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THREE ESSAYS ON MOBILE APPLICATION DEVELOPERS' PRODUCT
OFFERING STRATEGIES

BY

DEGAN YU

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

IN

OPERATIONS AND SUPPLY CHAIN MANAGEMENT

UNIVERSITY OF RHODE ISLAND

2017

DOCTOR OF PHILOSOPHY DISSERTATION

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2017

ABSTRACT

In the recent years, mobile applications (apps) have been immensely changing the way of communication, socialization, work, and recreation through mobile devices. Mobile app developers usually face challenges in product offering decisions. In the dissertation, I study app developers' product offering decisions when facing the setting of multiple platforms, in-app purchase (IAP), and in-app advertisement (IADV). Various analytical models are constructed in order to capture different conditions faced by app developers. In the first essay, an analytical framework is developed to address a product offering problem for an app developer that introduces paid or free apps in a two-platform market. When offering a paid app, I find that the developer should launch the product in the more profitable platform that has relative advantages in user base and willingness to pay; whether the developer should launch the same app in a second platform depends on the app launching cost. I find that launching free apps in both platforms is a better choice as long as users are tolerant to advertisements in the app. If users are not tolerant to advertisements, the developer should launch the app in the more profitable platform. Furthermore, I find that if users' disutility sensitivity to advertisement is very small, it is better to introduce the free app rather than the paid app. In the second essay, I consider mobile app developers' product versioning decisions by focusing on IAP. Should a developer provide consumers an app (free or paid) with IAP option in one platform (e.g., Android or iOS), and how should it design and price the basic app and IAP? I find that the answer to the former question is "it depends", although in most situations offering IAP is a better choice. I also compare forward-looking and myopic strategies and show that the former always outperforms the latter. In the third

essay, I focus on app developers' decision on the frequency of displaying ads to current users; also, I examine app developers' revenue model selection. From app developers' perspective, I show that under information asymmetry condition it is possible to figure out the optimal ad frequency. Furthermore, "trial and then free without ads" (TF) strategy is always suboptimal. "Trial and then paid" (TP) strategy is dominant if market size is small; otherwise, "trial and then free with ads" (TFA) strategy is dominant. The welfare analysis suggests that any of the above three strategies can result in the highest total welfare depending on market size and users' attributes.

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I would like to thank the College of Business Administration at the University of Rhode Island. My gratitude particularly goes to Dr. Shaw Chen, Dr. Douglas Hales, Dr. Koray Ozpolat, Dr. Dara Schniederjans, Dr. Su-Hsiu Wu, Prof. Joe Estrella, Prof. Brian Walsh, Dr. Bing-Xuan Lin, Dr. Scott Lloyd, Dr. Clay Sink, Dr. Xiaowei Xu, Dr. Shingo Goto, and Dr. Ruby Dholakia. Their advices, accessibility, support and kindness have been quite helpful. I am also thankful to my doctoral cohorts who helped alleviate the feeling of loneliness during my pursuit for the Ph.D. degree.

I am very grateful to all of the friends that I have met in the past decades. The experiences with the friends have become an indispensable part of my life. It is hard to mention everybody here but one is never forgettable: Dr. Tong Yu. Tong's generous helps are pivotal for my career transition from industry to academia, for my graduate studies, and for my pursuit to becoming a scholar; his advices have been life-long invaluable.

Last and most importantly, I am deeply indebted to my family. Without their love and support, I would not be able to step to this position in my academic career. In particular, my parents, Ziyun Chen and Dabo Yu, always have their faith in me and allow me to choose my own way of life with a very high degree of freedom. My uncle Dr. Paul K.L. Yu provided me a lot of valuable advices especially during my graduate studies in the United States. In the past years, my wife Jing (Ann) Tang was the one who was always willing to wait for me in the place, and would give me support and encouragement whenever I needed. It is her who let me truly know life is not just academic research.

PREFACE

This dissertation has been written to fulfill partial requirement of the degree of doctorate degree in Business Administration (with specialization in Operations & Supply Chain Management) at the University of Rhode Island. Manuscript format is in use in this dissertation. The dissertation addresses a number of decision issues faced by mobile app developers. Specifically, the first essay mainly considers the selection decision of mobile app platforms (e.g., iOS and/or Android). The second essay attempts to address the question of whether a developer should offer in-app purchase (IAP) on top of a basic app (either paid or free version). The third essay focuses on revenue model choice (either paid or free with ads after a trial period); it also considers decision issues regarding app ranking effort and ad frequency faced by app developers. All three essays target European Journal of Operational Research. At this moment, the first essay is being prepared for the 2nd round review at European Journal of Operational Research, while the second and the third essays are being prepared for submission.

The research topic of this dissertation originated from my work in Dr. Yuwen Chen's seminar namely Analytical Modeling in Spring 2014. In his seminar, I was assigned to read, discuss and/or review some analytical modeling papers. The topics of the papers mostly relate to new product development and digital economy, which are Dr. Chen's research expertise. Under Dr. Chen's excellent guidance, my analytical modeling skills got advanced; and, inspired by an article discussed in his seminar, I came up with an interesting research idea. The refinement of this idea along with my further pursuit has made it the first essay of my dissertation. The first essay was undertaken approximately from Spring 2014 to Winter 2017. During this period, I also

attempted to explore new research opportunities for the second and third essays of the dissertation.

I was very fortunate to have Dr. Yuwen Chen, Dr. John Ni (who served as my inside committee member before he left URI) and Dr. Gulver Karamemis as my inside committee members. Their guidance and support have been tremendously helpful. This dissertation has also benefited from Dr. Jyh-Hone Wang, Dr. Valerie Maier Speredelozzi, and Dr. Dara Schniederjans (who served as my committee member in the early period). Their discussions, comments and suggestions directed me to improve the dissertation. I would never forget the hand-written comments offered by Dr. Speredelozzi -- very impressive. The dissertation, particularly the first two essays, has benefited from the participants in a number of conferences (e.g., annual meetings of POMS, INFORMS, DSI) and URI CBA Brown Bags. To my family, colleagues, and friends: I would not be able to finish the dissertation without your help and support.

I hope you enjoy your reading.

Degan Yu

Rhode Island, July 25, 2017

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Manuscripts

MANUSCRIPT – I

*Currently being revised for the 2nd round review at European Journal of Operational
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Application Developers’ Product Offering Strategies in Multi-Platform Markets

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Abstract

Mobile Application (app) developers usually face challenges in product offering decisions. In this research, I develop an analytical framework to address a product offering problem for an app developer that introduces paid or free apps in a two-platform market. When offering a paid app, I find that the developer should launch the product in the more profitable platform that has relative advantages in user base and willingness to pay; whether the developer should launch the same app in a second platform depends on the app launching cost. I find that launching free apps in both platforms is a better choice as long as users are tolerant to advertisements in the app. If users are not tolerant to advertisements, the developer should launch the app in the more profitable platform. Furthermore, I find that if users' disutility sensitivity to advertisement is very small, it is better to introduce the free app rather than the paid app. The results from paid app analysis also apply to the computer software industry.

Keywords: Software/Application, Mobile App Platforms, Consumer Utility, Product Offering, App Developer.

1. Introduction

Since the launch of Apple's first iPhone in 2007, the touch-screen technology and larger screen size have transformed smartphones into pocket computers that can execute complex processing operations (including banking, photography, filmmaking, and gaming). The subsequent growth of the smartphone market stems not only from better hardware components and operating systems but also from the proliferation of mobile apps. In 2016, the numbers of apps (or application programs) available on Android and iOS reached 2.2 million and 2 million, respectively (Statista, 2016). The market value of the global mobile application (app) industry was estimated at US\$ 19.7 billion in 2014 and is expected to triple by 2020 (Transparency Market Research, 2015). As the number of mobile apps grows, the app industry attracts an increasing number of traditional software developers to join its market. By 2014, about half of the computer software developers worldwide (around 8.7 million in number) had entered the mobile app industry (Evans Data Corporation, 2014). Currently, there are several platforms in the mobile phone industry: Android, iOS, Windows, and Blackberry, among others. The first two platforms have a combined market share of 97%, and most new apps are often launched in Android and/or iOS first (International Data Corporation, 2015).

Apps including mobile apps, computer software, and web applications (e.g., games on Facebook, Twitter or MSN) are designed to run on specific devices and written for specific platforms or operating systems. Users can download apps through app stores (mostly online) either for free or at a price. App sales and advertisements have become major revenue sources for app developers. While users pay for most computer apps and some mobile apps, few apps are truly advertisement-free. In the mobile app market, most free apps come with intrusive pop-up advertisements. In mobile app markets, users also have different disutility sensitivities in response to advertisement and app price. Chambers, Kouvelis, & Semple's (2006) report suggests that users of free apps are normally less sensitive to the annoyance from advertisements. Revenues from app sales and advertisements are not the only factors app developers should focus on. The sizes of the user bases vary in different platforms, and the developer must evaluate whether it is profitable to develop an app for a specific platform. For instance, the market share of Android is six times that of iOS (International Data Corporation, 2015), and Windows' market share is 14 times as large as Mac's (Net Application, 2015). In addition, the willingness to pay (WTP) also varies across platforms. It was reported that iOS users are willing to spend about twice as much for apps as Android users (Evans, 2014). As a result, decisions concerning product offering in multiple platforms are challenging to and pivotal for app developers' success (e.g., Shapiro and Varian, 1998; Lee and Raghu, 2014).

Previous research suggests that product offerings for information goods differ from those for tangible products due to cost complementarity: information goods have a substantial setup cost and very low marginal cost (Weber, 2008). A number of studies have examined the information product offering strategy by focusing on factors such as product versioning (Weber, 2008) and pricing (Bala and Carr, 2009). However, the existing research focuses only on a single platform, failing to address developers' product offering decisions when facing multiple platforms, which is prevalent in the app/software industry.

Moreover, platforms connect two groups of market participants (i.e., end users and developers) and are thus considered as two-sided markets. Over the last decade, research on two-sided markets has evolved alongside the rapid development of the Internet economy. Analytical frameworks developed for two-sided markets have studied topics such as pricing and commitment (e.g., Hagiu, 2006; Weber, 2008), versioning (e.g., Dogan, Mokerjee, & Radhakrishnam, 2011; Bhargava, Kim, & Sun, 2013), and platform competition (e.g., Rochet and Tirole, 2003; Armstrong, 2006; Belleflamme and Peitz, 2010). Yet, most prior studies focused only on the business strategies from platform firms' perspectives, and there is scant analytical research from developers' perspectives. In other words, developers' product offering decisions when facing multiple platforms remained unexplored. An individual platform's user base is not the only factor determining whether developers should enter that platform. The difficulty of app development in a specific platform, users' WTP, and developers' current skill sets also influence developers' product offering decisions (Pappas, 2013). In this paper, I explore developers' decisions on app offering, including platform selection, pricing and product quality choice while considering app development difficulty, available prior quality, app launching cost, market size, and users' WTP.

This study addresses product offering strategy from the perspective of an app developer that faces two app platforms. As my model incorporates different user bases and different WTPs in two platforms, I focus on the app developer's platform selection, pricing and product quality decisions, assuming that the developer has predetermined whether to offer a paid or free app to achieve a parsimonious model. Specifically, I address the following research questions: (1) What platform(s) should a developer launch its app product in when the revenue comes from app sales (or advertisement)?, (2) What product quality level should the developer pursue?, (3) How do user bases, users' WTP, and other factors determine the developer's optimal platform selection, pricing and quality decisions?, and (4) What are the optimal conditions for different strategies?

This article makes several contributions to existing literature. First, it enriches product

offering literature by investigating two-sided markets from developers' perspectives in a multi-platform scenario. I provide optimal conditions and platform decisions for each strategy (in single and two platforms) by offering either a paid or a free app. The app developer should always enter the more profitable platform; entering the less profitable platform depends on the app launching cost. Second, I identify platforms' WTPs and consumer densities as key factors that determine platform entry strategy for paid app offering, and user's disutility sensitivity to advertisement as an extra factor to consider surrounding free apps. Third, I provide sensitivity analysis regarding other factors that impact quality and pricing decisions. The platform that has the higher WTP should always price the app higher if the developer decides to launch the product. Due to different user densities in different platforms, a platform with a higher app price may derive either a higher or lower realized demand. Furthermore, our findings from paid app analysis can be extended to the computer software industry.

The remainder of this article is organized as follows. Section 2 reviews literature relevant to our research. Section 3 presents our model, followed by an analysis of the model and summary of our major findings in Section 4. A numerical analysis is conducted in Section 5. The article closes with a summary of the conclusions, academic and managerial implications, model limitations, and directions for future research.

2. Literature Review

There are several streams of literature related to this research, including research on two-sided markets, multi-platform contexts, and product offering strategy.

Starting with Katz and Shapiro's (1985) early study on network effects, researchers on two-sided markets have considered problems where increasing the number of participants from one group leads to higher obtainable utility for participants from another group. As research on this topic has expanded over the last two decades, most studies have focused on business strategies from the perspective of the platform companies (e.g., Rochet and Tirole, 2003; Armstrong, 2006; Eisenmann, Parker, & Vanalstyne, 2011; Bhargava et al., 2013); analytical research from developers' perspective is still lacking. In an exception, Belleflamme and Peitz (2010) created analytical models to capture seller investment incentives in platform competition. However, their study assumed that the sellers have already chosen the platform. In contrast, my paper focuses on developers' strategies on platform selection and product offering. A recent study by Avinadav, Chernononog, & Perlman (2015) considers a competition game among app developers on one platform (retailer); in a Stackelberg game a developer with certain risk preferences decides both price and quality level for its app product. Different from the model of

Avinadav et al. (2015), my paper addresses platform selection as well as revenue from both app sales and advertisements.

Several recent studies investigated developers' decisions in a multi-platform context. For instance, Ku and Cho (2011) performed an empirical study of 245 ERP software developers on the determinants of complementors' platform selections. Jonnalageddaa and Saranga (2017) examined commonality decisions in manufactured product design for multiple markets. Bresnahan, Orsini, & Yin (2015) empirically estimated user preferences and developer's profitability. The results I got echo Bresnahan et al.'s (2015) finding that the most popular mobile apps tend to be offered in multiple platforms. Departing from Bresnahan et al.'s (2015) study, my analytical model focuses on app developers' platform selections and product offering decisions in a two-platform setting.

Another related stream of literature explores multiple product offering. Moorthy and Png (1992) suggested that it is optimal for a firm to sequentially offer products in high-end and then low-end markets if substitution and/or cannibalization effects exist. Desai (2001) analyzed the cannibalization effect in product line design under monopoly and two-competitor settings. Cattani and Heese (2009) investigated the offering strategy of two complementary products when competition between two firms that focus on different markets exists. Chen, Carrillo, Vakharia, & Sin (2010) explored product offering strategy for multifunctional products. For information goods whose marginal cost is negligible, the product offering is slightly different from that for tangible products. Weber (2008) adopted a locational approach to product differentiation and created a two-period model for vertically and/or horizontally versioning information products in a single-market setting. In the current paper, I apply circular markets (Weber, 2008) to model consumers in different platforms and utilize a similar cost function in Desai (2001) as product development cost. My research also offers one app product in multiple platforms that can be seen as a product offering decision in different horizontal market segments.

3. Model

In this section, I first introduce several assumptions regarding two app platforms and users' utility functions, and then develop the app developers' profit function for the subsequent analysis.

I assume that there are two platforms in a software/app market and each platform has its separate user base. An app developer offers software/applications in one or two platforms. Sales of paid apps and/or advertisements from free apps are the primary sources of revenue for the developer. Further, I assume that an app product is valued by consumers in terms of its aggregate

quality attributes or characteristics (Rosen, 1974). To simplify, I assume that the developer offers the app with the same quality (q) across the two platforms. Additionally, I assume that consumers are heterogeneous in terms of value preference and uniformly distributed within each platform. It has been proven that the uniform distribution assumption does not change the qualitative directions of analytical results (Calvo-Armengol and Zenou, 2002).

To capture characteristics of the app industry, I extend Desai's (2001) model and Weber's (2008) circular city model to a two-platform setting (Platform i , $i = 1, 2$). To accommodate the fact that platforms may have differently sized user bases, I assume that both platforms have users representing the same market size (which is 2) but with different densities (Platform 1's density is 1 and Platform 2's density is $r > 1$). In Fig. 1, I expand the circular market with size of 2 to a two-dimensional plane. Two end points on both sides of point 0 are one unit distant from point 0. These two end points are identical when we draw the market in a circular market. The thickness of the horizontal base line represents the user density. I normalize the first platform's density to 1 and allow the second platform with a different density (r). This setup allows the maximal distance disutility in both platforms to remain the same, regardless of the size of user bases in the respective platforms. The total number of users is the product of market size and user density (2 and $2r$ for Platforms 1 and 2, respectively). I assume that a consumer's valuation for an app product is $\theta_i q$, where q is the product quality determined by the developer and θ_i represents the consumers' valuation for the product at Platform i (See Table 1 for notations). The quality is an aggregate measure of a bundle of attributes that typically relate to the product's performance and functionality (Desai, 2001; Weber, 2008). For instance, the quality of the Google Maps app can be determined by attributes including user-friendliness, data accuracy, speed of location search and positioning, and algorithm success of route planning. I also assume that a quality upper bound, $\hat{q}$, which is constrained by either the developer's capability or external factors such as technology limits (Chambers et al., 2006). For example, an alarm is a simple app that represents an upper limit regarding how many features it can contain under the current state of technology.

Fig. 1. App Developer's Decisions in Two Platforms. An app developer faces two platforms (Platforms 1 and 2) with market spread in two-dimension (The cylinder on the right of platform 1 is shown when users are spread in circular market form). Two platforms may have different WTPs (θ_i) and user densities (1 in Platform 1 and r in Platform 2, shown in the thickness of horizontal lines). If the developer launches a paid app, the price (p_i), quality (q) decisions, and distance disutility (k) determine the realized demand (m_i). If it is a free app, then both p_1 and p_2 are replaced by ts (not shown in the figure).

Table 1. Variable Notations

Parameters	
θ_i	User's willingness-to-pay in platform i , $i = 1,2$
s	Unit revenue for in-app advertisement in each platform
k	User's distance disutility parameter
β	Development difficulty parameter of product
q_0	Prior quality level that the developer can reuse
$\hat{q}$	Upper limit of quality level q
t	App users' disutility sensitivity parameter for free apps, $0 < t < 1$
b	Developer's launching cost parameter for an app in each platform, $b > 0$
Decision Variables	
p_i	App product's price in platform i , $i = 1,2$
q	App product's quality
Other Variables	
u_i	User's utility in platform i , $i = 1,2$
x_i	Product's distance from the consumer's ideal point in platform i , $i = 1,2$
m_i	Realized demand in platform i , $i = 1,2$
1 or r	User densities in Platforms 1 and 2 are 1 and r , respectively
z_i	$z_i = 1$ if the developer launches the app in Platform i ; otherwise, $z_i = 0$

To determine the demand of the app product offered by the developer, I first evaluate individual users' utility. I adopt a linear demand function in our model. A user's utility is determined by the user's valuation for product quality, product price, and transportation disutility. The product price is a decision variable for the developer, while the user's transportation disutility depends on the unit transportation disutility (parameter k) as well as the distance between the product's position and the user's location. Without losing generality, I assume that the app product is positioned at point 0 on both platforms. Thus, a consumer in Platform i who is x_i distance away from the app product's position (point 0) has a net utility of $u_i = \theta_i q - p_i - kx_i$. As illustrated in Fig. 1, given a determined price p_i of the app product, the longer x_i is, the lower u_i one can derive. Consumers would not buy an app product if the

net utility is not positive. This non-negative net utility is the participation constraint. Hence, the farthest consumers that the developer can capture are the users who derive zero net utility on both sides of point 0. Take Platform 1 in Fig. 1 as an example, where the locations of the farthest consumers at price p_i are x_1^R and x_1^L in the right and left sides of point 0, respectively. The line segments on both sides are $x_i^R = x_i^L = x_i = \frac{\theta_i q - p_i}{k}$. Therefore, for the app developer, the realized demand in Platforms 1 (2) can be expressed as $m_1 = \frac{2(\theta_1 q - p_1)}{k}$ ($m_2 = \frac{2r(\theta_2 q - p_2)}{k}$).

In other cases, the app products can be offered free to users. While developers can gain revenues from in-app advertisements in these free apps, users might be annoyed by these advertisements. A recent study by Ghose and Han (2014) has shown that free app products with in-app ad options could reduce potential market demand. I assume that a developer is able to generate the same ad revenue per user (denoted by s) in both platforms. App users, however, are less sensitive to the unit ad revenue (in free apps) than the change of app price (in paid apps). The reason is that free app users do not need to pay out of their pockets for the service. Henceforth, I use Chambers et al.'s (2006) model structure for users' price sensitivity and assume that the user disutility for in-app advertisement is ts , where $0 < t < 1$. Similar to the price in the paid version, the more ads in the app, the higher level of disutility the users will experience. Note that s is exogenously determined by the market conditions and is not a decision variable for an app developer. Therefore, the realized demand in platforms 1 and 2 when offering free app are $m_1 = \frac{2(\theta_1 q - ts)}{k}$ and $m_2 = \frac{2r(\theta_2 q - ts)}{k}$, respectively.

To model the developer's app development cost structure, I extend Desai's (2001) model structure by including a parameter of prior quality level from other similar past products so that the firm can reuse the previous codes and thus save the development cost (denoted by q_0). For example, to develop apps for iOS or Android, a developer needs to be proficient in Java and/or Objective-C programming languages (Hamlyn, 2014), which can be considered the prior quality level possessed by the developer from previous projects. Similarly, the developer can reuse some app components that have been developed in previously launched apps. The reuse of prior quality levels is ubiquitous in the information goods industry and a topic of prior research (e.g., Bollinger and Pflieger, 1990; Lim, 1994; Favaro, Favaro, & Favaro, 1998). I model the developer's development cost for the app product as $\beta(q - q_0)^2$, where the positive constant β represents the app development difficulty parameter and is inherent to the developer's capability. I also assume that launching an app in one platform costs the developer bq ($b \geq 0$); hence, if the developer launches an app in two platforms, it will incur $2bq$ development costs. I am aware that it is possible to write codes for one app that can be translated into both iOS and

Android platforms; however, that type of coding works only for simple apps. For complex apps, coding in the specific language of each platform is necessary (Stack Overflow, 2013). Our cost structure can accommodate for complex apps (large b) with separate programs as well as for simple apps with one program (b is close to zero).

Note that I assume the developer launches either free or paid app on any platform it chooses to enter¹. Based on the above discussion, the profit objective functions for the developer to launch free and paid apps are shown as below ($z_i = 1$ indicates the developer launches app in Platform i ; and 0 otherwise):

$$\max_{q, z_1, z_2, p_1, p_2} \frac{2z_1 p_1 (\theta_1 q - p_1)}{k} + \frac{2z_2 p_2 r (\theta_2 q - p_2)}{k} - \beta(q - q_0)^2 - bq \sum_{i=1}^2 z_i \quad (\text{paid app}) \quad (1)$$

where $z_i = \{0,1\}$ and $p_i > 0$ for $i = 1, 2$, and $q_0 \leq q \leq \hat{q}$;

$$\max_{q, z_1, z_2} \frac{2z_1 s (\theta_1 q - st)}{k} + \frac{2z_2 sr (\theta_2 q - st)}{k} - \beta(q - q_0)^2 - bq \sum_{i=1}^2 z_i \quad (\text{free app}) \quad (2)$$

where $z_i = \{0,1\}$ for $i = 1, 2$, and $q_0 \leq q \leq \hat{q}$.

In the above profit functions, the first two components (e.g., $\frac{2z_1 p_1 (\theta_1 q - p_1)}{k}$ and $\frac{2z_2 p_2 r (\theta_2 q - p_2)}{k}$) are revenues gained by the developer; components $\beta(q - q_0)^2$ and $bq \sum_{i=1}^2 z_i$ represent app development cost and launching cost, respectively. Since $z_1 + z_2 \geq 1$, if the developer decides to launch an app, there are six cases for paid app and free app strategies (See Table 2).

4. Analysis

In this section I present our approach to solving the model and discuss the overall result; I then discuss cases for paid and free strategies separately; and finally, I compare the results from the two app strategies.

The objective functions above are quadratic with respect to product quality (q) and/or price (p_i). Since the developer has determined to launch either a paid or free version of its app product, the decisions for the developer are twofold: whether the developer should provide an

¹ I am aware that some app developers offer paid and free apps simultaneously; some even offer in-app purchase functionalities in free apps. Several papers have discussed software and app versioning issues for developers (Krishnan and Gupta, 2001). However, those papers only focus on one platform setting with an assumption that consumers have identical marginal disutility for different versions of a product. My paper focuses on developers' decisions facing multiple platforms. As my model is already very complex, I do not include product versioning in our model. Additionally, I also do not consider in-app purchases in our model since in-app purchases are subject to users' downloading first.

app in one or two platforms, and what quality and price(s) (if paid version) should be set for the app. Based on the combination of different scenarios, we have six cases for analysis (see Table 2). Since ad revenue per user (s) is exogenously determined by the market, the developer has three decision variables (2 pricing and 1 quality decisions) in Case P12; two decision variables (1 pricing and 1 quality decisions) in Cases P1 and P2; and one decision variable (1 quality decision) in Cases F1, F2, and F12.

Table 2. There are six strategies, denoted as Cases F1, F2, and F12 (free app cases) and P1, P2, and P12 (paid app cases). Decision variables include q , p_1 and/or p_2 ; s is exogenous unit ad revenue for free app, and “—” means that the developer does not provide app in that platform.

Ad/Paid Version in Product Offering in Six Cases						
<i>Free App Cases</i>				<i>Paid App Cases</i>		
Case	F1	F2	F12	P1	P2	P12
Platform 1	s	— —	s	p_1	— —	p_1
Platform 2	— —	s	s	— —	p_2	p_2

For the objective function in each case, I first examine the strict concavity condition and then solve the optimal solution for each case when the strict concavity condition holds. By substituting the optimal solution into the objective function we can obtain the optimal total profit (denoted by π_n , $n \in \{F1, F2, F12, P1, P2, P12\}$). Table 3 summarizes the optimal solutions and results for each case. The optimal conditions for each case are listed in Table 4. Note that the optimal solution and result for each case in Table 3 are derived only when the corresponding concavity and feasibility condition (in Table 4) hold.

Table 3. This table lists the optimal price(s), quality, and total profit for each case when the concavity condition holds.

Optimal Matrix for the Six Cases

Case	Price/Ad Revenue	Quality	Total Profit
F1	(s, --)	$q_0 - \frac{b}{2\beta} + \frac{s\theta_1}{k\beta}$	$\frac{1}{4k^2\beta} (b^2k^2 - 4bk(kq_0\beta + s\theta_1) + 4s(s\theta_1^2 + 2k\beta(q_0\theta_1 - st)))$
F2	(-- , s)	$q_0 - \frac{b}{2\beta} + \frac{sr\theta_2}{k\beta}$	$\frac{1}{4k^2\beta} (b^2k^2 - 4bk(kq_0\beta + rs\theta_2) + 4rs(rs\theta_2^2 + 2k\beta(q_0\theta_2 - st)))$
F12	(s, s)	$q_0 - \frac{b}{\beta} + \frac{s(\theta_1 + r\theta_2)}{k\beta}$	$\frac{1}{k^2\beta} (b^2k^2 - 2bk(kq_0\beta + s(\theta_1 + r\theta_2)) + s(s(\theta_1 + r\theta_2)^2 + 2k\beta(q_0\theta_1 + q_0r\theta_2 - (1+r)st)))$
P1	$(\frac{k(2q_0\beta - b)\theta_1}{4k\beta - 2\theta_1^2}, --)$	$\frac{k(2q_0\beta - b)}{2k\beta - \theta_1^2}$	$\frac{bk(b - 4q_0\beta) + 2q_0^2\beta\theta_1^2}{4k\beta - 2\theta_1^2}$
P2	$(--, \frac{k(2q_0\beta - b)\theta_2}{4k\beta - 2r\theta_2^2})$	$\frac{k(2q_0\beta - b)}{2k\beta - r\theta_2^2}$	$\frac{bk(b - 4q_0\beta) + 2q_0^2r\beta\theta_2^2}{4k\beta - 2r\theta_2^2}$
P12	$(\frac{k(q_0\beta - b)\theta_1}{2k\beta - \theta_1^2 - r\theta_2^2}, \frac{k(q_0\beta - b)\theta_2}{2k\beta - \theta_1^2 - r\theta_2^2})$	$\frac{2k(q_0\beta - b)}{2k\beta - \theta_1^2 - r\theta_2^2}$	$\frac{2b^2k - 4bkq_0\beta + q_0^2\beta(\theta_1^2 + r\theta_2^2)}{2k\beta - \theta_1^2 - r\theta_2^2}$

Table 4. This table lists the combined condition of concavity and feasibility for the six cases.

$$\theta_1 > \theta_2, \quad 0 < t < 1, \quad t_1 = \frac{\theta_1(2kq_0\beta + 2s\theta_1 - bk)}{2ks\beta}, \quad t_2 = \frac{\theta_2(2kq_0\beta + 2rs\theta_2 - bk)}{2ks\beta}, \quad t_3 = \frac{(bk - 2s\theta_1)(bk - 4kq_0\beta - 2s\theta_1)}{8ks^2\beta},$$

$$t_4 = \frac{(bk - 2rs\theta_2)(bk - 4kq_0\beta - 2rs\theta_2)}{8krs^2\beta}, \quad t_5 = \frac{(bk + s\theta_1(\theta_1 + r\theta_2))(bk - 2kq_0\beta + s\theta_1(\theta_1 + r\theta_2))}{2k(1+r)s^2\beta}, \quad t_6 = \frac{k\theta_2}{s(\theta_1 - \theta_2)}.$$

Combined Conditions of Concavity and Feasibility for the Six Cases

Case	Combined conditions of concavity and feasibility
F1	$b < \frac{2s\theta_1}{k} \text{ \& } t < \min\{t_1, t_3\}$
F2	$b < \frac{2rs\theta_2}{k} \text{ \& } t < \min\{t_2, t_4\}$
F12	$t < \min\{t_5, t_6\} \text{ \& } q_0\beta + \frac{s\theta_1}{k} - \frac{(k+st)\beta}{\theta_2} + \frac{rs\theta_2}{k} \leq b \leq q_0\beta + \frac{s(\theta_2(\theta_1+r\theta_2)-kt\beta)}{k\theta_2}$
P1	$\beta > \frac{\theta_1^2}{2k} \text{ \& } b < 2q_0\beta - \sqrt{2kq_0^2\beta(2k\beta - \theta_1^2)}/k$
P2	$\beta > \frac{r\theta_2^2}{2k} \text{ \& } b < 2q_0\beta - \sqrt{2kq_0^2\beta(2k\beta - r\theta_2^2)}/k$
P12	$\beta > \frac{\theta_1^2 + r\theta_2^2}{2k} \text{ \& } b < q_0\beta - \sqrt{kq_0^2\beta(2k\beta - \theta_1^2 - r\theta_2^2)}/(\sqrt{2}k)$

When strict concavity condition holds in any specific case, its optimal solutions can be obtained by setting first-order conditions to zero. The jointly strict concavity condition is $\beta > \frac{\theta_1^2 + r\theta_2^2}{2k}$ for Case P12, $\beta > \frac{\theta_1^2}{2k}$ for Case P1, and $\beta > \frac{r\theta_2^2}{2k}$ for Case P2, respectively, while Cases F1, F2 and F12 are always strictly concave at any β value.

When the jointly strict concavity does not hold, we have to find optimal solutions indirectly. The fact that the profit function is strictly concave in a single variable (q , p_1 or p_2) but not jointly strictly concave in all variables stems from the interaction between quality and price variables. I find that at any given quality level, the objective function is always strictly concave in price decision(s). Therefore, we can reduce the optimization problem by first solving for the optimal prices as a function of quality (step one), and then substituting the result back to solve for the optimal quality (step two) (Petruzzi and Dada, 1999). Using this approach, I find that the profit function derived at step two is strictly convex in quality for all cases (P1, P2, and P12). Note that the quality level has to be within $[q_0, \hat{q}]$. The optimal quality should be set as $\hat{q}$ when strict concavity does not hold for Cases P1, P2, and P12 since the profit function is monotonically increasing with respect to quality. Using Case P12 as an example, when β is greater than $\frac{\theta_1^2 + r\theta_2^2}{2k}$ (unbounded situation), the optimal solution (p_1, p_2, q) is $(\frac{k(q_0\beta - b)\theta_1}{2k\beta - \theta_1^2 - r\theta_2^2}, \frac{2r(q_0\beta - b)\theta_2}{2k\beta - \theta_1^2 - r\theta_2^2}, \frac{2k(q_0\beta - b)}{2k\beta - \theta_1^2 - r\theta_2^2})$. If β is smaller than $\frac{\theta_1^2 + r\theta_2^2}{2k}$ (bounded situation) or the optimal

quality is higher than $\hat{q}$, the optimal solution (p_1, p_2, q) should be $(\frac{\hat{q}\theta_1}{2}, \frac{\hat{q}\theta_2}{2}, \hat{q})$. Table 5 summarizes the optimal solutions and results for bounded cases P1, P2 and P12.

Table 5. The optimal price(s) and total profit for the three paid app cases when the concavity condition fails to hold or the optimal quality exceeds the quality upper bound $\hat{q}$.

Optimal Matrix for the Bounded Paid Cases

Case	Price/Unit Ad Revenue	Total Profit
P1	$(\frac{\hat{q}\theta_1}{2}, --)$	$\frac{\hat{q}^2\theta_1^2}{2k} - b\hat{q} - (q_0 - \hat{q})^2\beta$
P2	$(--, \frac{\hat{q}\theta_2}{2})$	$\frac{\hat{q}^2r\theta_2^2}{2k} - b\hat{q} - (q_0 - \hat{q})^2\beta$
P12	$(\frac{\hat{q}\theta_1}{2}, \frac{\hat{q}\theta_2}{2})$	$\frac{\hat{q}^2(\theta_1^2 + r\theta_2^2) - 4bk\hat{q} - 2k(q_0 - \hat{q})^2\beta}{2k}$

Overall, I find that in a two-platform market, a developer's product offering strategy is influenced by app development difficulty level (β), app launching cost parameter (b), consumers' marginal disutility (k), consumers' disutility sensitivity to apps (t for free apps and 1 for paid apps, respectively), consumers' willingness to pay (θ_i), consumer densities (1 or r), and quality upper bound ($\hat{q}$). Further, I present my major findings and insights in the following sections. Noting that Platforms 1 and 2 are quasi-symmetric in terms of analysis results, I assume $r > 1$ and $\theta_1^2 < r\theta_2^2$ for brevity. The above assumption represents the case that Platform 2 is more profitable (or attractive to app developers) than Platform 1; consequently, Case P1 is always dominated by Case P2 in paid app cases. With this assumption, I omit lengthy and symmetric analytical discussion when $\theta_1^2 > r\theta_2^2$. This assumption makes our analysis consistent with the app industry (iOS as Platform 1 vs. Android as Platform 2), where the market size of Android is approximately six times that of iOS (International Data Corporation, 2015) and users in iOS are willing to pay about twice as much as those in Android (e.g., Evans, 2014).

4.1 Paid Apps

As noted earlier, for paid app cases, the developer's profit function might not be concave. Denote $\overline{\pi_n}$ as the optimal total profit for Case n ($n \in \{P1, P2, P12\}$) when the optimal quality is $\hat{q}$. I show that either $\overline{\pi_{P2}}$ or $\overline{\pi_{P12}}$ can be dominant among all $\overline{\pi_n}$ s. Therefore, I offer Proposition 1 (proofs for all propositions are provided in Appendices):

Proposition 1. *Optimal Strategy for Bounded Paid App Offering:* For all paid strategies that optimal app quality is set as $\hat{q}$: (1) offering paid versions in both platforms (Case P12) is optimal

as long as $b < \frac{\hat{q}\theta_1^2}{2k}$; (2) otherwise, offering a paid version only in the more profitable platform (Case P2) is optimal. ■

Proposition 1 implies that in the cases that an app developer strives to exploit the maximal level of quality with the purpose of gaining app sales revenues from end users, the optimal strategy would be launching the app in both platforms (for example, iOS and Android) if the additional cost of launching the app is low enough. However, if an app's release in each platform requires a high investment, it becomes optimal to serve only the platform that is relatively more profitable (i.e., Platform 2 based on our assumption). Notably, the threshold value of app launching cost ($b < \frac{\hat{q}\theta_1^2}{2k}$) in deciding whether to serve both platforms represents the WTP of users in the less profitable platform (i.e., Platform 1). In other words, a developer should always launch its app product in the more profitable platform; offering the same app in another platform would depend on the developer's opinion of whether the platform is profitable enough. Note that offering apps in only Platform 1 is never optimal due to $\theta_1^2 < r\theta_2^2$. In the case when $\theta_1^2 > r\theta_2^2$, Platform 1 will become the first platform choice for the developer. Note also that although iOS users are, on average, willing to pay more than Android users, the ratio of θ_1/θ_2 can be different for different apps. Many apps are available in both iOS and Android platforms ($b < \frac{\hat{q}\theta_1^2}{2k}$). However, some paid apps such as Press (a RSS reader) and Tasker (a root access tool) are only available in Android due to their Android-users orientation (for these developers, $\theta_2 \gg \theta_1$). Some apps such as Super Mario Run game and iAnnotate (PDF note taker) are only available in iOS probably because the developers think iOS users have higher WTP (for these developers, $\theta_1 \gg \theta_2$).

On the other hand, in the cases when quality is set lower than the quality upper bound ($\hat{q}$) for the app product in achieving optimality, the concavity condition should hold. Similar to the bounded situation, I show that either Case P2 or P12 can be dominant among the three paid app cases. Therefore, I offer Proposition 2:

Proposition 2. *Optimal Strategy for Unbounded Paid App Offering:* For all paid strategies in which optimal app quality is lower than $\hat{q}$, (i) offering a paid version exclusively in the more profitable platform (Case P2) is optimal only when $r < \frac{2bk\beta(3b-4q_0\beta)+(b-2q_0\beta)^2\theta_1^2}{b(3b-4q_0\beta)\theta_2^2}$; (ii) otherwise, offering paid versions in both platforms (Case P12) is optimal. Case P1 is never optimal. ■

Proposition 2 suggests that the developer should launch its app product in the more profitable platform. However, whether it should offer the same version of the app in the other

platform depends on the user density (r) in the more profitable platform. Specifically, launching only in Platform 2 (Case P2) dominates launching in both platforms (Case P12) when Platform 2's user density is small ($r < \frac{2bk\beta(3b-4q_0\beta)+(b-2q_0\beta)^2\theta_1^2}{b(3b-4q_0\beta)\theta_2^2}$). Note that given our assumption of $r > 1$, if $\frac{2bk\beta(3b-4q_0\beta)+(b-2q_0\beta)^2\theta_1^2}{b(3b-4q_0\beta)\theta_2^2}$ is less than 1, then Case P12 will be optimal in all conditions. This proposition also implies that when the more profitable platform has a substantial user density ($r > \frac{2bk\beta(3b-4q_0\beta)+(b-2q_0\beta)^2\theta_1^2}{b(3b-4q_0\beta)\theta_2^2}$), launching in two platforms is always preferable. Case P1 in unbounded paid app is never optimal due to $\theta_1^2 < r\theta_2^2$.

I am interested in how the developer optimizes product prices in different platforms. By comparing results in different platforms, I find that the developer should always set the app price higher in the platform that has a higher WTP; that is, if $\theta_1 > \theta_2$, then $p_1^* > p_2^*$, and vice versa. In the mobile app market, I notice that for those paid apps offered in both iOS and Android, the price is often lower in Android than in iOS (Evans, 2014). At those optimal prices, a higher price does not always lead to a lower realized demand (represented by $m_{i,j}$); to be more specific, I find that $m_{1,p_1^*} < m_{2,p_2^*}$ only when $\theta_1 < r\theta_2$, and $m_{1,p_1^*} > m_{2,p_2^*}$ otherwise. For example, since Android's user base is six times that of iOS', the app developer can actually generate reasonable revenues by capturing more users from Android even when it is offered at a lower price in Android than it is in iOS. By comparing Cases P1, P2, and P12, I find that the optimal prices, quality and derived demands in dual-platform (P12) are always higher than those in single platforms (P1 and P2), which means $q_{P12}^* > q_{Pi}^*$, $p_{i,P12}^* > p_{i,Pi}^*$, and $m_{i,P12} > m_{i,Pi}$, where $i \in \{1,2\}$.

Propositions 1 and 2 apply to the paid mobile app market as well as the computer software market. In the computer software market, it was reported that the WTPs in both platforms (Windows and Mac) are similar ($\theta_1 \cong \theta_2$) and that Windows' market is 14 times as large as Mac's ($r = 14$) (netmarketshare.com, 2016). In the real market, I also find that for many developers such as Adobe, launching in both platforms is their strategy due to a larger r (Proposition 2). However, for developers of apps such as Minitab (a statistical tool), the parameter setting might be the reason that the app is still not offered to be directly installed on Mac.

4.2 Free Apps

When a developer chooses to offer a free app and gain advertisement revenue, there are three cases: platform 1 only (Case F1), platform 2 only (Case F2), and both platforms (Case

F12). Different from paid strategies in section 4.1 where Case P1 is dominated by Case P2, in free strategies all three cases (F1, F2, and F12) can be dominant under certain conditions, and the optimal condition depends on free app users' disutility sensitivity. Proposition 3 provides details of our finding.

Proposition 3. Optimal Strategy for Unbounded Free App Offering: For all free strategies in which optimal app quality is lower than $\hat{q}$, (i) offering free versions in both platforms (Case F12) is optimal as long as free app users' disutility sensitivity is $t < \min\{\varphi_1, \varphi_2\}$; (ii) offering a free version in only Platform 2 (Case F2) is optimal as long as $\varphi_2 < t < \varphi_3$; and (iii) offering a free version in only Platform 1 (Case F1) is optimal as long as $t > \max\{\varphi_1, \varphi_3\}$, where: $\varphi_1 = \frac{(bk-2rs\theta_2)(3bk-4kq_0\beta-2s(\theta_1+r\theta_2))}{8krs^2\beta}$, $\varphi_2 = \frac{(bk-2s\theta_1)(3bk-4kq_0\beta-2s(\theta_1+2r\theta_2))}{8ks^2\beta}$ and $\varphi_3 = \frac{(r\theta_2-\theta_1)(2kq_0\beta-bk+s(\theta_1+r\theta_2))}{2k(r-1)s\beta}$. ■

Note that while the order among φ_1 , φ_2 , and φ_3 is uncertain, there might be six scenarios of order for the three critical values of t . As a result, when none of the conditions (i), (ii), and (iii) hold, the developer needs to compare π_{F1} , π_{F2} , and π_{F12} to find the optimal case. Fig. 2 demonstrates that while the favorable conditions for Cases F1, F2, and F12 align with Proposition 3, the optimal conditions of Cases F1, F2, and F12 are different.

Fig. 2. Threshold of App Users Disutility Sensitivity for Free App Offering Strategy Dominance. This figure shows how free app users' disutility sensitivities impact the dominance of free app offering strategies. In each scenario, the order among φ_1 , φ_2 and φ_3 is sure. With that order, in each separate space, one case is dominant or it is unclear which strategy is dominant. For instance, in scenario (a), when $t < \varphi_1$, Case F12 is dominant; when $\varphi_1 < t < \varphi_2$, analytically no case is certainly dominant. "(F1,F2,F12)" means that the developer needs to compare profits of all cases to find the optimal strategy.

Proposition 3 implies that the developer should examine users' disutility sensitivity (t)

when determining the optimal product offering strategy for a free app. Specifically, if app users are highly tolerant of advertisements (i.e., t is very low), the developer most likely will offer the free app in both platforms (iOS and Android). By doing so, the developer can capture as many users as possible while still placing advertisements in the app targeting those users, thereby gaining as much revenue as possible. However, if app users are intolerant of advertisements (i.e., t is close to 1), the developer should focus on only one platform. I observe that some free apps in the current marketplace (e.g., Facebook) are available in both iOS and Android, while other free apps that are offered in only iOS (e.g., Fresh Air, a weather app) or Android (e.g., Muzei, a live wallpaper app), and I believe that developers of different free apps may face different user segments and market conditions.

In addition, I find that the free app's optimal quality and derived demand in dual-platform strategy (Case F12) are always higher than those in single-platform strategies (Cases F1 and F2), as in $q_{F12}^* > q_{Fi}^*$ and $m_{i,F12} > m_{i,Fi}$, where $i \in \{1,2\}$. In contrast to single-platform strategies, two-platform strategies enable the developer to generate higher gross revenue from larger realized demand; however, the additional cost of launching the app in an extra platform cannot guarantee that F12 is always a dominant strategy. In other words, when choosing whether to enter an additional platform, the developer has to make sure the additional costs of launching the same app in that platform will be offset by the incremental advertisement revenues, which is affected by users' tolerance to advertisements.

Since the app quality in bounded free app cases is set at the same level in all of the three cases (i.e., $q_{Fi}^* = \hat{q}$ ($i \in \{1,2,12\}$)), developers need only decide on platform selection. Proposition 4 summarizes our finding for free strategies when app quality reaches the upper limit.

Proposition 4. Optimal Strategy for Bounded Free App Offering: For all free strategies in which optimal app quality is $\hat{q}$, launching in both platforms (Case F12) is optimal as long as app launching cost is $b < \frac{2\hat{q}s(\theta_1+r\theta_2)-2(1+r)s^2t-k(\hat{q}-q_0)^2\beta}{2k\hat{q}}$ and free app users' disutility sensitivity is $t < \min\{\hat{q}\theta_1/s, \hat{q}\theta_2/s\}$; otherwise, launching in only Platform 1 (Case F1) is dominant when $r < \frac{\theta_1\hat{q}-st}{\theta_2\hat{q}-st}$, and launching in Platform 2 (Case F2) is dominant when $r > \frac{\theta_1\hat{q}-st}{\theta_2\hat{q}-st}$. ■

Proposition 4 implies that when $q = \hat{q}$, both high users' tolerance of advertisements in free apps ($t < \min\{\hat{q}\theta_1/s, \hat{q}\theta_2/s\}$, thus rendering a higher realized demand) and low app launching costs ($b < \frac{2\hat{q}s(\theta_1+r\theta_2)-2(1+r)s^2t-k(\hat{q}-q_0)^2\beta}{2k\hat{q}}$, thus extra revenue from another platform will cover app launching costs) are required for the developer to launch a free app in

both platforms. When those two conditions do not hold, the optimal strategy depends on the user densities and WTPs in the two platforms. Our result shows that a two-platform strategy (bounded Case F12) results in higher quality level and higher realized demand compared to that in any one-platform strategy (bounded Cases F1 and F2); that is, $q_{F12} > q_{Fi}$ and $m_{i,F12} > m_{i,Fi}$ where $i \in \{1,2\}$.

Thus far, I have investigated the strategies for either paid or free apps separately. In the next subsection I relax the assumption that the developer has a pre-determined paid or free strategy in launching its app. I am interested in how app users' tolerance to advertisements affects a developer's strategic platform choice. For analytical tractability, I only consider the situation when the developer has already chosen which platform(s) to serve (i.e., either one or two platforms) and now needs to decide whether to offer the product as a free or paid app.

4.3 When Should Developers Adopt Free or Paid Strategies?

In this section, I compare corresponding free and paid contexts when the quality levels are lower than the quality upper bound ($q^* < \hat{q}$). Regarding the contexts in which free or paid app options are optimal, I offer Proposition 5:

Proposition 5. Releasing a free version of an app (compared to a paid version) is a better choice if users' disutility sensitivity to advertisements (t) is smaller than φ_4 for two-platform strategies ($\pi_{F12} > \pi_{P12}$), φ_5 when launching the app in only Platform 1 ($\pi_{F1} > \pi_{P1}$), and φ_6 when launching the app in only Platform 2 ($\pi_{F2} > \pi_{P2}$), where: $\varphi_4 =$

$$\frac{(2k^3\beta(b-q_0\beta)^2-(kq_0\beta+s\theta_1-bk)^2(2k\beta-\theta_1^2-r\theta_2^2)-2rs(kq_0\beta+s\theta_1-bk)\theta_2(2k\beta-\theta_1^2-r\theta_2^2)+r^2s^2\theta_2^2(2k\beta-\theta_1^2-r\theta_2^2))}{(2k(1+r)s^2\beta(2k\beta-\theta_1^2-r\theta_2^2))},$$

$$\varphi_5 = \frac{\theta_1(\theta_1^3+8ks\beta\theta_1(s-q_0\theta_1)-b^2k^2\theta_1-4s^2-4k^2q_0\beta^2(q_0\theta_1-4s)+4bk(s\theta_1^2+k\beta(q_0\theta_1-2s)))}{8ks^2\beta(2k\beta-\theta_1^2)}, \quad \text{and} \quad \varphi_6 =$$

$$\frac{\theta_2(8krs\beta\theta_2(s-q_0\theta_2)-b^2k^2\theta_2-4r^2\theta_2^3-4k^2q_0\beta^2(q_0\theta_2-4s)+4bk(rs\theta_2^2+k\beta(q_0\theta_2-2s)))}{8ks^2\beta(2k\beta-r\theta_2^2)}. \blacksquare$$

Note that t represents both the disutility sensitivity for free apps and the disutility gap between paid users and free-with-ads users; the lower t is, the higher the disutility gap users perceive between paid and free apps. Proposition 5 implies that for the same group of app users more tolerant to the disutility arisen from pop-up advertisements yet relatively sensitive to a paid app's price, a developer gains more advantage from providing a free version rather than a paid version. Proposition 5 also indicates that developers choosing to launch a free app instead of a paid app in both platforms, a larger disutility gap is required (i.e., smaller t). Note that if

any of φ_4 , φ_5 , or φ_6 is smaller than zero, the paid case dominates the corresponding free case; if any of φ_4 , φ_5 , or φ_6 is greater than 1, the free case dominates the corresponding paid case. Many apps (e.g., Facebook) are free in both iOS and Android; some free apps that are offered in only Android (e.g., Solid Explorer) or iOS (e.g., Fresh Air).

4.4 Static Sensitivity Analysis

I conduct a static sensitivity analysis in order to examine the impact of each parameter on corresponding optimal metrics for the unbounded cases. The sensitivity analysis results shown in Table 6 present several additional insights. First, both user density and users' WTP (θ_i) are positively related to all of the optimal metrics, including product quality (q), price (p_i), demand in each platform (m_i), total cost (TC), and total profit (π). Second, for free cases, prior quality level (q_0) is positively related to all of the other resulting variables; however, for paid cases, the positive association between prior app quality and total cost (or total profit) becomes uncertain. Third, users' disutility sensitivity to advertisement (t) negative impacts a free app's demand and the total profit generated by a developer, yet it does not affect the optimal quality of the app. Also, the app launching cost parameter (b) is negatively associated with app quality, realized demand, and total profit. Lastly, for paid cases, users' marginal disutility (k) negatively affects all of the resulting variables; however, for free cases, it remains unclear whether users' marginal disutility can negatively influence demand and total profit. Also, app development difficulty parameter (β) has negative impact on quality, realized demands, and total cost in free cases; however, its impacts on quality, realized demands, and total cost are uncertain in paid cases.

Table 6. This table summarizes the results of a static sensitivity analysis. The parameters in the first column are the changing variables. For each row, I increase ($\uparrow$ as increasing) the value of the changing variable while keeping other parameters’ values unchanged, and examine how every resulting outcome reacts. For example, increasing unit ad revenue ($s \uparrow$) will result in an increase of optimal total profit ($\pi^* \uparrow$) in Case F1. The symbol “ $\uparrow$ ($\downarrow$)” means increase (decrease), “ $\sim$ ” means “indeterminate”, “NC” means no impact, and “NA” means “not applicable”.

Comparative Statics

Free Cases					F1					F2					F12				
	q^*	m_1^*	TC^*	π^*		q^*	m_2^*	TC^*	π^*		q^*	m_1^*	m_2^*	TC^*	π^*				
$s \uparrow$	$\uparrow$	$\sim$	$\uparrow$	$\sim$		$\uparrow$	$\sim$	$\uparrow$	$\sim$		$\uparrow$	$\sim$	$\sim$	$\uparrow$	$\sim$				
$q_0 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$		$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$		$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$				
$\beta \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$				
$k \uparrow$	$\downarrow$	$\sim$	$\downarrow$	$\sim$		$\downarrow$	$\sim$	$\downarrow$	$\sim$		$\downarrow$	$\sim$	$\sim$	$\downarrow$	$\sim$				
$r \uparrow$	NA	NA	NA	NA		$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$		$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$				
$\theta_1 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$		NA	NA	NA	NA		$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$				
$\theta_2 \uparrow$	NA	NA	NA	NA		$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$		$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$				
$b \uparrow$	$\downarrow$	$\downarrow$	$\sim$	$\downarrow$		$\downarrow$	$\downarrow$	$\sim$	$\downarrow$		$\downarrow$	$\downarrow$	$\downarrow$	$\sim$	$\downarrow$				
$t \uparrow$	NC	$\downarrow$	NC	$\downarrow$		NC	$\downarrow$	NC	$\downarrow$		NC	$\downarrow$	$\downarrow$	NC	$\downarrow$				
Paid Cases					P1					P2					P12				
	q^*	p_1^*	m_1^*	TC^*	π^*	q^*	p_2^*	m_2^*	TC^*	π^*	q^*	p_1^*	p_2^*	m_1^*	m_2^*	TC^*	π^*		
$q_0 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\sim$	$\sim$	$\uparrow$	$\uparrow$	$\uparrow$	$\sim$	$\sim$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\sim$	$\sim$		
$\beta \uparrow$	$\sim$	$\sim$	$\sim$	$\sim$	$\downarrow$	$\sim$	$\sim$	$\sim$	$\sim$	$\downarrow$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$	$\downarrow$		
$k \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$		
$r \uparrow$	NA	NA	NA	NA	NA	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$		
$\theta_1 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	NA	NA	NA	NA	NA	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$		
$\theta_2 \uparrow$	NA	NA	NA	NA	NA	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$		
$b \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\sim$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\sim$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\sim$	$\downarrow$		

5. Numerical Examples

In this section I conduct numerical examples to investigate the dynamics of app offering problems. In the mobile app industry, I consider iOS and Android as the two platforms that a developer considers. Given that Android’s market size is about six times larger than iOS’ (International Data Corporation, 2015), I treat iOS and Android as Platforms 1 and 2, respectively, and set $r = 6$. According to Evans (2014), iOS users are willing to spend about twice the amount as Android users. I thus simply set $\theta_1 = 20$ and $\theta_2 = 10$, respectively. For simplicity, I assume that users’ marginal disutility parameter (k) is 50. I use Facetune as an example app. Facetune is an easy to use app that edits portrait and selfie pictures. The developer of Facetune charges \$3.99 and \$2.99 for iOS and Android users, respectively. Assume the developer’s prior quality level (q_0) and quality upper bound ($\hat{q}$) are 10 and 20, respectively; app development difficulty level (β) is 12 and app launching cost parameter (b) is 1. The result shows that optimal total profit in Case P12 is the highest among the three paid app cases. Given that $\theta_1^2 < r\theta_2^2$ in this setting, the above result is consistent with Proposition 2 (i.e., the optimal

strategy is to provide paid versions of the app in both platforms because $r > \frac{2bk\beta(3b-4q_0\beta)+(b-2q_0\beta)^2\theta_1^2}{b(3b-4q_0\beta)\theta_2^2}$). In Fig. 3, I plot the dominant regions of paid app cases by varying user density in Platform 2 (r) and app launching cost parameters (b). When $r < 4$ only Cases P1 and P12 can be optimal; when $r > 4$ only Cases P1 and P12 can be optimal ($r = 4$ is the dividing line of $\theta_1^2 = r\theta_2^2$). I observe that Case P12's favorable condition occurs when the app launching cost (b) is low; when the app launching cost is high, a single-platform option (Cases P1 or P2) is preferred. I also suggest that the firm sets its quality level to its upper bound ($\hat{q}$) when user density in Platform 2 is higher than a certain level, where bounded cases are optimal. Fig. 3 also indicates that when the user density in Platform 2 is low and the app launching cost is high, the app developer should not develop the app.

Fig. 3. Dominant Strategies and Favorite Conditions for Paid App Offering. This figure illustrates the dominance among the three paid app cases along with changing values of user density in Platform 2 (r) and app launching cost parameter (b). I set $q_0 = 10$, $\hat{q} = 20$, $\beta = 12$, $\theta_1 = 20$, $\theta_2 = 10$, and $k = 50$. P2B and P12B are P2 and P12 with $q^* = \hat{q}$, respectively.

In the second example, I perform numerical analysis for free app strategies (i.e., Cases F1, F2, and F12). I set the parameter values as follows: $r = 6$, $\theta_1 = 20$, $\theta_2 = 10$, $\beta = 12$, $k = 30$, $q_0 = 10$, $\hat{q} = 15$, and $t = 0.6$. I plot these three cases in a plane in which the unit ad revenue (s) ranges between 3 and 60 and the app launching cost (b) ranges between 5 and 60. In Fig. 4 I find that Case F12 is favored when s is large and b is small, and Case F2 is favored when b is large at any given value of s . The thick border line separates between Cases F12 and F2. In Fig. 4, the dashed line shows where the optimal quality reaches its upper bound; this

dashed line shifts to the right if the quality upper bound becomes larger. The developer should design its app to its quality upper bound when s is relatively large and b is relatively small. Note that, in the two single-platform cases (Cases F1 and F2), one case is always dominated by another (Case F1 is dominated by Case F2 in the setting) when we present the optimal case in a two-dimension plane.

Fig. 4. Dominant Strategies and Favorite Conditions for Free App Offering. This figure illustrates how app launching cost parameter (b) and unit ad revenue (s) influence the optimal total profits for the three free app cases. I set $r = 6$, $\theta_1 = 20$ and $\theta_2 = 10$, $\beta = 12$, $k = 30$, $q_0 = 10$, $\hat{q} = 15$, and $t = 0.6$. F2B and F12B are F2 and F12 with $q^* = \hat{q}$, respectively. Case F1 is dominated by Case F2 in this setting.

In the last example, I compare two-platform strategies between paid and free app offerings (Cases F12 and P12). I explore situations in which a developer should launch a paid or free version for its app product. I allow users' willingness-to-pay (WTP) to vary in each platform (θ_i) but fix the values of other parameters as follows: $r = 6$, $s = \$30$, $t = 0.5$, $\beta = 12$, $k = 30$, $q_0 = 5$, $\hat{q} = 50$, and $b = 10$. Fig. 5 demonstrates the dynamics with this setting. I find that users' WTP in each platform is positively associated with total profit in both Cases F12 and P12, since higher WTP leads to higher quality level and realized demand for the developer. Furthermore, I find that when users in Platform 1 are less willing to pay ($\theta_1 < 9$ in this example), offering a free app (Case F12) is the better choice (Case P12); otherwise, when users in that platform are very willing to pay ($\theta_1 > 9$), the developer should adopt a paid strategy. In short, the choice between paid and free strategies depends on the relative WTP levels between

the two platforms. It is significant that users' WTP positively impacts developer's total profit; conversely, users' disutility sensitivity to advertisement or app price negatively affects the developer's total profit. Therefore, the above results are consistent with Proposition 5.

Fig. 5. Users' Willingness-to-Pay and Dominant Two-Platform Strategies. Focusing on two-platform strategies, this figure presents how users' willingness-to-pay in Platforms 1 and 2 (θ_1 and θ_2 , respectively) influences the profits of Case F12 (offering free app in both platforms) and Case P12 (offering paid app in both platforms). Assume that $r = 6$, $s = \$30$, $t = 0.5$, $\beta = 12$, $k = 30$, $q_0 = 5$, $\hat{q} = 50$, and $b = 10$. With this setting, approximately, when $\theta_1 > 9$, P12 is dominant; when $\theta_1 < 9$, F12 is dominant; when θ_1 is very small, neither case is feasible.

6. Conclusion and Future Extension

In this paper, I develop a stylized model for analyzing developers' app offering strategy when facing two platforms. My model incorporates several characteristics of the app market: different willingness-to-pays across platforms, different sizes of user base, different users' disutility sensitivities to free and paid apps, app launching costs in each platform, and potential app quality upper bounds. The major contribution of this paper is that it enriches product-offering literature for two-sided markets by providing decision rules from the perspective of app developers. The results I got can also apply to a context in which advertisement revenue is

absent and only a paid app is offered (e.g., computer app market). From my analysis and numerical examples, I derive several substantive insights.

First, I find that in paid app contexts, a developer should launch its app product in the more profitable platform; whether the developer should also launch the same app in the other platform depends on the app launching cost. Specifically, when the quality of the app developed reaches its upper limit, launching in both platforms is always optimal as long as the app launching cost is low. Also, when the app quality does not reach the upper bound, launching in two platforms is always preferable as long as the more profitable platform has significant advantage (user base and/or WTP) relative to the other. For free app strategies, I find that launching in two platforms is a better choice if app users are highly tolerant of advertisements embedded within apps. I suggest that developers identify the more profitable (or attractive) platform when deciding on their app offering strategies. A platform's attractiveness can be determined through users' willingness to pay and user base size in the potential platform. In other words, the developer should balance revenues (from either app sales or in-app advertising) from entering a platform and the app launching cost.

Second, I show that if users' disutility sensitivity to advertisement is minimal compared with sensitivity to app price, it is better to provide free apps rather than paid apps (Proposition 5). Also, among all cases in which free apps are offered, the developer should launch free apps in both platforms when users have low disutility sensitivity to in-app advertisement (Proposition 3). In practice, since an app developer usually decides whether to charge end-users and how much to charge, knowing user disutility sensitivity to in-app advertisement is critical to an app developer's decision. Note that this disutility sensitivity for different app categories and user segments may vary significantly. I observe that there are more free apps for gaming than for free apps that provide great productivity (e.g., PDF note taking app or professional video editing app); hence, app developers should understand their target segments and users well since the market conditions for different apps vary greatly.

Finally, I provide analytical results of optimal app quality and prices for any cases considered in my model. I find that factors such as user density and willingness to pay in different platforms, app development and launching cost parameters, inherited quality level, quality upper bound, unit advertisement revenue, and consumers' disutility sensitivity to advertisement affect the optimal quality and pricing decisions. However, the platform selection decision primarily hinged on users' densities and willingness to pay in different platforms, and on consumers' disutility sensitivity to advertisements. The results derived also suggest that the paid app in a platform with higher willingness-to-pay should be priced higher. If the developer

decides to offer an app (paid or free) in both platforms, the app's quality and prices (only applied to paid app case) should be set higher.

There are several limitations of the present paper that may point to directions for future research. First, I assume that a developer considers providing only one app (free or paid) in any platform. A multi-platform and multi-product offering strategy can be a future research extension. Second, in-app purchases have become an important revenue source for app developers (Hyam, 2016), and many apps are offered as a "freemium" in which users have an option to upgrade once they try using and subsequently enjoy the app. A two-stage decision model may be suitable for app offering strategies with in-app purchase options. Third, my model assumes equivalent app launching cost for different platforms. In practice, the launching cost could be platform specific for a developer. Complicated with different user bases and WTPs across multiple platforms (e.g., Windows, Mac, and Linux in computer market), platform selection for app development may have different criteria. Fourth, due to the complexity of my current model, the present research does not address the situation in which a developer offers a free app in one platform but a paid version in another platform. Future studies that examine this scenario would be a valued direction for future examination. Last, I assume a developer as a monopolist within the marketplace in the model. Future research could also consider developer competition to extend my model.

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Appendices

Appendix A: Proof for Proposition 1.

Among all of the six cases, in Cases P1 (offering paid version in only Platform 1), P2 (offering paid version in only Platform 2) and P12 (offering paid version in both platforms) strict concavity condition may not hold within certain β ranges. For these three cases, when strict concavity conditions fail to hold, the optimal total profits obtained are shown in Table 5, i.e.: $\overline{\pi}_{P1} = \frac{\hat{q}^2\theta_1^2}{2k} - b\hat{q} - (q_0 - \hat{q})^2\beta$, $\overline{\pi}_{P2} = \frac{\hat{q}^2r\theta_2^2}{2k} - b\hat{q} - (q_0 - \hat{q})^2\beta$, and $\overline{\pi}_{P12} = \frac{\hat{q}^2(\theta_1^2 + r\theta_2^2) - 4bk\hat{q} - 2k(q_0 - \hat{q})^2\beta}{2k}$.

Since I have assumed “ $\theta_1^2 < r\theta_2^2$ ” (i.e., Platform 2 is more attractive), $\overline{\pi}_{P2} - \overline{\pi}_{P1} = \frac{\hat{q}^2(r\theta_2^2 - \theta_1^2)}{2k} > 0$ always holds, meaning Case P1 is always dominated by Case P2.

Additionally, I have $\overline{\pi}_{P12} - \overline{\pi}_{P2} = \frac{1}{2}\hat{q}(\frac{\hat{q}\theta_1^2}{k} - 2b)$. Hence, if $\frac{\hat{q}\theta_1^2}{k} - 2b > 0$ (or $b < \frac{\hat{q}\theta_1^2}{2k}$), $\overline{\pi}_{P12} - \overline{\pi}_{P2} > 0$; otherwise, $\overline{\pi}_{P12} - \overline{\pi}_{P2} \leq 0$ if $b \geq \frac{\hat{q}\theta_1^2}{2k}$.

In summary, among the three paid version cases when optimal app quality level is bounded at the upper bound level ($\hat{q}$), either Case P12 (when $b < \frac{\hat{q}\theta_1^2}{2k}$) or Case P2 (when $b \geq \frac{\hat{q}\theta_1^2}{2k}$) is optimal.

Appendix B: Proof for Proposition 2.

There are three paid app strategies in our research setting: Cases P1 (paid version in only Platform 1), P2 (paid version in only Platform 2) and P12 (paid version in both platforms). The fact that the optimal app quality is lower than quality upper bound (i.e., $q^* = \hat{q}$) indicates that the concavity condition is satisfied in these cases. Hence, the optimal total profits obtained are shown in Table 3, i.e.: $\pi_{P1} = \frac{bk(b - 4q_0\beta) + 2q_0^2\beta\theta_1^2}{4k\beta - 2\theta_1^2}$, $\pi_{P2} = \frac{bk(b - 4q_0\beta) + 2q_0^2r\beta\theta_2^2}{4k\beta - 2r\theta_2^2}$, and $\pi_{P12} = \frac{2b^2k - 4bkq_0\beta + q_0^2\beta(\theta_1^2 + r\theta_2^2)}{2k\beta - \theta_1^2 - r\theta_2^2}$.

Since I have assumed “ $\theta_1^2 < r\theta_2^2$ ” (i.e., Platform 2 is more attractive), I deduce that $\pi_{P2} - \pi_{P1} = \frac{k(b - 2q_0\beta)^2(r\theta_2^2 - \theta_1^2)}{2(2k\beta - \theta_1^2)(2k\beta - r\theta_2^2)} > 0$ always holds, meaning that Case P1 is always dominated by Case P2.

Additionally, I offer that: $\pi_{P12} - \pi_{P2} = \frac{2b^2k - 4bkq_0\beta + q_0^2\beta(\theta_1^2 + r\theta_2^2)}{2k\beta - \theta_1^2 - r\theta_2^2} - \frac{bk(b - 4q_0\beta) + 2q_0^2r\beta\theta_2^2}{4k\beta - 2r\theta_2^2}$

$$= \frac{k(b-2q_0\beta)^2}{2(-2k\beta+r\theta_2^2)} - \frac{2k(b-q_0\beta)^2}{-2k\beta+\theta_1^2+r\theta_2^2}.$$

To make $\pi_{P12} - \pi_{P2} > 0$, $\frac{k(b-2q_0\beta)^2}{2(-2k\beta+r\theta_2^2)} + \frac{2k(b-q_0\beta)^2}{-2k\beta+\theta_1^2+r\theta_2^2} > 0$ must hold. The above inequality can be transformed to $-(b-2q_0\beta)^2(2k\beta-\theta_1^2-r\theta_2^2) + 2(b-q_0\beta)^2(2(2k\beta-r\theta_2^2)) > 0 \Rightarrow 6b^2k^2\beta - 8bk^2q_0\beta^2 + b^2k\theta_1^2 - 4bkq_0\beta\theta_1^2 + 4kq_0^2\beta^2\theta_1^2 - 3b^2kr\theta_2^2 + 4bkq_0r\beta\theta_2^2 > 0$
 $\Rightarrow (b-2q_0\beta)^2\theta_1^2 + b(3b-4q_0\beta)(2k\beta-r\theta_2^2) > 0$ Eq. (B.1).

Since $2k\beta - r\theta_2^2 > 0$ (concavity condition for Case P2), Eq. (B.1) holds if $b > \frac{4q_0\beta}{3}$. However, given $b < q_0\beta - \sqrt{kq_0^2\beta(2k\beta-\theta_1^2-r\theta_2^2)}/(\sqrt{2}k)$ (feasibility condition for Case P12), $b > \frac{4q_0\beta}{3}$ would never occur. Hence, if $b < \frac{4q_0\beta}{3}$, Eq. (B.1) can hold only when $r > \frac{2bk\beta(3b-4q_0\beta)+(b-2q_0\beta)^2\theta_1^2}{b(3b-4q_0\beta)\theta_2^2}$. In other words, $\pi_{P2} > \pi_{P12}$ only when $b < \frac{4q_0\beta}{3}$ and $r < \frac{2bk\beta(3b-4q_0\beta)+(b-2q_0\beta)^2\theta_1^2}{b(3b-4q_0\beta)\theta_2^2}$. Further, $b < \frac{4q_0\beta}{3}$ always hold as $b < q_0\beta$ (feasibility condition for Case P12). So $\pi_{P2} > \pi_{P12}$ only when $r < \frac{2bk\beta(3b-4q_0\beta)+(b-2q_0\beta)^2\theta_1^2}{b(3b-4q_0\beta)\theta_2^2}$.

In summary, for paid app strategies when optimal app quality does not reach quality upper bound, Case P2 is dominant if $r < \frac{2bk\beta(3b-4q_0\beta)+(b-2q_0\beta)^2\theta_1^2}{b(3b-4q_0\beta)\theta_2^2}$; otherwise, Case P12 is dominant.

Appendix C: Proof for Proposition 3.

There are three free app strategies in our research setting: Cases F1 (free version in only Platform 1), F2 (free version in only Platform 2) and F12 (free version in both platforms). The objective functions of these three cases are all strictly concave. Assuming “ $r > 1$ and $\theta_1^2 < r\theta_2^2$ ” and that the optimal quality does not exceed the quality upper bound ($\hat{q}$), I obtain the optimal total profits which are shown in Table 3, i.e.:

$$\pi_{F1} = \frac{1}{4k^2\beta}(b^2k^2 - 4bk(kq_0\beta + s\theta_1) + 4s(s\theta_1^2 + 2k\beta(q_0\theta_1 - st))),$$

$$\pi_{F2} = \frac{1}{4k^2\beta}(b^2k^2 - 4bk(kq_0\beta + rs\theta_2) + 4rs(rs\theta_2^2 + 2k\beta(q_0\theta_2 - st))), \text{ and}$$

$$\pi_{F12} = \frac{1}{k^2\beta}(b^2k^2 - 2bk(kq_0\beta + s(\theta_1 + r\theta_2)) + s(s(\theta_1 + r\theta_2)^2 + 2k\beta(q_0\theta_1 + q_0r\theta_2 - (1+r)st))).$$

Thus, $\pi_{F12} - \pi_{F1} = \frac{3b^2k^2 + 4rs(2k\beta(q_0\theta_2 - st) + s\theta_2(2\theta_1 + r\theta_2)) - 4bk(kq_0\beta + s(\theta_1 + 2r\theta_2))}{4k^2\beta}$; and so

$$\pi_{F12} > \pi_{F1} \text{ if } t \leq \frac{(bk-2rs\theta_2)(3bk-4kq_0\beta-4s\theta_1-2rs\theta_2)}{8krs^2\beta} = \varphi_1.$$

$$\text{Also, } \pi_{F12} - \pi_{F2} = \frac{3b^2k^2-4bk(kq_0\beta+2s\theta_1+rs\theta_2)+4s(2k\beta(q_0\theta_1-st)+s\theta_1(\theta_1+2r\theta_2))}{4k^2\beta} ; \text{ so}$$

$$\pi_{F12} > \pi_{F2} \text{ if } t \leq \frac{(2s\theta_1-bk)(4kq_0\beta-3bk+2s(\theta_1+2r\theta_2))}{8ks^2\beta} = \varphi_2.$$

$$\text{In addition, } \pi_{F2} - \pi_{F1} = \frac{s(s(r\theta_2-\theta_1)(\theta_1+r\theta_2)+k(2(r-1)st\beta+(2q_0\beta-b)(r\theta_2-\theta_1)))}{k^2\beta} ; \text{ thus}$$

$$\pi_{F2} > \pi_{F1} \text{ if } t \leq \frac{(r\theta_2-\theta_1)(2kq_0\beta-bk+s(\theta_1+r\theta_2))}{2k(r-1)s\beta} = \varphi_3.$$

In summary, Case F12 is dominant if $t < \min\{\varphi_1, \varphi_2\}$; Case F2 is dominant if $\varphi_2 < t < \varphi_3$; and Case F1 is dominant if $t > \max\{\varphi_1, \varphi_3\}$.

Appendix D: Proof for Proposition 4.

Similar to the situation in Proposition 3, there are three free app strategies when $q^* = \hat{q}$ in our research setting: Cases F1 (free version in only Platform 1), F2 (free version in only Platform 2) and F12 (free version in both platforms). I obtain the optimal total profits which are shown in Table 5 (i.e. $\overline{\pi}_{F1} = \frac{2s(\hat{q}\theta_1-st)}{k} - (\hat{q} - q_0)^2\beta - b\hat{q}$, $\overline{\pi}_{F2} = \frac{2rs(\hat{q}\theta_2-st)}{k} - (\hat{q} - q_0)^2\beta - b\hat{q}$, and

$$\overline{\pi}_{F12} = \frac{2s(\hat{q}\theta_1-st)}{k} + \frac{2rs(\hat{q}\theta_2-st)}{k} - (\hat{q} - q_0)^2\beta - 2b\hat{q}.$$

Thus, $\overline{\pi}_{F12} - \overline{\pi}_{F1} = \frac{2rs(\hat{q}\theta_2-st)}{k} - b\hat{q} = \overline{\pi}_{F2} + (\hat{q} - q_0)^2\beta > 0$, and $\overline{\pi}_{F12} - \overline{\pi}_{F2} = \frac{2s(\hat{q}\theta_1-st)}{k} - b\hat{q} = \overline{\pi}_{F1} + (\hat{q} - q_0)^2\beta > 0$. Therefore, when all free cases are bounded (i.e., $q^* = \hat{q}$), Case F12 is dominant as long as Case F12 is feasible (i.e., $b < \frac{2\hat{q}s(\theta_1+r\theta_2)-2(1+r)s^2t-k(\hat{q}-q_0)^2\beta}{2k\hat{q}}$ and $t < \min\{\hat{q}\theta_1/s, \hat{q}\theta_2/s\}$).

When $b > \frac{2\hat{q}s(\theta_1+r\theta_2)-2(1+r)s^2t-k(\hat{q}-q_0)^2\beta}{2k\hat{q}}$ and/or $t > \min\{\hat{q}\theta_1/s, \hat{q}\theta_2/s\}$, Case

F12 is infeasible. In this situation, I need only compare Cases F1 and F2. $\overline{\pi}_{F2} - \overline{\pi}_{F1} = \frac{2s((r-1)st+\hat{q}(r\theta_2-\theta_1))}{k}$. Thus, $\overline{\pi}_{F2} - \overline{\pi}_{F1} > 0$ if $r > \frac{\theta_1\hat{q}-st}{\theta_2\hat{q}-st}$ and $\overline{\pi}_{F2} - \overline{\pi}_{F1} < 0$ if $r < \frac{\theta_1\hat{q}-st}{\theta_2\hat{q}-st}$.

Appendix E: Proof for Proposition 5.

In this proposition, I compare optimal total profits between Case F12 (offering free versions in both platforms) and Case P12 (offering paid versions in both platforms), between Case F1 (offering free version in only Platform 1) and Case P1 (offering paid version in only Platform 1), and between Case F2 (offering free version in only Platform 2) and Case P2 (offering paid version in only Platform 2).

For two-platform strategies, I compare Case F12 against Case P12. Since

$$\begin{aligned}\pi_{F12} &= \frac{1}{k^2\beta} (b^2k^2 - 2bk(kq_0\beta + s(\theta_1 + r\theta_2)) + s(s(\theta_1 + r\theta_2)^2 + 2k\beta(q_0\theta_1 + q_0r\theta_2 - \\ &(1+r)st))) \text{ and } \pi_{P12} = \frac{2b^2k - 4bkq_0\beta + q_0^2\beta(\theta_1^2 + r\theta_2^2)}{2k\beta - \theta_1^2 - r\theta_2^2}, \\ \pi_{F12} - \pi_{P12} &= k^2 \left(\frac{(kq_0\beta + s\theta_1 - bk)^2}{\beta} - 2k(1+r)s^2t + \frac{2rs(kq_0\beta + s\theta_1 - bk)\theta_2}{\beta} + \frac{r^2s^2\theta_2^2}{\beta} - \right. \\ &\left. \frac{2k^3(b - q_0\beta)^2}{2k\beta - \theta_1^2 - r\theta_2^2} \right). \text{ Multiplying with } \beta(2k\beta - \theta_1^2 - r\theta_2^2)/k^2, \text{ I calculate } -2k^3\beta(b - q_0\beta)^2 + \\ &(kq_0\beta + s\theta_1 - bk)^2(2k\beta - \theta_1^2 - r\theta_2^2) - 2k(1+r)s^2t\beta(2k\beta - \theta_1^2 - r\theta_2^2) + 2rs(kq_0\beta + \\ &s\theta_1 - bk)\theta_2(2k\beta - \theta_1^2 - r\theta_2^2) + r^2s^2\theta_2^2(2k\beta - \theta_1^2 - r\theta_2^2). \text{ Note that the coefficient of } t \text{ is} \\ &\text{negative. Therefore, to make } \pi_{F12} - \pi_{P12} > 0, \quad t < \\ &\frac{(2k^3\beta(b - q_0\beta)^2 - (kq_0\beta + s\theta_1 - bk)^2(2k\beta - \theta_1^2 - r\theta_2^2) - 2rs(kq_0\beta + s\theta_1 - bk)\theta_2(2k\beta - \theta_1^2 - r\theta_2^2) + r^2s^2\theta_2^2(2k\beta - \theta_1^2 - r\theta_2^2))}{(2k(1+r)s^2\beta(2k\beta - \theta_1^2 - r\theta_2^2))} = \varphi_4\end{aligned}$$

must hold; otherwise, if t exceeds this threshold value, then $\pi_{F12} - \pi_{P12} < 0$. In short, by checking app users' disutility sensitivity parameter (t), the developer can decide to offer the app for either free or paid in both platforms.

Similarly, for single-platform strategies, I compare Case F1 against Case P1. Since

$$\begin{aligned}\pi_{F1} &= \frac{b^2k^2 - 4bk(kq_0\beta + s\theta_1) + 4s(s\theta_1^2 + 2k\beta(q_0\theta_1 - st))}{4k^2\beta} \text{ and } \pi_{P1} = \frac{bk(b - 4q_0\beta) + 2q_0^2\beta\theta_1^2}{4k\beta - 2\theta_1^2}, \quad \pi_{F1} - \pi_{P1} = \\ &\frac{(b - 2q_0\beta)^2}{4\beta} - \frac{s(b - 2q_0\beta)\theta_1}{k\beta} + \frac{s^2\theta_1^2}{k^2\beta} - \frac{k(b - 2q_0\beta)^2}{2(2k\beta - \theta_1^2)} - \frac{2s^2t}{k}. \text{ Solving for } \pi_{F1} - \pi_{P1} > 0 \text{ with respect} \\ &\text{to } t, \text{ I calculate } t < \frac{\theta_1(\theta_1^3 + 8ks\beta\theta_1(s - q_0\theta_1) - b^2k^2\theta_1 - 4s^2 - 4k^2q_0\beta^2(q_0\theta_1 - 4s) + 4bk(s\theta_1^2 + k\beta(q_0\theta_1 - 2s)))}{8ks^2\beta(2k\beta - \theta_1^2)} = \varphi_5. \text{ Therefore, if } t \text{ is smaller} \\ &\text{than this threshold value, then } \pi_{F1} > \pi_{P1}. \text{ Otherwise, } \pi_{F1} < \pi_{P1}.\end{aligned}$$

$$\begin{aligned}\text{Further, I compare Case F2 against Case P2. Since } \pi_{F2} &= \frac{b^2k^2 - 4bk(kq_0\beta + rs\theta_2) + 4rs(rs\theta_2^2 + 2k\beta(-st + q_0\theta_2))}{4k^2\beta} \text{ and } \pi_{P2} = \frac{bk(b - 4q_0\beta) + 2q_0^2r\beta\theta_2^2}{4k\beta - 2r\theta_2^2}, \quad \pi_{F2} - \pi_{P2} = \\ &\frac{(b - 2q_0\beta)^2}{4\beta} - \frac{2rs^2t}{k} + \frac{rs(2q_0\beta - b)\theta_2}{k\beta} + \frac{r^2s^2\theta_2^2}{k^2\beta} - \frac{k(b - 2q_0\beta)^2}{2(2k\beta - r\theta_2^2)}. \text{ Solving for } \pi_{F2} - \pi_{P2} > 0 \text{ with} \\ &\text{respect to } t, \text{ I calculate } t < \frac{\theta_2(8krs\beta\theta_2(s - q_0\theta_2) - b^2k^2\theta_2 - 4r^2\theta_2^3 - 4k^2q_0\beta^2(q_0\theta_2 - 4s) + 4bk(rs\theta_2^2 + k\beta(q_0\theta_2 - 2s)))}{8ks^2\beta(2k\beta - r\theta_2^2)} = \varphi_6. \text{ Therefore, if } t \text{ is} \\ &\text{smaller than this threshold value, then } \pi_{F2} > \pi_{P2}. \text{ Otherwise, } \pi_{F2} < \pi_{P2}.\end{aligned}$$

In summary, for two-platform product offering, a free app strategy is more advantageous than a paid app strategy as long as $t < \varphi_4$; for one-platform product offering, a free app strategy is also more beneficial than a paid app strategy as long as $t < \varphi_5$ (for app offering in Platform 1) or $t < \varphi_6$ (for app offering in Platform 2).

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In-App Purchase: To Offer or Not to Offer?

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Abstract

In this article, I consider mobile app developers' product versioning decisions by focusing on in-app purchase (IAP). Should a developer provide consumers an app (free or paid) with IAP option in one platform (e.g., Android or iOS), and how should it design and price the basic app and IAP? I find that the answer to the former question is "it depends", although in most situations offering IAP is a better choice. I also compare forward-looking and myopic strategies and show that the former always outperforms the latter.

Keywords: App Developer, Mobile App, In-App Purchase (IAP), Product Offering, Pricing Strategy.

1. Introduction

Nowadays mobile applications (apps) have become indispensable in our daily digital lives. App users not only use the initial functionalities or content of the apps (e.g., productivity apps) but sometimes also utilize additional content or service available in the apps (e.g., equipment that empowers a player in a virtual game or extra photo editing feature), which are so-called “in app purchase (IAP)”². Apple’s developer website lists several examples of IAP: (1) a basic versioned app with premium features, (2) a magazine app that allows consumers to buy and download new issues, and (3) a game app that provides players new levels to explore or allows them to buy virtual equipment. For example, a free Android video editing app named PowerDirector allows users to use basic functionalities; however, users will need to pay \$0.99 - 5.99 per item if they want to use extra features such as watermarks. Though very new, global IAP market’s sales revenue is predicted to hit \$14 billion in 2015 and will surpass \$36 billion by 2017 (Statista, 2016). The prevalence of IAP can partly be attributed to the fact that an increase in app price significantly decreases the number of app downloads in the app store (Casto, 2016). Since users are more sensitive to paid apps than free apps (Yu et al., 2017), IAP becomes a more effective tool for “converting casual app users into paying customers” as it allows users to buy additional features after experiencing the benefits of an app (Petty and van der Meulen, 2012; Garg and Telang, 2013). A market survey published by Petty and van der Meulen (2012) finds that in 2011 only 5% of mobile apps had featured IAP, but that percentage has increased to 30% in 2016, and IAP sales have increased from 10% to 41% of app store revenues in the same period.

Due to its novelty, IAP has not drawn much attention from researchers. There appeared a few empirical papers related to IAP in recent years. Garg and Telang (2013) find that free apps attract over ten times more volume of downloads than paid apps and IAP brings extra revenue for app developers. Jung, Baek, & Lee (2012) compare three software app (not mobile app) offering strategies: free with IAP, limited-time free with paid conversion, and pro-and-lite (paid-with-full-feature and free-with-limited-feature). Ghose and Han (2014) find that apps with IAP have higher demand relative to those without; in contrast, apps with in-app advertisement choice have lower demand (download) compared to those without. Yet, analytical research on this topic is particularly lacking. I am interested in the following question: In what situation should a mobile app developer offer IAP?

As defined, IAP phenomenon essentially is somehow akin to product upgrade or product

² In accordance with Apple, Inc., IAP is referred to as functionalities that include premium content, virtual products, and subscriptions that require direct payment from consumers.

improvement, which can be viewed as vertical differentiation from the perspective of app offering. However, upgraded product is completely an extension of the prior versioned product while IAP can be very different from the initial app functionality. Moreover, in IAP consumers usually do not wait to make purchase until a new version (i.e., IAP in this case) of an app product is released. This is different from many prior studies on product upgrade which assume that consumers can purchase either the earlier or upgraded version of a product (Bala and Carr, 2009). That being said, app users who buy IAP have to download the basic app first while new consumers in product upgrade scenario can buy the newer version directly. In addition, in-app advertisement offers another revenue source for app developers. All of the above features distinguish IAP from business practices in product upgrade and offerings and so the existing models of product offering (e.g., Dhebar, 1994; Bhattacharya et al., 2003; Bala and Carr, 2009) cannot be simply applied to the IAP problem.

The objective of this research is to build an analytical framework for app developers' decision making on product offering facing the IAP option. To achieve this goal, I create a two-period model. I consider a monopolistic developer who has an either-or choice between free-with-advertisement and paid app in the first period; the developer determines the quality level as well as price of the basic versioned app (BA). The developer also considers whether to offer IAP based on the BA in the second period. If IAP is offered, the developer determines the quality level and price for the IAP. In the analysis, I am particularly interested in the following questions: What product offering strategy should an app developer adopt when facing IAP option? Will IAP strategy always outperform non-IAP strategy? What quality level should the developer set for both BA and IAP? How much should the developer charge for both BA and IAP?

The present research makes a contribution in three areas. First, it adds to the existing literature on product offering strategy for information good developers. Prior analytical studies on product offering strategies from the app developers' perspective are mostly under-explored (Yu, et al., 2017). To the best of our knowledge, this paper is the first to create an analytical model that addresses the IAP decision faced by an app developer. Second, I provide guidance for practitioners on decisions regarding the choice of whether to offer IAP. We show that strategy of whether to offer IAP based on BA is dependent on different conditions, although in most cases offering IAP is a better choice. Moreover, I find that once the choice is determined, the developer can identify the optimal quality level and prices of the apps (BA and IAP). Finally, the result derived also suggests app developers to be forward-looking rather than myopic in deciding their product offering strategies.

The paper proceeds as follows. Section 2 summarizes the relevant literature, followed

by major assumptions and modeling framework in Section 3. Section 4 analyzes the problem based upon our model along with a summary of our findings. Corresponding technical notes of the analysis are provided in appendices. In Section 5 I conduct analyses of several numerical examples. The article is concluded with a final discussion and concluding remarks in Section 6.

2. Related Literature

In this section I discuss the relevant literature. The current research is mostly related to those about product offering strategies, particularly product improvement for information goods, and utility modeling.

Present literature regarding product offering strategies mostly focuses on new product development (Zirger and Maidique, 1990; Chen et al., 2010, Krishnan and Zhu, 2006), product improvement and pricing (Kornish, 2001; Fudenberg and Tirole, 1998; Bala and Carr, 2009), product bundling (Chen, 1997; Bakos and Brynjolfsson, 1999), and multi-platform selection (Yu et al., 2017). This paper focuses on app offering analysis regarding BA and IAP in the mobile app market that previous models did not address. Regarding the cost of products offered, many prior studies treat variable cost as negligible (zero) or linearly increasing with respect to product quality (e.g., Weber, 2008). Whereas some others adopt nonlinear functions to capture development cost (e.g., Desai, 2001). Jones and Mendelson (2011) point out that higher power of the nonlinear function implies higher product development complexity. In a recent research by Chellappa and Mehra (2017), the authors offer insights of cost drivers that can impact a developer's quality and pricing decisions in product versioning. The cost drivers they discuss include users' usage costs, versioning costs, and usage cost shifting from users to the developer due to cloud hosting. Since app products can be duplicated once developed, I assume zero variable cost (Weber, 2008) and nonlinear product development cost (Desai, 2001) in terms of quality level.

Research on vertical versioning was introduced by Mussa and Rosen (1978). Among prior studies, product improvement as one type of vertical versioning has also been examined vastly (e.g., Bhargava and Choudhary, 2008; Anderson and Dana, 2009). In the models for product improvement and pricing, a major assumption is which period consumers will purchase the initial product and/or improved version. A rational consumer will self-select the product version that can maximize her net utility after paying for the "price" (Chellappa and Mehra, 2017). Some studies make a "leapfrogging" assumption that only new users buy the improved version but existing users would not upgrade the product from the current version they are using (e.g., Dhebar, 1994; Bhattacharya et al., 2003). Bala and Carr (2009) relax this assumption by

allowing existing users to buy the upgraded version; additionally, new consumers who have not bought the current version can buy the new version directly. Given the difference between upgrade and IAP as explained above, I assume app users have to download BA (free or paid) first, then consider buying IAP if IAP is available and adds extra utility. Furthermore, Dogan et al. (2011) examine software developers' decision on how much to upgrade the initial version in a two-period setting and assume that the demand for the upgraded version is influenced by the features of the initial version. They find that the amount of a developer's upgrade effort is impacted by the demand variability. Yu et al. (2017) study app developers' one-period quality and pricing decisions for app products in a multi-platform setting. They show that user base is also a determinant for a developer's platform selection and pricing decision for its app products. In this present research I extend Yu et al.'s (2017) model to a two-period but in one-platform setting in order to focus on IAP's impact on decisions ; meanwhile, consistent with Dogan et al. (2011), I assume that the demand for IAP is based on the demand for the BA.

Another set of literature relevant to our research is Hotelling-type utility models. Tracing back to Hotelling (1929), there have been numerous scholars adopting Hotelling locational method to determine demand from heterogenous consumers (e.g., Irmén and Thisse, 1998; Huang et al., 2013; Pun, 2013). Hotelling-type models assume that consumers with heterogeneous preferences (or tastes) within the marketplace would derive different valuations to a product with a typical quality and price. Weber's (2008) circular market model is a special type of Hotelling utility model that avoids arbitrariness of endpoints existing in the linear Hotelling market model. In the current paper I also use the circular market model to derive product demand. The uniqueness and advantages of the circular market model in this paper include: (1) the user base for IAP is the exact realized demand of BA; and (2) the quality of IAP level in the second period should be higher than that of BA. In addition, many prior studies assume that users' disutility sensitivity to price is constant and normalize it to one (Chambers et al., 2006). In app industry, consumers might be less sensitive to free apps' ad-related disutility than that of paid apps' price (Yu, et al., 2017). In the current research, I also assume that free and paid app users have different disutility sensitivities.

3. Model

Assume that a monopolistic app developer considers whether to offer an app with IAP³ functionality in a mobile platform. The developer can offer either free or paid basic versioned

³ In real world, an app developer can develop many IAPs (each with different features). To keep the model tractable, I only consider one (non-repetitive purchase) IAP.

app (BA). The free (paid) version can bring in advertisement revenues (app sales revenues) for the developer. With IAP in consideration, the sales revenue from IAP is another stream of revenue for the developer. In this paper, I focus only on one platform (e.g., either Android or iOS platform). The developer determines the quality level of BA (free or paid), denoted by q (see Table 1 for parameter notation). The quality is a one-dimensional measure that aggregates a bundle of attributes typically related to the app's performance and/or functionality (Weber, 2008; Yu et al., 2017). For example, an online game app's quality can be related to attributes such as complexity of the game, user-friendliness, data accuracy, etc. As q is constrained by factors such as the current state of technology and the app itself, I assume there exists an upper bound of quality ($\hat{q}$) (e.g., Chambers et al., 2006; Yu et al., 2017). For instance, a calculator is a simple app for which there exists an upper bound regarding how many functionalities that can be added to this type of apps.

Table 1. Variable Notation

Parameters	
s	Unit advertisement revenue for a free app
θ	User valuation to an app offered in the platform
k	User's unit transportation disutility
β	Development difficulty parameter of product
q_0	Prior quality level that the developer can reuse
$\hat{q}$	Upper limit of quality level q
t	App users' disutility sensitivity parameter for free apps, $0 < t < 1$
Decision Variables	
q	Initial app's quality level
q_i	IAP's quality level
p	Price for the paid version of an initial app
p_i	Price of IAP
Other Parameters	
u	Consumer's net utility
x	Product's distance from the consumer's ideal point
m	Realized demand of downloaded app
m_i	Realized demand of in-app purchase (IAP)

I assume the app market is a circular space with a size that is normalized to two (Weber, 2008; Yu et al., 2017). In Fig. 1, I expand the market space to a two-dimensional plane. The two end points on both sides of point 0 are identical if we draw the space in a circular market; and they are one unit away from point 0. Consumers who locate in this market are heterogeneous and uniformly distributed in terms of their valuation to app products (θ). According to Calvo-Armengol and Zenou (2002), the uniform distribution assumption should not change the qualitative directions of analytical results for my model.

Fig. 1. Demand Modeling.

Let θq be a representative user's valuation where the BA's quality position perfectly fits with the user. A user whose location is x units from the product's position will incur some transportation disutility, kx , where k denotes unit transportation disutility. Therefore, the user's net utility for buying the BA is: $u = \theta q - p - kx$, where p denotes the BA's price when the developer offers a paid app. In the case when the app is free, the average ad revenue per user gained by the app developer is s , which is exogenously determined by the market. In general, advertisements bring users more annoyance and disutility from advertisement (more electricity and bandwidth consumption, memory space, effort to turn off ads, etc.). Additionally, it has been reported that consumers have lower disutility sensitivity to free apps relative to paid apps (e.g., Chambers et al., 2006; Yu, et al., 2017); hence, I assume the average disutility for free-app users in using the app is ts , where t ($0 < t < 1$) is users' disutility sensitivity to ads in free apps. If the developer offers a free app, then a user's net utility will be: $u = \theta q - ts - kx$. As illustrated in Fig. 1, given a price of the app, the higher price or the further the user is away from

the app product, the less net utility she realizes. Therefore, the farthest users captured by the developer are those who gain zero net utility. We thus derive the realized demand as $\frac{2(\theta q - p)}{k}$ for paid BA (or $\frac{2(\theta q - ts)}{k}$ for free BA). Note that the BA's quality (q) is developed based on the available prior quality (q_0). The developer can provide net utility of $q_0\theta - p$ (for paid BAs) or $q_0\theta - ts$ (for free BAs) even if it invests nothing (i.e., $q = q_0$) in developing the app product. I define this net utility as *residual utility* as below.

Definition 1: *Residual Utility (RU)*, mathematically expressed as $q_0\theta - p$ (for paid apps) or $q_0\theta - ts$ (for free apps), refers to the net utility gained by a user in using an app product with such a low quality level that $q \cong q_0$ with a “price” of p (for paid app) or ts (for free app). ■

I assume the app development cost to be $\beta(q - q_0)^2$ (Desai, 2001), which is nonlinearly increasing function of the quality differential ($q - q_0$) and β is app development difficulty parameter. Here q_0 represents the skillsets (e.g., Java and/or Objective-C programming) that the developer has obtained for app development and/or app components that have been developed by the developer and can be reused (Yu, et al., 2017).

Sometime after the BA launch, the developer captures a group of users. Since IAP revenues come only from those who download the app, I use a two-period model when IAP offering is considered. Specifically, the app download (paid or free) occurs in Period 1 and the purchase of IAP takes place in Period 2. Thus, users who purchase IAP will be only a subset of those who have used the BA. Let q_i ($q_i > q$) and p_i denote the IAP's quality and price, respectively. The realized demand for the IAP would be $\frac{2(\theta q_i - p_i - p)}{k}$ for paid BA case (or $\frac{2(\theta q_i - p_i - ts)}{k}$ for free BA case). The IAP's quality must be higher than that of BA for the same app, hence the development cost of IAP can be expressed as: $\beta(q_i - q_0)^2 - \beta(q - q_0)^2$. I use a discount factor δ ($0 < \delta < 1$) for the sales revenue and development cost of IAP, which occurs in Period 2⁴. In most situations the discount factor approaches 1 as the time between download BA and buying IAP is not far apart. As a profit oriented entity, the developer's objective is to maximize its total profit. Therefore, the objective function for the profit-maximizing developer who offers IAP in Period 2 can be set up as in Eq. (1) and Eq. (2).

⁴ Though there is no short-cut for periods 1 and 2, the time of buying BA still occurs before that of buying IAP. Therefore, this δ represents the discount factor between these two periods.

$$\max \left\{ \frac{2p(\theta q - p)}{k} + \delta \frac{2p_i(\theta q_i - p - p_i)}{k} \right\} - \beta(q - q_0)^2 - \delta\{\beta(q_i - q_0)^2 - \beta(q - q_0)^2\} \quad (\text{Paid BA case}) \quad (1)$$

$$\max \left\{ \frac{2s(\theta q - ts)}{k} + \delta \frac{2p_i(\theta q_i - ts - p_i)}{k} \right\} - \beta(q - q_0)^2 - \delta\{\beta(q_i - q_0)^2 - \beta(q - q_0)^2\} \quad (\text{Free BA case})(2)$$

$$\text{s.t. } q_0 < q < q_i \leq \hat{q}, 0 < t < 1, 0 < \delta < 1, p, p_i, \beta, q_0, \theta, s, k > 0.$$

The component $\frac{2p(\theta q - p)}{k} + \delta \frac{2p_i(\theta q_i - p - p_i)}{k}$ in paid IAP case (or $\frac{2s(\theta q - ts)}{k} + \delta \frac{2p_i(\theta q_i - ts - p_i)}{k}$ in free IAP case) above represents total revenues, while $\beta(q - q_0)^2 + \delta\{\beta(q_i - q_0)^2 - \beta(q - q_0)^2\}$ represents total development cost. The decision variables for the developer include: BA price p (if a paid BA is offered), BA quality q , IAP price p_i , and IAP quality q_i .

If the developer does not offer IAP in the second period, the objective function for the developer can be expressed as below.

$$\max \frac{2p(\theta q - p)}{k} - \beta(q - q_0)^2 \quad (\text{Paid BA only})(3)$$

$$\max \frac{2s(\theta q - ts)}{k} - \beta(q - q_0)^2 \quad (\text{Free BA only})(4)$$

$$\text{s.t. } 0 < p < 1, q_0 < q \leq \hat{q}, p, \beta, q_0, \theta, s, k > 0.$$

I construct four cases as follows: (F) free BA only, (P) paid BA only, (FI) free BA + IAP, and (PI) paid BA + IAP. Note that the developer is a forward-looking decision maker in FI and PI since all decisions are made in Period 1. However, the developer might be a myopic decision maker, which means it launches the app without considering IAP offering in Period 1 (cases F and P). The myopic developer would like to offer IAP in Period 2 if this strategy can generate extra profit. We therefore have two myopic cases: (FI_m) with free BA first then IAP later, and (PI_m) with paid BA first then IAP later. In the analysis section we discuss forward looking cases in 4.1 and 4.2, and myopic cases in 4.3.

4. Analysis

As mentioned above, I separate our model into four cases (F, P, FI and PI). Both cases FI and PI have IAP issue but neither case F nor P has IAP issue. Fig. 2 presents the decision tree faced by the developer. Table 2 displays decision variables in each case. I notice that the

objective function (i.e., total profit) is quadratic with respect to product quality (q or q_i) and/or price (p or p_i) in each case. I further check the concavity condition for each case. The result shows that when strict concavity condition is met, the case might be solvable through the following procedure: first solve FOCs, and then check whether the solution is feasible (for example, q^* or q_i^* must be no greater than $\hat{q}$). In the situation when concavity condition does not hold or the optimal quality exceeds quality upper bound, the profit is convexly increasing in quality. Since the objective function is a quadratic form of quality, in these situations the optimal quality (either q^* or q_i^*) should be $\hat{q}$ and p and/or p_i are solved accordingly afterwards. I call cases in the former situation unbounded cases and those in the latter bounded cases ($q < \hat{q}$).

Table 2. Decision Variables in Each Case

	<u>Forward-looking Cases</u>				<u>Myopic Cases</u>	
	F	P	FI	PI	FIm	PIIm
Decision Variables	q	q, p	q, q_i, p_i	q, q_i, p, p_i	$(q)(q_i, p_i)$	$(q, p) (q_i, p_i)$

Fig. 2. Decision Tree for a Developer's App Offering Strategy. This figure shows the route for a developer's decision making process of app offering. In each node among (1) – (4), there are offering strategies of free and paid BAs for the developer's choice, as listed in the following. Node (1): “free BA only with $q = \hat{q}$ ” (bounded case F), “paid BA only with $q = \hat{q}$ ” (bounded case P); Node (2): “free BA only with $q < \hat{q}$ ” (unbounded case F), “paid BA only with $q < \hat{q}$ ” (unbounded case P); Node (3): “free BA + IAP with $q_i = \hat{q}$ ” (bounded cases FI or FI_m), “paid BA + IAP with $q_i = \hat{q}$ ” (bounded cases PI or PI_m); Node (4): “free BA + IAP with $q_i < \hat{q}$ ” (unbounded cases FI or FI_m), “paid BA + IAP with $q_i < \hat{q}$ ” (unbounded cases PI or PI_m).

Following the above procedure, I first solve each case when the strict concavity condition holds. Table 3 lists the optimal solutions for three unbounded cases (F, P and FI). The concavity conditions for each case are presented in Table 4.

Table 3. Optimal Solution Matrix for Unbounded Situation

Case	Price (p) or (p, p_i)	Quality (q_i) or (q, q_i)	Total Profit
F	$(s, -)$	$(q_0 + \frac{s\theta}{k\beta}, -)$	$\frac{s(s\theta^2 + 2k\beta(q_0\theta - st))}{k^2\beta}$
P	$(\frac{kq_0\beta\theta}{2k\beta - \theta^2}, -)$	$(\frac{2kq_0\beta}{2k\beta - \theta^2}, -)$	$\frac{q_0^2\beta\theta^2}{2k\beta - \theta^2}$
FI	$(s, \frac{k\beta(q_0\theta - st)}{2k\beta - \theta^2})$	$(q_0 + \frac{s\theta}{k\beta - k\beta\delta}, \frac{2kq_0\beta - st\theta}{2k\beta - \theta^2})$	$\frac{s^2\theta^2}{k^2\beta(1 - \delta)} - \frac{2s^2t}{k} - q_0^2\beta\delta + \frac{2q_0s\theta}{k} + \frac{\beta\delta(2kq_0^2\beta + st(st - 2q_0\theta))}{2k\beta - \theta^2}$
PI	No unbounded solution exists in case PI		

Table 4. Concavity and Feasibility Condition for Cases F, P, FI and PI

F	$s < \frac{2kq_0\beta\theta}{2kt\beta-\theta^2}$ if $\beta > \frac{\theta^2}{2kt}$
P	$\beta > \frac{\theta^2}{2k}$
FI	$\beta > \frac{\theta^2}{2k}$ & $q_0\theta - ts > \frac{2sk\beta+\theta^2}{k\beta(1-\delta)}$ & $\{q_0\theta - ts < \frac{s\theta^2(2k\beta-\theta^2)}{k\beta(1-\delta)(k\beta-\theta^2)} \text{ if } \beta < \frac{\theta^2}{k} \}$
PI	NA

When the condition for strict concavity is not met, I use an indirect method to solve the model by following Petruzzi and Dada (1999). I find that, at any given quality level, the objective function is always strictly concave in price decisions, and case F is always strictly concave but case P is strictly concave only when $\beta > \frac{\theta^2}{2k}$. When $\beta < \frac{\theta^2}{2k}$, the objective function of case P is convex in quality q ; in this situation, the developer should set $q = \hat{q}$ and thus the optimal p can be solved accordingly. If the developer sets $q = \hat{q}$ in Period 1, since the quality upper bound is reached, IAP option will not be considered and case PI would not exist. In case FI, when $\beta < \frac{\theta^2}{2k}$, the objective function is not strictly concave (bounded situation). In that situation, I solve the profit maximization problem by setting $q_i = \hat{q}$ and then solve p_i and q accordingly as p_i and q are strictly concave at any q_i . I find that, in case PI, interior optimal solution ($q_i^* < \hat{q}$) exists when $\beta > \frac{\theta^2}{2k}$; but in case PI, the quality of IAP should be set at its quality upper bound ($\hat{q}$) because the optimal solution for case PI obtained by solving FOCs is not feasible due to $q_i^* < q^*$ when case PI's strict concavity condition holds. This implies that only the bounded solution ($q_i^* = \hat{q}$) should be considered for case PI. Thus I propose Lemma 1 below.

Table 5. Optimal Solution Matrix for Bounded Situation

Case	Price (p) or (p, p_i)	Quality (q) or (q, q_i)	Total Profit
F	($s, -$)	($\hat{q}, -$)	$\frac{2s(\hat{q}\theta - st)}{k} - \beta(\hat{q} - q_0)^2$
P	($\frac{\hat{q}\theta}{2}, -$)	($\hat{q}, -$)	$\frac{\hat{q}^2\theta^2}{2k} - \beta(\hat{q} - q_0)^2$
FI	($s, \frac{1}{2}(\hat{q}\theta - st)$)	($q_0 + \frac{s\theta}{k\beta - k\beta\delta}\hat{q}$)	$\frac{2s^2\theta^2 - 2k^2(q_0 - \hat{q})^2\beta^2(1-\delta)\delta + k\beta(1-\delta)(\hat{q}^2\delta\theta^2 - s^2t(4-t\delta) + s(4q_0\theta - 2\hat{q}t\delta\theta))}{2k^2\beta(1-\delta)}$
PI	$\left(\frac{k\beta\theta(1-\delta)(2q_0-\hat{q}\delta)}{k\beta(4-\delta)(1-\delta)-2\theta^2}, \frac{k\beta\theta(1-\delta)(2\hat{q}-q_0)-\hat{q}\theta^3}{k\beta(4-\delta)(1-\delta)-2\theta^2}\right)$	$\left(\frac{kq_0\beta(4-\delta)(1-\delta)-\hat{q}\delta\theta^2}{k\beta(4-\delta)(1-\delta)-2\theta^2}, \hat{q}\right)$	$\frac{2k\beta(q_0^2 - q_0\hat{q}(3-\delta)\delta + \hat{q}^2(2-\delta)\delta)\theta^2 - k^2(q_0 - \hat{q})^2\beta^2(4-\delta)(1-\delta)\delta - \hat{q}^2\delta\theta^4}{k(k\beta(4-\delta)(1-\delta) - 2\theta^2)}$

Lemma 1. In the "paid BA + IAP" strategy (PI), offering IAP with a quality lower than the upper bound ($\hat{q}$) is never optimal. ■

Lemma 1 offers an analytical explanation for the phenomenon that many paid apps with IAP option are well designed. In reality, paid app users have higher willingness-to-pay (WTP, denoted by θ) than those who prefer download for free. As a result, the optimal strategy in offering “paid BA + IAP” should be designing BA at a medium quality but IAP at a maximum quality. The BA quality is medium so that it can attract more users to pay BA in the first period. Since not all users need to use all features at a high app quality, having a too high BA quality will significantly decrease its user base given the price of BA (p^*) will be high too. I found such paid apps with IAP offerings especially in productivity, photo and video categories. For example, Photo Lab PRO HD is a full-feature photo editor priced at \$4.99 on iOS App store; its IAP options like Old Movie Film Effect and Oil Paint Brush Texture costs \$15 or above; Receipt Scanner 2 on iOS is priced at \$3.99, but its IAP providing device-to-device synchronization is \$4.99. With such a high price, the additional features in the IAP must be well designed for specific app users.

As shown in Fig. 2, for a developer, in Period 1 it decides either free or paid version for launching a BA; meanwhile it can choose to (1) set the BA with a quality level of upper bound which disallows the developer to offer IAP technically, or (2) set it lower than upper bound so that there is a room for offering IAP in Period 2. This is applicable for both forward-looking and myopic developers.

According to the decision tree in Fig. 2, I find it too intricate to identify the global app offering strategy if all decision points of (1) - (4) are considered at the same time. For tractability purpose, I make a further assumption. That is, the developer has decided to launch either free or paid BA with consideration for developing IAP in the later period (i.e., setting the BA’s quality $q < \hat{q}$). Thus, the developer only needs to consider decision nodes of (2), (3) and (4). In the following subsections, I discuss app offering decisions for a forward-looking developer first. The major findings are summarized in 4.1. I then discuss decisions for a myopic developer’s IAP offering by summarizing the major findings in 4.2. Finally, I compare forward-looking and myopic strategies and summarize the findings in 4.3. Proofs for all propositions are provided in Appendices A through F.

4.1 Forward-Looking Strategies for App Offering

A forward-looking developer will simultaneously consider decision choices in both periods. Since it has already chosen either free or paid version for launching the BA, it needs to compare strategies of “BA only” and “BA + IAP” for the situation of either version. Specifically, for free BA offering, the following decision choices are considered: unbounded case F (free BA

only with $q < \hat{q}$), unbounded case FI (free BA + IAP with $q_i^* < \hat{q}$), and bounded case FI (free BA + IAP with $q_i^* = \hat{q}$); while for paid BA offering, the following strategies are considered: unbounded case P (paid BA only with $q < \hat{q}$), and bounded case PI (paid BA + IAP with $q_i^* = \hat{q}$)⁵.

4.1.1 Free BA Plus IAP

Since the developer is profit oriented, I compare the total profit gained by the developer in different offering strategies. I find that for a developer adopting free BA strategy and offering IAP in the later period, the decision on whether to offer IAP depends on how challenging the IAP component to be developed (β) is and whether the IAP's quality (q_i) has reached the quality upper bound $\hat{q}$. I summarize my findings regarding free strategies for "BA + IAP" offering in Proposition 1(a) and (b) for the situation when the developer leaves room to offer IAP in the later period, and Proposition 2 for the situation when the developer leaves no room for future IAP offering.

Proposition 1. (a) When the IAP is challenging to develop (i.e., $\beta > \frac{\theta^2}{2k}$), offering IAP on top of free BA (unbounded case FI), if feasible (see Table 4), is always better off than not offering IAP (unbounded case F) as long as the optimal quality of IAP is lower than quality upper limit ($q_i^* < \hat{q}$). (b) When the optimal quality of IAP has to be set at its upper limit ($q_i^* = \hat{q}$), then as long as the feasibility condition holds (see Table 6), offering IAP on top of free BA (bounded case FI) outperforms the strategy that does not offer IAP (unbounded case F) when time discount factor (δ) is so large that $\delta > \frac{2k^2(\hat{q}-q_0)^2\beta^2-2s^2\theta^2-k\beta(\hat{q}\theta-ts)^2}{2k^2(\hat{q}-q_0)^2\beta^2-k\beta(\hat{q}\theta-ts)^2}$. ■

Proposition 1(a) implies that in the situation when the app is not easy to develop but there is room for designing IAP on top of the launched BA, the developer should always provide the IAP as long as it is feasible to do that. The feasibility conditions for the unbounded case FI are: $q_0\theta - ts > \frac{2sk\beta+\theta^2}{k\beta(1-\delta)}$ and $q_0\theta - ts < \frac{s\theta^2(2k\beta-\theta^2)}{k\beta(1-\delta)(k\beta-\theta^2)}$ (if $\beta > \frac{\theta^2}{k}$) (see Table 4). Note that " $q_0\theta - ts$ " is defined as residual utility (RU; see Definition 1). Hence, when the IAP is not easy to develop (i.e., $\beta > \frac{\theta^2}{2k}$) as well as the developer is able to provide high enough RU (i.e., $RU > \frac{2sk\beta+\theta^2}{k\beta(1-\delta)}$), offering IAP is a better option than not offering IAP. However, if the IAP is too challenging to develop (i.e., $\beta > \frac{\theta^2}{k}$), offering IAP can only be feasible when RU is not too

⁵ Since unbounded solution for PI does not exist (as discussed in Lemma 1), there are only two cases for comparison in paid BA situation.

high (i.e., $RU < \frac{s\theta^2(2k\beta - \theta^2)}{k\beta(1-\delta)(k\beta - \theta^2)}$). This makes sense since higher difficulty of IAP development (larger β) requires “charging” users more (larger s) for cost compensation, thereby lowering RU. Even though the average ad revenue per user (s) is predetermined by the market, the developer would not offer IAP if the average ad revenue per user is too low. This finding may explain why many mobile apps with IAP options (e.g., PowerDirector) are free even though those apps seem not easy to design.

In addition, my finding as summarized in Proposition 1(b) suggests that the developer should exploit the quality upper bound through offering IAP if the app is easy to develop (i.e., $\beta < \frac{\theta^2}{2k}$) and time discount factor is large (i.e., $\delta > \frac{2k^2(\hat{q}-q_0)^2\beta^2 - 2s^2\theta^2 - k\beta(\hat{q}\theta - ts)^2}{2k^2(\hat{q}-q_0)^2\beta^2 - k\beta(\hat{q}\theta - ts)^2}$). Larger time discount factor implies shorter time interval between BA and IAP offerings. Proposition 1(b) accords with the intuition because longer time interval between the two periods would require higher profitability for investing in the later period (offering IAP in Period 2 in this setting); Hence, offering IAP is preferable when time discount factor is large. Of course, the developer should adopt the strategy of offering IAP depending on whether it is feasible to do that (see feasibility condition in Table 6). In fact, I observe that some simple apps are free with IAP options (e.g., 7 Minute Workout).

Proposition 2. For free BA situation, offering “BA + IAP” with $q_i^* = \hat{q}$ (bounded case FI), as long as feasible, is more profitable than offering “BA only” with $q^* = \hat{q}$ (bounded case F). ■

For a developer adopting free BA strategies, there are two options for exploiting the quality upper bound (as Fig. 2 illustrates): launching “BA only” with $q = \hat{q}$ (bounded case F) without any choice of offering IAP, and offering “BA + IAP” with $q_i^* = \hat{q}$ (bounded case FI). I show that exploiting the quality upper bound in two periods (bounded case FI), as long as feasible, is always more profitable than doing this in only one period (bounded case F). The result suggests that embedding IAP option is a better choice if the developer wants to provide users all possible functions/features (i.e., product with quality upper bound). Here is the explanation. For an app with high quality, it is possible for a developer to “charge” users a high “price”. Since “free BA + IAP” strategy offers a pricing tool while the average ad revenue per user in “free BA only” offering is determined by the market, “free BA + IAP” is a better choice. In reality, I observe that many high-quality free apps are offered with IAP option rather than without IAP option. For instance, apps such as high-end movie editing tools (e.g., PowerDirector) and complex games (e.g., Pokemon Go) are provided for free. However, these apps are all featured with IAP.

4.1.2 Paid BA Plus IAP

As discussed previously in Lemma 1, the strategy that launches IAP on top of paid BA without setting the IAP's quality at the upper limit is always suboptimal. Further, the analytical result shows that the condition under which either "paid BA only" or "paid BA + IAP" is too complicated. This is not surprising since in bounded cases the dominance relationship depends on the level of quality upper bound. Due to the uninterpretable result, I do not discuss the details in this paper.

So far in this section, I assume the developer is a forward-looking decision maker. In some situations a developer may not consider the decision for Period 2 when it sets a strategy for app offering in Period 1. I summarize my major findings related to myopic app offering strategies in Section 4.2.

4.2 App Offering Strategies for Myopic Decisions

For a developer who is myopic in app offering decisions and does not plan to provide IAP in the later period, it can choose to offer the BA with quality level of the upper limit ($\hat{q}$) or lower than the upper limit. If optimal level of BA's quality can be set as lower than $\hat{q}$, I show that the optimal qualities are $q_0 + \frac{s\theta}{k\beta}$ for "free BA only" strategy and $\frac{2kq_0\beta}{2k\beta - \theta^2}$ for "paid BA only" strategy, respectively. A myopic developer will also offer IAP in Period 2 if it can gain some extra revenues by doing this. The results show that it is infeasible for a myopic developer to offer IAP in Period 2 without setting the IAP quality at the upper limit; and, for any feasible situation of IAP offering the developer should set the price and quality of the IAP as listed in Proposition 3.

Proposition 3. For a myopic developer, if feasible ⁶, it should set (p_i, q_i) as

⁶ Feasibility conditions:

Unbounded Case Flm: $\beta > \frac{\theta^2}{2k}$ & $q_0\theta - ts > 0$ & $s < \frac{kq_0\beta\theta}{k(2+t)\beta - \theta^2}$ & $\{0 < t < \min\{1, \frac{k\beta}{k\beta - \theta^2} - 1 + \frac{q_0\theta}{s} + \frac{\theta^2}{k\beta}\} \text{ if } \theta^2 < k\beta \}$ & $\{\max\{0, \frac{k\beta}{k\beta - \theta^2} - 1 + \frac{q_0\theta}{s} + \frac{\theta^2}{k\beta}\} < t < 1 \text{ if } k\beta < \theta^2 < 2k\beta \}$;

Bounded Case Flm: $s < \frac{k(2q_0 - \hat{q})\beta\theta}{kt\beta - 2\theta^2}$ & $\left\{ 0 < t < \frac{k\hat{q}s\beta\theta - \sqrt{2k^3(q_0 - \hat{q})^2s^2\beta^3 - 2ks^4\beta\theta^2}}{ks^2\beta} \text{ if } 0 < \frac{k\hat{q}s\beta\theta - \sqrt{2k^3(q_0 - \hat{q})^2s^2\beta^3 - 2ks^4\beta\theta^2}}{ks^2\beta} < 1 \right\}$ or $\left\{ \frac{k\hat{q}s\beta\theta + \sqrt{2k^3(q_0 - \hat{q})^2s^2\beta^3 - 2ks^4\beta\theta^2}}{ks^2\beta} < t < 1 \text{ if } \frac{k\hat{q}s\beta\theta - \sqrt{2k^3(q_0 - \hat{q})^2s^2\beta^3 - 2ks^4\beta\theta^2}}{ks^2\beta} < 1 \right\}$

Bounded Case Plm:

$(\frac{k\beta(q_0\theta-st)}{2k\beta-\theta^2}, \frac{2kq_0\beta-st\theta}{2k\beta-\theta^2})$ in unbounded “free BA + IAP” case (or $(\frac{1}{2}(\hat{q}\theta - st), \hat{q})$ in bounded “free BA + IAP” case), and (p_i, q_i) as $(\frac{1}{2}\theta(\hat{q} - \frac{kq_0\beta}{2k\beta-\theta^2}), \hat{q})$ in “paid BA + IAP” case. ■

Proposition 3 provides a guideline of IAP pricing and quality setting for developers who in Period 1 did not consider the choice of offering IAP at all. Specifically, it is possible for a myopic developer to gain extra revenues via offering IAP in both unbounded and bounded “free BA + IAP” cases; whereas in “paid BA + IAP” cases, the only way to achieve optimality is to set the IAP quality at the upper limit ($q_i^* = \hat{q}$). The implication is that if the app developer has not thought of IAP offering when launching a paid app, then it will be infeasible for the developer to offer IAP in the later period unless the market condition changes (i.e., some model parameters have evolved). Over time, the state of technology will advance, thereby making offering new features of the app product possible. For example, due to the development for the features of time lapse and image stabilization, consumers are willing to pay more for photo editors if such features are available as IAP. In this case, users’ WTP (θ) actually increases alongside the increase of quality upper limit ($\hat{q}$). The change of market condition is beyond my model assumption. Given the results for myopic cases are similar to those for forward-looking cases, I do not discuss the interpretations of Proposition 3 extensively for brevity.

However, I am interested in which one is superior between forward-looking and myopic strategies. To investigate this, I separately compare case FI (forward-looking offering of “free BA + IAP”) against case FIm (myopic offering of “free BA and then IAP”), and case PI (forward-looking offering of “paid BA + IAP”) against case PIm (myopic offering of “paid BA and then IAP”). I summarize my findings based on the comparison results in Proposition 4.

Proposition 4. For a developer who offers IAP, forward-looking strategy always outperforms myopic strategy no matter whether the developer launches the BA for free or paid. ■

Proposition 4 suggests that forward-looking decisions are always better off compared to those that are myopic. Note that for “paid BA + IAP” offerings, the myopic strategy is infeasible. In fact, even if the myopic strategy is feasible, it is inferior against forward-looking strategy as long as the feasibility condition for forward-looking strategy holds. This finding is intuitive

$$\max\left\{\frac{kq_0\beta(4k\beta - 3\theta^2)(2k\beta - \theta^2) - \sqrt{2}\sqrt{k^3q_0^2\beta^3\theta^2(-2k\beta + \theta^2)^2}}{(2k\beta - \theta^2)^3}, \frac{kq_0\beta}{2k\beta - \theta^2}, q_0\right\} < \hat{q}$$

$$< \frac{kq_0\beta(4k\beta - 3\theta^2)(2k\beta - \theta^2) + \sqrt{2}\sqrt{k^3q_0^2\beta^3\theta^2(-2k\beta + \theta^2)^2}}{(2k\beta - \theta^2)^3}$$

since making decision by simultaneously considering future periods should be a better approach than that by focusing only on current period decision. The findings suggest that a developer should follow a forward-looking strategy when making decisions for its app offering; however, as discussed in Proposition 3, if it does not take future decisions into consideration in the early period, offering IAP should still be considered in the later period as long as feasible and profitable. I notice that indeed some app developers are myopic in setting quality and price for its app product. Take 7 Minute Workout for instance, the developer did some tests by charging users some pennies at the very beginning; it then offered the app for free without considering what to do next. However, some days later the developer realized it should include IAP options (Hall, 2015).

5. Numerical Analysis

In this section I conduct numerical examples to investigate the dynamics of IAP offering problems. In mobile app industry, I simply set $s = \$10$, $\theta = \$8$, $\delta = 0.8$, $t = 0.7$, $k = 5$, and $q_0 = 11$, respectively. I examine how app development difficulty (β) and quality upper limit ($\hat{q}$) impact the dominance of offering or not offering IAP on top of a free BA in this setting. Figure 3 illustrates the dynamics of the strategy dominance. As shown, when both app development difficulty (β) and quality upper limit ($\hat{q}$) are large, case FI (offering IAP based on free BA) is dominant. Note that given the values that I have set for the other parameters, the concavity condition for case FI (i.e., $\beta > \frac{\theta^2}{2k}$) is satisfied when β exceeds 6.4. According to Proposition 1 (a), when case FI is concave, this strategy is more profitable than that in case F if case FI is (1) feasible (i.e., $q_0\theta - ts > \frac{2sk\beta + \theta^2}{k\beta(1-\delta)}$ and $q_0\theta - ts < \frac{s\theta^2(2k\beta - \theta^2)}{k\beta(1-\delta)(k\beta - \theta^2)}$ (if $\beta < \frac{\theta^2}{k}$)) as well as (2) quality upper limit ($\hat{q}$) is high such that $q_i^* < \hat{q}$. In this value setting, $q_0\theta - ts > \frac{2sk\beta + \theta^2}{k\beta(1-\delta)}$ always holds; however, in the region where β is smaller than 12.8, $\beta < \frac{\theta^2}{k}$ happens. In this scenario, case FI would be infeasible if $q_0\theta - ts > \frac{s\theta^2(2k\beta - \theta^2)}{k\beta(1-\delta)(k\beta - \theta^2)}$. Therefore, the developer should either offer IAP with maximum quality level on top of a launched BA (bounded case FI) or not offering IAP at all (case F), depending on the condition faced by the developer. That is why we see that in some area bounded case FI or case F are dominant even though $\beta > 12.8$.

Figure 3. Dominant Strategies and Favorite Conditions for Free App Offering. This figure illustrates the dominance relationship between case F (“free BA only”) and case FI (“free app + IAP”) along with changing values of app development difficulty (β) and quality upper limit ($\hat{q}$). I set $s = \$10$, $\theta = \$8$, $\delta = 0.8$, $t = 0.7$, $k = 5$, and $q_0 = 11$, respectively. FIB represents FI $q_i^* = \hat{q}$.

In fact, when $\beta < 6.4$ (i.e., $\beta < \frac{\theta^2}{2k}$), case FI is non-concave anymore, indicating the quality upper bound is the optimal level for the IAP's quality (i.e., $q_i^* = \hat{q}$). In this situation, bounded case FI (offering IAP on top of free BA with $q_i^* = \hat{q}$) can be dominant only when conditions of $\delta > \frac{2k^2(\hat{q}-q_0)^2\beta^2-2s^2\theta^2-k\beta(\hat{q}\theta-ts)^2}{2k^2(\hat{q}-q_0)^2\beta^2-k\beta(\hat{q}\theta-ts)^2}$ and bounded case FI's feasibility are simultaneously met. Note that bounded case FI's feasibility condition is more constrained (see Table 6). As a consequence, there is only a small region where the bounded case FI is dominant. Nevertheless, bounded case FI is favorable only when the same conditions, i.e., $\delta > \frac{2k^2(\hat{q}-q_0)^2\beta^2-2s^2\theta^2-k\beta(\hat{q}\theta-ts)^2}{2k^2(\hat{q}-q_0)^2\beta^2-k\beta(\hat{q}\theta-ts)^2}$ and bounded case FI's feasibility condition, are simultaneously satisfied. The above results are consistent with Proposition 2 (b).

Table 6. Feasibility Condition for Bounded Cases F, P, FI and PI

F	$\beta < \frac{2s^2(\hat{q}\theta - ts)}{k(\hat{q} - q_0)^2}$
P	$\beta < \frac{\hat{q}^2\theta^2}{2k(\hat{q} - q_0)^2}$
FI	$-\frac{s\theta^2}{k\beta - k\beta\delta} < q_0\theta - ts < (\hat{q} - q_0)\theta - \frac{2s\theta^2}{k\beta(1-\delta)} \quad \& \quad s < \frac{k\beta(1-\delta)(\hat{q}-q_0)}{\theta} \quad \& \quad t <$ $\min\left\{\frac{\hat{q}\theta}{s}, \frac{1}{ks^2\beta(1-\delta)\delta} \left(ks\beta(1-\delta)(2s + \hat{q}\delta\theta) - \right. \right.$ $\left. \sqrt{2} \sqrt{ks^2\beta(1-\delta)(k^2(\hat{q} - q_0)^2\beta^2(1-\delta)\delta^2 - s^2\delta\theta^2 - 2ks\beta(1-\delta)(s + (\hat{q} - q_0)\delta\theta))} \right\} \quad (\text{or } t >$ $\frac{1}{ks^2\beta(1-\delta)\delta} (ks\beta(1-\delta)(2s + \hat{q}\delta\theta) +$ $\sqrt{2} \sqrt{ks^2\beta(1-\delta)(k^2(\hat{q} - q_0)^2\beta^2(1-\delta)\delta^2 - s^2\delta\theta^2 - 2ks\beta(1-\delta)(s + (\hat{q} - q_0)\delta\theta))}) \quad (\text{if}$ $ks^2\beta(1-\delta)(k^2(\hat{q} - q_0)^2\beta^2(1-\delta)\delta^2 - s^2\delta\theta^2 - 2ks\beta(1-\delta)(s + (\hat{q} - q_0)\delta\theta)) > 0)$
PI	$k > \frac{2\theta^2}{\beta(4-5\delta+\delta^2)} \quad \& \quad \max\left\{\frac{\delta\hat{q}}{2}, \frac{k\hat{q}\beta\delta(k\beta(4-\delta)(1-\delta)-(3-\delta)\theta^2)-\sqrt{k\hat{q}^2\beta\delta\theta^2(k\beta(4-\delta)(1-\delta)-2\theta^2)(2k\beta(1-\delta)-\theta^2)}}{k\beta(k\beta(4-\delta)(1-\delta)\delta-2\theta^2)}\right\} <$ $q_0 < \min\left\{2\hat{q} - \frac{\hat{q}\theta^2}{k\beta(1-\delta)}, \frac{k\hat{q}\beta\delta(k\beta(4-\delta)(1-\delta)-(3-\delta)\theta^2)+\sqrt{k\hat{q}^2\beta\delta\theta^2(k\beta(4-\delta)(1-\delta)-2\theta^2)(2k\beta(1-\delta)-\theta^2)}}{k\beta(k\beta(4-\delta)(1-\delta)\delta-2\theta^2)}\right\}$

In the second example, I investigate the scenario when quality upper bound is reached in free app offering. As mentioned previously, there are two cases for this scenario: bounded case F (“free BA only” with $q = \hat{q}$) and bounded case FI (“free BA + IAP” with $q_i = \hat{q}$). To generate some data, I set $s = \$10$, $\theta = \$8$, $\delta = 0.8$, $\beta = 8$, $k = 5$, and $\hat{q} = 100$, respectively, allowing the values of the time discount factor (t) and prior quality (q_0) to be varying. Under this value setting, I compare the two cases in terms of optimal total profit. Fig. 4 demonstrates the dominance relationship between the two cases. As observed, when prior quality (q_0) is smaller than 90, the bounded case FI is dominant. When prior quality (q_0) exceeds 90, bounded case FI becomes infeasible and while bounded case F is still feasible; hence, the bounded case F is dominant.

Fig. 4. Dominance Relationship between Bounded Cases F & FI. This figure illustrates the dominance relationship between bounded case F (“free BA only” with $q^* = \hat{q}$) and case FI (“free app + IAP” with $q_i^* = \hat{q}$) along with changing values of time discount factor (t) and prior quality (q_0). I set $s = \$10$, $\theta = \$8$, $\delta = 0.8$, $\beta = 8$, $k = 5$, and $\hat{q} = 100$, respectively. FB and FIB represent bounded cases F and FI, respectively.

6. Conclusion

In this research, I investigate mobile application (app) developers’ decisions on whether to offer in app purchase (IAP) on top of a launched app with either a free or paid version. I build a two-period model which considers several characteristics of the app market: app users’ willingness-to-pay, different users’ disutility sensitivities to free and paid apps, app quality, average ad revenue per user, time discount factor, and potential app quality upper bound. This paper mainly contributes in that it enriches the literature of information good versioning in the context of IAP, a topic that has not been well addressed by analytical studies. The several insights derived from our analytical and numerical analyses have their managerial implications.

First, I find that in the free app scenario, when the app is not easy to develop, then as long as the IAP’s quality is lower than the quality upper bound, launching IAP on top this basic app is a better choice. Due to the high difficulty level of app development, offering the IAP will require a relatively small investment when the IAP does not need to be designed with an overall quality that has exploited the quality upper limit. In this situation, launching IAP is always preferable. The IAP development will require a relatively large investment if the developer

wants to provide a well-designed IAP such that the quality reaches the upper limit. In this situation, offering IAP on top of this basic app is not absolutely preferable. In fact, the developer can also exploit the quality upper bound when launching the basic versioned app at the very beginning. However, I show that this strategy is always inferior compared to the strategy that offers IAP with a quality of the upper limit on top of a basic app.

Second, I find that in the paid app scenario offering IAP based on the basic app is better off only under some complicated conditions. This is consistent with our intuition because a developer can actually utilize the pricing tool to achieve its profitability objective when launching the basic app at the very beginning. In general, users paying for the app out of their pocket require high-quality of the app. Therefore, a developer adopting a “paid” strategy should provide the market with a well-designed app product (e.g., offering as many features/functionalities as possible). Indeed, our analytical result shows that offering IAP with a quality lower than the quality upper limit on top of a paid versioned app is never optimal. Due to the “high-quality” requirement, the choice of offering IAP based on a paid app has a concern of high product development cost; hence, more conditions must be examined if a profit-maximizing firm considers launching IAP on top a paid app.

Last, I show that in app offering facing IAP option, a forward-looking decision is always preferable than a myopic decision under the same scenario. In practice, an app developer might be either forward-looking or myopic. For such developers who did not foresee IAP when offering its basic app, it is also possible to gain some extra revenues through adding IAP features under certain condition in the later period. However, this is not the better choice; conversely, if a developer takes into consideration of IAP option when launching the basic app, it should be able to gain more total revenue than just focusing on the current period in decision making.

This paper has several limitations which point to some potential directions for future research. First, for tractability I assume that a developer has decided on launching an app for either paid or free when considering whether to feature the app with IAP option. In practice, a developer can also offer a basic app with both paid and free versions at the same time and set relatively high quality for the paid version. Future research may relax this assumption so that the model is more generalizable. Second, in this article I consider the scenario that there is only one IAP option for a developer. In reality, the developer may offer a menu of IAP options. This typically is versioning the IAP component in order to capture more IAP users. Future studies that consider this situation should be quite interesting. Third, I assume the quality upper limit is unchanged within the planning horizon in this research. Extended models that allows evolving quality upper limit are encouraged in the future. Finally, this research does not consider the

competition among app developers. Future research could also consider extending our research by addressing this issue.

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Appendices

Appendix A. Proof for Lemma 1.

The objective function (total profit) in case PI includes decision variables: qualities of BA and IAP (q and q_i), and prices of BA and IAP (p and p_i). The objective function is always concave in prices; whereas it can be either concave or convex in qualities. The optimal quality that is lower than upper limit can only occur in the situation of concavity. Assuming the objective function is concave, I obtain the optimal qualities of the initial app and IAP as follows:

$$q^* = \frac{kq_0\beta(k\beta(4-\delta)(1-\delta)-(2-\delta)\theta^2)}{k^2\beta^2(4-\delta)(1-\delta)-2k\beta(2-\delta)\theta^2+\theta^4}, \text{ and } q_i^* = \frac{kq_0\beta(k\beta(4-\delta)(1-\delta)-(3-\delta)\theta^2)}{k^2\beta^2(4-\delta)(1-\delta)-2k\beta(2-\delta)\theta^2+\theta^4}.$$

I further check that $q_i^* - q^* = -\frac{kq_0\beta\theta^2}{k^2\beta^2(4-\delta)(1-\delta)-2k\beta(2-\delta)\theta^2+\theta^4}$. Note that when concavity condition holds, $k^2\beta^2(4-\delta)(1-\delta) - 2k\beta(2-\delta)\theta^2 + \theta^4 > 0$ (4th principal minor must be positive). Hence $q_i^* - q^* < 0$. However, this is infeasible since the quality of IAP should be higher than that of BA. Therefore, the optimal quality that is lower than upper limit can never happen. In other words, the optimal quality of IAP can only be bounded to the upper limit (i.e., $q_i^* = \hat{q}$).

In summary, in case PI (“paid app + IAP”), offering the IAP with a quality lower than the upper limit is never optimal.

Appendix B. Proof for Proposition 1.

Denote π_i ($\bar{\pi}_i$) as the optimal total profit for unbounded (bounded) case i . I use the same notations throughout the proofs for all of the propositions.

Proof for Proposition 1(a):

When product development difficulty (β) for the IAP is such high that $\beta > \frac{\theta^2}{2k}$, the objective function for case FI (“free app + IAP” strategy) is strictly concave. Given the optimal quality of IAP solved is lower than the upper limit ($q_i^* < \hat{q}$), I obtain the optimal total profit for case FI (see Table 3), which is: $\pi_{FI} = \frac{s^2\theta^2}{k^2\beta(1-\delta)} - \frac{2s^2t}{k} - q_0^2\beta\delta + \frac{2q_0s\theta}{k} + \frac{\beta\delta(2kq_0^2\beta+st(st-2q_0\theta))}{2k\beta-\theta^2}$. Also, from Table 3, we know the optimal total profit for case F (“free app only” strategy) is: $\pi_F = \frac{s(s\theta^2+2k\beta(q_0\theta-st))}{k^2\beta}$. Hence, $\pi_{FI} - \pi_F = \frac{\delta((2k\beta-\theta^2)s^2\theta^2+k^2\beta^2(1-\delta)(st-q_0\theta)^2)}{k^2\beta(1-\delta)(2k\beta-\theta^2)}$. Obviously, both numerator and denominator are positive. So $\pi_{FI} - \pi_F > 0$. That is to say, as long as unbounded case FI is feasible, it always dominates unbounded case F.

Proof for Proposition 1(b):

When the optimal quality of IAP is equal to upper limit ($q_i^* = \hat{q}$), I obtain the optimal total profits for cases F and FI (see Table 5), which are: $\pi_F = \frac{s(s\theta^2 + 2k\beta(q_0\theta - st))}{k^2\beta}$ and $\overline{\pi_{FI}} =$

$$\frac{2s^2\theta^2 - 2k^2(q_0 - \hat{q})^2\beta^2(1-\delta)\delta + k\beta(1-\delta)(\hat{q}^2\delta\theta^2 - s^2t(4-t\delta) + s(4q_0\theta - 2\hat{q}t\delta\theta))}{2k^2\beta(1-\delta)}, \quad \text{respectively.} \quad \text{Thus,} \quad \overline{\pi_{FI}} - \pi_F =$$

$$\frac{\delta(2s^2\theta^2 - 2k^2(q_0 - \hat{q})^2\beta^2(1-\delta) + k\beta(1-\delta)(st - \hat{q}\theta)^2)}{2k^2\beta(1-\delta)}. \quad \text{To ensure } \overline{\pi_{FI}} - \pi_F > 0, \text{ users' disutility sensitivity to advertisement (} t \text{) must satisfy the following condition: } t < \hat{q}\theta -$$

$$\frac{\sqrt{2}\sqrt{k s^2 \beta (1-\delta) (k^2 (q_0 - \hat{q})^2 \beta^2 (1-\delta) - s^2 \theta^2)}}{k \beta (1-\delta) s^2}.$$

Appendix C. Proof for Proposition 2.

In free BA offering strategies, the developer can exploit the quality upper limit by either providing “only BA with $q^* = \hat{q}$ ” (bounded case F) or offering “BA + IAP with $q_i^* = \hat{q}$ ” (bounded case FI). I obtain the total profits for bounded cases F and FI respectively as below:

$$\overline{\pi_F} = \frac{2s(\hat{q}\theta - st)}{k} - \beta(\hat{q} - q_0)^2, \text{ and}$$

$$\overline{\pi_{FI}} = \frac{2s^2\theta^2 - 2k^2(q_0 - \hat{q})^2\beta^2(1-\delta)\delta + k\beta(1-\delta)(\hat{q}^2\delta\theta^2 - s^2t(4-t\delta) + s(4q_0\theta - 2\hat{q}t\delta\theta))}{2k^2\beta(1-\delta)}.$$

Hence,

$$\overline{\pi_{FI}} - \overline{\pi_F} = \frac{-2k^2(\hat{q} - q_0)^2\beta^2(1-\delta)^2 - 2s^2\theta^2 - k\beta(1-\delta)(s^2t^2\delta + 2s(2q_0 - \hat{q}(2+t\delta))\theta + \hat{q}^2\delta\theta^2)}{-2k^2\beta(1-\delta)}. \quad \text{Since the denominator is}$$

negative, the numerator must be negative to make $\overline{\pi_{FI}} - \overline{\pi_F} > 0$. Denote the numerator $-2k^2(\hat{q} - q_0)^2\beta^2(1-\delta)^2 - 2s^2\theta^2 - k\beta(1-\delta)(s^2t^2\delta + 2s(2q_0 - \hat{q}(2+t\delta))\theta + \hat{q}^2\delta\theta^2)$ as M. M can be expanded as: $-2k^2q_0^2\beta^2 + 4k^2q_0\hat{q}\beta^2 - 2k^2\hat{q}^2\beta^2 - ks^2t^2\beta\delta + 4k^2q_0^2\beta^2\delta - 8k^2q_0\hat{q}\beta^2\delta + 4k^2\hat{q}^2\beta^2\delta + ks^2t^2\beta\delta^2 - 2k^2q_0^2\beta^2\delta^2 + 4k^2q_0\hat{q}\beta^2\delta^2 - 2k^2\hat{q}^2\beta^2\delta^2 - 4kq_0s\beta\theta + 4k\hat{q}s\beta\theta + 4kq_0s\beta\delta\theta - 4k\hat{q}s\beta\delta\theta + 2k\hat{q}st\beta\delta\theta - 2k\hat{q}st\beta\delta^2\theta - 2s^2\theta^2 - k\hat{q}^2\beta\delta\theta^2 + k\hat{q}^2\beta\delta^2\theta^2$.

Factoring all of the t^2 terms, I get $ks^2t^2\beta(-1+\delta)\delta$. Note that the coefficient of t^2 is negative. Thus, I solve for $M < 0$ with respect to t and get: $t < t_1$ or $t > t_2$ where $t_1 =$

$$\hat{q}s^3\theta - \frac{\sqrt{2}\sqrt{-ks^2\beta(1-\delta)\delta(k(\hat{q}-q_0)\beta(1-\delta)-s\theta)^2}}{k\beta(1-\delta)\delta s^2} \quad \text{and} \quad t_2 = \hat{q}s^3\theta +$$

$$\frac{\sqrt{2}\sqrt{-ks^2\beta(1-\delta)\delta(k(\hat{q}-q_0)\beta(1-\delta)-s\theta)^2}}{k\beta(1-\delta)\delta s^2}. \quad \text{Note that the term inside the square roots, i.e., } -ks^2\beta(1-$$

$\delta)\delta(k(\hat{q} - q_0)\beta(1 - \delta) - s\theta)^2$, is always negative. This indicates that t_1 and t_2 are not real numbers. Therefore, $M < 0$ should always holds, and so $\overline{\pi_{FI}} - \overline{\pi_F} > 0$ always holds.

Appendix D. Proof for Proposition 3.

For a myopic developer who wants to offer IAP, there are two situations: (1) free BA first and IAP later; and (2) paid BA first and IAP later. I check that for the first situation there might be either unbounded (unbounded case FI_m where $q_i^* < \hat{q}$) or bounded (bounded case FI_m where $q_i^* = \hat{q}$) scenario; whereas for the second situation only bounded scenario (case PI_m where $q_i^* = \hat{q}$) can take place (similar to Proposition 3). For each scenario, I can solve the optimal price and quality of IAP. Specifically, (p_i^*, q_i^*) are $(\frac{k\beta(q_0\theta-st)}{2k\beta-\theta^2}, \frac{2kq_0\beta-st\theta}{2k\beta-\theta^2})$ in unbounded free BA case, $(\frac{1}{2}(\hat{q}\theta-st), \hat{q})$ in bounded free BA case, and $(\frac{1}{2}\theta(\hat{q}-\frac{kq_0\beta}{2k\beta-\theta^2}), \hat{q})$ in paid BA case.

Appendix E. Proof for Proposition 4.

In this proposition, I compare forward-looking and myopic strategies of IAP offering. There are three pairs of total profits (forward-looking vs. myopic) for comparison: (1) π_{FI} vs. $\pi_{FI\text{m}}$, (2) $\overline{\pi_{FI}}$ vs. $\overline{\pi_{FI\text{m}}}$, and (3) $\overline{\pi_{PI}}$ vs. $\overline{\pi_{PI\text{m}}}$. The above optimal total profits I obtained are listed as below:

$$\begin{aligned}\pi_{FI} &= \frac{s^2\theta^2}{k^2\beta(1-\delta)} - \frac{2s^2t}{k} - q_0^2\beta\delta + \frac{2q_0s\theta}{k} + \frac{\beta\delta(2kq_0^2\beta+st(st-2q_0\theta))}{2k\beta-\theta^2}, \\ \pi_{FI\text{m}} &= \frac{2ks\beta\theta^2(s(1+t+\delta)-q_0\theta)-s^2(1+\delta)\theta^4+k^2\beta^2(q_0\theta-st)(s(4-t\delta)+q_0\delta\theta)}{k^2\beta(2k\beta-\theta^2)}, \\ \overline{\pi_{FI}} &= \frac{2s^2\theta^2-2k^2(q_0-\hat{q})^2\beta^2(1-\delta)\delta+k\beta(1-\delta)(\hat{q}^2\delta\theta^2-s^2t(4-t\delta)+s(4q_0\theta-2\hat{q}t\delta\theta))}{2k^2\beta(1-\delta)}, \\ \overline{\pi_{FI\text{m}}} &= \frac{k\beta(-2k(q_0-\hat{q})^2\beta\delta-s^2t(4-t\delta))+2ks\beta(2q_0-\hat{q}t\delta)\theta+(k\hat{q}^2\beta\delta+2s^2(1+\delta))\theta^2}{2k^2\beta}, \\ \overline{\pi_{PI}} &= \frac{2k\beta(q_0^2-q_0\hat{q}(3-\delta)\delta+\hat{q}^2(2-\delta)\delta)\theta^2-k^2(q_0-\hat{q})^2\beta^2(4-\delta)(1-\delta)\delta-\hat{q}^2\delta\theta^4}{k(k\beta(4-\delta)(1-\delta)-2\theta^2)}, \text{ and} \\ \overline{\pi_{PI\text{m}}} &= \frac{k^2\beta^2(12\hat{q}^2\delta-20q_0\hat{q}\delta+q_0^2(4+9\delta))\theta^2-2k\beta(q_0^2+3\hat{q}(\hat{q}-q_0)\delta)\theta^4+\hat{q}^2\delta\theta^6-8k^3(q_0-\hat{q})^2\beta^3\delta}{2k(2k\beta-\theta^2)^2}.\end{aligned}$$

First, I compared π_{FI} and $\pi_{FI\text{m}}$. Since $\pi_{FI} - \pi_{FI\text{m}} = \frac{s^2\delta^2\theta^2}{k^2\beta(1-\delta)} > 0$, FI (forward-looking offering of “free BA + IAP” with $q_i^* < \hat{q}$) always dominates FI_m (myopic offering of “free BA + IAP” with $q_i^* < \hat{q}$) as long as concavity and feasibility condition for case FI holds.

Second, I compare $\overline{\pi_{FI}}$ and $\overline{\pi_{FI\text{m}}}$. Since $\overline{\pi_{FI}} - \overline{\pi_{FI\text{m}}} = \frac{s^2\delta^2\theta^2}{k^2\beta(1-\delta)} > 0$, bounded case FI (forward-looking offering of “free BA + IAP” with $q_i^* = \hat{q}$) always dominates bounded case FI_m (myopic offering of “free BA + IAP” with $q_i^* = \hat{q}$) as long as feasibility and/or concavity condition for case FI holds.

Third, I compare $\overline{\pi_{PI}}$ and $\overline{\pi_{PIIm}}$. $\overline{\pi_{PI}} - \overline{\pi_{PIIm}} = \frac{((\beta\delta^2\theta^2(k^2(q_0-2\hat{q})^2\beta^2(1-\delta)+2k\beta(q_0^2(6-\delta)-2\hat{q}^2(1-\delta)-q_0\hat{q}(3+\delta))\theta^2+\hat{q}(4q_0+\hat{q}(1-\delta))\theta^4))}{(2(k\beta(4-\delta)(1-\delta)-2\theta^2)(2k\beta-\theta^2))}$. Since $(k\beta(4-\delta)(1-\delta)-2\theta^2) < 0$ (concavity condition for bounded FBI), the denominator is negative. To make $\overline{\pi_{PI}} - \overline{\pi_{PIIm}} < 0$, the numerator must be positive. So I examine this condition next. The numerator $\left((\beta\delta^2\theta^2(k^2(q_0-2\hat{q})^2\beta^2(1-\delta)+2k\beta(q_0^2(6-\delta)-2\hat{q}^2(1-\delta)-q_0\hat{q}(3+\delta))\theta^2+\hat{q}(4q_0+\hat{q}(1-\delta))\theta^4)\right) = -k^2q_0^2\beta^2 + 4k^2q_0\hat{q}\beta^2 - 4k^2\hat{q}^2\beta^2 + k^2q_0^2\beta^2\delta - 4k^2q_0\hat{q}\beta^2\delta + 4k^2\hat{q}^2\beta^2\delta - 12kq_0^2\beta\theta^2 + 6kq_0\hat{q}\beta\theta^2 + 4k\hat{q}^2\beta\theta^2 + 2kq_0^2\beta\delta\theta^2 + 2kq_0\hat{q}\beta\delta\theta^2 - 4k\hat{q}^2\beta\delta\theta^2 - 4q_0\hat{q}\theta^4 - \hat{q}^2\theta^4 + \hat{q}^2\delta\theta^4$. Now we check whether the following inequality holds:

$$-k^2q_0^2\beta^2 + 4k^2q_0\hat{q}\beta^2 - 4k^2\hat{q}^2\beta^2 + k^2q_0^2\beta^2\delta - 4k^2q_0\hat{q}\beta^2\delta + 4k^2\hat{q}^2\beta^2\delta - 12kq_0^2\beta\theta^2 + 6kq_0\hat{q}\beta\theta^2 + 4k\hat{q}^2\beta\theta^2 + 2kq_0^2\beta\delta\theta^2 + 2kq_0\hat{q}\beta\delta\theta^2 - 4k\hat{q}^2\beta\delta\theta^2 - 4q_0\hat{q}\theta^4 - \hat{q}^2\theta^4 + \hat{q}^2\delta\theta^4 > 0 \quad (1).$$

Putting together all the q_0^2 terms in inequality (1), I get $-k^2q_0^2\beta^2 + k^2q_0^2\beta^2\delta - 12kq_0^2\beta\theta^2 + 2kq_0^2\beta\delta\theta^2 = -k\beta(k\beta(1-\delta) + 2(6-\delta)\theta^2)q_0^2$. Note that the coefficient of q_0^2 in inequality (1) is negative. Solving inequality (1) w.r.t. q_0 , I obtain:

$$\frac{\hat{q}(k\beta(1-\delta)+2\theta^2)(2k\beta-\theta^2)-\sqrt{2}\sqrt{\hat{q}^2\theta^2(2k\beta-\theta^2)^2(-k\beta(4-\delta)(1-\delta)+2\theta^2)}}{k\beta(k\beta(1-\delta)+2(6-\delta)\theta^2)} < q_0 < \frac{\hat{q}(k\beta(1-\delta)+2\theta^2)(2k\beta-\theta^2)+\sqrt{2}\sqrt{\hat{q}^2\theta^2(2k\beta-\theta^2)^2(-k\beta(4-\delta)(1-\delta)+2\theta^2)}}{k\beta(k\beta(1-\delta)+2(6-\delta)\theta^2)} \quad (2).$$

Thus, if inequality (2) holds, inequality (1) also holds and $\overline{\pi_{PI}} - \overline{\pi_{PIIm}} < 0$. However, since $(-k\beta(4-\delta)(1-\delta) + 2\theta^2) < 0$ (concavity condition for bounded case PI), the two critical values in inequality (2) are not real numbers. Therefore, $\overline{\pi_{PI}} - \overline{\pi_{PIIm}} < 0$ can never happen. In other words, the opposite direction of inequality (1) should always hold, i.e.,

$$-k^2q_0^2\beta^2 + 4k^2q_0\hat{q}\beta^2 - 4k^2\hat{q}^2\beta^2 + k^2q_0^2\beta^2\delta - 4k^2q_0\hat{q}\beta^2\delta + 4k^2\hat{q}^2\beta^2\delta - 12kq_0^2\beta\theta^2 + 6kq_0\hat{q}\beta\theta^2 + 4k\hat{q}^2\beta\theta^2 + 2kq_0^2\beta\delta\theta^2 + 2kq_0\hat{q}\beta\delta\theta^2 - 4k\hat{q}^2\beta\delta\theta^2 - 4q_0\hat{q}\theta^4 - \hat{q}^2\theta^4 + \hat{q}^2\delta\theta^4 < 0 \quad (3).$$

Hence, $\overline{\pi_{PI}} - \overline{\pi_{PIIm}} > 0$ always holds, indicating bounded case PI (forward-looking offering of “paid BA + IAP” with $q_i^* = \hat{q}$) always dominates bounded case PIm (myopic offering of “paid BA + IAP” with $q_i^* = \hat{q}$).

In summary, for a developer who offers IAP, forward-looking strategy always outperforms myopic strategy no matter whether the developer launches the initial app for free or paid.

Free or Paid: Should Mobile App Developers Charge Users After Trial Period?

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Abstract

The choice of revenue models is critical for a mobile application (app) developer's success. In this paper, I examine the revenue model selection for a profit oriented app developer. From app developers' perspective, I show that under information asymmetry condition "trial and then free without ads" (TF) strategy is always suboptimal. "Trial and then paid" (TP) strategy is dominant if users are highly (not very) sensitive to app price as well as market size is small (big); otherwise, "trial and then free with ads" (TFA) strategy is dominant. The welfare analysis suggests that the optimal strategy is dependent on market size and users' attributes. Additionally, this paper examines developer's decisions on ranking effort in the trial period and ad frequency in the later period.

Keywords: App Developer, App Ranking, In-App Advertisement (IADV), Ad Frequency, Product Offering.

1. Introduction

As indispensable software for mobile devices, mobile applications (apps) have been vastly changing the way of communication, work, socialization, and recreation, and the number of apps has been growing rapidly in the recent years. As of June 2016, the number of apps available on the two leading platforms (Android and iOS) are 2.2 million and 2 million respectively (Statista, 2016). Since the screen of smartphones is small while there are a large number of apps available on the same platform, consumers usually choose to download those displayed on the top-ranking list (Garg and Telang, 2013). It is also observed that the majority of apps offered in app platforms are free and most of the free apps have embedded advertisements (ads) (Eberhardt, 2014), which are so-called in-app advertisements (IADVs)⁷. Empirical evidence suggests that consumers embrace free apps more than paid apps although the former have IADVs (e.g., Garg and Telang, 2013). However, users' engagement with an installed app is negatively impacted by the placement frequency of ads (Armour, 2017). In the worst situation, a user would choose to uninstall an app if she is drastically annoyed by frequent pop-up ads. Therefore, an app developer should try to boost the ranking of its app and carefully decide whether to charge users for the app. However, there are some factors that a developer should consider when making decisions regarding ranking effort and pricing strategies of the app.

First, there are many ways that may help boost an app's ranking (e.g., utilize multiple traffic sources, drive downloads from current users, require users to score the app rating, etc.). However, as most ranking efforts require investment, a developer should evaluate investment in ranking effort and the revenue gained from the effort. Second, app users are heterogeneous in terms of their preferences (Lee and Raghu, 2014). As users can only know how much they would use an app after a usage period (Wei and Nault, 2013), there is difference between app users' willingness to download an app and willingness to use an installed app. Users' willingness to download an app is mostly impacted by the app's perceived quality, ranking position, and app price (Yu et al., 2017a; Ifrach and Johari, 2014). Whereas users' willingness to engage with the installed app can be affected by the ex-post quality of the app and the price required to pay for playing it.

There are several ways that app developers may launch apps: let users try the app for a short period and then decide to offer an option such as keeping the app free but embedding ads into it, charging users without adding any additional features, or offering more features with in-

⁷ Ghose and Han (2014) use "IADV" to stand for in app advertisement.

app purchase (IAP) options. The advantages of trial usage lie in several folds. First, it enables a developer to capture a big enough user base and helps increase the app's ranking. Second, it allows the developer to collect users' information which are valuable for making decisions on how to recoup revenues from the captured users. Third, the trial period allows the developer to test and debug its app product. The problem faced by app developers is that although users' willingness-to-download is not difficult to estimate, the information of users' willingness-to-play is not easy to obtain when an app is launched. To our knowledge, most existing analytical models for information product offerings assume information symmetry, i.e., users' information is known to the developer at the very beginning (e.g., Weber, 2008, Wei and Nault, 2013). As exceptions, Chellapa and Shivendu (2010) and Chellappa and Mehra (2017) consider both information symmetry and asymmetry assumptions in their models. The above two articles do not consider free and paid offerings simultaneously for a developer's product offering strategies. As Yu et al. (2017a) and Yu et al. (2017b) have pointed out, consumers' disutility sensitivity to free apps might be different from that to paid apps. So a developer must be careful when choosing between free and paid offering strategies. I note that in Yu et al.'s (2017a, b) models, the average ad revenue per user is assumed to be predetermined by the market. In reality, a developer can actually determine how frequently to display ads in a free app, thereby influencing the average ad revenue per user (Ivanov, 2015). To summarize, the above discussion suggests that it is not suitable to simply apply existing analytical models to the free-trial and then fee-or-free setting.

The objective of the current research is to investigate app developers' decisions on ranking effort and whether to charge users for the app after a trial period. I build a two-period model by considering the following decision variables for the app developer: app quality, app ranking effort, ad placement frequency, and app price. For tractability purpose, I do not consider quality improvement⁸. Also, I only consider the situation that an app is offered for either free with/without ads or paid without ads. In specific, in the first period ("trial period" or "Period T" hereinafter), users download and install an app, and are allowed to use it for free. In this period, the developer does not show users any ads but will collect users' information. In the second period ("usage period" or "Period U" hereinafter) the current users engage with the app. In this period, based on the users' information obtained in Period T, the developer will decide whether to charge the current users; even if it does not charge users in this period, it would display ads to the current users. I define the former strategy as "trial and then paid (TP)" while the latter

⁸ In Yu et al. (2017b), the authors consider in app purchase (IAP) issue in a two-period model, where IAP has higher quality than the basic app.

“trial and then free with ads (TFA)”. In this setting, the developer’s revenue comes from two periods: the revenue generated in Period T, and the revenue paid off from advertisers for users’ clicks on the ads or users’ direct payment in Period U. I am particularly interested in the following questions: (1) How frequently should a developer display ads to its users if it adopts TFA strategy? (2) How much should the developer charge users if it adopts TP strategy? (3) How much effort should the developer put into boosting the app ranking? (4) What quality level should the developer set for the app? (5) Which strategy can result in relatively high total welfare?

This research potentially makes contributions to both academic research and industry practice. First, this paper enriches the literature on decision making for app developers’ product offering. The present study extends Yu et al. (2017a) and Yu et al. (2017b) by relaxing the assumption that average ad revenue per user is exogenously given. In the current research, I consider the scenario when a developer can determine average ad revenue per user via setting ad frequency. Moreover, I include app ranking effort in the model. To the best of my knowledge, this is the first article that considers both IADV display frequency and app ranking effort in an analytical model. Third, the findings from the analysis have managerial implications. Specifically, I show that from a developer’s perspective, after a trial period, the option of either switching to “paid” app or keeping the app “free” but embedding ads in it can be optimal depending on both users’ disutility sensitivity to app price and market size; whereas keeping the app “free” without embedding ads in it is always suboptimal. I also show that, from policy makers’ perspective, total welfare realized in either “free” or “paid” offering can be higher dependent on users’ willingness to play and market size. The model and analytical results can also address similar decision problems that are prevalent in information goods industry (e.g., online content products).

This article is structured as follows. In the next section, I summarize relevant literature. Section 3 presents the assumptions and model structure. Section 4 provides my analysis based on the model I created and summarizes the major findings. In section 5, I conduct numerical analyses. The article is concluded with remarks in Section 6 drawing a big picture of what I have done and pointing out future directions.

2. Relevant Literature

There are several literature streams that are closely related to this paper, including those about information product offering strategies, particularly the choice of revenue models. I provide a brief review of each literature stream and discuss how my study is related to them.

One set of articles closely tied to the present research is about product offering strategies

of information goods. Prior analytical studies on this topic have investigated new product development (Krishanan and Zhu, 2006; Chen et al., 2010), product versioning and pricing (Bala and Carr, 2009; Wei and Nault, 2013), product development cost structure (Jones and Mendelson, 2011; Chellappa and Mehra, 2017), competition game (Chambers, et al., 2006; Jones and Mendelson, 2011), and product delivery systems (Guo et al., 2015). For example, Guo et al. (2015) study delivery systems (push vs. pull) of online contents faced by producers and consumers. Offering decisions for mobile apps, one type of information goods mostly appearing in the recent years, have also caught scholars' attention; yet, analytical research on this topic is still lacking (Yu et al., 2017a). In mobile app industry, the app products are experience goods and users have to install an app before using it. Also, there are multiple app platforms and revenue sources for app developers' selection. Moreover, an app's product visibility, reflected as app ranking, is critical for a developer to capture users. All the above characteristics as a combination makes mobile app offering decisions require a distinct research setting. In fact, due to the unique characteristics of mobile apps, most existing analytical models regarding product offering strategies cannot be simply applied to this industry (Yu et al., 2017a). Yu et al. (2017a) investigate app developers' platform selection strategies when launching either paid or free version of an app, while Yu et al. (2017b) consider app developers' product versioning decisions by focusing on in-app purchase. This paper differentiates from Yu et al. (2017a, b) in several aspects. First, I consider the scenario when a developer can determine ad placement frequency which has not been addressed by Yu et al. (2017a, b). Second, I consider a developer's ranking effort for its app product which also has not been studied by Yu et al. (2017a, b). Third, in the current two-period model I assume that users' willingness-to-download is known but their willingness-to-play is unknown to a developer when an app is launched; the developer is able to obtain users' information of willingness-to-play during the trial period.

Further, there are a number of articles looking into the choice of revenue models for information good producers. Pauwels and Weiss' (2008) empirical study shows that charging users for online content after a free period can be a good choice if the target users are professionals. Many other articles have empirically documented that charging users for information goods is challenging (Shampanier et al., 2007; Ghose and Han, 2014). This stems largely from consumers' sensitivity to the price of information goods (Yu et al., 2017a). Due to this fact, the "free" strategy, usually accompanying with ads, has been prevalent in this industry. However, the profitability of free strategy is hinged on certain market conditions. For example, Goldfarb and Tucker (2011) find that matching an ad to online content and increasing an ad's obtrusiveness can independently increase users' purchase intent; however, doing both at the

same time actually reduces users' purchase intent. In a recent study by Lambrecht and Misra (2017), the authors address the problem of how online content producers price their product and make the trade-off between advertising and subscription when consumers' valuations change overtime. Although the current paper also considers app developers' choice of offering the app either-free-or-paid, my work differs from Lambrecht and Misra (2017) in that I take into account the product development cost. Goldfarb and Tucker's (2011) findings provide empirical evidence for my assumptions that users' information collected in the trial period is critical for a developer's offering decision in the later period and that ad frequency must be carefully determined.

3. Model

This section demonstrates my two-period model structure in terms of user utility, cost for boosting app ranking, app development cost, and total profit function.

Suppose a "monopolistic" app developer is considering whether to provide users an app product in a platform (e.g., iOS platform) with quality level of q (see Table 1 for notations). App quality is referred to as an aggregate measure of quality characteristics or attributes (Rosen, 1974; Yu et al., 2017a). The quality attributes include aspects such as an app's functionalities, user friendliness, trustworthiness by users (user review), and so forth (Kuehnhausen and Frost, 2013; Yu et al., 2017a). The developer will offer users a trial period and decide whether to keep the app free with ads ("trial and then free with ads" strategy) or charge users ("trial and then paid" strategy). Suppose the developer would put an effort of h ($h > 0$) in boosting the app's ranking r ($r \in \mathbb{N}^+$)⁹. In practice, an app's ranking is impacted by factors including the quality perceived by users and the ranking effort made by the developer. These factors influence downloads for the app and ultimately determine the app's ranking position. I therefore simply assume that app ranking, app quality and ranking effort are related to each other such that

$$r = \frac{a}{hq} \quad (1),$$

where a is a positive constant.

⁹ Note that smaller r means higher ranking. For example, app A ranked second has higher ranking position than app B ranked 5th.

Table 1. Notations

Parameters	
s	Ad revenue per click ($s > 0$)
v	Revenue per user gained by the developer in Period T ($v > 0$)
c	Average cost per user for download and usage in Period T ($c > 0$)
θ_d	Users' willingness to trial use the app ($\theta_d \sim U(0,1)$)
θ_u	Users' willingness to use after the trial period ($\theta_u \sim U\left(\frac{D_d}{N}, 1\right)$)
γ	Cost parameter of ranking effort ($\gamma > 0$)
β	App development difficulty (or cost parameter for app development, $\beta > 0$)
N	Total number of potential users (or market size, $N \in \mathbb{N}^+$)
δ	Time discount factor ($0 < \delta < 1$)
a	A positive constant ($a > 0$)
t	Users' disutility sensitivity to app price
Decision Variables	
q	App quality ($q > 0$)
h	Developer's effort on boosting the app ranking (or termed "ranking effort", $h > 0$)
f	Ad frequency ($f > 0$)
p	App price ($p > 0$)
Other Parameters	
r	App ranking ($r = \frac{a}{hq}$, $r \in \mathbb{N}^+$)
U_d	Net utility gained by a user by trial using the app
U_u	Net utility gained by a user in Period U
D_d	Realized demand in Period T
D_u	Realized demand in Period U
π_d	Profit gained by the developer in Period T
π_u	Profit gained by the developer in Period U

Further, I assume the total number of potential users for this app is N ($N \in \mathbb{N}^+$), and the users are uniformly distributed with $\theta_d \sim U(0,1)$ and $\theta_u \sim U\left(\frac{D_d}{N}, 1\right)$, where θ_d and θ_u represent users' willingness to use the trial and willingness to play the app respectively, and D_d

is the number of users who use trial of the app (i.e., realized demand in Period T). According to Calvo-Armengol and Zenou (2002), the uniform distribution assumption should not change the qualitative directions of analytical results for the model. Before downloading the app, a user can only perceive the quality of the app, which can be a function of attributes such as the app's description, other users' reviews, the app's ratings, and so forth. She downloads an app mainly based on her perception of the app quality as well as the app's ranking. Further assume that by using trial of the app, a user gains net utility

$$U_d = \theta_d/r - c \quad (2),$$

where the positive constant c denotes the costs regarding download, install, and play the app during Period T. Combining equations (1) and (2) we have

$$U_d = \theta_d hq/a - c \quad (3).$$

Since only those users who can gain positive net utility would try the app, the demand realized by the developer in Period T is: $D_d = N(1 - \frac{ac}{hq})$. Fig. 1 provides a visual illustration of demand modeling.

Fig. 1. Realized Demand in Both Periods. An app developer offers an app with quality of q for free in a trial period (Period T), and then keeps the app free by displaying ads with a frequency of (f) or charges users at a price of p in the usage period (Period U). In Period T, the developer also makes effort in boosting app ranking (h) which will determine the app's ranking. Users are uniformly distributed in terms of willingness to download (WTD, θ_d) and willingness to play (WTP, θ_u). Users evaluate the app based on both app quality and app ranking when choosing to download it. A representative user locating in a position of the market will realize net utility of U_d (or U_u) by participating in each period. Those who can realize positive net utility are captured by the developer (highlighted with shadow). c : trial usage cost, a : positive constant, and t : users' disutility sensitivity to app price.

In Period U, the current users are able to reevaluate the app quality based on their experience in using the app during Period T. For simplicity, I assume that both perceived and experienced qualities of the app have the same level (both are equal to q). However, I assume users' willingness to play (θ_u) can be different from the willingness to download (θ_d), which acts as a proxy of users' reevaluation ($\theta_u q$) of the app quality. In this period, the developer decides whether to (1) charge users a price p , or (2) keep the app free but display ads with a frequency of f . I assume further the developer gains ad revenue per click (denoted by s). Also, according to Yu et al. (2017a), users have higher disutility sensitivity to paid version than to free version of the same app. In Yu et al.'s (2017a) research setting, it is implicitly assumed that users know free apps are embedded with certain amount of ads requiring click when using the apps. In the current paper, I assume that users will know how much she must "pay" to play the app either by spending real money or by bearing annoyance from ads. Hence, I relax Yu et al.'s (2017a) assumption by assuming that users' disutility sensitivity to app price (denoted by t) can be either higher or lower than users' disutility sensitivity to ads (which is normalized to 1).

Hence, by playing the installed app in Period U, a user can gain net utility: $U_u = \theta_u q - fs$ (TFA strategy) or $U_u = \theta_u q - tp$ (TP strategy). Thus similarly, the demand realized by the developer in Period U is $D_u = N(1 - \frac{ac}{hq})(1 - \frac{fs}{q})$ and $D_u = N(1 - \frac{ac}{hq})(1 - \frac{tp}{q})$ for TFA and TP strategies, respectively.

Next, I model the cost structure for the developer. In the current research setting, I assume that ad placement related cost is negligible; hence, there are two cost components needed to be incurred: cost for ranking effort and app development. I adopt quadratic form that has been used by many prior studies (e.g., Desai, 2001) in modeling the above cost components. In specific, the cost for ranking efforts is: γr^2 , where the positive constant γ is the cost parameter of ranking effort; and the app development cost is: βq^2 , where the positive constant β is the difficulty parameter of the app development. All costs are assumed to be incurred in Period T.

Revenues for the developer come from both periods. In Period T, users provide some information when downloading and using the app. The information such as users' demographics and usage habits gathered in Period T may be sold to other parties or be utilized for the decisions for Period U and/or other business activities for the developer itself, thereby generating some revenues (Seneviratne et al., 2015; Meng, et al., 2016). So I assume that the developer can gain revenue per user (denoted by v) in Period T. In Period U, depending on either-free-or-paid strategy set, the developer gains ad revenue per user (fs) for TFA strategy (or p for TP strategy). Thus, the profit gained by the developer in the two periods are expressed as below.

$$\text{Period T: } \pi_d = N \left(1 - \frac{ac}{hq}\right) v - \gamma h^2 - \beta q^2 \quad (3)$$

$$\text{Period U: } \pi_u = N \left(1 - \frac{ac}{hq}\right) \left(1 - \frac{fs}{q}\right) fs \quad (\text{TFA strategy}) \quad (4)$$

$$\pi_u = N \left(1 - \frac{ac}{hq}\right) \left(1 - \frac{tp}{q}\right) p \quad (\text{TP strategy}) \quad (5)$$

Denote δ ($0 < \delta < 1$) as the time discount factor. Therefore, the objective for the profit-maximizing developer is to achieve optimal total profit from both periods. That is:

$$\max_{q, h, f} \left(N \left(1 - \frac{ac}{hq}\right) v - \gamma h^2 - \beta q^2 \right) + \delta \left(N \left(1 - \frac{ac}{hq}\right) \left(1 - \frac{fs}{q}\right) fs \right) \quad (\text{TFA strategy}) \quad (6), \text{ or}$$

$$\max_{q, h, p} \left(N \left(1 - \frac{ac}{hq}\right) v - \gamma h^2 - \beta q^2 \right) + \delta \left(N \left(1 - \frac{ac}{hq}\right) \left(1 - \frac{tp}{q}\right) p \right) \quad (\text{TP strategy}) \quad (7),$$

$$\text{s.t. } 0 < 1 - \frac{ac}{hq} \leq 1, 1 - \frac{fs}{q} \leq 1, 0 < \delta < 1, N \in \mathbb{N}^+, \text{ and } a, c, v, \gamma, f, s, t, q, h, \beta > 0.$$

In the above system, the component $N\left(1 - \frac{ac}{hq}\right)v - \gamma h^2 - \beta q^2$ represents profit generated in Period T; while the component $N\left(1 - \frac{ac}{hq}\right)\left(1 - \frac{fs}{q}\right)fs$ (or $N\left(1 - \frac{ac}{hq}\right)\left(1 - \frac{tp}{q}\right)p$) represents profit generated in Period U. Components γh^2 and βq^2 are the costs for ranking efforts and app development, respectively. Note that both the cost components are incurred in Period T. As for the developer, the decision variables include q, h and f (or p). It is worth to note that for $f = p = 0$, both systems (6) and (7) can be simplified as the same model as follows. In fact, in this special case, the developer does not display any ads or charge users after the trial period. I call this strategy “trial and then free without ads” (TF).

$$\max_{q,h} \left(N\left(1 - \frac{ac}{hq}\right)v - \gamma h^2 - \beta q^2 \right) \quad (\text{TF strategy}) \quad (8)$$

$$\text{s.t. } 0 < 1 - \frac{ac}{hq} \leq 1, \quad N \in \mathbb{N}^+, \quad \text{and } a, c, v, \gamma, q, h, \beta > 0.$$

4. Analysis

Now I analyze the models described in the previous section and examine the analytical results that correspond to my research questions. I also conduct analyses of welfare-efficient prices for the several offering strategies.

In the current research setting, the developer makes decisions on quality level (q) and ranking effort (h) for the app in Period T; in Period U, the developer decides a frequency (f) for the ads shown to users or the price (p) for the app. I checked that the objective function in system (6) is strictly concave in q and h . I therefore solved system (6) with respect to q and h , and then substituted (q^*, h^*) back into system (6). I further checked that the objective function is strictly concave in f , so I solved the system with respect to f . After this step, we can simply substitute (q^*, h^*, f^*) into the objective function in system (6) to obtain the optimal total profit (π_{TFA}) for TFA strategy. For system (7), the model for TP strategy, I found similar property of concavity. Thus, I can solve the model following the steps that I solved for system (6). System (8) is a special case of system (6) or (7), so it is easy to solve for the optimal metrics. Table 2 lists the optimal solution and the corresponding optimal total profit for each strategy.

Table 2. Optimal Matrix for Each Strategy

TFA	
Quality (q)	$\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}$
Ranking Effort (h)	$\frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}$
Ad Fr. (f)	$\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}s}$
Total Profit (π)	$Nv - \frac{a^{3/4}c^{3/4}\gamma^{3/8}N^{3/4}\delta}{22^{3/4}\beta^{1/8}v^{1/4}} + \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{5/4}v^{1/4}\delta}{42^{1/4}\beta^{3/8}} - 2\sqrt{2}\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4}\sqrt{N}\sqrt{v}$
TP	
Quality (q)	$\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}$
Ranking Effort (h)	$\frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}$
Price (p)	$\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}t}$
Total Profit (π)	$Nv - \frac{22^{1/4}a^{3/4}c^{3/4}\gamma^{3/8}N^{3/4}\delta}{8t\beta^{1/8}v^{1/4}} + \frac{2^{3/4}a^{1/4}c^{1/4}\gamma^{1/8}N^{5/4}v^{1/4}\delta}{8t\beta^{3/8}} - 2\sqrt{2}\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4}\sqrt{N}\sqrt{v}$
TF	
Quality (q)	$\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}$
Ranking Effort (h)	$\frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}$
Total Profit (π)	$Nv - 2\sqrt{2}\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4}\sqrt{N}\sqrt{v}$

4.1 “Trial and then Free with Ads” Strategy

If a developer chooses to continue the free offering and embed ads into the app after the trial period (TFA strategy), how should the developer make decisions? In this strategy, if the developer determines the ad frequency based on users’ information gathered during the trial

period, I show that it should set quality and ad frequency as well as ranking effort in accordance with Proposition 1. All proofs are provided in appendix.

Proposition 1. *Optimal app quality, ranking effort, and ad frequency for a developer in TFA strategy:* If users' information of willingness-to-play is unknown to a developer before launching an app (i.e., information asymmetry), the developer should set $q^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}$, $h^* = \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}$, and $f^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}s}$. ■

Furthermore, as summarized in Lemma 1, our result suggests that in this strategy, by setting ad frequency $f^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}s}$, the developer will capture only half of the users that have been captured in the trial period.

Lemma 1. For a developer adopting TFA strategy under information asymmetry condition, to achieve maximal total profit, it should only focus on half of the users captured in the trial period (i.e., $D_u^* = \frac{1}{2}D_t^*$). ■

Here is the interpretation for Lemma 1: In Period U, the developer can decide how frequently to place ads. If it displays ads to users too frequently, most users will not play or will even uninstall the app. On the other hand, if the developer shows ads to users not often, most users will be willing to engage with the app, but the ad revenue gained by the developer would be low since there will not be too many clicks on the ads.

Moreover, under information asymmetry condition, if the developer adopts TFA strategy, then the ad frequency should be set based on the optimal app quality and the per-click ad revenue. Lemma 2 summarizes this relation.

Lemma 2. If users' information of willingness-to-play is unknown to a developer before launching an app, there is a relation among the optimal ad frequency, the optimal app quality and per-click ad revenue, which is $f^* = \frac{q^*}{2s}$. ■

Lemma 2 actually answers one of the research questions: How frequently should a developer display ads to the users? The finding shows that it depends on both app quality (q) and per-click ad revenue (s). In specific, if the app quality is high, implying the app is well designed (large q), users can bear annoyance from relatively frequent ads (allowing large f). This is consistent with our intuition. For example, an app with functionalities or features that align with users' preferences can entice current users to play the app; as a result, it is feasible to display more ads. On the other hand, if the ads are designed with attributes of overly annoyances (e.g., an ad showed to consumers is long each time, which is likely to bring about high average ad revenue per user for the developer), users would engage less often with the app. As a result,

the developer should display the ads less frequently (small f). For instance, although longer ad is likely to bring about higher average ad revenue per user (large s), in general users would be easy to get upset if the display time of an ad is too long, which suggests that the ad frequency (f) should not be set too high.

4.2 “Trial and then Paid” Strategy

As an alternative, the developer can choose to charge the current users without placing any ads after a trial period. I show that under information asymmetry condition, the developer should set quality and ad frequency as well as ranking effort in accordance with Proposition 2.

Proposition 2. *Optimal app quality, ranking effort, and app price for a developer in TP strategy:*

If users’ information of willingness-to-play is unknown to a developer before launching an app,

the developer should set $q^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}$, $h^* = \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}$, and $p^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}t}$. ■

Proposition 2 indicates that under information asymmetry condition, it is possible for a developer adopting TP strategy to identify optimal decisions on the quality level, ranking effort and the price for its app product.

Further, I show in Lemma 3 that to take TP strategy the app price should not be set just based on the app quality. Rather, the developer should carefully examine users’ disutility sensitivity to app price before making a pricing decision. Typically, the more sensitive users are to app price, the lower price should be set. Lemma 3 is similar to Lemma 2. At the first glance, the only difference is: when switching from a trial to a paid version of the app, the price set is dependent on how sensitive the current users would react to the price; whereas when switching to a free app with embedded ads, ad frequency is dependent on the per-click ad revenue. As a matter of fact, per-click ad revenue can be treated as a proxy of users’ disutility sensitivity to the app’s “price” (the annoyance from ads in the research context).

Lemma 3. If users’ information of willingness-to-play is unknown to a developer before launching an app, there is a relation among the optimal app price, the optimal app quality and users’ disutility sensitivity to app price, which is $p^* = \frac{q^*}{2t}$. ■

4.3 Fee or Free?

In this subsection, I identify the dominant strategy for the developer. As discussed earlier, from the developer’s perspective, “trial and then free without ads” (TF) strategy is a special case

of either “trial and then free with ads” (TFA) or “trial and then paid” strategy, so TF is a suboptimal strategy. I further show that either TFA or TP strategy can be dominant under certain conditions. Proposition 3 summarizes our finding.

Proposition 3. *Dominance of product offering strategies:* If users’ information of willingness-to-play is unknown to a developer before launching an app, “trial and then paid” (TP) strategy is dominant if users’ sensitivity to app price $t > 1$ ($t < 1$) as well as market size $N < \frac{2ac\sqrt{\beta g}}{v}$ ($N > \frac{2ac\sqrt{\beta g}}{v}$); otherwise, “trial and then free with ads” (TFA) strategy is dominant. ■

Proposition 3 indicates that under information asymmetry condition, it is better off for a developer to charge users directly when market size is small as well as users are more sensitive to app price than to ads. In this situation, offering users the app for free with ads cannot result in enough revenues for the developer to compensate the costs for app development and ranking effort. Rather, it is more profitable to charge users directly. On the other hand, when market size is large as well as users are less sensitive to app price than to ads, it is highly possible to charge users a high price without worrying about chasing away users. As has been found, it is more profitable to charge users for online content after a free period if the target users are professional (e.g., Pauwels and Weiss, 2008). In general, professional users are insensitive to app price. Our finding suggests that if a developer wants to charge the current professional users it must make sure that the user base captured is big enough. Jiang and Sarkar (2009) and Prasad et al. (2003) provide an explanation for the “free” strategy: it increases the speed of diffusion. The increasing diffusion of an information product (app in our research context) actually can increase the net present value of future revenue generation from sources such as direct payments from users and/or advertising. Based on my analytical result, when market size is large while users are relatively insensitive to ads as opposed to app price, the developer should adopt “trial and then free with ads” (TFA) strategy. This is consistent with Jiang and Sarkar’s (2009) and Prasad et al.’s (2003) since the diffusion speed could be even faster in a market with a large number of potential users who are insensitive to ads.

4.4 Welfare Analysis

In this subsection, I conduct welfare analyses in order to see which strategy can actually result in higher total welfare gained by an app developer and its users. According to Jones and Mendelson (2011), “with information goods, the welfare-efficient price of any product, regardless of its quality, is zero.” I want to investigate whether the welfare can be improved if an app developer embeds ads within its free app or charges users for the app. I define total

welfare as the sum of net utility gained by app users and profit gained by the developer in both periods. Thus, for each offering strategy, we can calculate the total welfare that it can result in. I first compare the total welfares between TF and TFA strategies. The finding is summarized in Proposition 4.

Proposition 4. *Total welfares between TF and TFA strategies:* If users' information of willingness-to-play (θ_u) is unknown to a developer before launching an app, then when $\theta_u < \frac{1}{2}$ ($\theta_u > \frac{1}{2}$) and market size $N > \frac{2ac\sqrt{\beta g}}{v}$ ($N < \frac{2ac\sqrt{\beta g}}{v}$), TFA strategy results in more total welfare than TF strategy. ■

Based on the analytical results, under information asymmetry condition, in a large (small) app market and users' willingness-to-play is low (high), strategy of placing ads can lead to higher total welfare compared to not displaying ads. The result suggests that Jones and Mendelson's (2011) conclusion holds only when market size is large (small) but users' willingness-to-play is high (low). If this condition does not hold, total welfare can actually be improved by "charging" users indirectly via displaying ads. Proposition 4 provides an insight to the platform companies such as Apple Inc. In practice, a platform company would like to improve the total welfare of the whole ecosystem so that it can be more sustainable. With such insights, the platform company can know better on how to incentivize third party developers' pricing and ad placement decisions with a goal of improving total welfare.

I further compare total welfares between TP and TFA strategies. The finding is summarized in Proposition 5.

Proposition 5. *Total welfares between TP and TFA strategies:* If users' information of willingness-to-play is unknown to a developer before launching an app, then when users' disutility sensitivity to app price $t > \frac{2}{1-2\theta_u}$ ($t < \frac{2}{1-2\theta_u}$) and market size $N > \frac{2ac\sqrt{\beta \gamma}}{v}$ ($N < \frac{2ac\sqrt{\beta \gamma}}{v}$), TP strategy results in higher total welfare than TFA strategy. ■

Proposition 5 suggests that, under certain conditions, there could be a space for improving total welfare by charging users directly. Specifically, when users are highly sensitive (insensitive) to app price and market size is large (small), total welfare can actually be increased by charging users directly than letting users use the app for free but requiring them to click on ads or wait until the ads disappear in Period U. It is worth mentioning that the implicit condition based on $t > \frac{2}{1-2\theta_u}$ (or $t < \frac{2}{1-2\theta_u}$) is $t > 1$, meaning users are highly sensitive to app price. The underlying implication is: When users are insensitive to app price, then regardless of what size of the market, TP offering results in greater total welfare than TF offering. This finding

offers another insight to policy makers.

5. Numerical Examples

In this section, I conduct numerical analysis. I compare TFA and TP strategies. In a mobile app market for a typical app, I simply set $a = 5$, $c = \$10$, $\beta = 2$, $\gamma = 2$, $\delta = 0.8$, $s = \$2$, $v = \$2$, respectively. I examine how market size (N) and users' disutility sensitivity to app price (t) impact the dominance of “switching from free to paid” and “keeping the app free with ads” in this value setting. Fig. 2 depicts the dynamics of the strategy dominance. As illustrated, when market size (N) is large while users' disutility sensitivity to app price (t) is low (lower than 1), TP strategy (“trial and then paid”) is dominant. Whereas when market size (N) is large while users' disutility sensitivity to app price (t) is high (greater than 1), TFA strategy (“trial and then free with ads”) is dominant. Note that in this value setting, when market size (N) is small while users' disutility sensitivity to app price (t) is high (greater than 1), both TP and TFA strategies are infeasible. The analysis result in this numerical example is consistent with Proposition 3.

Fig. 2. Dominance: TPA versus TP Strategy. This figure illustrates how market size (N) and users' disutility sensitivity to app price (t) influence the dominance relationship in terms of the optimal total profits between TFA and TP strategies. I set $a = 5$, $c = \$10$, $\beta = 2$, $\gamma = 2$, $\delta = 0.8$, $s = \$2$, $v = \$2$, respectively.

6. Concluding Remarks

Mobile applications (apps) are a type of information goods that are evaluated by users' perception and experience. In general, a user's perception of an app impacts her decision on downloading it; whereas her experience in using it will influence her willingness to engage with

the app later on. In addition, consumers most likely choose to download the app appearing on top ranking list due to the small screen size of mobile devices and the huge number of mobile apps available. In this paper, I examine app developers' product offering strategies by focusing on the decision issue of whether to charge users after a trial period. I develop a two-period model which considers several characteristics of the app market: app users' willingness-to-download and willingness-to-play, different users' disutility sensitivities to ads and app price, app quality, per-click ad revenue, time discount factor, and app ranking effort. The major contribution of the current research is that it enriches the literature on mobile app offering strategies. In particular, I extended Yu et al.'s (2017a, b) research by considering app ranking effort, app price, and ad frequency of in-app advertisements that can impact a developer's choice of revenue models. Since the model developed in this research can be applied to the setting of whether a producer should charge users directly or offer users free with ads after a trial period, the model and analytical results can address similar decision problems that are prevalent in information goods industry (e.g., online content products). Under information asymmetry condition, there are several insights based on the analyses that have managerial implications.

First of all, regardless of what type of revenue models a developer has chosen, it is possible to figure out the optimal app quality, app ranking effort and ad frequency/app price. Furthermore, in "trial and then free with ads" (TFA) strategy, the optimal ad frequency is determined by both the optimal app quality and per-click ad revenue; while in "trial and then paid" (TP) strategy, the optimal price is determined by both the optimal app quality and users' disutility sensitivity to app price. In general, a developer should display more frequent ads or price the app higher for a relatively high quality app; however, it should display less frequent ads or price the app lower if per-click ad revenue or users' disutility sensitivity to app price is relatively high.

Second, I find that taking the strategy of "free without ads" after a trial period is always suboptimal, and, the choice between "free with ads" and "paid" strategies after a trial period is dependent on both users' disutility sensitivity to app price and market size. Intuitively, if users are less sensitive to app price, charging users should be a better choice. Yet, the result shows that in this situation, if market size is large, then "free with ads" is even more profitable. Also surprisingly, when users are highly sensitive to app price, charging users a price for the app could be a better choice if market size is small. The finding supplements Yu et al.'s (2017a) conclusion regarding developers' revenue model choice by offering another insight that market size can be another determinant in their choice.

Finally, I also find that total welfare realized in either "free" or "paid" offering can be

higher conditional on users' willingness to play and market size. Specifically, for a market with a small (large) base of high-type (low-type) users (i.e., high/low willingness-to-play users), free app offering with embedded ads can result in higher total welfare than not displaying ads to users. Further, when users are highly sensitive to app price, charging users directly can result in higher total welfare than offering users the app for free with ads conditional on market size; however, if users are insensitive to app price, offering the app free with ads always result in higher total welfare than charging users directly. The findings suggest that Mendelson's (2011) viewpoint that "with information goods, the welfare-efficient price of any product, regardless of its quality, is zero" might hold only under certain conditions. These findings also offer insights to policy makers. For instance, a mobile app platform company (e.g., Apple Inc.) should set an incentive scheme suitable for revenue model choices for different app developers who face different market segments. As such, the ecosystem of its platform can be sustainable.

There are several limitations that might inspire future research on this topic from different directions. First, due to the complexity of the current model, most of the analytical results I derived are based upon the assumption that users' willingness to play are unknown to a developer when the app is launched (i.e., "information asymmetry" assumption). Although "information asymmetry" is what we observe in most cases in reality, analyses based on an "information symmetry" assumption may also be considered as an extension of this research. To do that, I believe the current model structure has to be largely adjusted or even be totally different. Second, in the current study I focus on a developer's choice of fee-or-free after offering users a trial period. In the mobile app industry, there are several other revenue models that a developer can also consider, such as: paid, freemium (free for basic version and then paid for additional features), paymium (paid for basic version and then paid for additional features), and subscription. Further analysis of such revenue models can be explored. One must be aware that for the above revenue models, the scenarios of product versioning and/or repetitive purchases would make the analyses more complicated. In the present research, I assume that a developer displays ads to all users with the same frequency. However, another future direction of the study would be interesting, which is to investigate the developer's problem while considering "customized" ad frequencies for different users.

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Appendices

Appendix A. Proof for Proposition 1

The objective function (total profit) for TFA strategy is demonstrated in system (6), which is: $\left(N\left(1 - \frac{ac}{hq}\right)v - \gamma h^2 - \beta q^2\right) + \delta\left(N\left(1 - \frac{ac}{hq}\right)\left(1 - \frac{fs}{q}\right)fs\right)$. This function is strictly concave under information asymmetry assumption in decision variables of app quality (q), ranking effort (h), and ad frequency (f). Under the information asymmetry condition, the developer makes decisions on app quality (q) and ranking effort (h) in Period T; and then decides ad frequency (f) in Period U. Therefore, I adopt two-step method to solve system (6). Specifically, in step 1 I solve the model w.r.t. q and h by setting FOCs equal to zero. I therefore obtain optimal solution $q^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}$ and $h^* = \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}$. In step 2, I substitute $(q^*, h^*) = \left(\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}, \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}\right)$ back into the objective function, and then solve the model w.r.t. f . I then get $f^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}s}$.

By now, I have identified the optimal decisions of app quality, ranking effort and ad frequency for the developer who adopts TFA strategy, which is: $(q^*, h^*, f^*) = \left(\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}, \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}, \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}s}\right)$.

Appendix B. Proof for Proposition 2

The objective function (total profit) for TP strategy is presented in system (7), which is: $\left(N\left(1 - \frac{ac}{hq}\right)v - \gamma h^2 - \beta q^2\right) + \delta\left(N\left(1 - \frac{ac}{hq}\right)\left(1 - \frac{tp}{q}\right)p\right)$. This function is strictly concave under information asymmetry assumption in decision variables of app quality (q), ranking effort (h), and app price (p). Under the information asymmetry condition, the developer makes decisions on app quality (q) and ranking effort (h) in Period T; and then decides app price (p) in Period U. Therefore, I adopt two-step method to solve system (7). Specifically, in step 1 I solve the model w.r.t. q and h by setting FOCs equal to zero. I therefore obtain optimal solution $q^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}$ and $h^* = \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}$. In step 2, I substitute $(q^*, h^*) = \left(\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}, \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}\right)$ back into the objective function, and then solve the model w.r.t. p . I then get $p^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}t}$.

By now, I have identified the optimal decisions of app quality, ranking effort and app price for the developer who adopts TP strategy, which is: $(q^*, h^*, p^*) = (\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}, \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}, \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}t})$.

Appendix C. Proof for Proposition 3

To identify the global optimal strategy among TF, TFA and TP strategies, I compare the optimal total profit for the three strategies. As I have discussed in Model section, TF strategy is a special case of either TFA or TP, hence TF strategy is always suboptimal. In fact, it is noticed that, under the information asymmetry assumption, the developer only decides app quality (q) and ranking effort (h) in Period T. Since the three strategies have the same profit function for Period T as seen in system (6), (7), and (8), the optimal solution for q and h are the same, i.e., $(q^*, h^*) = (\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}, \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}})$. Because TF strategy does not have profit function for Period U while the other two strategies have, TF strategy is always dominated by TFA and TP strategies.

Therefore, I only need to compare TFA and TP strategies. As I have derived, the optimal total profits for the two strategies are (see Table 2):

$$\pi_{TFA} = Nv - \frac{a^{3/4}c^{3/4}\gamma^{3/8}N^{3/4}\delta}{22^{3/4}\beta^{1/8}v^{1/4}} + \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{5/4}v^{1/4}\delta}{42^{1/4}\beta^{3/8}} - 2\sqrt{2}\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4}\sqrt{N}\sqrt{v} \text{ and}$$

$$\pi_{TP} = Nv - \frac{22^{1/4}a^{3/4}c^{3/4}\gamma^{3/8}N^{3/4}\delta}{8t\beta^{1/8}v^{1/4}} + \frac{2^{3/4}a^{1/4}c^{1/4}\gamma^{1/8}N^{5/4}v^{1/4}\delta}{8t\beta^{3/8}} - 2\sqrt{2}\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4}\sqrt{N}\sqrt{v}.$$

Hence, $\pi_{TP} - \pi_{TFA} = \frac{(t-1)(22^{1/4}a^{3/4}\beta^{1/4}c^{3/4}\gamma^{3/8}N^{3/4} - 2^{3/4}a^{1/4}c^{1/4}\gamma^{1/8}N^{5/4}\sqrt{v})\delta}{8\beta^{3/8}tv^{1/4}}$. Thus, (1) if $t - 1 > 0$, $22^{1/4}a^{3/4}\beta^{1/4}c^{3/4}\gamma^{3/8}N^{3/4} - 2^{3/4}a^{1/4}c^{1/4}\gamma^{1/8}N^{5/4}\sqrt{v} > 0$ (i.e., $N < \frac{2ac\sqrt{\beta g}}{v}$) must hold to make $\pi_{TP} > \pi_{TFA}$; (2) if $t - 1 < 0$, $22^{1/4}a^{3/4}\beta^{1/4}c^{3/4}\gamma^{3/8}N^{3/4} - 2^{3/4}a^{1/4}c^{1/4}\gamma^{1/8}N^{5/4}\sqrt{v} < 0$ (i.e., $N > \frac{2ac\sqrt{\beta g}}{v}$) must hold to make $\pi_{TP} > \pi_{TFA}$;

By now, I can conclude that under information asymmetry assumption, “trial and then paid” (TP) strategy is dominant if users’ sensitivity to app price $t > 1$ ($t < 1$) as well as market size $N < \frac{2ac\sqrt{\beta g}}{v}$ ($N > \frac{2ac\sqrt{\beta g}}{v}$); otherwise, “trial and then free with ads” (TFA) strategy is dominant; also, TF strategy is always suboptimal.

Appendix D. Proof for Proposition 4

In our research setting, a developer makes decisions on app quality and ranking effort during the trial period; it then decides whether to embed ads by allowing users to play the app for free. In this setting, by setting the same (q^*, h^*) , the welfares gained by users and the

developer are equivalent between TFA and TF strategies in Period T. So I only compare welfares in Period U between the two strategies.

For TFA strategy, the total welfare realized in Period U includes two components: net utility gained by users and profit gained by the developer. To calculate total net utilities gained by users in Period U, I substitute (q^*, h^*, f^*) into the following function:

$$\frac{N}{2} * (1 - \frac{ac}{hq})(1 - \frac{fs}{q})(\theta_u * q - f * s), \text{ where } \frac{N}{2} * (1 - \frac{ac}{hq})(1 - \frac{fs}{q}) \text{ represents the total number of users captured in Period U and } \theta_u * q - f * s \text{ represents the net utility gained by each user.}$$

Next, I substitute (q^*, h^*, f^*) into the developer's profit function for Period U (i.e., Eq. (4)) in order to calculate the profit gained by the developer in Period U. By adding up the two components, I get the total welfare realized in Period U for TFA strategy as below: $W_{TFA} = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{3/4}(\sqrt{2}\sqrt{N}\sqrt{v}-2\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4})(1+2\theta_u)}{82^{3/4}\beta^{3/8}v^{1/4}}$.

For TF strategy, the developer gains zero profit in Period U since it does not display ads. On the other hand, all users who have installed the app will engage with the app since there is no cost in doing this. Hence the total welfare function realized in Period U for TF strategy is:

$$W_{TF} = \frac{N}{2} * (1 - \frac{ac}{hq}) * \theta_u * q. \text{ By substituting } (q^*, h^*) \text{ into this function I get: } W_{TF} = \frac{(2^{3/4}a^{1/4}c^{1/4}\gamma^{1/8}N^{5/4}\sqrt{v}-22^{1/4}a^{3/4}\beta^{1/4}c^{3/4}\gamma^{3/8}N^{3/4})\theta_u}{4\beta^{3/8}v^{1/4}}.$$

Thus, $W_{TFA} - W_{TF} = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{3/4}(\sqrt{2}\sqrt{N}\sqrt{v}-2\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4})(1-2\theta_u)}{82^{3/4}\beta^{3/8}v^{1/4}}$. Therefore, if $\theta_u < \frac{1}{2}$ ($\theta_u > \frac{1}{2}$) and $N > \frac{2ac\sqrt{\beta g}}{v}$ ($N < \frac{2ac\sqrt{\beta g}}{v}$), $W_{TFA} > W_{TF}$. We can thus conclude that if users' information of willingness-to-play (θ_u) is unknown to a developer before launching an app, then when $\theta_u < \frac{1}{2}$ ($\theta_u > \frac{1}{2}$) and market size $N > \frac{2ac\sqrt{\beta g}}{v}$ ($N < \frac{2ac\sqrt{\beta g}}{v}$), TFA strategy results in more total welfare than TF strategy.

Appendix F. Proof for Proposition 5

In our research setting, a developer makes decisions on app quality and ranking effort during the trial period; it then decides whether to either keep free with ads (TFA) or charge users directly (TP). In this setting, by setting the same (q^*, h^*) , the welfares gained by users and the developer are equivalent between TFA and TP strategies in Period T. Hence, to compare the total welfares realized in TFA and TP strategies, we only need to compare the welfares in Period U.

Similar to the proof in Proposition 4, we can calculate the optimal total welfares for the TFA and TP strategies respectively as below:

$$W_{TFA} = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{3/4}(\sqrt{2}\sqrt{N}\sqrt{v}-2\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4})(1+2\theta_u)}{82^{3/4}\beta^{3/8}v^{1/4}}, \text{ and}$$

$$W_{TP} = \frac{(2^{3/4}a^{1/4}c^{1/4}\gamma^{1/8}N^{5/4}\sqrt{v}-22^{1/4}a^{3/4}\beta^{1/4}c^{3/4}\gamma^{3/8}N^{3/4})\theta_u}{4\beta^{3/8}v^{1/4}}. \text{ Thus,}$$

$$W_{TP} - W_{TFA} = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{3/4}(\sqrt{2}\sqrt{N}\sqrt{v}-2\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4})(2-t+2t\theta_u)}{82^{3/4}\beta^{3/8}tv^{1/4}}.$$

Therefore, (1) if $2 - t + 2t\theta_u > 0$ (or $t > \frac{2}{1-2\theta_u}$), then $\sqrt{2}\sqrt{N}\sqrt{v} - 2\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4} > 0$ (or $N > \frac{2ac\sqrt{\beta\gamma}}{v}$) must hold to have $W_{TP} > W_{TFA}$; (2) if $2 - t + 2t\theta_u < 0$ (or $t < \frac{2}{1-2\theta_u}$), then $N < \frac{2ac\sqrt{\beta\gamma}}{v}$ must hold to have $W_{TP} > W_{TFA}$.

By now, we can conclude that, under information asymmetry condition, when users' disutility sensitivity to app price $t > \frac{2}{1-2\theta_u}$ ($t < \frac{2}{1-2\theta_u}$) and market size $N > \frac{2ac\sqrt{\beta\gamma}}{v}$ ($N < \frac{2ac\sqrt{\beta\gamma}}{v}$), TP strategy results in higher total welfare than TFA strategy.

Appendix G. Proof for Lemma 1

In TFA strategy, there are two periods in the planning timespan: trial period (Period T) and usage period (Period U). Under information asymmetry assumption, as listed in Table 2, the optimal solution to the model for TFA strategy is $(q^*, h^*, f^*) = (\frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}, \frac{a^{1/4}\beta^{1/8}c^{1/4}N^{1/4}v^{1/4}}{2^{1/4}\gamma^{3/8}}, \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}s})$. The realized demands in Periods T and U are $D_d = N(1 - \frac{ac}{hq})$ and $D_u = N(1 - \frac{ac}{hq})(1 - \frac{fs}{q})$, respectively. By substituting (q^*, h^*, f^*) into D_d and D_u we have $D_d = N - \frac{\sqrt{2}\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4}\sqrt{N}}{\sqrt{v}}$ and $D_u = \frac{N}{2} - \frac{\sqrt{a}\beta^{1/4}\sqrt{c}\gamma^{1/4}\sqrt{N}}{\sqrt{2}\sqrt{v}}$. Hence, $\frac{D_u}{D_d} = \frac{1}{2}$.

Thus, we can conclude that, for a developer adopting TFA strategy under information asymmetry condition, to achieve maximal total profit, it should only focus on half of the users captured in the trial period.

Appendix H. Proof for Lemma 2

In the model for TFA strategy (i.e., system (6)), as we have obtained under the information asymmetry assumption (see Table 2), the optimal ad frequency and app quality are

$$f^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}s} \text{ and } q^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}, \text{ respectively. Hence, } \frac{f^*}{q^*} = \frac{1}{2s} \text{ (or}$$

$f^* = \frac{q^*}{2s}$). It is therefore concluded that, if users' information of willingness-to-play is unknown to a developer before launching an app, there is a relation among the optimal ad frequency, the optimal app quality and per-click ad revenue, which is $f^* = \frac{q^*}{2s}$.

Appendix I. Proof for Lemma 3

Similar to the proof for Lemma 2, in the model for TP strategy (i.e., system (7)), the optimal app price and app quality are $p^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{22^{1/4}\beta^{3/8}t}$ and $q^* = \frac{a^{1/4}c^{1/4}\gamma^{1/8}N^{1/4}v^{1/4}}{2^{1/4}\beta^{3/8}}$, respectively (see Table 2). Hence, $\frac{p^*}{q^*} = \frac{1}{2t}$ (or $p^* = \frac{q^*}{2t}$). We can therefore conclude that, if users' information of willingness-to-play is unknown to a developer before launching an app, there is a relation among the optimal app price, the optimal app quality and users' disutility sensitivity to app price, which is $p^* = \frac{q^*}{2t}$.