Intellectual Property and Similar Assets; an Economic and Decision Analysis

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Intellectual Property and Similar Assets; an Economic and Decision Analysis

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ABSTRACT

Valuing Intellectual Property (IPR) and similar assets brought upon accounting practice and economic analysis severe problems in evaluating business decisions and management science of intangible business assets. At no time has the principles of accounting and accounting research determined rules, methods and principles without error in this risky analysis. Proposed here based on evidence methods based on estimation in accounting provided for better methods valuing intellectual property and other intangibles on financial statements. A rich history in such methods is the subject here.

Key Terms

Intellectual Property; Decision Analysis; Estimation in Financial Accounting; Valuation

Introduction

Financial researchers such as Dechow (1994; Dechow and Schrand, 2004) indicated that employing accrual-based accounting methods creates the capability of accounting-based earnings projections to control and continuously improve the measures of firm performance reflected in analysts’ earnings forecasts. The argument was that cash flow accuracy is expected to suffer from matching, realization, and other timing problems concerning the timing of the recognition of costs and revenues. Accuracy of financial earnings predictions was studied by Brandon and Jarrett (1974), Jarrett and Khumawala (1987), Jarrett (1983, 1992), and Lambert, Matolcsy, and Wyatt (2015). They compared methods of forecasting accounting earnings seeking to learn how forecast models can be compared and possibly improved to produce more accurate results as to cash flow. Questions posed included sources of accuracy, but accrual accounting alone was not considered the most important source of inaccurate results. However, no one established a theoretical link between sources of inaccuracy and the matching principle and the accuracy of financial analysts’ forecasts even though many studied the problem (e.g., Jarrett, 1989, 1990; Clement, 1999; Gu and Wu, 2003; Ramnath, Rock, and Shane, 2008; Groysberg, Healy, Nohria, and Serafeim, 2011). Accounting reports containing these forecasts of cash flow and rates of return are in addition subject to fluctuations in the interpretation of timing principles utilized by accountants. However, Gu and Wang (2005) brought up the possibility of another source of inaccuracy in the forecast of rates of return, cash flow, and earnings. Beneish, et al. (2013, 2015, 2016) created a model that uses financial ratios calculated with accounting data of a specific company to check if it is likely that the reported earnings for a firm were manipulated, the goal being to estimate earnings better in financial reports. Last, Lev and Gu (2016) produced evidence from large-sample empirical
analysis that financial documents continuously deteriorate in relevance to investors’ decisions. Further, they detail why accounting reporting is losing relevance in today’s decisions related to capital budgeting and the abandonment option.

More recently, Rees (2015) studied financial analysts’ forecasts of earnings, pointing out the incentives and constraints that impact on analysts’ forecasting proficiency. His results pointed to the seemingly poor performance of published analysts’ forecasts often resulting from difficulties experienced when one attempts to improve forecasts, and the evidence that is apparently consistent with the ability of analysts’ forecast to provide a basis for earning abnormal returns. The final point is not without criticism because investment practices based on publicly available information could be used to demonstrate market inefficiency. Furthermore, given the relatively inaccurate nature of forecasts, additional study of the nature of these inaccuracies is the purpose of the conclusion suggested in the current studies.

Currently, we examine how the presence of the abandonment option using normal capital budgeting methods to determine whether there is a relationship among the various capital budgeting options, financial leverage, and estimating earnings by analysts. We begin by studying capital budgeting with the abandonment option; later, most corporations use capital budgeting procedures to coordinate and motivate activities throughout their organization. It is well understood that the budgeting process is dynamic and flexible, involving the information flow throughout the organization that determines the investment and abandonment decisions at the individual stages. We now examine how an abandonment option influences the optimal timing of information and vice versa. In particular, we compare timely information, where the manager acquires perfect pre-contract project information. We examine how the future revenues from intangible assets may affect the level of financial leverage of a firm when not all is known about the economic value of intangible assets.

In the absence of the real option, the following trade-off arises: If information is timely, the investment decision can be based on perfect information. Alternatively, if information about intangible assets is not considered in the abandonment option, the timing and decision concerning the abandon option may very well be estimated incorrectly. The incorrect information is the product of the misreporting of factual events associated with intangible assets, and the error associated with incorrect analysts’ forecasts. This turns into the estimation problem in financial accounting and, in turn, applies it to the relation of analysts’ forecasts and the bias in estimating earnings and cash flow present in evaluating capital decisions.

The Capital Budgeting Methodology

Berger, Ofek, and Swary (1996) established the link among analysts’ forecasts, cash flow, the expected capital asset pricing model (CAPM) return, and the present value of cash flow, which includes forecasts of earning rather than the distributable cash flow. In addition, Wong (2009) examined the relation between the abandonment option’s potential effect on a firm’s decision analysis and the eventual analytics employed to determine the optimal decision and operating leverage. Furthermore, McDonald (2003) analyzed abandonment options, divestment options, expansion options, and growth options previously examined in a survey by Triantis and Borison (2001). These and many more studies revealed that they use real options to the general problems associated with capital budgeting.

Analysts’ earnings forecasts enable analysts to estimate the present value of cash flow (PVCF). According to Berger, Ofek, and Swary (1996), the advantage is that analysts’ forecasts of earning do not incorporate the value of the abandonment option. If business forecasts of distributable cash flows, cash flows from non-ongoing concern events would be included in the forecasts. Thus,

...
earnings may not be the same as cash flows. Hence, we adjust because capital expenditures are not equivalent to depreciation and the growth in working capital is not subtracted from earnings. No longer is it required to adjust for capital structure changes in the environment that such changes cannot be foreseen. Borrowing again from Berger, Ofek, and Swary (1996), their equation constructs the PVCF evolving from the analyst’s discounted forecasts. Included in the equation is the sum of the present value of analysts’ predicted going-concern cash flows discounted by analyst forecast of year $t$ after-interest earnings and expected CAPM (Intellectual Property and Similar Assets; an Economic and Decision Analysis) capital asset pricing model return, consensus forecast of five-year earnings growth, the terminal growth rate of earnings, the number of years for which earnings are forecast, and a year index. The CAPM adjustment includes the reduction to the present value of analysts’ earnings. The second adjustment to PVCF is the working capital adjustment, which is a reduction to the present value of analysts’ earnings forecasts to adjust for growth in working capital. Finally, the expected CAPM return is defined as

$$r = rf + \beta_e \times (rm - rf),$$

(1)

Where

$rf = \text{risk-free rate},$

$\beta_e = \text{the firm’s beta or systematic risk (from the CRSP beta file)},$

(rm – rf) = \text{risk premium of the stock market minus the risk-free rate}.$

In implementing Equation (1), we assume that the relevant investment horizon is short term. Therefore, a useful solution is to use the one-month Treasury bill rate as a proxy for the risk-free rate and a risk premium (the arithmetic mean from a long period of time from between the return on the S&P 500 and the return on the Treasury bills).

The problem with the above approach is the variable analyst forecasts of earnings. In part, this is a solution to the problems noted by Pappas (1977) in response to work by Brief and Owen (1968, 1969, 1970; Brief, 1977; Barnea and Sadan, 1974; Jarrett, 1983, 1992), who used their work in developing models to adjust analysts’ earnings forecasts in evaluating the abandonment option. Studies concerning analysts’ forecasts are well known and include a huge number. In general, as stated by many others in the field of financial accounting, earnings forecasts are dependent on the principles of financial accounting, which produces the data for modeling trends and seasonality (or modeling components). The accuracy of analysts’ forecasts has a long history and includes studies by Clement (1999), Gu and Wu (2003), Ramnath, Rock, and Shane (2008), Groysberg, Healy, Nohria, and Serafeim (2011), and Makridakis, Spiliotis, and Assimakopoulos (2017). The last paper suggested that machine learning models may have better results than self-prepared models for forecasting. The aforementioned studies focused on the relationship between analysts’ forecasts and the magnitude and value of intangible assets. Intangible assets were not considered in the forecasting method discussed by the researchers in their many and detailed studies. The value of intangible assets produces a great source of error if they are not considered in the forecasting methods utilized by analysts in the production of cash flow, rates of return earning per share (EPS) forecasts. When adjustments for intangible assets are included in the analyst’s forecasts, Gu and Wang (2005, p. 673) stated that “The rise of intangible assets in size and contribution to corporate growth over the last...
two decades poses an interesting dilemma for analysts. Most intangible assets are not recognized in financial statement, and current accounting rules do not require firms to report separate measures for intangibles.” Intangibles include trademarks, brand names, patents, and similar properties that have value but are generally not listed in the financial reports of firms. Many of these items are technology based and are very important in financial decisions such as in M&As. They are an INTRICAL PART OF GROWTH in the growth of firms and therefore are shown to be related in the statistical sense to the overall estimates made by accounting and analysts.

In another study concerning analysts’ forecasts, Matolcsy and Wyatt (2006) found an association between EPS forecast, growth rates forecast error, and measures of technological conditions in the firm’s industry. They found that as the forecast horizon increases, the technological conditions and current EPS are statistically associated with analysts’ forecasts. Long horizon creates the conditions for within one to conclude that interactions between technological conditions and current EPS are associated with analysts’ EPS and growth forecasts. This conclusion aligns itself with Jung, Shane, and Yang (2012), who suggested that analysts’ growth forecasts effect efforts to evaluate intangible assets and produce optimistically biased long-term forecasts. Because intangible assets that are often technology based and take up more of the balance sheet of many firms, it is likely that analysts’ forecasts may produce less accurate predictions of earnings, cash flow, and rate of return. The conclusions of Dechow (1994) thus become less important. Balance sheets usually have little or no involvement with the value of intangibles although there are some practices by accounting that are still used. Thus in the remaining portions of this analysis, we propose a method by which one can estimate earnings such that the value of intangible assets is valued and earnings estimate are not biased by serious errors of omission such that the capital budgeting model expressed earlier in equations by Berger, Olek, and Swary (1996, p. 264) are not unduly biased.

**Intellectual Property and Traditional Accounting Methodology**

As noted by Brief and Owen (1969, 1970, 1977), Jarrett (1971, 1974, 1983), Roberts and Roberts (1970), and Barnea and Sadan (1974), the timing of recognition of revenue for intellectual property rights (IPR) in financial statements of ten are not featured in M&A activity. The Financial Accounting Standards Board (FASB) provides for such activities; however, they are often ignored due to their evasiveness or are not fully informational in the normally structured rules. Recognizing future performance is a goal of matching and timing but is unrelated to recognizing cash flow and similar items in the historical performance of a firm. Nonprofit entities often do not use accrual rules at all because the goal of these are related to achieving high rates of return. Often IPR for nonprofits would differ from the same item for profit-maximizing entities because the goal of seeking high rates of return does not enter the strategic planning process for nonprofits (World Trade Organization, 2016). The purpose here is to consider IPR as intangible assets, as a product of intellect that law protects from unauthorized use by those not responsible for the IPR. Hence, IPR are characterized as the protection of distinguished signs such as trademarks for goods and services, patents, and other similar items that are under protection from unauthorized use. This includes art, music, creations by authors including the authorship of computer software and similar items such as discoveries, inventions, phrases, symbols, and design. Obviously, a writer and conductor of music such as Leonard Bernstein and Daniel Barenboim would have created intellectual property (IP) that differs greatly from physicists such as Lise Meitner, Niels Bohr, or Albert Einstein.

Presently, accounting suggests two methods to determine the value of IPR to produce better estimates of from accounting analysts’ forecasts. The convention of the “lower of cost or market” is based on the rule of conservatism in valuing assets to anticipate future losses instead of future gains. The policy tends to underestimate rather than overstate the value of net assets and could therefore lead to an understatement of income,
cash flow, earnings, and rates of return. The purpose of this study and its conclusive result is to neither understate nor overstate cash flow so as to produce a rate of return on cash flow that is commensurate with the goal of producing accurate prediction of cash flow and its rate of return for financial and decision-making purposes. Stated differently, the purpose is not to violate accounting policy but to ensure the M&A that cash flow is estimated properly. Traditionally, when accounting writes policy about intangible assets as a residual, by “residual” they mean a buyer is ready to value a firm in excess of the value of the tangible assets. This value is often referred to as “goodwill” (White, Sandhi, and Fried, 1994), which is an imperfect method. This notion of goodwill is estimated as a residual value. The valuation of intangible property is also imperfect since it is considered part of the solution of a bargaining process during mergers and acquisitions. In this case, the buyer and seller may have different market power, which greatly affects the residual of the bargaining process and produces an imperfect or biased estimate of the value of the intangible assets. One may examine the case of the sale of Superman by struggling comic book artists to a much larger corporate power who could market the character to comic books, television, and the film industry. The nearly destitute conditions of the original artists who created the intangible product could never cope with the business and marketing (power) of those who purchased the name Superman. Thus, goodwill becomes a vague valuation system that justifies the bringing of data analysis and science into the valuation process.

Another solution suggested during the M&A process is to simply list the patents, trademarks, brands, and similar items of IP in the financial reporting of the firm. Following this initiative and suggestion of the accounting principles board provide little aid concerning the economic value of IPR and products for a firm during the M&A events. In the final step of the problem, the evaluation may conclude influence relating to the biases of the reading of the financial reports. Such biases of IPR occurred often with works of Meitner, Bohr, and Einstein. Whereas, at least Bohr and Einstein received Nobel Prizes which did include wealth, Meitner perhaps due to her gender and religious preference never received the same award. For orchestral conductors and composers of music, there is no economic award from the Nobel Prize committee. Accountants forecast the overall rate of return for a firm but do not ignore the convention of “conservatism.” Accounting practice values the IPR for a firm each year for each and every IPR under consideration. The principle of goodwill is not to be used during M&A activity to account for the value of IPR. IP may induce not only greater asset values but also affects the rate of return on cash flow because the denominator of the rate of return will change. (To understand the gravity of ignoring or improperly valuing IPR, see Jarrett, 2016, 2017a, 2017b.) This result debated previously (Brief and Owen, 1969; Brief, 1977; Pappas, 1977) indicated that including earnings risk may not fully reflect all risks in estimating earnings, but it at least reflects that part of risk from the variation in earnings.

Furthermore, Helliar, Lonie, Power, and Sinclair (2001) summarized attitudes of managers toward risk in the following way. The abandonment option may be extremely important when considering the survival of a firm or nonprofit entity. Survival is often the goal of the abandonment option indicating that risks that are taken in special situations such as catastrophes when the survival of whole areas of an industry may be under threat (Shleifer and Vishny, 1992; Liu and Liu, 2011) may be different from those taken in more usual environments. An entity in decline may avoid innovative options and concentrate on immediate short-term options rather than riskier longer-term projects with more difficult goals to be accomplished. In addition, the choice may rapidly increase the rate upward of the process decline and result in managers becoming more risk averse and not employing greater use of intangible assets and IP.

Earnings Estimation Including the Estimation and Monetary Values for Intangible Assets
In this section, we illustrate the size of the bias in estimating earnings when the monetary equivalent of values of intangible assets is not considered by analysts in estimating future earnings. Note that misestimating future earnings affects PVCF resulting in errors in assessing the abandonment option. Intangible assets including patents, trademarks, copyrights, and similar items are usually overlooked and/or not estimated properly in many financial statements. These statements are considered fundamental information in determining PVCF in abandonment decisions, mergers, acquisitions, and similar financial decision analysis and analytics.

To illustrate the case of monetarizing property rights and other intangibles often referred to by the acronym IPR, let us consider the specific problem of a firm abandoning or selling IPR through a direct acquisition and the effect on debt as part of its holdings. Obviously, the ratio of common equity to total capital stock will be changed during the financial operation. In turn, the effects of financial leverage on total financial risk will also be part of the problem. The rate of return to common shareholders is related to the measure of financial risk utilized in any decision of this type. We assume that the firm is motivated to finance the acquisition by leverage instead of issuing new common share or a strict loan from a financial institution or similar institution as the result of an economic optimization policy. Define T as the sum of debt and common stock. To illustrate simply, preferred share and other financial instruments are valued at zero to avoid complications that may hinder the explanation. S is the monetary value of outstanding common, and D is amount of debt. X is the amount of earning in a future time period. X is a random variable, and E(X) is the mean of the random variable, V(X) the variance, and S(X) the square root or standard deviation. The cost of the debt per dollar is I, the interest rate. The mean earning per dollar of S is

\[ E(Y) = \frac{E(X)}{S} = \frac{E(X)}{(T-D)} \]  

Note that Y is also a random variable with mean E(Y). Mean (or expected) earning is defined as follows:

\[ E(X') = E(X) - iD \text{ for } D > 0; \]  
\[ E(X') = E(X), \text{ for } D = 0 \]  
\[ \text{Hence, } E(X') = E(X), \text{ for } D = 0 \]  
\[ \text{The variance of total earnings is } V(X') = V(X) \text{ for } D \geq 0 (iD \text{ and is a constant}) \]

The financial decision-optimum to fund the purchase is an example of decision analytics where the decisions are to substitute debt for common stock or not to substitute debt. Using data analytical language, for this decision problem, the states of nature are defined by

\[ E(X) > iD \text{ or } E(X) \leq iD \]  

We define the opportunity loss function as an integral approximation the firm’s view toward choosing a no optimal decision. No loss occurs when earnings are great than the cost of debt since management will benefit from the strategy of leverage financing.

As an example, consider cash flow to be greater than the cost of debt management and in turn the loss function would change reflecting the goal of optimum-decision analytics. The basic structure of the acquisition strategy would not change except for the substitution of cash flow for earnings. To calculate the opportunity loss function associated with this strategy, we estimate some probability density function (PDF) that approximates the PDF for future earnings. Before we consider all PDFs, let the firm focus on the normal distribution or T-distribution having a very large number of degrees of freedom, which approximates the standard normal distribution. The opportunity loss at breakeven (X b) becomes

\[ X_b = E(X') - Z \left( \frac{V(X')}{S(X')} \right) \]  

Z refers to the normal fiducial deviate, and S (X’) is the standard deviation. By rearrangement, we find E(X) = E (X’) – iD. The next step is to determine the size and distribution of the loss function for the distribution of future earnings, which is all in line with objectives of the timing of the realization revenues discussed before Jarrett (1971, 1992). In Table 1, we preview one of three methods to estimate the
monetary value of IPR. The $E(X)$ is $4,200, and the $S(X)$ increases by given amounts ($100).
Column 3 contains the cost of debt of $3,200. The $Z$ (the normal deviate) calculation is accomplished column 4 with column 5 containing the cumulative normal probability. In turn, the IPR monetary value is simply the normal probability multiplied by $E(X)$ and is contained in column 6. The IPR$ is thus calculated for a variety of circumstances.

Table 1

A second example of estimating the monetary value of IPR (Table 2), $E(X)$, and column 1 is constant from row to row, column 2, $S(X)$ remains the same ($600) from row to row, and column 3 the cost of debt changes from row to row due to the change in the interest rate and other costs associated with debt. Column 4, the standard normal deviate, $Z$, decreases in value from row to row, and column 5, the cumulative probability from the normal curve decreases from row to row. The dollar value of the IPR will continually decrease from the top row to the bottom row in Table 2.

Table 2

As one last example, in Table 3, we alter the example by comparing the monetary value of IPR when the cost of debt and debt: equity ratio in columns 1 and 2 of Table 3 change. In turn, both columns 3 and 4 (cost of debt and net cash) change from row to row. The $Z$-statistics and normal probabilities change, and the monetary value of IPR changes from row to row with the highest in row 1 and descending thereafter.

Table 3

In summary, three sets of examples demonstrate that estimation theory in financial accounting is a fundamental portion of correcting financial reporting data such that analysts now have a complete set of data to work with when making earnings forecasts and other decisions. Our finding does not dispute that of others.

Additional Evidence Concerning Estimation Theory and Methods

Estimation and timing of the recognition and matching of costs and revenues are dependent on the underlying analysis of data that corroborates its use. Berger et al. (1996) obtained data from the International Brokers Estimate System (IBES) that have forecasts of earnings and growth in earnings. In Table 4, we provide their descriptive information on the sample information obtained. The information obtained describes the distribution of PVCF for three separate forecast methods. In analyzing these data, we calculated the skewness coefficient and presented the results in the expanded table. The analytics indicate the symmetry in the distributions of the PVCF data.

Table 4

As we see from Table 4, the distribution of the sample data is probably very close to symmetrical and, in turn, likely to be distributed similar to a normal distribution process. If not exactly normally distributed, there are many ways one can estimate the distribution of the PVCF data bringing more credibility to the process. One last point concerning the distribution of PVCF in Table 4 concerns the kurtosis in the sample data summarized above. Westfall (2014) notes that its only unambiguous interpretation is in terms of tail extremity—that is, either existing outliers (for the sample kurtosis) or propensity to produce outliers (for the kurtosis of a probability distribution). The logic is simple: Kurtosis is the average (or expected value) of the standardized data raised to the fourth power. Any standardized values that are less than one (i.e., data within one standard deviation of the mean, where the “peak” would be) contribute virtually nothing to kurtosis, because raising a number that is less than one to the fourth power makes it closer to zero. The only data values (observed or observable) that contribute to kurtosis in any meaningful way are those outside the region of the peak—that is, the outliers. Therefore, kurtosis measures outliers only; it measures nothing about the “peak.” Without the original data, one cannot measure the exact kurtoses for the data. However, one can observe that the mean of the data and minimum and maximum values do not differ by huge amounts. Hence,
the exact likelihood of long tails in the
distribution of data about the mean do not exists.
The likelihood is, therefore, such an observation
indicates that if at all, the measures of kurtoses
would be relatively small and approach a normal
distribution when examining the population
from which the sample was chosen. Hence, the
normal approximation when the sample size is
large as in the cases observed indicates the
validity of the normal approximation. This is
also the case if one has evidence that the data are
distributed according to another probability
distribution function and that one could be used
in evaluating the value of IPR.

Introduction of More Technology with
Artificial Intelligence (AI)

To define technology improvements
with reference to AI is not limited to machine
learning. AI will impact many, if not all,
segments of daily life in the next decade, with
applications in a wide range of industries such as
health care, transportation, insurance, transport
and logistics (supply chain), and even customer
service. The need for AI specialists exists in just
about every field as companies seek to give
computers the ability to think, learn, and adapt.
IP and the investment in intangible assets
providing sources of revenue and income, which
enable investors in securing wealth from their
investments in firms, contain IP and other
intangibles such as the use of estimation theory
in financial accounting, the abandonment option,
and the valuing of intangible and similar assets.
The problem for firms involved in various
income-generating activities is to provide
information concerning their economic value
from accounting records kept in their financial
statements. We noticed how financial analysts’
forecast of future cash flow, revenues, and
earned income are often misestimates of these
items. With methods described in this analysis,
one foresees the creation of mathematical
models involving decision making under
uncertainty to create optimal solutions using
machine capabilities and learning that creates
the AI environment. The AI solution will yield
to decision makers a complete environment,
which will minimize or eliminate the misuse of
financial information and eliminate personal bias
in the decision environment (see Chang, Kao,
Mashruwala, and Sorensen, 2017). In addition,
Fricker (2007) applied multivariate statistical
quality control (MSQC) methods. In addition,
MSQC applications by Fricker, Knitt and Hu
(2008) continued earlier studies by focusing on
direction all sensitive procedures in bio
surveillance.

Summary and Conclusion

Firms entering into decisions in times of
financial distress are often confronted with
failure and survival. These decisions concern the
abandonment of assets. The problems associated
with valuing intangible assets and IPR are
similar to those involved in decisions about
M&A. The firm’s environment may be different
in each case, but the problems associated with
predicting cash flow and earnings by analysts
still prevail. This study suggests ways of
estimating the earnings and PVCF when
considering the effects of IPR and other
intangible assets in the process. The proposal
studied meets the requirements of the estimation
type in financial accounting, which is
consistent with accounting conservatism and
goals of financial accounting. Additional
methods exist for estimating the value of
intangibles, which include using the distribution
of financial earnings when the normal
distribution does not apply. This will be the
focus of new and additional research in the AI
environment of the near future.

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**Table 1**

**Monetarization of IPR with Changes in Standard Deviation of Earnings**

<table>
<thead>
<tr>
<th>E(X)</th>
<th>S(X)</th>
<th>Cost of Debt</th>
<th>Z-Score</th>
<th>Cum. Prob.</th>
<th>$IPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,200</td>
<td>400</td>
<td>3,200</td>
<td>2.50000</td>
<td>0.993790</td>
<td>4,174</td>
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<tr>
<td>4,200</td>
<td>500</td>
<td>3,200</td>
<td>2.00000</td>
<td>0.977250</td>
<td>4,104</td>
</tr>
<tr>
<td>4,200</td>
<td>600</td>
<td>3,200</td>
<td>1.66667</td>
<td>0.952210</td>
<td>3,999</td>
</tr>
<tr>
<td>4,200</td>
<td>700</td>
<td>3,200</td>
<td>1.42857</td>
<td>0.923436</td>
<td>3,878</td>
</tr>
<tr>
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<td>3,200</td>
<td>1.25000</td>
<td>0.894350</td>
<td>3,756</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td>4,200</td>
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<td>1.11111</td>
<td>0.866740</td>
<td>3,640</td>
</tr>
<tr>
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<td>1,000</td>
<td>3,200</td>
<td>1.00000</td>
<td>0.841345</td>
<td>3,534</td>
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</table>
Table 2

IPR Monetarization with Changes in Interest Rates and Cost of Debt

<table>
<thead>
<tr>
<th>E(X)</th>
<th>S(X)</th>
<th>Debt Cost</th>
<th>Z</th>
<th>Normal Probability</th>
<th>$IPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,100</td>
<td>600</td>
<td>500</td>
<td>2.66667</td>
<td>0.996170</td>
<td>2,091.96</td>
</tr>
<tr>
<td>2,100</td>
<td>600</td>
<td>1,200</td>
<td>1.50000</td>
<td>0.933193</td>
<td>1,959.70</td>
</tr>
<tr>
<td>2,100</td>
<td>600</td>
<td>1,400</td>
<td>1.16667</td>
<td>0.878327</td>
<td>1,844.49</td>
</tr>
<tr>
<td>2,100</td>
<td>600</td>
<td>1,600</td>
<td>0.83333</td>
<td>0.797672</td>
<td>1,675.11</td>
</tr>
<tr>
<td>2,100</td>
<td>600</td>
<td>1,800</td>
<td>0.50000</td>
<td>0.691462</td>
<td>1,452.07</td>
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<tr>
<td>2,100</td>
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<td>2,000</td>
<td>0.16667</td>
<td>0.566184</td>
<td>1,188.99</td>
</tr>
</tbody>
</table>

Table 3

Monetary Equivalent versus Capital Structure

( Equity = $200,000 )

<table>
<thead>
<tr>
<th>Debt</th>
<th>D:E Ratio</th>
<th>Debt Cost</th>
<th>Cash Inflow</th>
<th>S(X)</th>
<th>Z</th>
<th>Normal Probability</th>
<th>$IPR</th>
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</thead>
<tbody>
<tr>
<td>50,000</td>
<td>0.25</td>
<td>2,000</td>
<td>2,300</td>
<td>230</td>
<td>1.304</td>
<td>0.903942</td>
<td>180,788</td>
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<tr>
<td>60,000</td>
<td>0.30</td>
<td>2,400</td>
<td>1,900</td>
<td>190</td>
<td>−2.632</td>
<td>0.004249</td>
<td>850</td>
</tr>
<tr>
<td>70,000</td>
<td>0.35</td>
<td>2,800</td>
<td>1,500</td>
<td>150</td>
<td>−8.667</td>
<td>0.000000</td>
<td>0</td>
</tr>
<tr>
<td>80,000</td>
<td>0.40</td>
<td>3,200</td>
<td>1,100</td>
<td>110</td>
<td>−19.091</td>
<td>0.000000</td>
<td>0</td>
</tr>
<tr>
<td>90,000</td>
<td>0.45</td>
<td>3,600</td>
<td>700</td>
<td>70</td>
<td>−41.429</td>
<td>0.000000</td>
<td>0</td>
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<tr>
<td>Table 4</td>
<td>Distributions of PVCF from Berger, et al. (1996) p. 269</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Data from IBES</td>
<td>Sample Studies By Berger et al.</td>
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<tr>
<td>Minimum</td>
<td>Median</td>
<td>Mean</td>
<td>Maximum</td>
<td>Std. Dev.</td>
<td>Skewness</td>
<td></td>
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<tr>
<td>PVCF from Forecast/ETF</td>
<td>-0.645</td>
<td>0.128</td>
<td>0.144</td>
<td>0.622</td>
<td>0.08</td>
<td>0.200</td>
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<tr>
<td>PVCF from Growth</td>
<td>0.037</td>
<td>0.42</td>
<td>0.413</td>
<td>1.161</td>
<td>0.086</td>
<td>-0.081</td>
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<tr>
<td>PVCF from Terminal Growth</td>
<td>0.084</td>
<td>0.437</td>
<td>0.443</td>
<td>1.521</td>
<td>0.12</td>
<td>0.050</td>
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</tr>
</tbody>
</table>

If skewness is positive, the data are positively skewed or skewed right, meaning that the right tail of the distribution is longer than the left. If skewness is negative, the data are negatively skewed or skewed left, meaning that the left tail is longer. If skewness = 0, the data are perfectly symmetrical.

Any threshold or rule of thumb is arbitrary, but here is one: If the skewness is greater than 1.0 (or less than -1.0), the skewness is substantial and the distribution is far from symmetrical.