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Multi-Phase Multi-Physics Finite Element Model Updating of Piezoelectric Transducer

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MULTI-PHASE MULTI-PHYSICS FINITE ELEMENT MODEL UPDATING
OF PIEZOELECTRIC TRANSDUCER

BY
LIANG SU

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
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ABSTRACT

The overall objective of this dissertation is to propose a quick and accurate method of updating transducer models. When one makes measurements on a piezoelectric transducer, oftentimes only the impedance function is measured as the experimental data. Thus, to perform model updating for transducers, two major tasks will be covered: (i) developing and verifying an efficient method for estimating the electric impedance function of a transducer, and (ii) developing and testing a FE model updating method for piezoelectric transducers.

The proposed method to estimate the impedance function of a transducer is a Laplace domain method. It expresses both the voltage and current in their partial-fraction forms in the Laplace domain, and obtains the impedance function of the transducer from the ratio of the voltage and current. The Prony-SS method is employed to extract the poles and residues of the voltage and current signals. Compared with traditional methods, the proposed method uses the transient signals, and will not suffer any leakage problems or resolution issues. In addition, this method requires only very short signals to obtain the impedance function, and is excellent for rejecting noise.

This proposed model-updating method is a multi-physics FE model-updating method, including the correction of the elastic material properties based on a short-circuit model, and the correction of dielectric and piezoelectric parameters based on an open-circuit model. The fundamental updating algorithm employed in both steps is the cross-model cross-mode (CMCM) method. In addition to its accuracy and efficiency, this method has the advantages of both the direct matrix methods and indirect physical property-adjustment methods. Implementing the CMCM algorithm requires a knowledge of both the measured modal frequencies and the corresponding mode shapes, but a measured impedance function could

provide only modal frequencies for short- and open-circuit transducers. When dealing with the incomplete modal information, an iterative procedure is taken. In each iteration, the “measured” mode shapes are approximated by the mode shapes obtained from the previous iteration’s updated FE model.

In this study, we employed a tube transducer, which is made of piezoceramic material, to develop and test new methods of estimating the impedance function and updating piezoelectric constitutive properties. Both computer simulations and lab experiments have been conducted to verify the accuracy and efficiency of the proposed methods.

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PREFACE

This dissertation follows the University of Rhode Island Graduate School guidelines for the preparation of a dissertation in standard format. The material presented in this thesis is divided into eight chapters.

- Chapter 1 serves to provide an introduction to the studies within this dissertation, as well as a review of the relevant research in FE model updating.
- Chapter 2 is an introduction to piezoelectric transducers, especially tube transducers. We begin with a brief overview of transducers and piezoelectric material. Both full and reduced constitutive equations are introduced to explain the performance of thin-tube transducers, and elaborate the relationship between piezoelectric constitutive properties. In addition, we address the analytical solution for ring (single DOF vibrators) and the FE solution for tube (multi DOF vibrators), and their electrical representations, i.e., equivalent circuits, are extended from those solutions.
- Chapter 3 presents the electromechanically coupled FE formulations and the procedures for modeling tube transducers with 2D and 3D elements using a commercial package. Furthermore, we present numerical verifications of the FE model, including comparisons with an approximate analytical solution, among two commercial FE packages, and with physical measurements.
- Chapters 4 and 5 show techniques and numerical studies for the estimation of impedance functions. In addition to a review of the traditional methods for estimating the impedance function of a piezoelectric transducer, in Chapter 4, we also newly propose an improved method using the Laplace domain. The newly proposed method decomposes the signals into their complex exponential components using the Prony-SS method, and it is particularly effective

for handling measured voltage and current signals that have been contaminated by noise. In Chapter 5, we discuss a numerical study of the estimation of the impedance function. To test the effectiveness of the proposed Laplace domain method, both numerical simulations and lab experiment are included. The impedance function that was estimated from the proposed method is compared with those obtained using traditional methods. The effectiveness of noise cancellation for both the proposed and traditional Fourier-based method is also investigated.

- In Chapter 6, we describe procedures for updating piezoelectric transducers. Besides a brief review of traditional model updating methods, a new approach which is based on the extended cross-model cross-mode method is proposed. The proposed model-updating method is a two-step method, including the correction of the elastic material properties based on a short-circuit model in the first step, and the correction of dielectric and piezoelectric parameters based on an open-circuit model in the second step. Chapter 7 includes two examples of the numerical demonstration of model-updating methods: a simulated transducer and a real transducer. The simulated transducer is employed to test the accuracy of the proposed method, and ultimately, the effectiveness of the proposed method is tested on a real transducer, with the impedance function of the transducer being estimated from the experiment.
- Conclusions and suggestions for further work are given in Chapter 8.

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CHAPTER 1

Introduction

1.1 Statement of the Problem

The accuracy of the finite-element (FE) model of a piezoelectric transducer depends on the accuracy of the material's constitutive properties, but the values provided in the manufacturer's specification sheets for piezoelectric materials are often incomplete or inaccurate (Abboud et al., 1998). Because of the polycrystalline nature of piezoceramics and the statistical variations in their composition and production-related influences, the standard tolerance range of piezoelectric material is $\pm 20\%$ (Ltd, 2011). In addition, depolarization influences the accuracy of the material properties, which may result from operating temperatures that are too near the Curie temperature, high static-pressure cycling in deep-water applications, high alternating electric fields, or a slight extent from the passage of time (Sherman and Butler, 2007). Therefore, the properties of different transducers made with the same piezoelectric material may vary, and it is necessary to obtain a quick and accurate method of updating transducer models.

FE model updating has been defined as the systematic adjustment of FE models using the experimental data. When one makes measurements on a piezoelectric transducer, oftentimes only the impedance function is measured as the experimental data. Thus, to perform model updating for transducers, two major tasks will be covered: (i) developing and verifying an efficient method for estimating the electric impedance function of a transducer, and (ii) developing and testing a FE model updating method for piezoelectric transducers.

The electrical impedance function is a function of frequency, and it is generally defined as the total resistance offered by a device or circuit to the flow of an alternating current at specific frequencies. As an electrical feature, it is relatively

easy to be estimated, and includes many important transducer properties such as the resonance frequency, capacitance, bandwidth, and damping. Traditionally, there are two methods for estimating the impedance function: (i) stepped harmonic analysis (i.e., single frequency) method, and (ii) Fourier-based analysis method using transient signals. The stepped harmonic analysis involves repeatedly applying a harmonic input signal to obtain the corresponding steady-state response, then computing the impedance of the given frequency. The Fourier-based method computes the impedance function from the complex ratio of the Fast Fourier Transform (FFT) of the input and output time-domain signals. However, both methods above have drawbacks. The stepped harmonic analysis method is time consuming and requires specialized equipment, while the Fourier-based analysis method suffers from problems related to frequency leakage and resolution (Lis and Schmidt, 2004; Lis et al., 2005; Su et al., 2014).

In terms of model-updating technologies, the traditional methods have been classified into two major types: direct matrix methods and indirect physical property adjustment methods (Friswell and Mottershead, 1995). The first of these two groups generally involves non-iterative methods, all of which are based on computing changes that are made directly to the mass and stiffness matrices. Such changes may have succeeded in generating modified models that have properties close to those measured in the tests, but the resulting models become abstract “representation” models, and cannot be interpreted in a physical way. The second group comprises methods that are in many ways more acceptable in that the parameters that they adjust are much closer to physically realizable quantities. The methods in this second group, which seek to find correction factors for each individual FE or for each design parameter related to each FE, have generally been viewed as the main hope for the development of updating technology. The pub-

lished model-updating methods that were implemented for ultrasound transducers belong to the second group of methods (Piranda et al., 1998; Piranda et al., 2001). Because these methods are all iterative, a much greater computation effort is required, and the solutions for the physical properties are often prone to diverge (Hu et al., 2007).

1.2 Methodology and Procedures

In this study, we employed a tube transducer, which is made of piezoceramic material, to develop and test new methods of estimating the impedance function and updating piezoelectric constitutive properties. The effect of the backing layer and the transducer housing is not involved in this study, and the general procedure is also applicable to other types of piezoelectric transducers.

Figure 1. Real transducer (Left), Finite element model of transducer (Right)

Figure. 1 shows a real tube transducer and its FE model, which is built using the commercial FE package, ANSYS. Because the FE model is axially symmetric, we can build it using either a three-dimensional (3D) element or by revolving its two-dimensional (2D) element radial cross section (a rectangle in the x - y plane) around the z -axis. In this study, we conduct modal, harmonic, and transient (dynamic response) analyses based on this FE model. The modal analysis is employed to obtain the modal frequencies and mode shapes of the piezoelectric

tube transducer under both open- and short-circuit electric boundary conditions; the harmonic analysis is used to obtain the standard impedance function, and the transient analysis is to obtain the electric-charge time signal, while the piezoelectric tube transducer is subjected to an input electric-potential time signal. The ultimate purpose of the transient analysis is to obtain the impedance function of the transducer.

The newly developed method to estimate the impedance function of a transducer is a Laplace domain method. It expresses both the voltage and current in their partial-fraction forms in the Laplace domain, and obtains the impedance function of the transducer from the ratio of the voltage and current. The Prony-SS method is employed to extract the poles and residues of the voltage and current signals (Hu et al., 2013). Compared with traditional methods, the proposed Prony-based method uses the transient signals, and will not suffer any leakage problems or resolution issues. In addition, this method requires only very short signals to obtain the impedance function, and is excellent for rejecting noise. Both computer simulations and lab experiment have been conducted to verify the accuracy and efficiency of this Laplace domain method.

The proposed model-updating method is the multi-physics FE model-updating method. This method is a two-step method, including the correction of the elastic material properties based on a short-circuit model in the first step, and the correction of dielectric and piezoelectric parameters based on an open-circuit model in the second step. The fundamental updating algorithm employed in both steps is the CMCM method (Hu et al., 2007; Hu and Li, 2008). In addition to its accuracy and efficiency, this method has the advantages of both the direct matrix methods and indirect physical property-adjustment methods. Implementing the CMCM algorithm requires a knowledge of both the measured modal frequencies and the

corresponding mode shapes, but a measured impedance function could provide only modal frequencies for short- and open-circuit transducers. When dealing with the incomplete modal information, an iterative procedure is taken. In each iteration, the “measured” mode shapes are approximated by the mode shapes obtained from the previous iteration’s updated FE model.

Specific to piezoelectric tube transducers poling in the radial direction, which obeys the thin-tube assumption, elastic material parameters associated with a tube’s thickness direction would not affect the functionality of the piezoelectric tube. In turn, the corresponding impedance function depends only on four independent material parameters: two elastic parameters Y_{11}^E and ν_{12} (or s_{11}^E and s_{12}^E), the permittivity under constant stress ε_{33}^T , and the piezoelectric parameter d_{31} . Specifically, only the values of Y_{11}^E and ν_{12} (or s_{11}^E and s_{12}^E) determine the resonance frequencies of the impedance function, which are the modal frequencies of the transducer in the short-circuit situation, the value of ε_{33}^T determines the unclamped capacitance, and the value of d_{31} affects the anti-resonant frequencies of the impedance function, which are the modal frequencies of the transducer in the open-circuit condition. Likewise, one could update those two elastic properties Y_{11}^E and ν_{12} (or s_{11}^E and s_{12}^E) in the short-circuit model, then update ε_{33}^T and d_{31} in the open-circuit model.

However, with respect to References.(Hu et al., 2007; Piefort, 2001), the piezoelectric material’s parameters that meet the requirements of the CMCM method are e-form group parameters. Therefore, the d-form parameters, Y_{11}^E , ν_{12} (or s_{11}^E , s_{12}^E), ε_{33}^T , and d_{31} need to be converted to e-form parameters to finish the updating procedure. For most commercial FE method (FEM) packages such as ANSYS, the input e-form parameters of models (for both 2D and 3D models) are the e-form parameters of full constitutive equations, but are not reduced equations.

The 10 independent parameters in e-form that are used to fully characterize the constitutive equations of PZT-type materials are c_{11}^E , c_{33}^E , c_{12}^E , c_{13}^E , c_{44}^E , ε_{33}^S , ε_{11}^S , e_{31} , e_{33} , and e_{15} . We can easily determine that it is not possible to obtain these 10 e-form parameters with only four d-form parameters. Thus, to keep only those four parameters while using a commercial package, this study employs two simple ways to manipulate the situation: (1) a mathematical simplification that involves setting all insensitive parameters to be equal to zero, and (2) a physical simplification by using an isotropic material model. One must recognize that setting insensitive coefficients equal to zero may easily violate the underlying physics and cause singularity in mathematics.

CHAPTER 2

Piezoelectric Transducers

2.1 Introduction

Transducers refer to devices or agencies that convert energy from one form to another. To convert electrical energy to acoustical energy, or vice versa, we use an electroacoustic transducer (Burdic, 1991). The electroacoustic transducer may be used operated in either transmitting (projector) or receiving mode (hydrophone), or both. In transmitting mode, an ultrasonic wave is generated by an applied electrical voltage or current, while in receiving mode, an electrical signal is generated by an incoming acoustical wave. While loudspeakers and microphones are transducers that are used as sources and receivers of sound in air, the counterparts in water are projectors and hydrophones for sources and receivers, respectively. A transducer may also be operated in either continuous-wave mode, where the transducer is harmonically operated at a specified frequency, or in transient mode, where the transducer is used to send out pulses (Kocbach, 2000).

Transducers were first used to obtain the direct measurement of the speed of sound in the fresh water of Lake Geneva in Switzerland in 1826. At that time, there were no electroacoustic transducer to generate sound in the water. Instead, the projector was a mechano-acoustic transducer, i.e., the striking of a bell under water. In 1915, Paul Langevin et al. began work in France using an electrostatic transducer as a projector and a waterproofed carbon microphone as a hydrophone (Sherman and Butler, 2007). They realized that the use of the piezoelectric effect in quartz had the potential for realizing improved transducers. Then, the search for new man-made transduction materials was further prompted by the potential for application during both World Wars and the Cold War. In 1944, piezoelectricity was discovered by A.R. von Hippel in permanently polarized barium titanate

ceramics, and in 1954, even stronger piezoelectricity was achieved in polarized lead zirconate titanate ceramics (PZT), which are still being used in most underwater sound transducers (Jaffe et al., 1955). The development of underwater electroacoustic transducers expanded rapidly during the twentieth century, and it continued to be a growing field of knowledge (Sherman and Butler, 2007).

The various applications of transducers over a wide frequency range (from about 1 Hz to over 1 MHz) requires many different transducer designs. In naval applications, acoustic communication, echo ranging, and passive listening all require different projectors to transmit sound, and different hydrophones to receive sound. Generally, hydrophones and projectors are used in large groups of up to 1000 or more transducers closely packed in planar, cylindrical, or spherical arrays mounted on naval ships. Transducers also play an important role in other applications. For example, in oceanography, transducers help to finish seafloor mapping and determinate seafloor characteristics. In ocean engineering, they help to identify the locations of oil and gas deposits under the oceans, and with the laying of underwater cables or pipelines. Several research projects employ transducers to acquire data related to a wide variety of topics such as the Acoustic Thermometry of Ocean Climate project (ATOC) and the Sound Surveillance System, which has been used to study the behavior of sperm whales, and to detect earthquakes and volcanic eruptions under the sea (Sherman and Butler, 2007).

2.1.1 Tube Transducers

The cylindrical shell is the basic shape of transducers that are widely used in many engineering applications. Three possible modes of cylindrical transducers are shown in Figure. 2.

Figure. 2(a) shows a cylindrical shell that operates in the radial mode. The inner and outer cylindrical surfaces are coated with conducting material, and the

Figure 2. Three possible modes of cylindrical shell transducers

poling field is applied between these surfaces. This geometry is characterized by moderate sensitivity and high output capacitance. Because of the large capacitance, the design is capable of driving long cables directly without suffering much loss in sensitivity.

Figure. 2(b) shows the tangential mode of operation. Conducting stripes that are formed longitudinally on the ceramic cylindrical surface divide the cylinder into an even number of curved segments. Alternate stripes are electrically connected and a poling field applied in the circumferential direction. The cylindrical segments are thus electrically in parallel but mechanically in series. This configuration is characterized by high sensitivity and low capacitance, and is an excellent design where high sensitivity is required, and where suitable electronic isolation from the cable can be provided close to the hydrophone element.

The configuration shown in Figure. 2(c) operates in the longitudinal mode. The ends of the ceramic cylinder are made to conduct electrically and the poling field is applied in the direction parallel to the cylindrical axis. This configuration has a somewhat higher sensitivity than the radial mode, and as with the tangential mode, the electrical impedance of the device is high (Burdic, 1991).

The present study focuses on the radial mode transducer. Only the active

element (thin piezoelectric tube) which is sandwiched between a front layer and a backing layer is considered, and the effect of assembling and the transducer housing is not involved in the analysis. As a varying voltage is applied over the electrodes, the cylindrical transducer will begin to vibrate. This vibration is related to the geometry of this tube and the material parameters.

The present study focuses on the radial-mode transducer. We consider only the active element (thin piezoelectric tube), which is sandwiched between a front layer and a backing layer, and the effects of the assembly and the transducer housing were not considered in the analysis. As a varying voltage is applied over the electrodes, the cylindrical transducer will begin to vibrate. This vibration is related to the geometry of the tube and the material parameters.

2.1.2 Piezoelectric Material

At present, piezoelectric materials are the most commonly used material when developing transducers. When mechanical stresses are applied to a piezoelectric material solid, a voltage is produced between its surfaces, called a piezoelectric effect. Conversely, when a voltage is applied across certain surfaces of the solid, the solid undergoes a mechanical distortion, which is called the inverse piezoelectric effect. Most of the piezoelectric materials are crystalline solids. They can be single crystals, which are either formed naturally or by synthetic processes, or polycrystalline materials such as ferroelectric ceramics, which can be rendered piezoelectric in any chosen polar direction by the poling treatment. During the manufacture of piezoceramics, a suitable ferroelectric material is first fabricated into the desired shape. The piezoceramic element is then heated to an elevated temperature (the Curie point) while in the presence of a strong DC field.

The best known piezoelectric material is lead zirconate titanate (PZT) which is an intermetallic inorganic compound. It is a white solid that is insoluble in

all solvents, and it has a perovskite-type structure. The remanent polarization gives PZT an approximately linear response to an alternating electric field. It is very stable and large enough to give a strong piezoelectric effect. However, this also causes depolarization to significantly affect the performance of PZT transducers. This depolarization may result from operating temperatures that are too near the Curie temperature, high static pressure cycling in deep-water applications, high alternating electric fields, or a slight extent from the passage of time. Thus, while the properties of true piezoelectrics are determined by their internal crystal structure and cannot be changed, the piezoelectric properties of polarized electrostrictive materials depend on the level of remanence achieved during the polarization process, and may be changed by the operating conditions. Therefore, although PZT material has many advantages compared to other materials for underwater transducer applications, its limitations must also be considered during the design process (Piefort, 2001).

2.2 Constitutive Equations

2.2.1 Full Constitutive Equations

The linear piezoelectric effect in a piezoelectric medium can be described using a set of constitutive equations. When it is expressed in the so-called e-form, one has:

$$\mathbf{T} = \mathbf{c}^E \mathbf{S} - \mathbf{e}^t \mathbf{E} \quad (1)$$

$$\mathbf{D} = \mathbf{e} \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E} \quad (2)$$

where $\mathbf{T} \in \mathbb{R}^{6 \times 1}$ is stress vector, $\mathbf{S} \in \mathbb{R}^{6 \times 1}$ strain vector, $\mathbf{E} \in \mathbb{R}^{3 \times 1}$ electric field vector, $\mathbf{D} \in \mathbb{R}^{3 \times 1}$ electric displacement vector, $\mathbf{c}^E \in \mathbb{R}^{6 \times 6}$ elastic stiffness matrix, $\mathbf{e} \in \mathbb{R}^{3 \times 6}$ piezoelectric stress matrix (superscript t stands for the transpose) and $\boldsymbol{\epsilon}^S \in \mathbb{R}^{3 \times 3}$ permittivity matrix. The superscript of $\mathbf{c}^E$ indicates that the elastic compliance coefficients matrix is obtained with $\mathbf{E}$ being held constant, and the

superscript of ϵ^S indicates that the permittivity coefficients matrix is obtained with $\mathbf{S}$ being held constant (Sherman and Butler, 2007; Piefort, 2001).

Generally, the coefficient matrices in Eqs. 1 and 2 include 45 independent coefficients. However, for polarized piezoelectric ceramic material (such as PZT) which belongs to the 4 mm (C4V) crystal class (IEE, 1987) , many of the coefficients are zero or related, leaving only 10 independent coefficients, and the corresponding equations are explicitly expressed in the e-form as:

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{Bmatrix} = \begin{bmatrix} c_{11}^E & c_{12}^E & c_{13}^E & 0 & 0 & 0 \\ c_{12}^E & c_{11}^E & c_{13}^E & 0 & 0 & 0 \\ c_{13}^E & c_{13}^E & c_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}^E \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (3)$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} + \begin{bmatrix} \epsilon_{11}^S & 0 & 0 \\ 0 & \epsilon_{11}^S & 0 \\ 0 & 0 & \epsilon_{33}^S \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (4)$$

where $c_{66}^E = (c_{11}^E - c_{12}^E)/2$. The subscript 3 is the direction in which ceramic element is polarized. Subscripts 1 and 2 represent directions perpendicular to poling direction, and 4, 5 and 6 refer to shear directions that follow IEEE convention, namely, $23 \rightarrow 4$, $13 \rightarrow 5$, and $12 \rightarrow 6$ (Wik, 2015).

A widely used alternative and equivalent representation consists in writing the constitutive equations in the so-called d-form:

$$\mathbf{S} = \mathbf{s}^E \mathbf{T} + \mathbf{d}^t \mathbf{E} \quad (5)$$

$$\mathbf{D} = \mathbf{d} \mathbf{T} + \epsilon^T \mathbf{E} \quad (6)$$

where $\mathbf{s}^E \in \mathbb{R}^{6 \times 6}$ is elastic compliance matrix, $\mathbf{d} \in \mathbb{R}^{3 \times 6}$ piezoelectric strain matrix and $\epsilon^T \in \mathbb{R}^{3 \times 3}$ permittivity matrix at constant stress. Combined into one

equation, the expanded form for 4 mm crystal class is explicitly expressed as (Balmes and Deraemaeker, 2013):

$$\left\{ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ D_1 \\ D_2 \\ D_3 \end{matrix} \right\} = \left[\begin{matrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 & 0 & 0 & d_{31} \\ s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 & 0 & 0 & d_{31} \\ s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & 0 & 0 & 0 & d_{33} \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & 0 & s_{44}^E & 0 & d_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{15} & 0 & \varepsilon_{11}^T & 0 & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 & 0 & \varepsilon_{11}^T & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 & 0 & 0 & \varepsilon_{33}^T \end{matrix} \right] \left\{ \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ E_1 \\ E_2 \\ E_3 \end{matrix} \right\} \quad (7)$$

From the constitutive equations of d-form and e-form, it is straightforward to obtain the following relationships:

$$\mathbf{c}^E = (\mathbf{s}^E)^{-1} \quad (8)$$

$$\mathbf{e} = \mathbf{d} (\mathbf{s}^E)^{-1} \quad (9)$$

and

$$\boldsymbol{\varepsilon}^S = \boldsymbol{\varepsilon}^T - \mathbf{e} \mathbf{d}^t \quad (10)$$

Furthermore, Eq.10 can also be expressed as either

$$\boldsymbol{\varepsilon}^S = \boldsymbol{\varepsilon}^T - \mathbf{d} \mathbf{c}^E \mathbf{d}^t \quad (11)$$

or

$$\boldsymbol{\varepsilon}^S = \boldsymbol{\varepsilon}^T - \mathbf{e} \mathbf{s}^E \mathbf{e}^t \quad (12)$$

For quantifying the elastic parameters, many people prefer to use engineering parameters, instead of the compliance or stiffness parameters. For a transverse isotropic material whose properties are the same in the plane perpendicular to the poling direction, the compliance matrix $\mathbf{s}^E$ can be expressed in terms of engineering

parameters as:

$$\mathbf{s}^E = \begin{bmatrix} \frac{1}{Y_{11}^E} & -\frac{\nu_{12}}{Y_{11}^E} & -\frac{\nu_{13}}{Y_{33}^E} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{Y_{11}^E} & \frac{1}{Y_{11}^E} & -\frac{\nu_{13}}{Y_{33}^E} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{Y_{33}^E} & -\frac{\nu_{13}}{Y_{33}^E} & \frac{1}{Y_{33}^E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu_{12})}{Y_{11}^E} \end{bmatrix} \quad (13)$$

where Y_{11}^E and Y_{33}^E are the Young's modulus in 1-direction and 3-direction, respectively; G is the shear modulus on the plane whose normal is in 1-direction (or 2-direction); and ν_{12} and ν_{13} are the Poisson's ratios that correspond to the contraction in 2-direction and 3-direction when the extension is in 1-direction.

In summary, the material parameters of PZT include 2 independent dielectric permittivities, 3 independent piezoelectric constants and 5 independent elastic constants. These material parameters are listed in Table.1

Table 1. Independent parameters of PZT

e-form	d-form
Elastic Stiffness	Elastic Compliance Engineering Parameter
c_{11}^E	s_{11}^E Y_{11}^E
c_{12}^E	s_{12}^E Y_{33}^E
c_{13}^E	s_{13}^E ν_{12}^E
c_{33}^E	s_{33}^E ν_{13}^E
c_{44}^E	s_{44}^E G
Piezoelectric stress constant	Piezoelectric strain constant
e_{31}	d_{31}
e_{33}	d_{33}
e_{15}	d_{15}
Dielectric permittivity at constant strain	Dielectric permittivity at constant stress
ϵ_{11}^S	ϵ_{11}^T
ϵ_{33}^S	ϵ_{33}^T

2.2.2 Reduced Constitutive Equations for a Thin Cylinder

When the piezoceramic transducer polarized in the thickness direction is modeled as a very thin cylinder, the normal stress in the thickness (radial) direction and the respective transverse shear stress components are negligible (Erturk and Inman, 2011):

$$T_3 = T_4 = T_5 = 0 \quad (14)$$

If the electric field components in the axis-1 and axis-2 are zero,

$$E_1 = E_2 = 0 \quad (15)$$

then Eq.7 becomes the reduced constitutive equations

$$\begin{Bmatrix} S_1 \\ S_2 \\ S_6 \\ D_3 \end{Bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & 0 & d_{31} \\ s_{12}^E & s_{11}^E & 0 & d_{31} \\ 0 & 0 & s_{66}^E & 0 \\ d_{31} & d_{31} & 0 & \varepsilon_{33}^T \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_6 \\ E_3 \end{Bmatrix} \quad (16)$$

From the above equation, there are only four independent material parameters which will affect the dynamic characteristics of a piezoceramic thin tube transducer: s_{11} , s_{12} , ε_{33}^T and d_{31} , noting that $s_{66}^E = 2(s_{11}^E - s_{12}^E)$ is not an independent parameter. Since $s_{11}^E = 1/Y_{11}^E$ and $s_{12}^E = -\nu_{12}/Y_{11}^E$, the four independent material parameters can also be: Y_{11}^E , ν_{12} , ε_{33}^T and d_{31} .

In an attempt to show the corresponding e-form, one can first rearrange Eq.16 into:

$$\begin{bmatrix} s_{11}^E & s_{12}^E & 0 & 0 \\ s_{12}^E & s_{11}^E & 0 & 0 \\ 0 & 0 & s_{66}^E & 0 \\ -d_{31} & -d_{31} & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_6 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -d_{31} \\ 0 & 1 & 0 & -d_{31} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \varepsilon_{33}^T \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_6 \\ E_3 \end{Bmatrix} \quad (17)$$

Let the e-form of the reduced constitutive equations be denoted by

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_6 \\ D_3 \end{Bmatrix} = \begin{bmatrix} \bar{c}_{11}^E & \bar{c}_{12}^E & 0 & -\bar{e}_{31} \\ \bar{c}_{12}^E & \bar{c}_{11}^E & 0 & -\bar{e}_{31} \\ 0 & 0 & \bar{c}_{66}^E & 0 \\ \bar{e}_{31} & \bar{e}_{31} & 0 & \bar{\varepsilon}_{33}^S \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_6 \\ E_3 \end{Bmatrix} \quad (18)$$

From the above two equations, one has

$$\begin{bmatrix} \bar{c}_{11}^E & \bar{c}_{12}^E & 0 & -\bar{e}_{31} \\ \bar{c}_{12}^E & \bar{c}_{11}^E & 0 & -\bar{e}_{31} \\ 0 & 0 & \bar{c}_{66}^E & 0 \\ \bar{e}_{31} & \bar{e}_{31} & 0 & \bar{\varepsilon}_{33}^S \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & 0 & 0 \\ s_{12}^E & s_{11}^E & 0 & 0 \\ 0 & 0 & s_{66}^E & 0 \\ -d_{31} & -d_{31} & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & -d_{31} \\ 0 & 1 & 0 & -d_{31} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \varepsilon_{33}^T \end{bmatrix} \quad (19)$$

After some algebra, one obtains the reduced e-form parameters to be:

$$\bar{c}_{11}^E = \frac{s_{11}^E}{(s_{11}^E + s_{12}^E)(s_{11}^E - s_{12}^E)} = \frac{Y_{11}^E}{(1 + \nu_{12})(1 - \nu_{12})} \quad (20)$$

$$\bar{c}_{12}^E = \frac{-s_{12}^E}{(s_{11}^E + s_{12}^E)(s_{11}^E - s_{12}^E)} = \frac{Y_{11}^E \nu_{12}}{(1 + \nu_{12})(1 - \nu_{12})} \quad (21)$$

$$\bar{c}_{66}^E = \frac{1}{s_{66}^E} = \frac{Y_{11}^E}{2(1 + \nu_{12})} \quad (22)$$

$$\bar{e}_{31}^E = \frac{d_{31}}{s_{11}^E + s_{12}^E} = \frac{d_{31} Y_{11}^E}{1 - \nu_{12}} \quad (23)$$

$$\bar{\varepsilon}_{33}^S = \varepsilon_{33}^T - \frac{2d_{31}^2}{s_{11}^E + s_{12}^E} = \varepsilon_{33}^T - \frac{2d_{31}^2 Y_{11}^E}{1 - \nu_{12}} \quad (24)$$

It is realized that the reduced d-form parameters are identical to those of the original constitutive equations, but the reduced e-form parameters are completely different from those of the original constitutive equations. For the finite element model updating in this study, the purpose is to update the material parameters based on the experimentally obtained impedance function of a piezoceramic thin tube transducer. Only four independent material parameters are necessary, or could be updatable, for a piezoceramic thin tube transducer; and it would be better to update the d-form parameters for the obvious reason mentioned above.

2.3 Piezoelectric Vibrators

The piezoceramic transducer to be studied is a thin tube type, where the length and diameter of the tube is much larger than its thickness. If the length of this transducer is also very small, then it becomes a piezoceramic “ring” transducer. Traditionally, a ring transducer has been treated as a SDOF (single-degree-of-freedom) system in the analytical derivation for its impedance (or admittance) function.

2.3.1 Ring Vibrator

A ring vibrator has been often treated as a 3-1 SDOF vibrator (see Appendix A), where the only strain/stress occurs in the circumference direction (also 1-direction) and the poling direction (3-direction) could be the radial direction (or longitudinal direction). One can refer its detailed derivation for the impedance (or admittance) function to Reference.(Mason, 2013), while the following presentation provides a brief review.

Analytical Impedance Function for a Ring Transducer

The ring transducer poling in radial direction is treated as a 3-1 SDOF vibrator which has the reduced constitutive equation in its d-form to be:

$$S_1 = s_{11}^E T_1 + d_{31} E_3 \quad (25)$$

$$D_3 = d_{31} T_1 + \varepsilon_{33}^T E_3 \quad (26)$$

and its corresponding e-form is shown to be:

$$T_1 = \frac{1}{s_{11}^E} S_1 - \frac{d_{31}}{s_{11}^E} E_3 \quad (27)$$

$$D_3 = \frac{d_{31}}{s_{11}^E} S_1 + \left(\varepsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E} \right) E_3 \quad (28)$$

Thus, the laterally clamped (in one dimension) permittivity

$$\varepsilon_{33}^S = \varepsilon_{33}^T (1 - k_{31}^2) \quad (29)$$

where the coupling coefficient $k_{31}^2 = d_{31}^2 / s_{11}^E \varepsilon_{33}^T$.

With the assumptions of negligibly small cross-sectional dimensions, the equation of motion for a ring transducer (see Figure.3) can be derived from Newton's second law applied to a segment. This incremental segment which is not balanced by external radial forces causes a radial acceleration as a whole. The resultant force on this incremental segment is

$$F_r = F \delta \theta = T_1 \ell t \delta \theta \quad (30)$$

Figure 3. Illustration of (a) whole ring vibrator, (b) an incremental segment

where ℓ and t are the width and thickness of the ring transducer; and $\delta\theta$ is the angle associated with the incremental segment. From Newton's second law, it gives

$$\rho \ell t a \delta\theta \frac{\partial^2 u}{\partial t^2} = -T_1 \ell t \delta\theta \quad (31)$$

where ρ is the density of the ceramic, a is the average radius of the ring, and u the radial displacement of the segment.

Since the elongation or contraction of circumference is linearly proportional to the change of radius, one has $S_1 = u/a$. Substituting T_1 by Eq. 27 into Eq. 31, one shows

$$\frac{\partial^2 u}{\partial t^2} + \frac{1}{a^2 \rho s_{11}^E} u = \frac{d_{31}}{a \rho s_{11}^E} E_3 \quad (32)$$

The transfer function (in the Laplace domain) from E_3 to u , denoted $T_{u,E_3}(s)$ thus is shown to be

$$T_{u,E_3}(s) = \frac{d_{31}}{a \rho s_{11}^E (s^2 + \omega_1^2)} \quad (33)$$

where

$$\omega_1^2 = \frac{1}{a^2 \rho s_{11}^E} \quad (34)$$

is the free resonance of the unloaded ring ($E_3 = 0$). Since $S_1 = u/a$, the transfer

function from E_3 to S_1 , denoted $T_{S_1,E_3}(s)$, thus is

$$T_{S_1,E_3}(s) = T_{S_1,u}(s) T_{u,E_3}(s) = \frac{d_{31}}{a^2 \rho s_{11}^E (s^2 + \omega_1^2)} \quad (35)$$

From Eq. 28, one shows the transfer function from E_3 to D_3 to be

$$T_{D_3,E_3}(s) = \frac{d_{31}}{s_{11}^E} T_{S_1,E_3}(s) + \varepsilon_{33}^T (1 - k_{31}^2) = \frac{d_{31}^2}{a^2 \rho (s_{11}^E)^2 (s^2 + \omega_1^2)} + \varepsilon_{33}^T (1 - k_{31}^2) \quad (36)$$

From $E_3 = V/t$ where V is the electric potential and the current, $I = 2\pi a \ell \dot{D}_3$, one has the corresponding transfer functions

$$T_{E_3,V}(s) = 1/t \quad (37)$$

and

$$T_{I,D_3}(s) = 2\pi a \ell s \quad (38)$$

From Eqs. 36, 37 and 38, one obtains

$$T_{I,V}(s) = \frac{2\pi a \ell s}{t} \left[\frac{d_{31}^2}{a^2 \rho (s_{11}^E)^2 (s^2 + \omega_1^2)} + \varepsilon_{33}^T (1 - k_{31}^2) \right] \quad (39)$$

The admittance function $Y(\omega)$, in the frequency domain, of the ring transducer is defined as the transfer function $T_{I,V}(s)$ while substituting s by $j\omega$:

$$Y(\omega) = \frac{I(\omega)}{V(\omega)} = j\omega \left(\frac{2\pi a \ell}{t} \right) \left[\frac{d_{31}^2}{a^2 \rho (s_{11}^E)^2 (\omega_1^2 - \omega^2)} + \varepsilon_{33}^T (1 - k_{31}^2) \right] \quad (40)$$

Substituting for ω_1 (see Eq. 34) and k_{31} (see Eq. 29), Eq. 40 becomes

$$Y(\omega) = j\omega \left(\frac{2\pi a \ell}{t} \right) \left[\frac{k_{31}^2 \varepsilon_{33}^T \omega_1^2}{\omega_1^2 - \omega^2} + \varepsilon_{33}^T (1 - k_{31}^2) \right] \quad (41)$$

The reciprocal of the admittance function $Y(\omega)$ is the impedance function $Z(\omega)$.

Resonance frequency ω_r and antiresonance frequency ω_a are obtained from the pole and zero of the admittance, given by (Mason, 2013; Wilson, 1988)

$$\omega_r^2 = \omega_1^2 = \frac{1}{a^2 \rho s_{11}^E} \quad (42)$$

and

$$\omega_a^2 = \frac{\omega_r^2}{1 - k_{31}^2} \quad (43)$$

respectively. Conversely, the coupling coefficient k_{31} is given by

$$k_{31}^2 = 1 - \left(\frac{\omega_r}{\omega_a} \right)^2 \quad (44)$$

Single-degree-of-freedom Equivalent Circuits

Eq. 41 can be expressed as the sum of the blocked electrical admittance $Y_E(\omega)$ and the motional admittance $Y_M(\omega)$:

$$Y(\omega) = Y_E(\omega) + Y_M(\omega) \quad (45)$$

The blocked electrical admittance is

$$Y_E(\omega) = j \omega \left(\frac{2\pi a \ell}{t} \right) [\varepsilon_{33}^T (1 - k_{31}^2)] = j \omega C_0 \quad (46)$$

where C_0 is the clamped capacitance

$$C_0 = C_f (1 - k_{31}^2) \quad (47)$$

and the free (or total) capacitance of the ring is

$$C_f = \frac{2\pi a \ell \varepsilon_{33}^T}{t} \quad (48)$$

The motional admittance $Y_M(\omega)$ is

$$Y_M(\omega) = j \omega \left(\frac{2\pi a \ell}{t} \right) \frac{k_{31}^2 \varepsilon_{33}^T \omega_1^2}{\omega_1^2 - \omega^2} = j \omega C_1 \left(\frac{\omega_1^2}{\omega_1^2 - \omega^2} \right) \quad (49)$$

where the motional capacitance C_1 is

$$C_1 = C_f k_{31}^2 \quad (50)$$

or

$$C_1 = \left(\frac{2\pi a \ell \varepsilon_{33}^T}{t} \right) \frac{d_{31}^2}{s_{11}^E \varepsilon_{33}^T} = \frac{a s_{11}^E}{2\pi \ell t} \left(\frac{2\pi \ell d_{31}}{s_{11}^E} \right)^2 = C_m N^2 \quad (51)$$

in which the mechanical compliance of the ring C_m is

$$C_m = \frac{as_{11}^E}{2\pi\ell t} \quad (52)$$

and the turns ratio N is

$$N = \frac{2\pi\ell d_{31}}{s_{11}^E} \quad (53)$$

Referring to the simple equivalent circuit of Figure. 4, using Eq. 51 for C_1 , the motional inductance L_1 can be derived from $\omega_1^2 = (1/L_1 C_1)$ as:

$$L_1 = \frac{1}{\omega_1^2 C_1} = \frac{1}{\omega_1^2 C_m N^2} \quad (54)$$

Substituting Eq. 34 for ω_1^2 and Eq. 52 for C_m into Eq. 54, one obtains

$$L_1 = \frac{M}{N^2} \quad (55)$$

where $M = 2\pi a t \ell \rho$ is the total mass of the ring.

Figure 4. Simple equivalent circuit

From C_m and M and the turns ratio N , an equivalent electrical circuit is illustrated in Figure. 5, where the elements on the right side of the idealized transformer are the components of the mechanical impedance Z_m for the unloaded ring, where liberty has been taken to insert a damping factor R_m .

Figure 5. Equivalent circuit of ceramic ring

The equivalent circuit is an electrical alternative representation of transducer that can be combined with power amplifier circuits, and other electrical circuits. It is sometimes more convenient to fit the impedance plots to lumped circuit models in order to predict the electrical behaviour of the transducer.

2.3.2 Thin Tube Vibrator

This dissertation focuses on the study of a thin-wall tube transducer (see Figure. 6). The main difference of a tube transducer from a ring transducer is the axial length ℓ of the tube is not small relative to its diameter. There is no simple analytical solution for the admittance function of the tube transducer because multiple complex vibration modes will contribute to the admittance function. In practice, finite element models are often built to obtain the vibration modes, as well as the admittance (or impedance) function.

Finite-element Results for Tube Transducers

Figure. 7 displays three vibration modes of a tube transducer that are computed from an axisymmetrical FE model. The figure on the right shows the 3D mode shapes, and the corresponding cross section is shown on the left. Figure. 8 is the corresponding impedance function obtained from the FE model. There are

Figure 6. Thin-wall tube transducer

multiple local maxima and minima in the impedance function, indicating that multiple vibration modes contribute to the impedance function. In theory, the frequencies associated with the local minima and maxima should agree well with the modal frequencies that correspond to the short-circuit model and open-circuit model, respectively (Piefort, 2001).

Mode-driven Equivalent Circuit

For a SDOF ring vibrator, one rewrites the admittance function Eq. 45 as:

$$Y(\omega) = Y_0(\omega) + Y_1(\omega) \quad (56)$$

From Eq. 46 and Eq. 49, one has

$$Y_0(\omega) = j \omega C_0 \quad (57)$$

and

$$Y_1(\omega) = j \omega C_1 \left(\frac{\omega_1^2}{\omega_1^2 - \omega^2} \right) \quad (58)$$

Since $\omega_1^2 = 1/(L_1 C_1)$, Eq. 58 can also be written as

$$Y_1(\omega) = j \omega C_1 \left(\frac{1}{1 - C_1 L_1 \omega^2} \right) \quad (59)$$

Figure 7. Three finite element vibration modes of a tube transducer

Figure 8. The impedance function of a tube transducer

The above admittance function $Y_1(\omega)$ can also be obtained from the reciprocal of the impedance function $Z_1(\omega)$:

$$Z_1(\omega) = j \omega L_1 + \frac{1}{j \omega C_1} \quad (60)$$

For an N-degree-of-freedom (NDOF) vibrator, applying the principle of mode superposition — voltage is the input and current is the output, one can write the corresponding admittance function to be

$$Y(\omega) = Y_0(\omega) + \sum_{n=1}^N Y_n(\omega) \quad (61)$$

where

$$Y_n(\omega) = \frac{1}{Z_n(\omega)} \quad (62)$$

and

$$Z_n(\omega) = j \omega L_n + \frac{1}{j \omega C_n} = j \left(\omega L_n - \frac{1}{\omega C_n} \right) \quad (63)$$

in which C_n and L_n are the n th motional capacitance and n th motional inductance, respectively. They are related to the n th modal (resonant) frequency ω_n of a transducer through $\omega_n^2 = 1/(L_n C_n)$.

Eq. 61 to Eq. 63 implies that the transducer can be represented by an equivalent circuit, which includes N parallel modal branches, as shown in Figure. 9. This equivalent circuit is often called a mode-driven equivalent circuit (Ballato, 1990). Each branch represents one mechanical mode, and C_0 is the clamped capacitance, which is equal to the capacitance of the transducer without the piezoelectric effect.

Figure 9. Modes driven equivalent circuit

CHAPTER 3

Finite Element Model of Piezoelectric Structure

The mathematical model of a physical system often results in partial differential equations that cannot be solved analytically because of the complexity of the boundary conditions or domain. To study a complex model, the FE method is often employed. The FE method subdivides a complex system into its individual components or elements whose behavior is readily understood, and it then rebuilds the original system by assembling such components. Nowadays, the FE method has become one of the most important analysis tools in engineering.

Since the late 1960s, the work by Eer Nisse (1967) and Tiersten (1967) have established variational principles for piezoelectric media. Allik & Hughes (1970), who proposed a tetrahedral element that accounted for piezoelectricity, contributed greatly to the FE modelling of structures with embedded piezoelectricity. In 1986, piezoelectric elements were for the first time included in the commercial FE program ANSYS. Later, piezoelectric finite elements have also been included in other commercially available FE packages such as ABAQUS and COMSOL (Kocbach, 2000; Manual, 2005; Multiphysics and Module, 2014). These commercial software packages provide a wide range of simulation options that control the complexity of both the modelling and analysis of a system. It allows users to easily build an FE model with piezoelectric elements, and simplifies the analysis procedure. In the present study, we adopted primarily ANSYS to perform FE modelling using piezoelectric media.

Many studies have focused on the FE analysis for various piezoelectric structures (Hwang and Park, 1993; Kim and Lee, 2007; Lewis et al., 2009). In this study, we focus only on piezoelectric tube transducers that were polarized in the

radial direction. The specific analyses included in this study are modal and transient (dynamic response) analysis. We employed modal analysis to obtain the modal frequencies and mode shapes of the piezoelectric tube transducer under both open- and short-circuit electric boundary conditions. We used the transient analysis to obtain the electric charge time signal while the piezoelectric tube transducer is subjected to an input electric-potential time signal. The ultimate purpose of the transient analysis is to obtain the impedance function of the transducer.

3.1 Finite Element Formulation

3.1.1 Piezoelectric Element

For piezoelectric elements, both displacements and electric potentials exist at the nodal locations. The displacement field $\hat{\mathbf{u}}$ and electrical potentials $\hat{\phi}$ over an element are approximated within the element as

$$\hat{\mathbf{u}} = \mathcal{N}_u \mathbf{u} \quad (64)$$

$$\hat{\phi} = \mathcal{N}_\phi \phi \quad (65)$$

where $\mathcal{N}_u$ and $\mathcal{N}_\phi$ are the array of interpolating functions, and $\mathbf{u}$ and ϕ are nodal quantities. The body forces and charges as well as the surface forces and charges are interpolated in a similar manner (Kohnke, 1999).

The displacement field $\hat{\mathbf{u}}$ and the electric potential $\hat{\phi}$ are related to the strain field $\mathbf{S}$ and the electric field $\mathbf{E}$ by

$$\mathbf{S} = \mathcal{D} \hat{\mathbf{u}} \quad (66)$$

and

$$\mathbf{E} = -\nabla \hat{\phi} \quad (67)$$

where ∇ is the gradient operator and $\mathcal{D}$ is the derivation operator defined as:

$$\mathcal{D} = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ 0 & 0 & \partial_z \\ 0 & \partial_z & \partial_y \\ \partial_z & 0 & \partial_x \\ \partial_y & \partial_x & 0 \end{bmatrix} \quad (68)$$

Therefore, the strain field $\mathbf{S}$ and the electric field $\mathbf{E}$ are related to the nodal displacements and potential by the shape-functions derivatives $\mathcal{B}_u$ and $\mathcal{B}_\phi$, which are defined by

$$\mathbf{S} = \mathcal{D}\mathcal{N}_u \mathbf{u} = \mathcal{B}_u \mathbf{u} \quad (69)$$

$$\mathbf{E} = -\nabla \mathcal{N}_\phi \phi = -\mathcal{B}_\phi \phi \quad (70)$$

The dynamic equations of a piezoelectric continuum can be derived from the Hamilton principle, in which the Lagrangian and the virtual work are properly adapted to include the electrical contributions as well as the mechanical ones (Piefort, 2001; Allik and Hughes, 1970; Lerch, 1990; Tzou and Tseng, 1990). Without further elaboration, for a piezoelectric element, the resulting mechanical equilibrium equation and the electric-flux conservation equation can be written in the form

$$\mathbf{M}_{uu} \ddot{\mathbf{u}} + \mathbf{K}_{uu} \mathbf{u} + \mathbf{K}_{u\phi} \phi = \mathbf{f} \quad (71)$$

and

$$\mathbf{K}_{\phi u} \mathbf{u} + \mathbf{K}_{\phi\phi} \phi = \mathbf{g} \quad (72)$$

respectively, where

$$\mathbf{M}_{uu} = \int_V \rho \mathcal{N}_u^t \mathcal{N}_u dV \quad (73)$$

is the element mass matrix (no inertia terms exist for the electrical flux conservation equation), ρ is the mass density,

$$\mathbf{K}_{uu} = \int_V \mathcal{B}_u^t \mathbf{c}^E \mathcal{B}_u dV \quad (74)$$

is the element displacement stiffness matrix,

$$\mathbf{K}_{u\phi} = \mathbf{K}_{\phi u}^t = \int_V \mathcal{B}_u^t \mathbf{e}^t \mathcal{B}_\phi dV \quad (75)$$

is the element piezoelectric coupling matrix,

$$\mathbf{K}_{\phi\phi} = - \int_V \mathcal{B}_\phi^t \boldsymbol{\varepsilon}^S \mathcal{B}_\phi dV \quad (76)$$

is the element dielectric “stiffness” matrix, $\mathbf{f}$ is the element mechanical force vector, and $\mathbf{g}$ is the element electric charge vector. In addition, V denotes the volume of the element (Allik and Hughes, 1970).

3.1.2 Assembled System Equation

In the present study, the total volume V_p is divided into P volumes $V_p^{(j)}$, $j = 1, \dots, P$. Any variable A defined in element j will be written as $A^{(j)}$ as follows. The relation between the local node-displacement vector $\mathbf{u}^{(j)}$ and the global node-displacement vector $\mathbf{U}$ is given through a transformation matrix $\mathbf{L}_u^{(j)}$, such that

$$\mathbf{u}^{(j)} = \mathbf{L}_u^{(j)} \mathbf{U} \quad (77)$$

and the relation between the local potential vector $\boldsymbol{\phi}^{(j)}$ and the global potential vector $\boldsymbol{\Phi}$ is given through a transformation matrix $\mathbf{L}_\phi^{(j)}$, such that

$$\boldsymbol{\phi}^{(j)} = \mathbf{L}_\phi^{(j)} \boldsymbol{\Phi} \quad (78)$$

The system equations for the whole structure are obtained by the summation of the contribution from each FE, which can be written in terms of nodal quantities:

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}} \\ \ddot{\boldsymbol{\Phi}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{UU} & \mathbf{K}_{U\Phi} \\ \mathbf{K}_{\Phi U} & \mathbf{K}_{\Phi\Phi} \end{bmatrix} \begin{Bmatrix} \mathbf{U} \\ \boldsymbol{\Phi} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{G} \end{Bmatrix} \quad (79)$$

where the assembled matrices are given by:

$$\mathbf{M} = \sum_{j=1}^P \mathbf{L}_u^{(j)t} \mathbf{M}_{uu}^{(j)} \mathbf{L}_u^{(j)} \quad (80)$$

$$\mathbf{K}_{UU} = \sum_{j=1}^P \mathbf{L}_u^{(j)t} \mathbf{K}_{uu}^{(j)} \mathbf{L}_u^{(j)} \quad (81)$$

$$\mathbf{K}_{U\Phi} = \sum_{j=1}^P \mathbf{L}_u^{(j)t} \mathbf{K}_{u\phi}^{(j)} \mathbf{L}_\phi^{(j)} \quad (82)$$

$$\mathbf{K}_{\Phi U} = \sum_{j=1}^P \mathbf{L}_\phi^{(j)t} \mathbf{K}_{\phi u}^{(j)} \mathbf{L}_u^{(j)} \quad (83)$$

$$\mathbf{K}_{\Phi\Phi} = \sum_{j=1}^P \mathbf{L}_\phi^{(j)t} \mathbf{K}_{\phi\phi}^{(j)} \mathbf{L}_\phi^{(j)} \quad (84)$$

$$\mathbf{F} = \sum_{j=1}^P \mathbf{L}_u^{(j)t} \mathbf{f}^{(j)} \quad (85)$$

and

$$\mathbf{G} = \sum_{j=1}^P \mathbf{L}_\phi^{(j)t} \mathbf{g}^{(j)} \quad (86)$$

Eq. 79 couples the mechanical variables $\mathbf{U}$ and the electrical potentials Φ ; $\mathbf{F}$ represents the external forces applied to the structure, and $\mathbf{G}$ represents the electric charges brought to the electrodes (Piefort, 2001).

3.1.3 Open- and Short-circuited Electrical Boundary Conditions

If the electric potential Φ is controlled, the governing equations become

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}_{UU}\mathbf{U} = \mathbf{F} - \mathbf{K}_{U\Phi}\Phi \quad (87)$$

where the second term in the right hand side represents the equivalent piezoelectric loads. From Eq. 87, one sees that the eigenvalues problem of the system with short-circuited electrodes, namely $\Phi = \mathbf{0}$, is

$$(\mathbf{K}_{UU} - \omega^2 \mathbf{M})\mathbf{U} = \mathbf{0} \quad (88)$$

It can be seen from Eq. 87 that the modal frequencies, ω , and mode shapes of the short-circuit model are the same as if there was no piezoelectric electromechanical coupling.

Conversely, open electrodes correspond to a charge condition $\mathbf{G} = \mathbf{0}$. In this case, from Eq.79, one writes

$$\mathbf{\Phi} = -(\mathbf{K}_{\Phi\Phi}^{-1}\mathbf{K}_{\Phi U})\mathbf{U} \quad (89)$$

Upon substituting Eq. 89 into Eq. 87, it yields

$$\mathbf{M}\ddot{\mathbf{U}} + (\mathbf{K}_{UU} - \mathbf{K}_{U\Phi}\mathbf{K}_{\Phi\Phi}^{-1}\mathbf{K}_{\Phi U})\mathbf{U} = \mathbf{F} \quad (90)$$

which shows that the global stiffness matrix depends on the electrical boundary conditions. From Eq. 90, one sees that the eigenvalues problem of the system with open-circuited electrodes is

$$(\tilde{\mathbf{K}}_{UU} - \omega_a^2\mathbf{M})\mathbf{U} = \mathbf{0} \quad (91)$$

where

$$\tilde{\mathbf{K}}_{UU} = \mathbf{K}_{UU} - \mathbf{K}_{U\Phi}\mathbf{K}_{\Phi\Phi}^{-1}\mathbf{K}_{\Phi U} \quad (92)$$

and ω_a denotes the modal frequencies of the open-circuit model. Note that $\tilde{\mathbf{K}}_{UU}$ is the condensed electro-elastic stiffness matrix which amounts to performing a static condensation to eliminate the degrees of freedom associated with $\mathbf{\Phi}$. In physics, the piezoelectric electromechanical coupling increases the overall stiffness of the system if the electrodes are left open as additional electric potential energy has been stored. For a tube transducer, the corresponding modal frequencies of an open-circuited electrodes, ω_a , are the anti-resonant frequencies of the transducer (Piefort, 2001).

3.2 Cylindrical Transducer Modelling and Analysis

The commercial FE packages such as ANSYS have the capability to perform a fully coupled piezoelectric analysis. In this study, both 2D axisymmetric plane piezoelectric elements and 3D solid piezoelectric elements are suitable for modeling the axisymmetric piezoelectric tube.

3.2.1 Modelling and Analysis of 2D Element Model

A specific 2D coupled-field plane element of ANSYS is the PLANE13 element which will be utilized in this study for modeling axisymmetric transducers. Shown in Figure. 10 is the PLANE13 element which is defined by four nodes, with three degrees of freedom per node for piezoelectric analysis, including x -direction displacement, y -direction displacement and voltage (Kohnke, 1999). Please refer to Appendix B for more detailed ANSYS information about element settings, properties assignment, piezoelectric analysis and data extraction.

Figure 10. Sketch of PLANE13 element

Throughout this dissertation, the specific numerical model to be studied is a simple tube transducer with the inner diameter $D_i = 16 \times 10^{-3}$ m, outer diameter $D_o = 19 \times 10^{-3}$ m and length $\ell = 20 \times 10^{-3}$ m. While using axial symmetric plane elements, one can conveniently build this tube transducer model by revolving its cross section (a rectangle in the x - y plane) around the z -axis (see Figure. 11). The poling direction is radially outwards from the axis of symmetry, namely z -

axis. Since the order of the stresses in ANSYS differs from those typically used in electrical applications (IEEE standard), care must be taken while inputting the corresponding coefficients matrices for the tube transducer model.

Figure 11. Finite element model of a tube transducer: (a) 3D model, and (b) cross section

The material properties could be specified in either e-form or d-form. The 10 parameters in the e-form are c_{11}^E , c_{33}^E , c_{12}^E , c_{13}^E , c_{44}^E , ε_{33}^S , ε_{11}^S , e_{31} , e_{33} and e_{15} , and the 10 material properties given in the d-form are s_{11}^E , s_{33}^E , s_{12}^E , s_{13}^E , s_{44}^E , ε_{33}^T , ε_{11}^T , d_{31} , d_{33} and d_{15} . ANSYS allows the user to choose either e-form or d-form to build piezoelectric FE models. In the present numerical example, the material properties of APC850 (Navy II) has been chosen, and its d-form parameters are listed in Table 2, where $\varepsilon_0 = 8.854187817 \times 10^{-12} F/m$.

Modal properties, including modal frequencies and mode shapes, are important dynamic characteristics of a transducer. For a piezoelectric tube transducer, we are interested in knowing its modal properties for both short-circuit and open-circuit models. For the open-circuit FE modelling, the potentials on the inner surface are limited to zero, but are unspecified on the outer surface. For the

Table 2. Properties of PZT5A material

Material constant	Symbol	Value
Elastic compliance	s_{13}^E	$-5.74 \times 10^{-12} \text{ m}^2/\text{N}$
	s_{33}^E	$18.5 \times 10^{-12} \text{ m}^2/\text{N}$
	s_{44}^E	$46 \times 10^{-12} \text{ m}^2/\text{N}$
	s_{11}^E	$15.8 \times 10^{-12} \text{ m}^2/\text{N}$
	s_{12}^E	$-4.9 \times 10^{-12} \text{ m}^2/\text{N}$
Piezoelectric strain constant	d_{31}	$-175 \times 10^{-12} \text{ C/N}$
	d_{33}	$400 \times 10^{-12} \text{ C/N}$
	d_{15}	$590 \times 10^{-12} \text{ C/N}$
Permittivity (under constant stress)	$\varepsilon_{11}^T/\varepsilon_0$	1851
	$\varepsilon_{33}^T/\varepsilon_0$	1594

short-circuit model, the potentials on both the inner and outer surfaces are set to zero. Let $z = 0$ be the location at the middle of the tube in the axial direction. The mechanical boundary conditions are considered to be free at both ends, i.e., at $z = -L/2$ and $z = L/2$. First, we performed the modal analysis for the short-circuit model. The obtained mode shapes are classified into two particular groups: symmetric and anti-symmetric to $z = 0$. The symmetric mode shapes and their corresponding modal frequencies are shown in Figure. 12(a), and the antisymmetric mode shapes and their corresponding modal frequencies are shown in Figure. 12(b). In addition, we performed the modal analysis for the open-circuit model. The resulting mode shapes are also symmetric and anti-symmetric to $z = 0$. Figure. 13(a) and Figure. 13(b) show the symmetric and antisymmetric modes, respectively.

ANSYS has a built-in capability to compute the impedance function of a transducer by using the harmonic analysis. In the harmonic analysis, we applied a unit amplitude voltage having a frequency sweeping from 0 to 120 KHz with an increment of 200 Hz in the radial direction, and Figure. 14 shows the obtained impedance function. From the impedance function plot, the frequencies corre-

(a)

(b)

Figure 12. The mode shapes and modal frequencies of the short-circuit model: (a) symmetric modes, and (b) antisymmetric modes.

Figure 13. The mode shapes and modal frequencies of the open-circuit model: (a) symmetric modes, and (b) antisymmetric modes.

Figure 14. The impedance function of the tube transducer by using the harmonic analysis

sponding to the local minima and local maxima are commonly referred to as the resonance frequencies and anti-resonance frequencies, respectively, of multi-mode transducers. In theory, the resonance and anti-resonance frequencies should agree well with the modal frequencies obtained from the short- and open-circuit models, respectively. Only the modal frequencies corresponding to the symmetric modes of the short- and open-circuit models (see Figure. 12 and 13) are plotted as the vertical lines in Figure. 14. Clearly, they are in line with the resonance and anti-resonance frequencies of the impedance function.

Because the axially uniform voltage input is symmetric to $z = 0$, only symmetric modes can be excited. Consequently, only symmetric modes “participate” in the mechanical vibration and the associated electrical impedance function. Furthermore, because of the axially uniform nature of the input, the mode that has the most uniform mode shape would dominate the mechanical vibration. In fact, the most uniform mode shape for the short-circuit model has a modal frequency of 79.35 KHz (see Figure. 12), which is the global minimum of the impedance function, and the most uniform mode shape for the open-circuit model has the modal frequency 95.09 KHz (see Figure. 13), which is the global maximum of the impedance function.

3.2.2 Modelling and Analysis of 3D Element Model

In this study, the specific 3D element type employed to build up the piezoelectric cylinder is SOLID226 of ANSYS, as shown in Figure. 15. This type of element contains 20 nodes with four degrees-of-freedom (DoFs) per node, including the x -direction displacement, y -direction displacement, z -direction displacement, and voltage (Kohnke, 1999). Please refer to Appendix B for more detailed ANSYS information about element settings, properties assignment, piezoelectric analyses, and data extraction.

Figure 15. Sketch of SOLID226 element

Figure 16. 3D tube model built by SOLID226 element

The 3D tube model that was built using SOLID226 element is shown in Figure.16. Because building the 3D model follows the same geometry, boundary conditions, and material properties, as utilized for the 2D model, similar modal frequencies are expected from the 2D and 3D models. A comparison of the first three

modal frequencies between the 2D and 3D models for both short- and open-circuit models is presented in Table 3, which shows an excellent agreement between these two models. In Table 3, the relative error was defined as the 2D result minus the 3D result, and it was then normalized by the 3D result. Furthermore, Figure. 17 shows the excellent agreement of impedance functions obtained from the 2D and 3D models.

Table 3. The comparison of modal frequencies between 2D and 3D models

Circuit	Mode number	Modal frequency (Hz.)		Relative error
		2D model	3D model	
Short	1	50290.60	50293.68	-0.006%
	2	53409.10	53470.08	-0.114%
	3	79350.58	79355.57	-0.006%
Open	1	51149.01	51147.52	0.003%
	2	53992.38	54054.97	-0.116%
	3	95087.15	95134.09	-0.049%

3.2.3 Constitutive Equations of 2D and 3D Models

The distinction of the constitutive equations for 2D (axisymmetrical) and 3D models is very important, as it is often not well understood and many errors can arise from the confusion between 2D and 3D properties of piezoelectric materials. Note however that the d_{ij} , s_{ij}^E and ε_{ii}^T coefficients are equal for 2D and 3D constitutive equations. It is therefore preferable to handle the material properties of piezoelectric materials in the d-form.

In calculations for axisymmetric structures, it is preferable to use a cylindrical coordinate system where the symmetry of the problem can be exploited. Let the three-dimensional cylindrical coordinate system be denoted by (R, θ, Z) , where R is in the radial direction, θ the circumferential direction, and Z axial direction.

Figure 17. Comparison of impedance functions obtained from 2D and 3D models

The strain vector for the cylindrical coordinate system is given by:

$$\mathbf{S} = \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} = \begin{Bmatrix} S_R \\ S_\theta \\ S_Z \\ 2S_{\theta Z} \\ 2S_{RZ} \\ 2S_{R\theta} \end{Bmatrix} = \mathbf{L}_u \mathbf{u} \quad (93)$$

where

$$\mathbf{L}_u = \begin{bmatrix} \frac{\partial}{\partial R} & 0 & 0 \\ \frac{1}{R} & \frac{1}{R} \frac{\partial}{\partial \theta} & 0 \\ 0 & 0 & \frac{\partial}{\partial Z} \\ 0 & \frac{\partial}{\partial Z} & \frac{1}{R} \frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial Z} & 0 & \frac{\partial}{\partial R} \\ \frac{1}{R} \frac{\partial}{\partial \theta} & \frac{\partial}{\partial R} - \frac{1}{R} & 0 \end{bmatrix} \quad (94)$$

and

$$\mathbf{u} = \begin{Bmatrix} u_R \\ u_\theta \\ u_Z \end{Bmatrix} \quad (95)$$

The electric field vector for a cylindrical coordinate system is defined by

$$\mathbf{E} = \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \begin{Bmatrix} E_R \\ E_\theta \\ E_Z \end{Bmatrix} = - \begin{Bmatrix} \frac{\partial}{\partial R} \\ \frac{1}{R} \frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial Z} \end{Bmatrix} \phi = -\mathbf{L}_\phi \phi \quad (96)$$

where $\mathbf{L}_\phi$ is defined by the last transition.

When only axially symmetric vibrations are considered, this axisymmetrical constraint suggests

$$\frac{\partial}{\partial \theta} = 0 \quad (97)$$

Additionally, let axially symmetric torsional (circumferential) vibrations be excluded, i.e.,

$$u_\theta = 0 \quad (98)$$

Thus, only two coordinates are needed to describe the problem in the axisymmetric case: R and Z . This class of problems has been often referred to as 2D axisymmetric problems.

Imposing Eqs. 97 and 98 on Eqs. 93 and 96 yields

$$S_4 = S_{\theta Z} = 0 \quad (99)$$

$$S_6 = S_{R\theta} = 0 \quad (100)$$

$$E_2 = E_\theta = 0 \quad (101)$$

Only four of the six strain components and two of the three components of the electric field need to be included in the analysis. This results in the following definition for the strain vector and the electric field vector in the two-dimensional axisymmetric case:

$$\mathbf{S} = \begin{Bmatrix} S_R \\ S_\theta \\ S_Z \\ 2S_{RZ} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial R} & 0 \\ \frac{1}{R} & 0 \\ 0 & \frac{\partial}{\partial Z} \\ \frac{\partial}{\partial Z} & \frac{\partial}{\partial R} \end{bmatrix} \begin{Bmatrix} u_R \\ u_Z \end{Bmatrix} = \mathbf{L}_u \mathbf{u} \quad (102)$$

and

$$\mathbf{E} = \begin{Bmatrix} E_R \\ E_Z \end{Bmatrix} = - \begin{Bmatrix} \frac{\partial}{\partial R} \\ \frac{\partial}{\partial Z} \end{Bmatrix} \phi = -\mathbf{L}_\phi \phi \quad (103)$$

The derivation operator matrices $\mathbf{L}_u$ and $\mathbf{L}_\phi$ therefore have the following definitions in this case

$$\mathbf{L}_u = \begin{bmatrix} \frac{\partial}{\partial R} & 0 \\ \frac{1}{R} & 0 \\ 0 & \frac{\partial}{\partial Z} \\ \frac{\partial}{\partial Z} & \frac{\partial}{\partial R} \end{bmatrix} \quad (104)$$

and

$$\mathbf{L}_\phi = \begin{Bmatrix} \frac{\partial}{\partial R} \\ \frac{\partial}{\partial Z} \end{Bmatrix} \quad (105)$$

In terms of the d-form, the reduced constitutive equations for the axisymmetric 2D element become

$$\begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_5 \\ D_1 \\ D_3 \end{Bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & d_{31} \\ s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & d_{31} \\ s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & d_{33} \\ 0 & 0 & 0 & s_{44}^E & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & \varepsilon_{11}^T & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & \varepsilon_{33}^T \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_5 \\ E_1 \\ E_3 \end{Bmatrix} \quad (106)$$

To derive the corresponding e-form, one can first rearrange Eq. 106 into:

$$\begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\ s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 \\ s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 \\ 0 & 0 & 0 & -d_{15} & 1 & 0 \\ -d_{31} & -d_{31} & -d_{33} & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_5 \\ D_1 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -d_{31} \\ 0 & 1 & 0 & 0 & 0 & -d_{31} \\ 0 & 0 & 1 & 0 & 0 & -d_{33} \\ 0 & 0 & 0 & 1 & -d_{15} & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_{11}^T & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon_{33}^T \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_5 \\ E_1 \\ E_3 \end{Bmatrix} \quad (107)$$

Let the e-form of the reduced constitutive equations be denoted by

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_5 \\ D_1 \\ D_3 \end{Bmatrix} = \bar{\mathbf{A}} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_5 \\ E_1 \\ E_3 \end{Bmatrix} \quad (108)$$

From the above two equations, one has

$$\bar{\mathbf{A}} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\ s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 \\ s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 \\ 0 & 0 & 0 & -d_{15} & 1 & 0 \\ -d_{31} & -d_{31} & -d_{33} & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -d_{31} \\ 0 & 1 & 0 & 0 & 0 & -d_{31} \\ 0 & 0 & 1 & 0 & 0 & -d_{33} \\ 0 & 0 & 0 & 1 & -d_{15} & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_{11}^T & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon_{33}^T \end{bmatrix} \quad (109)$$

The resulting stiffness matrix $\bar{\mathbf{c}}^E$ is a 4×4 matrix:

$$\bar{\mathbf{c}}^E = \begin{bmatrix} \bar{c}_{11}^E & \bar{c}_{12}^E & \bar{c}_{13}^E & 0 \\ \bar{c}_{12}^E & \bar{c}_{11}^E & \bar{c}_{13}^E & 0 \\ \bar{c}_{13}^E & \bar{c}_{13}^E & \bar{c}_{33}^E & 0 \\ 0 & 0 & 0 & \bar{c}_{44}^E \end{bmatrix} \quad (110)$$

Note that the stiffness coefficient $\bar{c}_{ij}$ of the 2D formulation differs from the stiffness coefficient c_{ij} of the 3D constitutive equations.

3.3 Numerical Verifications

While using finite element models to conduct research, the numerical verification of them is always essential. In the following presentation, the verification process includes: (1) the comparison with an approximate analytical solution, (2) the comparison among two commercial finite element packages, and (3) the comparison with physical measurements.

3.3.1 Comparison with Analytical Solution

One can compare the impedance function of the FE model with the analytical impedance function based on a ring transducer which has been presented in Chapter 2. While using the analytical formula, Eq. 40, the following parameters are utilized: radius $a = \frac{D_o + D_i}{4} = 8.75 \times 10^{-3}$ m, length $\ell = 20 \times 10^{-3}$ m, permittivity $\varepsilon_{33}^T = 1.594 \times 10^{-8}$ (F/m), density $\rho = 7600$ (kg/m³), piezoelectric coefficient $d_{31} = -175 \times 10^{-12}$ (C/N) and compliance $s_{11}^E = 15.8 \times 10^{-12}$ (m²/N).

Shown in Figure. 18 is the analytical impedance function, comparing to the impedance function from the finite element model. The analytical impedance function, which has been derived from a mathematical model of a single-degree-of-freedom vibrator, contains only a “breathing” vibration mode. In contrast, the impedance function from the finite element model has been contributed by multiple vibration modes. Due to the imposed simplifications for the derivation of the analytical impedance function, the significant discrepancy between these two impedance functions is not surprising.

For better matching the impedance function computed from a finite element model to the analytical impedance function, a “constrained” finite element model has been built. This constrained model does not allow movements in the axial direction for all elements. The resulting impedance function of the constrained model — contributed only by a uniformly breathing mode as well — is shown in Figure. 19. Although there is still discrepancy between the analytical impedance function and the impedance function from the constrained finite element model, the difference has become much smaller.

Figure 18. Comparison of the impedance function of the FE model with the analytical impedance function based on a ring transducer

Figure 19. Comparison of the impedance function of the constrained FE model with the analytical impedance function based on a ring transducer

3.3.2 Verification by ABAQUS Model

Unless state otherwise, all FE models throughout this dissertation will be built by using ANSYS. For independently verifying the correctness of the corresponding ANSYS model, a finite element model has been constructed by using ABAQUS. Examining on several specific sinusoidal input (voltage) signals, excellent agreement between ANSYS and ABAQUS models has been found while comparing the output (charge) time histories. A good agreement for the impedance function is shown in Figure. 20.

Figure 20. The comparison of impedance functions obtained from ANSYS and ABAQUS

3.3.3 Comparison with Experimental Data

An ultimate verification for a finite element model should be done with the lab data. To this end, an experiment has been conducted to measure the impedance function of a real transducer with the same geometrical dimensions as those utilized in the finite element model. Shown in Figure. 21 is the comparison of the impedance

functions obtained from ANSYS and the lab measurement. By and large, they match reasonably well with each other. One obvious reason for the discrepancy between the impedance functions obtained from ANSYS and the lab measurement is that the piezoelectric material properties used in ANSYS model could not be the same as those of the real transducer. In fact, the ultimate goal of this dissertation is to perform FE model updating on the piezoelectric material properties so that the impedance functions obtained from ANSYS and the lab measurement can match better. The algorithm and procedure to carry out the multi-physics FE model updating on the piezoelectric material properties will be addressed in Chapter 6.

Figure 21. The comparison of impedance functions obtained from ANSYS and lab measurement

CHAPTER 4

Methods of Estimating Impedance Function

One of the most important characteristics of transducers is its impedance function, which is usually obtained from processing the measured voltage and current signals. In addition to a review of the traditional methods for estimating the impedance function of a piezoelectric transducer (Lis and Schmidt, 2004; Umchid et al., 2009), in this chapter, we newly propose an improved method. This newly proposed method, which is operated in the Laplace domain, is particularly effective for handling measured voltage and current signals that have been contaminated by noise.

4.1 Traditional Methods

Traditionally, there are two kinds of methods for estimating the impedance function: (i) stepped harmonic analysis (i.e. single frequency) method, and (ii) Fourier-based analysis method using transient signals. The stepped harmonic analysis is to repeatedly apply a harmonic input signal to get the corresponding steady state response, then compute the impedance of the given frequency (Lis and Schmidt, 2004). The Fourier method computes the impedance function from the complex ratio of the FFT of the input and output time domain signals. Both methods have drawbacks. The stepped harmonic analysis method is very time consuming; and the Fourier-based analysis method always suffers from the problems related to the periodic assumption of both input and output signals, such as frequency leakage and resolution.

4.2 Proposed Method

This study proposes a new method to estimate the impedance function via the Laplace domain in order to overcome the drawbacks of the traditional methods. The first step of the proposed method is to decompose both input and output time domain signals into complex exponential components. Because the input voltage signal $v(t_k)$ can be controlled, one can choose to use an exponentially decayed voltage signal:

$$v(t_k) = \gamma_v e^{\lambda_v t_k} \quad (111)$$

where $\lambda_v, \gamma_v = \text{constants}$. For the sake of a clear presentation, let the output signal (from FE model) be a transient charge signal, $c(t_k)$. Decomposed into complex exponential components, $c(t_k)$ can be written into

$$c(t_k) = \sum_{n=1}^p \gamma_n e^{\lambda_n t_k} \quad (112)$$

where p is the number of the decomposition terms, and the complex exponentials λ_n and complex amplitudes γ_n are computed coefficients. Because $c(t_k)$ is a real signal, λ_n and γ_n must either be real numbers or occur in complex conjugate pairs. Let $\lambda_n = -a_n + j\omega_n$, then a_n is the damping factor in seconds⁻¹ and ω_n is the frequency in radians. Since the Laplace property $\mathcal{L}(e^{\lambda_v t}) = 1/(s - \lambda_v)$, the Laplace transform of the input voltage $v(t) = \gamma_v e^{\lambda_v t}$ is shown to be

$$V(s) \equiv \mathcal{L}(v(t)) = \frac{\gamma_v}{s - \lambda_v} \quad (113)$$

Similarly, the Laplace transform of the output charge $c(t)$ is:

$$C(s) \equiv \mathcal{L}(c(t)) = \sum_{n=1}^p \frac{\gamma_n}{s - \lambda_n} \quad (114)$$

One can find the complex amplitudes and exponentials of a signal in the time domain are really the poles and residues in the Laplace domain associated with

the signal. In turn, both input and output signals in the Laplace domain can be readily established in a partial fraction form.

To calculate the impedance function, one needs to get the current signal, $i(t)$, from the derivative of the charge with respect time, that is $i(t) = dc(t)/dt$. By using the Laplace property $\mathcal{L}(df(t)/dt) = sF(s) - f(0)$, the Laplace transform of the current $i(t)$ becomes

$$I(s) = sC(s) = \sum_{n=1}^P \frac{s\gamma_n}{s - \lambda_n} \quad (115)$$

if the initial charge $c(0)$ is equal to 0.

The impedance function in the s domain can be readily obtained from Eqs. 113 and 115:

$$Z(s) = \frac{V(s)}{I(s)} \quad (116)$$

Finally, substituting s by $j\omega$ in the above equation, one obtains the impedance function of ω .

4.3 Complex Exponential Decomposition

To implement the proposed method, an essential task is to decompose the signal into its complex exponential components. An arbitrary signal $y(t)$ can always be decomposed into a finite number of complex exponential components, called Prony's series.

$$y(t) = \sum_{n=1}^p \gamma_n e^{\lambda_n t} \quad 0 \leq t < T \quad (117)$$

where p is the number of terms. For digital signal analysis, the continuous $y(t)$ is not known and usually equally spaced samples are available. Denoting sampling interval Δt , $t_k = k\Delta t$ and $y_k \equiv y(t_k)$, where $k = 0, \dots, m$, one writes the discrete Prony's series as

$$y_k = \sum_{n=1}^p \gamma_n z_n^k \quad (118)$$

where $z_n = e^{\lambda_n \Delta t}$.

In Eq. 117, the exponential functions $e^{\lambda_n t}$, $n = 1, \dots, p$ form a basis on the open interval $0 \leq t < T$. Clearly, the Prony series (Eq. 117) can also be viewed as the general solution of a p th-order homogenous linear ordinary differential equation with constant real-valued coefficients a_n :

$$\sum_{n=0}^p a_n y^{(n)}(t) = 0, \quad \text{for } 0 \leq t < T \quad (119)$$

where $y^{(n)}(t) = d^n y / dt^n$. And the exponents λ_n ($n = 1, \dots, p$) in Eq. 117 are the roots of the characteristic equation associated with Eq. 119:

$$\sum_{n=0}^p a_n \lambda^n = 0 \quad (120)$$

If only the equally-spaced sampled data y_k ($k = 0, 1, \dots, N-1$) are given, then Eq. 119 becomes a p th-order difference equation:

$$\sum_{n=0}^p b_n y_{k+n} = 0, \quad \text{for } k = 0, 1, \dots, N-p-1 \quad (121)$$

where b_n are real-valued constants. Without losing the generality, let $b_p = 1$. From Eqs. 118 and 121, the characteristic equation associated with Eq. 121 can be obtained to be:

$$\sum_{n=0}^p b_n z^n = 0 \quad (122)$$

Note that the linear difference equation and its characteristic equation have the same coefficients b_n .

4.3.1 Prony's method

Traditionally, Prony's method has often been utilized to decompose a signal into complex exponential components. In brief, Prony's method is a technique for fitting an exponential model to a few equally spaced measured data points (Prony, 1795). While the original Prony's method presented a method that exactly

fits as many purely damped exponentials as required to fit the available data points, the modern version of Prony’s method generalizes to damped sinusoidal models. In addition, it utilizes the least-squares analysis to approximately fit an exponential model for cases where there are more data points than needed to fit the assumed number of exponential terms. However, because Prony’s method suffers from an ill-conditioned problem with respect to determining the roots of the high-order polynomial that is encountered, the estimation of Prony’s method may have a significant error (Hu et al., 2013).

4.3.2 Prony-SS method

To avoid dealing with the ill-conditioned problem, in this study, we employed an alternative approach that uses a first-order matrix homogenous difference equation (state-space model) to replace the high-order homogenous difference equation, and the approach is called the Prony-SS method (Hu et al., 2013). Although the Prony-SS method and Prony’s method appear to be theoretically identical, they are drastically different with respect to crucial numerical issues, including conditioning and stability. While Prony’s method is very sensitive to the sampling rate and round-off error, the Prony-SS method is not. While Prony’s method has trouble dealing with noise embedded in the signal, the Prony-SS method can handle noisy signals properly because it has a built-in noise-rejection mechanism that uses truncated singular-value decomposition. While root-finding of a high-order polynomial, which is a classical ill-conditioned problem, is a required step in Prony’s method, the Prony-SS method completely avoids it (Hu et al., 2013).

The Prony-SS method begins with a first-order matrix difference equation, which is a state-space model, to replace a high-order difference equation used in Prony’s method. The three steps of the Prony-SS method are: (1) obtain a realization for the state matrix of the first-order matrix difference equation from

the Hankel matrices composed by the sampled data; (2) compute the eigenvalues of the realization matrix, and then obtain the estimates of the frequency and damping factor of each exponential term, and (3) solve for a set of linear equations to yield the estimates of the exponential amplitude and sinusoidal initial phase of each component (Hu et al., 2013).

By introducing

$$x_1 = y^{(0)}, x_2 = y^{(1)}, \dots, x_p = y^{(p-1)} \quad (123)$$

and assuming $a_p = 1$, Eq. 117 is converted into a first-order matrix differential equation as

$$\dot{\mathbf{x}}(t) = \mathbf{F}\mathbf{x}(t) \quad (124)$$

where $\dot{\mathbf{x}}(t) = d\mathbf{x}(t)/dt$,

$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 & \cdots & -a_{p-2} & -a_{p-1} \end{bmatrix} \in \mathbb{R}^{p \times p} \quad (125)$$

and

$$\mathbf{x}(t) = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{Bmatrix} \in \mathbb{R}^p \quad (126)$$

The solution of Eq. 124 has the form $\mathbf{x}(t) = \mathbf{v}e^{\lambda t}$ where λ is a constant and $\mathbf{v}$ is a time-independent p -element column vector. When $\mathbf{x}(t) = \mathbf{v}e^{\lambda t}$ is substituted into Eq. 124, it yields the algebraic eigenvalue equation:

$$\mathbf{F}\mathbf{v} = \lambda\mathbf{v} \quad (127)$$

Theoretically, the p eigenvalues of the matrix $\mathbf{F}$ are exactly the p roots of the characteristic equation of Eq. 120. Instead of determining the roots of Eq. 120, the Prony-SS method is to compute the eigenvalues of the matrix $\mathbf{F}$.

The elements of vector $\mathbf{x}$ in Eq. 126 are often termed the state variables of the linear dynamic system, and they serve as the “coordinates” of the system. However, there are an infinite number of ways of selecting the coordinates. Denoting an arbitrary nonsingular transformation matrix by $\Phi \in \mathbb{R}^{p \times p}$, and another set of coordinates by $\mathbf{q} \in \mathbb{R}^p$, one can formulate the transformation relationship between $\mathbf{x}$ and $\mathbf{q}$ as:

$$\mathbf{x}(t) = \Phi \mathbf{q}(t) \quad (128)$$

Now, substitute $\mathbf{x}(t) = \Phi \mathbf{q}(t)$ and $\dot{\mathbf{x}}(t) = \Phi \dot{\mathbf{q}}(t)$ into Eq. 124 and premultiply both sides by Φ^{-1} , the resulting equation becomes

$$\dot{\mathbf{q}}(t) = \mathbf{G} \mathbf{q}(t) \quad (129)$$

where $\mathbf{G} = \Phi^{-1} \mathbf{F} \Phi$. Note that Eq. 129 and Eq. 124 share the same form. Because $\mathbf{G}$ is a similarity transformation of $\mathbf{F}$, they must have the same eigenvalues.

Let the eigenvalues and the corresponding eigenvectors of $\mathbf{G}$ be denoted by $\lambda_1, \lambda_2, \dots, \lambda_p$ and $\mathbf{u}_{(1)}, \mathbf{u}_{(2)}, \dots, \mathbf{u}_{(p)}$, respectively. Because $\mathbf{u}_{(1)} e^{\lambda_1 t}, \mathbf{u}_{(2)} e^{\lambda_2 t}, \dots, \mathbf{u}_{(p)} e^{\lambda_p t}$ form a basis for solutions of Eq. 129, the general solution of Eq. 129 can be written as:

$$\mathbf{q}(t) = \sum_{n=1}^p g_n \mathbf{u}_{(n)} e^{\lambda_n t} \quad (130)$$

where g_n are arbitrary constants. In Eq. 130, each element of $\mathbf{q}(t)$ is a linear combination of $e^{\lambda_n t}$, $n = 1, \dots, p$, so is $y(t)$ in Eq. 117. Therefore, $y(t)$ is a linear combination of the elements of $\mathbf{q}(t)$, that is,

$$y(t) = \sum_{n=1}^p c_n q_n(t) \quad (131)$$

or

$$y(t) = \mathbf{c}^t \mathbf{q} \quad (132)$$

where $q_n(t)$ is the n th element of $\mathbf{q}(t)$ and $\mathbf{c} \in \mathbb{R}^p$ is a column vector.

The discrete version of Eq. 129 is a first-order homogeneous matrix difference equation which can be written as:

$$\mathbf{q}_{k+1} = \mathbf{A}\mathbf{q}_k \quad (133)$$

where $\mathbf{q}_k \equiv \mathbf{q}(t_k)$ and $\mathbf{A} = \exp(\mathbf{G}\Delta t)$. From Eq. 132, at $t = t_k$, one has

$$y_k = \mathbf{c}^t \mathbf{q}_k = \mathbf{c}^t \mathbf{A}^k \mathbf{q}_0 \quad (134)$$

Here, the specific goal is using the given discrete signal y_k , $k = 0, \dots, N-1$, to obtain a realization of $\mathbf{A}$. Next, the eigenvalues of $\mathbf{A}$, and those of $\mathbf{F}$ (*i.e.* λ_n , $n = 1, \dots, p$), can be computed without going through the procedure of determining polynomial coefficients and zeros (Hu et al., 2013).

The Hankel matrix $\mathbf{H}(k) \in \mathbb{R}^{\xi \times \eta}$ was defined as

$$\mathbf{H}(k) = \begin{bmatrix} y_k & y_{k+1} & \cdots & y_{k+\eta-1} \\ y_{k+1} & y_{k+2} & \cdots & y_{k+\eta} \\ \vdots & \vdots & \ddots & \vdots \\ y_{k+\xi-1} & y_{k+\xi} & \cdots & y_{k+\xi+\eta-2} \end{bmatrix} \quad (135)$$

Upon substituting Eq. 134 into Eq. 135, it can be shown that $\mathbf{H}(k)$ is the product of three matrices:

$$\mathbf{H}(k) = \mathcal{P}_\xi \mathbf{A}^k \mathcal{Q}_\eta \quad (136)$$

where

$$\mathcal{P}_\xi = \begin{bmatrix} \mathbf{c}^t \\ \mathbf{c}^t \mathbf{A} \\ \mathbf{c}^t \mathbf{A}^2 \\ \vdots \\ \mathbf{c}^t \mathbf{A}^{\xi-1} \end{bmatrix} \in \mathbb{R}^{\xi \times p} \quad (137)$$

and

$$\mathcal{Q}_\eta = [\mathbf{q}_0 \quad \mathbf{A}\mathbf{q}_0 \quad \mathbf{A}^2\mathbf{q}_0 \quad \cdots \quad \mathbf{A}^{\eta-1}\mathbf{q}_0] \in \mathbb{R}^{p \times \eta} \quad (138)$$

A realization of $\mathbf{A}$ can be obtained from $\mathbf{H}(0)$ and $\mathbf{H}(1)$. Substituting $k = 0$ and $k = 1$ into Eq. 136 yields

$$\mathbf{H}(0) = \mathcal{P}_\xi \mathcal{Q}_\eta \quad (139)$$

and

$$\mathbf{H}(1) = \mathcal{P}_\xi \mathbf{A} \mathcal{Q}_\eta \quad (140)$$

Applying the singular value decomposition of $\mathbf{H}(0)$, one writes (Zhou and Doyle, 1996):

$$\mathbf{H}(0) = [\mathbf{U}_1 \quad \mathbf{U}_2] \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix} = \mathbf{U}_1 \boldsymbol{\Sigma}_1 \mathbf{V}_1^T \quad (141)$$

In theory, the model order of the dynamic system, p , is equal to the number of non-zero singular values in Eq. 141, that is, $\mathbf{U}_1 \in \mathbb{R}^{\xi \times p}$, $\boldsymbol{\Sigma}_1 \in \mathbb{R}^{p \times p}$ and $\mathbf{V}_1 \in \mathbb{R}^{p \times \eta}$, and the rank of $\mathbf{H}(0)$ is p . The singular values should go to zero when the rank of the matrix is exceeded, but for measured data due to random errors and small inconsistencies in the data, the singular values will not become zero but will become very small. While choosing the size of $\mathbf{H}(0)$, both ξ and η must be greater than p .

A direct comparison between Eqs. 139 and 141 suggests that $\mathcal{P}_\xi$ is related to $\mathbf{U}_1$, and $\mathcal{Q}_\eta$ to $\mathbf{V}_1^T$. A possible “balanced” choice is $\mathcal{P}_\xi = \mathbf{U}_1 \boldsymbol{\Sigma}_1^{1/2}$ and $\mathcal{Q}_\eta = \boldsymbol{\Sigma}_1^{1/2} \mathbf{V}_1^T$. Substituting $\mathcal{P}_\xi = \mathbf{U}_1 \boldsymbol{\Sigma}_1^{1/2}$ and $\mathcal{Q}_\eta = \boldsymbol{\Sigma}_1^{1/2} \mathbf{V}_1^T$ into Eq. 140, one obtains:

$$\mathbf{H}(1) = \mathbf{U}_1 \boldsymbol{\Sigma}_1^{1/2} \mathbf{A} \boldsymbol{\Sigma}_1^{1/2} \mathbf{V}_1^T \quad (142)$$

Premultiplying $\boldsymbol{\Sigma}_1^{-1/2} \mathbf{U}_1^T$ and postmultiplying $\mathbf{V}_1 \boldsymbol{\Sigma}_1^{-1/2}$ at Eq. 142 yields a realization of $\mathbf{A}$ as:

$$\mathbf{A} = \boldsymbol{\Sigma}_1^{-1/2} \mathbf{U}_1^T \mathbf{H}(1) \mathbf{V}_1 \boldsymbol{\Sigma}_1^{-1/2} \quad (143)$$

While the computed eigenvalues of $\mathbf{A}$ are z_n , $n = 1, \dots, p$, the corresponding λ_n in Eq. 117 are $\lambda_n = \ln(z_n)/\Delta t$. Finally, the complex coefficients γ_n can be computed

by solving a second set of linear equations (Hu et al., 2013):

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_p^0 \\ z_1^1 & z_2^1 & \cdots & z_p^1 \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{K-1} & z_2^{K-1} & \cdots & z_p^{K-1} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_p \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{K-1} \end{bmatrix} \quad (144)$$

Multi-signal Prony-SS method

While the above Prony-SS method has been derived to deal with a single signal decomposition, it can also be extended to perform the multiple signals (Hu et al., 2016). We denote the column vector $\mathbf{y}_k \in \mathbb{R}^{N \times 1}$ for the signal data, where N is the number of signals. The multiple-signal vector can be written as:

$$\mathbf{y}_k = \sum_{n=1}^p \mathbf{\Gamma}_n z_n^k \quad (145)$$

where

$$\mathbf{\Gamma}_n = \begin{bmatrix} \gamma_{n,1} \\ \gamma_{n,2} \\ \vdots \\ \gamma_{n,N} \end{bmatrix} \in \mathbb{R}^N \quad (146)$$

The extension from the single-signal Prony-SS method to multiple signals is quite straightforward. The first two steps of the proposed method are the same as the single-signal Prony-SS's step described above. We first obtained a realization for the state matrix of a first-order matrix difference equation, and we then compute the eigenvalues of the realization matrix to obtain the estimates of the frequency and damping factor of each exponential term. The distinction between the single-signal and multiple-signal Prony-SS method is that the elements in Eq. 135 are replaced by corresponding block matrices, that is,

$$\mathcal{H}(k) = \begin{bmatrix} \mathbf{y}_k & \mathbf{y}_{k+1} & \cdots & \mathbf{y}_{k+\eta-1} \\ \mathbf{y}_{k+1} & \mathbf{y}_{k+2} & \cdots & \mathbf{y}_{k+\eta} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_{k+\xi-1} & \mathbf{y}_{k+\xi} & \cdots & \mathbf{y}_{k+\xi+\eta-2} \end{bmatrix} \in \mathbb{R}^{\xi \times \eta} \quad (147)$$

Further, when considering multiple signals, Eq. 134 becomes

$$\mathbf{y}_k = \mathbf{C}^t \mathbf{A}^k \mathbf{q}_0 \quad (148)$$

where $\mathbf{C}^t$ is $\in \mathbb{R}^{N \times p}$ with $\mathbf{c}_n^t \in \mathbb{R}^{1 \times p}$, $n = 1, \dots, N$ as its rows and $\mathbf{c}_n^t$ is the row vector corresponding to the n th signal. Upon substituting Eq. 148 into Eq. 147, it can be shown that $\mathcal{H}(k)$ is the product of three matrices:

$$\mathcal{H}(k) = \hat{\mathcal{P}}_\xi \mathbf{A}^k \mathcal{Q}_\eta \quad (149)$$

where

$$\hat{\mathcal{P}}_\xi = \left\{ \begin{array}{c} \mathbf{C}^t \\ \mathbf{C}^t \mathbf{A} \\ \mathbf{C}^t \mathbf{A}^2 \\ \vdots \\ \mathbf{C}^t \mathbf{A}^{\xi-1} \end{array} \right\} \in \mathbb{R}^{N\xi \times p} \quad (150)$$

and $\mathcal{Q}_\eta$ stay the same as Eq. 138.

Then using the same step illustrated in single signal, a realization of $\mathbf{A}$ can be obtained from $\mathcal{H}(0)$ and $\mathcal{H}(1)$:

$$\mathbf{A} = \hat{\Sigma}_1^{-1/2} \hat{\mathbf{U}}_1^t \mathcal{H}(1) \hat{\mathbf{V}}_1 \hat{\Sigma}_1^{-1/2} \quad (151)$$

where

$$\mathcal{H}(0) = \hat{\mathcal{P}}_\xi \mathcal{Q}_\eta \quad (152)$$

and

$$\mathcal{H}(1) = \hat{\mathcal{P}}_\xi \mathbf{A} \mathcal{Q}_\eta \quad (153)$$

$\hat{\mathbf{U}}_1^T$, $\hat{\mathbf{V}}_1$ and $\hat{\Sigma}_1$ can be obtained by applying the singular value decomposition of $\mathcal{H}(0)$, one writes (Zhou and Doyle, 1996):

$$\mathcal{H}(0) = [\hat{\mathbf{U}}_1 \quad \hat{\mathbf{U}}_2] \begin{bmatrix} \hat{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{V}}_1^t \\ \hat{\mathbf{V}}_2^t \end{bmatrix} = \hat{\mathbf{U}}_1 \hat{\Sigma}_1 \hat{\mathbf{V}}_1^t \quad (154)$$

In theory, the model order of the dynamic system, p , is equal to the number of non-zero singular values in Eq. (154), that is, $\hat{\mathbf{U}}_1 \in \mathbb{R}^{\xi \times p}$, $\hat{\mathbf{\Sigma}}_1 \in \mathbb{R}^{p \times p}$ and $\hat{\mathbf{V}}_1 \in \mathbb{R}^{p \times \eta}$, and the rank of $\mathcal{H}(0)$ is p . While choosing the size of $\mathcal{H}(0)$, both ξ and η must be greater than p . The computed eigenvalues of $\mathbf{A}$ are z_n , $n = 1, \dots, p$, and the corresponding λ_n in Eq. 112 are $\lambda_n = \ln(z_n)/\Delta t$.

Once z_n , $n = 1, \dots, p$, have been computed, the next step involves the solution of a second set of linear equations. The matrix form of equations is written as:

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_p^0 \\ z_1^1 & z_2^1 & \cdots & z_p^1 \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{K-1} & z_2^{K-1} & \cdots & z_p^{K-1} \end{bmatrix} \begin{bmatrix} \mathbf{\Gamma}_1^t \\ \mathbf{\Gamma}_2^t \\ \vdots \\ \mathbf{\Gamma}_p^t \end{bmatrix} = \begin{bmatrix} \mathbf{y}_0^t \\ \mathbf{y}_1^t \\ \vdots \\ \mathbf{y}_{K-1}^t \end{bmatrix} \quad (155)$$

where

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,N} \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{p,1} & \gamma_{p,2} & \cdots & \gamma_{p,N} \end{bmatrix} \quad (156)$$

The residues Γ_n , $n = 1, \dots, p$ can be calculated using Eq. 155 and , consequently, the amplitudes and phase angles.

When estimating the impedance function, it has been theoretically proven that the poles of the output signal of a linear system will contain the poles of the input signal (Hu et al., 2016). To guarantee that the estimated poles of the output (current) signal contain the input (voltage) poles, it is necessary to process both input and output signals simultaneously by using the multi-signal Prony-SS method. An intended feature of the multi-signal Prony-SS method is that all input and output signals share the same model – a p th-order difference equation. With this feature, a consistent set of “global” poles (natural frequencies and damping factors) exists among all signals.

4.4 Noise Rejection and Damping Modelling

It is inevitable that the measured input and output signals must contain various noises, and a realistic transducer must include damping (energy dissipation) mechanism. This section addresses the noise handling by using the traditional frequency domain method and the proposed method. As the proposed method has the capability of estimating the damping ratios associated with multiple vibration modes, this section also includes using a Rayleigh damping for the finite element model based on the estimated damping ratios of dominant vibration modes.

4.4.1 Noise-cancelling Methods Statement

Figure. 22 shows a schematic representation of a measurement system to both input and output noise.

Figure 22. Measurement system with noise sources

To remove the noise, the traditional frequency-domain method used to estimate the frequency-response function (reciprocal of impedance function) of a linear system is based on the calculation of the averaged auto-power spectra

$$\tilde{G}_{\hat{P}\hat{P}}(\omega) = \frac{1}{N} \sum_{n=1}^N \hat{P}_n(\omega) \hat{P}_n^*(\omega) \quad (157)$$

and

$$\tilde{G}_{\hat{U}\hat{U}}(\omega) = \frac{1}{N} \sum_{n=1}^N \hat{U}_n(\omega) \hat{U}_n^*(\omega) \quad (158)$$

together with the cross-power spectra

$$\tilde{G}_{\hat{U}\hat{P}}(\omega) = \frac{1}{N} \sum_{n=1}^N \hat{U}_n(\omega) \hat{P}_n^*(\omega) \quad (159)$$

and

$$\tilde{G}_{\hat{P}\hat{U}}(\omega) = \frac{1}{N} \sum_{n=1}^N \hat{P}_n(\omega) \hat{U}_n^*(\omega) \quad (160)$$

where N is the number of segments, and $\hat{P}_n$ and $\hat{U}_n$ are the Fourier transforms of measured time-domain signals of a segment $\hat{p}_n$ and $\hat{u}_n$, respectively, while $(.)^*$ denotes the complex conjugate. Using the averaged spectra of the measured input and output signals, the frequency-response function (reciprocal of the impedance function) of the system can be estimated by the following estimator:

$$H(\omega) = \frac{1}{Z(\omega)} = \sqrt{\frac{\tilde{G}_{\hat{U}\hat{P}}(\omega) \tilde{G}_{\hat{P}\hat{U}}(\omega)}{\tilde{G}_{\hat{P}\hat{P}}(\omega) \tilde{G}_{\hat{U}\hat{U}}(\omega)}} \quad (161)$$

Note that because of limitations of the Fourier transform, this Fourier-based denoising method suffers from the problems related to the periodic assumption, such as frequency leakage and resolution. In principle, the averaging procedure usually requires the amount of data to be very large (Craig and Kurdila, 2006).

Noise Cancellation of Laplace Method

The noise cancellation of the proposed method is based on neglecting the insignificant exponential components of the signal. Assume that s_k ($k = 1, \dots, i+1$) is a singular value of the Hankel matrix, sorted from high to low, Eq. 141 can be written as

$$\begin{aligned} \mathbf{U}_1 \mathbf{\Sigma}_1 \mathbf{V}_1^t &= [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \cdots \quad \mathbf{u}_{(i+1)}] \cdot \begin{bmatrix} s_1 & & & \\ & s_2 & & \\ & & \ddots & \\ & & & s_{i+1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_1^t \\ \mathbf{v}_2^t \\ \vdots \\ \mathbf{v}_{m-i}^t \end{bmatrix} \\ &= \mathbf{u}_1 s_1 \mathbf{v}_1^t + \mathbf{u}_2 s_2 \mathbf{v}_2^t + \cdots + \mathbf{u}_{i+1} s_{i+1} \mathbf{v}_{i+1}^t \end{aligned} \quad (162)$$

where m is the amount of data and i is the number of rows of Hanker matrix. We can find the value of s_k that determines the effect of a component on the entire

signal. Because the noisy components are much smaller relative to the system components, one can take the first n components as the response from the system, and the rest are from noise. Therefore, Eq. 162 can be written into

$$\begin{aligned}\mathbf{U}_1 \boldsymbol{\Sigma}_1 \mathbf{V}_1^t &= [\mathbf{U}_{system} | \mathbf{U}_{noise}] \begin{bmatrix} \mathbf{S}_{system} & 0 \\ 0 & \mathbf{S}_{noise} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{system}^t \\ \mathbf{V}_{noise}^t \end{bmatrix} \\ &= \mathbf{u}_1 s_1 \mathbf{v}_1^t + \cdots + \mathbf{u}_n s_n \mathbf{v}_n^t + \cdots + \mathbf{u}_{i+1} s_{i+1} \mathbf{v}_{i+1}^t\end{aligned}\quad (163)$$

We can neglect the components ranked after n to achieve noise cancelling. It is important to note that the number of rows should not be too small, as that will affect the rank of system, while the number should not be too big as that will affect the efficiency of noise cancelling. Therefore, the noise cancelling of the proposed method depends on the choice of the noise threshold n , which is the number of exponential components produced by the system but noise. A conventional way of choosing the noise threshold is to find a significant gap in the normalized singular values. Generally, there is a sharp drop between the system and noise components, and therefore, we should take the n , which is the number of singular values before the sharp drop, as the rank of the system.

4.4.2 Damping Estimation

The proposed method is capable of estimating the damping ratio, ξ_n , of the impedance function at resonance frequencies. After computing the eigenvalues z_n of the transition matrix $\mathbf{A}$ using Eq. 143, the corresponding λ_n can be calculated by

$$\lambda_n = \frac{\ln(z_n)}{\Delta t} \quad (164)$$

and the damping ratio

$$\xi_n = \frac{a_n}{|\lambda_n|} \quad (165)$$

where a_n is the real part of λ_n .

When using FE commercial packages, we often cannot specify damping coefficients directly, but employ them while setting the harmonic analysis or transient analysis. The Rayleigh damping matrix, $\mathbf{C}$, is a widely used damping model, and is defined as (Cai et al., 2002):

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \quad (166)$$

where $\mathbf{M}$ is the system mass matrix, $\mathbf{K}$ is the system stiffness matrix, and α and β are two coefficients that are related to the damping ratios by

$$\xi_n = \frac{1}{2} \left(\frac{\alpha}{\omega_n} + \beta \omega_n \right) \quad (167)$$

where ω_n is the n th modal frequency of the system. When the two damping ratios, ξ_1 and ξ_2 , which are associated with frequencies ω_1 and ω_2 , are estimated using the proposed method, the values of α and β can be calculated by

$$\alpha = \frac{2\xi_1\omega_1\omega_2^2 - 2\xi_2\omega_1^2\omega_2}{\omega_2^2 - \omega_1^2} \quad (168)$$

and

$$\beta = \frac{2\xi_1\omega_1 - 2\xi_2\omega_2}{\omega_1^2 - \omega_2^2} \quad (169)$$

respectively.

CHAPTER 5

Numerical Study of Estimating Impedance Function

In this chapter, we present a numerical study of the estimation of the impedance function of a piezoelectric transducer. To test the effectiveness of the proposed Laplace domain method, both the numerical simulation and the lab experiment are included. The impedance functions estimated from the proposed method are compared to those obtained from traditional methods. In addition, the effectiveness of noise cancelation for both the proposed and the traditional Fourier-based methods is also investigated.

5.1 Simulation Study

The simulation model is a 2D FE model, shown in Figure. 11. The impedance function estimated by the stepped single-frequency method which repeats hundreds or thousands of single-frequency analysis is considered to be accurate. Some commercial FE packages, including ANSYS, have the built-in capability to compute the impedance function of a transducer using the stepped harmonic (single-frequency) analysis (Moaveni, 2003). By using ANSYS with the frequencies ranged from 0 to 120 KHz with an increment 200 Hz, the impedance function of the present FE model from the harmonic analysis is shown in Figure. 23.

5.1.1 Estimation with Laplace- and Fourier-based Methods

The drawback of the stepped single-frequency method is that it is very time-consuming, especially compared to traditional Fourier-based methods. Fourier-based methods usually carry out the FFT of the transient signals of voltage and current (or charge) that are obtained from a single test. In this study, the proposed Laplace domain method is to be evaluated against both the stepped single-

Figure 23. The impedance function from ANSYS

frequency and a Fourier-based method.

For the numerical simulation study, the transient time signals of the voltage and charge are generated by the transient dynamic analysis of ANSYS. The sampling rate of the time signals is taken to be $f_s = 10^4$ KHz, or the time interval $\Delta t = 1 \times 10^{-7}$ s. Thus, the present analysis targets modal frequencies below the Nyquist frequency 5×10^3 KHz. The input (voltage) signal was generated with the form $v(t) = \gamma_v e^{\lambda_v t}$, with $\gamma_v = 500$ V and $\lambda_v = -1 \times 10^{-3}$, for a total duration of 5×10^{-3} s. The output signal is the electric charge response on the outer surface of the transducer. The time histories of the voltage and charge are shown in Figure. 24.

To properly estimate the impedance function, Fourier-based methods and the proposed Laplace method have significantly different requirements with respect to the duration of the output signal. While applying a Fourier-based method, the

Figure 24. The voltage and charge signals

input and output signals must be long enough to achieve the desired frequency resolution. To achieve the frequency resolution of 200 Hz in the Fourier analysis, the duration of both voltage and charge signals must be 5×10^{-3} s long. In contrast, to extract the poles and residues associated with the charge signal, the proposed Laplace-domain method would employ the initial segment from 0 to 1×10^{-4} s only for the charge signal.

Figure 25. The comparison of the estimated impedance functions

Figure. 25 shows the comparison of the estimated impedance functions from the proposed method and the Fourier-based method, against that from the stepped harmonic method. One can easily conclude the following:

1. The impedance function estimated from the proposed method agrees well with that from the stepped single frequency method.
2. The proposed method significantly outperforms the Fourier-based method, especially at the region of high frequency. Note that the damping factor for this FE model has been set to be equal to 0.

In summary, the proposed method significantly outperforms the Fourier-based method. Furthermore, the signal required for the proposed method is much shorter than that for the Fourier-based method. The impedance function estimated by the proposed method also has a much higher resolution than that of the Fourier-based method.

5.1.2 Estimating Impedance Function in Noisy Environment

To simulate a more realistic environment, we included the Rayleigh damping with $\alpha = 0$ and $\beta = 1 \times 10^{-7}$ for the FE model, and added 10% random Gaussian white noise to the clean signals. The level of the additive white noise is quantified by a stated percentage, which is defined as the ratio of the standard deviation of the Gaussian white noise (σ_n) to that of the noise-free signal (σ_s): $\sigma_n/\sigma_s \times 100\%$. Let the signal-to-noise ratio be defined as $S/N = 20\log_{10}(\sigma_s/\sigma_n)$. A 10% noisy signal corresponds to $S/N = 20$.

Figure 26. Impedance functions estimated from noisy signals by FFT methods

Shown in Figure. 26 are two impedance functions estimated from noisy signals, including: (1) direct FFT approach which obtains the impedance function based on the ratio of the FFT of the voltage to that of the current, and (2) auto/cross power spectra approach, i.e. Eq. 161. For comparison purpose, Figure. 26 also shows the impedance function that is obtained from the stepped harmonic analysis. While using the power spectra approach, we use 20 signals, each having 5×10^4 data points, to calculate the averaged impedance function.

Next, the noise rejection of the proposed method by using a truncated singular value decomposition technique is carried out. The first 100 singular values of $\mathbf{H}(0)_{500 \times 500}$ for the charge signal, which are normalized by the first singular value, are shown in Figure. 27.

Figure 27. Singular Values of Hankel Matrix

Clearly, the singular value curve in Figure. 27 has a significant gap between its 26th and 27th singular values. As the gap may represent the separation between the true signal and noise, the model order for this signal is set equal to 26. By implementing the proposed method with model order 26, we obtain the impedance function of the simulation model shown in Figure. 28. One notices that a good agreement has been achieved between the stepped single-frequency method and the proposed method using only 1000 data points. When 3000 data points are employed in the proposed method, it results in an even better match (see Figure. 29).

Figure 28. The impedance function using 1000 data points (proposed method)

Figure 29. The impedance function using 3000 data points (proposed method)

5.2 Laboratory Test

Ultimately, the effectiveness of the proposed method must be tested for a real transducer (see Figure. 6). The laboratory test was setup (see Figure. 30) for estimating the impedance function of a real piezoelectric tube. In this experiment, the transducer tube was mounted on a “soft” base (a sponge pad) to simulate a free-free mechanical boundary condition.

Figure 30. The experiment setup for estimating the impedance function

The key components of the hardware system include:

1. Source of excitation input: Arbitrary Waveform Signal Generator (AWG5000B);
2. Data-acquisition hardware: Multifunction I/O Boards for PCI Bus Computers (PCI-6110E);
3. Dynamic analyzer: a computer with the signal analysis software (MATLAB) and FE software (ANSYS).

For detailed information about these pieces of equipment, one can refer to References.(Mul, 1998; Tektronix, 2008).

The corresponding circuit diagram of the experiment is shown in Figure. 31, where the resistance $R_1 = 3140$ ohm, V_1 is the input voltage generated by the

Figure 31. The circuit diagram of the experiment

signal generator, and V_2 is the response voltage on the piezoelectric tube. The impedance Z of the piezoelectric tube can be calculated by

$$Z = \frac{V_2}{V_1 - V_2} \times R_1 \quad (170)$$

The experimental procedure requires three steps to estimate the impedance function:

1. Generate the input signal V_1 on a computer and download it to the generator (AWG5000);
2. Conduct the experiment and collect the response signal V_2 ;
3. Perform the necessary signal-processing tasks to obtain the impedance function.

Both the stepped harmonic signals and transient signals have been utilized as the input V_1 . The harmonic input signals were generated repeatedly at uniformly

spaced frequencies with intervals of 0.24 KHz. The transient signal was an exponential decay signal with a maximum of 1V and a decay ratio of -3×10^3 . Figure. 32 shows the transient input and its corresponding output.

Figure 32. Transient input and the corresponding output signals

Using the transient signals, the impedance functions estimated by the direct FFT and proposed methods are shown in Figure. 33. Also included is the impedance function estimated by the stepped harmonic method. It is clear that the proposed method outperforms the traditional Fourier-based method.

Figure 33. The comparison of the impedance functions using experimental data

CHAPTER 6

Finite Element Model Updating of Piezoelectric Structure

6.1 Introduction

Model updating is defined as the process of correcting the numerical values of individual parameters in a mathematical model using data obtained from an associated experimental model such that the updated model more correctly describes the dynamic properties of the subject structure. This chapter briefly reviews traditional finite element model updating methods for elastic systems. In particular, the CMCM method is presented and extended to apply for the finite element model updating when only modal frequencies are available. Based on the extended CMCM method, this chapter newly develops an approach for the multi-physics FE model updating of piezoelectric systems. In this chapter, the newly developed method is limited to update the material parameters of a piezoelectric tube transducer while the only information available is the impedance function of the transducer.

6.2 Traditional Model Updating Methods

Traditionally, model updating methods can be classified into two major groups: direct methods and iterative methods.

6.2.1 Direct Methods

Direct methods are generally of non-iterative methods, all of which are based on computing changes made directly to the mass and stiffness matrices. Typical direct methods include Lagrange multiplier methods, matrix mixing methods and control theory methods (Friswell and Mottershead, 1995). These methods have the advantage of not requiring any iterations, thus eliminating the possibilities of divergence and excessive computation. And abstract “representation” models

could be built up to reproduce the measured data exactly.

One major drawback of direct methods is that the updated mass and stiffness matrices have little physical meaning and cannot be related to physical parameters to the finite elements in the original model. Another drawback of direct methods is the need to expand the mode shape vectors to the full set of finite element model degrees of freedom. Because the quality of mode shape measurements generally cannot be considered to be completely accurate using current measurement technology, this expansion will introduce errors in the data required by the updating algorithm (Friswell and Mottershead, 1995).

6.2.2 Iterative Methods

In many ways, iterative methods are more acceptable in that the parameters they adjust are much closer to physically realizable quantities, and have generally been viewed as the main hope for updating technology. In the iterative methods, the basic procedure consists of solving an optimization problem, in which the discrepancies between the analytical (numerical) and measured dynamic characteristics are minimized by adjusting the unknown model properties.

Typically, the dynamic characteristics that are employed are modal parameters (denoted as $\mathbf{z}$), including both modal frequencies and mode shapes, and the unknown model properties are the updating parameters (denoted as $\boldsymbol{\theta}$), which are commonly the dimensionless correction factors for each element. Suppose there are more measured modal parameters than unknown updating parameters, then the estimate for the updating parameters can be obtained using a least-squares method. Because the modal parameters are always non-linear functions of the updating parameters, the least-squares problem is solved in an iterative way.

The iterative methods are generally based on the use of a Taylor series of the modal parameters that are expanded as a function of the updating parameters,

and this series is often truncated to produce a linear approximation of the form

$$\delta \mathbf{z} = \mathbf{S}_{en} \delta \boldsymbol{\theta} \quad (171)$$

where $\delta \boldsymbol{\theta}$ is the perturbation in the updating parameters, $\delta \mathbf{z}$ is the error in the analytical and measured modal parameters, and $\mathbf{S}_{en}$ is the sensitivity matrix. The sensitivity matrix $\mathbf{S}_{en}$, which contains the first derivatives of the modal parameters with respect to the updating parameters, is calculated analytically with formulas based on the equation of motion. The calculation of these derivatives is computationally intensive. In principle, the sensitivity matrix, $\mathbf{S}_{en}$, must be reevaluated at each iteration based on the latest estimate of the updating parameters. Considering Eq. 171, the use of $\delta \mathbf{z}$ indicates that the measured and analytical modes must be paired correctly. Because the mass distribution of the analytical model and that of the actual structure may be different, the measured and analytical mode shapes must be scaled consistently. Furthermore, because of the iterative procedure and a possible ill-conditioned $\mathbf{S}_{en}$, the solutions are often prone to diverge (Hu et al., 2007).

Up to the present, most published model updating methods for ultrasound transducers belong to the category of iterative methods in the traditional sense (Piranda et al., 1998; Piranda et al., 2001; Araújo et al., 2009; Kwok et al., 1997).

6.3 Cross-model Cross-mode (CMCM) Method

The model updating method employed in this study is the cross-model cross-mode (CMCM) method, which involves solving a set of linear simultaneous equations for the physically meaningful correction factors. Each linear equation of the CMCM method is formulated based on the product terms from two same/different modes associated with the mathematical and experimental models, respectively. In addition to its accuracy and efficiency, the CMCM method has several other

advantages over traditional model updating methods, including: (1) it is a direct, as well as a physical property adjustment, model updating method. While it is a non-iterative method and therefore very cost-effective in computational time, it also has the advantage of preserving the initial model configuration and physical connectivity of the updated model; (2) it uses both the mode shapes and modal frequencies for the simultaneous updating of the mass and stiffness matrices, whereas it does not require the measured mode be paired with the same mode of the analytical model; (3) the implementation of the new updating approach needs only the first few measured modes being arbitrarily scaled; (4) the method is a systematic approach and theoretically it is readily applicable to various kinds of structural or mechanical systems (Hu et al., 2007).

6.3.1 Procedure of CMCM Method

In the development of the CMCM method, it is assumed that the stiffness and mass matrices of the structure, denoted by $\mathbf{K}$ and $\mathbf{M}$, are obtained from the finite-element model. It is intended to “correct” or “update” the stiffness and mass matrix coefficients by modal measurements, including a few mode shapes and their corresponding frequencies.

The i th eigenvalue and eigenvector associated with $\mathbf{K}$ and $\mathbf{M}$ is expressed as

$$\mathbf{K}\Phi_i = \lambda_i\mathbf{M}\Phi_i \quad (172)$$

Assume that the stiffness matrix $\mathbf{K}^*$ of the actual (experimental) model is a modification of $\mathbf{K}$ to be formulated as

$$\mathbf{K}^* = \mathbf{K} + \sum_{n=1}^{N_e} \alpha_n \mathbf{K}_n \quad (173)$$

where $\mathbf{K}_n$ is the stiffness matrix corresponding to the n th element; N_e is the number of “elements”; and α_n are correction factors to be determined. Herein, for simplicity in presentation, it is assumed that each element involves a parameter

to be updated, such as the Youngs modulus of each element. A more general formulation of Eq. 173 can have the number of unknowns, or correction terms, to be less, or greater, than N_e .

Likewise, one writes the corresponding expression for a corrected mass matrix $\mathbf{M}^*$ as

$$\mathbf{M}^* = \mathbf{M} + \sum_{n=1}^{N_e} \beta_n \mathbf{M}_n \quad (174)$$

in which $\mathbf{M}_n$ is the mass matrix corresponding to the n th element; and β_n are correction coefficients to be determined. Again, it is assumed that each element involves a mass quantity to be corrected, such as the mass density of each element. Clearly, Eqs. 173 and 174 are applicable to general finite-element formulations with any element type where individual mass and stiffness matrices may be very large.

Express the j th eigenvalue and eigenvector associated with $\mathbf{K}^*$ and $\mathbf{M}^*$ as

$$\mathbf{K}^* \Phi_j^* = \lambda_j^* \mathbf{M}^* \Phi_j^* \quad (175)$$

In the following development, it is assumed that a few of λ_j^* and Φ_j^* are known measurements available from modal testing.

Premultiplying Eq. 172 by Φ_j^{*t} and Eq. 175 by Φ_i^t yields

$$\Phi_j^{*t} \mathbf{K} \Phi_i = \lambda_i \Phi_j^{*t} \mathbf{M} \Phi_i \quad (176)$$

and

$$\Phi_i^t \mathbf{K}^* \Phi_j^* = \lambda_j^* \Phi_i^t \mathbf{M}^* \Phi_j^* \quad (177)$$

respectively. Because $\mathbf{M}$ is a symmetric matrix, one has $[\Phi_j^{*t} \mathbf{M} \Phi_i]^t = \Phi_i^t \mathbf{M} \Phi_j^*$.

Also noting the transpose of a scalar equals to itself, i.e. $[\Phi_j^{*t} \mathbf{M} \Phi_i]^t = \Phi_j^{*t} \mathbf{M} \Phi_i$.

One thus has

$$\Phi_j^{*t} \mathbf{M} \Phi_i = \Phi_i^t \mathbf{M} \Phi_j^* \quad (178)$$

Similarly, since $\mathbf{K}$ is a symmetric matrix, one shows that

$$\Phi_j^{*t} \mathbf{K} \Phi_i = \Phi_i^t \mathbf{K} \Phi_j^* \quad (179)$$

Dividing Eq. 177 by Eq. 176, and using the scalar identities of Eqs. 178 and 179, one obtains

$$\frac{\Phi_i^t \mathbf{K}^* \Phi_j^*}{\Phi_i^t \mathbf{K} \Phi_j^*} = \frac{\lambda_j^*}{\lambda_i} \frac{\Phi_i^t \mathbf{M}^* \Phi_j^*}{\Phi_i^t \mathbf{M} \Phi_j^*} \quad (180)$$

Substituting Eqs. 173 and 174 into the above equation yields

$$1 + \sum_{n=1}^{N_e} \alpha_n A_{n,ij} = \frac{\lambda_j^*}{\lambda_i} (1 + \sum_{n=1}^{N_e} \beta_n B_{n,ij}) \quad (181)$$

where

$$A_{n,ij} = \frac{\Phi_i^t \mathbf{K}_n \Phi_j^*}{\Phi_i^t \mathbf{K} \Phi_j^*} \quad (182)$$

and

$$B_{n,ij} = \frac{\Phi_i^t \mathbf{M}_n \Phi_j^*}{\Phi_i^t \mathbf{M} \Phi_j^*} \quad (183)$$

Using a new index m to replace ij , Eq. 181 becomes

$$1 + \sum_{n=1}^{N_e} \alpha_n A_{n,m} = b_m (1 + \sum_{n=1}^{N_e} \beta_n B_{n,m}) \quad (184)$$

where

$$b_m = \frac{\lambda_j^*}{\lambda_i} \quad (185)$$

Rearranging Eq. 184, one obtains

$$\sum_{n=1}^{N_e} \alpha_n A_{n,m} - b_m \sum_{n=1}^{N_e} \beta_n B_{n,m} = b_m - 1 \quad (186)$$

or

$$\sum_{n=1}^{N_e} \alpha_n A_{n,m} + \sum_{n=1}^{N_e} \beta_n E_{n,m} = f_m \quad (187)$$

where $E_{n,m} = -b_m B_{n,m}$, $f_m = b_m - 1$. When N_i reliable modes are available from the finite-element model, and N_j modes are measured from the corresponding real structure, totally $N_m = N_i \times N_j$ CMCM equations can be formed from Eq. 187.

Written in a matrix form, one has

$$\boldsymbol{\alpha} \mathbf{A} + \boldsymbol{\beta} \mathbf{E} = \mathbf{f} \quad (188)$$

in which $\mathbf{A}$ and $\mathbf{E}$ are $N_m \times N_e$ matrix; $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are column vectors of size N_e ; and $\mathbf{f}$ is a column vector of size N_m . Furthermore, one can rewrite Eq. 188 as

$$\mathbf{G}\boldsymbol{\gamma} = \mathbf{f} \quad (189)$$

where

$$\mathbf{G} = \begin{bmatrix} \mathbf{A} & \mathbf{E} \end{bmatrix}, \boldsymbol{\gamma} = \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} \quad (190)$$

A singular value decomposition (SVD) approach can be used to solve for $\mathbf{G}\boldsymbol{\gamma} = \mathbf{f}$ where the system can be just-determined, under-determined, over-determined, or mixed-determined. Decompose the rectangular matrix $\mathbf{G}$ as the product of three matrices

$$\mathbf{G} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^t \quad (191)$$

where the matrices $\mathbf{U}$ and $\mathbf{V}$, real or complex unitary matrices with $\mathbf{U}^t\mathbf{U} = \mathbf{I}$, $\mathbf{V}^t\mathbf{V} = \mathbf{I}$, and the matrix $\boldsymbol{\Sigma}$, a diagonal matrix whose diagonal elements are singular values. These singular values are usually arranged in order of decreasing size. Some of the singular values may be zero. One can partition $\boldsymbol{\Sigma}$ into a submatrix $\boldsymbol{\Sigma}_p$ of p nonzero singular values and zero matrices as

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (192)$$

where $\boldsymbol{\Sigma}_p$ is a $p \times p$ diagonal matrix. The decomposition then becomes $\hat{\mathbf{G}} = \mathbf{U}_p\boldsymbol{\Sigma}_p\mathbf{V}_p^t$, where $\mathbf{U}_p$ and $\mathbf{V}_p$ consist of the first p columns of $\mathbf{U}$ and $\mathbf{V}$. The natural solution of the inverse problem has been shown to be (Hu et al., 2007; Hu and Li, 2008)

$$\hat{\boldsymbol{\gamma}} = \hat{\mathbf{G}}^{-1}\mathbf{f} = \mathbf{V}_p\boldsymbol{\Sigma}_p^{-1}\mathbf{U}_p^t\mathbf{f} \quad (193)$$

6.3.2 Extended CMCM Method

In dealing with spatially incomplete situations, one usually applies either model reduction or modal expansion schemes. The case of no mode shapes at all is

an extreme case of incompleteness. A possible approach to deal with the extreme incompleteness case — only measured modal frequencies available — is to extend the original CMCM method involving an iterative procedure (see the flowchart in Figure.34) (Li et al., 2008). It involves repeatedly applying the following two steps:

1. When no measured mode shapes are available, a plausible approximation for a measured mode shape is the corresponding analytical mode shape from the FE model.
2. Using the measured modal frequencies and the corresponding approximated measured mode shapes (from Step 1), one can proceed the original CM-CM method to compute correction coefficients, and rebuild the updated FE model.

The iteration will continue until a converged solution with a preset tolerance is achieved.

6.4 Model Updating of Piezoceramic Tube by CMCM Method

This section develops a procedure to apply the extended CMCM method to perform the multi-physics model updating for piezoceramic tube transducers. It is assumed that the mass density and geometry of a piezoceramic tube transducer could be measured accurately, and thus do not need to be corrected. The quantities require to be updated are the constitutive material properties of the piezoelectric tube. The only modal information available to perform the model updating is the measured impedance function of the piezoelectric tube transducer.

6.4.1 Multi-physics Model Updating

The term multi-physics model updating indicates that the parameters to be updated belong to more than one category of physics. For a piezoelectric transduc-

Figure 34. Flowchart of the iterative extended CMCM method with measured modal frequencies only

er, there are three groups of parameters, elastic parameters, dielectric parameters and piezoelectric parameters. Referring to a measured impedance function, elastic parameters would solely decide the resonance frequencies which are the modal frequencies of the transducer in short circuit situation; dielectric parameters (permittivity) determine the unclamped capacitance; and all three groups of parameters affect the anti-resonance frequencies which are the modal frequencies of the transducer in open circuit condition. Therefore, one can proceed the multi-physics parameter updating as follows:

1. From the resonant frequencies of the measured impedance function, update the elastic parameters based on a short-circuit FE model.
2. From the measured impedance value at a very low frequency region, update the dielectric parameters (permittivity) in the poling direction.
3. From the anti-resonant frequencies of the measured impedance function, update the piezoelectric parameters based on a open-circuit FE model.

6.4.2 Model Updating Based on Reduced Constitutive Equations

Theoretically, the equations of motion for a specific piezoelectric structure, such as a thin tube transducer, can be derived by applying either the reduced constitutive equations or the 3D full constitutive equations. Also, the extended CMC method can perform the multi-physics model updating for piezoelectric transducers based on the equations of motion derived from either of the constitutive equations. However, one should not update the parameters associated with the reduced constitutive equations while the equations of motion of the finite element models have been formulated based on the 3D full constitutive equations, vice versa.

Updating of Elastic Parameters

This section presents the implementation of the extended CMCM method on a thin cylinder transducer based on the reduced constitutive equations. The first step is to update the elastic parameters from a short-circuit model. From Eqs. 74 and 81, the global stiffness matrix of a short-circuit model $\mathbf{K}_{UU}$ could be written as

$$\mathbf{K}_{UU} = \sum_{j=1}^P \mathbf{L}_u^{(j)t} \left[\int_V \mathbf{B}_u^t \mathbf{c}^E \mathbf{B}_u dV \right]_j \mathbf{L}_u^{(j)} \quad (194)$$

where P is the number of elements, $\mathbf{L}_u^{(j)}$ is the localization matrix of the j th element, and the term within the brackets is the stiffness matrix associated with the j th element.

For a thin cylindrical transducer polarized in the radial direction, its reduced constitutive equations have been shown previously in Eq. 18 to be in a form of coefficients $\bar{c}_{11}^E$ and $\bar{c}_{12}^E$. Considering that all the elements are made of the same polarized piezoelectric PZT material, since the operation of Eq. 194 is linear in terms of the stiffness coefficients, one shows

$$\mathbf{K}_{UU} = \mathbf{K}_1 + \mathbf{K}_2 \quad (195)$$

where $\mathbf{K}_1 = \bar{c}_{11}^E \mathbf{K}_{\bar{c}_{11}^E}$, $\mathbf{K}_2 = \bar{c}_{12}^E \mathbf{K}_{\bar{c}_{12}^E}$, $\mathbf{K}_{\bar{c}_{11}^E}$ and $\mathbf{K}_{\bar{c}_{12}^E}$ are the change of $\mathbf{K}_{UU}$ due to the unit changes of $\bar{c}_{11}^E$ and $\bar{c}_{12}^E$, respectively. Sequentially, in the finite element model updating formulation for the stiffness coefficients $\bar{c}_{11}^E$ and $\bar{c}_{12}^E$, the updated global stiffness matrix of a short circuit model $\mathbf{K}_{UU}^*$ can be written as

$$\mathbf{K}_{UU}^* = (1 + \alpha_1) \bar{c}_{11}^E \mathbf{K}_{\bar{c}_{11}^E} + (1 + \alpha_2) \bar{c}_{12}^E \mathbf{K}_{\bar{c}_{12}^E} = \mathbf{K}_{UU} + \sum_{i=1}^2 \alpha_i \mathbf{K}_i \quad (196)$$

where α_1 and α_2 are the correction coefficient of $\bar{c}_{11}^E$ and $\bar{c}_{12}^E$ respectively. The form of Eq. 196 is consistent with Eq. 173 that has been formulated for the development of the CMCM method, thus one can proceed the extended CMCM method to

compute α_1 and α_2 when two or more resonant frequencies are extractable from the measured impedance function. After obtaining α_1 and α_2 , the updated elastic parameters $\bar{c}_{11}^{E*}$ and $\bar{c}_{12}^{E*}$ are

$$\bar{c}_{11}^{E*} = (1 + \alpha_1)\bar{c}_{11}^E \quad (197)$$

and

$$\bar{c}_{12}^{E*} = (1 + \alpha_2)\bar{c}_{12}^E \quad (198)$$

respectively.

Updating of Dielectric Parameters (Permittivity)

When a piezoceramic transducer is excited at a very low frequency f_L , it can be treated as a capacitor. Thus, the electrical impedance function $Z(f)$ at the frequency f_L is completely determined by the capacitance, C , at the electrical terminals:

$$Z_L = \frac{1}{2\pi f_L C} \quad (199)$$

where $Z_L \equiv Z(f_L)$. With the inner and outer surfaces fully covered by conducting electrodes, the capacitance of cylinder in the radial direction is given by (Burdic, 1991)

$$C = \frac{2\pi\epsilon_{33}\ell}{\ln(D_o/D_i)} \quad (200)$$

where D_i is the inner diameter, D_o the outer diameter, ℓ the axial length and ϵ_{33} the permittivity in the poling direction.

If the mechanical boundary condition is free-free during the experiment of estimating the impedance function of a transducer, the related permittivity of this unclamped transducer is the permittivity at constant stress, noted by ϵ_{33}^T . From Eqs. 199 and 200, the permittivity at constant stress of cylinder transducer is estimated by

$$\epsilon_{33}^{T*} = \frac{\ln(D_o/D_i)}{4\pi^2\ell Z_L f_L} \quad (201)$$

Note that Eqs. 201 is valid for updating the permittivity ε_{33}^T regardless of the equations of motion of the finite element model having been formulated based on the reduced constitutive equations or the 3D full constitutive equations.

Updating of Piezoelectric Parameters

After the elastic parameters $\bar{c}_{11}^{E*}$ and $\bar{c}_{12}^{E*}$ and the permittivity parameter ε_{33}^{T*} have been obtained, the open-circuited FE model, together with the anti-resonant frequencies of the measured impedance function, can be employed to update the piezoelectric parameter d_{31} (or $\bar{e}_{31}$) of a thin-walled cylindrical transducer poling in the 3-direction. As shown in Eq. 92, the global stiffness matrix of an open-circuited FE model consists of displacement stiffness matrix, $\mathbf{K}_{UU}$, piezoelectric coupling matrix, $\mathbf{K}_{\Phi U}$ and dielectric “stiffness” matrix, $\mathbf{K}_{\Phi\Phi}$. Considering that all the elements are made of the same polarized piezoelectric PZT material, one writes

$$\mathbf{K}_{U\Phi} = \mathbf{K}_{\Phi U}^t = \sum_{j=1}^P \mathbf{L}_u^{(j)t} \left[\int_V \mathbf{B}_u^t \mathbf{e}^t \mathbf{B}_\phi dV \right]_j \mathbf{L}_\phi^{(j)} \quad (202)$$

and

$$\mathbf{K}_{\Phi\Phi} = \sum_{j=1}^P \mathbf{L}_\phi^{(j)t} \left[\int_V \mathbf{B}_\phi^t \boldsymbol{\varepsilon}^S \mathbf{B}_\phi dV \right]_j \mathbf{L}_\phi^{(j)} \quad (203)$$

For a thin-wall tube transducer, since the operations of Eq. 202 and 203 are linear in $\bar{e}_{31}$ and $\bar{\varepsilon}_{33}^S$, respectively, one can express

$$\mathbf{K}_{U\Phi} = \mathbf{K}_{\Phi U}^t = \bar{e}_{31} \mathbf{K}_{\bar{e}_{31}} \quad (204)$$

and

$$\mathbf{K}_{\Phi\Phi} = \bar{\varepsilon}_{33}^S \mathbf{K}_{\bar{\varepsilon}_{33}^S} \quad (205)$$

where $\mathbf{K}_{\bar{e}_{31}}$ is the change of $\mathbf{K}_{U\Phi}$ due to the unit change of $\bar{e}_{31}$, and $\mathbf{K}_{\bar{\varepsilon}_{33}^S}$ is the change of $\mathbf{K}_{\Phi\Phi}$ due to the unit change of $\bar{\varepsilon}_{33}^S$.

Consequently, the condensed electro-elastic stiffness matrix in Eq. 92 becomes

$$\tilde{\mathbf{K}}_{UU} = \mathbf{K}_{UU} + \mathbf{K}_3 \quad (206)$$

where

$$\mathbf{K}_3 = -\bar{a} \mathbf{K}_{\bar{e}_{31}} (\mathbf{K}_{\bar{\varepsilon}_{33}^S})^{-1} \mathbf{K}_{\bar{e}_{31}}^t \quad (207)$$

in which

$$\bar{a} = \frac{(\bar{e}_{31})^2}{\bar{\varepsilon}_{33}^S} \quad (208)$$

In the finite element model updating formulation for the condensed electro-elastic stiffness matrix, one writes

$$\tilde{\mathbf{K}}_{UU}^* = \mathbf{K}_{UU}^* + (1 + \alpha_3) \mathbf{K}_3 \quad (209)$$

where α_3 is the correction coefficient of $\bar{a}$, i.e. the correction coefficient for $(\bar{e}_{31})^2/\bar{\varepsilon}_{33}^S$.

As the form of Eq. 209 is consistent with Eq. 173, one can proceed the extended CMCM method to compute α_3 when one or more modal frequencies of an open-circuit model (anti-resonant frequencies of the transducer) are available. The updated $\bar{a}$ is calculated by

$$\bar{a}^* = (1 + \alpha_3) \bar{a} \quad (210)$$

Although α_3 can be used to update $(\bar{e}_{31})^2/\bar{\varepsilon}_{33}^S$, updating $\bar{e}_{31}$ and $\bar{\varepsilon}_{33}^S$ individually requires at least one more equation.

For a unclamped thin cylindrical transducer polarized in the radial direction, from $\bar{e}_{31} = d_{31}/(s_{11}^E + s_{12}^E)$ (see Eq. 23) and $\bar{\varepsilon}_{33}^S = \varepsilon_{33}^T - 2d_{31}^2/(s_{11}^E + s_{12}^E)$ (see Eq. 24), one writes

$$\bar{\varepsilon}_{33}^S = \varepsilon_{33}^T - 2(s_{11}^E + s_{12}^E)(\bar{e}_{31})^2 \quad (211)$$

Furthermore, from $\bar{c}_{11}^E = s_{11}^E/((s_{11}^E + s_{12}^E)(s_{11}^E - s_{12}^E))$ (see Eq. 20) and $\bar{c}_{12}^E = -s_{12}^E/((s_{11}^E + s_{12}^E)(s_{11}^E - s_{12}^E))$ (see Eq. 21), one writes

$$s_{11}^E + s_{12}^E = \frac{1}{\bar{c}_{11}^E + \bar{c}_{12}^E} \quad (212)$$

After the updated quantities ε_{33}^{T*} , $\bar{c}_{11}^{E*}$, $\bar{c}_{12}^{E*}$ and $\bar{a}^*$ have been obtained, from Eqs. 208, 211 and 212, one shows the updated $\bar{\varepsilon}_{33}^S$ and $\bar{e}_{31}$ to be

$$\bar{\varepsilon}_{33}^{S*} = \frac{(\bar{c}_{11}^{E*} + \bar{c}_{12}^{E*})\varepsilon_{33}^{T*}}{\bar{c}_{11}^{E*} + \bar{c}_{12}^{E*} + 2\bar{a}^*} \quad (213)$$

and

$$\bar{e}_{31}^* = -\sqrt{\bar{a}^* \bar{\varepsilon}_{33}^{S*}} \quad (214)$$

respectively.

6.4.3 Model Updating Based on 3D Constitutive Equations

Because most commercial finite element packages do not use the reduced constitutive equations to generate the equations of motion for piezoelectric structures, one can not update the parameters associated with the reduced constitutive equations: $\bar{c}_{11}^E$, $\bar{c}_{12}^E$, $\bar{e}_{31}$ and $\bar{\varepsilon}_{33}^S$, while using commercial finite element packages. When a finite element model for a piezoceramic structure is built by a commercial package based on full 3D constitutive equations, the user must input all 10 material parameters of piezoceramic associated with the 3D constitutive equations.

As described in Chapter 2, while using the d-form constitutive equations, only 4 parameters, namely s_{11}^E , s_{12}^E , d_{31} and ε_{33}^T , among the 10 independent material parameters are sensitive to the impedance function of a thin-wall tube piezoceramic transducer. In other words, parameters s_{33}^E , s_{13}^E , s_{44}^E , ε_{11}^T , d_{33} and d_{15} will not affect the impedance function of a thin-wall tube piezoceramic transducer. On the one hand, while using the impedance function of a thin-wall tube piezoceramic transducer, it is not possible to update all 10 material parameters. On the other hand, only 4 sensitive parameters are required to be updated for accurately obtaining its impedance function.

While using the commercial package ANSYS to build a finite element model for a thin-wall tube piezoceramic transducer, the user must input s_{11}^E , s_{12}^E , d_{31} and

ε_{33}^T as accurate as possible in order to obtain an accurate impedance function of this transducer. Because parameters s_{33}^E , s_{13}^E , s_{44}^E , ε_{11}^T , d_{33} and d_{15} are insensitive to the impedance function of this transducer, one has the freedom to input arbitrary values for these 6 parameters. There are 2 intuitive ways to manipulate these 6 insensitive parameters for simplifying the model: (1) a mathematical simplification by setting all insensitive parameters equal to zero, and (2) a physical simplification by using isotropic material model.

Mathematical Simplification

A mathematical simplification for a thin-wall tube piezoceramic transducer is to set all 6 insensitive parameters equal to zero:

$$s_{33}^E = s_{13}^E = s_{44}^E = \varepsilon_{11}^T = d_{33} = d_{15} = 0 \quad (215)$$

The 3D constitutive equations in the d-form then becomes

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & 0 & 0 & 0 & 0 & 0 & 0 & d_{31} \\ s_{12}^E & s_{11}^E & 0 & 0 & 0 & 0 & 0 & 0 & d_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ d_{31} & d_{31} & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_{33}^T \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (216)$$

Immediately, Eq. 216 can be rewritten in the following form

$$\begin{bmatrix} S_1 \\ S_2 \\ S_6 \\ D_3 \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & 0 & d_{31} \\ s_{12}^E & s_{11}^E & 0 & d_{31} \\ 0 & 0 & s_{66}^E & 0 \\ d_{31} & d_{31} & 0 & \varepsilon_{33}^T \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_6 \\ E_3 \end{bmatrix} \quad (217)$$

Although Eq. 217 has the same e-form as that shown in Eq. 16 of the reduced constitutive equations for a thin-wall tube transducer. One must recognize that setting material coefficients equal to zero is completely different from setting some stress or strain terms equal to zero. Setting insensitive coefficients equal to zero may easily violate the underlying physics and cause singularity in mathematics.

Physical Simplification

One physical simplification for a thin-wall tube transducer is to assign those 6 insensitive parameters s_{13}^E , s_{33}^E , s_{44}^E , ε_{11}^T , d_{33} and d_{15} as follows:

$$s_{13}^E = s_{12}^E, s_{33}^E = s_{11}^E, s_{44}^E = s_{66}^E \quad (218)$$

and

$$\varepsilon_{11}^T = d_{33} = d_{15} = 0 \quad (219)$$

Eq. 218 implies the related engineering parameters as

$$Y_{33}^E = Y_{11}^E, \nu_{13} = \nu_{12}, G = \frac{Y_{11}^E}{2(1 + \nu_{12})} \quad (220)$$

From the physical point of view, Eq. 218 suggests that the elastic material properties are characterized as an isotropic material. The conditions in Eq. 219 imply the following: (1) $\varepsilon_{11}^T = 0$, there is no current caused by the electric field in the non-poling directions (*i.e.*, 1- and 2-direction); (2) $d_{33} = 0$, there is no electrical polarization in the poling direction caused by the stress applied in the poling direction; and (3) $d_{15} = 0$, there is no electrical polarization in the non-poling direction caused by the shear stress applied in the non-poling directions (Ltd, 2011).

In the remaining part of this section, it intends to show the corresponding e-form parameters based on the above physical simplification. For a isotropic material, the stiffness matrix is written as:

$$\mathbf{c} = \begin{bmatrix} c_{11}^E & c_{12}^E & c_{12}^E & 0 & 0 & 0 \\ c_{12}^E & c_{11}^E & c_{12}^E & 0 & 0 & 0 \\ c_{12}^E & c_{12}^E & c_{11}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{66}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{66}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}^E \end{bmatrix} \quad (221)$$

where $c_{66}^E = (c_{11}^E - c_{12}^E)/2$. The relationships among the stiffness, engineering and compliance parameters for isotropic material are as follows (Vignjevic et al., 2008):

$$c_{11}^E = \frac{(1 - \nu_{12})Y_{11}^E}{(1 + \nu_{12})(1 - 2\nu_{12})} = \frac{s_{11}^E + s_{12}^E}{(s_{11}^E - s_{12}^E)(s_{11}^E + 2s_{12}^E)} \quad (222)$$

$$c_{12}^E = \frac{\nu_{12}Y_{11}^E}{(1 + \nu_{12})(1 - 2\nu_{12})} = -\frac{s_{12}^E}{(s_{11}^E - s_{12}^E)(s_{11}^E + 2s_{12}^E)} \quad (223)$$

$$c_{66}^E = \frac{Y_{11}^E}{2(1 + \nu_{12})} = \frac{1}{2(s_{11}^E - s_{12}^E)} \quad (224)$$

With $\varepsilon_{11}^T = d_{33} = d_{15} = 0$, the piezoelectric strain matrix and dielectric matrix at constant stress become

$$\mathbf{d} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ d_{31} & d_{31} & 0 & 0 & 0 & 0 \end{bmatrix} \quad (225)$$

and

$$\boldsymbol{\varepsilon}^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{33}^T \end{bmatrix} \quad (226)$$

Using $\mathbf{e} = \mathbf{d}(\mathbf{s}^E)^{-1}$ (see Eq. 9):

$$\begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ d_{31} & d_{31} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{11}^E & c_{12}^E & c_{12}^E & 0 & 0 & 0 \\ c_{12}^E & c_{11}^E & c_{12}^E & 0 & 0 & 0 \\ c_{12}^E & c_{12}^E & c_{11}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{66}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{66}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}^E \end{bmatrix} \quad (227)$$

one shows

$$e_{31} = d_{31}(c_{11}^E + c_{12}^E) \quad (228)$$

and

$$e_{33} = 2d_{31}c_{12}^E \quad (229)$$

From Eqs. 228 and 229, it yields

$$e_{33} = \frac{2c_{12}^E}{c_{11}^E + c_{12}^E} e_{31} \quad (230)$$

Applying $\boldsymbol{\varepsilon}^S = \boldsymbol{\varepsilon}^T - \mathbf{e}\mathbf{d}^t$ (see Eq. 10),

$$\begin{bmatrix} \varepsilon_{11}^S & 0 & 0 \\ 0 & \varepsilon_{11}^S & 0 \\ 0 & 0 & \varepsilon_{33}^S \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{33}^T \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (231)$$

one obtains the following dielectric permittivity relationship

$$\varepsilon_{33}^S = \varepsilon_{33}^T - 2e_{31}d_{31} \quad (232)$$

Substituting Eq. 228 into Eq. 232 yields

$$\varepsilon_{33}^S = \varepsilon_{33}^T - 2d_{31}^2(c_{11}^E + c_{12}^E) \quad (233)$$

The above physical simplification satisfies the general physics of a tube transducer, and requires only four independent sensitive parameters: s_{11}^E , s_{12}^E , d_{31} and ε_{33}^T , in the finite element modeling based on the full 3D constitutive equations. In the finite element model formulation for the equations of motion, the stiffness matrix of the dynamic structure is needed. The CMC model updating method also updates the stiffness matrix of the structure. Thus, one must update the e-form material coefficients. There are 6 non-zero e-form parameters in the above physical simplification, including: c_{11}^E , c_{12}^E , c_{66}^E , e_{31} , e_{33} and ε_{33}^S .

Updating of Elastic Parameters

Considering that all the elements are made of the same polarized piezoelectric PZT material, since the operation of Eq. 194 is linear in terms of the stiffness coefficients c_{11}^E , c_{12}^E , c_{13}^E , c_{33}^E and c_{44}^E , any 3D structure shows

$$\mathbf{K}_{UU} = c_{11}^E \mathbf{K}_{c_{11}^E} + c_{12}^E \mathbf{K}_{c_{12}^E} + c_{13}^E \mathbf{K}_{c_{13}^E} + c_{33}^E \mathbf{K}_{c_{33}^E} + c_{44}^E \mathbf{K}_{c_{44}^E} \quad (234)$$

where $\mathbf{K}_{c_{ij}^E}$ can be interpreted as the change of $\mathbf{K}_{UU}$ due to the unit change of c_{ij}^E .

Imposing the physical simplification in Eq. 218 yields:

$$c_{44}^E = \frac{1}{2}(c_{11}^E - c_{12}^E) \quad (235)$$

$$c_{11}^E = c_{33}^E \quad (236)$$

and

$$c_{12}^E = c_{13}^E \quad (237)$$

Therefore, Eq. 234 can be rewritten as

$$\mathbf{K}_{UU} = c_{11}^E \widehat{\mathbf{K}}_{c_{11}^E} + c_{12}^E \widehat{\mathbf{K}}_{c_{12}^E} \quad (238)$$

where

$$\widehat{\mathbf{K}}_{c_{11}^E} = \mathbf{K}_{c_{11}^E} + \mathbf{K}_{c_{33}^E} + \frac{1}{2}\mathbf{K}_{c_{44}^E} \quad (239)$$

and

$$\widehat{\mathbf{K}}_{c_{12}^E} = \mathbf{K}_{c_{12}^E} + \mathbf{K}_{c_{13}^E} - \frac{1}{2}\mathbf{K}_{c_{44}^E} \quad (240)$$

In the finite element model updating formulation, the updated global stiffness matrix of a short circuit model $\mathbf{K}_{UU}^*$ then can be written as

$$\mathbf{K}_{UU}^* = (1 + \alpha_1)c_{11}^E \widehat{\mathbf{K}}_{c_{11}^E} + (1 + \alpha_2)c_{12}^E \widehat{\mathbf{K}}_{c_{12}^E} = \mathbf{K}_{UU} + \sum_{i=1}^2 \alpha_i \widehat{\mathbf{K}}_i \quad (241)$$

where α_1 and α_2 are the correction coefficient of c_{11}^E and c_{12}^E respectively. The form of Eq. 241 is consistent with Eq. 173 that has been formulated for the development of the CMCM method, thus one can proceed the extended CMCM method to compute α_1 and α_2 .

Updating of Piezoelectric Parameters

From Eqs. 202, the linear operations on the piezoelectric stress matrix $\mathbf{e}$ for full 3D constitutive equations yields

$$\mathbf{K}_{U\Phi} = \mathbf{K}_{\Phi U}^t = e_{31}\mathbf{K}_{e_{31}} + e_{33}\mathbf{K}_{e_{33}} + e_{15}\mathbf{K}_{e_{15}} \quad (242)$$

where $\mathbf{K}_{e_{31}}$, $\mathbf{K}_{e_{33}}$ and $\mathbf{K}_{e_{15}}$ are the changing of global stiffness due to the unit change of e_{31} , e_{33} and e_{15} , respectively. According to the the proposed physical simplification, $e_{15} = 0$ and $e_{33} = [2c_{12}^E / (c_{11}^E + c_{12}^E)] e_{31}$, Eq.242 becomes

$$\mathbf{K}_{U\Phi} = e_{31} \hat{\mathbf{K}}_{e_{31}}^E \quad (243)$$

where

$$\hat{\mathbf{K}}_{e_{31}}^E = \mathbf{K}_{e_{31}} + \left(\frac{2c_{12}^E}{c_{11}^E + c_{12}^E} \right) \mathbf{K}_{e_{33}} \quad (244)$$

Similarly, from Eq. 203, the linear operations on the permittivity matrix $\boldsymbol{\varepsilon}$ for full 3D constitutive equations yields

$$\mathbf{K}_{\Phi\Phi} = \varepsilon_{11}^S \mathbf{K}_{\varepsilon_{11}^S} + \varepsilon_{33}^S \mathbf{K}_{\varepsilon_{33}^S} \quad (245)$$

where $\mathbf{K}_{\varepsilon_{11}^S}$ and $\mathbf{K}_{\varepsilon_{33}^S}$ are the changing of global stiffness due to the unit change of ε_{11}^S and ε_{33}^S , respectively. Since $\varepsilon_{11}^S = 0$ due to the imposed physical simplification, Eq. 245 becomes

$$\mathbf{K}_{\Phi\Phi} = \varepsilon_{33}^S \mathbf{K}_{\varepsilon_{33}^S} \quad (246)$$

Consequently, the condensed eletro-elastic stiffness matrix in Eq. 92 becomes

$$\tilde{\mathbf{K}}_{UU} = \mathbf{K}_{UU} + \mathbf{K}_3 \quad (247)$$

where

$$\mathbf{K}_3 = -a \hat{\mathbf{K}}_{e_{31}} (\mathbf{K}_{\varepsilon_{33}^S})^{-1} \hat{\mathbf{K}}_{e_{31}}^t \quad (248)$$

in which

$$a = \frac{(e_{31})^2}{\varepsilon_{33}^S} \quad (249)$$

After the updated global stiffness matrix of a short-circuit model $\mathbf{K}_{UU}^*$ has been obtained, the finite element model updating formulation for the updated global stiffness matrix of an open-circuit model $\tilde{\mathbf{K}}_{UU}^*$ then can be written as

$$\tilde{\mathbf{K}}_{UU}^* = \mathbf{K}_{UU}^* + (1 + \alpha_3) \mathbf{K}_3 \quad (250)$$

where α_3 are the correction coefficients of a . The form of Eq. 250 agrees with Eq. 173, thus the extended CMCM method can be employed to compute α_3 .

Summary of Updating Material Parameters

In summary, the values of c_{11}^{E*} , c_{12}^{E*} , ε_{33}^{T*} and a^* were first updated at the three phases of the proposed multi-physics finite element model updating method. Next, similar to the derivation of Eqs. 213 and 214, ε_{33}^{S*} and e_{31}^* can be calculated by

$$\varepsilon_{33}^{S*} = \frac{(c_{11}^{E*} + c_{12}^{E*})\varepsilon_{33}^{T*}}{c_{11}^{E*} + c_{12}^{E*} + 2a^*} \quad (251)$$

and

$$e_{31}^* = -\sqrt{a^* \varepsilon_{33}^{S*}} \quad (252)$$

respectively.

For completeness, also presented below are the updated d-form coefficients and engineering parameters which are related to the updated e-form coefficients. Directly from Eq. 228 that $e_{31} = d_{31}(c_{11}^E + c_{12}^E)$, the updated piezoelectric strain constant, d_{31}^* , can be calculated by

$$d_{31}^* = \frac{e_{31}^*}{c_{11}^{E*} + c_{12}^{E*}} \quad (253)$$

Recalling $c_{11}^E = (1 - \nu_{12})Y_{11}^E/[(1 + \nu_{12})(1 - 2\nu_{12})]$ (see Eq. 222) and $c_{12}^E = \nu_{12}Y_{11}^E/[(1 + \nu_{12})(1 - 2\nu_{12})]$ (see Eq. 223), one gets $c_{11}^E/c_{12}^E = (1 - \nu_{12})/\nu_{12}$. From this ratio relationship, one writes the updated Poisson ratio in terms of c_{11}^{E*} and c_{12}^{E*} as

$$\nu_{12}^* = \frac{c_{12}^{E*}}{c_{11}^{E*} + c_{12}^{E*}} \quad (254)$$

Furthermore, one obtains the updated Young's modulus Y_{11}^{E*} in terms of ν_{12}^* and c_{12}^{E*} as

$$Y_{11}^{E*} = \frac{c_{12}^{E*}}{\nu_{12}^*}(1 + \nu_{12}^*)(1 - 2\nu_{12}^*) \quad (255)$$

Once Y_{11}^{E*} and ν_{12}^* are available, one writes

$$s_{11}^{E*} = \frac{1}{Y_{11}^{E*}} \quad (256)$$

and

$$s_{12}^{E*} = -\frac{\nu_{12}^*}{Y_{11}^{E*}} \quad (257)$$

CHAPTER 7

Numerical Study of Model Updating for Tube Transducer

Chosen for the numerical demonstration are two examples: a simulated transducer and a real transducer. Because the target values of parameters are known for the simulated transducer and can be compared with the updated values, the simulated transducer is employed to test the accuracy of the proposed method. Ultimately, the effectiveness of the proposed method must be tested on a real transducer, with the impedance function of the transducer being estimated experimentally. Theoretically, one can use either a 2D or 3D FE model for model updating using the extended multi-physics CCM method. However, the allowed input for the material property in most commercial softwares is for the 3D constitutive equations, not corresponding to the 2D reduced constitutive equations under the axisymmetric assumption; therefore, this study focuses on using the 3D constitutive equations for the model updating of tube transducer.

7.1 Model Updating of Simulated Tube Transducer

In the first example, both the original analytical model and the target model are generated from using the same FE model, but with different sets of material coefficients. It is assumed that the analytical model is with a set of “wrong” coefficients. The goal is to correct those wrong coefficients from the measured modal information which is simulated from the same FE model with “right” coefficients.

The ANSYS FE models in this example are built using the 3D solid element type: SOLID226. These models have the same geometry and poling direction as the transducer shown in Figure. 16. Because engineering parameters are more commonly provided in manufacture specification sheets, they are utilized as the intended updating quantities for the elastic material properties. According to

the theoretical derivation in Chapter 2, as well as the sensitivity study shown in Appendix C, the four parameters that are sensitive to the impedance function of a tube transducer are: Y_{11}^E , ν_{12} , ε_{33}^T , and d_{31} . The original and target values of these four parameters are listed in Table 4. The relative error provided in Table 4 is defined as the difference between the original value and the target value, and it is then normalized by the target value. Figure. 35 shows a comparison of the impedance function of the original model and that of the target model. There are three modes that significantly contributing to the impedance functions. For the impedance function of the target model, the three major resonant frequencies are 51.4, 55, and 82.6 KHz, and the three anti-resonant frequencies are 52.4, 55.8, and 101.6 KHz.

Table 4. Material properties of the original and target models

Parameter	Unit	Original Value	Target Value	Relative Error
Y_{11}^E	$10^{10} (N/m^2)$	6.3	6.66	5.4054%
ν_{12}	—	0.31	0.3333	6.9907%
d_{31}	$10^{-12} (C/N)$	−175	−210	16.6667%
$\varepsilon_{33}^T/\varepsilon_0$	—	1576	2294	31.2990%

7.1.1 Updating of Material Parameters

Given the three modal frequencies of the target short-circuit model: 51.4, 55 and 82.6 KHz, and those of the target open-circuit model: 52.4, 55.8 and 101.6 KHz, the task is to apply the extended CMC method to perform the multi-physics model updating for the analytical piezoceramic tube transducer model.

Updating of Elastic Parameters

The first step is to update the elastic parameters of the analytical piezoceramic tube transducer model, using the short-circuit model (or pure mechanical model). As described in Chapter 6, a physical simplification is to assume an isotropic

Figure 35. The comparison of the impedance function between the original and target models

material in the finite element formulation. Given the Young's modulus $Y_{11}^E = 6.3 \times 10^{10} \text{ N/m}^2$ and Poisson's ratio $\nu_{12} = 0.31$, the following stiffness parameters are computed:

$$\begin{aligned} c_{11}^E &= c_{33}^E = \frac{(1 - \nu_{12})Y_{11}^E}{(1 + \nu_{12})(1 - 2\nu_{12})} = 8.73 \times 10^{10} \text{ N/m}^2 \\ c_{12}^E &= c_{13}^E = \frac{\nu_{12}Y_{11}^E}{(1 + \nu_{12})(1 - 2\nu_{12})} = 3.92 \times 10^{10} \text{ N/m}^2 \\ c_{44}^E &= c_{66}^E = \frac{Y_{11}^E}{2(1 + \nu_{12})} = 2.415 \times 10^{10} \text{ N/m}^2 \end{aligned}$$

For formulating the CMCM equations in Eq. 241, one first needs to extract $\hat{\mathbf{K}}_{c_{11}^E}$ and $\hat{\mathbf{K}}_{c_{12}^E}$. The ways to extract $\hat{\mathbf{K}}_{c_{11}^E}$ and $\hat{\mathbf{K}}_{c_{12}^E}$ are basically identical. Eq. 239 shows that $\hat{\mathbf{K}}_{c_{11}^E}$ is a linear combination of $\mathbf{K}_{c_{11}^E}$, $\mathbf{K}_{c_{33}^E}$ and $\mathbf{K}_{c_{44}^E}$. While using ANSYS, a possible way of extracting $\mathbf{K}_{c_{11}^E}$ is the same as that of extracting $\mathbf{K}_{c_{33}^E}$ or $\mathbf{K}_{c_{44}^E}$.

The following presents a way of obtaining $\mathbf{K}_{c_{11}^E}$ numerically. First, build a short-circuit FE model with distinct values of the five stiffness parameters: c_{11}^E , c_{12}^E , c_{13}^E , c_{33}^E and c_{44}^E , and denote the corresponding numerical global stiffness matrix to be $\mathbf{K}_{UU}^{(1)}$. Second, build another short-circuit FE model with the same values for these stiffness parameters, except doubling the value of c_{11}^E , and denote the corresponding numerical global stiffness matrix to be $\mathbf{K}_{UU}^{(2)}$. Recall that $\mathbf{K}_{c_{11}^E}$ is defined as the change of $\mathbf{K}_{UU}$ (global stiffness matrix for the short-circuit model) due to the unit change of c_{11}^E . Thus one possible way of obtaining $\mathbf{K}_{c_{11}^E}$ numerically is by performing the following operation:

$$\mathbf{K}_{c_{11}^E} = \frac{\mathbf{K}_{UU}^{(2)} - \mathbf{K}_{UU}^{(1)}}{c_{11}^E} \quad (258)$$

Given the three modal frequencies of the target short-circuit model: 51.4, 55 and 82.6 KHz, and two submatrices: $\hat{\mathbf{K}}_{c_{11}^E}$ and $\hat{\mathbf{K}}_{c_{12}^E}$, the extended CMCM method is applied to estimate α_1 and α_2 which are the correction coefficients of c_{11}^E and c_{12}^E , respectively.

Figure 36. Iteration procedure of α_1 and α_2 on simulation model

Figure. 36 shows the convergence of α_1 and α_2 . After 14 iteration steps, these values converge to 0.1455 and 0.2755, respectively, and match the target values accurately. The updated elastic parameters c_{11}^{E*} and c_{12}^{E*} can be calculated by

$$c_{11}^{E*} = (1 + \alpha_1)c_{11}^E = 10 \times 10^{10} N/m^2$$

and

$$c_{12}^{E*} = (1 + \alpha_2)c_{12}^E = 5.0004 \times 10^{10} N/m^2$$

respectively.

Updating of Permittivity

Because the mechanical boundary condition is free-free, the related permittivity of this unclamped transducer is the permittivity at constant stress. According

to Eq. 201, with the impedance, $Z_L = 26804\Omega$, from the impedance function at 400 Hz, the permittivity at constant stress is

$$\varepsilon_{33}^{T*} = \frac{\ln(D_o/D_i)}{4\pi^2\ell Z_L f_L} = 2293.3\varepsilon_0$$

where the vacuum permittivity $\varepsilon_0 = 8.854187817 \times 10^{-12}C/m$.

Updating of Piezoelectric Parameters

After updating the elastic parameters and permittivity, the open-circuited FE model, together with the anti-resonant frequencies of the target model, can be employed to update the piezoelectric parameter d_{31} (or e_{31}). Given the original piezoelectric constant $d_{31} = -175 \times 10^{-12}C/N$ and permittivity at constant stress $\varepsilon_{33}^T = 1576\varepsilon_0$, the original e-form parameters are computed:

$$\begin{aligned} e_{31} &= (c_{11}^{E*} + c_{12}^{E*})d_{31} = -26.25N/V \cdot m \\ e_{33} &= 2c_{12}^{E*}d_{31} = -17.5N/V \cdot m \\ \varepsilon_{33}^S &= \varepsilon_{33}^T - 2d_{31}^2(c_{11}^{E*} + c_{12}^{E*}) = 538\varepsilon_0 \end{aligned}$$

For formulating the CMCM equations in Eq. 250, one needs to compute the original linear variable

$$a = \frac{(e_{31})^2}{\varepsilon_{33}^S} = 1.2808/\varepsilon_0$$

and extract submatrices $\hat{\mathbf{K}}_{e_{31}}$ and $\mathbf{K}_{\varepsilon_{33}^S}$. Eq. 244 shows that $\hat{\mathbf{K}}_{e_{31}}$ is a linear combination of $\mathbf{K}_{e_{33}}$ and $\mathbf{K}_{e_{31}}$. While using ANSYS, the way of extracting $\mathbf{K}_{\varepsilon_{33}^S}$, $\mathbf{K}_{e_{31}}$ and $\mathbf{K}_{e_{31}}$ is the same as that of extracting $\mathbf{K}_{c_{11}^E}$.

Given the three modal frequencies of the target open-circuit model: 52.4, 55.8, and 101.6 KHz, and two submatrices: $\hat{\mathbf{K}}_{e_{31}}$ and $\mathbf{K}_{\varepsilon_{33}^S}$, the extended CMCM method is applied to estimate α_3 , which is the correction coefficient of a , i.e., the correction coefficient for $(e_{31})^2/\varepsilon_{33}^S$.

Figure. 37 shows the convergence of the correction coefficient of α_3 . During this procedure, the value of the coefficient drops sharply to the point of -0.0284

Figure 37. Iteration procedure of α_3 on simulation model

at the first iteration. After that, it continues to decline slightly until it converges to -0.0315 after 300 steps. The updated parameter a^* is

$$a^* = (1 + \alpha_3)a = 1.2404/\varepsilon_0$$

7.1.2 Summary of the Updating of Model Parameters

The values of c_{11}^{E*} , c_{12}^{E*} , ε_{33}^{T*} and a^* were first updated in three phases of the multi-physics FE model updating method. Next, according to Eqs. 251 and 252, the updated permittivity at constant strain, ε_{33}^{S*} , and the piezoelectric stress constant, e_{31}^* , can be calculated by

$$\varepsilon_{33}^{S*} = \frac{(c_{11}^{E*} + c_{12}^{E*})\varepsilon_{33}^{T*}}{c_{11}^{E*} + c_{12}^{E*} + 2a^*} = 800\varepsilon_0$$

and

$$e_{31}^* = -\sqrt{a^* \varepsilon_{33}^{S*}} = -31.5 N/V \cdot m$$

respectively.

For completeness, also presented below are the updated d-form coefficients and engineering parameters which are related to the updated e-form coefficients. According to Eq. 253, the updated piezoelectric strain constant, d_{31}^* , can be calculated by

$$d_{31}^* = \frac{e_{31}^*}{c_{11}^{E*} + c_{12}^{E*}} = -209.96 \times 10^{-12} C/N$$

Furthermore, the updated Poisson ratio, ν_{12}^* , and Young's modulus, Y_{11}^{E*} , can be calculated by

$$\nu_{12}^* = \frac{c_{12}^{E*}}{c_{11}^{E*} + c_{12}^{E*}} = 0.3333$$

and

$$Y_{11}^{E*} = \frac{c_{12}^{E*}}{\nu_{12}^*} (1 + \nu_{12}^*) (1 - 2\nu_{12}^*) = 6.66 \times 10^{10} N/m^2$$

respectively.

In summary, the comparison between the original values, target values, and updated values is given in Table 5. The relative error provided in Table 5 is defined as the difference between the updated value and the target value, normalized by the target value. The corresponding impedance functions obtained from the target and updated models are shown in Figure. 38. We found that the updated parameters from the proposed method match perfectly with the target values, and the proposed method can handle well the model updating of cylinder transducers.

7.2 Model Updating of Real Tube Transducer

In the second example, the real transducer, shown in Figure. 6, is employed which has the same geometry and boundary conditions as utilized for the simulation example. Lab experiment of estimating impedance function for it was con-

Table 5. The comparison of properties between original, target and updated simulation models

Parameters	Unit	Original Value	Target Value	Updated Value	Relative error
Y_{11}^E	$(10^{10}N/m^2)$	6.3	6.66	6.66	0%
ν_{12}	—	0.31	0.3333	0.3333	0%
d_{31}	$(10^{-12}C/N)$	-175	-209.96	-210	0.019%
$\varepsilon_{33}^T/\varepsilon_0$	—	1576	2293.3	2294	0.0305%

Figure 38. The comparison of the impedance functions between the target and updated models

ducted in Chapter 5. And the original material properties of its FE model were obtained from the manufacturer's specification sheet of APC material 850 (Navy Equivalent Navy II, APC International, Ltd (Ltd, 2011)): $Y_{11}^E = 6.3 \times 10^{10} N/m^2$, $\nu_{12} = 0.31$, $d_{31} = -175 \times 10^{-12} C/N$, $\varepsilon_{33}^T = 1900\varepsilon_0$.

Figure 39. The comparison of the impedance functions between the original model and the experiment

Figure. 39 shows the impedance functions that were obtained from the original FE model and experiment. With the frequencies at the local minimum of the impedance function for the experiment being 49.89, 52.99, and 78.75 KHz, the elastic parameters for the real transducer can be updated using the same procedure as in the simulation example.

Figure 40. Iteration procedure of α_1 and α_2 on experiment model

Figure. 40 shows the convergence of the correction coefficients of α_1 and α_2 . After 14 iteration steps, these values converge to 0.025 and 0.1013, respectively. Then, the updated elastic parameters c_{11}^{E*} and c_{12}^{E*} can be calculated by

$$c_{11}^{E*} = (1 + \alpha_1)c_{11}^E = 8.9503 \times 10^{10} N/m^2$$

and

$$c_{12}^{E*} = (1 + \alpha_2)c_{12}^E = 4.3209 \times 10^{10} N/m^2$$

respectively.

Because the transducer in the experiment is placed on the sponge (free-free mechanical boundary condition), the permittivity is the permittivity at constant stress. According to Eq. 201, with the impedance, $Z_L = 9617\Omega$, from impedance function at 1 KHz, the permittivity at constant stress is

$$\varepsilon_{33}^{T*} = \frac{\ln(D_o/D_i)}{4\pi^2\ell Z_L f_L} = 2556\varepsilon_0$$

With the frequencies at the local maximum of the impedance function of the experiment being 50.8, 53.94, and 94.7 KHz, the piezoelectric parameter for the real transducer can also be updated using the same procedure as in the simulation example.

Figure 41. Iteration procedure of α_3 on experiment model

Figure. 41 shows the convergence of the correction coefficient α_3 . After five

iterations, α_3 converges to 0.4556, and the updated parameter $a^* = 0.7996/\varepsilon_0$.

According to Eqs. 251 and 252, after updating c_{11}^{E*} , c_{12}^{E*} , ε_{33}^{T*} , and a^* , the updated permittivity at constant strain, ε_{33}^{S*} , and the piezoelectric stress constant, e_{31}^* , can be calculated by

$$\varepsilon_{33}^{S*} = \frac{(c_{11}^{E*} + c_{12}^{E*})\varepsilon_{33}^{T*}}{c_{11}^{E*} + c_{12}^{E*} + 2a^*} = 1082\varepsilon_0$$

and

$$e_{31}^* = -\sqrt{a^* \varepsilon_{33}^{S*}} = -29.42 \text{ N/V}\cdot\text{m}$$

respectively.

By converting to d-form coefficients and engineering parameters, the updated piezoelectric strain constant, d_{31}^* , Poisson ratio, ν_{12}^* , and Young's modulus, Y_{11}^{E*} , can be calculated by

$$d_{31}^* = \frac{e_{31}^*}{c_{11}^{E*} + c_{12}^{E*}} = -221.7 \times 10^{-12} \text{ C/N}$$

,

$$\nu_{12}^* = \frac{c_{12}^{E*}}{c_{11}^{E*} + c_{12}^{E*}} = 0.3256$$

and

$$Y_{11}^{E*} = \frac{c_{12}^{E*}}{\nu_{12}^*} (1 + \nu_{12}^*) (1 - 2\nu_{12}^*) = 6.1367 \times 10^{10} \text{ N/m}^2$$

respectively. A comparison of the original values and updated values is shown in Table 6. The relative error presented in Table 6 is defined as the difference between the original value and the updated value, then normalized by the updated value. Figure. 42 shows the resulting impedance function of the updated model vs. the experimental estimation. We obtained the Rayleigh damping coefficients $\alpha \approx 0$ and $\beta = -3.996$ from the experiment using Eqs.168 and 169. Again, this result suggests that the proposed method can also handle the model updating of real transducers.

Table 6. The comparison of properties between original and updated experiment models

Parameters	Unit	Original	Updated	Relative error
Y_{11}^E	$10^{10} N/m^2$	6.3	6.1367	-2.66%
ν_{12}	—	0.31	0.3256	4.7912%
d_{31}	$10^{-12} C/N$	-175	-221.7	21.0645%
$\varepsilon_{33}^T/\varepsilon_0$	—	1900	2556	25.67%

Figure 42. The comparison of the impedance function between the updated model and the experiment

CHAPTER 8

Conclusions

Finite element model updating has been defined as the systematic adjustment of finite element models using experimental data. In this study, for the finite element model updating of a piezoelectric tube transducer, the experimental data has been limited to be the measured electric impedance function of the tube transducer. This study covered two major tasks: (i) developing and verifying an efficient method for estimating the electric impedance function of a transducer, and (ii) developing and testing a finite element model updating method for piezoelectric tube transducers. Although this study has focused on the correction of the material parameters of tube type of piezoelectric transducers, the general procedure is also applicable to other types of piezoelectric transducers. The newly developed method on estimating the impedance function of a transducer is a Laplace domain method. The proposed model updating method is a two-step method, including the correction of the elastic material properties based on a short circuit model in the first step, and the correction of dielectric and piezoelectric parameters based on an open-circuit model in the second step. The fundamental updating algorithm employed in both steps is the CMCM method. Both computer simulations and lab experiments have been conducted to verify the correctness and accuracy of the proposed methods on the two tasks mentioned above.

Major findings and contributions are summarized as follows:

Modal analysis of the finite element models This study shed new light on the modal analysis of the short- and open-circuit finite element models associated with piezoelectric tube transducers. It has been demonstrated that the resonant and anti-resonant frequencies of an impedance function matched

well with the modal frequencies of the short- and open-circuit models, respectively. Mode shapes of a piezoelectric tube transducer could not be measured at lab, but they are necessary information to conduct the CMCM model updating method. Due to the uniform voltage (electromotive force) along the axial direction, only symmetric modes (to the axial center of the tube transducer) would “participate” in the mechanical vibration. In turn, only symmetric modes would affect the impedance function of the tube transducer. This study verified that the most dominant mode identified from the impedance function was the mode with axially uniform mode shape. Basically, three symmetric modes would participate in the impedance function of piezoelectric tube transducer: breathing mode, bending mode and axial mode. These three mode forms did not have deformation in the tube’s thickness direction (i.e., poling direction or 3-direction). Consequently, elastic material parameters associated with the 3-direction would not affect the impedance function of piezoelectric tube transducers.

Estimating the impedance function Processing both the input (voltage) and output (charge or current) signals simultaneously, the proposed multi-signal Prony-SS method was able to extract the poles and residues of the input and output signals. After expressing both the voltage and current in their partial fraction forms in the Laplace domain, the impedance function of the transducer was obtained from the ratio of the voltage and current as a function of frequency, ω , per substituting the Laplace variable s by $j\omega$. In comparison to traditional Fourier-based methods, the proposed Prony-based method did not suffer any leakage problems or resolution issues. Another advantage of the proposed method is that it requires only very short signals to obtain the impedance function. The accuracy and efficiency of the proposed

method has been demonstrated by using both computer simulation and lab experiment data. This study also demonstrated the excellent capability of the proposed method on rejecting noise.

Reduced constitutive equations There are ten material parameters to fully character the constitutive equations of PZT-type material. However, the reduced constitutive equations associated with piezoelectric tube transducers contains only four independent material parameters: two elastic parameters Y_{11}^E and ν_{12} (or s_{11}^E and s_{12}^E), the permittivity in constant stress ε_{33}^T , and the piezoelectric parameter d_{31} . In other words, the impedance function of a piezoelectric tube transducer should depend only on these four material parameters. Through the sensitivity study for each material parameter, this study also numerically verified that the impedance function of a piezoelectric tube transducer depends only on those four material parameters. Specifically, the values of Y_{11}^E and ν_{12} determined the resonant frequencies in the impedance function; the value of ε_{33}^T decided the impedance at the low frequency region; and the value of d_{31} determined the anti-resonant frequencies in the impedance function.

Implementation of ANSYS finite element models This study adopted the commercial finite element software ANSYS to build piezoelectric tube transducers. However, ANSYS would not allow the user to build a finite element model based on the reduced constitutive equations. In other words, one must specify all 10 material parameters in the d-form or e-form for the full 3-dimensional constitutive equations. For using the CMCM method, it is required to express the global stiffness matrix as a linear combination of only those parameters to be updated, and those parameters must be associated with the e-form. Thus, particular steps have been taken to formulate the

global stiffness matrix from ANSYS to be a linear form of elastic parameters c_{11}^E and c_{12}^E . In this study, the submatrices associated with c_{11}^E and c_{12}^E were successfully extracted while using ANSYS. Similarly, submatrices associated with ε_{33}^S and e_{31} were extracted as well.

Multi-physics finite element model updating Although there have been many papers conducting the finite element model updating for civil and mechanical systems, only a few articles addressed the model updating for piezoelectric transducers. This study has been the literature to conduct the multi-physics finite element model updating using a two-step approach. Because a short-circuit transducer is equivalent to a pure mechanical oscillator, it was reasonable to update the elastic parameters of a piezoelectric transducer in the first step based on a short-circuit finite element model. In the second step for updating the electric and piezoelectric parameters, it was necessary to employ an open-circuit finite element model. Implementing the CMM algorithm required the knowledge of both the measured modal frequencies and the corresponding mode shapes, but a measured impedance function could provide only modal frequencies for short- and open-circuit transducers. In dealing with the incomplete modal information, an iterative procedure was taken. In each iteration, the “measured” mode shapes were approximated by the mode shapes obtained from the previous iteration’s updated finite element model. After a sufficient number of iterations, a converged solution (model) was always obtained. The high performance of the proposed two-step approach has been demonstrated through both computer simulations and lab experiment.

APPENDIX A

Simple 33- and 31-mode Vibrators

If DC voltages are externally applied, the transducer responds with a mechanical deformation. Similarly, an AC field will cause the transducer to vibrate as a vibrator. To better understand piezoelectric vibrators, we introduced two typical one-dimensional vibrators, 33-mode and 31-mode vibrators (Sherman and Butler, 2007).

Shown in Figure. A.1 is an idealized 33-mode vibrator. A piezoelectric ceramic

Figure A.1. The piezoelectric longitudinal vibrator in 33 mode

bar of length L is fixed at one end, and is attached to a mass M at the other end. This piezoelectric bar is polarized using electrodes on the ends to establish the polar axis parallel to the length of the bar. An alternating voltage, V , is applied between the same electrodes, creating an alternating electric field, E_3 , which is parallel to the polarization. Assume that the stresses T_1 , T_2 and electric fields E_1 , E_2 in no poling directions are both neglected, and the purely longitudinal electric field does not excite shear stresses, (i.e., $T_4 = T_5 = T_6 = 0$). Therefore, the

governing state equations for this kind of transducer can be written as

$$S_1 = s_{13}^E T_3 + d_{31} E_3 \quad (\text{A.1})$$

$$S_2 = s_{13}^E T_3 + d_{31} E_3 \quad (\text{A.2})$$

$$S_3 = s_{33}^E T_3 + d_{33} E_3 \quad (\text{A.3})$$

$$D_3 = d_{33} T_3 + \varepsilon_{33}^T E_3 \quad (\text{A.4})$$

These lateral strains, $S_1 = S_2$, are caused by a Poisson ratio effect that is modified by the piezoelectric strain, and they play no role in the operation of the transducer because the sides are not usually in contact with the acoustic medium in real transducers.

Another type of longitudinal vibrator uses the same ceramic bar in a different way, and is called a 31-mode vibrator. This vibrator has lower coupling, but has the advantage of being less susceptible to depoling by static-pressure cycling because the polarization is perpendicular to the static stress. We consider the bar to have lateral dimensions h and w , and to have been polarized using electrodes on the sides of area wL , as shown in Figure. A.2.

The polar axis is now perpendicular to the length, and is parallel to the side with dimension h . One end of the bar is fixed and the other end is attached to the mass as before. The only non-zero stress component is still parallel to the length of the bar, but it is now called T_1 , with $T_2 = T_3 = 0$. The driving voltage is applied between the electrodes used for polarizing, and E_3 is the only electric field component. In this case, the governing equations become

$$S_1 = s_{11}^E T_1 + d_{31} E_3 \quad (\text{A.5})$$

Figure A.2. The piezoelectric longitudinal vibrator in 31 mode

$$S_2 = s_{12}^E T_1 + d_{31} E_3 \quad (\text{A.6})$$

$$S_3 = s_{13}^E T_1 + d_{33} E_3 \quad (\text{A.7})$$

$$D_3 = d_{31} T_1 + \varepsilon_{33}^T E_3 \quad (\text{A.8})$$

APPENDIX B

Modeling and Interfacing with ANSYS

B.1 Modeling for Piezoelectric Structures with ANSYS

ANSYS is one of the most commonly used commercial FE packages in many engineering fields. It is used to simplify the development of FE models as users can operate step-by-step to determine whether a specific process is relevant to the problem being addressed. There are six main steps in the development of FE models using ANSYS:

1. Define the element types;
2. Define the material properties;
3. Define the model geometry;
4. Generate the mesh;
5. Apply the constraints;
6. Start the analysis.

In terms of the piezoelectric analysis, ANSYS provides several powerful elements to improve the efficiency of engineering designs. Some important elements and their key options for piezoelectric material are as follows:

2-D Axial Symmetric Elements:

- Plane13, KEYOPT(1) = 7, coupled-field quadrilateral solid;
- Plane223, KEYOPT(1) = 1001, coupled-field 8-node quadrilateral.

3-D Elements:

- Solid5, KEYOPT(1) = 0 or 3 coupled-field brick;
- Solid98, KEYOPT(1) = 0 or 3 coupled-field tetrahedron;
- Solid226, KEYOPT(1) = 1001, coupled-field 20-node brick;
- Solid227, KEYOPT(1) = 1001, coupled-field 10-node tetrahedron.

Note that the permittivity of Plane223, Solid226, and Solid227 can be specified either as PERX, PERY, and PERZ on the MP command, respectively, or by specifying the terms of the anisotropic permittivity matrix using the TB, DPER, and TBDATA commands, respectively. If we use the MP command to specify the permittivity, the permittivity input will be interpreted as the permittivity at constant strain. If we use the TB, DPER command, we can specify the permittivity matrix either at constant strain ϵ^S (TBOPT = 0) or at constant stress ϵ^T (TBOPT = 1). The latter input will be internally converted to the permittivity at constant strain ϵ^S using the piezoelectric strain and stress matrices. The values that are inputted on either MP, PERX or TB, DPER will always be interpreted as relative permittivity values.

It is important to note the difference between the notation standards of ANSYS and IEEE. For most published piezoelectric materials, the order used for the piezoelectric matrix is x, y, z, yz, xz, xy, based on IEEE standards (Voigt notation) (IEE, 1987; Helnwein, 2001), while the ANSYS input order is x, y, z, xy, yz, xz. This means that we need to transform the IEEE standards matrix to the ANSYS input order by switching the row data for the shear terms. A comparison of notations used by ANSYS and IEEE for tensor indices is given in Table B.1. Therefore, we can obtain the ANSYS parameter matrix from the IEEE standard matrix by switching 4(yz) and 6(xy) ($4 \longleftrightarrow 6$), and then switching 5(xz) and new 6(yz) ($5 \longleftrightarrow 6$) (Kohnke, 1999).

Table B.1. Tensor indices notations

	IEEE standard	ANSYS
1	x	x
2	y	y
3	z	z
4	yz	xy
5	xz	yz
6	xy	xz

B.2 Interfacing with ANSYS

The ANSYS program writes several binary files to store data during the analysis. There are many different binary files, and the ones used in this study include:

- Jobname.MODE file: stores data related to a modal analysis (modal shapes and mode frequencies);
- Jobname.FULL file: stores the full stiffness-mass matrices.

The data in these binary files are written in Harwell-Boeing format, which is the most popular mechanism for the text-file exchange of sparse matrix data. For portability, its matrix data is held in an 80-column, fixed-length format. Each matrix begins with a multiple-line header block, which is followed by two, three, or four data blocks. The header block contains summary information regarding the storage formats and space requirements. From the header block alone, the user can determine how much space will be required to store the matrix. Information on the size of the representation in lines is given for ease in overlooking unwanted data. For more information, one can refer to References.(ANSYS, 2005; Duff et al., 1992).

The procedure for exporting ANSYS models into MATLAB matrices can be divided into 2 steps:

- 1, Generating binary files from ANSYS

A modal analysis must be launched if a mass matrix is needed. Otherwise, a simple static analysis is sufficient for writing the stiffness matrix into a binary file. The command WRFULL makes the analysis stop after writing the files, meaning that ANSYS will not actually do the analysis. This command can be useful and help to avoid long calculations.

2, Reading and converting Harwell-Boeing format with MATLAB

The matrices written by ANSYS are compatible with the Harwell-Boeing (H-B) format, and must be converted into MATLAB format. A library and stand-alone utility developed by the Berkeley Benchmarking and Optimization (BeBOP) Group can be used to convert between the Harwell-Boeing, Matrix Market, and MATLAB sparse matrix formats. A version for complex matrices is also available (Tex, 2013).

APPENDIX C

Parameter Sensitivity of Piezoelectric Tube Transducer

There are 10 independent parameters for piezoelectric materials (such as PZT), which can be divided into three groups: five elastic parameters, two dielectric parameters, and three piezoelectric parameters.

C.1 Sensitivity of d-form Parameters

In d-form, the five elastic parameters are s_{11}^E , s_{12}^E , s_{13}^E , s_{33}^E , and s_{44}^E , while the two dielectric parameters are ε_{11}^T and ε_{33}^T , and the three piezoelectric parameters are d_{31} , d_{33} , and d_{15} . Of the five elastic parameters, as proven in Eq. 16, only two parameters, i.e., s_{11}^E and s_{12}^E , theoretically affect the mechanical and electric behaviors of tube transducers. We can use the sensitivity study of the FE model to prove this theoretical deduction. When the elastic parameters (s_{11}^E , s_{33}^E , s_{12}^E , s_{13}^E and s_{44}^E) are doubled respectively, Table C.1 shows the sensitivity of the impedance function to elastic parameters. Using the engineering parameters Y_{11}^E , Y_{33}^E , ν_{12} , ν_{13} , and G to substitute s_{11}^E , s_{33}^E , s_{12}^E , s_{13}^E , and s_{44}^E as elastic parameters, respectively, the sensitivity study of the impedance function is as shown in Table C.2. Then, we verified numerically the theoretical deduction, i.e., that only two elastic parameters Y_{11}^E , ν_{12} or s_{11}^E , s_{12}^E in d-form can affect the impedance function.

In terms of the dielectric and piezoelectric parameters, in theory, only one dielectric parameter ε_{33}^T and one piezoelectric parameter d_{31} affect the electric behaviors of transducers. Dividing the parameters by two respectively, the sensitivity study of the impedance function is as shown in Table C.3. We can easily see that the sensitivity study result agrees well with the theoretical deduction. The permittivity at constant stress ε_{33}^T determines the impedance at the low-frequency region, and the piezoelectric strain constant d_{31} influences the anti-resonance frequencies.

Table C.1. Elastic parameters sensitivity in the d-form

Table C.2. Elastic parameters sensitivity in the d-form (engineering parameters)

Table C.3. Dielectric and piezoelectric parameters sensitivity in the d-form

C.2 Sensitivity of e-form Parameters

In contrast, we can also choose e-form parameters, c_{11}^E , c_{33}^E , c_{12}^E , c_{13}^E , c_{44}^E , ε_{33}^S , ε_{11}^S , e_{31} , e_{33} , and e_{15} , to develop the FE models. Tables C.4 and C.5 show the results of the sensitivity study when we double the elastic parameters and divide the dielectric and piezoelectric parameters by two, respectively.

We can easily find that e-form parameters, c_{11}^E , c_{12}^E , c_{13}^E , c_{33}^E , e_{31} , e_{33} , and ε_{33}^S all affect the impedance function of transducers. Because only four parameters determine the vibration of a thin-tube transducer in the d-form, we can use an infinite number of e-form parameter groups to develop FE models that share the same impedance function.

Table C.4. Elastic parameters sensitivity in the e-form

Table C.5. Dielectric and piezoelectric parameters sensitivity in the e-form

APPENDIX D

Model Updating with Measured Mode Shapes

While the CMCM method can achieve the model updating for thin-tube transducers using the impedance function through an iteration procedure, it can also update all independent parameters directly with mode shapes that are difficult to measure experimentally but easy to obtain from FE models. Although this method cannot currently be used in model updating for real transducers, in future, it will enable model updating without iterations.

D.1 Model Updating of Elastic Parameters

The global stiffness matrix of a short-circuit model in linear combination form with coefficients c_{11}^E , c_{12}^E , c_{13}^E , c_{33}^E , and c_{44}^E is shown in Eq. 234. Consistent with the formulation for the development of the CMCM method, the updated global stiffness matrix can be written as

$$\begin{aligned}\mathbf{K}_{UU}^* &= (1 + \alpha_1)c_{11}^E \mathbf{K}_{c_{11}^E} + (1 + \alpha_2)c_{12}^E \mathbf{K}_{c_{12}^E} + (1 + \alpha_3)c_{13}^E \mathbf{K}_{c_{13}^E} \\ &\quad + (1 + \alpha_4)c_{33}^E \mathbf{K}_{c_{33}^E} + (1 + \alpha_5)c_{44}^E \mathbf{K}_{c_{44}^E} \\ &= \mathbf{K}_{UU} + \sum_{i=1}^5 \alpha_i \mathbf{K}_i\end{aligned}\tag{D.1}$$

where α_1 , α_2 , α_3 , α_4 , and α_5 are the correction coefficients of $c_{11}^E, c_{12}^E, c_{13}^E, c_{33}^E$, and c_{44}^E respectively. Therefore, we can perform the CMCM method using target mode shapes to compute the correction coefficients ($\alpha_1, \alpha_2, \alpha_3, \alpha_4$, and α_5) directly.

D.2 Model Updating of Electric-coupling Parameters

Assuming ε_{11}^S to be zero and substituting Eq. 242 and 245 into 92, the global stiffness matrix in the open-circuit model can be written as

$$\tilde{\mathbf{K}}_{UU} = \mathbf{K}_{UU} + \mathbf{K}_3\tag{D.2}$$

where

$$\mathbf{K}_3 = b_1 \mathbf{K}_6 + b_2 \mathbf{K}_7 + b_3 \mathbf{K}_8 + \sqrt{b_1 b_2} \mathbf{K}_9 + \sqrt{b_1 b_3} \mathbf{K}_{10} + \sqrt{b_2 b_3} \mathbf{K}_{11} \quad (\text{D.3})$$

in which $b_1 = e_{31}^2/\varepsilon_{33}^S$, $b_2 = e_{33}^2/\varepsilon_{33}^S$, $b_3 = e_{15}^2/\varepsilon_{33}^S$, and the submatrices are

$$\mathbf{K}_6 = -\mathbf{K}_{e_{31}} \mathbf{K}_{\varepsilon_{33}^S}^{-1} \mathbf{K}_{e_{31}}^t \quad (\text{D.4})$$

$$\mathbf{K}_7 = -\mathbf{K}_{e_{33}} \mathbf{K}_{\varepsilon_{33}^S}^{-1} \mathbf{K}_{e_{33}}^t \quad (\text{D.5})$$

$$\mathbf{K}_8 = -\mathbf{K}_{e_{15}} \mathbf{K}_{\varepsilon_{33}^S}^{-1} \mathbf{K}_{e_{15}}^t \quad (\text{D.6})$$

$$\mathbf{K}_9 = -\mathbf{K}_{e_{31}} \mathbf{K}_{\varepsilon_{33}^S}^{-1} \mathbf{K}_{e_{33}}^t - \mathbf{K}_{e_{33}} \mathbf{K}_{\varepsilon_{33}^S}^{-1} \mathbf{K}_{e_{31}}^t \quad (\text{D.7})$$

$$\mathbf{K}_{10} = -\mathbf{K}_{e_{15}} \mathbf{K}_{\varepsilon_{33}^S}^{-1} \mathbf{K}_{e_{33}}^t - \mathbf{K}_{e_{33}} \mathbf{K}_{\varepsilon_{33}^S}^{-1} \mathbf{K}_{e_{15}}^t \quad (\text{D.8})$$

$$\mathbf{K}_{11} = -\mathbf{K}_{e_{31}} \mathbf{K}_{\varepsilon_{33}^S}^{-1} \mathbf{K}_{e_{15}}^t - \mathbf{K}_{e_{15}} \mathbf{K}_{\varepsilon_{33}^S}^{-1} \mathbf{K}_{e_{31}}^t \quad (\text{D.9})$$

Consistent with the formulation for the development of the CMCM method, the updated global stiffness matrix in the open-circuit model can be written as

$$\begin{aligned} \tilde{\mathbf{K}}_{UU}^* &= \mathbf{K}_{UU}^* + (1 + \alpha_6) b_1 \mathbf{K}_6 + (1 + \alpha_7) b_2 \mathbf{K}_7 + (1 + \alpha_8) b_3 \mathbf{K}_8 \\ &\quad + \sqrt{(1 + \alpha_6) b_1 (1 + \alpha_7) b_2} \mathbf{K}_9 + \sqrt{(1 + \alpha_6) b_1 (1 + \alpha_8) b_3} \mathbf{K}_{10} + \sqrt{(1 + \alpha_7) b_2 (1 + \alpha_8) b_3} \mathbf{K}_{11} \end{aligned} \quad (\text{D.10})$$

where α_6 , α_7 , and α_8 are the correction coefficients of b_1 , b_2 , and b_3 respectively.

The form of Eq. D.10 agrees with Eq. 173; thus, we can perform the CMCM method to compute α_6 , α_7 , and α_8 . Although α_6 , α_7 , and α_8 can be used to update $e_{31}^2/\varepsilon_{33}^S$, $e_{33}^2/\varepsilon_{33}^S$, and $e_{15}^2/\varepsilon_{33}^S$, respectively, updating e_{31} , e_{33} , e_{15} , and ε_{33}^S individually requires at least one more equation. For an unclamped thin cylindrical transducer polarized in the radial direction, from $\boldsymbol{\varepsilon}^S = \boldsymbol{\varepsilon}^T - \mathbf{e} \mathbf{s}^E \mathbf{e}^t$ (see Eq. 12), the additional equation is

$$\varepsilon_{33}^{S*} = \varepsilon_{33}^{T*} - \frac{2(s_{11}^* + s_{12}^*)(e_{31}^*)^2 + 4s_{13}^* e_{33}^* e_{31}^* + s_{33}^* (e_{33}^*)^2}{\varepsilon_0} \quad (\text{D.11})$$

where ε_{33}^{T*} is the permittivity at constant stress, which can be calculated from the measured capacitance of the transducer at low frequencies.

D.3 Numerical Study

We performed a simulation example to test the accuracy of the proposed method. In this example, both the original model and the target model are generated by using the same 3D FE model with different sets of coefficients. Using d-form parameters ($c_{11}^E, c_{12}^E, c_{13}^E, c_{33}^E, c_{44}^E, \varepsilon_{33}^S, e_{33}, e_{31}$, and e_{15}) to build FE models, the result of model updating is as shown in Table D.1. This result suggests that the proposed method can handle well the model updating of tube transducers.

Table D.1. The comparison of properties between original, target and updated models

Parameters	Original	Target	Updated
c_{11}^E	9.2e10	11.04e10	11.0399e10
c_{12}^E	4.4e10	5.104e10	5.1041e10
c_{13}^E	4.2e10	4.62e10	4.62e10
c_{33}^E	8e10	8.48e10	8.4798e10
c_{44}^E	2.2e10	2.53e10	2.5302e10
ε_{33}^S	538	800	799.393
e_{33}	17.3	30	30.0132
e_{31}	-5.363	-10	-9.989
e_{15}	12.8	15	15.013

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