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Dynamics and Impacts of Roll Vortices in the Hurricane Boundary Layer

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DYNAMICS AND IMPACTS OF ROLL VORTICES IN THE
HURRICANE BOUNDARY LAYER

BY

KUN GAO

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ABSTRACT

This Ph.D. dissertation presents a process-oriented study focusing on the coherent large eddies, commonly known as roll vortices (rolls), in the hurricane boundary layer (HBL). We develop a new methodology for explicit representation of the vertical fluxes induced by rolls in the hurricane model. In this method a two-dimensional high-resolution roll-resolving model SRM (Single-Grid Roll-Resolving Model) is embedded at multiple horizontal grid points in the hurricane model. Such numerical design explicitly resolves the two-way interactions between the small-scale rolls and the large-scale hurricane flow. The dynamics of rolls and their impacts on the hurricane structure and intensity are investigated through a series of numerical experiments, conducted with the SRM embedded either in an axisymmetric HBL model (Chapters 1 and 2) or the three-dimensional full-physics hurricane model COAMPS-TC (Chapter 3).

Chapter 1 focuses on rolls in the linear phase. The effects of mean wind and stratification in the HBL on the characteristics of rolls are investigated. We identify two important factors associated with the HBL mean wind that affect the characteristics of rolls. The dynamical HBL height affects the wavelength of rolls, and the magnitude of the mean wind shear affects the growth rate of rolls. The mixed layer height in the stratification profile is another important factor affecting the characteristics of rolls. Provided the mean wind profiles are the same, the rolls have a larger growth rate under a higher mixed layer, and these rolls can trigger internal wave beams that are more inclined from the vertical direction and reach into a higher level.

Chapter 2 focuses on the properties of rolls in the equilibrium state, including their structures, vertical momentum fluxes and their effects on the HBL mean wind. We find that the mixed layer height is important in affecting the magnitude of the rolls and the structure of the internal waves triggered in the stably stratified layer above. The cross-roll momentum flux is dependent on the mean wind shear, but the along-roll momentum flux is not. Therefore, there is no physical basis for applying the K-theory to represent the roll-induced momentum fluxes. The rolls induce more significant changes in the mean radial wind than in the mean azimuthal wind. It is found that rolls affect the mean radial wind by redistributing the azimuthal momentum vertically in the HBL.

Chapter 3 focuses the effects of rolls on the development of a stationary and axisymmetric hurricane. It is found that the roll-induced wind changes in the HBL lead to the changes in the structure and intensity of the entire hurricane. The roll-induced vertical transport of the azimuthal momentum flux is primarily responsible for these changes. By enhancing the vertical momentum exchange, the rolls trigger a chain of dynamical responses within the HBL, increasing the mass convergence and inducing a more active deep eyewall convection, which leads to the enhanced hurricane intensity.

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DEDICATION

To my wife, parents and parents-in-law

PREFACE

This dissertation is written in manuscript format and consists of three scientific journal articles. The first chapter was published in the Journal of Atmospheric Sciences, November 2014. The second chapter has been submitted to the Journal of Atmospheric Sciences. The third chapter will be submitted to the Monthly Weather Review for publication.

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CHAPTER 1

On the Generation of Roll Vortices due to the Inflection Point Instability of the Hurricane Boundary Layer Flow

by

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Abstract

Horizontal roll vortices, or rolls, are frequently observed in the hurricane boundary layer (HBL). Previous studies suggest that these rolls can be generated by the inflection point instability of the HBL flow. In this study we investigate the formation of rolls due to this mechanism in the axisymmetric HBL, using a numerical approach that explicitly resolves rolls. The effects of mean HBL wind and stratification distributions on rolls are evaluated. We identify two important factors of the mean HBL wind that affect the characteristics of rolls. The dynamical HBL height affects the wavelength of rolls, and the magnitude of the mean wind shear affects the growth rate of rolls. As a result, under neutrally stratified HBL, the wavelength of rolls increases with the radius (out of the radius of maximum wind), while the growth rate of rolls decreases. The stratification also plays an important role in the generation of rolls. The stable stratification suppresses the growth of rolls because of the negative work done by the buoyancy force. Nonuniform stratification with a mixed layer has less suppressing effect on rolls. Rolls can trigger internal waves in the stably stratified layer, which have both vertically propagating and decaying properties. We derive analytical solutions for the internal waves, which relate the properties of the internal waves to the boundary layer rolls. We find the properties of the internal waves are affected by the mixed layer height.

1.1. Introduction

Recent observations suggest that horizontal roll vortices, or rolls, frequently occur in the hurricane boundary layer (HBL). Most of these observations were obtained by Doppler radar from landfalling hurricanes (Wurman and Winslow 1998; Morrison et al. 2005; Lorsolo et al. 2008; Ellis and Businger 2010). In the radar observations, rolls are detected because they induce organized perturbations in the wind field. The wavelength of rolls estimated from the radar observations is highly variable, ranging from a few hundred meters to a few kilometers. For example, Lorsolo et al. (2008) observed rolls in Hurricane Isabel (2003) and Frances (2004) with wavelength ranging from 200-650 m. Morrison et al. (2005) found the majority of rolls during the landfalls of four hurricanes had wavelength in the range of 1-2 km. Synthetic aperture radar observations identified rolls with wavelength of 3-6 km (Katsaros et al. 2000). Zhang et al. (2008) provided the first in-situ measurement of the roll-induced vertical fluxes in the HBL and showed that the rolls enhanced the total turbulent momentum and moisture fluxes in the HBL. However, the mixing effects of rolls are not represented in current hurricane models, which may limit their forecast accuracy. In order to represent the roll induced vertical momentum and energy transports in hurricane numerical models, we must better understand their formation mechanism.

The formation mechanism and characteristics of rolls in the HBL were studied based on theoretical and numerical approaches. Foster (2005) applied linear and nonlinear analyses to the HBL momentum equations and argued that rolls are expected to be a common feature in the HBL. He demonstrated that the inflection points in the basic-state wind profiles cause the instability and lead to the generation of rolls. Such mechanism is

analogous to the classical Ekman boundary layer instability (Faller 1965; Lilly 1966; Brown 1970, 1972). Since this type of instability is related to the inflection point in the boundary layer wind profile, it is commonly referred to as the inflection point instability. Wavelengths of the rolls estimated by Foster (2005) were similar to those observed by Morrison et al. (2005). Nolan (2005) used both a nonlinear axisymmetric model and a linear instability analysis approach to investigate the instability in the HBL. He also found inflection points in the HBL wind profiles are responsible for the formation of rolls, which have wavelengths of 3-5 km. Zhu (2008) used the WRF-LES (Large Eddy Simulation version of Weather Research Forecast model) nested within the WRF mesoscale model to simulate large eddies in a landfalling hurricane. He found roll-like perturbations existed in a statically stable boundary layer environment, suggesting these features were generated by the inflection point instability rather than the thermal instability. Nakanishi and Niino (2012) conducted an LES study at two different locations in the idealized HBL. By applying the EOF analysis to the model results, they identified rolls generated by the inflection point instability with spatial scales similar to those found by Foster (2005).

These previous studies investigated the rolls in neutral or near-neutral stratification (Foster 2005; Nolan 2005), or the rolls at specific locations within the HBL (Zhu 2008; Nakanishi and Niino 2012), but they did not systematically examine the conditions favorable for the formation of rolls by the inflection point instability, and the impacts of the mean wind and stratification on the characteristics of rolls. In this study, we aim to gain a more comprehensive understanding of the formation of rolls under various HBL

conditions, and identify the important mean flow factors affecting the characteristics of rolls. Specifically, this study addresses the following questions:

- i) What factors affect the spatial and temporal characteristics of the rolls generated by the inflection point instability?
- ii) What are the spatial variations of the rolls in the HBL?
- iii) What are the effects of stratification on the rolls?

To answer these questions, we use a numerical approach that explicitly resolves the rolls in the HBL. We examine the formation and characteristics of rolls under various ambient environments by a set of idealized numerical experiments.

1.2. Method

1.2.1 Modeling approach

We assume that rolls can be separated from the large-scale HBL flow because of their small spatial scale. Based on this assumption, we decompose the HBL flow variables into the mean and perturbations, as $a = \bar{a} + a'$, where a can represent the velocity vector, potential temperature, etc.. Similar flow decomposition approach was applied by Ginis et al. (2004) and Foster (2005) for studying boundary layer eddies or rolls under hurricane condition. In this study, the mean component represents the large-scale HBL flow, which consists of the primary cyclonic circulation and the secondary circulation induced by surface friction; the perturbations represent the rolls, which are the small-scale features generated by the inflection point instability of the mean HBL flow.

For simplicity, the mean HBL flow is assumed axisymmetric and therefore can be described in a cylindrical coordinate system (r, λ, z) . Observations suggest that rolls have

quasi-two-dimensional structures and they are elongated approximately in the mean wind direction. We use a local Cartesian coordinate system (x, y, z) for the rolls at large radius in the hurricane (radius larger than the radius of the maximum wind). (x, y, z) is set up in such a way that the y axis is parallel to the direction in which the rolls are aligned. Hereafter, we will refer to the y axis (x axis) as the along-roll axis (cross-roll axis). The along-roll variations of the perturbations are assumed negligible, i.e., $\partial a' / \partial y = 0$, where a' represents the variables associated with roll motions. This assumption allows reducing the three-dimensional equations for the rolls into two-dimensional equations written in x - z plane. To distinguish the velocity components projected onto the two different coordinate systems, we use upper case letters (U, V, W) to represent the wind components in the cylindrical coordinate system and lower case letters (u, v, w) to represent the wind components in the local Cartesian coordinate system. If the angle between the along-roll direction (y) and the azimuthal direction (λ) is defined as ε (Fig. 1.1), the wind components in the two coordinate systems can be transformed as follows:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos \varepsilon & \sin \varepsilon \\ -\sin \varepsilon & \cos \varepsilon \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} \quad \text{and} \quad w = W .$$

The complete sets of equations for the mean flow and rolls are presented in appendix A. For the purpose of this study, the governing equations for the mean flow and rolls are further simplified. We consider the evolution of the rolls to take place in two phases: the linear phase and the nonlinear phase. During the linear phase, rolls are formed by the instability of the mean flow and have the following characters: i) roll velocities grow exponentially with time, but remain at least one order smaller than the mean winds, and thus the nonlinear terms in their governing equations are negligible; ii) the averaged

fluxes induced by rolls are too weak to cause any significant modifications to the mean flow, and therefore we can assume the mean flow remains unchanged during the linear phase. The evolution of rolls enters the nonlinear phase after roll velocities reach sufficiently large magnitude (comparable to the mean winds), during which the nonlinear effects and the roll-induced fluxes are important. In this paper, we focus on the generation and initial evolution of rolls, and therefore only consider the linear phase. Thus some terms in the governing equations of the mean flow and rolls can be neglected, as discussed below.

1.2.2 The basic-state HBL flow

The basic-state HBL flow is used as the background environment for the formation of rolls. The basic-state HBL wind profiles are obtained by resolving the steady-state mean wind equations, neglecting time dependency and roll-induced forcing terms in (A4)-(A6). While the basic-state potential temperature profiles are prescribed analytically. We assume that the basic-state HBL wind and potential temperature profiles remain unchanged over time during the linear phase of rolls.

The governing equations for the basic-state HBL winds are

$$\bar{U} \frac{\partial \bar{U}}{\partial r} + \bar{W} \frac{\partial \bar{U}}{\partial z} - \frac{\bar{V}^2}{r} - f\bar{V} = -\frac{1}{\rho_0} \frac{\partial \bar{P}}{\partial r} + \bar{K}_m \frac{\partial^2 \bar{U}}{\partial z^2}, \quad (1.1)$$

$$\bar{U} \frac{\partial \bar{V}}{\partial r} + \bar{W} \frac{\partial \bar{V}}{\partial z} + \frac{\bar{U}\bar{V}}{r} + f\bar{U} = \bar{K}_m \frac{\partial^2 \bar{V}}{\partial z^2}, \text{ and} \quad (1.2)$$

$$\frac{\partial \bar{U}}{\partial r} + \frac{\bar{U}}{r} + \frac{\partial \bar{W}}{\partial z} = 0, \quad (1.3)$$

where $\bar{U}$, $\bar{V}$ and $\bar{W}$ are the mean wind components in the radial, azimuthal and vertical directions respectively, and $\bar{K}_m$ is the mean turbulent viscosity.

The hurricane is assumed to be on the f -plane centered at $20^\circ N$. At the upper boundary ($z = H$, where H is the vertical extent of the atmospheric layer we consider), we assume the wind is under gradient wind balance, i.e., $\bar{V} = V_g$ and $\bar{U} = 0$, where V_g is the gradient wind, satisfying

$$-\frac{V_g^2}{r} - fV_g = -\frac{1}{\rho_0} \frac{\partial \bar{P}}{\partial r}. \quad (1.4)$$

We use the Holland (1980) parametric model to specify the radial distribution of V_g at the upper boundary and assume the pressure gradient force doesn't vary vertically in the shallow atmosphere layer we consider. The radial V_g profile we use has maximum wind speed of 39 m s^{-1} , radius of maximum wind (RMW) of 40 km, and parameter B of 1.3 (parameter B controls how rapidly V_g decreases with radius). This profile represents a generic hurricane and has been used in other HBL studies (Kepert 2001, 2012; Kepert and Wang 2001; Foster 2005, 2009). The distribution of V_g inside of the RMW is modified following the formulation used by Kepert and Wang (2001). Fig. 1.2 shows the radial distribution V_g and the inertial stability parameter I for the storm we consider in this study. The inertial stability parameter I is given by

$$I = \sqrt{(f + 2V_g/r)(f + V_g/r + \partial V_g/\partial r)} \quad (\text{Kepert 2001}).$$

The vertical distribution of the velocity and turbulent viscosity near the lower boundary (from the surface to the lowest model level) is determined using the Monin-Obukhov similarity theory as described in Ginis et al. (2004). The only difference from Ginis et al. (2004) is that we calculate the roughness length using the formulation proposed by Moon et al. (2007). This formulation limits the drag coefficient below 0.003 at high wind speed, which is within the error bars

estimated by Powell et al. (2003) based on in-situ observations and consistent with the results from the laboratory experiments conducted by Donelan et al. (2004).

One critical parameter that affects the basic-state HBL wind distribution is the turbulent viscosity ($\bar{K}_m$). We calculate $\bar{K}_m$ using a first-order scheme as recommended by Zhang and Drennan (2012) and Kepert (2012). In this scheme, hereafter called the Louis scheme following Kepert (2012), $\bar{K}_m$ is parameterized in the form of

$$\bar{K}_m = l^2 \cdot S \cdot f(Ri), \quad (1.5)$$

where l is the mixing length, S is the magnitude of the vertical wind shear, and Ri is the gradient Richardson number. We apply the neutral version of the Louis scheme, i.e.

$f(Ri) = 1$. l has the form suggested by Blackadar (1962), written as

$$\frac{1}{l} = \frac{1}{\kappa z} + \frac{1}{l_\infty}, \quad (1.6)$$

where κ is the von Karman constant ($\kappa = 0.4$) and l_∞ is asymptotic mixing length.

The model described above is hereafter called the basic-state HBL model. Fig. 1.3 shows an example of the basic-state HBL wind solution with the choice of $l_\infty = 80$ m. The overall structure of the boundary layer is consistent with observations (Zhang et al. 2011) and similar to the simulation in Kepert (2012) using the same l_∞ (see Fig. 1.3 in Kepert 2012). Representative wind profiles at selected locations, 1RMW ($r = 40$ km), 2RMW ($r = 80$ km) and 3RMW ($r = 120$ km), are shown in Fig. 1.4. Both the radial and azimuthal wind profiles have inflection points (only the inflection points in the radial wind profiles are marked in Fig. 1.4). The vertical wind shear reaches extreme value at the inflection point. Compared with the inflection point in the azimuthal wind profile, the inflection point in the radial wind profile has considerably larger wind shear.

1.2.3 Equations for rolls in the linear phase

The nonlinear terms in the roll equations are negligible during the linear phase of rolls. Sensitivity experiments suggest that the Coriolis and curvature terms in the roll equations do not affect the solutions in any significant way, and thus can also be neglected, which is consistent with Foster (2005). By neglecting these terms, we decouple the along-roll velocity component (v') from the cross-roll motions (u', w'). This effectively filters out the parallel instability, which is another type of dynamical instability that can operate in Ekman-like boundary layer flow (Lilly 1966). Given these considerations, the equations governing the linear-phase rolls are

$$\frac{\partial \eta'}{\partial t} = -\bar{u} \frac{\partial \eta'}{\partial x} + \frac{\partial \psi'}{\partial x} \frac{\partial^2 \bar{u}}{\partial z^2} + \frac{g}{\theta_0} \frac{\partial \theta'}{\partial x} + \bar{K}_m \nabla^2 \eta', \quad (1.7)$$

$$\frac{\partial v'}{\partial t} = -\bar{u} \frac{\partial v'}{\partial x} - \frac{\partial \psi'}{\partial x} \frac{\partial \bar{v}}{\partial z} + \bar{K}_m \nabla^2 v', \quad (1.8)$$

$$\frac{\partial \theta'}{\partial t} = -\bar{u} \frac{\partial \theta'}{\partial x} - \frac{\partial \psi'}{\partial x} \frac{\partial \bar{\theta}}{\partial z} + \bar{K}_h \nabla^2 \theta', \text{ and} \quad (1.9)$$

$$\eta' = \nabla^2 \psi', \quad (1.10)$$

where $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial z^2$. Variables with the prime sign are associated with roll motions: ψ' is the stream function (cross-roll velocity $u' = -\partial \psi' / \partial z$, vertical velocity $w' = \partial \psi' / \partial x$), η' is the along-roll vorticity ($\eta' = \partial w' / \partial x - \partial u' / \partial z$), v' is the along-roll velocity, θ' is the potential temperature perturbations. Variables with the overbar are associated with the mean flow: $\bar{u}$ and $\bar{v}$ are the mean winds in the cross-roll (x) and along-roll (y) directions respectively, $\bar{\theta}$ is the mean potential temperature, $\bar{K}_m$ and $\bar{K}_h$ are the mean turbulent viscosity and diffusivity respectively. The above set of equations describes the evolution of rolls during the linear phase at a specific location in the HBL.

The mean flow parameters ($\bar{u}$, $\bar{v}$, $\bar{\theta}$, $\bar{K}_m$ and $\bar{K}_h$) are given by the basic-state HBL flow.

For simplicity, we assume $\bar{K}_h$ is equal to $\bar{K}_m$.

Periodic conditions are applied at the lateral boundaries. At the upper and lower boundaries, we set $\eta' = \theta' = w' = 0$ ($\eta' = w' = 0$ implies $\partial u' / \partial z = 0$) and $\partial v' / \partial z = 0$. We also tested the no-slip conditions ($u' = v' = 0$) and found that away from the lower and upper boundaries the structure of rolls is not affected. An infinitesimal perturbation in the vorticity field (η') is turned on as the initial condition for the perturbations. The model described above is hereafter referred to as the single-grid rolls-resolving model (SRM). It resolves rolls based on the mean flow profiles at a single grid of the basic-state HBL model.

1.3. Experimental design

Three groups of experiments are designed to investigate the impacts of the basic-state wind and stratification on the formation and characteristics of rolls. In the numerical experiments presented below, the SRM is embedded at every grid point of the basic-state HBL model, from 1RMW to 3RMW. The horizontal and vertical resolutions and the domain sizes for both models are listed in Table 1.1. The calculations in each experiment are done in two steps. During the first step, the basic-state HBL model resolves (1.1)-(1.3) to derive the basic-state wind distribution. During the second step, the SRM resolves (1.7)-(1.10) based on the basic-state wind profiles and the prescribed potential temperature profiles. Both theoretical analysis (Foster 2005) and three-dimensional LES study (Nakanishi and Niino 2012) suggested that the angle between the along-roll axis and the azimuthal wind is relatively small. Therefore, for simplicity we assume $\varepsilon = 0$, i.e.,

the along-roll (cross-roll) axis is in the local azimuthal (radial) wind direction ($\bar{u} = \bar{U}$ and $\bar{v} = \bar{V}$), in all groups of experiments.

Group N is designed to investigate the effect of basic-state HBL winds. In this group of experiments, we neglect the effect of stratification on the rolls by assuming neutral stratification, i.e., $\partial \bar{\theta} / \partial z = 0$ in the entire atmosphere layer we consider. Similar to the modeling results of Kepert (2012), our basic-state HBL model simulations indicate that an increase of l_{∞} results in an increase of the boundary layer height and a decrease of the wind shear. Therefore we use different values of l_{∞} (listed in Table 1.2) to vary the distribution of basic-state HBL winds in Group N.

Group S and Group M are designed to investigate the effect of stratification on roll formation and characteristics. The basic-state HBL winds used in Group S and Group M are the same as in experiment N3 ($l_{\infty} = 40$ m) in Group N, but the potential temperature profiles are varied. GPS dropsondes measurements collected in hurricanes (Zhang et al. 2011) suggest that the potential temperature profiles in the HBL usually contain a stably stratified layer ($d\theta / dz > 0$), a relatively well-mixed layer ($d\theta / dz \sim 0$) and a near-surface unstably stratified ($d\theta / dz < 0$) layer (Fig. 1.5). In Group S and M, two different types of idealized potential temperature profiles are used to investigate the effects of stratification. Since we are only concerned with the inflection point instability, we do not consider the unstably stratified layer in the potential temperature profiles to exclude the convective instability. Group S is aimed to investigate the effect of stable stratification. Potential temperature profiles with constant lapse rate ($\partial \bar{\theta} / \partial z$) are used in Group S, and the lapse rate is varied (Table 1.3). There is no mixed layer in the potential temperature profiles used in Group S. The same potential temperature profile is applied at all the

locations from 1RMW to 3RMW. Group M focuses on the effect of the mixed layer. In contrast to Group S, we use potential temperature profile that contains a well-mixed layer ($\partial \bar{\theta} / \partial z = 0$) and a stably stratified layer above in Group M, as shown in Fig. 1.6. The lapse rate in the stably stratified layer is fixed at 5 K km^{-1} and the mixed layer height z_i is varied from 200 m to 600 m (Table 1.4). The value of the lapse rate and the range of the mixed layer heights we use in Group M are consistent with those observed in hurricane conditions (Fig. 1.5).

1.4. Effect of mean wind on rolls

1.4.1 Basic structure of rolls

Rolls are formed in all experiments of Group N due to the inflection point instability. They have similar structures, but their wavelengths and growth rates vary with the changes of the mean wind profiles. The typical structure of rolls is shown in Fig. 1.7. Since rolls in the linear phase grow exponentially with time, the variables shown in Fig. 1.7 are normalized to remove the time-dependent part. The length scale δ we use to nondimensionalize the horizontal and vertical axes is the dynamical HBL height scale suggested by Kepert (2001). It is defined as $\delta = \sqrt{2K/I}$, where I is the inertial stability parameter and K is the turbulent viscosity. Here we use the maximum value in the $\bar{K}_m$ profile to calculate δ . The rolls shown in Fig. 1.7 have similar structure to those derived by Foster (2005). One noticeable feature is the tilted streamlines near the surface ($z/\delta < 3$). The tilted streamlines create asymmetry in the distribution of $w'u'$ (the product of vertical velocity w' and cross-roll velocity u' , Fig. 1.7d): $w'u'$ is mostly

negative near the surface. While at upper levels ($z/\delta > 3$), the streamlines and $w'u'$ are more vertically symmetric.

Next, we consider the kinetic energy budget for rolls to help us better understand the role of wind shear and stratification in the roll formation. Here we define the kinetic energy of rolls as $e' = 0.5(u'^2 + w'^2)$. v' is excluded from the energy budget consideration because it has no influence on the cross-roll overturning circulation (u', w'). The equation for e' is derived from equations (1.7)-(1.10) and can be written as

$$\frac{d}{dt} \int \bar{e}' dz = \int \left(-\overline{w'u'} \frac{\partial \bar{u}}{\partial z} + \frac{g}{\theta_0} \overline{w'\theta'} - \bar{K}_m \overline{\eta'^2} \right) dz, \quad (1.11)$$

where $\bar{e}'$ is the cross-roll averaged kinetic energy and $\int dz$ denotes the vertical integration over the atmospheric layer we consider. The three terms on the right-hand-side represent the shear production, buoyancy work, and the kinetic energy dissipation respectively. In neutral stratification, the only energy source for the rolls is the shear production. Fig. 1.8 shows the vertical distribution of the shear production for the rolls in Fig. 1.7. Such distribution is representative for all rolls in Group N. The mean wind shear ($\partial \bar{u} / \partial z$) reaches the maximum value at the inflection point. Correspondingly, the averaged cross-roll momentum flux ($\overline{w'u'}$) also reaches maximum value at the height of the inflection point. The tilted near-surface streamlines make the averaged momentum flux negative (opposite to the sign of the mean wind shear), and thus make the shear production term positive. Therefore, we can conclude that the near-surface tilted streamlines are necessary for rolls to extract the kinetic energy from the mean wind.

1.4.2 Factors affecting growth rate and wavelength of rolls

We next explore the characteristics of rolls, specifically their wavelength and growth rate, and identify mean wind factors that affect these characteristics. The SRM solutions indicate roll variables are in the form of normal mode. For example, the vertical velocity w' can be written as $w' = \hat{w}(z)\exp i(kx - \omega t)$, where k is the horizontal wavenumber, $\hat{w}(z)$ and ω are complex, $\omega = \omega_r + i\omega_i$, ω_r is the angular frequency and ω_i is the growth rate. We derive the wavelength of rolls ($2\pi/k$) by applying Fourier analysis on the horizontal distribution of vertical velocity (w'). The wavelength corresponding to the peak spectral density is considered as the wavelength of rolls. In the linear phase, the wavelength of rolls is vertically coherent and doesn't vary with time. The growth rate is estimated from the time series of the domain-averaged kinetic energy, using $\omega_i = 0.5 \cdot d \log(\langle e' \rangle) / dt$, where $\langle e' \rangle$ is kinetic energy averaged over the entire SRM domain.

Fig. 1.9 shows the wavelength of rolls and the dynamical HBL height scale δ from all experiments in Group N. The wavelength of rolls increases with radius, which is consistent with the theoretical study of Foster (2005). The increase of roll wavelength is associated with the increase of the dynamical HBL height scale δ (Fig. 1.9b). The ratios of the roll wavelength to δ in all experiments are in the range of 15-20. According to our basic-state HBL wind simulations, larger δ means that the shear layer (the layer in which the wind shear $\partial \bar{u} / \partial z$ is relatively large) is higher. Since the rolls gain the kinetic energy within the shear layer, the basic-state wind profiles with larger δ favor rolls with larger vertical and horizontal extents. This explains why the wavelength of rolls becomes larger with increasing radius.

Fig. 1.10 shows the growth rate of rolls and the bulk wind shear from all experiments in Group N. The bulk wind shear represents the magnitude of mean wind shear ($\partial \bar{u} / \partial z$) and is defined as $\bar{u}_{\max} / \delta$, where $\bar{u}_{\max}$ is the maximum cross-roll mean wind. The growth rate of rolls decreases with radius (Fig. 1.10a) and it is closely related to the bulk shear (Fig. 1.10b). This is because the mean wind shear is the only energy source for the rolls in the neutrally stratified HBL. Due to the decrease of the radial wind speed and the increase of the boundary layer height, the magnitude of the wind shear decreases with radius. As a result, the growth rate of rolls decreases with radius.

1.5. Effects of stratification on rolls

1.5.1 Effect of the stable stratification

The growth rates of rolls in the Group S experiments are shown in Fig. 1.11a. The major effect caused by the stable stratification is the reduction the growth rate of rolls. At a given location, the growth rate decreases with increasing lapse rate. When the stratification is sufficiently strong, the growth rate can be reduced to zero, which means the inflection point instability is completely suppressed. We find the classic Richardson number (Ri) criterion¹ (Miles 1961) can be used to determine whether or not the inflection point instability can operate under stable stratification. Ri is defined as

$$Ri = N^2 / \left(\partial \bar{u} / \partial z \right)^2, \text{ where } N^2 = g / \theta_0 \cdot \partial \bar{\theta} / \partial z. \text{ Since } N^2 \text{ is a constant in Group S, Ri}$$

¹ The Richardson number criterion states that for a stratified shear flow, the necessary (but not sufficient) condition for the flow to be unstable is the minimum Ri is less than 0.25, and the sufficient (but not necessary) condition for the flow to become stable is the minimum Ri is greater than 0.25.

reaches minimum at the inflection point where the wind shear reaches maximum. The minimum Ri at each location is plotted in Fig. 1.11b. As seen in Fig. 1.11, rolls can only be generated when minimum Ri is less than 0.25, while rolls cannot be generated when the minimum Ri is greater than 0.25.

The effect of stable stratification can be explained by the buoyancy work in the kinetic energy budget equation (1.11). The correlation between vertical velocity w' and potential temperature perturbation θ' determines the sign of the buoyancy work. Typical distributions of θ' , as well as the product of w' and θ' ($w'\theta'$) are shown in Fig. 1.12. In the lower part of rolls w' and θ' are negatively correlated, which indicates that the rolls tend to move heavier air parcels upwards and lighter air parcels downwards. As a result, the buoyancy work term in (1.11) is negative, causing the reduction of the growth rate. Typical vertical distributions of the buoyancy work and the shear production in Group S are shown in Fig. 1.13. As the ambient lapse rate increases, the ratio of buoyancy work to shear production increases too (not shown). This means stronger stratification results in stronger negative buoyancy work, and thus explains why the growth rate of rolls decreases with increasing stratification. When the strength of stable stratification reaches a critical value (indicated by the critical Ri , 0.25), rolls cannot be generated, i.e., the inflection point instability is completely suppressed.

1.5.2 Effect of the mixed layer

Fig. 1.14a shows the growth rate of rolls in Group M. Overall the mixed layer relaxes the suppressing effect of stratification on the inflection point instability. In the presence of a mixed layer, rolls can be formed at the locations where the inflection point instability is completely suppressed without the mixed layer (e.g., experiment S5 in Group S). The

roll growth rate increases with increasing mixed layer height (z_i), suggesting the buoyancy work weakens as z_i increases. When the mixed layer is sufficiently high (e.g., experiments M5), the growth rate of rolls is almost the same as in the experiment when stratification is not considered (experiment N3 in Group N).

The typical structure of rolls in the Group M experiments is shown in Fig. 1.15. The near-surface part of the rolls appears to be similar to those in Group N (Fig. 1.7) and Group S (Fig. 1.12). However, one noticeable change appears in the upper part where the roll streamlines become inclined from the vertical axis ($z/\delta > 5$). Such inclined streamlines suggest the generation of vertically propagating internal waves. These internal waves are generated because the rolls keep perturbing the stably stratified layer while they propagate horizontally and grow in time. The internal waves have the same horizontal wavenumber (k), angular frequency (ω_r) and growth rate (ω_i) as the rolls. As shown in Fig. 1.15, the rolls and internal waves are phase-locked and coupled together. In the following, we use an analytical model to explore the linkage between the rolls and internal waves, and the effect of the mixed layer height on the internal wave properties.

We consider a region ($z > z_i$) where the mean wind is weak ($\bar{u} \sim 0$) and N^2 is constant. This region will be referred to as region $\Re$. The mean wind shear and turbulent viscosity ($\bar{K}_m$) are negligible in region $\Re$. Below region $\Re$, the mean wind shear is relatively stronger and rolls are generated due to the inflection point instability. In region $\Re$, we can neglect terms in (1.7)-(1.10) related to $\bar{u}$ and $\bar{K}_m$. Such approximation allows us to derive an analytical solution for internal waves in region $\Re$. Let's consider the internal wave solution in the form of normal mode. For example, the stream function ψ'

is written as $\psi' = \hat{\psi}(z)\exp i(kx - \omega t)$, where $\omega = \omega_r + i\omega_i$. k , ω_r and ω_i of the internal waves in region $\Re$ are determined by the rolls generated below region $\Re$. By substituting the normal mode solutions into (1.7)-(1.10), we can derive the equation for $\hat{\psi}(z)$ in region $\Re$,

$$\frac{d^2 \hat{\psi}}{dz^2} + m^2 \hat{\psi} = 0, \quad (1.12)$$

in which

$$m^2 = \frac{N^2}{c^2} - k^2, \quad (1.13)$$

where $c = \omega / k$. (1.12) has solution in the form of $\hat{\psi} = \psi_0 \exp(imz)$, where ψ_0 is an arbitrary constant. Since c has both the real and imaginary parts, m is a complex number, i.e., $m = m_r + im_i$. If $N^2 = 0$ (e.g., Group N), according to (1.13), $m_r = 0$ and $m_i = |k|$, and therefore $\hat{\psi} = \exp(-|k|z)$, which means $\hat{\psi}(z)$ decays exponentially with height. But if $N^2 \neq 0$ (e.g., Group M and Group S), m has both non-zero real and imaginary parts, suggesting the internal waves have both vertically propagating and vertically decaying properties.

To characterize the properties of internal waves, we introduce two non-dimensional parameters: $\Gamma = m / k$ and $D = |m_r / m_i|$. These nondimensional parameters were first introduced by Sutherland (1994). The real part of Γ , i.e., m_r / k , measures the inclination angle (α) of the internal wave phase lines from the vertical direction, where α is given by $\alpha = \arctan(\Gamma_r)$. D is the ratio of the vertical e-folding length to the vertical wavelength. Following Sutherland (1994), D is referred to as the penetration ratio. Larger D suggests the internal waves have less vertically decaying property and more vertically

propagating property. For example, if m_i is zero and D is infinitely large, the internal waves propagate vertically without any decay. After introducing Γ , (1.13) becomes

$$\Gamma^2 = \frac{N^2}{\omega^2} - 1. \quad (1.14)$$

Taking square root of (1.14), we get

$$\Gamma_{\pm} = \pm \left(\sqrt{\frac{C_1 + \sqrt{C_1^2 + C_2^2}}{2}} + i \cdot \text{sgn}(C_2) \sqrt{\frac{-C_1 + \sqrt{C_1^2 + C_2^2}}{2}} \right), \quad (1.15)$$

where $C_1 = N^2(\omega_r^2 - \omega_i^2) / (\omega_r^2 + \omega_i^2)^2 - 1$, $C_2 = -2N^2\omega_r\omega_i / (\omega_r^2 + \omega_i^2)^2$, and sgn is the sign

function. ω_r and ω_i are both positive, and therefore $\text{sgn}(C_2) = -1$. We only allow

vertically bounded solutions, i.e. $m_i > 0$. This requires $\Gamma_i < 0$ because the horizontal

wavenumber k is always negative (rolls always propagate in the negative x direction). So

we only keep Γ_+ in (1.15). Thus we have

$$\Gamma_r = \sqrt{\frac{C_1 + \sqrt{C_1^2 + C_2^2}}{2}}, \quad (1.16)$$

$$\Gamma_i = -\sqrt{\frac{-C_1 + \sqrt{C_1^2 + C_2^2}}{2}}, \text{ and} \quad (1.17)$$

$$D = \frac{\Gamma_r}{\Gamma_i} = \frac{C_1 + \sqrt{C_1^2 + C_2^2}}{C_2}. \quad (1.18)$$

The solutions (1.16) and (1.18) allow to determine the inclination angle (α) and the

penetration ratio (D) of the internal waves generated by rolls with known angular

frequency (ω_r) and growth rate (ω_i). To validate the analytical solutions, we compare

the inclination angles estimated based on (1.16) with the inclination angles of the upper

level structures derived by the SRM in Group M. The roll growth rate is calculated using

the method described in section 4b. The roll angular frequency is derived from Fourier analysis of the time series of the vertical velocity (w') at a fixed point within the mixed layer. The angular frequency corresponding to the peak of the spectrum is considered to be the angular frequency of the rolls. As seen in Fig. 1.15, the inclination angles in the analytical and numerical solutions match very well.

Fig. 1.16 shows the inclination angle (α) and the penetration ratio (D), derived based on (1.16) and (1.18), as functions of the mixed layer height at RMW. As the mixed layer height increases, the inclination angle of the internal wave phase lines also increases. This implies that the phase lines of internal waves are more inclined from the vertical direction when the mixed layer gets higher. The penetration ratio also increases with the increased mixed layer height, suggesting that the internal waves generated by the rolls have less vertically decaying property and more vertically propagating property under non-uniform stratification with a higher mixed layer.

1.6. Discussion

In the three groups of experiments described above, we assume the angle between the along-roll axis and the azimuthal direction (ε) is zero. To explore the effect of this assumption on the roll characteristics, we performed an additional group of experiments to test how sensitive the rolls are to the choice of ε . In these experiments, we used the basic-state HBL wind distribution in experiment N3 ($l_\infty = 40\text{m}$), and assumed that the stratification is neutral. We varied ε from -10° to 15° with an interval of 5° (positive ε means the along-roll axis is to the left of the azimuthal direction). Fig. 1.17 shows the growth rates of rolls as functions of angle ε at different locations. The results suggest the angle ε at which rolls reach the largest growth rate is within 10° (slightly to the left of

the local azimuthal wind direction), which is consistent with Foster (2005). The assumption $\varepsilon = 0$ underestimates the roll growth rate by a small factor (10% at most). The roll wavelengths change insignificantly from 0° to 10° (not shown).

In Group N, we vary the basic-state HBL wind distribution by changing the asymptotic mixing length (l_∞). An alternative way to vary the basic-state HBL wind distribution is to change the gradient wind (V_g) distribution at the upper boundary of the basic-state HBL model. Following Kepert and Wang (2001, hereafter referred to as KW01), in addition to the storm used in Group N (i.e., Storm I in KW01), we considered another two storms: a less initially stable storm (Storm II in KW01) and a more intense storm (Storm III in KW01). Similar to the effect of l_∞ in Group N, the radial distribution of V_g affected the dynamical boundary layer height and the wind shear, and thus affected the wavelength and growth rate of rolls. We found the overall radial tendencies of the wavelength and growth rate of rolls under Storm II and Storm III were qualitatively similar to those under Storm I.

According to our analysis, the spatial scale of rolls is proportional to the dynamical HBL height scale δ , which depends on the inertial stability parameter I and the turbulent viscosity $\bar{K}_m$. The rolls obtained in Group N have generally larger wavelengths than those in Foster (2005), primarily because we applied a larger turbulent viscosity to set up the basic-state HBL winds. In fact, by reducing l_∞ in Group N (e.g. $l_\infty = 5$ m), we can derive rolls with the wavelength similar to those in Foster (2005). The dependency of rolls on the HBL height may explain the difference between the rolls derived by Nolan (2005) and Foster (2005). Rolls derived by Nolan (2005) have wavelengths in the range of 3-5 km, much larger than the rolls derived by Foster (2005). However, this doesn't

necessarily mean the roll formation mechanisms are different in these two studies. The difference in the wavelengths between Nolan (2005) and Foster (2005) likely originates from the different choices of the gradient wind distribution and the turbulent viscosity, both affecting the dynamical HBL height and ultimately the wavelength of rolls. The dependency of roll wavelength on the dynamical HBL height may also give some explanation to the variability of the roll wavelength in the observations. The variability of observed roll wavelength could result from the variability of dynamical height associated with the mean HBL wind distribution.

Previous theoretical studies (Foster 2005; Nolan 2005) primarily focused on the effect of the mean HBL wind structure on the rolls, but the effect of stratification has not been fully investigated. Nolan (2005) used neutral stratification throughout the atmospheric layer he considered. Foster (2005) included the near-surface unstably stratified layer but he did not consider the stably stratified layer above the mixed layer. The unstably stratified layer Foster (2005) considered did not appear to affect roll characteristics significantly. This study emphasizes the effects of stratification on rolls. We find that the stable stratification in the HBL may suppress the inflection point instability under some conditions. This may explain why rolls do not always exist in the HBL observations (e.g., Morison et al. 2005, French et al. 2007) even though the inflection point in the radial wind profile is always present. Moreover, we find the stably stratified layer provides the environment for rolls to couple with internal waves. The characteristics of rolls and internal waves are affected by the mixed layer height, suggesting the mixed layer height is a critical length scale.

Spatial resolutions of the majority of current hurricane models are too coarse to resolve rolls and therefore their effects are not explicitly included. To properly parameterize the effects of rolls, process-oriented studies need to be conducted to study their dynamics, such as their formation and nonlinear evolution. One possible limitation of our numerical approach is the assumption of two-dimensional structure of rolls. Although this assumption can be justified because rolls tend to be elongated along the mean wind direction within the HBL, a high-resolution three-dimensional LES model would be the best tool for studying rolls in hurricanes. However, it is not practical at present because of computer limitations. Utilization of the SRM (Single-grid Roll-resolving Model) imbedded into a HBL model allows to simplify the problem and dramatically decrease the amount of computation. This numerical approach can serve as a useful tool to study both the thermal and inflection point instabilities in the HBL, and the effects of the environmental factors such as the mean wind and stratification. Because the equations describing the HBL mean flow and the SRM are coupled, this approach can be further extended to study the nonlinear phase of rolls, their interactions with the internal waves, as well as their interactions with the mean HBL flow.

1.7. Conclusions

We have investigated the role of the mean wind (large-scale wind) and the stratification on the inflection point instability in the hurricane boundary layer (HBL). The study was performed using a two dimensional Single-grid Roll-resolving Model (SRM) to resolve linear-phase rolls under various HBL conditions. The key findings in this study are summarized as follows. Rolls generated by the inflection point instability are characterized by tilted streamlines in the vicinity of the inflection point of the radial

wind profile. Kinetic energy budget consideration reveals that the tilted streamlines are critical for the rolls to extract kinetic energy from the mean wind. We have identified two important factors of the mean HBL winds that affect the characteristics of rolls: the dynamical HBL height affects the wavelength of rolls, and the magnitude of the mean wind shear affects the growth rate of rolls. Therefore, under neutrally stratified HBL, the wavelength of rolls increases with the distance from the storm center (outside of the RMW), while their growth rate decreases. The stable stratification in the HBL can suppress the growth of rolls, and rolls can be generated only if the minimum gradient Richardson number is less than 0.25. The nonuniform stratification with a mixed layer has less suppressing effect on rolls. If the mixed layer is sufficiently high, the stably stratified layer above has minor effect on the growth of rolls.

Rolls generated by the inflection point instability can trigger internal waves in the stably stratified layer, which have the same horizontal wavelength, growth rate and angular frequency as rolls. These internal waves have both vertically propagating and decaying properties. We derived analytical solutions for internal waves, which relate the properties of the internal waves to the properties of boundary layer rolls. We find as the mixed layer height increases, the phase lines of the internal waves are more inclined from the vertical direction, and the internal waves have less vertically decaying property and more vertically propagating property.

References

- Blackadar, A. K., 1962: The vertical distribution of wind and turbulent exchange in a neutral atmosphere. *J. Geophys. Res.*, **67**, 3095–3102.
- Brown, R. A., 1970: A secondary flow model of the planetary boundary layer. *J. Atmos. Sci.*, **27**, 742–757.
- Brown, R. A., 1972: On the inflection point instability of a stratified Ekman boundary layer. *J. Atmos. Sci.*, **29**, 850–859.
- Brown R. A., 1980: Longitudinal instability and secondary flows in the planetary boundary layer: a review. *Rev. Geophys. Space Phys.*, **18**, 683–697.
- Ellis R. and S. Businger, 2010: Helical circulation in the typhoon boundary layer. *J. Geophys. Res.*, **115**, D06205.
- Etling, D., and S. Raasch, 1987: Numerical simulation of vortex roll development during a cold air outbreak. *Dyn. Atmos. Oceans*, **10**, 277–290.
- Etling, D., and R. A. Brown, 1993: Roll vortices in the planetary boundary layer: A review. *Bound.-Layer Meteor.*, **65**, 215–248.
- Faller, A. J., 1965: Large Eddies in the Atmospheric Boundary Layer and Their Possible Role in the Formation of Cloud Rows. *J. Atmos. Sci.*, **22**, 176–184.
- Foster, R., 1996: An analytic model for planetary boundary roll vortices. Ph. D. dissertation, University of Washington, 193 pp.
- Foster, R., 2005: Why rolls are prevalent in the hurricane boundary layer. *J. Atmos. Sci.*, **62**, 2647–2661.
- Foster, R., 2009: Boundary-layer similarity under an axisymmetric, gradient wind vortex. *Boundary-Layer Meteorol*, **131**, 321–344.

- Foster, R. 2013: Signature of large aspect ratio roll vortices in synthetic aperture radar images of tropical cyclones. *Oceanography*, **26**, 58–67.
- Gao, K. and I., Ginis, 2014: On the Generation of Roll Vortices due to the Inflection Point Instability of the Hurricane Boundary Layer Flow. *J. Atmos. Sci.*, **71**, 4292–4307.
- Glendening, J.W., 1996: Lineal eddy features under strong shear conditions, *J. Atmos. Sci.*, **53**: 3430-3449.
- Gopalakrishnan, S.G., F. Marks, J.A. Zhang, X. Zhang, J.-W. Bao, and V. Tallapragada, 2013: A study of the impacts of vertical diffusion on the structure and intensity of tropical cyclones using the high resolution HWRF system, *J. Atmos. Sci.*, **70**, 524-541.
- Holland, G. J., 1980: An analytic model of the wind and pressure profiles in hurricanes. *Mon. Wea. Rev.*, **108**, 1212–1218.
- Kepert, J. D., 2001: The dynamics of boundary layer jets within the tropical cyclone core. Part I: Linear theory. *J. Atmos. Sci.*, **58**, 2469-2484.
- Kepert, J. D. and Y. Wang, 2001: The dynamics of boundary layer jets within the tropical cyclone core. Part II: Nonlinear enhancement. *J. Atmos. Sci.*, **58**, 2485–2501.
- Kepert, J. D., 2012: Choosing a boundary layer parameterization for tropical cyclone modeling. *Mon. Wea. Rev.*, **140**, 1427–1445.
- Lilly, D., 1966: On the instability of Ekman boundary flow. *J. Atmos. Sci.*, **23**, 481-494.
- Lorsolo, S., J. L. Schroeder, P. Dodge, and F. Marks, 2008: An observational study of hurricane boundary layer small-scale coherent structures. *Mon. Wea. Rev.*, **136**, 2871-2893.

- Moon, I.-J., I. Ginis, T. Hara, and B. Thomas, 2007: A physics-based parameterization of air-sea momentum flux at high wind speeds and its impact on hurricane intensity predictions. *Mon. Wea. Rev.*, **135**, 2869–2878.
- Morrison, I., S. Businger, F. Marks, P. Dodge, and J. Businger, 2005: An observational case for the prevalence of roll vortices in the hurricane boundary layer. *J. Atmos. Sci.*, **62**, 2662–2673.
- Nakanishi, M. and H. Niino, 2012: Large-Eddy Simulation of roll vortices in a hurricane boundary layer. *J. Atmos. Sci.*, **69**, 3558–3575.
- Nolan, D. S., 2005: Instabilities in hurricane-like boundary layers. *Dyn. Atmos. Oceans*, **40**, 209–236.
- Sutherland, B. R., 2010: *Internal Gravity Waves*. Cambridge University Press, 378 pp.
- Wurman, J. and J. Winslow, 1998: Intense sub-kilometer boundary layer rolls in Hurricane Fran. *Science*, **280**, 555–557.
- Zhang J. A., K. B. Katsaros, P. G. Black, S. Lehner, J. R. French and W. M. Drennan, 2008: Effects of roll vortices on turbulent fluxes in the hurricane boundary layer. *Bound.-Layer Meteor.*, **128**, 173–189.
- Zhang, J. A., R. F. Rogers, D. S. Nolan, and F. D. Marks Jr., 2011: On the characteristic height scales of the hurricane boundary layer. *Mon. Wea. Rev.*, **139**, 2523–2535.
- Zhang, J. A. and W. M. Drennan, 2012: An Observational Study of Vertical Eddy Diffusivity in the Hurricane Boundary Layer. *J. Atmos. Sci.*, **69**, 3223–3236.
- Zhu, P., 2008: Simulation and parameterization of the turbulent transport in the hurricane boundary layer by large eddies, *J. Geophys. Res.*, **113**, D17104.

Table 1.1 Numerical parameters for the basic-state HBL model and the SRM.

	Basic-state HBL model	SRM
Vertical extent	3 km	3 km
Horizontal extent	1000 km	15.36 km
Vertical resolution	30 m	30 m
Horizontal resolution	10 km	30 m

Table 1.2 Values of asymptotic mixing length (l_{∞}) used in Group N.

Experiment Name	l_{∞} (m)
N1	10
N2	20
N3	40
N4	60
N5	80

Table 1.3 Values of lapse rate ($\partial \bar{\theta} / \partial z$) used in Group S.

Experiment Name	$\partial \bar{\theta} / \partial z$ (K km ⁻¹)
S1	1
S2	2
S3	3
S4	4
S5	5

Table 1.4 Values of mixed layer height (z_i) used in Group M.

Experiment Name	z_i (m)
M1	200
M2	300
M3	400
M4	500
M5	600

Figure 1.1 Diagram illustrating the cylindrical coordinate system (r, λ, z) for the mean flow and local Cartesian coordinate system (x, y, z) for rolls at radius $r = R$ (z axis is not shown). ε is the angle between the along-roll direction (y) and the azimuthal direction (λ). Positive ε means the along-roll direction is to the left of the azimuthal direction.

Figure 1.2 Gradient wind V_g (solid line) and inertial stability parameter I (dashed line) as functions of radius (normalized by the radius of maximum wind).

Figure 1.3 Height-radius distribution of the basic-state hurricane boundary layer variables with asymptotic mixing length $l_\infty = 80$ m. Black contours are (a) multiples of -2 m s^{-1} , (b) multiples of 10 m s^{-1} , (c) multiples of 0.05 m s^{-1} , and (d) multiples of $20 \text{ m}^2 \text{ s}^{-1}$.

Figure 1.4 Vertical profiles of (a) radial wind and (b) azimuthal wind at 1RMW, 2RMW and 3RMW in the hurricane boundary layer with asymptotic mixing length $l_{\infty} = 80$ m.

The inflection points with the largest radial wind shear are indicated by the black triangles in (a).

Figure 1.5 Vertical profiles of (a) potential temperature and (b) lapse rate in the hurricane boundary layer from a composite dataset based on GPS dropsondes measurements (Zhang et al. 2011). The approximate locations of these profiles are indicated in the figure (RMW = 40 km).

Figure 1.6 Idealized potential temperature profile used in Group M. The profile consists of two layers, the mixed layer and the stably stratified layer. The potential temperature is vertically uniform in the mixed layer and increases linearly with height in the stably stratified layer. The height of the mixed layer (z_i) is indicated by the dotted line.

Figure 1.7 Typical structure of rolls under neutral stratification. The colored backgrounds represent fields for (a) u' , (b) w' , (c) v' and (d) $w'u'$. The three velocity components are nondimensionalized by the maximum value of w' . $w'u'$ is nondimensionalized by its own maximum value. The x and z axes are nondimensionalized by the dynamical HBL height scale δ . Contours are streamlines: solid (dashed) contours represent clockwise (counterclockwise) circulations. The rolls shown are at RMW in experiment N3 in Group N with $\delta = 200$ m.

Figure 1.8 Typical distributions of (a) mean wind shear $\partial \bar{u} / \partial z$, (b) averaged cross-roll momentum flux $\overline{w'u'}$, and (c) shear production $-\overline{w'u'} \cdot \partial \bar{u} / \partial z$ for rolls at RMW in experiment N3, as shown in Fig. 1.6. The averaged cross-roll momentum flux and shear production shown are normalized by their maximum values respectively. The vertical axis is nondimensionalized by δ ($\delta = 200$ m). The height of the inflection point is indicated by the black triangle in (a).

Figure 1.9 (a) Wavelength of rolls and (b) the dynamical HBL height scale (δ) as functions of radius in Group N.

Figure 1.10 (a) Growth rate of rolls and (b) the bulk shear as functions of radius in Group N.

Figure 1.11 (a) Growth rate of rolls and (b) the minimum Richardson number as functions of radius in Group S. Growth rate of rolls in experiment N3 (dashdot line) is also shown in (a). The critical Richardson number (0.25) is indicated by the dashed line in (b).

Figure 1.12 Typical structure of rolls in Group S. The colored backgrounds represent fields for (a) potential temperature perturbation θ' and (b) product of vertical velocity and potential temperature perturbation $w'\theta'$. These variables are normalized by their maximum values respectively. The x and z axes are nondimensionalized by δ . Contours are streamlines: solid (dashed) contours represent clockwise (counterclockwise) circulations. The rolls shown are at RMW ($\delta = 200$ m) in experiment S3 with lapse rate of 3 K km^{-1} .

Figure 1.13 Vertical distributions of buoyancy work (solid lines) and shear production (dashed lines) in experiment S1, S3 and S5 at RMW. The values shown are nondimensional. Each pair of shear production and buoyancy work profiles is nondimensionalized by dividing the maximum value of the shear production. The vertical axis is nondimensionalized by δ ($\delta = 200$ m).

Figure 1.14 Growth rate of rolls as function of radius in Group M. For comparison, growth rates of rolls in experiment S5 (dashed line) and experiment N3 (dashdot line) are also shown.

Figure 1.15 Typical structure of rolls in Group M. The colored background represent potential temperature perturbations θ' , normalized by its maximum value. The x and z axes are nondimensionalized by δ . Contours are streamlines: solid (dashed) contours represent clockwise (counterclockwise) circulations. The rolls shown are at RMW ($\delta = 200$ m) in experiment M5 with the mixed layer height $z_i = 600$ m. The angle between the red dashed line and the vertical axis is the inclination angle (α) calculated based on (1.16) in the text.

Figure 1.16 (a) Inclination angle and (b) the penetration ratio of the internal waves as functions the mixed layer height at RMW calculated based on (1.16) and (1.18) in the text. The roll growth rates and angular frequencies from experiment S5 and M1-M5 are used.

Figure 1.17 Growth rates of rolls as functions of angle ϵ at different locations. In these experiments, the basic-state HBL wind distribution is the same as in experiment N3 ($l_\infty = 40$ m) and no stratification is considered. Angle ϵ is defined as the angle between the along-roll direction and the azimuthal direction.

CHAPTER 2

On the Equilibrium-State Roll Vortices and Their Effects in the Hurricane Boundary Layer

by

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Abstract

In this study, we simulate numerically roll vortices (rolls) generated by the inflection point instability in the hurricane boundary layer (HBL). Our approach is based on embedding a two-dimensional high-resolution Single-Grid Roll-Resolving Model (SRM) at selected horizontal grid point(s) of an axisymmetric HBL model. The results from a set of idealized experiments indicate that the mixed layer height is an important factor affecting the magnitude of the rolls and the structure of the internal waves triggered in the stably stratified layer above. The roll-induced cross-roll and along-roll momentum fluxes are determined by different mechanisms. While the cross-roll momentum flux is dependent on the cross-roll mean wind shear, the along-roll momentum flux is typically uncorrelated with the along-roll mean wind shear. Therefore, there is no physical basis for applying the K-theory to represent the roll-induced momentum fluxes. The rolls induce more significant changes in the mean radial wind than in the mean azimuthal wind. Specifically, rolls reduce the inflow near surface, enhance the inflow at upper levels and increase the inflow layer height. Based on a linear dynamical HBL model, we find that rolls affect the mean radial wind by redistributing the angular momentum (or the azimuthal momentum) vertically in the HBL.

2.1. Introduction

Roll vortices (hereafter rolls) refer to the coherent counter-rotating vortices in the atmospheric boundary layer, which are approximately aligned in the mean wind direction (Etling and Brown 1993). Rolls have been reported to frequently occur in the hurricane boundary layer (hereafter HBL) in observational studies (Wurman and Winslow 1998; Morrison et al. 2005; Lorsolo et al., 2008; Zhang et al. 2008; Ellis and Businger 2010). Most of the observations were made by Doppler radars during landfalling hurricanes and helped to quantify some characteristics of rolls. For example, Morrison et al. (2005) estimated the average horizontal wavelength ~ 1.5 km, the average depth ~ 700 m, the average vertical velocity $\sim 3 \text{ m s}^{-1}$ and the average horizontal wind speed perturbations $\sim 7 \text{ m s}^{-1}$. Zhang et al. (2008) reported the first in-situ measurements of the roll-induced vertical fluxes and showed that the rolls significantly enhanced the total momentum and moisture transports in the HBL. Because of the large vertical extent and strong vertical motions of rolls, the non-local effects induced by rolls may not be adequately represented by the existing turbulence parameterizations in hurricane models (Foster 2005). That is possibly an important factor limiting the current hurricane model forecast skill.

The formation mechanism and characteristics of rolls in the classical Ekman boundary layer have been extensively studied (e.g., Faller 1965; Lilly 1966; Brown 1970, 1972; Etling and Raasch 1987), and summarized in the literature (e.g., Brown 1980; Etling and Brown 1993). Thus we only review here the previous theoretical and numerical studies that focused on rolls in the HBL. Foster (2005) applied stability analyses to the basic-state HBL flow and demonstrated that the inflection points in the radial wind profiles cause the instability and lead to the formation of rolls. This generation mechanism is

often referred to as the inflection point instability. In a more recent study, Foster (2013) found that rolls with $O(10\text{ km})$ wavelength can be formed by the nonlinear wave-wave interaction. Nolan (2005) used both a stability analysis approach and a nonlinear axisymmetric model, and also found that inflection points in the HBL wind profiles are responsible for the formation of rolls. Subsequent numerical studies based on Large Eddy Simulation (LES) concurred that rolls in the HBL can be generated by the inflection point instability. Zhu (2008) nested a LES model within the WRF regional model to simulate a landfalling hurricane. He found roll-like perturbations in a statically stable boundary layer, which were likely generated by the dynamical instability rather than the convective instability. Nakanishi and Niino (2012) applied an LES model for the idealized HBL and simulated rolls generated by the inflection point instability with the spatial scales similar to those found by Foster (2005). In a recent study by the authors (Gao and Ginis 2014), rolls were resolved by applying a two-dimensional high-resolution Single-grid Roll-resolving Model (SRM) embedded into an axisymmetric HBL model. They found that rolls generated by the inflection point instability are affected by the stratification during the linear phase, in which the rolls grow exponentially with time. Particularly, the mixed layer height affects the growth rate of rolls and the structure of the internal waves in the stably stratified layer, which are triggered by the rolls.

In this study, we extend the approach by Gao and Ginis (2014) to investigate the rolls in the nonlinear phase, in which rolls have finite magnitude and can affect the mean flow. Specifically, we address the following questions:

- i) How does the mixed layer height affect the rolls and the internal waves triggered in the stably stratified layer above?

ii) Is the commonly used K -theory suitable for representing the roll-induced momentum fluxes?

iii) What is the effect of rolls on the mean wind in the HBL?

2.2. Method

2.2.1 Modeling approach

Our modeling approach is based on the assumption that rolls can be separated from the large-scale flow because of their small spatial scale. The total resolvable flow in the HBL is split into two components: the mean flow and the perturbations. Taking velocity $\mathbf{v}$ as an example, we assume $\mathbf{v} = \bar{\mathbf{v}} + \mathbf{v}'$, where $\bar{\mathbf{v}}$ represents the mean wind and $\mathbf{v}'$ represents the wind perturbations. In this study, the mean flow refers to the large-scale flow in the HBL, which consists of the primary cyclonic circulation and the secondary circulation induced by the surface friction; the perturbations refer to the small-scale features formed due to the dynamical instability of the mean flow, which consist of the boundary layer rolls and the internal waves triggered by them. Based on the above flow-separation assumption, the governing equations for the mean flow and the perturbations can be derived, as shown in appendix A. The equations for the mean flow and the perturbations are solved by two different numerical models, while the unresolved small-scale turbulences are parameterized using the traditional method.

Two different coordinate systems are used to describe the mean flow and the perturbations, respectively. The HBL mean flow is assumed axisymmetric and therefore can be described in a cylindrical coordinate system (r, λ, z) . For the perturbations, similar

to Foster (2005), we use a local Cartesian coordinate system (x, y, z) ¹ (Fig. 2.1). The system (x, y, z) is set up in such a way that the y axis is parallel to the direction in which the rolls are aligned. We will hereafter refer to the y (x) axis as the along-roll (cross-roll) axis. The along-roll variations of the perturbations are assumed negligible; that is, $\partial a' / \partial y' = 0$, where a' represents the perturbations. To distinguish the wind components projected onto the two different coordinate systems, upper case letters (U, V, W) will represent the wind components in the cylindrical coordinates and lower case letters (u, v, w) will represent the wind components in the local Cartesian coordinate system. If the angle between the along-roll direction (y) and the azimuthal direction (λ) is defined as ε (Fig. 2.1), the wind components in the two coordinate systems can be transformed as follows:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos \varepsilon & \sin \varepsilon \\ -\sin \varepsilon & \cos \varepsilon \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} \text{ and } w = W. \quad (2.1)$$

2.2.2 The HBL model

The governing equations for the mean wind in the HBL are

$$\frac{\partial \bar{U}}{\partial t} + \bar{U} \frac{\partial \bar{U}}{\partial r} + \bar{W} \frac{\partial \bar{U}}{\partial z} - \frac{\bar{V}^2}{r} - f\bar{V} = -\frac{1}{\rho_0} \frac{\partial \bar{P}}{\partial r} + \frac{\partial}{\partial z} \left(\bar{K} \frac{\partial \bar{U}}{\partial z} \right) - \frac{\partial \overline{W'U'}}{\partial z}, \quad (2.2)$$

$$\frac{\partial \bar{V}}{\partial t} + \bar{U} \frac{\partial \bar{V}}{\partial r} + \bar{W} \frac{\partial \bar{V}}{\partial z} + \frac{\bar{U}\bar{V}}{r} + f\bar{U} = \frac{\partial}{\partial z} \left(\bar{K} \frac{\partial \bar{V}}{\partial z} \right) - \frac{\partial \overline{W'V'}}{\partial z}, \quad (2.3)$$

$$\frac{\partial \bar{U}}{\partial r} + \frac{\bar{U}}{r} + \frac{\partial \bar{W}}{\partial z} = 0, \quad (2.4)$$

¹ The approximation of using the local Cartesian coordinates instead of the cylindrical coordinates is not valid at small radii. Therefore, we only consider perturbations outside of the radius of maximum wind in this study.

where $\bar{U}$, $\bar{V}$ and $\bar{W}$ are the mean wind components in the radial, azimuthal and vertical directions, respectively; $\overline{W'U'}$ and $\overline{W'V'}$ are the radial and azimuthal momentum fluxes induced by the perturbations, respectively; $\bar{K}$ is the parameterized turbulent diffusivity (the parameterization of K is discussed in section 2.2.4). Similar equations are used to describe the HBL mean wind in other studies (e.g., Foster 2009; Kepert 2012). A major difference here is that we explicitly consider the momentum tendencies induced by the resolved perturbations in (2.2) and (2.3).

The numerical model solving equations (2.2)-(2.4) is hereafter referred to as the HBL model. At the upper boundary ($z = H$, where H is the vertical extent of the atmospheric layer in the HBL model and set to 3 km), we assume that the mean wind is under the gradient wind balance, i.e., $\bar{V} = V_g$ and $\bar{U} = 0$, where V_g is the gradient wind, satisfying

$$-\frac{V_g^2}{r} - fV_g = -\frac{1}{\rho_0} \frac{\partial \bar{P}}{\partial r}. \quad (2.5)$$

The radial distribution of V_g is prescribed, and the pressure gradient force derived from (2.5) is assumed vertically uniform. The Holland (1980) parametric model is used to specify the radial distribution of V_g . The hurricane is assumed to be on the f -plane at $20^\circ N$ with maximum V_g of 39 m s^{-1} , the radius of maximum wind (RMW) of 40 km, and the parameter B of 1.3 (parameter B controls how rapidly V_g decreases with radius larger than the RMW). V_g inside of the RMW is specified following the formulation in Kepert and Wang (2001). Fig. 2.2 shows the radial distribution of V_g and the inertial stability parameter given by $I = \sqrt{(f + 2V_g/r)(f + V_g/r + \partial V_g/\partial r)}$ (Kepert 2001). The surface layer is parameterized using the Monin-Obukhov similarity theory with the roughness

length formulation proposed by Moon et al. (2007), which limits the drag coefficient below 0.003 at high wind speeds.

Another important variable is the mean virtual potential temperature ($\bar{\theta}_v$). Since the idealized HBL model used in this study cannot fully capture the physical processes that determine $\bar{\theta}_v$, its vertical profile is specified using the analytical formula described in section 2.3.

2.2.3 The Single-Grid Roll-Resolving Model (SRM)

The governing equations describing the perturbations at a horizontal grid point in the HBL model are

$$\frac{\partial \eta'}{\partial t} + u' \frac{\partial \eta'}{\partial x} + w' \frac{\partial \eta'}{\partial z} = -\bar{u} \frac{\partial \eta'}{\partial x} + w' \frac{\partial^2 \bar{u}}{\partial z^2} + \frac{g}{\theta_{v0}} \frac{\partial \theta'_v}{\partial x} + T_{\eta'}, \quad (2.6)$$

$$\eta' = \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial z^2}, \quad (2.7)$$

$$\frac{\partial v'}{\partial t} + u' \frac{\partial v'}{\partial x} + w' \frac{\partial v'}{\partial z} = -\bar{u} \frac{\partial v'}{\partial x} - w' \frac{\partial \bar{v}}{\partial z} + T_{v'}, \quad (2.8)$$

$$\frac{\partial \theta'_v}{\partial t} + u' \frac{\partial \theta'_v}{\partial x} + w' \frac{\partial \theta'_v}{\partial z} = -\bar{u} \frac{\partial \theta'_v}{\partial x} - w' \frac{\partial \bar{\theta}_v}{\partial z} + T_{\theta'_v}, \quad (2.9)$$

Variables with the prime sign are associated with the perturbations: (u' , v' , w') are the velocity components in the local Cartesian coordinates, ψ' is the stream function ($u' = -\partial \psi' / \partial z$, $w' = \partial \psi' / \partial x$), η' is the along-roll vorticity ($\eta' = \partial w' / \partial x - \partial u' / \partial z$), and θ'_v is the virtual potential temperature perturbation. Variables with the overbar are associated with the mean flow: $\bar{u}$ and $\bar{v}$ are the mean winds projected onto the cross-roll

(x) and along-roll (y) directions, respectively, and they are provided by the HBL model.

θ_{v0} in (2.6) is a constant and set to 300 K. $T_{\eta'}$, $T_{v'}$ and $T_{\theta'_v}$ represent the turbulent diffusions; take $T_{\theta'_v}$ for example, it is in the form of

$T_{\theta'_v} = \partial(K\partial\theta'_v/\partial x)/\partial x + \partial(K\partial\theta'_v/\partial z)/\partial z$, where K is the turbulent diffusivity. The Coriolis and centrifugal terms in (2.6) and (2.8) are neglected because we found that these terms did not affect the solutions in any significant way, which is consistent with Foster (2005). The numerical model resolving (2.6)-(2.9) is hereafter referred to as the Single-Grid Roll-Resolving Model (SRM). Its function is to resolve the perturbations at a single horizontal grid of the HBL model. No-slip condition is applied at the upper and lower boundaries of the SRM domain (that is $u' = v' = w' = \theta'_v = 0$), and the periodic condition is applied at the lateral boundaries.

2.2.4 Turbulent diffusivity parameterization

The turbulent diffusivity K is parameterized by a first order scheme as in Zhang and Drennan (2012) and Kepert (2012). In this scheme, K is given in the form of $K = l^2 Sf(Ri)$, where l is the mixing length, S is the strain rate, and Ri is the gradient Richardson number.

- i) l has the form suggested by Blackdar (1962), $l^{-1} = (\kappa z)^{-1} + l_{\infty}^{-1}$, where κ is the von Karman constant ($\kappa = 0.4$), and l_{∞} is asymptotic mixing length set to 30 m. The value of l_{∞} is relatively small compared to the value estimated based on in-situ observations by Zhang and Drennan (2012). The reason is that we assume the fluxes induced by large eddies, such as rolls, are resolved explicitly by the SRM and only the small-scale turbulence is parameterized.

ii) S is given by $S^2 = 1/2(\partial u_i / \partial x_j + \partial u_j / \partial x_i)^2$, where $u_i = \bar{u}_i + u'_i$.

iii) $f(Ri) = \sqrt{1 - Ri}$ and Ri is defined as $Ri = g\theta_{v0}^{-1}S^{-2}\partial\theta_v / \partial z$, where $\theta_v = \bar{\theta}_v + \theta'_v$.

The calculations of K at a horizontal grid point of the HBL model are as follows. If SRM is not embedded, $\bar{K}$ in the mean wind equations (2.2) and (2.3) are calculated with S equal to the mean wind shear, given by $S^2 = (\partial\bar{U} / \partial z)^2 + (\partial\bar{V} / \partial z)^2$. If SRM is embedded, the total turbulent diffusivity K in roll equations (2.6), (2.8) and (2.9) are calculated with S given by $S^2 = 2(\partial u / \partial x)^2 + 2(\partial w / \partial z)^2 + (\partial u / \partial z + \partial w / \partial x)^2 + (\partial v / \partial x)^2 + (\partial v / \partial z)^2$, where the wind components are the total resolved winds projected onto the local Cartesian coordinates, for example, $u = \bar{u}(z) + u'(x, z)$; the horizontally-averaged K in the SRM domain, that is $\bar{K}$, is applied in (2.2) and (2.3).

2.2.5 The HBL-SRM coupled system

The HBL model and the SRM are dynamically coupled and integrated in time simultaneously. The SRM can be embedded into the HBL model at multiple horizontal grid points (outside of the RMW). The HBL model provides the mean wind profiles to the SRM at selected grid points, and SRM provides the HBL model with the vertical momentum fluxes induced by the resolved perturbations. The numerical parameters for the HBL model and the SRM are listed in Table 2.1. Note that the vertical extend of the SRM domain (6 km) is set to be higher than the vertical extend of the HBL model domain (3 km). The Rayleigh damping is applied in the upper 3 km of the SRM domain to suppress the vertically propagating internal waves and prevent them from affecting the boundary layer solutions.

In each numerical experiment, the HBL model is run first for 24 hours without the effects of perturbations to derive a steady-state mean wind distribution. Then the SRM is embedded at selected grid points in the HBL model and the two models are run in the coupled mode. An infinitesimal perturbation in the along-roll vorticity field (η') is introduced as the initial condition for the perturbations.

2.3. Experimental design

In the primary set of experiments, hereafter M200 - M600, we investigate how the stratification, particularly the mixed layer height, affects the characteristics of the perturbations in the HBL. In these experiments the SRM is only embedded at the RMW in the HBL model. The initial mean wind profiles are kept the same¹, but the initial $\bar{\theta}_v$ profiles are varied and prescribed analytically based on the composite GPS dropsondes measurements described in Zhang et al. (2011). The formula for the vertical gradient of $\bar{\theta}_v$ is given by

$$\frac{\partial \bar{\theta}_v}{\partial z} = A \cdot \tanh^2\left(\frac{z}{h}\right), \quad (2.10)$$

where A is the background value for $\partial \bar{\theta}_v / \partial z$, and h is a depth scale controlling the mixed layer height. The vertical profile of $\bar{\theta}_v$ can be derived by integrating $\partial \bar{\theta}_v / \partial z$ vertically.

Fig. 2.3 shows a typical analytical profile of $\partial \bar{\theta}_v / \partial z$, as well as several observed profiles from Zhang et al. (2011) for comparison. The analytical $\partial \bar{\theta}_v / \partial z$ profile gradually

¹ The initial $\bar{\theta}_v$ profiles can affect the initial mean wind profiles through affecting the static stability factor in the K parameterization. However, it is found that such influence is very weak.

increases from 0 to the background value $A = 5.5 \text{ K km}^{-1}$, similar to the observed profiles¹. Fig. 2.4 shows the vertical profiles of $\bar{\theta}_v$ and $\partial\bar{\theta}_v/\partial z$ in the primary set of experiments. The mixed layer height, defined as the height where $\partial\bar{\theta}_v/\partial z$ is equal to 3 K km^{-1} (Zhang et al. 2011), varies from 200 m to 600 m, with an increment of 100 m. These prescribed $\bar{\theta}_v$ profiles are held unchanged with time. Fig. 2.5 shows the initial mean wind profiles at the RMW in the primary set of experiments, which are derived by running the HBL model for 24 hours without the effects of perturbations.

In an additional experiment, SRM is embedded at all horizontal grid points (except those inside of the RMW) in the HBL model to investigate the impact of the rolls on the overall mean wind distributions in the HBL. In this experiment, the same $\bar{\theta}_v$ profile is used at all locations with the mixed layer height set to 600 m. A relatively high mixed layer is used to ensure that rolls can reach maximum magnitude if they are generated.

In all experiments the angle ε (Fig. 2.1) between the y axis and the λ axis is calculated based on the mean wind profiles in the HBL. Previous observations and theoretical studies (Morrison et al. 2005; Foster 2005) suggest that rolls tend to align in the direction of the depth-averaged wind. Here we assume that the along-roll axis (y axis) is in the direction of the depth-averaged wind vector below 1 km. The calculated angle ε varies

¹ The observed $\bar{\theta}_v$ profiles (Fig. 2.3) suggest that a shallow unstably stratified layer ($\partial\bar{\theta}_v/\partial z < 0$) exists near the surface ($z < 100 \text{ m}$). To test the effect of the unstably stratified layer on the characteristics of the rolls, we conducted a series of sensitivity experiments. The height of the unstably stratified layer and the lapse rate within this layer were constrained by the observed values (Zhang et al. 2011). The effects on the characteristics of rolls, including their basic structures, kinetic energy and fluxes, were found to be insignificant. Thus for the sake of simplicity, we chose to initialize all the experiments presented in this paper without the unstably stratified layer.

with distance from $\sim 4^\circ$ (at RMW) to $\sim 10^\circ$ (at 3 RMW), which is consistent with Foster (2005).

2.4. Effect of mixed layer height on rolls and internal waves

2.4.1 Kinetic energy of the perturbations

To describe the strength of the perturbations, we introduce the overturning kinetic energy as $e' = 0.5(u'^2 + w'^2)$. The along-roll velocity component v' is excluded from the definition of e' because v' has no direct influence on the overturning circulations (u', w') in the x - z plane [see (2.6)]. We further introduce the domain-averaged overturning kinetic energy $\langle \bar{e}' \rangle$, where $\bar{}$ represents horizontal averaging in the cross-roll direction, and $\langle \rangle$ represents vertical averaging in the lower 3 km layer of the SRM domain. The perturbations in the upper 3 km layer where the Rayleigh damping is applied will not be considered. The equation for $\langle \bar{e}' \rangle$ are derived from the equations governing the perturbations (see appendix B), and written as

$$\frac{d\langle \bar{e}' \rangle}{dt} = \underbrace{\langle -\bar{w}'u' \frac{\partial \bar{u}}{\partial z} \rangle}_S + \underbrace{\langle \frac{g}{\theta_{v0}} \bar{w}'\theta'_v \rangle}_B + \underbrace{\langle \bar{d} \rangle}_D - \underbrace{\frac{1}{H} \bar{w}'p'_{z=H}}_P - \underbrace{\frac{1}{H} \bar{w}'e'_{z=H}}_T, \quad (2.11)$$

where $H = 3$ km; the terms on the right-hand-side are the production and loss terms for

$\langle \bar{e}' \rangle$: S - shear production; B - buoyancy work; D - turbulent dissipation, where

$d = u'[\partial(K\partial u' / \partial x) / \partial x + \partial(K\partial u' / \partial z) / \partial z] + w'[\partial(K\partial w' / \partial x) / \partial x + \partial(K\partial w' / \partial z) / \partial z]$; P - loss

of the kinetic energy through the pressure work ($\bar{w}'p'$) at $z = 3$ km (at the lower

boundary, $\bar{w}'p' = 0$); T - loss of kinetic energy through the net vertical transport ($\bar{w}'e'$) at

$z = 3$ km (at the lower boundary, $\bar{w}'e' = 0$).

Fig. 2.6 shows the time series of $\langle \bar{e}' \rangle$ in M200-M600. During the linear phase $\langle \bar{e}' \rangle$ grows exponentially ($\langle \bar{e}' \rangle$ is nearly zero in the beginning of each experiment). Once the perturbations reach finite amplitude, they quickly reach a quasi-equilibrium state, which is defined as the period during which $\langle \bar{e}' \rangle$ is maintained at a near-steady level. The characteristics of the perturbations in the linear phase were investigated by Gao and Ginis (2014). In this study, we focus on the perturbations during the quasi-equilibrium state. In experiments M300-M600, $\langle \bar{e}' \rangle$ oscillates around the near-steady level in the quasi-equilibrium state. The frequency of the oscillation is almost identical to the local inertial frequency I , corresponding to the period of ~ 1.2 hour. This implies that the oscillation of $\langle \bar{e}' \rangle$ originates from the inertial oscillation of the mean wind. The magnitude of $\langle \bar{e}' \rangle$, which characterizes the strength of the perturbations, increases for a higher mixed layer.

Next we investigate how the perturbations in the quasi-equilibrium state are maintained. Fig. 2.7 shows the time series of the budget terms in (2.11) in M300 and M600. In both cases, shear production (S) is the only source term for $\langle \bar{e}' \rangle$, and it is balanced by the combination of the buoyancy work (B), turbulent dissipation (D) and pressure work (P) in the quasi-equilibrium state. The vertical transport term (T) has no net contribution (not shown). In the experiment with a higher mixed layer (M600), the shear production term is larger, suggesting that the rolls generated by the inflection point instability can extract kinetic energy more effectively from the cross-roll mean wind. Moreover, the pressure work term becomes more significant in balancing the shear production in experiment M600. In fact $\overline{w'p'}$ is equivalent to the vertical energy flux transported by the internal waves at $z = 3$ km. This suggests that under a higher mixed

layer, the internal waves are more effectively triggered and they can carry more kinetic energy upwards.

2.4.2 Basic structure of the perturbations

Fig. 2.8 and Fig. 2.9 show the representative structures of perturbations in M300 and M600, respectively, in the quasi-equilibrium-state. Rolls exist at the lower levels (roughly $z < 1$ km), which have vertical velocities (w') up to ~ 3 m s⁻¹ and along-roll velocities (v') up to ~ 10 m s⁻¹. The roll horizontal wavelength, defined as the distance between two nearby w' peaks, is ~ 3 km. These values are within the range estimated from the radar observations (Morrison et al. 2005; Ellis and Businger 2010). The roll structures are also similar to those derived analytically by Foster (2005). Consistent with Foster's solution, the vertical velocity of rolls (Fig. 2.8 and Fig. 2.9) is asymmetric near the surface (roughly $z < 0.5$ km): the updrafts are narrower and stronger than the downdrafts. According to Foster (1996), the asymmetry in w' is mainly due to the formation of higher-wavenumber harmonics. The spatial spectral analysis of w' in Fig. 2.8 and Fig. 2.9 reveals that the higher-wavenumber harmonics indeed exist in our numerical solutions.

The most apparent differences between the perturbations in M300 and M600 (Fig. 2.8 and Fig. 2.9) are at upper levels ($z > 1$ km): the contour lines of ψ' , u' , w' and θ'_v are noticeably inclined from the vertical axis in M600. This is the signature of vertically propagating internal waves. As discussed by Gao and Ginis (2014), the internal waves are generated because the rolls keep perturbing the stably stratified layer above while they propagate horizontally. The internal waves are phase-locked with the rolls, and have the same horizontal wavenumber, horizontal phase speed and angular frequency as the rolls.

The absence of wave signature in M300 (Fig. 2.8) suggests that internal waves are not effectively triggered under a relatively low mixed layer.

2.4.3 Vertical fluxes induced by the perturbations

Fig. 2.10 shows the cross-roll ($\overline{w'u'}$), along-roll ($\overline{w'v'}$) momentum fluxes, and the virtual potential temperature flux ($\overline{w'\theta'_v}$) induced by the perturbations and averaged over two inertial periods in the quasi-equilibrium state. These fluxes are mostly concentrated at the lower levels ($z < 1$ km) and induced by rolls in M200-M600. The momentum fluxes in different experiments have similar vertical distributions, but their magnitudes increase as the mixed layer height increases. This is the result of stronger perturbations under higher mixed layers. At upper levels ($z > 1$ km), $\overline{w'u'}$ is non-zero and vertically uniform in the experiments with relatively high mixed layers (M500 and M600), which is associated with the vertically propagating internal waves. $\overline{w'\theta'_v}$ at upper levels is almost zero (Fig. 2.10), which is consistent with the common understanding of the internal wave transport properties. The ambient environment for the internal waves in this study is vertically uniform, and thus these waves can propagate freely without breaking. Therefore the momentum flux induced by the internal waves has no net effect on the mean flow. However, in the real atmosphere, internal waves may break via various mechanisms (Sutherland 2010), and the internal wave momentum may be deposited to the mean flow. Therefore, the generation of internal waves implies that the effect of rolls in the hurricane may not be limited to the HBL.

2.5. Roll-induced momentum fluxes

We next examine the correlations between the roll-induced momentum fluxes and the mean wind shear, and address the question whether the roll-induced momentum fluxes can be represented by the K -theory commonly used in hurricane models (Kepert 2012). The K -theory assumes that the vertical fluxes induced by the unresolved boundary layer motions depends on the vertical gradient of the mean (or resolved) variables, and can be represented as $\overline{w'\phi'} = -K_\phi \partial \bar{\phi} / \partial z$, where ϕ can be an arbitrary variable and K_ϕ is the parameterized diffusivity for ϕ . Following Glendening (1996), we define the effective K for the cross-roll momentum (K_u) and along-roll momentum (K_v) as follows:

$$K_u = -\frac{\overline{w'u'}}{\partial \bar{u} / \partial z}, \quad (2.12)$$

$$K_v = -\frac{\overline{w'v'}}{\partial \bar{v} / \partial z}, \quad (2.13)$$

where the roll-induced momentum fluxes and the mean wind shear are time-averaged over two inertial periods in the quasi-equilibrium state.

We find that K_u and K_v have very different distributions in the mid boundary layer region (0.1-0.6 km), where the roll-induced fluxes are most significant (Fig. 2.11). In all experiments, K_u is always positive and has finite values, while K_v reaches infinity at ~0.4 km and has negative values between 0.4-0.6 km. To investigate this in more detail, Fig. 2.12 shows the profiles of the mean wind shear and the roll-induced momentum fluxes in M600. While $\overline{w'u'}$ is negatively correlated with $\partial \bar{u} / \partial z$, as suggested by the distribution of K_u (Fig. 2.11a), it is not the case for $\overline{w'v'}$: $\partial \bar{v} / \partial z$ changes monotonically with height, but $\overline{w'v'}$ does not. Moreover, between 0.4-0.6 km (corresponding to the layer with

negative K_v), $\overline{w'v'}$ has same sign as $\partial \bar{v} / \partial z$, which means that $\overline{w'v'}$ is counter-gradient. Therefore, the vertical distribution of the along-roll momentum flux conflicts with the assumption of K -theory that the flux is always down-gradient, and thus cannot be represented by the K -theory.

The physical explanation for the different correlations of the cross-roll and along-roll momentum fluxes with the mean wind shear is as follows. Rolls, which are generated by the inflection point instability, gain their kinetic energy from the cross-roll mean wind $\bar{u}(z)$. In order for the shear production in (2.11) be positive, $\overline{w'u'}$ must have an opposite sign with $\partial \bar{u} / \partial z$. As discussed in Gao and Ginis (2014), the streamlines of rolls tend to tilt in such a way that $\overline{w'u'}$ is negatively correlated with $\partial \bar{u} / \partial z$. However, there is no such constraint on $\overline{w'v'}$. Physically, $\overline{w'v'}$ represents the net vertical transport of along-roll momentum by the overturning circulations of rolls. The vertical distribution of $\overline{w'v'}$ does not directly correlate with the along-roll mean wind shear $\partial \bar{v} / \partial z$. The along-roll mean wind (approximately the azimuthal mean wind, see Fig.13b) is relatively slow near the surface because of the surface friction and relatively fast at higher levels. The roll updrafts move the slower-moving air parcels upward (this process creates negative v' , see Fig. 2.8b and Fig. 2.9b) and the roll downdrafts move the faster-moving air parcels downward (this process creates positive v'). As a result, w' and v' are negatively correlated, making $\overline{w'v'}$ negative throughout the vertical layer below 0.6 km.

2.6. Effect of rolls on the HBL mean wind

In this section, we discuss how rolls affect the HBL mean wind and investigate the underlying physical mechanisms. The HBL model results indicate that after roll-induced momentum fluxes are introduced, the HBL mean wind profiles are adjusted to a new balanced state (corresponding to the quasi-equilibrium state of the rolls) within a few inertial cycles. We only focus on the well-adjusted mean wind, derived by averaging the mean wind profiles over two inertial cycles in the quasi-equilibrium state. For convenience, we define the balanced state under the impact of rolls as the final state and the balanced state without the impact of rolls as the initial state. Fig. 2.13 shows the mean wind profiles in the final and initial states at RMW in M200-M600, as well as the differences of the mean wind profiles in the two states. These differences can be considered as the net changes induced by rolls. The changes of the mean wind profiles in all experiments are qualitatively similar. Rolls in the experiments with higher mixed layers cause more significant modifications of the mean wind, because they generate stronger momentum fluxes. In the radial direction (Fig. 2.13a), rolls weaken the inflow near surface, enhance the inflow at upper levels, and increase the height of the inflow layer. In the azimuthal direction (Fig. 2.13b), the most apparent feature is a small weakening of the super-gradient jet (at ~ 0.4 km). Overall, the changes in the radial wind are more significant than those in the azimuthal wind (Fig. 2.13c,d).

To explain the impact of rolls, we first consider the sub-grid momentum tendencies in the mean wind equations (2.2)-(2.3) in the initial and final states. In the HBL model, there are two types of sub-grid tendencies: the turbulent (parameterized) and roll-induced (explicitly resolved by the SRM). Hereafter, the sum of the roll-induced and turbulent

momentum tendencies or fluxes will be referred to as the total sub-grid momentum tendencies or fluxes. Fig. 2.14 shows the sub-grid momentum fluxes and tendencies in the initial and final states in M600. Because the angle ε is small, there are minor differences between the cross-roll momentum flux $\overline{w'u'}$ (along-roll momentum flux $\overline{w'v'}$) and the radial momentum flux $\overline{W'U'}$ (azimuthal momentum flux $\overline{W'V'}$).

- i) In the radial direction, the roll-induced and the turbulent momentum fluxes have very similar vertical distributions (Fig. 2.14a). This is mainly because both fluxes depend on the mean wind shear. While the turbulent flux is reduced in the final state, the total sub-grid flux is increased due to the roll-induced flux. Nevertheless, the total radial momentum tendency in the final state is quite similar to that in the initial state (Fig. 2.14c).
- ii) In the azimuthal direction, the roll-induced and turbulent momentum fluxes have very different vertical distributions (Fig. 2.14b). Particularly, near the surface ($z < 0.2$ km), the roll-induced flux and the turbulent flux have opposite vertical trends. As discussed in section 2.5, this difference is due to the fact that the roll-induced along-roll momentum flux does not depend on the local mean wind shear. The turbulent and roll-induced azimuthal momentum tendencies also have different distributions (Fig. 2.14d). While the turbulent tendency is mostly negative, the roll-induced tendency is positive near the surface and negative at upper levels, and the vertically-integrated roll-induced tendency is zero. This indicates the fundamental difference between the turbulent and roll-induced azimuthal momentum tendencies: while the turbulence has a net dissipation effect on the azimuthal momentum, the rolls only have a vertical redistribution effect.

In order to gain a physical insight of how the rolls modify the mean wind, we now consider a linear model describing the basic dynamical balances in the HBL. Following Kepert (2001), the mean wind is decomposed into two components: the gradient wind and departures from the gradient wind, i.e., $\bar{V} = V_g + V_d$ and $\bar{U} = U_d$, where V_g is the gradient wind and (U_d, V_d) are the wind departures. Based on this decomposition, linear equations governing U_d and V_d can be derived (Kepert 2001). Hereafter, we denote (U_d, V_d) in the initial state as (U_0, V_0) and in the final state as (U_1, V_1) . Accordingly, the turbulent diffusivity $\bar{K}$ in the initial state is denoted as K_0 , and in the final state as K_1 . The linear equations for the wind departures in the initial state can be written as

$$-(f + \frac{2V_g}{r})V_0 = \frac{\partial}{\partial z}(K_0 \frac{\partial U_0}{\partial z}), \quad (2.14)$$

$$(f + \frac{V_g}{r} + \frac{\partial V_g}{\partial r})U_0 = \frac{\partial}{\partial z}(K_0 \frac{\partial V_0}{\partial z}). \quad (2.15)$$

The terms on the right-hand-sides (RHS) represent the turbulent momentum tendencies. According to Kepert (2001), (2.14) describes the balance between the radial acceleration due to the gradient wind imbalance and the turbulent radial momentum tendency, and (2.15) describes the balance between the radial advection of the absolute angular momentum (defined as $1/2fr^2 + V_g r$) and the turbulent angular momentum dissipation. Similarly, the linear equations for the wind departures in the final state can be written as

$$-(f + \frac{2V_g}{r})V_1 = \frac{\partial}{\partial z}(K_1 \frac{\partial U_1}{\partial z}) - \frac{\partial \overline{WU'}}{\partial z}, \quad (2.16)$$

$$(f + \frac{V_g}{r} + \frac{\partial V_g}{\partial r})U_1 = \frac{\partial}{\partial z}(K_1 \frac{\partial V_1}{\partial z}) - \frac{\partial \overline{WV'}}{\partial z}. \quad (2.17)$$

Compared with (2.14)-(2.15), the additional terms on the RHS are the roll-induced momentum tendencies, which represent the effect of rolls on the radial momentum (2.16) and the angular momentum (2.17). By subtracting (2.14)-(2.15) from (2.16)-(2.17), we have

$$-(f + \frac{2V_g}{r})\delta V = \delta \text{RHS}_U, \quad (2.18)$$

$$(f + \frac{V_g}{r} + \frac{\partial V_g}{\partial r})\delta U = \delta \text{RHS}_V, \quad (2.19)$$

where δU and δV represent the mean wind changes from the initial state to final state (for example, $\delta U = U_f - U_0$); δRHS_U and δRHS_V represent the differences between the total sub-grid tendencies in the final state and those in the initial state.

To explore whether the linear model (2.18)-(2.19) can be used to interpret the roll-induced mean wind changes, we compare the mean wind changes (δU and δV) estimated based on the linear model with those directly calculated by the HBL model (Fig. 2.13c,d). δRHS_U and δRHS_V used in (2.18)-(2.19) are calculated based on the total sub-grid tendencies the HBL model (as shown in Fig. 2.14), and then δU and δV can be estimated based on (2.18)-(2.19). Fig. 2.15 shows the comparison of the mean wind changes estimated based on the linear model and the actual mean wind changes in M600. The linear model solution indicates that the radial wind change is more significant than the azimuthal wind change, which is in agreement with the HBL model, and it also captures the radial wind change reasonably well. The estimated azimuthal wind change by the linear model, although similar in magnitude, does not match well with that in the HBL

model. This is because the linear model neglects the vertical advection terms, which are important in determining the mean azimuthal wind profiles (Kepert 2001).

Since the linear model successfully captures the key features of the mean wind changes seen in the HBL model, we can use it to elucidate the physical mechanisms by which rolls affect the mean wind profiles. In the azimuthal direction, rolls redistribute the angular momentum vertically (Fig. 2.14d). At lower levels (below ~ 0.3 km), the turbulent angular momentum dissipation is partially balanced by the transport of angular momentum by rolls, and therefore less radial transport of the absolute angular momentum is needed; as a result, the inflow is reduced (Fig. 2.15a). At higher levels (roughly 0.3-0.8 km), additional angular momentum loss is created due to the redistribution effect of rolls, which needs to be compensated by the radial transport of the absolute angular momentum; as a result, the inflow is enhanced (Fig. 2.15a). In the radial direction, the relatively small change of the total sub-grid momentum tendency (Fig. 2.14c) results in the relatively small change of the azimuthal wind.

Finally, we investigate the impact of rolls on the overall HBL mean wind structure based on the results of the additional experiment. We consider the results after a 30-hour integration to ensure the perturbations at all locations reach the quasi-equilibrium state. The mean wind distribution under the impact of rolls is derived by averaging the mean wind profiles during the last 10 hours. In this experiment, rolls are generated from radius = 40 km (RMW) to radius = 110 km (2.75 RMW). At larger radii, rolls are not generated because the inflection point instability is suppressed by the stratification (Gao and Ginis 2014). Fig. 2.16 shows height-radius distribution of the mean radial and azimuthal wind changes induced by rolls. Consistent with the results of the primary experiments, rolls

induce more significant changes in the radial wind. Particularly, rolls weaken the inflow near the surface, and enhance the inflow at upper levels; the net effect is that the height of the inflow layer is increased. Previous modeling studies suggest that the HBL inflow plays an important role in affecting the intensity of the hurricane (e.g., Gopalakrishnan et al. 2013). By altering the HBL inflow magnitude and structure, the rolls may thus contribute to the hurricane intensity changes. To investigate this effect, a more advanced model that considers the entire hurricane is needed.

2.7. Conclusions

In this study, we applied a novel numerical approach to study rolls generated by the inflection point instability in the hurricane boundary layer. Our approach is based on embedding a two-dimensional high-resolution Single-Grid Roll-Resolving Model (SRM) at selected horizontal grid point(s) of the axisymmetric HBL model. The SRM resolves roll motions at selected locations and the roll-induced momentum fluxes are passed to the HBL model. Using this coupled HBL-SRM numerical system, we have investigated 1) the effect the mixed layer height on the characteristics of rolls and the internal waves triggered by rolls, 2) the vertical distributions of the roll-induced momentum fluxes and 3) the effects of rolls on the HBL mean wind. The key findings are summarized as follows.

The mixed layer height is an important environmental factor affecting the characteristics of rolls. Provided that the initial mean wind profiles are the same, rolls can reach a stronger magnitude under a higher mixed layer. Moreover, under a higher mixed layer, rolls can more efficiently trigger vertically propagating internal waves, which can radiate momentum out of the HBL.

We have shown that the vertical distributions of the roll-induced cross-roll and along-roll momentum flux are determined by different mechanisms. While the cross-roll momentum flux is dependent on the cross-roll mean wind shear, the along-roll momentum flux is typically uncorrelated with the along-roll mean wind shear. Therefore, there is no physical basis for applying the K -theory to represent the roll-induced momentum fluxes.

We have demonstrated that rolls can induce more significant changes in the mean radial wind than in the mean azimuthal wind. Specifically, rolls reduce the inflow near surface, increase the inflow at upper levels, and broaden the inflow layer. Based on a linear dynamical HBL model, we have explained the physical mechanism by which rolls affect the HBL mean wind. It is found that rolls affect the mean radial wind by redistributing the angular momentum (or the azimuthal momentum) vertically in the HBL.

References

- Blackadar, A. K., 1962: The vertical distribution of wind and turbulent exchange in a neutral atmosphere. *J. Geophys. Res.*, **67**, 3095–3102.
- Brown, R. A., 1970: A secondary flow model of the planetary boundary layer. *J. Atmos. Sci.*, **27**, 742–757.
- Brown, R. A., 1972: On the inflection point instability of a stratified Ekman boundary layer. *J. Atmos. Sci.*, **29**, 850–859.
- Donelan, M. A., B. K. Haus, N. Reul, W. J. Plant, M. Stiassnie, H. C. Graber, O. B. Brown, and E. S. Saltzman, 2004: On the limiting aerodynamic roughness of the ocean in very strong winds. *Geophys. Res. Lett.*, **31**, L18306, doi:10.1029/2004GL019460.
- Ellis R. and S. Businger, 2010: Helical circulation in the typhoon boundary layer. *J. Geophys. Res.*, **115**, D06205.
- Faller, A. J., 1965: Large Eddies in the Atmospheric Boundary Layer and Their Possible Role in the Formation of Cloud Rows. *J. Atmos. Sci.*, **22**, 176–184.
- Foster, R.C., 2005: Why rolls are prevalent in the hurricane boundary layer. *J. Atmos. Sci.*, **62**, 2647–2661.
- Foster, R.C., 2009: Boundary-layer similarity under an axisymmetric, gradient wind vortex. *Boundary-Layer Meteorol*, **131**, 321–344.
- Foster, R.C., 2013: Signature of large aspect ratio roll vortices in synthetic aperture radar images of tropical cyclones. *Oceanography*, **26(2)**, 58–67.

- Ginis, I., A.P. Khain and E. Morozovsky, 2004: Effects of large eddies on the structure of the marine boundary layer under strong wind conditions, *J. Atmos. Sci.*, **61**, 3049-3064.
- Holland, G. J., 1980: An analytic model of the wind and pressure profiles in hurricanes. *Mon. Wea. Rev.*, **108**, 1212–1218.
- Katsaros, K. B., P. W. Vachon, P. G. Black, P. P. Dodge and E. W. Uhlhorn (2000): Wind fields from SAR: Could they improve our understanding of storm dynamics? *Johns Hopkins APL Technical Digest*, **21**, 86–93.
- Kepert, J. D., 2001: The dynamics of boundary layer jets within the tropical cyclone core. Part I: Linear theory. *J. Atmos. Sci.*, **58**, 2469-2484.
- Kepert, J. D. and Y. Wang, 2001: The dynamics of boundary layer jets within the tropical cyclone core. Part II: Nonlinear enhancement. *J. Atmos. Sci.*, **58**, 2485–2501.
- Kepert, J. D., 2012: Choosing a boundary layer parameterization for tropical cyclone modeling. *Mon. Wea. Rev.*, **140**, 1427–1445.
- Lilly, D., 1966: On the instability of Ekman boundary flow. *J. Atmos. Sci.*, **23**, 481-494.
- Lorsolo, S., J. L. Schroeder, P. Dodge, and F. Marks, 2008: An observational study of hurricane boundary layer small-scale coherent structures. *Mon. Wea. Rev.*, **136**, 2871-2893.
- Miles, J.W., 1961: On the stability of heterogenous shear flows. *J. Fluid Mech.*, **10**, 496-508.

- Moon, I.-J., I. Ginis, T. Hara, and B. Thomas, 2007: A physics-based parameterization of air-sea momentum flux at high wind speeds and its impact on hurricane intensity predictions. *Mon. Wea. Rev.*, **135**, 2869–2878.
- Morrison, I., S. Businger, F. Marks, P. Dodge, and J. Businger, 2005: An observational case for the prevalence of roll vortices in the hurricane boundary layer. *J. Atmos. Sci.*, **62**, 2662–2673.
- Nakanishi, M. and H. Niino, 2012: Large-Eddy Simulation of roll vortices in a hurricane boundary layer. *J. Atmos. Sci.*, **69**, 3558–3575.
- Nolan, D. S., 2005: Instabilities in hurricane-like boundary layers. *Dyn. Atmos. Oceans*, **40**, 209–236.
- Powell, P., J. Vickery, and T. A. Reinhold, 2003: Reduced drag coefficient for high wind speeds in tropical cyclone. *Nature*, **422**, 279–283.
- Sutherland, B. R., C. P. Caulfield and W. R. Peltier, 1994: Internal gravity wave generation and hydrodynamic instability. *J. Atmos. Sci.*, **51**, 3261–3280.
- Wurman, J. and J. Winslow, 1998: Intense sub-kilometer boundary layer rolls in Hurricane Fran. *Science*, **280**, 555–557.
- Zhang J. A., K. B. Katsaros, P. G. Black, S. Lehner, J. R. French and W. M. Drennan, 2008: Effects of roll vortices on turbulent fluxes in the hurricane boundary layer. *Bound.-Layer Meteor.*, **128**, 173–189.
- Zhang, J. A., R. F. Rogers, D. S. Nolan, and F. D. Marks Jr., 2011: On the characteristic height scales of the hurricane boundary layer. *Mon. Wea. Rev.*, **139**, 2523–2535.

Zhang, J. A. and W. M. Drennan, 2012: An Observational Study of Vertical Eddy Diffusivity in the Hurricane Boundary Layer. *J. Atmos. Sci.*, **69**, 3223–3236.

Zhu, P., 2008: Simulation and parameterization of the turbulent transport in the hurricane boundary layer by large eddies, *J. Geophys. Res.*, **113**, D17104.

Table 2.1 Numerical parameters for the HBL model and the SRM.

	HBL Model	SRM
Vertical extent	3 km	6 km
Horizontal extent	1000 km	15.36 km
Vertical grid spacing	30 m	30 m
Horizontal grid spacing	10 km	30 m
Time step	1 s	1 s

Figure 2.1 A diagram illustrating the cylindrical coordinate system (r, λ, z) for the mean flow and the local Cartesian coordinate system (x, y, z) for the perturbations (z axis is not shown) at $r = R$. ε is the angle between the along-roll direction y and the azimuthal direction λ . Positive ε means the along-roll direction is to the left of the azimuthal direction.

Figure 2.2 The gradient wind (solid line) and inertial stability parameter (dashed line) as functions of radius (normalized by the radius of maximum wind).

Figure 2.3 Vertical profiles of the vertical gradient of the virtual potential temperature in the HBL. Color-dashed lines: typical profiles from the composite observational dataset (Zhang et al. 2011). Black-solid line: the profile specified based on (2.10) in the text, with $A = 5.5 \text{ K km}^{-1}$ and $h = 300 \text{ m}$.

Figure 2.4 Vertical profiles of (a) the initial mean virtual potential temperature and (b) its vertical gradient used in the primary set of experiments M200-M600. The heights of the mixed layer in these profiles are indicated in the figure.

Figure 2.5 Vertical profiles of the initial mean (a) radial wind and (b) azimuthal wind at the RMW. The inflection point with the largest radial wind shear is indicated by the triangle in (a).

Figure 2.6 Time evolution of the domain-averaged overturning kinetic energy of the perturbations in experiments M200-M600. The insert shows the kinetic energy at 1-2 hr on a log scale.

Figure 2.7 Time evolution of the overturning kinetic energy budget terms in equation (2.11) in the text: S - shear production, B - buoyancy work, D - turbulent dissipation and P - pressure work. (a) M300 and (b) M600.

Figure 2.8 Typical structure of the perturbations in M300. The colored backgrounds represent (a) cross-roll velocity u' , (b) along-roll velocity v' , (c) vertical velocity w' and (d) virtual potential temperature perturbation θ'_v . The contour lines represent the streamlines: solid (dashed) contours correspond to clockwise (counterclockwise) circulations.

Figure 2.9 Same as Fig. 2.8, but in M600.

Figure 2.10 Vertical profiles of the time-averaged (a) cross-roll momentum flux $\overline{w'u'}$, (b) along-roll momentum flux $\overline{w'v'}$ and (c) virtual potential temperature flux $\overline{w'\theta'_v}$ in the quasi-equilibrium state.

Figure 2.11 Profiles of the effective K calculated based on the time-averaged roll-induced momentum fluxes and the mean wind shear. (a) K_u is for the cross-roll momentum and (b) K_v is for the along-roll momentum.

Figure 2.12 Vertical profiles of the time-averaged (a) mean wind shears ($\partial \bar{u} / \partial z$ multiplied by 1.5 - solid line, $\partial \bar{v} / \partial z$ - dashed line) and (b) roll-induced momentum fluxes ($\overline{w'u'}$ - solid line, $\overline{w'v'}$ - dashed line) in M600.

Figure 2.13 Comparison of the mean wind profiles with and without the effect of rolls at RMW in M200-M600. The mean radial and azimuthal wind profiles under (without) the effect of rolls are shown as solid (dashed) lines in (a) and (b). The corresponding differences of the mean wind profiles are shown in (c) and (d).

Figure 2.14 The sub-grid (with respect to the HBL model) momentum fluxes and tendencies in the initial and final states in M600. The initial state only has the turbulent component (black solid lines). The final state has two components: the turbulent component (black dashed lines) and the roll-induced component (red dashed lines); the total is shown by red solid lines.

Figure 2.15 The estimated mean wind changes based on the linear model (solid lines) and the actual mean wind changes in M600 (dashed lines).

Figure 2.16 Radius-height distribution of (a) radial and (b) azimuthal wind changes induced by rolls in the additional experiment. Contours represent the initial mean radial and azimuthal wind distributions (before rolls are introduced). The contour interval is (a) 2 m s^{-1} (0 m s^{-1} shown in bold) and (b) 5 m s^{-1} (40 m s^{-1} shown in bold).

CHAPTER 3

Effects of Boundary Layer Roll Vortices on the Development of the Axisymmetric Tropical Cyclone

by

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Abstract

In this study, we numerically investigate the effects of roll vortices (rolls) on the development of an axisymmetric tropical cyclone (TC). A two-dimensional high-resolution single-grid roll-resolving model is embedded at multiple horizontal grid points in a three-dimensional full-physics TC model (COAMPS-TC). It is found that the roll-induced wind changes in the TC boundary layer lead to the changes in the structure and intensity of the entire TC. The roll-induced vertical transport of the tangential momentum flux is primarily responsible for these changes. By enhancing the vertical momentum exchange, the rolls trigger a chain of dynamical responses within the TC boundary layer, increasing the mass convergence and inducing a more active deep eyewall convection, which leads to enhanced TC intensity.

3.1. Introduction

The vertical momentum, heat and moisture transports caused by the turbulent motions in the boundary layer (BL) are critical in the development and maintenance of a tropical cyclone (TC). The BL parameterizations used to represent the vertical turbulent fluxes in current TC models are mostly adapted from those developed for low wind conditions. These schemes are typically based on the K -theory and assume that the vertical fluxes induced by the unresolved motions are down-gradient and depend on the vertical gradients of the mean (or resolved) variables. The general form for the parameterized vertical fluxes can be written as $-K_\phi(\partial\phi/\partial z)$, where ϕ represents an arbitrary variable resolved by the TC model and K_ϕ is the turbulent diffusivity for ϕ . A hierarchy of parameterizations for K_ϕ with different levels of complexity has been used in existing TC models and these parameterizations are well summarized by Kepert (2012). Several modeling studies have suggested that the turbulent diffusivity parameterizations can critically affect the TC development and its dynamical structure (e.g., Smith et al. 2010; Gopalakrishnan et al. 2013).

Recent observational, numerical and theoretical studies have conclusively demonstrated that roll vortices (rolls) are prevalent in the TCBL (Katsaros et al. 2000; Morrison et al. 2005; Foster 2005; Ellis and Businger 2010; Nakanishi and Niino 2012; Gao and Ginis 2014, 2015). Rolls consist of overturning circulations in the plane roughly perpendicular to the mean wind direction. When rolls are present, they could significantly contribute to the vertical transports throughout the TCBL. However, the parameterizations based on the K -theory may not properly capture the vertical transports

caused by rolls. Gao and Ginis (2015) found that the vertical distributions of the roll-induced momentum fluxes projected onto the cross-roll and along-roll directions are fundamentally different: while the roll-induced cross-roll momentum flux is dependent on the mean wind shear and down-gradient, the along-roll momentum flux is largely counter-gradient. Therefore, there is no physical basis for applying the K-theory to represent the roll-induced momentum fluxes. Some BL parameterizations additionally consider a “non-local” component by adding a correction (r_ϕ) to the vertical gradient of ϕ (Hong and Pan 1996). In this case, the total flux is written as $-K_\phi(\partial\phi/\partial z - r_\phi)$. The “non-local” flux component is dependent on the surface flux of ϕ and represents the contribution of the large-scale eddies on the total flux of ϕ . However, these “non-local” schemes may not reasonably capture the fluxes induced by rolls because rolls are primarily forced by the shear instability (Gao and Ginis 2014, 2015) in the mid boundary layer and the roll-induced fluxes are not directly dependent on the surface condition.

The roll-induced fluxes are potentially important in affecting the TC development. By including the roll-induced momentum fluxes in an idealized TCBL model, Gao and Ginis (2015) found that rolls affect the magnitude and structure of the radial inflow, which implies that rolls may play an important role in affecting the time evolution of the entire TC. In this study, we aim to investigate the effect of rolls on the structure and intensity of the TC by applying the modeling approach proposed by Gao and Ginis (2014, 2015) in a three-dimensional full-physics TC model.

3.2. Method

3.2.1 Overview of the modeling approach

Our modeling approach is based on embedding a two-dimensional small-scale model (single-grid roll-resolving model, or SRM), which is capable of resolving the structures of rolls, at multiple horizontal grid points in the relative large-scale TC model. The fundamental assumption behind this approach is that the rolls are assumed mathematically separable from the large-scale TC flow because of their small spatial scales (~ 1 km). Based on this assumption, the governing equations for rolls at a single horizontal grid of the large-scale TC model can be derived (Gao and Ginis 2014, 2015). The SRM resolves the roll motions at a particular horizontal grid point of the large-scale TC model and provides the roll-induced vertical fluxes to the TC model.

3.2.2 The Single-Grid Roll-Resolving Model (SRM)

Here we describe some details of the SRM used in this study. The roll motions at a single horizontal grid point in the TC model are described in a local Cartesian coordinate system (x, y, z) , with y axis parallel to the direction in which the rolls are aligned. The x axis is thus referred to as the along-roll (cross-roll) axis. The along-roll variations of the rolls are assumed negligible. Following are the equations describing rolls at an arbitrary horizontal grid point (X_i, Y_j) in the TC model, which are numerically solved by the SRM.

$$\frac{\partial \eta'}{\partial t} + u' \frac{\partial \eta'}{\partial x} + w' \frac{\partial \eta'}{\partial z} = -\bar{u} \frac{\partial \eta'}{\partial x} + w' \frac{\partial^2 \bar{u}}{\partial z^2} + g \frac{\partial}{\partial x} \left(\frac{\theta'}{\theta_0} + 0.61 q' \right) + T_{\eta'} \quad (3.1)$$

$$\eta' = \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial z^2} \quad (3.2)$$

$$\frac{\partial v'}{\partial t} + u' \frac{\partial v'}{\partial x} + w' \frac{\partial v'}{\partial z} = -\bar{u} \frac{\partial v'}{\partial x} - w' \frac{\partial \bar{v}}{\partial z} + T_{v'} \quad (3.3)$$

$$\frac{\partial \theta'}{\partial t} + u' \frac{\partial \theta'}{\partial x} + w' \frac{\partial \theta'}{\partial z} = -\bar{u} \frac{\partial \theta'}{\partial x} - w' \frac{\partial \bar{\theta}}{\partial z} + T_{\theta'} \quad (3.4)$$

$$\frac{\partial q'}{\partial t} + u' \frac{\partial q'}{\partial x} + w' \frac{\partial q'}{\partial z} = -\bar{u} \frac{\partial q'}{\partial x} - w' \frac{\partial \bar{q}}{\partial z} + T_{q'} \quad (3.5)$$

(3.1) - (3.2) describe the overturning circulations of rolls (u' , w') in the x - z plane, where ψ' is the stream function (cross-roll velocity $u' = -\partial \psi' / \partial z$, vertical velocity $w' = \partial \psi' / \partial x$) and η' is the along-roll vorticity ($\eta' = \partial w' / \partial x - \partial u' / \partial z$). (3.3) – (3.5) describe the along-roll wind perturbation v' , the potential temperature perturbation θ' , and the water vapor mixing ratio perturbation q' caused by the overturning circulations of rolls, respectively. Variables with the overbar represent the mean flow variables and they are provided by the TC model at the horizontal grid point (X_i, Y_j): $\bar{u}$ and $\bar{v}$ are the mean winds projected onto the cross-roll (x) and along-roll (y) directions respectively, $\bar{\theta}$ is the mean potential temperature and $\bar{q}$ is the mean water vapor mixing ratio. The mean flow variables ($\bar{u}, \bar{v}, \bar{\theta}, \bar{q}$) are assumed horizontally uniform in the relative small SRM domain. Terms like $T_{\eta'}$ represent the turbulent diffusion terms, which is in the form of $T_{\eta'} = \partial(v_t \partial \eta' / \partial x) / \partial x + \partial(v_t \partial \eta' / \partial z) / \partial z$, where v_t is the turbulent diffusivity for rolls. A scheme commonly used in LES models (Sullivan et al. 1994) is used to parameterize v_t in the SRM, which is in the form of

$$v_t = (C_s \Delta)^2 (2S_{ij}S_{ij} - \frac{g}{\theta_{v0}} \frac{\partial \theta_v}{\partial z})^{1/2},$$

where C_s is the Smagorinsky constant ($C_s = 0.18$); Δ is the grid spacing of the SRM; S_{ij} is the resolved strain rate tensor, given by

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),$$

where u_i and θ_v are the total resolved velocity and virtual potential temperature (e.g., $u_i = \bar{u}_i + u'_i$), respectively.

SRM uses a very fine grid spacing (30 m in both horizontal and vertical) and a small time step (1 s) in order to resolve the small-scale roll motions. The two-dimensional domain of the SRM is 3 km high in the vertical (from 0 to 3 km) and 15.36 km wide in the horizontal, which is large enough to contain at least a few pairs of rolls. Periodic conditions are applied at the lateral boundaries, and no-slip boundary conditions are applied at upper and lower boundaries of the SRM domain.

3.2.3 Implementing the SRM into COAMPS-TC

The three-dimensional TC model used in this study is COAMPS-TC, which is the U.S. Navy's tropical cyclone prediction system (Doyle et al. 2014). The strategy of embedding the SRM mesh into the COAMPS-TC mesh is illustrated in Fig. 3.1. The larger-size mesh with a coarse-resolution represents the mesh in COAMPS-TC, and the smaller-size mesh with a high-resolution represents the mesh in the SRM. In Fig. 3.1, the SRM mesh is embedded at two horizontal grid points (X_1, Y_1) and (X_2, Y_2). In practice, multiple locations can be selected. At each horizontal grid point, the SRM mesh is oriented

perpendicular to the direction of the wind vector averaged within the lowest 1 km layer, that is the along-roll axis is in the direction of the vertically-averaged wind vector.

SRM and COAMPS-TC exchange information at each of the selected horizontal grid point: COAMPS-TC provides SRM with the vertical profiles of the horizontal winds (U , V), potential temperature (Θ) and water vapor mixing ratio (Q) as the mean flow variables for rolls; and SRM provides COAMPS-TC with the roll-induced vertical fluxes of momentum ($\overline{w'u'}$ and $\overline{w'v'}$), potential temperature ($\overline{w'\theta'}$), and water vapor mixing ratio ($\overline{w'q'}$), which are all horizontally averaged in the SRM domain. Considering that COAMPS-TC uses a larger time step than SRM, the roll-induced fluxes are time averaged during one COAMPS-TC time step.

With SRM embedded, the total vertical boundary layer fluxes in COAMPS-TC have two components: the parameterized turbulent fluxes and explicitly resolved roll-induced fluxes, written as

$$-K_\phi \frac{\partial \phi}{\partial z} + \overline{w'\phi'}, \quad (3.6)$$

where K_ϕ is the turbulent diffusivity, and ϕ can represent the winds (U , V), potential temperature (Θ) and water vapor mixing ratio (Q). The first component in (3.6) represents the fluxes induced by the unresolved small-scale turbulent motions and assumed to satisfy the down-gradient assumption; the second component in (3.6) represents the roll-induced fluxes calculated by SRM. There are several options available in COAMPS-TC for parameterizing the turbulent diffusivity K_ϕ . The K_ϕ scheme used in this study is presented in section 3.3.

3.3. Experimental design

For simplicity, we assume that the TC is stationary and axisymmetric on the f -plane (20°N) and over the ocean with constant sea surface temperature (30°C). The horizontal domain is 1000 km wide in both E-W and N-S directions with uniform 5 km grid spacing in the Cartesian coordinates. Periodic boundary conditions are applied at the horizontal boundaries. 40 vertical levels are applied, which extend from 10 m to approximately 32 km with 18 levels below 3 km. The time step is set to 5 s. As for the physical parameterizations, no radiation and cumulus schemes are used for the purpose of this study; the microphysical parameterization is based on Rutledge and Hobbs (1983); the surface layer parameterization follows Wang et al. (2002) and limits the values of the drag coefficient under high wind speeds; the turbulent diffusivity parameterization is based on a 1.5 order scheme (Mellor and Yamada 1982). The turbulent diffusivity is written as

$$K_{m,h} = S_{m,h} l \sqrt{e}, \quad (3.7)$$

where K_m and K_h represent the turbulent diffusivity for momentum and scalars, respectively; $S_{m,h}$ represent the polynomial functions of the flux Richardson number; e is the turbulent kinetic energy [the prognostic equation for e is shown in Hodor (1997)]; l is the mixing length, written as,

$$\frac{1}{l} = \frac{1}{\kappa z} + \frac{1}{\lambda}, \quad (3.8)$$

where λ is the asymptotic mixing length. The value of λ directly affects the magnitude of the turbulent diffusivity.

The COAMPS-TC is first run without SRM to spin up a TC-like vortex. The initial wind field contains a weak axisymmetric cyclonic vortex with a maximum tangential wind speed of $\sim 20 \text{ m s}^{-1}$ at radius = 90 km. The wind speed is vertically uniform from the surface to $z = 10 \text{ km}$, and then decreases sinusoidally to zero at $z = 20 \text{ km}$. The initial unperturbed temperature and the moisture profiles (Hodur 1997) are horizontally uniform, and the initial pressure and potential temperature perturbations are set to be in balance with the initial wind field. In the spin-up run, λ has a form described in Mellor and Yamada (1982), written as

$$\lambda = \alpha \frac{\int z e dz}{\int e dz}$$

and α is a constant.

After a spin-up period of 70 h, the initial weak vortex is developed into a typical TC structure (Fig. 3.2). Because the TC is stationary and nearly axisymmetric, we will focus on the azimuthally averaged fields. Hereafter $\langle \rangle$ denotes the azimuthally averaged variable. By the end of the spinup, the TC has the maximum tangential wind ($\langle V \rangle$) of 37 m s^{-1} at radius = 25 km and $z \sim 600 \text{ m}$ (Fig. 3.2). It has a shallow radial inflow (negative $\langle U \rangle$) layer below $z = 1 \text{ km}$ and a radial outflow at the upper levels $z = 12\text{-}14 \text{ km}$. The deep eyewall convection is characterized by strong upward vertical motion ($\langle W \rangle$) and large adiabatic heating rate ($\langle H \rangle$) in the vicinity of radius = 25 km.

The spunup TC is used as the initial condition for the experiments with the effects of rolls and the elapsed time is reset to 0 h. A summary of these experiments is provided in Table 3.1. In all experiments, except CTRL, the SRM is embedded at selected horizontal grid points. Since the fluxes caused by the large eddies (rolls) in the TCBL are explicitly

resolved and only the fluxes caused by the small-scale turbulences are parameterized, we use a relatively small constant value (40 m) for λ in (3.8). All other COAMPS-TC configurations are the same as in the spin-up run. Experiments ROLL and CTRL are designed to demonstrate how rolls affect the structure and intensity of the TC. The only difference between the two experiments is that ROLL has SRM embedded into COAMPS-TC, but CTRL does not. Fig. 3.3 shows the horizontal grid points where SRM is embedded in experiment ROLL.

Two groups of sensitivity experiments are also conducted. In Group A we investigate which components of the roll-induced fluxes are most important in affecting the development of the TC. In each experiment, only one component of the roll-induced fluxes is provided to COAMPS-TC. In ROLL-U, for example, at every horizontal grid point only the roll-induced radial momentum flux is passed to COAMPS-TC. In Group B we investigate the effects of rolls formed at different radius ranges on the development of the TC. In each of these experiments, SRM is only embedded at the horizontal grid points within a 5 km range of the specified radius. In ROLL-R20, for example, SRM is only embedded at the horizontal grid points within the radius range of 17.5-22.5 km.

3.4. Characteristics of rolls

This section focuses on the characteristics of rolls in experiment ROLL. Fig. 3.4 shows a series of consecutive snapshots of the roll-induced cross-roll momentum flux ($\overline{w'u'}$) at $z = 300$ m. Each pixel represents a single horizontal grid point in the COAMPS-TC domain. The grid points where rolls are generated have non-zero fluxes. Rolls are

first formed at some horizontal grid points at the outer (~50 km) and inner (~20 km) radii, and with time they are formed at more and more locations.

In order to understand their formation mechanism we consider the cross-roll averaged equation for the overturning kinetic energy of rolls (the derivations is given in appendix B),

$$\frac{\partial \bar{e}'}{\partial t} = -\overline{w'u'} \frac{\partial \bar{u}}{\partial z} + \frac{g}{\theta_{v0}} \overline{w'\theta'_v} + \bar{d} - \frac{\partial \overline{w'e'}}{\partial z} - \frac{\partial \overline{w'p'}}{\partial z},$$

where the overturning kinetic energy e' is defined as $e' = 0.5(u'^2 + w'^2)$. The terms on the right-hand-side represent: (i) cross-roll mean shear production, (ii) buoyancy work, (iii) dissipation, (iv) vertical advection and (v) pressure redistribution, respectively. The shear production and buoyancy work terms are directly affected by the mean flow. Fig. 3.5 shows the Hovmöller diagrams of the azimuthally-averaged cross-roll mean shear ($\langle \partial \bar{u} / \partial z \rangle$) and the roll-induced cross-roll momentum flux ($\langle \overline{w'u'} \rangle$) at 25, 35 and 45 km radii. The black lines represent the heights of the inflection point where $\langle \partial \bar{u} / \partial z \rangle$ reaches the maximum value. At all selected locations, $\langle \overline{w'u'} \rangle$ negatively correlates with $\langle \partial \bar{u} / \partial z \rangle$, and $\langle \overline{w'u'} \rangle$ reaches maximum magnitude in the vicinity of the inflection point. Such distribution of $\langle \overline{w'u'} \rangle$ is an important signature of the rolls generated by the inflection point instability, as discussed in Gao and Ginis (2014). Fig. 3.6 shows the vertical profiles of the cross-roll mean shear production terms at 25, 35, and 45 km radii, as well as the buoyancy work terms, which are azimuthally and time (6-7 h) averaged. The shear production terms are clearly the dominant source terms for the overturning kinetic energy of rolls at these radii. The buoyancy work terms have mostly negative

contributions. Therefore, the rolls are generated by the inflection point instability of the TCBL flow as discussed by Foster (2005) and Gao and Ginis (2014, 2015). As in Gao and Ginis (2014 and 2015), both the mean wind and the stratification in the TCBL affect the formation of rolls: the cross-roll mean wind provides the kinetic energy source, while the mean stratification has mostly a suppressing effect.

The rolls at different locations have similar structures. Fig. 3.7 shows the typical structures of rolls resolved by the SRM. The structures are also similar to those derived by Gao and Ginis (2015) in the idealized TCBL model and Foster's analytical solutions (2005). The rolls are roughly aligned in the local azimuthal direction. The angle between the along-roll axis and the local azimuthal axis is generally smaller than 15° . The spatial scales and velocity magnitudes vary with time and location, mostly because the mean flow that determines the characteristics of rolls changes. The horizontal wavelengths of rolls (defined as the distance between two nearby w' peaks) are in the range 2-5 km, the maximum along-roll wind perturbations are in the range 5-10 m s^{-1} , and the maximum vertical velocities are in the range 1-3 m s^{-1} . These values are consistent with the observations (Katsaros et al. 2000; Morrison et al. 2005; Ellis and Businger 2010).

Fig. 3.8 shows the vertical profiles of the azimuthally-averaged roll-induced vertical fluxes at radius = 25 km, as well as the parameterized turbulent fluxes for comparison. The profiles of the roll-induced radial and tangential momentum fluxes are consistent with those in Gao and Ginis (2015). The shapes of the vertical distributions of roll-induced radial momentum flux and the turbulent radial momentum flux are similar below $z = 1$ km. This is because for the rolls generated by the inflection point instability the cross-roll momentum flux, which is approximately the radial momentum flux, is

negatively correlated with the mean wind shear (Fig. 3.5). On the contrary, the vertical distributions of the roll-induced tangential momentum flux and the turbulent tangential momentum flux are very different. This is because the roll-induced along-roll momentum flux, which is approximately the tangential momentum flux, is not dependent on the mean wind shear, as discussed in Gao and Ginis (2015). The roll-induced heat and moisture fluxes have much stronger magnitudes than the corresponding parameterized turbulent fluxes, suggesting that rolls tend to enhance the vertical mixing of heat and moisture in the TCBL. The effect of these roll-induced fluxes on the development of the TC is discussed next.

3.5. Effect of rolls on the TC structure and intensity

To investigate the effect of rolls on the TC structure and intensity we compare experiments ROLL and CTRL. Fig. 3.9 shows the cross-sections of the differences, hereafter referred to as the “changes”, in the azimuthally-averaged wind fields averaged during three sequential 1h time periods. During the 4-5 h time period (Fig. 3.9a-c), the most significant radial and tangential wind changes are limited to the lower levels (roughly $z < 4$ km), suggesting that rolls first trigger the changes in the TCBL. During the 5-6 h time period (Fig. 3.9d-e), the magnitudes of the wind changes are increased, and more wind changes appear at upper levels (roughly $z > 4$ km). The TC in ROLL has a stronger TCBL inflow ($z < 2$ km) and upper level outflow ($z = 12-16$ km), a stronger tangential wind speed above $z = 4$ km, and a stronger eyewall updraft (entire vertical column). During the 6-7 h time period (Fig. 3.9g-i), the wind changes are similar to those during 5-6 h, but their magnitudes are further increased. The time evolution of the wind changes shown in Fig. 3.9 suggest that rolls first affect the winds in the TCBL and then

result in a more intense TC.

Before we proceed to explore how rolls contribute to the TC intensity, it is necessary to assess the relative contribution of rolls at different radii, and identify which component(s) of the roll-induced fluxes is (are) affecting the TC intensity the most. We use the maximum $\langle V \rangle$ at $z = 5$ km (far away from the TCBL) to represent of the overall TC intensity. Fig. 3.10 shows the time series of the maximum $\langle V \rangle$ at $z = 5$ km in group A and B experiments, as well as in CTRL and ROLL, for comparison. Interestingly, only the TC in ROLL-V experiment has a similar intensification rate as the TC in ROLL experiment, while the TC intensification rates in other group A experiments are similar to that in CTRL experiment (Fig. 3.10a). This suggests that the roll-induced tangential momentum flux is primarily responsible for the enhanced TC intensification. All the TCs in group B experiments have stronger intensification rates than the TC in CTRL, indicating that rolls at different radii (within the range considered in this study) all have positive contributions to the TC intensity (Fig. 3.10b). The TCs in the group B experiments are weaker than the TC in ROLL, suggesting the enhanced TC intensity in ROLL is very likely due to the combined contributions of rolls at different radii.

3.6. Dynamical interpretation

In this section, we analyze the physical mechanism through which rolls affect the TC structure and intensity. Specifically, we focus on (a) how the roll-induced vertical fluxes, particularly the tangential momentum flux, affect the TCBL and (b) how the roll-induced changes in the TCBL affect the entire TC. Our analysis is based on ROLL-R45 because

the rolls in this experiment are generated in a limited radius range, and therefore it is easier to identify their local and nonlocal impacts.

3.6.1 Effect of rolls in the TCBL

To gain a physical insight on the impacts of roll-induced momentum fluxes in the TCBL, we analyze the azimuthally-averaged momentum budgets, which are written as follows (the horizontal diffusion terms are weak and therefore neglected).

$$\frac{d\langle U \rangle}{dt} = - \underbrace{\left\langle \frac{1}{\rho} \frac{\partial P}{\partial r} \right\rangle + \left(f + \frac{\langle V \rangle}{r} \right) \langle V \rangle}_{\text{IMB}} - \underbrace{\left\langle \frac{\partial \overline{W'U'}}{\partial z} \right\rangle + \left\langle \frac{\partial}{\partial z} \left(K_m \frac{\partial U}{\partial z} \right) \right\rangle}_{\text{UBL}} \quad (3.9)$$

$$\frac{\partial \langle V \rangle}{\partial t} = - \underbrace{\left(f + \frac{\langle V \rangle}{r} + \frac{\partial \langle V \rangle}{\partial r} \right) \langle U \rangle}_{\text{ANG}} - \underbrace{\left\langle \frac{\partial \overline{W'V'}}{\partial z} \right\rangle + \left\langle \frac{\partial}{\partial z} \left(K_m \frac{\partial V}{\partial z} \right) \right\rangle}_{\text{VBL}} - \langle W \rangle \frac{\partial \langle V \rangle}{\partial z} \quad (3.10)$$

The physical meaning of the terms in (3.9) is (from left to right): the net acceleration in the radial direction (that is the material derivative of $\langle U \rangle$); IMB – the gradient wind imbalance, which is the imbalance between the pressure gradient force and the sum of Coriolis and centrifugal forces; and UBL – the total sub-grid momentum tendencies in the radial direction, which consists of the roll-induced tendency (hereafter $\langle R_U \rangle$) and the turbulent tendency (hereafter $\langle T_U \rangle$). The physical meaning of the terms in (3.10) is (from left to right): the local time-changing rate of $\langle V \rangle$; ANG – the acceleration due to the radial advection of the absolute angular momentum (defined as $1/2 fr^2 + Vr$); VBL – the total sub-grid momentum tendencies in the tangential direction, which consists of the roll-induced tendency (hereafter $\langle R_V \rangle$) and the turbulent tendency (hereafter $\langle T_V \rangle$); and the vertical advection effect.

We first consider the sub-grid momentum tendencies in ROLL-R45. Fig. 3.11 shows the Hovmöller diagrams of the azimuthally-averaged sub-grid momentum tendencies at radius = 45 km. $\langle R_v \rangle$ is positive at lower levels and negative at upper levels, suggesting that rolls tends to redistribute vertically the tangential momentum (or angular momentum). Particularly, at the lower levels ($z < 0.4$ km), $\langle R_v \rangle$ has opposite sign to $\langle T_v \rangle$. The reason for this is that the roll-induced tangential momentum flux (approximately the along-roll momentum flux) does not depend on the tangential wind shear, which is essentially different from the parameterized turbulent tangential momentum flux (Gao and Ginis 2015). $\langle R_v \rangle$ has a much larger magnitude than $\langle R_u \rangle$, which explains why the roll-induced tangential momentum flux is more important than the radial momentum flux in affecting the TC intensity.

To further explore how the roll-induced tangential momentum tendency $\langle R_v \rangle$ affects the TCBL dynamical structures, we derive the differences between the momentum budget terms in (3.9) and (3.10) in ROLL-R45 and CTRL. The vertical distribution and time evolution of these differences, denoted by δ , are shown in Fig. 3.12, which help reveal the chain of responses in the TCBL to $\langle R_v \rangle$ at the location where rolls are generated.

- i) The vertical distribution of δVBL suggests that rolls redistribute the tangential momentum vertically, and thus have a direct effect on the local $\langle V \rangle$ profile. Rolls tend to increase $\langle V \rangle$ at lower levels and decrease $\langle V \rangle$ at the upper levels [see equation (3.10)].
- ii) The vertical distribution of δIMB reflects the effects of rolls on the local $\langle V \rangle$ profile: δIMB is positive near surface where rolls tend to increase $\langle V \rangle$, and δIMB is negative at upper levels where rolls tend to decrease $\langle V \rangle$.

iii) The distribution of $\delta d\langle U \rangle / dt$ is similar to δIMB , suggesting that the change in IMB directly affects the net radial acceleration, and therefore affects the local $\langle U \rangle$ profile.

iv) The changes in $\langle U \rangle$ and $\langle V \rangle$ result in the change in ANG term (note negative δANG). Under the impact of rolls, the local wind profiles adjust to a state in which δANG is nearly in balance with δVBL .

Fig. 3.13 summarizes schematically how $\langle R_v \rangle$ affects the local TCBL dynamical structures. It should be noted that even though the four steps are well separated in the diagram, they occur simultaneously in the TC simulation.

The effect of rolls on the TCBL wind is not limited to the location where the rolls are generated. Fig. 3.14 shows the height-radius distribution of the radial ($\delta \langle U \rangle$) and tangential ($\delta \langle V \rangle$) wind changes (ROLL-R45 – CTRL), averaged during 3-4 h. While the roll-induced momentum fluxes only exist in the vicinity of radius = 45 km in ROLL-R45, the wind changes are spread in the radial and vertical directions. During this time period, there are no changes in the upper-level vortex structure above 3 km (not shown). Thus the observed wind changes are only due to the TCBL local response to the roll-induced momentum fluxes and the advection by the TC mean wind circulation.

3.6.2 Effect of rolls on the deep eyewall convection

We now explore how the wind changes in TCBL induced by rolls contribute to the intensity of the entire TC. Kepert and Nolan (2014) pointed out that the mass convergence in the TCBL is important in localizing the deep convection and affecting its intensity. Here we hypothesize that rolls affect the entire TC through a similar mechanism: the roll-induced radial wind changes in the TCBL enhance the deep eyewall

convection by enhancing the mass convergence in the TCBL. The enhanced deep eyewall convection therefore contributes to the intensity of the entire TC. The following analysis aims to validate above hypothesis by considering the two main aspects of the deep convection: the upward vertical velocity ($\langle W \rangle$) and the diabatic heating rate ($\langle H \rangle$) associated with the thermo-dynamical processes.

We first demonstrate that the change in eyewall updraft ($\delta \langle W \rangle$) is primarily caused by the change of TCBL mass convergence triggered by rolls. The vertical velocity in COAMPS-TC is calculated as a function of the buoyancy of the air and some other factors, but is not calculated directly based on the mass convergence. To highlight the contribution of the TCBL mass convergence on the eyewall updraft, we diagnose the vertical velocity at the lower levels of the TC ($z < 3$ km) based on the following formula:

$$\langle W \rangle_e = \int_0^z \left(-\frac{1}{r} \frac{\partial \langle U \rangle r}{\partial r} \right) dz, \quad (3.11)$$

where $\langle W \rangle_e$ represents the diagnosed vertical velocity. According to (3.11), $\langle W \rangle_e$ is only determined by the mass convergence, which only depends on the distribution of the radial wind. $\langle W \rangle_e$ in ROLL-R45 and CTRL can be diagnosed based on the lower-level $\langle U \rangle$ distributions in these experiments, and $\delta \langle W \rangle_e$ (ROLL-R45 – CTRL) can then be derived.

Fig. 3.15 shows the cross-sections of $\delta \langle W \rangle_e$ and $\delta \langle W \rangle$, averaged during 3-4 h (same period as in Fig. 3.14). The black contours represent $\langle W \rangle_e$ (Fig. 3.15a) and $\langle W \rangle$ (Fig. 3.15b) in CTRL, and the bold black contours roughly mark the boundary of the eyewall updraft. The distribution and magnitude of $\delta \langle W \rangle_e$ diagnosed based on the mass convergence is very similar to $\delta \langle W \rangle$ calculated directly from the COAMPS-TC

simulation. The magnitude of $\langle W \rangle_e$ is slightly weaker than $\langle W \rangle$, which is likely because the buoyancy associated with latent heat release in the deep convection also contributes to $\langle W \rangle$, which is unaccounted in (3.11). The good agreement between $\delta \langle W \rangle_e$ and $\delta \langle W \rangle$ confirms that radial wind change triggered by rolls can indeed contribute to the eyewall updraft through affecting the TCBL mass convergence. Fig. 3.15c shows the change in the diabatic heating rate ($\delta \langle H \rangle$) calculated from the COAMPS-TC simulations (ROLL-R45 – CTRL). $\delta \langle H \rangle$ has a similar distribution as $\delta \langle W \rangle_e$ and $\delta \langle W \rangle$, suggesting that the change in the latent heat release is due to the change of updraft and the associated change of moist air supply to the condensation levels.

Fig. 3.16 shows the time evolution of $\delta \langle W \rangle_e$, $\delta \langle W \rangle$ and $\delta \langle H \rangle$ (ROLL-R45 – CTRL), vertically averaged within the lowest 3 km. The contours represent the vertically-averaged $\langle W \rangle_e$, $\langle W \rangle$ and $\langle H \rangle$ in CTRL, and the bold contours roughly mark the radial boundaries of the deep eyewall convection. Fig. 3.16b,c demonstrates that the deep eyewall convection in ROLL-R45 is more active than in CTRL. The time evolutions of $\delta \langle W \rangle_e$ and $\delta \langle W \rangle$ are similar, confirming that the change in the TCBL mass convergence induced by rolls is largely responsible for the change in the eyewall updraft. The magnitude of $\delta \langle W \rangle_e$ is apparently weaker than $\delta \langle W \rangle$ after time = 6 h, which is very likely because the change in the latent heat release also contributes to $\delta \langle W \rangle$.

The more active eyewall convection in ROLL-R45 is responsible for a stronger TC. The same mechanism is responsible for the stronger TCs in other group B experiments. The effects of rolls at all radii contribute to the strongest TC in ROLL (Fig. 3.10b). To illustrate this, Fig. 3.17 shows Hovmöller diagrams of $\delta \langle W \rangle_e$, $\delta \langle W \rangle$ and $\delta \langle H \rangle$ in

ROLL. Before ~ 4.5 h, rolls are mostly generated at the grid points near radius = 45 km, similar to ROLL-R45, and therefore the magnitudes of $\delta\langle W \rangle_e$, $\delta\langle W \rangle$ and $\delta\langle H \rangle$ in Fig. 3.17 are very similar to those in Fig. 3.16. At later times, rolls forming at other locations (Fig. 3.4) additionally contribute to the mass convergence in the TCBL (therefore $\delta\langle W \rangle_e$ in Fig. 3.17 is stronger than in Fig. 3.16), and further enhance the deep eyewall convection (therefore $\delta\langle W \rangle$ and $\delta\langle H \rangle$ in Fig. 3.17 are stronger than in Fig. 3.16).

3.7. Conclusions

In this study, we investigated the effects of boundary layer roll vortices (rolls) on the structure and intensity of the tropical cyclone (TC) by applying the modeling approach developed in Gao and Ginis (2014, 2015), in which the small-scale roll motions are explicitly resolved. In this approach, a two-dimensional high-resolution single-grid roll-resolving model (SRM) is embedded at multiple horizontal grid points in a three-dimensional full-physics TC model (COAMPS-TC). The numerical experiments focused on the development of a stationary and axisymmetric TC, in which the SRM was embedded within the inner core region of the TC. The rolls simulated in this study are mostly generated by the inflection point instability and gained their kinetic energy from the mean wind shear. We find that the roll-induced wind changes in the boundary layer lead to an increase of the TC intensity. The rolls affect the TC intensity mainly via the induced vertical transport of the tangential momentum. We have also identified the dynamical mechanism through which rolls contribute to the structure and intensity of the entire TC. By enhancing the vertical tangential momentum exchange, the rolls trigger a chain of dynamical responses in the boundary layer, increasing the mass convergence and inducing a more active deep convection, which leads to the enhanced TC intensity.

References

- Doyle, J.D., R.M. Hodur, S. Chen, Y. Jin, J.R. Moskaitis, S. Wang, E.A. Hendricks, H. Jin, and T.A. Smith, 2014: Tropical cyclone prediction using COAMPS-TC. *Oceanography*, **27**, 104-115.
- Ellis R. and S. Businger, 2010: Helical circulation in the typhoon boundary layer. *J. Geophys. Res.*, **115**, D06205.
- Foster, R.C., 2005: Why rolls are prevalent in the hurricane boundary layer. *J. Atmos. Sci.*, **62**, 2647-2661.
- Foster, R.C., 2013: Signature of large aspect ratio roll vortices in synthetic aperture radar images of tropical cyclones. *Oceanography*, **26**, 58-67.
- Gao, K. and I. Ginis, 2014: On the generation of roll vortices due the inflection point instability of the hurricane boundary layer flow. *J. Atmos. Sci.*, **71**, 4292-4307.
- Gao, K. and I. Ginis, 2015: On the equilibrium-state roll vortices and their effects in the hurricane boundary layer. *J. Atmos. Sci.*, under review.
- Gopalakrishnan, S.G., F. Marks, J.A. Zhang, X. Zhang, J.-W. Bao, and V. Tallapragada, 2013: A study of the impacts of vertical diffusion on the structure and intensity of tropical cyclones using the high resolution HWRF system. *J. Atmos. Sci.*, **70**, 524-541.
- Grabowski, W., 2004: An Improved Framework for Superparameterization. *J. Atmos. Sci.*, **61**, 1940-1952.
- Hodur, R., 1997: The Naval Research Laboratory's Coupled Ocean/Atmosphere Mesoscale Prediction System (COAMPS). *Mon. Wea. Rev.*, **125**, 1414-1430.

- Katsaros, K. B., P. W. Vachon, P. G. Black, P. P. Dodge and E. W. Uhlhorn (2000): Wind fields from SAR: Could they improve our understanding of storm dynamics? *Johns Hopkins APL Technical Digest*, **21**, 86–93.
- Kepert, J. D., 2010: Tropical cyclone structure and dynamics. Global Perspectives on Tropical Cyclones – From Science to Mitigation. *World Scientific Publishing*, 3-53.
- Kepert, J. D., 2012: Choosing a boundary layer parameterization for tropical cyclone modeling. *Mon. Wea. Rev.*, **140**, 1427–1445.
- Kepert, J.D. and D.S. Nolan, 2014: Reply to “Comments on ‘How Does the Boundary Layer Contribute to Eyewall Replacement Cycles in Axisymmetric Tropical Cyclones?’”. *J. Atmos. Sci.*, **71**, 4692-4704.
- Lorsolo, S., J. L. Schroeder, P. Dodge, and F. Marks, 2008: An observational study of hurricane boundary layer small-scale coherent structures. *Mon. Wea. Rev.*, **136**, 2871-2893.
- Rutledge, S. A., and P. V. Hobbs, 1983: The Mesoscale and Microscale Structure and Organization of Clouds and Precipitation in Midlatitude Cyclones. VIII: A Model for the “Seeder-Feeder” Process in Warm-Frontal Rainbands. *J. Atmos. Sci.*, **40**, 1185-1206.
- Smith, R. K. and G. L. Thomsen, 2010: Dependence of tropical-cyclone intensification on the boundary layer representation in a numerical model. *Quart. J. Roy Met. Soc.*, **136**, 1671-1685.
- Sullivan, P. P., J. C. McWilliams, and C.-H. Moeng, 1994: A subgrid-scale model for large-eddy simulation of planetary boundary-layer flows. *Boundary Layer Meteorol.*, **71**, 247-276.

- Mellor, G. L., and T. Yamada, 1982: Development of a turbulence closure model for geophysical fluid problems, *Rev. Geophys.*, **20**, 851-875.
- Morrison, I., S. Businger, F. Marks, P. Dodge, and J. Businger, 2005: An observational case for the prevalence of roll vortices in the hurricane boundary layer. *J. Atmos. Sci.*, **62**, 2662-2673.
- Nakanishi, M. and H. Niino, 2012: Large-Eddy Simulation of roll vortices in a hurricane boundary layer. *J. Atmos. Sci.*, **69**, 3558–3575.
- Nolan, D. S., 2005: Instabilities in hurricane-like boundary layers. *Dyn. Atmos. Oceans*, **40**, 209-236.
- Wang, S., Q. Wang, and J. Doyle, 2002: Some improvement of Louis surface flux parameterization. Preprints, *15th Symp. on Boundary Layers and Turbulence*, Wageningen, Netherlands, Amer. Meteor. Soc., 547-550.
- Wang, Y., 2012: Recent research progress on tropical cyclone structure and intensity. *Tropical cyclone Res. Rev.*, **1**, 254-275.
- Wurman, J. and J. Winslow, 1998: Intense sub-kilometer boundary layer rolls in Hurricane Fran. *Science*, **280**, 555-557.
- Zhang J. A., K. B. Katsaros, P. G. Black, S. Lehner, J. R. French and W. M. Drennan, 2008: Effects of roll vortices on turbulent fluxes in the hurricane boundary layer. *Bound.-Layer Meteor.*, **128**, 173–189.
- Zhu, P., 2008: Simulation and parameterization of the turbulent transport in the hurricane boundary layer by large eddies. *J. Geophys. Res.*, **113**, D17104.

Table 3.1 A summary of the numerical experiments performed in this study.

	Experiment	Description
	CTRL	Control experiment without the effects of rolls.
	ROLL	SRM embedded at the horizontal grid points in the radius range 15–50 km.
Group A	ROLL-U	Only the roll-induced radial momentum flux is applied to the TC.
	ROLL-V	Only the roll-induced tangential momentum flux is applied to the TC.
	ROLL- Θ	Only the roll-induced potential temperature flux is applied to the TC.
	ROLL-Q	Only the roll-induced water vapor mixing ratio flux is applied to the TC.
Group B	ROLL-R20	SRM embedded at the grid points in the radius range 17.5–22.5 km.
	ROLL-R25	SRM embedded at the grid points in the radius range 22.5–27.5 km.
	ROLL-R30	SRM embedded at the grid points in the radius range 27.5–32.5 km.
	ROLL-R35	SRM embedded at the grid points in the radius range 32.5–37.5 km.
	ROLL-R40	SRM embedded at the grid points in the radius range 37.5–42.5 km.
	ROLL-R45	SRM embedded at the grid points in the radius range 42.5–47.5 km.

Figure 3.1 Schematic diagram illustrating how SRM is embedded at selected horizontal grid points in the COAMPS-TC domain. The larger-size mesh with relative coarse resolution represents the mesh for COAMPS-TC, and the smaller-size mesh with relative high resolution represents the mesh for the SRM. In this diagram, the SRM meshes are embedded at two horizontal grid points in the COAMPS-TC mesh, labeled as (X_1, Y_1) and (X_2, Y_2) .

Figure 3.2 The azimuthally-averaged cross-sections of the TC after the 70 h spin-up: (a) radial wind (m s⁻¹), (b) tangential wind (m s⁻¹), (c) vertical wind (m s⁻¹), (d) potential temperature (K), (e) water vapor mixing ratio (g kg⁻¹) and (f) the diabatic heating rate (K s⁻¹).

Figure 3.3 A map showing the horizontal grid points where SRM is embedded in experiment ROLL. The background color represents the tangential wind at 3 km height. + represents the horizontal grid point where SRM is embedded. The two dashed contours represent radius = 15 km and 50 km, respectively.

Figure 3.4 The horizontal distributions of the roll-induced cross-roll momentum flux ($\text{m}^2 \text{s}^{-2}$) at (a) 4 h, (b) 5 h, (c) 6 h, (d) 7 h in experiment ROLL. The height is 300 m. Each pixel resents a single horizontal grid point in the COAMPS-TC domain. The horizontal grid points where rolls are formed have non-zero flux.

Figure 3.5 Upper-panel: Hovmöller diagrams of the azimuthally-averaged (a) cross-roll mean wind shear and (b) roll-induced cross-roll momentum flux at radius = 25 km. The black lines show the height of the inflection point in the cross-roll mean wind profile. Middle-panel: As in (a)-(b), respectively, but for radius = 35 km. Lower-panel: As in (a)-(b), respectively, but for radius = 45 km.

Figure 3.6 Vertical distributions of the azimuthally-averaged kinetic energy budget terms for rolls at three radii. The solid lines represent the cross-roll mean shear production and the dashed lines represent the buoyancy work. Different colors represent different radii (indicated in the figure).

Figure 3.7 Representative structures of the rolls resolved by the SRM. The rolls shown are at the horizontal grid point corresponding to $(X, Y) = (25, 0)$ km in Fig. 3.4 at time = 6 h. The colored backgrounds represent (a) cross-roll velocity, (b) along-roll velocity, (c) potential temperature perturbation and (d) water vapor mixing ratio perturbation, respectively. Contours in (a)-(d) represent the vertical velocity: solid (dashed) contours represent upward (downward) velocities; the dash-dot contours represent 0 m s^{-1} ; the contour interval is 0.2 m s^{-1} .

Figure 3.8 Representative vertical profiles of the roll-induced fluxes (red) and the parameterized turbulent fluxes (black): (a) radial momentum flux, (b) tangential momentum flux, (c) potential temperature flux and (d) the water vapor mixing ratio flux. These profiles are time (6-7 h) and azimuthally averaged, and at radius = 25 km.

Figure 3.9 Upper-panel: Height-radius distributions of the changes (ROLL – CTRL) in the azimuthally-averaged (a) radial (b) tangential and (c) vertical wind fields, time averaged during 4-5 h. Middle-panel: As in (a)-(c), but time averaged during 5-6 h. Lower-panel: As in (a)-(c), but time averaged during 6-7 h. The contours represent the azimuthally-averaged TC wind fields in experiment CTRL: in (a), (d) and (g), the solid contour is 10 m s^{-1} and the dashed contour is -15 m s^{-1} ; in (b), (e) and (h), the contours are 15, 25, 35 and 45 m s^{-1} , respectively; in (c), (f) and (i), the contours are 1 and 2 m s^{-1} , respectively.

Figure 3.10 (a) Time series of the maximum azimuthally-averaged tangential wind at $z = 5$ km in the group A experiments. (b) As in (a), but for the group B experiments.

Figure 3.11 Hovmöller diagrams of the azimuthally-averaged sub-grid momentum budget terms in experiment ROLL-R45 at radius = 45 km: (a) roll-induced radial momentum tendency, (b) turbulent radial momentum tendency, (c) roll-induced tangential momentum tendency and (d) turbulent tangential momentum tendency.

Figure 3.12 Hovmöller diagrams of the changes (ROLL-R45 – CTRL) in (a) the net radial acceleration term, (b) the gradient wind imbalance term, (c) the radial advection of absolute angular momentum term (with a negative sign) and (d) the total sub-grid tangential momentum tendency term.

Figure 3.13 Schematic diagram illustrating the chain of responses in the TCBL to the roll-induced tangential momentum tendency at the location where rolls are generated (see the text for explanation).

Figure 3.14 Height-radius distributions of the changes (ROLL-R45 – CTRL) in azimuthally-averaged (a) radial and (b) tangential winds, time averaged during 3–4 h. The contours in (a) represent $\langle U \rangle$ in CTRL (bold contour – 0 m s⁻¹; dashed contours – inflow; solid contours – outflow; contour interval – 5 m s⁻¹) and the contours in (b) represent $\langle V \rangle$ in CTRL (bold contour – 40 m s⁻¹; contour interval – 10 m s⁻¹).

Figure 3.15 Height-radius distributions of the changes (ROLL-R45 – CTRL) in the azimuthally- and time- (3-4 h) averaged (a) vertical velocity diagnosed based on mass convergence (b) actual vertical velocity from COAMPS-TC and (c) diabatic heating rate. The contours represent (a) $\langle W \rangle_e$, (b) $\langle W \rangle$ and (c) $\langle H \rangle$ in experiment CTRL: in (a) and (b) the bold contour is 0.1 m s^{-1} and the contour interval is 0.5 m s^{-1} ; in (c) the bold contour is 0.01 K s^{-1} and the contour interval is 0.03 K s^{-1} .

Figure 3.16 Hovmöller diagrams of the changes (ROLL-R45 – CTRL) in azimuthally- and vertically- (within the lowest 3 km) averaged (a) vertical velocity diagnosed based on mass convergence (b) actual vertical velocity from COAMPS-TC and (c) diabatic heating rate. The contours represent (a) $\langle W \rangle_e$, (b) $\langle W \rangle$ and (c) $\langle H \rangle$ in experiment CTRL: in (a) and (b) the bold contour is 0.1 m s^{-1} and the contour interval is 0.5 m s^{-1} ; in (c) the bold contour is 0.01 K s^{-1} and the contour interval is 0.03 K s^{-1} .

Figure 3.17 As in Fig. 3.16, but for ROLL – CTRL.

APPENDIX A

Equations for the Mean HBL Flow and Roll Vortices

The governing equations for the dry atmosphere in the hurricane boundary layer under Boussinesq assumption are

$$\frac{\partial \bar{v}}{\partial t} + \bar{v} \cdot \nabla \bar{v} + f \hat{k} \times \bar{v} = -\frac{1}{\rho_0} \nabla p + \frac{\theta'}{\theta_0} \bar{g} + K_m \nabla^2 \bar{v}, \quad (\text{A1})$$

$$\nabla \cdot \bar{v} = 0, \text{ and} \quad (\text{A2})$$

$$\frac{\partial \theta}{\partial t} + \bar{v} \cdot \nabla \theta = K_h \nabla^2 \theta, \quad (\text{A3})$$

where $\bar{v}$ is the velocity vector, θ is the potential temperature. $K_m \nabla^2 \bar{v}$ and $K_h \nabla^2 \theta$ in the above equations represent mixing effect of small-scale turbulences in the boundary layer, where K_m and K_h are turbulent viscosity and diffusivity respectively and need to be parameterized. Based on the flow-separation assumption described in section 2, the governing equations for the mean flow (denoted by the overbar sign) and rolls (denotes by the prime sign) can be derived from (A1)-(A3). These equations are presented below.

a. Equations for the mean flow

We assume the large-scale hurricane flow is stationary and axisymmetric. The governing equations for the mean flow in the cylindrical coordinates (r, λ, z) are

$$\frac{\partial \bar{U}}{\partial t} + \bar{U} \frac{\partial \bar{U}}{\partial r} + \bar{W} \frac{\partial \bar{U}}{\partial z} - \frac{\bar{V}^2}{r} - f \bar{V} = -\frac{1}{\rho_0} \frac{\partial \bar{P}}{\partial r} - \frac{\partial \bar{W}' U'}{\partial z} + \bar{K}_m \frac{\partial^2 \bar{U}}{\partial z^2}, \quad (\text{A4})$$

$$\frac{\partial \bar{V}}{\partial t} + \bar{U} \frac{\partial \bar{V}}{\partial r} + \bar{W} \frac{\partial \bar{V}}{\partial z} + \frac{\bar{U} \bar{V}}{r} + f \bar{U} = -\frac{\partial \bar{W}' V'}{\partial z} + \bar{K}_m \frac{\partial^2 \bar{V}}{\partial z^2}, \quad (\text{A5})$$

$$\frac{\partial \bar{U}}{\partial r} + \frac{\bar{U}}{r} + \frac{\partial \bar{W}}{\partial z} = 0, \text{ and} \quad (\text{A6})$$

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{U} \frac{\partial \bar{\theta}}{\partial r} + \bar{W} \frac{\partial \bar{\theta}}{\partial z} = -\frac{\partial \overline{W'\theta'}}{\partial z} + \bar{K}_h \frac{\partial^2 \bar{\theta}}{\partial z^2}, \quad (\text{A7})$$

where $\bar{U}$, $\bar{V}$ and $\bar{W}$ are the mean wind components in the radial, azimuthal and vertical directions respectively. Terms like $\overline{W'U'}$ are the horizontally averaged fluxes induced by rolls.

b. Equations for rolls

The perturbation equations in the cylindrical coordinates are

$$\frac{\partial U'}{\partial t} + U' \frac{\partial U'}{\partial r} + \frac{V'}{r} \frac{\partial U'}{\partial \lambda} + W' \frac{\partial U'}{\partial z} = -\bar{U} \frac{\partial U'}{\partial r} - \frac{\bar{V}}{r} \frac{\partial U'}{\partial \lambda} - W' \frac{\partial \bar{U}}{\partial z} + \frac{2\bar{V}}{r} V' + fV' - \frac{1}{\rho_0} \frac{\partial p'}{\partial r} + K_m \nabla_r^2 U',$$

$$\frac{\partial V'}{\partial t} + U' \frac{\partial V'}{\partial r} + \frac{V'}{r} \frac{\partial V'}{\partial \lambda} + W' \frac{\partial V'}{\partial z} = -\bar{U} \frac{\partial V'}{\partial r} - \frac{\bar{V}}{r} \frac{\partial V'}{\partial \lambda} - W' \frac{\partial \bar{V}}{\partial z} - \frac{\bar{V}}{r} U' - \frac{\bar{U}}{r} V' - fU' - \frac{1}{r\rho_0} \frac{\partial p'}{\partial \lambda} + K_m \nabla_r^2 V',$$

$$\frac{\partial W'}{\partial t} + U' \frac{\partial W'}{\partial r} + \frac{V'}{r} \frac{\partial W'}{\partial \lambda} + W' \frac{\partial W'}{\partial z} = -\bar{U} \frac{\partial W'}{\partial r} - \frac{\bar{V}}{r} \frac{\partial W'}{\partial \lambda} - \frac{1}{\rho_0} \frac{\partial p'}{\partial z} + g \frac{\theta'}{\theta_0} + K_m \nabla_r^2 W',$$

$$\frac{\partial U'}{\partial r} + \frac{U'}{r} + \frac{1}{r} \frac{\partial V'}{\partial \lambda} + \frac{\partial W'}{\partial z} = 0, \text{ and}$$

$$\frac{\partial \theta'}{\partial t} + U' \frac{\partial \theta'}{\partial r} + \frac{V'}{r} \frac{\partial \theta'}{\partial \lambda} + W' \frac{\partial \theta'}{\partial z} = -\bar{U} \frac{\partial \theta'}{\partial r} - \frac{\bar{V}}{r} \frac{\partial \theta'}{\partial \lambda} - W' \frac{\partial \bar{\theta}}{\partial z} + K_h \nabla_r^2 \theta',$$

where $\nabla_r^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \lambda^2} + \frac{\partial^2}{\partial z^2}$, U' , V' and W' are the wind perturbations in the

radial, azimuthal and vertical directions respectively.

Similar to Foster (2005), we use a local Cartesian coordinate system (x, y, z) instead of the cylindrical coordinates at large radius (e.g. radius larger than the radius of maximum wind). We assume rolls are aligned along the y axis and the along-roll variations are negligible. The governing equations for rolls in the local Cartesian coordinates at radius $r = R$ are

$$\frac{\partial u'}{\partial t} + u' \frac{\partial u'}{\partial x} + w' \frac{\partial u'}{\partial z} = -\bar{u} \frac{\partial u'}{\partial x} - w' \frac{\partial \bar{u}}{\partial z} - \frac{1}{\rho_0} \frac{\partial p'}{\partial x} + \frac{1}{R} [(\bar{v} \sin \varepsilon) u' + (2\bar{v} \cos \varepsilon + \bar{u} \sin \varepsilon) v'] + f v' + K_m \nabla^2 u', \quad (\text{A8})$$

$$\frac{\partial v'}{\partial t} + u' \frac{\partial v'}{\partial x} + w' \frac{\partial v'}{\partial z} = -\bar{u} \frac{\partial v'}{\partial x} - w' \frac{\partial \bar{v}}{\partial z} - \frac{1}{R} [(2\bar{u} \sin \varepsilon + \bar{v} \cos \varepsilon)u' + (\bar{u} \cos \varepsilon)v'] - fu' + K_m \nabla^2 v', \quad (\text{A9})$$

$$\frac{\partial w'}{\partial t} + u' \frac{\partial w'}{\partial x} + w' \frac{\partial w'}{\partial z} = -\bar{u} \frac{\partial w'}{\partial x} - \frac{1}{\rho_0} \frac{\partial p'}{\partial z} + g \frac{\theta'}{\theta_0} + K_m \nabla^2 w', \quad (\text{A10})$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0, \text{ and} \quad (\text{A11})$$

$$\frac{\partial \theta'}{\partial t} + u' \frac{\partial \theta'}{\partial x} + w' \frac{\partial \theta'}{\partial z} = -\bar{u} \frac{\partial \theta'}{\partial x} - w' \frac{\partial \bar{\theta}}{\partial z} + K_h \nabla^2 \theta', \quad (\text{A12})$$

where $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial z^2$, (u', v', w') are the roll velocities in the local Cartesian coordinates, $\bar{u}$ and $\bar{v}$ are the mean winds in the x and y directions respectively.

Next we introduce the along-roll vorticity η' and the stream function ψ' to simplify the roll equations. The along-roll vorticity η' ($\eta' = \partial w' / \partial x - \partial u' / \partial z$) is related to stream function ψ' ($u' = -\partial \psi' / \partial z$ and $w' = \partial \psi' / \partial x$) by the Poisson's equation:

$$\eta' = \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial \psi'}{\partial z^2}. \quad (\text{A13})$$

The equation for η' can be derived from (A13) and (A15), which is

$$\frac{\partial \eta'}{\partial t} + u' \frac{\partial \eta'}{\partial x} + w' \frac{\partial \eta'}{\partial z} = -\bar{u} \frac{\partial \eta'}{\partial x} + w' \frac{\partial^2 \bar{u}}{\partial z^2} + \frac{g}{\theta_0} \frac{\partial \theta'}{\partial x} - \frac{1}{R} \frac{\partial [(\bar{v} \sin \varepsilon)u' + (2\bar{v} \cos \varepsilon + \bar{u} \sin \varepsilon)v']}{\partial z} - f \frac{\partial v'}{\partial z} + K_m \nabla^2 \eta'. \quad (\text{A14})$$

The new set of equations for rolls are given by (A9), (A12), (A13) and (A14).

The numerical methods applied for solving the mean flow and roll equations are described in Ginis et al. (2004).

APPENDIX B

Overturning Kinetic Energy Budget Equation for the Perturbations

Here we show the derivations of the overturning kinetic energy budget equation for the perturbations. Without introducing the stream function and the along-roll vorticity, the equations governing the overturning circulation (u' , w') of the perturbations are

$$\frac{\partial u'}{\partial t} + u' \frac{\partial u'}{\partial x} + w' \frac{\partial u'}{\partial z} = -\bar{u} \frac{\partial u'}{\partial x} - w' \frac{\partial \bar{u}}{\partial z} - \frac{\partial p'}{\partial x} + T_{u'} , \quad (\text{B1})$$

$$\frac{\partial w'}{\partial t} + u' \frac{\partial w'}{\partial x} + w' \frac{\partial w'}{\partial z} = -\bar{u} \frac{\partial w'}{\partial x} - \frac{\partial p'}{\partial z} + \frac{g}{\theta_{v0}} \theta'_v + T_{w'} , \quad (\text{B2})$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 . \quad (\text{B3})$$

$T_{u'}$ and $T_{w'}$ represent the turbulent diffusions. Take $T_{u'}$ for example, it is in the form of

$T_{u'} = \partial(K \partial u' / \partial x) / \partial x + \partial(K \partial w' / \partial z) / \partial z$, where K is the parameterized turbulent diffusivity.

(B1)-(B3) are equivalent to (2.6)-(2.7) in the main text.

By combining (B1) multiplied by u' and (B2) multiplied by w' , and considering (B3), we can derive the governing equation for the overturning kinetic energy e' , which is given by

$$\frac{\partial e'}{\partial t} + \frac{\partial u' e'}{\partial x} + \frac{\partial w' e'}{\partial z} = -\frac{\partial \bar{u} e'}{\partial x} - \frac{\partial u' p'}{\partial x} - \frac{\partial w' p'}{\partial z} - w' u' \frac{\partial \bar{u}}{\partial z} + \frac{g}{\theta_{v0}} w' \theta'_v + d , \quad (\text{B4})$$

where

$$e' = \frac{1}{2} (u'^2 + w'^2)$$

and

$$d = \frac{\partial}{\partial x} \left(K \frac{\partial u'}{\partial x} \right) u' + \frac{\partial}{\partial z} \left(K \frac{\partial u'}{\partial z} \right) u' + \frac{\partial}{\partial x} \left(K \frac{\partial w'}{\partial x} \right) w' + \frac{\partial}{\partial z} \left(K \frac{\partial w'}{\partial z} \right) w' .$$

By averaging (B4) in the x (cross-roll) direction, we can rule out the terms with the x derivative and get

$$\frac{\partial \bar{e}'}{\partial t} + \frac{\partial \bar{w}'e'}{\partial z} = -\frac{\partial \bar{w}'p'}{\partial z} - \bar{w}'u' \frac{\partial \bar{u}}{\partial z} + \frac{g}{\theta_{v0}} \bar{w}'\theta'_v + \bar{d} . \quad (\text{B5})$$

In this study, the additional Rayleigh damping is applied to damp out the internal waves above $z = 3$ km, and therefore the perturbations in these levels will not be considered. By averaging (B5) vertically over the range 0-3 km, and considering $\bar{w}'p'_{z=0} = \bar{w}'e'_{z=0} = 0$, we eventually get

$$\frac{d \langle \bar{e}' \rangle}{dt} = \langle -\bar{w}'u' \frac{\partial \bar{u}}{\partial z} \rangle + \langle \frac{g}{\theta_{v0}} \bar{w}'\theta'_v \rangle + \langle \bar{d} \rangle - \frac{1}{H} \bar{w}'p'_{z=H} - \frac{1}{H} \bar{w}'e'_{z=H} , \quad (\text{B6})$$

where $H = 3$ km.