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STRENGTH IN NUMBERS:
HOW COMPUTATIONAL ESTIMATION IMPACTS MATHEMATICS
ACHIEVEMENT OF HIGH SCHOOL STUDENTS WITH AND WITHOUT
DISABILITIES

BY

BRIAN M. THOMSEN

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY IN EDUCATION

UNIVERSITY OF RHODE ISLAND

AND

RHODE ISLAND COLLEGE

2017

DOCTOR OF PHILOSOPHY DISSERTATION

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2017

Abstract

Too many students are entering post-secondary education lacking foundational mathematics skills that are critical to performance on placement examinations. As a result, students are forced to take remedial courses that are often non-credit bearing and hinder their progress toward graduation. Research suggests that a lack of number sense may contribute to poor performance on standardized assessments. Number sense consists of multiple skills and concepts embedded within a concrete understanding of how numbers are represented. One concept featured in number sense is computational estimation, an interaction of mental computation, number concepts, and technical arithmetic skills which is performed quickly (without any recording tools) and which results in answers that are reasonably close to a correctly computed result.

This experimental study measured the impact of an intervention featuring supplemental activities in computational estimation delivered in game format. Students in tenth grade solved real-life mathematics questions independently and collaboratively, without any recording tools, with the goal of forming reasonable estimates. Over six weeks, students earned points for answers that fell within an appropriate range. Results of this study suggest that students without disabilities significantly improved their performance on standardized assessment questions featuring rounding, but did not outperform control groups in overall performance on questions encouraging the use of computational estimation. Students with disabilities did not demonstrate improved performance in any areas, suggesting the length of the study may have been too short for students who require more time to grasp new concepts and skills.

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Chapter 1

Framing the Study

Prior to this research study, I worked with hundreds of high school students who struggled to make effective progress in mathematics. As a special educator, I provided services for students with disabilities in the general education setting and in substantially separate classrooms. My experiences exposed me to a variety of problem-solving approaches and levels of ability within a community of general and special education students. Overwhelmingly, my students demonstrated inadequate foundational skills and poor concept of numbers. They avoided algebra problems with fractions, computed decimals inaccurately without the use of a calculator, and rarely demonstrated an understanding of what a percentage represents. To try and ease their lack of understanding, I began implementing daily number challenges with a handful of students who I worked with consistently because of their difficulty accessing the mathematics curriculum. These tasks ranged from finding percentages of numbers to adding and subtracting decimals and fractions. After just a few days, it was evident to me that not only did the majority of my students lack adequate foundational skills, they were heavily reliant on procedures using pencils and paper or calculators that were often incorrect. Few of them understood how to solve simple problems using mental computation, forcing them to engage in written procedures that were often inaccurate or incomplete. As a result of my observations, I consulted with their general education teachers and asked if I could work more closely with certain students who demonstrated poor foundational skills and were not making effective progress in class.

After gaining approval from teachers, I began administering daily problems to groups of students that targeted fractions, decimals, or percent, and asked them to solve the problems in their heads without the use of any tools. I explained to them that I was not looking for exact answers; instead, I wanted to assess whether they could think through problems efficiently and arrive at reasonable conclusions. Within a few days, I realized that these students did not have any sense of how numbers are represented, simplified, or computed. Without the use of tools to help them find solutions, they had no problem-solving strategies available to them. Nevertheless, I continued offering daily problems and encouraged them to use mental computation strategies we discussed as a group following each activity. We talked about the value of rounding and understanding place value to simplify computations. We discussed parts of wholes including the meaning of denominators, the significance of the number one with decimals, and the concept of 100 percent. As days marched on, I began to see independent applications of these strategies and an improvement in the answers my students produced. They estimated fractions of numbers more appropriately, determined percentages of values more reasonably, and recognized connections between decimals and whole numbers more thoughtfully. I also observed students who were more engaged in mathematics, often walking into my classroom asking what the daily challenge would be that day. Was this a result of an increase in confidence or did they just prefer playing an estimation game rather than the traditional learning experiences focused on practice and procedures? Perhaps these struggling students finally found some success in mathematics and wanted to build on this progress. I was eager to learn more about the significance of the improvements I was seeing. Could students elevate their performance in mathematics

classes because of this increase in confidence and skills? Could they increase their mathematics achievement? Unfortunately, I did not collect reliable data on these students, and I accepted an administrative position the following year that removed me from the classroom. These circumstances prompted me to start researching computational estimation and, ultimately, design a research study to determine its impact on student achievement in mathematics.

The results of my investigation into research on number sense and computational estimation paired with my observations in the classroom indicated to me that too many students across the country, especially those with learning disabilities, lack number sense (Schneider & Siegler, 2010; Vosniadou, Vamvakoussi, & Skopeliti, 2008; Ortiz, 2009; Landerl, Bevan, & Butterworth, 2004; Gersten & Chard, 1999). Their dislike of mathematics and poor performance on standardized assessments may stem from inadequate foundational skills necessary to solve problems and endure challenging content. In an attempt to learn more, I created a number sense experience for high school students that fosters a collaborative learning environment where students help one another solve real-world mathematics problems that focus on the effective uses of computational estimation. My goal was to trigger students' thinking about how expressions with numbers can be composed, simplified, and computed to solve problems, while making them aware of the relevance of mathematics to their own lives. I believe that if students consistently work with classmates on challenges that mirror situations they may encounter outside the classroom, they are more likely to engage in the learning process, thereby developing their number sense and raising their achievement in

mathematics. Improved achievement would likely lessen the need for remediation and help students prepare for success in college and their careers.

Statement of Problem

Research suggests that many high school students who choose to pursue post-secondary education perform poorly on placement tests and, consequently, are required to take foundational courses that are often non-credit bearing (National Center for Educational Statistics (NCES), 2015; Bailey, Jeong, & Cho, 2010; Barry & Dannenberg, 2016). Nearly every post-secondary public institution uses some form of standardized mathematics assessment for placement, and the vast majority of schools use these assessments to determine whether remediation is necessary (Fields & Parsad, 2012; The College Board, 2017). If students do not earn adequate scores, they are forced to enroll in classes centered on foundational skills they should have previously learned in school. According to a study from Columbia University, 59% of first year students attending community colleges require remedial courses in mathematics (Bailey et al., 2010). Of these 13 million students, only five percent who enrolled in developmental mathematics courses earned college-level mathematics credit within their first year, and 80% did not earn credits even after three years (Bailey et al., 2010). A report from *Education Reform Now* indicates that U.S. students paid 1.5 billion dollars in 2011-12 for remedial coursework in their first year of college, primarily in mathematics (Barry & Dannenberg, 2016). A discrepancy in the skills higher education representatives believe are critical to success and what placement tests actually assess is an important factor in this discussion and will be investigated in Chapter 2; however, high school teachers need to consider

why so many students are performing poorly on mathematics placement tests and discuss how they should address this pattern.

It should not be a surprise that many students are failing to meet the standards of post-secondary placement tests. According to a report from the most recent National Assessment of Educational Progress (NAEP), only 25% of twelfth graders scored at or above proficiency in mathematics, including only four percent of students with disabilities (NCES, 2015). Even more, 81% of students with disabilities were labeled “Below Basic.” Long-term NAEP trends indicate that twelfth grade students actually have not made significant gains in mathematics since 1973 (NCES, 2015). Even students preparing to enter high school are not meeting the national standard. Results from the 2015 NAEP indicate that 59% of eighth graders scored below proficiency in mathematics including 93% of students with disabilities (NCES, 2015). Why are so many students below the standard? Based on my observations as well as the content of remedial coursework in college (Bailey, Jeong, & Cho, 2010), students lack foundational skills and concepts that are critical to higher level understanding. According to Tighe (2014), research is needed to design interventions that help adult students with numeracy. Stigler, Givvin, and Thompson (2009) reported that community college developmental-level mathematics students lack conceptual understanding of key numeracy components that is essential for accessing more advanced concepts. Although there is some good news regarding student achievement in younger grades that will be outlined in Chapter 2, there are few interventions for high school and college students who are not demonstrating adequate number sense.

While mathematics achievement is an ongoing challenge for all educational stakeholders, there is also concern for students who have high levels of mathematics anxiety. Research suggests that these students are more likely to demonstrate lower mathematics achievement and are less likely to enter a mathematics-related career (Bekdemir, 2010; Scarpello, 2007). This anxiety can be particularly challenging for students with disabilities who endure learning challenges that impact their processing and retrieval during assessments (Whitaker-Sena et al., 2007; Zeidner & Matthews, 2005). Middle school students have actually demonstrated a decline in school engagement, particularly in mathematics (Martin, 2007; 2008), and some students often avoid mathematics activities in the classroom and real life because of a genuine fear of the subject (Bekdemir, 2010; Gresham, 2008; Vinson, 2001; Zettle & Raines, 2002). What is happening in classrooms that is contributing to these perceptions?

Challenges in mathematics are evident to most members of the educational community and leaders have tried to implement solutions. The National Council of Teachers of Mathematics (NCTM) revised a new set of standards in 2000 (NCTM, 1989; 2000) and endorsed the Common Core State Standards for Mathematics (CCSSM) in 2010 (Common Core Standards Initiative, 2010). It also published a recent “Call to Action” (NCTM, 2014) imploring teachers to adopt new instructional methods using technology and collaboration. The United States Department of Education (USDOE) triggered a nationwide effort to increase and improve Science, Technology, Engineering, and Mathematics (STEM) education to spark interest and confidence in mathematics-related subject areas (2011), and incentives for potential teachers in STEM fields range from federal loan forgiveness (USDOE, 2017) to significant scholarships and financial

awards (Higher Education Services Corporation, 2017; Teach for America, 2017). The nation is addressing the problems, and perhaps future studies will reveal increased engagement and higher achievement; but I believe a closer look at student learning experiences in mathematics classrooms, particularly in high school, is what will drive effective change. The mathematics community needs to discuss students' lack of foundational understanding, as well as the absence of student-centered instruction that engages all learners, including those who are low-performing or reluctant to participate.

Theoretical Framework

Students should be at the center of the learning process in school tackling relevant tasks individually and collaboratively with their peers and teachers. Philosophers like Dewey (1938), Piaget (1970), Vygotsky (1978), and Bruner (1996) advocated for students to engage in meaningful experiences that inspire them to question, explore, and reflect. This student-centered approach helped spark the theory of Constructivism that shapes this research study. Constructivism can be defined as

a self-regulatory process of struggling with the conflict between existing personal models of the world and discrepant new insights, constructing new representations and models of reality as a human meaning-making venture with culturally developed tools and symbols, and further negotiating such meaning through cooperative social activity, discourse, and debate. (Fosnot, 1996, p. ix)

Beginning with the works of Piaget, Constructivists support the claim that knowledge is “a mapping of actions and conceptual operations that had proven viable in the knowing subject's experience” (von Glasersfeld, 1996, p. 4). In short, as people accumulate and

reflect on more experiences, they acquire knowledge. The goal is then to transfer this knowledge to a range of situations (Bruner, 1961). This theory led me to consider mathematics classes in my school and how students rarely construct their own knowledge collaboratively. So much of the instruction is teacher-centered and students learn to mimic problem-solving strategies rather than initiate their own plans. The learning process is much more powerful if students take the lead in finding solutions to problems with support from their peers and teachers. Vygotsky (1978) endorsed the concept of experiential learning, but he believed social interaction, language, and cultural symbols were the critical components to development. He, along with other Constructivists, identified dialogue as a critical component to learning, specifically the way students and teachers converse, question, explain, and negotiate meaning (Vygotsky, 1978; Bruner, 1996; Fosnot, 1996). The learning experience creates the Zone of Proximal Development; that is, “learning awakens a variety of internal development processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers” (Vygotsky, 1978, p. 90). Vygotsky’s discoveries prompted the theory of Social Constructivism. This theory describes learning and knowing as a social process where individuals negotiate understanding through experience and discourse with people who share common goals (Vygotsky, 1987; Bruner, 1996; Brophy, 2002). The emphasis on discourse prompted me to create a learning tool that fosters engagement but encourages collaboration and discussion. Students might sit passively in some classrooms listening to teachers describe processes for completing tasks, but lasting understanding is constructed socially among peers with teacher guidance.

Statement of Purpose

The purpose of this study is to determine whether structured, collaborative mathematics activities that feature computational estimation impact high school student performance on standardized assessment questions that target this particular skill. Too many students are graduating from high school without an understanding of how to simplify and compute numerical expressions to solve problems, a critical set of skills consistently targeted in daily life, the classroom, and on placement tests (Reys & Bestgen, 1981; NCES, 2015). While there is an abundance of research outlining what teachers should do to build number sense at the elementary level (Gersten, Jordan, & Flojo, 2005; Wu, 2011; Andrews & Sayers, 2015), embedded interventions for high school students who demonstrate inadequate number sense are scarce. The Common Core State Standards for Mathematics (2010) target foundational skills and concepts through grade six, including the ability to add, subtract, multiply, and divide, as well as manipulate fractions, decimals, and percent appropriately. In addition, students learn about rounding, place value, and ratio and proportion. As students move into middle and high school, these skills and concepts are not explicitly featured. Although they are frequently embedded in advanced mathematical concepts, consistent experiences to build or refine foundational understanding is not a reality for most students. The Massachusetts Curriculum Frameworks (Massachusetts Department of Elementary and Secondary Education, 2011) echoes the importance of building number sense through grade five, but there is little emphasis as students enter the middle grades. It is understandable why educational leaders might feel students should have a strong sense of how numbers work before finishing elementary school, but high school mathematics

teachers know this is often not the reality. Perhaps some students require more time and consistent practice with foundational skills and concepts, particularly those who encounter challenges through the learning process. Maybe students have gaps in their education that impacted their progress. Perhaps the learning experiences in elementary school were inadequate because of teachers who were unprepared to provide meaningful instruction. It is my opinion that middle and high school mathematics educators should embed daily number sense activities into their curricula that will sharpen students' computational abilities, engage them in thought-provoking problems, and provide experiences in relevant mathematical situations that may bolster their overall mathematics achievement.

Research indicates that elementary students who demonstrate strong number sense earn higher achievement scores than students who exhibit lower abilities working with numbers (Geary, 2013; Jordan, Glutting, Ramineni, & Watkins, 2010; Aubrey, Dahl, & Godfrey, 2006). Few studies featuring the impact of number sense on achievement exist however because it is a complex topic encompassing a variety of skills that should be targeted individually (Sowder & Schappelle, 1989). One of the skills exemplified in students with strong number sense is computational estimation (Reys & Bestgen, 1981; Case & Sowder, 1990; Booth & Siegler, 2006, Sowder & Schappelle, 1989).

Computational estimation is an interaction of number concepts and arithmetic skills that is performed mentally and results in reasonable answers (Reys & Bestgen, 1981).

Students who process this interaction appropriately are more likely to demonstrate strong number sense (Reys & Bestgen, 1981). Although researchers claim that an ability to use computational estimation to solve problems indicates an understanding of several

components of number sense (Sowder, 1988; Reys & Bestgen, 1981), there is a gap in the research that correlates computational estimation with mathematics achievement. This study seeks to determine whether high school students can improve their computational estimation abilities through daily problem solving, discussing, and reflecting. As part of their mathematics class, one group of students will engage in daily computational estimation activities that require collaboration and decision-making. The other group will engage in typical standardized assessment practice. Through an experimental design, I will compare results of pre- and posttests as well as performance on an authentic, high-stakes standardized assessment. I hope to further the research on high school students' computational estimation skills and begin to understand whether daily practice impacts mathematics achievement. If students do not make progress after participating in the intervention, I hope they benefit from collaborating with their peers to solve problems they will encounter in their lives outside of school.

Research Questions and Hypotheses

RQ1: Does a six-week collaborative learning activity featuring computational estimation improve high school students' abilities to answer posttest questions that target this particular skill?

H1: A six-week collaborative learning activity featuring computational estimation improves high school students' abilities to answer posttest questions that target this particular skill.

H0: A six-week collaborative learning activity featuring computational estimation does not improve high school students' abilities to answer posttest questions that target this particular skill.

RQ2: Does a six-week collaborative learning activity targeting computational estimation improve students' abilities to answer posttest questions that target this particular skill compared to students who engage in daily practice with sample standardized test questions targeting multiple skill areas?

H2: A six-week collaborative learning activity targeting computational estimation improves students' abilities to answer posttest questions that target this particular skill compared to students who engage in daily practice with sample standardized test questions targeting multiple skill areas.

H0: A six-week collaborative learning activity targeting computational estimation does not improve students' abilities to answer posttest questions that target this particular skill compared to students who engage in daily practice with sample standardized test questions targeting multiple skill areas.

RQ3: Do students who engage in a six-week collaborative learning activity targeting computational estimation earn higher scores on questions embedded within a standardized mathematics assessment that target this particular skill compared to students who participate in daily practice with sample standardized test questions targeting multiple skill areas?

H3: Students who engage in a six-week collaborative learning activity targeting computational estimation will earn higher scores on questions embedded within a

standardized assessment that target this particular skill compared to students who participate in daily practice with sample standardized test questions targeting multiple areas.

H0: Students who engage in a six-week collaborative learning activity targeting computational estimation will not earn higher scores on questions embedded within a standardized assessment that target this particular skill compared to students who participate in daily practice with sample standardized test questions targeting multiple areas.

RQ4: How does a six-week learning activity featuring computational estimation impact performance on posttest questions that target this particular skill for students with disabilities?

H4: Students with disabilities who engage in a six-week learning activity featuring computational estimation will improve their performance on posttest questions that target this particular skill.

H0: Students with disabilities who engage in a six-week learning activity featuring computational estimation will not improve their performance on posttest questions that target this particular skill.

Assumptions

There are assumptions readers should consider when reading this study. It is assumed that treatment group students will participate appropriately during the daily mathematics activities. Meaningful collaboration is a significant part of this study so activities are designed to inspire student discussion through problems relevant to their

lives. Teachers will be asked to circulate the room and encourage students to share their ideas and discuss thoughtful ways to problem-solve. Teachers will also coordinate discussion following the activity each day to highlight different problem-solving approaches.

It is also assumed that all students will put forth their best efforts on the pre- and posttests as well as the Massachusetts Comprehensive Assessment System (MCAS) examination. Although the pre- and posttests are relatively low-risk for students, a passing score on this particular MCAS assessment administered to tenth graders is a graduation requirement, so it is anticipated that these results will accurately reflect student abilities.

Finally, it is assumed that the participating teachers will not have a significant influence on performance. Both teachers are delivering the same curriculum to the same level of students, but a different instructional approach is certainly something to consider. To try and counter this variable, results from treatment and control groups taught by the teachers will be included separately so that it is assumed that the intervention is the only element making a difference.

Definition of Terms

This study will repeatedly discuss four terms that readers should clearly understand: number sense, computational estimation, foundational skills (and concepts), and mathematics achievement. Operational definitions will be included in Chapter 3, but a conceptual understanding is important prior to the review of literature. It should be noted that prominent mathematicians caution teachers and researchers to focus on one

definition of number sense because it includes so many critical skills, so the following definition is the description from NCTM regarding what students should be able to do with the standard "Numbers and Operations." These skills coincide with the many definitions of number sense researchers have tried to provide (Howden, 1989; Case, 1998; Fennell, 2008).

Number Sense (Numbers and Operations) - students will be able to:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- Understand meanings of operations and how they relate to one another;
- Compute fluently and make reasonable estimates (National Research Council, 2001; NCTM, 2000).

Computational Estimation - an interaction of mental computation, number concepts, and technical arithmetic skills such as rounding and applying place value which is performed quickly (without any recording tools) and which results in answers that are reasonably close to a correctly computed result (Reys & Bestgen, 1981, p. 119).

Foundational Skills (and Concepts) - the ability to relate a quantity to the numerical symbol that represents it, and to manipulate quantities and make calculations (Geary, Hoard, Nugent, & Bailey, 2013). Five building blocks include an understanding of numbers, the place value system, whole number operations, fractions and decimals, and problem-solving (Wilson, 2009).

Mathematics Achievement - performance on standardized mathematics assessments (i.e. NAEP, MCAS, Scholastic Aptitude Test (SAT), Accuplacer)

Chapter 2

Review of Literature

Several learning theories and mathematical investigations impacted the design of this research study. Chapter Two will examine some of this literature to provide a solid foundation for the reader in understanding the justification for and purpose of this experiment. The beginning of the literature review focuses on theory, specifically the significance of a student-centered classroom and social constructivist approach to teaching and learning. Following the theoretical framework is a dense description and highlight of research targeting number sense and computational estimation, the two concepts that sparked my curiosity in mathematics achievement among high school students. I will then describe other factors that are important to understand when considering the results of this study, including mental calculation, mathematics anxiety, peer collaboration, relevancy, standardized assessments, and post-secondary outcomes. Weaved throughout this review of literature are best practices and teaching implications for students with disabilities. This comprehensive review should paint a broad picture of the mathematics landscape in the United States and help explain why new pedagogical strategies are critical to continued progress for all students.

Social Constructivism and the Student-Centered Classroom

The role of students as discoverers rather than listeners in the classroom is critical to the learning process, but it is not a new idea. Philosophers like Dewey (1938), Piaget (1970), Vygotsky (1978), and Bruner (1996) advocated for students to engage in meaningful experiences that encouraged them to question, construct, apply, and reflect. This student-centered approach undergirds the theory of constructivism that inspired the

design of this research study. Every day, students from different backgrounds enter classrooms eager to share stories and ask questions relevant to their lives, but too often they sit quietly and absorb the information they are told is important. Teachers often find themselves explaining content to students rather than facilitating discovery because it mirrors their experiences in classrooms. But when teachers consistently transfer their knowledge to students through lecture and replication, is it feasible to assume accurate procedures on an assessment indicate conceptual understanding? According to Shor (1992), this top-down approach to teaching is not impactful because too many critical learning opportunities are missed (Shor, 1992).

It is futile to present a body of content to students rather than expose them to concepts that enable them to construct meaning with their peers and apply it to their own lives. Shor (1992) claimed that, “in a curriculum that encourages questioning, the teacher avoids a unilateral transfer of knowledge.” He argued that it is crucial for students to create their own meaning of issues through questioning, listening, and discussing. Teachers introduce situations and students actively deconstruct them. “Empowered students make meaning and act from reflection, instead of memorizing facts and values handed to them” (Shor, 1992). This habit of telling students what they need to know rather than creating opportunities for them to discover concepts impacts all students, particularly those with disabilities (Rose, Harbour, Johnston, Daley & Abarbanell, 2006). In fact, most Individual Education Plans (IEP)s include multi-modal teaching strategies that teachers must use, so providing support for teachers to design lessons should be a priority (Rose et al., 2006).

When teachers design lessons, it is appropriate for them to consider *how* students learn as well as what they learn. A classroom environment filled with collaboration, discussion, and group projects encourages students to share ideas and opinions and settle disagreements through reason and evidence. Teachers act as facilitators and frequently question students' approaches to solving problems and provide guidance for finding solutions. When appropriate, these progressive lesson designs provide hands-on activities to reach students with different learning styles and make experiences more authentic. The goal is to mirror challenges in society and bolster cognitive development through purposeful, organized interactions. Students and their experiences are just as important as the curriculum, and it is the responsibility of the teacher to connect them. If we really want our students to develop a passion for learning, improve their achievement, and become informed members of their communities, we must create opportunities to do so in the classroom.

Dewey and the student experience.

John Dewey was among the first philosophers to advocate for students positioned at the center of the classroom experience. He claimed that students and their experiences are just as important as curriculum, and it is the responsibility of the teacher to connect them. Dewey believed children need to act, observe what happens as a result of their actions, and reflect on what was (and was not) effective in order to truly learn and develop (1916). "I have taken for granted the soundness of the principle that education in order to accomplish its ends both for the individual learner and for society must be based upon experience" (Dewey, 1938, p. 89). Dewey stressed that these educational experiences had to arouse interest, enjoyment, and challenge in the immediate

experiences of the student to be worthwhile (Dewey, 1938). The role of the teacher is critical to maximizing these experiences. Teachers must carefully construct learning opportunities that engage students and help them recognize the consequences of their actions in a variety of applications. Current mathematics researchers such as Dan Meyer (2014) take active learning even farther by suggesting students choose not only how to arrive at answers, but what initial questions they have regarding given situations. Teachers must facilitate these inquiries to maintain focus, but this approach piques student interest by empowering students and makes the problem-solving process more meaningful. Jo Boaler (2015) promotes helping students create a "mathematical mindset" that celebrates mistakes as steps toward improved solutions. These pedagogical tools are enriching the classroom experience and changing mathematics education, and they evolved from philosophers like Dewey.

Dewey (1938) claimed that "he [the teacher] must survey the capacities and needs of the particular set of individuals with whom he is dealing and must at the same time arrange the conditions which provide the subject-matter or content for experiences that satisfy these needs and develop these capacities" (p. 58). The teaching profession is not easy according to Dewey. Not only must teachers create active experiences connected to curriculum within the classroom, they must recognize the abilities of all students and differentiate their instruction to meet their needs. Simply delivering information is not the teachers' job. They must know who the learners are in their classroom and design activities that most efficiently provoke thinking and understanding. Dewey recognized this over a century ago and it is crucial that teachers understand his approach to learning.

Piaget and constructivism.

Jean Piaget echoed the power of experience Dewey emphasized claiming “each time one prematurely teaches a child something he could have discovered himself, that child is kept from inventing it and consequently from understanding it completely” (Piaget, 1970, p. 715). People learn through experience and natural development, according to Piaget, and they cognitively compare what they already know with what they encounter. The outcomes often impact how they approach situations in the future and whether different tactics result in more favorable results. This idea that students should discover knowledge rather than receive it sparked the philosophy of constructivism and challenged traditional learning theories that do not highlight the importance of the student experience.

Constructivism can be defined as

a self-regulatory process of struggling with the conflict between existing personal models of the world and discrepant new insights, constructing new representations and models of reality as a human meaning-making venture with culturally developed tools and symbols, and further negotiating such meaning through cooperative social activity, discourse, and debate. (Fosnot, 1996, p. ix)

Beginning with the works of Piaget, constructivists support the claim that knowledge is “a mapping of actions and conceptual operations that had proven viable in the knowing subject’s experience” (von Glasersfeld, 1996, p. 4). Cobb (1994) applied the constructivist approach to mathematics education explaining that “students actively construct their mathematical ways of knowing as they strive to be effective by restoring

coherence to the worlds of their personal experience." Not only do students need to create their own meaning of mathematics, they need to apply their understanding to personal experiences in order for it to make sense. This approach to learning can be contrasted by another theory of learning that stems from a sociocultural perspective (von Glasersfeld, 1996). Through this lens, mathematical activity is socially and culturally situated. That is, students understand mathematics as they experience it in their lives, whether through worksheets in school, purchases in stores, or chores at home (Carraher, Carraher, & Schliemann, 1985). According to Cobb (1994), the goal of educators and philosophers should be to combine these approaches to learning so students are actively constructing mathematics that is applicable to their lives inside and outside of school. The coordination of these two philosophies mirrors some of the ideas of Vygotsky and social constructivism.

Vygotsky, Bruner, and social constructivism.

Lev Vygotsky (1978) endorsed the concept of constructing learning as well, but he believed social interaction, language, and cultural symbols are the critical components to understanding and development. He identified dialogue as a featured component to learning, specifically the way students and teachers converse, question, explain, and negotiate meaning (Vygotsky, 1978; Fosnot, 1996). The learning experience "awakens a variety of internal development processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers" (Vygotsky, 1978, p. 90). Vygotsky's discoveries prompted the theory of social constructivism. This theory describes learning and knowing as a social process where individuals negotiate understanding through experience and discourse with people who share common goals

(Vygotsky, 1987; Bruner, 1996; von Glasersfeld, 1996; Brophy, 2002). Human beings create their own society with those that surround them and construct knowledge through social interaction and cultural understanding (Kukla, 2000). Bruner (1961) echoed the importance of language and claimed that humans organize and categorize information through a coding system they construct through experience and social interaction.

Vygotsky also emphasized the gains students can make through social interaction in school resulting in a Zone of Proximal Development. Vygotsky (1986) stated that “it is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (p. 86). The Zone of Proximal Development applies to the teacher/student relationship, but also the interactions between peers who are more advanced and those who tend to struggle. It is the teacher's responsibility to design opportunities that engage higher performing students while building the skills of learners who demonstrate more difficulties in comprehension. “Facilitating the child’s movement to the next step of development involves exposure to the next higher level of thought and conflict requiring active application of the current level of thought to problematic situations” (Kohlberg & Mayer, 1972, p. 459). Bruner (1960) discussed the concept of a spiral curriculum, where teachers introduce simplified ideas and then revisit them later at more complex levels. Through carefully planned learning experiences that gradually increase in complexity and encourage collaboration of learners at different levels, all students can participate and make cognitive gains. Inclusion classrooms are successful because higher performing students can provide ideas and feedback to students who are unable to independently match their understanding.

These higher performing students improve their own practices through these reflective experiences and are hopefully challenged by their teachers to further their comprehension (Woodward & Brown, 2006).

Implications of theory for this research study.

These seminal philosophers of education depicted a classroom environment centered on purposeful student participation, collaboration through discourse, and discovery. They believed that teachers are responsible for coordinating these learning experiences based on the curriculum as well as the needs and interests of their students. This research study will promote and analyze educational experiences through this student-centered lens, particularly guided by social constructivist theory. Students make gains developmentally when they learn from their teachers and from one another. Whether learning to throw a football or tackle a mathematics problem, students need to act, reflect on what they did, refine their skills based on their realizations and feedback, and apply new understanding to novel situations that embed and generalize this knowledge. The role of educators should be to purposefully design experiences within the classroom that encourage these key components to meaningful learning opportunities. As a researcher, I decided to play the role of the lesson designer in this study, thereby relying on the participating teachers to facilitate discussion and reflection with their students. The learning tool I designed encourages (and relies on) collaboration and discourse, so it is my hope that the experiences were enlightening and meaningful. The theory behind the design of this study centers on the idea that real understanding is constructed socially among peers with targeted teacher design and guidance.

Targeted Mathematics Skills and Concepts

Educator and social constructivist, Paul Cobb (1994), defines mathematical learning as the “process of active construction that occurs when [students] engage in classroom mathematical practices, frequently while interacting with others” (p. 41). Cobb is not alone in his portrait of the mathematics classroom, a place where students actively problem-solve through discussion and reflection. Jo Boaler (2015) emphasizes the importance of actively making sense of mathematics through mistakes, feedback, and revision. Ball and Hill (2009) advocate for active learning and "the importance of teachers being able to hear their students and to build bridges between their thinking and fundamental ideas and practices of the discipline". The following sections will outline several components of an active mathematics classroom that are supported in research and embody this research study. The most significant topics include number sense and computational estimation, but other factors in mathematics education are discussed because they play an important role in the classroom as well as in this experiment.

Number sense.

The National Council of Teachers of Mathematics (NCTM) identified five components that characterize number sense: number meaning, number relationships, number magnitude, operations involving numbers, and referents for numbers and quantities (1989). These skills and concepts are considered vital because they contribute to general intuitions about numbers and lay the foundation for more advanced skills. According to the Cockcroft Report (1982), when students have a strong sense of numeracy they feel confident and comfortable working with numbers to solve everyday problems and they appreciate how numbers communicate information through graphs, charts, or

other appropriate means (Cockcroft, 1982). Howden (1989) explained that "number sense can be defined as good intuition about numbers and relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of concepts, and relating them in ways that are not limited by traditional algorithms" (p. 11). These descriptions, like many others, are loaded with specific skills and conceptual understanding, but they are not clear about what teachers should target in the classroom. Case (1998) provided a specific summary of what number sense looks like, and admitted that it is far easier to recognize than define.

Students with strong number sense move seamlessly between the real world of quantities and the mathematical world of numbers and numerical expressions. They can invent their own procedures for conducting numerical operations. They can represent the same number in multiple ways depending on the context and purpose of this representation. They can recognize benchmark numbers and number patterns: especially ones that derive from the deep structure of the number system. They have a good sense of numerical magnitude and can recognize gross numerical errors that is, errors that are off by an order of magnitude. Finally, they can think or talk in a sensible way about the general properties of a numerical problem or expression-- without doing any precise computation. (p. 1)

More recently, Fennell (2008) weighed in on the importance of number sense and described a list of foundational skills and concepts that are essential to its acquisition.

These experiences include, but are certainly not limited to, working with place value, composing and decomposing numbers, understanding how addition, subtraction, multiplication, and division work, acquiring basic facts, and

developing fluency with whole-number operations. Number sense also requires an understanding of how the commutative, associative, and distributive properties work and how they are used in learning basic-fact combinations, adding columns of numbers, and seeing how the multiplication algorithm works. This work must extend to fractions, decimals, and related percents as students move through the elementary grades into middle school. (p. 3)

While an understanding of these characteristics of number sense is vital to mathematics education, the term itself is not narrowly defined and does not easily guide instruction. It consists of several quantitative skills and concepts converging through years of instruction and application to help people reach reasonable conclusions in mathematical scenarios. Ideally, students acquire these skills and concepts throughout elementary and middle school so they are ready to tackle more rigorous content in high school; however, there are several factors that can hinder students from owning these skills, including elementary school teachers who fail to build solid computational skills (Ma, 2010), high student absenteeism (Mac Iver & Mac Iver, 2015; NCES, 2017), or a lack of support from home (O'Sullivan, Chen & Fish, 2014; Vukovic, Roberts, & Wright, 2013).

The research on why students may not build number sense at the elementary level is crucial and should inform revised standards and action plans, but regardless of what researchers discover, it is evident that many students in middle and high school right now are not demonstrating adequate number sense. In addition to my own findings as a mathematics teacher and researcher, there are indicators of students' limited number sense in several studies (Schneider & Siegler, 2010; Vosniadou, Vamvakoussi, & Skopeliti, 2008) as well as in achievement scores (NCES, 2015).

Figure 1. Grade 8 2015 NAEP scores. This figure illustrates grade 8 scores in five mathematics categories for students with and without disabilities (NCES, 2015).

Figure 2. Grade 12 2015 NAEP scores. This figure illustrates grade 12 scores in five mathematics categories for students with and without disabilities (NCES, 2015)

It should be clear that the previous scores in Figure 1 and Figure 2 in each mathematical category were not compared to determine statistically significant differences and do not prove that number sense is the reason for overall low achievement, but it is relevant to highlight that "Number Properties & Operations" scores are the lowest for all students in both grade eight and grade 12. The pattern is slightly different for students with disabilities, but clearly working with numbers is not a strength for the average student. There are also certainly aspects of number sense in the other categories, most notably in "Data Analysis, Probability, and Statistics" as well as "Measurement", and those categories are also areas of struggle for the average student. Do these scores indicate that the questions targeting number sense are more difficult than those involving

algebra and geometry, or do they convey a deficiency in students' abilities to work with numbers?

Based on my experience in high school mathematics classrooms and achievement scores that are not meeting the standard, there is a problem with the way our students understand numbers, particularly once they reach high school. And because number sense consists of multiple skills and concepts, it is challenging for teachers to identify what their students are lacking and develop strategies that will make up for the learning they may have missed. Should high school teachers review how to multiply fractions? Should they review how to convert a percent to a decimal? Would a review just reinforce previously taught procedures without improving conceptual understanding? Given the uncertainty in students' backgrounds and abilities, what should high school mathematics teachers do to help their students build a stronger sense of how numbers work?

In 1989, leading mathematics educators and researchers joined cognitive psychologists at a conference in San Diego to discuss number sense, particularly, how to define it and how students can acquire it (Sowder & Schappelle, 1989). Although participants represented just a fraction of the many researchers who contributed to studies on number sense, their meeting indicates the concern they had for building this overarching ownership of computational skills. This gathering, documented by prominent mathematics researcher Judith Sowder, was one highlight among decades of research citing number sense as a critical component of achievement in mathematics (Cockcroft, 1982; Sowder, 1988, 1992; NCTM, 1989, 2000; Reys, Reys, & McIntosh, 1999; Faulkner, 2009; Boaler, J. 2015). While substantive discussion at the San Diego conference provoked new insight and further investigation into mathematics education,

two key takeaways emerged that confirmed my frustration but helped focus my research study: there are several components of number sense and the term itself may be impossible to define and measure (Sowder & Schappelle, 1989).

The more I considered the skills and conceptual understanding my students lacked, the more I realized what these researchers confirmed - number sense is a dense concept and may be too big to address entirely. Trafton argued that defining number sense may not be as useful as “pursuing those aspects of number sense that have direct relation to how children process numbers in computational situations” (Sowder & Schappelle, 1989, p. 30). According to Resnick (1987), “Number sense resists the precise forms we have come to associate with the setting of specified objectives for schooling” (p. 3). Silver described number sense as a “paralyzing large phenomenon that we don’t quite know how to get a handle on...I’m not arguing that you want to get really narrow about what that means, but that it might be helpful to think about those pieces sometimes, rather than trying to think about the whole area of number sense” (Sowder & Schappelle, 1989, p. 28-29). Those particular skills and concepts that embody number sense have proven important in several studies. For example, number comparison (Bugden & Ansari, 2011) and number line estimation (Booth & Siegler, 2006) have shown to be significant predictors of mathematics achievement. Siegler et al. (2012) found that knowledge of fractions and whole number division predict performance in algebra. Wu (2005) discussed the importance of understanding rational numbers in middle school for success in high school and college mathematics courses. Considering the difficulty in designing research aimed at measuring number sense as a whole, this study will target one component researchers have observed in students with strong

number sense: proficiency in computational estimation (Reys & Bestgen, 1981; Trafton, 1986; Paulos, 1988; Bobis, 1991; Case & Sowder, 1990).

Computational estimation.

For the purpose of this study, the definition of computational estimation is described as “an interaction of mental computation, number concepts, and technical arithmetic skills such as rounding and place value. It is a mental process which is performed quickly (without any recording tools) and which results in answers that are reasonably close to a correctly computed result” (Reys & Bestgen, 1981, p. 119). According to Reys and Bestgen (1981) computational estimation “is an essential basic skill with lifelong applications and should be an integral part of every mathematics program” (p. 118). I will discuss the relevance of computational estimation to everyday life later in this chapter, but first I will justify its benefits in the classroom. The most significant benefit of targeting computational estimation in the classroom is that it may improve students' overall calculation skills and general number sense (Beishuizen, van Putten & van Mulken, 1997; National Research Council, 2001; Fennell, 2008; Cochran & Dugger, 2013). Given the difficulty of tackling the huge concept of number sense, teachers should target skills that are likely to bolster it. Authors of the Cockcroft Report (1982) voiced concern that teachers are not paying enough attention to the wider aspects of numeracy and are instead content with a student's ability to perform basic arithmetic computations. Steen (1999) argued that educators need to move beyond the traditional arithmetic to algebra pathway and focus on skills in numeracy such as estimation and mental calculation. When designed appropriately, these skills can improve students' overall sense of how numbers are organized.

Another benefit of bolstering computational estimation skills involves more efficient ways to solve problems and find answers that make sense. When students demonstrate an ability to make computational estimations, it enables them to determine the reasonable closeness of their solutions when solving problems. This skill of determining whether solutions are reasonable is discussed in The Common Core State Standards (2010) as well as the Massachusetts Curriculum Frameworks for Mathematics (2017). Paulos (1988) argued the importance of students understanding actual and relative sizes of numbers and demonstrating caution when answers are contrary to logic. Even if students have strong computation skills, they should be able to identify errors based on their ability to estimate a solution. Fennell explained, “As students estimate, talk about numbers, compute, use mental math, and judge the reasonableness of their results, they become more flexible in working with numbers” (2008, p. 3). When students consistently use computational estimation to solve problems in class prior to checking their work with written procedures or a calculator, they may utilize this skill on standardized assessments when answering multiple choice questions. If lessons targeting computational estimation can help students build number sense, increase their performance on standardized tests, and provide relevant learning experiences that highlight the importance of mathematics, it is worthwhile to design these opportunities in the classroom and assess the results.

While there is an abundance of research describing computational estimation as something strong mathematics students own, the content-heavy curricula that has driven middle and high school mathematics classrooms have failed to emphasize its importance (Trafton, 1986; Paulos, 1988, NCTM, 1989; 2000; The Common Core Standards

Initiative, 2010; Massachusetts Curriculum Frameworks, 2017). Computational estimation is not explicitly featured beyond elementary school because the focus turns to algebra, geometry, and advanced number properties. The Common Core Standards (2010) target skills in computation using estimation and mental strategies through grade six, but as students move into middle and high school, concepts and skills that are critical to computational estimation are implied and not explicitly taught (see Figure 3).

Although it is embedded in advanced mathematical concepts, daily practice with computational estimation is rare. The Massachusetts Curriculum Frameworks (2017) echoes the importance of computational estimation through grade five, but there is little emphasis as students enter middle and high school. It is understandable to expect students to have a solid understanding of numbers upon entering the middle grades given the focus on working with numbers through elementary school, but it is not the reality. Too many students never gained a sense of how numbers work and are unable to make reasonable computational estimations, particularly those who encounter more challenges through the learning process (Ortiz, 2009; Landerl, Bevan, & Butterworth, 2004).

Perhaps middle and high school mathematics educators should embed daily computational estimation activities into their curricula that will engage students in thought-provoking problems, facilitate peer discussion of how to solve these problems, and provide practice for relevant mathematical situations they will encounter beyond the classroom.

	Counting and Cardinality	Operations and Algebraic Thinking	Number and Operations in Base Ten	Number and Operations - Fractions	Measurement and Data	Geometry	Ratios and Proportional Relationships	The Number System	Expressions and Equations	Statistics and Probability	Functions	Number and Quantity	Algebra	Modeling
K														
1														
2														
3														
4														
5														
6														
7														
8														
HS														

Figure 3 – CCSSM Domains. This figure illustrates the CCSSM (2010) domains by grade level.

Although computational estimation is a critical skill to learn, there are factors that impact student understanding. For example, when should teachers introduce this set of interrelated skills and concepts to their students? According to B. Reys, it may be too late for students in seventh grade or higher to learn adequate estimation skills (Sowder & Schappelle, 1989). This may indeed be true and is one of the risks I took designing my research study, but as a high school educator I believe it is my responsibility to implement learning experiences that target the skill of computational estimation that so many researchers believe is a critical component of number sense. High school is also a time that students take high-stakes standardized assessments, so any skills that can help them maximize their performance is worthwhile to target. This research design

integrated ten minute warm-up activities to engage students in making estimations. This time may be sufficient, or perhaps I will find that students need more than a ten minute daily challenge to develop computational estimation skills. No matter the results, hopefully this study will contribute to the research on best practices for high school students to improve their number sense and raise their mathematics achievement.

Another factor impacting student understanding of computational estimation is the reality of finding exact answers in mathematics education (Reys, Bestgen, Rybolt, & Wyatt, 1980). Students learn procedures for solving problems and the results are almost always exact answers rounded to specific decimal places or simplified fractions. When students are asked to estimate, the expectation is they will use alternative skills and conceptual understandings to arrive at a reasonable answer. These skills are arguably more difficult and more indicative of mathematical comprehension than procedural understanding, but do students know how to do this? Will they try to perform the procedure and change the answer slightly instead of accessing skills such as rounding or using base-ten logic? Estimating involves using different skills than students are accustomed to and this difficulty can be challenging for educators to overcome. Consider the following situation:

If presented with a problem where 108 of something must be multiplied by 45 of something else, the majority of students will either use a calculator or write the numbers vertically and use a procedure to solve. These are perfectly adequate methods to use. But what if a calculator and a pencil are not easily available and the context of the problem to be solved implies an exact answer is unnecessary? Will students have a method for solving? Are they able to produce an estimation that is remotely close? If we

want to claim that our students have a strong understanding of mathematics, they should be able to provide a reasonable answer without the use of any tools. For example, the students could round the numbers to 100 and 50 and produce an answer of 5000.

Another strategy may involve multiplying 100 by 45 and then multiplying 8 by 45 and adding those values together. This would result in a more precise solution if the context of the problem calls for it. Using these two strategies without any tools demonstrates a stronger understanding of numbers than punching values into a calculator or following a memorized procedure. These strategies involve knowing how to round appropriately, how to multiply powers of ten, what the distributive property entails, and how context impacts the reasonableness of solutions. If students can use these skills and concepts to find answers, they are proving they have a deeper understanding of how mathematics works. This should be a feature of the mathematics classroom.

As students discover more ways to arrive at solutions, they increase their flexibility in working with numbers and choosing problem-solving approaches. No longer will they have to rely on a memorized procedure without a full understanding of what they are doing. Students will be able to recognize multiple ways to solve tasks and then collaborate with their peers to share understanding and construct even more strategies. According to Reys et al. (1980), as students improve their ability to work with numbers, they develop estimation techniques independently and through collaboration with peers. This can be especially helpful for students with disabilities who may struggle to independently find alternate problem-solving methods. Through discussion and active construction, students with disabilities can model what they see from their peers and gain a stronger understanding of problem-solving strategies that work.

As mentioned previously in the "Number Sense" section of this paper, computational estimation is just a piece of an overall sense of how numbers work. R. Reys highlighted a concern that critical topics such as computational estimation may be forgotten if simply left under the large umbrella of number sense (Sowder & Schappelle, 1989). In my experience as a mathematics educator at the high school and college levels, I agree that estimation has been forgotten, not only within the concept of number sense, but amid the dominance of procedures and content. Educational leaders provide teachers with standards, frameworks, and professional development, but is this guidance overshadowing the simple and practical aspects of mathematics? I believe teachers should consistently explore elements of number sense such as computational estimation and integrate them into daily mathematics lessons so students can focus on how numbers work rather than what procedures they should use. As I comb through studies and data on mathematics achievement, I am more convinced than ever that number sense, and particularly computational estimation, is critical to classroom engagement, real-world application, and improved mathematical achievement among our students. We need to decide how we want to incorporate these lifelong skills and be certain that all students are gaining exposure and understanding.

Mental calculation.

According to the Cockroft Report (1982), excessive concentration on the purely mechanical skills of arithmetic will not assist the development of understanding. Simply stated, just because students can remember to move a decimal point two places or carry a one after multiplying to find accurate solutions does not mean they understand what they are doing. Kamii and Dominick (1997) noted that "when we try to teach children to

make relationships between numbers (logico-mathematical knowledge) by teaching them algorithms (social-conventional knowledge), we redirect their attention from trying to make sense of numbers to remembering procedures” (p. 59). Gravemeijer (2003) is critical of teaching algorithms “in readymade form” that students do not understand, advocating instead “instructional sequences in which the students act like mathematicians of the past and reinvent procedures and algorithms” (p. 121) as a means of promoting growth in mathematical understanding. Similar to students who can decode words but do not comprehend the meaning of a sentence, some mathematics students appear to know what they are doing when performing calculations but do not have an overall understanding of the problems they are solving. It may be more telling to ask students to get rid of the pencil and paper and solve computation problems in their heads. While this may not provide as much accuracy, it will show whether students understand the values they are computing.

Mental Calculation (sometimes referred to as arithmetic reasoning) has been the focus of several studies (Hickendorff, van Putten, Verhelst, & Heiser, 2010; Fuson, 1992; Steffe, Cobb, & von Glasersfeld, 1988), and it relates directly to computational estimation (Reys & Bestgen, 1981). McIntosh and And (1997) argued that solving problems using mental computation forces students to think about and understand the numbers they are working with in order to generate strategies. Sowder (1990, p.19) asserts that “mental computation should not be delayed until after formal written algorithms have been mastered. In fact delaying it until that time encourages students to mentally use the algorithms meant only for pencil-and-paper calculations.” McIntosh, De Nardi, and Swan (1994) recommend educators teach mental computation strategies and

encourage students to share their thought processes through consistent 10 to 15 minute activities. They argue that it is important not to demand all students use one particular way to compute (i.e. rounding to nearest ten) because they may misuse it if they do not fully understand. Based on these findings and my own experience, it certainly makes sense to discuss multiple methods of solving so students are exposed to possibilities and can decide whether the strategies are appropriate for them. For example, Figure 4 shows five approaches to multiplying 15×12 mentally that students can use to demonstrate flexible thinking and an understanding of how to manipulate numbers. These solutions can all be applied mentally and they demonstrate a command of simplifying numbers.

$15 \times 10 = 150$	$30 \times 12 = 360$	$12 \times 15 =$	$12 \times 5 = 60$	$12 \times 12 = 144$
$15 \times 2 = 30$	$360 \div 2 = 180$	6×30	$12 \times 10 = 120$	$12 \times 3 = 36$
$150 + 30 = 180$		$6 \times 30 = 180$	$120 + 60 = 180$	$144 + 36 = 180$

Figure 4. Five ways to multiply. This figure illustrates five ways to multiply 15 by 12 (Boaler, 2015).

Changing the Perception

Mathematics anxiety.

Students' lack of enthusiasm for mathematics has been evident for decades. In the 1980s, several reports based on NAEP data indicated student confidence and enjoyment in mathematics decline as they move from elementary school to high school (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Dossey, Mullis, Lindquist, & Chambers, 1988). These findings helped to motivate the NCTM (1989) to highlight the

importance of helping students understand the value of mathematics and building their self-confidence. Concurrently, the National Research Council (1989) released a report identifying the immediate need to change public belief and attitude toward mathematics in order to improve the future of the field. Clearly, the educational leaders at this time were concerned with students' perception of mathematics. Fast-forward 25 years and the data is not drastically different. According to one study in a typical American suburb high school (see Figure 5), mathematics is the most hated subject by far among students (Wiggins, 2014).

Figure 5. Least favorite subjects. This figure shows a group of students' least favorite school subjects (Wiggins, 2014).

Interestingly, mathematics is also one of students' favorite courses in high school. According to Wiggins (2014), this is not surprising. Students typically either love mathematics or hate it. It is encouraging to hear that some students love mathematics and look forward to the challenges it presents; however, teachers need to focus on *all* students, including those who hate mathematics and are less likely to reach their academic goals as a result.

Figure 6. Favorite subjects. This figure highlights students' favorite subjects in schools (Wiggins, 2014).

Students fear mathematics as a result of prior learning experiences and a lack of confidence in the subject matter (Brady & Bowd, 2005; Bramald, Hardman, & Leat,

1995; Scarpello, 2007). Middle school students have demonstrated a decline in school engagement, particularly in mathematics (Martin, 2007; 2008), and other studies reveal students often avoid mathematics activities in the classroom and real life because of a genuine fear of the subject (Bekdemir, 2010; Gresham, 2008; Vinson, 2001; Zettle & Raines, 2002). This is particularly evident among students with disabilities (Woodward & Brown, 2006). These children begin struggling in elementary school and rarely catch up to their peers. Consequently, they learn to dislike mathematics and avoid it as often as possible. This lack of enthusiasm and frequent avoidance accelerates the problem, and students sometimes never recover. Even adults fear mathematics and are often reluctant to engage in tasks because of a lack of confidence (Markus, 1996). Students who see their parents avoid mathematics and articulate fear or dislike of the subject are likely not encouraged or supported outside the classroom (Fan & Chen, 2001). Evidently, leaders in mathematics still have gains to make in emboldening confidence among our students, especially at the high school level.

Mathematics anxiety plays a key role in students' perception. Although individuals experience debilitating anxiety in many different contexts (e.g., social or classroom), anxiety in learning is commonly reduced to test anxiety and has two elements - cognitive and emotional (Stipek, 2002). In the mathematics context, the cognitive element is apparent and includes having negative thoughts or low expectations for learning mathematics and performing well on exams (Meece, Wigfield, & Eccles, 1990; Wigfield & Meece, 1988). Something is happening after elementary school in students' perception of and achievement in mathematics. Is it a coincidence that the instruction appears to change from student-centered applications of numbers to teacher-led

explanations of content? I do believe that students need exposure to higher level concepts in mathematics, but my experience indicates that teachers make more of an effort to cover content and meet standards than engage the students who struggle.

Research on mathematics anxiety has consistently revealed that it correlates with poor mathematics achievement (Hembree, 1990; Hsiu-Zu et al., 2000). In her meta-analysis, Ma (2010) found that there is a significant negative correlation between mathematics anxiety and mathematics achievement among secondary students regardless of gender and ethnicity. And much of this anxiety surfaces as a result of low self-efficacy. Self-efficacy can be defined as "people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" (Bandura, 1986). Researchers have argued that self-efficacy affects whether students choose challenging or easy activities, set higher or lower goals, exert more or less effort, and persist through obstacles or give up easily (Pajares, 2005; Schunk & Pajares, 2002; Zimmerman, 2000). As a result, students with high self-efficacy tend to learn and achieve more on a given task and are more motivated and actively engaged in their learning (Stipek, 2002; Zimmerman, 2000). This outcome makes sense when considering that students either love mathematics or they hate it. If students' perceptions of their own learning are low, they are more likely to disengage; therefore, teachers need to employ new strategies to change this. I have already discussed the importance of building their computational estimation and overall number sense to increase their confidence and ability to solve problems, but there are general pedagogical methods teachers can use to effect change such as facilitating peer collaboration and relevant learning experiences.

Peer collaboration.

Routman (2000) states that “all learning involves conversation. The ongoing dialogue, internal and external, that occurs as we read, write, listen, compose, observe, refine, interpret, and analyze is how we learn.” Research has shown that students benefit from small-group learning (Williamson, 2006; Wenzel, 2000). When students have a common goal and each group member takes responsibility for contributing meaningfully, the learning experience is enhanced. “Students who work in cooperative groups with other students are more motivated and successful, especially with regard to reasoning and critical thinking skills, than those that do not” (Wenzel, 2000). While there is an abundance of research describing the benefits of peer collaboration, it is important to consider the importance of designing learning activities that promote meaningful interactions. As Dewey (1938) discussed, it is the responsibility of the teacher to design collaborative learning opportunities that maximize the potential for thought-provoking dialogue. Peer collaboration is only as effective as the teacher who designs it, and those of us who employ a social constructivist approach to learning must recognize that social interaction involving active thinking and discussion must be arranged appropriately.

Problem-solving and decision-making are more efficient when they involve input from multiple perspectives. Working in groups allows students to observe a variety of methods to solve problems and build on these to reach solutions (Lee, 2006; Panitz, 2000; Williamson, 2006). Routman (2000) claimed that

much of what I know, I know because I have questioned and thought about ideas with others, tried things out, modified stances, talked with colleagues. Always, conversations play a major role in my thinking, learning, teaching, and changing.

So it is with all learners. I would argue that when no conversations are going on, as in whole class “skill and drill,” it’s not learning that’s taking place but rather rote memorization (p. xxxvi).

Students should be encouraged to work in groups and learn from one another not only to develop strategies to solve specific mathematical problems, but to learn collaboration skills that are necessary beyond school (Markus, 1996). One of the most common and critical skills current employers are looking for in professional candidates is collaboration. If educators want to sufficiently meet current mathematical standards (CCSSI, 2010; NCTM, 2000), design learning activities that are engaging and relevant, and prepare students for life after school, they will have to move from a traditional approach where expert teachers present information to passive students (Tyner-Mullings, 2012) to a productive culture filled with collaborative communities working to make sense of the world (Moses & Cobb, 2001; Sfard, 2001). This is a major part of this research study and should be featured appropriately in every mathematics classroom.

Relevancy.

A learning environment designed with students tackling new challenges through peer interaction and discussion is crucial to constructing knowledge. But what about those students who see little value in the learning experience? The Common Core Standards (2010) addressed this concern advocating for instructional practices focused on conception, explanation, and application. “The standards were created to ensure that all students graduate from high school with the skills and knowledge necessary to succeed in college, career, and life, regardless of where they live” (Common Core

Initiative, 2010). Educational leaders are challenging teachers across the nation to make content more relevant to students, and research supports this teaching approach.

Students are more likely to engage in learning if they find the material relevant to their lives (Andriessen, Phalet, & Lens, 2006; Simons, Dewitte, & Lens, 2004; Meece & Kurtz-Costes, 2001). Relevance, often referred to as authentic instruction, plays a critical role in education today, particularly in mathematics. According to Newman, Bryk, and Nagoaka (2001), authentic instruction is a combination of instruction and assessment that challenges students to comprehend complex ideas beyond the walls of the classroom. Burden (2000) explained that relevancy includes highlighting the significance of concepts, providing real world examples, and developing opportunities for students to take information they previously learned and apply it to more advanced concepts. Authentic instruction and relevancy, as defined by these authors, are integral components to the 21st century classroom. If students are unclear of their need to know the mathematics presented to them, perhaps they will avoid the learning process, particularly those students who find it challenging. As so many students express anxiety and low confidence in mathematics, teachers need to find ways to include them. This research study will use real-world problems to try and engage more students and provide examples of the importance of mathematics in their teenage lives. Through relevant daily challenges, students will find meaning in their learning and pride in their accomplishments.

One positive factor in understanding computational estimation is its relevancy to students. Regardless of occupation or educational status, an ability to perform computational estimations is a common feature of everyday life (Booth & Siegler, 2006).

Whether it be creating a personal budget or comparing sales at a grocery store, students will encounter situations where estimating values is functional and efficient. For example, it is more practical for students to estimate how much money they spend on gasoline in one month than add a detailed list of individual trips to the station. If students are shopping for clothes, it is beneficial for them to understand discounts and estimate cost between similar products. Teachers need to use practical applications of computational estimation that engage students in lessons designed to benefit them inside and outside the classroom.

Why Number Sense Matters

Standardized assessments.

Standardized testing has permeated education since the 19th century providing measurable tools to assess student certification and school accountability (Madaus, Clark, & O'Leary, 2003; Kilpatrick, 1992). Although these tests have impacted policy for years, they became high-stakes for students and schools when the report, *A Nation at Risk* (National Commission on Excellence in Education, 1983) called for more rigorous high school graduation requirements. Several states decided to assess students when considering grade-level promotion, course placement, high school graduation status, and college entrance (Wilson, 2007). Reliance on standardized testing reached the national level most significantly through the No Child Left Behind Act (NCLB, 2002). States were mandated to assess students annually in reading and mathematics in grades 3-8 and once in grades 10-12 and report their results to ensure students of all demographics were making effective gains in achievement. Currently, standardized testing is a major component to the educational experience. From high school graduation requirements to

college placement examinations, students (and teachers) cannot seem to escape the reliance on standardized assessments.

In Massachusetts, high school students are required to pass the MCAS tests in English Language Arts (ELA), mathematics, and science (Massachusetts Department of Elementary and Secondary Education, 2017). Most colleges use these MCAS scores, in addition to SAT results and Grade Point Average (which are often comprised of tests), to determine if students are eligible to attend their institutions. And if students are accepted to a school, they often have to take a placement test in order to determine the course level they should take (Fields & Parsad, 2012; The College Board, n.d.). The high stakes assessments students must take in high school and college are critical, and they have can have serious consequences for learners who struggle to gain mastery in mathematical concepts, battle test anxiety, or have missed chunks of their schooling. This is why it is essential that students improve their achievement scores in mathematics - not to improve public perception of education, but to help students access advanced educational opportunities and enroll in classes that are challenging and interesting rather than foundational and repetitive.

This section of my research study is not intended to negatively portray standardized testing. Although I do feel that stakeholders put too much stock in test results, there is valuable information that teachers can learn from student achievement. For example, as a special educator who led eligibility determination meetings for current and potential special education students, I used standardized assessments to help understand students' abilities. If a student was performing poorly in a mathematics class but demonstrated average ability on an assessment, I used that information to investigate

further why the student was not making effective progress. Clearly the student was capable of understanding and solving mathematical problems based on the assessment results, so perhaps there was another factor impacting the learning experience. As long as we combine test results with other modes of assessment to determine student ability and potential, standardized testing can be a powerful tool.

According to the Cockcroft Report (1982), standardized assessments measure only some aspects of mathematical attainment. They do not target perseverance or attitudes, and most often do not assess a student's ability to solve mathematical problems that are unfamiliar. Standardized assessments can help uncover some blatant skills or concepts that should be revisited, but they should be grouped with other forms of formative and summative assessments in order to provide a complete picture of student ability. Nevertheless, these tests are currently a major part of the educational experience and they have serious consequences for our students.

Post-secondary impact.

Every couple of years, achievement data is released to the public indicating that many students are not meeting standards in mathematics (NCES, 2015). While this is valuable information that mobilizes educational leaders to try and effect positive change, this is not the most concerning collection of data. According to Barry and Dannenberg (2016), over half a million college freshmen are forced to enroll in remedial classes during their first year of school. First-time full-time bachelor degree students who take a remedial course are 74% more likely to drop out than their peers. First-time full-time associate's degree students who take a remedial course are 12% more likely to drop out than their peers (Barry & Dannenberg, 2016). According to a study from Columbia

University, 59% of first year students attending community colleges require remedial courses in mathematics (Bailey, T., Jeong, D. W., & Cho, S. W. (2010), and of these 13 million students, only 5% earn college level credit within their first year, and 80% will not earn credits even after three years (Bailey, T., Jeong, D.W., & Cho, S. W. (2010).

Other reports indicate that less than 25% of remedial students at community colleges earn a certificate or degree within eight years (Bailey, 2009), and just 27% of students enrolled in remedial math eventually earn a bachelor's degree compared to 57% of students who do not require remediation (Wirt, 2004). With such a high percentage of college students dropping out after paying for classes that mirror those they took in grade school, the educational community needs to target the symptoms and develop new approaches to what students need to know in mathematics.

In addition to facing the disappointment of not graduating from college, many students endure the lingering financial burden of college courses. Families pay a combined \$1.5 billion and borrow \$380 million for classes that do not award credit toward graduation. This results in the average student paying \$3000 extra and borrowing \$1000 for remedial coursework (Barry & Dannenberg, 2016). The National Council of State Legislatures (2008) issued a report indicating that states and students in the U.S. spend about 2.3 billion dollars on all remedial college courses each year, primarily in mathematics. While current educational and political leaders encourage more students to attend college, are they failing to consider whether these students possess college-readiness skills? Are the assessments colleges rely on effective ways to determine whether students require remediation? More research is necessary to determine feasible

solutions to this issue that impacts a significant number of students and families every year.

Future Research

While international assessments from primary grades through high school may not mean much in isolation, they are clearly identifying a trend that indicates real problems for our nation, particularly our most vulnerable students and families. One common thread among data, studies, and this researcher's observations could be students' lack of number sense. A consistent feature of students who do have a strong sense of how numbers work is an ability to perform appropriate computational estimations. When students can form estimates mentally using a variety of computational skills and an understanding of number concepts, perhaps they can improve their performance on standardized assessments and eliminate the need for remediation when attending post-secondary institutions. The following study will test this theory and provide guidance for future research in mathematics achievement.

Chapter 3

Methodology

Throughout my teaching career, I have valued student performance beyond traditional classroom and standardized assessments. As a special educator, many of my students struggled to demonstrate what they knew on tests because of the learning challenges they faced. Whether it be a learning disability in reading or mathematics, difficulties with attention or communication, or challenges with emotional functioning, student performance on tests often failed to reflect what students knew, or what they were capable of knowing. Nevertheless, testing is a major part of the student experience and it impacts the path toward college and career. As a result, I designed a quantitative study to measure the effectiveness of an instructional tool on student achievement at my high school. Instead of bombarding students with mathematical procedures and examples of test questions, I aimed to provide relevant scenarios for students to think about independently and discuss collaboratively with their peers. The mathematical problems targeted fractions, decimals, and percent, and encouraged a variety of solution strategies that students could implement. My short term goal is to raise student achievement on standardized assessments through meaningful learning experiences rather than memorization techniques and test practice. In the long term, I want students to improve their overall number sense, raise their self-confidence in mathematics, and increase their likelihood of accessing post-secondary education without having to enroll in remedial coursework.

This study features lessons that target computational estimation, a critical component of number sense that researchers and educators believe will improve

measures on a range of assessments (Sowder, 1992; Reys & Bestgen, 1981). I worked with general and special education students for several years in high school mathematics classrooms and my experience provided me much of the information I needed to create this learning tool. I know my students struggle with fractions, decimals, and percent, and I know they often work individually, rarely learning from anyone other than their teachers. Through my own informal instruction, I watched students improve their understanding of numbers through computational estimation activities and now I want to determine whether this growth correlates with higher scores on standardized assessments. Teachers and administrators at my school are currently working to develop mathematical learning experiences (i.e. project-based learning) that increase student engagement and raise achievement scores. While I feel this is a significant step forward for all students, particularly those who demonstrate difficulty with traditional teacher-led instruction, I recognize how challenging it is for teachers to change their practices. In conjunction with joining the effort to improve overall mathematics experiences for students, I believe we should design short, targeted lessons that activate specific skills and concepts in order to improve students' overall sense of number. If teachers observe engaged students who demonstrate improved foundational skills and conceptual understanding, perhaps they will build on this momentum and move toward a more exciting, student-centered instructional approach.

My decision to design a quantitative study was not easily reached. I worked for seven years with general and special education students in an inclusive setting trying to explain algebraic and geometric concepts despite the difficulty I saw in their ability to compute using fractions, decimals, and percent. How were they supposed to find

volumes or surface areas if they did not fully understand the concept of division or multiplying decimals? Their strategies were procedural and often irrational, and I questioned whether the algebra and geometry content was even worth explaining before tackling their poor sense of number. As I considered my research study, I initially thought a qualitative approach would be appropriate because it would allow me to consistently observe and interact with participants to determine any common themes (Fraenkel, Wallen, Hyun, 2012). I considered designing a study in grounded theory to collect data through one-on-one interviews, focus group interviews, and participant observations (Fraenkel, Wallen, Hyun, 2012). I utilized these data collection strategies for two summers as a graduate student working with high school students at risk of meeting graduation requirements in mathematics. I was familiar with these experiences and I thought further investigation into student number sense through a grounded theory design would help me design lessons to increase student achievement. This type of methodology allows the researcher to interact with participants in a natural setting and use multiple sources of data to develop concrete theories on learning (Creswell, 2009). After gathering information I would generate a theory that might help me professionally and pave the way for a future study. While this qualitative design would no doubt benefit my understanding of teaching and learning, I felt my experiences in classrooms and as a research assistant with struggling students provided me enough information to design an experiment targeting computational estimation, an element of number sense I believe is crucial to an advanced understanding of mathematics. I feel strongly that if teachers provide opportunities for students to engage in computational estimation, they would expand their problem-solving strategies and increase their achievement scores. If I did

not see an improvement in student mathematics performance following my intervention, perhaps an in-depth qualitative study would be warranted to determine if a different targeted concept or skill would be more effective. Of course there are many reasons why students may not demonstrate improved performance following my intervention, but a deeper look at my theory of computational estimation may be appropriate, and this deeper investigation would likely follow a grounded theory design.

Quasi-Experimental Design

To measure the impact of daily experiences in computational estimation, I designed a quasi-experimental study consisting of four intact high school mathematics classrooms. Specifically, I used a Static Pretest-Posttest Control Group Design with a secondary assessment (see Figure 7) to measure growth over time while controlling for differences in the abilities of groups using a pretest as a covariate (Fraenkel, Wallen, & Hyun, 2012).

				O – Pretest/Posttest X – Intervention C – Control A – Assessment
Treatment Group	O	X	O	A
Control Group	O	C	O	A

Figure 7. Static-Group Pretest-Posttest Design with Secondary Assessment. This figure demonstrates the design of this research study.

Although using intact classrooms raised the threat to internal validity because the assignments were not random (Fraenkel, Wallen, & Hyun, 2012; Creswell, 2009), there were no other practical options to administer the learning tool to large groups of students. This more convenient method, while less impactful, prompted me to use a pretest and control groups to understand whether students engaging in typical mathematics lessons made similar gains/losses on a posttest as well as a high-stakes standardized assessment. This design allowed me to control for student ability prior to the study and determine how treatment groups compared to their peers. Additionally, this method had the potential to directly impact student performance on a high-stakes assessment. While I considered the notion that students in treatment groups could perform worse, I deemed the potential negative impact of engaging in daily computational estimation activities on achievement as minor.

As a special educator interested in discovering ways to reach students with learning challenges, I am also interested in looking at the performance of students with disabilities to see if their achievement improves as a result of my intervention. Although the sample size of students with disabilities is small because I am using intact groups, I want to analyze their achievement and spark ideas for a future study targeting this population of students. As a result, I included a second independent variable (disability status) to determine any statistical patterns in performance. It will likely be difficult to convincingly attribute my findings to the overall population of students with disabilities given the small sample (Huck, 2012); however, there is valuable information that should be discussed based on posttests and the standardized mathematics assessment.

Research Design

Variables.

An independent variable represents one way comparison groups differ from one another prior to collecting data (Huck, 2012). The independent variables in this experiment consist of the learning intervention (treatment variable) and student disability status. The intervention was administered to two of the four groups with an intent to compare posttest results and scores on the MCAS mathematics assessment. I also wanted to see whether students with disabilities benefited from the focus on computational estimation as well as exposure to other students' problem-solving strategies. When analyzing the data I collected, I intended to compare results of students with disabilities in treatment and control groups as well as any differences within groups based on disability status. Because of sample size, I chose to only compare the pretest, posttest, and achievement scores of students with disabilities who participated in the intervention.

A dependent variable is a characteristic of the participants that a researcher is interested in analyzing, is not possessed to an equal degree by the participants, and is the target of data collection (Huck, 2012). In most studies (such as this one), the dependent variable is closely connected to the measuring instrument used to collect data. In this research study, the dependent variable is mathematics achievement based on a posttest as well as scores on questions embedded within a standardized assessment. The posttest only features questions that encourage the use of computational estimation, while the standardized assessment includes a collection of standardized questions of which only ten can be answered using computational estimation. This assessment is of interest to

determine whether students apply estimation strategies when they are not explicitly encouraged to on specific questions.

A covariate variable is a measured difference among participants that acts as a control (Huck, 2012). In most studies, participants differ in ways that may be difficult to know or explain. A covariate variable measures participants in an attempt to make groups more similar, thereby increasing the study's power and reducing the probability of Type II error (Huck, 2012). In this study, the covariate variable is a pretest consisting of ten questions targeting computational estimation. An analysis of covariance using pretest scores improved my ability to determine if posttest scores are results of the intervention or student ability prior to the study.

Extraneous variables are independent variables that are not controlled in a research study. They can potentially impact the dependent variable and should be discussed in a study for readers' consideration (Fraenkel, Wallen, & Hyun, 2012). There are many extraneous variables within classroom research that are not discussed in this study, but Figure 8 lists some of the more significant ones that may impact the outcome of this experiment.

Independent Variables	Dependent Variable	Covariate	Extraneous Variables
<ul style="list-style-type: none"> • Learning Intervention • Students with Disabilities 	<ul style="list-style-type: none"> • Mathematics Achievement 	<ul style="list-style-type: none"> • Pretest 	<ul style="list-style-type: none"> • Time of Day • Teacher Instruction • Student Participation

Figure 8. Variables of Interest. This figure lists examples of the variables used in this study.

Research questions.

Although research questions were listed in Chapter One, it is helpful to state them again in the Methods section to remind readers of the purpose of the study and the expectations of the researcher.

RQ1: Does a six-week collaborative learning activity featuring computational estimation improve high school students' abilities to answer posttest questions that target this particular skill?

RQ2: Does a six-week collaborative learning activity targeting computational estimation improve students' abilities to answer posttest questions that target this particular skill compared to students who engage in daily practice with sample test questions targeting multiple skill areas?

RQ3: Do students who engage in a six-week collaborative learning activity targeting computational estimation earn higher scores on questions embedded within a standardized mathematics assessment that target this particular skill compared to students who participate in daily practice with sample test questions targeting multiple skill areas?

RQ4: How does a six-week learning activity featuring computational estimation impact performance on posttest questions that target this particular skill for students with disabilities?

Selection of Participants

Population.

The target population of this research study is tenth grade students on track to earn a high school diploma in Massachusetts. Student achievement scores on the MCAS mathematics examination at the participating high school are commensurate with average scores throughout the state. Stating a target population is worthwhile to readers so they know the researcher's ideal choice of generalization; however, the target population is rarely the most realistic for several reasons (Fraenkel, Wallen, & Hun, 2012). For example, students encounter various forms of mathematics instruction in different school districts throughout elementary, middle, and high school which may impact achievement. Schools have different student/teacher ratios that can contribute to learning experiences. Some students face socioeconomic challenges or privileges within their communities that impact overall achievement. These are just a few of the differences that make a target population of tenth grade students in Massachusetts challenging. As a result, an accessible population is more reasonable and more likely to reflect sample characteristics and outcomes (Fraenkel, Wallen, & Hun, 2012). The accessible population in this research study is tenth graders in the participating high school.

The setting of this research study is a high school in southeastern Massachusetts consisting of nearly 1700 students (see Table 1). According to 2016 reports, 91% of students graduate within four years including 70% of students with disabilities. Information gathered from the 2014-15 cohort of students indicated 74% of students attended a college or university following graduation, including 57% of students with

disabilities. Of the 57% of students with disabilities, 62% attended community college in Massachusetts.

Table 1	
<i>2016-17 Student Demographics (1700 Students)</i>	
Ethnicity	Percentage
White	75
Hispanic	12
African American	6
Asian	4
Students with Disabilities	14

The principal of the participating high school approved this research study through a signed consent letter submitted to the Rhode Island College Institutional Review Board. The researcher also met with the head of the mathematics department to explain the intervention and the impact on the students.

Sampling.

Prior to the start of the study, students and their parents had to complete consent forms approved by the Rhode Island College Institutional Review Board. Students were told that they did not have to participate in the study and they could drop out at any time if they wished. The 75 students who chose to participate in this study were tenth graders

enrolled in four Geometry classes at a high school ranging from grades 9 - 12. The classes were chosen based on convenience sampling as they were already intact at this high school and were led by teachers who taught two sections of Geometry. This nonrandom sampling method was not ideal because it may not be representative of a population; however, it was the only practical option for my research design (Fraenkel, Wallen, & Hun, 2012). Overall, the sample closely matched demographics of the accessible population of students in the high school. I also made sure to select two teachers who each led an intervention and control group classroom allowing me to replicate the study and conduct it with more validity (Fraenkel, Wallen, & Hun, 2012).

The experiment was originally intended to target 87 students, but 16 students either missed a substantial number of school days during the intervention, did not complete consent forms, or left school prior to the study's completion. Of the 75 students who participated, 14 received special education services for a variety of disability-related needs. The original target was 16 special education students; however, two of these students did not participate.

Table 2		
<i>Research Participants</i>		
Group	Total Students	Students with Disabilities
Intervention I	16	2
Control I	20	4
Intervention II	18	6
Control II	21	2

Instrumentation

Pre- and posttests were composed of questions from prior MCAS Mathematics examinations administered from 2013 to 2016 (see Appendix A). They were approved by a group of mathematics educators at the participating school to ensure that the selected questions had a high potential for using computational estimation strategies and number sense. Both the pretest and posttest contained the same questions, but students were not provided answers to any of the questions following the pretest and they were not allowed to keep copies of the questions. Students in both the treatment and control groups participated in the pretest to control any variability in performance (Fraenkel, Wallen, & Hun, 2012).

The 2017 MCAS examination was used to measure student performance following the intervention. The MCAS assessment measures student achievement in

tenth grade in the areas listed in Table 4. The test consists of multiple-choice, short answer, and open response questions, and is administered in two sessions to students each spring. Potential scores on this assessment range from 200 to 280 and a qualifying score to be eligible for a high school diploma is 220. Technical reports indicate that the MCAS assessment has a reliability coefficient of .92 which suggests it is a reliable test (Massachusetts Department of Elementary & Secondary Education, 2013). Multiple investigations indicate a comprehensive presentation of validity evidence associated with the MCAS program including sections on test design and development, test administration, scoring, scaling and equating, item analysis, reliability, and score reporting (Massachusetts Department of Elementary & Secondary Education, 2013).

The learning intervention I created is composed of relevant mathematical questions targeting computational estimation. A group of teachers at the participating high school reviewed the questions and solutions to make sure computational estimation was an appropriate strategy. The relevancy of questions was developed over time using feedback from students in a variety of classrooms. My high school and college mathematics students provided input regarding whether they could relate the situations presented in the problems to their own lives. After revision and trial, I settled on 24 questions I was confident would engage students and challenge them to construct reasonable estimates (see Appendix B). The intervention was also designed as an individual and group competition to increase student engagement and effort. As I will discuss in the next section, the intervention is a game that students play at the beginning of each class involving computational estimation and collaboration.

Procedures

Prior to the start of the experiment, both teachers attended two half-hour training sessions to learn how the intervention should be administered. Teachers also received copies of the intervention to review prior to beginning the experiment. The first day, teachers administered a pretest to all four classes composed of prior MCAS examination questions that promoted the use of computational estimation. Students had 20 minutes to complete nine multiple choice questions and one short answer question. Students with accommodations for extra time were offered the opportunity, but no one utilized this option. Upon students completing the pretests, teachers collected them without discussing the answers or any strategies for solving any of the problems. On day two, teachers explained the guidelines of the learning intervention to the treatment groups and provided an opportunity to practice one. This gave students a chance to familiarize themselves with procedures and the rules of the game. Students in control groups began their daily warm-ups of sample MCAS questions that is typical practice in Geometry classes at this participating high school as students prepare for the upcoming MCAS assessment.

As students in the treatment groups arrived to class on day three of the experiment, they knew to take their seats and clear their desks in preparation for the start of the game. Teachers distributed green pieces of paper to each student and asked that they write their names on the lines provided. Next, students were presented a mathematics question visually and orally and given one minute to provide an estimate. A one-minute countdown clock was visible to all students at the front of the classroom. Students were not allowed to use a writing utensil or calculator to solve the problem and

they were not permitted to talk with any other students before providing an answer. Teachers walked around the room during the minute to make sure students were following directions. After one minute elapsed, students used pencils provided by the teachers to write their estimations on the green pieces of paper. Students immediately submitted their written responses to their teachers.

Once all answers were submitted, students moved into designated groups of four or five students. These groups were posted digitally and they changed each week so students were exposed to different peers and possibly different estimation strategies. Once students were grouped, teachers distributed one orange piece of paper to each group labeled with a group number. Teachers then presented the same mathematical question visually and orally and allowed students three minutes to discuss effective ways to solve the problem. A three-minute countdown clock was visible to all students at the front of the classroom. At the conclusion of three minutes, groups agreed on one final estimation to submit to their teachers. This collaborative opportunity allowed students to discuss and refine their methods. Once the groups agreed on one estimation, they submitted their responses to their teachers.

Following submission of group answers, teachers opened discussion on how students reached their results. Students lead this dialogue with prompting, but teachers were encouraged to discuss estimating strategies if students were inaccurate or did not participate. After all groups shared their strategies and at the teachers' discretion, a range of estimations were presented digitally with corresponding point values for more reasonable results (see Figure 1). Teachers had the option of discussing reasonable responses further or answering student questions, but the revealed scores typically ended

the activity. In total, the intervention lasted ten minutes each day. Teachers presented these challenges four days each week for six weeks (one day per week was open for assessments or holidays) for a total of 24 lessons. The first three days of each week, students had an opportunity to earn up to ten points (five points for individual answer and five points for group answer). The last day of each week, the point values were doubled because of more complex problems, so students had an opportunity to earn up to 20 points. Figure 9 shows an example of points awarded for one of the intervention problems. All students scored a minimum of two points each day no matter how far their answers were from the acceptable ranges. At the beginning of each week, teachers revealed the point standings of teams as well as the top five individuals.

Figure 9. Learning Intervention Point System. This figure describes the points students could earn on one particular problem.

Upon completion of the intervention, students in the treatment and control groups were administered a posttest composed of the same MCAS examination questions provided during the pretest. Students again had 20 minutes to complete the posttest and were not allowed to use a calculator. Students with disabilities were offered extra time if this accommodation was part of their education plans, but no students utilized this option. After students completed the posttest, they submitted them to their teachers marking the end of the experiment in the classroom. The following week, students participated in a mandatory statewide MCAS mathematics examination marking the end of the research study.

Data collection.

This experiment took place in four college preparatory Geometry classes that were scheduled to meet every day for 65 minutes throughout the school year. These courses are designed for students at similar ability levels who are on a diploma track and require instruction at a moderate pace. The six week intervention began March 27, 2017 and concluded May 17, 2017 with administration of the MCAS mathematics assessment. A brief schedule of events is outlined in Table 3.

Table 3	
<i>Data Collection Schedule</i>	
<u>Date</u>	<u>Task</u>
March 27	Pretest
March 28	Start of Six Week Intervention
May 11	Conclusion of Six Week Intervention
May 12	Posttest
May 16 & 17	MCAS Mathematics Assessment
August 15	Release of MCAS Mathematics Scores

All four classes completed the same pretest on March 27th. Students who were not in class this day were given an opportunity to complete the pretest on March 28th at the end of their class period. Any students who missed the first two days were excluded from the study. Students wrote their names on pretests and handed them in to their teachers. Teachers corrected the pretests and submitted scores to the researcher coded with corresponding numbers that were developed prior to the start of the study to maintain anonymity. Scores ranging from zero to ten were entered into a Microsoft Excel document for each student.

Data Analysis

Chapter Four will include a detailed description of data analysis procedures and findings, but the following section will briefly discuss the methodological approach. The pretest/posttest design established more control in this study given the nonrandom samples and extraneous variables that may have impacted results. Pretest scores served as a covariate variable that helped make the groups more closely aligned in terms of achievement prior to the study. Through an analysis of covariance (ANCOVA), I was able to determine whether significant differences were evident between the means of the samples (Fraenkel, Wallen, & Hun, 2012; Creswell, 2009). In addition to using ANCOVA to compare treatment and control groups, I also ran *t*-tests for correlated means to determine whether the same groups made significant gains on the posttest (Fraenkel, Wallen, & Hun, 2012). After analyzing data using all pre- and posttest questions featuring computational estimation, I broke down the questions into categories to determine whether students performed significantly better on certain skills or concepts. I was able to gather data on questions that encouraged the use of rounding as well as percent and I included the results in my findings.

Each participant in this study also completed the tenth grade MCAS examination following the posttest as part of their graduation requirement in Massachusetts. Upon receiving the scores, I compared the means of ten questions embedded in the assessment that could have been solved using computational estimation (see Appendix C). Through analyses of covariance using pretest scores as the covariate variable and paired-sample *t*-tests for students in the treatment groups, I was able to draw conclusions regarding the impact of the intervention. I also compared student performance on six questions that

featured rounding to supplement the findings from the pre-and posttests. Although this analysis lacked power because of the small number of problems, the results are relevant to further investigation into mathematics achievement.

A particular interest in this study centered on the achievement of students with disabilities. The sample of students with disabilities was relatively small so I used descriptive statistics to show how students performed following the intervention. The findings lacked statistical significance, perhaps because of the sample size; however, they provided more information to researchers and teachers for future studies.

Limitations

Validity.

Validity is defined as "the appropriateness, correctness, meaningfulness, and usefulness of the specific inferences researchers make based on the data they collect" (Fraenkel, Wallen, & Hyun, 2012). Most researchers will discuss two forms of validity that can impact research studies - internal and external (Fraenkel, Wallen, & Hyun, 2012). Internal validity refers to the strength of the relationship between two or more variables, while external validity concerns the generalizability of the findings (Fraenkel, Wallen, & Hyun, 2012). Both of these threats to a meaningful study need to be considered prior to the research design to maximize impact and credibility. This section will begin with the internal validity of this study.

A quasi-experimental research design almost always impacts the internal validity of a study because it is composed of nonrandom samples. This design can cause issues with subject characteristics, location, and even data collector characteristics (Fraenkel,

Wallen, & Hyun, 2012). Subject characteristics, or the ways that participants in each group may differ, can increase the chance of factors other than the independent variable contributing to results. For example, in this study, four classrooms were chosen to participate as intervention or control groups. While the classes are all designed for students of the same ability level according to the course descriptions, there are many ways the participants can differ such as ability level, prior experience in school, or attitude, just to name a few. Convenience sampling can also cause problems with internal validity in regards to the location of the classroom, or the time of day the class meets. In this study, one class met at 7:15 each morning while the other classes were directly before and after lunch. It is difficult to determine whether students were less engaged because they were barely awake or hungry. Data collector characteristics are also important to consider in convenience sampling because teachers may have different approaches to instruction, classroom management, and student engagement. In this study, both teachers led a control and intervention group in an attempt to counter this validity threat, but their characteristics are crucial when analyzing the results.

One of the most critical factors of internal validity is instrumentation. In order to limit some of the threats to internal validity related to instrumentation in this study, I employed standardized instruments of measurement (Fraenkel, Wallen, & Hyun, 2012). The pretest, posttest, and standardized assessment included questions from comprehensive assessments developed by leading mathematics teachers and administrators in Massachusetts. Although the pretest and posttest were only comprised of ten questions that targeted computational estimation, the format and appearance were identical to the MCAS mathematics assessment. Not only did this validate the findings in

my study, it exposed students to questions they needed to answer in order to earn a high school diploma.

One other factor that contributed to the internal validity was the loss of subjects during the study or, mortality. Several students, including some with disabilities that were crucial to an already small sample of students, were unable to participate for a variety of reasons. As a result, some participants that were originally intended to provide valuable data were excluded. Because my study targets students with learning challenges, I was unable to include a number of these particular students because they did not attend class regularly, never submitted consent forms, or were removed from their classes. Mortality is difficult to control for in fluid classrooms but can be significant in generalizability and bias (Fraenkel, Wallen, & Hyun, 2012). Not only were fewer students included to make inferences regarding the population, some of the more vulnerable learners' experiences were not included. This should be considered when discussing the findings of this research study.

A quasi-experimental research design also impacts the external validity of a study because nonrandom samples do not always represent the intended population (Fraenkel, Wallen, & Hun, 2012; Creswell, 2009). This method makes the generalizability of an intervention less powerful (Fraenkel, Wallen, & Hyun, 2012; Creswell, 2009; Huck, 2012). Because I am targeting a problem at my school that I have identified with mathematics teachers and administration, convenience sampling through a quasi-experimental design was the most appropriate approach to this study. The groups I used already existed because they were classes created at the beginning of the school year. Although they were assembled based on past performance and recommendations from

previous teachers to align ability levels as much as possible, this design impacted subject characteristics and, consequently, the power of this study. I was aware of these concerns when designing this experiment so I employed a level of control to limit threats to external validity and generalize my findings to a population of tenth grade students at comparable high schools in Massachusetts. This level of control consisted of a pretest to enable an analysis of covariance (ANCOVA) that controlled for student ability (Fraenkel, Wallen, & Hyun, 2012; Creswell, 2009). This element increased the power of this study through a leveling of groups based on pretest data. When using nonrandom samples, a covariate variable is a powerful way to control for threats to external validity.

Another method to counter the external validity threats to nonrandom sampling is replication. This tactic involves repeating the study with different groups of subjects in different situations (Fraenkel, Wallen, & Hyun, 2012). In this study, replication was utilized when a second teacher was asked to lead an intervention and control class. Although including two more convenience samples does not replace the power of random samples, it provides more confidence if the findings are similar in both teachers' classes. Nevertheless, more replication needs to occur before making any definitive conclusions about the effectiveness of the intervention. This is the intent of this researcher as well as the teachers and administrators of the participating high school.

Reliability.

Reliability refers to the consistency of an instrument from one administration to the next. For example, if a student takes an assessment twice, the test would be deemed reliable if the student earned similar scores each time. This is different from validity in that it only concerns consistency. If students earn similar scores on a mathematics

achievement test twice but these scores do not accurately predict their grades in a mathematics course, the test would be considered reliable but not valid. As for the reliability of my study, I used standardized questions from prior MCAS examinations that were constructed by teams of mathematics teachers, administrators, and researchers across the state of Massachusetts. Previous technical reports indicate the MCAS Mathematics assessments have a reliability coefficient of .92 (Massachusetts Department of Elementary and Secondary Education, 2013), making them consistent measures of mathematics achievement. I administered ten MCAS questions from previous assessments in a test/retest format over a six-week period of time. Although these questions were not part of the same MCAS test and did not encompass all mathematical concepts typically assessed, they were standardized questions approved by mathematics teachers at the participating school to target computational estimation.

Summary of Methods

This research study will utilize a quasi-experimental design to determine whether 10th graders in a high school in Massachusetts participating in an intervention targeting computational estimation will significantly improve their mathematics achievement. Chapter 4 will reveal the findings of this study including anecdotal data from teachers regarding their perceptions of student performance and attitude. Chapter 5 will include a discussion of the findings and limitations of the study as well as suggestions for further research.

Chapter 4

Findings

This chapter will begin with a brief overview of the problem and research questions followed by a description of the data analysis process. Each finding will be stated and then supported with detailed statistics presented in tables to clearly and efficiently report student performance. Although this is a quantitative study, anecdotal data collected from meetings with teachers following the study will be reported to provide a more detailed picture of the participants' experiences. The final chapter will provide a summary of the study, conclusions and interpretations of the findings, limitations of the study, and suggestions for future research.

Problem Statement and Research Questions

Research indicates that many high school students who pursue post-secondary education are performing poorly on placement tests. As a result, these students are required to take remedial courses that cost them money but are often non-credit bearing (NCES, 2015; Bailey, Jeong, & Cho, 2010; Barry & Dannenberg, 2016). Despite efforts to revise standards with an emphasis on conceptual understanding, students are still displaying challenges with foundational mathematical skills. It is crucial for teachers to design learning opportunities for students that target grade-level content but continue to develop their number sense. One feature of number sense that leading mathematics researchers and educators feel is critical is computational estimation (Beishuizen, van Putten & van Mulken, 1997; National Research Council, 2001; Fennell, 2008; Cochran & Dugger, M.H., 2013). The following research questions helped guide this investigation to determine whether a specific intervention had an impact on mathematics achievement:

RQ1: Does a six-week collaborative learning activity featuring computational estimation improve high school students' abilities to answer posttest questions that target this particular skill?

H1: A six-week collaborative learning activity featuring computational estimation improves high school students' abilities to answer posttest questions that target this particular skill.

H0: A six-week collaborative learning activity featuring computational estimation does not improve high school students' abilities to answer posttest questions that target this particular skill.

RQ2: Does a six-week collaborative learning activity targeting computational estimation improve students' abilities to answer posttest questions that target this particular skill compared to students who engage in daily practice with sample standardized test questions targeting multiple skill areas?

H2: A six-week collaborative learning activity targeting computational estimation improves students' abilities to answer posttest questions that target this particular skill compared to students who engage in daily practice with sample standardized test questions targeting multiple skill areas.

H0: A six-week collaborative learning activity targeting computational estimation does not improve students' abilities to answer posttest questions that target this particular skill compared to students who engage in daily practice with sample standardized test questions targeting multiple skill areas.

RQ3: Do students who engage in a six-week collaborative learning activity targeting computational estimation earn higher scores on questions embedded within a standardized mathematics assessment that target this particular skill compared to students who participate in daily practice with sample standardized test questions targeting multiple skill areas?

H3: Students who engage in a six-week collaborative learning activity targeting computational estimation will earn higher scores on questions embedded within a standardized assessment that target this particular skill compared to students who participate in daily practice with sample standardized test questions targeting multiple areas.

H0: Students who engage in a six-week collaborative learning activity targeting computational estimation will not earn higher scores on questions embedded within a standardized assessment that target this particular skill compared to students who participate in daily practice with sample standardized test questions targeting multiple areas.

RQ4: How does a six-week learning activity featuring computational estimation impact performance on posttest questions that target this particular skill for students with disabilities?

H4: Students with disabilities who engage in a six-week learning activity featuring computational estimation will improve their performance on posttest questions that target this particular skill.

H0: Students with disabilities who engage in a six-week learning activity featuring computational estimation will not improve their performance on posttest questions that target this particular skill.

Overview of Data Analysis

All analyses of quantitative data were conducted using Microsoft Excel and IBM SPSS. Student identification numbers were grouped with pretest, posttest, and selected MCAS question results and exported to SPSS for analysis. All of the research questions involved comparing means; therefore, descriptive statistics, paired sample *t*-tests, and ANCOVA were used to analyze data. It was determined that too few students with disabilities ($n = 8$) took part in the intervention so descriptive statistics will be included to report findings and suggest whether future investigation is warranted.

In order to properly conduct analyses of covariance using a pretest as a covariate, certain criterion needed to be met. The first condition states that the covariate was measured before the start of the experiment. This criteria was met because all students took the pretest prior to the start of the intervention. The second condition states that the covariate was measured reliably. This criteria was met because pretest questions were selected from prior MCAS examinations and mathematics teachers approved the content of the questions to match the skills targeted in the intervention. The third condition states that there must be linearity among the dependent variables and the covariate. An analysis using a scatterplot in SPSS indicated that there is indeed a linear relationship. The fourth and final condition states that there can be no relationship between the covariate and the dependent variable. This is known as homogeneity of regression and is critical to the validity of this study. An analysis in SPSS indicated there is not a significant interaction

between the covariate and the dependent variables, thus meeting the criteria for an analysis of covariance.

Major Findings

- Finding 1: Students who participated in a six-week collaborative learning activity targeting computational estimation did not demonstrate statistically significant improvement in their abilities to answer posttest questions that targeted this particular skill; therefore, we failed to reject the null hypothesis. There is evidence of improved performance when considering students without disabilities from one class that should be reported, including an increase in overall posttest scores as well as posttest questions that target rounding and percent. These results are not statistically significant; however, they will be reported to provide evidence for a discussion in Chapter 5.
- Finding 2: Students who engaged in a six-week collaborative learning activity featuring computational estimation did not earn statistically significant higher scores on posttest questions that target this particular skill than students who engaged in daily practice with sample test questions targeting multiple areas; therefore, we failed to reject the null hypothesis. There is no significant evidence to report regarding students without disabilities or categories of test items (e.g. rounding, percent).
- Finding 3: Students who engaged in a six-week collaborative learning activity targeting computational estimation did not earn higher scores on questions embedded within a standardized assessment that target this particular skill compared to students who participated in daily practice with sample test questions

targeting multiple areas; therefore, we failed to reject the null hypothesis. There is statistically significant evidence, however, that students without disabilities in the treatment group outperformed students without disabilities in the control group on test items that featured rounding. This finding contributes to the pattern of disability status and improved performance that will be discussed in Chapter 5.

- Finding 4: Students with disabilities who engaged in a six-week learning activity featuring computational estimation did not improve their performance on test questions that target this particular skill; therefore, we failed to reject the null hypothesis. In fact, there is evidence that students with disabilities may have had more difficulty than their peers trying to implement computational estimation into their approaches to problem-solving. Given the small sample size, there is no statistically significant evidence of poorer performance; however, descriptive statistics indicate further research is critical to determine the impact this intervention had on students with disabilities.

Summary of Findings

Finding 1 – single group performance.

Finding 1 helps to answer RQ 1 which seeks to determine whether students participating in a six-week intervention targeting skills in computational estimation improved their performance on posttest questions that targeted this particular skill. Students ($n = 34$) completed a pretest prior to the intervention and an identical posttest immediately following the six-week program. Table 4 describes the means for both tests which indicate a slight improvement from pretest to posttest.

Table 4

Descriptive Statistics of All Intervention Students

Test	Mean	Std. Deviation	N
Pretest	5.24	2.523	34
Posttest	5.32	2.531	34

Although sample means are nearly identical, a paired sample t -test (Table 5) was conducted to confirm that the means were not significantly different. Not surprisingly, the results indicate that there exists no statistically significant difference between pretest and posttest scores ($p = .823$).

Table 5

Paired Samples t - test of all Intervention Students

Pair	Mean Difference	T	Df	Sig. (2-tailed)
Pretest-Posttest	-.088	-.225	33	.823

Given the influence that teaching has on student performance, results from two classrooms with two different teachers were also collected. In addition, test items featuring percent and rounding are included to provide more specific data and paint a clearer picture of student performance. Several skills and concepts embedded in number sense were considered for analysis in test items, but rounding and finding percentages were the most common skills that emerged and could be analyzed meaningfully from the questions. Table 6 includes statistics from students enrolled in class with Teacher 1.

Although scores did not increase with statistical significance, total posttest scores as well as responses to selected questions involving percent and rounding indicate improved performance.

Table 6

Paired Sample t-tests for Teacher 1 Students

	Test	Mean	Standard Deviation	N	T	Sig. (2-tailed)
Pair 1	Pretest	4.81	2.588	16	-1.459	.165
	Posttest	5.63	2.187	16		
Pair 2	Pretest	2.00	1.751	16	-1.142	.271
	Percent					
	Posttest	2.50	1.211	16		
	Percent					
Pair 3	Pretest Round	3.13	1.360	16	-1.379	.188
	Posttest	3.69	1.493	16		
	Round					

The data is even more interesting when students with disabilities are removed from the results. Table 7 presents the same paired data from Teacher 1 for students without disabilities. Although the mean differences are not statistically significant, there exists a pattern of information worth discussing in Chapter 5 and perhaps targeting in future studies.

Table 7

Paired Sample t-tests for Teacher 1 Students Without Disabilities

	Test	Mean	Standard Deviation	N	t	Sig. (2-tailed)
Pair 1	Pretest	4.64	2.468	14	-1.883	.082
	Posttest	5.71	2.301	14		
Pair 2	Pretest	2.00	1.710	14	-1.385	.189
	Percent					
	Posttest	2.64	1.216	14		
Pair 3	Percent				-2.121	.054
	Pretest Round	3.00	1.359	14		
	Posttest	3.86	1.512	14		
	Round					

Table 8 includes paired data for all students enrolled in class with Teacher 2.

Although there was no statistical significance of any of the scores listed below, it is evident that students did not perform as well on the posttest following the intervention.

Table 8

Paired Sample t-tests for Teacher 2 Intervention Students

	Test	Mean	Standard Deviation	N	t	Sig. (2-tailed)
Pair 1	Pretest	5.61	2.477	18	1.070	.299
	Posttest	5.06	2.838	18		
Pair 2	Pretest	2.50	1.581	18	.736	.472
	Percent					
	Posttest	2.22	1.517	18		
Pair 3	Percent				1.279	.218
	Pretest Round	3.72	1.602	18		
	Posttest	3.33	1.879	18		
	Round					

The pretest and posttest scores in all three categories are more similar when only comparing students without disabilities (see Table 9). Although the data does not suggest anything statistically significant, a pattern regarding disability status and performance is evident in the results. This pattern will emerge again later in this chapter and will be discussed in Chapter 5.

Table 9

Paired Sample t-tests for Teacher 2 Students Without Disabilities

	Test	Mean	Standard Deviation	N	t	Sig. (2-tailed)
Pair 1	Pretest	5.69	2.780	13	.137	.893
	Posttest	5.62	3.070	13		
Pair 2	Pretest	2.54	1.664	13	-.185	.856
	Percent					
	Posttest	2.62	1.325	13		
Pair 3	Percent				.433	.673
	Pretest Round	3.77	1.833	13		
	Posttest	3.62	1.981	13		
	Round					

Finding 2 – intervention and control group performance.

Finding 2 helps to answer RQ 2 which seeks to determine whether intervention groups participating in a six-week learning activity targeting computational estimation outperformed control groups engaged in a six-week program featuring practice with sample test questions targeting multiple skills. To strengthen the validity of this experiment, control and intervention groups with the same teacher were paired to account for teacher impact on performance. The following results are separated into two parts to display findings from Teacher 1 and Teacher 2.

Table 10

Descriptive Statistics for Teacher 1 - Posttest

Group	Mean	Std. Deviation	N
Control	5.50	1.762	20
Intervention	5.63	2.187	16

Table 10 provides descriptive statistics of the posttest for the control group and the intervention group enrolled in classes with Teacher 1. Although the mean scores are similar, it should be noted that a pretest was administered to control for any variability in performance prior to the experiment; therefore, ANCOVA was an appropriate method to determine whether statistically significant differences were present. After all criteria were met for integrating a covariate into the analysis, ANCOVA was used to determine how much of an impact the intervention had on posttest scores, and whether that impact was significant. Table 11 displays data computed in SPSS that identifies factors relevant to the two groups. The line labeled “Intervention” informs us that the significance factor ($p = .458$) is not lower than .05 indicating that the intervention is not a significant predictor of posttest scores, even when controlling for the pretest. The pretest, which is the covariate in this analysis, is significant ($p = .001$) which means that students’ ability prior to the intervention explained more than 28% of the variance in scores. This type of impact indicates that a pretest was an important factor to control for in order to more closely align the groups.

Table 11

One-way Analysis of Covariance of Teacher 1 Posttest Scores

Source	df	SS	MS	F	p	R Squared
Corrected Model	2	35.883 ^a	17.941	6.646	.004	.287
Pretest	1	35.848	35.848	13.279	.001	.287
Intervention	1	1.525	1.525	.565	.458	.017

a. R Squared = .287 (Adjusted R Squared = .244)

Table 12 identifies descriptive statistics of control and intervention groups led by Teacher 2. Again, the means are similar but ANCOVA will inform us whether they are significantly different when factoring in pretest scores.

Table 12

Teacher 2 Descriptive Statistics - Posttest

Group	Mean	Std. Deviation	N
Control	4.86	2.220	21
Intervention	5.06	2.838	18

Table 13 indicates that the impact of the intervention with this teacher ($p = .517$) is not lower than .05; therefore, there is not a significant difference between the posttest scores of each group, even when controlling for pretest results.

Table 13

One-way Analysis of Covariance of Teacher 2 Posttest Scores

Source	df	SS	MS	F	P	R Squared
Corrected Model	2	75.727 ^a	37.864	8.510	.001	.321
Pretest	1	75.346	75.346	16.935	.000	.320
Intervention	1	1.909	1.909	.429	.517	.012

a. R Squared = .321 (Adjusted R Squared = .283)

Finding 3 – group comparison of MCAS computational estimation.

Finding 3 helps to answer RQ3 regarding computational estimation questions embedded in the administered MCAS examination. Student performance on these particular questions within an MCAS examination are most appropriate to measure because it would be revealing to see if intervention students applied computational estimation strategies during a high-stakes assessment. Two mathematics teachers and the researcher isolated a total of ten questions from the MCAS examination that could be solved using computational estimation. It should be noted that these ten questions do not completely align with the questions from the pre- and posttests because students were permitted to use a calculator on six of the ten questions according to MCAS guidelines. Students were not allowed to use a calculator on the pre- and posttests. Once again, means were compared using ANCOVA to determine whether control and intervention groups from two teachers were significantly different.

Table 14 lists descriptive statistics for control and intervention groups from Teacher 1. The mean scores are similar and notably higher than the means from the pretests and posttests but again, a calculator was permitted on six of these problems.

Table 14

Teacher 1 Descriptive Statistics for CE Questions on MCAS

Group	Mean	Std. Deviation	N
Control	6.60	1.957	20
Intervention	7.00	1.826	16

Using the pretest as a covariate, groups were compared through ANCOVA to determine whether significant differences exist on these ten particular items. According to the “Intervention” row of data (Table 15), the treatment did not significantly impact scores on these test items ($p = .876$).

Table 15

Teacher 1 Analysis of Covariance for CE Questions on MCAS

Source	df	SS	MS	F	P	R Squared
Corrected Model	2	27.989 ^a	13.995	4.799	.015	.225
Pretest	1	26.567	26.567	9.110	.005	.216
Intervention	1	.072	.072	.025	.876	.001

a. R Squared = .225

Descriptive statistics for intervention and control groups with Teacher 2 (Table 16) show a difference in means, so ANCOVA was necessary to determine whether this difference is statistically significant. Table 17 shows that the intervention ($p = .309$) was not a statistically significant factor in performance on these test items.

Table 16

Teacher 2 Descriptive Statistics for CE Questions on MCAS

Group	Mean	Std. Deviation	N
Control	6.24	1.700	21
Intervention	7.17	1.917	18

Table 17

Teacher 2 Analysis of Covariance for CE Questions on MCAS

Source	Df	SS	MS	F	p	R Squared
Corrected Model	2	32.800 ^a	16.400	6.159	.005	.255
Pretest	1	24.443	24.443	9.179	.005	.203
Intervention	1	2.839	2.839	1.066	.309	.029

a. R Squared = .255 (Adjusted R Squared = .214)

When the MCAS test items were broken down into categories of rounding and percent, one statistically significant finding emerged that contributes to a pattern forming throughout these results. Students without disabilities in the intervention group outperformed students without disabilities in the control group when using scores from pretest rounding problems as a covariate (see Table 19). Although the power of this

finding is not strong because only three questions from the MCAS examination were considered, the means are so different that the category of rounding should be investigated further to determine how much of an impact this intervention has on that particular skill. This finding also relates to previous observations concerning students without disabilities. This pattern will be discussed further in Chapter 5.

Table 18

Total Students Without Disabilities for Rounding Questions on MCAS

Group	Mean	Std. Deviation	N
Control	1.85	.881	26
Intervention	1.34	.838	35

Table 19

Analysis of Covariance for Students Without Disabilities on MCAS Rounding

Source	Df	SS	MS	F	P	R Squared	Observed Power
Corrected Model	2	15.846	7.923	14.727	.000	.337	.999
Pretest Rounding	1	12.067	12.067	22.429	.000	.279	.996
Intervention	1	2.333	2.333	4.337	.042	.070	.535

Finding 4 – students with disabilities and mathematics achievement.

A limited number of students with disabilities were able to participate making comparative group results less powerful; however, there is relevant information to report

and consider regarding the students with disabilities who participated in the intervention.

The following section will outline pretest, posttest, and MCAS performance for students with disabilities to determine significant differences and inspire further research.

Discussion of these results will take place in Chapter 5.

Table 20

Descriptive Statistics of Students With Disabilities

Test	Mean	Std. Deviation	N
Pretest	4.75	1.982	8
Posttest	3.63	1.923	8

Table 20 displays student means with a total of eight participants included among both intervention groups ($n = 8$). Students with disabilities earned lower scores on the posttest than the pretest indicating that the intervention may have had a negative impact on their performance; however, the small sample size decreased the power of the comparison resulting in a paired-samples analysis that was not statistically significant ($p = .094$).

Table 21 describes mean statistics for the control and intervention groups. Although the intervention group scored higher on the posttest, the potential to find statistically significant differences was lowered because of the small sample size. As anticipated, an analysis of covariance indicated that the scores are not statistically significant ($p = .518$).

Table 21

Descriptive Statistics for All Students with Disabilities - Posttest

Group	Mean	Std. Deviation	N
Control	3.17	1.722	6
Intervention	3.63	1.923	8

Finally, descriptive statistics for MCAS questions featuring computational estimation are listed in Table 22. An analysis of covariance indicates that the intervention is not a significant predictor of performance ($p = .940$).

Table 22

Descriptive Statistics for Students With Disabilities – MCAS CE

Group	Mean	Std. Deviation	N
Control	5.50	1.643	6
Intervention	6.50	1.773	8

Strength in Numbers Game Data

The intervention in this study challenged students to make reasonable estimates independently and then collaboratively in small groups. Students earned point values if their estimates fell within targeted ranges, and these point values were tallied over the six-week intervention to determine individual and group winners. One interesting outcome regarding the point values was the drastic difference in the average points students earned on each question independently compared to the points they earned in groups. Independently, students averaged 2.69 points per question, which falls just below

the three points awarded to students who provided a reasonable estimate.

Collaboratively, students averaged 4.32 points per question, which approaches the five point value for estimates that are nearly accurate. Readers should note that one question per week involved applying multiple problem-solving strategies so the point values were doubled. Also, students who were not as confident tended to rely on higher performing students for group answers. But teachers reported that the discussions they heard during the collaborative part of the game were productive and students talked about more efficient ways to solve problems in order to earn the maximum number of points.

Qualitative Data

Qualitative data collected from participating teachers following the study suggested that students in treatment classes enjoyed the daily computational estimation challenges. The competitive individual and group format, time limits on questions, and higher point values for more reasonable responses kept students engaged and eager to view the standings each week. One teacher reported that a student asked if any other classes were participating in the game so she could determine how well her group was doing beyond her own classroom. The other teacher explained that students walked into class each Monday morning asking if the standings were ready and if any changes had occurred over the previous week. Both teachers confirmed that regardless of the improvement in student achievement, this intervention was a worthy experience for them and for their students.

While feedback from the participating teachers was positive, there was concern regarding the time it took for students to employ effective estimation strategies. The first two weeks of the study were challenging for students. According to Teacher 1 at the end

of week one, "My students have not bought into this yet. The majority tried to find 18% of 67 in their heads rather than 20% of 70." Teacher 2 reported that students preferred to add decimals in their heads rather than round to the nearest whole numbers. Both teachers felt that most students reverted to traditional problem-solving strategies in their heads but had difficulty completing the processes without any tools. They did not try to make the problems easier for themselves because explicit procedures was what they knew. But as week three arrived, students began discussing ways to round and break apart numbers throughout the problem-solving process and simpler strategies evolved. Teacher 1 indicated that one student solved the problem in Figure 11 by rounding up to 210, dividing by 7 to get 30, and then lowered the estimate to 27. Teacher 2 described students solving the problem in Figure 12 by breaking 12.15 into 10 and 2 and multiplying each value by 16 before adding them together to settle on 192. Students demonstrated that they had an understanding of how to work with numbers – they just did not utilize it until they were forced to find answers in their heads.

Figure 10 – Question #11. This is a question from the Strength in Numbers intervention highlighted in this study.

You decide to look for a job after school so you can pay your phone bill and put gas in your car. A department store at the mall offers you 16 hours per week for a wage of \$12.15/hour.

If you are paid every two weeks, about how much money will you earn in your first paycheck before taxes?
(Whole Number)

1:00
Stop

3:00

Figure 11 – Question #10. This is a question from the Strength in Numbers intervention highlighted in this study.

Summary of Results

There is evidence in this study indicating that the six-week learning activity featuring computational estimation through this particular design is not a significant predictor of mathematics achievement for all students as well as students with disabilities. Paired-sample t-tests and analyses of covariance provided the evidence necessary to reach these conclusions. Despite a lack of statistical evidence to suggest the intervention was effective, there are conclusions and ideas for further research that could impact data analysis. These reactions will be discussed in Chapter 5.

Chapter 5

Discussion

The purpose of this study was to determine whether a six-week computational estimation intervention increases mathematics achievement for general and special education students in high school. Computational estimation is a skill featured within the broader concept of number sense and is considered vital to an improved understanding of how numbers are applied to a variety of situations (Reys & Bestgen, 1981; Paulos, 1988; Fennell, 2008). Prominent researchers have suggested that partitioning the concept of number sense into more manageable parts is an effective approach to building student understanding (Resnick, 1987; Silver, 1989; Trafton, 1989). This recommendation, paired with this researcher's observations in classrooms, prompted the decision to feature computational estimation in this study. The following discussion will address the findings of this study integrated with the theory and research addressed in the first two chapters, as well as a thorough analysis of the limitations of the design and implications for further research and practice.

Analysis of Findings

The research questions underpinning this study centered on student achievement in mathematics following a six-week intervention featuring computational estimation. I sought to determine how this intervention impacted performance on questions where computational estimation is an appropriate strategy to utilize. Even more, I wanted to know how the intervention impacted the performance of students with disabilities who may have difficulty implementing problem-solving strategies on assessments. The general findings of this study indicating that the intervention had no statistically

significant effects on mathematics achievement for all students with and without disabilities suggest that it did not meet the expectations based on prior experiences and research; however, readers should first consider relevant patterns within the results as well as the following takeaways and significant limitations of this research design before drawing conclusions based on achievement scores alone. This information could encourage mathematics teachers to implement this intervention more effectively given the lessons learned from this initial experiment.

Achievement in Computational Estimation

The results of this study indicate that when grouped together, students with and without disabilities in control and intervention groups performed similarly on a posttest and an MCAS examination; however, there is one statistically significant finding that should be discussed regarding non-disabled student performance in rounding, as well as other notable results that should be included in the conversation. We should begin with the statistically significant finding of students without disabilities on MCAS questions featuring rounding.

Three of the ten questions that could be answered using computational estimation on the MCAS examination encouraged rounding strategies that students with adequate number sense would likely utilize (see figures). All three questions were included on the non-calculator section of the test, leaving only rounding or written procedures as the likely options for solving. Students without disabilities performed significantly better on these three test items indicating a possible link between the six weeks of computational estimation experience and correct answers on the MCAS examination. During the intervention, students were encouraged to use an element of rounding on 21 of the 24

questions asked, so perhaps rounding is a component of computational estimation that resonated with many of them even though it was not the primary focus of this study. Although there were no statistically significant differences between intervention and control groups on questions featuring rounding on the posttest, students in the treatment group did demonstrate a strong performance on these items. Further investigation should include assessment items in rounding to gain a more concrete understanding of the intervention's impact on students' ability to round numbers in order to solve problems more efficiently.

Another notable result from this study concerns non-disabled student performance in Teacher 1's intervention group. Students without disabilities made gains on posttest scores as well as questions targeting percent and rounding. Although they were not statistically significant, a greater number of questions related to percent and rounding may have prompted a different result. Only five questions on the pre- and posttest targeted percent and six questions encouraged the use of rounding. Perhaps if more data was collected, the analysis would have been more powerful and statistically significant results would have been more likely. Once again, a closer look at skills or concepts embedded within computational estimation may have been a more effective way to determine the impact of this intervention.

One final result that is worthy of discussing involves the performance of students with disabilities. It is reasonable to conclude that non-disabled students may have benefited from this intervention, particularly in their abilities to round. More data should be collected before making this claim; however, there is evidence supporting further investigation. Readers should reach a conclusion that more investigation is necessary

when considering students with disabilities, but there is evidence suggesting that this intervention may have hindered their performance on questions encouraging computational estimation. Descriptive statistics for all eight students with disabilities indicated that they performed worse on the posttest than the pretest by more than one full point. Six of the eight students with disabilities scored lower on the posttest, including two students who dropped by three and six points respectively. Given the anecdotal data provided by the teachers regarding the difficulties students with disabilities seemed to have grasping the concept of estimating in their heads rather than relying on a calculator or written procedures, this decline in performance is worth investigating further. Perhaps students with disabilities, who often need more time to understand and apply new concepts and skills, did not benefit from an intervention that forced them to change their problem-solving approaches in six weeks. Perhaps these students need more time, or maybe their reliance on procedures is not appropriate to change at the high school level. These are possible conclusions that I want to investigate further because they are critical to mathematics instruction, particularly for teachers who want to veer from a more traditional approach to learning.

An initial takeaway that readers should consider regarding this intervention involves the lack of statistically significant findings in student achievement compared to groups that participated in more traditional warm-up activities. While these results were initially disappointing, I believe they should be viewed encouragingly by mathematics educators. Six weeks prior to a statewide assessment that is a graduation requirement for all students, two classes began a daily warm-up program targeting computational estimation, and the students' scores on the assessment were statistically no different than

their non-participating peers. In fact, there is evidence that students without disabilities improved their performance on questions where rounding was a possible solving strategy. Too often teachers are reluctant to engage in unfamiliar learning activities because of comfort in their approaches as well as a fear of change (OECD, 2009; Brown, Hanley, Darby, & Calder, 2007; Hiebert et al., 2003). This study should serve as one example of teachers taking a risk resulting in no apparent negative consequences for them or their students. And given the limitations of this study that are discussed in the next section, there are opportunities to improve this experiment and possibly the achievement of their students.

Anecdotal Data

Although there is no concrete data supporting a conclusion that students preferred the intervention game to traditional warm-up activities involving review and test preparation, the teachers reported that there was a general excitement in their treatment classrooms and students seemed to enjoy the beginning of their mathematics classes each day. Perhaps this type of engaging learning opportunity is a positive step toward changing students' often dismal view of the mathematics classroom (Wiggins, 2014; Scarpello, 2007; Brady & Bowd, 2005; Bramald, Hardman, & Leat, 1995). Even students who traditionally struggle with mathematics and fear looking incompetent in front of their classmates may enjoy this activity because their peers do not see their answers and they can collaborate as little as they want in their groups if they are not feeling confident. Perhaps these students can learn new problem-solving approaches and improve their attitudes after experiencing some success with the subject.

A second takeaway from information provided by teachers involves the potential benefits of a collaborative learning environment. Teachers reported that when students moved to their groups to discuss estimation strategies, several approaches materialized that may not have in a teacher-led, individualized learning session. For example, when students needed to find 25% of a certain value to estimate an answer to a problem, one teacher observed some students dividing the value by four, some students finding 50% and then cutting it in half, and others finding 10% twice and then half of 10% to reach their conclusions. While any of these methods are appropriate, one may be easier for certain students who never thought of solving in that particular way. Students can often be useful resources because they provide different perspectives that may never emerge in a more traditional, teacher-centered classroom (Worley & Naresh, 2014). These perspectives were not measured in this study; however, anecdotal evidence from participating teachers indicates that they changed the learning experience for students who needed exposure to more feasible problem-solving strategies.

Limitations

Length of study.

One major limitation of this research study was its length. Considering that computational estimation is a skill not typically featured in the classroom (Steen, 1999; Paulos, 1988; Trafton, 1986), treatment participants were engaging in something relatively new that takes time to understand and apply (Cochran & Dugger, 2013). This realization was supported by both teachers in the study who noted that their students did not demonstrate appropriate problem-solving strategies, particularly at the beginning of the intervention. They also reported that students with disabilities had an especially

difficult time trying to estimate solutions without the use of any tools. Perhaps it would have been beneficial to introduce computational estimation prior to the intervention to maximize the six-week learning experience for students. Ideally, students would spend an entire school year tackling problems using computational estimation and perhaps other skills or concepts embedded in number sense that are integrated throughout all instruction and content. But given the pressure to cover content that several teachers feel is critical to higher achievement, as well as a lack of evidence that daily targeted warm-ups benefit students, I chose to design a six-week experiment leading up to a high-stakes assessment that was relatively unencumbering. This type of supplemental experience, which I will discuss in more detail throughout the next section, is often highlighted in educational research because of its minor disruption to daily lessons. After careful review of teacher feedback and students' posttests, I suspect that a longer study could capture improved results.

There are studies that suggest short interventions can significantly improve student achievement in mathematics (Hanover Research, 2014), but there is also evidence to the contrary. In an experiment at the elementary level featuring a six-week number sense intervention with a particular focus on place value, students engaged in 20 minute daily lessons but showed no statistically significant gains at the conclusion of the study (Stella & Flemming, 2011). The researchers identified the length of their study as a limitation and recommended a longer intervention for future research. In an experiment featuring a high school mathematics course contextualized in agriculture and technology (Parr, Edwards, & Leising, 2009), the researchers concluded that one semester was not

enough time to cause significant change and recommended a full year of implementation in order to determine its effectiveness.

Given the teacher feedback in this study regarding students' limited estimation skills and the several examples of studies that required more time to impact participants, I have come to the conclusion that the design of this study did not maximize the improvement potential of the intervention students. Students needed more time to discuss estimation strategies and practice problem-solving in their heads in order to make significant progress. According to both of the participating teachers, by the time many of the students started discussing and utilizing effective problem-solving strategies, the intervention was ending. Further research in computational estimation and mathematics achievement should likely include at least 12 weeks of lessons and ideally more to assure a reasonable opportunity for all learners to acquire this complex interaction of skills and concepts; however, even this increased time practicing computational estimation as an add-on learning experience may not result in higher achievement.

Integrating computational estimation.

Another limitation of this study exists regarding an integration of computational estimation into teaching and learning. Students' participation in supplemental exercises featuring computational estimation at the beginning of each class helped them begin to understand how to make sense of numbers in isolation, but it did not provide opportunities to integrate these skills and concepts into the course content. When constructing estimates during the game, students understood that utilizing skills such as rounding was necessary to arrive at reasonable answers. But did they apply the same strategies when tackling problems in class or on assessments? Perhaps the lack of

integration throughout the rest of their mathematics lessons hindered their understanding of how and when to apply these approaches. For example, if students were trying to find the volume of a right rectangular pyramid during a class project or on an assessment, would they apply computational estimation prior to using tools or procedures that may lead to results that are more precise? The findings of this study indicate that they may not utilize the strategies they learned in the intervention to simplify the problem-solving process and construct reasonable answers prior to conducting procedures that attend to precision. This absence of integration may explain their similar pre- and posttest scores as well as their comparable results to the control groups on the posttest and MCAS examination. Students may not have even considered using computational estimation solving problems outside of the intervention.

There is limited research on integrating skills and concepts in mathematics courses compared to explicitly teaching them through supplemental activities. If teachers try to find strategies to improve skill acquisition, the majority of recommendations include add-on programs that promise to enhance the desired skill (Hanover Research, 2014). While these solutions may help students increase their performance on targeted assessments, do they transfer to tasks embedded in class lessons or assessments? Throughout this research study, I stressed the importance of social constructivism – a theory founded on a belief that students must construct and discover knowledge through relevant experiences, and apply what they learn to novel situations (Vygotsky, 1987; Bruner, 1996; von Glasersfeld, 1996; Brophy, 2002). Supplemental activities may provide relevant experiences for students, and they may encourage them to construct new ideas depending on their design, but they may not provide integrated opportunities to

promote student application to new challenges. This was a limitation of the intervention in this research study. Students engaged in real-life experiences and constructed problem-solving strategies collaboratively to reach conclusions, but they did not apply these strategies to novel situations. A more complete design would have included coordinated applications within class content and assessments to foster a transfer of the skills and concepts they discovered in the intervention. Future researchers need to account for this and design studies that thoughtfully integrate the targeted skills or concepts into other mathematical experiences.

Assessment.

After comparing the concepts, skills, and content embedded in intervention and assessment questions, notable differences were evident that could have impacted the findings. Table D1 in Appendix D maps the questions that targeted rounding, fractions, decimals, or percent. The intervention questions overwhelmingly featured rounding while evenly assessing fractions, decimals and percent. The pre- and posttest included rounding in more than half the questions but featured percent more than decimals and fractions. An even larger discrepancy exists within the MCAS questions that targeted each concept or skill less than a third of the time. Table D2 in Appendix D maps the content of each question, specifically the categories of money, distance/time/size, and specific items (i.e. three-point shots). The intervention questions primarily included amounts of money and values of distance/time/size, with only one question using a specific item as the unit of measure. The pre- and posttest and the MCAS examination included only one question each that featured money, and nearly half of the questions on

the tests included an analysis of items. This misalignment is worth considering because students may have felt more confident in their abilities to manipulate values of money but only encountered one item in this category on the assessments. The same can be stated for rounding and percent. These items were frequent during the intervention and on the pre- and posttest but rarely appeared on the MCAS examination. Future studies in this area should consider this limitation and make adjustments. Perhaps a closer look at MCAS examinations over recent years will reveal patterns of content, concept, and skill, and researchers can enhance the intervention to align more closely with the assessments.

Another limitation regarding the assessments in this study involves the use of pencil and paper on the pre- and posttests. Even though students tackled six weeks of problems with no tools other than their brains, they were allowed to use pencils on the assessments which may have influenced their problem-solving approaches. Students may have resorted to traditional procedures they previously learned rather than computational estimation that promotes a thoughtful approach intended to narrow the solution range. As a result, further investigation should include changes to the assessment tool.

A pre- and posttest that does not allow students to use pencil and paper would likely be a smart choice to enhance this learning experience. Perhaps a digital device to record student answers or dark sheets of paper with light text color would eliminate students' ability to solve problems procedurally using a pencil. While scoring posttests, it was evident that some students resorted to more traditional problem-solving approaches so it was impossible to determine the impact of the intervention on their achievement. This approach would also be interesting to researchers concerning the performance of control groups on the assessments. Would they make any gains if teachers did not

include mental calculation in their instruction? Perhaps there would be a significant difference in performance between intervention and control groups because mental computation is so rare in the high school mathematics classroom.

Participants with disabilities.

A final limitation that I anticipated but could not overcome was the number of students with disabilities who participated in the study. The intended sample included more students in each group but a lack of permission coupled with a lack of attendance impacted some of the expected participants. Even so, students with disabilities almost always make up a small percentage of classes so using only two treatment and control groups was not sufficient for gathering an adequate sample. This is a common challenge to collecting data from this population of students because a large number of classes need to be included. Although the small sample of students with disabilities is a limitation to this study, descriptive statistics provide relevant information to learn more about this population of students.

Suggestions for Future Research

Computational estimation is a critical skill embedded in the broader concept of number sense that mathematics researchers and educators believe is evident in high-achieving students (Reys & Bestgen, 1981; Sowder & Schappelle, 1989). Unfortunately, teachers rarely target this skill in their instruction, particularly at the high school level. As a result, students often lack the ability to think about numbers creatively and employ problem-solving strategies that stem from understanding rather than learned procedures. While it is helpful to discuss the importance of computational estimation and identify

where it is lacking, future research needs to demonstrate its impact on student achievement in order to effectively change instruction in the classroom. This research study provided evidence that students were engaged in a computational estimation challenge, collaborated with peers thoughtfully, and did not perform differently on a high-stakes assessment than students enrolled in more traditional classes. If the length of the study is increased, the skill is integrated into the curriculum, the assessment tools are reimagined, and more students with disabilities are able to participate, evidence of increases in achievement scores may be plausible. This study offers some important information to build upon; however, further studies are essential in order to effect real change in mathematics education.

Appendix A

Pretest/Posttest from MCAS (2016a) Test Questions

1. The length, in centimeters, of a rectangle is represented by an expression, as shown in the diagram below.

$$2 + \sqrt{45}$$

Based on the diagram, which of the following is closest to the length, in centimeters, of the rectangle?

- A. 8.3
- B. 8.7
- C. 9.1
- D. 9.5

2. The first 2,450 people to attend a baseball game received a free hat. A total of 19,544 people attended the game. Which of the following is closest to the fraction of people attending the game who received a free hat?

- A. $\frac{1}{20}$
- B. $\frac{1}{8}$
- C. $\frac{1}{5}$
- D. $\frac{1}{4}$

3. Leah took a 5-day car trip. The table below shows the number of miles she drove on each day of her trip

Leah's Car Trip

Day of Trip	1	2	3	4	5
Miles Driven	297	179	203	131	192

Of the total number of miles that Leah drove on her trip, which of the following is closest to the percentage she drove on day 1?

- A. 15%
- B. 20%
- C. 25%
- D. 30%

4. A total of 29,183 votes were cast in an election. The winning candidate in the election received 61.3% of the votes. Which of the following is closest to the number of votes received by the winning candidate?

- A. 21,000
- B. 18,000
- C. 15,000
- D. 9,000

5. Which of the following is closest to the value of the expression below?

$$3.14(7.9)^2$$

- A. 150
- B. 200
- C. 250
- D. 300

6. The bowling scores for 9 friends are shown in the box below.

110	62	80
132	126	194
95	78	95

The mean score is 108 and the median score is 95. Which statement **best** explains why the mean score is greater than the median score?

- A. The score of 95 occurs twice.
 - B. The data set includes only a few scores.
 - C. The minimum score is well below the other scores.
 - D. The maximum score is well above the other scores.
7. A farmer harvested a total of 364 pumpkins. The pumpkins had an average weight of 10.9 pounds. Which of the following is closest to the total weight, in pounds, of the pumpkins the farmer harvested?
- A. 3,000
 - B. 3,300
 - C. 4,000
 - D. 4,400

8. Jaya is buying a new car that has a price of \$28,495. She is required to pay a sales tax that is 6.25% of the car's price. Which of the following estimates is closest to the amount of sales tax Jaya will pay for the car?

A. \$1,200
B. \$1,400
C. \$1,800
D. \$2,100

9. The circle graph below shows the percentages of the types of coins in a collection.

There are 700 coins in the collection. What is the total number of Standing Liberty quarters in the collection?

Short Answer _____

10. A student is knitting sweaters to give as gifts. The time it takes the student to knit each sweater is 10% less than the time it took the student to knit each previous sweater. It took the student 14 hours to knit the first sweater.

Which of the following is closest to the time it will take the student to knit the **third** sweater?

- A. 10.2 hours
- B. 11.3 hours
- C. 12.6 hours
- D. 16.9 hours

Appendix B

Strength in Numbers: A Learning Intervention

Percentage Questions

1. If your electric bill was \$73.27 last month and you are anticipating a 25% increase this month because you are running air conditioning, **estimate** what your bill will be for the upcoming month.
2. One night your dishwasher starts making horrible noises so you decide it is time to buy a new one. An appliance store down the street from you is advertising 45% off the list price of all items. If you find the dishwasher you want and it is listed at \$410.00, **estimate** how much will you pay after the discount if you also factor in a 6.25% sales tax?
3. A new online clothing company named “37 North” offers a 37 percent discount if you purchase at least 5 items at one time. If you decide to buy 5 items for a total of \$286.43, **about** how much will your total order cost with the discount?
4. A new iPhone is priced at \$299.99 with the signing of a 2 year contract. If Verizon is running a promotion offering a 45% price reduction, **estimate** how much the final cost will be given a 6.25% sales tax.
5. If a high school basketball team successfully hit 174 three-point shots in a season which resulted in a 36% success rate, **estimate** how many three-point shots the team attempted?

6. You are purchasing a framed picture for your friend's birthday that costs \$18.19. The store is currently offering a special where you can purchase a picture twice as big for an additional 25% of the price. **Approximate** the new price if you agree to this deal?

Decimal Questions

1. You decide to look for a job after school so you can pay your phone bill and put gas in your car. A department store at the mall offers you 16 hours per week for a wage of \$12.15/hour. If you are paid every two weeks, **about** how much money will you earn in your first paycheck before taxes?

2. Your car averages 28.35 miles/gallon of gasoline on the highway which is far better than when you drive on side roads. If your tank holds 15.1 gallons of gasoline, **about** how many highway miles can you travel on one full tank of gasoline?

3. You and your friends decide to join the track team at school. Every Monday and Wednesday you run 3.1 miles. Every Tuesday and Thursday you run 5.5 miles. On Fridays at your track meets you run 6.2 miles. **Approximate** how many miles you average per day during the school week.

4. The top 5 runners in the 100 meter dash at the Olympics in Rio had the following times:

9.81 seconds

9.89 seconds

9.91 seconds

9.93 seconds

9.94 seconds

Estimate the average time of the five runners.

5. If you join class council at your school and sell 373 tickets to a school dance for \$8.75 per ticket, **about** how much money will you make for your class?

6. The total snowfall amounts (in inches) in Boston for the last 10 years are listed in the table below. **Estimate** the total amount of snowfall Boston has gotten over this time period.

2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
1.02	8.34	23.71	13.24	38.34	6.77	5.00	21.79	34.28	9.52

Fraction Questions

1. You are asked to make brownies for a fundraiser at your school. The ingredients in one box calls for $2\frac{1}{3}$ cups of sugar to make 12 brownies. If you are expecting about 100 people to attend the fundraiser, **estimate** how many cups of sugar will you need to ensure everyone can have one brownie?

2. **About** how wide is a rectangular strip of land with length $3\frac{2}{5}$ miles and area $18\frac{1}{2}$ square miles? (Area = length x width)

3. If the directions from your house to your friend's apartment state the following distances for each street, **about** how far will you travel from your house to your friend's apartment?

$\frac{1}{8}$ mile west

$\frac{2}{3}$ mile south

$1\frac{1}{2}$ miles east

$3\frac{1}{3}$ miles north

$2\frac{4}{5}$ miles west

4. You and 6 of your friends decide to go out to dinner for your 16th birthday. If the entire meal costs \$187.45 before tax, **about** how much will you each pay if you split the bill 7 ways?

5. A bus trip from Providence to Los Angeles will take just under 2 days and is about 2969 miles. If you decide to get off the bus $\frac{5}{6}$ of the way there to visit a friend, **about** how many miles did you travel?

6. You decide to go out to lunch with 5 friends and the bill comes to \$193.81 including tax and gratuity (tip). If you decide to split the bill evenly between the 6 of you, **about** how much did each of you pay?

Multi-skill Problems

1. In preparation for your math course you take a trip to the store to grab some school supplies. You purchase 3 notebooks for \$2.75 a piece, a pack of pens for \$4.60, a calculator for \$9.99, and a stack of folders for \$1.50. If you factor in a 6.25% sales tax, **estimate** how much money you will spend.

2. Your school just received a huge grant from the state and your principal chose you to plan the tile floor layout in your classroom. The tiles you like best are 12 inches x 24 inches. Your classroom floor is a perfect rectangle measuring 22 feet x 32 feet. If the tiles are \$2.28 apiece, **estimate** the total cost? (12 inches = 1 foot)

3. At the start of April break the price of gasoline drops to \$1.74/gallon, so you decide to get in the car with some friends and drive to Daytona Beach, Florida. The distance from Providence to Daytona Beach is 1211.3 miles. If your car averages 23 miles/gallon on fuel, **about** how much money will it cost to drive one way?

4. An online video game distributor is offering a deal for high school students. If you purchase 3 games at the original price of \$35.99 apiece, you can buy a fourth game for $\frac{1}{5}$ of the original price. If you factor in a 12% membership fee, **estimate** how much will you spend in total for the four games?

5. A new iPhone is priced at \$299.99 with the signing of a 2 year contract. If Verizon is running a promotion offering a 45% price reduction, **estimate** how much the final cost will be given a 6.25% sales tax.

6. After taking your MCAS Mathematics test, you and 5 of your friends decide to celebrate by going out to dinner. The bill comes to \$171.18 before tax. You owe one of your friends some money so you decide to pay for her meal in addition to your own. If you factor in a 16% gratuity (tip) and are paying $\frac{2}{3}$ of the bill, **about** how much money will you spend? (Do not factor in the tax.)

Appendix C

MCAS Computational Estimation Questions

- 2 The length, in centimeters, of a diagonal of a rectangle is represented by the expression below.

$$\sqrt{11^2 + 14^2}$$

Which of the following is closest to the length of the diagonal?

- A. 5 centimeters
- B. 7 centimeters
- C. 18 centimeters
- D. 25 centimeters

- 9 The approximate areas of four oceans are shown in the table below.

Ocean Areas

Ocean	Area (square kilometers)
Atlantic	76,762,000
Pacific	155,557,000
Indian	68,556,000
Arctic	14,056,000

The area of Lake Superior is approximately 82,100 square kilometers. Based on the table, which ocean has an area that is closest to 1,000 times the area of Lake Superior?

- A. Atlantic
- B. Pacific
- C. Indian
- D. Arctic

- 5 An art museum has two types of pieces of art: paintings and sculptures.

- The museum has 2,009 paintings.
- The museum has 492 sculptures.

Which of the following is closest to the fraction of the museum's pieces of art that are paintings?

- A. $\frac{20}{25}$
- B. $\frac{20}{24}$
- C. $\frac{21}{24}$
- D. $\frac{21}{25}$

- 13 The volume of a cube is 24 cubic inches. Which of the following estimates is closest to the length of each edge of the cube?

- A. 4.9 inches
- B. 3.1 inches
- C. 2.9 inches
- D. 2.5 inches

- 22 Shirley is saving money to buy a computer.

- The computer she will buy costs \$1,200.
- She has already saved \$300.

Shirley will save another \$60 each week until she has saved enough money to buy the computer.

How many weeks will it take Shirley to save enough money to buy the computer?

- A. 5
- B. 10
- C. 15
- D. 20

- 27 A store sells different-colored pens. The circle graph below represents all pens for sale at the store.

There are 200 black pens for sale at the store. How many blue pens are for sale at the store?

- A. 70
- B. 115
- C. 175
- D. 190

- 25 The number of chaperones needed for a school field trip is directly proportional to the number of students going on the field trip. If 96 students are going on the field trip, 12 chaperones are needed.

How many chaperones are needed if 120 students are going on the field trip?

- A. 15
- B. 20
- C. 24
- D. 36

- 31 The length of a rectangular patio is three times its width. The area of the patio is 432 square feet. What is the **length**, in feet, of the patio?

- A. 12
- B. 18
- C. 24
- D. 36

- 34 A café uses three different types of bread and three different fillings to make sandwiches. The table below shows the number of sandwiches the café made yesterday.

		Sandwich Filling		
		Turkey	Cheese	Veggie
Type of Bread	Rye	40	15	15
	Wheat	24	20	6
	Sourdough	16	5	9

What percent of the **turkey** sandwiches made yesterday were made with wheat bread?

- A. 16%
- B. 24%
- C. 30%
- D. 48%

- 40 The diagram below shows a trapezoid and some of its dimensions.

What is the area, in square centimeters, of the trapezoid?

- A. 56
- B. 72
- C. 112
- D. 144

Appendix D

Concept/Skill and Content Question Maps

Table D1			
<i>Concept and Skill Question Map</i>			
<u>Concept/Skill</u>	<u>Pre- and Posttest</u>	<u>Intervention</u>	<u>MCAS</u>
Rounding	2, 3, 4, 5, 7, 8	1, 2, 3, 6, 7, 8, 9, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 24	2, 5, 9
Percent	3, 4, 8, 9, 10	1, 2, 3, 4, 5, 6, 19, 22, 23, 24	27, 34
Fraction	2	13, 14, 15, 16, 17, 18, 21, 22, 24	5, 31
Decimal	1, 5, 7	7, 8, 9, 10, 11, 12, 19, 20, 21	13

Table D2			
<i>Content Question Map</i>			
<u>Content</u>	<u>Pre- and Posttest</u>	<u>Intervention</u>	<u>MCAS</u>
Money	8	1, 2, 3, 4, 6, 7, 11, 16, 18, 19, 20, 21, 22, 23, 24	22
Distance/Time/Size	1, 3, 10	8, 9, 10, 12, 13, 14, 15, 17, 20, 21	2, 9, 13, 31, 40
Item	2, 4, 6, 7, 9	5	5, 25, 27, 34
<i>Note.</i> Some questions did not match a category and were omitted from the table.			

Bibliography

- Andrews, P. & Sayers, J. (2015). Identifying opportunities for grade one children to acquire foundational number sense: Developing a framework for cross cultural classroom analyses. *Early Childhood Education Journal*, 43(4), 257-267.
- Andriessen, I., Phalet, K., and Lens, W. (2006). Future goal setting, task motivation, and learning of minority and non-minority students in Dutch schools. *British Journal of Educational Psychology*, 76, 827–850.
- Aubrey, C., Dahl, S., & Godfrey, R. (2006). Early mathematics development and later achievement: Further evidence. *Mathematics Education Research Journal*, 18(1), 27–46.
- Bailey, T. (2009). Rethinking remedial education in community college. *Community College Research Center*, New York.
- Bailey, T., Jeong, D. W., & Cho, S. W. (2010). Referral, enrollment, and completion in developmental education sequences in community colleges. *Economics of Education Review*, 29(2), 255-270.
- Ball, D. L., Hill, H. (2009). The curious—and crucial—case of mathematical knowledge for teaching. *Phi Delta Kappan*, 91(2), 68–71.
- Ball, D. L., Sleep, L., Boerst, T., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *Elementary School Journal*, 109(5), 458-474.

- Bandura A. (1986). *Social foundations of thought and action: a social cognitive theory*. Englewood Cliffs: Prentice Hall.
- Barry, M.N. & Dannenberg, M. (2016). Out of pocket: The high cost of inadequate high schools and high school student achievement on college affordability. *Education Reform Now*. Retrieved from <https://edreformnow.org/wp-content/uploads/2016/04/EdReformNow-O-O-P-Embargoed-Final.pdf>
- Beishuizen, M., van Putten, C. M., & van Mulken, F. (1997). Mental arithmetic and strategy use with indirect number problems up to one hundred. *Learning and Instruction*, 7(1), 87- 106.
- Bekdemir, M. (2010). The pre-service teachers' mathematics anxiety related to depth of negative experiences in mathematics classroom while they were students. *Educational Studies in Mathematics*, 75(3), 311-328.
- Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco, CA: Jossey-Bass.
- Bobis, J. (1991). The effect of instruction on the development of computation estimation strategies. *Mathematics Education Research Journal*, 3, 7-29.
- Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology*, 42(1), 189-201.

- Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, 79, 1016–1031.
- Brady, P., & Bowd, A. (2005). Mathematics anxiety, prior experience and confidence to teach mathematics among pre-service education students. *Teachers and Teaching: Theory and Practice*, 11, 37-46.
- Bramald, R., Hardman, F., & Leat, D. (1995). Initial teacher trainees and their views of teaching and learning. *Teaching and Teacher Education*, 11, 23-31.
- Brophy, J. (ed.) (2002). *Social constructivist teaching: Affordances and constraints*. Boston: Elsevier.
- Brown, T., Hanley, U., Darby, S., & Calder, N. (2007). Teachers' conceptions of learning philosophies: Discussing context and contextualising discussion. *Journal of Mathematics Teacher Education*, 10(3), 183-200.
- Bruner, J. S. (1960). *The Process of education*. Cambridge, Mass.: Harvard University Press.
- Bruner, J. S. (1961). The act of discovery. *Harvard Educational Review*, 31, 21-32.
- Bruner, J. S. (1996). *The culture of education*. Cambridge, M.A.: Harvard University Press.
- Bugden, S., & Ansari, D. (2011). Individual differences in children's mathematical competence are related to the intentional but not automatic processing of Arabic numerals. *Cognition*, 118, 35–47.

- Burden, P. R. (2000). *Powerful classroom management strategies: Motivating students to learn*. Thousand Oaks, CA: Corwin Press
- Carpenter, T. P., Kepner, H., Corbitt, M. K., Lindquist, M. M., and Reys, R. E. (1980). Results and implications of the second NAEP Mathematics Assessment: Elementary School. *Arithmetic Teacher*, 27, (p. 10–12, 44–47).
- Carraher, T.N., Carraher, D.W., & Schliemann, A.D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology*, 3(1), 21-29.
- Case, R. (1998). A psychological model of number sense and its development. Annual meeting of the American Educational Research Association, San Diego, CA.
- Case, R., & Sowder, J.T. (1990). The development of computational estimation: A neopiagetian analysis. *Cognition and Instruction*, 7(2), 79-104.
- Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23(7), 13–20.
- Cochran, J., & Dugger, M. H. (2013). Taking the guesswork out of computational estimation. *Mathematics Educator*, 23(1), 60-73
- Cockcroft, W.H. (1982). Mathematics counts: Report of the committee of inquiry into the teaching of mathematics in schools under the chairmanship of Dr WH Cockcroft. Retrieved from <http://www.educationengland.org.uk/documents/cockcroft/cockcroft1982.html>
- Creswell, J.W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (3rd ed.). Thousand Oaks, CA: Sage.

- Dewey, J. (1916). *Democracy and education*. New York: The Free Press.
- Dewey, J. (1938). *Experience and education*. New York, NY: Touchstone.
- Dossey, J. A., Mullis, I. V. S., Lindquist, M. M., & Chambers, D. L. (1988). *The mathematics report card: Are we measuring up? Trends and achievement based on the national assessment*. Princeton, NJ: Educational Testing Service.
- Fan, X. & Chen, M. (2001). Parental involvement and students' academic achievement: A metaanalysis. *Educational Psychology Review*, 13 (1), 1-22.
- Faulkner, V. N. (2009). The components of number sense: An instructional model for teachers. *Teaching Exceptional Children*, 41(5), 24–30.
- Fennell, S. (2008). Number Sense - Right Now! NCTM News Bulletin, March Ed.
- Fields, R., & Parsad, B. (2012). *Tests and Cut Scores Used for Student Placement in Postsecondary Education: Fall 2011*. Washington, DC: National Assessment Governing Board.
- Flores, M. M., & Kaylor, M. (2007). The effects of a direct instruction program on the fraction performance of middle school students at-risk for failure in mathematics. *Journal of Instructional Psychology*, 34(2), 84-94.
- Fosnot, C. (1996). *Constructivism: Theory, perspectives, and practice*. New York: Teacher's College Press.
- Fraenkel, J.R., Wallen, N.E., & Hyun, H.H. (2011). *How to design and evaluate research in education* (8th ed.). New York, NY: McGraw-Hill.

- Fuson, K. C. (1992). Research on whole number addition and subtraction in Grouws, D. (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (p. 243-275). New York: Macmillan & Co.
- Geary D. C., Hoard M. K., Nugent L., Bailey D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PLoS ONE*, 8, e54651.
- Geary, D. (2013). Early foundations for mathematics learning and their relations to learning disabilities. *Current Directions in Psychological Science*, 22(1), 23–27.
- Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education*, 33(1), 18–28.
- Gersten, R., Jordan, N., & Flojo, J. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, 38(4), 293–304.
- Gravemeijer, K. (2003). Facts and algorithms as products of students' own mathematical activity. In Kilpatrick, J., Martin, W. G., & Schifter, D. (Eds.), *A research companion to Principles and Standards for school mathematics*, pp. 114–122. Reston, VA: National Council of Teachers of Mathematics.
- Gresham, G. (2008). Mathematics anxiety and mathematics teacher efficacy in elementary pre-service teachers. *Teaching Education*, 19, 171-184.

Hanover Research (2014). Best Practices in Math Interventions. Retrieved from:

https://www.mbaea.org/documents/filelibrary/numeracy/Best_Practices_in_Math_Intervention_53D80FEED7650.pdf

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal of Research in Mathematics Education*, 21(2), 33-46.

Hickendorff, M., van Putten, C. M., Verhelst, N. D., & Heiser, W. J. (2010). Individual differences in strategy use on division problems: Mental versus written computation. *Journal Of Educational Psychology*, 102(2), 438-452.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., . . . Stigler, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study*. Washington, DC: US Department of Education, National Center for Education Statistics.

Higher Education Services Corporation, (2017). *NYS math and science teaching incentive program*. Retrieved from <https://www.hesc.ny.gov/pay-for-college/financial-aid/types-of-financial-aid/nys-grants-scholarships-awards/nys-math-and-science-teaching-incentive-scholarships.html>.

Howden, H. (1989). Teacher number sense. *Arithmetic Teacher*, 36(6), 6-11.

Hsiu-Zu, H., Senturk, D., Lam, A.G., Zimmer, J.M., Hong, S., Okamoto, Y., Chiu, S., Nakazawa, Y. & Wang, C. (2000). The affective and cognitive dimensions of math anxiety: A cross-national study,” *Journal for Research in Mathematics Education*, 31(3), 362-379.

- Huck, S.W. (2012). *Reading statistics and research* (6th ed.). Boston, MA: Pearson Education.
- Inhelder, B., Piaget, J., Parsons, A., & Milgram, S. (1958). *The growth of logical thinking: from childhood to adolescence*. New York: Basic Books.
- Jordan, N. C., Glutting, J., Ramineni, C., & Watkins, M. W. (2010). Validating a number sense screening tool for use in kindergarten and first grade: Prediction of mathematics proficiency in third grade. *School Psychology Review*, 39(2), 181-195.
- Kamii, C. & Dominick, A. (1997). To teach or not to teach algorithms. *Journal of Mathematical Behavior*, 16(1), 51–61.
- Kilpatrick, J. (1992). A history of research in mathematics education. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 3-38). New York: Macmillan
- Kohlberg, L. & Mayer, R. (1972). Development as the aim of education. *Harvard Educational Review*, 42 (4), 449-496.
- Kukla, A. (2000). *Social constructivism and the philosophy of science*. New York: Routledge.
- Landerl K., Bevan A., & Butterworth B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8-9 year old students. *Cognition*, 93, 99-125.
- Lee, H. (2006). The Power of Groupthink [Abstract]. *Prevention*, 58. Retrieved June 10, 2008, from www.apps2.sxu.edu:2255/ehost/detail

- Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers understanding of fundamental mathematics in China and the United States*. Hillsdale, NJ: Erlbaum.
- Mac Iver, M. A., Mac Iver, D. J. (2015). “STEMming” the swell of absenteeism in the middle years: Impacts of an urban district summer robotics program. *Urban Education*. Advance Online Publication.
- Madaus, G., Clarke, M., & O’Leary, M. (2003). A century of standardized mathematics testing. In G. M. A. Stanic, & J. Kilpatrick (Eds.). *A history of school mathematics* (Vol. 2, pp. 1311-1433). Reston, VA: National Council of Teachers of Mathematics.
- Markus, N. L. (1996). *Adult numeracy themes*. Columbus: Ohio State Department of Education.
- Martin, A. J. (2007). Examining a multidimensional model of student motivation and engagement using a construct validation approach. *British Journal of Educational Psychology*, 77, 413–40.
- Martin, A.J. (2008). *The motivation and engagement scale*. Sydney: Lifelong Achievement Group.
- Massachusetts Department of Elementary and Secondary Education (2011). *Massachusetts curriculum frameworks for mathematics*. Retrieved from: <http://www.doe.mass.edu/frameworks/current.html>.

Massachusetts Department of Elementary and Secondary Education. (2013). *2013 MCAS and MCAS Alt technical report*. Malden, MA.

Massachusetts Department of Elementary and Secondary Education. (2016a). *Massachusetts comprehensive assessment system: Test questions* [Data file]. Retrieved from <http://www.doe.mass.edu/mcas/testitems.html>

Massachusetts Department of Elementary and Secondary Education. (2016b). *School and district profiles* [Data file]. Retrieved from http://profiles.doe.mass.edu/state_report/mcas.aspx

Massachusetts Department of Elementary and Secondary Education. (2017). *Massachusetts comprehensive assessment system: Graduation requirements* [Data file]. Retrieved from <http://www.doe.mass.edu/mcas/graduation.html>

McIntosh, A., & And, O. (1997). Mental computation in the middle grades: The importance of thinking strategies. *Mathematics Teaching In The Middle School*, 2(5), 322-27.

McIntosh, A., De Nardi, E., & Swan, P. A. (1994). *Think mathematically! : How to teach mental maths in the primary classroom*. Melbourne: Longman Cheshire.

Meece, J. L., & Kurtz-Costes, B. (2001). Introduction: The schooling of ethnic minority children and youth. *Educational Psychologist*, 36, 1–7.

Meece, J. L., Wigfield, A., and Eccles, J. S. (1990). Predictors of math anxiety and its consequences for young adolescents' course enrollment intentions and performances in mathematics. *Journal of Educational Psychology*, 82, 60-70.

- Meyer, D. (2014, August 14). Developing the question: Good work! [Web log comment]. Retrieved from <https://blog.mrmeyer.com/2014/developing-the-question-good-work/>
- Moses, R. P. & Cobb, C. E. (2001). Radical equations: Math literacy and civil rights. Retrieved from <http://ebookw.net/2009/04/06/radical-equations-math-literacy-and-civil-rights.html>
- National Center for Education Statistics. *National Assessment of Educational Progress* [Data file]. Retrieved from <http://nces.ed.gov/nationsreportcard/naepdata/>
- National Conference of State Legislatures. (n.d.). *Hot topics in higher education: Reforming remedial education*. Retrieved from <http://www.ncsl.org/research/education/improving-college-completion-reforming-remedial.aspx>
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.
- National Governors Association Center for Best Practices & Council of Chief State School Officers (2010). *Common Core state standards mathematics*. Washington,

DC: National Governors Association Center for Best Practices & Council of Chief State School Officers.

National Research Council's Mathematical Sciences Education Board, Board on Mathematical Sciences, & Committee on the Mathematical Sciences in the Year 2000. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Newmann, F.M., Bryk, A.S., & Nagaoka, J.K. (2001). *Authentic intellectual work and standardized tests: Conflict or coexistence?* Chicago, IL: Consortium on Chicago School Research.

No Child Left Behind Act of 2001, P.L. 107-110, 20 U.S.C. § 6319 (2002).

Organisation for Economic Cooperation and Development (OECD). (2009). *Creating effective teaching and learning environments: First results from TALIS*. Paris: OECD Publishing.

Ortiz, M.R. (2009). *Manual de dificultades de aprendizaje*. Madrid: Pirámide.

O'Sullivan, R. H., Chen, Y., & Fish, M. C. (2014). Parental Mathematics Homework Involvement of Low-Income Families with Middle School Students. *School Community Journal*, 24(2), 165-188.

Pajares, F. (2005). *Gender differences in mathematics self-efficacy beliefs*. In A. M. Gallagher & J. C. Kaufman (Eds.), *Gender differences in mathematics: An*

integrative psychological approach (p. 294–315). New York: Cambridge University Press.

Panitz, T. (2000). Using cooperative learning 100% of the time in mathematics classes establishes a student-centered interactive learning and environment. 3-12. Retrieved June 10, 2008, from Academic Search Premier database.

Parr, B. A., Edwards, M. C., & Leising, J. G. (2008). Does a curriculum integration intervention to improve the mathematics achievement of students diminish their acquisition of technical competence? An experimental study in agricultural mechanics. *Journal of Agricultural Education*, 49(1), 61-71.

Paulos, J. A. (1988). *Innumeracy: Mathematical illiteracy and its consequences*. New York: Vintage Books.

Piaget, J. (1970). *Structuralism*. New York: Basic Books.

Resnick, L. (1987). *Education and learning to think*. Washington D.C. National Academy Press.

Reys, R. E.; Bestgen, B. J.; Rybolt, J. F.; and Wyatt, J. W. (1980). Identification and characterization of computational estimation processes. Washington, D.C.: National Institute of Education.

Reys, R. & Bestgen. (1981). Teaching and assessing computational estimation skills. *The Elementary School Journal*, 82(2), 116-127.

- Reys, R. E., Reys, B., & McIntosh, A. (1999). Assessing number sense of students in Australia, Sweden, Taiwan, and the United States. *School Science & Mathematics*, 99(2), 61-70.
- Rogoff, B. (1998). Cognition as a collaborative process. In D. Kuhn & R.S. Siegler (eds.) *Handbook of child psychology*, pp. 679-744 New York: John Wiley
- Rose, D. H., Harbour, W. S., Johnston, C. S., Daley, S. G., & Abarbanell, L. (2006). Universal design for learning in postsecondary education: Reflections on principles and their application. *Journal of Postsecondary Education and Disability*, 19(2), 135-151.
- Routman, R. (2000). *Conversations: Strategies for teaching, learning, and evaluating*. Portsmouth, NH: Heinemann.
- Scarpello, G. (2007). Helping students get past math anxiety. Techniques: *Connecting Education and Careers*, 82 (6) 34-35.
- Schneider, M. & Siegler, R.S. (2010). Representations of the magnitudes of fractions. *Journal of Experimental Psychology: Human Performance and Perception*, 36(5), 1227-1238.
- Schunk, D. H., & Pajares, F. (2002). The development of academic self-efficacy. In A. Wigfield & J. S. Eccles (Eds.), *Development of achievement motivation: A volume in the educational psychology series* (pp. 15–31). San Diego: Academic Press.

- Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics*, 46(1), 13–57.
- Shor, I. (1992). Empowering education: Critical teaching for social change. Chicago: University of Chicago Press.
- Siegler, R., Duncan, G., Davis-Kean, P., Duckworth, K., Claessens, A., Engel, M..., Chen, M., (2012). Early predictors of high school mathematics achievement. *Psychological Science*, 23(7), 691-697.
- Simons, J., Dewitte, S., & Lens, W. (2004). The role of different types of instrumentality in motivation, study strategies, and performance: know why you learn, so you'll know what you learn! *British Journal of Educational Psychology*, 74, 343-360.
- Sowder, J. (1988). Mental computation and number comparison: Their roles in the development of number sense and computational estimation. *Number concepts and operations in the middle grades*, 2, 182-197.
- Sowder, J. T., & Schappelle, B. P. (1989). *Establishing Foundations for Research on Number Sense and Related Topics: Report of a Conference (San Diego, California, February 16-17, 1989)*.
- Sowder, J. T. (1990). Mental computation and number sense. *The Arithmetic Teacher*, 37(7), 18–20.

- Sowder, J. (1992). Estimation and number sense. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 371–389). New York: Macmillan.
- Steen, L.A. (1999). Numeracy: the new literacy for a data-drenched society. *Educational Leadership*, 57(2), 8-13.
- Steffe, L.P., Cobb, P., & von Glasersfeld, E. (1988). *Construction of arithmetical meaning and strategies*. New York: Springer-Verlag.
- Stella, M. E. & Fleming, M. R. (2011). *Clarity in mathematics instruction: The impact of teaching number sense and place value skills on elementary school students*. Retrieved from Eric (ED529621)
- Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2009). What community college developmental mathematics students understand about mathematics. *The Carnegie Foundation for the Advancement of Teaching: Problem Solution Exploration Papers*.
- Stipek, D. (2002). *Motivation to learn: From theory to practice (4th edition)*. Needham Heights, MA: Allyn & Bacon.
- Teach For America. (2017) Scholarships and grants. Retrieved from:
<https://www.teachforamerica.org/join-tfa/leading-classroom/salary-benefits-aid>
- The College Board. (n.d.). What are college placement tests? Retrieved from
<https://bigfuture.collegeboard.org/find-colleges/academic-life/what-are-college-placement-tests>

- Tighe, E. L. (2014). Broadening our perspectives on adult literacy, numeracy, and problem-solving skills with PIAAC Data: A commentary on The Centre for Literacy's 2014 Summer Institute. *Journal of Research and Practice for Adult Literacy, Secondary, and Basic Education*, 3(3), 63-69.
- Trafton, P. (1986). Teaching computational estimation: Establishing an estimation mind set. In H.L. Schoen & M.J. Zweng (Eds.). *Estimation and mental computation* (p. 16-30). NCTM. Reston, VA.
- Tyner-Mullings, A.R. (2012). *Redefining success: How CPESS students reached the goals that mattered*. In Hantzopoulos and TynerMullings, Critical Small Schools.
- United States Department of Education. (2017). Federal Student Aid. Retrieved from: <https://studentaid.ed.gov/sa/repay-loans/forgiveness-cancellation/teacher>
- United States Department of Education. (2011). Science, Technology, Engineering and Math: Education for Global Leadership. Retrieved from: <https://www.ed.gov/stem>
- United States. National Commission on Excellence in Education. (1983). *A nation at risk : The imperative for educational reform : A report to the Nation and the Secretary of Education, United States Department of Education*. Washington, D.C.: The Commission: [Supt. of Docs., U.S. G.P.O. distributor]
- Vinson, B.M. (2001). A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives. *Early Childhood Education Journal*, 29(2), 89-94.

- Von Glasersfeld, E. V. (1996). Introduction: Aspects of constructivism. In Fosnot, C.T. (Ed.), *Constructivism: Theory, perspectives, and practice* (pp. 3-7). New York, NY: Teachers College Press.
- Vosniadou, S., Vamvakoussi, X., & Skopeliti, I. (2008). The framework theory approach to the problem of conceptual change, In S. Vosniadou (Ed.). *International Handbook of Research on Conceptual Change*. New York: Routledge.
- Vukovic, R.K., Roberts, S.O. & Wright, L.G. (2013). From Parental Involvement to Children's Mathematical Performance: The Role of Mathematics Anxiety. *Early Education and Development*, 24(4), 446-467.
- Vygotsky, L.S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Weiss, I., & Pasley, J. (2004). What is high-quality instruction? *Educational Leadership*, 61(5), 24-28.
- Wenzel, T. (2000). Cooperative student activities as learning devices. *Analytical Chemistry*, 72, 293-296.
- Whitaker-Sena, J. D., Lowe, P. A., & Lee, S. W. (2007). Significant predictors of test anxiety among students with and without learning disabilities. *Journal of Learning Disabilities*, 40, 360–376.
- Wigfield, A. & Meece, J. (1988). Math anxiety in elementary and secondary education students. *Journal of Educational Psychology*, 80(2), 210-216.

- Wiggins, G. (2014). Fixing the high school - student survey, part 1. Retrieved from <https://grantwiggins.wordpress.com/2014/05/21/fixing-the-high-school>
- Williamson, V. (2006). Group and individual work. *Mathematics teaching incorporating micromath*, 195, 42-45. Retrieved June 10, 2008, from Academic Search Premier database.
- Wilson, L. D. (2007). High-stakes testing in mathematics. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (Vol. 2, pp. 1099-1110). Charlotte, NC: Information Age.
- Wilson, J. S. (2009). Elementary school mathematics priorities. *AASA Journal of Scholarship & Practice*, 6, 40–49.
- Wirt, J. (2004). The condition of education. National Center for Education Statistics, U.S. Government Printing Office.
- Woodward, J., & Brown, C. (2006). Meeting the curricular needs of academically low achieving students in middle grade mathematics. *Journal of Special Education*, 40, 151–159.
- Worley, J., & Naresh, N. (2014). Heterogeneous peer-tutoring: An intervention that fosters collaborations and empowers learners. *Middle School Journal*, 46(2), 26-32.
- Wu, H. (2005). Key mathematical ideas in grades 5-8. *Report of a conference (University of California, September 12, 2005)*

- Wu, H. (2011). Teaching Fractions According to the Common Core Standards. Retrieved from <http://math.berkeley.edu/~wu/CCSS-Fractions.pdf>
- Zeidner, M., & Matthews, G. (2005). Evaluation anxiety: Current theory and research. In A. J. Elliot, & C. S. Dweck (Eds.), *Handbook of competence and motivation* (pp. 141–163). New York, NY: Guilford.
- Zettle, R., & Raines, S. (2002). The relationship of trait and test anxiety with mathematics anxiety. *College Student Journal*, 34, 246-258.
- Zimmerman, B. J. (2000). Self-efficacy: An essential motive to learn. *Contemporary Educational Psychology*, 25, 82–91.