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MODELING THE FOCUS OF ATTENTION AS A RANDOM PROCESS

Staci A. Wasserman
University of Rhode Island, staci@my.uri.edu

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MODELING THE FOCUS OF ATTENTION AS A
RANDOM PROCESS

BY

STACI A. WASSERMAN

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
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OF

STACI A. WASSERMAN

APPROVED:

Thesis Committee:

Major Professor Manbir Sodhi

David Taggart

Godi Fischer

Nasser H. Zawia

DEAN OF THE GRADUATE SCHOOL

UNIVERSITY OF RHODE ISLAND

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ABSTRACT

This research investigates the feasibility of modeling visual attention (as represented through eye movements) as a stochastic process. A stochastic model of attention would provide a foundation for research involving probabilistic predictions of attention allocation which could be used in a variety of domains. The following hypotheses are examined as part of this research.

1. *The visual trace of participants when asked to fixate on a single point can be modeled as a stochastic process (Supported).*
2. *The visual trace of participants will fluctuate when performing an additional cognitive task, but can still be modeled stochastically with additional parameter considerations (Supported).*

In order to determine whether attention could be modeled as a stochastic process, eye-tracking data were collected and analyzed. The experiment contains only a single focal point with no distractors. The goal of this experiment is to determine how the eyes move when attention is singularly focused. This experiment does not attempt to determine how attention is captured or distracted, but rather to understand the foundational elements of attention that can be ascertained from the inherent movement of the eyes.

To determine whether the data could be modeled as a stochastic process, different tests are used to compare the empirical cumulative distribution function to the hypothesized theoretical distribution. It was hypothesized that the saccade occurrences follow a Poisson process, but only 46% of the 52 runs provided support that the data

could be modeled as a Poisson process. There was no significant difference between the control and n-back runs. Overall, there is not enough evidence to support that the saccades follow a Poisson process. The Wiener process and random walk are hypothesized to relate to the gaze pattern or visual trace. For the Wiener process, the length of movement in the horizontal and vertical directions was assessed for normality. The hypothesis that the data followed a Wiener process was supported by 100% of the 53 runs in both the horizontal and vertical directions. Thus, this data was able to be modeled as a stochastic process, specifically a Wiener process, which supported hypothesis 1.

This analysis was extended to consider whether the distribution changed as the differences in position increased to two samples, three samples, four samples, five samples, and ten samples. As the number of time samples between eye position difference calculations increased, the results still strongly supported that the data followed a normal distribution. However, the variance proportions did not increase as expected by a Wiener process. This suggests that as the distance between time samples increases, at some point, the differences in position will no longer follow a Wiener process. The additional parameter assessment comparison for the Wiener process showed that in both the horizontal and vertical directions, the variances of the distributions differed significantly between the control and n-back runs. The variances for the n-back runs were consistently larger than those for the control runs. This provides support to hypothesis 2.

Additional analyses regarding whether the gaze path followed a random walk were executed. The distribution of the angle or direction of movement was analyzed.

In comparison to a uniform distribution, when saccades were removed, 60% of the 53 runs failed to reject the null hypothesis. For the data with saccades, only 21% of the 53 runs failed to reject the null hypothesis. Thus, there was a significant difference in the distributions of the angle of eye movement between the data with and without saccades. When saccades were removed, there is more support for the gaze path following a random walk, compared to when saccades are included in the assessment.

This research and resulting conclusions are important in starting to explain the involuntary eye movements which occur when a participant is singularly focused. These results provide strong evidence that this underlying movement can be represented as a Wiener process. For any experiment considering eye movements, the inherent movement of the eyes should be considered in the analysis.

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I would like to dedicate this work to my mom, who always inspired me to pursue my goals and encouraged me to believe in myself. Without her, I would not be the person I am today. I would like to acknowledge my advisor, Manbir Sodhi, and thank him for his assistance and guidance. I would also like to thank Meghan Steinhaus for writing the C Sharp program for the eye tracker data collection.

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CHAPTER 1

INTRODUCTION

Attention is the focusing of sensory, motor, and/or mental resources on aspects of the environment in order to acquire knowledge [1]. It is required for the “conscious perception of any object” [2]. Two types of movements guide attention: endogenous movements, which are driven by the goals and intentions of the observer (i.e., top-down or goal-directed attention), and exogenous movements, which are driven by stimulus properties in the visual environment (i.e., bottom-up or stimulus-driven attention) [1, 3, and 4]. In the attention literature, eye movement data are used as an indication of the focus of attention. Two types of eye movements typically examined in relation to attention are saccades and fixations. Saccades are the fast movements that redirect the eye to a new part of the surroundings, and fixations are intervals between saccades in which gaze is held almost stationary and visual information is taken into the visual system [5].

This research investigates the feasibility of modeling visual attention (as represented through eye movements) as a stochastic process. For the modeling effort, the Poisson process, the Wiener process, and the random walk are considered. It is hypothesized that the saccade occurrences can be modeled as a Poisson process, and the visual trace or gaze path can be characterized by the Wiener process or a random walk. In addition, this research explores the effects of additional cognitive activity on visual trace. The following hypotheses are examined.

1. *The visual trace of participants when asked to fixate on a single point can be modeled as a stochastic process.*
2. *The visual trace of participants will fluctuate when performing an additional cognitive task, but can still be modeled stochastically with additional parameter considerations.*

In order to determine whether attention can be modeled as a stochastic process, eye-tracking data were collected and analyzed. The data included horizontal and vertical eye position within the experimental field and pupil size. For this study, a limited number of gaze paths were utilized to support the analysis. The experiment contains only a single focal point with no distractors. The goal of this experiment is to determine how the eyes move when attention is singularly focused. This research does not attempt to determine how attention is captured or distracted, but rather to understand the foundational elements of attention that can be ascertained from eye movement data.

Participants were solicited from the University of Rhode Island (URI) student population. The study took place in the Eye Lab in Gilbreth Hall at URI utilizing the ISCAN software available. A C-Sharp program displayed a single black dot in the center of a screen, and the participants were asked to look at the dot for two minutes, during two distinct types of trials. Each participant repeated each of the two types of trials three times (total of 6 runs = 12 minutes). On one trial, participants performed an additional task requiring cognitive processing, i.e., the n-back task. In the n-back task, a series of verbal stimuli (e.g., a series of numbers) is presented and the participant is asked to indicate when the currently presented stimulus is the same as the one

presented n trials previously [6]. The other trial simply asked that participants focus on the black dot. Once the data was collected through the eye tracker software and C-Sharp program, it was analyzed to support assessment of the hypothesis that eye movements, when directed to focus on a single point, can be modeled as a random process. The inherent properties of eye movements are examined without distractors or additional required tasks (i.e., visual search for a specified object within the scene). This is important in determining how the eyes move involuntarily while singularly focused. The purpose of including a cognitive task is to determine its effect on the focus of visual attention.

Chapter 2 provides background on research regarding attention, details of various attention experiments, and theories and models of attention described in the current literature. In addition, an overview of random variables, random processes, and stochastic modeling is included in Chapter 2. Chapter 3 details the experiment including the experimental setup and execution. Chapter 4 describes the data analysis, modeling efforts, and resulting model of the data. Chapter 5 contains conclusions and recommendations for future research.

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CHAPTER 2

LITERATURE REVIEW

2.1. Overview of Attention Literature

The research of attention is important to many fields of study, and thus, the current literature extends to a variety of areas, including: biological and cognitive aspects of attention [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]; visual search and detection [4, 7, 12, 13, 14, 15]; attention allocation and control [1, 4, 7, 10, 12, 13, 14, 16, 17]; limitations of attention [1, 6, 8, 10, 11, 12, 17, 18, 19]; a variety of attention experiments [10, 11, 12, 15, 17, 20, 21, 22, 23]; and theories and models of attention [13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29].

Throughout the literature, eye position is used as an indirect measurement of where attention is currently deployed in space. This eye position is then translated into two primary classes of eye movements, fixations and saccades. Fixations are a type of eye movement in which the gaze is held almost stationary and visual information is obtained [3]. Saccades are fast movements (typically defined as movements greater than $300^{\circ}/\text{sec}$) that redirect the eye to a new part of the surroundings [3]. In much of the literature, the emphasis of the experiments, analysis, and modeling is based on fixations since they represent where a person's attention is focused. In this research, the focus is on saccades. Since, in addition to playing an important role in redirecting the eye to a new position, saccades may provide further insight into the underlying nature of eye movements.

2.2. Attention Experiments

Most of the current literature seeks to utilize attention experiments to describe to what extent attention is controlled by top-down (driven by observer's goals and intentions) versus bottom-up guidance (driven by external stimuli). Two types of experiments typically used to describe attention allocation and guidance include: capture of attention experiments and experiments that require visual search and detection amongst distractors. Capture of attention experiments often seek to determine to which specific stimuli (e.g., abrupt onset, specific colors, etc.) the visual system is particularly sensitive. These experiments help to explain the extent to which attention is allocated utilizing bottom-up guidance and the strength of specific stimuli to capture attention.

Visual search experiments address top-down attention by requiring the participant to orient attention and detect a particular stimulus based on pre-specified goals. Orienting is the process of aligning attention with a source of sensory input or an internal semantic structure stored in memory [4]. Detection of a stimulus means that the stimulus has reached a level of the nervous system at which it is now possible for the subject to report its presence [4]. These search experiments address bottom-up guidance, in addition to the goal-directed search guided by top-down mechanisms, by examining the difficulty of detecting the target amongst different combinations of distractors.

Egeth and Yantis (1997) reviewed various experiments related to attentional control and attentional capture. Attentional control describes the extent to which deployment of attention is a result of an individual's deliberate state of attentional readiness (i.e., goal-directed control) or is captured by specific aspects of the image

(i.e., stimulus-driven control) [10]. Their paper focuses on two types of external stimuli, feature singletons and abrupt visual onsets, to determine the extent to which each captures attention. By exploring attentional control in addition to capture, the paper explores not only the effect of these stimuli on bottom-up guidance, but also the extent to which top-down guidance influences the ability of a specific stimulus to capture one's attention.

Feature singletons are stimuli that differ substantially from their background in one or more visual attributes (e.g., color, orientation) [10]. Egeth and Yantis comment on the amount of conflicting evidence in the literature regarding whether singletons capture attention (see Pashler (1988), Theeuwes (1991a), and Joseph and Optican (1996) for studies supporting that singletons capture attention; and Jonides and Yantis (1988) and Theeuwes (1990), and Hillstrom and Yantis (1994) for examples of studies that conclude that singletons do not capture attention) [10]. In an attempt to reconcile the conflicting conclusions regarding feature singletons and attentional capture, Bacon and Egeth (1994) suggest that these results are the manifestations of two different attentional strategies adopted by participants [10]. Under some circumstances, subjects direct attention to the location exhibiting the largest local feature contrast as they search for the feature singleton [10]. In other stimulus conditions, subjects instead direct attention to the locations that match some task-defined visual feature (e.g., locations of blue items since the task is to identify a blue circle) [10]. This suggests that based upon a participant's goals or intentions, a participant may be able to resist or minimize the effect of a feature singleton on attentional capture.

With abrupt visual onsets, there is a consensus that abrupt onsets capture attention. The extent to which attention is captured may depend on a number of variables. For example, Jonides (1981) showed that peripheral cues always drew attention (whether or not they were informative about the target location), while central cues only captured attention when they were informative [10]. Yantis and Jonides (1990) found that capture, which occurs in the absence of any relevant attentional set, is prevented when subjects are induced to focus attention on a different spatial location in advance of each trial [10]. This suggests that an individual can potentially resist or reduce the effects of abrupt visual onsets capturing attention, if attention is focused in advance.

Mack *et al.* (2002) describe a series of experiments that explore the power of specific stimuli to capture attention when an inattentive state is produced. One inattentive state utilized was the attentional blink. In order to investigate whether an attentional blink occurred, a rapid serial presentation of alphanumeric characters was presented to an observer [11]. The observer was asked to report two consecutively presented characters designated as targets. An attentional blink occurs when the observer fails to detect the second target. It is called an attentional blink because the failure is attributed to the demands placed on attentional processing imposed by the first target which makes the processes for detecting the second target temporarily unavailable [11]. The experiments produced evidence that a complex, familiar, and meaningful stimuli (i.e., one's own name) is able to capture attention and be perceived under a variety of conditions in which other stimuli are not. Mack *et al.* concluded that under conditions of inattention, it is the meaning of the stimulus which is ascertained

or processed without attention that captures attention and subsequently brings that stimulus into awareness [11].

Posner *et al.* (1980) examined the relationship of orienting and detecting in the task of reporting the presence of a visual signal by reviewing experiments from the literature as well as executing their own experiments to validate or extend upon those conclusions [12]. The objective of one experiment was to compare detection latencies when a stimulus location was cued on each trial (mixed block) to a non-cued situation in which subjects prepared for one location for a block of trials (pure blocks) [12]. Each stimulus trial consisted of a visual warning signal, a stimulus (LED), the subject's response, feedback based on the response, and an inter-trial interval. For the mixed block trials, 80% of the trials contained a digit (1, 2, 3, or 4) as a warning signal [12]. The digit indicated the most probable location at which the subject could expect the stimulus to occur [12]. The remaining 20% included a warning signal of a plus sign, indicating that the four locations were equally likely [12]. For the pure blocks, the warning signal was always a plus sign followed by an equal (each of the four locations is equally probable) or unequal block condition [12]. In the unequal condition, participants were informed of the most likely stimulus location at the beginning of each unequal block [12]. When subjects were cued on each trial, they showed stronger expectancy effects than when a probable position was held constant for a block, indicating the active nature of the expectancy [12]. Knowledge regarding the location of the stimulus produced benefits when it was used actively (cued) but not when it was used to maintain a general set (blocked) [12].

Another experimental task described in Posner *et al.* (1980) involved a measure of reaction time to the onset of a stimulus LED. Again, warning signals of either a plus sign or a digit (1, 2, 3, or 4) were presented to indicate likely stimulus locations [12]. The plus sign indicated that each position was equally likely to contain the stimulus [12]. The digit indicated the most likely position during that block, while the next most likely position remained constant for three consecutive blocks [12]. Subjects were asked to remember these positions throughout the block and try to prepare accordingly [12]. Statistical analysis showed that for the most likely target positions, subjects exhibited significantly faster reaction times [12]. When the second most likely position was adjacent to the most likely position, the reaction time resembled that of the most likely target position [12]. However, when it was separated by a position from the most likely target position, the reaction time resembled that of the least likely position [12]. The results suggest that it is not possible for subjects to split their attentional mechanism between two positions separated in space when trying to detect a target [12]. Furthermore, the results indicate severe limits in the ability of subjects to assign attention to a secondary focus in addition to a primary focus [12]. The authors conclude that there is no evidence of an ability to divide attention [12].

The conclusions of Posner *et al.*, regarding the ability to divide attention, dealt with an experiment that was solely based on visual stimuli. In this research, attention is again divided, but between a visual focus and a secondary aural task. One goal of this research is to determine how the model of attention changes when participants are asked to perform this secondary task. These changes in the focus of attention while

performing a secondary task may provide insight into the extent of one's ability to effectively divide attention between a visual focus and a secondary aural task.

2.3. Attention Theories and Models

The models in the current literature focus primarily on attentional capture, visual search, and detection. Certain models rely solely on saliency-based, bottom-up attention [23, 25, 27, 28, 29], while others incorporate both bottom-up, image-based saliency cues and top-down, task-dependent cues [13, 14, 16, 22, 24]. Absent from the literature are models which focus on only top-down guidance, without search or distractors, which is the focus of this research. Since the participants are presented with a singular visual task (focus on the single black dot on the screen) with no other images on the screen, only top-down attention is utilized. Auditory stimuli are also presented during the n-back task, but the stimuli consist of a steady stream of numbers with no abrupt changes or onsets to draw attention away from the screen.

“For the last decade the attention literature has been embroiled in a debate over the nature of visual spatial attention that focuses on the ‘thing’ that attention selects;” many have attempted to determine whether attention selects regions of space (space-based attention) or specific objects within a region (object-based attention) [13]. The Contour Detector (CODE) Theory of Visual Attention (TVA), CTVA, integrates space-based and object-based approaches to attention by merging the CODE theory of perceptual grouping by proximity with TVA [13]. CTVA's representation of space and objects derives from the CODE theory of perceptual grouping [13]. CODE theory utilizes two representations of space, one which represents the locations of items and the other representing objects and groups of objects [13]. CODE assumes that the

location of each item is represented by its own distribution, and the CODE surface is formed by bottom-up processes from the summation of those distributions [13]. By applying a threshold set by top-down processes to the CODE surface, items residing in the same above-threshold region are categorized into perceptual groups [13]. When applying the CODE theory to attention, the CODE surface can be viewed as distributions of item features where the height represents the probability of sampling those features [13]. CODE provides a sampling of visual features as an input to TVA, which then chooses among categorizations of perceptual inputs to select where attention will be allocated [13].

Pomplun *et al.* (2003) developed the Area Activation Model, which is a computational model that predicts the statistical distribution of saccadic endpoints in visual search tasks [15]. The distribution is based on the assumption that saccades in visual search tend to foveate the display areas that provide the maximum amount of task-relevant information for the subsequent fixation. Navalpakkam and Itti (2005) describe another computational model for task-specific guidance of visual attention. Their model is based on a biologically motivated architecture which emphasizes the aspects important to biological vision [16]. The model is divided into four important steps in guiding visual attention in real-world scenes. The model first determines the task-relevance of an entity. Then, the model biases attention for the low-level visual features indicative of the desired targets. The model uses these low-level features to recognize targets. Throughout the process, the model incrementally builds a visual map of the task-relevance at every scene location and stores this information within the memory of the model.

Rutishauser and Koch (2007) developed a generative model that reproduces eye movements during a visual search task. It calculates the conditional probabilities that, given a specified target, observers fixate on or near an item sharing a specific feature with that target [22]. The model consists of three modules: (1) feature extraction and representation of the visual scene, (2) saccade planning, and (3) target detection [22]. The probabilities generated by the model are used to infer which visual features were biased by top-down attention [22].

Itti and Koch (2011) established a combined model of attentional selection and object recognition [24]. This model is based on a framework that subjects selectively direct attention to objects within a scene using both bottom-up, image-based saliency cues and top-down, task-dependent cues [24]. The model uses a bottom-up feature extraction pathway to select informative image regions from an incoming visual scene [24]. A trained knowledge base hierarchically represents object classes and encodes each object within an object class [24]. This encoding includes both expected visual features and a set of critical points on these objects, which the model then uses for object recognition [24].

Torralla (2003) describes a contextual cueing model for attentional guidance based on global scene configuration [25]. This model utilizes statistical correlations between global scene structure and object properties to facilitate search in complex scenes [25]. In addition, it provides estimates of the likelihood of finding an object in a given scene and its most likely position [25]. In Rimey and Brown (1990), the focus of the model is attention allocation [26]. They consider an augmented hidden Markov model in which allocation of visual processing is controlled preferentially within a

scene, leaving open just the resources being managed [26]. As in the attention experiments, the models focus on aspects of attention such as visual search, detection of features, and attention allocation. The model considered in this research focuses on the inherent elements of eye movements when attention is singularly focused. The papers listed in the bibliography contain additional information about attentional theories and models.

2.4. Random Variables and Probability Distributions

A random variable, X , is a function which maps the sample space, S , into a subset of numbers on the real line [30]. This subset of numbers forms a new sample space S_X , which for a discrete random variable includes a finite or countably infinite set of points, $X(s_i) = x_i$ for $i = 1, 2, \dots$ [30]. To characterize the properties of a random variable, a probability mass function (for discrete random variables) or a cumulative distribution function (CDF) can be used. These functions summarize the probabilities associated with the random variable assuming certain values and contain all the information available about a random variable before its value is determined by an experiment [31]. The probability mass function (PMF) is defined in [32] as:

$$p_X[x_i] = P[X(s) = x_i]$$

For a discrete random variable, the probability mass function can be equivalently defined as the discrete probability density function (PDF) as in Hoel (1984) by:

$$f(x) = P[X = x]$$

For a continuous random variable, the probability density function contains the same information as that contained in a probability mass function for a discrete random variable. The probability density function is a function, $f(x)$, which possesses

the following properties: (1) $f(x) \geq 0$, (2) $\int_{-\infty}^{\infty} f(x)dx = 1$, and (3) $\int_a^b f(x)dx = P(a < X < b)$ where a and b are any two values of x satisfying $a < b$ [33]. The cumulative distribution function represents the probability that X takes on a value less than or equal to x , and is defined as:

$$F_X(x) = P(X \leq x), -\infty < x < \infty$$

The cumulative distribution functions for discrete and continuous random variables are defined in terms of the probability mass or density functions as follows:

$$F_X(x) = \sum_{t \leq x} f(t), \text{ if } X \text{ is discrete}$$

$$F_X(x) = \int_{-\infty}^x f(t)dt, \text{ if } X \text{ is continuous}$$

Often when it is clear to which random variable the distribution function applies, the subscript is removed, and thus, $F_X(x)$ becomes $F(x)$. These two terms can be viewed as interchangeable throughout this paper.

This section describes a few of the most important distributions in order to illustrate some of the statistical concepts discussed. The first distribution, which is perhaps the easiest to understand and represent, is the uniform distribution. In the uniform distribution, within the range of possible outcomes, each outcome is equally likely to occur. Its distribution functions are defined as follows:

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$$

The normal distribution, or Gaussian distribution, is probably the most important distribution due to its wide applicability throughout a variety of domains. It is defined by two parameters, a location parameter, μ , and a scale parameter, σ . The location parameter represents the center or mean of the ‘bell-shaped’ curve of the distribution, whereas the scale parameter represents the spread or standard deviation of the distribution. Thus, both parameters are important in defining the related distribution functions:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x - \mu)^2}{2\sigma^2}\right), -\infty < x < \infty$$

$$F(x) = \int_{-\infty}^x \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(t - \mu)^2}{2\sigma^2}\right) dt$$

Since the integral for $F(x)$ cannot be solved explicitly, the value for the cumulative distribution function for the normal curve is found from a table representing the standard normal distribution. In the standard normal distribution, $\mu = 0$ and $\sigma = 1$, resulting in the following distribution functions:

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right), -\infty < x < \infty$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp\left(\frac{-t^2}{2}\right) dt$$

For the standard normal distribution, $F(x)$ is often represented by $\Phi(x)$, and the value for $\Phi(1)$ represents the area depicted in Figure 2-1.

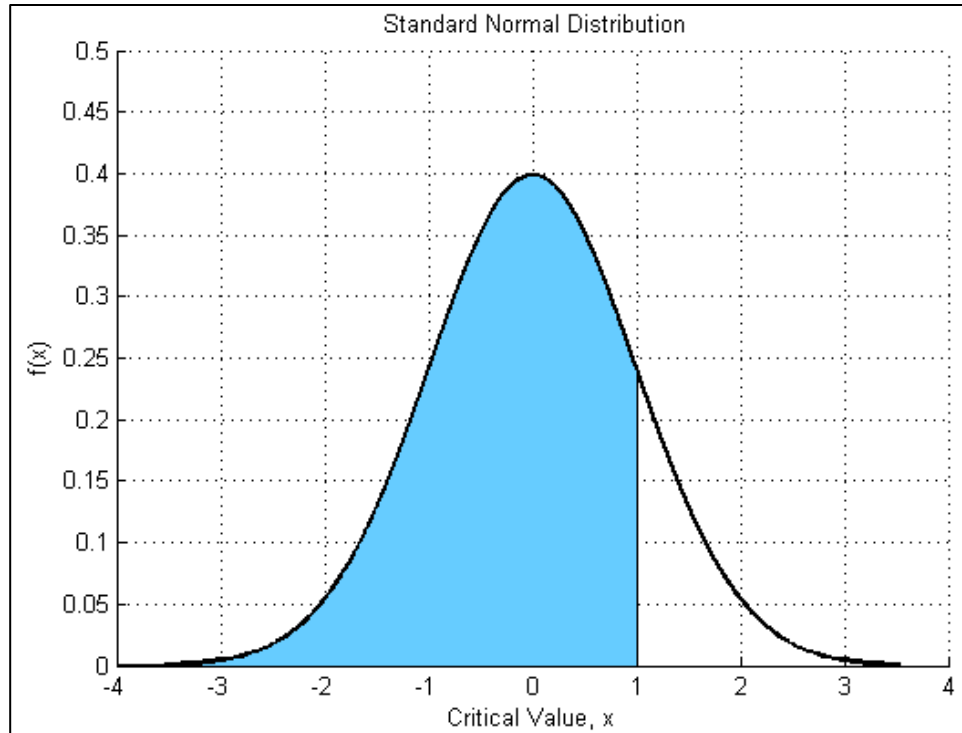


Figure 2-1. Standard Normal Distribution - Shaded area represents $\Phi(1) = 0.84$

To obtain the value for the general normal distribution, the following transformation is used to determine which value of $\Phi(x)$ to look up in the table:

$$F(x) = \Phi\left(\frac{x - \mu}{\sigma}\right)$$

Another distribution of relevance to this research is the Poisson distribution. A random variable, X , is a Poisson random variable with parameter $\lambda > 0$, if its probability mass function is defined as:

$$p_X[k] = P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}, k = 0, 1, 2, \dots$$

The cumulative distribution function of a Poisson random variable is defined as:

$$F_X(x) = e^{-\lambda} \sum_{k=0}^n \frac{\lambda^k}{k!}, n \leq x < n + 1$$

The expected value, $E[X]$, of a random variable is the average value of the outcomes of a large number of experimental trials [30]. For a discrete random variable, it is defined by the equation:

$$E[X] = \sum_i x_i p_X[x_i]$$

Equivalently, for a continuous random variable:

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx$$

For a uniform random variable, $E[X] = \frac{a+b}{2}$. The expected value for a normal random variable is equal to the mean or location parameter, $E[X] = \mu$. For a Poisson random variable, $E[X] = \lambda$.

The variability of a random variable is measured by its variance, defined by:

$$\text{var}(X) = E[(X - E[X])^2] = E[X^2] - E^2[X]$$

For a uniform random variable, the variance is equal to $\frac{(b-a)^2}{12}$. The variance for the normal distribution is again easily obtained from one of its parameters, with $\text{var}(X) = \sigma^2$. One important characteristic of a Poisson random variable is that its variance is equivalent to its expected value; thus, $\text{var}[X] = \lambda$.

2.5. Overview of Random Processes

Suppose a characteristic of a system is observed at discrete points in time, $0, 1, 2, \dots$ and let X_t be a random variable, which represents the value of the system characteristic at time, t . A discrete-time stochastic process or random process is simply a description of the relation between the random variables X_0, X_1, X_2, \dots [34]. A

continuous-time stochastic process is a stochastic process in which the state of the system can be viewed at any time, not just at discrete instants in time [34]. Stochastic processes are distinguished by their state space, or the range of possible values for the random variables X_t , by their index set T , and by the dependence relations among the random variables X_t [31].

Another way to think about a random process is to consider a conceptual random process generator as described in Kay (2006). In this conceptual random process generator, the input consists of an infinite sequence of random variables along with their probabilistic description (e.g., probability mass function), and the output is an infinite sequence of numbers, or the random process [30]. In other words, a random process is an infinite sequence of random variables, with one random variable for each time instant, and each realization of the random process takes on a value represented by an infinite sequence of random variables [30].

A point process is a type of random process that models the occurrences of some phenomenon at the time epochs $\{t_i\}$ where i refers to an index set [35]. For a point process, there are four equivalent descriptions of the sample paths: (1) counting measures, (2) non-decreasing, integer-valued step functions, (3) sequences of points, and (4) sequences of intervals (see Figure 2-2) [35]. A temporal point process is composed of a time-series of binary events that occur in continuous time [35]. At each point in time, the point process can take on only one of two possible values indicating whether an event occurred [35]. As with any random signal, the data observed as part of a point process can be expressed in terms of multiple collections of random variables [36]. For example, let Y_1, Y_2, \dots be random variables describing the

occurrence times of a point process. A realization of this point process is the event $Y_1 = y_1, Y_2 = y_2, \dots$ for some collection of times $0 < y_1 < y_2 < \dots$ [36].

Any point process can be completely characterized by its conditional intensity function, $\lambda(t|H_t)$ defined as:

$$\lambda(t|H_t) = \lim_{\Delta t \rightarrow 0} \frac{\Pr(\Delta N_{(t, t+\Delta t]} = 1 | H_t)}{\Delta t}$$

where $\Pr(\Delta N_{(t, t+\Delta t]} = 1 | H_t)$ is the instantaneous conditional probability of an event and H_t is history of events up to time t [35]. A Poisson process of intensity, or rate, $\lambda > 0$ is an integer-valued stochastic process $\{X(t); t \geq 0\}$ for which the following properties hold:

(1) for any time points $t_0 = 0 < t_1 < t_2 < \dots < t_n$, the process increments:

$X(t_1) - X(t_0), X(t_2) - X(t_1), \dots, X(t_n) - X(t_{n-1})$ are independent random variables

(2) for $s \geq 0$ and $t > 0$, the random variable $X(s + t) - X(s)$ has the Poisson distribution:

$$\Pr\{X(s + t) - X(s) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, k = 0, 1, \dots$$

(3) $X(0) = 0$ [31].

A Poisson point process $N((s, t])$ counts the number of events occurring in an interval $(s, t]$ [31]. A Poisson counting process, or more simply a Poisson process $X(t)$, counts the number of events occurring up to time t [31].

Figure 2-2, adapted from Pinsky and Karlin (2011), illustrates a typical sample path of a Poisson process. The waiting times, W_n , represent the time up until event n has occurred. These follow the gamma distribution, with probability density function:

$$f_{W_n}(t) = \frac{\lambda^n t^{n-1}}{(n-1)!} \exp(-\lambda t), n = 1, 2, \dots, t \geq 0$$

In particular, W_1 , the time to the first event, is exponentially distributed:

$$f_{W_1}(t) = \lambda e^{-\lambda t}, t \geq 0$$

The sojourn times (i.e., the times between events), S_0, S_1, \dots, S_{n-1} , are independent random variables, each having the exponential probability density function:

$$f_{S_k}(s) = \lambda e^{-\lambda s}, s \geq 0$$

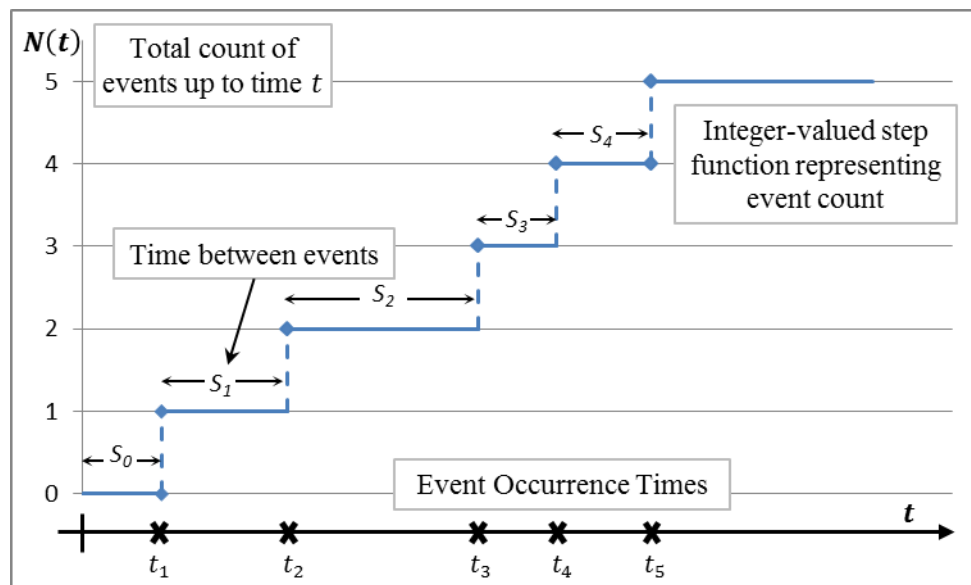


Figure 2-2. Typical sample path of a Poisson process (adapted from [31])

Additionally, events of a Poisson process occur randomly in time, the first at T_1 or W_1 , the second at T_2 , and so on, with the n^{th} event occurring at T_n . The random variable T_i denotes the time at which the i^{th} event occurs, and the values t_i of T_i are called the points of occurrence [32]. These points, occurring randomly over a specified time period, follow a uniform distribution.

Another type of stochastic process of interest is a Wiener Process, which is the mathematical model for Brownian motion. Brownian motion describes the random motion of particles suspended in a fluid resulting from their collision with the atoms within the fluid. A stochastic process $B(t)$, $-\infty < t < \infty$, is a Wiener process with parameter σ^2 , if it satisfies the following properties as defined in [37]:

- (1) $B(0) = 0$

- (2) $B(t) - B(s)$ has a normal distribution with mean equal to zero and a variance of $\sigma^2(t - s)$ for $s \leq t$

- (3) $B(t_2) - B(t_1), \dots, B(t_n) - B(t_{n-1})$ are independent for $t_1 \leq t_2 \leq \dots \leq t_n$

A random walk is an extension of the Wiener process for which, the direction of change in position is also of importance. For example, define a sequence of independent, identically distributed discrete random variables by $\{X_k\}_{k=1}^{\infty}$. For each positive integer n , let S_n denote the sum of the sequence of X_k 's up to X_n : $S_n = X_1 + X_2 + \dots + X_n$. This sequence $\{S_n\}_{n=1}^{\infty}$ is a random walk. In a simple random walk, the X_k 's follow a Bernoulli distribution in which:

$$f(x) = \begin{cases} p, & x = +1 \\ q, & x = -1 \\ 0, & \text{otherwise} \end{cases}$$

where $p + q = 1$. If $p = q = 0.5$, this is a simple symmetric random walk. The more general symmetric random walk is the process obtained by summing up independent, uniformly distributed, $(A \cup A^{-1})$ -valued random variables, X_n . The random walk can also be studied in multiple dimensions. When considering a two-dimensional random walk, in which a step can be taken in any direction or angle, d , the direction is chosen

randomly with probability $\frac{1}{d}$. Thus, the angle or direction of the random walk follows a uniform distribution.

If the X_k 's follow a normal distribution with mean equal to zero and variance equal to σ_k^2 , the sequence $\{S_n\}_{n=1}^{\infty}$ is a random walk with Gaussian steps. When adding Gaussian random variables, the resulting sum is also normally distributed with mean and variance equal to the respective sums. If W is normally distributed with mean μ_W and variance σ_W^2 , this can be represented as: $W \sim N(\mu_W, \sigma_W^2)$. If in addition, $Y \sim N(\mu_Y, \sigma_Y^2)$ and $Z = X + Y$, then $Z \sim N(\mu_W + \mu_Y, \sigma_W^2 + \sigma_Y^2)$. A random walk with Gaussian steps is normally distributed with mean equal to zero and $\sigma_n^2 = \sum_{i=1}^n \sigma_i^2$.

Each of the stochastic processes described in this section are considered in the data analysis and modeling of the eye tracker data. The Poisson process is considered as a representation of the saccade occurrences. The Wiener process and random walk are considered for the gaze pattern or eye movements that occur during the run.

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CHAPTER 3

EXPERIMENT

3.1. Material

The required equipment for this experiment includes:

1. the ISCAN ETL-500 eye tracking equipment,
2. a dedicated computer connected to the eye tracker and a video device to collect eye movement data,
3. a video recorder,
4. a second computer to run the C-Sharp program,
5. a table to house all eye scanning equipment (each participant was seated at this table), and
6. a projector and projector screen.

All of this equipment belongs to the eye-tracking lab in Gilbreth Hall where the experiments were executed. The remote ISCAN eye-tracker was set up on the table so that the participant's eye was 96 inches from the projector screen. The projector was set up so that the projected image had a width of 63 inches and a height of 35 inches. This configuration was consistent for all test subjects.

3.2. Description of Experiment

In order to determine whether attention can be modeled as a stochastic process, eye-tracking data was collected and analyzed. The data included horizontal and vertical eye position within the experimental field and pupil size. For this study, a

limited number of gaze paths (nine participants) were collected to support the data analysis. Participants were solicited from the URI student population. The study was performed in the Eye Lab in Gilbreth Hall and utilized the ISCAN software available. A C-Sharp program displayed a single black dot in the center of a screen, and collected data from the eye tracker system. The participants were asked to look at the dot for two minutes, on two separate types of trials. Each participant repeated each of the two trial types three times (total of six runs per participant).

On one trial, participants were required to perform an additional task requiring cognitive processing, i.e., the n-back task. In the n-back task, a series of verbal stimuli (e.g., a series of numbers) were presented and the participant was asked to indicate when the currently presented stimulus is the same as the one presented n trials previously [1]. For this data collection, $n = 2$ (i.e., participants would indicate that the same stimulus was presented on the string “141” after hearing the second “1”). On the other trial, the participants were simply asked to focus on the black dot in the center of the screen. Data was collected from the eye tracker software through the C-Sharp program and analyzed to support assessment of the hypothesis that eye movements, when a participant is asked to focus on a single point, can be modeled as a random process (see Chapter 4 for more details on the modeling process). The purpose of including the additional cognitive task is to determine its effect on the focus of visual attention and resulting gaze path.

The data collection occurred in May-June 2013. This experiment utilized the ISCAN ETL-500 remote eye tracking system with ISCAN Raw Eye Movement Data Acquisition (DQW) Software which collected samples at 240-Hz. The RK-826 PCI

Pupil Corneal Reflection Tracking System and the RK-630 PCI Autocalibration System are the hardware installed in the system. The RK-826 system tracks pupil position and size with the dark-pupil-to-corneal reflection method [2]. This system utilizes an infrared light source that illuminates the eye so that the pupil acts like a sink and the surrounding areas reflect the light source back to the camera [2]. The pupil is recognized and tracked as the dark image and the corneal position as the bright spot off which the infrared light source reflects; this allows the tracking of eye movements [2]. Figure 3-1 shows an example of an eye-tracking image in which the pupil and corneal reflection thresholds are set to the appropriate levels. This image is an example of the clarity and markings one would ideally like to see before starting the collection of eye tracking data.

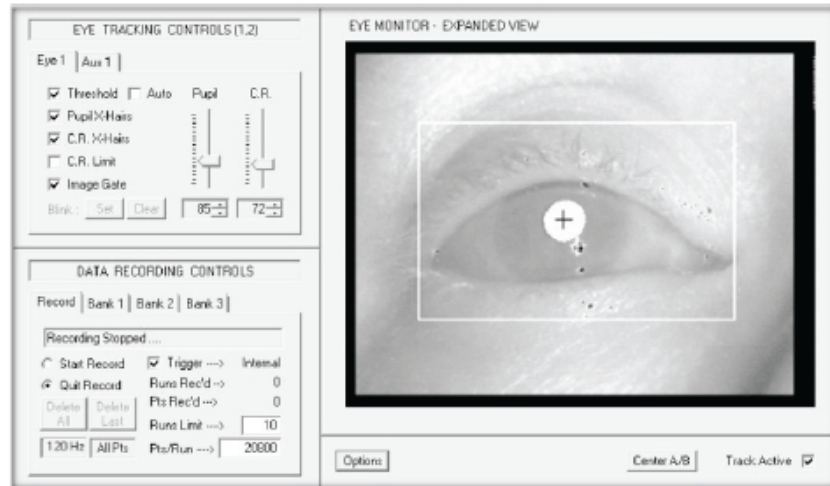


Figure 3-1. Ideal Eye Tracking Image (thresholds adjusted to appropriate levels) [3]

The RK-630 PCI Autocalibration System utilizes a five-point calibration procedure to calculate the subject's point of regard (POR) with respect to the scene being viewed [4]. The DQW software displays graphical representations of the pupil

size and eye positions. The video eye image display over which the eye is tracked is represented by 512 points in both the horizontal and vertical dimensions, with the upper left corner representing the point (0, 0) and the bottom right, (511, 511). The output data consisted of a matrix of horizontal and vertical coordinates representing eye position along with pupil size for each sample.

Figure 3-2 illustrates the geometry needed to calculate the visual span or degrees of visual movement of a participant, which is sufficient when determining the amount of movement (in terms of degrees) for solely horizontal or solely vertical movement.

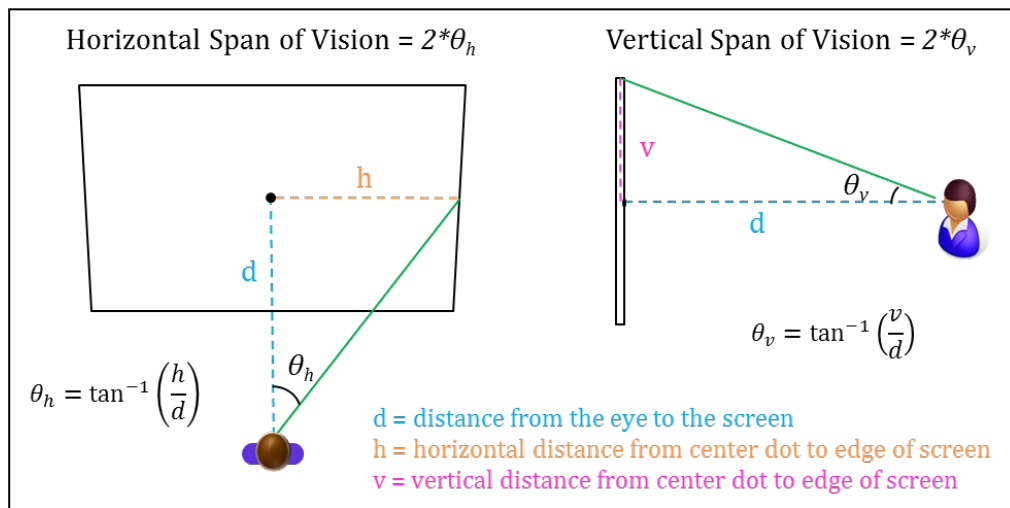


Figure 3-2. Illustration of the visual span (horizontal and vertical) based on positioning of the participant from the screen and the size of the projected image on the screen

Since the focus of the eye does not always start in a position perpendicular to the screen, the angle of movement must be calculated utilizing three-dimensional geometry. Figure 3-3 illustrates the three-dimensional geometry utilized to determine the overall amount of movement between eye position samples (in degrees).

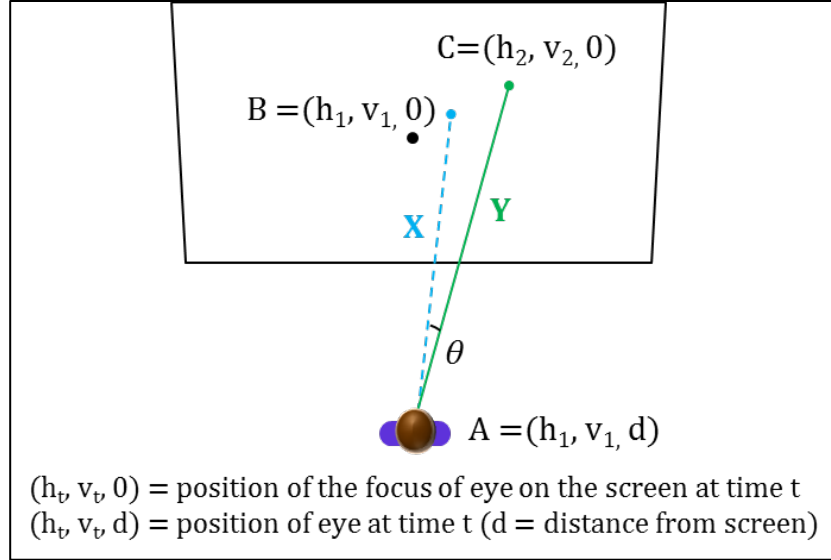


Figure 3-3. Three-dimensional geometry utilized to calculate amount of eye movement between samples (used to identify saccades)

Each point within the space is defined as an ordered triple of real numbers, i.e., (h, v, d) where h = horizontal position of the eye on the screen, v = vertical position of the eye on the screen, and d = distance of the eye from the screen. With the points defined as in Figure 3-3, the vectors \mathbf{X} and \mathbf{Y} are defined by the following equations.

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \overrightarrow{AB} = \begin{bmatrix} h_1 - h_1 \\ v_1 - v_1 \\ 0 - d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \overrightarrow{AC} = \begin{bmatrix} h_2 - h_1 \\ v_2 - v_1 \\ 0 - d \end{bmatrix} = \begin{bmatrix} h_2 - h_1 \\ v_2 - v_1 \\ -d \end{bmatrix}$$

The angle θ between two non-zero vectors \mathbf{X} and \mathbf{Y} is then calculated using the following equation:

$$\cos \theta = \frac{\mathbf{X} \cdot \mathbf{Y}}{\|\mathbf{X}\| \|\mathbf{Y}\|}, 0 \leq \theta \leq \pi$$

where $\mathbf{X} \cdot \mathbf{Y}$ represents the dot product and is defined as: $\mathbf{X} \cdot \mathbf{Y} = x_1 y_1 + x_2 y_2 + x_3 y_3$ and $\|\mathbf{X}\|$ represents the magnitudes of \mathbf{X} and is defined as: $\|\mathbf{X}\| = (\mathbf{X} \cdot \mathbf{X})^{1/2}$.

For example, let $h_1 = 50$, $v_1 = 60$, $h_2 = 70$, $v_2 = 85$, and $d = 175$.

$$\mathbf{X} = \begin{bmatrix} 0 \\ 0 \\ -175 \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} h_2 - h_1 \\ v_2 - v_1 \\ -d \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ -175 \end{bmatrix}$$

$$\mathbf{X} \cdot \mathbf{Y} = (0 * 10) + (0 * 15) + (-175 * -175) = 30,625$$

$$\|\mathbf{X}\| = (0^2 + 0^2 + (-175)^2)^{1/2} = 175$$

$$\|\mathbf{Y}\| = (10^2 + 15^2 + (-175)^2)^{1/2} = 175.926$$

$$\cos \theta = \frac{30625}{(175) * (175.926)} = 0.9947$$

$$\theta = \cos^{-1}(0.9947) = 5.88^\circ$$

This angle is then used to calculate the angular velocity of the eye movement by dividing by the time between samples.

3.3. Experimental Procedure

The sequence of events for the experiment is detailed in the following procedure.

1. Start the eye tracker system.
2. Open and run the C-sharp program “SingleDotFocus”.
3. Perform the eye calibration.
 - a. Ask the participant to sit in the chair facing the large projector screen and place his/her chin on the chin rest, and look straight ahead.
 - b. Select ‘track active’ and make the necessary adjustments to the eye tracker system to obtain a clear picture of the eye and pupil (as shown in Figure 3-1).

- c. Present screen with five dots (one central dot and one on each corner).
Assure that each dot aligns with the dots on the eye tracker software calibration screen.
 - d. Ask the participant to look at each dot in the sequence (e.g., center, top left, top right, bottom left, bottom right, back to center) while advancing through the corresponding dots in the calibration screen, so that the eye tracker system can be calibrated.
 - e. Assure that the system is properly calibrated before proceeding to experiment. If the system calibration is inaccurate, repeat calibration.
4. Present the screen containing a single center dot and ask participants to focus on the dot for two minutes on two different types of trials.
 - a. On one trial, the participant will simply focus on the black dot.
 - b. On the other trial, a series of numbers will be presented to the participant and he/she will be asked to indicate when the currently presented number is the same as the number presented two trials previously (e.g., for the sequence of numbers “3-7-3”, a report of same would occur after the second “3” is heard).
 5. Repeat each type of trial three times (order of trial type will be randomized), for a total of six runs per participant.

During the n-back trials, each time a participant signaled that the currently presented number was the same as the one presently two trials previously, it was manually documented. No associated time stamp was recorded. This additional cognitive task was included in order to determine its effect of gaze pattern. The

purpose of this experiment was not to assess a participant's speed or accuracy in identifying the repeated numbers, and thus, an actual performance assessment of the n-back task extended beyond the scope of this research.

List of References

- [1] A. M. Owen, K. M. McMillan, A. R. Laird, and E. Bullmore, "N-back working memory paradigm: A meta-analysis of normative functional neuroimaging studies," *Human Brain Mapping*, vol. 25, no.1, pp. 46-59, 2005.
- [2] ISCAN, *Operating Instructions: ETL-200 Primate Eye Tracking Laboratory*, Woburn, MA, August 2011.
- [3] ISCAN, *ISCAN Operating Instructions: RK-826 PCI Pupil/Corneal Reflection Tracking System*, Burlington, MA, October 2003.
- [4] ISCAN, *ISCAN Operating Instructions: RK-630 PCI Autocalibration System*, Burlington, MA, July 2003.

CHAPTER 4

ANALYSIS AND RESULTS

4.1. Eye-tracker data analysis

Data collected for each participant was analyzed using MATLAB (see Appendix A for MATLAB scripts used for analysis). To ascertain the amount of movement occurring during the trial, plots of eye positions over time were constructed. At the beginning of each run, the central dot positions (in addition to the calibration dot positions) were recorded. Utilizing these positions, for each sample, an eye position distance from center, d_{fc} , was calculated using a basic distance formula as follows:

$$d_{fc} = \sqrt{(e_v - c_v)^2 + (e_h - c_h)^2}$$

where e_v = vertical eye position, e_h = horizontal eye position, c_v = central dot vertical position, c_h = central dot horizontal eye position. A subgroup of plots for selected runs of two participants is shown below for illustrative purposes. In Figure 4-1 and Figure 4-3, a series of spikes indicates that the participant's focus shifted from the center dot. The shifts of focus, or saccades, are of primary interest for the development of a model to fit this data.

In addition, the difference from the central focus of each participant was calculated and used to illustrate where the drifts occurred. Though participants were instructed to focus on the central dot, their actual focus did not align exactly with the central dot location. Thus, a central focal location was determined for each participant and used to generate Figure 4-2 and Figure 4-4. However, in order to calculate a

central position, blinks were removed (to avoid skewing the data unfairly), and the data was plotted without blinks. It is clear from the following plots that there is a slight difference in the pattern of drifts, but the overall shapes are similar. In addition, there is a reduction of noise in the data when the blinks were removed and the distance was determined using the central focus of the participant.

It was also important to consider the variation in the amount of noise between participants. Initially, a distance from center threshold was used to identify drifts. However, due to the differences in variation, thresholds were also considered in terms of the number of standard deviations for each participant. In order to classify eye movements from the eye tracker data, various event detection algorithms were reviewed.

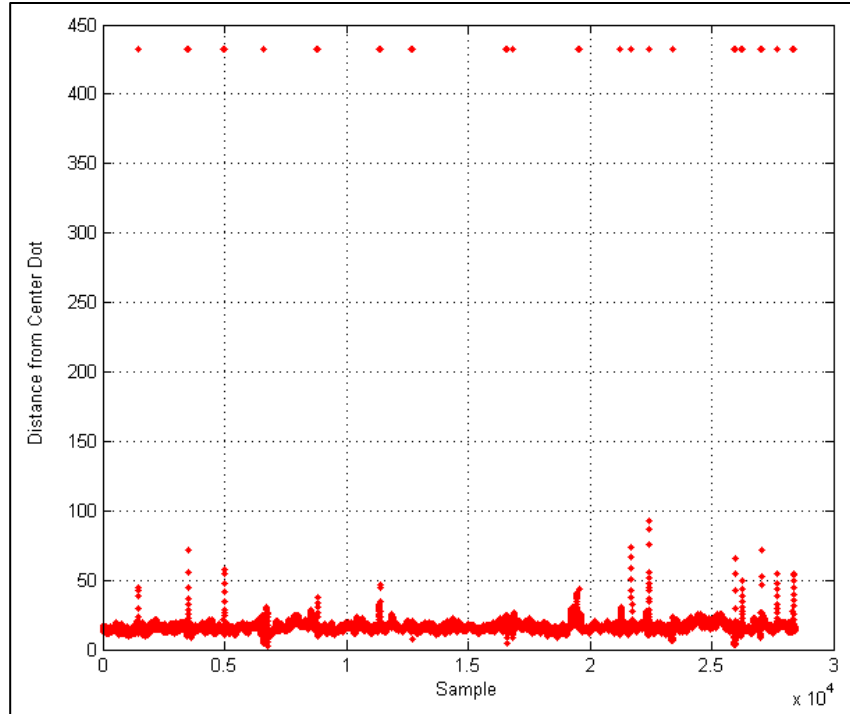


Figure 4-1. Distance from Center Dot - Participant 1, Control Run 1

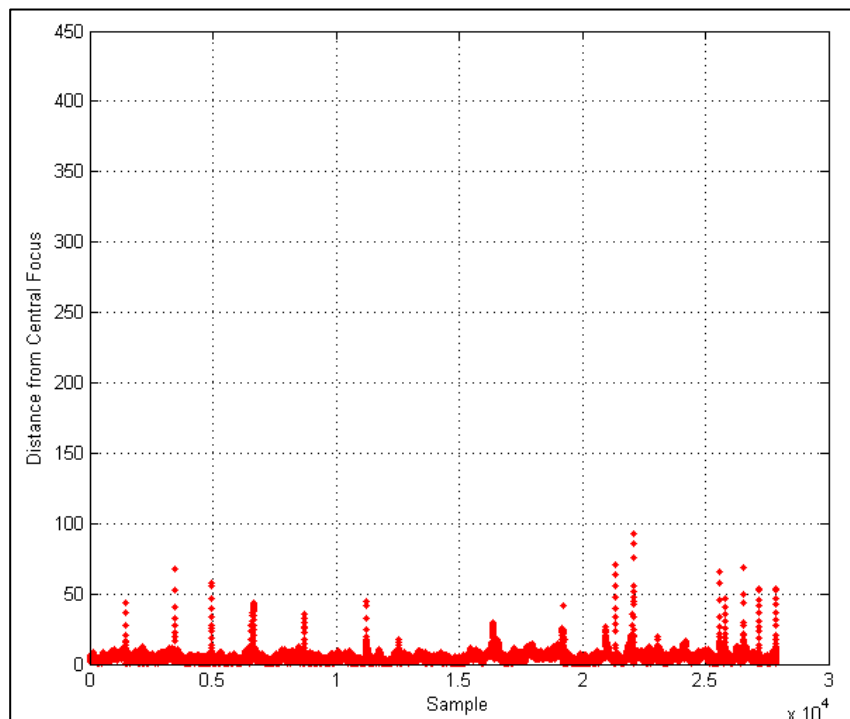


Figure 4-2. Distance from Central Focus - Participant 1, Control Run 1

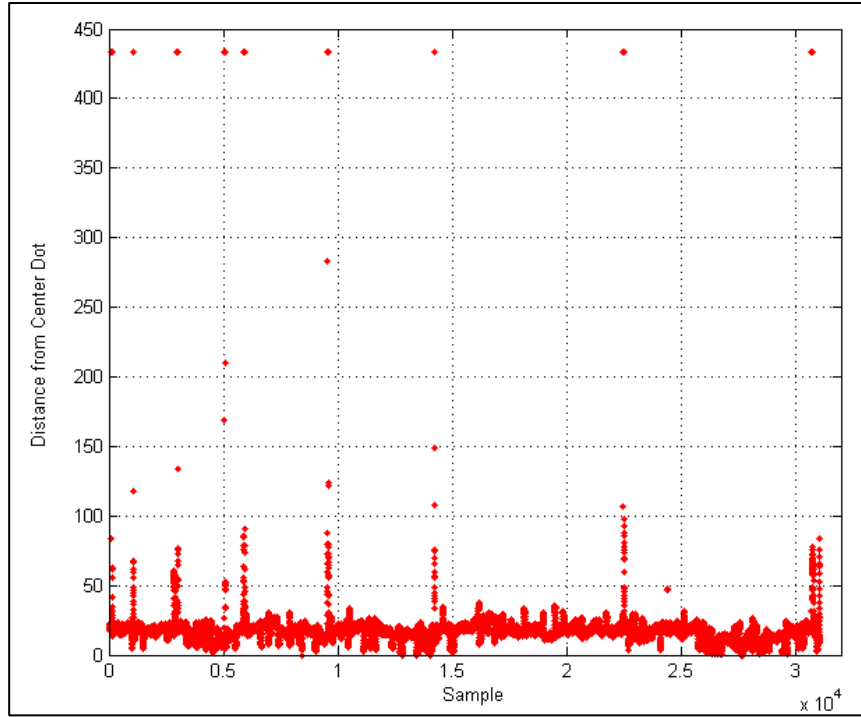


Figure 4-3. Distance from Center Dot - Participant 5, n-back Run 1

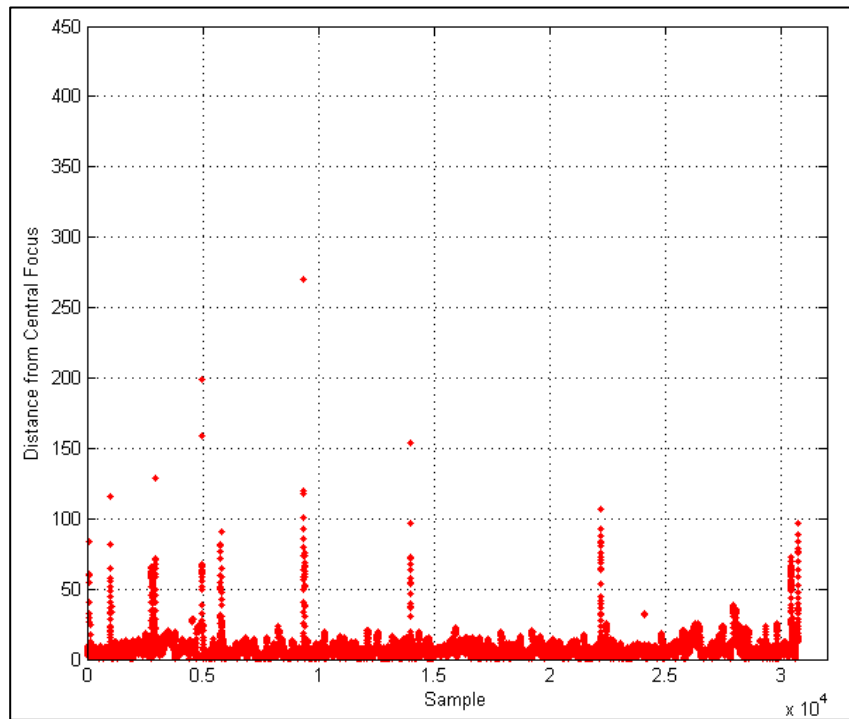


Figure 4-4. Distance from Central Focus - Participant 5, n-back Run 1

4.2. Event Detection Algorithms

Salvucci and Goldberg [1] classify event detection algorithms in terms of two temporal and three spatial criteria. The temporal criteria consider the algorithm's use of duration information and its local adaptivity [1]. The incorporation of local adaptivity allows the interpretation of a given data point to be influenced by the interpretation of temporally adjacent points [1]. This is important for event detection, i.e., when determining points belonging to a fixation, removing data surrounding blinks, and identifying the onset and offset of a saccade. Thus, this concept will be considered, but not a focal point in the following discussion.

The spatial criteria divide algorithms into categories based on their use of dispersion, velocity, or area-of-interest (AOI) information to detect events [1]. Dispersion-based algorithms typically identify samples as belonging to a fixation if the samples are located within a spatially limited region for a minimum period of time (i.e., the fixation duration) [2]. Dispersion-based algorithms utilize the fact that the distance between samples is different for fixations and saccades. These algorithms also take into account that a fixation has a minimum duration of 50-100 msec [3].

Velocity-based algorithms compute the angular velocity between each data point and use a velocity threshold to distinguish whether a point belongs to a fixation or a saccade [3]. These algorithms take advantage of the fact that the velocity of a fixation is different from that of a saccade [3]. Area-based algorithms identify points within given AOIs that represent the relevant visual targets [1]. Since in this task there is only a single focal point, area-based algorithms utilizing AOI information are irrelevant. Thus, the remainder of the section will focus on dispersion and velocity-based methods.

Dispersion-based algorithms identify fixations as groups of consecutive points of regard (POR) within a particular dispersion or maximum separation [4]. There are a variety of methods that can be used to calculate dispersion. For example, the dispersion can be measured as the distance between each point in a fixation and every other point or it can be calculated as the distance between successive points [4]. The Dispersion-Threshold Identification (I-DT) algorithm utilizes a moving window of consecutive data points and calculates the dispersion of points within the window [1]. The dispersion is compared to a pre-specified dispersion threshold (e.g., 1 degree), and if the dispersion exceeds this value, the window is identified as not representing a fixation [1]. The window moves one data point further and continues to identify and group points that belong to a fixation [1]. Additionally, I-DT takes into account a minimum fixation duration threshold. If the groups of points within a fixation window do not span the minimum fixation duration threshold, the window is discarded since it no longer represents a fixation [1].

Dispersion-based algorithms typically focus on identifying fixations whereas velocity-based algorithms are better suited to saccade identification. The Velocity-Threshold Identification (I-VT) algorithm computes the angular velocity between each data point and uses a velocity threshold to distinguish whether points belong to a fixation or a saccade [3]. The angular velocity is simply calculated by dividing the angular dispersion by the time between sample points. Thus, for consecutive data points, at a 240-Hz sampling rate as used in this experiment, the points are 1/240 second apart, and the angular velocity is calculated by dividing the angle by 1/240.

There are a few areas of concern in implementing any of the event detection algorithms described. Throughout the literature, the thresholds for identifying saccades and fixations based on angular dispersion and angular velocity vary significantly. For example, dispersion thresholds range from 0.1° to 1.3° and duration thresholds range from 50 milliseconds to 400 milliseconds for the cutoff of a fixation [1, 2, 3, and 4]. Velocity-based thresholds fluctuate from $20^{\circ}/\text{second}$ to greater than $300^{\circ}/\text{second}$ for distinguishing between a fixation and a saccade [1, 2, 3, and 4]. In addition, methods for detecting events utilizing velocity vary as well. Whereas some simply set a threshold, others consist of a series of values, which must be reached to signify a saccade. For example, one algorithm first searches for saccades by the point at which the velocity has exceeded $160^{\circ}/\text{second}$, and then searches backward in time to find the first point where the threshold of $20^{\circ}/\text{second}$ is exceeded, and defines this point as the actual starting point of the saccade [5].

Another issue in analyzing eye tracker data in order to detect saccades results from the noise within the data. Various methods exist for filtering the data, but then one risks filtering out important pieces of information. Other methods suggest adaptive thresholds, which can take into account the variation between runs or participants by utilizing the standard deviation [6]. The final issue of concern, when attempting to identify saccades, is the sampling rate of the eye tracker system. Larsson (2010) compared event detection algorithm results for a variety of sampling frequencies ranging from 50 Hz to 1250 Hz [3]. Larsson concluded that a sampling frequency of at least 250 Hz is required to obtain reasonable results when computing

angular velocity [3]. Thus, with the maximum available sampling frequency of 240 Hz for this data collection, accurately identifying saccades may be an issue.

4.3. Saccade Detection Results and Graphs

Before the data could be modeled, it was necessary to determine where and at what rate saccades were occurring. Based on the review of event detection algorithms, it was determined that a velocity-based algorithm would be used. A variety of thresholds and filtering methods were explored, before selecting a threshold value of 300° /second. An adaptive threshold was also considered to account for participants with larger amounts of variation, but ultimately, was not used in the saccade identification.

A simple algorithm which identified saccades based predominately on the velocity threshold was developed. The points that exceeded the threshold were categorized as potential saccades. Further review assessed whether or not these high velocity events occurred surrounding a blink. The dark-pupil-to-corneal reflection method used by the eye tracker is not able to accurately obtain the eye's position while it is opening (or closing) as part of a blink. Thus, if the data occurred within twenty samples before or after a blink (identified by a zero in the positional data), it was discarded from the set of potential saccades. The remaining potential saccades were then identified as saccades and used for the following analyses. The data was analyzed using MATLAB with the pseudo code shown in **Error! Reference source not found.**

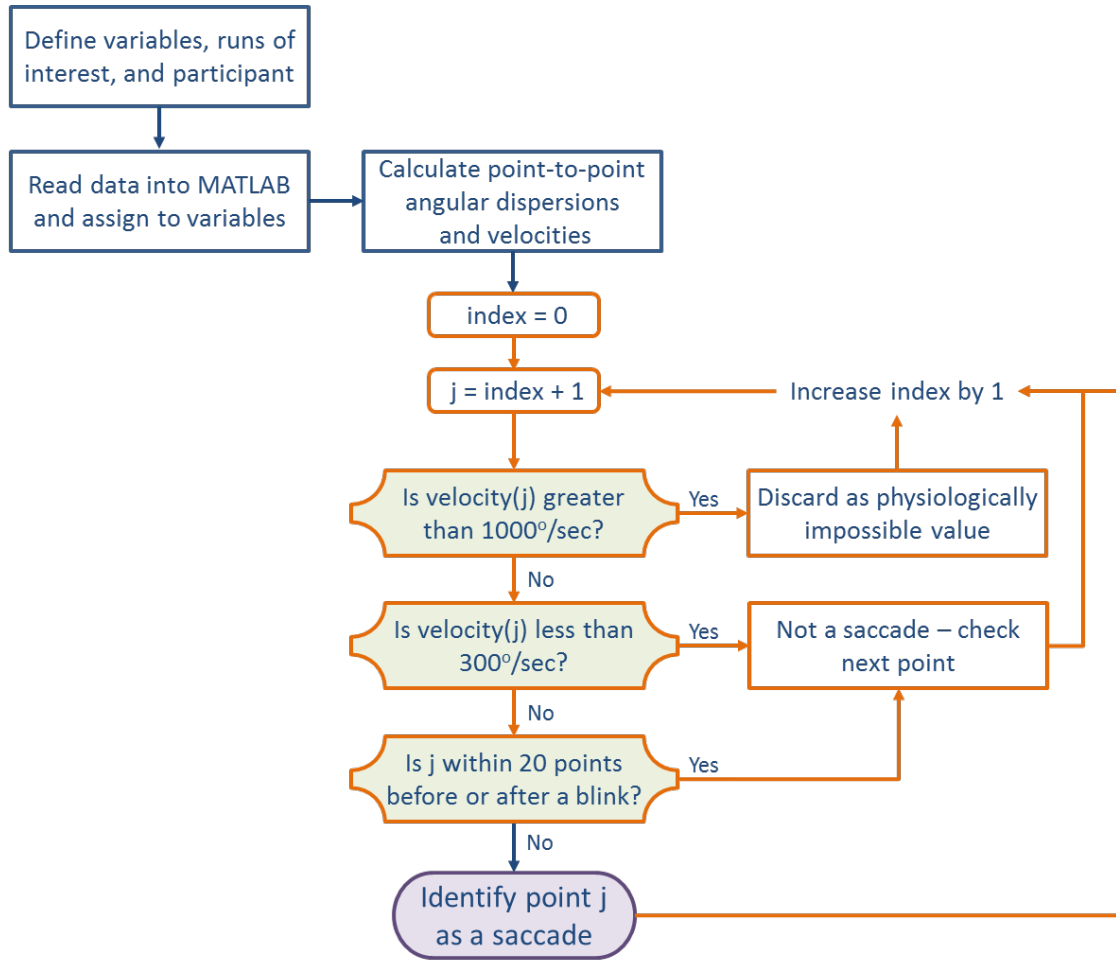


Figure 4-5. Saccade Detection Algorithm

The number of saccades within each run was calculated and the times at which the saccades occurred were obtained. The number of data points for each run both with and without saccades is shown in Table 4-1. Note that blinks and physiologically impossible values have been removed from the data prior to the total data counts. Though for saccade identification 20 samples before and after blinks were considered, only ten samples before and after each blink were removed. For saccade detection, it was important that the adjustment of the eye before and after a blink was not identified as a saccade. For blink removal, it was only necessary to remove the data points that

might be part of the actual opening or closing of the eye. For the majority of the runs, removing blinks and the ten samples before and after blinks resulted in less than 10% of the data points being removed.

	Saccades	Control 1	Control 2	Control 3	N-back 1	N-back 2	N-back 3
P1	Included	27495	26003	26592	27736	28612	28353
	Removed	27493	26001	26592	27510	28450	27954
P2	Included	24017	24724	22505	15175	21503	20821
	Removed	23983	23318	21779	15122	21442	20793
P3	Included	26843	27250	27424	29043	26434	25060
	Removed	25140	26245	25862	27962	24815	23624
P4	Included	27238	27691	27028	29085	30364	30260
	Removed	27145	27529	25651	28758	30141	28131
P5	Included	27621	27815	26807	30564	28010	N/A
	Removed	27615	27790	26497	30554	28008	N/A
P6	Included	27437	28002	26891	27341	29438	29576
	Removed	26301	27818	26520	25890	29294	29327
P7	Included	26412	26249	26600	26273	26032	29035
	Removed	23889	24283	24543	25045	23778	27040
P8	Included	27853	27183	26762	28578	28757	28450
	Removed	26958	26375	26019	27750	27064	26149
P9	Included	27046	26804	27133	26520	26356	26278
	Removed	24629	24436	25209	24742	24652	24834

Table 4-1. Number of data points in each run for each participant (before and after saccades have been removed from the data)

The following plots illustrate the saccade counts versus time for each of the participants. Each subplot shows either the three control runs or three n-back runs of that participant. Participant 5 runs have been removed due to a data recording issue.

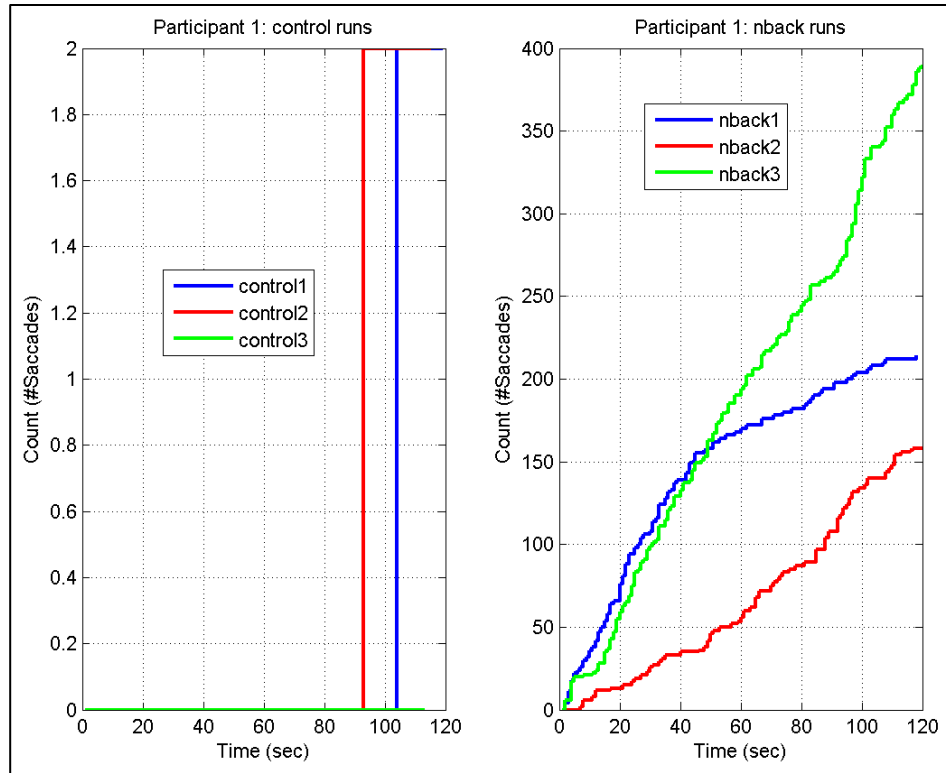


Figure 4-6. Saccade Count versus Time – Participant 1 Runs

Participant 1 had significantly fewer saccades than the other participants (with the exception of Participant 2’s n-back runs). For the control runs, Participant 1 exhibited no more than two saccades during the two-minute period. The number of saccades increased for the n-back runs resulting in approximately 150-400 saccades during the two-minute period.

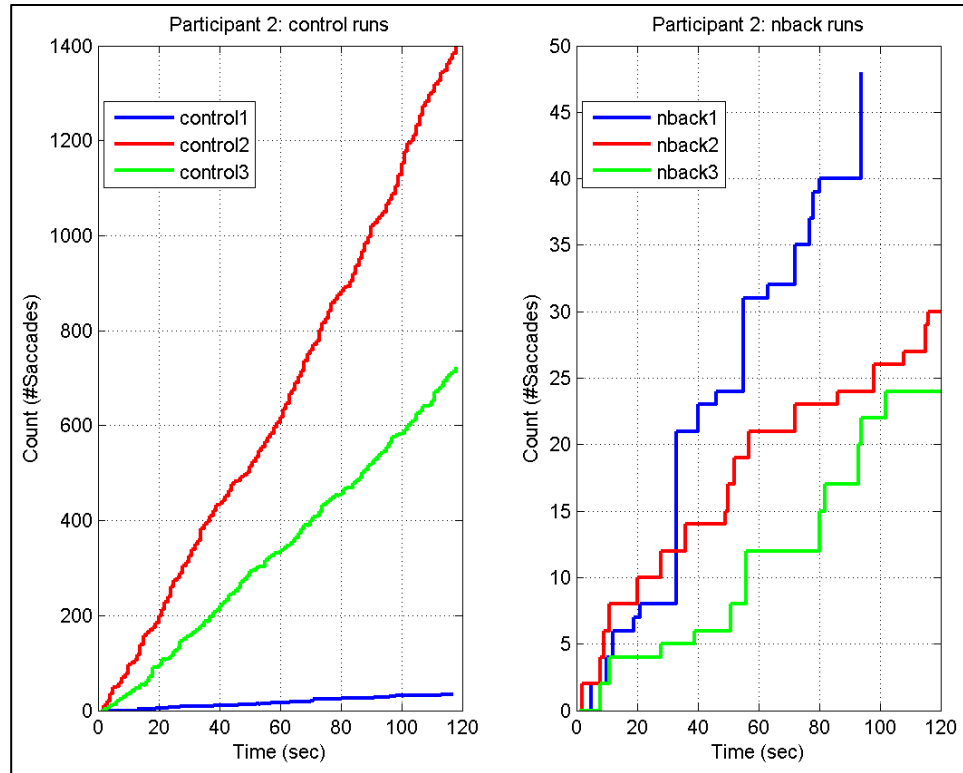


Figure 4-7. Saccade Count versus Time – Participant 2 Runs

Participant 2 exhibited more saccades during the control runs than during the n-back runs. However, the number of saccades varied greatly, from less than 50 saccades in the first control run, up to approximately 1400 saccades during the second control run. For the n-back runs, participant 2’s number of saccades decreased to 25-50 saccades during the two-minute period. The results from these two participants differed the most from the results of the other participants in terms of patterns and quantity of saccades.

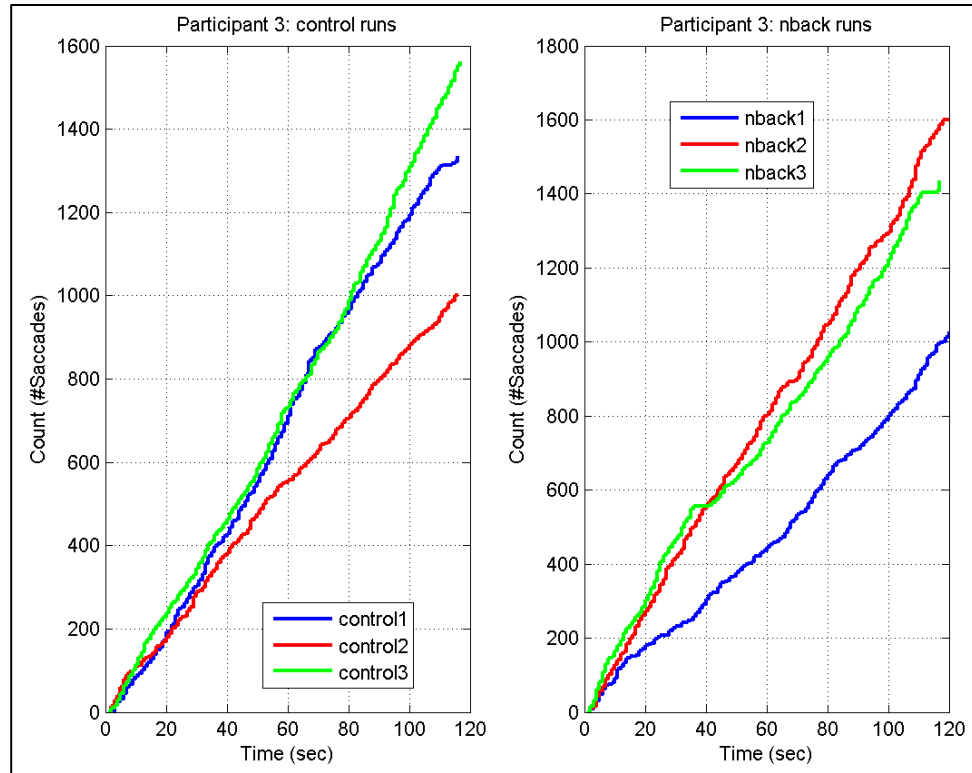


Figure 4-8. Saccade Count versus Time – Participant 3 Runs

For participant 3, the saccade pattern between the control runs and the n-back runs did not differ much, nor did the quantity of saccades. Both types of runs varied between 1000 and 1600 saccades during the two-minute period. Participant 4 also showed similar patterns of saccades between the control runs and n-back runs. However, while the first and second runs resulted in 200 saccades or less for the control runs, and less than 300 saccades for the n-back runs, this number significantly increased to almost 1400 saccades for control run 3 and almost 2000 saccades for n-back run 3.

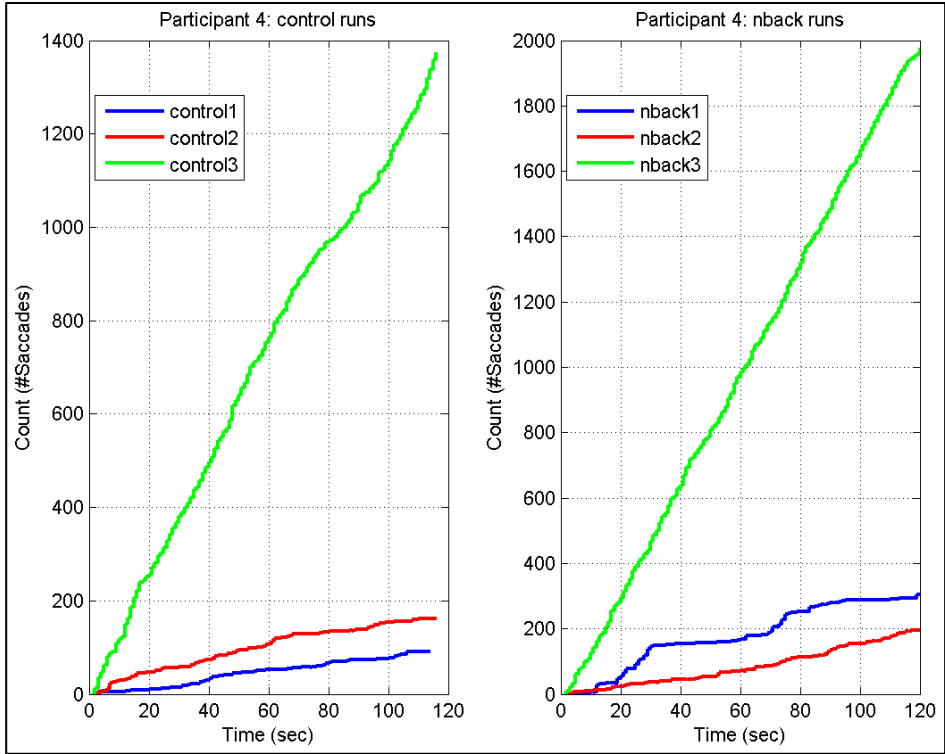


Figure 4-9. Saccade Count versus Time – Participant 4 Runs

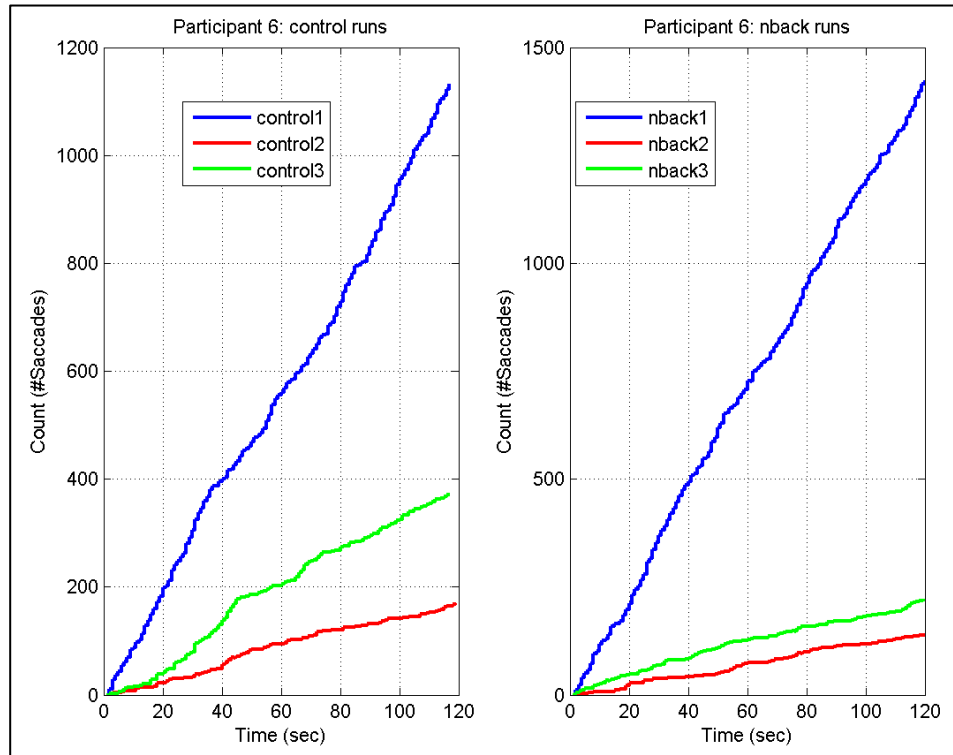


Figure 4-10. Saccade Count versus Time – Participant 6 Runs

Participant 6 followed a similar pattern as participant 4, in that control runs 2 and 3 exhibited about 175 saccades and 375 saccades, respectively, whereas in control run 1, approximately 1100 saccades were detected. For n-back runs 2 and 3, participant 6 exhibited less than 200 saccades each, but in n-back run 1 that number shot up to over 1400 saccades.

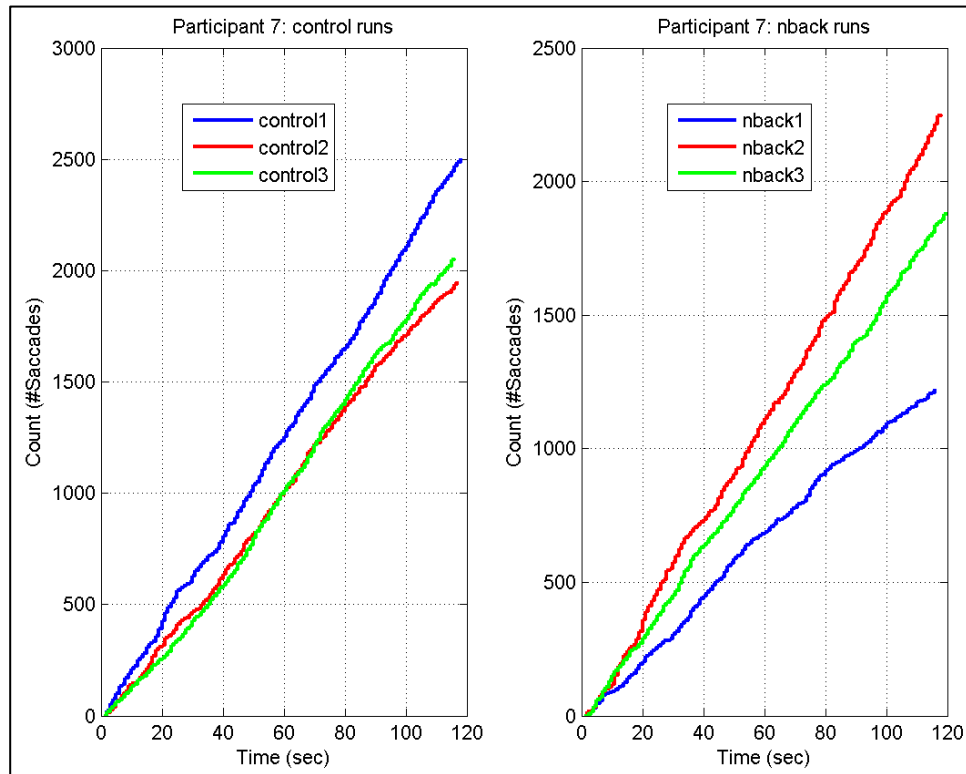


Figure 4-11. Saccade Count versus Time – Participant 7 Runs

For participant 7 there were more saccades and less variation exhibited in the control runs compared to the n-back runs. In the control runs, the number of saccades ranged from slightly under 2000 to 2500 saccades, whereas for the n-back runs the values ranged from about 1200 saccades to 2200 saccades.

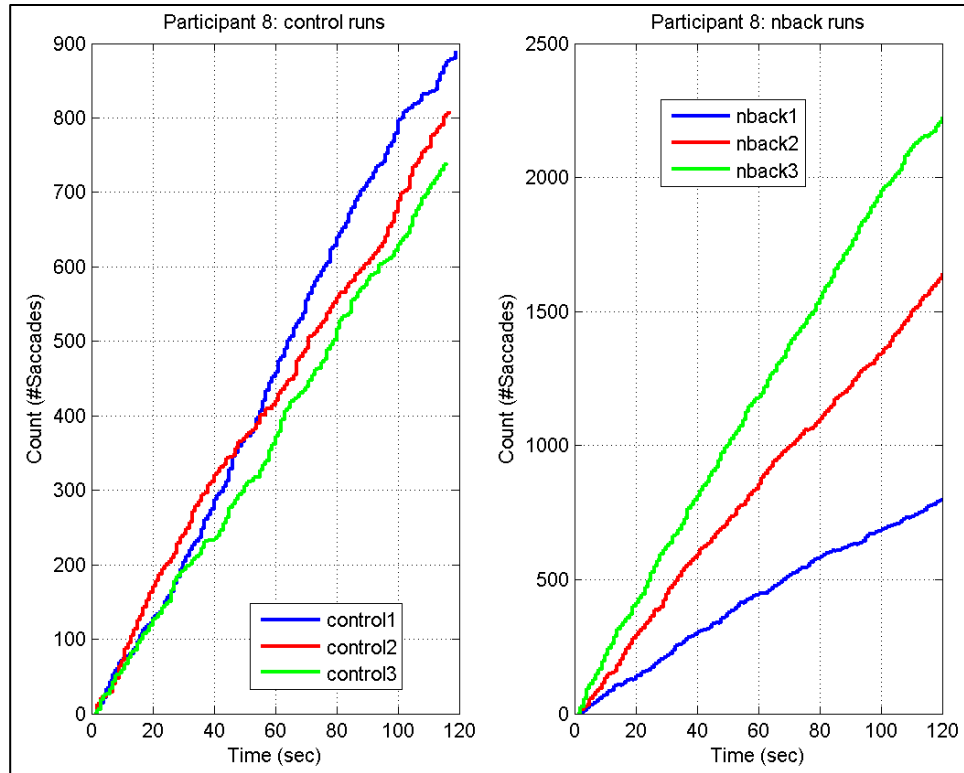


Figure 4-12. Saccade Count versus Time – Participant 8 Runs

For participant 8, approximately 750-900 saccades occurred during the two-minute period of the control runs, whereas the n-back runs resulted in between 750 and 2200 saccades. Thus, the quantity and the variation in the control runs were considerably smaller than that exhibited in the n-back runs. The pattern of saccades for participant 9 was opposite of the pattern observed for participant 8. Participant 9 exhibited more saccades in the control runs, with approximately 1900 to 2400 saccades, than in the n-back runs, where the number of saccades ranged from 1400 to 1800 saccades. The two types of runs exhibited similar amounts of variation.

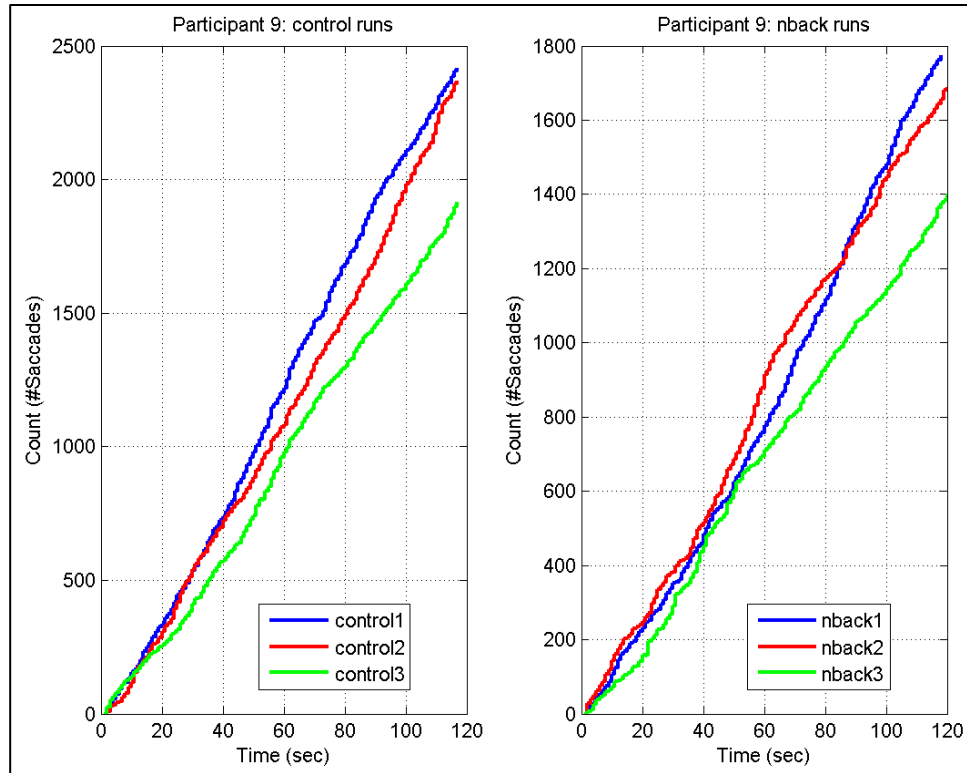


Figure 4-13. Saccade Count versus Time – Participant 9 Runs

Overall, there was not a consistent pattern among the saccade counts across participants, or even within participants for the same type of run. Some participants exhibited more consistent patterns of saccades across all runs, whereas others differed significantly between the control and n-back runs. To further assess the distribution of saccades, histograms of saccade counts were generated using ten-second bins. The following figures illustrate a subset of these histograms. Figures 4-14 through 4-16 illustrate Participant 1’s n-back histograms. These plots do not appear to consistently follow one specified distribution.

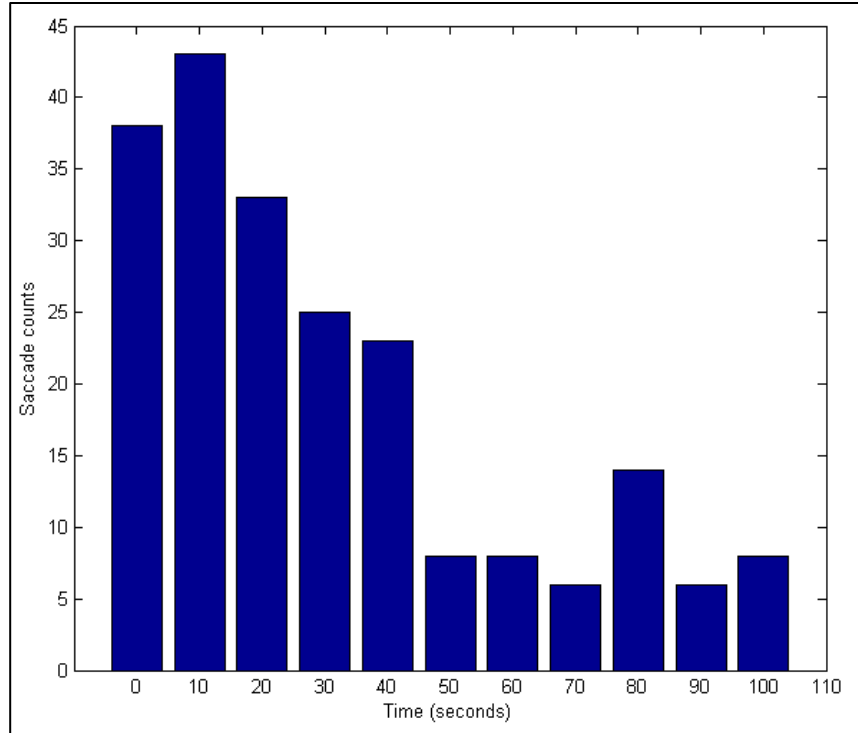


Figure 4-14. Histogram of Saccade Counts (participant 1, n-back 1)

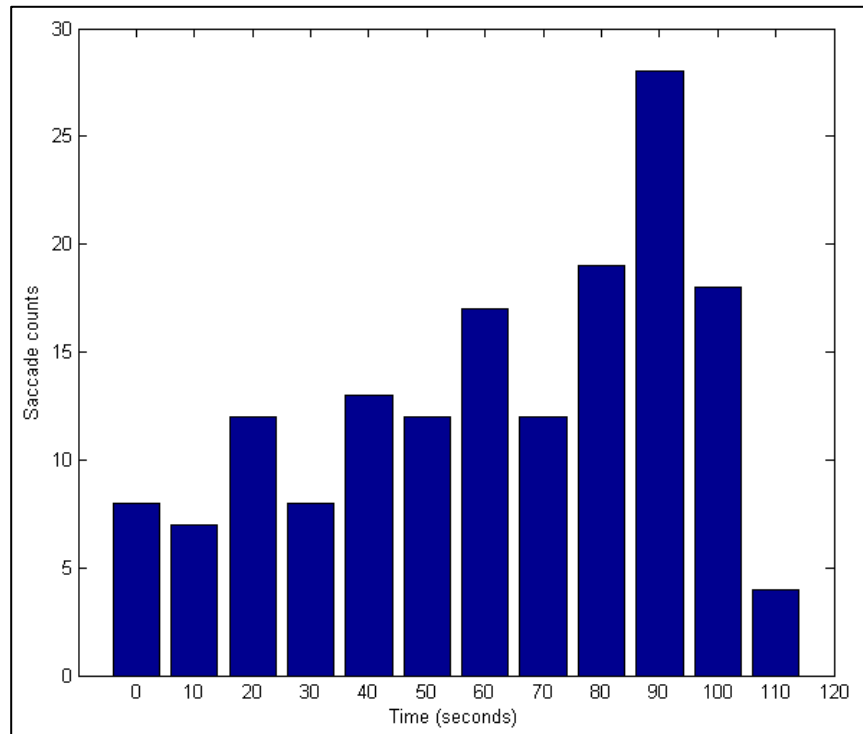


Figure 4-15. Histogram of Saccade Counts (participant 1, n-back 2)

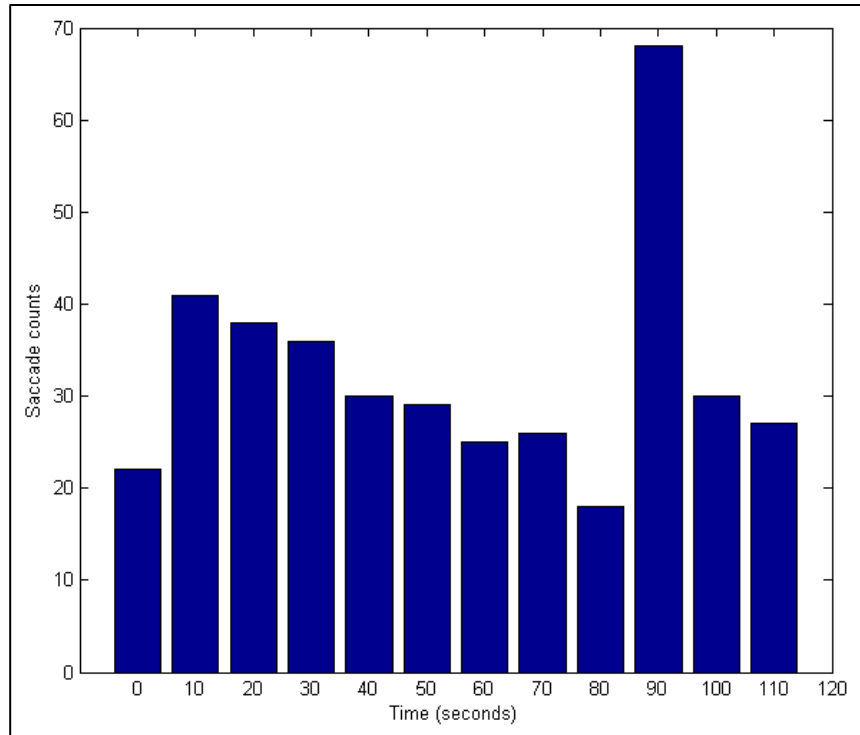


Figure 4-16. Histogram of Saccade Counts (participant 1, n-back 3)

Participant 7 did exhibit a fairly consistent pattern across all runs (Figures 4-17 through 4-22). These graphs contain about 150-250 saccades per ten-second bin, and appear uniform across both the control and n-back runs. Additional analysis of the distribution of saccades is examined in the next section.

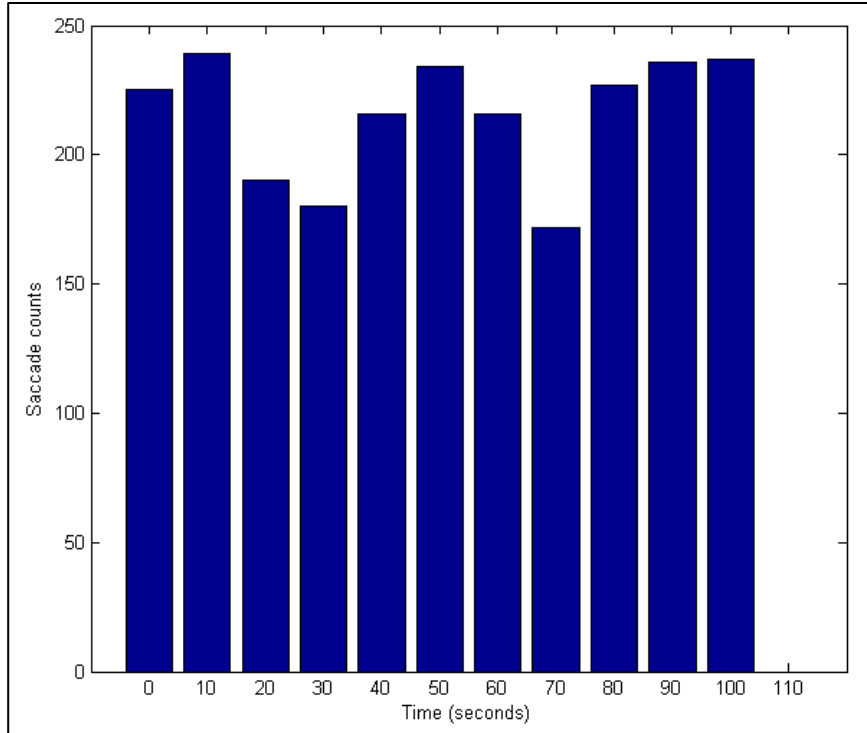


Figure 4-17. Histogram of Saccade Counts (participant 7, control 1)

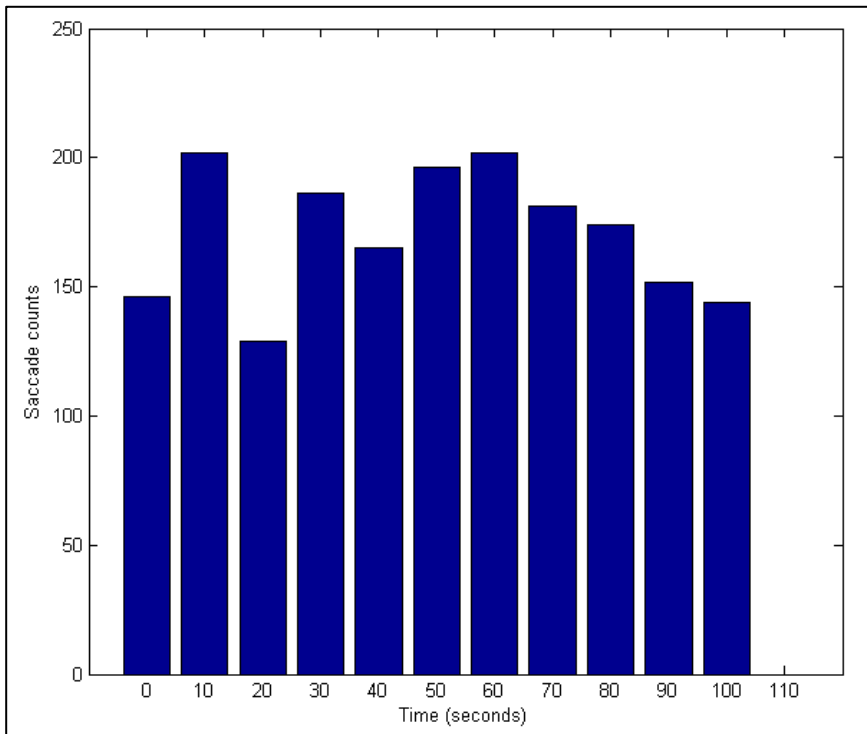


Figure 4-18. Histogram of Saccade Counts (participant 7, control 2)

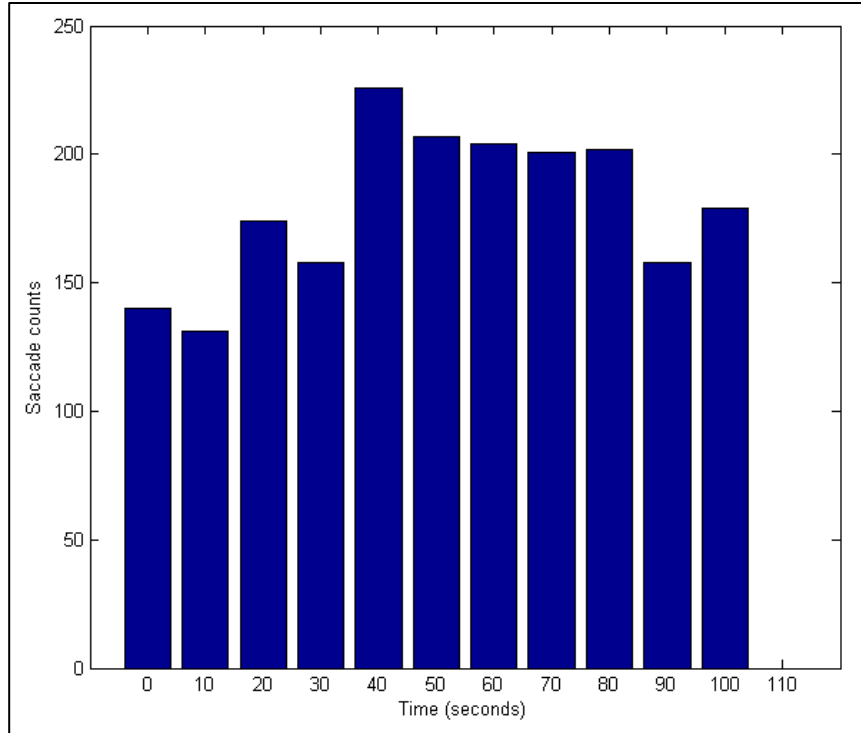


Figure 4-19. Histogram of Saccade Counts (participant 7, control 3)

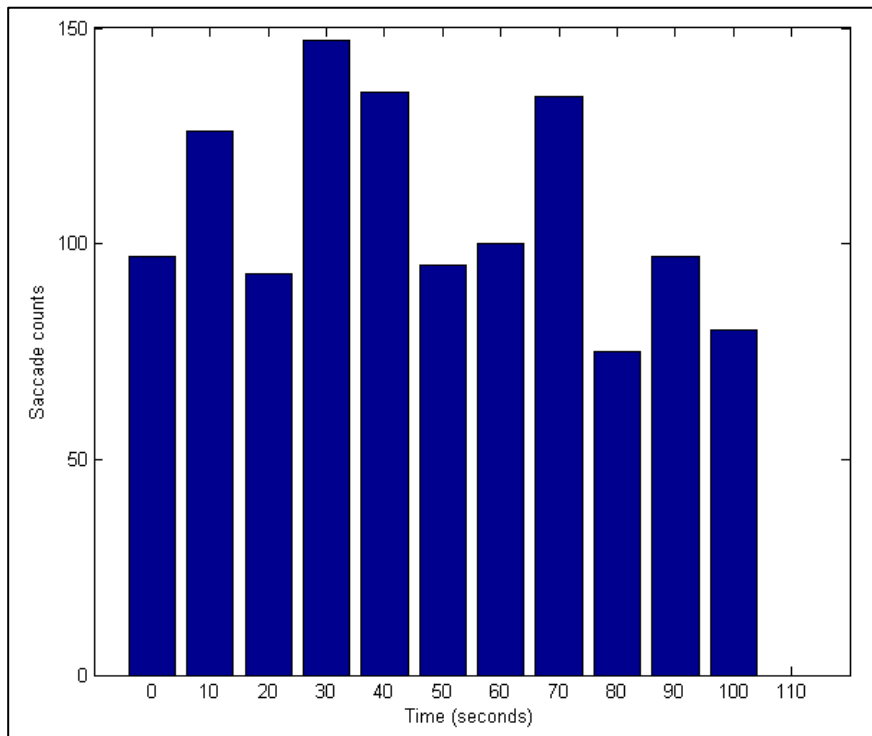


Figure 4-20. Histogram of Saccade Counts (participant 7, n-back 1)

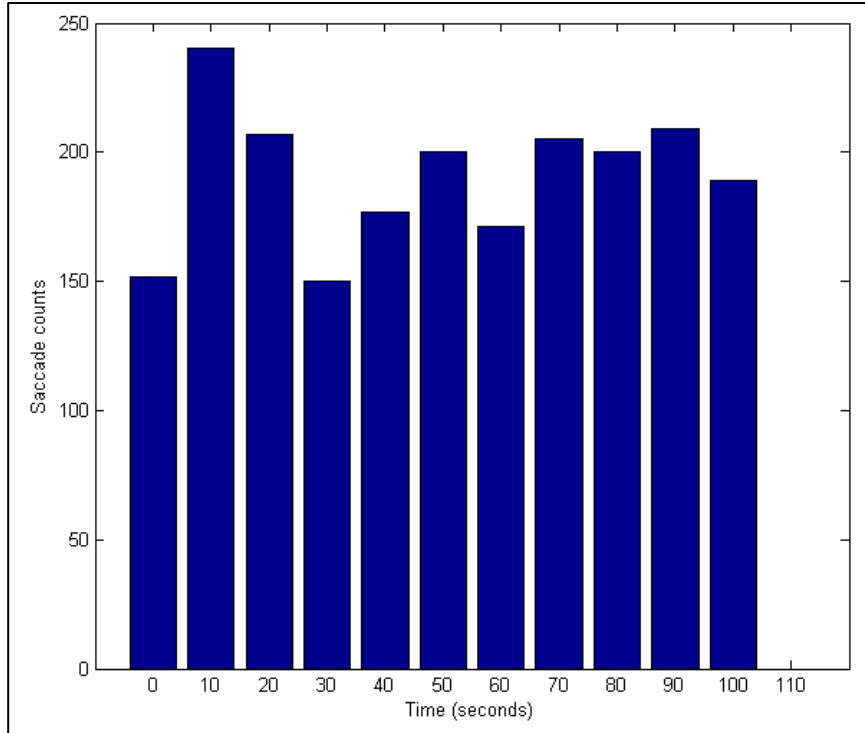


Figure 4-21. Histogram of Saccade Counts (participant 7, n-back 2)

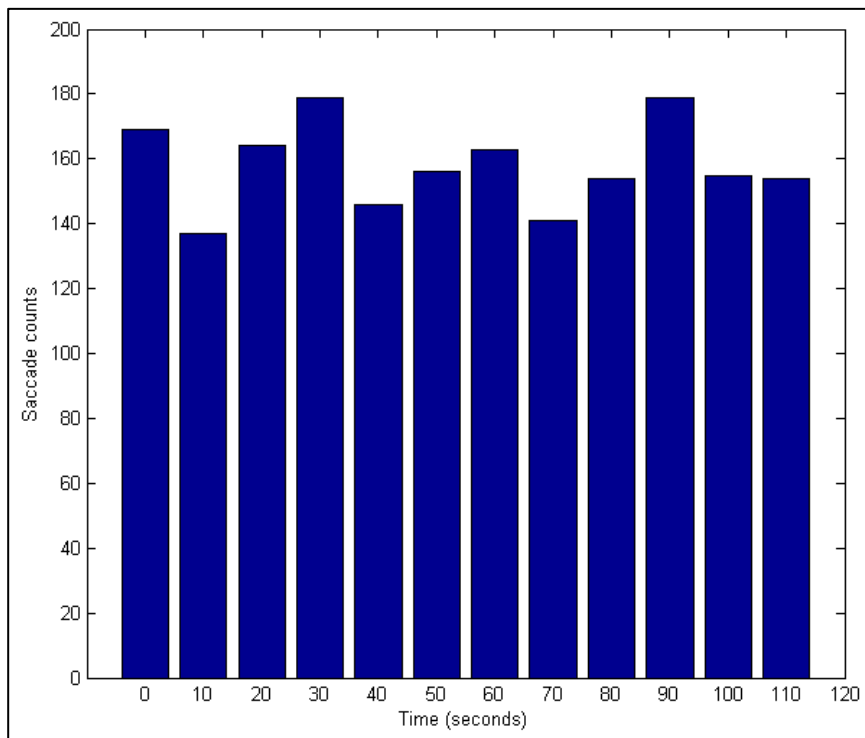


Figure 4-22. Histogram of Saccade Counts (participant 7, n-back 3)

4.4. Modeling and Simulation Results

This section considers the various stochastic models described in section 2.5, and their applicability to different aspects of visual attention. The first process considered is a Poisson process, and how accurately it describes the saccade patterns of the participants. As described in Section 2.5, the events of a Poisson process, X , occur randomly in time, and thus, follow a uniform distribution. In addition, $X(0) = 0$ and event occurrences in disjoint time intervals are independent. At the beginning of a run, no saccades have yet occurred, and thus the condition that $X(0) = 0$ is satisfied. Since saccade occurrences are not influenced by past saccades, the requirement for independence of events between disjoint time intervals is also satisfied.

In order to test whether the times at which the events occur follow a uniform distribution, the Kolmogorov-Smirnov test was used. A Kolmogorov-Smirnov plot is a graph of the empirical cumulative distribution function compared to the theoretical cumulative distribution function. The empirical distribution was obtained from the event times at which the saccades occurred. For any x , the empirical distribution function, $F_n(x)$, represents the proportion of observations less than or equal x and is defined as:

$$F_n(x) = \begin{cases} 0, & x < X(1) \\ \frac{1}{n}, & X(i) \leq x < X(i+1), i = 1, \dots, n-1 \\ 1, & x \geq X(n) \end{cases}$$

where n = the total number of observations or events. The theoretical cumulative distribution function, $F(x)$, represents the hypothesized distribution that the empirical data is expected to follow. As such, $F(x) = P(X \leq x)$, the probability of an observation value less than or equal to x . If the hypothesized model fails to account

for some aspect of the event behavior, then the lack of fit will be reflected in the Kolmogorov-Smirnov plot as a significant deviation from the theoretical cumulative distribution function.

The Kolmogorov-Smirnov test statistic was then calculated to obtain a numerical estimate of lack of fit. This test statistic equals the largest absolute vertical difference between the empirical and theoretical cumulative distribution functions. This test is useful in determining whether a model accurately describes the structure of the data. For this test, the null hypothesis is that the empirical data follow the hypothesized distribution. Therefore, since a significance level of 0.05 is used, any value exceeding 0.05, would fail to reject the null hypothesis, and thus, provide evidence to support that the empirical data follows the hypothesized distribution.

In Table 4-2, the Kolmogorov-Smirnov test is assessing whether or not the saccade occurrence times follow a uniform distribution, and thus, a Poisson process. The cells highlighted in magenta represent those showing support for the saccade occurrences following a uniform distribution. Of the 52 runs, only 21% failed to reject the null hypothesis, indicating evidence towards the data following a Poisson process. Thus, it is not likely that the saccades follow a Poisson process.

Participant	Kolmogorov-Smirnov Test: p-value for each run					
	Control 1	Control 2	Control 3	N-back 1	N-back 2	N-back 3
1	<0.0001	<0.0001	N/A	<0.0001	0.0004	0.0048
2	0.8547	<0.0001	0.0202	0.0047	<0.0001	0.2353
3	0.0002	0.0021	<0.0001	<0.0001	0.0273	<0.0001
4	0.2295	<0.0001	0.0014	<0.0001	<0.0001	0.1339
5	0.2061	<0.0001	0.0001	0.0664	<0.0001	N/A
6	0.0123	0.0400	0.0003	0.0264	0.1799	0.0417
7	0.2448	0.0105	<0.0001	<0.0001	0.0782	0.3555
8	0.0085	0.0025	0.3348	0.0005	0.0058	<0.0001
9	0.0060	<0.0001	0.0004	<0.0001	<0.0001	0.0006

Table 4-2. Kolmogorov-Smirnov Test for a uniform distribution of saccade event times to indicate whether the saccades follow a Poisson process

The following series of plots show a subset of the Kolmogorov-Smirnov plots testing whether the saccade occurrence times follow a uniform distribution. One aspect of concern is the runs in which the p-value does not appear to correspond with the plot. For example, there are certain plots with small p-values in which it appears that the empirical distribution closely follows the theoretical distribution and others with the opposite issue. These discrepancies may suggest that the Kolmogorov-Smirnov test is not the ideal test for this data. The Kolmogorov-Smirnov test requires that the cumulative distribution function be pre-determined [7]. It is not accurate if the parameters of the cumulative distribution function are estimated from the data [7]. In this case, the parameters were estimated from the data, and this inaccuracy may be represented in the discrepancies exhibited between the graphical representations and the p-values.

Figure 4-23 shows an example where the p-value is quite large, 0.235, indicating a failure to reject the null hypothesis, but a large discrepancy, 0.154, exists between the empirical and theoretical distributions. Another example of this discrepancy is

shown in Figure 4-24 where the maximum difference is 0.241 and the p-value equals 0.206. Figures 4-25 and 4-26 illustrate additional plots in which the p-values are large, indicating a failure to reject the null hypothesis. In these plots, however, the empirical distribution function appears to closely follow the theoretical one. The maximum differences between the distributions is about 0.02 for both plots; thus, corresponding appropriately to the p-value. Figures 4-27 and 4-28 illustrate plots with small p-values, indicating a rejection of the null hypothesis. In this set of plots, the empirical distribution appears to closely follow the theoretical one (with maximum differences of about 0.035), but the p-value does not agree with the graphical representation.

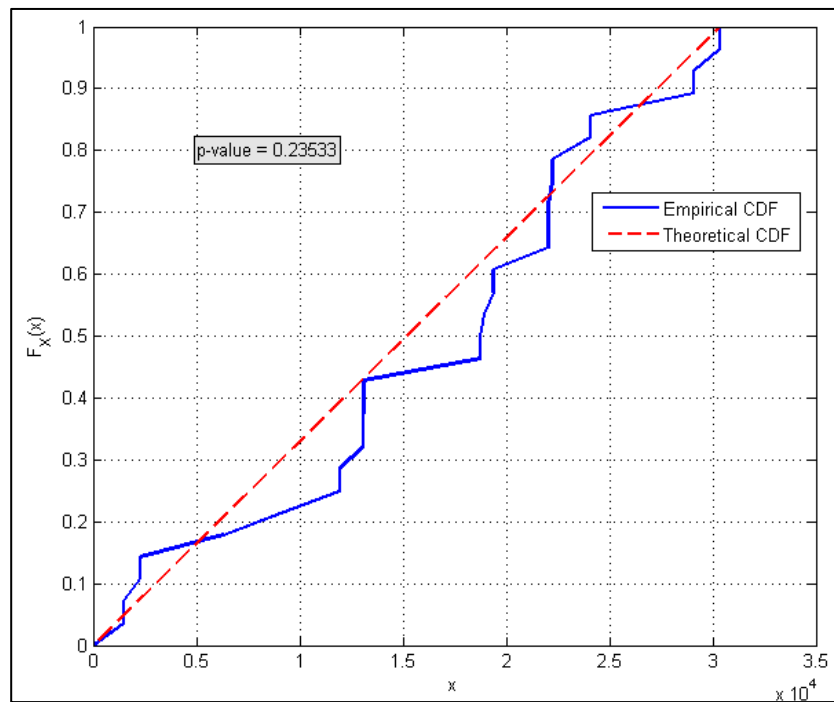


Figure 4-23. Distribution of Saccade Occurrence Times (participant 2, n-back 3): Illustrates large p-value with large discrepancy between empirical and theoretical cumulative distribution function

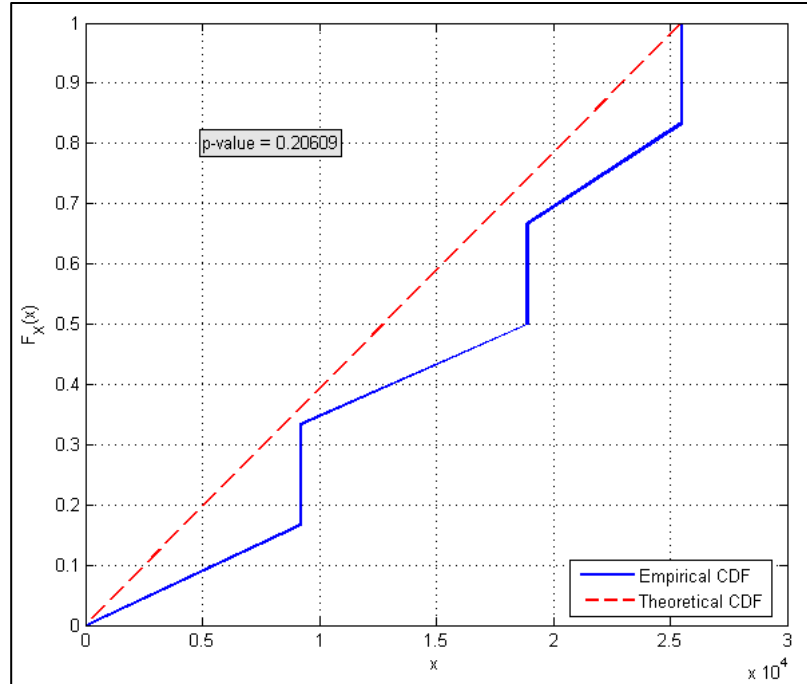


Figure 4-24. Distribution of Saccade Occurrence Times (participant 5, control 1): Illustrates large p-value with large discrepancy between empirical and theoretical cumulative distribution function

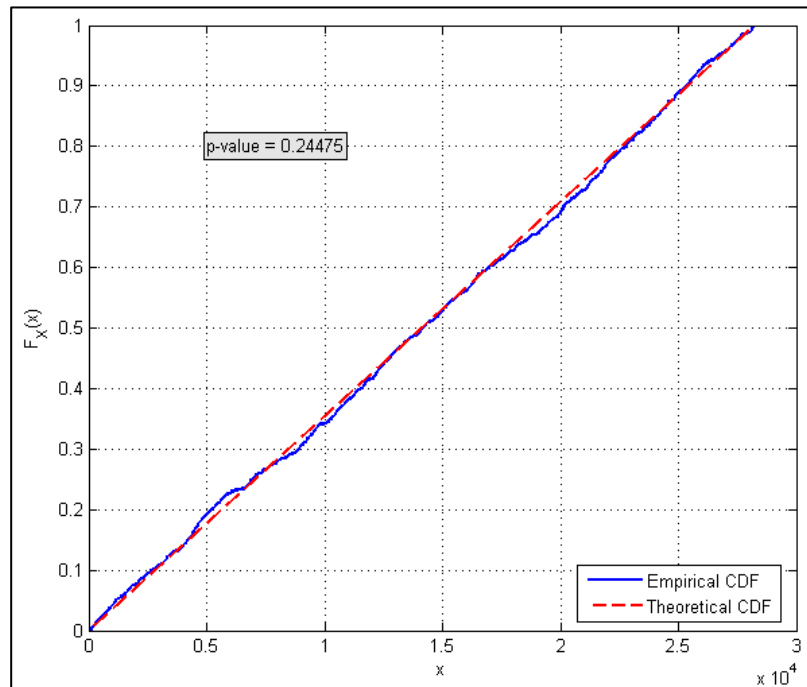


Figure 4-25. Distribution of Saccade Occurrence Times (participant 7, control 1): Agreement between p-value and graphical representation – empirical distribution follows the theoretical distribution

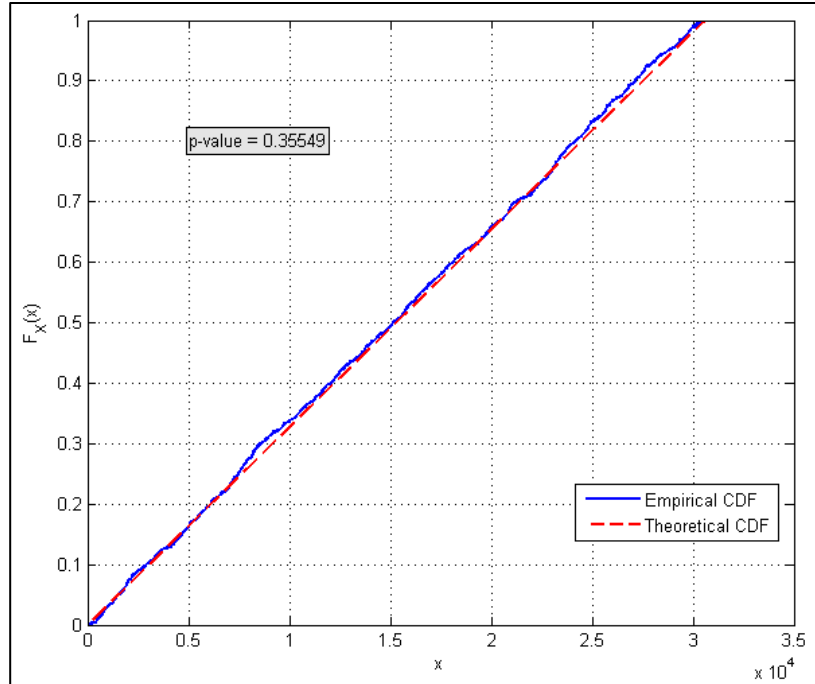


Figure 4-26. Distribution of Saccade Occurrence Times (participant 7, n-back 3): Agreement between p-value and graphical representation – empirical distribution follows the theoretical distribution

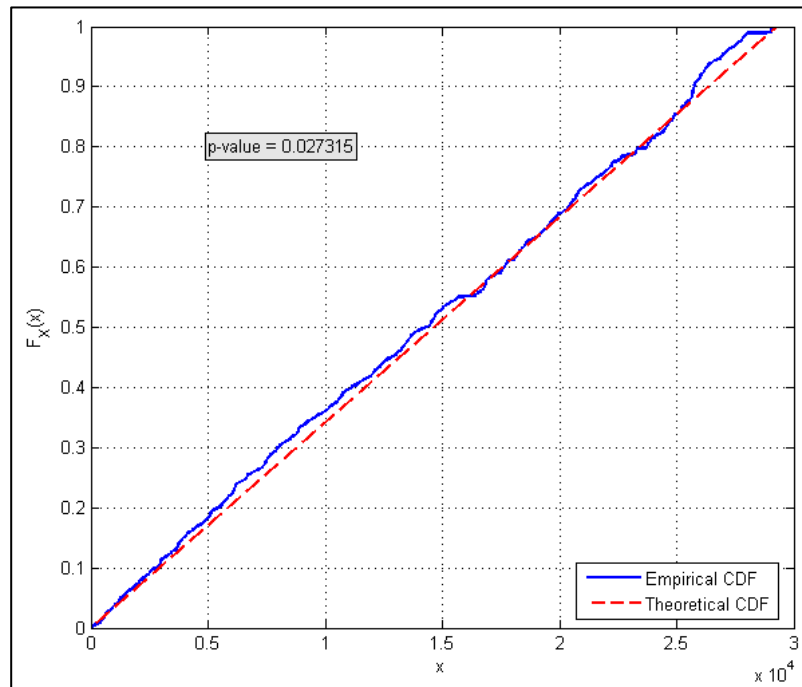


Figure 4-27. Distribution of Saccade Occurrence Times (participant 3, n-back 2): Illustrates small p-value (reject that data comes from the theoretical distribution) with small discrepancy between empirical and theoretical distributions

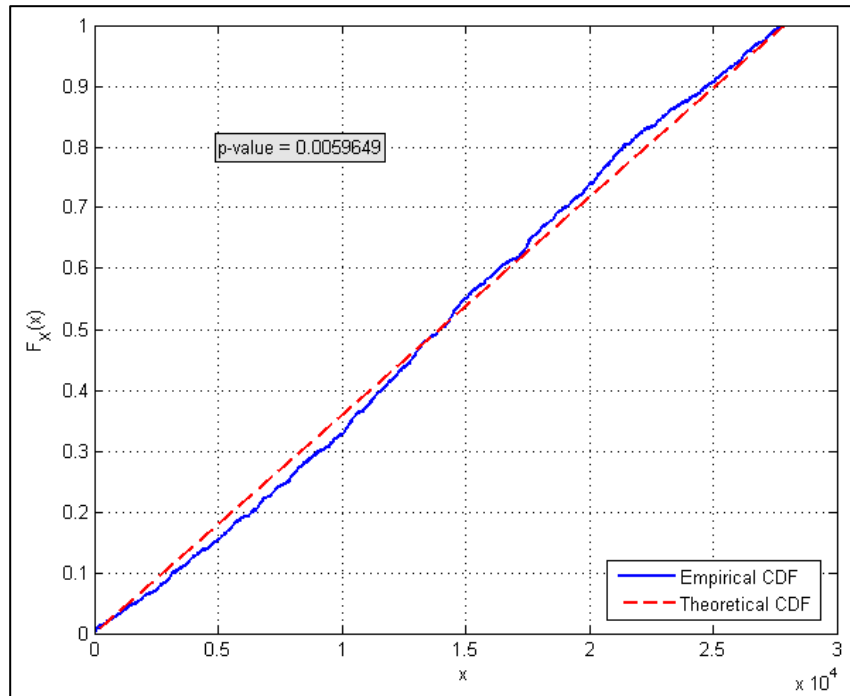


Figure 4-28. Distribution of Saccade Occurrence Times (participant 9, control 1): Illustrates small p-value (reject that data comes from the theoretical distribution) with small discrepancy between empirical and theoretical distributions

Due to the numerous discrepancies between the p-values and the graphical representations, additional distribution tests were explored. The Lilliefors test is a goodness-of-fit test suitable when a fully specified null distribution is unknown and the parameters must be estimated from the data [7]. The Lilliefors test statistic is the same as the Kolmogorov-Smirnov test, but the theoretical cumulative distribution function parameters are estimated from the data [7]. The table of critical values is calculated differently for the Lilliefors test. For each value of N , a large number of samples are drawn from the estimated distribution, and then used to determine the distribution of critical values [8]. One limitation of the Lilliefors test is that it is only able to test whether the empirical data comes from a distribution in the normal family,

as opposed to the Kolmogorov-Smirnov test, which allows a comparison of any two distribution functions [7]. Thus, instead of analyzing the distribution of the saccade event times, the distribution of the time between saccades was compared to an exponential distribution.

As noted previously, another indicator that a point process is Poisson is that its sojourn times (i.e., the time between events) follow an exponential distribution. The empirical cumulative distribution function was assessed against an exponential distribution at a 95% confidence level using the `lillietest` function in MATLAB. Thus, an additional limitation of this test was that a dataset required at least four sample points, and that p-values above 0.5 and below 0.001 were rounded to 0.5 and 0.001, respectively. Data which encountered data collection issues or contained less than four data points (i.e., less than four saccades), are shown as N/A in the table of p-values (see Table 4-3).

Razali and Wah (2011) compared the power of four formal tests of normality including the Kolmogorov-Smirnov and Lilliefors test. Power comparisons were obtained via Monte Carlo simulation of sample data generated from alternative distributions [9]. Samples sizes ranged from 10 to 2000 [9]. For small sample sizes, the power of each test varied considerably, but at a sample size of 2000, all tests obtained power values exceeding 80% [9]. In this study, the number of samples exceeds 20,000 for each run. Thus, for the distribution tests utilized, the power of each test is more than sufficient, with values exceeding 99% [9].

Participant	Lilliefors Test: p-value for each run					
	Control 1	Control 2	Control 3	N-back 1	N-back 2	N-back 3
1	N/A	N/A	N/A	<0.001	0.0026	0.0021
2	0.0163	>0.50	0.0420	<0.001	<0.001	0.0012
3	0.3730	0.0191	>0.50	0.1025	0.3007	0.0188
4	0.0149	<0.001	0.2799	<0.001	0.0117	0.2111
5	0.0054	0.0175	0.0131	0.0363	N/A	N/A
6	0.1648	<0.001	<0.001	0.0400	<0.001	0.0037
7	>0.50	0.4858	0.1394	>0.50	0.3355	0.0137
8	0.1420	0.2004	0.1097	0.1379	>0.50	0.3629
9	0.1626	0.1503	>0.50	>0.50	0.0112	0.0892

Table 4-3. Lilliefors Test for an exponential distribution of saccade sojourn times to indicate whether the saccades follow a Poisson process

The p-values appeared to better correspond to the graphical representations in this case, not only for the exponential distribution comparison, but also when reviewing the uniform distribution comparison plots previously illustrated. The following subset of figures illustrates both those empirical distributions that follow the theoretical one and those that do not. For this test, 46% of the 52 runs (assuming the three runs with an insufficient number of saccades do not follow an exponential distribution) resulted in evidence which supports that the saccades follow a Poisson process. The overall conclusion is still that the saccade occurrences do not follow a Poisson process.

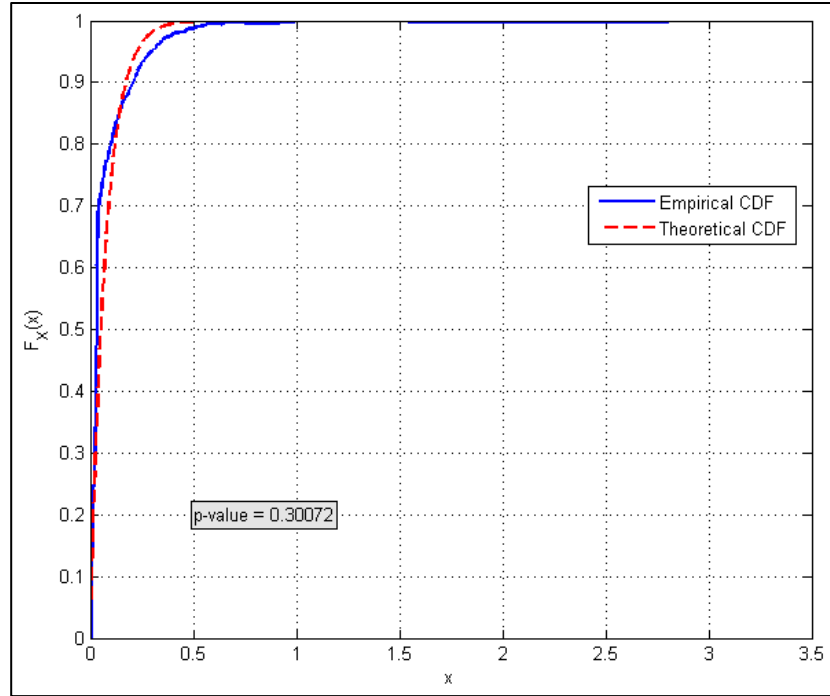


Figure 4-29. Distribution of Saccade Sojourn Times (participant 3, n-back 2): Empirical distribution follows the theoretical distribution

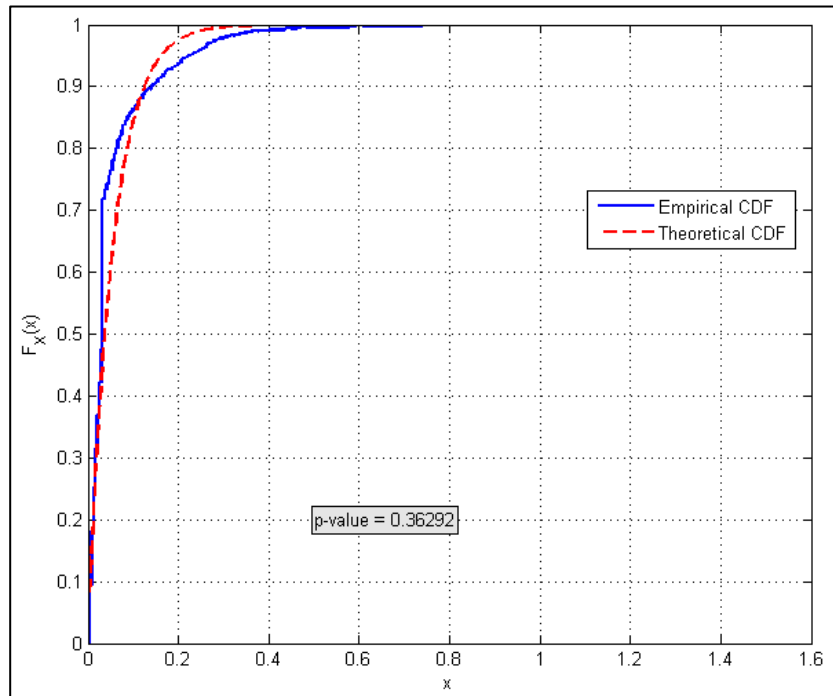


Figure 4-30. Distribution of Saccade Sojourn Times (participant 8, n-back 3): Empirical distribution follows the theoretical distribution

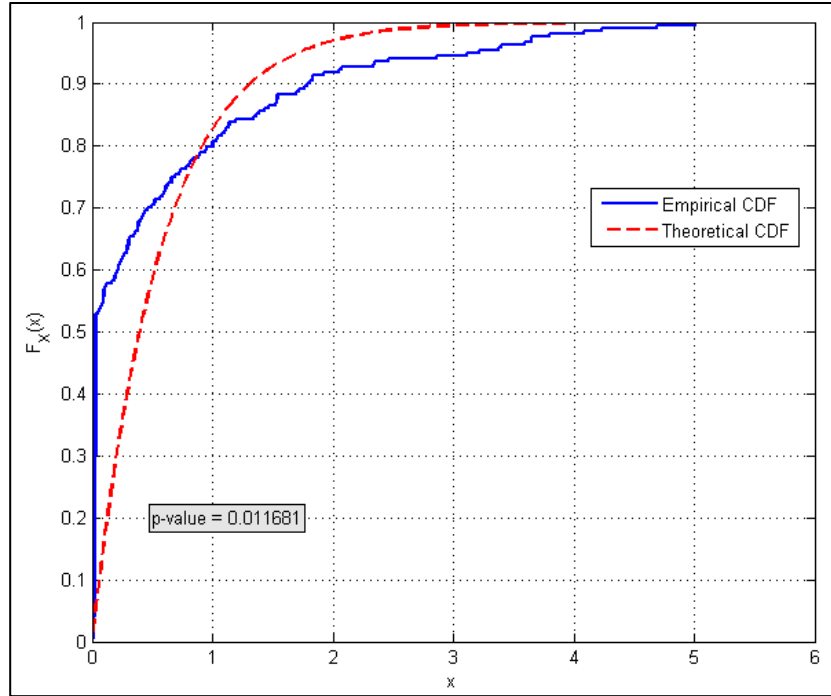


Figure 4-31. Distribution of Saccade Sojourn Times (participant 4, n-back 2): Empirical distribution does not follow the theoretical distribution

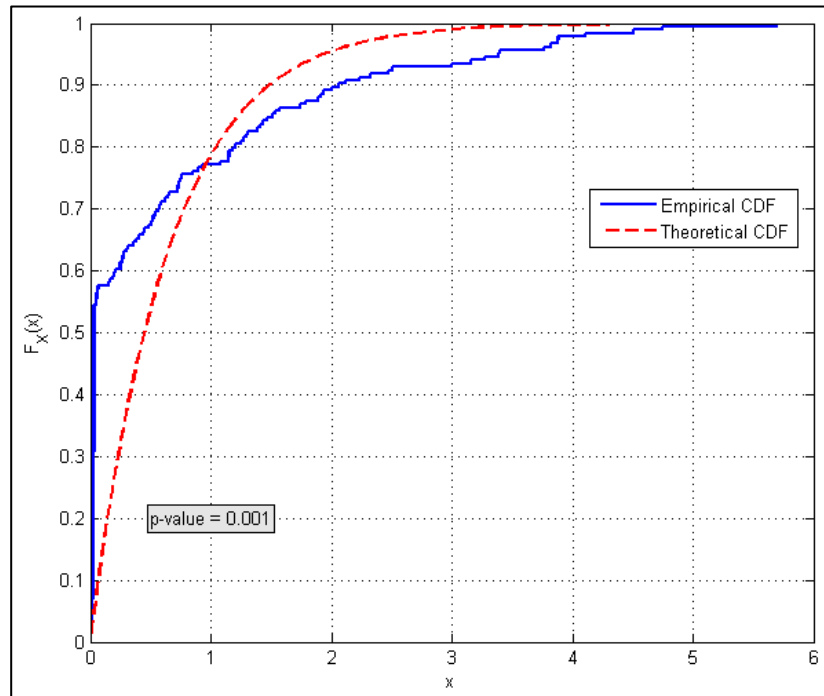


Figure 4-32. Distribution of Saccade Sojourn Times (participant 6, control 2): Empirical distribution does not follow the theoretical distribution

Table 4-4 summarizes the results in relation to the null hypothesis, H_0 : The sojourn times follow an exponential distribution. By definition of a Poisson process, the times between events should follow an exponential distribution. Thus, failure to reject the null hypothesis provides evidence to support the idea that the saccade occurrences follow a Poisson process. Overall, 46% of the 52 runs failed to reject the null hypothesis, providing some evidence that the saccades may follow a Poisson process. However, this does not provide enough evidence to support that the saccades follow a Poisson process. There was not a substantial difference between the control runs and the n-back runs with 53% of control and 38% of n-back runs failing to reject the null hypothesis.

	Control	N-back	Total
Reject H_0	12	16	28
Fail to Reject H_0	14	10	24

Table 4-4. Summary of Lilliefors Distribution Tests – Poisson Process

Another stochastic process considered is the Wiener process. In this section, the goal is to determine whether the gaze path, as defined by the eye movements made between sample times, follows a Wiener process. As defined in section 2.5, a stochastic process $B(t)$, $-\infty < t < \infty$, is a Wiener process with parameter σ^2 , if:

- (1) $B(0) = 0$,
- (2) $B(t_2) - B(t_1), \dots, B(t_n) - B(t_{n-1})$ are independent for $t_1 \leq t_2 \leq \dots \leq t_n$,
- (3) $B(t) - B(s)$ has a normal distribution with mean equal to zero and a variance of $\sigma^2(t - s)$ for $s \leq t$.

As noted before, the first two conditions are clearly satisfied, and thus, testing the third condition is of interest here. For both the horizontal and vertical positions, $B(t) - B(s)$ was calculated for consecutive time samples, after removing blinks and saccades. In addition, data values corresponding to angular velocities exceeding $1000^\circ/\text{second}$ were removed since these data represent a movement of the eye that is not physiologically possible [2]. The distribution of the differences in position was then tested for normality at a 95% confidence level using the `lillietest` function in MATLAB. For the Lilliefors test, all runs (i.e., for all participants, in both the horizontal and vertical dimensions, and both with and without saccades) resulted in p-values equal to 0.5.

	Horizontal	Vertical
Reject H_0	0	0
Fail to Reject H_0	53	53

Table 4-5. Summary of Lilliefors Distribution Tests for a Wiener process

For both the horizontal differences and vertical differences in movement between samples, 100% of the 53 runs failed to reject the null hypothesis. This provides strong evidence that the gaze path in each dimension can be modeled as Brownian motion.

Due to the limitations in minimum and maximum p-values, additional tests for normality were considered. The Jarque-Bera test assesses the hypothesis that the sample data follow a normal distribution with unknown mean and variance, against the alternative that the data does not come from a normal distribution. The test statistic for this test is: $JB = \frac{n}{6} \left(s^2 + \frac{(k-3)^2}{4} \right)$ where n = sample size, s = skewness of the sample data, and k = kurtosis of the sample data [7]. The `jbtest` function in MATLAB was

used. This function replaces the chi-square approximation of the test statistic with a more accurate algorithm that interpolates p-values from a table of quantiles [7]. The resulting Jarque-Bera test p-values for the difference in position, both with and without saccades, are shown in Table 4-6.

Participant	Run	Saccades removed		Saccades included	
		Horizontal	Vertical	Horizontal	Vertical
1	Control 1	0.5000	0.4883	0.5000	0.5000
	Control 2	0.4357	0.3542	0.5000	0.3542
	Control 3	0.5000	0.3201	0.5000	0.3201
	N-back 1	0.3591	0.3612	0.1511	0.1691
	N-back 2	0.2582	0.4920	0.1646	0.2207
	N-back 3	0.3599	0.4881	0.5000	0.4222
2	Control 1	0.4070	0.2823	0.4341	0.2848
	Control 2	0.1747	0.1793	0.0808	0.1214
	Control 3	0.5000	0.1549	0.0996	0.1336
	N-back 1	0.5000	0.5000	0.2631	0.2288
	N-back 2	0.1003	0.5000	0.1115	0.3927
	N-back 3	0.2358	0.2680	0.1715	0.2554
3	Control 1	0.5000	0.2082	0.0884	0.1533
	Control 2	0.5000	0.3414	0.2664	0.2323
	Control 3	0.4767	0.5000	0.0815	0.1707
	N-back 1	0.5000	0.5000	0.3533	0.2354
	N-back 2	0.3017	0.4336	0.0731	0.1966
	N-back 3	0.5000	0.0955	0.0705	0.2678
4	Control 1	0.5000	0.3223	0.1393	0.3590
	Control 2	0.4211	0.3080	0.1199	0.2713
	Control 3	0.5000	0.1994	0.0901	0.0833
	N-back 1	0.2024	0.5000	0.2877	0.2109
	N-back 2	0.5000	0.2840	0.1255	0.1601
	N-back 3	0.4217	0.1818	0.1130	0.0826
5	Control 1	0.2987	0.5000	0.4113	0.5000
	Control 2	0.3315	0.2987	0.1760	0.2643
	Control 3	0.2840	0.4983	0.1194	0.1916
	N-back 1	0.5000	0.3339	0.5000	0.3339
	N-back 2	0.2704	0.3084	0.2704	0.3084

Table 4-6. P-values for the Jarque-Bera Test for a normal distribution to indicate whether the gaze path follows a Wiener process

Participant	Run	Saccades removed		Saccades included	
		Horizontal	Vertical	Horizontal	Vertical
6	Control 1	0.4588	0.2993	0.0727	0.1868
	Control 2	0.5000	0.5000	0.3179	0.2182
	Control 3	0.3481	0.2921	0.1380	0.3134
	N-back 1	0.0804	0.1765	0.0703	0.1039
	N-back 2	0.3203	0.3015	0.3403	0.3665
	N-back 3	0.5000	0.2261	0.0999	0.1656
7	Control 1	0.0844	0.3635	0.0624	0.3246
	Control 2	0.0906	0.2321	0.0716	0.1496
	Control 3	0.1343	0.1659	0.0686	0.1214
	N-back 1	0.1336	0.1444	0.0818	0.1266
	N-back 2	0.0833	0.1368	0.0608	0.1358
	N-back 3	0.1299	0.1398	0.0670	0.1435
8	Control 1	0.4276	0.3779	0.1018	0.1586
	Control 2	0.2207	0.2484	0.2337	0.0968
	Control 3	0.3877	0.2254	0.2279	0.0782
	N-back 1	0.3336	0.2914	0.1605	0.2118
	N-back 2	0.3389	0.3107	0.0791	0.3803
	N-back 3	0.1522	0.3788	0.0659	0.1643
9	Control 1	0.1334	0.1245	0.0891	0.0860
	Control 2	0.1239	0.1203	0.0737	0.1149
	Control 3	0.2497	0.1335	0.0770	0.0732
	N-back 1	0.1262	0.1462	0.0747	0.0771
	N-back 2	0.1027	0.0878	0.0886	0.0896
	N-back 3	0.1067	0.0916	0.0792	0.0709

Table 4-6 (cont.) P-values for the Jarque-Bera Test for a normal distribution to indicate whether the gaze path follows a Wiener process

In terms of overall rejection or failure to reject the null hypothesis, the results of the Jarque-Bera test resulted in the same conclusions as those from the Lilliefors test. Though there was more variation in the p-values, 100% of the 53 still failed to reject the null hypothesis, which provided support that the data could be modeled as a Wiener process. The following plots illustrate the comparisons between the empirical distribution and the theoretical distribution, a normal distribution with mean equal to zero and a standard deviation estimated from the data.

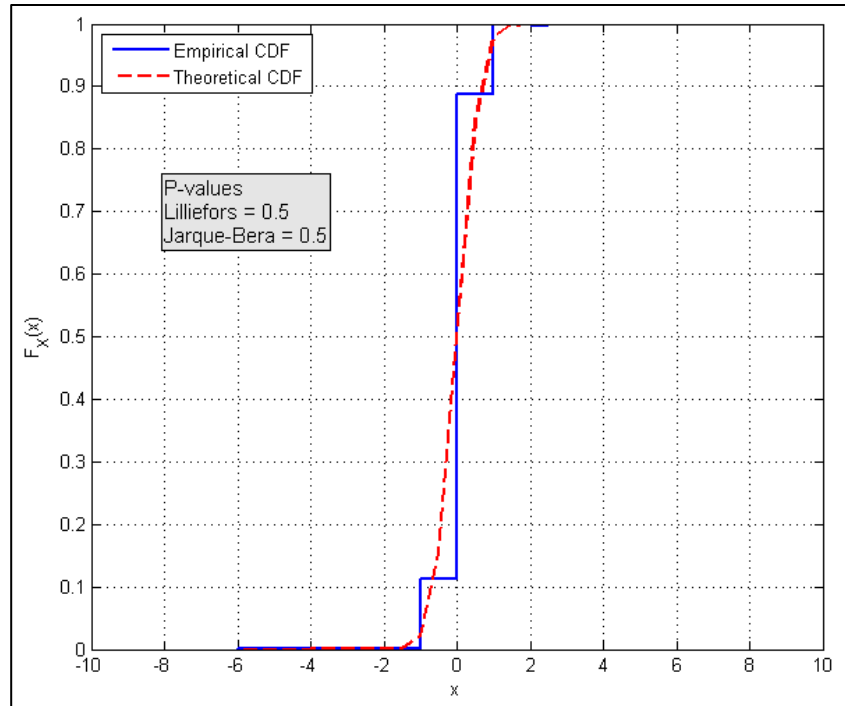


Figure 4-33. Length of Horizontal Changes in Eye Position (participant 1, control 1): Empirical distribution follows theoretical distribution

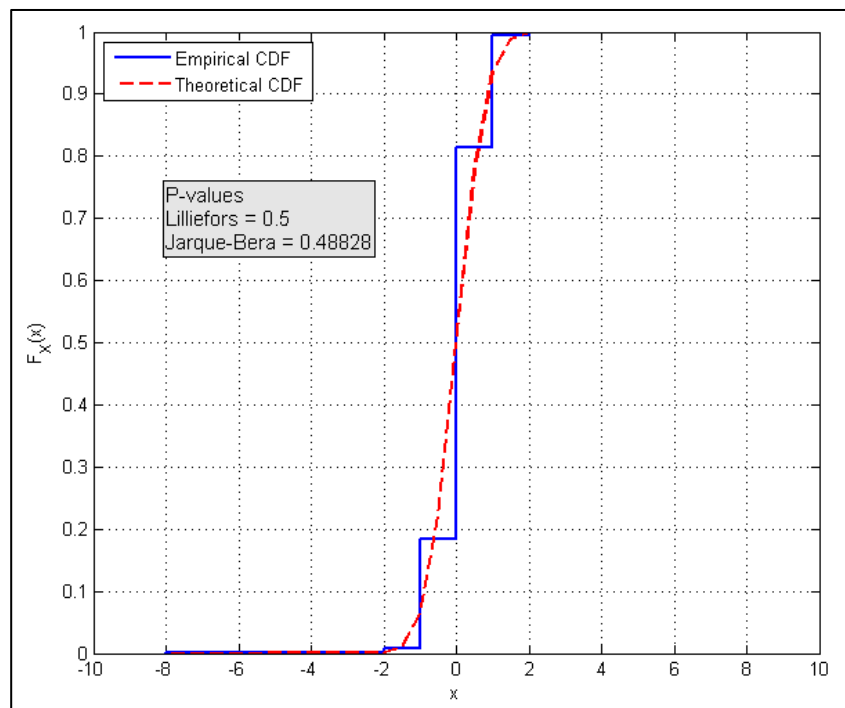


Figure 4-34. Length of Vertical Changes in Eye Position (participant 1, control 1): Empirical distribution follows theoretical distribution

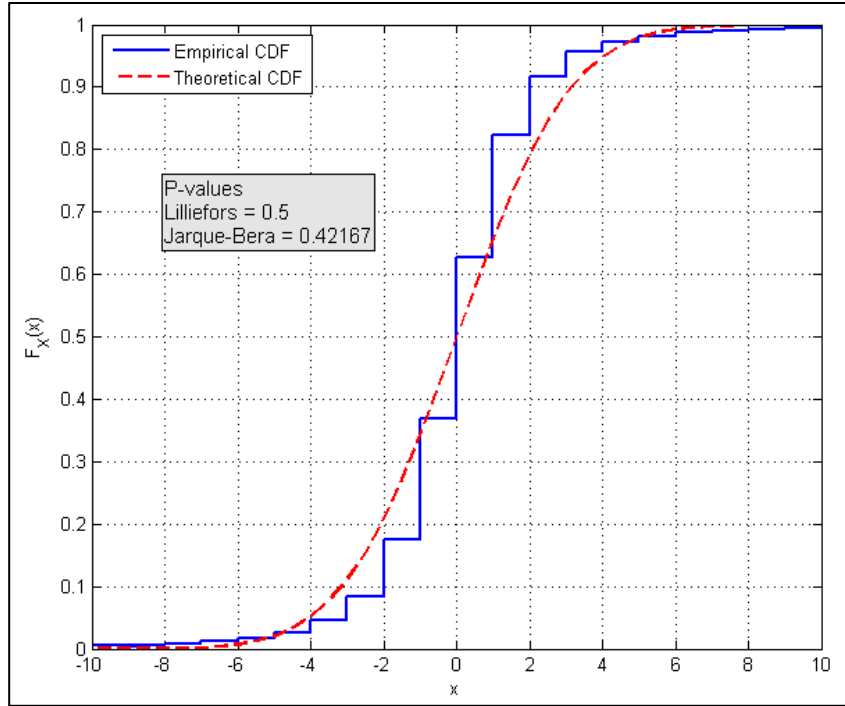


Figure 4-35. Length of Horizontal Changes in Eye Position (participant 4, n-back 3): Empirical distribution follows theoretical distribution

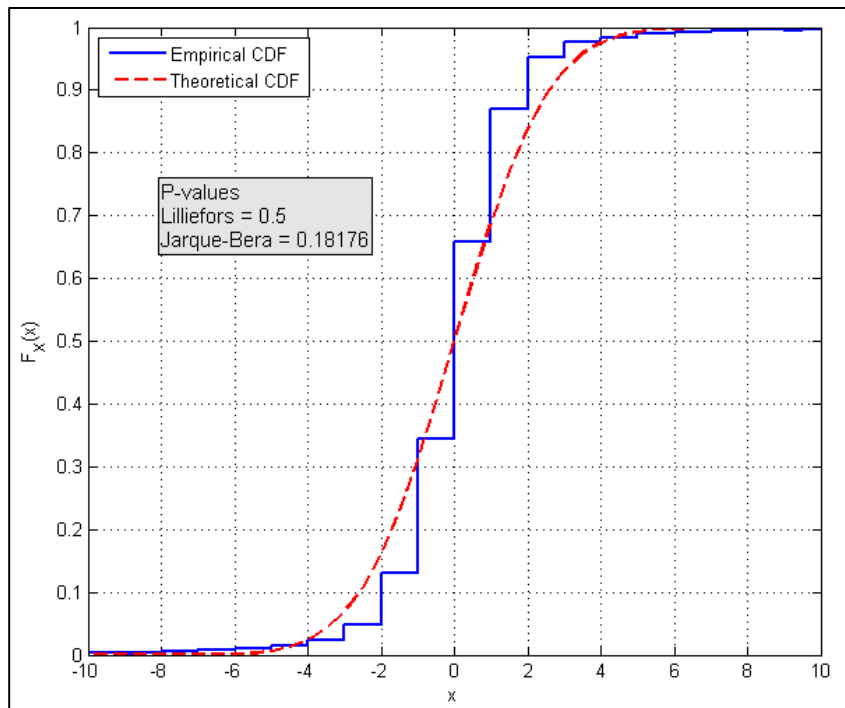


Figure 4-36. Length of Vertical Changes in Eye Position (participant 4, n-back 3): Empirical distribution follows theoretical distribution

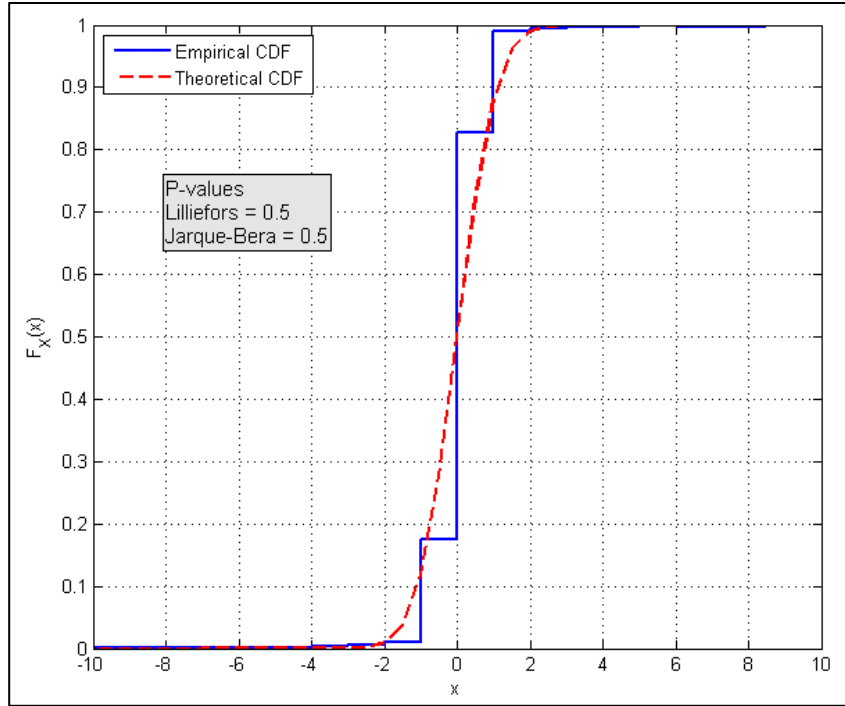


Figure 4-37. Length of Horizontal Changes in Eye Position (participant 6, control 2): Empirical distribution follows theoretical distribution

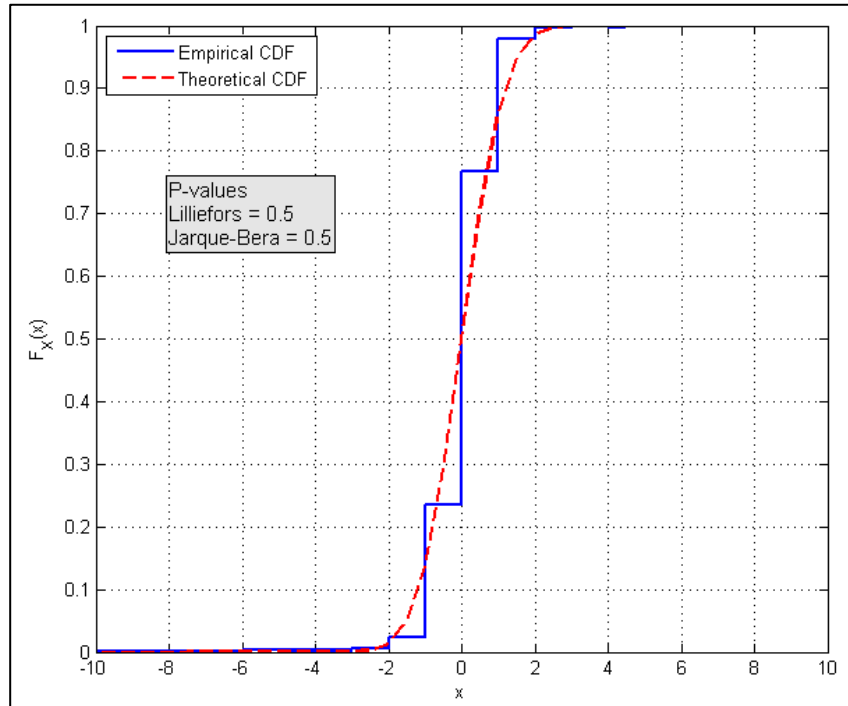


Figure 4-38. Length of Vertical Changes in Eye Position (participant 6, control 2): Empirical distribution follows theoretical distribution

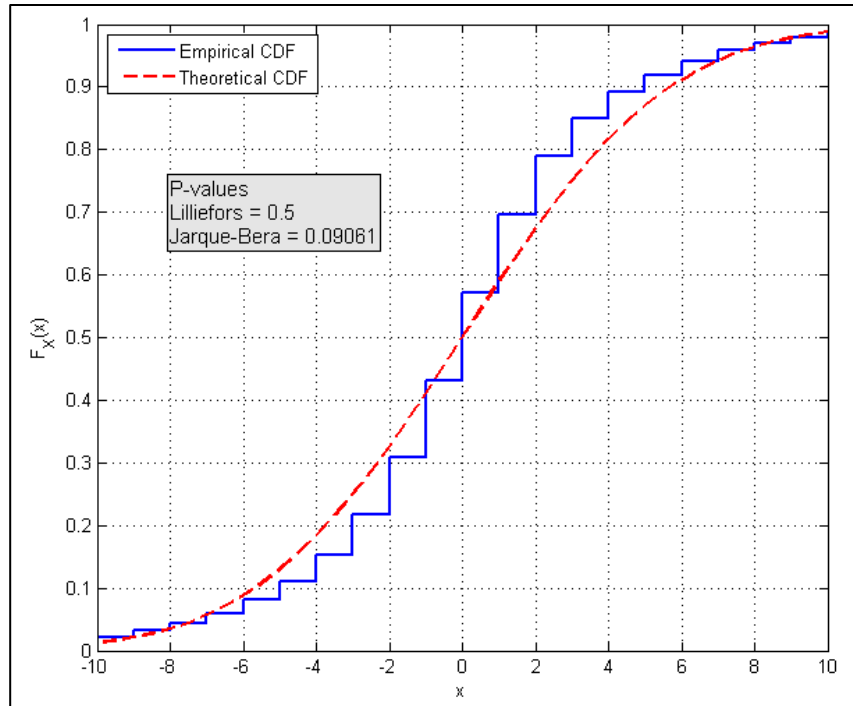


Figure 4-39. Length of Horizontal Changes in Eye Position (participant 7, control 2): Empirical distribution follows theoretical distribution

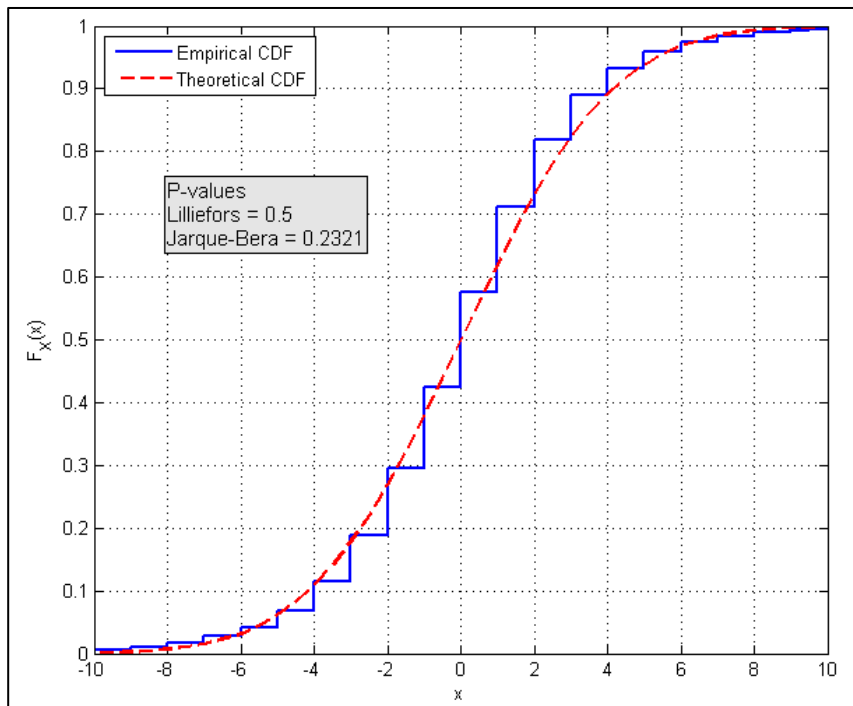


Figure 4-40. Length of Vertical Changes in Eye Position (participant 7, control 2): Empirical distribution follows theoretical distribution

The plots illustrate the differences in p-values between the Lilliefors and Jarque-Bera tests. Though the overall conclusions drawn were always the same, the Jarque-Bera test provided additional variation in the p-values that corresponded better with the graphical depiction and reflected the differences in the distribution functions. A summary of the Jarque-Bera test conclusions is shown in Table 4-7. Overall, 100% of the 53 runs provide support that the eye movements follow Brownian motion.

Saccades:	Horizontal Direction		Vertical Direction	
	Removed	Included	Removed	Included
Reject H_0	0	0	0	0
Fail to Reject H_0	53	53	53	53

Table 4-7. Summary of Jarque-Bera Distribution Tests for a Wiener process; Note that for the columns labeled removed or included, this refers to whether saccades were removed from the data before the analysis

This analysis was extended to consider whether the distribution changed as the differences in position increased to two samples, three samples, four samples, five samples, and ten samples. For a Wiener process, $B(t) - B(s)$ should follow a normal distribution with mean equal to zero and variance equal to $\sigma^2(t - s)$. Thus, as the distance between t and s increases, it is expected that the variance will increase accordingly. For instance, if $(t - s) = 3$, then the proportion of variances between this three sample case and the consecutive sample case (i.e., $(t - s) = 1$) should be equal to three. The results are summarized in Table 4-8 and Table 4-9.

	<i>Reject H_0</i>		<i>Fail to Reject H_0</i>	
	<i>Horizontal</i>	<i>Vertical</i>	<i>Horizontal</i>	<i>Vertical</i>
$(t - s) = 1$	0	0	53	53
$(t - s) = 2$	4	4	49	49
$(t - s) = 3$	2	1	51	52
$(t - s) = 4$	5	4	48	49
$(t - s) = 5$	5	5	48	48
$(t - s) = 10$	16	13	37	40

**Table 4-8. Summary of Jarque-Bera Distribution Tests (with saccades removed)
Comparison of different amounts of time between samples**

	<i>Reject H_0</i>		<i>Fail to Reject H_0</i>	
	<i>Horizontal</i>	<i>Vertical</i>	<i>Horizontal</i>	<i>Vertical</i>
$(t - s) = 1$	0	0	53	53
$(t - s) = 2$	2	3	51	50
$(t - s) = 3$	1	1	52	52
$(t - s) = 4$	3	3	50	50
$(t - s) = 5$	7	1	46	52
$(t - s) = 10$	17	11	36	42

**Table 4-9. Summary of Jarque-Bera Distribution Tests (with saccades included)
Comparison of different amounts of time between samples**

As the number of time samples between eye positions increased in the calculation of horizontal and vertical differences, the results still strongly supported that the data follow a normal distribution. For the number of time samples up to four, more than 90% of the runs failed to reject the null hypothesis. This number dropped slightly to 86% for five samples between positional difference calculations, but still showed strong support that the data followed a normal distribution. At ten samples, a little more than 70% of the runs failed to reject the null hypothesis. However, the variance proportions did not increase as expected by a Wiener process as seen in Table 4-10. Table 4-10 shows the average variance proportion by distance between time

samples. The proportions are grouped by direction of movement and saccade inclusion (or removal).

Saccades:	Horizontal Direction		Vertical Direction	
	Removed	Included	Removed	Included
$(t - s) = 2$	2.149	2.092	2.557	2.333
$(t - s) = 3$	3.324	3.280	4.138	3.724
$(t - s) = 4$	4.677	4.603	5.824	5.194
$(t - s) = 5$	6.149	5.998	7.624	6.731
$(t - s) = 10$	11.707	11.920	13.382	12.160

Table 4-10. Average proportion of variances between the time sample differences shown and the consecutive time sample case, i.e., $(t - s) = 1$

In the vertical dimension, there was more variation than in the horizontal dimension. For $(t - s) = 2$ or 3, the variance proportion values were quite close to the expected values. However, as the distance between time samples increased up to ten samples, the smallest average proportion observed was 11.707 when it should have been equal to 10. This suggests that as the distance between samples increases, at some point, the differences in position will no longer follow a Wiener process.

Additional analysis of the parameter fits for the mean and standard deviation of the distributions was executed. A statistical summary of the results is provided in Appendix B for each type of run and for the analysis both with and without saccades. For each comparison, the mean and variance for each pair of datasets were assessed. In order to compare the variance, the MATLAB function `vartest2` was used. This function tests the null hypothesis that the variances of the two distributions are equal. This result was used in comparing the means using `ttest2` in MATLAB. This function tests the null hypothesis that the means of the two datasets are equal, and an

equal or unequal variance assumption must be defined. The results from the various comparisons are shown in Table 4-11. For the individual Jarque-Bera p-values and parameter fit values by participant, see tables B-3 through B-11 in Appendix B.

Comparison description	P-value: Mean test	P-value: Variance test
Control vs. N-back: Horizontal (without saccades)	0.3478	<0.0001
Control vs. N-back: Horizontal (with saccades)	0.3346	<0.0001
Control vs. N-back: Vertical (without saccades)	0.2706	<0.0001
Control vs. N-back: Vertical (with saccades)	0.3817	<0.0001
With vs. Without Saccades: Horizontal (control)	0.9999	<0.0001
With vs. Without Saccades: Horizontal (n-back)	0.7324	<0.0001
With vs. Without Saccades: Vertical (control)	0.9999	<0.0001
With vs. Without Saccades: Vertical (n-back)	0.9914	<0.0001
With vs. Without Saccades: Horizontal (both run types)	0.9419	<0.0001
With vs. Without Saccades: Vertical (both run types)	0.5633	<0.0001
Horizontal vs. Vertical: Control (without saccades)	0.0155	<0.0001
Horizontal vs. Vertical: Control (with saccades)	0.0163	<0.0001
Horizontal vs. Vertical: N-back (without saccades)	0.0748	<0.0001
Horizontal vs. Vertical: N-back (with saccades)	0.3931	<0.0001

Table 4-11. Hypothesis testing for differences in means and variances between different subsets of the data

All the tests resulted in a rejection of the null hypothesis that the variances were equal. For the means, when comparing with versus without saccades, all the tests failed to reject the null hypothesis that the means were equal. Thus, for the comparison of with versus without saccades, there was no difference in means but there was a difference in variances. The runs with saccades consistently resulted in larger variances than those without the saccades. For the horizontal versus vertical differences in positions, the control runs rejected the hypothesis that the means are

equal, but the n-back runs failed to reject the null hypothesis. Hence, there was a difference in the variances between the horizontal and vertical dimensions, and between the means for the control runs. For the comparison of means between the control and n-back runs, in both dimensions, there was a failure to reject the null hypothesis. There was a difference in variances between the control and n-back runs, but there was no difference in means. The variances for the n-back runs were consistently larger than those for the control runs.

Additional analyses regarding whether the gaze path followed a random walk were executed. The distribution of the angle or direction of movement was analyzed. In comparison to a uniform distribution, when saccades were removed, 60% of the 53 runs failed to reject the null hypothesis. This provides support that the angle of eye movement, in two dimensions, followed a uniform distribution. For the data with saccades, only 21% of the 53 runs failed to reject the null hypothesis. Thus, there was a significant difference in the distributions of the angle of eye movement between the data with and without saccades. When saccades were removed, there is more support for the gaze path following a random walk, compared to when saccades are included in the assessment.

Recall that S_n represents the position of the eye at the end of n movements. The distribution of the sequence of $\{S_k\}_{k=1}^n$ was also examined. However, the resulting empirical distribution functions were varied, and without a consistent pattern. The distributions were first tested against a normal distribution and then a uniform distribution, but neither showed promising results. Thus, from this data, no conclusions can be drawn regarding the distribution of S_n .

For each of the distribution tests, simulation runs were also executed and assessed for validation. Multiple sets of 1000 random variables were generated in MATLAB for each of the distributions tested, i.e., uniform distribution, exponential distribution, and normal distribution. For the uniform distribution, each set of random variables was distributed over the same specified range, [0, 120], to represent the two-minute test runs. For the exponential distribution, the mean parameter value was varied over 40 different data points based on the range of mean values in the collected data set. The same approach was followed for determining the standard deviation values for the normal random variables generated. The mean value for the normal random variables was set at zero for all the simulation runs.

The tests used to assess the uniform, exponential, and normal distributions for the collected data were then used on the simulated data for comparison. For each test, the random variables corresponding to the same distribution as the test were assessed, as well as a set of random variables corresponding to a different distribution. First, the uniform theoretical distribution was tested against the set of uniform random variables and the set of exponential random variables using the Kolmogorov-Smirnov test. Thus, the null hypothesis is that the data (i.e., simulated set of random variables) follow a uniform distribution. The results are summarized in Table 4-12. The cells highlighted in yellow represent that the correct conclusion was drawn, i.e., reject the null hypothesis that the exponential random variables follow a uniform distribution. Ideally, all of the data points would fall within these cells.

	Uniform random variables	Exponential random variables
Reject H_0	8	200
Fail to Reject H_0	192	0

Table 4-12. Kolmogorov-Smirnov test results; simulated sets of both uniform and exponential random variables were compared to a uniform distribution function

The exponential distribution was assessed using the Lilliefors test. For the exponential distribution, a set of exponential random variables and a set of uniform random variables were tested. The null hypothesis is that the data follow an exponential distribution, and the results are summarized in Table 4-13.

	Exponential random variables	Uniform random variables
Reject H_0	11	200
Fail to Reject H_0	189	0

Table 4-13. Lilliefors test results; simulated sets of both uniform and exponential random variables were compared to an exponential distribution function

The last set of tests assessed how a simulated set of random variables (generated from a normal distribution or from an exponential distribution) compared to a normal distribution. The null hypothesis is that the data follow a normal distribution with mean equal to zero and a standard deviation estimated from the data. The results are summarized in Table 4-14 and Table 4-15 for the Lilliefors and Jarque-Bera tests, respectively.

	Normal random variables	Exponential random variables
Reject H_0	8	200
Fail to Reject H_0	192	0

Table 4-14. Lilliefors test results; simulated sets of both normal and exponential random variables were compared to a normal distribution function

	Normal random variables	Exponential random variables
Reject H_0	25	200
Fail to Reject H_0	175	0

Table 4-15. Jarque-Bera test results; simulated sets of both normal and exponential random variables were compared to a normal distribution function

In all the simulation results, the test never identified a set of random variables as following the specified distribution, when in fact it was generated from a different distribution. For instance, of the 200 trials testing the set of exponential random variables against a normal distribution, all 200 trials rejected the null hypothesis that the data followed a normal distribution. This was consistent across all distribution tests. Thus, these tests generated no false positives. This is important because it assures that those which failed to reject the null hypothesis, were actually variables generated from the same distribution being tested.

On the other hand, there were instances in which a set of random variables (e.g., normal random variables) generated from the specified distribution rejected the null hypothesis that the data followed that specified distribution (e.g., the normal distribution). Thus, there are instances in which the tests may be too rigid or the data, though generated from the specified distribution, still results in test statistics exceeding the acceptable value. It appears that the Jarque-Bera test was more

conservative than the Lilliefors test in failing to reject the null hypothesis when the data was indeed generated from a normal distribution. Thus, for the summary of results when testing for Brownian motion, the results from the Jarque-Bera test are used. Overall, the results from the simulated sets of data provide evidence that when a test failed to reject the null hypothesis, that data did indeed follow the specified distribution.

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CHAPTER 5

CONCLUSIONS

Eye tracker data was collected on nine participants for a series of runs. Two types of runs were considered, control and n-back runs. In both types of runs, the visual task was simple: focus on a single dot on a screen for two minutes. The n-back runs extended the task to include an auditory task in which a stream of numbers was presented to the participant. While focusing on the dot, and listening to the number stream, participants were asked to identify when the current number matched the number presented two steps previously. The goal of this research was to identify how the eyes moved when a participant is simply focusing on a single location. Unlike most of the experiments in the literature, the goal was not to identify how distractors affected eye movements or how one performed in a visual search task, but rather to provide a foundational model for basic eye movement.

Three stochastic processes were considered in terms of modeling the focus of attention. The first process considered was a Poisson process, and how accurately it describes the saccade patterns of the participants. First, the saccade occurrence times were assessed using a Kolmogorov-Smirnov test. This resulted in 21% of the 52 runs supporting the hypothesis that the data followed a Poisson process. However, due to discrepancies between the graphical representation and the test statistics, additional distribution test statistics were considered. The Lilliefors test was then used to assess the times between saccades (i.e., the sojourn times). The sojourn times were compared

to an exponential distribution; for this test, 46% of the 52 runs resulted in evidence, which supports that the saccades follow a Poisson process. There was not a substantial difference between the control runs and the n-back runs with 53% of control and 38% of n-back runs failing to reject the null hypothesis. Overall, the evidence was not strong enough to support that the saccades followed a Poisson process.

One goal of this research was to determine the distribution of saccades. Different algorithms for saccade detection were considered as well as how noise affected the saccade detection. A saccade detection algorithm was implemented and histograms of saccade occurrences were analyzed. However, there was insufficient evidence to support that the saccade occurrences consistently follow one distribution across participants. Some participants exhibited similar patterns of saccades throughout while others varied considerably within the six runs. Thus, future work could extend the data collection and assessment to determine the distribution of the saccade occurrences.

Another stochastic process considered was the Wiener process. The goal was to determine whether the gaze path, as defined by the eye movements made between sample times, follows a Wiener process. For this process, the length of eye movements made between samples, $B(t) - B(s)$, follows a normal distribution with mean equal to zero and a variance of $\sigma^2(t - s)$ for $s \leq t$. For both the horizontal and vertical positions, $B(t) - B(s)$ was calculated for consecutive time samples, after removing blinks and saccades, and then tested for normality. For the consecutive time sample case, 100% of the 53 runs provided support to the hypothesis that the data followed a normal distribution. This provides strong evidence that the gaze path in each dimension follows Brownian motion.

The analysis extended to consider whether the distribution changed as the differences in position increased to two samples, three samples, four samples, five samples, and ten samples. As the number of time samples between eye position differences calculations, the results still strongly supported that the data followed a normal distribution. For the number of time samples up to four, more than 90% of the runs failed to reject the null hypothesis. This number dropped slightly, to 86%, for five samples between positional difference calculations, but still showed strong support that the data followed a normal distribution. At ten samples, a little more than 70% of the runs failed to reject the null hypothesis. However, the variance proportions did not increase as expected by a Wiener process. This suggests that as the distance between samples increases, at some point, the differences in position will no longer follow a Wiener process.

Additional analysis of the parameter fits for the mean and standard deviation of the distributions was executed. All the tests resulted in a rejection of the null hypothesis that the variances were equal. For the comparison of with versus without saccades, there was no difference in means, but the runs with saccades resulted in consistently larger variances than those without the saccades. For the horizontal versus vertical differences in positions, the control runs rejected the hypothesis that the means are equal, but the n-back runs failed to reject the null hypothesis. For the comparison between the control and n-back runs, there was a difference in variances between the control and n-back runs, but there was no difference in means. The variances for the n-back runs were consistently larger than those for the control runs.

Additional analyses regarding whether the gaze path followed a random walk were executed. The distribution of the angle or direction of movement was analyzed. In comparison to a uniform distribution, when saccades were removed, 60% of the 53 runs failed to reject the null hypothesis. This provides support that the angle of eye movement, in two dimensions, followed a uniform distribution. For the data with saccades, only 21% of the 53 runs failed to reject the null hypothesis. Thus, there was a significant difference in the distributions of the angle of eye movement between the data with and without saccades. When saccades were removed, there is more support for the gaze path following a random walk, compared to when saccades are included in the assessment.

Overall, both hypotheses were supported. Focus of attention, as represented by eye movement data, was able to be modeled as a stochastic process. The strongest evidence resulted from modeling differences in the horizontal and vertical eye movement as Brownian motion or the Wiener process. For these tests, 100% of the runs failed to reject the null hypothesis that the data followed a normal distribution, and thus, supported the hypothesis that the data followed a Wiener process. Additionally, there were differences exhibited between the variances for the parameter estimates between the control and n-back runs in both dimensions.

This research and resulting conclusions are important in starting to explain the involuntary eye movements, which occur when a participant is singularly focused. These results provide strong evidence that this underlying movement can be represented as a Wiener process. For any experiment considering eye movements, the inherent movement of the eyes should be considered in the analysis. For instance,

consider an experiment in which one is trying to determine how attention is shifted when searching for a specified object. If the experimenter does not consider the eye movement that occurs involuntarily, inaccurate conclusions may be drawn about the items viewed prior to identifying the item of interest. Involuntary saccades or shifts of attention may be identified as important steps towards reaching the goal of object detection, when in actuality they are simply involuntary movements of the eyes. In any attention experiment, it may be important for the experimenter to consider initial experiments, which identify these basic eye movements. This would allow experimenters to estimate parameters for the process and consider how this underlying movement affects the results.

Future work could build upon this model by extending the experiments to consider additional factors. Runs with motion or additional visual distractors could be included to determine how the model is affected. Obvious extensions could also include more participants or longer test runs. In addition, a more demanding cognitive task could be included to further assess the degree of eye movement differences encountered. Cognitive task performance in terms of reaction time and speed could also be assessed to determine the impact on gaze pattern.

Future research could consider the application of these results to various domains. For example, one could study the gaze pattern differences in participants with attention deficit disorder (ADD). It would be interesting to compare how focus of attention, as represented by gaze pattern, changes for participants diagnosed with ADD. This could extend to studying how medication for ADD affects the gaze pattern, which could potentially provide insight into the effectiveness of different types and

doses of ADD medication. In any more complex experiment, extension to an application, or predictive analysis of the focus of attention, it is important to consider the model of the inherent eye movement pattern. This research provides initial evidence of the structure of this eye movement pattern.

APPENDIX A: MATLAB SCRIPT

The following MATLAB script was utilized for testing the distribution of the saccade data. Each section contains brief comments describing the purpose of that piece of code. Comments are shown in green. For clarity, when dividing the scripts into pages, sections of script focusing on a specified task (e.g., a for loop which reads in a series of data files) are illustrated as part of the same page instead of dividing the scripts where the page breaks actually occur. Each participant was assessed by changing the participant number in the script.

```
% Eye Tracker Data - Saccade Distribution Tests (page 1 of 6)

%% Define run data to obtain
folder = 'C:\Documents\Graduate
School\Data_collection\participant_runs\';
participant = '1';
datatype = {'control';'nback'};
run_num = {'1';'2';'3'};

%% Set up variables
din = 96; % distance from eye to projector screen in inches
% convert d to pixels (or spaces on screen)

width = 63; % width of displayed image on screen (in inches)
up_rt_h = 422; center_h = 250;
in_per_pix_h = (width/2)/(up_rt_h-center_h);

height = 35; % height of displayed image on screen (in inches)
up_rt_v = 239; center_v = 350;
in_per_pix_v = (height/2)/(center_v-up_rt_v);

in_per_pix = mean([in_per_pix_h in_per_pix_v]);
d = din/in_per_pix; % distance from screen converted to pixels

dt = 240; % number of samples in 1 sec
```

Figure A-1. MATLAB script for analysis of the saccade distribution

```

% Eye Tracker Data - Saccade Distribution Tests (page 2 of 6)

%% Read csv data and put into variables and calculate point-to-point
% angles and angular velocities
cd([folder, '\', 'p', participant, '_data'])
for t = 1:length(datatype)
    for rn = 1:length(run_num)
        filename = strcat('p', participant, '_', datatype{t, :}, ...
                           run_num{rn, :}, '_data.csv');
        data = csvread(filename);
        nm = strcat(datatype{t, :}, run_num{rn, :});
        horiz.(nm) = data(:,1);
        vert.(nm) = data(:,2);
        pos = [horiz.(nm) vert.(nm) zeros(length(horiz.(nm))(:),1)];
        A = [horiz.(nm) vert.(nm) d*ones(length(vert.(nm))(:),1)];
        for i = 1:length(A)-1
            X = (pos(i,:) - A(i,:))';
            Y = (pos(i+1,:) - A(i,:))';
            ang.(nm)(i,1) = acosd((dot(X,Y))/...
                                   (sqrt(dot(X,X))*sqrt(dot(Y,Y))));
            vel.(nm)(i,1) = ang.(nm)(i,1)*dt;
        end
    end
end

%% Velocity Threshold -- Identify Occurrences of Saccades vs. Time
% based on crossing of threshold -- set values to 0 or 1 (when
% velocity threshold is crossed)

run = {'control1'; 'control2'; 'control3'; 'nback1'; 'nback2'; 'nback3'};

vt = 300; % saccade velocity threshold
vmax = 1000; % based on physical limitations
for k = 1:length(run)
    ind_v.(run{k,:}) = find(vel.(run{k,:}) >= vt & vel.(run{k,:}) <= vmax);
    rlng_fix = zeros(length(vel.(run{k,:})),1);
    rlng_fix(ind_v.(run{k,:}),1) = 1;
    scid.(run{k,:}).fixed = rlng_fix;
end

```

Figure A-1 (continued). MATLAB script for analysis of the saccade distribution

```

% Eye Tracker Data - Saccade Distribution Tests (page 3 of 6)

%% Identify saccades from potential saccades
for m = 1:length(run)
    samp = (1:length(horiz.(run{m,:})))'; posh = horiz.(run{m,:});
    posv = vert.(run{m,:}); veln = [-1; vel.(run{m,:})];
    sc = [0; scid.(run{m,:}).fixed];
    scnew.(run{m,:}) = sc;
    indsc = find(sc==1);
    for b = 1:length(indsc)
        if indsc(b,1)<=20
            if any(posv(1:indsc(b,1)+20,1)==0)
                scnew.(run{m,:})(indsc(b,1))=0;
            end
        elseif indsc(b,1)+20>length(posv)
            if any(posv(indsc(b,1)-20:end)==0)
                scnew.(run{m,:})(indsc(b,1))=0;
            end
        else
            if any(posv(indsc(b,1)-20:indsc(b,1)+20,1)==0)
                scnew.(run{m,:})(indsc(b,1)) = 0;
            end
        end
    end
    end
    for c = 1:length(sc)
        ct(c,1) = sum(scnew.(run{m,:})(1:c));
    end
    scn = scnew.(run{m,:});
    spdata.(run{m,:}) = [samp samp/240 posh posv veln sc scn ct];
    sdta.(run{m,:}) = [samp/240 scn];
    clear ct
end

%% Develop histograms with counts of saccades in each 10-second bin
for tt = 1:length(run)
    figure
    edges = 0:2400:length(spdata.(run{tt,:})(:,7));
    ctn = find(spdata.(run{tt,:})(:,7)==1);
    n = histc(ctn,edges);
    bar(edges/240,n)
    xlabel('Time (seconds)'); ylabel('Saccade counts');
    tstrng = ['Histogram of Saccade Counts: Participant ', ...
            participant, ' - ', run{tt,:}];
    title(tstrng)
    sstrng = ['p',participant,run{tt,:},'_histogram_of_saccades']
    saveas(gcf,sstrng,'fig'); saveas(gcf,sstrng,'png')
end

```

Figure A-1 (continued). MATLAB script for analysis of the saccade distribution

```

% Eye Tracker Data - Saccade Distribution Tests (page 4 of 6)

%% Generate plots of saccade counts versus time
clr = {'b','r','g'}; datatype = {'control';'nback'};

for t = 1:length(datatype)
    subplot(1, 2, t)
    for rn = 1:3
        nm = strcat(datatype{t,:},num2str(rn));
        i = 1:240:length(spdata.(nm)); i = i';
        stairs(spdata.(nm)(i,8),clr{rn},'LineWidth',2)
        hold on
        grid on
        xlabel('Time (sec)'); xlim([0 120]);
        ylabel('Count (#Saccades)')
        tstrng = ['Participant ',participant,': ', ...
                datatype{t,:},' runs'];
        title(tstrng)
        legend([datatype{t,:},num2str(1)],[datatype{t,:}, ...
                num2str(2)],[datatype{t,:},num2str(3)],'Location','best')
    end
    saveas(gcf,['p',participant,'_num_saccades_all_runs'],'fig')
    saveas(gcf,['p',participant,'_num_saccades_all_runs'],'png')
end

%% Create variable to represent time between saccades (or sojourn times)
for sj = 1:length(run)
    ind = find(spdata.(run{sj,:})(:,7)==1);
    sdta = spdata.(run{sj,:})(ind,2);
    stimes.(run{sj,:})(1,1) = sdta(1,1);
    for sjt = 2:length(sdta)
        stimes.(run{sj,:})(sjt,1) = sdta(sjt,1)-sdta(sjt-1,1);
    end
end
end

```

Figure A-1 (continued). MATLAB script for analysis of the saccade distribution

```

% Eye Tracker Data - Saccade Distribution Tests (page 5 of 6)

%% Kolmogorov-Smirnov test - Uniform distribution of event times
% Test to determine whether the event times (i.e., the times at which
% a saccade occurs) are uniformly distributed over the length of the
% run as required by a Poisson point process
for k = 1:length(run)
    figure
    ind = find(spdata.(run{k,:})(:,7)==1);
    xtst = spdata.(run{k,:})(ind,1);
    [fval,xval]=ecdf(spdata.(run{k,:})(:,1), 'frequency', ...
                    spdata.(run{k,:})(:,7));
    plot(xval,fval, 'LineWidth', 2)
    xx = 1:max(xval); xx = xx';
    thcdf = unifcdf(xx,0,max(xval));
    hold on
    plot(xx,thcdf, 'r--', 'LineWidth', 2)
    hold on
    legend('Empirical CDF', 'Theoretical CDF', 'Location', 'Best')
    grid on
    [hval.(run{k,:}),pval.(run{k,:})] = kstest(xtst,[xx,thcdf]);
    xlabel('x'); ylabel('F_X(x)');
    text(5000,0.8,['p-value = ',num2str(pval.(run{k,:}))],...
         'BackgroundColor',[0.9 0.9 0.9], 'EdgeColor', 'k')
    tstrng = ['Distribution of Saccade Event Times: Participant ', ...
             participant, ' - ', run{k,:}];
    title(tstrng)
    saveas(gcf,['p',participant,run{k,:},...
               '_CDF_comparison_saccades_ks_pval'],'fig')
    saveas(gcf,['p',participant,run{k,:},...
               '_CDF_comparison_saccades_ks_pval'],'png')
    close
end

```

Figure A-1 (continued). MATLAB script for analysis of the saccade distribution

```

% Eye Tracker Data - Saccade Distribution Tests (page 6 of 6)

%% Lilliefors test - Exponential distribution of sojourn times
% Test to determine whether the sojourn times (i.e., time between
% saccade occurrences) follow an exponential distribution as required
% by the points of a Poisson process
for sj = 1:length(run)
    [fval,xval] = ecdf(stimes.(run{sj,:}));
    thcdf = expcdf(xval,mean(stimes.(run{sj,:})));
    stairs(xval,fval,'b','LineWidth',2)
    hold on
    plot(xval,thcdf,'r--','LineWidth',2)
    legend('Empirical CDF','Theoretical CDF','Location','Best')
    grid on
    xval = xval'; thcdf = thcdf';
    [hval.(run{sj,:}),pval.(run{sj,:})] = lillietest(xval,0.05,'exp');
    xlabel('x'); ylabel('F_X(x)');
    text(0.5,0.2,['p-value = ',num2str(pval.(run{sj,:}))],...
        'BackgroundColor',[0.9 0.9 0.9],'EdgeColor','k')
    tstrng = ['Distribution of Sojourn Times: Participant ',...
        participant,' - ',run{sj,:}];
    title(tstrng)
    saveas(gcf,['p',participant,run{sj,:},...
        '_CDF_comparison_sojourns_ll_pval'],'fig')
    saveas(gcf,['p',participant,run{sj,:},...
        '_CDF_comparison_sojourns_ll_pval'],'png')
close
end

```

Figure A-1 (continued). MATLAB script for analysis of the saccade distribution

For the assessment of gaze path, much of the initial code as well as the methods used are the same as shown above. The first page is identical and thus, is not repeated here. The code starts after that first page has occurred, and thus labeling begins at page 2. For this page 2, compared to the above, there are only slight differences. Additional calculations were added to obtain values for horizontal, vertical, and overall differences in position between time samples. The other pages contain more extensive changes in the code and thus, are not summarized here.

```

% Eye Tracker Data - Gaze Path Assessment (page 2 of 6)

%% Read csv data and put into variables and calculate point-to-point
% angles and angular velocities
for t = 1:length(datatype)
    for rn = 1:length(run_num)
        filename = strcat(folder, 'p', participant, '_data\', 'p', ...
                           participant, '_', datatype{t, :}, run_num{rn, :}, ...
                           '_data.csv');
        data = csvread(filename);
        nm = strcat(datatype{t, :}, run_num{rn, :});
        horiz.(nm) = data(:,1);
        vert.(nm) = data(:,2);
        for i = 1:length(horiz.(nm))-1
            hor_diff.(nm)(i,1) = (horiz.(nm)(i+1,1)-horiz.(nm)(i,1));
            ver_diff.(nm)(i,1) = (vert.(nm)(i+1,1)-vert.(nm)(i,1));
            dist.(nm)(i,1) = sqrt(hor_diff.(nm)(i,1)^2+...
                                   ver_diff.(nm)(i,1)^2);

            end
        pos = [horiz.(nm) vert.(nm) zeros(length(horiz.(nm)(:)),1)];
        A = [horiz.(nm) vert.(nm) d*ones(length(vert.(nm)(:)),1)];
        for i = 1:length(A)-1
            X = (pos(i,:) - A(i,:))';
            Y = (pos(i+1,:) - A(i,:))';
            ang.(nm)(i,1) =
                acosd((dot(X,Y))/(sqrt(dot(X,X))*sqrt(dot(Y,Y))));
            vel.(nm)(i,1) = ang.(nm)(i,1)*dt;
        end
    end
end

%% Velocity Threshold -- Identify Occurrences of Saccades vs. Time
run = {'control1'; 'control2'; 'control3'; 'nback1'; 'nback2'; 'nback3'};
vt = 300; vmax = 1000;

for k = 1:length(run)
    ind_v.(run{k,:}) = find(vel.(run{k,:}) >= vt & ...
                            vel.(run{k,:}) <= vmax);
    rlng_fix = zeros(length(vel.(run{k,:})),1);
    rlng_fix(ind_v.(run{k,:}),1) = 1;
    scid.(run{k,:}) = rlng_fix;
end

```

Figure A-2. MATLAB script for Brownian motion assessment

```

% Eye Tracker Data - Gaze Path Assessment (page 3 of 6)
%% Find locations of saccades
for m = 1:length(run)
    posv = vert.(run{m,:});
    horiz_ns.(run{m,:}) = [0; hor_diff.(run{m,:})];
    vert_ns.(run{m,:}) = [0; ver_diff.(run{m,:})];
    dist_ns.(run{m,:}) = [0; dist.(run{m,:})];
    sc = [0; scid.(run{m,:})]; scnew.(run{m,:}) = sc;
    indsc = find(sc==1);
    for b = 1:length(indsc)
        if indsc(b,1)<=20
            if any(posv(1:indsc(b,1)+20,1)==0)
                scnew.(run{m,:})(indsc(b),1)=0;
            end
        elseif indsc(b,1)+20>length(posv)
            if any(posv(indsc(b,1)-20:end)==0)
                scnew.(run{m,:})(indsc(b),1)=0;
            end
        else
            if any(posv(indsc(b,1)-20:indsc(b,1)+20,1)==0)
                scnew.(run{m,:})(indsc(b),1) = 0;
            end
        end
    end
end
end

```

Figure A-2 (continued). MATLAB script for Brownian motion assessment


```

% Eye Tracker Data - Gaze Path Assessment (page 4 of 6)

%% Find pattern with blinks and saccades removed
hnew = horiz; vnew = vert;
for q = 1:length(run)
    rb = 11;
    while rb < length(hnew.(run{q,:}))-10
        if hnew.(run{q,:})(rb)==0 && vert.(run{q,:})(rb)==0
            if horiz.(run{q,:})(rb-1)==0 || horiz.(run{q,:})(rb+1)==0
                % set data around blinks to 0
                hnew.(run{q,:})(rb-10:rb+10)=0;
                vnew.(run{q,:})(rb-10:rb+10)=0;
            else
                hnew.(run{q,:})(rb-samp_num:rb+samp_num)=0;
                vnew.(run{q,:})(rb-samp_num:rb+samp_num)=0;
            end
        end
        rb = rb + 1;
    end
    hdiff = [0; hor_diff.(run{q,:})];
    vdifff = [0; ver_diff.(run{q,:})];
    odiff = [0; dist.(run{q,:})]; velnew = [0; vel.(run{q,:})];
    ind_ns = find(scnew.(run{q,:})==0 & hnew.(run{q,:})>0 & ...
        vnew.(run{q,:})>0 & velnew<=1000);
    horiz_ns.(run{q,:}) = hdiff(ind_ns,1);
    vert_ns.(run{q,:}) = vdifff(ind_ns,1);
    dist_ns.(run{q,:}) = odiff(ind_ns,1);
    ind_nb = find(hnew.(run{q,:})>0 & vnew.(run{q,:})>0 & ...
        velnew<=1000);
    horiz_nb.(run{q,:}) = hdiff(ind_nb,1);
    vert_nb.(run{q,:}) = vdifff(ind_nb,1);
    dist_nb.(run{q,:}) = odiff(ind_nb,1);
    dist_nb.(run{q,:}) = odiff(ind_nb,1);
end

```

Figure A-2 (continued). MATLAB script for Brownian motion assessment

```

% Eye Tracker Data - Gaze Path Assessment (page 5 of 6)
%% Tests for normality - With blinks and saccades removed

% Comment two out of three of the following lines for each run
% through this code; run through the code for each line

dir = horiz_ns; tval = 'Horizontal'; sval = 'horiz';
% dir = vert_ns; tval = 'Vertical'; sval = 'vert';
% dir = dist_ns; tval = 'Overall'; sval = 'overall';

for tn = 1:length(run)
    figure
    [fval,xval] = ecdf(dir.(run{tn,:}));
    stairs(xval,fval,'LineWidth',2)
    sdev = std(dir.(run{tn,:}));
    xx = min(xval):0.5:max(xval);
    thcdf = normcdf(xx,0,sdev);
    hold on
    plot(xx,thcdf,'r--','LineWidth',2)
    hold on
    legend('Empirical CDF','Theoretical CDF','Location','NorthWest')
    grid on
    xlabel('x'); ylabel('F_X(x)'); xlim([-10 10]);
    [h_ll.(run{tn,:}),p_ll.(run{tn,:})] =
        lillietest(xval,0.05,'norm');
    [h_jb.(run{tn,:}),p_jb.(run{tn,:})] = jbstest(xval);
    text(-8,0.7,{'P-values';['Lilliefors = ',...
        num2str(p_ll.(run{tn,:}))];['Jarque-Bera = ',...
        num2str(p_jb.(run{tn,:}))]},'BackgroundColor',...
        [0.9 0.9 0.9],'EdgeColor','k','FontSize',10.5)
    tstrng = [tval, ' Path Distances: ', 'Participant ',...
        participant, ' - ', run{tn,:}];
    title(tstrng)
    saveas(gcf,['p',participant,run{tn,:},'_CDF_comp_',sval,...
        '_dist_both_ns'],'fig')
    saveas(gcf,['p',participant,run{tn,:},'_CDF_comp_',sval,...
        '_dist_both_ns'],'png')
end

```

Figure A-2 (continued). MATLAB script for Brownian motion assessment

```

% Eye Tracker Data - Gaze Path Assessment (page 6 of 6)
%% Tests for normality - Only blinks are removed

% Comment two out of three of the following lines for each run
% through this code; run through the code for each line - directory
% is changed for each set of graphs
dir = horiz_nb; tval = 'Horizontal'; sval = 'horiz';
% dir = vert_nb; tval = 'Vertical'; sval = 'vert';
% dir = dist_nb; tval = 'Overall'; sval = 'overall';

for tn = 1:length(run)
    figure
    [fval,xval] = ecdf(dir.(run{tn,:}));
    stairs(xval,fval,'LineWidth',2)
    sdev = std(dir.(run{tn,:}));
    xx = min(xval):0.5:max(xval);
    thcdf = normcdf(xx,0,sdev);
    hold on
    plot(xx,thcdf,'r--','LineWidth',2)
    hold on
    legend('Empirical CDF','Theoretical CDF','Location','NorthWest')
    grid on
    xlabel('x'); ylabel('F_X(x)'); xlim([-10 10]);
    [h_ll.(run{tn,:}),p_ll.(run{tn,:})] =
        lillietest(xval,0.05,'norm');
    [h_jb.(run{tn,:}),p_jb.(run{tn,:})] = jbstest(xval);
    text(-8,0.7,{'P-values';['Lilliefors = ',...
        num2str(p_ll.(run{tn,:}))];['Jarque-Bera = ',...
        num2str(p_jb.(run{tn,:}))]},'BackgroundColor',...
        [0.9 0.9 0.9],'EdgeColor','k','FontSize',10.5)
    tstrng = [tval, ' Path Distances: ', 'Participant ',...
        participant, ' - ', run{tn,:}];
    title(tstrng)
    saveas(gcf,['p',participant,run{tn,:},'_CDF_comp_',sval,...
        '_dist_both_ns'],'fig')
    saveas(gcf,['p',participant,run{tn,:},'_CDF_comp_',sval,...
        '_dist_both_ns'],'png')
end

close all

%% Estimate parameters for comparison
for i = 1:length(run)
    [muh_ns(i,1),sigh_ns(i,1)] = normfit(horiz_ns.(run{i,:}));
    [muv_ns(i,1),sigv_ns(i,1)] = normfit(vert_ns.(run{i,:}));
    [muo_ns(i,1),sigo_ns(i,1)] = normfit(dist_ns.(run{i,:}));
    [muh_nb(i,1),sigh_nb(i,1)] = normfit(horiz_nb.(run{i,:}));
    [muv_nb(i,1),sigv_nb(i,1)] = normfit(vert_nb.(run{i,:}));
    [muo_nb(i,1),sigo_nb(i,1)] = normfit(dist_nb.(run{i,:}));
end

```

Figure A-2 (continued). MATLAB script for Brownian motion assessment

For the assessment of the random walk, much of the script was the same as that for the Brownian motion assessment. However, an additional variable needed to be calculated and tested. Again page 1 from figure A-1 remained the same. For page 2 in Figure A-2, the following lines were the only changes made. These were added to the initial for loop used for reading data into MATLAB and defining variables.

```
hor_length = hor_diff.(nm); ver_length = ver_diff.(nm);  
ang_dir.(nm) = atand(ver_length./hor_length);
```

Page 3 in Figure A-2 was again repeated almost identically with only three added lines. In the for loop which identifies variables with blinks and saccades removed, the following three lines were added.

```
adiff = [0; ang_dir.(run{q,:})];  
ang_dir_nb.(run{q,:}) = adiff(ind_nb,1);  
ang_dir_ns.(run{q,:}) = adiff(ind_ns,1);
```

The distribution of both these variables, equivalent to the angle of eye movement with saccades (`ang_dir_nb`) and without saccades (`ang_dir_ns`), were then tested using the Kolmogorov-Smirnov test as described in Figure A-1. The only changes were to inconsequential items such as titles and file names. The remainder of the code is shown in Figure A-3.

```

% Eye Tracker Data - Random Walk Assessment

%% Sum up horizontal and vertical distances to obtain new variable
% S_n, which represents a random walk
for s = 1:length(run)
    for n = 1:length(horiz_ns.(run{s,:}))
        sn_horiz_ns.(run{s,:})(n,1) = sum(horiz_ns.(run{s,:})(1:n,1));
        sn_vert_ns.(run{s,:})(n,1) = sum(vert_ns.(run{s,:})(1:n,1));
    end
    for m = 1:length(horiz_nb.(run{s,:}))
        sn_horiz_nb.(run{s,:})(m,1) = sum(horiz_nb.(run{s,:})(1:m,1));
        sn_vert_nb.(run{s,:})(m,1) = sum(vert_nb.(run{s,:})(1:m,1));
    end
end

%% Test for distribution of summed values

% Comment all but one of the following lines for each run through
% this code; run through the code for each line - commented out thcdf
% and kstest were also explored and thus included in this code

dir = sn_horiz_ns; sval = 'sn_h_ns';
% dir = sn_horiz_nb; sval = 'sn_h_nb';
% dir = sn_vert_ns; sval = 'sn_v_ns';
% dir = sn_vert_nb; sval = 'sn_v_nb';

for tn = 1:length(run)
    figure
    [fval,xval] = ecdf(dir.(run{tn,:}));
    stairs(xval,fval,'LineWidth',2)
    xx = min(xval):0.5:max(xval); xx = xx';
    sdev = std(dir.(run{tn,:}));
    thcdf = normcdf(xx,0,sdev);
    % thcdf = unifcdf(xx,min(xval),max(xval));
    hold on
    plot(xx,thcdf,'r--','LineWidth',2)
    hold on
    legend('Empirical CDF','Theoretical CDF','Location','NorthWest')
    grid on
    xlabel('x'); ylabel('F_X(x)'); ylim([0 1]);
    [h_jb.(run{tn,:}), p_jb.(run{tn,:})] = jbttest(xval);
    % [h_ks.(run{tn,:}), p_ks.(run{tn,:})] = kstest(xval,[xx,thcdf]);
    text(mean(xval),0.3,['p-value = ',num2str(p_jb.(run{tn,:}))],...
        'BackgroundColor',[0.9 0.9 0.9],'EdgeColor','k')
    tstrng = ['Gaze Path Distribution: Participant ',participant,...
        ' - ', run{tn,:}];
    title(tstrng)
end

```

Figure A-3. MATLAB script for Random Walk assessment

APPENDIX B: PARAMETER ASSESSMENT SUMMARY

A statistical summary of the results is provided below for each type of run and for the analysis both with and without saccades.

Parameter	Run Type	Minimum	Maximum	Mean	Median
Horizontal Mean	Control	-0.0460	0.0135	-0.0034	-0.0007
	N-back	-0.0184	0.0256	-0.0003	0.0004
Horizontal Standard Deviation	Control	0.4919	4.4372	1.8798	1.5976
	N-back	0.7170	4.4759	2.0122	1.5861
Vertical Mean	Control	-0.0245	0.0421	0.0002	0.0014
	N-back	-0.1028	0.0158	-0.0113	-0.0008
Vertical Standard Deviation	Control	0.6280	3.6273	1.8272	1.7089
	N-back	0.7861	4.1807	2.0009	1.6973

Table B-1: Summary Statistics of Parameter Fits for data without saccades

Parameter	Run Type	Minimum	Maximum	Mean	Median
Horizontal Mean	Control	-0.0458	0.0079	-0.0031	-0.0004
	N-back	-0.0335	0.0131	-0.0012	0.0020
Horizontal Standard Deviation	Control	0.5062	6.4903	2.9758	2.7528
	N-back	0.7180	6.4506	3.0977	2.9249
Vertical Mean	Control	-0.0187	0.0501	-0.0001	-0.0001
	N-back	-0.1060	0.0145	-0.0093	-0.0008
Vertical Standard Deviation	Control	0.6280	5.4379	2.6482	2.8265
	N-back	0.7962	5.0790	2.7261	2.6570

Table B-2: Summary Statistics of Parameter Fits for data with saccades

The following tables are the results of testing whether the empirical data follow a normal distribution. The empirical data is the difference in horizontal and vertical eye position for the consecutive time sample case. Each table summarizes the results for one participant and contains the parameter fit values and Jarque-Bera p-values by run.

Run	Direction	Saccades Removed			Saccades Included		
		$\hat{\mu}$	$\hat{\sigma}$	p-value	$\hat{\mu}$	$\hat{\sigma}$	p-value
Control 1	Horizontal	-0.0004	0.4919	0.5000	-0.0004	0.5062	0.5000
	Vertical	-0.0009	0.6537	0.4883	-0.0009	0.6582	0.5000
Control 2	Horizontal	-0.0091	0.5341	0.4357	-0.0091	0.5428	0.5000
	Vertical	0.0125	0.6925	0.3542	0.0125	0.6945	0.3542
Control 3	Horizontal	-0.0020	0.5064	0.5000	-0.0020	0.5064	0.5000
	Vertical	-0.0010	0.6280	0.3201	-0.0010	0.6280	0.3201
N-back 1	Horizontal	-0.0032	0.7657	0.3591	0.0010	1.4937	0.1511
	Vertical	0.0081	1.0070	0.3612	0.0097	1.3710	0.1691
N-back 2	Horizontal	-0.0005	0.7900	0.2582	-0.0004	1.2525	0.1646
	Vertical	0.0058	0.8925	0.4920	0.0050	1.0950	0.2207
N-back 3	Horizontal	0.0025	1.1280	0.3599	0.0048	1.9399	0.5000
	Vertical	0.0101	1.0945	0.4881	0.0091	1.5326	0.4222

Table B-3: Participant 1 -- Parameter fit values and Jarque-Bera p-values for testing the empirical data against the normal distribution

Run	Direction	Saccades Removed			Saccades Included		
		$\hat{\mu}$	$\hat{\sigma}$	p-value	$\hat{\mu}$	$\hat{\sigma}$	p-value
Control 1	Horizontal	-0.0049	0.6422	0.4070	-0.0050	0.7866	0.4341
	Vertical	-0.0178	0.7079	0.2823	-0.0177	0.7711	0.2848
Control 2	Horizontal	-0.0265	2.8862	0.1747	-0.0178	4.5786	0.0808
	Vertical	0.0042	2.5136	0.1793	-0.0047	3.5932	0.1214
Control 3	Horizontal	-0.0460	1.9874	0.5000	-0.0458	3.1406	0.0996
	Vertical	0.0421	1.9775	0.1549	0.0501	2.8265	0.1336
N-back 1	Horizontal	-0.0184	0.9541	0.5000	-0.0335	1.2449	0.2631
	Vertical	-0.1028	1.2233	0.5000	-0.1060	1.3474	0.2288
N-back 2	Horizontal	-0.0147	0.9828	0.1003	-0.0180	1.2130	0.1115
	Vertical	-0.0562	1.0569	0.5000	-0.0553	1.1842	0.3927
N-back 3	Horizontal	-0.0134	0.7645	0.2358	-0.0135	0.9021	0.1715
	Vertical	-0.0650	1.0367	0.2680	-0.0640	1.0824	0.2554

Table B-4: Participant 2 -- Parameter fit values and Jarque-Bera p-values for testing the empirical data against the normal distribution

Run	Direction	Saccades Removed			Saccades Included		
		$\hat{\mu}$	$\hat{\sigma}$	p-value	$\hat{\mu}$	$\hat{\sigma}$	p-value
Control 1	Horizontal	0.0081	1.8818	0.5000	0.0037	3.5018	0.0884
	Vertical	0.0014	1.6904	0.2082	0.0007	2.8836	0.1533
Control 2	Horizontal	0.0036	1.5976	0.5000	0.0033	2.9677	0.2664
	Vertical	-0.0163	1.7089	0.3414	-0.0140	2.8277	0.2323
Control 3	Horizontal	-0.0024	2.0752	0.4767	0.0079	3.7422	0.0815
	Vertical	-0.0019	1.8261	0.5000	-0.0132	3.3581	0.1707
N-back 1	Horizontal	-0.0001	1.7596	0.5000	0.0018	3.0853	0.3533
	Vertical	0.0064	1.6684	0.5000	0.0069	2.7224	0.2354
N-back 2	Horizontal	0.0100	2.3622	0.3017	0.0052	4.0739	0.0731
	Vertical	-0.0090	2.1509	0.4336	-0.0084	3.6601	0.1966
N-back 3	Horizontal	-0.0149	2.1927	0.5000	0.0030	3.8877	0.0705
	Vertical	-0.0014	1.9012	0.0955	-0.0029	3.3923	0.2678

Table B-5: Participant 3 -- Parameter fit values and Jarque-Bera p-values for testing the empirical data against the normal distribution

Run	Direction	Saccades Removed			Saccades Included		
		$\hat{\mu}$	$\hat{\sigma}$	p-value	$\hat{\mu}$	$\hat{\sigma}$	p-value
Control 1	Horizontal	-0.0007	0.7041	0.5000	0.0002	1.1895	0.1393
	Vertical	0.0032	1.4450	0.3223	0.0032	1.6328	0.3590
Control 2	Horizontal	-0.0016	0.7381	0.4211	-0.0004	1.3827	0.1199
	Vertical	0.0053	1.1642	0.3080	0.0042	1.5810	0.2713
Control 3	Horizontal	0.0053	2.1757	0.5000	-0.0007	3.9835	0.0901
	Vertical	0.0074	1.8502	0.1994	0.0057	3.3664	0.0833
N-back 1	Horizontal	0.0256	1.3847	0.2024	0.0108	2.0900	0.2877
	Vertical	0.0025	1.5335	0.5000	-0.0008	1.8965	0.2109
N-back 2	Horizontal	-0.0001	0.8786	0.5000	0.0004	1.6177	0.1255
	Vertical	0.0026	1.6158	0.2840	0.0028	1.9589	0.1601
N-back 3	Horizontal	0.0089	2.4600	0.4217	0.0022	4.6431	0.1130
	Vertical	0.0046	2.0206	0.1818	0.0084	4.0941	0.0826

Table B-6: Participant 4 -- Parameter fit values and Jarque-Bera p-values for testing the empirical data against the normal distribution

Run	Direction	Saccades Removed			Saccades Included		
		$\hat{\mu}$	$\hat{\sigma}$	p-value	$\hat{\mu}$	$\hat{\sigma}$	p-value
Control 1	Horizontal	0.0028	0.7990	0.2987	0.0028	0.8196	0.4113
	Vertical	0.0021	0.6902	0.5000	0.0022	0.7027	0.5000
Control 2	Horizontal	0.0016	0.8450	0.3315	0.0021	0.9376	0.1760
	Vertical	0.0016	0.7796	0.2987	0.0022	0.8248	0.2643
Control 3	Horizontal	-0.0023	2.0480	0.2840	-0.0006	2.5733	0.1194
	Vertical	0.0004	1.5312	0.4983	0.0011	1.7691	0.1916
N-back 1	Horizontal	0.0060	0.7312	0.5000	0.0061	0.7875	0.5000
	Vertical	0.0122	0.9002	0.3339	0.0122	0.9160	0.3339
N-back 2	Horizontal	0.0035	0.7170	0.2704	0.0038	0.7180	0.2704
	Vertical	0.0051	0.7861	0.3084	0.0062	0.7962	0.3084

Table B-7: Participant 5 -- Parameter fit values and Jarque-Bera p-values for testing the empirical data against the normal distribution

Run	Direction	Saccades Removed			Saccades Included		
		$\hat{\mu}$	$\hat{\sigma}$	p-value	$\hat{\mu}$	$\hat{\sigma}$	p-value
Control 1	Horizontal	0.0122	2.1533	0.4588	0.0051	3.7847	0.0727
	Vertical	0.0053	1.7801	0.2993	-0.0053	2.8196	0.1868
Control 2	Horizontal	0.0020	0.8467	0.5000	0.0022	1.5220	0.3179
	Vertical	-0.0158	0.9127	0.5000	-0.0139	1.1505	0.2182
Control 3	Horizontal	-0.0112	1.1203	0.3481	-0.0083	2.0745	0.1380
	Vertical	-0.0191	1.1432	0.2921	-0.0187	1.6637	0.3134
N-back 1	Horizontal	0.0026	3.4100	0.0804	0.0020	4.9946	0.0703
	Vertical	0.0015	2.6848	0.1765	-0.0007	3.6660	0.1039
N-back 2	Horizontal	-0.0068	0.7848	0.3203	-0.0075	1.2968	0.3403
	Vertical	-0.0194	0.9682	0.3015	-0.0183	1.1406	0.3665
N-back 3	Horizontal	-0.0164	1.0563	0.5000	-0.0179	1.8187	0.0999
	Vertical	-0.0193	1.0123	0.2261	-0.0183	1.2954	0.1656

Table B-8: Participant 6 -- Parameter fit values and Jarque-Bera p-values for testing the empirical data against the normal distribution

Run	Direction	Saccades Removed			Saccades Included		
		$\hat{\mu}$	$\hat{\sigma}$	p-value	$\hat{\mu}$	$\hat{\sigma}$	p-value
Control 1	Horizontal	0.0135	4.2281	0.0844	-0.0061	6.4903	0.0624
	Vertical	-0.0234	3.4840	0.3635	-0.0134	4.6092	0.3246
Control 2	Horizontal	-0.0275	4.4372	0.0906	-0.0069	6.2041	0.0716
	Vertical	-0.0130	3.2408	0.2321	-0.0158	3.9840	0.1496
Control 3	Horizontal	-0.0020	3.6856	0.1343	0.0016	5.5872	0.0686
	Vertical	-0.0014	3.5847	0.1659	-0.0060	4.4963	0.1214
N-back 1	Horizontal	-0.0060	2.4956	0.1336	-0.0153	4.0969	0.0818
	Vertical	-0.0143	2.6259	0.1444	-0.0125	3.4928	0.1266
N-back 2	Horizontal	0.0032	4.4759	0.0833	0.0113	6.4506	0.0608
	Vertical	-0.0209	3.7152	0.1368	-0.0130	4.6964	0.1358
N-back 3	Horizontal	0.0218	3.5505	0.1299	0.0131	5.2158	0.0670
	Vertical	-0.0366	4.1305	0.1398	-0.0160	5.0529	0.1435

Table B-9: Participant 7 -- Parameter fit values and Jarque-Bera p-values for testing the empirical data against the normal distribution

Run	Direction	Saccades Removed			Saccades Included		
		$\hat{\mu}$	$\hat{\sigma}$	p-value	$\hat{\mu}$	$\hat{\sigma}$	p-value
Control 1	Horizontal	0.0107	1.5268	0.4276	0.0048	2.7528	0.1018
	Vertical	0.0169	1.9356	0.3779	0.0107	2.9753	0.1586
Control 2	Horizontal	0.0003	1.3304	0.2207	0.0026	2.6373	0.2337
	Vertical	0.0084	1.5494	0.2484	0.0085	2.6977	0.0968
Control 3	Horizontal	-0.0074	1.4395	0.3877	-0.0042	2.4997	0.2279
	Vertical	0.0280	1.7898	0.2254	0.0210	3.0853	0.0782
N-back 1	Horizontal	0.0003	1.4126	0.3336	0.0022	2.7645	0.1605
	Vertical	0.0158	1.7262	0.2914	0.0145	2.5917	0.2118
N-back 2	Horizontal	0.0014	2.4139	0.3389	0.0021	4.2094	0.0791
	Vertical	-0.0004	2.1845	0.3107	0.0020	3.3757	0.3803
N-back 3	Horizontal	0.0063	3.4181	0.1522	-0.0013	5.5262	0.0659
	Vertical	0.0010	2.8700	0.3788	-0.0017	4.1678	0.1643

Table B-10: Participant 8 -- Parameter fit values and Jarque-Bera p-values for testing the empirical data against the normal distribution

Run	Direction	Saccades Removed			Saccades Included		
		$\hat{\mu}$	$\hat{\sigma}$	p-value	$\hat{\mu}$	$\hat{\sigma}$	p-value
Control 1	Horizontal	0.0067	3.2404	0.1334	0.0046	5.3199	0.0891
	Vertical	-0.0245	3.2479	0.1245	-0.0049	5.3876	0.0860
Control 2	Horizontal	0.0023	3.5644	0.1239	-0.0034	5.5267	0.0737
	Vertical	-0.0021	3.6273	0.1203	-0.0001	5.4379	0.1149
Control 3	Horizontal	-0.0160	3.2702	0.2497	-0.0142	4.7893	0.0770
	Vertical	0.0040	3.1811	0.1335	0.0055	5.0758	0.0732
N-back 1	Horizontal	-0.0066	3.5243	0.1262	0.0046	5.0795	0.0747
	Vertical	-0.0114	3.2776	0.1462	-0.0040	4.4956	0.0771
N-back 2	Horizontal	0.0005	3.8445	0.1027	0.0044	5.0386	0.0886
	Vertical	-0.0109	3.7597	0.0878	0.0017	4.7744	0.0896
N-back 3	Horizontal	0.0010	4.0591	0.1067	-0.0031	5.0985	0.0792
	Vertical	-0.0013	4.1807	0.0916	0.0028	5.0790	0.0709

Table B-11: Participant 9 -- Parameter fit values and Jarque-Bera p-values for testing the empirical data against the normal distribution

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