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WaveQ3D: Fast and Accurate Acoustic Transmission Loss (TL) Eigenrays, in Littoral Environments

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WAVEQ3D: FAST AND ACCURATE ACOUSTIC TRANSMISSION LOSS (TL)

EIGENRAYS, IN LITTORAL ENVIRONMENTS

BY

SEAN M. REILLY

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

IN OCEAN ENGINEERING

UNIVERSITY OF RHODE ISLAND

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DOCTOR OF PHILOSOPHY DISSERTATION

OF

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2016

Abstract

This study defines a new 3D Gaussian ray bundling acoustic transmission loss model in geodetic coordinates: latitude, longitude, and altitude. This approach is designed to lower the computation burden of computing accurate environmental effects in sonar training application by eliminating the need to transform the ocean environment into a collection of Nx2D Cartesian radials. This approach also improves model accuracy by incorporating real world 3D effects, like horizontal refraction, into the model. This study starts with derivations for a 3D variant of Gaussian ray bundles in this coordinate system. To verify the accuracy of this approach, acoustic propagation predictions of transmission loss, time of arrival, and propagation direction are compared to analytic solutions and other models. To validate the model's ability to predict real world phenomena, predictions of transmission loss and propagation direction are compared to at-sea measurements, in an environment where strong horizontal refraction effect have been observed. This model has been integrated into U.S. Navy active sonar training system applications, where testing has demonstrated its ability to improve transmission loss calculation speed without sacrificing accuracy.

Acknowledgments

The author would like to acknowledge all of the collaborators who have assisted in this research over the years. In 1994, Dr. Roy Deavenport (Naval Undersea Warfare Center Division Newport) first encouraged me to explore the development of a 3D Gaussian beam theory to support the modeling of the environmental impulse response for active sonar applications. The theory for reflection from a 3-D slope was developed by Dr. Michael Goodrich (Alion Science and Technology), who also made significant contributions to the development of test cases as part of an internal research and development effort funded by Alion Science and Technology in 2009. Efforts to test and productize this model were funded from 2011-2015 by the High Frequency Active Sonar Training (HiFAST) project at the U.S. Office of Naval Research (ONR), where Mr. Michael Vaccaro led the charge to evolve this unproven concept into a Navy training capability. Mr. David Thibaudeau provided testing support and Mr. Ted Burns acted as our lead system integrator during the HiFAST effort at Aegis Technology Group. Thanks to Dr. Thomas Yudichak (Applied Research Laboratory at the University of Texas in Austin), who provided years of Navy certification experience and keen insights during his independent verification and validation effort. Special thanks are due to Jonathan Glass, and his team at Naval Air Warfare Center Training System Division, who suffered through the first integration of this model into a real Navy training system. Finally, the author would like to thank Dr. Gopu

Potty and Dr. James Miller who brought their academic rigor and perspective to this effort.

Dedication

To my wife, Joan, my inspiration in all things.

Preface

Many underwater acoustic propagation models transform the three dimensional (3D) environment into a collection of two dimensional (2D) radials, and then compute transmission loss as a function of range and depth along each of those radials. This approach, often called Nx2D, is used in tactical decision aids to compute millions of range, depth, and bearing combinations in just a few minutes. Sonar training applications have an additional requirement that the results must be computed in real time for presentation to the trainees. In littoral environments, the number of acoustic contacts can be in the hundreds, and the Nx2D approach places a large computation burden on the training system. The primary goal of this research is to develop a transmission loss model that reduces the computational burden on training systems without sacrificing accuracy. Improving the speed of this computation reduce acquisition costs by reducing reliance on massively parallel computing systems. This study also seeks to provide the acoustics research community with a tool that can predict 3D effects in applications, like geophysical parameter inversion, where execution speed is important.

This dissertation follows the University of Rhode Island Graduate School guidelines for the preparation of a dissertation in manuscript format. There are three chapters that represent formally published papers:

- Manuscript 1 derives the theory behind the model, and provides key test results

including: ray path refraction accuracy using a Munk profile, Gaussian beam projection into the shadow zone for an n^2 linear profile, and horizontal refraction from a 3D analytic wedge.

- Manuscript 2 compares model predictions to at-sea measurements of horizontal refraction in a sloped environment.
- Manuscript 3 discusses the application of this model in deployable sonar training systems.
- Manuscript 4 discusses the accuracy of this model in the deep sound channel.

Appendix A is an unpublished model accuracy test report, delivered to the High Frequency Active Sonar Training (HiFAST) project at the U.S. Office of Naval Research (ONR), in 2012.

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Manuscript 1

Computing Acoustic Transmission Loss Using 3D Gaussian Ray Bundles in
Geodetic Coordinates

by

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1.1 Abstract

This paper defines a new 3-D Gaussian ray bundling model in geodetic coordinates: latitude, longitude, and altitude. Derivations are provided for 3-D refraction, 3-D interface reflection, 3-D eigenray detection, and a 3-D variant of CASS/GRAB Gaussian ray bundles. This approach allows environmental parameters and their derivatives are computed directly in latitude, longitude, and depth directions without reducing the problem to a series of Nx2-D Cartesian projections. Our model supports 3-D effects such as great circle routes and horizontal refraction in sloped environments. Key test results are included for ray path refraction accuracy using a Munk profile, Gaussian beam projection into the shadow zone for an n^2 linear profile, and horizontal refraction from a 3-D analytic wedge. Testing to date indicates that this approach has accuracy at least as good as CASS/GRAB, but with improved execution speed benefits for large numbers of targets, and 3-D transmission loss effects.

1.2 Introduction

Many underwater acoustic propagation models transform the three dimensional (3-D) environment into a collection of two dimensional (2-D) radials, and then compute transmission loss as a function of range and depth along each of those radials. This approach, often called Nx2-D, is used in tactical decision aids to compute millions of range, depth, and bearing combinations in just a few minutes. Sonar training applications have an additional requirement that the results must be computed in

real time for presentation to the trainees. In littoral environments, the number of acoustic contacts can be in the hundreds, and transmission loss calculations place a large computation burden on the training system. The primary goal of this research is to develop a transmission loss model that reduces the computational burden on training systems without sacrificing accuracy. Research into improving the speed of this computation may reduce acquisition costs by reducing reliance on massively parallel computing systems. The High Fidelity Active Sonar Training (HiFAST) Project at the U.S. Office of Naval Research funded this research to provide fast and accurate acoustic transmission loss predictions for hardware-in-the loop (HWIL) active sonar training applications.

There are several ongoing efforts to deliver 3-D versions of Parabolic Equation^{25;24} and Normal Mode⁴ models. However, these models require large numbers of modes at frequencies above 1000 Hz, and this impedes their ability to provide real-time, active sonar results on low cost computer hardware. This paper develops a new acoustic transmission loss model using 3-D ray theory in geodetic coordinates. Unlike other efforts to model ray theory in geodetic coordinates,²³ our model focuses on localized propagation in littoral environments, instead of propagation on a global scale. This difference requires our model to support not only 3-D refraction by the speed of sound, but also geodetic implementations of interface reflection, eigenray detection, and transmission loss computation.

Databases of 3-D environmental parameters are usually provided in geodetic coordinates: latitude, longitude, and altitude (or depth). Instead of transforming the 3-D environment into a collection of Nx2-D radials, we maintain the 3-D nature of

the environment by using spherical polar coordinates (r, θ, ϕ) to solve the acoustic eikonal equation. The impulse response of the environment is modeled as a series of acoustic wavefronts that propagate away from the source as a function of time. To improve transmission loss accuracy in neighborhood of shadow zones and caustics, this derivation includes the development of a 3-D variant of the Gaussian Ray Bundling (GRAB) model.^{45;17} Each wavefront has the ability to compute transmission loss to large numbers of acoustic contacts and share the overhead of the wavefront propagation computation across those contacts. Eigenrays are computed in reaction to a collision of the wavefront with acoustic contacts.

Section 2 of this paper develops the underlying equations used to implement our model. Section 3 provides some of the key testing results used to verify model accuracy. Even though this model was developed with the primary objective of supporting the sonar training applications, the model inherently 3-D in nature and supports the physics of out-of-plane propagation effects. Hence, this model could be a useful tool to explore 3-D propagation effects in other research studies.^{41;18;3}

1.3 Derivation

This section develops an implementation of 3-D refraction in spherical coordinates, interface reflection in this coordinate system, eigenray detection, and Gaussian ray bundle transmission loss.

Figure 1.1: Acoustic rays are a vector field normal to the wavefront at each point in space.

1.3.1 3-D ray propagation in spherical coordinates

Ray theory is a high frequency approximation of the wave equation that decomposes the acoustic impulse response of the environment into surfaces of constant travel time (t) from the source (Fig. 1.1). The rays are a vector field $\vec{r}$ that is normal to these surfaces at each point in space, and the path of these rays through the medium defines the direction of propagation. The fundamental equations of ray theory are derived by seeking power series solutions²¹ to the Helmholtz equation.

$$\nabla^2 p(\vec{r}) + \frac{\omega^2}{c^2(\vec{r})} p(\vec{r}) = -\delta(\vec{r} - \vec{r}_0) \quad (1.1)$$

$$p(\vec{r}) = e^{i\omega t(\vec{r})} \sum_{j=0}^{\infty} \frac{A_j(\vec{r})}{(i\omega)^j} \quad (1.2)$$

where $p(\vec{r})$ is the acoustic pressure as a function of location, $\vec{r}_0$ is the source position, $c(\vec{r})$ is the speed of sound in water, $t(\vec{r})$ is the travel time, ω is the angular frequency, and $A_j(\vec{r})$ are the components of acoustic amplitude. Equating terms of like order in

ω yields an infinite sequence of equations.

$$O(\omega^2) : \left| \vec{\nabla} t \right|^2 = \frac{1}{c^2(\vec{r})} \quad (1.3)$$

$$O(\omega) : 2\vec{\nabla} A_0 \cdot \vec{\nabla} t + (\nabla^2 t) A_0 = 0 \quad (1.4)$$

$$O(\omega^{1-j}) : 2\vec{\nabla} A_j \cdot \vec{\nabla} t + (\nabla^2 t) A_j = -\nabla^2 A_{j-1} \quad \text{for } j=1,2,\dots \quad (1.5)$$

The eikonal equation (1.3) defines the relationship between the direction of propagation and the speed of sound in water. The first transport equation (1.4) relates the spreading loss of the acoustic field to divergence in the propagation direction. The remaining transport equations (1.5) relate the spreading loss of the acoustic field to diffraction effects. Eqs. (1.3) and (1.4) are an exact solution of the wave equation in the geometric limit, that is, when the sound speed gradient along the direction of motion changes slowly compared to the acoustic wavelength. This accuracy breaks down at lower frequencies where diffraction becomes a significant feature of acoustic propagation.

Eq. (1.3) can be solved by recognizing that the direction of propagation is related to the gradient of the travel time.

$$\hat{n} = \frac{d\vec{r}}{ds} = c\vec{\nabla} t \quad (1.6)$$

where $\hat{n}$ is the direction of propagation and s is the distance along the ray path. Applying Eq. (1.6) to Eq. (1.3) yields a second order ordinary differential equation in terms of $\vec{r}$, c , and s .

$$\frac{d}{ds} \left(\frac{1}{c} \frac{d\vec{r}}{ds} \right) = -\frac{1}{c^2} \vec{\nabla} c \quad (1.7)$$

Introducing the temporary variable $\vec{\xi}$ reduces Eq. (1.7) to a pair of simultaneous first order equations.

$$\frac{d\vec{\xi}}{ds} = -\frac{1}{c^2} \vec{\nabla} c \quad (1.8)$$

$$\frac{d\vec{r}}{ds} = c \vec{\xi} \quad (1.9)$$

Ray tracing is the process of using Eqs. (1.8) and (1.9) to update the location and direction of each point on the wavefront given previous values for $(\vec{r}, \vec{\xi})$, and a finite step size Δs . The initial value for $\vec{r}$ uses the location of the acoustic source. Each ray path then corresponds to a set of $\vec{\xi}$ values that have been discretized in the depression/elevation (μ) and the azimuthal steering (φ) directions. Although Eqs. (1.8) and (1.9) are independent of frequency, loss along the paths includes the frequency dependent effects of seawater absorption and interface reflection. Combining Eqs. (1.6) and (1.9) exposes the temporary variable $\vec{\xi}$ as the direction of propagation divided by the speed of sound. This is equivalent to the wave number vector $\vec{k}$ divided by the angular frequency ω .

$$\vec{\xi} = \frac{\hat{n}}{c} = \frac{\vec{k}}{\omega} \quad (1.10)$$

Instead of solving the ray equations in units of arc-length, our approach uses a change of variables $ds = c dt$ to transform equations (1.8) and (1.9) into functions of time.

$$\frac{d\vec{\xi}}{dt} = -\frac{1}{c} \vec{\nabla} c \quad (1.11)$$

$$\frac{d\vec{r}}{dt} = c^2 \vec{\xi} \quad (1.12)$$

In this form, the ray tracing equations represent the time evolution of acoustic wavefronts. Propagation of the wavefront in the time domain models the impulse response

of the environment, which is a useful form for broadband modeling.

Converting Eqs. (1.11) and (1.12) into spherical coordinates require a representation of $d\vec{\xi}/dt$ and $d\vec{r}/dt$ in that coordinate system. This derivation uses arrows for vectors with magnitude and direction (such as $\vec{r}$), carets for unit length vectors (such as $\hat{r}$), and plain text for magnitude parameters (such as r). The vector form of the time derivatives is found by recognizing that $\vec{r}$ only has radial components, while $\vec{\xi}$ has components in all three dimensions

$$\vec{r}(t) = r(t)\hat{r}(t) \quad (1.13)$$

$$\vec{\xi}(t) = \alpha(t)\hat{r}(t) + \beta(t)\hat{\theta}(t) + \gamma(t)\hat{\phi}(t) \quad (1.14)$$

$$\frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} \quad (1.15)$$

$$\frac{d\vec{\xi}}{dt} = \frac{d\alpha}{dt}\hat{r} + \alpha\frac{d\hat{r}}{dt} + \frac{d\beta}{dt}\hat{\theta} + \beta\frac{d\hat{\theta}}{dt} + \frac{d\gamma}{dt}\hat{\phi} + \gamma\frac{d\hat{\phi}}{dt} \quad (1.16)$$

where $\alpha(t)$, $\beta(t)$, and $\gamma(t)$ are the r , θ , and ϕ components of $\vec{\xi}$. The time derivatives of $\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$ are computed using a conversion into Cartesian coordinates.

$$\hat{r}(t) = \sin\theta(t)\cos\phi(t)\hat{i} + \sin\theta(t)\sin\phi(t)\hat{j} + \cos\theta(t)\hat{k} \quad (1.17)$$

$$\hat{\theta}(t) = \cos\theta(t)\cos\phi(t)\hat{i} + \cos\theta(t)\sin\phi(t)\hat{j} - \sin\theta(t)\hat{k} \quad (1.18)$$

$$\hat{\phi}(t) = -\sin\phi(t)\hat{i} + \cos\phi(t)\hat{j} \quad (1.19)$$

The chain rule, when applied to Eqs. (1.17), (1.18), and (1.19), yields the time derivatives in spherical coordinates.

$$\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{\theta} + \sin\theta\frac{d\phi}{dt}\hat{\phi} \quad (1.20)$$

$$\frac{d\hat{\theta}}{dt} = -\frac{d\theta}{dt}\hat{r} + \cos\theta\frac{d\phi}{dt}\hat{\phi} \quad (1.21)$$

$$\frac{d\hat{\phi}}{dt} = -\sin\theta \frac{d\phi}{dt} \hat{r} \quad (1.22)$$

Applying Eqs. (1.20) through (1.22) to Eqs. (1.15) and (1.16) transforms Eqs. (1.11) and (1.12) into spherical coordinates.

$$\frac{d\vec{r}}{dt} = \frac{dr}{dt} \hat{r} + r \frac{d\theta}{dt} \hat{\theta} + r \sin\theta \frac{d\phi}{dt} \hat{\phi} \quad (1.23)$$

$$\begin{aligned} \frac{d\vec{\xi}}{dt} = & \left[\frac{d\alpha}{dt} - \beta \frac{d\theta}{dt} + \gamma \sin\theta \frac{d\phi}{dt} \right] \hat{r} + \left[\frac{d\beta}{dt} + \alpha \frac{d\theta}{dt} - \gamma \cos\theta \frac{d\phi}{dt} \right] \hat{\theta} \\ & + \left[\frac{d\gamma}{dt} + (\alpha \sin\theta + \beta \cos\theta) \frac{d\phi}{dt} \right] \hat{\phi} \end{aligned} \quad (1.24)$$

Matching terms for $\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$ yields a system of six scalar first-order differential equations.

$$\frac{dr}{dt} = c^2 \alpha \quad (1.25)$$

$$r \frac{d\theta}{dt} = c^2 \beta \quad (1.26)$$

$$r \sin\theta \frac{d\phi}{dt} = c^2 \gamma \quad (1.27)$$

$$\frac{d\alpha}{dt} - \beta \frac{d\theta}{dt} - \gamma \sin\theta \frac{d\phi}{dt} = -\frac{1}{c} \frac{dc}{dr} \quad (1.28)$$

$$\frac{d\beta}{dt} + \alpha \frac{d\theta}{dt} - \gamma \cos\theta \frac{d\phi}{dt} = -\frac{1}{cr} \frac{dc}{d\theta} \quad (1.29)$$

$$\frac{d\gamma}{dt} + (\alpha \sin\theta + \beta \cos\theta) \frac{d\phi}{dt} = -\frac{1}{cr \sin\theta} \frac{dc}{d\phi} \quad (1.30)$$

When Eqs. (1.25) through (1.27) are combined with Eqs. (1.28) through (1.30), the system is reduced to a state where all of the coordinate derivatives appear only once.

$$\frac{dr}{dt} = c^2 \alpha \quad (1.31)$$

$$\frac{d\theta}{dt} = \frac{c^2 \beta}{r} \quad (1.32)$$

$$\frac{d\phi}{dt} = \frac{c^2 \gamma}{r \sin\theta} \quad (1.33)$$

$$\frac{d\alpha}{dt} = -\frac{1}{c} \frac{dc}{dr} + \frac{c^2}{r} (\beta^2 + \gamma^2) \quad (1.34)$$

$$\frac{d\beta}{dt} = -\frac{1}{cr} \frac{dc}{d\theta} - \frac{c^2}{r} (\alpha\beta + \gamma^2 \cot\theta) \quad (1.35)$$

$$\frac{d\gamma}{dt} = -\frac{1}{cr \sin\theta} \frac{dc}{d\phi} - \frac{c^2\gamma}{r} (\alpha + \beta \cot\theta) \quad (1.36)$$

In our model, the numerical integration of Eqs. (1.31) through (1.36) uses an explicit, third order, Adams-Bashforth (AB3) algorithm.⁴⁹ AB3 approximates the next iteration in time from the three previous iterations.

$$\vec{f}(t_{n+1}) = \vec{f}(t_n) + \Delta t \left[\frac{23}{12} \frac{d\vec{f}}{dt}(t_n) - \frac{16}{12} \frac{d\vec{f}}{dt}(t_{n-1}) + \frac{5}{12} \frac{d\vec{f}}{dt}(t_{n-2}) \right] \quad (1.37)$$

where Δt is the time step, and $\vec{f}$ is a vector of the positions, directions, and their derivatives.

$$\vec{f} = \left[r, \theta, \phi, \alpha, \beta, \gamma, \frac{dr}{dt}, \frac{d\theta}{dt}, \frac{d\phi}{dt}, \frac{d\alpha}{dt}, \frac{d\beta}{dt}, \frac{d\gamma}{dt} \right] \quad (1.38)$$

When past values are cached instead of re-calculated, AB3 is much faster than other integrators with similar accuracy. However, because AB3 is not self-starting, a third order Runge-Kutta (RK3) algorithm³⁷ is used whenever the ray parameters must be initialized, or re-initialized as part of reflection.

Eqs. (1.31) through (1.38) represent the time evolution of acoustic wavefronts that propagate through the ocean on a spherical Earth. Computing a local radius of curvature can be used to take the non-spherical shape of the Earth⁴⁶ into account for specific operating areas. Broadband effects along each ray path are modeled as the frequency dependent accumulation of losses from seawater absorption and interface reflection. The ray paths include horizontal refraction effects and they automatically follow great circle routes (the shortest distance between points on a spherical

Earth) as they traverse latitudes and longitudes. Environmental parameters and their derivatives are computed directly in the latitude, longitude, and depth directions.

This approach was selected to avoid the computation burden of converting 3-D environmental data into Nx2-D radials. However, this choice comes at the price of propagation equations that are much more complex than their Cartesian equivalents. Counterintuitively, the inclusion of spherical coordinate terms in Eqs. (1.31) through (1.36) has little impact on computational speed when the application relies on interpolations of gridded environmental data. On modern computers with built-in math coprocessors, the search operations inherent in interpolation are much more computationally expensive than algebraic or trigonometric functions. In our testing, gridded interpolation of sound velocity and bathymetry consumed over 70% of the overall computation time.

1.3.2 3-D interface reflections

Propagation in this 3-D spherical coordinate system requires the derivation of a compatible model for interface reflection. In the real world, reflection from 3-D bathymetry causes ray paths to bend in the horizontal direction.^{41;18;3} Fig. 1.2 illustrates this phenomenon using a simple case where the ray paths follow a curved path after several reflections from a sloped bottom. Each time that the ray path encounters the bottom, the tilt of the normal turns the ray down the slope. Eventually the ray stops traveling up toward the apex and turns down slope. Seen from above, the ray paths appear to bend in the horizontal direction, even though no actual re-

fraction has taken place. Rays with steep launch angle have more bottom reflections and turn down slope faster than shallow-angle rays.

Figure 1.2: Reflection from 3-D bathymetry causes ray paths to bend in the horizontal direction.

Interface reflection starts with a recognition that the next location in the iteration of Eq. (1.31) will put the wavefront location above the ocean surface or below the bottom. However, estimating the precise location of the point of impact requires an estimate of the point in time when the incident ray strikes the interface. Our derivation for estimating that time uses the geometry illustrated in Fig. 1.3 where $\vec{I}$ is the incident ray along direction $\hat{I}$, $\vec{R}$ is the reflected ray along direction $\hat{R}$, ζ is the incident grazing angle, and $\hat{s}$ is the surface normal. If the interface slope is

Figure 1.3: Geometry for estimating time of impact and reflection direction from a 3-D slope.

nearly constant across the length of the incident ray, then the ratio of the time steps is equivalent to the ratio of the distances normal to the surface

$$d_1 \equiv -\vec{I} \cdot \hat{s} = -\left(\frac{d\vec{r}}{dt} \cdot \hat{s}\right) \Delta t \quad (1.39)$$

$$d_2 \equiv -h\hat{r} \cdot \hat{s} \quad (1.40)$$

$$\frac{\delta t}{\Delta t} = \frac{d_2}{d_1} = \frac{h\hat{r} \cdot \hat{s}}{\left(\frac{d\vec{r}}{dt} \cdot \hat{s}\right) \Delta t} \quad (1.41)$$

$$\delta t = \frac{h\hat{r} \cdot \hat{s}}{\frac{d\vec{r}}{dt} \cdot \hat{s}} \quad (1.42)$$

where h is the incident ray height above bottom, d_1 is the component of $\vec{I}$ normal to the interface, d_2 is the fraction of d_1 needed to reach the interface, Δt is the normal time step, δt is the time step needed to reach the interface, and $\frac{d\vec{r}}{dt}$ is taken from Eqs. (1.31) through (1.33). At the ocean surface, Eq. (1.42) simplifies to

$$\delta t_{surface} = \frac{h}{\frac{dr}{dt}} \quad (1.43)$$

where h is the incident ray depth below the surface.

The direction of reflection uses the fact that incident and reflected rays share a common grazing angle. This means that the two vectors labeled $\vec{B}$ in Fig. 1.3 must have the same length. Vector addition leads to a relationship between $\hat{I}$, $\hat{R}$, and $\vec{B}$.

$$\hat{R} = 2\vec{B} - \hat{I} \quad (1.44)$$

$\vec{B}$ can also be computed by subtracting the normal component of $\hat{I}$ from $\hat{I}$.

$$\vec{B} = \hat{I} - (\hat{I} \cdot \hat{s})\hat{s} \quad (1.45)$$

Combining Eq. (1.44) and (1.45) yields a vector expression for the reflected direction.

$$\hat{R} = \hat{I} - 2(\hat{I} \cdot \hat{s})\hat{s} \quad (1.46)$$

For ocean surface reflections, Eq. (1.46) negates the sign of the radial component while leaving the θ and ϕ direction components unchanged.

This type of horizontal refraction can have a significant impact on real-world transmission loss in littoral environments,^{41;18;3} but it is not supported by Nx2-D transmission loss models. Fig. 1.4, illustrates a collection of ray paths launched at azimuths from 200° to 268° from a common depression/elevation angle of -12° . Fig. 1.5, provides a side view of the same scenario, but focuses on a single path launched at an azimuth of 222° and traveling right to left. In both images, bathymetry from the ETOPO1 database¹⁵ is displayed as a greyscale surface with white depth contours every 50 m. Refraction in the water is modeled using 3-D climatology data for January from the World Ocean Atlas.⁴⁸ The ray paths, illustrated as black lines, represent 25 s of propagation in 100 ms steps. As illustrated by Fig. 1.5, each bottom reflection

Figure 1.4: 3-D refraction around a seamount (top down view, 50 m contours).

changes the depression/elevation angle of the ray path. The depression/elevation angle increases as the ray travels up slope, and decrease as it travels down slope. For each of these reflections, the ray path is also diverted away from the seamount peak in the horizontal. As shown in Fig. 1.4, this broadens the acoustic shadow created by the seamount.⁷

1.3.3 3-D eigenray path detection

If we think of Eqs. (1.31) through (1.38) as the time evolution of acoustic wavefronts, then eigenray arrivals can be thought of as collisions of those wavefronts with acoustic

Figure 1.5: 3-D refraction around a seamount (side view, 50 m contours).

targets. Fig. 1.6 is a side view of such a collision where $\vec{r}_p$ is the position of a single acoustic contact, $\vec{r}_{njk}$ is the position of a point on the wavefront, d_{njk} is the distance from target to each point on wavefront, Δt is the target offset along the direction of propagation, $\Delta\mu$ is the target offset in the depression/elevation direction, and $\Delta\varphi$ is the target offset in the azimuthal direction.

A point on the wavefront $\vec{r}_{njk}$ is the closest point of approach (CPA) for a specific target if it has the smallest distance to that target relative to the 26 wavefront points immediately surround it in the t , μ , and ϕ directions. The offset in each of these directions is computed by expressing d_p^2 , the square of the distance to this target, as

Figure 1.6: Eigenray estimation geometry (side view: φ_k direction not shown).

a second order Taylor series, relative to the CPA, in vector form.

$$\vec{\rho} \equiv (\rho_1, \rho_2, \rho_3) \equiv (\Delta t, \Delta \mu, \Delta \varphi) \quad (1.47)$$

$$d_p^2 \approx \epsilon + \vec{g} \cdot \vec{\rho} + \frac{1}{2} \vec{\rho} \cdot \mathbf{H} \vec{\rho} \quad (1.48)$$

$$\epsilon \equiv d^2|_{CPA} \quad (1.49)$$

$$\vec{g} \equiv \frac{\partial d^2}{\partial \vec{\rho}}|_{CPA} \quad (1.50)$$

$$\mathbf{H} \equiv \frac{\partial^2 d^2}{\partial \vec{\rho}^2}|_{CPA} \quad (1.51)$$

where $\vec{\rho}$ is the target offset from CPA in vector form, $\vec{g}$ is the gradient of squared distance at CPA (3 elements), and $\mathbf{H}$ is the Hessian matrix of squared distance at CPA (3x3). One way to solve this equation would be to search for a value of $\vec{\rho}$ for which Eq. (1.48) was zero. However, since d_p^2 is positive definite, searching for

the minimum value of d_p^2 , indicated by a zero in the first derivative, also solves this problem.

$$\frac{\partial d_p^2}{\partial \vec{\rho}} = \vec{g} + \mathbf{H} \vec{\rho} = 0 \quad (1.52)$$

$$\mathbf{H} \vec{\rho} = -\vec{g} \quad (1.53)$$

$$\vec{\rho} = -\mathbf{H}^{-1} \vec{g} \quad (1.54)$$

Eq. (1.54) reduces the offset estimation problem to the calculation of the gradient of distance, the calculation of the Hessian matrix, and a 3x3 matrix inversion. The 26 wavefront points immediately surrounding the CPA provide the data needed to use a centered difference equation for first derivatives, and it provides the 3 points in each direction needed for second derivatives.

Some eigenray products are computed directly from this offset vector.

$$t_p = t_n + \delta t \quad (1.55)$$

$$\mu_p = \mu_j + \delta \mu \quad (1.56)$$

$$\varphi_p = \varphi_k + \delta \varphi \quad (1.57)$$

where t_p is the travel time from the source, μ_p is the depression/elevation launch angle at source, and φ_p is the azimuthal launch angle at source. The direction at the target is found by forward solving a 2nd order Taylor series for $\vec{\xi}$ in the neighborhood of the CPA. The eigenray data is completed by the calculation of transmission loss, which is discussed in the next section.

The computation of d_p^2 in spherical coordinates is much less efficient than an equivalent calculation in Cartesian coordinates. However, the impact of this difference

is minimized when the number of targets is small compared to the number ray tracing points. This is a good assumption for sonar training systems, but it makes our model much slower than existing methods for calculating transmission loss over millions of range, depth, and bearing combinations in tactical decision aids.

1.3.4 3-D Gaussian ray bundles

In conventional ray theory, the spreading of acoustic transmission loss is estimated by measuring the changes in ensonified area between ray paths. The intensity across the wavefront is inversely proportional to the change in a surface area segment compared to its area at the source.²¹ The Gaussian beam approach uses a parabolic equation approximation normal to each ray to compute spreading in the form of a Gaussian profile for each ray.^{9;35;34;5} Gaussian ray bundling models, such as CASS/GRAB, use the distance between rays (from conventional ray theory) to define the size of the Gaussian profile for each ray.

In 2-D Gaussian beam models, the intensity at the target location is a summation of contributions from rays above and below the eigenray target. To extend Gaussian ray bundles to three dimensions, we use an approximation that computes independent factors in the μ and φ directions (Fig. 1.7).

$$G(\vec{r}_p) = \left(\sum_{j'=j-J}^{j+J} g_{j'}(\vec{r}_p) \right) \left(\sum_{k'=k-K}^{k+K} g_{k'}(\vec{r}_p) \right) \quad (1.58)$$

where $G(\vec{r}_p)$ is the total Gaussian ray bundle intensity at the eigenray target, (j, k) are the index numbers of the cell containing the eigenray target, $g_{j'}$ are the Gaussian ray bundle contributions along depression/elevation direction, $g_{k'}$ are the Gaussian ray

Figure 1.7: Gaussian ray nearest neighbors (front view: t_n direction not shown).

bundle contributions along the azimuthal direction, $2J + 1$ is the number of beams used in the depression/elevation summation, and $2K + 1$ is the number of beams used in the azimuthal summation. Computing a product of independent factors is equivalent to assuming that the width of the Gaussian ray bundles in the μ direction represents a local average in the φ direction, and vice versa.

The intensity of each Gaussian ray bundle contribution is a function of the width of each beam and the distance along the wavefront to the eigenray target, normalized

to the average distance across the beam at the source

$$g_{j'}(\vec{r}_p) = \frac{N_{j'}}{\sqrt{2\pi w_{j'}^2}} \exp\left(-\frac{d_{j'}^2}{2w_{j'}^2}\right) \quad (1.59)$$

$$g_{k'}(\vec{r}_p) = \frac{N_{k'}}{\sqrt{2\pi w_{k'}^2}} \exp\left(-\frac{d_{k'}^2}{2w_{k'}^2}\right) \quad (1.60)$$

$$N_{j'} = \frac{\int_{\varphi_{k'}}^{\varphi_{k'+1}} (\mu_{j'+1} - \mu_{j'}) d\varphi}{\int_{\varphi_{k'}}^{\varphi_{k'+1}} d\varphi} = \mu_{j'+1} - \mu_{j'} \quad (1.61)$$

$$N_{k'} = \frac{\int_{\mu_{j'}}^{\mu_{j'+1}} (\varphi_{k'+1} - \varphi_{k'}) \cos(\mu) d\mu}{\int_{\mu_{j'}}^{\mu_{j'+1}} d\mu} = \frac{\sin(\mu_{j'+1}) - \sin(\mu_{j'})}{\mu_{j'+1} - \mu_{j'}} (\varphi_{k'+1} - \varphi_{k'}) \quad (1.62)$$

where $w_{j'}$ and $w_{k'}$ are the half-widths of the Gaussian ray bundle in the μ and φ directions, and $d_{j'}$ and $d_{k'}$ are the distances in the μ and φ directions from the Gaussian ray bundle center to the target.

GRAB⁴⁵ models the frequency dependent component of the beamwidth by giving each beam a minimum width.

$$w'_j(f) = \max(w_j, 2\pi\lambda) \quad (1.63)$$

where λ is the wavelength of the signal being modeled, w_j is the half cell width of beam j , and $w'_j(f)$ is the adjusted width of beam j . GRAB models beams centered on each ray and then between each ray to create a minimum overlap of 50% between Gaussian ray bundles.

A physical interpretation of Eq. (1.63) is that the λ term is the frequency dependent Gaussian spreading that GRAB expects for rays that are infinitely close together. The w_j terms can be interpreted as the Gaussian width created by discretely sampling the launch angles. Instead of using the maximum of these two contributions,

our model convolves these two sources of spreading and adds their Gaussian widths as the sum of squares.

$$(w'_j(f))^2 = (2w_j)^2 + (2\pi\lambda)^2 \quad (1.64)$$

Eq. (1.64) produces results that are similar to (1.63), but there is a smooth transition between the domains dominated by the w_j and λ terms. The factor of 2 in w_j has been artificially introduced to produce the same 50% overlap as GRAB without doubling the number of beam calculations. Normalizing Eq. (1.59) and (1.60) by the combined effect of both spreading sources conserves energy across the wavefront.

1.4 Test Results

Although the primary goal of this research is to develop a transmission loss model that reduces computational burden, the training systems also require an accurate representation of real-world phenomena. This section presents the results of several key accuracy tests including for ray path refraction accuracy using a Munk profile (Section 3.1), Gaussian beam projection into the shadow zone for an n^2 linear profile (Section 3.2), and horizontal refraction from a 3-D analytic wedge (Section 3.3). An comparison to CASS/GRAB executions times is provided in Section 3.4.

1.4.1 Refraction accuracy benchmark

Because our model's calculation of transmission loss is closely tied to the location of ray paths, refraction accuracy is an important element of its overall accuracy. Because the use of spherical coordinates incorporates the radius of the Earth (a

large number) into the radial coordinate in Eqs. 1.31 through 1.36, we need to address concerns that our approach would suffer from numerical accuracy problems. To evaluate the refraction accuracy, we compare an analytic solution for the Munk profile²⁸ representation of a deep sound channel to ray paths generated by our model. The version of the Munk profile used in this test is defined by Eqs. (1.65) and Eq. (1.66)

$$z' = 2 \frac{z - z_1}{B} \quad (1.65)$$

$$c(z) = c_1 \left[1 + \epsilon \left(z' - 1 + e^{-z'} \right) \right] \quad (1.66)$$

where z' is the normalized depth (positive is down), z_1 is the depth of the deep sound channel axis (1300 m), B is a depth scaling factor (1300 m), c_1 is the sound speed on deep sound channel axis (1500 m/s), and ϵ is the profile scaling factor (7.37×10^{-3}). Fig. 1.8 illustrates the ray paths computed using using a 100 ms time increment and 1° separated depression/elevation launch angles from -14° to 14° .

Munk's paper²⁸ characterized ray paths using their cycle range, the range required to complete one period of upward and downward refraction. To create an analytic solution for this test, Snell's Law, Eq. (1.67) is integrated numerically.

$$a = \frac{\cos \eta(z)}{c(z)} = \text{constant} \quad (1.67)$$

$$\frac{dH}{dz} = \frac{\cos \eta(z)}{\sin \eta(z)} \quad (1.68)$$

$$\Delta H = \int_{z_s}^{z_t} \frac{ac(z)}{\sqrt{1 - (ac(z))^2}} dz \quad (1.69)$$

where $\eta(z)$ is the depression/elevation angle along the ray path; a is the ray parameter (constant for each launch angle); H is the horizontal range; z_s is the source depth; and

Figure 1.8: Munk profile (left panel) and modeled ray paths (right panel).

z_t is the target depth. Although these integrals only apply between the source and the first vertex or reflection, paths out to any range can be constructed by repeating this process after the vertex or reflection.

Pekeris' modified index of refraction, shown in Eq. (1.70), allows Cartesian models, like the Munk profile, to incorporate earth curvature effects into their calculations.³³ This process can also be inverted, as shown in Eq. (1.71), to allow spherical models, like ours, be compared to flat earth benchmarks.

$$n(z) = \frac{r}{R} \frac{n(r)}{n(R)}, \quad (1.70)$$

$$c(r) = \frac{r}{R} c(z), \quad (1.71)$$

where R is the radius of earth's curvature in this area of operations; r is the radial distance from the center of curvature (positive is up); $z = R - r$ is the depth below the ocean surface (positive is down); $n(r)$ is original index of refraction in spherical coordinates, $n(z)$ is adjusted index of refraction in Cartesian coordinates, $c(z)$ is the benchmark's speed of sound in Cartesian coordinates, $c(r)$ is the adjusted index of refraction in spherical coordinates.

Fig. 1.9 compares our model's cycle range to those computed using Eqs. (1.69). Our model deviates from the analytic solution by a maximum of -8.62 m at a range of 129.95 km (0.007% error). However, 50 out of 58 samples (86%) exhibit errors less than ± 2 m (0.002% error). Ray paths that are initially launched toward the surface, where the sound speed gradient is highest, have consistently larger errors than paths that were launched down. Because the use of spherical coordinates incorporates the radius of the Earth (a large number) into all radial values, we initially had concerns that our approach would suffer from numerical accuracy problems. Our results for refraction accuracy indicate that those concerns are unfounded. At this time, it does not appear that incorporation the radius of the Earth into the radial coordinate in Eqs. 1.31 through 1.36 causes significant accuracy problems.

1.4.2 2-D transmission loss benchmark

To compare the accuracy of our transmission loss estimates to existing 2-D Gaussian beam models, coherent transmission loss is calculated at the edge of a shadow zone

Figure 1.9: Cycle range difference between model and analytic solution for Munk profile ray traces.

using the Pedersen and Gordon n^2 linear test case.³²

$$c(z) = \frac{c_0}{\sqrt{1 + \frac{2g_0}{c_0} z}} \quad (1.72)$$

where c_0 is the sound speed at the ocean surface (1550 m/s) and g_0 is the sound speed gradient at the ocean surface (1.2 s^{-1}). Fig. 1.10 illustrates ray paths; both the surface and direct paths encounter a shadow zone at ranges beyond 880 m. Conventional ray theory predicts that no energy enters the shadow zone. This benchmark has been used in several other 2-D Gaussian beam models^{45;34} to validate their ability to predict the smooth transition predicted by the analytic solution.

Figure 1.10: Pederson profile (left panel) and modeled ray paths (right panel).

Fig. 1.11 illustrates the computed transmission losses for this scenario at 2000 Hz. The Fast Field Program (FFP) wavenumber integration technique^{14;6} generates the solution labeled *theory* in this figure. The *GRAB* solution is computed using the Comprehensive Acoustic System Simulation Model (CASS) version 4.2.¹⁷ These solutions are compared to our model, labeled "New" in Fig. 1.10. In this comparison Eq. (1.71) is used to remove the effect of the Earth's curvature from the "New" results. Prior to the shadow zone, all three models produce similar results. Although the loss predicted by our model is slightly higher than FFP in the shadow zone, its results are similar to the ones produced by GRAB. In both cases, the slight rise

Figure 1.11: Gaussian beam projection into the shadow zone for an n^2 linear profile at 2000 Hz.

in the coherent transmission loss appears to be caused by phase inaccuracies in the multi-path travel times predicted in the shadow zone.

1.4.3 3-D transmission loss benchmark

To demonstrate 3-D effects in transmission loss, we examine the analytic solutions for a 3-D wedge. In this scenario, receivers are at the same distance from the wedge apex as the source, but offset in range across the slope. In an 2-D model, these receivers appear to exist in an environment of constant depth. Because the 3-D

solution horizontally refracts acoustic energy down the slope, the 3-D solution has higher transmission loss, as a function of range across the slope, than the 2-D model.

Figure 1.12: Geometry for method of images in a 3-D wedge.

Using the method of images, we assume that each reflection gives rise to a source image, and that these images lie on a circle centered on the apex of the wedge. This

derivation is similar to the Deane/Buckingham model¹³, but it simplifies that model by assuming that interface reflection coefficients are limited to 1. This simplification creates an analytic solution that is accurate at higher frequencies.

Fig. 1.12 is a cross-slope view of the 3-D wedge showing each of the image sources and each virtual interface. In this illustration, surface interfaces are shown with a dashed line, bottom interfaces are shown with a dot-dashed line, and source images are shown as dots along the circumference of a circle whose radius is defined by the original distance of the source from the apex. The complex pressure at each receiver location is a sum of spherical wave contributions from each source image. If we assume that the reflection coefficient is +1 at the bottom and -1 at the surface.

$$p_q = \sum_{n=-n_{max}}^{n_{max}} \sum_{m=n-1}^n (-1)^m e^{ikR_{n,m,q}/R_{n,m,q}} \quad (1.73)$$

where n is the number of bottom reflections for source image, negative if above surface; m is the number of surface reflections for source image, negative if above surface; n_{max} is the maximum number of bottom bounces; $\vec{s}_{n,m}$ is location of each source image; q is the index number for each receiver; $\vec{r}_q$ is the location of each receiver; $R_{n,m,q}$ is the slant range from each source image to each receiver; c is the speed of sound in water; f is the signal frequency; k is the acoustic wave number $= 2\pi f/c$; and p_q is the total complex pressure for each receiver.

To compute $R_{n,m,q}$, we define a cylindrical coordinate system whose axis travels along the wedge apex: R_s is the slant range of original source from the wedge apex; α_s is the angle of original source down from the ocean surface; $\alpha_{n,m}$ is the angle of

each source image, relative to the ocean surface, negative if above surface; R_q is the slant range of each receiver from the wedge apex; α_q is the angle of each receiver down from the ocean surface; and y_q is the cross-slope distance of each receiver relative the vertical source/origin plane.

An inspection of the geometry in Fig. 1.12 allow us to compute $\alpha_{n,m}$ and $R_{n,m,q}$ for the 3-D wedge.

$$\alpha_{n,m} = 2n\alpha_w + (-1)^{n+m}\alpha_s \quad (1.74)$$

$$R_{n,m,q} = \sqrt{(R_s \cos \alpha_{n,m} - R_q \cos \alpha_q)^2 + y_q^2 + (R_s \sin \alpha_{n,m} - R_q \sin \alpha_q)^2} \quad (1.75)$$

Source images outside of the range $\alpha_{n,m} \in [-\pi, \pi]$ result in imaginary images that contribute to the diffracted component of the acoustic field. For small wedge angles and locations far from the apex, the diffracted components are negligible and need not be considered.¹³

An equivalent solution for an environment of constant depth uses an alternate version of $R_{n,m,q}$.

$$z_{n,m} = 2nz_w + (-1)^{n+m}z_s \quad (1.76)$$

$$R'_{n,m,q} = \sqrt{(x_s - x_q)^2 + y_q^2 + (z_{n,m} - z_q)^2} \quad (1.77)$$

where x_s, z_s is the range and depth of original source relative to ocean surface; x_q, z_q is the range and depth of each receiver relative to ocean surface; y_q is the cross-slope distance of the receiver relative the vertical source/origin plane; $z_{n,m}$ is the depth of each source image; and z_w is the water depth;

Because our model uses geodetic coordinates, the simple wedge used in our analytic solution can only be approximated in this comparison. On a round earth, an interface with constant slope is a curved surface instead of a plane. To minimize the impact of this curvature, the wide wedge angle scenario is used to shorten the range over which 3-D effects can be observed. The source and receivers are placed at a depth of 100 meters at the Equator. The water depth at this point is set to 200 meters and the bottom slope is a constant 21° , sloping down to the north, at all latitudes and longitudes. This definition orients the wedge in Fig. 1.12 such that the x-direction is north, the y-direction is east, and the z-direction is down. Receivers are placed east of the source, along the y-direction, at varying cross slope ranges.

Fig. 1.13 compares of our model to the analytic solution for a 3-D wedge. It also illustrates the analytic solution and the CASS/GRAB v4.2 prediction for an equivalent 2-D ocean with a constant depth of 200 meters. As predicted, the analytic solution for a simple 3-D wedge has higher transmission loss as a function of range across the slope than 2-D models of this same scenario. Our 3-D model accurately predicts this effect.

1.4.4 Computational efficiency

A key premise of this paper is that, when target/sensor geometries are constantly evolving, it is more computationally efficient to perform acoustic transmission loss in the latitude, longitude, altitude coordinates of the underlying environmental databases, than it is to convert the 3-D environments into a series of Nx2-D radials. To evaluate

Figure 1.13: Incoherent transmission loss at 2000 Hz for 3-D wedge and flat bottom.

this premise, a CASS scenario that interpolates radials from a 3-D data set was modified to compute transmission loss for a variable number of targets. The "std14" test that is distributed with CASS includes a grid of sound speeds and bottom depths, in latitude and longitude coordinates, for an area between 16:12N to 24:36N and 164:42W to 155:24W. The source is located at 19:31.2N 160:30.0W, at a depth of 200 m. We modified this test to include multiple targets, defined at a depth of 100 m and a range 100 km from the source, evenly spaced in azimuth. CASS constructs a radial for each target, and then computes transmission loss using a depression/elevation ray fan of $+89^\circ$ to -89° using 1° increments. An equivalent result from our model

propagates a wavefront for 80 seconds using a 100 ms time step, using 181 depression/elevation angles -90° to $+90^\circ$, and 25 azimuthal launch angles from 0° to 360° . Execution time is measured as a function of the number of targets.

Figure 1.14: Comparison to CASS/GRAB executions times as a function of number of targets.

Fig. 1.14 illustrates the computational speeds of the models, run on a Dell Latitude Laptop E6520 Intel i5-2540M CPU @2.60GHz, with a variable number of targets from 0 to 100, in increments of 10. The ordinal axis illustrates the time required to compute transmission loss for all targets on this hardware. The time required to compute transmission loss is roughly linear for both models. The GRAB model requires approximately 476 ms per target. The measured speed of our model is

approximately 3.07 seconds plus 42 ms per target. For small numbers of targets, GRAB is faster, because our model has to propagate the wavefront in all directions for 80 seconds, regardless of the number of targets. GRAB only needs to construct 2-D radials if a target is actually present. However, as the number of targets grows large, our model is faster because it has a much less computational overhead on a per target basis. The crossover point for this scenario appears to be approximately 4 targets. For 100 targets, our model is over 6 times faster than GRAB and 10 times faster than the speed of sound.

1.5 Conclusions

This paper has defined a 3-D Gaussian ray bundling model based on the same latitude, longitude, altitude coordinates used in the underlying environmental databases. The development of our model incorporates an implementation of 3-D refraction, 3-D interface reflection, 3-D eigenray detection, and a 3-D variant of Gaussian ray bundles. Testing to date indicates that this approach has accuracy at least as good as CASS/GRAB v4.2, but with improved execution speed benefits for large numbers of targets, and 3-D transmission loss effects.

Wavefront Queue 3-D (WaveQ3D) is a C++ implementation of this model. WaveQ3D is freely available to the research community as an open-source product distributed as part of the Under Sea Modeling Library (USML). Formal releases and test results are distributed through the Ocean Acoustics Library,³¹ a web site used by the U.S. Office of Naval Research as a means of publishing software of general use to the international

ocean acoustics community. Software developers can also participate directly in the WaveQ3D development process through the Under Sea Modeling Library project on the GitHub repository hosting service.⁴³ Documentation on the application programmer’s interface (API) for this software and additional test results are also available from both sources.

1.6 Acknowledgments

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Manuscript 2

Investigation of horizontal refraction on Florida Straits continental shelf using a
three-dimensional Gaussian ray bundling model

by

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2.1 Abstract

Acoustic transmission loss measurements from the Calibration Operations (CALOPS) experiment for the Shallow Water Array Performance (SWAP) program included horizontally refracted returns that were as much as 30 degrees away from the true bearing between source and receiver. In many cases, the in-shore refracted path was as much as 20 dB stronger than the true bearing path. In this study CALOPS transmission loss measurements at 415 Hz are compared to predictions from a 3D Gaussian Ray bundling model. The geoacoustic model that provides good model-data comparison is consistent with the geologic and sediment core data collected at the location but differs slightly from the bottom model used at lower frequencies (206 Hz and 52.5 Hz) in a previous study.

2.2 Introduction

Several investigators have recently studied the presence of strong 3D propagation effects in experimental data on the continental shelf in the Florida Straits area.^{30;22;41;19;20;4}

Acoustic transmission loss measurements from the Calibration Operations (CALOPS) experiment for the Shallow Water Array Performance (SWAP) program included horizontally refracted returns that were as much as 30 degrees away from the true bearing between source and receiver.¹⁹ In many cases, the in-shore refracted path was as much as 20 dB stronger than the true bearing path. CALOPS transmission loss measurements at 206 Hz²⁰ and 52.5 Hz⁴ have already been analyzed using

3D normal model/parabolic equation hybrid models. In the present study, measurements at 415 Hz are used to evaluate the 3D capabilities of the Wavefront Queue 3D (WaveQ3D) transmission loss model. WaveQ3D is a 3D Gaussian ray bundling model that implements propagation in geodetic coordinates.³⁸ This model supports 3D effects including horizontal refraction in sloped environments, and in this study we investigate this capability of the model.

2.3 Experiment

Figure 2.1 provides an overview of the geometry of the CALOPS experiment and the environment for this location, as specified in the Heaney¹⁹ and Ballard⁴ studies. The experiment used a horizontal line array of 120 elements, with half wavelength spacing at 450 Hz (1.75 m). This receiver is located on the bottom at 26:01:18N 79:59:26W and oriented such that the broadside beam points toward 8° relative to true north. As illustrate by the white dashed line in Figure 2.1(a), the source in Run 1N was towed along a heading of 8° true away from the receiver. This source, towed at a depth of 100 m, used a combination of 60 second long CW pulses, with frequencies of 24, 52.5, 106, 206, and 415 Hz and a 30-s multi-band set of five linear frequency modulated (LFM) pulses in the frequency bands 20-50, 50-100, 120-180, 200-300, and 320-420 Hz. The source levels varied from 170.5 $dB//\mu Pa@1m$ at 52.5 Hz to 171.0 $dB//\mu Pa@1m$ at 415 Hz. Transmission loss measurements were made at ranges between 3 and 80 km

The measured signal level is extracted from the frequency spectrum peak for each

Figure 2.1: (a) Area of operations showing source track (white dotted line). The sand-limestone boundary from Ballard⁴ is shown as black continuous line. The black dashed line is the modified boundary proposed by this study. (b) average sound velocity profiles along 350 m, 250 m, and 130 m contours (c) bottom loss for sand and limestone.

received ping. The output of this process is a series of 14 CW pings at 415 Hz followed by a period where 14 LFM pings at 320-420 Hz bleed into the CW frequency band. To isolate the CW contributions, we search for the output peaks of a 14 point uniformly weighted, normalized FIR filter. This filter yields the average level in each group of 14 CW pings. Noise levels are estimated from neighboring frequency bands. Results are rejected if they have a signal-to-noise ratio of less than 10 dB.

The bathymetry in Figure 2.1(a) is based on 3 arc-second resolution from the U.S.

Coastal Relief Model (CRM).¹² The grey scale has contours at 25 m increments. The black squares are locations where expendable bathythermographs (XBTs) were used to estimate the in-situ sound velocity profile. For this analysis, we averaged XBT measurements across the 350 m, 250 m, and 130 m contours and extended them to the bottom using Data Interpolating Empirical Orthogonal Functions (DINEOF).^{1;2} Panel (b) illustrates the averaged sound speeds, which increase in deeper areas under the influence of the warm waters of the Florida Current. To create a 3D profile, the averaged results are interpolated onto a 3D grid of latitude/longitude/depth locations based on their distance perpendicular to the source ship track.

The bottom loss shown in Figure 2.1(c) is a plane wave reflection coefficient derived from Ballard's analysis of geophysical measurements in this area.⁴ The sand bottom loss has a compressional wave speed of 1676 m/s , compressional attenuation of 0.01 (dB/λ), and a density of 1.70 (g/cm^3). The limestone bottom loss has a compressional wave speed of 3000 m/s , compressional attenuation of 0.10 (dB/λ), and a density of 2.40 (g/cm^3), a shear speed of 1430 m/s , and a shear attenuation of 0.20 (dB/λ). The limestone bottom has high bottom loss, but the carbonate sand sediments are almost perfectly reflective at the low grazing angles that influence long range propagation.

Ballard reports that the bottom is bare limestone below the 236 m isobaths, where loose sediments have been scoured away by the Florida Current. At shallower depths, carbonate sand layers cover the bottom. Ballard also reports an area near 26:12N 79:58W where echo sounder measurements along the source track indicate that a deep pool of sediment has formed between two sea mounts. Ballard's boundary between the

sand and limestone areas is shown by the black line in panel (a). Ballard states that the reported location of this boundary likely varies along the shelf and its position is difficult to fully characterize with the limited data available.⁴ Our modeling results discussed in Section III indicate that the geoacoustic model which provides good model-data comparison has this boundary shifted slightly towards deeper waters as shown in Figure 2.1(a) (dashed lines).

2.4 Comparison of measurements with model predictions

The environmental conditions and geometry discussed in Section II were used for the 3D modeling of the acoustic propagation for comparison with the measurements. Results from two iterations of modeling are presented in this section. The only difference in inputs between these two model runs is the location of the sand-limestone boundary. The first model run was performed with the location of the sand-limestone boundary as discussed in Ballard⁴ i.e. along the 236 m isobath. These model results at 415 Hz did not compare well with the observations. After a rigorous parametric study we identified the location of the sand-limestone boundary plays a critical role in determining the transmission loss. Hence we repeated our model runs with a modified location of the sand-limestone boundary which produced better agreement with the data. The details of these two model runs and the comparisons of the model results with the data are described in more detail in this Section.

To model this scenario in WaveQ3D, wavefronts are propagated from the receiver

to a series of target locations along the source track. WaveQ3D models propagation as the time evolution of ray paths that are launched across a fan of depression/elevation and azimuthal launch angles. Transmission loss is modeled as a sum of Gaussian contributions across the wavefront at each point where the wavefront intersects with a target.³⁸

The model indicates that ray paths are trapped along the bottom by the downward refracting nature of the sound velocity profile; this results in large numbers of bottom reflections along every path. Rays that are launched east of the ship's track travel at nearly constant azimuth, but they suffer from significant attenuation each time that they reflect off of the limestone. Ray paths west of the track (up the slope) are horizontally refracted back toward the source, but suffer from little or no reflection loss, because of the sandy bottom.

At each source location, WaveQ3D computes a series of eigenrays that each includes transmission loss amplitude and phase, depression/elevation and azimuthal launch angles, depression/elevation and azimuthal arrival angles, and travel time. To model the detection process, eigenrays are scaled by the beam pattern gain for each receiver beam and incoherently summed to estimate the average receive level. The strongest beam in the direct and in-shore regions is reported as the transmission loss and bearing at each range. Figure 2.2(a) illustrates transmission loss as a function of array bearing for each source range. The contributions between 0 and 40 km at a bearing of 8° are referred to by Heaney¹⁹ as the direct path contributions, even though they suffer from multiple bottom bounces. The in-shore paths, created by

horizontal refraction off of the continental slope, come in at a bearing of -80° at 3 km and shift to -18° at ranges of 60 km.

Figure 2.2: Compare modeled horizontal refraction to measured data.

Figure 2.2 (b) and (c) shows the comparison between measured and modeled results. The model accurately predicts the presence of the in-shore path, but over estimates its arrival angle at ranges below 20 km. The modeled transmission loss values for the direct paths are slightly weaker than the measured levels at ranges up to 40 km, but the in-shore levels appear to follow the measured transmission loss. At ranges beyond 40 km, the model significantly under predicts the strength of in-shore paths.

Figure 2.3: Modeled horizontal refraction with thin sediment layers.

We investigated the sensitivity of the modeling result to different parameters and found that the modeled transmission loss levels are most sensitive to the placement of the boundary between the sandy slope and the limestone shelf. Figure 2.3 illustrates results for an adjusted placement of the sand/limestone boundary that is a better fit to measured transmission loss. The first adjustment, shown as a dashed line in Figure 2.1(a), places an additional sand pool in the area around the receiver. This adjustment is supported by the fact that array was placed in a relatively flat region which is several kilometers wide, and echo sounder measurements indicate a sand layer of up to a 5 m thickness in the area along the source track and south of 26:02N.⁴ The

second adjustment shifts the sand/limestone boundary north of 26:27N from a depth of 236 m to 255 m. In this area, a sand layer could overlay the limestone without contradicting the echo sounder measurements, which indicate little or no coverage along the ship track in this area.

As shown in Figure 2.3, the agreement between transmission loss measurements is very good when the spatial distribution of the bottom properties is slightly modified (consistent with the core data and echo sounder measurements). The addition of a sand pool near the receiver significantly improves the fit of the modeled direct path transmission loss to measurements. It also significantly increases the strength of the modeled in-shore paths for ranges between 10 and 50 km. The movement of sand/limestone boundary north of 26:02N reduces the rate at which transmission loss decreases as a function of range beyond 50 km. In the high frequency approximation, bottom loss values are limited to an interface plane-wave reflection coefficient. It should be noted that the 415 Hz measurements used in this study may be more sensitive to surficial sediments than measurements at 52.5 Hz and 206 Hz. This sensitivity difference could explain our need to change the effective location of the boundary between sand and limestone bottom loss provinces.

2.5 Acknowledgments

This work would not have been possible without the sea test data and sage advice provided by Dr. Kevin Heaney, Dr. Megan Ballard, and their associates. The authors would like to also thank Dr. James Miller for his help and insight on the sensitivity

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Manuscript 3

How the U.S Navy is Migrating from Legacy/Large Footprint to Low Cost/Small
Footprint Sonar Simulation Systems

by

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3.1 Abstract

This paper describes the migration process undertaken by the US Navy to migrate from legacy, large footprint mainframe computer-based sonar simulation systems to next-generation sonar simulation systems with a smaller footprint, lower costs and better accuracy than the legacy models. The paper will describe the development efforts to create a faster and more accurate acoustic transmission loss (TL) and reverberation model for sonar simulation/stimulation systems in littoral environments based on ray theory for active sonar frequencies (above 1000 Hz). The paper also describes how the next-generation model augments ray theory with Gaussian beam techniques (based on the Gaussian Ray Bundling or GRAB), which enables simulation of frequencies as low as 150 Hz. The paper will detail the integration challenges faced by the US Navy to migrate from the legacy models to the next-generation sonar simulation model into the Navy's Live Virtual Constructive Modeling and Simulation (LVCMS) product line that includes PACT3, BATTT, and EFAAS simulators/simulations. The paper will also describe the results of these integration efforts, including the ability to provide trainees with improved training via more complex scenarios in the LVCMS training suite without increasing their hardware costs or footprint.

3.2 Introduction

The Live, Virtual, Constructive Modeling and Simulation (LVCMS)/Anti-Submarine Warfare Virtual At-Sea Training Systems (ASW VAST) is a suite of networked, PC-based deployable trainers designed to support integrated and coordinated ASW tactical training using Joint Semi Automated Forces (JSAF) Navy Training Baseline (NTB) simulation.²⁹ The current training system configuration is comprised of personal computers (PC), each PC system representing a mission training station in a particular aircraft (i.e. SH-608, SH-60F, P-3C) or shipboard system that, when in a networked environment, allow for integrated mission training. Currently, the LVCMS and ASW VAST family of training systems include:

- Mission Rehearsal Tactical Team Trainer (MRT3),
- Bravo Aircrew Tactical Team Trainer (BATTT),
- Bravo Romeo Acoustic Stimulation System (BRASS),
- High Fidelity Active Sonar Training Command (HIFAST CMD),
- P-3 Air Crew Tactical Team Trainer (PACT3),
- Virtual Maintenance Performance Aide (VMPA),
- Virtual Anti-Submarine Warfare (ASW)/Anti-Surface Warfare (ASUW) Tactical Air Controller (VASTAC),
- Effective Active Acoustic Simulation/Stimulation (EFAAS),

- Sonobuoy Acoustic Training System (SATS),
- Common Distributed Mission Training Station (CDMTS), and
- Tactical Warfare Instructor Support Environment (TACWISE).

Each of these systems supports a distributed simulation interface to the Fleet Synthetic Training (FST). FST exercises use distributed simulation to conduct in-port exercises at the Joint (FST-J), Group Command (FST-GC), Warfare Commander (FST-WC), and Unit (FST-U) levels. These exercises are used as part of Fleet Readiness Training Plan (F RTP) to certify the readiness of ships, submarines, aircraft, and commanders for deployment. When Navy assets are at-sea, they conduct operations; when they are in port, they train for the next operation. The Navy Continuous Training Environment (NCTE) is the hardware/software architecture that brings together the training systems for individual ships, submarines, aircraft, and commanders into a common virtual environment. Each combat system builds their own training system, and the NCTE integrates these systems across wide area networks.

Coordinated anti-submarine warfare (ASW) depends on a complex mosaic of diverse capabilities in a highly variable physical environment.²⁷ No single ASW platform, system, or weapon will work all the time. The undersea environment demand a multi-disciplinary approach, subsuming intelligence, oceanography, surveillance and cueing, multiple sensors and sensor technologies, coordinated multi-platform operations, and underwater weapons. Low-cost, deployable ASW training increase the opportunities for sonar technicians and commanders, from the Air Tactical Officers all the way up to the Theater ASW Commander, to practice playing together as a

Figure 3.1: Fleet Synthetic Training Concept

team.

One of the key technological hurdles to the engineering of such systems is creating sonar simulations that realistically represent the acoustic characteristics of the ocean environment, for dynamic scenarios, on limited hardware, in real-time. Sonar realism is a key element in teaching sonar technicians to recognize and react to the acoustic phenomena that allow them to detect, track, classify, localize, and engage submarine targets. In littoral environments, the sonar is often cluttered with large numbers of false targets, and accurate portrayal of these contacts is critical in teaching sonar technicians to discard false detections. False targets can be a particularly difficult problem for active sonar systems because these systems provide more limited classification data than passive systems. In this case, improving the quantity of acoustic contacts directly impacts the quality of the training. Training realism suffers when

only a small number of acoustic contacts are presented to the sonar operator. Unfortunately, most efforts to increase the number of realistic contacts have relied on the use of massively parallel computing systems with high acquisition costs.

Figure 3.2: Mission Rehearsal Tactical Team Trainer (MRT3)

The Wavefront Queue 3-D (WaveQ3D) model is a research effort to create fast and accurate acoustic transmission loss (TL) eigenrays, in littoral environments, for sonar simulation/stimulation systems. The new model is based on ray theory because Parabolic Equation and Normal Mode models run prohibitively slow at active sonar frequencies above 1000 Hz, where the number of propagating modes is large. To extend applicability to lower frequencies, the new model augments ray theory with Gaussian beam techniques based on the Gaussian Ray Bundling (GRAB),⁴⁵ which is certified for use down to 150 Hz. Our computation speed goal is to model one-way transmission loss for 100 targets on a single core of an average laptop.

3.3 Wavefront Queue 3-D Model

The key innovation in the development of the new model was the recognition that the requirements of sonar training systems are much different from those of tactical

decision aids. Sonar tactical decision aids (TDAs) are used at-sea to provide the sonar team with an ability to analyze the impact of ocean conditions on the likelihood of submarine detection. Because the behavior of the target cannot be predicted ahead of time, the TDAs must be optimized to perform their calculations over a wide variety of ranges, bearing, and depths. For many models, this optimization requires a transformation of oceanographic data onto a series of 2-D radials. This transformation allows the transmission loss calculations to use a simple Cartesian coordinate system and produce results at multiple ranges and depths. The amount of data to be analyzed is large, but such predictions are only generated a few times per day.

Sonar training systems must solve a very different problem. The ground truth locations of the targets are known by the sonar simulation, the behaviors of those targets react to trainee actions in real-time, and the sonar displays must display those reactions in real-time. The computational benefit of transforming into a series of 2-D radials is lost if each of those radials is only used to create a single transmission loss for the current location of each target. If the sonar simulation attempts to regain this advantage by interpolating between a small number of fixed radials, accuracy is lost. The amount of data to be analyzed is much smaller than in the TDA case, but predictions must be generated in real time for a dynamically evolving scenario.

The new model takes a new approach by leaving the data in latitude, longitude, and depth coordinates, and changing the transmission loss equations to operate in this 3-D coordinate system. The oceanographic data for large areas is cached in memory and re-used for multiple sensors. As illustrated in Figure 3.3, oceanographic

Figure 3.3: Extract 2-D radial and compute transmission loss

data from multiple sources, including open access data from the National Oceanic and Atmospheric Administration (NOAA), U.S. Navy standard databases like the Oceanographic and Atmospheric Master Library (OAML), dynamically evolving gridded data from NCTE, and live forecasts from the Fleet Numerical Meteorology and Oceanography Center (FNMOC), is combined to meet the needs of specific training exercises. Motion of the sensors does not require a change to the underlying environment. The premise of this approach is that, when scenario geometries are constantly evolving, it is more computationally efficient to perform acoustic transmission loss (TL) in spherical Earth coordinates than it is to convert the world into a series of 2-D Cartesian slices.

WaveQ3D models acoustic propagation as collection of wavefronts that move through the environment in increments of time. A single wavefront supports all

Figure 3.4: Caching data in latitude, longitude, and depth coordinates

of the acoustics contacts relevant to a given sensor. When the wavefront collides with an acoustic target, the model generates an eigenray that specifies the transmission loss, travel time, and source/receiver angles of the ensonification. The new model can collect the eigenrays for all acoustic contacts into a single container, or it can pass them back to the simulation as they happen. This second mode of operation allows the training simulation to start signal processing on nearby targets before processing has been completed on distant targets.

In conventional ray theory, the spreading of acoustic propagation loss is estimated by measuring the changes in ensonified area between ray paths. The intensity across the wavefront is inversely proportional to the change in a surface area segment compared to its area at the source. The Gaussian beam approach uses dynamic ray

equations to compute the divergence of the acoustic field normal to the path of propagation. The U.S. Navy’s Comprehensive Acoustic Simulation System / Gaussian Ray Bundle (CASS/GRAB) model takes a different approach and estimates the divergence directly from the shapes of the wavefront. This reduces the computational cost of transmission loss calculations. The WaveQ3D model adapts the 2-D CASS/GRAB approach for estimating divergence into 3-D environment.

3.4 Speed and Accuracy Testing

To evaluate the speed potential of the new model, a scenario was developed to compare speeds to existing Navy models for high fidelity TDAs. One challenge in developing such a scenario was finding a case in which the speed of the TDA model included the formation of 2-D radials. Fortunately, one of test routines that come with CASS/GRAB is a case where the bathymetry and sound speed are extracted directly from grids specified in latitude and longitude coordinates (Express Test 14). This test was modified to create a variable number of targets, 100 km from the source, evenly spaced in azimuth. An equivalent scenario was then created using WaveQ3D. The timing tests were run on a Dell Latitude Laptop with an E6520 Intel i5-2540M CPU running at 2.60GHz. As shown in Figure 3.5, CASS/GRAB ran faster than the new model when the number of targets was small. This happens because all WaveQ3D runs had the overhead of wavefront propagation, regardless of the number of contacts in the water. However, when the number of contacts became significant, the new model ran much faster than CASS/GRAB. Although the overhead of

wavefront propagation was non-trivial, the new model required very little additional computation time for each additional contact.

Figure 3.5: Execution speed comparison

WaveQ3D is currently undergoing a series of tests to evaluate its realism. These tests are based on the premise that improvements in execution speed would not support the training objectives if the new model did not support sonar realism. These tests take several forms:

- Verification relative to analytic solutions for simple conditions,
- Verification relative to results of other standard models,
- Validation relative to at-sea measurements,

- Validation by sonar training experts as part of integration into actual training systems.

One of the key tests (Section A.5.5) is a classic benchmark that looks at the accuracy of transmission loss propagation into an acoustic shadow zone.³² In this scenario, illustrate by the left side of Figure 3.5, a strong downward refracting environment limits the amount of energy reaching distant targets. The right side of Figure 3.5 compares the transmission loss results of WaveQ3D, CASS/GRAB, and a full field calculation using the Fast Field Program (FFP) wavenumber integration model.¹⁴ Note that, at all ranges, the FFP result is consistent with an ideal wave equation solution except for the presence of some minor implementation jitter in the ranges above 880 m. Prior to the shadow zone, all three models produce similar results. In the region beyond 840 m, the WaveQ3D and CASS/GRAB transmission loss values are similar to each other, but slightly higher than FFP. Given the wide use of CASS/GRAB in tactical decision aids, the ability to produce results that are no worse than those used for operational planning is considered a major milestone for WaveQ3D.

The High Fidelity Active Sonar Training (HiFAST) Project at the U.S. Office of Naval Research has funded an effort at the Advanced Research Laboratory at the University of Texas in Austin to independently evaluate the accuracy of the WaveQ3D model relative to other U.S. Navy standards. The results of that study are still pending.

Figure 3.6: Accuracy comparison

3.5 Integration into Training System

Our first test case for the integration of The new model into a real training system was the replacement of the FeyRay model in the active processing module (AP) for LVCMS/ASW VAST. Figure 3.7 provides a high-level overview of the AP module process. The Acoustic Server for the trainer of interest creates an AP module process for each active sensor. When a new scenario is started, the LOAD_GAMING_AREA message instructs Environmental Manager objects for each data type to request data from the Dynamic Ocean Generator (DOG). This data includes

- Bathymetry - Latitude, Longitude, Depth
- Sound Velocity Profile - Latitude, Longitude, Depth, Speed of Sound at Depth
- Bottom Type - Latitude, Longitude, Bottom Type

- Wind - Latitude, Longitude, Wind Speed, Wind Direction
- Ocean Wave - Latitude, Longitude, Wave Height
- Ocean Current - Latitude, Longitude, Current Speed, Current Direction

The legacy implementation computes transmission loss, using the FeyRay model, along a predetermined number of radials. Interpolation is then used to create a series of multipath echoes for each target. Signal processing in the AP module converts the echoes from each target into an audio result. The echoes for all targets are combined with each other and with data from the noise and reverberation models. The resulting audio is then sent back to the Acoustic Server, which forwards it to the sonar for processing and presentation to the trainee.

Figure 3.7: AP module Processing

Integrating the new model into the AP module required the following changes:

- A method was added to the Dynamic Ocean Generator (DOG) that allowed clients to request information for the whole gaming area in latitude, longitude, depth coordinates. This was already the native coordinate system of the data, but the DOG had no request mechanism for this format.
- A singleton Environment Manager was created to store and manage environmental data, in a WaveQ3D compatible format, for the whole gaming area. Multiple processing threads, for multiple sensors, can share this single representation of the environmental data.
- The function that creates the requests for transmission loss was modified to create a single request for all targets, instead of a separate request for each target.
- WaveQ3D specific configuration options were added to the AP modules setup process.

Most of the engineer effort focused on fitting the new model into the existing architecture, and then testing the accuracy and robustness of the integration. Figure 3.8 provides some examples of screen shots taken during integration testing. In this example, a DICASS active sonobuoy was used to detect a 15 knot surface target using a CW waveform in the Bravo Acoustic Tactical Team Trainer (BATTT). The horizontal axis represents range from the sensor, and the vertical axis represents Doppler shift. A shallow water area with strong reverberation was selected, as evidenced by the long green return around zero Doppler. The targets speed relative to the sensor

caused the target echo to be Doppler shifted from the reverberation, and this appears as the return highlighted by the plot annotations.

Figure 3.8: Comparison of sonar displays on BATTT

To quantify the differences between the two models, difference in the transmission loss, travel time, and source/receiver angles of each eigenray were measured and analyzed.⁸ Table 3.1 presents the differences between the WaveQ3D and FeyRay models, for the scenario illustrated in Figure 3.8. Our analysis indicates that differences between the two models were primarily due to differences in the sound velocity profiles in the 2-D and 3-D environments.

Table 3.1: Differences in WaveQ3D eigenrays, relative to FeyRay.

Path	Time	Launch	Arrival	TL
Direct	0.2 ms	0.104	0.09	0.44 dB
Surface	0.0 ms	0.245	0.450	0.65 dB
Bottom	0.1 ms	0.646	3.192	0.53 dB

3.6 Conclusions

The LVCMS/ASW VAST suite of training systems increase the opportunities for sonar technicians and commanders, from the Air Tactical Officers all the up to the Theater ASW Commander, to practice playing together as a team. A new acoustic transmission model, WaveQ3D, has been developed to improve the scope and complexity of this training without sacrificing fidelity and without increasing hardware costs or footprint. Future work includes:

- Development of a reverberation model based on WaveQ3D,
- Replacing the legacy model in all LVCMS/ASW VAST signal processing modules,
- Continuing to support accuracy testing, and
- Improving other sonar training systems using this technology.

The WaveQ3D model is an open-source product distributed as part of the Under Sea Modeling Library (USML), in accordance with the BSD 2-Clause License. Official

releases are posted to the Ocean Acoustics Library,¹ sponsored by the U.S. Office of Naval Research. Third parties are also free to contribute to the development of this model by forking the Git repository.²

3.7 Acknowledgments

This model was developed as part of Sean Reilly’s PhD studies at the Ocean Engineering Department of the University of Rhode Island, under the direction of Dr. Gopu Potty and Dr. James Miller. Testing and productization of this model were funded by Michael Vaccaro as part of the High Fidelity Active Sonar Training (HiFAST) Project at the U.S. Office of Naval Research.

3.8 About the Authors

Mr. Sean M. Reilly is a Ph.D. student at the University of Rhode Island and serves as the Senior System Architect for the Air Force Modeling and Simulation Training Toolkit (AFMSTT) at The Aegis Technologies Group. He received his M.S. and B.S. in Physics at the University of Connecticut, with a specialty in computational Physics. Sean was previously the System Architect for the Navy Continuous Training Environment (NCTE), as well as working on the High Fidelity Active Sonar Training (HiFAST) project at Office of Naval Research to develop a new 3D underwater acoustic propagation loss model for real-time, active sonar, simulation/stimulation

¹<http://oalib.hlsresearch.com/Rays/USML/usml.frontpage>

²<https://github.com/campreilly/UnderSeaModelingLibrary>

systems in littoral environments.

Mr. Jonathan Glass is a Program Manager of the Live Virtual Constructive Modeling and Simulation (LVCMS) training suite at the Naval Air Warfare Center Training Systems Division in Orlando, FL. He has over 20 years of experience as a Sonar Modeling and Acoustic simulation environments.

Manuscript 4

Evaluating accuracy limits of Gaussian ray bundling model in the deep sound
channel

by

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4.1 Abstract

Acoustic tomography uses propagation models to predict the multi-path arrival structure of one-way transmissions received from a distant source. Tomographic inversion repeatedly perturbs the ocean sound speed and re-computes the arrival structure until only trivial differences remain between the measured and modeled results. WaveQ3D is a new 3D Gaussian ray bundling model that is designed to reduce the computation burden of propagation loss calculations. This feature may have significant benefit in tomographic inversion, where thousands of modeling runs may be required to converge on a solution. However, speed is only a benefit if the model accurately predict the multi-path arrival structure for a given profile. This paper investigates WaveQ3D's ability to predict the multi-path arrival structure for the Munk profile, an idealized representation of deep sound channel conditions in the North Pacific.

4.2 Introduction

Acoustic tomography uses propagation models to predict the multi-path arrival structure of one-way transmissions received from a distant source.⁴⁰ Tomographic inversion repeatedly perturbs the ocean sound speed and re-computes the arrival structure until only trivial differences remain between the measured and modeled results. WaveQ3D is a new 3D Gaussian ray bundling model that is designed to reduce the computation burden of propagation loss calculations. This feature may have significant benefit in tomographic inversion, where thousands of modeling runs may be required to converge

on a solution. However, speed is only a benefit if the model accurately predict the multi-path arrival structure for a given profile. This paper investigates WaveQ3D's ability to predict the multi-path arrival structure for the Munk profile, an idealized representation of deep sound channel conditions in the North Pacific.²⁸

$$z' = 2 \frac{z - z_1}{B} \quad (4.1)$$

$$c(\rho) = c_1 \left[1 + \epsilon \left(z' - 1 + e^{-z'} \right) \right] \quad (4.2)$$

where z' is the normalized depth (positive is down), z_1 is the depth of the deep sound channel axis (1300 meters below ocean surface), B is a depth scaling factor (1300 meters), c_1 is the sound speed on the deep sound channel axis (1500 m/s), and ϵ is a profile scaling factor (7.37×10^{-3}).

4.3 Benchmark solution for Snell's Law in spherical media

WaveQ3D computes propagation on a spherical section of the Earth using geodetic coordinates: latitude, longitude, and altitude. This approach speeds up 3D applications by using environmental parameters and their derivatives directly in the latitude, longitude, and depth directions without reducing the problem to a series of Nx2D Cartesian projections. Snell's Law for spherical media (Eq. (4.3)) allows us to build an benchmark propagation solution that is also based on a spherical Earth.¹⁰

$$m = \frac{\rho \cos \eta}{c(\rho)} = \text{constant} \quad (4.3)$$

Figure 4.1: (a) Munk profile, an idealized representation of deep sound channel conditions in the North Pacific. (left panel) Ray paths trapped in the deep sound channel.

$$\frac{d\theta}{d\rho} = \frac{1 \cos \eta(\rho)}{\rho \sin \eta(\rho)} \quad (4.4)$$

$$\theta = \int_{\rho_s}^{\rho_t} \frac{mc(\rho)}{\rho \sqrt{\rho^2 - (mc(\rho))^2}} d\rho \quad (4.5)$$

$$\tau = \int_{\rho_s}^{\rho_t} \frac{\rho}{c(\rho) \sqrt{\rho^2 - (mc(\rho))^2}} d\rho \quad (4.6)$$

where η is the depression/elevation angle along the ray path; m is the ray parameter for spherical media (constant for each launch angle), τ is the travel time along the ray path; ρ_s is the radial coordinate for the source depth; and ρ_t is the radial coordinate

Figure 4.2: (a) Munk profile (b) eigenrays to a target at 200 km, computing using Snell’s Law for spherical media, legend indicates launch angle.

for the target depth. Although these integrals only apply between the source and the first vertex or reflection, paths out to any range can be constructed by repeating this process after the vertex or reflection. In Figure 4.1, our spherical coordinates benchmark solution for ray paths in a deep sound channel are computed by applying Eq. (4.5) to the Munk profile.²⁸ In this implementation, integrals are evaluated discretely using the MATLABTM *quadgk* implementation of an adaptive Gauss-Kronrod quadrature.³⁹

The eigenrays are the discrete set of paths that connect the source to a target at the same depth. To find eigenrays for the benchmark solution, we break the each

Table 4.1: Eigenrays to a target at 200 km, computing using Snell’s Law in spherical media.

ID	Launch Angle (deg)	Travel Time (sec)
1	13.2195	132.8836
2	-8.1769	133.2306
3	-5.2455	133.2876
4	5.2455	133.2876
5	1.7881	133.2995

propagation path into a series of source depth crossings, and calculate the range at which each crossing occurs. After an even number of crossings, paths that left the source traveling up will arrive at their target from below. At the end of each cycle, the ranges for each path increase monotonically as a function of increasing launch angle. An eigenray is present when these ranges span the target location. Interpolation allows us to invert range as a function of launch angle into launch angle at the target range. The same process is used for Eq. 4.6, the travel time integral . To find all eigenrays, the process is repeated for odd numbers of crossings, and paths that leave the source traveling down. Figure 4.2 and Table 4.1 illustrate the eigenrays computed using this technique. Note that in Figure 4.2, distances are converted to range along the surface of the earth by multiplying $\Delta\theta$ by the standard radius of the FAI Sphere (6371.0 km).¹⁶

4.4 Equivalent benchmark solution in Cartesian coordinates

To evaluate the impact of numerical error in the discrete evaluation of Eqs. (4.5) and (4.6), an equivalent benchmark solution is also computed in Cartesian coordinates. The Earth Flattening Transform²⁶ allows acoustic models that are based on depth and range to incorporate Earth curvature effects into their calculations. To understand the Earth Flattening Transformation, we start by looking at propagation on a range-independent spherical Earth, an environment where environmental parameters do not depend on latitude or longitude. In this environment, propagation can be modeled using the 2D Helmholtz equation in cylindrical coordinates.

$$\frac{\partial^2 \psi}{\partial \rho^2} + \frac{1}{\rho^2} \frac{\partial^2 \psi}{\partial \theta^2} + \left(\frac{2\pi f}{c(\rho)} \right)^2 \psi = 0 \quad (4.7)$$

where f is frequency of signal; $c(\rho)$ is speed of sound as a function of depth in this coordinate system. We define an equivalent flat Earth using a change of variables (x, z) .

$$x = R \theta \quad (4.8)$$

$$z = R - \rho \quad (4.9)$$

$$c'(z) = c(R - z) \quad (4.10)$$

where R is radius of the earth; x is range as change in distance along the surface of the earth; z is depth below the ocean surface (mean sea level); and $c'(z)$ is speed of sound as a function of depth in this coordinate system. To model acoustic propagation in the (x, z) system, we use another change of variables (x, u) to transform Eq. 4.7 into a Cartesian form.

$$\rho = Re^{-\frac{u}{R}} \quad (4.11)$$

$$c'(\rho) = \frac{\rho}{R}c(u) \quad (4.12)$$

$$d\rho = -e^{-\frac{u}{R}}du = -\frac{\rho}{R}du \quad (4.13)$$

$$d\theta = \frac{1}{R}dx \quad (4.14)$$

$$\left(-\frac{R}{\rho}\right)^2 \frac{\partial^2 \psi}{\partial u^2} + \left(\frac{R}{\rho}\right)^2 \frac{\partial^2 \psi}{\partial x^2} + \left(\frac{R}{\rho} \frac{2\pi f}{c'(u)}\right)^2 \psi = 0 \quad (4.15)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial u^2} + \left(\frac{2\pi f}{c'(u)}\right)^2 \psi = 0 \quad (4.16)$$

where ρ is depth non-linear function of ρ ; $c'(u)$ is speed of sound as a function of depth in this coordinate system. The relationship between the (x, z) and (x, u) coordinate systems is given by

$$u = -R \ln \left(1 - \frac{z}{R}\right) \quad (4.17)$$

$$c'(u) = c'(z) / \left(1 - \frac{z}{R}\right) \quad (4.18)$$

The relationship between z and u is often approximated.

$$u = -R \left[\left(-\frac{z}{R}\right) - \frac{1}{2} \left(-\frac{z}{R}\right)^2 + \frac{1}{3} \left(-\frac{z}{R}\right)^3 - \dots \right] \quad (4.19)$$

$$u_1 \equiv z \quad (4.20)$$

$$u_2 \equiv z \left[1 + \frac{z}{2R} \right] \quad (4.21)$$

where u_1 is the first order approximation; and u_2 is the second order approximation.

The analysis solution in (x, u) terms is implemented using Snell's Law in Cartesian coordinates.

$$a = \frac{\cos \eta(u)}{c'(u)} = \text{constant} \quad (4.22)$$

$$\frac{dx}{du} = \frac{\cos \eta(u)}{\sin \eta(u)} \quad (4.23)$$

$$x = \int_{u_s}^{u_t} \frac{ac'(u)}{\sqrt{1 - (ac'(u))^2}} du \quad (4.24)$$

$$t = \int_{u_s}^{u_t} \frac{1}{c'(u)\sqrt{1 - (ac'(u))^2}} du \quad (4.25)$$

where $\eta(u)$ is the depression/elevation angle along the ray path; a is the ray parameter in Cartesian coordinates (constant for each launch angle); t is the travel time in Cartesian coordinates; u_s is the source depth; and u_t is the target depth. In theory, there should be no difference between the results computed using Eqns. (4.5) to (4.6) and those computed using Eqns. (4.24) to (4.25).

Table 4.2: Largest errors for each versions of the Earth Flattening Transform.

ID	Range Error (m)	Time Error (msec)
no transform	+690.1	+464.4
1st order	-28.72	-18.92
2nd order	+0.2538	+0.1637
no approx	+0.2706	+0.1747

Figures 4.3 and 4.4 illustrates the difference between the Cartesian and spherical benchmark solutions to Snell's Law, using different versions of the Earth Flattening

Figure 4.3: Range difference between Cartesian and spherical benchmark solutions to Snell's Law, using difference versions of the Earth Flattening Transform.

Transform. The range error in Figure 4.3 is computed by measuring the range to the first source depth crossing, as a function of launch angle. The time error in Figure 4.4 is computed by comparing travel times at this same point. Each curve labeled "no transform" represents the error without the Earth Flattening Transform, that is when $u = z$ and $c'(u) = c'(z)$. The curves labeled "1st order" uses a combination of Eq. (4.18) and the approximation in Eq. (4.20). Similarly, the curves labeled "2nd order" uses the approximation represented by Eqs. (4.18) and (4.21). Each curve labeled "no approx" uses Eqs. (4.18) and (4.17), the exact form of the Earth Flattening Transform. Note that the results for "2nd order" are so close to the "no approx"

Figure 4.4: Travel time difference between Cartesian and spherical benchmark solutions to Snell’s Law, using difference versions of the Earth Flattening Transform.

curve, that the two curves can not be distinguished at this scale. The largest errors for each curve are represented in Table 4.2.

In theory, Snell’s Law solutions derived from the Flat Earth Transformation with no approximations should be identical to those computed in spherical coordinates. From Table 4.2, we conclude that numerical precision limitations in the adaptive Gauss-Kronrod quadrature introduce a error of up to 0.27 m in range and 0.17 ms in travel time to each up/down cycle of the ray trace.

4.5 WaveQ3D accuracy

To evaluate the accuracy limits of WaveQ3D in this application, we propagated a wavefront to a target 200 km north of the source. To mimic the conditions from our benchmark solutions, we limited the ray fan to $\pm 14^\circ$ in the vertical and $\pm 0.2^\circ$ in the horizontal. Spacing between rays was set to 0.1° in both directions. The wavefronts were propagated in time with a step size that varied from 25 to 100 ms.

Figure 4.5: Range difference between WaveQ3D and Snell's Law in spherical media, for 25 ms time step, after 4 complete cycles. Includes comparison to Flat Earth Transform with no approximations.

Figures 4.5 and 4.4 illustrates the difference between the WaveQ3D results with

Figure 4.6: Travel time difference between WaveQ3D and Snell’s Law in spherical media, for 25 ms time step. Includes comparison to Flat Earth Transform with no approximations.

Table 4.3: Largest errors for each step size in WaveQ3D

Step (ms)	Range Error (m)	Time Error (msec)	Exec Speed (s)
100	-26.48	-0.9723	0.5
50	-3.315	-0.1394	0.9
25	-0.4146	-0.0385	1.8
flat earth	+1.135	-0.0239	n/a

a time step of 10 ms and spherical benchmark solutions to Snell’s Law. To estimate range error in the vicinity of the 200 km target, Figure 4.5 changes the definition

or range error to be the difference after each solution completes 4 cycles. The time error is computed as the difference in eigenray travel times at the 200 km target. These figures includes a comparison to Flat Earth Transform equivalent with no approximations to illustrate the scale of these error relative to numerical precision limitations in the adaptive Gauss-Kronrod quadrature.

The largest errors for each time step setting are represented in Table 4.3. In addition to illustrating the dependence of errors on time step size, this table also includes the execution time needed to compute WaveQ3D eigenray results for this scenario on a single core of a Dell Latitude E6540 laptop with an Intel(R) i7-4810MQ CPU @ 2.80 GHz. Execution speed information is provided for acoustic tomography researchers who may be interested in the relationship between accuracy and computational efficiency for inversion applications.

4.6 Conclusions

For a target at 200 km, the differences between WaveQ3D and the benchmark solutions for Snell’s law in spherical media less than $1.6 \times 10^{-4}\%$ in range and $2.9 \times 10^{-5}\%$ in travel time, for a time step of 25 ms. However, these differences are nearly identical to the errors introduced by numerical precision limitations in the adaptive Gauss-Kronrod quadrature used in the benchmark solution. For a time step of 25 ms, there appears to be no significant difference between WaveQ3D accuracy and that of the benchmark.

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Appendix A

Verification Tests for Hybrid Gaussian Beams in Spherical/Time Coordinates

by

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A.1 Abstract

The theory paper defined a new undersea acoustic propagation loss model that is specifically designed to support real-time, sonar simulation/stimulation systems, in littoral environments, at active sonar frequencies. This paper seeks to verify the model's implementation by comparing the modeled results to analytic solutions.

A.2 Introduction

The theory paper³⁸ defined the theory for a new undersea acoustic propagation loss model that is specifically designed to support real-time, sonar simulation/stimulation systems, in littoral environments, at active sonar frequencies. The WaveQ3D model implements this theory using a circular queue of time domain wavefronts, in a fully 3-D ocean environment, with a computationally-efficient form of C++ vector processing.⁴⁴ This follow-up paper analyzes the capabilities and limitations of this implementation. The Capability Maturity Model Integration (CMMI)¹¹ separates testing needed for such an effort into two phases:

- **Verification** testing ensures that selected work products meet their specified requirements. For this analysis, the WaveQ3D model is decomposed into its component parts (such as ray tracing, reflection, eigenray finding, and propagation loss), and the results from each parts are compared to analytic solutions.
- **Validation** testing demonstrates that a product or product component fulfills its intended use when placed in its intended environment. For the WaveQ3D

model, this will consist of comparisons to real-world results in a subsequent paper.

Decomposing the testing in this way is designed to ensure that any conclusions drawn from the modeled results rest on a firm foundation of understanding.

A.3 Ray Tracing Tests

The ray paths in this model use a third order Adams-Bashforth (AB3) marching solution⁴⁹ to create a circular queue of time domain wavefronts. The evolution of the wavefront shape, as it passed through the 3-D ocean environment, is defined by the following equations.

$$\frac{dr}{dt} = c^2 \alpha , \quad (\text{A.1})$$

$$\frac{d\theta}{dt} = \frac{c^2 \beta}{r} , \quad (\text{A.2})$$

$$\frac{d\phi}{dt} = \frac{c^2 \gamma}{r \sin \theta} , \quad (\text{A.3})$$

$$\frac{d\alpha}{dt} = -\frac{1}{c} \frac{dc}{dr} + \frac{c^2}{r} (\beta^2 + \gamma^2) , \quad (\text{A.4})$$

$$\frac{d\beta}{dt} = -\frac{1}{c r} \frac{dc}{d\theta} - \frac{c^2}{r} (\alpha \beta + \gamma^2 \cot \theta) . \quad (\text{A.5})$$

$$\frac{d\gamma}{dt} = -\frac{1}{c r \sin \theta} \frac{dc}{d\phi} - \frac{c^2 \gamma}{r} (\alpha + \beta \cot \theta) , \quad (\text{A.6})$$

where c is the speed of sound as a function of location; t is the travel time; (r, θ, ϕ) are the spherical earth coordinates of the modeled ray path as a function of time; and (α, β, γ) are the spherical earth coordinates of the normalized ray direction as a function of time. The tests discussed in this section analyze the accuracy of Eqs. (A.1) through (A.6) in scenarios where the rays do not encounter any boundaries.

A.3.1 Comparisons to “flat earth” benchmarks

Because WaveQ3D is one of the few models to use non-Cartesian coordinate system, comparisons to other work often require translation before differences can be analyzed. This section analyzes the accuracy of the translation between spherical and Cartesian coordinate models. Cartesian coordinate propagation models frequently use a modified index of refraction³³ to incorporate earth curvature effects into their calculations.

$$n'(r) = \frac{r}{R} \frac{n(r)}{n(R)}, \quad (\text{A.7})$$

$$c'(z) = \frac{c(z)}{1 - z/R}, \quad (\text{A.8})$$

where R is the radius of earth’s curvature in this area of operations; r is the radial distance from the center of curvature (positive is up); $z = R - r$ is the below the ocean surface (positive is down); $n(z), n'(z)$ are the original and modified index of refraction, and $c(z), c'(z)$ are the original and modified speed of sound. When a testing benchmark is specified in Cartesian coordinate, the inverse of this process must be implemented to create an equivalent environment in spherical earth coordinates.

$$c'(r) = \frac{r}{R} c(r), \quad (\text{A.9})$$

where $c(r)$ is the benchmark’s original $c(z)$ sound speed converted to a function of r .

The Munk profile²⁸ was used to evaluate the accuracy of testing Cartesian benchmarks in a model based on spherical earth coordinates. The Munk profile is an idealized representation of a typical deep sound channel, and it was chosen for its

ability to support long range paths without interface reflection.

$$z' = 2 \frac{z - z_1}{B} , \quad (\text{A.10})$$

$$c(z) = c_1 \left[1 + \epsilon \left(z' - 1 + e^{-z'} \right) \right] , \quad (\text{A.11})$$

where z' is the normalized depth (positive is down), z_1 is the depth of the deep sound channel axis (1300 meters), B is a depth scaling factor (1300 meters), c_1 is the sound speed on deep sound channel axis (1500 m/s), and ϵ is the profile scaling factor (7.37×10^{-3}). The specific Munk profile parameters used in this test were selected to match Fig. 3.19 in Jensen, Kuperman, et. al.^{21;36} An example of the ray paths for this profile are illustrated in Fig. A.1. This figure was created by WaveQ3D with a 100 ms time increment and 1° separated depression/elevation launch angles from -14° to 14° . Launch angles greater than 14.38° encounter interface reflections.

Munk's paper²⁸ characterized the analytic solution for ray paths using their cycle range, the range required to complete one period of upward and downward refraction. The cycle range is equal for positive and negative launch angles. The cycle range was used as the metric for this test because it could be cast into identical units in both spherical and Cartesian coordinates.

To create an analytic equivalent to a typical Cartesian model, Snell's Law (Eq. A.12) was integrated numerically, using the MATLABTM `quadgk()` implementation of an adaptive Gauss-Kronrod quadrature.³⁹

$$a = \frac{\cos \eta(z)}{c(z)} = \text{constant} , \quad (\text{A.12})$$

$$\frac{dH}{dz} = \frac{\cos \eta(z)}{\sin \eta(z)} , \quad (\text{A.13})$$

Figure A.1: Modeled ray paths for the Munk profile.

$$\Delta H = \int_{z_s}^{z_t} \frac{ac(z)}{\sqrt{1 - (ac(z))^2}} dz , \quad (\text{A.14})$$

$$\Delta t = \int_{z_s}^{z_t} \frac{1}{c(z)\sqrt{1 - (ac(z))^2}} dz , \quad (\text{A.15})$$

where $\eta(z)$ is the depression/elevation angle along the ray path; a is the ray parameter (constant for each launch angle); H is the horizontal range; t is the travel time; z_s is the source depth; and z_t is the target depth. Although these integrals only apply between the source and the first vertex or reflection, paths out to any range can be constructed by repeating this process after the vertex or reflection.

To create equivalent conditions for a model based on spherical earth coordinates,

we modified the original sound speed using Eq. (A.9), and then performed a similar integration in spherical coordinates. Snell's law in spherical media includes an extra factor of r that is not present in the Cartesian coordinates.¹⁰ The slope of the ray path also includes an extra factor of r in spherical coordinates.

$$p = \frac{r \cos \eta(r)}{c(r)} = \text{constant} \quad (\text{A.16})$$

$$\frac{r d\theta}{dr} = \frac{\cos \eta(r)}{\sin \eta(r)} \quad (\text{A.17})$$

$$\Delta\theta = \int_{r_s}^{r_t} \frac{pc(r)}{r \sqrt{r^2 - (pc(r))^2}} dr \quad (\text{A.18})$$

$$\Delta t = \int_{r_s}^{r_t} \frac{r}{c(r) \sqrt{r^2 - (pc(r))^2}} dr \quad (\text{A.19})$$

where $\eta(r)$ is the depression/elevation angle along the ray path; p is the ray parameter for spherical media (constant for each launch angle), $\Delta\theta$ is the horizontal range in solid angle units; r_s is the radial coordinate for the source depth; and r_t is the radial coordinate for the target depth.

The difference in the cycle ranges computed using these two analytic solutions is shown in A.2. The results were computed for two complete periods, with launch angles from 0° to 14° , as a function of the spherical coordinates cycle range solution. The 0° launch angle has difference of -5.14 meters at a range of 96.16 km. The 14° launch angle has a difference of -38.34 meters at range of 129.95 km. The intermediate angles have monotonically increasing values between those two extremes.

Since both solutions are analytic, we conclude that difference must be a fundamental property of ray paths computations in the two coordinate systems. It is equally valid to attribute these results to

Figure A.2: earth-flattening accuracy for the Munk profile.

- Errors created by a spherical model working with a Cartesian environment, or
- Errors created by a Cartesian model working with a spherical environment.

The second case is more interesting for this study, because it represents a widely accepted (but seldom mentioned) lower limit on ray path range accuracy, approximately 0.03% of the total range.

A.3.2 Ray path accuracy in a deep sound channel

In this test, the refraction accuracy of the WaveQ3D model was computed for a “flat earth” Munk profile defined in spherical coordinates by combining Eqs. (A.10) and (A.11) with Eq. (A.9). Fig. A.3 illustrates the difference between individual WaveQ3D rays and the spherical analytic solution defined by Eqs. (A.16) through (A.18). Cycle ranges were computed for both the first and second period of the SOFAR cycle. Because both solutions were computed in spherical coordinates, using the same sound speed profile, these error can not be attributed to the use of a modified index of refraction.

With a 100 ms step size, the WaveQ3D result deviates from the analytic solution by a maximum of -8.62 meters at cycle range of 129.95 km (0.007% error). However, 50 out of 58 samples (86%) exhibited errors less than ± 2 meters (0.002% error). Ray paths that were initially launched toward the surface, where the sound speed gradient is highest, had consistently larger errors than paths that were launched down. The fact that these errors were all significantly less than those of associated with the earth curvature correction (Fig. A.2) suggests that WaveQ3D model’s cycle range estimate meets or exceeds the accuracy of Cartesian models used on a spherical earth.

To estimate the impact of WaveQ3D options on this result, the maximum error was also computed as a function of time step size. The circles in Fig. A.4 represent maximum errors, across launch angles, for time steps of 25, 50, 100, 150, 200, 250, 300, 350, and 400 ms. The connecting lines smoothly interpolate between these discrete values. From this, we conclude that the accuracy of WaveQ3D cycle range estimates is

Figure A.3: WaveQ3D errors for the Munk profile.

slightly weaker than a power law; a doubling of the time step decreased the accuracy by approximately a factor of 10. Step sizes as large as 150 yielded results that were at least as accurate as errors associated with the modified index of refraction. Given that some environments may have stronger gradients than the Munk profile, we conclude that a 100 ms step size should be adequate for most long range applications.

Figure A.4: Path accuracy sensitivity to step size for the Munk profile.

A.3.3 Ray path accuracy in an extreme downward refraction environment

In this test, the refraction accuracy of the WaveQ3D model was computed for the extreme n^2 linear test case developed by Pedersen and Gordon.³²

$$c(z) = \frac{c_0}{\sqrt{1 + \frac{2g_0}{c_0} z}} \quad (\text{A.20})$$

where c_0 is the sound speed at the ocean surface (1550 m/s); g_0 is the sound speed gradient at the ocean surface (1.2 s^{-1}). At shallow depths, this profile matches

observed conditions from the Pacific, in an area of extreme velocity gradient. But at depths greater than about 60 meters, it predicts theoretically useful, but physically unrealistic sound speeds. This profile was selected for this study because of its wide use by other authors in the testing of Gaussian beam model behavior at the edge of a shadow zone.^{34;45} (Note that the specific values for this profile parameters were selected to match the MKS representation of Eq. 3.47 in Jensen, Kuperman, et. al.²¹ instead of the English units used in the original Pedersen and Gordon paper.)

Figure A.5: Modeled ray paths for the Pedersen/Gordon profile.

The ray paths to be tested for this profile are illustrated in Fig. A.5. This figure was created by WaveQ3D with a 100 ms time increment and 1° separated depres-

sion/elevation launch angles from 20° to 50° . A “flat earth” adjustment was applied to Eqn. A.20 to allow these results to be similar to those computed in Cartesian coordinates. Launch angles greater than 51.21° encounter interface reflections. The rays launched at angles between 44° and 50° travel through a caustic and cross the direct path rays. Because the ray paths were not periodic, we redefined the cycle range as the horizontal range needed to travel up to the first vertex and back to the source depth. Figure A.6 illustrates the difference between individual WaveQ3D rays and the spherical analytic solution defined by Eqs. (A.16) through (A.18). The WaveQ3D model had a maximum error of 1.3 m at cycle range of 2595.1 m (0.05% error), but there appeared to be a slight bias. We discovered that many of the launch angles had cycle range errors right around -0.5 m.

The sensitivity of these results to time step size is illustrated in Fig. A.7. The 150 ms step size had errors of about 2 meters. Errors for smaller time steps approached zero as expected. Larger time steps increase as a power law until the step size was larger than 350 ms, where the error quickly grew to hundreds of meters. This sudden change in error appears to be the result of an inability of model to properly sample the sound velocity profile field at the larger step sizes. From this, we conclude that our earlier 100 ms time step recommendation continues to be valid for this case.

A.3.4 Ray path accuracy along great circle routes

In this test, an ocean with a small amount of downward refraction was used to verify the WaveQ3D model’s ability to propagate rays along great circle routes. Acoustic

Figure A.6: Path accuracy sensitivity to step size for the Pedersen/Gordon profile.

rays traveling at a constant depth follow these paths because they are the shortest distance between two points along the earth's surface. The amount of downward refraction needed to parallel the earth's surface was computed by inverting Eq. A.9

$$c(r) = \frac{r}{R} c_0. \quad (\text{A.21})$$

where c_0 was set to 1500 m/s.

This test launched four horizontal rays from 45N 45W, at a depth of 1000 meters, with azimuths of 0°, 30°, 60°, and 90°. The rays propagated for 1000 s (about 1500 km) with a time step of 100 ms. Fig. A.8 illustrates the resulting ray paths as a function of latitude and longitude. The solid lines represent acoustic ray paths while

Figure A.7: Ray path accuracy as a function of step size for the Pedersen/Gordon profile.

the dashed lines represent the rhumb line paths, paths that would have been taken if latitude/longitude were a Cartesian system. The accuracy of the great circle routes were computed by converting the latitude, longitude, and altitude of each ray back into a great circle azimuth at the point of origin⁴⁷

$$\varphi_a(t) = \arctan \left[\frac{\cos \chi(t) \sin(\phi(0) - \phi(t))}{\cos \chi(0) \sin \chi(t) - \sin \chi(0) \cos \chi(t) \cos(\phi(0) - \phi(t))} \right], \quad (\text{A.22})$$

where $\chi(t), \phi(t)$ are latitude and longitude as a function of time; and $\varphi_a(t)$ is the analytic solution for the azimuthal launch angle for a target at $\chi(t), \phi(t)$. The rays traveled at constant depth with a maximum deviation from $\varphi_a(t)$ of 2.81×10^{-10} degrees.

where $\hat{s}$ is the surface normal; $\hat{r}$ is the unit vector in the radial direction; h is the incident ray height above bottom; δt is the time step needed to reach the interface; and $\frac{dr}{dt}$ is the radial ray tracing component defined by Eq. (A.1). The direction of the 3-D reflection is computed using

$$\hat{R} = \hat{I} - 2(\hat{I} \cdot \hat{s})\hat{s} \quad (\text{A.24})$$

where $\hat{I}$ is the incident ray path direction; and $\hat{R}$ is the reflected ray path direction. The WaveQ3D model also applies a second order Taylor expansion to each component of the position, normalized direction, and sound speed to improve the accuracy of their values at the time of collision. The tests discussed in this section analyze the accuracy of this reflection process in an isovelocity ocean.

A.4.1 Reflection accuracy with a flat bottom

This test constructed a geometry in which the changes in latitude and travel time for multiple interface bounces could be calculated analytically. The path of a downwardly steered ray, given a constant bottom depth and latitude change, is illustrated by Fig. A.9 and the equations

$$\zeta_s = \arcsin\left(\frac{R^2 - R_b^2 + L^2}{2RL}\right), \quad (\text{A.25})$$

$$\zeta_b = \zeta_s - \Delta\theta/2, \quad (\text{A.26})$$

$$t = L/c, \quad (\text{A.27})$$

where R is the radius for the ocean surface; D is the water depth; $R_b = R - D$ is the radius for the ocean bottom; $\Delta\theta$ is the latitude change between surface bounces; L

Figure A.9: Flat bottom reflection geometry.

is the path length from surface to bottom; ζ_s is the grazing angle at the surface (also the ray launch angle); ζ_b is the grazing angle at bottom; c is the sound speed; and t is the travel time between the surface and the bottom.

$\Delta\theta$ was set 0.2° and the ocean depth was set to 1000 meters, which caused the remaining parameters take on the analytic values shown in Table A.1. A single WaveQ3D ray was launched at a depression/elevation angle of -5.183617057° (down), and a time step of 100 ms, which produced the path shown in Fig. A.10. After four complete cycles of bottom and surface reflection, a distance of about 89 km, the latitude for the point of reflection deviated from the analytic result by less than 3.9×10^{-7} degrees (about 40 cm). The travel time differed from the analytic result by

Table A.1: Flat bottom expected values.

Parameter	Analytic Result
c	1500 m/s
D	1000 m
$\Delta\theta$	0.2°
ζ_s	5.183617057°
ζ_b	5.083617057°
L	11,175.841460125 m
t	7.450560973 s

less than 2.9×10^{-5} s. From this, we conclude that the WaveQ3D reflection process has an acceptable reflection accuracy when the interfaces are flat.

A.4.2 Reflection accuracy with a sloped bottom

This test looked at the ability of WaveQ3D to predict the direction of reflection from a bottom that has a 1 degree up-slope in the latitude direction. At each bottom reflection, the depression/elevation angle of the WaveQ3D ray path should decrease by 2° , just like the analytic result. Launching a ray at the same depression/elevation angle as the last test (-5.183617057° , down) produced the results shown in Fig. A.11. Note that the time step for this test was set to 1 ms to make it easier to compute depression/elevation angle just before, and just after, each collision. The maximum deviation of any of the three bottom reflections from their analytic reflection direction result was 1×10^{-5} degrees. From this, we conclude that WaveQ3D produces acceptable

Figure A.10: Flat bottom reflection test results.

reflections for sloped bottoms.

A.4.3 Out-of-plane reflection from gridded bathymetry

ETOPO1 gridded bathymetry¹⁵ from the Malta Escarpment was used to qualitatively test out-of-plane reflection from realistic bathymetry features. Out-of-plane reflection is a real-world phenomena that can have a significant impact on shallow water experiments.⁴¹ To isolate the testing to reflection effects, the speed of sound was fixed at 1500 m/s at all locations. WaveQ3D used a 100 ms step size to compute a single path, illustrated in Figure A.12. In this figure, bottom bathymetry contours are represented

Figure A.11: Analytic slope reflection test results.

as dashed lines. A ray launched from 35:59N 16:00E, at a depth of 10 meters, with a depression/elevation angle of -20° (down), and an azimuth of 270° traveled along the path illustrated by the solid black line. The closed circles along this path represent places where bottom reflections occurred; the open circles represent surface reflections. The decrease in spacing between the shallow water dots illustrates the type of depression/elevation angle change (Fig. A.11) expected for sloped bottoms. What was new in this test was the fact that ray paths were reflected into a new azimuthal direction each time that they interacted with the bottom. These out-of-plane reflections resulted in a down slope ray path that was offset by more than 21.9 km from

the up slope path, after 14 bounces off of the bottom. From this, we conclude that WaveQ3D results will have a significant contributions from out-of-plane ray paths whenever there are multiple interactions with complex bottom bathymetry features.

Figure A.12: Reflection on the Malta Escarpment.

A.5 Eigenray and Propagation Loss Tests

The tests discussed in this section compare WaveQ3D's eigenray and propagation loss calculation to analytic solutions.

The WaveQ3D eigenray estimation process establishes the relative geometry be-

tween rays paths and targets. That geometry then allows the calculation of travel time (t), source angle (μ), and target angle (φ) for each multi-path arrival. WaveQ3D computes these eigenray products by searching for the offsets which minimize the squared distance from targets to points on the wavefront. Once the CPA has been determined, the 3-D offset to the target is computed using

$$\vec{\rho} \equiv (\rho_1, \rho_2, \rho_3) \equiv (\delta t, \delta \mu, \delta \varphi) , \quad (\text{A.28})$$

$$\vec{g} \equiv \frac{\partial d^2}{\partial \vec{\rho}} \Big|_{CPA} , \quad (\text{A.29})$$

$$\mathbf{H} \equiv \frac{\partial^2 d^2}{\partial \vec{\rho}^2} \Big|_{CPA} \quad (\text{A.30})$$

$$\vec{\rho} = -\mathbf{H}^{-1} \vec{g} . \quad (\text{A.31})$$

where $\vec{\rho}$ is the offset from CPA in vector form; d^2 is the squared distance from each point on the wavefront to the target; $\vec{g}$ is the gradient of squared distance, evaluated at CPA (3 elements), and $\mathbf{H}$ is the Hessian matrix of squared distance, evaluated at CPA (3x3).

The propagation loss at the target location is a summation of contributions from the rays that surround the eigenray target. To create a 3-D acoustic field across the wavefront, WaveQ3D uses independent Gaussian beams in the μ and φ directions and ignores the cross terms.

$$G(\vec{r}_p) = \left(\sum_{j'=j-J}^{j+J} g_{j'}(\vec{r}_p) \right) \left(\sum_{k'=k-K}^{k+K} g_{k'}(\vec{r}_p) \right) , \quad (\text{A.32})$$

$$g_{j'}(\vec{r}_p) = \frac{(\mu_{j'+1} - \mu_{j'})}{\sqrt{2\pi w_{j'}^2}} \exp \left(-\frac{d_{j'}^2}{2w_{j'}^2} \right) , \quad (\text{A.33})$$

$$g_{k'}(\vec{r}_p) = \frac{(\sin(\mu_{j'+1}) - \sin(\mu_{j'})) (\varphi_{k'+1} - \varphi_{k'})}{\sqrt{2\pi w_{k'}^2}} \exp \left(-\frac{d_{k'}^2}{2w_{k'}^2} \right) , \quad (\text{A.34})$$

where $G(\vec{r}_p)$ is the total Gaussian beam intensity at the eigenray target; (j, k) are the index numbers of wavefront cell containing the eigenray target; $g_{j'}$ are the Gaussian beam contributions along depression/elevation direction; $g_{k'}$ are the Gaussian beam contributions along the azimuthal direction; $2J+1$ are the number of significant beams in the depression/elevation direction; and $2K+1$ are the number of significant beams in the azimuthal direction; w_j and w_k are the half-widths of the Gaussian beam in the μ and φ directions; and d_j^2 and d_k^2 are the distance in the μ and φ directions from the Gaussian beam center to the eigenray target. The WaveQ3D implementation divides the wavefront into ray families based on the number of surface reflections, bottom reflections, and caustic encounters. Within each ray family, propagation loss contributions are added across the wavefront until an edge is hit or the accumulated loss result changes by less than 0.01 dB.

WaveQ3D treats the beam width calculation as a convolution between Weinberg's frequency dependent "minimum width" term⁴⁵ and a second Gaussian that represents the spatial spreading created by the sampling of the wavefront.

$$(w'_{j,k}(f))^2 = (2w_{j,k})^2 + (2\pi\lambda)^2 . \quad (\text{A.35})$$

where λ is the wavelength of the signal being modeled; $w_{j,k}$ is the cell width of beam j or k , and $w'_{j,k}(f)$ is the adjusted width of beam j or k . The factor of 2 in the $w_{j,k}$ term creates a minimum overlap of 50% between neighboring beams. Normalizing Eqs. (A.33) and (A.34) by the combined effect of both spreading sources conserves energy across the wavefront.

A.5.1 Eigenray accuracy for a simple geometry

This test constructs a short range geometry in which travel time, source angle, and target angle can be computed analytically for direct path, surface reflected, and bottom reflected paths on a spherical earth. The geometry for this test is illustrated in Fig. A.13.

Figure A.13: Flat bottom eigenray test geometry.

$$r = R - d, \quad (\text{A.36})$$

$$L_1 = 2r \sin(\Delta\theta/2), \quad (\text{A.37})$$

$$\mu_1 = 90^\circ - \arccos\left(\frac{L_1}{2r}\right), \quad (\text{A.38})$$

$$L_2 = 2\sqrt{(L_1/2)^2 + [R - r \cos(\Delta\theta/2)]^2}, \quad (\text{A.39})$$

$$\mu_2 = 90^\circ - \arccos\left(\frac{d}{L_2/2}\right), \quad (\text{A.40})$$

$$L_3 = 2\sqrt{(L_1/2)^2 + [r \cos(\Delta\theta/2) - (R - D)]^2}, \quad (\text{A.41})$$

$$\mu_3 = \mu_1 + \arccos\left(\frac{L_1/2}{L_3/2}\right), \quad (\text{A.42})$$

$$t_n = L_n/c, \quad (\text{A.43})$$

where c is the sound speed; D is the water depth; d is both the source and target depth; r is the distance of the source/target from the center of curvature; L_1, L_2, L_3 are the path lengths for the direct, surface reflected, and bottom reflected paths; t_1, t_2, t_3 are the travel times; and μ_1, μ_2, μ_3 are the source/target depression/elevation angles.

WaveQ3D was tested using an ocean depth of 3000 m, a $\Delta\theta$ value of 0.02° , and the source/target depth of 1000 m. Eqs. (A.36) through (A.43) were then used to compute the values shown in Table A.2.

WaveQ3D eigenrays were calculated using a ray fan with depression/elevation angles from -60° to $+60^\circ$, azimuth angles from -4° to $+4^\circ$, angle spacings of 1° in both directions, and a time step of 100 ms. This geometry was specifically designed to stress the model by forcing it to extrapolate the bottom bounce path from a location outside of the ray fan. The other two paths are firmly inside of the ray fan.

For the direct path, the maximum difference between the modeled times/angles and their analytic counterparts were 2.2×10^{-5} ms and 1.1×10^{-4} degrees. These values represented eigenray accuracy limitations purely derived from Eqs. (A.28) through (A.31). The equivalent measurements for the surface reflected path yielded differences of 2.2×10^{-3} ms and 4.8×10^{-3} degrees. The reduced accuracy of the surface reflected

Table A.2: Expected eigenray values for a simple geometry.

Parameter	Analytic Result
D	3000 m
d	1000 m
$\Delta\theta$	0.02°
t_1	1.484019 s
t_2	1.995103 s
t_3	3.051677 s
μ_1	-0.010000°
μ_2	41.936232°
μ_3	-60.912572°

paths appears to be due to limitations in the interface reflection process. Even though the bottom reflected path was extrapolated from outside of the ray fan, it still achieved accuracies of 1.7 ms and 0.91° . We believe that any of these eigenray accuracies should be adequate for most sim/stim applications.

A.5.2 Eigenray accuracy for Lloyd's mirror on spherical earth

On a flat earth, the Lloyd's mirror geometry generates exactly two paths: a direct path and a surface reflection. However (as shown in Fig. A.14) isovelocity ray paths actually form unexpected caustics when the curvature of the earth is incorporated. Fig. A.15 illustrates that these caustics are formed by focusing from the concave

surface of the earth. Incorporating these effects into our eigenray tests is important because of WaveQ3D's use of spherical earth coordinates.

Figure A.14: Isovelocity paths in spherical coordinates.

Fig. A.16 defines the geometry used to compute an analytic eigenray solution. In this figure, d_1, d_2 are the source and target depths; r_1, r_2 are the source and target distance from the center of curvature; $\Delta\theta$ is the latitude change from source to target; $\Delta\theta_1$ is the latitude change from the source to the point of reflection; $\Delta\theta_2 = \Delta\theta - \Delta\theta_1$ is the latitude change from the reflection point to the target; β is the reflection angle relative to the normal; a is the length of the direct path; and $b_1 + b_2$ is the length of the surface reflected path.

Figure A.15: Isovelocity paths in Cartesian coordinates.

The analytic solutions for the surface reflected path requires finding values of $\Delta\theta_1$ which solve the transcendental equation (derived in the Section A.6.1)

$$r_1 \sin(\Delta\theta_1) - r_2 \sin(\Delta\theta - \Delta\theta_1) + \frac{r_1 r_2}{R} \sin(\Delta\theta - 2\Delta\theta_1) = 0. \quad (\text{A.44})$$

Once the roots of Eq. (A.44) are known, the analytic solution for surface reflected eigenrays can be computed using

$$b_1^2 = R^2 + r_1^2 - 2Rr_1 \cos(\Delta\theta_1), \quad (\text{A.45})$$

$$b_2^2 = R^2 + r_2^2 - 2Rr_2 \cos(\Delta\theta_2), \quad (\text{A.46})$$

$$t_s = \frac{b_1 + b_2}{c_0}, \quad (\text{A.47})$$

Figure A.16: Ocean surface as concave reflector.

$$\mu_{s,source} = -\arcsin\left(\frac{b_1^2 + r_1^2 - R^2}{2b_1r_1}\right), \quad (\text{A.48})$$

$$\mu_{s,target} = \arcsin\left(\frac{b_2^2 + r_2^2 - R^2}{2b_2r_2}\right), \quad (\text{A.49})$$

where t_s is the surface-reflected travel time from source to target; $\mu_{s,source}$ is the surface-reflected depression/elevation angle at source; and $\mu_{s,target}$ is the surface-reflected depression/elevation angle at target.

Fig. A.17 illustrates the roots of Eq. (A.44) for a source depth of 200 meters. The horizontal axis is the ratio of $\Delta\theta_1/\Delta\theta$. The vertical axis defines target depths from 0 to 500 meters. The contours on this plot represent the values for the left hand side of Eq. (A.44), multiplied by 10^5 , for a target $\Delta\theta$ of 1.2° . The zero contour illustrates

Figure A.17: Roots of surface reflection transcendental equation.

the location of the roots at each target depth. For example, we found that there are three roots for $\Delta\theta_1$ at values of $\Delta\theta$ times 0.190047203712437, 0.425088688451783, and 0.88486312787168 when a target depth was 150 meters. The physical interpretation of the multiple roots is that there are multiple surface-reflection paths focused onto the target location by the concave surface of the earth.

The analytic eigenray products for a target at a depth of 150 m are shown in Table A.3. WaveQ3D was run with a time step of 100 ms and a depression/elevation launch angle spacing of 0.05° . The travel times computed by WaveQ3D matched

Table A.3: Expected eigenray values for target at 1.2° and 150 m.

Path	Travel time	Launch angle	Target angle
Direct Path	89.05102557 s	-0.578554378°	+0.621445622°
Surface 1	89.05369537 s	+0.337347599°	+0.406539112°
Surface 2	89.05379297 s	-0.053251329°	+0.233038477°
Surface 3	89.05320459 s	-0.433973977°	-0.48969753°

the analytic result to within 1.2×10^{-5} s, and the angles were accurate to within 0.012°. Note that the launch angle spacing was much tighter than other tests because WaveQ3D’s eigenray searching logic is limited to finding one ray path between any two launch angles. A larger launch angle increment in this test would have caused the model to fail to find the ”Surface 3” path. But with this context, we felt that WaveQ3D was quite accurate in predicting the travel times and angles for the equivalent of a Lloyd’s mirror geometry on a spherical earth.

A.5.3 Eigenray robustness for Lloyd’s mirror on spherical earth

This test extends the results of the previous section by comparing modeled travel times and ray path angles, for the spherical equivalent of Lloyd’s mirror, at a variety of depths and ranges. This test used a isovelocity speed of sound of 1500 m/s, a frequency of 2000 Hz, a source depth of 200 m, target depths from 0 to 1000 m, 181 tangent spaced depression/elevation angles (explained in the Section A.6.2), 1° spaced azimuth angles from -4° to +4°, and a time step of 100 ms. The maximum

range was limited to $\Delta\theta = 0.8^\circ$ to ensure that only a single surface reflected path was produced at each target location (see Fig. A.14). This choice allows the test to be run with a ray spacing that was much more typical than the fine scale D/E angles used in the previous section.

Figure A.18: Eigenray errors for Lloyd's mirror direct path.

The eigenray errors for direct path are summarized in Fig. A.18. Each plot shows the absolute value of the difference between the WaveQ3D model and the analytic solution, as a function of range, for targets at depths of 0, 10, 100, and 1000 m. Beyond a range of 0.1° (about 11 km), the direct path model had maximum errors in

travel time error and angle of 0.0087 ms and 0.024° . In each case, these errors were larger at short ranges. The short range errors were most pronounced near the ocean surface and at the 1000 m depth.

The eigenray errors for the surface reflected path are shown in Fig. A.19. Once again, the errors were largest at short ranges, both near the ocean surface and at the 1000 m depth. Beyond a range of 0.1° , the surface reflected path model has a maximum travel time error of 0.0041 ms, a maximum source angle error of 0.055° and a maximum target angle error of 0.047° . In each case, the errors are larger at short ranges.

The inaccuracies near the surface were attributed to undersampling by the 100 ms time step, as illustrated in Fig. A.20. The solid lines in this figure represent direct path rays that are moving out in range and up toward the surface from the source at 200 m. The dashed lines present the same ray paths after reflection from the ocean surface. The dotted lines that connect them represent discontinuities between the direct path and surface reflected segments of the wavefront. At short ranges, these discontinuities cause the edges of ray families to be far from targets that are near the interface; extrapolation led to inaccuracy. Shortening the time step mitigates this source of inaccuracy. At longer ranges, this source of inaccuracy is automatically reduced by the fact that the edges of the ray families get closer to the surface as the rays become more horizontal.

The errors at large depths were attributed to the tangent spaced depression/elevation angles used in this test. Using a more uniform scheme for depression/elevation angles would have mitigated this source of inaccuracy.

Figure A.19: Eigenray errors for Lloyd's mirror surface reflected path.

A.5.4 Propagation loss accuracy for Lloyd's Mirror

Near the surface, WaveQ3D must extrapolate eigenrays from distances that are up to 1.5 paths away from the interface. The tests in this section were designed to expose the impact of this limitations on propagation loss accuracy. The analytic solutions for these test were derived in Cartesian coordinates using the method of images.

$$p(r, z) = \frac{e^{ikL_1}}{L_1} - \frac{e^{ikL_2}}{L_2} ; \quad (\text{A.50})$$

$$L_1 = \sqrt{r^2 + (z - z_s)^2} ; \quad (\text{A.51})$$

Figure A.20: Time-step effects near the ocean surface.

$$L_2 = \sqrt{r^2 + (z + z_s)^2}; \quad (\text{A.52})$$

where r is the target range; z is the target depth (positive is down); z_s is the source depth (positive is down); L_1 is the slant range to source; L_2 is the slant range to image source (above water); k is the acoustic wave number ($2\pi f/c$); and f is the signal frequency; c is the speed of sound; $p(r, z)$ is the complex pressure.

The propagation loss for a 200 meters deep target is shown as a function of range in Fig. A.21 and Fig. A.22. Results for a 10 km range target are shown as a function of depth in Fig. A.23. Both cases used a speed of sound of 1500 m/s, with a “flat

earth” adjustment from Eq. (A.9), a frequency of 2000 Hz, a source depth of 75 m, 181 tangent spaced depression/elevation angles, 1° spaced azimuth angles from -4° to $+4^\circ$, and a time step of 100 ms. Figure A.22 highlights fact that the errors in Fig. A.21 are most sever at range less than 2 km. Figure A.22 illustrates that fact that the largest errors, as a function of depth, are right below the ocean surface. We attribute the ocean surface results to small in the computation of travel time for the direct and surface reflected paths.

Figure A.21: Lloyd’s mirror propagation loss as a function of range.

To compare these results quantitatively, we will use a set of statistical measures

Figure A.22: Lloyd's mirror propagation loss errors at short ranges.

that are defined in detail in Appendix C. The differences between the WaveQ3D model and the analytic results are summarized in Table A.4. From this, we conclude that, although the WaveQ3D model's Lloyd's Mirror predictions clearly has limitations, those limitations will have a limited impact on its ability to accurately model propagation loss in a Lloyd's mirror environment.

Figure A.23: Lloyd's mirror propagation loss as a function of depth.

A.5.5 Eigneray and propagation loss accuracy in an extreme downward refraction environment

In this test, the eigneray and propagation loss accuracy of the WaveQ3D model were analyzed for the extreme n^2 linear test case developed by Pedersen and Gordon.³²

Two geometries were supported in this test:

- The shallow source geometry puts the source at a depth of 75 m and creates a series of targets at a depth of 75 m with ranges from 500-1000 m.

Table A.4: Lloyd’s Mirror propagation loss accuracy.

Scenario	bias	deviation	r^2
Lloyd’s Mirror vs. range	+0.42 dB	± 3.51 dB	87.2%
Lloyd’s Mirror vs. depth	+0.50 dB	± 3.45 dB	87.2%

- The deep source geometry puts the source at a depth of 1000 m and creates a series of targets at a depth of 800 m with ranges from 3000-3100 m.

Both cases used the profile defined in Eq. (A.20), with a “flat earth” correction using Eq. (A.9), and a frequency of 2000 Hz. The eigenrays for the WaveQ3D model were compared to both the GRAB model¹⁷ and analytic solutions. The comparison to GRAB, a U.S. Navy standard, was included to assess WaveQ3D error statistics against a well understood, high quality, Gaussian beam model.

Shallow source geometry

Figure A.24 is a ray trace for the shallow source geometry. This plot illustrates a ray fan with launch angles from 0° to 25° in 0.5° increments. The target locations are illustrated by the horizontal black line. There are two potential eigenrays for each target. The direct path and surface reflected components of the wavefront at 0.4 sec are illustrated with circle and square symbols. Rays launched below the critical angle (18.82°) form the direct path contribution after traveling through an upper vertex. (Because this geometry does not support the formation of a caustic, neither WaveQ3D nor GRAB applies a $-\pi/2$ phase shift to this path.) Rays above the critical angle hit the surface before ensonifying the target. Both ray families included targets that

were ensonified in their evanescent region, the region outside of the ray fan.

Figure A.24: Ray trace for shallow source.

Figures A.25 and A.26 compare the individual WaveQ3D and GRAB eigenrays for the direct and surface reflected paths. Analytic solutions for the travel time and rays angles were also computed using Eqn. (A.16) through (A.19). To highlight differences in travel time, a bulk time has been removed using the slope of the analytic solution for the direct path.

The GRAB model was configured using ray fan with launch angles from 0° to 25° in 0.1° increments. However, it is important to note that, using the sound speed

Figure A.25: Direct path eigenrays for shallow source.

at the source, GRAB automatically enhances the fan around the critical with 11 additional beams with spacings of 0° , $\pm 0.03125^\circ$, $\pm 0.0625^\circ$, $\pm 0.125^\circ$, $\pm 0.25^\circ$, and $\pm 0.5^\circ$. If these small rays spacings were not used near the critical ray, it would lead to un-realistically large gaps between the outer edge of each ray family and the critical ray. This is especially true for the surface reflected path in this geometry.

To achieve a similar effect, WaveQ3D was configured with 0.025° spaced depression/elevation angles from 0° to $+25^\circ$ and 1° spaced azimuth angles from -4° to $+4^\circ$. Because this geometry evolves so quickly in time, the WaveQ3D results were created

Figure A.26: Surface reflected eigenrays for shallow source.

using a 10 ms time step instead of the 100 ms value used in other tests.

Quantitative results for the shallow source's individual eigenrays are summarized in Table A.5. In this table, the maximum difference in travel time, source depression/elevation angle, and target depression/elevation angle are computed relative to the analytic solution. Those time and angle comparisons are limited to non-evanescent regions, where the the analytic solutions had real values. Because our analytic solution did not support propagation loss for individual eigenrays, WaveQ3D values were compared to GRAB in this table.

Table A.5: Individual eigenrays for shallow source.

Scenario	bias	deviation	r^2	time	source DE	target DE
GRAB Direct	-	-	-	0.74 ms	0.25°	0.25°
GRAB Surface	-	-	-	0.66 ms	0.02°	0.02°
WaveQ3D Direct	+0.03 dB	± 0.46 dB	99.9%	0.71 ms	0.00°	0.00°
WaveQ3D Surface	0.00 dB	± 1.40 dB	99.1%	0.47 ms	0.02°	0.99°

From these results, we conclude that GRAB and WaveQ3D have similar eigenray accuracy when compared to analytic solutions. In this case, the major difference appears to be differences in the way that GRAB and WaveQ3D handle depression/elevation angles near the shadow zone. GRAB's angles are a weighted sum of contribution from neighboring Gaussian beams. This gives GRAB a smooth roll-off from angles inside the ray fan to a constant value on the outside. GRAB's constant value is taken from the closest ray in horizontal range. In contrast, the WaveQ3D angles are computed from the geometry of eigenray offsets. Outside of the ray fan, WaveQ3D uses angles from the ray that is closest in slant range. Because the rays in this geometry are rapidly changing direction near the shadow zone, the difference between slant and horizontal range results in WaveQ3D target depression/elevation angles that are up to 4° different than the GRAB result. But because the analytic solution is not valid in this region, that difference is not reflected in Table A.5.

Differences in total propagation loss for the shallow source geometry are illustrated in Fig. A.27 and summarized in Table A.6. The analytic solution for this test were computing using the Fast Field Program (FFP) wavenumber integration tech-

Figure A.27: Propagation loss for shallow source.

nique.^{14;6} Note that, in all regions, this implementation of FFP was consistent with an ideal wave equation solution, except for the presence of some implementation jitter in the ranges above 880 m. In the region between 500-750 m, all three models produced almost identical results. As target passed into the shadow zone region, WaveQ3D and GRAB produced values that were similar to each other, but slightly higher than FFP. But we also noted that, if a coarser set of depression elevation launch angles was used for WaveQ3D, the surface reflected path would have disappeared prematurely, which would have manifested as shadow zone oscillations in the total propagation

loss. Taken as a whole, we feel confident that Wave3D propagation loss errors are similar to those of GRAB for this scenario.

Table A.6: Total propagation loss for shallow source.

Scenario	bias	deviation	r^2
GRAB	+0.71 dB	± 1.78 dB	92.5%
WaveQ3D	+0.80 dB	± 2.21 dB	88.7%
GRAB ≤ 0.75 km	+0.09 dB	± 0.58 dB	97.7%
WaveQ3D ≤ 0.75 km	-0.03 dB	± 0.47 dB	98.4%

Deep source geometry

Figure A.28 is a ray trace for the deep source geometry. This plot illustrates a ray fan with launch angles from 20° to 60° in 1° increments. The target locations are illustrated by the horizontal black line. There are three potential eigenrays for each target. Rays launched above the critical angle (51.21°) hit the surface before ensonifying the target. However, because all of the surface reflected paths are far from the targets, their contribution to the overall solution is weak. Starting at around 2.5 seconds of travel time, the outer edge of the wavefront folds back on itself and splits the remaining contributions into strong direct path and caustic ray families. The direct path, surface reflected, and caustic components of the wavefront at 2.5 sec are illustrated with circle, square, and diamond symbols on Fig. A.28.

Figures A.29 and A.30 compare the individual WaveQ3D and GRAB eigenrays for the deep source's direct and caustic paths. As before, a bulk time has been removed

Figure A.28: Ray trace for shallow source.

using the slope of the analytic solution for the direct path. In these results, the GRAB model was configured using ray fan with launch angles from 20° to 60° in 0.1° increments and rays near the caustic were automatically augmented. WaveQ3D used a ray spacing of 0.25° spacing to achieve a similar result.

The big difference between the models appears to be the fact that the WaveQ3D transmission loss has a more gradual roll-off into the shadow zones than GRAB. Quantitative comparisons for the deep source's individual eigenrays are summarized in Table A.7. The statistics for the WaveQ3D angles for the caustic path are skewed by

Figure A.29: Caustic eigenrays for deep source.

the fact that differences in launch angle resolutions causes inner edge of the WaveQ3D result to prematurely transition to its shadow zone result. In other regions, the agreement is much closer. It is also important to note that the depression/elevation angle deviations seen in the WaveQ3D results for the shallow source are less evident in the deep source case.

Differences in total propagation loss for the deep source geometry are illustrated in Fig. A.31 and summarized in Table A.8. In the region from 3000 to 3040 meters, the FFP has stronger oscillations than either the WaveQ3D or GRAB models. This

Figure A.30: Direct path eigenrays for deep source.

can be attributed to surface reflected contribution that is stronger in the FFP result than in either of the Gaussian beam models. At all other ranges, the WaveQ3D and GRAB results were both a very close fit to FFP's total propagation loss.

A WaveQ3D anomaly was discovered during the deep source geometry testing. Errors in the total propagation loss within the shadow zone were frequently seen when the launch angles finer than 0.01 degrees were used. What we discovered was that at this spacing, the contribution in the shadow zone was the result of a summation of over 100 Gaussian beams for both the direct and caustic paths. Small errors in the

Table A.7: Individual eigenrays for deep source.

Scenario	bias	deviation	r^2	time	source DE	target DE
GRAB Direct	-	-	-	0.65 ms	0.47°	0.54°
GRAB Caustic	-	-	-	3.77 ms	0.87°	0.98°
WaveQ3D Direct	+0.06 dB	± 3.17 dB	97.0%	0.64 ms	0.21°	0.24°
WaveQ3D Caustic	+1.15 dB	± 4.07 dB	94.4%	3.97 ms	1.24°	1.40°

Figure A.31: Propagation loss for deep source.

calculation of cell width and cell distance to the target appear to accumulate when the total loss is the result of many weak contributions. For the individual eigenrays,

Table A.8: Total propagation loss for deep source.

Scenario	bias	deviation	r ²
GRAB	-0.95 dB	±3.05 dB	95.4%
WaveQ3D	-0.22 dB	±1.94 dB	92.8%
GRAB ≥ 3.04 km	-1.45 dB	±3.70 dB	97.0%
WaveQ3D ≥ 3.04 km	-0.03 dB	±2.20 dB	94.4%

this anomaly results in a slight broadening of the transmission loss decay tails. But, since the shape of the outer edge of the total propagation loss depends on destructive interference between the paths, these errors often resulted in imperfect cancellation, on the order of -70 dB, in the region between 3090 and 3105 meters. Although these problems may have been aggravated by the extreme environment, developers should use extra caution when using WaveQ3D with super-fine ray spacings.

A.6 Derivations

A.6.1 Ray path derivation for concave ocean surface

The analytic solution for the direct-path eigenrays was derived from the laws of sines and cosines.

$$a^2 = r_1^2 + r_2^2 - 2r_1r_2\cos(\Delta\theta) , \quad (\text{A.53})$$

$$t_d = a/c_0 , \quad (\text{A.54})$$

$$\mu_{d,source} = -\arcsin\left(\frac{a^2 + r_1^2 - r_2^2}{2ar_1}\right) , \quad (\text{A.55})$$

$$\mu_{d,target} = \arcsin\left(\frac{a^2 + r_2^2 - r_1^2}{2ar_2}\right), \quad (\text{A.56})$$

where t_d is the direct-path travel time from source to target; $\mu_{d,source}$ is the direct-path depression/elevation angle at source; and $\mu_{d,target}$ is the direct-path depression/elevation angle at target.

The analytic solution for the surface reflected solution also starts with the law of cosines.

$$\cos \beta = \frac{R^2 + b_1^2 - r_1^2}{2Rb_1} = \frac{R^2 + b_2^2 - r_2^2}{2Rb_2}. \quad (\text{A.57})$$

This can be reduced to a simpler form using

$$R^2 + b_n^2 - r_n^2 = R^2 + (R^2 + r_n^2 - 2Rr_n \cos(\Delta\theta_n)) - r_1^2 = 2R(R - r_n \cos(\Delta\theta_n)) \quad (\text{A.58})$$

to yield

$$\cos \beta = \frac{R - r_1 \cos(\Delta\theta_1)}{b_1} = \frac{R - r_2 \cos(\Delta\theta_2)}{b_2}. \quad (\text{A.59})$$

When this is combined with the law of sines

$$\sin \beta = \frac{r_1 \sin(\Delta\theta_1)}{b_1} = \frac{r_2 \sin(\Delta\theta_2)}{b_2}, \quad (\text{A.60})$$

it yields an analytic relationship between the source/target depths and the angles $(\Delta\theta_1, \Delta\theta_2)$ at which reflections occur

$$\frac{\cos \beta}{\sin \beta} = \frac{R/r_1 - \cos(\Delta\theta_1)}{\sin(\Delta\theta_1)} = \frac{R/r_2 - \cos(\Delta\theta_2)}{\sin(\Delta\theta_2)}, \quad (\text{A.61})$$

$$\frac{R}{r_1} \sin(\Delta\theta_2) - \sin(\Delta\theta_2) \cos(\Delta\theta_1) = \frac{R}{r_2} \sin(\Delta\theta_1) - \cos(\Delta\theta_2) \sin(\Delta\theta_1), \quad (\text{A.62})$$

$$r_1 \sin(\Delta\theta_1) - r_2 \sin(\Delta\theta_2) + \frac{r_1 r_2}{R} \sin(\Delta\theta_2 - \Delta\theta_1) = 0. \quad (\text{A.63})$$

The analytic solutions for the surface reflected path requires finding values of $\Delta\theta_1$ which solve the transcendental equation

$$r_1 \sin(\Delta\theta_1) - r_2 \sin(\Delta\theta - \Delta\theta_1) + \frac{r_1 r_2}{R} \sin(\Delta\theta - 2\Delta\theta_1) = 0. \quad (\text{A.64})$$

A.6.2 Tangent spaced depression/elevation angles

Figure A.32: Tangent spaced beams.

Because sonar detection ranges are often much longer than the depths of interest, there is frequently a need to emphasis propagation paths near the horizontal. The GRAB model¹⁷ manages this requirement by automatically adding rays in the de-

pression/elevation directions needed to support caustics, surface ducts, and SOFAR channels. The WaveQ3D model currently does not support any such automatic ray adjustment; but it does support any ray spacing that is monotonically increasing.

Tangent spaced depression/elevation angles will be used frequently in the eigenray and propagation loss testing for WaveQ3D. As shown in the top panel of Fig. A.32, uniformly spaced rays in an isovelocity environment severely under-sample the direct path contributions at ranges beyond a few kilometers. When the ray paths are initialized such that the tangents of the launch angles are uniformly spaced (bottom panel), the long range contributions are better supported. Of course, this improvement comes at the expense of short range contributions. But, that trade-off often matches the requirements of sonar simulations.

To generate a set of N tangent spaced beams $\mu[n]$, over the interval $[\mu_1, \mu_N]$, with the densest spacing at μ_c , WaveQ3D uses the following algorithm

$$a_1 = \arctan\left(\frac{\mu_1 - \mu_c}{\sigma}\right) \quad (\text{A.65})$$

$$a_N = \arctan\left(\frac{\mu_N - \mu_c}{\sigma}\right) \quad (\text{A.66})$$

$$x[n] = a_1 + \frac{a_N - a_1}{N - 1}n \quad \text{for } n = 0, 1, 2, \dots, N-1 \quad (\text{A.67})$$

$$\mu[n] = \mu_c + \sigma \tan(x[n]) \quad (\text{A.68})$$

where σ is an arbitrary scaling factor. WaveQ3D testing frequently uses 181 tangent spaced depression/elevation rays, from -90° to $+90^\circ$, with $\mu_c = 0^\circ$ and $\sigma = 6$. This combination of factors yields a ray fan with maximum resolution of about 0.1° and 85% of its rays in the $\pm 20^\circ$ range.

A.6.3 Propagation loss error statistics

We would like to define quantitative differences between modeled and analytic propagation losses in a way that illuminates the suitability of the model for real-time, sonar simulation/stimulation systems. To that end, we define the following statistical measures

$$b[PL] = \frac{1}{N} \sum_{n=1}^N (PL_{model}[n] - PL_{theory}[n]) , \quad (A.69)$$

$$\psi^2[PL] = \frac{1}{N} \sum_{n=1}^N (PL_{model}[n] - PL_{theory}[n])^2 , \quad (A.70)$$

$$s[PL] = \sqrt{\psi^2[PL] - b^2[PL]} , \quad (A.71)$$

$$x[n] = PL_{theory}[n] - \frac{1}{N} \sum_{n'=1}^N (PL_{theory}[n']) , \quad (A.72)$$

$$y[n] = PL_{model}[n] - \frac{1}{N} \sum_{n'=1}^N (PL_{model}[n']) , \quad (A.73)$$

$$r^2[PL] = \frac{\left(\sum_{n=1}^N x[n]y[n] \right)^2}{\sum_{n=1}^N x^2[n] \sum_{n=1}^N y^2[n]} \times 100\% , \quad (A.74)$$

where $PL_{model}[n]$ and $PL_{theory}[n]$ are the samples in the model and theory in dB units; $b[PL]$ is the estimated bias between the model and the theory; $s[PL]$ is the estimated deviation between the model and the theory; and $r^2[PL]$ is the coefficient of determination between the model and the theory.

The active sonar equation can be expressed in the form

$$SE = FOM - 2PL , \quad (A.75)$$

$$FOM = SL + TS - NL + DI - DT , \quad (A.76)$$

where SE is the signal excess; PL is the propagation loss; FOM is the active figure of merit; SL is the source level; TS is the target strength; NL is the noise level; DI is the directivity index; and DT is the detection threshold.⁴² If we assume that errors in the figure of merit are handled separately, then $2b[PL]$ and $s^2[PL]$ are estimates of signal excess bias and variance. The coefficient of determination, $r^2[PL]$, estimates how well the modeled signal excess' shape is correlated to an ideal solution.

A.7 Summary

This paper represents an important milestone the development of the WaveQ3D model. A suite of tests were developed to quantitatively compare the ray tracing, reflection, eigenray finding, and propagation loss elements of the model to analytic solutions. Hopefully, this approach will help other researchers evaluate the capability and limitations of the WaveQ3D model and provide a firm foundation to move forward with further testing.

This testing also led to the development of a few general guidelines for the use of the WaveQ3D model.

- Using a time step size as coarse as 100 ms seems to produce accurate results for most applications. However, this step size should be decreased to 10 ms for applications that need high accuracy at ranges less than 2 km. This is particularly true for applications that are interested in effects near the ocean surface, or directly below the source.
- The selection of launch angles can have a significant effect on propagation loss

results. Like all models that as based on ray theory, WaveQ3D can miss features in the environment because of spatial under-sampling. Tangent spaced depression/elevation angles appear to improve WaveQ3D model performance for scenarios dominated by horizontal paths. Uniform spacing is suggested for applications that need high accuracy at ranges less than 2 km.

- The WaveQ3D model is designed to be computationally efficient when computing transmission loss for up to 100 targets, at multiple frequencies, in a fully 3-D environment. But because the WaveQ3D eigenray detection process is less efficient than an equivalent calculation in Cartesian coordinates, WaveQ3D can actually be much slower than other models when thousands of range/depth combinations are required. Applying the WaveQ3D model in 2-D environments is also not very efficient.

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