR) (From Jan 1948 to Dec 2014)

Figure 2.1 shows the UR data have asymmetrical cyclical movements, particularly during the severe downward cycles, which dominate the behavior over time of the UR data. Short and steep rises, ending in sharp peaks, are characteristic of general business contractions; long and gradual declines are characteristic of business expansions (Montgomery et al. 1998).

### 2.2 Initial Claims for Unemployment Insurance

The seasonally adjusted initial claims for unemployment insurance (IC) are released by the U.S. Department of Labor (http://www.dol.gov), which are available weekly. At the national level, data starting from Jan 1967 can be found in the website: http://workforcesecurity.doleta.gov/unemploy/claims.asp

Montgomery et al. (1998) showed that the IC data can be used as a leading indicator of UR, because they contain information on whether unemployment is rising or falling.

In order to align the weekly IC data with the monthly UR data, the IC data is constructed the same way as the UR data are collected. For month t, we take into consideration the week including the 12th of the month and the three preceding weeks, exactly the same interval used to calculate the UR for month t reported in official statistics. In that interval, I summarize the total of the four weeks initial claim to get the total of the monthly IC data. When there are more than four weeks between the reference week of month t and the following month t + 1, we do not use the number from that extra week. In Table 2.1, the week ending at 9/20/2014 (marked in yellow) is the extra week, which will neither be included in Sep 2014 month calculation, nor in Oct 2014 month calculation.

**Table 2.1** A visual description of the alignment procedure of initial jobless claim (IC)

| Week | IC | |
|---|---|---|
| **8/9/2014** | 312,000 | |
| **8/16/2014** | 299,000 | **Reference Week** |
| **8/23/2014** | 298,000 | |
| **8/30/2014** | 304,000 | |
| **9/6/2014** | 316,000 | |
| **9/13/2014** | 281,000 | **Reference Week** |
| **9/20/2014** | 295,000 | **Extra Week** |
| **9/27/2014** | 288,000 | |
| **10/4/2014** | 287,000 | |
| **10/11/2014** | 266,000 | |
| **10/18/2014** | 284,000 | **Reference Week** |
| **10/25/2014** | 288,000 | |

In Figure 2.2, we can see the IC data and the UR data have similar cyclical movements. During the general business recession periods, such as the 1990 oil price shock, the 2000 Dot Com bubble and the Dec 2007 ~Jun 2009 subprime mortgage crisis, we can see short and steep rises in both IC and UR, ending in sharp peaks. Following business expansions, UR and IC both show long and gradual declines. This similar cyclical pattern suggests that IC can be used as a leading indicator for UR forecasting.

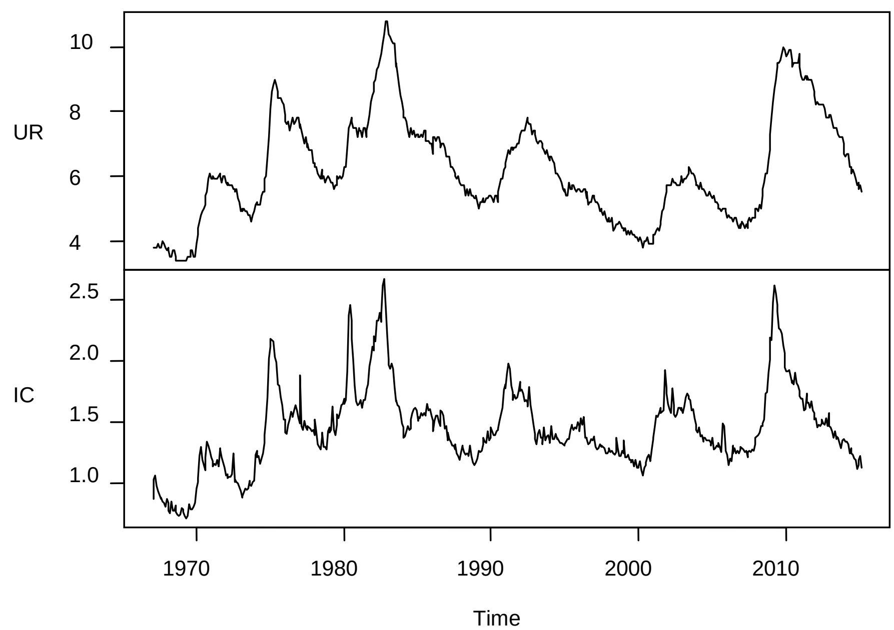


**Figure 2.2** Seasonally adjusted initial claims (IC) & unemployment rate (UR) (UR(%) and IC(x100,000) From Jan 1967 to Dec 2014)

### 2.3 Bureau New Job Openings

The economists are searching for other indicators, besides IC. D'Amuri and Marcucci (2009) tested the relevance of a Google job search index. They show that the Google job search index can also be a leading indicator for unemployment dynamics in the United States, which in turn can increase the precision of the forecasts.

Similar to the IC index, the Google job search index shows a positive correlation to the UR index. In another words, more people are looking for jobs are related to a higher unemployment rate. Inspired by the Google job index, Bureau's new job openings comes to my interest. More new job openings will create more employment opportunities, which seem to have strong negative correlation to the UR index.

Starting from Dec 2000, the Bureau of Labor Statistics launched the Job Openings and Labor Turnover Survey (JOLTS). Prior to JOLTS, there was no economic indicator of the unmet demand for labor with which to assess the presence or extent of labor shortages in the United States. This monthly survey is developed to address the need for data on job openings, hires, and separations. The availability of unfilled jobs is an important measure of the tightness of job markets. These data serve as demand-side indicators of labor at the national level (www.bls.gov, 2014).

The JOLTS survey covers all non-agricultural industries in the public and private sectors for the 50 States and the District of Columbia, which has a sample size of approximately 16,000 U.S. business establishments. For job openings, these are monthly observations; the reference period is the last business day of the month.

The seasonally adjusted national level job openings (JOB) data are available from Dec 2000 to Dec 2014. It can be downloaded from http://www.bls.gov/jlt/data.htm.

In Figure 2.3, the UR data and the JOB data look fare symmetric. The JOB index declined to a series low in July 2009, one month after the official end of the most recent recession. The UR index continued to increase after the end of the recession, reaching a high point in February 2010. The JOB data have trended upward since their series low in July 2009, and have surpassed the pre-recession peak (March 2007). In October 2014, there were 4.8 million job openings.

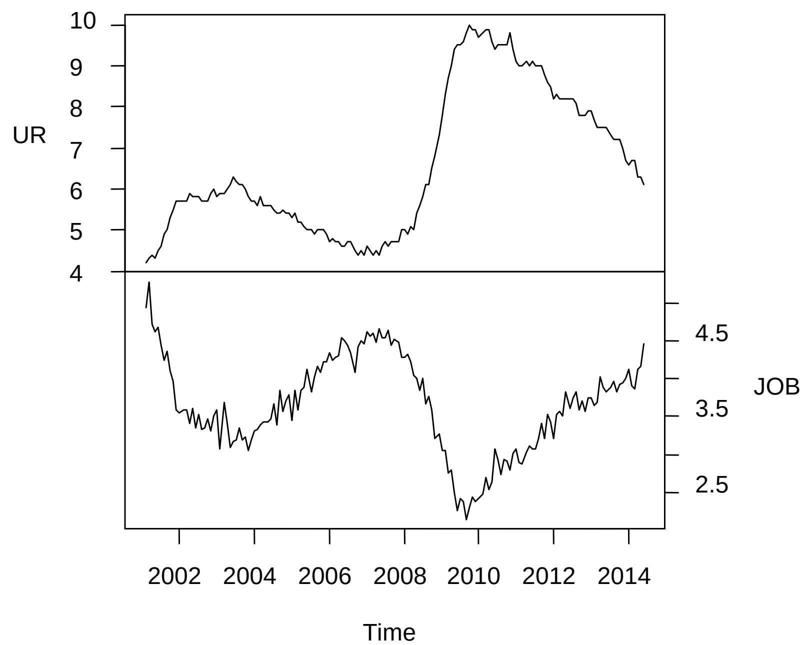


**Figure 2.3** Seasonally adjusted job openings (JOB) & unemployment rate (UR) (UR(%) and JOB(x1000,000) From Dec 2000 to Oct 2014)

In Figure 2.3, the correlation between the JOB and the UR index is clear, however, I haven't found any studies that propose the JOB index as a leading indicator. I think there are at least two reasons.

First, compared to UR, the JOB data are relatively new. People have been studying the UR index for a long time and most of the related researches are published before year 2000. In 1998, Montgomery et al. present a comparison summary of forecasting performance for a variety of linear and non-linear time series models for the UR index. The UR data started in 1948, comparing to it, the JOB data are much newer, which were published after Dec 2000.

Second, the release date of the JOB index has a time lag about two months in reality. For example, in Table 2.2, if we want to predict Dec 2014 UR in 1/1/2015, prior to its release date (1/9/2015), we can have Dec 2014 IC data ready (released on 12/18/2014). But we can only have Oct 2014 JOB data available. When people focus on the Dec 2014 UR, the Oct JOB data may be too old to be an interest.

**Table 2.2** A visual description of the index release date

| | Index Released Date | | |
|---|---|---|---|
| **Index Period** | **Unemployment Rate (UR)** | **Initial Jobless Claim (IC)** | **Job Openings (JOB)** |
| Dec 2014 index | 1/9/2015 | 12/18/2014 | |
| Nov 2014 index | 12/5/2015 | 11/20/2014 | 1/13/2014 |
| Oct 2014 index | | | 12/9/2014 |

The release date of the JOB index has two months lag. When forecasting the unemployment rate at time t, we can have the initial claim index at time t; but for job openings index, we can only have it at time t-2.

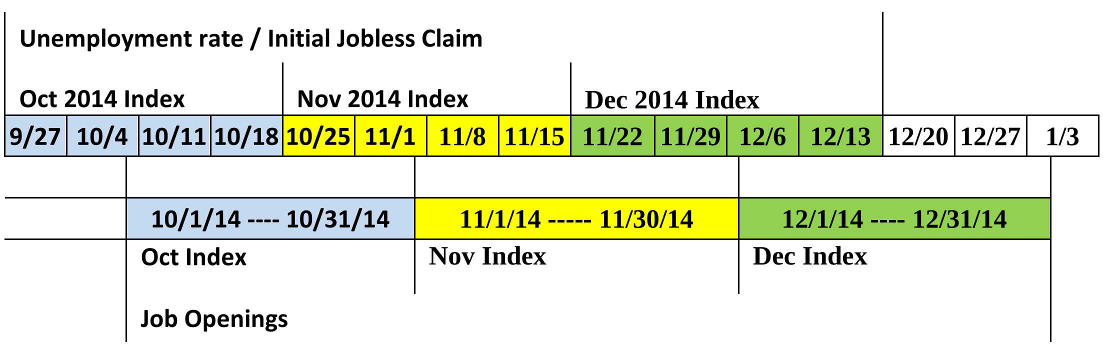


**Figure 2.4** A visual description of the index reference period

As mentioned before, the UR data and the IC data are collected based on the weekly records. For month t, we take into consideration the week including the 12th of the month and the three preceding weeks. However, the JOB data are collected monthly, and the reference period is the last business day of the month. From Figure

2.4, we can see that if I consider Oct 2014 JOB index as an indicator to forecast Dec 2014 UR index, which means I will use the JOB index from Oct 1st to Oct 30th 2014 as an indicator to predict the UR index from Nov 16th to Dec 13th.

According to the Dice-DFH Vacancy Duration Measure, an index created by University of Chicago economist Steven Davis, in 2014, U.S. employers are taking about 25 working days (5 weeks), on average, to fill vacant positions (Davis, 2014). From Oct 1st to Nov 16th, it is about 6 weeks apart between the JOB index and the UR index. It is reasonable to expect that the JOB index could be a good indicator to forecast UR.

### 2.4 Time Series Models

The UR data are time series data, so the time series models would be the first choice. Nowadays, one of the most used is the methodology based on autoregressive integrated moving average (ARIMA) model by Box and Jenkins (Box and Jenkins, 2008). This is mostly because it offers great flexibility in analyzing various time series and because of achieving accurate forecasts. Its other advantage is that this method uses only historical data of univariate time series to analyze its own trend and forecast future cycle (Peter and Silvia, 2012).

Montgomery et al. (1998) used ARIMA model as a benchmark model to present a comparison of forecasting performance for a variety of time series models for US unemployment rate. D’Amuri and Marcucci (2009) tested the relevance of a Google job search index for the UR index, with an ARIMA model as the benchmark. Tsay (2005) also chooses an ARIMA specification as the benchmark model for the US unemployment rate, when demonstrating the application of threshold autoregressive model.

In many applications, when related variables are available, people would like to make use of all relevant information in forecasting. Empirical studies on macroeconomic forecasting, such as Stock and Watson (1999), found that including leading indicators into the model can improve forecasting performance. In the econometric literature, ARIMA model with external variables (ARIMAX) and Vector autoregressive model (VAR) have a dominant place (Otter, 1990).

VAR is one of the most successful, flexible and easy to use models for the analysis of multivariate time series. It is a natural multivariate extension of the univariate autoregressive model. The VAR model has proven to be especially useful for describing the dynamic behavior of economic and financial time series and for forecasting. It often provides superior forecasts to those from univariate time series models and elaborate theory-based simultaneous equations models (Zivot, 2014).

D'Amuri and Marcucci (2009) tested the relevance of a Google job search index for the UR forecasting. They applied ARIMAX model both for Google job search index and for IC index as well as for comparison purposes.

Montgomery et al. (1998) applied a vector autoregressive model (VAR) for the UR and IC data. They showed that the IC index can be used as a leading indicator for the UR forecasting.

Inspired by the past researches, in my thesis, an ARIMA model is used as the benchmark model. Four ARIMAX and four VAR models are also successfully constructed with the JOB and the IC data.

## 2.5 Forecasting

Unlike structural models that relate the variable we want to forecast with a set of other variables, the time series model is not based on any economic theory. Time series models use the past movements of variables in order to forecast their future values. In term of forecasting, the reliability of the estimated equation should be based on the out-of-sample performance (Stock and Watson, 2003).

In this paper, the out-of-sample forecasting method is chosen. In the recursive forecasting, 8 steps ahead forecasting is calculated. In the rolling forecast, 1 step ahead to 4 steps ahead forecasting is computed. By comparing them with the actual UR values from Jul 2014 to Feb 2015, mean absolute forecast error (MAFE) and mean square forecast error (MSFE) and are calculated. A Diebold-Mariano (DM) test is also used for model comparison.

# CHAPTER 3

# METHODOLOGY

## 3.1 Univariate Time Series Models

A time series is a set of numbers that measures the status of some activity over equally spaced time interval. It is the historical record of some activity, with measurements taken at equally spaced intervals with a consistency in the activity and the method of measurement.

**White Noise Process**

A time series $\{\epsilon_t\}$ is called a white noise if $\{\epsilon_t\}$ is a sequence of independent and identically distributed random variables with finite mean and variance. In particular, if $\epsilon_t$ is normally distributed with mean zero and variance $\sigma^2$, then the series is called a Gaussian white noise.

The mean function is constant $E(\epsilon_t) = \mu_t = \mu$

The auto-covariance function $\begin{cases} \gamma_k = \sigma^2 & (k = 0) \\ \gamma_k = 0 & (k \neq 0) \end{cases}$

The auto-correlation function $\begin{cases} \rho_k = 1 & (k = 0) \\ \rho_k = 0 & (k \neq 0) \end{cases}$

White noise process is very important and basic stationary process. It is the building block for other more interesting time processes.

**Autoregressive (AR) Model**

Yule (1926) carried out the original work on autoregressive processes. The autoregressive model specifies that the output variable depends linearly on its own previous values. Specifically, a p^th order autoregressive process $\{Y_t\}$ satisfies the equation

$$Y_t = \theta_1 Y_{t-1} + \theta_2 Y_{t-1} + \cdots + \theta_p Y_{t-p} + \epsilon_t$$

The current value of the series $Y_t$ is a linear combination of the p most recent past values of itself plus a white noise term at time t. For every t, we assume $Y_{t-1}$, $Y_{t-2}$, ... $Y_{t-p}$ are independent.

The **Backshift** (or **Lag**) operator, indicated with **B** transforms a variable into its lagged version: $BY_t = Y_{t-1}$. And for iterated version: $B^k Y_t = Y_{t-k}$.

The AR model can be expressed as a polynomial of order p using the Backshift operator:

$$\Theta_p(B)Y_t = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_P B^p)Y_t = \epsilon_t$$

**Moving Average (MA) Model**

Sometimes, after estimation a model using autoregressive component, trend or seasonality (or even regression components), there can still be an autocorrelation in the residuals. An autocorrelation in the residuals can be modeled using the Moving Average terms. Moving average models were first considered by Slutsky (1927). A Moving Average of order q, or MA(q) can be written as

$$Y_t = \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \cdots + \phi_q \epsilon_{t-q}$$

This is a linear combination of the current white noise term and the past q white noise terms. Alternatively, using the backshift operator, an MA(q) can be written as

$$Y_t = \left(1 + \phi_1 B + \phi_2 B^2 + \cdots + \phi_q B^q\right)\epsilon_t = \Phi_q(B)\epsilon_t$$

**Autoregressive Moving Average (ARMA) Model**

In some applications, the AR or MA models discussed in the previous sections become cumbersome because one may need a high-order model with many parameters to adequately describe the dynamic structure of the data. To overcome this difficulty, the autoregressive moving average (ARMA) models are introduced. (Box, Jenkins, and Reinsel, 1994). Basically, an ARMA model combines the ideas of AR and MA models into a compact form so that the number of parameters used is kept small.

The general ARMA model was described in 1951 in the thesis of Peter whittle, "*Hypothesis testing in time series analysis*" and it was popularized in 1970 by Box and Jenkins. ARMA (p, q) can be written as

$$Y_t = \theta_1 Y_{t-1} + \theta_2 Y_{t-1} + \cdots + \theta_p Y_{t-p} + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \cdots + \phi_q \epsilon_{t-q}$$

By using the backshift operator, an ARMA (p, q) can be written as

$$(1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_P B^p) Y_t = \left(1 + \phi_1 B + \phi_2 B^2 + \cdots + \phi_q B^q\right) \epsilon_t$$

Or: $$\Theta_p(B) Y_t = \Phi_q(B) \epsilon_t$$

**Autoregressive Integrated Moving Average (ARIMA) Model**

A time series $\{ Y_t \}$ is said to follow an integrated autoregressive moving average (ARIMA) model if the d[th] difference $W_t = \nabla^d Y_t = (1 - B)^d Y_t$ is a stationary ARMA process. If $W_t$ follows an ARMA (p, q) model, we say that $\{ Y_t \}$ is an ARIMA(p, d, q) process, which can be written as:

$$W_t = \theta_1 W_{t-1} + \theta_2 W_{t-2} + \cdots + \theta_p W_{t-p} + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \cdots + \phi_q \epsilon_{t-q}$$

By using the backshift operator, an ARIMA (p, d, q) can be written as

$$(1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_P B^p)(1 - B)^d Y_t = \left(1 + \phi_1 B + \phi_2 B^2 + \cdots + \phi_q B^q\right) \epsilon_t$$

Or $$\Theta_p(B)(1 - B)^d Y_t = \Phi_q(B) \epsilon_t$$

Same as the ARMA model, the ARIMA model methodology was first introduced by Box and Jenkins in 1970. In time series analysis, the **Box–Jenkins** method, refer to the approach that applies ARMA or ARIMA models to find the best fit of a time-series model to past values of a time series.

**Autoregressive Integrated Moving Average with External Variables (ARIMAX) Model**

ARIMAX model is combining the predictive value of both the trailing time series values themselves ($Y_t$) and the trailing model errors ($\epsilon_t$) with the predictive value of external variables.

An ARIMA model with external variables, that is, ARIMAX model with $W_t = \nabla^d Y_t = (1 - B)^d Y_t$ can be written as

$$W_t = \theta_1 W_{t-1} + \theta_2 W_{t-2} + \cdots + \theta_p W_{t-p} + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \cdots + \phi_q \epsilon_{t-q}$$
$$+ \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots \beta_m X_{tm}$$

Where X’s are external variables and β’s are the coefficients of external variables.

**Seasonal Time Series Models**

Some financial time series such as quarterly earnings per share of a company exhibits certain cyclical or periodic behavior. Such a time series is called seasonal time series.

Analysis of seasonal time series has a long history. In some applications, seasonality is of secondary importance and is removed from the data, resulting in a seasonally adjusted time series that is then used to make inference. The procedure to remove seasonality from a time series is referred to as seasonal adjustment. Most economic data published by the US government are seasonally adjusted, such as the growth rate of gross domestic product and the unemployment rate (Tsay, 2005).

Seasonal ARMA(p,q) × $(P,Q)_s$ with period s can be defined as following expression obtained using the backshift operator:

$$\Theta_p(B)\Theta_P(B^S)Y_t = \Phi_q(B)\,\Phi_Q(B^S)\epsilon_t$$

Similar to non seasonal models, seasonal difference can be defined as $\nabla_s Y_t = Y_t - Y_{t-s} = (1 - B^S)Y_t$ . Seasonal ARIMA(p, d, q) × $(P, D, Q)_s$ with period s can be defined as following expression obtained using the backshift operator:

$$\Theta_p(B)\Theta_P(B^S)(1 - B^S)^D(1 - B)^d Y_t = \Phi_q(B)\,\Phi_Q(B^S)\epsilon_t$$

These models are easily over-parameterized, so care should be put in the choice of parameters.

**Vector Autoregression (VAR) Model**

VAR is an extension of univariate autoregressive models to multivariate time series data. VAR model is a multi-equation system where all the variables are treated as endogenous. There is one equation for each variable as dependent variable. Right-hand side of each equation includes lagged values of all dependent variables in the system, no contemporaneous variables.

Let $Y_t = (y_{1t,} y_{2t,} \dots, y_{nt})'$denote an (n×1) vector of time series variables. The basic *p*-lag vector autoregressive (VAR(p)) model has the form

$$Y_t = c + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \cdots + \Pi_p Y_{t-p} + \epsilon_t\,, \qquad t = 1, \dots, T$$

Where $\Pi_i$ are $(n \times n)$ coefficient matrices and $\epsilon_t$ is an (n×1) unobservable zero mean white noise vector process (serially uncorrelated or independent) with time invariant covariance matrix Σ.

For example, a bivariate VAR(2) model equation has the form

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \pi_{11}^1 & \pi_{12}^1 \\ \pi_{21}^1 & \pi_{22}^1 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} \pi_{11}^2 & \pi_{12}^2 \\ \pi_{21}^2 & \pi_{22}^2 \end{pmatrix} \begin{pmatrix} y_{1t-2} \\ y_{2t-2} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}$$

Or: $y_{1t} = c_1 + \pi_{11}^1 y_{1t-1} + \pi_{12}^1 y_{2t-1} + \pi_{11}^2 y_{1t-2} + \pi_{12}^2 y_{2t-2} + \epsilon_{1t}$

$$y_{2t} = c_2 + \pi_{21}^1 y_{1t-1} + \pi_{22}^1 y_{2t-1} + \pi_{21}^2 y_{1t-2} + \pi_{22}^2 y_{2t-2} + \epsilon_{2t}$$

Where $cov(\epsilon_{1t}, \epsilon_{2s}) = \sigma_{12}\ for\ t = s; 0\ otherwise.$ Hence cross correlation doesn't have to be zero. In each equation, they have the same regressors — lagged values of $y_{1t}$ and $y_{2t}$ .

**Structural VAR (SVAR)**

In VAR model, we assume there is no concurrent linear relationship between the component series. When this assumption is relaxed, we have the so called Structural VAR (SVAR). Consider for instant the SVAR(1,1)

$$\begin{cases} x_t = \gamma_x - b_{12} y_t + \theta_{11} x_{t-1} + \theta_{12} y_{t-1} + w_{x,t} \\ y_t = \gamma_y - b_{21} x_t + \theta_{21} x_{t-1} + \theta_{22} y_{t-1} + w_{y,t} \end{cases}$$

Both variables are endogenous, so we cannot estimate it as regular VAR. In matrix form we have:

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \gamma_x \\ \gamma_y \end{bmatrix} + \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} w_{x,t} \\ w_{y,t} \end{bmatrix}$$

Or in compact form: $BY_t = \gamma + \Theta Y_{t-1} + W_t$

In order to estimate the parameters, we have to calculate the so-called reduced form, obtained by solving for $Y_{t-1}$and $W_t$:

$$Y_t = B^{-1}\gamma + B^{-1}\Theta Y_{t-1} + B^{-1}W_t$$

$$= a_0 + AY_{t-1} + U_t$$

Given the parameters of the reduced form, it is impossible to identify and estimate the structural parameters $B$ without some restrictions. Some of most used are the exclusion condition $b_{12} = 0$, and the linear restriction $b_{12} + b_{21} = 1$.

The condition $b_{12} = 0$ is equivalent to requiring that B matrix is lower triangular, which means $x_t$ does not have a simultaneous effect on $y_t$.

The matrix of reduced form $B = \begin{bmatrix} 1 & 0 \\ b_{21} & 1 \end{bmatrix}$ , $B^{-1} = \begin{bmatrix} 1 & b_{12} \\ -b_{21} & 1 \end{bmatrix}$

So the bivariate SVAR becomes a recursive system:

$$\begin{cases} x_t = \gamma_x + \theta_{11} x_{t-1} + \theta_{12} y_{t-1} + w_{x,t} \\ y_t = \gamma_y - b_{21} x_t + \theta_{21} x_{t-1} + \theta_{22} y_{t-1} + w_{y,t} \end{cases}$$

**3.2 Construct and Validate an ARIMA Model**

**Box and Jenkins' Approach for ARIMA Modeling**

To construct an ARIMA model, I follow the three-stage iterative procedure of Box, Jenkins and Reinsel (1994). It includes the model specification (Tsay and Tiao 1984), parameter estimation (Ansley 1979; Hillmer and Tiao 1979), and Diagnostic checking (Ljung and Box 1978). Below I will describe these three steps one by one and followed with their related tests.

**Step 1: Model Specification and Unit Root Test**

Determining whether the series is stationary or not, we can use plots of the autocorrelation and partial autocorrelation functions of the dependent time series. If the time series is not stationary, it can often be converted to a stationary series by differencing. To reduce the risk of over-differencing, we will perform unit root tests.

**Unit Root Test for Stationarity**

**(1) Dickey-Fuller (DF) Unit Root Test**

To test whether a time series follows a random walk, we employ an AR(1) model

$$Y_t = \theta_1 Y_{t-1} + \epsilon_t \qquad \text{or} \qquad Y_t - Y_{t-1} = (\theta_1 - 1) Y_{t-1} + \epsilon_t$$

We consider the $H_0: \theta_1 = 1$ versus $H_a: \theta_1 < 1$. Clearly, if $\theta_1 = 1$, this time series is a random walk, which is not stationary. This is the well-known unit-root test problem (see Dickey and Fuller, 1979). A convenient test statistic is the t-ratio of the least squares estimate of $\theta_1$. For Equation (3.8a), the least squares method gives

$$\hat{\theta}_1 = \frac{\sum_{t=1}^{T} Y_{t-1} Y_t}{\sum_{t=1}^{T} Y_{t-1}^2}, \qquad \hat{\sigma}_\epsilon^2 = \frac{\sum_{t=1}^{T} (Y_t - \hat{\theta}_1 Y_{t-1})^2}{\mathrm{T}-1}$$

Where $Y_0 = 0$, and T is the sample size, the t-ratio is

$$\text{t-ratio} = \frac{\hat{\theta}_1 - 1}{std(\hat{\theta}_1)}$$

This is commonly referred to as the Dickey and Fuller test.

**(2) Augmented Dickey-Fuller (ADF) Unit Root Test**

Dickey and Fuller test can only test for the unit root in AR(1) process. To verify the existence of a unit root in an AR(p) process, one may perform the test $H_0: \beta = 1$ versus $H_a: \beta < 1$ using the regression

$$Y_t = c_t + \beta Y_{t-1} + \sum_{i=1}^{p-1} \theta_i \nabla \mathrm{Y}_{t-i} + \epsilon_t$$

Where $c_t$ is a deterministic function of time index of t. $\nabla \mathrm{Y}_j = \mathrm{Y}_j - \mathrm{Y}_{j-1}$ is the differenced series of $\mathrm{Y}_t$. Note that because of the first differencing, above equation can be rewritten as

$$\nabla Y_t = c_t + \delta Y_{t-1} + \sum_{i=1}^{p-1} \theta_i \nabla \mathrm{Y}_{t-i} + \epsilon_t$$

Where $\delta = \beta - 1$. We can then test the equivalent hypothesis $H_0: \delta = 0$ versus $H_a: \delta < 0$. The t-ratio of $(\hat{\delta})$ is

$$\text{ADF-test} = \frac{\hat{\delta}}{std(\hat{\delta})}$$

Where $\hat{\delta}$ denotes the least squares estimate of $\delta$. This is well-known augmented Dickey-Fuller unit root test. If $\delta = 0$, the series contains a unit root implying non stationary, whereas if $\delta < 0$, there is no unit root implying stationary.

**ADF Test with Generalized Least Squares (GLS) De-trending**

In 1996, Elliott, Rothenberg, and Stock performed a modified ADF test (known as the ADF-GLS test). Essentially, the test is an ADF test, except that the time series is transformed via a generalized least squares (GLS) regression before performing the test. Elliott, Rothenberg, and Stock have shown that, comparing to the standard versions of the ADF test, this test is significantly greater in terms of small sample size and power.

**Step 2: Parameter Estimation**

Finding appropriate values of *p* and *q* in the ARMA(*p*,*q*) model can be facilitated by plotting the partial autocorrelation functions for an estimate of *p*, and likewise using the autocorrelation functions for an estimate of $q$.

**Autocorrelation Functions (ACF) & Partial ACF (PACF)**

Autocorrelation is the linear dependence of a variable with itself at two points in time. For stationary processes, autocorrelation between any two observations only depends on the time lag h between them. By defining $Cov(Y_t, Y_{t+h}) = \gamma_h$, for h > = 0, Lag-h autocorrelation is given by

$$\rho_h = Corr(Y_t, Y_{t+h}) = \frac{\gamma_h}{\gamma_0}$$

The denominator $\gamma_0$ is the lag 0 covariance, the unconditional variance of the process. Correlation between two variables can result from a mutual linear dependence on other variables. Partial autocorrelation is the autocorrelation between $Y_t$ and $Y_{t+h}$ after removing any linear dependence on $Y_{t+1}, Y_{t+2}, \dots, Y_{t+h-1}$.

The theoretical ACF and PACF for AR, MA and ARMA conditional mean models are known, and quite different for each model. The differences in ACF and PACF among models are useful when selecting models. The following summarizes the ACF and PACF behavior for these models.

**Table 3.1** ACF and PACF behavior for AR, MA and ARMA models.

| Model | ACF | PACF |
|---|---|---|
| **AR(*p*)** | Tails off gradually | Cuts off after *p* lags |
| **MA(*q*)** | Cuts off after *q* lags | Tails off gradually |
| **ARMA(*p*,*q*)** | Tails off gradually | Tails off gradually |

**Model Selection Criteria**

When fitting models, it is possible to increase the likelihood by adding parameters, but doing so may result in over-fitting. Both the Akaike Information Criterion, AIC (Akaike, 1973) and the Schwartz / Bayesian Information Criterion, SIC / BIC (Schwartz, 1978) can resolve this problem by introducing a penalty term for the number of parameters in the model.

Suppose that we have a statistical model of some data. Let $L$ be the maximized value of the likelihood function for the model; let n be the sample size; let $k$ be the number of parameters in the model. Then the AIC and BIC value are:

$$AIC = 2k - 2\ln(L) \qquad (3.13)$$

$$BIC = k\ln(n) - 2\ln(L) \qquad (3.13)$$

Both criteria are based on the maximized value of the likelihood function, plus a penalty adjustment depending on the number of estimated parameters. Comparing to AIC, the penalty term in BIC is larger since $\ln(n) > 2 \; when \; n \geq 8$. Therefore, the difference between both criteria can be very large if sample size is large. Given a set of candidate models for the data, the preferred model is the one with the minimum AIC/BIC value.

In practical work, both criteria are usually examined. As just mentioned, the BIC penalizes additional parameters more strongly than the AIC. Thus, the BIC always choose a lag length that is shorter (or the same as) the one that minimizes the AIC. So one may consider the BIC as a lower bound and AIC as an upper bound for the appropriate lag length. In the case that they happen to agree, the choice is clear.

The AIC is similar to the BIC, but some authors believe that AIC is superior to BIC for a number of reasons. First, AIC is derived from principles of information. Second, the Bayesian approach requires a prior input but usually it is debatable. Third, AIC is asymptotically optimal in model selection in terms of the least squared mean error, but BIC is not asymptotically optimal (Burnham and Anderson, 2004; Yang, 2005). If AIC and BIC do not select the same model, Brockwell and Davis 2009 also recommend using AIC for finding $p$ and $q$.

**Step 3: Diagnostic Checking**

After the model being chosen, we can test whether the estimated model conforms to the specifications of a stationary univariate process. In particular, the residuals should meet white noise assumptions, as the residuals from the selected ARIMA model are assumed to be independent, homoskedastic, and usually normally distributed. Several diagnostic statistics and plots of the residuals can be used to examine the goodness of fit of the tentative model to the historical data.

**Residuals Diagnostic Checking**

**(1) Ljung-Box Test for Autocorrelation**

Instead of visual inspection of the sample autocorrelation plot, more formally, we can apply the Ljung-Box test (Ljung and Box, 1978) to the residual series to check for autocorrelation.

Suppose we have the first $L$ autocorrelation values $\hat{\rho}_k(\varepsilon)$ $(k = 1,2, \dots, L)$ from any ARMA (p, q) process. For a fixed sufficiently large $L$, the usual Ljung-Box Q statistic is given by

$$Q = N(N+2)\sum_{k=1}^{L}\frac{\hat{\rho}_k^2(\varepsilon)}{(N-k)} \sim \chi^2(L-p-q)$$

Where, N is the number of sample size, $L$ is the number of lags being tested. $\hat{\rho}_k^2(\varepsilon)$ is the squared sample autocorrelation of residual series $\{\varepsilon_t\}$ at lag k.

Under the null hypothesis that residuals are independently distributed, $\rho_1(\varepsilon) = \rho_2(\varepsilon) = \cdots = \rho_L(\varepsilon) = 0$. the test statistic Q follows the chi-square distribution with ($L$-$p$-$q$) degree of freedom. For significance level α, the critical region for rejection of the hypothesis of randomness is $Q > \chi^2_{(1-\alpha)}(L-p-q)$, where $\chi^2_{(1-\alpha)}(L-p-q)$ is the (1- α) quantile of the $\chi^2(L-p-q)$ distribution.

**(2) Shapiro-Wilk Test for Normality**

The residuals from ARIMA model should be normally distributed. We can check the normality assumption by using the Shapiro-Wilk test. The Shapiro–Wilk test was published in 1965 by Samuel Sanford Shapiro and Martin Wilk. This was the first test that was able to detect departure from normality due to either skewness or kurtosis, or both. It has become the preferred test because of its good power properties (Mendes and Pala, 2003). The test statistic is:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Where:

$x_{(i)}$ is the *i*th order statistic, i.e., the *i*th-smallest number in the sample;

$\bar{x} = (x_1 + \cdots + x_n)/n$ is the sample mean

The constants $a_i$ are given by $(a_1, \ldots, a_n) = \frac{m^T V^{-1}}{(m^T V^{-1} V^{-1} m)^{1/2}}$

where $m = (m_1, \ldots, m_n)^{\mathrm{T}}$, and $m_1, \ldots, m_n$ are the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution,

V is the covariance matrix of those order statistics.

The null-hypothesis of this test is that the population is normally distributed. The user may reject the null hypothesis if W is below a predetermined threshold. Or if the *p-value* is less than the chosen alpha level, then the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population.

**(3) Jarque-Bera Test for Normality**

The Jarque-Bera test is a goodness of fit measure of departure from normality (Jarque and Bera, 1980), which is based on the sample kurtosis (k) and skewness(s). The test statistics Jarque-Bera (JB) is defined as

$$JB = \frac{n}{6}\left(s^2 + \frac{(k-3)^2}{4}\right) \sim \chi^2_{(2)}$$

Where n is the number of observations and k is the number of estimated parameters. The statistic JB has an asymptotic chi-square distribution with 2 degrees of freedom, and can be used to test the hypothesis of skewness being zero and excess kurtosis being zero, since sample from a normal distribution have expected skewness of zero and expected excess kurtosis of zero.

**(4) Test for Homoskedasticity**

Volatility (i.e. time-varying variance) clustering, in which large changes tend to follow large changes, and small changes tend to follow small changes, has been well recognized in financial time series. This phenomenon is called the conditional heteroskedasticity, and can be modeled by autoregressive conditionally heteroskedasticity (ARCH) type models, including the ARCH model proposed by Engle (1982) and the later extension GARCH (generalized ARCH) model proposed by Bollerslev (1986), etc. Accordingly, when a time series exhibits the autoregressive conditionally heteroskedasticity, we say it has the ARCH effect or GARCH effect.

McLeod and Li (1983) proposed a formal test for ARCH effect based on the Ljung-Box test. It tests whether the first $L$ autocorrelations for the squared residuals are collectively small in magnitude.

Similar to the Ljung-Box test, for fixed sufficiently large $L$, the Ljung-Box Q-statistic of McLeod-Li test is given by

$$Q = N(N+2)\sum_{k=1}^{L}\frac{\hat{\rho}_k^2(\varepsilon^2)}{(N-k)} \sim \chi^2(L)$$

Where, N is the number of sample size, $L$ is the number of lags being tested. $\hat{\rho}_k^2(\varepsilon^2)$ is the squared sample autocorrelation of squared residual series at lag k.

Under the null hypothesis that no ARCH effect in the data, the test statistic Q follows the chi-square distribution with ($L$) degree of freedom. For significance level α, the critical region for rejection of the null hypothesis is $Q > \chi^2_{(1-\alpha)}(L)$, where $\chi^2_{(1-\alpha)}(L)$ is the (1- α) quantile of the $\chi^2(L)$ distribution.

**3.3 Construct and Validate an ARMAX Model**

ARIMAX model is combining the predictive value of both ARIMA model and the predictive value of external variables. For example, if a set of external variables serving as independent variables in a multiple regression are all significant, and the residuals are white noise, and then there would be no need for ARIMAX modeling. However, if the residuals have significant serial correlation, then ARIMAX model would be required to overcome time effects problem by adding some Auto-Regressive (AR) and Moving Average (MA) terms.

Traditionally, an ARIMAX model starts with a regression model. Then errors from the regression model are modeled with AR and MA term to remove serial correlation. Actually, this two steps process is not that simple. For example, when an additional external variable is added into an ARIMAX model, very likely, it will disrupt the white noise pattern of residuals from the previous ARIMAX model. On the other hand, adding in new AR and/ or MA terms may cause external variable(s) to be statistically insignificant.

**Prewhitening**

Let $\{Y_t\}$ be the time series that we want to predict (or forecast), and let $\{X_t\}$ be the covariate time series used to improve the forecasting performance.

When both series are auto correlated, it is very difficult to evaluate the linear association between them. Therefore, creating an ARIMAX model requires an additional step to remove autocorrelations among external variables.

This step is called "prewhitening". The series $\{X_t\}$, perhaps modeled with an ARIMA(p,d,q) is first written according to its representation and then pre-filter the series itself: $\tilde{X}_t = (1 - \theta_1 B - \theta_2 B^2 \ldots)\, X_t = \Theta(B)\, X_t$. By using the same filter, we "prewhiten" the Y series and then evaluate the CCF.

Once we were able to demonstrate that there is a significant correlation between two series after the pre-whitening process, we can estimate a regression model. When the residuals are still autocorrelated, we can expand the regression model adding an ARIMA structure. The resulting model is called ARIMAX.

**Residuals Diagnostic Checking**

The Ljung-Box test can be used to statistically evaluate the degree to which the residuals are serially correlated. If a model fit well, the residuals should not be correlated. If significant serial correlation exists among the residuals, it may be reduced by adding an appropriate combination of one or more significant AR and/or MA terms identified from the PACF and ACF, respectively.

The normality check can be done by the Shapiro-Wilk test. The QQ plot and the histogram plot will also be produced.

### 3.4 Construct and Validate a VAR Model

Building a VAR model involves three steps: (1) Check for co-integration. If the time series are co-integrated, then Vector Error Correction Model (VECM) needs to be applied. (2) Use some information criteria to identify the lag. (3) Use Portmanteau test for autocorrelation; apply Jarque-Bera Test for normality; choose ARCH test for homoskedasticity.

**Co-integration**

When modeling several unit-root non-stationary time series jointly, one may encounter the case of co-integration, which means two or more series that contain unit roots are related. For example, if $y_{1,t}$ and $y_{2,t}$ are co-integrated then the VAR model is not the most suitable representation for analysis because the co-integrating relations are not explicitly apparent.

A simple but instructive test for co-integration is the two step procedure proposed by Engle and Granger (1987). In the first step, fit with the following regression model: $y_{1,t} = \alpha + \beta y_{2,t} + \epsilon_t$ , then a univariate unit root test, such as ADF test will be performed on the residuals $\hat{\epsilon}_t$. Once the null hypothesis of a unit root has been rejected, the second step is to specify a Vector Error Correction Model (VECM), which run on the first differenced variables.

Corresponding to VAR(p), the VECM(p-1) form is written as:

$$\Delta Y_t = \mu_t + \Pi \mathrm{Y_{t-1}} + \Phi_1 \Delta \mathrm{Y_{t-2}} + \cdots + \Phi_{\mathrm{p-1}} \Delta \mathrm{Y_{t-p+1}} + \epsilon_{\mathrm{t}}$$

Where $\Delta$ is the differencing operator, $\Delta Y_t = Y_t - Y_{t-1}$ . To specify a VECM model, the lag order, the co-integration rank has to be determined.

The other common used test for co-integration is Phillips- Ouliaris test. Peter C. B. Phillips and Sam Ouliaris (1990) show that residual-based unit root tests applied to the estimated cointegrating residuals do not have the usual Dickey–Fuller distributions under the null hypothesis of no-cointegration. Because of the spurious regression phenomenon under the null hypothesis, the distribution of these tests have asymptotic distributions that depend on (1) the number of deterministic trend terms and (2) the number of variables with which co-integration is being tested. These distributions are known as Phillips–Ouliaris distributions and critical values have been tabulated.

**Lag Length Selection**

Before we can estimate a VAR model for the series we must specify the order p. The lag length for the VAR(p) model may be determined using model selection criteria. The general approach is to fit VAR(p) models with orders p = 1, ..., $p_{max}$ and choose the value of p which minimizes some model selection criteria. Model selection criteria for VAR(p) models have the form

$$IC(p) = ln|\bar{\Sigma}(p)| + f(T) \times \varphi(n,p)$$

Where $\bar{\Sigma}(p) = T^{-1}(\sum_{t=1}^{T} \hat{\epsilon}_t \hat{\epsilon}_t')$ is the residual covariance matrix without a degrees of freedom correction from a VAR(p) model. $f(T)$ is a sequence indexed by the sample size T; and $\varphi(n,p)$ is a penalty function which penalizes large VAR(p) models. The three most common information criteria are the Akaike (AIC), Schwarz-Bayesian (BIC) and Hannan-Quinn (HQ, 1979):

$$AIC(p) = ln|\bar{\Sigma}(p)| + \frac{2}{T} pn^2$$

$$BIC(p) = ln|\bar{\Sigma}(p)| + \frac{lnT}{T} pn^2$$

$$HQ(p) = ln|\bar{\Sigma}(p)| + \frac{2lnlnT}{T} pn^2$$

The AIC criterion asymptotically overestimates the order with positive probability, whereas the BIC and HQ criteria estimate the order consistently under fairly general conditions if the true order p is less than or equal to $p_{max}$. The key difference between the criteria is how severely each penalizes increases in model order. For more information on the use of model selection criteria in VAR models see Lutkepohl (1991) chapter four.

**Multivariate Portmanteau test**

In time series analysis, a version of portmanteau test is available for testing for autocorrelation in the residuals of a model. It tests whether any of a group of autocorrelations of the residual time series are different from zero. This test is the Ljung–Box test. The univariate Ljung-Box test has been generalized to the multivariate case by Hosking (1980, 1981) and Li and McLeod (1981). For a multivariate series, the null hypothesis is $H_0$: $\rho_1 = \ldots = \rho_m = 0$ and the alternative hypothesis$H_a$: $\rho_i \neq 0\ for\ some\ i \in \{1, \ldots, m\}$. Thus the statistic is used to test that there are no auto- and cross-correlations in the vector series $r_t$ . The test statistic assumes the form

$$Q_k(m) = T^2 \sum_{l=1}^{m} \frac{1}{T-l} tr(\hat{\Gamma}_l' \hat{\Gamma}_0^{-1\prime} \hat{\Gamma}_l \hat{\Gamma}_0^{-1})$$

Where T is the sample size, k is the dimension of $r_t$, and tr(A) is the trace of the matrix A, which is the sum of the diagonal elements of A. under the null hypothesis and some regularity conditions, $Q_k(m)$ follows asymptotically a chi-squared distribution with $k^2m$ degrees of freedom.

**Impulse Response Function (IRF)**

Generally, an impulse response refers to the reaction of any dynamic system in response to some external change. In particular, VAR‘s impulse responses mainly examine how the dependent variables react to shocks from each independent variable. Lutkepohl and Reimers (1992) stated that the traditional impulse response analysis requires orthogonalization of shocks. And the results vary with the ordering of the variables in the VAR. The higher correlations between the residuals are, the more important the variable ordering is. In order to overcome this problem, Pesaran and Shin (1998) developed the generalized impulse response functions which adjust the influence of a different ordering of the variables on impulse response functions.

More detailed, if everything else stays constant, the (i,j) element of the matrix $\Psi_k$ identifies the impact of a unit increase in the j-th variable's error (or innovation shock) at time t for the value of i-th variable at time t+k. Plotting these functions for different values of k can be very helpful to determine how individual shocks affect forecasts. R can calculate them easily and provide MC and bootstrap error bands.

### 3.5 Software

All analyses in this thesis are performed using the open source Software **R** for MS-Windows (version 64X 3.1.2). Additional library packages were used: **TSA, tseries, urca, vars, MTS**, **MVN, and forecast.**

# CHAPTER 4

# UNIVARIATE TIME SERIES MODELING

## 4.1Data Description

Three data are involved in this thesis: the seasonally adjusted U.S. national unemployment rate (UR), the seasonally adjusted job openings (JOB), and the seasonally adjusted initial claim for unemployment insurance (IC).

As mentioned in chapter 2, the JOB data are available from Dec 2000 to Dec 2014, and the release date of JOB data has two months lag. In align with the JOB data I pick the UR and the IC data from Feb 2001 to Feb 2015. Therefore, we have three time series datasets: $\{U_t\}$ for UR, $\{J_{t-2}\}$ for JOB and $\{I_t\}$ for IC, and they have same released period from Feb 2001 to Dec 2014. In this thesis, I choose data released from Feb 2001 to Jun 2014 for in sample model estimation, and leave 8 months data from Jul 2014 to Feb 2015 for the out of sample forecasting comparison.

**Table 4.1** The descriptive statistics for UR, JOB and IC

| Data | Obs. | Mean | Std. Dev. | Median | Min | Max | Skew | Kurtosis | Shapiro-Wilk |
|---|---|---|---|---|---|---|---|---|---|
| UR | 167 | 6.56 | 1.75 | 5.90 | 4.20 | 10.00 | 0.57 | -1.05 | P<0.01 |
| JOB | 167 | 3.69 | 0.65 | 3.69 | 2.15 | 5.27 | -0.17 | -0.56 | P=0.1098 |
| IC | 167 | 1.55 | 0.30 | 1.48 | 1.12 | 2.62 | 1.35 | 1.92 | P<0.01 |

Table 4.1 reports the descriptive statistics for UR, JOB and IC. The UR and IC data are right-skewed distributed, they are non-normal (Reject Shapiro-Wilk test at 1% level). The JOB data look normal with a p-value of 0.1098.

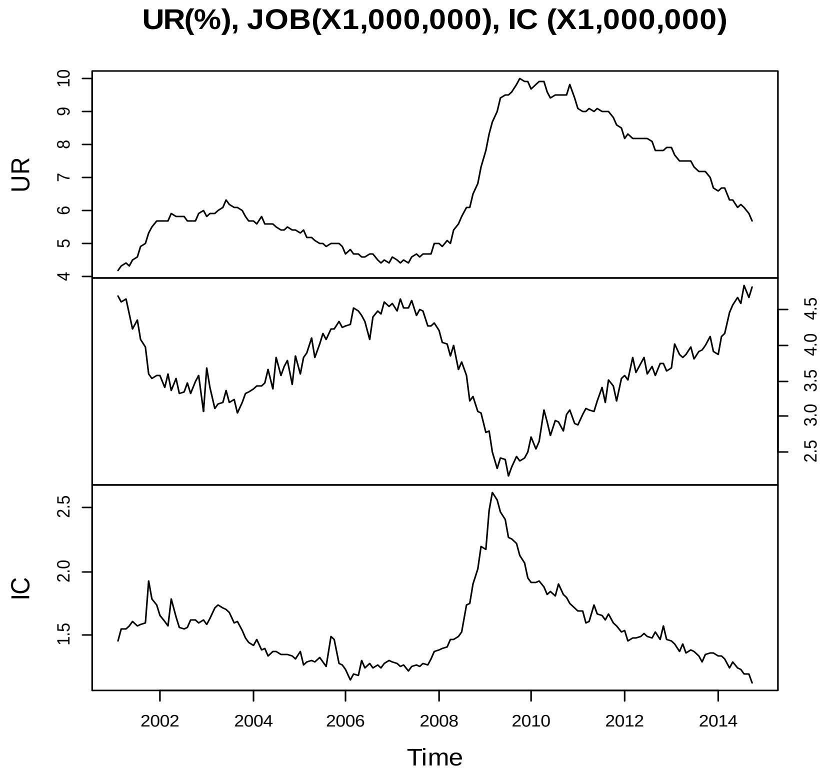


**Figure 4.1:** Seasonally adjusted Unemployment Rate (UR), Job Openings (JOB) & Initial Claim (IC) (From Feb 2001 to Oct 2014)

In Figure 4.1, we can see UR and JOB are fare symmetric; and UR and IC have very similar up and down movements.

**Table 4.2** Correlation of UR, JOB and IC

| Correlation | UR | JOB | IC |
|---|---|---|---|
| **UR** | 1.0000 | -0.7928 | 0.6442 |
| **JOB** | -0.7928 | 1.0000 | -0.7503 |
| **IC** | 0.6442 | -0.7503 | 1.0000 |

From Table 4.2, we can see that correlation between JOB and UR is very high (-0.7928), even higher than the correlation between IC and UR (0.6442). As we know, IC is a well-known indicator for UR. It suggests that the JOB data has a very good potential to be a good indicator for predicting UR.

## 4.2 Constructing an ARIMA Model for the UR data

### Unit root test for UR

Montgomery et al. (1998) concluded that the UR had no consistent trend at all, and they used ARIMA model as a benchmark model for forecasting the UR. In this thesis, I will also apply ARIMA model to the UR data.

Before proceeding with the estimation of an ARIMA model, I check the stationarity of the UR data by performing unit root tests. Augmented Dickey-Fuller test with GLS de-trending (ADF-GLS) is suggested by Elliott et al. (1996). This test is similar to the standard ADF test but it applies GLS de-trending before the series is tested with the ADF test. Compared with the standard ADF test, ADF-GLS test has the best overall performance in terms of small sample size and power.

**Table 4.3** Unit Root Test for UR

| ADF-GLS with a Constant Test | | |
|---|---|---|
| **Variable** | **test Stat.** | **Test Result** |
| $U_t$ | -1.0912 | Has Unit Root |
| $U_t$-$U_{t-1}$ | -2.0395 | Stationary |
| log($U_t$) | -0.9339 | Has Unit Root |
| logit($U_t$) | -0.9442 | Has Unit Root |

Notes: Critical values at 1, 5, 10% are -2.58 -1.94 -1.62

In Table 4.3, I report the unit root test results by performing ADF-GLS with a constant Test for the series $U_t$, $U_t$-$U_{t-1}$, log($U_t$) and logit($U_t$), where $\text{logit}(U_t) = \log\left(\frac{U_t/100}{1-U_t/100}\right)$. The log($U_t$) and logit($U_t$) are suggested by Koop and Potter (1999) to make the series unbounded. Looking at the $U_t$, log($U_t$) and logit($U_t$), they all fail to reject the null hypothesis of having unit root at 10% significant level. For $U_t$-$U_{t-1}$, it shows the stationarity. The test statistic -2.0395 is small than 5% critical value -1.94; so it rejects the null hypothesis at 5% significant level.

Montgomery et al. (1998) applied first differences of UR to build ARIMA model as a benchmark model. From the above ADF-GLS test results and following previous researches, I choose first differences of the UR to build up Benchmark ARIMA model.

**Constructing an ARIMA Model**

After a time series has been stationarized by differencing, next step, I will check how many AR or MA terms are needed to correct autocorrelations.

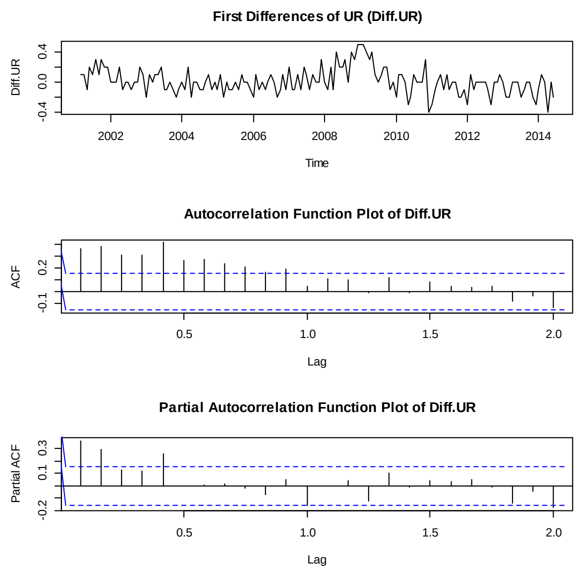


**Figure 4.2** Plots of First Differences of UR data, and related ACF and PACF

Figure 4.2 show the graph of the first differences of UR data (Diff.UR) and the related ACF and PACF plots. ACF plot tails off gradually after lag 5, PACF plot also cuts off after lag 5, which give us the sign of both AR and MA term are needed to remove the autocorrelation. As a result, I choose p=6 and q=6 as the maximum lag length to find the best model for Diff.UR.

In order to select the best model ARIMA model for Diff.UR, I start with a very short lag model ARIMA(1,1,1) and then successively add one lag separately for AR or MA term until it reaches maximum lag length 6 for both AR and MA, which is

ARIMA(6,1,6). Among the total 36 ARIMA models, I find ARIMA(4,1,2) comes with the lowest AIC and BIC.

**Table 4.4** AIC and BIC for ARIMA model selection

| Model | AIC | BIC |
|---|---|---|
| **ARIMA(4,1,1)** | -153.65 | -135.20 |
| **ARIMA(4,1,2)** | **-159.16** | **-137.63** |
| **ARIMA(4,1,3)** | -150.66 | -126.05 |
| **ARIMA(3,1,2)** | -153.14 | -134.69 |
| **ARIMA(5,1,2)** | -154.98 | -130.38 |

Table 4.4 shows the AIC and BIC for model ARIMA(4,1,2) and the models with one lag difference in AR and MA term. The ARIMA(4,1,2) has lowest AIC (-159.16) and the lowest BIC (-137.63). Since both AIC and BIC happen to agree, the choice of model ARIMA(4,1,2) is clear.

Table 4.5 shows the parameter statistics of model ARIMA(4,1,2). Except AR3 (P-value=0.3052), the other coefficients for AR1, AR2, AR4, MA1, MA2 are all significant different from 0. Do we need to remove AR3 due to the not statistically significant? It is conventional in dynamic analysis that if we determine the appropriate lag length to be q, we usually include all lags between 0 and q. It would be very unusual to encounter an economic model in which, for example, $Y_{t-1}$ and $Y_{t-3}$ would affect $Y_t$, but $Y_{t-2}$ would not. Therefore we would not usually omit $Y_{t-2}$ as a regressor even if its coefficient is not statistically significant (Parker, 2014).

**Table 4.5** Parameters Statistics of model ARIMA(4,1,2) for UR

| Parameter | Estimate | s.e. | t Value | Pr > \|t\| |
|---|---|---|---|---|
| AR1 | 1.8228 | 0.0770 | 23.6727 | < 0.0001 |
| AR2 | -1.0051 | 0.1624 | -6.1890 | <0.0001 |
| AR3 | -0.1678 | 0.1637 | -1.0250 | 0.3052 |
| AR4 | 0.2558 | 0.0790 | 3.2380 | 0.0012 |
| MA1 | -1.7095 | 0.0325 | -52.6000 | < 0.0001 |
| MA2 | 1.0000 | 0.0347 | 28.8184 | <0.0001 |

As a result, for UR, the ARIMA(4,1,2) model is created as below:

$$(1 + 1.8288B - 1.0051B^2 - 0.1678B^3 + 0.2558B^4)(1 - B)^1 Y_t =$$

$$(1 - 1.7095B + 1.0B^2)\epsilon_t \qquad (4.1)$$

$$\hat{\sigma}_\epsilon^2 = 0.01938.$$

**Diagnostic Check for ARIMA(4,1,2)**

After the model (model 4.1) being chosen for the UR data, we can test whether the residuals can meet white noise assumptions, as the residuals from the selected ARIMA(4,1,2) model are assumed to be independent, homoscedastic, and usually normally distributed.

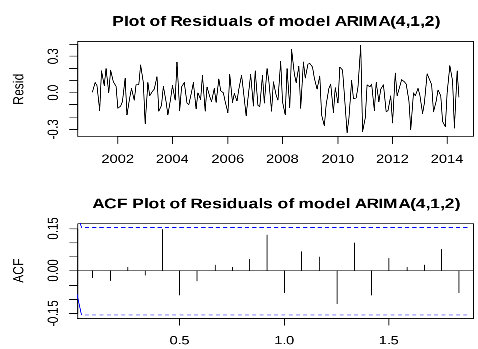


**Figure 4.3** Plot of residuals from ARIMA(4,1,2) and the residuals' ACF plot

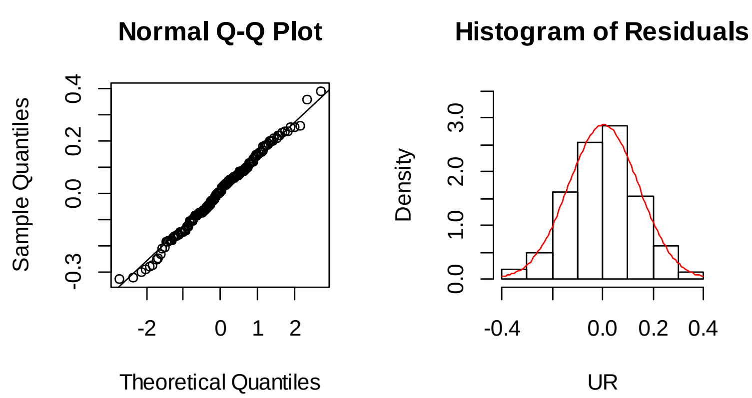


**Figure 4.4** Normal QQ Plot and Histogram of residuals from ARIMA(4,1,2)

In Figure 4.4, almost all the points are laid on the line in Normal QQ plot; and the shape of the Histogram appears "Bell Shaped" curve. So the residuals of the fitted model can be referred as normal.

Instead of visual inspection of the sample autocorrelation plot, more formally, we can apply the Ljung-Box test to the residual series to check for autocorrelation and we can use the Shapiro-Wilk test for the normality. For ARCH effect, we choose McLeod and Li test.

**Table 4.6** Independent and normality test for residuals of ARIMA(4,1,2)

| | **Residuals of ARIMA(4,1,2)** | |
|---|---|---|
| **Test** | Ljung-Box test | Shapiro-Wilk normality test |
| **$H_0$** | residuals are independent | residuals are normal distributed |
| **Test Stat.** | X-squared = 9.6663 | W = 0.9946 |
| **P-value** | P-value = 0.1394 (df=6) | P-value = 0.8234 |

In Table 4.6, Box-Ljung test fails to reject the null hypothesis of independence with a P-value of 0.1394. Shapiro-Wilk normality test fails to reject the null hypothesis of normality at P-value of 0.8234.

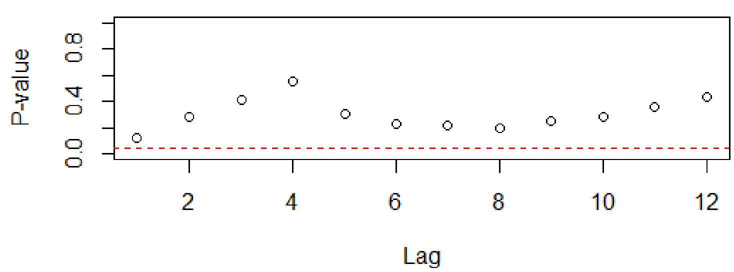


**Figure 4.5** McLeod and Li test for residuals of ARIMA(4,1,2)

In Figure 4.5, McLeod and Li test fails to reject the null hypothesis of no ARCH effect with P-values > 0.1 for all the lags from 1 to 12.

In conclusion, the residuals from fitted model are independent and normal distributed, so the UR data can be well represented by ARIMA(4,1,2) model.

### 4.3 Constructing an ARIMAX Model with the JOB data

Creating an ARIMAX model follows the same steps as for an ARIMA model. The Box-Jenkins methodology of model specification, estimation of parameters, and diagnostic check applies to ARIMAX models as well as ARIMA models. There is an additional step added to remove autocorrelations among external variables. This step is called prewhitening and is necessary when an external predictor is autocorrelated.

**Prewhitening of the UR data with the JOB data**

When "prewhitening", the series $\{X_t\}$, perhaps modeled with an ARIMA(p,d,q) is first written according to its representation and then pre-filter the series itself: $\tilde{X}_t = (1 - \theta_1 B - \theta_2 B^2 \dots)\, X_t = \Theta(B)\, X_t$. By using the same filter, we "Prewhiten" the Y series and then evaluate the Cross-Correlation Function (CCF).

As we mentioned in Chapter 3, in reality, the release date of JOB data has two months lag, so we choose JOB at time t-2 ($JOB_{t-2}$)as an external variable to predict UR at time t ($UR_t$) and construct ARIMAX model.

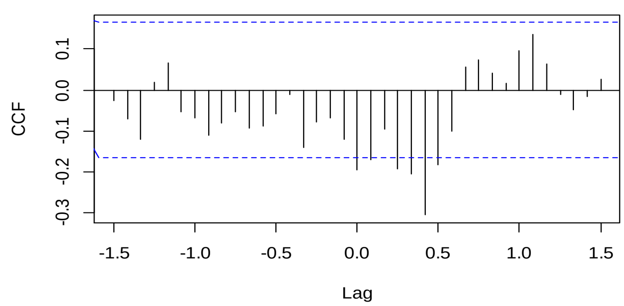


**Figure 4.6** CCF Plot of the prewhitened $UR_t$ and $JOB_{t-2}$ data

Figure 4.6 shows there are significant correlations between two series in after the pre-whitening process, so we can start to estimate a simple regression model.

**Table 4.7** linear regression model for $UR_t$ with $JOB_{t-2}$ as an external variable

| lm(formula = $UR_t$ ~ $JOB_{t-2}$) | | | | |
|---|---|---|---|---|
| **Parameter** | **Estimate** | **Std. Error** | **t value** | **Pr(>\|t\|)** |
| **(Intercept)** | 14.9544 | 0.487 | 30.71 | < 0.0001 |
| **$JOB_{t-2}$** | -2.2911 | 0.1313 | -17.44 | <0.0001 |

Table 4.7 shows a simple linear regression model by using $JOB_{t-2}$ as an external variable to predict $UR_t$:

$$UR_t = 14.9544 - 2.2911 \times JOB_{t-2} \qquad (4.2)$$

Figure 4.7 shows that the residuals from the regression model (model 4.2) are still auto-correlated, it is appropriate to expand the regression model by adding an ARIMA structure to construct ARIMAX model. In Figure 4.7, the ACF plot tails off gradually, PACF plot cuts off after lag 3. As a result, I choose p=6 and q=6 as the maximum lag length to find the best ARIMAX model.

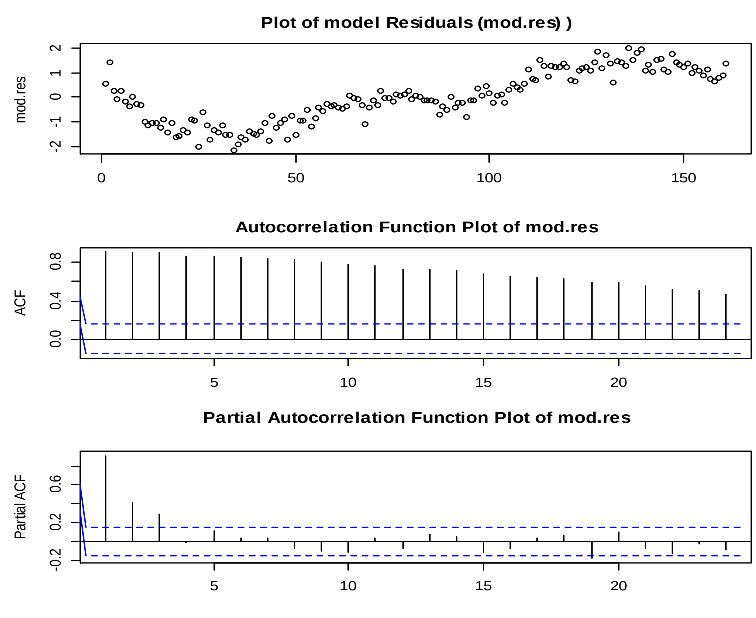


**Figure 4.7** Plots of the residuals of (model 4.2), and related ACF and PACF.

**Unit Root Test for Residuals from Model 4.2**

Before proceeding with estimation of ARIMAX model, I am checking the stationarity of the residuals of model 4.2 by performing ADF unit root tests.

**Table 4.8** Unit Root Test for residuals of model 4.2

| ADF Test (alternative hypothesis: stationary) | | |
|---|---|---|
| **Variable** | **Test Stat.** | **P-value** |
| **Residuals of model 4.2** | -3.6745 | 0.02863 |

In Table 4.8, The ADF unit root test result rejects the null hypothesis of having a unit root with p-value of 0.02863. The residuals are stationary, so we don't need to apply any transformation for data UR and JOB.

**Model Selection**

The UR and JOB data are all seasonally adjusted, so it is not necessary to add the seasonal term into ARIMAX model. To select the best ARIMAX model, I start with a very short lag model ARIMAX(1,0,1) and then successively add one lag separately for AR or MA term until it reaches maximum lag length 6 for both AR and MA, which is ARIMAX(6,0,6). Among the total 36 ARIMA models, I find ARIMAX(6,0,0) comes with the lowest AIC and BIC.

**Table 4.9** AIC and BIC of ARIMAX model for $UR_t$ with $JOB_{t-2}$

| Model | AIC | BIC |
|---|---|---|
| **ARIMAX(5,0,0)** | -145.90 | -124.33 |
| **ARIMAX(6,0,0)** | -158.60 | -133.95 |
| **ARIMAX(7,0,0)** | -156.78 | -129.05 |
| **ARIMAX(6,0,1)** | -156.84 | -129.11 |

Table 4.9 shows the AIC and BIC for model ARIMAX(6,0,0) and the models with one lag difference in AR and MA term. The ARIMAX(6,0,0) has the lowest AIC (-158.60) and the lowest BIC (-133.95). Since both AIC and BIC happen to agree, the choice of model ARIMAX(6,0,0) is clear.

Table 4.10 shows the parameter statistics of model ARIMAX(6,0,0), we can also call it ARX(6). Even though AR2, AR3, AR4 not statistically significant, as Parker suggested (2014), we would usually not omit them.

**Table 4.10** Parameters Statistics of model ARX(6) for $UR_t$ with $JOB_{t-2}$

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| |
|---|---|---|---|---|
| AR1 | 1.1465 | 0.0752 | 15.2460 | < 0.0001 |
| AR2 | 0.0349 | 0.1164 | 0.2998 | 0.764 |
| AR3 | -0.1381 | 0.1183 | -1.1674 | 0.243 |
| AR4 | -0.0184 | 0.1217 | -0.1512 | 0.8799 |
| AR5 | 0.2657 | 0.1216 | 2.1850 | 0.0289 |
| AR6 | -0.3029 | 0.0768 | -3.9440 | <0.0001 |
| Intercept | 6.6451 | 0.8400 | 7.9108 | < 0.0001 |
| $JOB_{t-2}$ | -0.1316 | 0.0555 | -2.3712 | 0.0177 |

As a result, the ARX(6)-$JOB_{t-2}$ model is created as below:

$$(1 + 1.1.1465B + 0.0349B^2 - 0.1381B^3 - 0.0184B^4 + 0.2657B^5 -$$

$$0.3029B^6)UR_t = 6.6451 - 0.1316JOB_{t-2} + \epsilon_t \qquad (4.3)$$

$$\hat{\sigma}_\epsilon^2 = 0.01909.$$

### Diagnostic Check for Model ARX(6)-$JOB_{t-2}$

After the model 4.3 being chosen for $UR_t$ data, with $JOB_{t-2}$ data as an external variable, we can test whether the residuals can meet white noise assumptions.

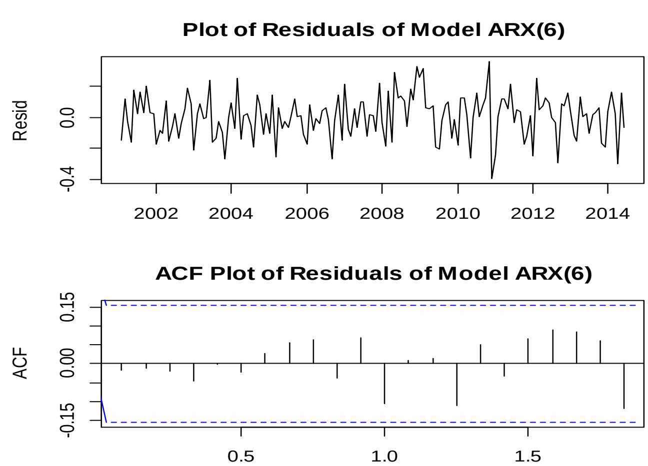


**Figure 4.8** Plot of residuals of ARX(6)-$JOB_{t-2}$ and the residuals' ACF plot

Figure 4.8 show the plot of residuals from selected model ARX(6)-JOB $_{t-2}$. The ACF plot shows no "Statistically Significant" correlation appears.

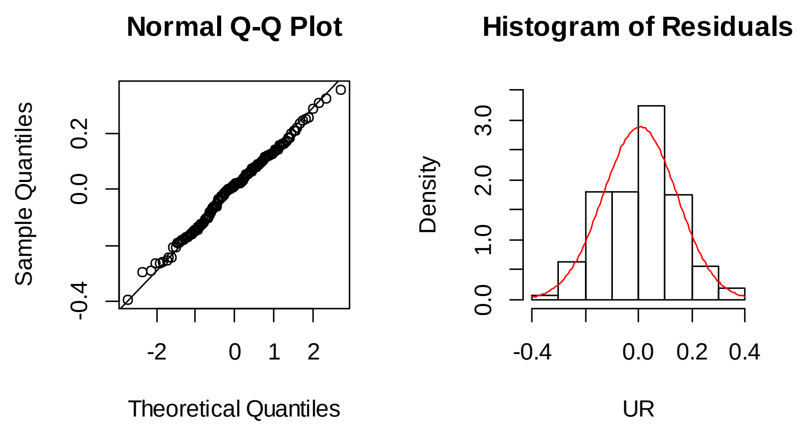


**Figure 4.9** Normal QQ Plot and Histogram of Residuals of Model ARX(6)-JOB $_{t-2}$

In Figure 4.9, almost all the points are laid on the line in Normal QQ plot; and the shape of the Histogram appears "Bell Shaped" curve. So the residuals of the fitted model can be referred as normal.

**Table 4.11** Independent and normality test for residuals of ARX(6)-JOB $_{t-2}$

| | **Residuals of ARX(6)-JOB $_{t-2}$** | |
|---|---|---|
| **Test** | Ljung-Box test | Shapiro-Wilk normality test |
| **$H_0$** | residuals are independent | residuals are normal distributed |
| **Test Stat.** | X-squared = 4.9288 | W = 0.9941 |
| **P-value** | p-value = 0.5530 (df=6) | p-value = 0.7609 |

In table 4.11, for Box-Ljung test, the null hypothesis of independence fail to be rejected (p-value =0.553). Shapiro-Wilk test has a test statistics of W=0.9941, leading to a P-value of 0.7609, and fails to reject the null hypothesis of normality.

In conclusion, the residuals from fitted model are independent and normal distributed. So the UR data can be well represented by model ARX(6)-JOB $_{t-2}$.

## 4.4 Constructing an ARIMAX Model with the IC data

Two models have been well constructed in Chapter 4.2 and 4.3: an benchmark model ARIMA(4,1,2) for UR, and an ARX(6)-JOB $_{t-2}$ model for UR with JOB $_{t-2}$ as an external variable. As mentioned in Chapter 3, IC is a well known indicator for UR prediction. For a better comparison, I am constructing an ARIMAX model with IC as an external variable.

### Prewhitening of the UR data with the IC data

Comparing to the UR data, the release date of IC data has no lag; so we choose IC at time t ($IC_t$) as an external variable to predict UR at time t ($UR_t$) and construct an ARIMAX model.

Figure 4.10 shows there are significant correlations between two series after the pre-whitening process, so we can start to estimate a regression model.

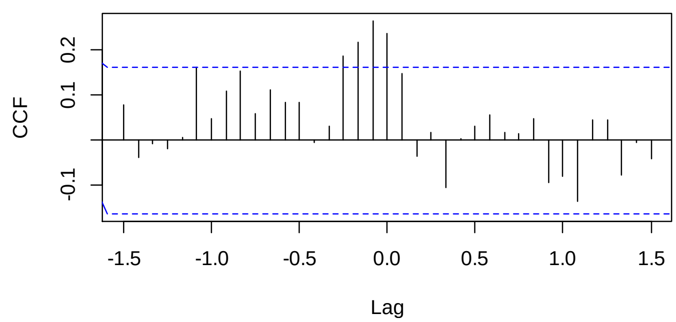


**Figure 4.10** CCF Plot of the prewhitened $UR_t$ and $IC_t$ data

Table 4.12 shows the simple linear regression model by using $IC_t$ as an external variable to predict $UR_t$: $UR_t = 0.4744 + 3.9166 \times IC_t$ (model 4.4)

**Table 4.12** linear regression model for $UR_t$ with $IC_t$ as an external variable

| lm(formula = $UR_t$ ~ $IC_t$) | | | | |
|---|---|---|---|---|
| **Parameter** | **Estimate** | **Std. Error** | **t value** | **Pr(>\|t\|)** |
| **(Intercept)** | 0.4744 | 0.5815 | 0.816 | 0.416 |
| **$IC_t$** | 3.9166 | 0.3665 | 10.687 | <0.001 |

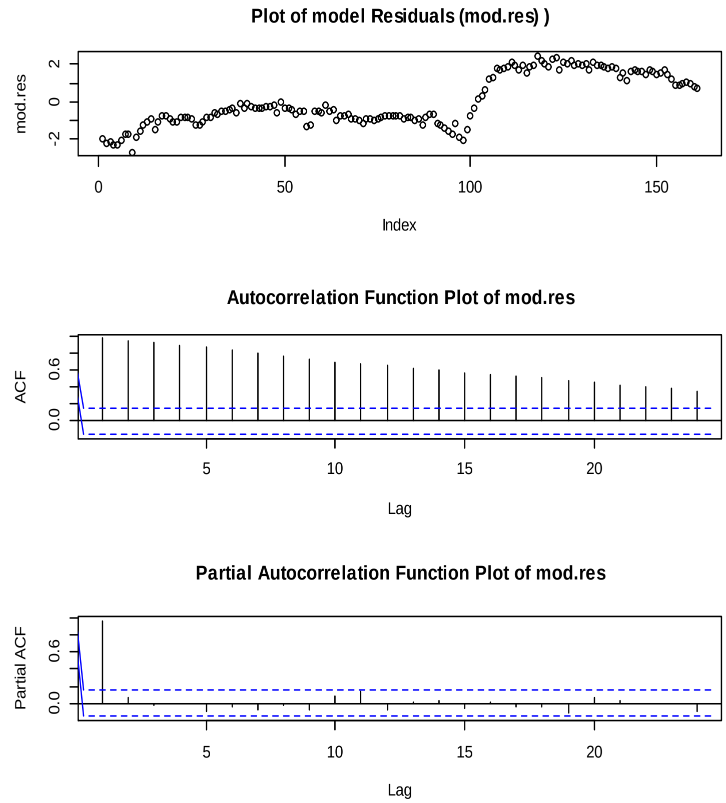


**Figure 4.11** Plots of the residuals of (model 4.4), and related ACF and PACF

Figure 4.11 shows that the residuals from the regression model (model 4.4) are auto-correlated; the ACF plot tails off gradually, PACF plot cuts off after lag 1. Therefore, time series model would be appropriate to remove the autocorrelation among the residuals.

### Unit Root Test for the Residuals from Model 4.4

Before proceeding with estimation of ARIMAX model, I am checking the stationarity of the residuals of model 4.4 by performing ADF unit root tests.

**Table 4.13** Unit Root Test for the residuals of model 4.2

| **ADF Test (**alternative hypothesis: stationary) | | |
|---|---|---|
| **Variable** | **Test Stat.** | **P-value** |
| **Residuals of model 4.2** | -2.0232, | 0.5666 |
| **First Differences of Residuals** | -3.8764 | 0.01705 |

In Table 4.13, The ADF unit root test results for the residuals from model 4.4 fail to reject the null hypothesis of having a unit root with p-value of 0.5666. But first differences of the residuals reject the null hypothesis with p-value of 0.01705. So the first differences of the residuals are stationary. The integrated model will be applied. Now we can start to expand the regression model by adding an ARIMA structure to construct ARIMAX model. I also choose p=6 and q=6 as the maximum lag length to find the best ARIMAX model.

**Model Selection**

In order to select the best ARIMAX model, I start with a very short lag model ARIMAX(1,1,1) and then successively add one lag separately for AR or MA term until it reaches maximum lag length 6 for both AR and MA, which is ARIMAX(6,1,6). Among the total 36 ARIMA models, ARIMAX(4,1,5) comes with the lowest AIC, and ARIMAX(4,1,4) comes with the lowest BIC.

**Table 4.14** AIC and BIC of ARIMAX model for UR with IC

| Model | AIC | BIC |
|---|---|---|
| **ARIMAX(3,1,5)** | -162.72 | -134.99 |
| **ARIMAX(4,1,5)** | -166.36 | -135.55 |
| **ARIMAX(5,1,5)** | -164.4 | -130.5 |
| **ARIMAX(4,1,4)** | -165.66 | -137.93 |
| **ARIMAX(4,1,6)** | -162.78 | -128.88 |

Table 4.14 shows the AIC and BIC for model selection. The ARIMAX(4,1,5) has the lowest AIC (-166.36) and but ARIMAX(4,1,4) has the lowest BIC (-137.93). In this thesis, both models will be selected in the Chapter 6 for forecasting comparison. Table 4.15 and 4.16 show the parameter statistics of model ARIMAX(4,1,4)-ICt and model ARIMAX(4,1,5)-ICt separately.

**Table 4.15** Parameters Statistics of model ARIMAX(4,1,4)-$IC_t$

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| |
|---|---|---|---|---|
| AR1 | 0.7151 | 0.1139 | 6.278314 | <0.0001 |
| AR2 | 0.0171 | 0.0956 | 0.17887 | 0.8834 |
| AR3 | 0.6392 | 0.0906 | 7.055188 | <0.0001 |
| AR4 | -0.5595 | 0.1063 | -5.26341 | <0.0001 |
| MA1 | -0.6774 | 0.1168 | -5.79966 | <0.0001 |
| MA2 | 0.1847 | 0.052 | 3.551923 | 0.01617 |
| MA3 | -0.8336 | 0.0577 | -14.4471 | 0.321 |
| MA4 | 0.8427 | 0.141 | 5.976596 | <0.0001 |
| $IC_t$ | 0.566 | 0.1445 | 3.916955 | <0.0001 |

**Table 4.16** Parameters Statistics of model ARIMAX(4,1,5)-$IC_t$

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| |
|---|---|---|---|---|
| AR1 | 0.5228 | 0.1647 | 3.174256 | 0.0015 |
| AR2 | 0.1051 | 0.0962 | 1.092516 | 0.2749 |
| AR3 | 0.6195 | 0.0761 | 8.140604 | <0.0001 |
| AR4 | -0.4969 | 0.1214 | -4.09308 | <0.0001 |
| MA1 | -0.4276 | 0.1761 | -2.42817 | 0.0152 |
| MA2 | 0.0287 | 0.0991 | 0.289606 | 0.7722 |
| MA3 | -0.7588 | 0.0694 | -10.9337 | <0.0001 |
| MA4 | 0.6689 | 0.143 | 4.677622 | <0.0001 |
| MA5 | 0.2124 | 0.1233 | 1.722628 | 0.0848 |
| $IC_t$ | 0.5652 | 0.1277 | 4.425998 | <0.0001 |

As a result, **ARIMAX(4,1,4)-ICt** model is created as below:

$$(1 + 0.7151 + 0.0171B^2 + 0.6392B^3 - 0.5595B^4)(1 - B)^1 UR_t = 0.566IC_t + (1 - 0.6774B + 0.1847B^2 - 0.8336B^3 + 0.8427B^4)\epsilon_t \qquad (4.5)$$

$$\hat{\sigma}_\epsilon^2 = 0.01789$$

**ARIMAX(4,1,5)-ICt** model is created as below:

$$(1 + 0.5228 + 0.1051B^2 + 0.6195B^3 - 0.4969B^4)(1 - B)^1 UR_t = 0.5652IC_t + (1 - 0.4276B + 0.0287B^2 - 0.7588B^3 + 0.6689B^4 + 0.2124B^5)\epsilon_t \qquad (4.6)$$

$$\hat{\sigma}_\epsilon^2 = 0.01754$$

**Diagnostic Check for Model ARIMAX(4,1,5)-$IC_t$**

The residuals of model ARIMAX(4,1,4)-$IC_t$ and ARIMAX(4,1,5)-$IC_t$ both show the white noise pattern and fit the model assumption of independent and normal distributed. For simlicity, the diagnostic procedure of ARIMAX(4,1,5)-$IC_t$ will be demonstrated as an example.

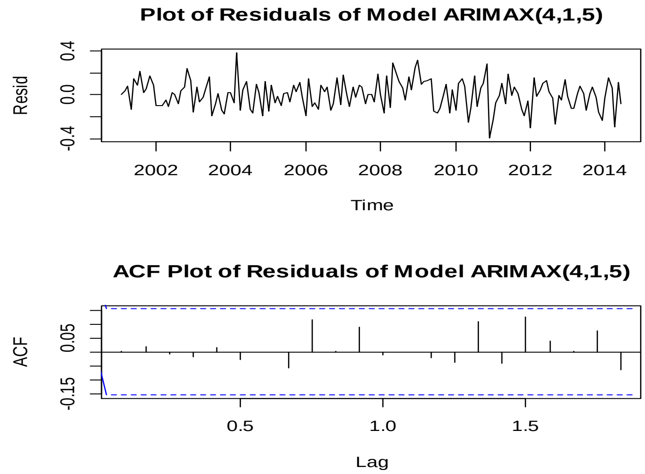


**Figure 4.12** Plot of residuals of ARIMAX(4,1,5)-$IC_t$ and the residuals' ACF plot

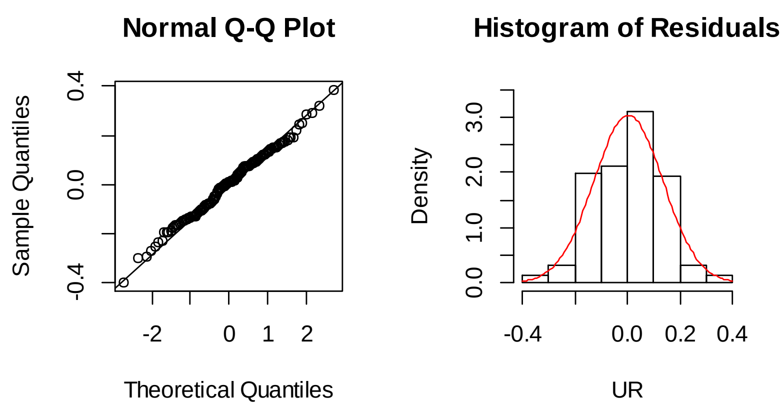


**Figure 4.13** Normal QQ Plot and Histogram of residuals of ARIMAX(4,1,5)-$IC_t$

Figure 4.12 shows the plot of residuals from selected model **ARIMAX(4,1,5)-$IC_t$.** The ACF plot shows no statistically significant correlation appears.

In Figure 4.13, almost all the points are laid on the line in Normal QQ plot; and the shape of the histogram appears "Bell Shaped" curve. So the residuals of the fitted model ARIMAX(4,1,5)-ICt can be considered approximately normal.

**Table 4.17** Independent and normality test for residuals of ARIMAX(4,1,5)-$IC_t$

| | Residuals of ARIMA(4,1,5) | |
|---|---|---|
| **Test** | Ljung-Box test | Shapiro-Wilk normality test |
| **$H_0$** | residuals are independent | residuals are normal distributed |
| **Test Stat.** | X-squared = 4.5857 | W = 0.9946 |
| **P-value** | p-value = 0.2048 (df=3) | p-value = 0.824 |

In table 4.17, Box-Ljung test fails to rejectthe null hypothesis of independence with P-value =0.2048. Shapiro-Wilk normality test has a test statistics of W=0.9946, leading to a P-value of 0.824, and fails to reject the null hypothesis of normality. These confirm that the residuals from fitted model ARIMAX(4,1,5)-$IC_t$ are independent and normal distributed.

### 4.5 Constructing an ARIMAX Model with JOB & IC data

In this section, both JOB and IC will be added in ARIMAX model for UR forecasting. In Chapter 4.3 and 4.4, it has been shown that there are correlations between UR and JOB, and between UR and IC after the pre-whitening process. Following, I can start to estimate a regression model.

**Table 4.18** linear regression model for $UR_t$ with $IC_t$ and $JOB_{t-2}$ as external variables

| lm(formula = $UR_t$ ~ $IC_t$+ $JOB_{t-2}$) | | | | |
|---|---|---|---|---|
| **Parameter** | **Estimate** | **Std. Error** | **t value** | **Pr(>\|t\|)** |
| **(Intercept)** | 13.0263 | 1.2536 | 10.391 | <0.0001 |
| **$JOB_{t-2}$** | -2.0563 | 0.1921 | -10.707 | <0.0001 |
| **$IC_t$** | 0.6862 | 0.4115 | 1.668 | 0.0974 |

Table 4.20 shows the linear regression model by using $JOB_{t-2}$ and $IC_t$ as an external variable to predict $UR_t$:

$$UR_t = 13.0263 - 2.0563 \times JOB_{t-2} + 0.6862IC_t \qquad (4.7)$$

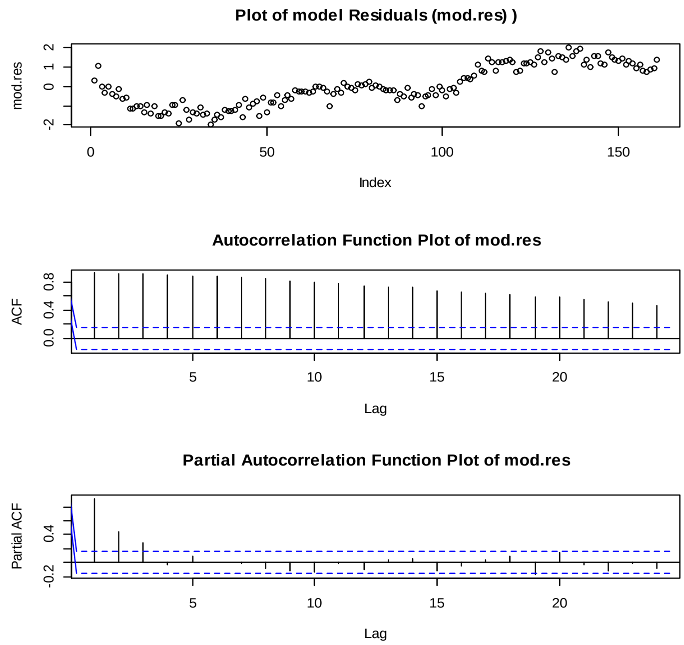


**Figure 4.14** Plots of the residuals of (model 4.7), and related ACF and PACF.

Figure 4.14 shows that the residuals from the regression model (model 4.7) are auto-correlated, the ACF plot tails off gradually, PACF plot cuts off after lag 3.

**Unit Root Test for Residuals from Model 4.7**

Before proceeding with estimation of ARIMAX model, the stationarity of the residuals of model 4.7 is checked by performing ADF unit root tests.

**Table 4.19** Unit Root Test for residuals of model 4.7

| **ADF Test (**alternative hypothesis: stationary) | | |
|---|---|---|
| **Variable** | **Test Stat.** | **P-value** |
| **Residuals of model 4.6** | -3.264 | 0.07967 |
| **First Differences of Residuals** | -6.2585 | <0.01 |

In Table 4.19, The ADF unit root test result for the residuals from model 4.6 rejects the null hypothesis of having a unit root at a 0.10 significant level. To avoid over-differencing, ARMAX model is first selected. I also choose p=6 and q=6 as the maximum lag length to find the best ARMAX model.

**Model Selection**

In order to select the best ARMAX model, I start with a very short lag model ARMAX(1,1) and then successively add one lag separately for AR or MA term until it reaches maximum lag length 6 for both AR and MA, which is ARIMAX(6,6). Among the total 36 ARIMA models, I find ARMAX(6,0), or ARX(6) comes with the lowest AIC and BIC (Table 4.20).

**Table 4.20** AIC and BIC of ARIMAX model for UR with JOB and IC

| Model | AIC | BIC |
|---|---|---|
| **ARMAX(5,0)** | -148.7 | -124.05 |
| **ARMAX(6,0)** | -164.36 | -136.63 |
| **ARMAX(7,0)** | -162.76 | -131.95 |
| **ARMAX(6,1)** | -162.76 | -131.95 |

Table 4.21 shows the parameter statistics of model AR(6). Both external variables JOB and IC are statistically significant at 5% level and 1% level.

**Table 4.21** Parameters Statistics of model ARX(6) with JOB & IC

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| |
|---|---|---|---|---|
| **AR1** | 1.0866 | 0.0782 | 13.89514 | < 0.01 |
| **AR2** | 0.0898 | 0.114 | 0.787719 | 0.4308 |
| **AR3** | 0.1503 | 0.1137 | 1.3219 | 0.1863 |
| **AR4** | 0.0437 | 0.1193 | 0.366303 | 0.7142 |
| **AR5** | 0.248 | 0.117 | 2.119658 | 0.03408 |
| **AR6** | -0.3302 | 0.0758 | -4.3562 | <0.01 |
| **Intercept** | 5.9564 | 0.8542 | 6.973074 | <0.01 |
| **$JOB_{t-2}$** | -0.1198 | 0.0553 | -2.16637 | 0.0303 |
| **$IC_t$** | 0.4243 | 0.1516 | 2.798813 | <0.01 |

As a result, ARX(6)-JOB&IC model is created as below:

$$(1 + 1.0866B + 0.0898B^2 - 0.1503B^3 - 0.0437B^4 + 0.248B^5 - 0.3302B^6)UR_t = 5.9564 - 0.1198JOB_{t-2} + 0.14243IC_t + \epsilon_t \quad (4.8)$$

$$\hat{\sigma}_\epsilon^2 = 0.01819.$$

### Diagnostic Check for Model ARX(6)-JOB&IC

After the model 4.8 being chosen with the JOB and IC data as external data, we can test whether the residuals can meet white noise assumptions.

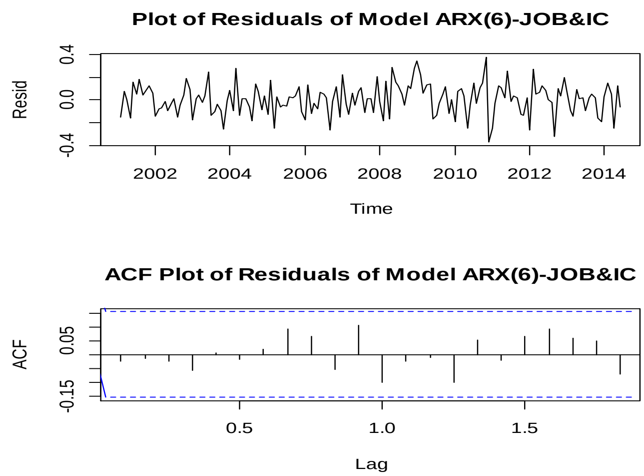


**Figure 4.15** Plot of residuals from ARX(6)-JOB & IC and the residuals' ACF plot

Figure 4.15 shows the plot of residuals from selected model ARX(6)-JOB & IC. The ACF plot shows no "Statistically Significant" correlation appears.

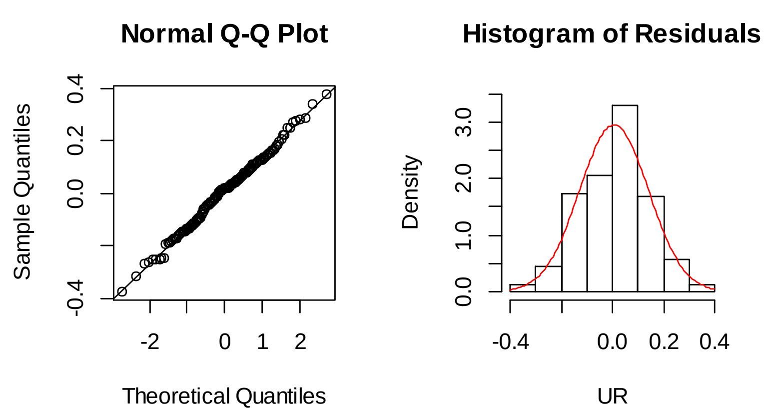


**Figure 4.16** Normal QQ Plot and Histogram of residuals of model ARX(6)-JOB&IC

In Figure 4.16, almost all the points are laid on the line in Normal QQ plot; and the shape of the Histogram appears "Bell Shaped" curve. So the residuals of the fitted model can be considered approximately normal.

**Table 4.22** Independent and normality test for residuals of ARX(6)-JOB&IC

| | **Residuals of ARX(6)-JOB&IC** | |
|---|---|---|
| **Test** | Ljung-Box test | Shapiro-Wilk normality test |
| **$H_0$** | residuals are independent | residuals are normal distributed |
| **Test Stat.** | X-squared = 7.3821 | W = 0.9947 |
| **P-value** | P-value = 0.287(df=6) | P-value = 0.835 |

In table 4.22, Box-Ljung test fails to reject the null hypothesis of independence at P-value of 0.2048. Shapiro-Wilk normality test has a test statistics of W=0.9946, leading to a P-value of 0.824, and fails to reject the null hypothesis of normality.

In conclusion, the residuals from fitted model ARX(6)-JOB&IC are independent and normal distributed. So the UR data can be well represented by model ARX(6)-JOB&IC.

### 4.6 ARIMAX Model Attempt with 1 month and 3 months Lag JOB data

According to the Dice-DFH Vacancy Duration Measure, in 2014, U.S. employers are taking about 25 working days, on average, to fill vacant positions (Davis, 2014). Some positions may be filled sooner; some may need more time to be filled. In Chapter 4.3 an ARX(6)-JOB $_{t-2}$ model has been well built for UR with JOB $_{t-2}$ as a external variable. JOB with 1 month (JOB $_{t-1}$) and 3 months lag (JOB $_{t-3}$) will be interests. Also, a half number of JOB data from t-2 with a half number of JOB data from t-3 is constructed as JOB $_{t-2\&3.}$ ARMAX model will be attempted with JOB $_{t-3}$ or JOB $_{t-2\&3}$ as an external data for UR forecasting.

Same as data JOB $_{t-2}$, after “pre-whitening”, there are significant correlations between JOB $_{t-1}$ and UR$_{t.}$ The residuals from the simple regression model “lm(UR~JOB)” show auto-correlated, so it is appropriate to construct ARIMAX model. With the lowest AIC (-153.56) and BIC (-138.15), ARMAX(2,1) model is selected. The P-value of t test is 0.3334 for JOB $_{t-3,}$ which means coefficient of JOB$_{t-3}$ is 0.4538, not significant different from 0. As a result, model ARMAX(2,1) - JOB$_{t-1}$ is not selected in Chapter 6 for forecasting comparison.

For data JOB $_{t-3}$, after going through the same model constructing procedure as JOB $_{t-2}$, with the lowest AIC (-151.91) and BIC (-127.25), ARX(6) model is selected. The P-value of t test is 0.3334 for JOB $_{t-3,}$ which means coefficient of JOB$_{t-3}$ is not significant different from 0. As a result, model ARX(6)- JOB$_{t-3}$ is not selected in Chapter 6 for forecasting comparison.

For data JOB $_{t-2\&3}$, after going through the same model constructing procedure as JOB $_{t-2}$, with the lowest AIC (-153.01) and BIC (-128.35), ARX(6) model is selected. The P-value of t test is 0.1536 for JOB $_{t-2\&3,}$ which means coefficient of JOB $_{t-2\&3}$ is not significant different from 0. As a result, model ARX(6)- JOB $_{t-2\&3}$ is not selected in Chapter 6 for forecasting comparison.

# CHAPTER 5

# MULTIVARIATE TIME SERIES MODELING

In Chapter 4, a univariate time series ARIMA model have been built as a benchmark model for the UR data. Considering the JOB and IC as external data, several ARIMAX model have been successfully constructed. In this chapter, the UR, JOB and IC data will be observed over time from one system.

As mentioned in Chapter 2, the release date of JOB data has two months lag in reality. When we have Dec 2104 UR and IC index, only Nov 2014 JOB data are available. In order to fit the real data availability, $JOB_{t-1}$ is chosen along with the $UR_t$ and the $IC_t$ for the VAR modeling. For doing so, I also choose the JOB data from Jan 2001 to May 2014, in align with UR and IC data from Feb 2001 to Jun 2014,

## 5.1 Bivariate VAR modeling with JOB data

In Table 5.1, both the $U_t$ and $JOB_{t-1}$ data are showed as unit root non-stationary. But their first differences are showed as stationary series.

**Table 5.1** Unit Root Test for $UR_t$ and $JOB_{t-1}$

| ADF-GLS Test with a Constant | | |
|---|---|---|
| **Variable** | **test Stat.** | **Test Result** |
| **$U_t$** | -1.0912 | Has Unit Root |
| **$JOB_{t-1}$** | -0.9033 | Has Unit Root |

Notes: critical values at 1, 5, 10% are -2.58 -1.94 -1.62

When modeling these two unit root time series jointly, there may be the case of co-integration, which means two series that contain unit roots may be related.

In Table 5.2, Engle and Granger's two steps procedures are first applied, the ADF test results fail to reject the null hypothesis of estimated residuals having unit root with a p-value of 0.4976 and 0.1848. Phillips-Ouliaris Cointegration test also fails to reject the null hypothesis of these two time series are not co-integrated with a p-value greater than 0.15.

**Table 5.2** Cointegration Test for $UR_t$ and $JOB_{t-1}$

| Engle and Granger Two Steps Procedure | | Phillips-Ouliaris Cointegration Test |
|---|---|---|
| Augmented Dickey-Fuller Test | | |
| Model: lm(JOB~UR) | Model: lm(UR~JOB) | |
| P-value=0.4136 | P-value=0.1061 | P-value > 0.15 |

### Lag Length Selection

Before we can estimate a VAR model for the $U_t$ and $JOB_{t-1}$ series, the order p must be specified. The lag length for the VAR(p) model will be determined by using three most common information criteria: the Akaike (AIC), the Schwarz-Bayesian (BIC) and the Hannan-Quinn (HQ).

**Table 5.3** VAR order selection for the $UR_t$ and $JOB_{t-1}$ series

| Criteria | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| AIC(n) | -7.16 | -7.41 | -7.54 | -7.54 | -7.60 | **-7.69** |
| HQ(n) | -7.10 | -7.31 | **-7.41** | -7.38 | -7.41 | -7.46 |
| BIC(n) | -7.01 | -7.17 | -7.23 | -7.15 | -7.13 | **-7.14** |

In table 5.3, the AIC criteria and HQ criteria prefer n=6 as the optimal lag number, the BIC criteria prefer n=3 as the optimal lag number. The Jarque-Bera multivariate-normality test results show that residuals of VAR(3) model reject the null hypothesis of normality with a p-value less than 0.0001, while VAR(6) model fails to reject the null hypothesis of normality with a p-value=0.4925. Therefore, VAR(6) model is chosen for further analysis. Table 5.4 shows the estimation results for equation $UR_t$, table 5.5 shows the estimation results for equation $JOB_{t-1}$.

**Table 5.4** Estimation results for equation UR of model VAR(6)-UR&JOB

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| | Sig. |
|---|---|---|---|---|---|
| UR.l1 | 1.0816 | 0.0820 | 13.1890 | 0.0000 | *** |
| JOB.l1 | -0.2109 | 0.0819 | -2.5760 | 0.0110 | * |
| UR.l2 | 0.0117 | 0.1192 | 0.0980 | 0.9217 | |
| JOB.l2 | 0.0661 | 0.0835 | 0.7910 | 0.4305 | |
| UR.l3 | -0.1458 | 0.1202 | -1.2130 | 0.2271 | |
| JOB.l3 | -0.0840 | 0.0863 | -0.9740 | 0.3319 | |
| UR.l4 | -0.0017 | 0.1212 | -0.0140 | 0.9886 | |
| JOB.l4 | -0.0324 | 0.0860 | -0.3770 | 0.7069 | |
| UR.l5 | 0.1918 | 0.1225 | 1.5650 | 0.1198 | |
| JOB.l5 | -0.0856 | 0.0818 | -1.0460 | 0.2974 | |
| UR.l6 | -0.1801 | 0.0844 | -2.1320 | 0.0347 | * |
| JOB.l6 | 0.2723 | 0.0739 | 3.6820 | 0.0003 | *** |
| const | 0.4772 | 0.3389 | 1.4080 | 0.1613 | |
| trend | 0.0009 | 0.0006 | 1.5210 | 0.1304 | |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

**Table 5.5** Estimation results for equation JOB of model VAR(6)-UR&JOB

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| | Sig. |
|---|---|---|---|---|---|
| UR.l1 | -0.2023 | 0.0882 | -2.2940 | 0.0233 | * |
| JOB.l1 | 0.2978 | 0.0881 | 3.3810 | 0.0009 | *** |
| UR.l2 | -0.0117 | 0.1282 | -0.0910 | 0.9273 | |
| JOB.l2 | 0.2472 | 0.0898 | 2.7520 | 0.0067 | ** |
| UR.l3 | 0.0807 | 0.1293 | 0.6240 | 0.5335 | |
| JOB.l3 | 0.3237 | 0.0928 | 3.4890 | 0.0007 | *** |
| UR.l4 | -0.2503 | 0.1303 | -1.9210 | 0.0568 | . |
| JOB.l4 | -0.0618 | 0.0925 | -0.6680 | 0.5049 | |
| UR.l5 | 0.2164 | 0.1318 | 1.6420 | 0.1028 | |
| JOB.l5 | 0.0048 | 0.0880 | 0.0540 | 0.9570 | |
| UR.l6 | 0.1044 | 0.0908 | 1.1500 | 0.2520 | |
| JOB.l6 | -0.0181 | 0.0795 | -0.2280 | 0.8203 | |
| const | 1.0548 | 0.3644 | 2.8950 | 0.0044 | ** |
| trend | 0.0015 | 0.0006 | 2.3110 | 0.0223 | * |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Using notation U=UR, J=JOB, VAR(6)-$UR_t$&$JOB_{t-1}$ model can be showed as :

$$\begin{aligned} U_t = {} & 1.0816U_{t-1} - 0.2109J_{t-2} + 0.0117U_{t-2} + 0.0661J_{t-3} - 0.1458U_{t-3} \\ & - 0.0840J_{t-4} - 0.0017U_{t-4} - 0.0324J_{t-5} + 0.1918U_{t-5} \\ & - 0.0856J_{t-6} - 0.1801U_{t-6} + 0.2723J_{t-7} + 0.4772 + 0.0009t + \epsilon_{1,t} \end{aligned}$$

$$\begin{aligned} J_{t-1} = {} & -0.2023U_{t-1} + 0.2978J_{t-2} - 0.0117U_{t-2} + 0.2472J_{t-3} + 0.0807U_{t-3} \\ & + 0.3237J_{t-4} - 0.2503U_{t-4} - 0.0618J_{t-5} + 0.2164U_{t-5} \\ & + 0.0048J_{t-6} + 0.1044U_{t-6} - 0.0181J_{t-7} + 1.0548 + 0.0015t \\ & + \epsilon_{2,t-1} \end{aligned}$$

$$cov(\varepsilon_t) = \begin{bmatrix} 0.0190 & -0.0056 \\ -0.0056 & 0.0220 \end{bmatrix} \qquad (5.1)$$

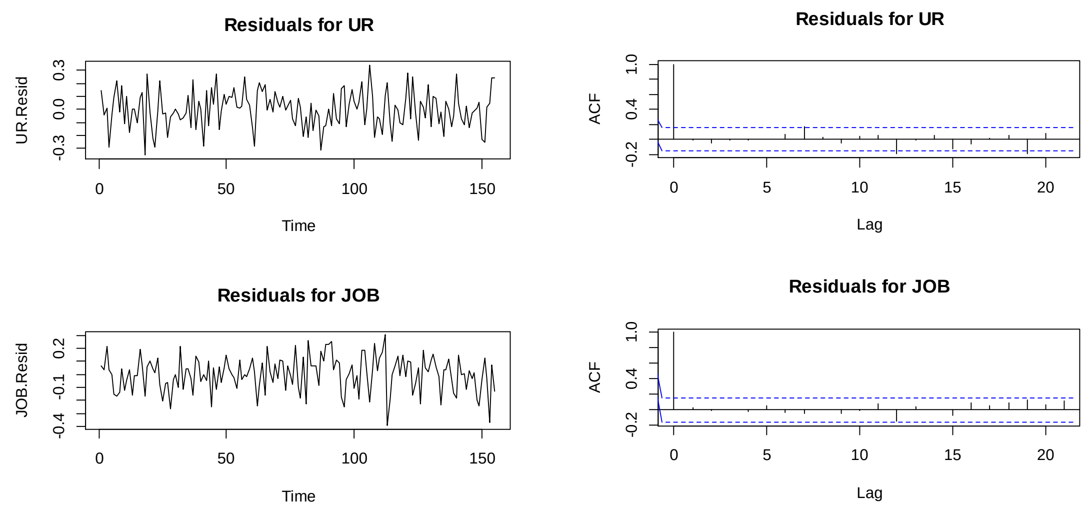


**Figure 5.1** Plot of residuals and related ACF plots of model VAR(6)- $UR_t$&$JOB_{t-1}$

Figure 5.1 show the plots of residuals and the ACF plots of the model residuals. Plots show no apparent sign of autocorrelation.

Instead of visual inspection of the residuals, more formally, we can apply the JB test for normality. In order to check for autocorrelation, we choose the Portmanteau Test. For ARCH effect, we choose the ARCH-LM test.

**Table 5.6** Test for Residuals of VAR(6)- $UR_t$&$JOB_{t-1}$

| **Test for Residuals of VAR(6)- $UR_t$&$JOB_{t-1}$** | | | |
|---|---|---|---|
| **Test** | **JB Test for normality** | **Portmanteau Test for autocorrelation** | **ARCH-LM (multivariate)** |
| **$H_0$** | **Residuals are normal distributed** | **Residuals are not autocorrelated** | **Residuals has no ARCH effect** |
| **Test Stat.** | Chi-squared = 3.4047 | Chi-squared = 41.7378 | Chi-squared = 60.314 |
| **DF** | df=4 | df=40 | df=45 |
| **P-value** | p-value = 0.4925 | p-value = 0.3952 | p-value = 0.0631 |

In table 5.6, the JB- test fails to reject the null hypothesis of normality. The Portmanteau test fails to reject the null hypothesis of autocorrelation at 10% level. The ARCH test shows no significant ARCH effect at $\alpha$=0.05 level. So VAR(6) model is a good fit for data UR and JOB.

In following further analyses, an R package for assessing multivariate normality (Package MVN) is used to provide a graphical approach, such as chi-square Q-Q plot. In Package MVN, Royston's test uses the Shapiro-Wilk/Shapiro-Francia statistic to test multivariate normality (Korkmaz, 2015). One may use the "qqplot = TRUE" option in "roystonTest" function to create a chi-square Q-Q plot for multivariate normality. Figure 5.2 shows almost all the points are laid on the line in QQ plot. Royston's Multivariate normality test fails to reject the null hypothesis of normality with a P-value of 0.3704.

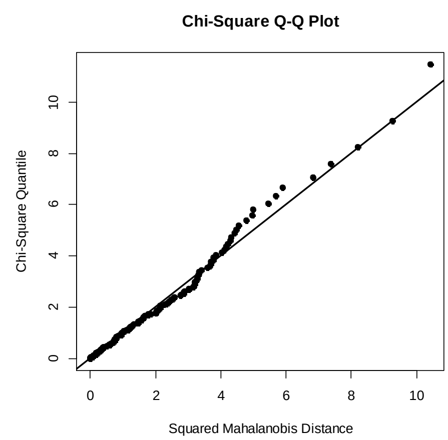


**Figure 5.2** Chi-square Q-Q plot of residuals of VAR(6)- $UR_t$&$JOB_{t-1}$

In table5.7, Granger causality test results show that the null hypotheses are both rejected at 0.01 significant level, we can believe that UR do granger-cause JOB, and JOB do granger-cause UR as well.

**Table 5.7** Granger causality test for model VAR(6)-$UR_t$&$JOB_{t-1}$

| **Granger causality $H_0$:** | **Test Stat.** | **D.F.** | **Pr > \|t\|** |
|---|---|---|---|
| **UR do not Granger-cause JOB** | F-Test = 3.4779 | (6, 282) | 0.0025 |
| **JOB do not Granger-cause UR** | F-Test = 5.6090 | (6, 282) | 1.62E-05 |

The impulse response function of VAR is to analysis dynamic affects of the system when the model received the impulse. As our VAR model, we have UR and

JOB two variables. In Figure 5.3, the left graph is the impulse response from UR. UR responses positively and keeps increasing from 1 to 10 months. JOB responses negatively and keeps decreasing from 1 to 10 months.

The right graph is the impulse response from JOB. UR responds negatively and decreases gradually from 1 to 10 months. JOB responds positively and it has a response bottom in the first month, and is fare persistent from 2 to 10 months.

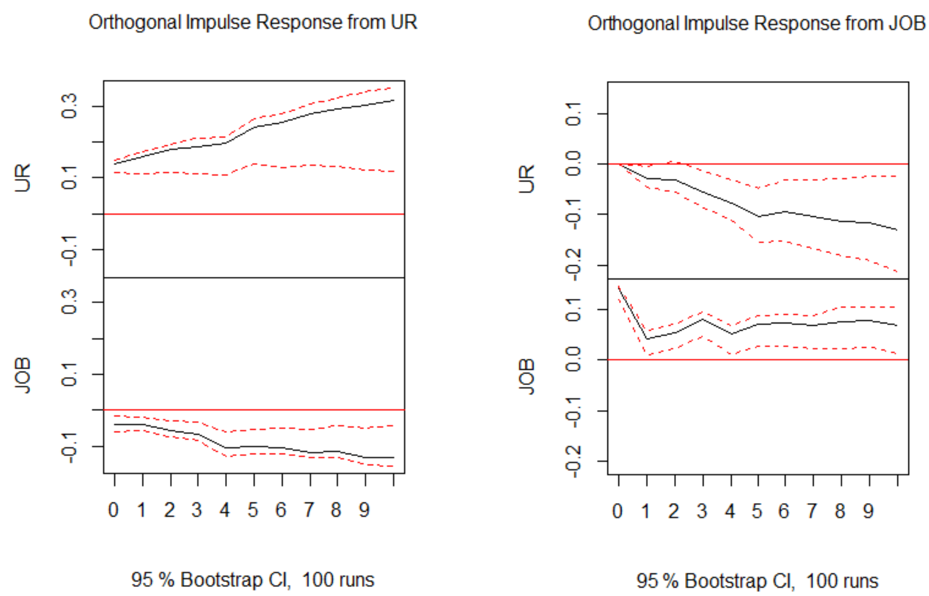


**Figure 5.3** Plots of Impulse response function from UR and JOB

### 5.2 Bivariate VAR modeling with IC data

In Table 5.8, the $U_t$ data are unit root non-stationary. While, the $IC_t$ data are showed as stationary series.

**Table 5.8** Unit Root Test for $UR_t$ and $IC_t$

| ADF-GLS Test with a Constant | | |
|---|---|---|
| **Variable** | **Test Stat.** | **Test Result** |
| **$U_t$** | -1.0912 | Has Unit Root |
| **$IC_t$** | -2.0376 | Stationary |

Notes: critical values at 1, 5, 10% are -2.58 -1.94 -1.62

In Table 5.9, Engle and Granger's two steps procedures are first applied, the ADF test results fail to reject the null hypothesis of estimated residuals having unit root with a p-value of 0.5666 and 0.3811. Phillips-Ouliaris Cointegration test also fails to reject the null hypothesis of these two series are not co-integrated with a p-value > 0.15.

**Table 5.9** Cointegration Test for $UR_t$ and $IC_t$

| Engle and Granger Two Steps Procedure | | Phillips-Ouliaris Cointegration Test |
|---|---|---|
| Augmented Dickey-Fuller Test | | |
| Model: lm(UR~IC) | Model: lm(IC~UR) | |
| P-value=0.5666 | P-value=0.3811 | P-value > 0.15 |

### Lag Length Selection

Before we can estimate a VAR model for the $UR_t$ and $IC_t$ series, the order p must be specified. In table 5.10, the HQ criteria and the BIC criteria prefer n=1, the AIC criteria prefer n=4 as the optimal lag number. Following, please see the estimation results for both VAR(4) and VAR(1) models.

**Table 5.10** VAR order selection for the $UR_t$ and $IC_t$ series

| Criteria | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| AIC(n) | -9.3728 | -9.4013 | -9.3853 | -9.4195 | -9.4149 | -9.3844 |
| HQ(n) | -9.3090 | -9.3056 | -9.2577 | -9.2600 | -9.2235 | -9.1611 |
| BIC(n) | -9.2157 | -9.1657 | -9.0712 | -9.0268 | -8.9437 | -8.8346 |

Table 5.11 shows the VAR(4) estimation results for equation $UR_t$. Table 5.12 shows the VAR(4) estimation results for equation $IC_t$. In both equations, estimates of coefficients for lag 1 and lag 4 are significant different from 0.

**Table 5.11** Estimation results for equation UR of model VAR(4)-$UR_t$&$IC_t$

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| | Sig. |
|---|---|---|---|---|---|
| UR.l1 | 0.7953 | 0.0873 | 9.1130 | 0.0000 | *** |
| IC.l1 | 0.7848 | 0.1597 | 4.9150 | 0.0000 | *** |
| UR.l2 | 0.0192 | 0.1095 | 0.1750 | 0.8611 | |
| IC.l2 | 0.2260 | 0.2051 | 1.1020 | 0.2723 | |
| UR.l3 | -0.0567 | 0.1116 | -0.5080 | 0.6124 | |
| IC.l3 | -0.0757 | 0.2016 | -0.3760 | 0.7078 | |
| UR.l4 | 0.1423 | 0.0836 | 1.7020 | 0.0908 | . |
| IC.l4 | -0.2765 | 0.1612 | -1.7150 | 0.0884 | . |
| const | -0.4625 | 0.0912 | -5.0730 | 0.0000 | *** |
| trend | 0.0014 | 0.0005 | 3.0050 | 0.0031 | ** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

**Table 5.12** Estimation results for equation IC of model VAR(4)-$UR_t$&$IC_t$

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| | Sig. |
|---|---|---|---|---|---|
| UR.l1 | 0.1200 | 0.0481 | 2.4930 | 0.0138 | * |
| IC.l1 | 0.8457 | 0.0881 | 9.6030 | 0.0000 | *** |
| UR.l2 | -0.0629 | 0.0604 | -1.0410 | 0.2995 | |
| IC.l2 | 0.0967 | 0.1131 | 0.8550 | 0.3941 | |
| UR.l3 | 0.0285 | 0.0616 | 0.4630 | 0.6441 | |
| IC.l3 | -0.0717 | 0.1112 | -0.6450 | 0.5201 | |
| UR.l4 | -0.0810 | 0.0461 | -1.7570 | 0.0810 | . |
| IC.l4 | 0.0170 | 0.0889 | 0.1910 | 0.8488 | |
| const | 0.1385 | 0.0503 | 2.7550 | 0.0066 | ** |
| trend | 0.00001 | 0.00025 | 0.0530 | 0.9581 | |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Using notation U=UR, IC=I, VAR(4)-$UR_t$&$IC_t$ model can be showed as :

$$U_t = 0.7953U_{t-1} + 0.7848I_{t-1} - 0.0192U_{t-2} + 0.2260I_{t-2} - 0.0567U_{t-3} - 0.0757I_{t-3} + 0.1423U_{t-4} - 0.2765I_{t-4} - 0.4625 + 0.0014t + \epsilon_{1,t}$$

$$I_t = 0.1200U_{t-1} + 0.8457I_{t-1} - 0.0629U_{t-2} + 0.0967I_{t-2} + 0.0285U_{t-3} - 0.0717I_{t-3} - 0.0810U_{t-4} + 0.0170I_{t-4} + 0.1385 + 0.00001t + \epsilon_{2,t}$$

$$cov(\varepsilon_t) = \begin{bmatrix} 0.01668 & 0.00380 \\ 0.00380 & 0.00507 \end{bmatrix} \qquad (5.2)$$

**Table 5.13** Estimation results for equation UR of model VAR(1)-$UR_t$&$IC_t$

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| | Sig. |
|---|---|---|---|---|---|
| UR.l1 | 0.9011 | 0.0127 | 71.1770 | 0.0000 | *** |
| IC.l1 | 0.5894 | 0.0584 | 10.0870 | 0.0000 | *** |
| const | -0.3830 | 0.0594 | -6.4450 | 0.0000 | *** |
| trend | 0.0016 | 0.0004 | 4.1590 | 0.0001 | *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

**Table 5.14** Estimation results for equation IC of model VAR(1)-$UR_t$&$IC_t$

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| | Sig. |
|---|---|---|---|---|---|
| UR.l1 | -0.0139 | 0.0070 | -1.9850 | 0.0489 | * |
| IC.l1 | 1.0242 | 0.0324 | 31.6050 | 0.0000 | *** |
| const | 0.0329 | 0.0330 | 0.9970 | 0.3202 | |
| trend | 0.00024 | 0.00201 | 1.1760 | 0.2413 | |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Table 5.13 shows the VAR(1) estimation results for equation $UR_t$. Table 5.14 shows the VAR(1) estimation results for equation $IC_t$. In both equations, estimates of coefficients for lag 1 are significant different from 0.

Using notation U=UR, IC=I, VAR(1)-$UR_t$ &$IC_t$ model can be showed as :

$$U_t = 0.9011U_{t-1} + 0.5894I_{t-1} - 0.3830 + 0.0016t + \epsilon_{1,t}$$

$$I_t = -0.0139U_{t-1} + 1.0242I_{t-1} + 0.0329 + 0.00024t + \epsilon_{2,t}$$

$$cov(\varepsilon_t) = \begin{bmatrix} 0.01703 & 0.00329 \\ 0.00329 & 0.00524 \end{bmatrix} \qquad (5.3)$$

I will choose both VAR(1) and VAR(4) model for $UR_t$ & $IC_t$ in Chapter 6 for model comparison. For simplicity, in following analyses, I only use VAR(1)-$UR_t$ &$IC_t$ model to demonstrate the procedures of the model estimation and validation.

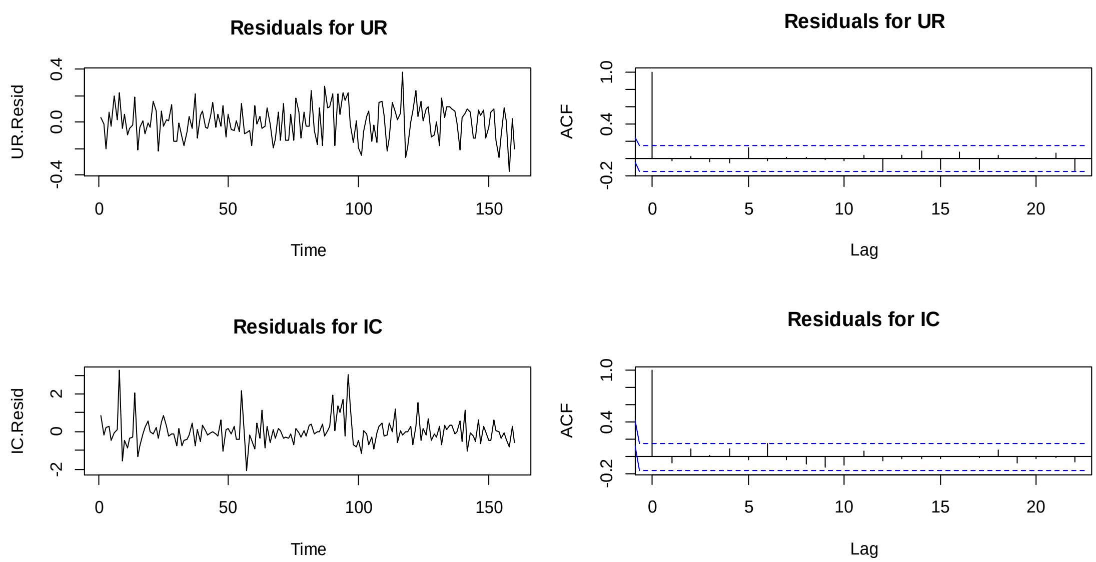


**Figure 5.4** Plots of residuals and related ACF plots of model VAR(1)-$UR_t$&$IC_t$

Figure 5.4 shows the plot of residuals and the ACF plots of the model residuals. Residuals for UR are within 0.2 range, but some residuals for IC are over 2.0 range. ACF Plots both show no apparent sign of autocorrelation.

Instead of visual inspection of the residuals, we apply the JB test for normality, the Portmanteau test for autocorrelation, the ARCH-LM test for ARCH effect,

**Table 5.15** Test for Residuals of VAR(1)-$UR_t$&$IC_t$

| Test for Residuals of VAR(1)-$UR_t$&$IC_t$ | | | |
|---|---|---|---|
| **Test** | **JB Test for normality** | **Portmanteau Test for autocorrelation** | **ARCH-LM (multivariate)** |
| **$H_0$** | **Residuals are normal distributed** | **Residuals are not autocorrelated** | **Residuals has no ARCH effect** |
| **Test Stat.** | Chi-squared = 199.41 | Chi-squared =65.0733 | Chi-squared = 46.4824 |
| **DF** | df=4 | df=60 | df=45 |
| **P-value** | p-value < 2.2e-16 | p-value = 0.3046 | p-value = 0.4111 |

In table 5.15, the JB- test rejects null hypothesis of normality with a p-value < 0.0001. The Portmanteau test fails to reject the null hypothesis of autocorrelation with a p-value of 0.3046. The ARCH test shows no ARCH effect. Since the residuals are not bi-normal distributed, the further investigation is needed.

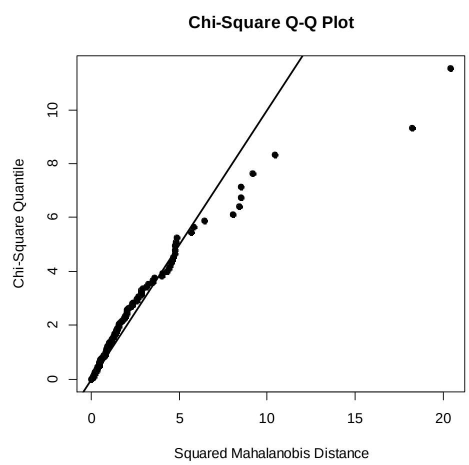


**Figure 5.5** Chi-square Q-Q plot of residuals of VAR(1)-$UR_t$&$IC_t$

Figure 5.5 shows chi-square Q-Q plot for multivariate normality generated by “roystonTest” function in R MVN package. It shows that some points are laid out of the line in QQ plot. Royston's multivariate normality test also rejects the null hypothesis of normality with a p-value < 0.0001.

In order to fit the multivariate normality assumption, Structural VAR models and Vector ARMA models have been tried. But all those models’ residuals do not fit the normality assumption either.

As noted by several authors (Burdenski, 2000 ; Stevens, 2012), if data have a multivariate normal distribution, then each of the variables have a univariate normal distribution; but the opposite does not have to be true. Hence, checking univariate plots could be very useful to diagnose the reason for deviation from multivariate normality.

As seen in Figure 5.6, almost all the residuals from UR model are laid in the line in QQ plot, whereas some residuals from IC model are laid out of the line.

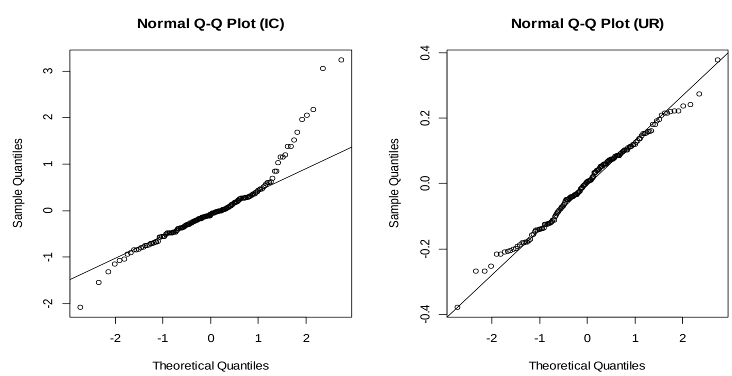


**Figure 5.6** Univariate QQ plots for residuals from model IC and UR

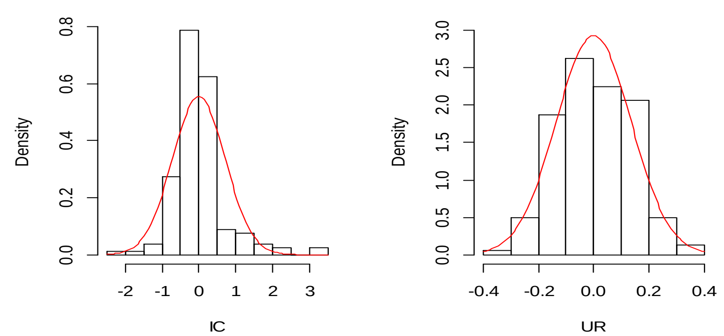


**Figure 5.7** Histograms with normal curves for residuals from model IC and UR

In Figure 5.7, the residuals from $UR_t$ model have approximately normal distributions, whereas the residuals from $IC_t$ model have a right-skewed distribution.

Thus, we can conclude that problems with multivariate normality arise from the skewed distribution of residuals from IC model. Since my interest in this thesis is for UR forecasting, and the VAR model for UR&IC is created as a referencing model, therefore VAR(1)-UR&IC and VAR(4)-UR&IC would still be included in Chapter 6 for forecasting comparison.

**Table 5.16** Granger causality test for model VAR(1)-$UR_t$&$IC_t$

| Granger causality $H_0$: | Test Stat. | D.F. | Pr > \|t\| |
|---|---|---|---|
| **UR do not Granger-cause IC** | F-Test = 3.9405 | (1, 312) | 0.04801 |
| **IC do not Granger-cause UR** | F-Test = 101.75 | (1, 312) | < 2.2e-16 |

Table 5.16 shows that the null hypothesis IC do not Granger-cause UR is rejected with a p-value lower than 0.0001. It proves that IC does Granger-cause UR. While null hypothesis of UR do not Granger-cause IC is also rejected at 5% significant level with a p-value of 0.048.

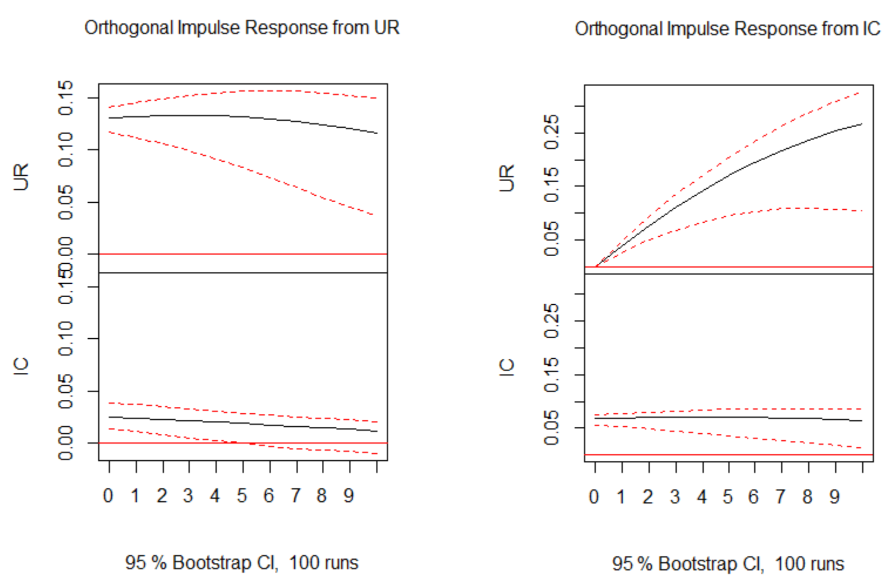


**Figure 5.8** Plots of Impulse response function from UR and IC

In Figure 5.8, the left graph is the impulse response from UR. UR responses positively and is fare persistent around 0.14 from 1 to 10 months. IC responses also positively and decreases very slowly. The right graph is the impulse response from IC. UR responds also positively, and keeps increasing from 1 to 10 months. IC responses positively and is fare persistent around 0.06.

### 5.3 Trivariate VAR modeling with both JOB and IC

When considering the $UR_t$ , $JOB_{t-1}$ and $IC_t$ data together in one system, the PO test is first checked. The test result fails to reject the null hypothesis of non co-integration with p-value > 0.15.

**Table 5.17** VAR order selection for the $UR_t$ , $JOB_{t-1}$ & $IC_t$ series

| Criteria | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| AIC(n) | -13.00 | -13.17 | -13.21 | -13.20 | -13.22 | **-13.23** |
| HQ(n) | -12.89 | **-12.98** | -12.95 | -12.87 | -12.81 | -12.75 |
| BIC(n) | **-12.71** | -12.70 | -12.56 | -12.38 | -12.22 | -12.05 |

In Table 5.17, the BIC criteria prefer n=1, the HQ criteria prefer n=2 and the AIC criteria prefer n=6 as the optimal lag number.

For the best BIC selected Trivariate VAR(1) model, the JB- test rejects the null hypothesis of normality with a p-value smaller than 0.0001. The Granger causality test fails to reject the null hypothesis of “JOB do not Granger-cause UR IC” with a p-value = 0.5719. Therefore, Trivariate VAR(1) is not a good fit.

For the best HQ selected Trivariate VAR(2) model, the JB- test rejects the null hypothesis of normality with a p-value smaller than 0.0001. The Granger causality test also fails to reject the null hypothesis of “JOB do not Granger-cause UR IC” with a p-value = 0.1955. Therefore, Trivariate VAR(2) is not a good fit.

For the best AIC selected Trivariate VAR(6) model, the Granger causality test rejects the null hypothesis of “JOB do not Granger-cause UR IC” with a p-value < 0.01. Hence, Trivariate VAR(6) is chosen for further investigation.

In table 5.18, the JB- test rejects the null hypothesis of normality with a p-value smaller than 0.0001, which is similar to the Bivariate VAR model for $UR_t$ & $IC_t$ . The Portmanteau test fails to reject the null hypothesis of autocorrelation with a p-value of 0.3644. The ARCH test shows no ARCH effect. Since the residuals are not multivariate normal distributed, the further investigation is needed.

**Table 5.18** Test for Residuals of VAR(6)- $UR_t$ , $JOB_{t-1}$ & $IC_t$

| Test for Residuals of VAR(6)-UR,JOB & IC | | | |
|---|---|---|---|
| **Test** | **JB Test for normality** | **Portmanteau Test for autocorrelation** | **ARCH-LM (multivariate)** |
| **$H_0$** | **Residuals are normal distributed** | **Residuals are not autocorrelated** | **Residuals has no ARCH effect** |
| **Test Stat.** | Chi-squared =54.985 | Chi-squared =94.0454 | Chi-squared =197.9173 |
| **DF** | df=6 | df=90 | df=180 |
| **P-value** | p-value < 0.0001 | p-value = 0.3644 | p-value = 0.1712 |

**Table 5.19** Estimation results for equation UR of Trivariate VAR(6)

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| | Sig. |
|---|---|---|---|---|---|
| UR.l1 | 0.7998 | 0.0904 | 8.8480 | <0.0001 | *** |
| JOB.l1 | -0.1192 | 0.0768 | -1.5510 | 0.1232 | |
| IC.l1 | 0.7306 | 0.1650 | 4.4280 | <0.0001 | *** |
| UR.l2 | 0.0202 | 0.1089 | 0.1850 | 0.8534 | |
| JOB.l2 | 0.0690 | 0.0781 | 0.8830 | 0.3786 | |
| IC.l2 | 0.1597 | 0.2053 | 0.7780 | 0.4380 | |
| UR.l3 | -0.0639 | 0.1096 | -0.5830 | 0.5610 | |
| JOB.l3 | -0.1389 | 0.0805 | -1.7250 | 0.0868 | . |
| IC.l3 | 0.0155 | 0.2044 | 0.0760 | 0.9397 | |
| UR.l4 | 0.0336 | 0.1105 | 0.3040 | 0.7619 | |
| JOB.l4 | -0.0381 | 0.0805 | -0.4730 | 0.6367 | |
| IC.l4 | -0.4711 | 0.2047 | -2.3010 | 0.0229 | * |
| UR.l5 | 0.2358 | 0.1124 | 2.0990 | 0.0377 | * |
| JOB.l5 | -0.0511 | 0.0764 | -0.6680 | 0.5050 | |
| IC.l5 | -0.1035 | 0.1988 | -0.5210 | 0.6034 | |
| UR.l6 | -0.1032 | 0.0886 | -1.1650 | 0.2460 | |
| JOB.l6 | 0.2656 | 0.0698 | 3.8040 | 0.0002 | *** |
| IC.l6 | 0.1605 | 0.1597 | 1.0050 | 0.3168 | |
| const | -0.2882 | 0.3916 | -0.7360 | 0.4631 | |
| trend | 0.0012 | 0.0006 | 1.9500 | 0.0533 | . |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

**Table 5.20** Estimation results for equation JOB of Trivariate VAR(6)

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| | Sig. |
|---|---|---|---|---|---|
| UR.l1 | -0.0091 | 0.1034 | -0.0870 | 0.9304 | |
| JOB.l1 | 0.2392 | 0.0879 | 2.7220 | 0.0073 | ** |
| IC.l1 | -0.5901 | 0.1887 | -3.1270 | 0.0022 | ** |
| UR.l2 | -0.0134 | 0.1246 | -0.1070 | 0.9147 | |
| JOB.l2 | 0.2559 | 0.0894 | 2.8620 | 0.0049 | ** |
| IC.l2 | -0.1075 | 0.2348 | -0.4580 | 0.6477 | |
| UR.l3 | 0.0050 | 0.1254 | 0.0400 | 0.9683 | |
| JOB.l3 | 0.3720 | 0.0921 | 4.0400 | 0.0001 | *** |
| IC.l3 | 0.1428 | 0.2338 | 0.6110 | 0.5423 | |
| UR.l4 | -0.2844 | 0.1264 | -2.2500 | 0.0261 | * |
| JOB.l4 | -0.0584 | 0.0921 | -0.6350 | 0.5267 | |
| IC.l4 | 0.3471 | 0.2341 | 1.4830 | 0.1405 | |
| UR.l5 | 0.1869 | 0.1285 | 1.4540 | 0.1483 | |
| JOB.l5 | -0.0209 | 0.0874 | -0.2390 | 0.8113 | |
| IC.l5 | -0.1262 | 0.2274 | -0.5550 | 0.5799 | |
| UR.l6 | 0.0628 | 0.1013 | 0.6190 | 0.5367 | |
| JOB.l6 | -0.0202 | 0.0799 | -0.2520 | 0.8012 | |
| IC.l6 | 0.0980 | 0.1827 | 0.5360 | 0.5927 | |
| const | 1.4288 | 0.4479 | 3.1900 | 0.0018 | ** |
| trend | 0.0015 | 0.0007 | 2.2830 | 0.0240 | * |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

**Table 5.21** Estimation results for equation IC of Trivariate VAR(6)

| Parameter | Estimate | S.E. | t Value | Pr > \|t\| | Sig. |
|---|---|---|---|---|---|
| UR.l1 | 0.1445 | 0.0517 | 2.7970 | 0.0059 | ** |
| JOB.l1 | -0.0482 | 0.0439 | -1.0980 | 0.2744 | |
| IC.l1 | 0.7991 | 0.0943 | 8.4720 | 0.0000 | *** |
| UR.l2 | -0.0731 | 0.0623 | -1.1740 | 0.2424 | |
| JOB.l2 | 0.0283 | 0.0447 | 0.6340 | 0.5270 | |
| IC.l2 | 0.0663 | 0.1174 | 0.5650 | 0.5731 | |
| UR.l3 | 0.0189 | 0.0627 | 0.3010 | 0.7640 | |
| JOB.l3 | -0.0916 | 0.0460 | -1.9900 | 0.0486 | * |
| IC.l3 | -0.1148 | 0.1169 | -0.9830 | 0.3276 | |
| UR.l4 | -0.0018 | 0.0632 | -0.0280 | 0.9776 | |
| JOB.l4 | -0.0301 | 0.0460 | -0.6540 | 0.5143 | |
| IC.l4 | -0.0053 | 0.1170 | -0.0450 | 0.9641 | |
| UR.l5 | -0.0369 | 0.0642 | -0.5750 | 0.5664 | |
| JOB.l5 | 0.0638 | 0.0437 | 1.4600 | 0.1467 | |
| IC.l5 | -0.0889 | 0.1136 | -0.7820 | 0.4355 | |
| UR.l6 | -0.0383 | 0.0507 | -0.7560 | 0.4512 | |
| JOB.l6 | 0.0355 | 0.0399 | 0.8880 | 0.3761 | |
| IC.l6 | 0.0630 | 0.0913 | 0.6900 | 0.4916 | |
| const | 0.4961 | 0.2239 | 2.2160 | 0.0284 | * |
| trend | -0.00002 | 0.00034 | -0.057 | 0.9547 | |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Table 5.19, 5.20, 5.21 show the Trivariate VAR(6) estimation results for equation UR, JOB and IC. Using notation U=UR, J=JOB, I=IC, this model is showed as:

$$\begin{aligned} U_t = {} & 0.7998U_{t-1} - 0.1192J_{t-2} + 0.7306I_{t-1} + 0.0202U_{t-2} + 0.0690J_{t-3} \\ & + 0.1597I_{t-2} - 0.0639U_{t-3} - 0.1389J_{t-4} + 0.0155I_{t-3} \\ & + 0.0336U_{t-4} - 0.0381J_{t-5} - 0.4711I_{t-4} + 0.2358U_{t-5} \\ & - 0.0511J_{t-6} - 0.1035I_{t-5} - 0.1032U_{t-6} + 0.2656J_{t-7} \\ & + 0.1605I_{t-6} - 0.2882 + 0.0012t + \epsilon_{1,t} \end{aligned}$$

$$\begin{aligned} J_{t-1} = {} & -0.0091U_{t-1} + 0.2392J_{t-2} - 0.5901I_{t-1} - 0.0134U_{t-2} + 0.2559J_{t-3} \\ & - 0.1075I_{t-2} - 0.0050U_{t-3} + 0.3720J_{t-4} - 0.1428I_{t-3} \\ & - 0.2844U_{t-4} - 0.0584J_{t-5} + 0.3471I_{t-4} + 0.1869U_{t-5} \\ & - 0.0209J_{t-6} - 0.1262I_{t-5} + 0.0628U_{t-6} - 0.0202J_{t-7} \\ & + 0.0980I_{t-6} + 1.4288 + 0.0015t + \epsilon_{2,t-1} \end{aligned}$$

$$\begin{aligned} I_t = {} & 0.1445U_{t-1} - 0.0482J_{t-2} + 0.7991I_{t-1} - 0.0731U_{t-2} + 0.0283J_{t-3} \\ & + 0.0663I_{t-2} + 0.0189U_{t-3} - 0.0916J_{t-4} - 0.1148I_{t-3} \\ & - 0.0018U_{t-4} - 0.0301J_{t-5} - 0.0053I_{t-4} - 0.0369U_{t-5} \\ & + 0.0638J_{t-6} - 0.0889I_{t-5} - 0.0383U_{t-6} + 0.0355J_{t-7} \\ & - 0.0630I_{t-6} - 0.4961 - 0.00002t + \epsilon_{3,t} \end{aligned}$$

$$cov(\varepsilon_t) = \begin{bmatrix} 0.0154 & -0.0026 & 0.0036 \\ -0.0026 & 0.0201 & -0.0008 \\ 0.0036 & -0.0008 & 0.0050 \end{bmatrix} \tag{5.4}$$

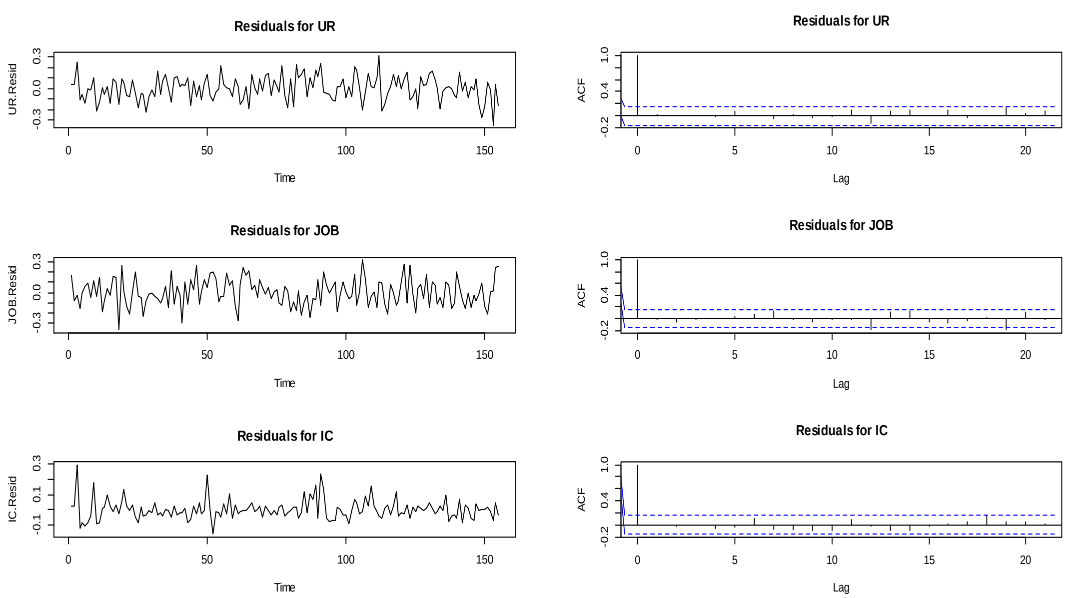


**Figure 5.9** Plots of residuals and related ACF plots of model VAR(6)-$UR_t$, $JOB_{t-1}$ & $IC_t$

Figure 5.9 shows the plot of residuals and the ACF plots of the model residuals. ACF Plots show no apparent sign of autocorrelation.

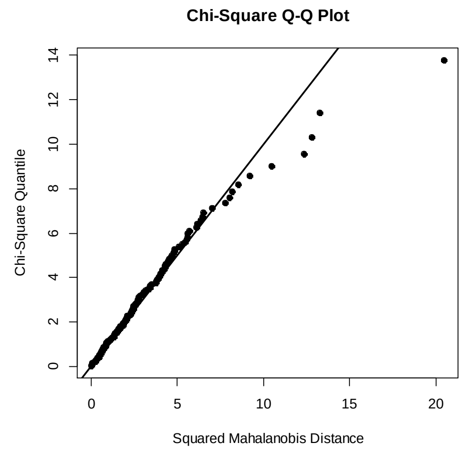


**Figure 5.10** Chi-square Q-Q plot of residuals of VAR(6)-$UR_t$, $JOB_{t-1}$ & $IC_t$

Figure 5.10 shows chi-square Q-Q plot for multivariate normality generated by "roystonTest" function in R MVN package. It shows that some points are laid out of the line in QQ plot. The Royston's multivariate normality test also rejects the null hypothesis of normality with a p-value < 0.0001.

Same as showed in the Table 5.18, the residuals of the Trivariate VAR(6) model are not multivariate normal distributed. In order to diagnose the reason of deviation from multivariate normality, the univariate plots are checked.

As seen in Figure 5.11, almost all the residuals from UR and JOB equation are laid in the line in QQ plot, whereas, some residuals from IC equation are laid out of the line.

In Figure 5.12, the residuals from UR and JOB equation have approximately normal distributions, whereas the residuals from IC model have a right-skewed distribution. Thus, we can conclude that problems with multivariate normality arise from the skewed distribution of residuals from IC equation. Since the main focus of this thesis is for UR forecasting, therefore Trivariate VAR(6) model is appropriate to be included in Chapter 6 for forecasting comparison.

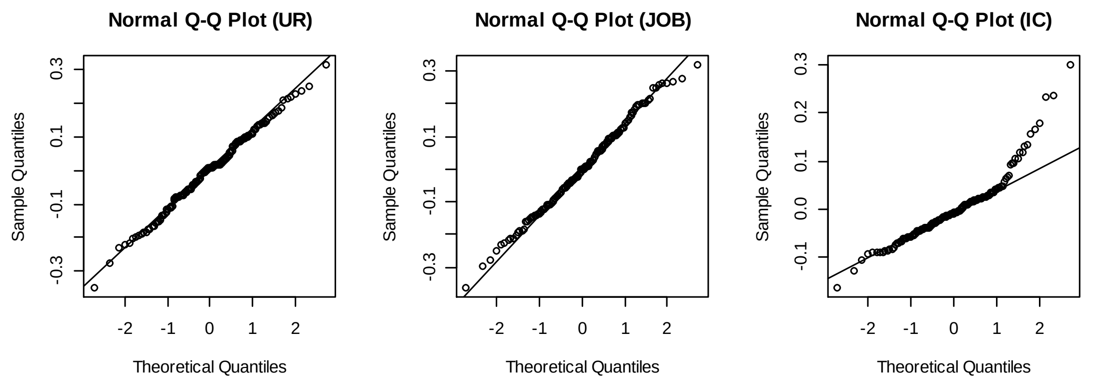


**Figure 5.11** Univariate QQ plots for residuals from equation UR, JOB and IC

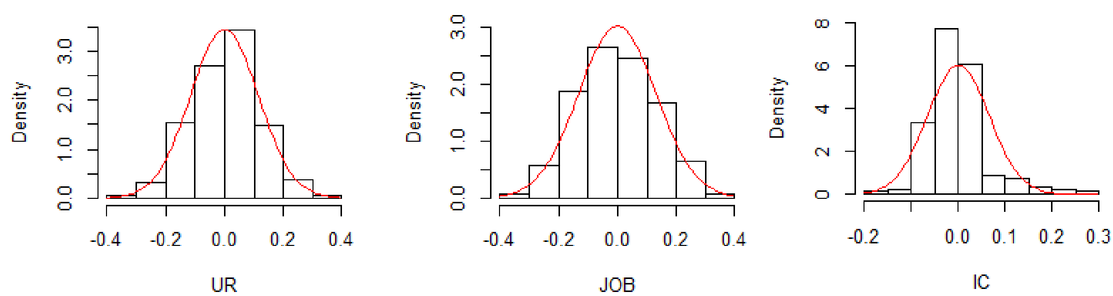


**Figure 5.12** Histograms for residuals from equation UR, JOB and IC

Table 5.22 shows that the null hypothesis JOB do not Granger-cause UR and IC is rejected with a p-value less than 0.01; and the null hypothesis IC do not Granger-cause UR and JOB is rejected with a p-value less than 0.001, which prove that both JOB and IC do Granger-cause UR.

**Table 5.22** Granger causality test for model VAR(6)-$UR_t$, $JOB_{t-1}$ & $IC_t$

| Granger causality $H_0$: | Test Stat. | D.F. | Pr > \|t\| |
|---|---|---|---|
| **UR do not Granger-cause JOB, IC** | F-Test = 1.5838 | (12, 405) | 0.0934 |
| **JOB do not Granger-cause UR, IC** | F-Test = 2.2542 | (12, 405) | 0.0091 |
| **IC do not Granger-cause UR, JOB** | F-Test = 4.3632 | (1, 312) | < 0.0001 |

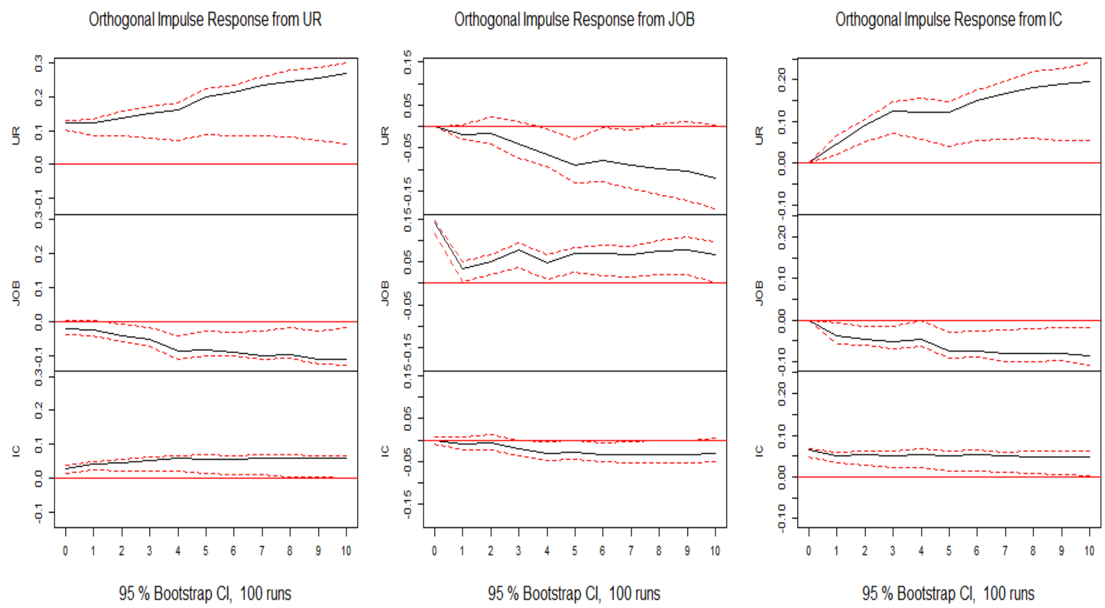


**Figure 5.13** Plots of Impulse response function from UR, JOB and IC

In Figure 5.13, the left graph is the impulse response from UR. UR responses positively and keep increasing from 1 to 10 months. JOB responses negatively and keep decreasing from 1 to 10 months. IC responses also positively and it is fare persistent.

The middle graph is the impulse response from JOB. UR responses negatively and keep decreasing from 1 to 10 months. JOB responds positively and it has a response bottom in the first month, and is fare persistent from 2 to 10 months. IC responses negatively and keep decreasing from 1 to 4 months, and then it is fare persistent from 5 to 10 months.

The right graph is the impulse response from IC. UR responses positively and keep increasing from 1 to 10 months. JOB responses negatively and keep gradually decreasing from 1 to 10 months. IC responses also positively and it is fare persistent.

# CHAPTER 6

# MODEL FORECASTING COMPARISON

Along with a bench mark ARIMA model, 4 univariate ARIMAX models and 4 multivariate VAR models have been constructed in Chapter 4 and Chapter 5 by using data from Feb 2001 to Jun 2014. Figure 6.4 shows the plots of all selected models.

Both recursive and rolling forecasting comparison are applied to evaluate these models within the out of sample period (Jul 2014 to Feb 2015). For simplicity, in the following analyses, the models' names (M1 ~M9) in Table 6.1 will be used as the notation of different models.

**Table 6.1** List of selected models

| Name | Model | Model # |
|---|---|---|
| UR | Actual UR Values | ---------- |
| M1 | ARIMA(4,1,2) | Model 4.1 |
| M2 | ARX(6)--$JOB_{t-2}$ | Model 4.3 |
| M3 | ARIMAX(4,1,5)--$IC_t$ | Model 4.5 |
| M4 | ARIMAX(4,1,4)--$IC_t$ | Model 4.6 |
| M5 | ARX(6)--$JOB_{t-2}$ +$IC_t$ | Model 4.8 |
| M6 | VAR(6)--$UR_t$+$JOB_{t-1}$ | Model 5.1 |
| M7 | VAR(1)--$UR_t$+$IC_t$ | Model 5.2 |
| M8 | VAR(4)--$UR_t$+$IC_t$ | Model 5.3 |
| M9 | VAR(6)--$UR_t$+$JOB_{t-1}$+$IC_t$ | Model 5.4 |

## 6.1 Model Forecasting Methods and DM Test

There are two types of forecasting methods: recursive forecasting and rolling forecasting. In recursive forecasting, a series of data is used to predict h times ahead. Each time a new value is collected the model needs to be re-estimated and the forecast updated. The rolling forecasting method involves a rolling window of size h that gets shifted each time a new value is collected.

For example: for an AR(1) model, the h times ahead recursive forecasting model can be showed as:

$$\begin{cases} \hat{Y}_{t+h} = f(Y_t) & h = 1 \\ \hat{Y}_{t+h} = f(\hat{Y}_{t+h-1}) & h > 1 \end{cases}$$

With a fixed value t, there is only one predicted value for every step ahead recursive forecasting. But in rolling forecasting, the value t is not fixed. Every time when t changes, there are a new set of h step(s) ahead forecasted values. With n different time t, for a specific step(s) h, there are n different forecasted values.

After obtain a set of forecasting data from all models, the accuracy of all selected models can be compared by using Mean Square Forecast Error (MSFE), Mean Absolute Forecast Error (MAFE) and Diebold-Mariano (DM) test (1995).

**Mean Square Forecast Error (MSFE)**

$$MSFE = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y_i)^2$$

**Mean Absolute Forecast Error (MAFE)**

$$MAFE = \frac{1}{n}\sum_{i=1}^{n}|\hat{Y}_i - Y_i|$$

Where $\hat{Y}$ is a vector of n forecasted values; Y is the vector of the true values; n is the size of the out-of-sample.

**Diebold-Mariano test (DM test)**

The DM test is test for the null of equal forecast accuracy between the benchmark and the competitor. The DM test is based on the loss differential between the benchmark (model 0) and its competitor (model k), i.e. $d_t = e_{0,t}^2 - e_{k,t}^2$ .To test the null of equal forecast accuracy $H_0 : E(d_t) = 0$ , the DM statistic is:

$$DM = P^{1/2} \frac{\bar{d}}{\hat{\sigma}DM}$$

Where $\bar{d}$ is the average loss differential, P is the out of sample size, and $\hat{\sigma}DM$ is the squared root of long run variance of $d_t$. The test statistics DM is asymptotically N(0, 1) distributed.

### 6.2 Recursive Forecasting Results

Recursive forecast simply adds more time to the initial forecast while keeping the same start date. The out-of-sample period is from Jul 2014 to Feb 2015 (Table 6.2). Table 6.3 shows the Absolute Forecasting Errors and their MAFE and MSFE.

**Table 6.2** Recursive forecasting results

| NAME | Model | Jul-14 | Aug-14 | Sep-14 | Oct-14 | Nov-14 | Dec-14 | Jan-15 | Feb-15 |
|---|---|---|---|---|---|---|---|---|---|
| UR | Actual Value | 6.2 | 6.1 | 5.9 | 5.7 | 5.8 | 5.6 | 5.7 | 5.5 |
| M1 | ARIMA(4,1,2) | 6.0407 | 5.9952 | 6.0055 | 6.0288 | 6.0533 | 6.0612 | 6.0497 | 6.0227 |
| M2 | ARX(6)--$JOB_{t-2}$ | 6.0613 | 5.9983 | 5.8754 | 5.8085 | 5.7496 | 5.6980 | 5.6568 | 5.5989 |
| M3 | ARIMAX(4,1,5)--$IC_t$ | 6.0945 | 5.9510 | 5.8619 | 5.8239 | 5.7372 | 5.7181 | 5.7483 | 5.6313 |
| M4 | ARIMAX(4,1,4)--$IC_t$ | 6.0995 | 5.9470 | 5.8812 | 5.8297 | 5.7475 | 5.7375 | 5.7587 | 5.6495 |
| M5 | ARX(6)--$JOB_{t-2}$ +$IC_t$ | 6.0738 | 5.9918 | 5.8710 | 5.7705 | 5.7236 | 5.6998 | 5.6663 | 5.5678 |
| M6 | VAR(6)--$UR_t$+$JOB_{t-1}$ | 6.0396 | 5.9288 | 5.7540 | 5.6580 | 5.5771 | 5.5652 | 5.5069 | 5.4882 |
| M7 | VAR(1)--$UR_t$+$IC_t$ | 6.1004 | 6.1125 | 6.1356 | 6.1690 | 6.2118 | 6.2632 | 6.3224 | 6.3887 |
| M8 | VAR(4)--$UR_t$+$IC_t$ | 6.1394 | 6.1008 | 6.0879 | 6.0725 | 6.0975 | 6.1267 | 6.1690 | 6.2151 |
| M9 | VAR(6)--$UR_t$+$JOB_{t-1}$+$IC_t$ | 6.0866 | 6.0060 | 5.8103 | 5.6809 | 5.5903 | 5.5747 | 5.5057 | 5.4702 |

**Table 6.3** Absolute Forecasting Errors of Recursive Forecast Results

| NAME | Model | Jul-2014 | Aug-2014 | Sep-2014 | Oct-2014 | Nov-2014 | Dec-2014 | Jan-2015 | Feb-2015 | MAFE | MSFE |
|---|---|---|---|---|---|---|---|---|---|---|---|
| M1 | ARIMA(4,1,2) | 0.1593 | 0.1048 | 0.1055 | 0.3288 | 0.2533 | 0.4612 | 0.3497 | 0.5227 | 0.2857 | 0.1035 |
| M2 | ARX(6)--$JOB_{t-2}$ | 0.1387 | 0.1017 | 0.0246 | 0.1085 | 0.0504 | 0.0980 | 0.0433 | 0.0989 | 0.0830 | 0.0082 |
| M3 | ARIMAX(4,1,5)--$IC_t$ | 0.1055 | 0.1490 | 0.0381 | 0.1239 | 0.0628 | 0.1181 | 0.0483 | 0.1313 | 0.0971 | 0.0109 |
| M4 | ARIMAX(4,1,4)--$IC_t$ | 0.1005 | 0.1531 | 0.0188 | 0.1297 | 0.0526 | 0.1375 | 0.0587 | 0.1495 | 0.1000 | 0.0123 |
| M5 | ARX(6)--$JOB_{t-2}$ +$IC_t$ | 0.1262 | 0.1082 | 0.0290 | 0.0705 | 0.0764 | 0.0998 | 0.0337 | 0.0678 | 0.0764 | 0.0069 |
| M6 | VAR(6)--$UR_t$+$JOB_{t-1}$ | 0.1604 | 0.1713 | 0.1460 | 0.0420 | 0.2229 | 0.0348 | 0.1931 | 0.0119 | 0.1228 | 0.0208 |
| M7 | VAR(1)--$UR_t$+$IC_t$ | 0.0996 | 0.0125 | 0.2356 | 0.4690 | 0.4118 | 0.6632 | 0.6224 | 0.8887 | 0.4254 | 0.2590 |
| M8 | VAR(4)--$UR_t$+$IC_t$ | 0.0606 | 0.0008 | 0.1879 | 0.3725 | 0.2975 | 0.5267 | 0.4690 | 0.7151 | 0.3288 | 0.1594 |
| M9 | VAR(6)--$UR_t$+$JOB_{t-1}$+$IC_t$ | 0.1134 | 0.0940 | 0.0897 | 0.0191 | 0.2097 | 0.0253 | 0.1943 | 0.0299 | 0.0969 | 0.0142 |

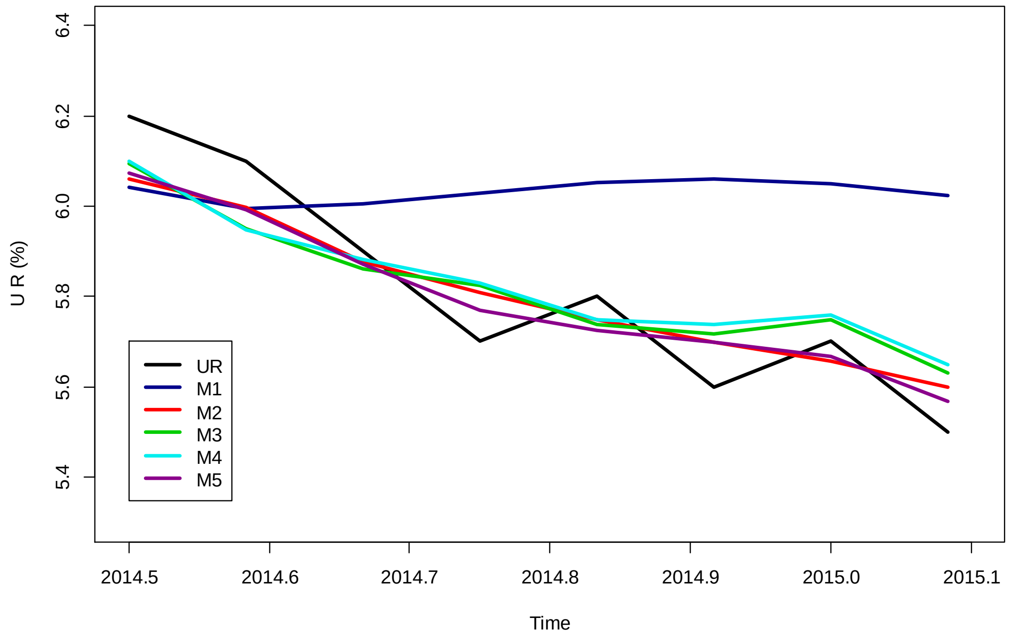


**Figure 6.1** Recursive forecasted values of ARIMAX models (Jul 2014 - Feb 2015)

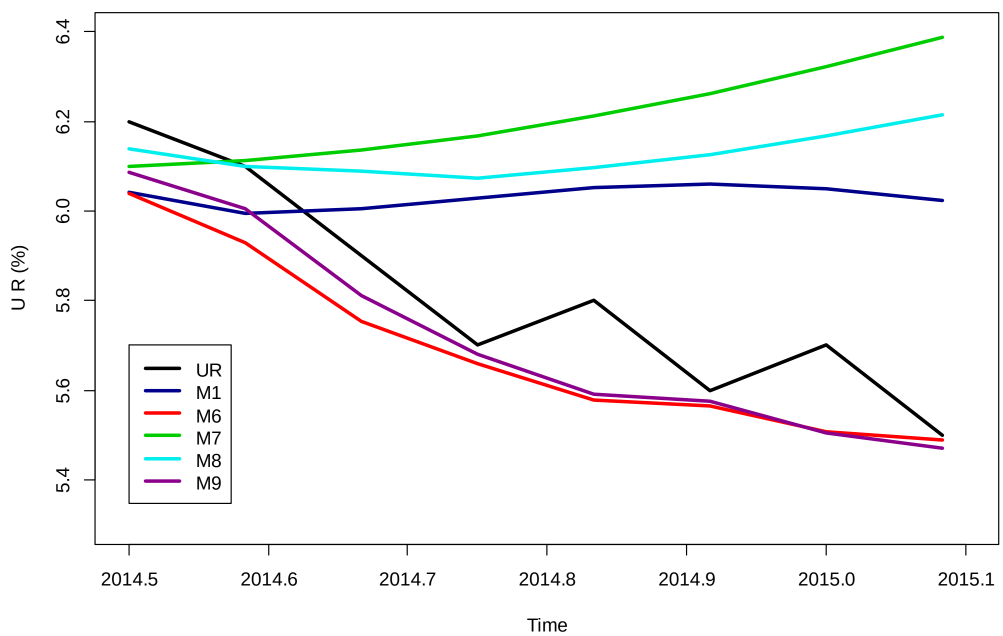


**Figure 6.2** Recursive forecasted values of VAR models (Jul 2014 - Feb 2015)

In Figure 6.1 and Table 6.2 & 6.3, comparing to Benchmark model M1, all the ARIMAX models have better predicting power with much smaller MAFE and MSFE.

Comparing to IC related model M3 or M4, JOB related ARIMAX model M2 has better performance with smaller MAFE and MSFE. The model with lowest MAFE and MSFE is M5, which includes both IC and JOB as external variables. JOB related ARIMAX model M2 has second lowest MAFE and MSFE.

In Figure 6.2 and Table 6.2 & 6.3, IC related VAR models (M7 and M8) fail to beat the benchmark model M1 with larger MAFE and MSFE, while JOB related VAR model M6 has better performance with lower MAFE and MSFE. The best VAR model is M9 including both JOB and UR.

Comparing to the VAR models, all the ARIMAX models have better performance with much smaller MAFE or MSFE. The reason is that ARIMAX model forecasting needs the inputs of external data, but VAR model only based on the historical data. These actual external variables may somehow adjust the predicting errors in the forecasting process. Nevertheless, in the real world, it is impossible to have such future values ready for ARIMAX models when forecasting 2 or more steps ahead, while VAR models can easily forecast many steps ahead as needed.

In Table 6.2, the recursive forecasting has only one sample for every step; but in rolling forecasting, we can have 8 predicted values for every step forecasting; so the average of all 8 forecast errors would be more helpful to evaluate the models.

### 6.3 Rolling Forecasting Results

A rolling forecast is a process ensures that the forecast always covers the same amount of time, so a rolling forecast window requires routine revisions. The out-of-sample period is from Jul 2014 to Feb 2015. So we have 8 months actual UR values in hand. The rolling window is from 1 step ahead to 4 steps ahead. For each model, every step forecasting will come with 8 predictions. By comparing the predicted value with the actual UR value, the Mean Squared Forecasting Error (MSFE, see Table 6.4) and Diebold-Mariano test (DM test, see Table 6.5) is also applied to compare the forecast accuracy between the benchmark model and the competitors.

**Table 6.4** Rolling forecast MSFE (Jul 2014-Feb 2015)

| NAME | Model | 1 Step Ahead | 2 Steps Ahead | 3 Steps Ahead | 4 Steps Ahead |
|---|---|---|---|---|---|
| M1 | ARIMA(4,1,2) | 0.0252 | 0.0442 | 0.0935 | 0.1161 |
| M2 | ARX(6)--$JOB_{t-2}$ | 0.0192 | 0.0132 | 0.0336 | 0.0257 |
| M3 | ARIMAX(4,1,5)--$IC_t$ | 0.0184 | 0.0204 | 0.0258 | 0.0317 |
| M4 | ARIMAX(4,1,4)--$IC_t$ | 0.0185 | 0.0212 | 0.0239 | 0.0355 |
| M5 | ARX(6)--$JOB_{t-2}$ +$IC_t$ | 0.0158 | 0.0119 | 0.0219 | 0.0192 |
| M6 | VAR(6)--$UR_t$+$JOB_{t-1}$ | 0.0150 | 0.0112 | 0.0114 | 0.0396 |
| M7 | VAR(1)--$UR_t$+$IC_t$ | 0.0249 | 0.0366 | 0.0845 | 0.1686 |
| M8 | VAR(4)--$UR_t$+$IC_t$ | 0.0195 | 0.0250 | 0.0591 | 0.1163 |
| M9 | VAR(6)--$UR_t$+$JOB_{t-1}$+$IC_t$ | 0.0130 | 0.0062 | 0.0102 | 0.0307 |

**Table 6.5** Rolling forecast DM one side test P-value (Jul 2014-Feb 2015)

| NAME | Model | 1 Step Ahead | 2 Steps Ahead | 3 Steps Ahead | 4 Steps Ahead |
|---|---|---|---|---|---|
| M1 | ARIMA(4,1,2) | **Benchmark model** | | | |
| M2 | ARX(6)--$JOB_{t-2}$ | 0.2113 | 0.1412 | 0.0763 | 0.0400 |
| M3 | ARIMAX(4,1,5)--$IC_t$ | 0.1729 | 0.2039 | 0.0296 | 0.0344 |
| M4 | ARIMAX(4,1,4)--$IC_t$ | 0.1401 | 0.1905 | 0.0143 | 0.0483 |
| M5 | ARX(6)--$JOB_{t-2}$ +$IC_t$ | 0.0977 | 0.1587 | 0.0369 | 0.0182 |
| M6 | VAR(6)--$UR_t$+$JOB_{t-1}$ | 0.1169 | 0.1686 | 0.0265 | 0.2255 |
| M7 | VAR(1)--$UR_t$+$IC_t$ | 0.4819 | 0.2742 | 0.0000 | 0.7811 |
| M8 | VAR(4)--$UR_t$+$IC_t$ | 0.2438 | 0.1714 | 0.0000 | 0.5011 |
| M9 | VAR(6)--$UR_t$+$JOB_{t-1}$+$IC_t$ | 0.1053 | 0.1557 | 0.0152 | 0.1684 |

In Table 6.4, comparing to benchmark model M1, JOB related Bivariate VAR model M6 has much better forecasting power with much smaller MAFE in all steps ahead forecasting. Comparing to IC related VAR model M7 and M8, JOB related VAR model M6 also has better forecasting performance with smaller MAFE in all steps ahead forecasting. Among all the VAR models, the Trivariate VAR model M9 has best forecasting power with the lowest MSFE in all 4 steps ahead forecasting.

In Table 6.5, comparing to Benchmark model M1, the DM one side test results show M5 and M9 is significantly greater with a p-value close to 0.10 for 1 steps ahead forecast. For 3 steps ahead forecast, all the models are significantly greater with a p-value less than 0.05.

Overall, comparing to benchmark model, JOB related Bivariate VAR model has much better predict power in UR forecasting. Comparing to the well known indicator IC related VAR models, the JOB related Bivariate VAR model has even better forecasting performance. The forecasting comparison results demonstrate that JOB index can improve UR forecasting accuracy, so it can be used as a good indicator.

In addition, the best model with smallest MSFE is the trivariate VAR model including both JOB and IC. Figure 6.3 shows this best model's recursive forecasting values with its 95% confident interval. In Figure 6.2, the JOB related model tends to under-predict UR, while IC related models tends to over-predict UR, therefore include both JOB and IC may moderate the variation of UR to reach the best forecasting results.

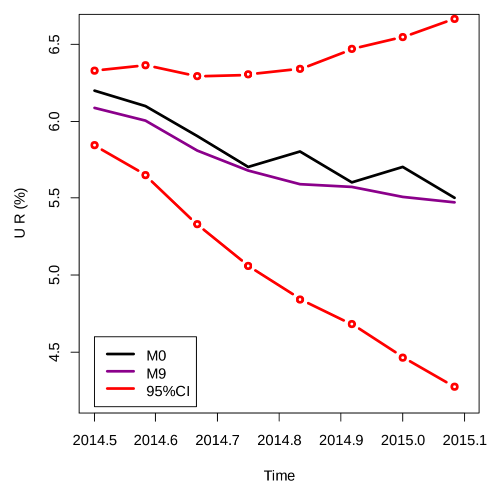


**Figure 6.3** Model M9 and its 95% confident interval (Jul 2014 - Feb 2015)

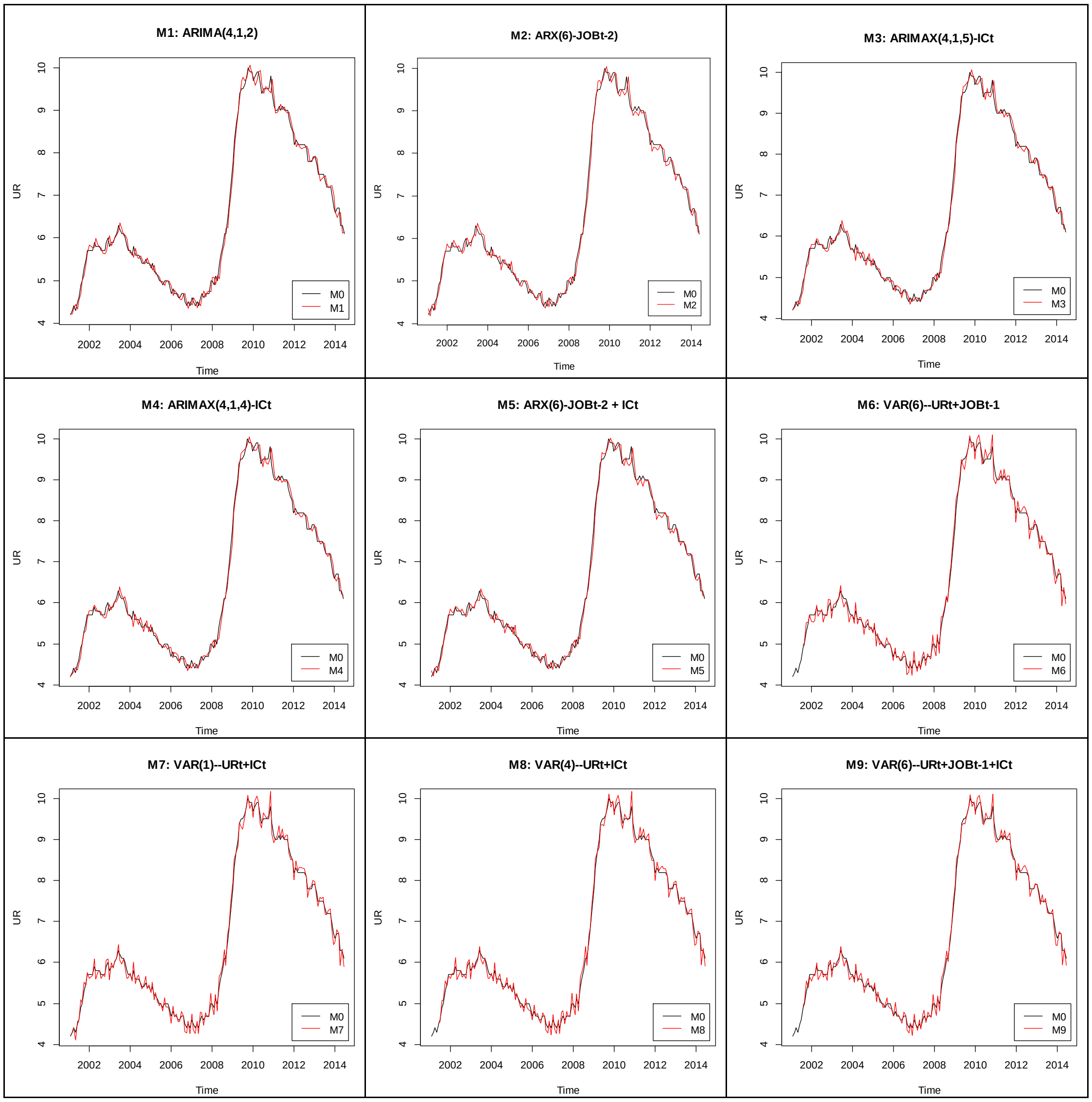


**Figure 6.4** the fitted values of all the selected models

# CHAPTER 7

# CONCLUSION

This thesis is focusing on the UR forecasting. The main purpose is to introduce the potential indicator-- JOB data for UR forecasting. For a better comparison, a well known indicator-- IC data are also included.

In order to forecast UR, an ARIMA model is firstly built as a benchmark model. By using JOB and IC as external inputs, four ARIMAX models have been well constructed. Also, four multivariate VAR models are successfully created with UR, JOB and IC data.

Recursive forecasting and rolling forecasting are both applied for model comparison along with Diebold-Mariano test (DM test).

Comparing to benchmark model, all the JOB related VAR models have much better predict accuracy with much smaller MAFE and MSFE in all forecasting. Comparing to the well known indicator IC related VAR model, the JOB related Bivariate VAR model has even better forecasting with smaller MSFE in all steps ahead forecasting. The best model belongs to trivariate VAR model with smallest MSFE.

In conclusion, JOB index can be a good indicator for UR prediction. When combining both the JOB and IC data, the UR forecasting accuracy can improve even more with smaller MSFE. Overall, the JOB index does have a big contribution to improvement of the UR forecasting.

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