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DECISION-MAKING FOR STRATEGIC SPARE PARTS PRICING LEVELS: AN EVALUATION OF CONSUMER PRODUCTS SUSTAINABILITY

Masoud Vaziri
University of Rhode Island, masoudvaziri@my.uri.edu

DECISION-MAKING FOR STRATEGIC SPARE PARTS PRICING LEVELS: AN EVALUATION OF CONSUMER PRODUCTS SUSTAINABILITY

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MASOUD VAZIRI

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MASOUD VAZIRI

APPROVED:

Thesis Committee:

Major Professor Manbir Sodhi

David G. Taggart

James J. Opaluch

Nasser H. Zawia
DEAN OF THE GRADUATE SCHOOL

UNIVERSITY OF RHODE ISLAND
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ABSTRACT

The aftermarket business is a highly profitable activity for companies, and they can earn considerable profits from selling spare parts. Spare parts demands are more uncertain and intermittent in comparison with finished goods and associated work in progress parts. In the aftermarket, the demand uncertainty of the spare parts for the OEMs is complicated by the fact that the other competitors, known as market players or will-fitters, supply substitutable parts usually with lower cost of production and deliver them to the market at cheaper prices. This uncertainty makes spare parts management challenging, and this study develops strategic approaches for spare parts price setting and inventory level control to further exploit the benefits of the spare parts business. This dissertation is divided into four main parts.

In the first part, a brief review of inventory system policies is provided. The review starts with an introduction to the inventory systems terminology and follows with a categorization of the inventory systems with the aim of developing spare parts inventory models. Moreover, a discussion about the computations related to the (Q,r) policy is provided. An algorithm is proposed to find the optimal re-order point/lot-size and a Monte Carlo simulation is designed to evaluate the mathematical optimization solutions including the new algorithm and the other classical methods. In the second part, a literature review related to spare parts management is presented. The literature review is organized in such a way that in the beginning the inventory control policies are introduced. Then the perspective of uniqueness of spare parts on the inventory management is illustrated. Next, spare parts clustering and demand are studied and forecasting methods are reviewed. The use of Game Theory for inventory systems

planning is studied. Also spare parts pricing as a strategic method to increase the profit of the suppliers is evaluated. In the third part, to investigate the profitability of spare parts business, the notion of renewal cost versus the replacement cost is proposed. The replacement cost of a product is defined as the current market price of the product and the renewal cost of a product is the acquisition cost of spares to completely renew the product excluding labor costs. These costs are calculated for some products with specific characteristics, and the ratio between the renewal cost and the replacement cost as a scale to evaluate the sustainability of the spare parts pricing is determined which declares that the spare parts pricing is unfair. In the last part, Game Theory as a tool to find ideal decision-making in spare parts management taking into account the interactions among spare parts manufacturers. According to definite assumptions, spare parts inventory games in the form of normal, cooperative and non-cooperative, non-zero-sum, evolutionary, and competitive fringes are studied. The proposed games study the OEMs' decision-making on spare parts pricing strategies, inventory levels, batch productions and re-manufacturing efforts.

The proposed strategic spare parts pricing methods as an alternative for regular pricing can factor in customers' willingness to purchase spare parts, demand uncertainty, market uncertainty, competitiveness of the parts in the market, stability of the cooperation or competition in price setting, marginal costs of designing an agreement for cooperation, and the marginal cost of production and inventory. Furthermore it is possible to add the notion of renewal cost and the replacement cost ratio to the price sustainability description and to include it in the suggested strategic pricing formulations as a factor that affects the demand and supply curves.

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CHAPTER 1

1. INTRODUCTION

1.1. MOTIVATION

The aftermarket business is a highly profitable activity for the companies, and they can earn considerable profits from selling spare parts. Spare parts functions are different from finished goods (products to be delivered to the market) or work in progress parts (as a source to smooth production rate) and demands for the spare parts are more uncertain and intermittent in comparison with them. This uncertainty makes spare parts management challenging, and companies develop strategic approaches to exploit the benefits of the spare parts business.

In a real world situation, Original Equipment Manufacturers (OEMs) manufacture final products and introduce them to the market. In after-sale services, they provide spare parts to maintain or repair the final products or equipment, which forces OEMs to deal with high level of inventory investment for customer satisfaction in after-sales services. For instance, GM has over 10 million square feet of spare parts storage space in the United States with hundreds of thousands of different parts, and the value of spare parts inventories for the United States military exceeds $100 billion (Muckstadt, 2004).

On the other hand, studies show roughly 50% of the customers of the America's biggest car manufacturers face unnecessary delays in after-sale services because dealers do not have the right spares on-hand (M. A. Cohen, Agrawal, & Agrawal, 2006). The suppliers manufacture final goods with specific level of quality and quantity rates. Based on different factors such as the number of products sold, quality of parts and quality of the maintenance, products will face failure, which gives rise to the need for the spare parts to keep the products in working condition. To keep products operative, different parts should be on-hand and each part has its own price and criticality factor. Therefore, to ensure timely repair of these products, an extensive supply chain system must be set up.

Spare parts, which are stocked in suppliers' inventories, satisfy the rising demands. Most inventory problems deal with a single supplier or decision maker who makes decisions on the purchase/production rate under certain assumptions on the demand, planning horizon, etc. Therefore, the resulting policies are indifferent to the other suppliers' decision-making. In the aftermarket, the demand uncertainty for the spare parts for the OEMs is complicated by the fact that the other competitors, known as market players or will-fitters, can supply similar parts, usually with lower quality and cost of production and deliver them to the market at cheaper prices. In this complex scenario, the OEM as a decision maker should decide on his spare parts production and inventory policies. However, these actions or strategies are influenced by other competitors' strategies including pricing and quality.

A Game theoretical approach can study the interactions among spare parts manufacturers, who are the players of the aftermarket business game, to find ideal decision-making on inventory levels, quality, Green manufacturing and pricing strategies. Figure 1 depicts the strategic spare part flow line (based on influence diagram) for the oil-leveling sensor for a BMW 320i vehicle. As we can see, the OEM manufactures the vehicle as a parent product, which has a certain designated life cycle. Each product, according to its complexity, consists of different numbers of parts, in this case the car has its own major components (powertrain), main groups (engine), subgroups (engine housing), and sub subgroups (oil pan) which each consists of different parts and our desired part belongs to this category.

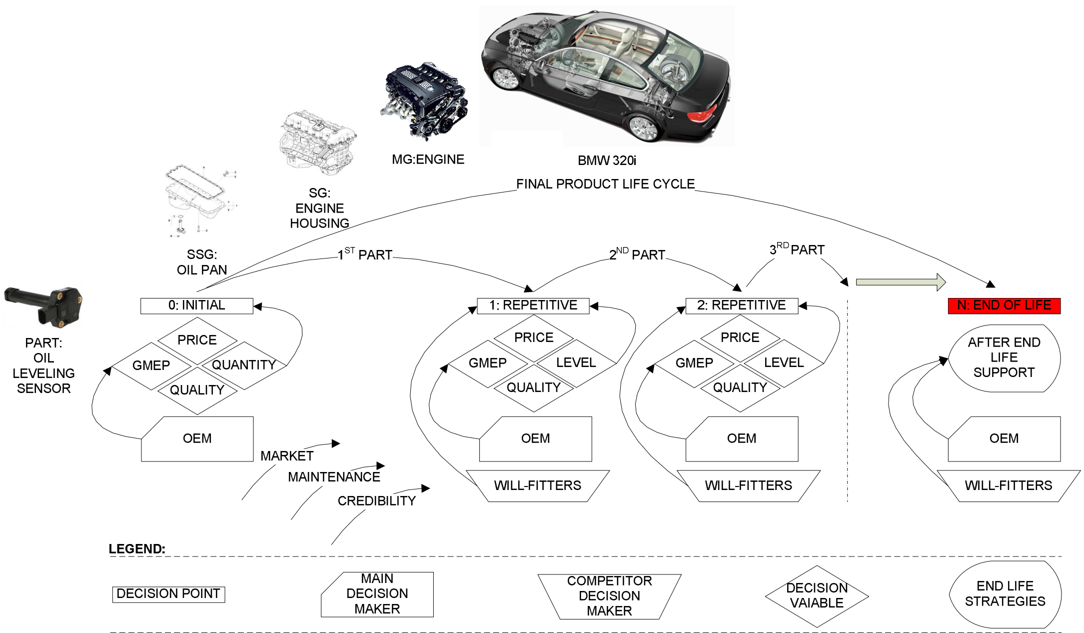


Figure 1: Strategic spare parts flow line (Oil leveling sensor-BMW 320i)

The OEM has to decide on the quality, production rate, Green manufacturing effort that is the use of recycled parts (in general the use of manufacturing methods to minimize emission of Greenhouse gases, use of non-renewable or toxic materials, and waste generation) and price of the original product and introduce it to the market, which is the initial phase of the product. Then, the product will face failure during its working period and this failure relates to its durability or life cycle, quality, working condition, maintenance quality or any unpredictable factors. In this phase because of defects and aging, failure happens that generates the need for spare parts and OEMs can satisfy this need, which is the repetitive phase of the product. However, other competitors intervene into the market and diminish the market share of the OEMs by supplying substitutable parts. This interaction creates the aftermarket game and players of the game are OEMs and will-fitters who have different strategies to take to manufacture and stock spare parts.

1.2. OBJECTIVES

The primary objectives of this thesis can be summarized as follows:

- To provide a literature review covering spare parts management and Game Theory
- To develop a novel method to evaluate the sustainability of spare parts prices;
- To develop a method for the strategic spare parts pricing and inventory level under uncertainty of the market;
- To develop a method to determine the OEM's spare part pricing level, Green manufacturing effort and inventory level in competition with will-fitters and uncertainty of the market type;
- To develop a method to study the stability of the OEM's strategic spare parts pricing and inventory level;

- To develop a method to investigate the competitive or cooperative spare parts pricing, Green manufacturing effort and inventory level strategies in different scenarios of centralized and decentralized inventory systems;
- To develop a method to study the OEM's spare parts price and inventory level determination in a price leadership market;

1.3. METHODOLOGY

The goal of this thesis is to evaluate sustainability of the spare parts pricing. This problem will be addressed by examining a newly introduced concept of the renewal cost vs. replacement cost of consumer products. Then we will study OEMs' decision-making in aftermarket business games in six steps, and the output of research answers fair strategic decision-making for spare parts pricing levels.

Aftermarkets in industries such as automobiles, white goods, industrial machineries and information technology companies have become four to five times larger than the original equipment businesses. To investigate the profitability of spare parts business (specifically in mentioned industries) the research comes up with the idea of renewal cost versus the replacement cost. The replacement cost of a product is defined as the current market price of the product and the renewal cost of a product is the acquisition cost of spares to completely renew the product excluding labor costs. Customers purchase products from OEMs and to keep them in working condition, they replace failure parts with spare parts. The price of products and its spares are set by

OEMs and our study looks for fair or sustainable spare parts pricing via investigation of replacement and renewal costs. This study follows the following procedure:

First, a review of inventory systems will be presented with the aim of developing spare parts inventory models.

Second, we review articles related to fair spare parts pricing and provide a survey for earlier literature to present a literature review related to the spare parts inventory management and Game theory. This review updates previous surveys and highlights the different issues considered and the methodologies used in spare part inventory modeling. A categorization from the perspectives of spare parts inventory control policies, uniqueness of spare parts, spare parts clustering and demand, inventory systems and Game Theory and spare parts pricing will be done.

Third, we check replacement and renewal costs for some products with specific characteristics, and compare the ratio between the renewal cost and the replacement cost as a scale to evaluate the sustainability of the spare parts pricing.

Fourth, from Game Theoretical perspective, the market, which puts intermittent demands on the spare parts, is a player of the aftermarket game. We will review previous methods of modeling the market as an agent or player from different aspects of the monopolistic or competitive situations, dummy player, and demand distributions to select proper models for the market.

Fifth, Price adjustment and Green manufacturing effort are two factors that contribute to fair spare parts pricing. Pricing strategy as a factor that can guarantee the competitiveness of companies in the market will be investigated. Meanwhile, Green manufacturing or re-manufacturing and its implementation on the production cost and credibility of the parts in the market will be considered in the study.

Sixth, payoffs of the OEMs and market players based on, inventory levels, re-manufacturing efforts and sale price will be formulated. Then, the resulting payoffs of the OEMs and will-fitters in cooperative or competitive environment will be studied by using Game Theoretical methods to investigate sustainability of the spare parts prices.

1.4. CONTRIBUTIONS

According to definite assumptions, spare parts inventory games in the form of normal, cooperative and non-cooperative, non-zero-sum, evolutionary, and competitive fringes will be investigated. The proposed games will study the OEMs' decision-making on spare parts pricing strategies, inventory levels, batch productions and re-manufacturing efforts. The outputs of the research contribute to spare parts inventory games, which are finite non-zero-sum games, answers fair strategic decision-making for spare parts pricing levels in the following format:

1. Comparison of renewal cost and replacement cost to evaluate the cost of spare parts.
2. Spare parts inventory level decision-making in a monopolistic market; a non-cooperative two-person game that can determine the production/purchase rate and inventory level.

3. Implication of the theory of games on spare parts inventory control policies; a non-cooperative three-person game that can determine pricing strategies, inventory levels and re-manufacturing efforts.
4. Evolutionary spare parts inventory games; a two-person game that can study stable pricing strategies and quality levels.
5. Cooperative spare parts inventory games; a co-operative three-person game that can determine cost allocations and inventory levels in case of cooperation between suppliers.
6. Competitive fringe spare parts inventory games; a non-cooperative multi-player game that investigates decision-making on the spare parts price and inventory level.

1.5. THESIS OUTLINE

The remainder of the thesis is organized as follows. Chapter 2 starts with a brief review of inventory system policies with the aim of developing spare parts inventory models. Then, a discussion about the errors of the (Q,r) policy is provided along with Monte Carlo simulation and proposed algorithm to find the optimal control variables. Chapter 3 provides a discussion on the importance of the aftermarket business and its profitability for the OEMs. A literature review related to the spare parts inventory management and Game theory is subsequently presented. Chapter 4 describes the novel measurement of the renewal cost versus the replacement cost. Then, the ratio between these costs as a scale to evaluate the sustainability of the spare parts pricing is determined. Chapter 5 describes spare parts inventory management as the spare parts inventory games. According to definite assumptions, spare parts inventory games in the form of normal, cooperative and non-cooperative, non-zero-sum, evolutionary, and competitive fringes are investigated. Conclusions are provided in Chapter 6.

CHAPTER 2

2. INVENTORY CONTROL MODELS

2.1. Introduction

In this chapter a brief review of inventory system policies is provided. The review starts with an introduction to the inventory systems terminology and follows by a general categorization of the inventory systems. Inventory systems can be categorized into deterministic and probabilistic policies and the most commonly used methods in each group are introduced. A discussion is presented about spare parts management and how to relate inventory control policies to spare parts management. Moreover, a discussion about the errors of the (Q,r) policy is provided and a Monte Carlo simulation is designed to evaluate the mathematical optimization solutions. An algorithm is proposed to find the optimal policy for reorder points and lot-size that minimizes the inventory cost and a comparison between the classical method, the most reliable algorithm for (Q,r) and the newly suggested algorithm is presented.

2.2. Terminology

An inventory system is a system that has three significant types of costs. All these costs are controllable and can be listed as follows:

- The cost of carrying inventories: It is the cost of investment in inventories, storage, handling the items, obsolescence, etc.;
- The cost of incurring shortages: It is the cost of lost sales, loss of good will, overtime payments, special administrative efforts (telephone calls, memos, letters), etc.;
- The cost of replenishing inventories: It is the cost of machine setups for production, preparing orders, handling shipments, etc.;

These costs are typically included by most production systems. In this system costs can be controlled by a variety of means including decision-making related to raw materials ordering, manufacturing semi-goods and finished goods and stocking goods which are ready for shipment.

The sum of those costs is known as the total cost. Interestingly, these costs are closely related to each other and increase (decrease) of one of them may results in decrease (increase) of the others. But the total cost can be controlled by means of suitable decision-making; in this case we say that costs are controllable.

According to the different costs and their controllability, inventory systems can be grouped into 4 types (Naddor, 1982):

- Type 1: Where carrying and shortage costs are controllable;
- Type 2: Where carrying and replenishing costs are controllable;
- Type 3: Where shortage and replenishing costs are controllable;
- Type 4: Where all costs are controllable;

Decision-making related to inventory systems seeks to minimize the total cost of the inventory. Mainly two types of decisions are concerned:

- When should the inventory be replenished?
    - The inventory should be replenished when the amount of inventory reaches to a specific level known as reorder point;
    - The inventory should be replenished after specific time intervals;
- How much should be added to the inventory?
    - The quantity to be ordered is a fixed value known as lot-size;
    - The quantity to be ordered will bring the a mount of inventory to a certain value known as inventory order level;

Also, the inventory problem can be considered as the balancing of the costs. For example, in some situations when carrying cost and replenishment cost are equal and balanced, the total cost will be minimized. When the time interval between placing an order and its addition to the inventory known as lead-time is significant, the inventory order level and reorder point are calculated respectively.

Based on the above discussion inventory policies can also be categorized as:

- Zero lead-time;
- Non-zero lead-time;

An analysis of an inventory system consists of four major steps:

- Determination of the properties of the system;
- Formulation of the inventory problem;
- Development of the model;
- Derivation of a solution of the system;

The establishment of the properties is the first step to analyze an inventory system. The properties of each system consist of four components:

- Demands: What is taken out of inventory;
- Replenishment: What is put into the system;
- Costs: carrying, Shortage and replenishment costs;
- Constraints: Various administrative, physical and other factors that place limitations on the rest of the properties;

The most important component is demand, because inventories are kept to meet the demand. Demands are not controllable in either ways of directly or indirectly, because people who are outside the inventory system make decision on the quantity of demands. However, the following properties related to demand can be studied:

- When do customers place an order;
- How much do they need;
- Is demand is higher at the beginning of the month than the end of it;
- Does any accurate information about future requirement exist;
- Shortage/backorders tolerated;

Demand size is the quantity required to satisfy the demand for the inventory. Demand size may be considered to be the same from one period to the other (constant demand), or otherwise it can be assumed to be variable. Demand is known when there is precise information about the demand size, and related inventory systems are called deterministic systems. Sometimes the demand size is not known but it is possible to find a probability distribution for them and these inventory systems are called probabilistic systems.

In probabilistic systems, probability of the occurrence of a demand size is assumed, or estimated. Demand rate is the demand size per unit of time. In probabilistic systems the average rate of demand is used. Demand pattern is the way that demand occurs in a period of time, in other words if we consider a period of time in which demand size occurs there are different ways of taking out the quantities and each way in known as demand pattern.

Replenishing is the quantities that are be added to the inventory based on a schedule according to the time they are ordered and they actually are added to the stock. The following three elements are important to replenishment:

- The schedule period: the time length between consecutive decisions. It can be prescribed and in this situation the only controllable variable is replenishment size. If it is not prescribed and there exists equal schedule periods, it is called constant scheduling periods;
- The replenishment size: the quantity scheduled to be added to stock. Replenishment period is the time length in which the replenishment is added to the stock and the replenishment rate is the ratio of the replenishment size and its period. If replenishment period is insignificant, the rate is infinite and can be said replenishment is instantaneous. If it is not instantaneous, the way it is added to stock over the period is important and studied as replenishment pattern;
- The lead-time: the time length between placing an order and its actual addition to the system. Lead-time most of the time is prescribed and constant which means it is similar for each decision;

As it was mentioned before costs in inventory systems consist of carrying, shortage and replenishment. Each of them has its own parameters:

- Carrying cost: The cost of carrying inventory per unit time. For many industries the fraction of carrying inventories is about 5-25% per year. The carrying cost has different elements including:
    - The cost of money tied up in inventory;
    - The cost of storage;
    - The cost of taxes on inventory;
    - The cost of obsolescence;
    - The cost of insurance of inventories;
- Shortage cost: The most difficult cost to calculate is the shortage cost. Most managers believe that it is impossible to calculate exactly the amount of shortage and they assume this cost is infinite, so they never let shortage occurs in the inventory. Mostly it depends on the quantity of shortage and the duration of time in which over time the shortage exists. The following elements are included in this type of cost:
    - Overtime costs;
    - Special clerical and administrative costs;
    - Loss of specific sales;
    - Loss of goodwill;
    - Loss of customers;
- Replenishment cost: The replenishment cost in general can be categorized into two groups:

- Costs of replenishment regarding ordering parts from the out-bound agency (the ordering cost). It may include clerical and administrative costs, transportation costs, unloading costs, and etc.;
- Costs of manufacturing parts within the under study organization or in-bound system (the setup cost). It may include labor setup costs, cost of material used during setup testing, cost of shutdown time during the setup that manufacturing stops, and etc.;

Also there are some constraints that their properties affect the inventory system. First of all, units can be continuous or discrete. Moreover demand can have some constraints including:

- Making up the shortage: In some cases it is impossible to make up the loss sales and in this situation the property of the demand has an important effect on the shortage cost;
- Negative demand: In some cases it is possible to return parts from the customer which is known as the negative demand;
- Dependent demand structure: If the demand for the next period is dependent to the previous periods that would be very complex to analyze the system;

Replenishment also has its own constraints. The major ones are:

- Space constraints: The amount of space available for sorting and stocking inventory is limited;
- Scheduling and reviewing constraints: The scheduling and reviewing periods can be prescribed which inserts constraints to the system;

- Inventory level: In some cases shortage is impossible so the level of inventory should be specific times of the average demand to assure that shortage is not happening;

2.3. Inventory Systems Categorization

Inventory systems can be categorized according to their related demand. Based on type of the demand including known or expected demands, inventory systems are divided into two groups:

1. Deterministic systems:
    - Lot-size systems: Orders are placed in lots of a fixed size so the goal is to balance carrying cost against replenishing cost. Replenishments are made whenever the inventory level reaches to zero and since the replenishment rate is infinite and there is no lead-time, no shortages can occur;
    - Order level systems: Since the scheduling is prescribed the cost of replenishment is not controllable so the goal is to balance the carrying cost against shortage cost. Lot-size systems and order level systems are identical except that in order level systems shortages are allowed and there is no prescribed scheduling period;
    - Order-level-lot-size system: The cost of carrying, shortage and replenishment can be balanced. The lot-size system is a special case of the order-level-lot-size system when the cost of backorder is infinite;

- Lot-size systems with various cost properties: In these systems it has been assumed that the cost of carrying and replenishment is not constant. Major conditions can be listed as follows:
    - Quantity discount: Where purchasing price is not constant and it depends on the quantity ordered and can be decreased while number of orders increases. This discount also can be continuous or discrete;
    - Price change anticipation: Where the price of the parts to be replenished anticipated increasing which can motivate the inventory systems to order them in advance and carry them. The price change can be known (the price increases a certain amount after specific time) or variable (the price changes in a probabilistic manner);
    - Carrying-cost functions: Carrying cost can follow different functions based on the types of the parts. This can be exclusively studied for perishable parts and expensive-storage parts;
- Deterministic systems with non-constant demand: Demands for these systems are known but it is not constant. It can be increasing demand or variable known demand during each period;

2. Probabilistic systems:
    - Probabilistic scheduling-period systems: In these systems demand is not known with certainty and the goal is to determine the optimal replenishment scheduling period and order level;

- Scheduling-period-order-level systems: Scheduling –period system tries to balance carrying cost and replenishment cos. In other words, since shortages are not allowed the order level should be large enough to meet the maximum demand in each period. The order-level system balances the carrying cost and backorder cost while the replenishment scheduling period is prescribed;
- Scheduling-period-order-level systems with lead-time: These systems are similar to previous category except that replenishment lead-time is considered;

- Probabilistic reorder-point-order-level systems: In these systems demand is not known with certainty and the goal is to determine the optimal replenishment scheduling period and order level;
    - Probabilistic order-level system: The goal is to balance carrying cost and replenishment while there is no lead-time;
    - Probabilistic order-level system with lead-time: It is similar to previous system except that replenishment lead-time exists;
    - Probabilistic reorder-point system: The goal is to balance carrying cost and backorder while there is no lead-time;
    - Probabilistic reorder-point system with lead-time: It is similar to previous system except that replenishment lead-time exists;
    - Probabilistic reorder-point-order level system: The goal is to balance carrying cost, backorder and replenishment cost while there is no lead-time;

- Probabilistic reorder-order level systems with lead-time: It is similar to previous system except that replenishment lead-time exists;

2.4. Deterministic Inventory Models

The following inventory models assume that the demand is known in advance. In this section two deterministic models which are widely used are introduced.

2.4.1. The Economic Order Quantity Model (EOQ)

The application of the mathematic to the factory management can be investigated through early work of Ford W.Harris 1913 by manufacturing lot-size determination known as EOQ model.

2.4.1.1. Solution

Solution for this problem includes balancing the setup and carrying costs. If the manufacturer produces more parts in each run he can reduce the setup cost more and in contrast if he produces and stocks more parts he would spend more cash on storing and holding parts in inventory. So the main question is how many parts to make at once in order to compromise among the above mentioned costs. The sum of the labor and material costs to ready a shop for manufacturing a part is defined as the setup cost.

Larger lots will decrease the setup cost and smaller lots would decrease the inventory cost. The balance between those two concerns can be answered by EOQ model. The lot-size mathematical formula is derived regarding following assumptions for the manufacturer:

- Production is instantaneous: The entire lot can be produced simultaneously and there is no capacity limitation;
- Delivery is immediate: In order to satisfy the demand, there is no time lag between production and availability of parts;
- Demand is deterministic: The quantity and timing of the demand is certain
- Demand is constant over time: Which means if the demand is 7 units over a week the daily demand is one;
- A production run needs a fixed setup cost: The setup cost is constant and indifferent from the lot-size or the factory condition;
- Products can be analyzed individually: Means there is only a single product or there is no interaction between products;

2.4.1.2. FORMULATION

The optimal production lot-size can be computed regarding mentioned assumptions. The following parameters are needed to generate the formula.

- D: Demand rate (in units per year);
- c: Unit production cost excluding setup or inventory costs (in dollars per unit);

- A: Fixed setup cost (in dollars);
- h: Holding cost (in dollars per unit per year) also can be represented as an annual interest (ir) on money tied up to the production $h = ir \times c$;
- Q: The lot-size or decision variable (in units);

Number of orders per year equals to D/Q and timing to place an order per year, know as order interval equals to Q/D which is a fraction of year. The total cost per year including inventory, setup and production costs would be formulated as follows (the cost is a function of the lot-size):

$$Y(Q) = \frac{hQ}{2} + \frac{AD}{Q} + cD \tag{1}$$

The lot-size that minimizes the total cost is:

$$Q^* = \sqrt{\frac{2AD}{h}} \tag{2}$$

Moreover, the optimal order interval can also be calculated ($T = Q/D$):

$$T^* = \sqrt{\frac{2A}{hD}} \tag{3}$$

This square root formula is known as EOQ and referred to as the economic lot-size. This formula tells us that there is a tradeoff between lot-size and inventory. Also the sum of holding and setup costs is insensitive to lot-size:

$$Y^* = \frac{hQ^*}{2} + \frac{AD}{Q^*} = \sqrt{2ADh} \tag{4}$$

Now consider that we use an arbitrary lot-size which is different from the optimal lot-size. The ratio of annual cost (holding and setup costs) can be written as:

$$\frac{Y(Q^{/})}{Y^{*}} = \frac{1}{2}\left(\frac{Q^{/}}{Q^{*}} + \frac{Q^{*}}{Q^{/}}\right) \tag{5}$$

Also, this can be extended for order interval too.

$$\frac{Y(T^{/})}{T^{*}} = \frac{1}{2}\left(\frac{T^{/}}{T^{*}} + \frac{T^{*}}{T^{/}}\right) \tag{6}$$

This tells us that 100% error in calculating the lot-size will result in 25% in inventory cost.

One of our assumptions was that the production is instantaneous, which means the production or replenishment is infinitely fast. Now, we can assume the production rate (P) is finite but deterministic. This model known as economic production lot (EPL), and the optimal level of lot-size is:

$$Q^{*} = \sqrt{\frac{2AD}{h(1 - \frac{D}{P})}} \tag{7}$$

If (P) is infinite we get the same result as before.

2.4.2. DYNAMIC LOT-SIZING

In order to implement more randomness into the inventory system mathematical model, relaxing the deterministic demand is studied as dynamic lot-sizing.

2.4.2.1. SOLUTION

The solution for this problem comes with the idea of finding the batch size over a random demand. The simplest possible solution would be producing exactly same amount of parts at the beginning of each week. This known as lot-for-lot rule which can be justified in some situations, but in general it forces a lot of setup cost into the system. The other possible solution can be producing fixed amount of parts each time the production is performed. This known as fixed order quantity and it is better than lot-for-lot policy because less setup cost is implemented. Although it is not optimal, cause considerable cost is forced to the system as carrying parts to next weeks. The optimal solution is the Wagner-Whitin method and its main approach is determination of the production batch size while demand is deterministic over the specific time horizon but it is time-varying in each time period. A continuous time model is not valid for a time-varying demand, so the demand should breaks into periods of days, weeks and etc. Depending on each system different schedule might be used from daily production for a high-volume system and fast changing demand to a monthly production for a low-volume system and slow changing demand.

2.4.2.2. FORMULATION

The basic goal is to satisfy the demand with minimal cost including inventory, holding and production costs. In order to facilitate the problem solving and model representation following notations are considered:

- t: Time period, the range of time periods is t=1,…,T which is the planning horizon. It is assumed that the intervals are weekly;
- D_t: Demand rate in week t (in units);
- c_t: Unit production cost in week t excluding setup or inventory costs (in dollars per unit);
- A_t: Fixed setup cost in week t (in dollars);
- h: Holding cost (in dollars per unit) to carry a part from week t to week t+1 also can be represented as an annual interest (ir) on money tied up to the production $\mathrm{h} = (\mathrm{ir} \times \mathrm{c})/52$;
- I_t: Inventory left over at the end of week t (in units);
- Q_t: The lot-size or decision variable in week t (in units);

Wagner-Whitin method states that under an optimal lot-sizing policy either the inventory carried to week t+1 from a previous week will be zero or the production quantity in week t+1 will be zero.

$$\mathrm{Z}_1^* = \mathrm{A}_1 \tag{8}$$

$$\mathrm{Z}_2^* = \min\begin{Bmatrix} \mathrm{A}_1 + \mathrm{h}_1\mathrm{D}_2 & \text{produce in week 1} \\ \mathrm{Z}_1^* + \mathrm{A}_2 & \text{produce in week 2} \end{Bmatrix} \tag{9}$$

$$\mathrm{Z}_\mathrm{T}^* = \min\begin{Bmatrix} \mathrm{A}_1 + \sum_{i=2}^{T}\sum_{j=1}^{i-1} h_j D_i & \text{produce in week 1} \\ \mathrm{Z}_1^* + \mathrm{A}_2 + \sum_{i=3}^{T}\sum_{j=2}^{i-1} h_j D_i & \text{produce in week 2} \\ \vdots & \vdots \\ \mathrm{Z}_{\mathrm{T}-1}^* + \mathrm{A}_\mathrm{T} & \text{produce in week T} \end{Bmatrix} \tag{10}$$

By introducing the optimal cost in week t as (Z_t^*) and optimal last week of production (j_t^*), an algorithm would be proposed to find the lot-sizing during different periods, Equations (8-10).

2.5. Probabilistic Inventory Models

Previous models assumed that the demand is known in advance but in realistic situations the demand is uncertain. There are two major approaches to face these kinds of problems:

- Model demand deterministically and then modify the solution regarding the uncertainty;
- Explicitly take into account the uncertainty into modeling;

Statistical inventory models are not new and they back to Wilson 1934 with two major parts:

- Order quantity determination, the amount of inventory that will be purchased or produced with each replenishment;
- Reorder point determination, the inventory level at which a replenishment would be triggered;

Generally three major situations can be considered regarding random demands:

- Periodic review model, in which we are interested in a single replenishment and only determining the order quantity, is an issue. The replenishment occurs periodically and it is known as the Newsvendor model;

- Base stock model, in which the inventory replenished one unit at a time so the target is to find the reorder point known as base stock level;
- Continuous review, in which the reorder point r and order quantity Q are determined during a random demand and parts arrive after a lead-time L which may cause the stock out situation;

2.5.1. The Newsvendor Model

Consider a situation where there is a sale season. Demand is uncertain and occurs prior to the sale, the inventory on shelves will be sold and if there is no part the sale will be lost. Moreover the cost of holding the inventory till next sale is high, so unsold items will be discounted steeply after the sale.

2.5.1.1. Solution

An appropriate production quantity would be chosen considering two sets of information:

- Anticipated demand;
- The cost of production too much or too little;

In order to develop a mathematical model, some assumptions are needed including:

- Products are separable, there are no interactions between products;
- Demand is random, it is characterized as a known probability distribution;

- Planning for a single period, inventory is not carried to the next period so the effect of current decision on future situation is negligible;
- Deliveries are made in advance of demand, all stock is available to meet demand;
- Cost of overage and underage are linear, the cost of having too much inventory or too little is proportional to the amount of overage and underage;

2.5.1.2. FORMULATION

In order to facilitate the problem solving and model representation following notations are used:

- X: Demand which is a random variable (in units);
- g(x): Probability density function (PDF) of demand;
- G(x): Cumulative distribution function (CDF) of demand;
- μ: Mean demand (in units);
- σ: Standard deviation of demand (in units);
- c_o: Cost per unit of overage (in dollars);
- c_s: Cost per unit of shortage (in dollars);
- Q: Production or order quantity or decision variable (in units);

To minimize sum of expected overage and shortage cost, an optimal order quantity will be chosen which satisfies the critical fractile:

$$G(Q^*) = \Phi\left(\frac{Q^* - \mu}{\sigma}\right) = \frac{c_s}{c_0 + c_s} \tag{11}$$

$$z = \left(\frac{Q^* - \mu}{\sigma}\right) \tag{12}$$

Where (Φ) is the CDF of the standard normal distribution and (z) is the value in the standard normal table or from the following formula in Excel:

$$\Phi(z) = \mathrm{NORMDIST}(z, 0,1, \mathrm{TRUE}) \tag{13}$$

In general speaking, for Newsvendor models three conclusions can be made:

- For uncertain demand, the optimal order quantity depends on the demand probability distribution and costs of overage and shortage;
- For normal distribution of demand, increase of mean leads to increase in order quantity;
- For normal distribution of demand, increasing variability of demand (i.e. standard deviation of demand) can increase or decrease the order quantity. If the critical fractile is greater than 0.5, the order quantity increases as the variability of demand increases. If the critical fractile is less than 0.5, the order quantity decreases as the variability of demand increases;

2.5.2. THE BASE STOCK MODEL

Consider a situation where there is a store who sells a particular part. Because of some difficulties like space and delivery seller decides to place an order when one single part is sold. But the replenishment takes time so seller needs to carry some parts in stock. The base stock model discusses about how much should be stocked, when the space available is limited.

2.5.2.1. Solution

In order to develop a mathematical model by use of continuous-time framework, some assumptions are needed including:

- Products can be analyzed separately - there are no interactions between products;
- Demand occurs one at a time - there is no batch order;
- Unfilled demand is backordered - there are no lost sales;
- Replenishment lead-times are fixed and known - there is no randomness in delivery lead-times;
- Replenishments are ordered one at a time, there is no motivation such as setup costs or minimum order size for batch replenishments;
- Demand can be considered following a continuous distribution;

2.5.2.2. Formulation

In order to facilitate the problem solving and model representation following notations are used:

- l: Replenishment lead-time (in days);

- X: Demand during replenishment lead-time (in units);
- g(x): Probability density function (PDF) of demand during replenishment lead-time;
- G(x): Cumulative distribution function (CDF) of demand during replenishment lead-time;
- Ө: Mean demand (in units) during lead-time;
- σ: Standard deviation of demand (in units) during lead-time, for Poisson distributed demand, standard deviation equals to $\sqrt{\theta}$;
- h: Cost to carry one unit of inventory for one year (in dollars per unit per year);
- b: Cost to carry one unit of backorder for one year (in dollars per unit per year);
- r: Reorder point (in units), decision variable;
- R: r+1 inventory position (in units);
- s: r-Ө safety stock level (in units);
- S(r): Fill rate (fraction of orders filled from stock);
- B(r): Average number of backorders;
- I(r): Average on-hand inventory level;

In base stock model, the inventory would be monitored and whenever the inventory level or inventory position drops to the reorder point, the replenishment will be placed. The optimal reorder point (r) that minimizes the inventory cost including holding plus backorder cost is calculated as following:

$$G(r^* + 1) = G(R^*) = \frac{b}{b + h} \tag{14}$$

Also, we assume that (G) is normal so this formula has the same fractile structure:

$$G(R^*) = \Phi\left(\frac{R^* - \mu}{\sigma}\right) = \frac{b}{b + h} \tag{15}$$

$$z = \left(\frac{R^* - \mu}{\sigma}\right) \tag{16}$$

In base stock model, service level, backorder level and inventory level are important and it is possible to determine each level regarding the normally distributed demand:

- Service level: $G(R^*)$ is the fraction of the demand that can be filled from stock, so it is called the fill rate and equals to the service level:

$$S(r) = G(R^*) \tag{17}$$

- Backorder level: This is a very important component for inventory control, because it measures the amount of unmet demand and also relates to the loss function. If (Φ) is the cdf and (ϕ) is the PDF of the standard normal distribution and (z) is the value in the standard normal table or from the following formula in Excel:

$$B(r) = (\theta - R)[1 - \Phi(z)] + \sigma\phi(z) \tag{18}$$

$$\Phi(z) = \text{NORMDIST}(z, 0, 1, \text{TRUE}) \tag{19}$$

$$\phi(z) = \text{NORMDIST}(z, 0,1, \text{FALSE}) \tag{20}$$

- Inventory level: The expected on hand inventory equals to:

$$I(r) = r + 1 - \theta + B(r) \tag{21}$$

2.5.3. THE (Q,R) MODEL

When demand for parts is inherently unpredictable (for example it is a function of machine breakdowns) and the setup cost is significant (means one-at-a-time replenishment is impractical), the manager should decide both how much inventory to carry (r) and how many parts to order (Q). The solution for this problem can be answered by (Q,r) model. Larger values of (Q) results in few replenishment but high average inventory level, and smaller values results in low average inventory, but a lot of replenishment per year. A higher reorder point (r) leads to high level of inventory and low chance of Stock out and vice versa. The replenishment quantity (Q) affects cycle stock, means holding inventory to avoid extra replenishment, the EOQ approach. The reorder point (r) affects safety stock, means holding inventory to avoid Stock out, the base stock model approach. The (Q,r) model is compromising among two models.

2.5.3.1. SOLUTION

In excess of assumptions for base stock model, we need two assume one of the following statements:

- There is a fixed cost for a replenishment order;
- There is a constraint on the annual replenishment numbers;

2.5.3.2. FORMULATION

In order to formulate the problem two different optimization functions can be used:

- Min (fixed setup cost + backorder cost + holding cost)
- Min (fixed setup cost + Stock out cost + holding cost)

Backorder cost assumes a charge per unit time when a customer demand is unmet, and Stock out cost assumes a fixed charge for each unmet demand. To develop the model following notations are used:

- D: Expected demand per year (in units);
- l: Replenishment lead-time (in days);
- X: Demand during replenishment lead-time (in units);
- g(x): Probability density function (PDF) of demand during replenishment lead-time;
- G(x): Cumulative distribution function (CDF) of demand during replenishment lead-time;
- ϴ: $Dl/365$ mean demand (in units) during lead-time;
- σ: Standard deviation of demand (in units) during lead-time, for normal distribution it is $\sqrt{\theta}$;
- A: Setup cost per replenishment (in dollars);

- c: Unit production cost (in dollars per unit);
- h: Cost to carry one unit of inventory for one year (in dollars per unit per year);
- k: Cost per Stock out (in dollars);
- b: Cost to carry one unit of backorder for one year (in dollars per unit per year);
- Q: Replenishment quantity (in units), decision variable;
- r: Reorder point (in units), decision variable;
- s: r-ϴ safety stock level (in units);
- F(Q,r): Order frequency (replenishment orders per year);
- S(Q,r): Fill rate (fraction of orders filled from stock);
- B(Q,r): Average number of backorders;
- I(Q,r): Average on-hand inventory level;

In (Q,r) model following costs are included in modeling: fixed setup cost, Stock out cost, backorder cost and holding cost.

As it was mentioned before two different approaches can be used to formulate the problem:

- Backorder cost approach: The total cost is the sum of setup, backorder and inventory carrying and the goal is to make a reasonable balance between setups, service and inventory. The optimal reorder quantity is calculated from Equation (2). The optimal reorder point equals to (based on backorder and holding costs) where Φ and z are calculate from Equation (13):

$$G(r^*) = \Phi\left(\frac{r^* - \mu}{\sigma}\right) = \frac{b}{b + h} \tag{22}$$

$$z = \left(\frac{r^* - \mu}{\sigma}\right) \tag{23}$$

- Stock out cost approach: The total cost is the sum of setup, stock out and inventory carrying and the goal is to make a reasonable balance between setups, service and inventory. The optimal reorder quantity is calculated from Equation (2). The optimal reorder point equals to (based on backorder and holding costs) where Φ and z are calculate from Equation (13):

$$G(r^*) = \Phi\left(\frac{r^* - \mu}{\sigma}\right) = \frac{kD}{kD + hQ} \tag{24}$$

$$z = \left(\frac{r^* - \mu}{\sigma}\right) \tag{25}$$

2.5.3.3. PROPOSED ALGORITHM

The (Q,r) policy has been used in industry and widely studied in the literature since Hadley and Whitin introduced this method in their classical textbook in 1963. The classical method as described in previous section, optimize the inventory cost based on the use of EOQ and base stock model to evaluate reorder point and lot-size. However this method does not determine the optimal policy that minimizes the inventory cost. In order to check the error of the classical method a simulation is designed. We assume that demand arrives as a Poisson process and arises on a unit-by-unit basis. Due to use of simulation, a decision tree of the (Q,r) policy for different values of the (Q) and (r) has been implemented and the surface of the inventory cost is graphed. The inventory cost is convex and the result of the simulation proves that the classical method is not able to determine the actual minimum inventory cost (Figure 2).

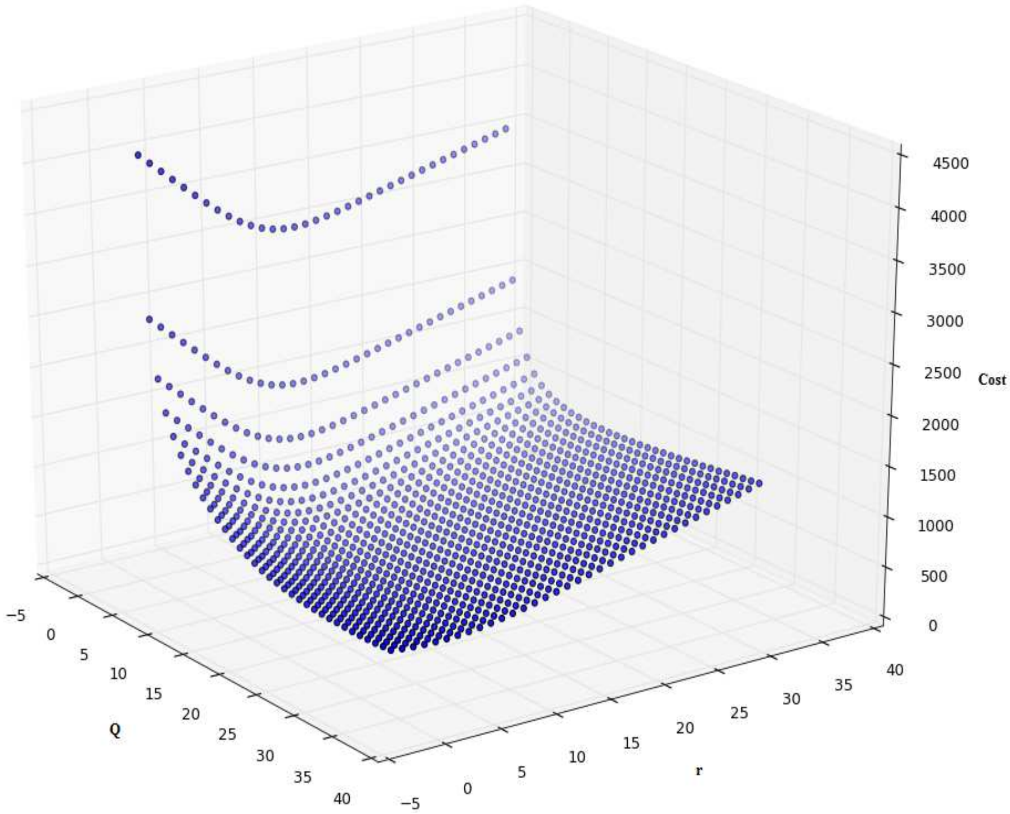


Figure 2: Inventory cost vs. Q & r - D=100, A=15, h=30, b=100, L=365

Classical method: Q=10, r=15 Cost=404.3 & Optimal solution: Q=14, r=9 Cost=320.7

A new algorithm is proposed to determine the optimal reorder point and lot-size which optimizes the cost of inventory. The suggested method is an iterative analytical method that uses curve fitting. The inventory cost which is the sum of setup and purchase order cost, backorder cost, and inventory carrying cost can be written into a mathematical formulation:

$$E[C(Q,r)] = A\frac{D}{Q} + bB(Q,r) + hI(Q,r) \tag{26}$$

According to (Zipkin, 2000), I(Q,r) can be written as:

$$I(Q,r) = \frac{Q+1}{2} + r - \lambda L + B(Q,r) \tag{27}$$

The loss function B(r) which represents the average backorder level in a base stock model with reorder point r, can be computed as following:

$$B(r) = \sum_{x=r+1}^{\infty} \big(x - (r+1)\big) f(x) \tag{28}$$

The loss function B(Q,r) for the (Q,r) model as the average of the backorder levels of the base stock model for reorder points from (r) to $(r + Q - 1)$:

$$B(Q,r) = \frac{1}{Q} \sum_{x=r}^{r+Q-1} B(x) \tag{29}$$

Now we can rewrite the cost function as Equation 30:

$$E[C(Q,r)] = A\frac{D}{Q} + b\left[\frac{1}{Q}\sum_{x=r}^{r+Q-1}\sum_{y=x+1}^{\infty}\big(y-(x+1)\big)f(y)\right] + h\left[\frac{Q+1}{2} + r - \lambda L + \frac{1}{Q}\sum_{x=r}^{r+Q-1}\sum_{y=x+1}^{\infty}\big(y-(x+1)\big)f(y)\right] \tag{30}$$

The optimal (Q,r) are the values that minimize the expected cost function. In other words differentiating E[C(Q,r)] with respect to (Q) and r will determine the optimal lot-size and reorder point.

$$\frac{\partial E[C(Q, r)]}{\partial Q} = 0 \rightarrow Q^{*} \tag{31}$$

$$\frac{\partial E[C(Q, r)]}{\partial r} = 0 \rightarrow r^{*} \tag{32}$$

As we can see it is difficult to calculate the derivatives analytically:

$$-\frac{AD}{Q^2} - \frac{b + h}{Q^2}\left(\frac{\partial B(Q, r)}{\partial Q}\right) + \frac{h}{2} = 0 \tag{33}$$

$$(b + h)\frac{\partial B(Q, r)}{\partial r} + h = 0 \tag{34}$$

It is sometimes difficult to use exact expressions in optimizations, so various approximation methods have been offered. In an approximation method, Zipkin approximated B(Q,r) with the base stock backorder B(r) as Equation (18). This simplification relaxes the $(\partial B(Q, r))/\partial Q$ term in Equation (33). In other words, the optimal order quantity simply is derived from Equation (2). Now, treating (Q) as a continuous variable and replacing B(Q,r) with B(r) in Equation (34) provides the optimal re-order point from the base stock model formulation Equations (14-16).

In our proposed algorithm, in order to solve the optimization problem without approximation we introduce the following method:

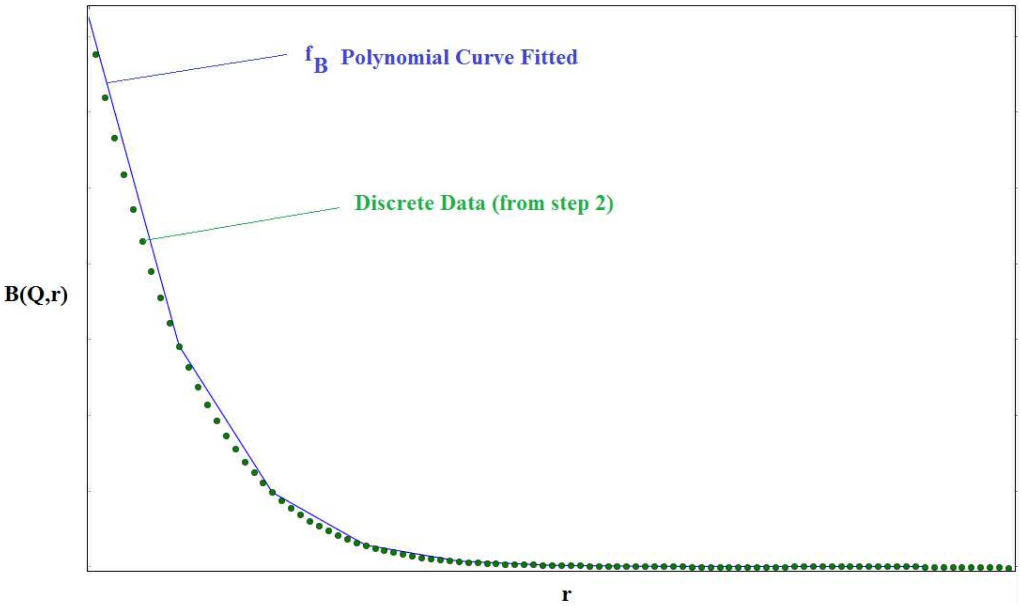
1. Calculate $Q_i^{*} = \sqrt{\frac{2AD}{h}}$;

2. For a given $Q^* = Q_i^*$ Graph B(Q^*,r) for different values of r or generate the trend of B(Q^*,r) versus r;

$$\frac{1}{Q}\sum_{x=r}^{r+Q-1}\sum_{y=x+1}^{\infty}\left(y-(x+1)\right)\frac{e^{-\lambda L}(\lambda L)^y}{y!} \quad \forall\, r \in [0,\infty) \tag{35}$$

3. Fit a curve to the B(Q^*,r) and generate the curve fitted function (f_B);

$$f_B = \text{Polyfit}(r, B(Q,r)) \therefore f_B(r) \tag{36}$$



4. Solve $(b+h)\frac{\partial f_B}{\partial r} + h = 0$ for r that determines r_i^*;
5. Calculate $E[C(Q_i^*, r_i^*)]$, if $E[C(Q_{i-1}^*, r_{i-1}^*)] < E[C(Q_i^*, r_i^*)]$ stop;
6. Q_i^*=Q_i^*+1 and i=i+1 go back to 2;

In this section we prepare a sensitivity analysis on the cost function based on variation of D, b, h, L, A and compare the result of our algorithm with the result of the classical method which is shown as $(x^`)$.

Table 1: Sensitivity analysis based on variation of annual demand (D)

D	Q`	r`	cost`	Q	r	cost
14	4	3	167.5	5	0	118.6
50	8	8	289.5	9	4	226.7
100	10	15	404.3	14	9	320.7
150	13	22	498	16	15	392.6
200	15	29	580.4	19	20	453.4

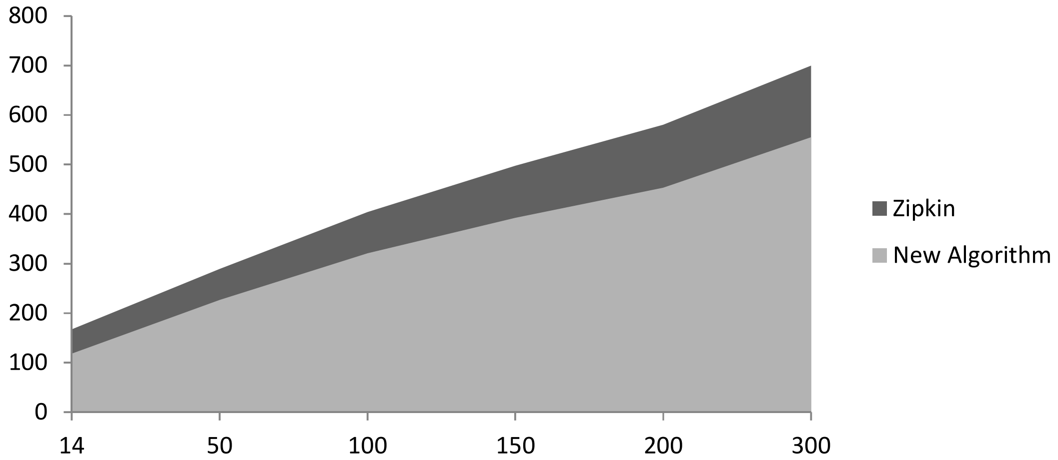


Table 2: Sensitivity analysis based on variation of backorder cost (b)

b	Q`	r`	cost`	Q	r	cost
100	4	3	167.5	5	0	118.6
500	4	4	197.6	5	2	161
1000	4	5	226.5	4	3	180.3
5000	4	6	256.5	4	4	213.7
10000	4	7	286	4	4	231.6

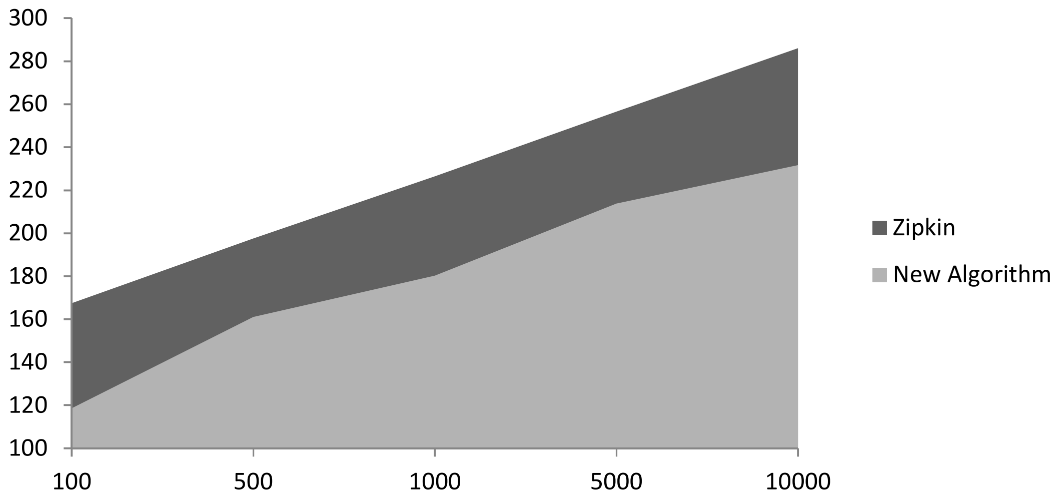


Table 3: Sensitivity analysis based on variation of holding cost (h)

h	Q`	r`	cost`	Q	r	cost
15	6	4	121.8	6	1	87.8
25	5	3	150.2	6	0	110.3
30	4	3	167.5	5	0	118.6
50	3	3	236.5	4	0	146.5
80	3	2	263.5	4	0	180.8

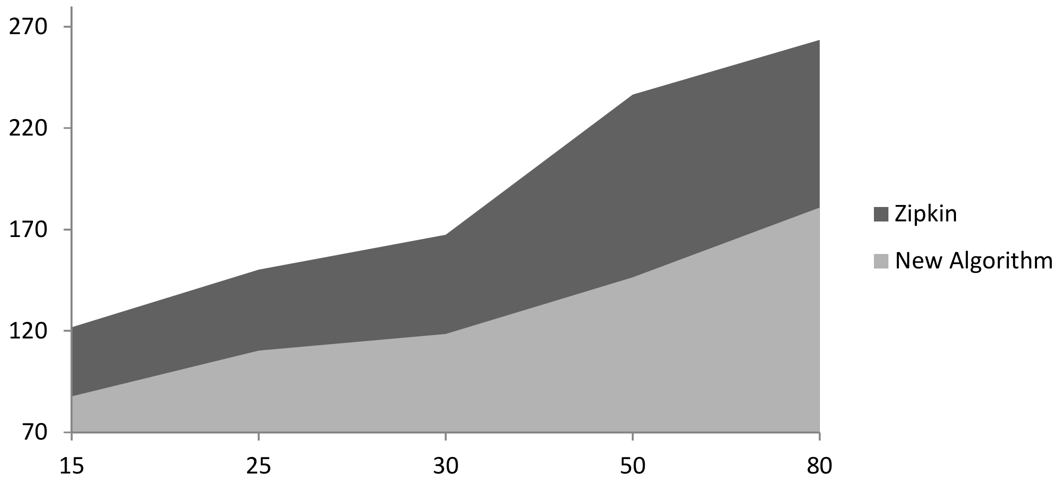


Table 4: Sensitivity analysis based on variation of lead-time (L)

L	Q`	r`	cost`	Q	r	cost
10	4	1	146.2	4	0	118.3
30	4	2	154.5	4	0	114.4
45	4	3	167.5	5	0	118.6
55	4	4	185.4	5	1	124.4
75	4	5	193.1	5	1	132.5

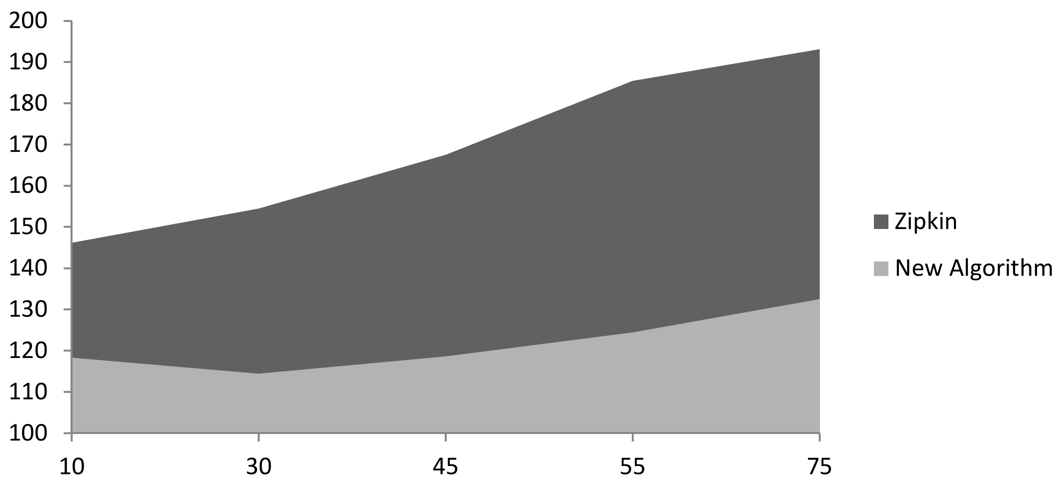
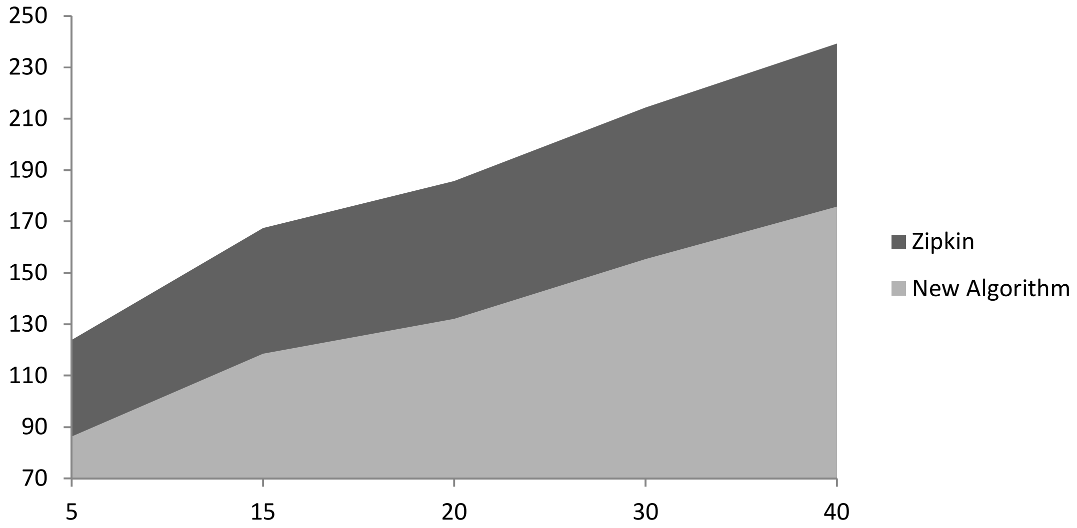


Table 5: Sensitivity analysis based on variation of setup cost (A)

A	Q`	r`	cost`	Q	r	cost
5	3	3	123.9	3	1	86.3
15	4	3	167.5	5	0	118.6
20	5	3	185.7	6	0	132.1
30	6	3	214.4	6	0	155.4
40	7	3	239.2	7	0	175.8



According to (Federgruen & Zheng, 1992) for a period of 30 years there has been no efficient algorithm for calculating the determination of an optimal (Q,r) policy. In the following we compare the result of our proposed algorithm for the continuous review (Q,r) policy with Federgruen policy, as the most reliable algorithm, and compare the results with the Monte Carlo simulation for the long time horizon (5000 years).

Table 6: Comparison of Federgruen Alg, Proposed Alg. and Simulation - D=50, L=1, h=10, b=25

	Proposed Algorithm			**Federgruen Algorithm**			**Simulation**		
A	**Q**	**r**	**cost**	**Q**	**r**	**cost**	**Q**	**r**	**cost**
1	7	51	96.15	7	50	95.46	7	50	95.56
5	12	49	116.1	12	48	115.3	12	48	115.58
25	23	44	171.1	23	44	171.1	23	44	171.24
100	40	38	289.3	40	38	289.3	40	38	289.48
1000	120	15	852.5	120	15	852.5	120	15	852.24

Table 7: Comparison of Federgruen Alg, Proposed Alg. and Simulation - D=50, L=1, h=10, b=100

	Proposed Algorithm			Federgruen Algorithm			Simulation		
A	**Q**	**r**	**cost**	**Q**	**r**	**cost**	**Q**	**r**	**cost**
1	5	58	143	7	56	142	7	56	142.79
5	11	55	165.3	12	54	165.2	11	55	165.23
25	20	52	227.4	20	52	227.4	20	52	227.39
100	36	49	357.7	37	48	357.8	36	49	357.77
1000	107	40	978.5	107	40	978.5	107	40	978.44

Table 8: Comparison of Federgruen Alg, Proposed Alg. and Simulation - D=50, L=1, h=25, b=25

	Proposed Algorithm			Federgruen Algorithm			Simulation		
A	**Q**	**r**	**cost**	**Q**	**r**	**cost**	**Q**	**r**	**cost**
1	6	46	153	6	46	153	6	46	153.44
5	11	44	177.2	11	44	177.2	11	44	177.49
25	19	40	245.5	19	40	245.5	19	40	245.09
100	31	34	394.8	31	34	394.8	31	34	394.87
1000	91	4	1131.8	91	4	1131.8	91	4	1131.77

The results of our sensitivity analysis that are provided in Tables 1-5 state that the classical method is not able to determine the minimum inventory cost and there is a considerable difference between the optimal inventory solution and the classical solution. However, our algorithm is able to provide the optimal re-order point and order quantity that leads to the minimum inventory cost.

The comparisons of our new algorithm, Federgruen algorithm and the simulation that are depicted in Tables 6-8 shows a general consistency of the optimal policy and minimum inventory cost in two methods and simulation. There are some cases that the suggested values for (Q) and (r) by algorithms are different from each other, but comparison of the results with the simulation declares that the error is less than 0.5% which is negligible.

2.6. SPARE PARTS MANAGEMENT

In this chapter we have reviewed inventory systems with the aim of developing spare parts inventory models. In order to achieve this goal, we need to consider spare parts characteristics from the managerial point of view.

Spare parts management looks for achieving the original products (parent products e.g. machines, equipment and etc.) availability at an optimum cost. The downtime of parent products is expensive and prohibited from the point of view of customers and non-availability of spare parts mostly contributes to 50% of the total downtime cost. Also, the cost of spare parts contributes to more than 50% of the total maintenance cost in most industries. Therefore, there is a paradox to complain about non-availability of the spare parts in contrast with increasing the locked up investment to stock spare parts to reach high level of availability.

The unique problem that deals with spare parts management is the element of uncertainty. This uncertainty comes from when a spare part is required or when a product fails and once it fails what is the quantity of parts required for replacement. It is a fact that the failure of a component is unpredictable so demand for spare parts is uncertain. On the other hand, most of the time demand for spare parts is low and spare parts are considered as slow moving items. This leads to low level of availability of the parts in the market and usually high lead-time of spare parts supply especially when the complexity of parts is high. Furthermore, the number and variety of spare parts are high (for a medium scale engineering industry can be around 15000 parts and for a large scale industry may be around 100000 parts), and the rate of consumption of spare parts for some parts are high and for some parts are low. For example in a case study for a system, over 80% of the cumulative annual demand was concentrated in about 8% of the items and over 50% of items contributes to less than 1% of the cumulative annual demand. In automotive industry it is common that the number of demands per dealer per year is less than one unit for some parts. Even for items with more than 20000 units demanded annually, considerable fraction of dealers demand the items less than one on average per year.

Regarding aforementioned characteristics for spare parts, a good inventory control policy should determine the ordering procedure and optimal level of inventory to provision spare parts in a right time with efficient cost. Under these circumstances suggested inventory control policies should factor in unpredictable demand for spare parts known as intermittent demand, considerable lead-time to supply parts and variety of parts. Based on these considerations suggested inventory models for spare parts management can be divided into two groups:

1. Negligible setup cost:
    - Newsvendor inventory policy;
    - Order-up-to level inventory policy;
2. Non-negligible setup cost:
    - Two parameters policies such as (Q,r) policy;
    - EOQ based inventory policies;

CHAPTER 3

3. REVIEW OF LITERATURE

3.1. Introduction

The spare part business is defined as the purchasing, warehousing, selling and delivering of spare parts to customers. Extended activities including customer services and handling warranty issues are also included within the definition of the spare part business (Suomala, Sievänen, & Paranko, 2002). Spare parts, for many companies producing durable products, is the most profitable function of the corporation (Wagner & Lindemann, 2008).

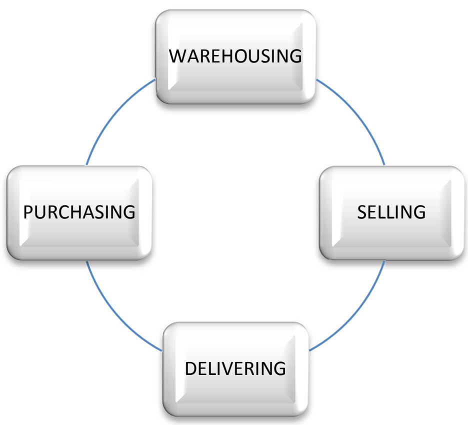


Figure 3: Spare parts business activities

Despite absence of reliable data, it is acknowledged that the spare parts business is very profitable. It is believed that spare parts contribute to one-third of the net sales and two-third of the profits (Suomala et al., 2002). Spare parts only contributes to 10%

of the global sales but can contribute to more than 50% of the net income for an average industrial company (Noeuvéglise & Chevenement, 2011).

Aftermarkets in industries such as automobiles, white goods, industrial machinery, and information technology have become four to five times larger than the original equipment businesses. In 2001, GM earned more profit from 9 billion US dollars sales in after-sales revenue than 150 billion US dollars income of car sales (M. A. Cohen et al., 2006). In 2005, the supply of aftermarket parts (that covers everything from replacement toner cartridges to engines of cruise ships) was a 400 billion US dollar business and recently this amount has reached to 700 billion US dollar (T. Gallagher, Mitchke, & Rogers, 2005). In 2006, the sale of spare parts and after-sales services in the United States was at 8% of annual gross domestic product (GDP), that means American costumers spent about 1 trillion US dollars annually on assets they already own (M. A. Cohen et al., 2006). In 2010, according to Rolls-Royce group annual report, Rolls-Royce engine-maker generated more than half of its revenue (more than 5.5 billion British pounds) from service activities.

The share of a company’s spare parts revenue is an indicator to show the importance of the spare parts business. In a case study for different firms, on average companies generate 13.3% of their revenues from the sale of spare parts (Wagner & Lindemann, 2008). In automotive industry, the profit margins of spare parts sales are three to four times higher than the margins in car sales. Some firms sell their primary products (i.e. machines) with the price close to the production cost with goal of attracting future demands for spare parts (Dennis & Kambil, 2003).

After-sale services are high-margin business, and they are considered as a huge portion of corporations profits. The profit of spare parts for manufacturing and engineering-driven firms including after-sale services is significant. It contributes to about 25% of all revenue, although it is 40% to 50% of all profits (Dennis & Kambil, 2003).

In spite of profitability of spare parts business, it is very challenging and expensive to handle spare parts inventory and customer satisfaction. In recent years the value and share of the spare parts inventory in manufacturing companies has increased significantly. Few years ago the value of the spare parts inventory for a manufacturing company was about 2 to 10 million US dollars, but now it is about 5 to 15 million US dollars, which shows a significant increase on spare parts inventory investment.

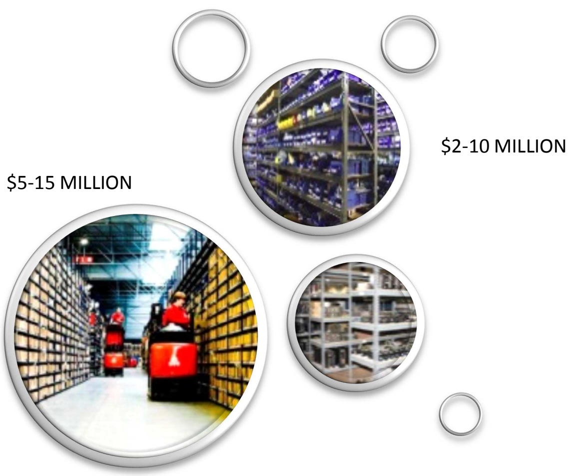


Figure 4: The value of the spare parts inventory for a typical manufacturing company

In aircraft industry for instance, the supply of spare parts for Boeing airplanes is a 7 billion US dollars per year business with more than 2000 suppliers. For another instance, TNT post group uses more than 3 million square feet of warehouse space to handle 120,000 tons of shipments and 34.6 million orders per year for Fiat spare parts in Europe and South America (Parker, 1999). However, it has to be mentioned that the maturation of technologies such as the Internet, WIFI, RFID, etc. have made it easier to track products over their life cycle, and have allowed product and component replacements to be planned strategically instead of mostly on an as-needed or emergency basis.

Furthermore, there are constant challenges between original equipment manufacturers (OEM) and market players or competitors called will-fitters. The lower-cost producers attack the profit of aftermarkets. Risks of after-market business for OEM can be listed as following:

- Buying non-OEM and used parts;
- Refurbishing instead of replacing parts;

Traditional OEM remedies, which are discounting the price of original parts and recovering lost profits by selling parts and services at higher margin, does not work efficiently anymore. Although, OEMs still have some advantages, according to (T. Gallagher et al., 2005) advantages of OEM compared to lower-cost producers are:

- Stronger relationship with customers;
- Better distribution system;
- Deeper engineering resources;
- Advanced technical support;
- Superior quality assurance;

On average, OEMs carry 10% of their annual sales as spare parts. Most of OEMs do not get the best out of those assets because most of the time their people and facilities are idle, and inventory turns is once or twice per year that means about 23% of their parts become obsolete yearly (M. A. Cohen et al., 2006).

The major problem of the OEMs to support service parts is the high risk of obsolescence and sudden increase of the prices. After the final phase of a product production, demand for parts decreases dramatically, which increases the price of the production even more than 300% (R. H. Teunter & Klein Haneveld, 2002). Spare parts are usually manufactured along with the parent product and the OEM keeps the stock of spares for replacement during warranty, post warranty and after sale services. Availability of the spare parts during the product life cycle; the time span before the end-of-production and the period between end-of-production and end-of-service is a major factor to keep OEMs competitive in the market. For instance, in the automotive market in Germany, the life cycle of the product is 15 years which is a long time to stock spare parts (Inderfurth & Mukherjee, 2008).

In spite of the aftermarket's obvious benefits, most organizations waste its potential. Companies should use systematic approaches to increase their benefits in the aftermarket business and this is possible by focusing on three following strategies (M. A. Cohen et al., 2006):

- Improve after-sales service quality levels;
- Reduce investment in service assets;
- Cut operating costs;

In previous survey paper in 2001 by (Kennedy, Wayne Patterson, & Fredendall, 2002) the spare parts management has been studied specifically in case of maintenance policies, where spare parts as repairable parts are being used to maintain equipment in working condition. To perform the study, the authors conducted a literature review based on the intermittent property of the spare parts demand and its effects on proposed inventory management policies. Also, the role of the modern technologies like Internet in tracking parts in supply chain is being notified as an important factor to improve the material and information flow in the supply chain management via faster and more up to date communication between customers, retailers and manufacturers.

Spare parts or service parts are distinguished in three control situations including (Botter & Fortuin, 2000):

- Service parts to maintain own (production) facilities and systems;
- Service parts to service (professional) systems installed at customer sites;
- Service parts to repair consumer products at service workshops;

Service parts can be divided into two categories:

- Repairable parts: Service parts that are repairable (both technically and economically). When failure happens, failed parts are replaced with a new part and sent to repair facility.
- Consumables: Service parts that are not repairable (both technically and economically). When failure happens, failed parts are replaced with a new part and scrapped.

In this literature review, first, the importance of the spare parts business is studied, then a review is provided to show related techniques and policies which are applied in spare parts inventory systems. In order to set up the review, the literature review is organized in such a way that in the beginning the inventory control policies are introduced. Then the perspective of uniqueness of spare parts on the inventory management is illustrated that makes them to be distinguished from finished good products or work in progress products. Next, spare parts clustering and demand are studied and forecasting methods are reviewed. The use of Game Theory for inventory systems planning is studied. Also spare parts pricing as a strategic method to increase the profit of the suppliers is evaluated.

3.2. Spare Parts Inventory Management

Spare part inventory management is considered as a special case of general inventory management with special characteristics, especially high-variety, low-volume demand and high risk of obsolescence. The main goal is to achieve adequate service level with minimum inventory investment and operating costs. Despite the importance of the spare parts business, the previous literature on spare parts management is limited.

There are some literature reviews including spare parts and maintenance models (Kennedy et al., 2002, Nahmias, 1981, Pierskalla & Voelker, 1976) (José Roberto do Regoa, 2011) that discussed about the maintenance inventories, the maintenance policies including procure, inspect, and repair or replace units for stochastically failing equipment, and finally demand forecasting and inventory control decisions on the different life cycle Stages of spare parts.

In general categorizing, spare parts management can be divided into two major groups:

- Planning and operational aspects (the determination of optimum spare parts level) consists of demand forecasting, service levels and inventory levels.
- Strategic and organizational aspect consists of outsourcing, locations, channels of distributions, supply chain type, information and communication technologies.

The need for spare parts arises when a component fails and must be replaced. The failure rate is not deterministic and it has a link to the quality of maintenance. This, in turn, causes an unpredictable demand for spare parts. Maintenance for each machine can be categorized into preventive and corrective groups. From a spare parts manufacturer's perspective, preventive maintenance can result in periodic but stochastic demand. On the other hand demand for corrective maintenance is deterministic under the assumption that only one failure can occur at any instant of time, but stochastic in the time of arrival. Therefore, in both cases the nature of demand is intermittent and forecasting methods can predict demands.

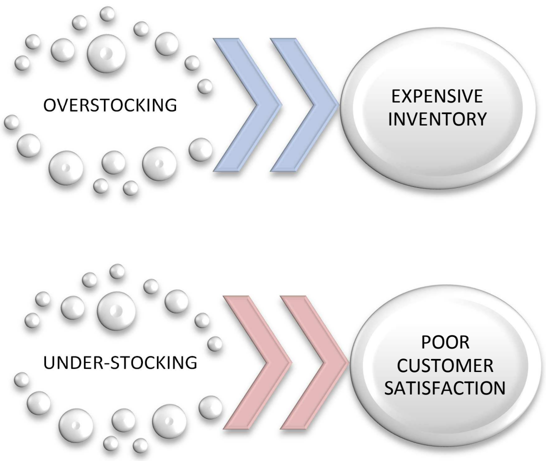


Figure 5: Stocking levels impact on inventory management

In this manner stocking level affects cost and liability of the suppliers. Over-stocking leads to expensive inventory planning and even obsolete inventory while under-stocking contributes to poor customer satisfaction. The body of literature in the field of planning is adequately extensive, while there have been few studies in the field of strategic and organizational matters including logistic system design and strategic concepts which lead to a service-to-profit supply chain (Wagner & Lindemann, 2008).

In order to determine the optimal level of inventory, different inventory management concepts have been developed that can be clustered according to their complexity as follows:

1. Simple repair shops: (Scudder, 1984) improved scheduling rules and spares stocking policies for a repair shop supporting multi-item repairable inventory system.

2. Multi-hub systems: (Wong, Cattrysse, & Van Oudheusden, 2005) presented a model for determining spare parts stocking level. This method applied for a single-item, multi-hub, multi-company, repairable inventory system to minimize total system cost. The total system cost is defined as the combination of inventory holding, downtime and transshipment costs.

3. Closed-loop supply chains: The goal is to study the return of products as a source of spare parts. The integration of product returns into business operations using information management, and its implication for the IBM company has been investigated by (Fleischmann, Van Nunen, & Gräve, 2003). (Spengler & Schröter, 2003) integrated production and recovery systems which benefits from an Internet-based information technology. The communication platform uses the system dynamics to evaluate spare-parts demand for the Agfa-Gevaert and Electrocycling GmbH.

4. Multi-echelon supply chains: Spare parts optimization as a part of Maintenance, Repair and Operation materials (MRO) needs an integrated approach that removes noise factors. Noise factors can be listed as the following:

- Bad coding;
- Lack of classification;
- Poor network practices including uncoupled warehouses and poor relation with suppliers;
- Poor data integrity known as non-centralized and non-real time data;

The multi-echelon technique for recoverable item control can be used to model the inventory and it has been tried for two realistic systems including a subway system and a mobile telephone company in Venezuela (Diaz, 2003).

Case studies on supply chain management for spare parts have been investigated in different industries such as:

- The computer industry (Ashayeri, Heuts, Jansen, & Szczerba, 1996), (Thonemann, Brown, & Hausman, 2002);
- The airline industry (Tedone, 1989);
- The metal industry (Suomala et al., 2002);
- The electronics industry (M. Cohen, Kamesam, Kleindorfer, Lee, & Tekerian, 1990);
- Power generation (Bailey & Helms, 2007);
- The military (Rustenburg, van Houtum, & Zijm, 2001);

Decision-making in strategy and design level focused on logistic systems, which is a long term procedure.

Spare parts maintenance has four major characteristics including:

1. Criticality: The criticality of a part is related to the critical consequences of the part failure on the whole process. Generally the criticality can be estimated by evaluating the down time costs of the process.

2. Specificity: Spare parts can be grouped into two major sets of standard parts which have widely usage, so there are several suppliers that provide them and specific parts which have low volume demand, so suppliers are reluctant to stock them and their availability is not good.

3. Demand pattern: This includes the demand size and its predictability. Demand volume for spare parts is usually low and irregular. Predictability is related to the failure process and can be estimated by statistical means. Parts can be grouped into two categories of parts with random failures and parts with predictable wear patterns.

4. Value of parts: High value parts are not intended for stock holding and low value parts have to be managed for an effective replenishment arrangement to decrease the administrative costs of ordering.

Decision-making in level of design and strategy focuses on the effect of the mentioned factors on logistic elements including (Huiskonen, 2001):

- Network structure: Determines the number of echelons and their locations in the supply chain.
- Positioning of materials: Defines how to position materials in the network.
- Responsibility of control: Discusses about cooperation and risk pooling among the suppliers.
- Control principles: Manage the material flow.

Also there is an emphasis to consider the whole supply chain including complex mix of materials, information and service labor in analysis. This analysis increases the coalition among different parties at planning Stages (Dennis & Kambil, 2003).

Inventory control management can be classified into general parts and spare parts. General parts usually have high and independent demands while spare parts have intermittent demand (Prof. Maurizio Faccio, 2010). Since Economic Order Quantity (EOQ) model, many inventory control policies have been developed for general parts inventory control management. Some classical models can be listed as following:

- Continuous review (Q,r);
- Periodic review (T,S);
- Base stock (B);

In (Q,r) policy, an order size of (Q) is placed when the inventory level reaches to r. In (T,S) policy, at every interval time T the inventory is reviewed and replenished to the level (S). Both continuous and periodic models are useful for high and stationary demands and the difference is that for the continuous review policies the interval time is variable and the replenishment amount is constant while for the periodic policies it is reverse. The base stock model is a special case of continuous review model, which reviews the inventory level and whenever the inventory level drops to $(B-1)$ it places and order of single unit. This policy is useful for items with low demands, which are similar to spare parts.

There have been considerable number of studies available for the general parts inventory management including (Love, 1979), (Silver, Pyke, Peterson, & others, 1998), (Muckstadt, 2004), (Sherbrooke, 2004), (W. J. Hopp & Spearman, 2008).

One of the first inventory models addressing intermittent demand was introduced by (Williams, 1982). They considered a model similar to (Q,r) with variable interval time based on a Gamma distribution. A periodic review model, that determines the inventory level based on adjusted demand distribution using Bayesian method, developed by (Popovi’c, 1987). (Aronis, Magou, Dekker, & Tagaras, 2004), (HILL, 1999) introduced a base stock model that determines the stock level based on using Bayesian model to forecast the demand. An expert inventory management system was developed by (Petrovic, Petrovic, Senborn, & Vujoševi’c, 1990) that considers additional subjective aspects for costs, lead-times and demand beyond traditional data. The distribution of the time between failures is exponential and subjective questions about reparability, repair time, cost and criticality of components are answered by users and users will get the lot-size and the expected inventory cost. A continuous review system (Q,r) is provided by (Jin & Liao, 2009) that minimizes the costs of purchase, storage, failure, and revision of control parameters during time intervals which follows an exponential distribution or constant failure rate. A similar model was developed later, which considers the intervals between failures as a Weibull distribution (Liao, Wang, Jin, & Repaka, 2008). An inventory control system for spare parts was proposed by (Lonardo, Anghinolfi, Paolucci, & Tonelli, 2008) that determines the level of spare parts by minimizing the total storage cost and assuming the demand as a normal distribution. The solution to this inventory optimization problem benefits from stochastic linear programming and it is validated by some tests over real historical data related to the orders and availability of 2704 spare parts in a period of four years that obtained from an Italian large manufacturing industry.

A heuristic method to obtain the order point and order level in (s,S) model by using Markov chains and assuming Poisson demand was developed by (Gomes & Wanke, 2008). The proposed method is compared with a conventional simulation showing that the results are the same. In spare parts inventory systems identical parts can be used in different equipment with different down times. Therefore, demand for spare parts can be classified into critical and non-critical demand. Based on a case study for semiconductor equipment, a (r,r,Q) inventory model for spare parts have been proposed that determines reorder point and reorder quantity according to criticality of the parts. The reorder point r and the critical level are equal and once the inventory level drops to the reorder level, new order with size Q will be released and the remaining stock is reserved for critical demand until the inventory replenished (Chang, Chou, & Huang, 2005). The critical or non-critical demand is assumed to be high enough to be modeled as the normal distribution. Service parts stock management seeks to increase availability of spare parts in the warehouses in right time and place to satisfy customer demands. Customer satisfaction can be calculated by the first fill rate value (FFRV). An inventory stock mix optimization problem has been formulated by (Lonardo et al., 2008) that determines the optimal safety stock levels for the spare parts that minimizes the total production and inventory costs while satisfying desired FFRV. There are some theoretical inventory models for spare parts. Among them, the most investigated policy is the so-called (S-1,S) model, a particular case of (s,S) models, that assumes that demands arrive as Poisson process (Feeney & Sherbrooke, 1964). When transactions are greater than one, the use of compound-Poisson models for demand has been proposed (Williams, 1984). These models are difficult to apply because they need assumptions to identify compound distribution parameters. In other words one parameter is needed for exponential distribution inter-arrival times of demand, and two parameters for the Gamma distribution for the demand size. Two different (s,Q) inventory models, known as a simple and advanced model, for spare parts in a confectionary producer production plant have been developed (Strijbosch, Heuts, & Van der Schoot, 2000). For the simple model it is assumed that demand is normally distributed over the lead-time and the advanced model utilizes the Gamma distribution for the demand. In order to find the effective inventory policy for a mail processing equipment manufacturer that stocks 30,000 distinct parts in a distribution center, a constrained optimization model with the goal of total inventory investment minimization subject to constraints on customer services has been developed (Wallace J Hopp, Spearman, & Zhang, 1997). Because of the size of the problem, the problem is not tractable to exact analysis and three different heuristic methods have been proposed to solve the optimization problem. An inventory control policy of a service part in its final phase for an appliance manufacturer is investigated (R. H. Teunter & Klein Haneveld, 2002). An order-up-to policy has been suggested that minimizes the total expected undiscounted costs of replenishment, inventory holding, backorder and disposal where demand is considered as a stationary Poisson process.

Three different options are introduced by (Inderfurth & Mukherjee, 2008) to supply spare parts during the product life cycle between end-of-production and end-of-service. First, setting up a large order with the final lot of production, second, setting up extra production runs, and third, implementing remanufacturing to manufacture spare parts from used parts. The authors solved the problem by proposing Decision

Tree, Dynamic Programming procedure and a heuristic method to find the optimal combination of three options. In a case study for controlling spare parts inventory level of an electronic equipment manufacturer, the optimal parameter S of an (S-1,S) inventory system is calculated by applying a Bayesian approach to forecast demands (Aronis et al., 2004). A heuristic method, which has close relation to the Greedy heuristic, based on duality theory, is developed by (Morris A Cohen, Kleindorfer, & Lee, 1989) that determine base stock policies for various spare parts in a facility that stocks various parts for set of products. As far as the usage of equipment is changing over time, it will result in intermittent demand for spare parts that fluctuate with time. Therefore, demand could be considered as a non-stationary Poisson process and an inventory policy similar to (S-1,S) inventory system has been proposed by (Bian, Guo, Yang, & Wang, 2013). Demands for spare parts for items which are no longer manufactured could be assumed as a Poisson process with failure rate that is decreasing exponentially. A dynamic programming formulation is developed by (Hill, Omar, & Smith, 1999) that determines replenishment policy which minimizes the total discounted setup cost, production cost, inventory holding cost and backorder cost over the time horizon. Inventory pooling known as lateral transshipments and direct deliveries can help companies to maintain high service levels with low cost. This strategy provides an inventory system which is insensitive to the lead-time distribution (Alfredsson & Verrijdt, 1999). The low cost information sharing and possible quick delivery of items with reasonable cost are two major factors that affect inventory management. The sharing and transshipment of items most of the times reduces overall cost of inventory systems (Grahovac & Chakravarty, 2001).

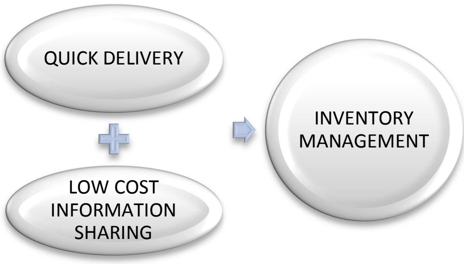


Figure 6: Inventory management crucial factors

Common ownerships that have reliable and precise information can benefit from pooling. For instance, a centralized inventory system that deals with several stores with common ownership can benefit a lot from the cooperation among different stores and retailers. In this situation cost allocation among the stores can be studied via three criteria of stability, justifiability and computability (B. C. Hartman & Dror, 1996).

Real-life spare parts network can benefit from lateral transshipment. A partial pooling system is developed by (Kranenburg & Van Houtum, 2009) that determines base stock level and lateral transshipment for warehouses by exploiting a heuristic procedure. The advantage of pooling in the area of repairable spare parts with lateral transshipment has been investigated broadly by (Lee, 1987), (Axsäter, 1990), (Sherbrooke, 1992), (Alfredsson & Verrijdt, 1999), (Grahovac & Chakravarty, 2001), (Kukreja, Schmidt, & Miller, 2001), (Kukreja et al., 2001, Wong et al., 2005, Wong, Oudheusden, & Cattrysse, 2007).

Among reviewed literatures, we can list following inventory control policies that have been widely used in spare parts management field:

1. Order-up-to level policy (Aronis et al., 2004, Ashayeri et al., 1996, Bian et al., 2013, M. A. Cohen & Lee, 1990, Feeney & Sherbrooke, 1964, Fleischmann et al., 2003, Gomes & Wanke, 2008, Hill et al., 1999,

Inderfurth & Mukherjee, 2008, F. Karsten, Slikker, & van Houtum, 2012, FJP Karsten, Slikker, & van Houtum, 2009, Lonardo et al., 2008, Rustenburg et al., 2001, Scudder, 1984, Tedone, 1989, R. H. Teunter & Klein Haneveld, 2002, Thonemann et al., 2002).

2. Two parameters inventory policy (Q,r) (Chang et al., 2005, Gerchak & Gupta, 1991, Bruce C Hartman, 1994, Jin & Liao, 2009, Liao et al., 2008, Williams, 1982).

3. Newsvendor policy (Dror & Hartman, 2010, Bruce C Hartman, Dror, & Shaked, 2000, Inderfurth & Mukherjee, 2008, Montrucchio & Scarsini, 2007, Müller, Scarsini, & Shaked, 2002, Q. Wang, 1991).

4. EOQ based policies (Dror & Hartman, 2010, Guardiola, Meca, & Puerto, 2009, W Heuvel & van den P, 2007, Wilco van den Heuvel, Borm, & Hamers, 2007, S. X. Li, Huang, & Ashley, 1996, Q. Wang, 1991, Whitin, 1955).

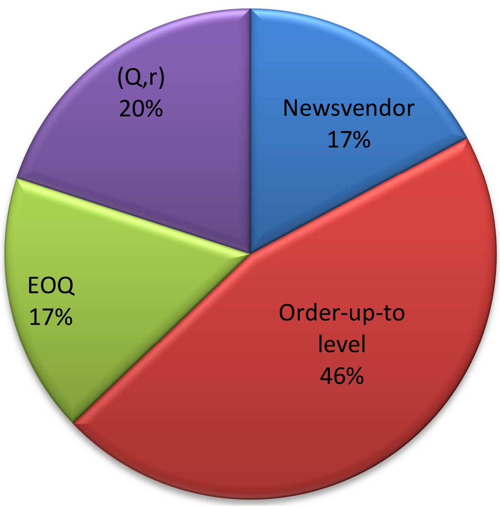


Figure 7: The usage frequency of inventory control policies in spare parts related literatures

3.3. Spare Parts vs. General Parts

Spare parts are being used to maintain or repair the final products or equipment which basically deals with high level of inventory investment and customer satisfaction. Different factors make spare parts inventories different from other types of inventories. The main factors are customers' satisfaction, variety of different parts and low demands that makes spare parts become unique.

The following factors affect the uniqueness of spare parts significantly (M. A. Cohen & Lee, 1990), (M. A. Cohen, Zheng, & Agrawal, 1997), (Muckstadt, 2004), (Kumar, 2004), (José Roberto do Regoa, 2011):

- Delays in repairing;
- Spare parts demand which mostly is intermittent;
- High risk of obsolescence due to complexity of products and their life cycles;

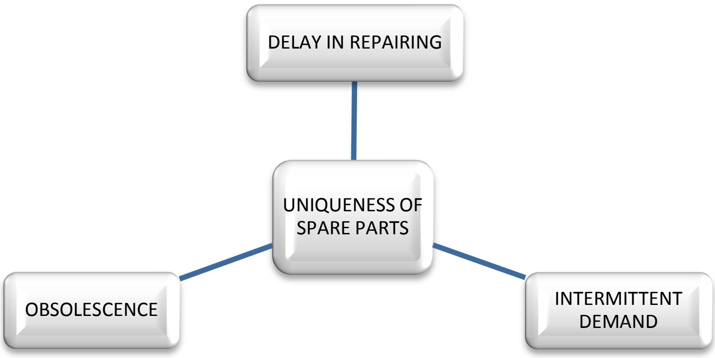


Figure 8: Spare parts uniqueness criteria

As long as spare parts inventories are not intermediate or final products, the policies that govern their inventories are not the same as work in progress (WIP) and finished goods (FG) (Kennedy et al., 2002). This difference significantly caused by two following factors: first, their functions are different. It means, WIP exists to smooth production rate and FG exists as a source of product to be delivered to the customer, but spare parts exist for maintenance to keep equipment in working condition. Second, inventory policies are different because WIP and FG rates can be changed to adjust the production rate but the level of spare parts depends on the use of machineries and the quality of maintenance.

Therefore demands for spare parts depend on the maintenance policy. There are two types of maintenance policies, scheduled or preventive and unplanned repair. For the first situation, demands are predictable but for the latter, they are unpredictable. Meanwhile, usually the cost of stock-outs is significant so the safety stock is necessary and the amount of stock pile can be determined according to the following categorization (Kennedy et al., 2002):

- Maintenance functions;
- Management issues;
- Age-based replacement;
- Multi-echelon problems;
- Obsolescence;
- Repairable parts;
- Special applications;

Maintenance functions are useful to give solutions to calculate optimal re-order point and quantity of the orders. In other words, these deal with when to place an order, how many units to be ordered while making decision between reducing the costs and increasing the availability (Mamer & Smith, 1982), (Seidel, 1983). Also (Hegde & Karmarkar, 1993) have studied the support costs and system availability from the customer point of view. In another study, providing spare parts kits based on the ratio of the expected usage and the cost of having spares at hand is investigated by (Robert, 1980). Furthermore, the problem of field repair kits which is providing spares and the tools for the repairing has been studied by (Mamer & Smith, 1982).

Management issues discuss about maintenance inventory very broadly. This comprehensive discussion starts from non-technical aspects based on (Moore, 1996):

- Reliability;
- Capacity objectives;
- Systematic strategy;
- And continues to technical aspects including:
- Control-based forecasting;
- Maximum likelihood estimation (Foote, 1995);
- Recursive methods to obtain probability distribution of machine down times (Gupta & Srinivasa Rao, 1996);
- Population models to group parts;
- Optimization models (M. A. Cohen, Kleindorfer, & Lee, 1986), (M. A. Cohen, Kleindorfer, Lee, & Pyke, 1992), (Haneveld & Teunter, 1997);
- Categorization techniques such as ABC, fast moving, slow moving and non-moving (FSN) and vital, essential and desirable (VED) to partition parts and criticality factor evaluations like analytic hierarchical process (AHP) (Gajpal, Ganesh, & Rajendran, 1994);

Replacement the items at the end of their pre-determined interval is a simple maintenance policy. The age replacement decision has been investigated for a system with a single component subjects to a random failure (Michael & Derek, 1996). Also this decision-making has been tried for extended models with many identical units by use of the optimal stocking policy. This policy benefits from Barlow-Proschan age replacement policy that is supported by the optimal (s,S) inventory policy (Zohrul Kabir & Al-Olayan, 1996).

Multi-echelon problems deal with where to place spare parts. (Muckstadt, 1973) Introduced the MOD-METRIC system. This system determines the stock level according to two different factors:

- Minimizing the expected backorder cost of the end product;
- Using the average re-supply time for the end product;

Also two-level inventories' stock level can be minimized by using heuristic methods (Vrat, 1984). It is a fact that number of stocking policies depends on the number of the stocking points or levels, so a branch and bound algorithm to find an optimal policy is useful (M. A. Cohen et al., 1986, 1992). In field service management the goal is to find a proper way to prioritize customers. In order to achieve this goal, multi-echelon problems can be implemented where a closed queuing network model is used to balance the high service rate to the customers while minimizing the cost of holding down the spare parts (Papadopoulos, 1996). The influence of the limited repair capacity on multi-echelon repairable item inventory systems has been studied by (Diaz & Fu, 1997) that considers different repair distributions where demand for spare parts are generated as Poisson or compound Poisson distribution.

The repair distributions are as the following:

- Single-class exponential: Single server with exponentially distributed service time.
- Single-class general: Single server with service time that is governed by some general distribution.
- Multi-class general: Multi servers with service time that is governed by some general distribution.

However, the authors recommended that using the double binomial negative distribution improves the accuracy of their model.

In order to determine the lower and upper bounds, multi-location and multi-period inventory systems have been studied. The lower bound determination is based on Lagrangean decomposition and an upper bound is based on dual relaxation (Karmarkar, 1981).

Spare parts are retained in inventory as an insurance against the machine downtimes because downtimes are expensive and processing the spares from suppliers can be very time consuming. Obsolescence is a problem for parts that are used rarely. The obsolescence cost is usually considered as a part of inventory holding cost and specifically contributes to those types of spares known as insurance which have a high probability of not being used during the system lifetime (Karmarkar, 1981, Kennedy et al., 2002). The effects of obsolescence are studied in an EOQ model which states that ignoring the cost of being obsolete as small as 20% would lead to an average increase of 15% in the inventory cost (Cobbaert & Van Oudheusden, 1996).

The ability to repair failed parts and enter them to the inventory system can be examined via repairable items. The demand for the serviceable stock can be analyzed with two different Poisson processes, one for repairable items and the other for non-repairable items (Allen & D'Esopo, 1968). Ready rate as a fraction of time that the customer back orders are zero can be used instead of expected backorder. The problem of inventory allocation among main assembly and sub-assemblies can be solved by using three methods (Silver, 1972):

- Dynamic programming;
- Marginal allocation (maximizing the ready rate);
- Lagrange multipliers;

Backward dynamic programming with a joint probability density function for both the demand and return can be used to calculate the optimum repair level, purchase level and scrap-down-to level (Simpson, 1978). A queuing analysis is implementable for the repairable inventory of a repair depot. The system service is calculated by availability, which is the probability that the spare inventory is not empty (Gross & Ince, 1978). The optimal level of inventory for repairable spare parts can be studied subject to budget constraints (Kohlas & Pasquier, 1981).

The main problem of spare parts inventory and sales is that they have low level of inventory turnover which is commonly about one to two times per year. This low rate leads to an obsolescence of 23% of the whole inventory (G. P. Cachon & Netessine, 2006). The obsolete parts are no longer can be sold which tides up with high cost of holding and warehouse. (Van Jaarsveld & Dekker, 2011) analyzed obsolescence of service parts in a practical environment. The authors proposed a method to estimate the risk of obsolescence of service parts using the behavior of identical parts in the past.

Spare parts management for some sort of special cases has been studied by different researchers. These special problems consist of some rare conditions which can be listed as the following:

- Regular and emergency orderings, spare parts are ordered at regular intervals and kept for emergency failurs (Kaio & Osaki, 1981);
- Effects of job lateness on the optimum repair parts, providing repair kits based on cyclic queue without the assumption of high availability of parts (Gross & Ince, 1978);
- Random number of parts, where the number of parts to be replaced is a random variable (Bruggeman & Van Dierdonck, 1985);
- Replenishment at the ends of phases, where parts are replace based on time-based maintenance (Vujossevi'c, Petrovi'c, & Senborn, 1990);
- Spare parts management for equipment with scheduled non-continuous usage, where systems of equipment are used on a periodic scheduled basis instead of continuous usage (Bridgman & Mount-Campbell, 1993);

3.4. Spare Parts Clustering and Demand

Optimization problem related to inventory management includes consideration of inventory costs, and service level by selecting inventory control parameters, allocating control resources and purchasing decisions. For this purpose, item classification is very useful. Classification of spare parts is an essential part of the inventory management because it affects the methods of the demand forecasting and inventory control policy (Huiskonen, 2001) (Boylan, Syntetos, & Karakostas, 2006).

Spare parts for industrial maintenance can be classified by Vital, Essential and Desirable (VED) (Gajpal et al., 1994) whereas consumer goods are classified in Pareto’s graph with categories of high, medium and low values (ABC) (Silver et al., 1998).

In another classification, spare parts are categorized into three categories of intermittent, slow moving and smooth by (Williams, 1982). His work has been resumed by (Eaves & Kingsman, 2004) and categorized in more details into smooth, irregular, slow moving, slightly intermittent and highly intermittent. The effective classification can simplify and optimize the inventory policy. For instance, (R. Q. Zhang, Hopp, & Supatgiat, 2001) by using a modified ABC classification reduced the cost of inventory by 30%.

The most commonly used classification method is the ABC classification which is useful where materials are fairly homogenous and differ from each other by unit price or demand volume. However, the ABC classification is a one-dimensional method that is not suitable for control policies with several factors. In this case, multi-dimensional classifications are useful. (Duchessi, Tayi, & Levy, 1988) introduced a two-dimensional classification method which combines inventory cost and part criticality as a criteria. (Flores & Whybark, 1987) introduced a multiple-criteria classification method. (M. Cohen et al., 1990) used a general grouping method. (Petrovi’c & Petrovi’c, 1992) developed a heuristic classification model based on several factors like availability of the system, essentiality, price, weight, the volume of the part, availability of the parts in the market, and the efficiency of repair. (Gajpal et al., 1994) improved the criticality analysis of the spare parts in the classification by using the analytic hierarchy process (AHP).

The life cycle of the spare parts is affected by finished goods life cycle. Their life cycle can be divided into three phases of initial, normal or repetitive and final (Fortuin, 1980). Therefore, the demand for spare parts depends on finished goods, and following factors would affect it (Fortuin & Martin, 1999):

- Size and age of the final products (sales, running fleet, installation base, etc.);
- Products maintenance characteristics (preventive, corrective, etc.);
- Parts characteristics and their defects (wear, accident, aging, etc.);

Demand for spare parts is volatile and unpredictable, so demand forecasting and inventory management is very challenging (W. Wang & Syntetos, 2011). Demand for spare parts arrives in irregular time intervals and with variable quantities. This characteristic can be evaluated by two following factors (Prof. Maurizio Faccio, 2010):

1) ADI - average inter-demand interval: average interval between two demands of the spare part;
2) CV - coefficient of variation: standard deviation of the demand divided by the average demand;

According to changes in values of ADI and CV, four typologies could be assumed (Ghobbar & Friend, 2003):

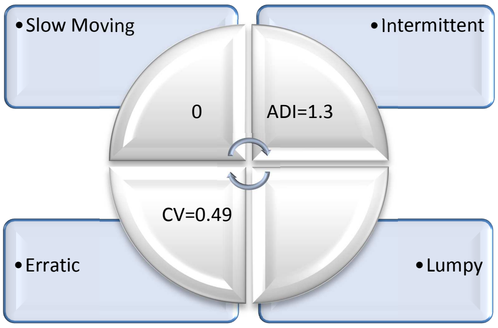


Figure 9 Patterns for the characterization of the spare parts demand

- Slow moving or smooth: They have low rotation rate (ADI=0 & CV=0);
- Intermittent: They have sporadic demand, which means there are a lot of period without demands but variability of the demand quantity is low (ADI=1.32 & CV=0);
- Erratic: The variability of the demand quantity is high but the interval time periods are constantly distributed (ADI=0 & CV=0.49);
- Lumpy: They have high variability in both demand quantity and interval times (ADI=1.32 & CV=0.49);

However, two additional factors should also be factored in this categorization, which are cost and criticality. The cost of purchase and maintenance and the criticality based on the risk of not completing a process that is assigned to equipment, are classified into low, moderate and high (Ben-Daya, Duffuaa, & Raouf, 2000). Spare parts demand is mostly intermittent or lumpy which means it occurs after a long variable periods without demand. The lack of parts leads to high losses, and demand forecasting can decrease the loss. The demand forecasting is necessary for inventory control and planning, although it has some errors (Love, 1979). Many forecasting methods as uncertainty reduction methods have been devised that may perform well when CV is low, but they perform poorly when demand is lumpy or intermittent. Lumpy demand for spare parts has been observed in different industries such as the automotive industry, durable goods spare parts in aircraft maintenance, and telecommunication systems. A classical reference for demand forecasting has been provided by (Wheelwright & Hyndman, 1998) and a related literature review has been provided by (Boylan et al., 2006) for last fifty years. According to (Prof. Maurizio Faccio, 2010), many different forecasting methods have been introduced in literatures such as Single Exponential Smoothing method (SES), Croston's method (CR), Syntetos-Boylan Aproximation (SBA), Moving Average method (MA), Weighted Moving Average method (WMA), Additive Winter method (AW), Multiplicative Winter method (MW), Bootstrap method (BT), Autoregressive and Moving Average methods (AMA), Poisson method (PM), Binomial method (BM), Grey Prediction method (GM) and Neural Networks (NN).

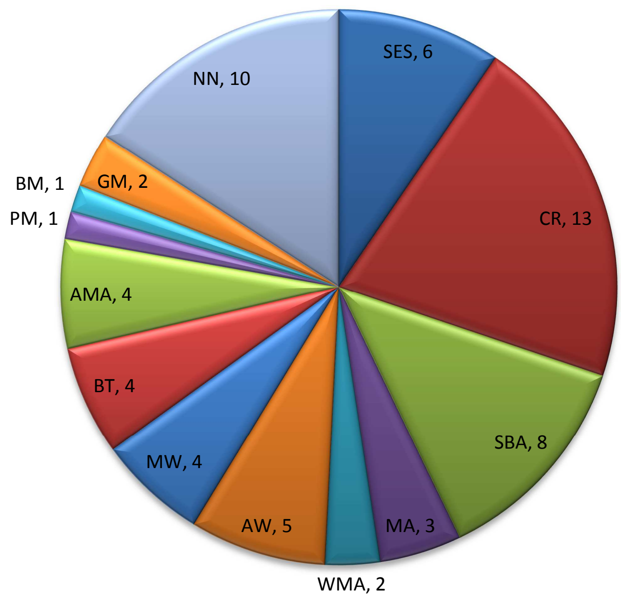


Figure 10: The usage frequency of forecasting methods in spare parts related literatures

One item which is important for the demand forecasting is the determination of the time bucket. The shorter time bucket results in a more intermittent demand. Several comparisons have been performed to select proper time buckets. It can be monthly (Eaves & Kingsman, 2004) (R. Teunter & Duncan, 2008) or weekly (Ghobbar & Friend, 2003) and even daily (Gutierrez, Solis, & Mukhopadhyay, 2008) and the issue of choosing the time bucket has not been discussed in the literature.

Methods used to forecast the demand and the optimal inventory level are determined using mathematical and operations research methods (Aronis et al., 2004). One of the most accurate methods for forecasting is the Single Demand Approach (SDA), which computes mean and variance of the demand during lead-time by use of three random variables. (Krever, Wunderink, Dekker, & Schorr, 2005) listed these random variables as follows:

1. Amounts demanded during lead-time;
2. Time intervals between demands
3. Lead-time;

Classical methods of demand forecasting like exponential smoothing are being used frequently for routine stock control systems which have large number of products. But for low demand items with intermittent demands, they are erroneous that result in an excessive inventory right after the demand occurs and lower before the demand occurs, but can be modified by separating estimation of intervals (Croston, 1972). A proposed correction (A. A. Syntetos & Boylan, 2001) is known as the Syntetos-Boylan Approximation (SBA) method. Several case studies have been done to establish the superiority of the SBA, Croston and double exponential smoothing techniques (Eaves & Kingsman, 2004, Ghobbar & Friend, 2003). Moreover, other more complicated models have been developed for the demand forecasting including; bootstrapping technique together with autocorrelation (Willemain, Smart, & Schwarz, 2004), bootstrapping with regression analysis (Hua, Zhang, Yang, & Tan, 2006), neural networks (Gutierrez et al., 2008) and Enhanced Fuzzy Neural Network (EFNN) method which uses the fuzzy logic method together with the Analytical Hierarchical Process (AHP) and Genetic Algorithm (GA) (S. Li & Kuo, 2008). Some inventory models known as reactive (Santoro, Freire, & others, 2008) do not use directly demand forecasting, and even for those models, a medium-term demand forecast is needed.

Demands for spare parts can also be more and more uncertain. The main reasons that cause this uncertainty are categorized into two groups of quick changes in customer’s preferences (for example in fashion industry demand for a specific color changes dramatically from time to time) and the structure of the supply chain (that means by moving to higher levels of supply chain the pattern of the demand will be more uncertain). This effect or phenomenon is known as Bullwip effect and several factors like erroneous demand forecast, long lead-times, supply shortage and backlog, price variations, etc. cause this effect (Inger, Braithwaite, & Christopher, 1995, Lee, Padmanabhan, & Whang, 1997). One of the most important forms of the demand variability is the simultaneous increase of the inventory level and decrease of the customer services, the supply chain management of a system with similar characteristic which has multi-echelons has been investigated by (Kalchschmidt, Zotteri, & Verganti, 2003). Spare parts demand is intermittent, also called lumpy, sporadic and erratic. It is characterized by infrequent demands, often of variable size with irregular intervals. Hence, researchers prefer to model demand from two perspectives, i.e. the demand size and inter-arrival times (Aris A Syntetos, Babai, & Altay, 2012). In this situation, the use of compound theoretical distributions that factors in the size-interval combination is very appealing. If time is considered as a discrete variable, demand arrives as a Bernoulli process with geometric inter-arrival distribution. If time is considered as a continuous variable, demand arrives as a Poisson process with exponential inter-arrival distribution. In order to model demand for spare parts various distributions have been used to represent time intervals and demand size. Most commonly used distributions in literatures are listed as follows:

- The compound Poisson distribution which is a combination of a Poisson distribution for demand occurrence and a geometric distribution for demand size, known as Stuttering Poisson (D. J. Gallagher, 1969), (Ward, 1978), (Watson, 1987);
- The combination of a Poisson distribution for demand occurrence and a normal distribution for demand sizes (Vereecke & Verstraeten, 1994);
- The combination of a Poisson distribution for demand occurrence and a logarithmic distribution for demand sizes, known as Poisson-Logarithmic process that yields a negative binomial distribution (NBD) (Quenouille, 1949);
- The gamma distribution as the continuous analogue of the NBD which covers wide range of distribution shapes (Burgin, 1975), (Burgin & Wild, 1967), (Johnston, 1980);

- The combination of a Bernoulli process for demand occurrence and a Logarithmic-Poisson distribution for demand sizes, known as log-zero-Poisson (Kwan, 1991);
- The combination of a Bernoulli process for demand occurrence and a normal distribution for demand sizes(Croston, 1972), (Croston, 1974);
- The Poisson distribution with unit-sized transactions (Silver et al., 1998), (Friend, 1960);

3.5. INVENTORY SYSTEMS AND GAME THEORY

The first application of Game Theory, cooperative and non-cooperative games goes back to (Von Neumann & Morgenstern, 1953). Supply chain management has both cooperative and non-cooperative interactions between different agents, and the cooperative games in supply chain management are called inventory games. Cooperative games can be categorized into deterministic and stochastic games that (Dror & Hartman, 2010) studied through EOQ and Newsvendor policies respectively. The EOQ model is designed for multi-item orders and known as the joint replenishment game. The latter game is based on classic-Newsvendor policy and known as the Newsvendor centralization game. Both of them have infinite repetition and used for single-Stage and stationary problems.

The application of the Game Theory in production and inventory management can be divided into two groups; players determine the condition of the market and market equilibrium can be found by studying players' interactions, and another group which consists of individual players who compete against each other; decision makers find the optimal decisions under these conditions (Q. Wang, 1991). The author first studied the application of static games in management science, then investigated the discount game (both quantity and price discount) in the buyer-seller environment. Results showed that quantity discount is always better for the seller. The rest of the research focuses on Newsboy game for substitutable parts with stochastic demand and in the end the market of repeated purchasing products is compared to the market of consumer durable products.

In first category, the presented models look for the market equilibria and investigate the existence, uniqueness and stability of the equilibrium and do not pay attention to the optimal decisions for players. This method is called Oligopolist theory and reviewed by (M. Shubik, 1981, 1984, 2006, M. Shubik & Levitan, 1980).

In the second category, the primary goal of the models is to find an optimal decision of the players and (Parlar, 1988) started working on this theory by considering two retailers who sell substitutable products with random demand, and their goal is to order an optimal number of parts to maximize their profits.

A recent literature review has been provided by (Dror & Hartman, 2010) that reviews the implementation of the cooperative games in inventory management. Similarly another survey is provided by (Fiestras-Janeiro, Garcia-Jurado, Meca, & Mosquera, 2011). According to the literature reviews, there are four different game setups:

- Players face deterministic demands and use economic order quantity policies;

- Players face stochastic demands and utilize single-order Newsvendor policies;
- Players face stochastic demands and use continuous review settings including penalty costs;
- Players face stochastic demands with different methods of game setup regarding spare parts application including infinite-horizon games, Erlang loss formula and queuing systems;

Supply chain management and inventory management can benefit from Game Theory. In general, Game Theory can improve or clarify interactions between different groups who are competing against each other. Cooperative and non-cooperative games are used to model several supply chains with single and multi-period settings (Chinchuluun, Karakitsiou, & Mavrommati, 2008), (G. P. Cachon & Netessine, 2006, Leng & Parlar, 2005). Game Theory is a useful tool to study supply chains, it can be used for decision-making where there are conflicts between multiple entities. The application of the Game Theory in supply chain management was first reviewed by (G. P. Cachon & Netessine, 2006, Gerard P Cachon, 2003) in which the authors focused on different Game theoretical methods. (Meca & Timmer, 2007) reviewed the application of the cooperative Game Theory to supply chain management. In another survey paper by (Gerard P Cachon, 2003, Leng & Parlar, 2005), a review based on classification of supply chain topics has been provided. Also, (Gerard P Cachon, 2003) presented the literature review on supply chain collaboration with contracts. According to (Leng & Parlar, 2005) numbers of studies related to supply chain and Game Theory have been doubled in last decade compared to previous decades.

For instance, the Game Theory has been used to analyze detailed supply chains (G. P. Cachon & Netessine, 2006) where cooperative and non-cooperative games are used to solve static and dynamic games. The existence of the equilibrium in non-cooperative games has been studied. Generally, extensive games have not been considered for supply chain games and only normal forms have been considered.

In order to investigate distribution systems where supplier has finite or infinite capacity (Dai, Chao, Fang, & Nuttle, 2005) Game Theory can also be useful. In this case a single period game between one supplier and two retailers is considered. Inventory control decisions can be made by retailers using Game Theory which depends on the existence of the Nash equilibrium. When the pure strategy could not be found, the Stackelberg method is implemented to find the optimal strategy in form of the leader-follower game. Also, supply chain management and inventory management for substitutable products with stochastic demand have been investigated (Avsar & Baykal-Gürsoy, 2002). An extensive survey for supply chain games has been done by (G. P. Cachon & Netessine, 2006) which basically looks for existence and uniqueness of the equilibrium in non-cooperative games, however they developed different game-theoretical techniques to study four types of games including:

- Non-cooperative static games;
- Dynamic games;
- Cooperative games;
- Signaling, screening and Bayesian games;

The goal of supply chain management is higher benefits, lower costs and better service quality and Game Theory is an effective tool to achieve this goal. In another review by (Leng & Parlar, 2005) supply chain games have been investigated in five areas:

1. Inventory games with fixed unit purchase cost (including games with horizontal and vertical competition among players, usually as a single period game).
2. Inventory games with quantity discounts (where the buyer as a player has an interest to increase the number of purchase quantity to benefit from lower unit price).
3. Production and pricing competition (efficiency of the supply chain depends on the production and pricing decisions where production equilibrium can be found by using Game Theory and Cournot method involving capacity decisions, service quality, product quality, and advertising).
4. Games with specific joint decisions on inventory (where each player should make two or more decisions at the same time).

One of the first studies of the buyer-seller interaction in supply chain was published by (Whitin, 1955). They examined a monopolistic market position with respect to the seller and discussed about the inventory level by using EOQ model where demand was a linear function of price. The assumption of the market as a monopolistic market compared to competitive market will result in different strategies and number of research has been done by (Abad, 1988, Cheng, 1990), (Kunreuther & Richard, 1971), (Susan X Li & Huang, 1995), (Susan X Li, Huang, & Ashley, 1995) about monopolistic markets. (S. X. Li et al., 1996) studied the buyer-seller relationship in the market by constructing cooperative and non-cooperative operations which are formulated as an EOQ inventory models with a Game Theory framework. In form of non-cooperative game the seller acts as a leader and buyer is the follower, the equilibrium of this game is consistent with the result of the EOQ model.

In another scenario, sellers and buyers cooperate to maximize their profits. The comparison of results reveals that the total payoff and the order quantity are higher in case of cooperation and the sale price is lower. In other words the quantity discount is more beneficial in the cooperative game (S. X. Li et al., 1996).

A supplier inventory problem is investigated by assigning two-person Game Theory solution. The supplier utility function of the game is set up from the side of the supplier. This game has two players (supplier & customer) that has no dominant strategy and can be solved as a mixed strategy problem (Mileff & Nehéz, 2006).

Multiple retailers who form a coalition place their joint orders to a single supplier; this interaction has been studied as a cooperative game which is the economic lot-sizing game (X. Chen & Zhang, 2007). This game has a non-empty core when inventory holding cost and back logging cost have linear functions. This approach is investigated based on linear programming duality which has an optimal dual solution that contributes to an allocation as the core. In similar research, an economic lot-sizing game has been suggested between several retailers with known demand for a limited period of time who can reduce their cost by placing joint order (Wilco van den Heuvel et al., 2007).

The contrast between cooperative and non-cooperative games has been investigated by (Hart & Mas-Colell, 1997) which states that non-cooperative games are strategy oriented methods while cooperative games study the outcomes and decide what is the optimal coalition to get the best payoff and distribute the cost among the players while satisfying the non-emptiness of the core.

In order to study cooperative games in supply chain management, the literature can be divided into two categories of deterministic and stochastic including; the deterministic joint-replenishment game (this game is based on an EOQ model) and the Newsvendor centralization game (this game basically relies on the classic newsvendor setting) (Dror & Hartman, 2010).

The EOQ game first represented by (Meca, Timmer, García-Jurado, & Borm, 2004), where the cooperation between different firms is structured and the proportional division method is used for the cost allocation. The basic inventory model is introduced and can be extended to more precise model as an inventory model. This method then followed by (Anily & Haviv, 2007). They considered an infinite-horizon deterministic problem and showed that this game has non-empty core where optimal replenishment policy is determined by power-of-two policies. Also, (Dror & Hartman, 2007) studied inventory games and cost allocation while using an EOQ model as an inventory policy. Similarly (W Heuvel & van den P, 2007) proceed to use this method for economic lot-size games. This method applied for a model with a fixed time horizon, known demand for a single item in a situation where backlogging is not allowed. (Guardiola et al., 2009) used EOQ games for production-inventory games. Joint replenishment can be addressed by the optimal power-of-two policies, which was introduced by (Roundy, 1985, 1986) and gives 98% cost effectiveness as the ratio of the lowest cost to the selected cost.

To study mentioned problem as a cooperative game, (J. Zhang, 2009) proposed a general sub-modular setup by use of the Lagrangian dual and strong duality to guarantee the non-emptiness of the core. In their setup, there is one-warehouse with multiple retailer inventory models and a joint cost function. Their goal is to find the best replenishment policy that minimizes the cost during an infinite time horizon.

The (Q,r) games first introduced by (Bruce C Hartman, 1994) were not concave but it had non-empty core. The proof of having non-empty core for a normally distributed demand has been investigated by (B. C. Hartman & Dror, 1996). A cost allocation for a centralized inventory who deals with different stores with in common ownership is studied. Three major characteristics of the game including stability (existence of the core), justifiability (logical relation between cost and benefit), and computability are satisfied during their analysis. (Gerchak & Gupta, 1991) applied a continuous (Q,r) model for a single-period inventory and proved that the total benefit is higher in the case of coalition and they applied different methods of allocation resulting that some stores are not satisfied with the share cost. (Robinson, 1993) showed that the best cost allocation that Gupta used is not stable and they introduced a new policy for allocation based on Shapley value which is stable but needs complicated computations in case of large number of players.

The Newsvendor game that considers different stores with single period demands for single item has been studied (Bruce C Hartman et al., 2000). There was a centralized inventory system with holding and penalty cost, the allocation cost defined by setting a centralized inventory cooperative game which has non-empty core and the existence of the non-empty core has been examined for demands with normal symmetric distribution and joint multi-variate normal distribution (Bruce C Hartman et al., 2000). Similar studies have been conducted by (Özen, Fransoo, Norde, & Slikker, 2008) assuming that where there are number of M warehouses and number of N retailers, the retailers can order single products and after their demands realization they can change their demand. In this environment the cost allocation between retailers is investigated as a cooperative Newsvendor game. A similar game has been presented by (Slikker, Fransoo, & Wouters, 2005) where transshipment among retailers modeled and the game has non-empty core, means players have incentives for cooperation. Also there are several studies that show the core of the Newsvendor game is non-empty (Müller et al., 2002), (Slikker, Fransoo, & Wouters, 2001). (Müller et al., 2002) proved that the Newsvendor game has non-empty core for all kind of random demands distribution. (Montrucchio & Scarsini, 2007) examined the Newsvendor solution for infinite number of retailers of single-item who attend a coalition and proved the game is balanced and the core exists.

A general framework for the analysis of decentralized distribution centers with number of N retailer and one or more central locations has been developed. The demand is stochastic and when demand is unsatisfied the retailer can use excess stocks at other retailers or central location. A cooperative framework for the sequential decision-making on inventory, shipping and cost allocation is introduced which has non-empty core and provide pure strategies based on Nash equilibrium (Anupindi, Bassok, & Zemel, 2001). In this situation each retailer as an independent agent looks for his own interest instead of the whole system profit. Cooperation can increase their benefits but sometimes it conflicts with individual benefits. The game studies possible scenarios of cooperation and competition. The solution consists of three different Stages of cooperation, cooperation-competition hybrid form known as coopetitive (Anupindi et al., 2001). This solution has been extended by (Granot & Sosic, 2003) where demand is stochastic for an identical item regarding that there is a three-Stage decentralized inventory system where in the first Stage before realization of the demand, each retailer orders his initial inventory level, then after realization decides on the level of inventory that he wants to share and finally residual inventories are allocated and transshipped for residual demands and the profit would be allocated.

Inventory centralization is not always beneficial and it can reduce the total performance. This idea has been studied by (Anupindi & Bassok, 1999) to investigate a car manufacturer and its two outlets and compromise when it is more beneficial for the manufacturer to consider outlets as one (centralization) or consider them as competitive dealers with independent demands. The study investigates the effect of lost sales in stock-out situation on manufacturer profit. In other words, they look for what is the effect of market search in cooperative decision-making. Market search is the percentage of the customers who face unsatisfied order at the local retailer and search for the product at the other retailer. Studies show that there is a threshold for the market search and above the specific amount, coalition would result in loss of the manufacturer or even total less payoff or benefit for the manufacturer and retailers as the whole system. Generally, decentralized strategy would be more beneficial in case of the high rate of market search.

In decentralized situation with stochastic demand one can compete first and then cooperate (Anupindi et al., 2001). It means retailers compete for transshipment while there is a non-empty core, then they cooperate and there is a pure Nash equilibrium and core solution. In industries like oil and gas sector there are several inventory plants that stock spare parts to support facilities operations. In this condition, risk pooling and cost allocation by means of centralized inventory solution can lead to considerable savings considering ordering and holding costs. Game Theory principles are very beneficial for this allocation. The comparison between centralized and decentralized inventory of spare parts has been provided showing that centralized system achieve more savings and five different approaches of cost allocation have been investigated (Guajardo & Rönnqvist, 2012).

The cost allocation in the context of repairable spare parts pooling has been studied regarding two different situations. First, participants cooperate in pooling without having any self-interest, and in second situation they participate in the game with interest of maximizing their benefits. Two strategies of core concept and Nash equilibrium have been examined to investigate two different problems respectively (Wong et al., 2007). The results show that in case of cooperation players can increase their payoffs, also the game with imperfect information is studied which indicates that having an agreement on downtime cost or service level can boost the required trust among players to convince them to cooperate (Wong et al., 2007).

When cost of spare parts inventory is high, companies can reduce the cost of their inventory by pooling their stock parts. This pooling can be defined as a cooperative cost game which reduces expected joint holding and downtime costs (FJP Karsten et al., 2009). The suggested non-empty core game is applicable even for the companies with non-identical demands, base stock levels and downtime costs. To be precise it investigates that there is a stable allocation while there is a non-identical demand and base stock level or there is a non-identical down time cost. When companies have non-identical downtime costs along with non-identical base stock levels or demand rate, it is possible to have an empty core game.

The stability of pooling arrangements is the main concern of the cooperation, and fair distribution of holding and downtime costs among participants have been studied by setting two different considerations; first for a setting with fixed stocking level and then for the optimized stock level (F. Karsten et al., 2012). In both cases a stable cost allocation has been provided which is equivalent to the result of the Erlang loss system. Furthermore, some realistic considerations have been considered as assumptions including; demand with Poisson process, perfect and immediate repair of failed parts, full pooling, constant repair lead-time, and emergency procedure in case of stock-out and infinite time horizon regarding the long life time of the machines.

The cost allocation is defined as the core of the game and five different methods of cost allocations have been implemented, including; Egalitarian which simply assigns equal share to each player (Tijs & Driessen, 1986), Proportional to demand which assigns the cost share for each player based on his demand proportion (Wong et al., 2007), Altruistic which assigns the cost share for each player based on the proportion of his stand alone share to the total share while they are all playing alone (Audy, D'Amours, & Rönnqvist, 2012), Shapely value which allocate the cost among players based on their marginal cost of entering the coalition (Wong et al., 2007), and Equal Profit Method (EPM) which looks for stable cost allocation with shares of as similar as possible (EPM) (Frisk, Göthe-Lundgren, Jörnsten, & Rönnqvist, 2010).

Companies who use complex machines need to stock low-demand expensive spare parts. Those companies can cooperate with each other to meet their demands. The cooperation can be practiced by keeping their own stock-points and let the others to use them through lateral transshipment. A diminished total cost of spare parts inventory is achievable and the distribution of inventory costs among companies would be determined by using cooperative Game Theory models (F. Karsten, Slikker, Houtum, & others, 2006).

A single firm can minimize its spare parts inventory costs by use of proper inventory management policies. In case of existence of collective firms, the joint inventory costs could be minimized by means of cooperation. A basic inventory model with deterministic demands for spare parts has been provided where several shops place their orders to a supplier in a cooperative manner. The savings of cooperation is allocated between shops by means of cooperative Game Theory (Meca et al., 2004).

Among reviewed literatures, there are few papers that studied spare parts inventory systems and Game Theory that can be listed as follows:

- Repairable parts: (Wong et al., 2007), (Guajardo & Rönnqvist, 2012), (FJP Karsten et al., 2009), (F. Karsten et al., 2012);
- Consumable parts: (Meca et al., 2004);
- Stochastic demand: Poisson process (Wong et al., 2007), (Guajardo & Rönnqvist, 2012), (F. Karsten et al., 2012), Poisson process/Erlang (FJP Karsten et al., 2009);
- Deterministic demand: (Meca et al., 2004);

3.6. Spare Parts Pricing

In some cases, number of spare parts is much larger than the primary products. Despite low number of spare parts sale (25% of total revenue for most OEMs), they stand for considerable portion of the suppliers profits (40-50% of all profits for most OEMs). In this environment, the price of the parts has more effects on the profit compared to the amount of sale or reduction in production cost.

It means the price of the parts is a main factor for profits. For instance, a consumer durable product manufacturer increased its profit by 30% with only 2.5% average increase of the products or an industrial equipment manufacturer benefits from 35% increase in its profit by only 3% increasing its price level (Marn & Rosiello, 1992). Also, consistent pricing will result in better customer satisfaction and their loyalty. The competitiveness of a company can be improved by three major activities, including decrease of cost of production; increase of the market share; and price adjustment known as the pricing strategy. In order to achieve a proper pricing strategy three major fields should be studied including; pricing strategies, pricing methodologies, and pricing tools. In a study for APL forklift manufacturer three pricing strategies have been investigated which are cost-based (uses the cost of the part then adding the standard mark-up value also known as cost-plus or mark-up pricing), market-based (it is based on the market willingness to buy the product or the comparison with other competitors prices) and value-based pricing (the customer decides on the value of the part and based on that, the cost of production and sale can be adjusted). Eight different methods were used including; spare parts pricing method, market adaptation, discount policy, life cycle pricing, kitting, price elasticity and spare parts competition and spreadsheet were used as a tool (Cullbrand & Levén, 2012).

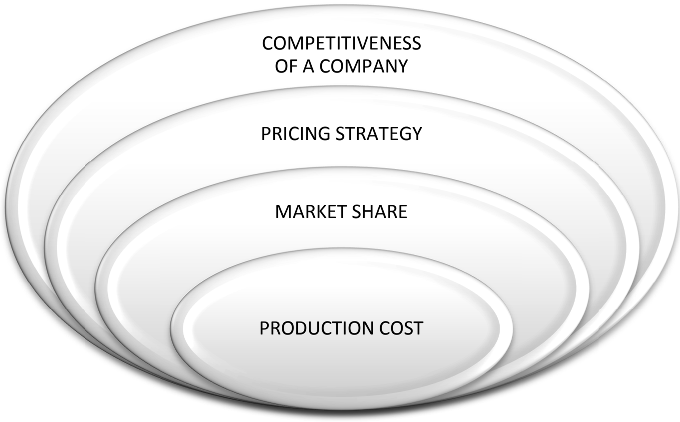


Figure 11: Competitiveness of a company influential factors

Despite the possible profit potential of selling spare parts, companies have neglected the proper price adjustment. The empirical research on spare parts pricing shows that only less than 20% of the companies benefit from systematic pricing strategies (Zinoecker, 2006), (Hinterhuber, 2004). The consolidation of pricing, production and distribution decisions in manufacturing environment has a potential to improve supply chain efficiencies. A literature review related to strategies that combine pricing decisions with production and inventory decisions have been provided by (Chan, Shen, Simchi-Levi, & Swann, 2004).

An optimal control theory model has been developed by (Kim & Park, 2008) that studies a company's strategy to determine spare parts price and warranty issues over the decision time horizon, i.e. the product's life cycle plus its end of life service period.

A market of multiple firms that face price-dependent, stochastic and substitutable demand has been investigated by (F. Y. Chen, Yan, & Yao, 2004). They proved that there is a pure-strategy Nash Equilibrium that determines the joint pricing/inventory decisions among the firms.

The simultaneous determination of pricing and inventory replenishment for a single item in the face of an uncertain demand has been developed by (Federgruen & Heching, 1999). They analyzed a periodic review inventory model in which demands are stochastic and independent in consecutive periods but dependent on the item's price.

Traditionally, OEMs have priced the spare parts based on the upper limit of the marketplace in which using cost-based pricing method is tempting. This method causes diminishing revenues and margins, customer dissatisfaction, increased competition, and lost market shares, because it leads to a lack of understanding the potential value of the parts (Vigoroso, 2005), (T. Gallagher et al., 2005).

Spare parts consist of thousands of components, so differentiated pricing strategies can be applied. One possible way of price segmentation is to differentiate spare parts prices based on the amount of competition. Companies can update their knowledge about how the spare parts are used and how the competitors enter to the market through field engineering and customer support. According to this concept, spare parts are divided into three groups; non-competition, some competition and heavy competition.

According to (Docters, 2003), the first step in pricing spare parts is creating spare parts matrix in line with part velocity (is how fast the spare parts move off from the inventory) and proprietary position (is how unique are the spare parts means they are inelastic when only one OEM exclusively provides them).

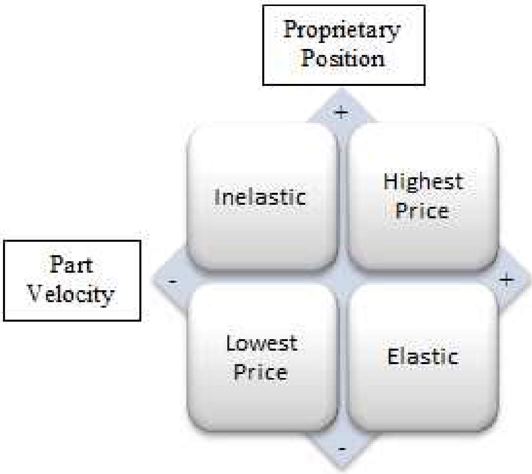


Figure 12: Spare parts price matrix

Three major methods of spare parts pricing have been suggested by (Vigoroso, 2005). The first method is to categorize spare parts based on complexity and competition. Intuitively, the spare parts with highest complexity and least competition can have the highest prices. The second method is the consistency-oriented pricing. In this method, spare are grouped into part families, and value driver for each family is defined and based on the value driver a pricing logic is built. As the value driver increases the prices increase. The third method is to price spare parts in comparison with a new product. The upper bound for repair of machinery including the labor cost and spare parts is about 50-70% of the new product's price.

CHAPTER 4

4. RENEWAL COST VS. REPLACEMENT COST

4.1. INTRODUCTION

To investigate the profitability of spare parts business, specifically in industries such as automobiles, white goods, industrial machineries and information technology, the research comes up with the idea of renewal cost versus the replacement cost. The replacement cost of a product is defined as the current market price of the product and the renewal cost of a product is the acquisition cost of spares to completely renew the product excluding labor costs. Customers purchase products from OEMs and to keep them in working condition, they replace failure parts with spare parts. The price of products and its spares are set by OEMs and our research looks for fair or sustainable spare parts pricing via investigation of replacement and renewal costs. In this chapter, these costs for some products with specific characteristics as listed below are calculated, and the ratio between the renewal cost and the replacement cost as a scale to evaluate the sustainability of the spare parts pricing is determined.

- High volume products;
- Products with lots of components;
- Products with a long lifetime;

One of the best products that can be fit in aforementioned characteristics is the passenger car. In this chapter, first the procedure of the data acquisition for spare parts and costs calculation for two BMW car models (328i & X6) are described. Then the same procedure is repeated for other similar consumer products and the ratio between renewal cost and replacement cost are provided.

4.2. Replacement Costs

The replacement cost of products is defined as the current market price of the product. The current prices for each vehicle have been acquired in two conditions. First the brand new price (replacement cost) and then the used vehicle price. The prices have been determined based on the KBB website data as listed in Table 9.

Table 9: Vehicles prices

Reference	BMW 328i	BMW X6
KBB Price for used vehicle (average)	$20,169.50	$41,000.00
KBB MSRP Price for new vehicle	$43,995.00	$60,495.00

4.3. Data Acquisition

In order to determine the price of the car and its parts, parts lists are collected according to the procedure that is explained in detail in the Appendix.

4.4. Cost Analysis

After gathering information about parts lists for each vehicle, the price list for each category or main group of vehicles has been developed. The total price list determines the renewal cost of each vehicle which is provided in following sections.

4.4.1. RESULTS PER CATEGORY

The following sections provide an overview of the collected data by dividing all MGs into four categories which are Powertrain, Chassis, Vehicle Body and Electrical System.

4.4.1.1. POWERTRAIN

The vehicle's powertrain incorporates the engine, fuel system, exhaust system, transmission system, gearshift, and drive shaft. It consists of 10 MGs, 36 SGs, 77 SSGs, and 1,503 parts for BMW 328i and 10 MGs, 43 SGs, 85 SSGs, and 1635 parts for BMW X6. The detailed price list is listed in Table 10.

Table 10: Powertrain price list

	BMW 328i				BMW X6			
Main Group	**SGs**	**SSGs**	**Parts**	**Costs**	**SGs**	**SSGs**	**Parts**	**Costs**
ENGINE	9	27	873	$29,602.39	11	27	812	$33,761.14
ENGINE ELECTRICAL SYSTEM	11	20	141	$5,124.17	9	13	125	$3,908.18
FUEL PREPARATION SYSTEM	2	4	76	$2,517.79	2	7	157	$5,475.42
FUEL SUPPLY	3	5	125	$2,479.79	3	5	87	$3,086.12
RADIATOR	1	5	69	$2,102.53	1	8	116	$4,077.28
EXHAUST SYSTEM	2	4	115	$6,399.82	3	6	86	$7,843.75
ENGINE AND TRANSMISSION SUSPENSION	2	2	32	$514.56	2	2	29	$1,032.65
AUTOMATIC TRANSMISSION	3	7	44	$10,337.60	4	8	116	$11,060.34

	BMW 328i				BMW X6			
Main Group	**SGs**	**SSGs**	**Parts**	**Costs**	**SGs**	**SSGs**	**Parts**	**Costs**
GEARSHIFT	1	1	2	$201.82	2	2	17	$1,181.57
DRIVE SHAFT	2	2	26	$975.28	2	3	48	$2,301.49
TRANSFER CASE E-VEHICLE TRANSMISSION	0	0	0	0	4	4	42	$4,599.73
Total Results	**36**	**77**	**1503**	**$60,255.7**	**43**	**85**	**1635**	**$78,327.67**

4.4.1.2. CHASSIS

The chassis incorporates the front axle, rear axle, steering system, brakes, and pedals. It consists of 5 MGs, 28 SGs, 39 SSGs, and 766 parts for BMW 328i and 5 MGs, 32 SGs, 44 SSGs, and 976 parts for BMW X6. The detailed price list is listed in Table 11.

Table 11: Chassis price list

	BMW 328i				BMW X6			
Main Group	**SGs**	**SSGs**	**Parts**	**Costs**	**SGs**	**SSGs**	**Parts**	**Costs**
FRONT AXLE	2	6	141	$3,980.32	4	9	224	$10,938.62
STEERING	6	9	95	$5,670.65	7	10	128	$14,170.40
REAR AXLE	6	10	246	$8,521.11	7	11	316	$17,395.76
BRAKES	12	12	260	$9,230.76	11	11	288	$10,419.05
PEDALS	2	2	24	$279.93	3	3	20	$404.24
Total Results	**28**	**39**	**766**	**$27,682.77**	**32**	**44**	**976**	**$53,328.07**

4.4.1.3. VEHICLE BODY

The Vehicle body incorporates the bodywork, exterior, interior, wheels, and other equipment. It consists of 7 MGs, 40 SGs, 116 SSGs, and 3,233 parts for BMW 328i and 7 MGs, 43 SGs, 118 SSGs, and 3328 parts for BMW X6. The detailed price list is listed in Table 12.

Table 12: Vehicle body price list

	BMW 328i				**BMW X6**			
Main Group	**SGs**	**SSGs**	**Parts**	**Costs**	**SGs**	**SSGs**	**Parts**	**Costs**
WHEELS	4	4	169	$2,842.14	5	6	76	$3,701.63
BODYWORK	10	27	772	$33,081.34	10	21	1161	$93,224.33
VEHICLE TRIM	18	58	1995	$26,805.39	19	63	1648	$51,883.46
SEATS	3	11	130	$12,845.62	3	15	284	$21,875.33
SLIDING ROOF FOLDING TOP	1	1	50	$1,435.94	2	2	43	$3,057.76
EQUIPMENT PARTS	2	10	44	$484.71	2	7	44	$369.08
RESTRAINT SYSTEM	2	5	73	$2,430.06	2	4	72	$2,591.88
Total Results	**40**	**116**	**3233**	**$79,925.20**	**43**	**118**	**3328**	**$176,703.47**

4.4.1.4. ELECTRICAL SYSTEM

The vehicle's electrical system incorporates the vehicle cable harness, instruments, lighting, heater and air conditioning, audio and communication systems, sensors and control units. It consists of 7 MGs, 43, SGs, 107 SSGs, and 1,107 parts 7

MGs, 47 SGs, 133 SSGs, and 1460 parts for BMW X6. The detailed price list is listed in Table 13.

Table 13: Electrical system price list

	BMW 328i				BMW X6			
Main Group	**SGs**	**SSGs**	**Parts**	**Costs**	**SGs**	**SSGs**	**Parts**	**Costs**
VEHICLE ELECTRICAL SYSTEM	9	62	582	$9,132.70	9	59	526	$13,389.73
INSTRUMENTS, MEASURING SYSTEM	1	1	3	$849.32	2	2	8	$2,491.10
LIGHTING	6	8	133	$3,833.83	6	10	246	$6,380.02
HEATER AND AIR CONDITIONING	13	16	195	$6,388.28	12	16	273	$11,405.76
AUDIO, NAVIGATION, ELECTRONIC SYSTEMS	9	13	132	$4,118.40	12	31	273	$15,291.19
DISTANCE SYSTEMS, CRUISE CONTROL	2	4	39	$1,562.86	2	6	103	$2,683.13
COMMUNICATION SYSTEMS	3	3	23	$1,212.85	4	9	31	$2,011.53
Total Results	**43**	**107**	**1107**	**$27,098.24**	**47**	**133**	**1460**	**$53,652.46**

4.4.2. TOTAL RESULT

The BMW 328i has 29 MGs, 147 SGs, 339 SSGs, and 6,609 parts, and the total cost of the parts of $194,961.96 versus BMW X6 which has 30MGs, 165 SGs, 380 SSGs, and 7399 parts, and the total cost of the parts of $362,011.67. In other words, the renewal cost of each vehicle equals to the total cost of the spare parts that are listed in Table 14.

Table 14: Total price list

	BMW 328i					BMW X6				
Category	**MGs**	**SGs**	**SSGs**	**Parts**	**Costs**	**MGs**	**SGs**	**SSGs**	**Parts**	**Costs**
Powertrain	10	36	77	1,503	$60,255.75	11	43	85	1,635	$78,327.67
Chassis	5	28	39	766	$27,682.77	5	32	44	976	$53,328.07
Vehicle Body	7	40	116	3,233	$79,925.20	7	43	118	3,328	$176,703.47
Electrical System	7	43	107	1,107	$27,098.24	7	47	133	1,460	$53,652.46
Total	**29**	**147**	**339**	**6,609**	**$194,961.96**	**30**	**165**	**380**	**7,399**	**$362,011.67**

4.5. Development of The Costs Comparison

Based on this estimate of the renewal cost and replacement cost of the chosen BMW cars, it is evident that the renewal cost of these products is excessively high. The comparison of the replacement cost and renewal cost for other types of products with variety of brands, models, the replacement cost and number of parts helps us to evaluate the sustainability of spare parts prices. Therefore, in the following more products are selected for the costs comparison. The chosen products for the study are among following categories:

- Passenger cars;
- Motorcycles;
- All-train vehicles;
- Refrigerators;
- Lawn mowers;

- Trimmers and edgers;
- Lawn tractors;
- Washers;

In Table 15 the comparison of the replacement cost and renewal cost for different products are listed:

Table 15: comparison of renewal cost and replacement cost

BRAND	MODEL	Replacement cost	Renewal cost	Number of parts
BMW	X6 35i	$60,495.00	$362,011.00	7,399
	328i	$43,995.00	$194,961.00	6,609
Honda	CB1000R	$10,999.00	$42,443.00	2,227
YAMAHA	V star 250	$3,990.00	$19,627.30	1,253
Suzuki	RM-Z250	$7,399.00	$32,996.00	1,271
Honda	TRX90X	$2,999.00	$13,011.00	1,027
GE	GSS20GEWWW	$1,285.90	$5,785.75	240
	GTS20ICNCWW	$580.00	$2,589.00	105
B&D	SPCM1936	$209.99	$1,016.00	180
	NST2018	$59.99	$230.00	51
Craftsman	917272751	$1,614.99	$9,770.00	761
GE	WCVH6800J	$1,027.12	$7,332.50	162

Figure 13 shows the ratio between renewal cost and replacement cost:

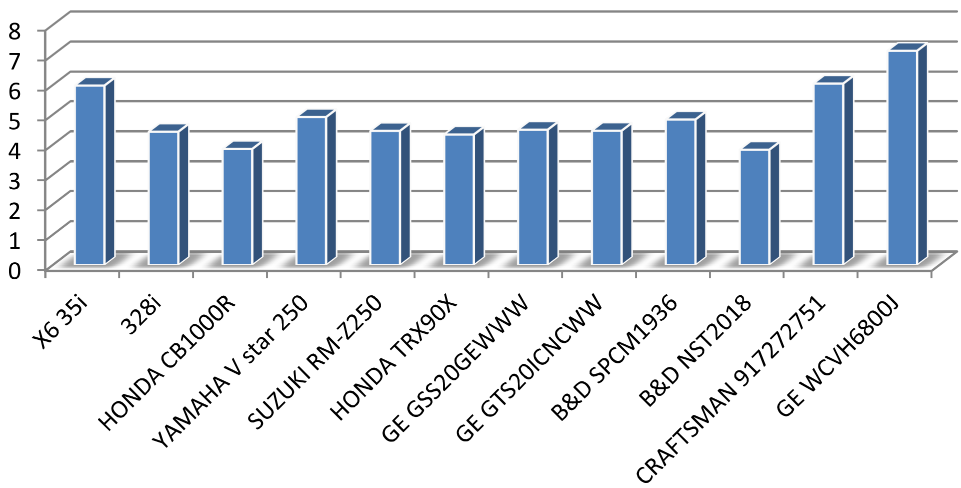


Figure 13: Ratio between renewal cost and replacement cost

The results show us the renewal cost of products is high and the ratio between the renewal cost and the replacement cost follows a certain pattern. The consistency of this ratio states that most OEMs implement cost-based or mark-up pricing to retrieve the final price of spare parts. According to (Vigoroso, 2005), cost-based pricing is popular and most OEMs use this method for spare parts price setting. Despite popularity of this method, it significant weak points (Hinterhuber, 2008, Kotler & Armstrong, 2010, Nagle & Holden, 2002, Noble & Gruca, 1999). The main disadvantages of cost-based pricing are under-pricing and over-pricing that leads to lower-than-average profitability, and ignoring competitiveness of the parts in the market. These factors are against the spare parts prices sustainability as a long term profitability of the OEMs without compromising consumer loyalty and satisfaction. This issue can be addressed in competition-based pricing or strategic pricing that will be discussed in the next chapter.

CHAPTER 5

5. SPARE PARTS INVENTORY GAMES

5.1. INTRODUCTION

Supply chain management and inventory management can benefit from Game Theory. In general, Game Theory can improve or clarify interactions between different groups who are competing against each other. Cooperative and non-cooperative games have been used to model supply chains with single and multi-period settings (Chinchuluun et al., 2008). Game Theory is a useful tool to study supply chains, for it can be used for decision-making when there are conflicts between multiple entities. For instance, it has been used to analyze detailed supply chains (G. P. Cachon & Netessine, 2006) where cooperative and non-cooperative games are used to solve static and dynamic games. The majority of related studies focus on the existence of the equilibrium in non-cooperative game.

Game Theory is the logical analysis of situations of conflicts and cooperation, such situations can be defined as a game in which (Straffin, 1993):

- There are at least two players;
- Each player has a number of possible strategies;
- The strategies chosen by each player determine the outcome of the game;
- Associated with each possible outcome of the game is a collection of numerical payoffs, one to each player;

Game Theory studies how players should play rationally based on set strategies and try choosing an action that gives them maximum profit. Game Theory is categorized into non-cooperative and cooperative games. Non-cooperative games use the notion of equilibrium to determine rational outcomes of the game. Most common related concepts are Nash Equilibrium, dominant strategy and sub game perfect equilibrium, which are defined as the following:

- Nash Equilibrium: chosen strategies by the players are in Nash Equilibrium if no player can benefit by unilaterally changing his/her strategy;
- Dominant strategy: dominant strategy results in the highest payoff no matter what the strategies of the other players are;
- Sub game perfect equilibrium: in extensive form, strategies are in the sub game perfect equilibrium if they constitute a Nash Equilibrium at every decision point;

In cooperative games, groups of players or all of the players form binding agreements to make coalitions. In cooperative games, determination of the solution concept depends on satisfying sets of assumptions known as axioms. The most common axioms are:

- Pareto optimality: the total utility allocated to the players must be equal to the total utility of the game;
- Individual rationality: the utility of each individual in coalition must be greater than his utility when playing alone;
- Kick-back: the allocated utility to each individual must be non-negative;

- Monotonicity: the allocated utility to each player should increase when the overall utility increases;

According to definite assumptions, spare parts inventory games in the form of normal, cooperative and non-cooperative, non-zero-sum, evolutionary, and competitive fringes will be investigated. The proposed games will study the OEMs' decision-making on spare parts pricing strategies, inventory levels, batch productions and re-manufacturing efforts.

5.2. OEM Against Market

5.2.1. Introduction

Spare parts, for many companies producing durable products, are the most profitable function of the corporation. The OEM, in a competitive and uncertain aftermarket, can benefit from Game Theory to manage his spare parts inventory. We study the spare parts inventory game as an N-person non-zero-sum single-shot game. Our game is restricted to a two-person (the OEM and the market), non-cooperative game setup. The market can be considered as an unreasoning entity whose strategic choices affect the payoff of the OEM, but which has no interest in the outcome of the game. This is modeled as the game against nature which means the OEM plays against the market. In our game, the OEM decides on his pricing strategy (in a competition against will-fitters to absorb more customers) and the order-up-to stock level. The OEM's inventory level strategy does not have a dominant level therefore the game has a mixed strategy solution. The solution of the mixed strategy determines the optimal strategy of the OEM to maximize his payoff in the aftermarket business. In other words, the theory of games in our spare parts inventory problem provides the optimal pricing strategy and the expected payoff distribution in relation to the expected market demand, and the OEM chooses his level of inventory with respect to the probability of the market's demand.

5.2.2. Inventory Game, OEM Against Nature

This game investigates spare part inventory problem as an N-person non-zero-sum single-shot game. In order to achieve this goal, the problem is restricted in two-person (the OEM and the market), non-cooperative game setup. The game has been set up from the OEM viewpoint, which means the solution of the game gives him the maximum payoff or minimum loss.

It has been assumed that the game is a non-cooperative game and the market is unkind and chooses hostile strategies. In spare parts stock control literature, it has been considered that the parts' failures are random and the Poisson process with constant failure rate represents spare parts demand. A more realistic modeling assumption declares that failure rate during the products' life span varies and it is not constant over the product life cycle and post product life cycle. This consideration justifies the non-stationary Poisson demand process assumption for spare parts. The OEM knows the demands for spare parts arrive as a non-stationary Poisson process, but he is not aware of the exact distribution of the failure intensity factors and can only forecast the bounds of the intensity factors. Also, the sale price changed by the OEM in comparison to the will-fitters sale prices has an influence on the demand intensity factors which is estimated by the OEM.

We consider the market as an unreasoning entity whose strategic choices affect the payoff of the manufacturers, but which has no interest in the outcome of the game. The aforementioned characteristic of the market enables us to consider the spare parts inventory game as a game against nature. Literature related to this includes: (Chinchuluun et al., 2008, Dror & Hartman, 2010, Meca et al., 2004, Mileff & Nehéz, 2006), but none of the research discussed the application of the game against nature in the spare parts inventory management. One of the recent study related to the application of the game against nature in the strategic decision-making is provided by (Beckenkamp, 2008). The author discussed the psychological aspect of decision-making in games against nature where the selected strategies improve the effects of Minimax-strategies in the cases of risk-aversion. In our study we model the spare parts inventory management as a game against nature which means the OEM competes with will-fitters on the sale prices, at the same time playing against the market to optimize spare part inventory levels.

5.2.3. THE MARKET DEMAND

Spare parts demand is intermittent or lumpy which means it often occurs after a long variable period without any demand. The lack of parts leads to high losses for suppliers, and demand forecasting methods can decrease the loss. Demand forecasting is very important, although it has some errors (Love, 1979). (Wheelwright & Hyndman, 1998) introduced a classical method for demand forecasting and (Boylan et al., 2006) has provided a related literature review covering forecasting over the last fifty years.

In our inventory game we assume that the OEM has limited information about the market's expected demand. The OEM knows that demands for spare parts arrive as a non-stationary Poisson process with varying failure rates or intensity factors, but the exact distribution of these factors over the time is not observed and only the upper and lower bounds of the intensity factors are forecasted.

In the aftermarket business, other than the OEM as an original manufacturer, there are other low cost manufacturers, known as will-fitters, who can manufacture the same parts and deliver them to the market. Based on the sale price of the manufacturers, the total demand for the spare parts will be allocated among suppliers. In other words, manufacturers compete with each other on their sale prices to absorb more customers, so the sale price is a decision variable for the OEM to optimize his payoff in the aftermarket.

Original parts may face failure because of defects and aging, and once they fail, demands for spare parts will arise. Due to the intermittent and slow moving characteristics of spare parts demand, we consider the demand for spare parts as a Poisson process. A Poisson process with an intensity factor or rate of (λ) is a stochastic process in which the inter-arrival time distribution is exponential with mean time of $(\mu = 1/\lambda)$ and the arrival distribution is Poisson with the rate of (λ). If λ is constant over time, the process is a stationary Poisson process and when (λ) changes over time, the process is a non-stationary Poisson process. In the case of spare parts management, the rate of demand depends on three factors: quality, usage and maintenance, which are not constant over time. Intuitively, we can assume that the demands are a non-stationary Poisson process in which the demand rate is a function of time $\lambda(t)$.

In this study, we assume that the OEM introduces a new product to the market and he wants to forecast demands for spare parts in the next future. The OEM considers that there are two major phases of the original parts failure: the initial phase (introduction phase) and the repetitive phase (growth, maturity and decline phase). Figure 14 graphs products in the market and demand for the spare parts during the product life span for automotive electronics industry (Inderfurth & Mukherjee, 2008).

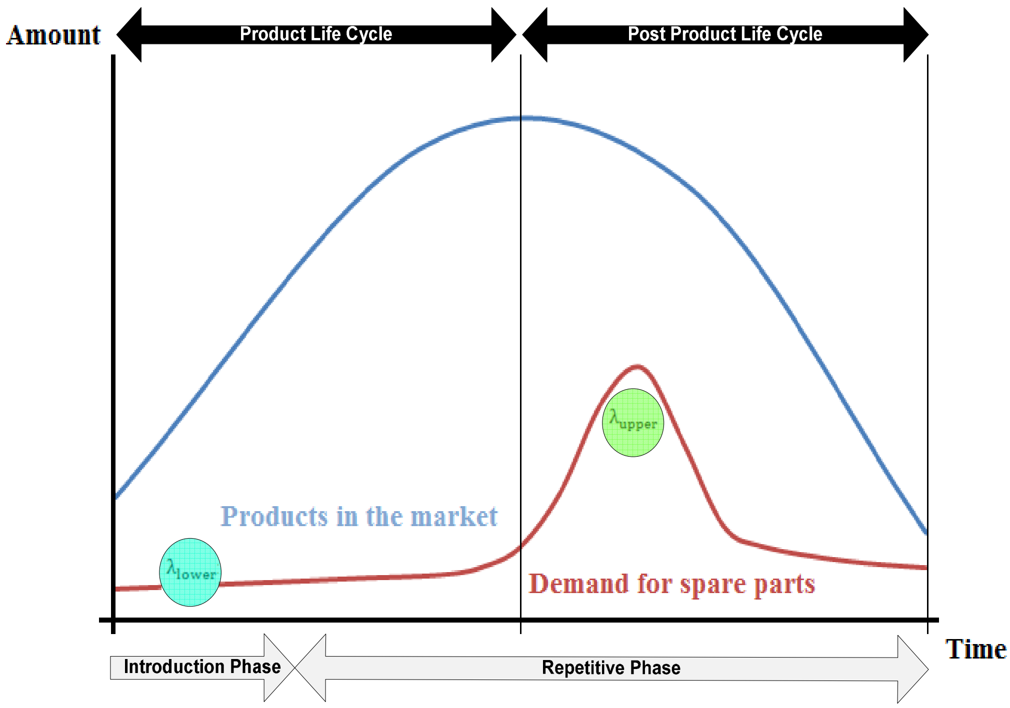


Figure 14: Demand for spare parts distribution during product life span

Modified from (Inderfurth & Mukherjee, 2008)

Demands for spare parts arrive as a non-stationary Poisson process with two levels of intensity factors:

- Upper bound or $(\lambda_{\mathrm{upper}})$: The repetitive phase of the original parts consists of the period right before the end of the original production and post product life cycle as the repetitive support, which has higher failure intensity;
- Lower bound or $(\lambda_{\mathrm{lower}})$: The initial phase of the original parts consists of the period right after the introduction of the original products to the market as the initial support, which has lower failure intensity;

Regarding the competition among the OEM and will-fitters based on the sale prices, and the demand intensity factors, the following parameters are known by the OEM which is listed in Table 16:

Table 16: The market demand

Variable spare part sale price	**Upper bound demand rate**	**Lower bound demand rate**
$\mathbf{c_s}$	$\boldsymbol{\lambda_{upper}}$	$\boldsymbol{\lambda_{lower}}$

5.2.4. THE OEM COST FUNCTION

The OEM strategies are determination of the sale price and the spare parts stock level in the order-up-to level inventory. The payoff of the OEM is the profit of the OEM $(\mathrm{K_B})$ which is the difference between the cost of production and inventory $(\mathrm{K_M})$ and the revenue $(\mathrm{K_R})$ which attained by selling spare parts. Let (X) be a random variable and Pr{X=x} determines the probability that the random variable (X) takes on a specific value (x) from some unspecified probability distribution. The expected value or mean of the random variable (X) is equal to $E[X]$ that is calculated in Equation (37):

$$E[X] = \sum_{x=1}^{\infty} x \Pr\{X = x\} \tag{37}$$

As it was mentioned in the last section, we assume that the demand arrives as a Poisson process. The Poisson distribution is given by Equation (38):

$$p(x) = \frac{(\lambda T)^x e^{-\lambda T}}{x!} \tag{38}$$

Where the mean E[X], from Equation (37) is found to be (λT). It is assumed that (λ) is the average annual demand for spare parts or the intensity factor and (T) is the average lead-time measured in years. The origin of the single-item inventory theory is a queuing theorem of Palm's (Sherbrooke, 2004). If the demand for an item is a Poisson process with an intensity factor of (λ) and if the lead-time for each failed unit is independently and identically distributed according to any distribution with mean (T) years, then the steady-state probability distribution of the number of failure units in the lead-time has a Poisson distribution with mean (λT). The most common inventory policy for low demand, high cost repairable items is the order-up-to level policy that is a one-for-one policy with a stock level of (S) and re-order point of $(S - 1)$.

There are two principal measures of item performance (i) the fill rate which is the percentage of demands that can be met at the time they are placed, and (ii) the backorders which is the number of unfilled demands that exist at a point in time. The expected fill rate and number of backorders are non-negative quantities, and they are calculated as Equations (39) and (40). Moreover, the expected number of parts in inventory is derived as Equation (41).

Expected fill rate:

$$\mathrm{EFR(S)} = \sum_{x=0}^{S-1} \Pr\{X = x\} \tag{39}$$

Expected backorder:

$$\mathrm{EBO(S)} = \sum_{x=S+1}^{\infty} (x - S)\Pr\{X = x\} \tag{40}$$

Expected inventory:

$$\mathrm{EI(S)} = \sum_{x=0}^{S} (S - x)\Pr\{X = x\} \tag{41}$$

The goal is to reach a low level of the backorder or a high level of fill rate with minimum investment on inventory. We must calculate the cost of production and inventory, as well as the revenue from selling the products. Equation (42) gives us the cost of production and inventory:

$$K_M = c_p \times S + p \times \mathrm{EBO(S)} + h \times \mathrm{EI(S)} \tag{42}$$

Equation (43) gives us the revenue of selling products:

$$K_R = c_s \times D \times \mathrm{EFR(S)} \tag{43}$$

Equation (44) gives us the OEM payoff:

$$\mathrm{K_B = K_R - K_M} \tag{44}$$

As we can see the cost of production and inventory and the revenue of selling products are functions of the sale price and inventory levels, which are the strategic actions of the OEM. Moreover there are different parameters that affect the cost of production and inventory. The parameters in Equation (42) and (43) are listed in Table 17.

Table 17: Cost of production and inventory parameters

Notation	Parameter definition
D	Demand (in units per period)
S	Spare parts inventory level (in units)
$\mathbf{c_p}$	Variable cost of production (in dollars per unit)
p	Penalty cost (in dollars per unit per period)
h	Holding cost (in dollars per unit per period)
$\mathbf{c_s}$	Sale price (in dollars per unit)

5.2.5. The Game Setup

Game Theory is applied to determine the sale price and the spare part stock level for an OEM who manufactures single-item spare parts, keeps them in inventory with order-up-to level inventory policy and sells them to the market. The game is set up between the OEM and the market and the solution of the game maximizes the profit of the OEM in the buyer-seller environment. The game has been set up from the OEMs perspective, which means the solution of the game gives him the maximum payoff or minimum loss. In order to achieve this goal, the problem is restricted to a two-person, non-cooperative game setup. Players choose their strategies simultaneously and the game is a static game that can be modeled and solved by finding the Nash Equilibrium. This configuration requires the following assumptions:

- Players play simultaneously;
- The OEM possesses the information of the original parts failure rates and can predict the allocated demand rates including: the upper bound intensity factor (λ_{upper}) and the lower bound intensity factor $(\lambda_{lower})$with respect to his selected sale price;
- The market as a nature has two choices of Poisson process demand types with upper and lower bounds intensity factors;
- The probability that the market plays with lower bound demand is (P) or $\mathrm{P(Market^{D_{lower}})}$;
- Respectively the probability that the market plays with upper bound demand would be $(1 - P)$;
- The OEM has several strategies which are order-up-to inventory levels (as discrete numbers) that varies from 1 to N;

The game setup can be shown in strategic or matrix form. Table 18 gives us the information of the game setup in the matrix form. This matrix known as the payoff matrix and the value of each cell is the payoff the OEM:

Table 18: The payoff matrix of inventory game

		Market (Nature)	
		D_{lower}	**D_{upper}**
OEM	**Order-up-to level**		
	S_1	$K_B(S_1,D_{lower})$	$K_B(S_1,D_{upper})$
	…	...	...
	S_N	$K_B(S_N,D_{lower})$	$K_B(S_N,D_{upper})$

Regarding the probability of market decision-making $P(Market^{D_{lower}})$, the expected utility of the OEM will be calculated from Equation (45):

$$EX(OEM^i) = P(Market^{D_{lower}}) \times U(OEM^i, Market^{D_{lower}}) + P(Market^{D_{upper}}) \times U(OEM^i, Market^{D_{upper}}) \ i: order - up - to\ levels = 1, \dots, N \tag{45}$$

Equation (46) tells us that the total probability of market decision-makings equals to 1.

$$P(Market^{D_{lower}}) + P(Market^{D_{upper}}) = 1 \tag{46}$$

The investigation of the payoff matrix declares there is no dominant strategy in this game that leads to lack of a pure strategy. Therefore, the game has a mixed strategy solution, which depends on the probability of the market's demand.

5.2.6. The Mixed Strategy Solution

In Game Theory, a game has a mixed strategy solution where a player has to choose his/her strategies over available sets of available actions randomly. A mixed strategy is a probability distribution that assigns to each available action a likelihood of being selected. In 1950, John Nash proved that each game (with a finite number of players and actions) has at least one equilibrium point known as Nash Equilibrium. This saddle point exists whenever there is a dominant strategy. In this game this can be explained based on the OEM payoff matrix where there is a specific level of inventory for the OEM that satisfies Equation (47). In case of existence of this specific level of inventory, the selected inventory level would be the dominant strategy for the OEM:

$$\begin{bmatrix} K_B(s_i^*, D_{lower}) > K_B(s_i \ , D_{upper}) \\ K_B(s_i^*, D_{upper}) > K_B(s_i \ , D_{upper}) \\ \forall\ s_i\ \&\ i \end{bmatrix} \tag{47}$$

In the decision-making problem, since there is no dominant level of inventory the OEM is making a choice between different alternatives. If the payoff for each alternative is known, the decision is made under certainty. If not, the decision is made under uncertainty. The major solution for decisions under uncertainty and risks are expected utility (EU) and subjective expected utility (SEU). Given a choice of an action and different possible payoffs in nature, SEU is calculated by multiplying the payoff for each option by the subjective probabilities. The decision maker chooses an action with the highest expected utility. The subjective expected utility (SEU) determines the inventory level for the OEM.

5.2.7. NUMERICAL STUDY

In order to demonstrate the decision-making of the OEM based on the implication of the game against nature and the mixed strategy solution, we consider a single-item spare part inventory game. The sample parameters of our spare parts management system are listed in Table 19. Also we assume that the average lead-time T equals to 1.

Table 19: Parameters of the sample spare part inventory

Notation	Parameter value
D	D_{lower}: $\boldsymbol{\lambda_{lower}}$ & D_{upper}: $\boldsymbol{\lambda_{upper}}$

Notation	Parameter value
D	D_{lower}: $\boldsymbol{\lambda_{lower}}$ & D_{upper}: $\boldsymbol{\lambda_{upper}}$
S	Decision variables (1 to 10)
$\mathbf{c_p}$	40
p	60
h	5
$\mathbf{c_s}$	Decision variables (90 to 130)

As mentioned in previous section, the OEM decision variables are the spare part order-up-to level and the sale price. Demand for spare parts arises when the original parts fail, then the emerging demand would be allocated among the OEM and will-fitters in the aftermarket business. The main factor that affects the allocation of the demand among the suppliers is the spare part sale price. We assume that the OEM can forecast the demand bounds (including the lower bound intensity factor and the upper bound intensity factor) with respect to its sale price. In Table 20 the forecasted demand rates versus the spare part sale prices are depicted.

Table 20: Parameters of the sample spare part inventory

Spare part sale price: $\mathbf{c_s}$	Upper bound demand rate: $\boldsymbol{\lambda_{upper}}$	Lower bound demand rate: $\boldsymbol{\lambda_{lower}}$
90	5.5	3.5
100	5	3
110	4.5	2.5
120	4	2
130	3.5	1.5

At each sale price level, the expected utility of the OEM for 10 different levels of inventory as a function of the probability that the market chooses to play with the lower bound intensity factor in the aftermarket business $\mathrm{P(Market^{D_{lower}})}$ has been calculated. According to the results of the game against nature, the optimal decision variables of the OEM are determined. Table 21 shows the result of our spare part inventory game:

Table 21: Parameters of the sample spare part inventory

Spare part sale price	Optimal order-up-to levels (higher-lower)	Share of lower order-up-to level	Guaranteed payoff	Maximum payoff
90	7- 6	76%	2	59
100	6- 5	88%	18	90
110	6- 5	91%	26	115
120	5- 4	78%	24	127
130	4- 3	83%	14	130

As we can see the Minimax of the OEM payoff or the guaranteed payoff of the game reaches to its maximum level at the sale price of 110. In other words, the optimal sale price for the OEM is 110 and the optimal inventory policy is to keep the order-up-to level of the spare part inventory at 6 and 5 for the 9% and 91% times of the production horizon respectively. Figure 15 depicts the trend of the OEM's guaranteed payoff versus the sale price.

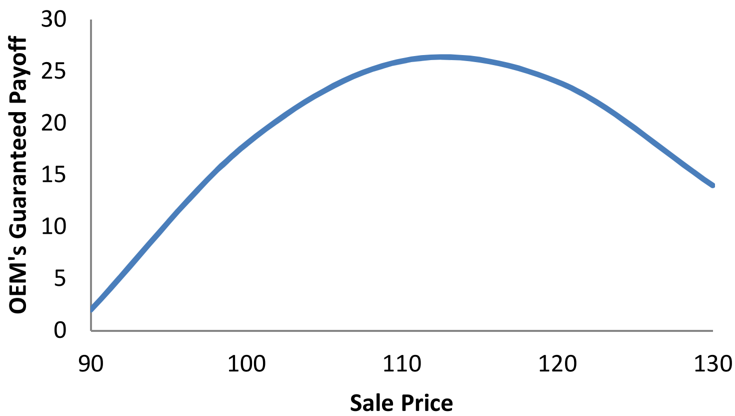


Figure 15: The OEM's guaranteed payoff vs. The sale price

In the following the detailed description of the optimal solution of the game is presented. In Table 22, the resulting expected fill rate, expected backorder and expected inventory level with respect to different values of the stock levels for the lower bound and upper bound of the demand are listed.

Table 22: Numerical example for a single-item inventory

Mean annual demand (λ) Average lead-time (T) Item cost (c_p)	2.5 1 40			4.5 1 40		
S	**EFR(S)**	**EBO(S)**	**EI(S)**	**EFR(S)**	**EBO(S)**	**EI(S)**
0	0.00	2.50	0.00	0.00	4.50	0.00
1	0.08	1.58	0.08	0.01	3.51	0.01
2	0.29	0.87	0.37	0.06	2.57	0.07
3	0.54	0.41	0.91	0.17	1.75	0.25
4	0.76	0.17	1.67	0.34	1.09	0.59
5	0.89	0.06	2.56	0.53	0.62	1.12
6	0.96	0.02	3.52	0.70	0.32	1.82
7	0.99	0.01	4.51	0.83	0.15	2.65
8	1.00	0.00	5.50	0.91	0.07	3.57
9	1.00	0.00	6.50	0.96	0.03	4.53
10	1.00	0.00	7.50	0.98	0.01	5.51

According to EBO(S), EI(S) and related stock levels, the cost of inventory including the holding and backorder costs is calculated. Figure 16 shows the OEM's inventory cost versus the spare part order-up-to levels.

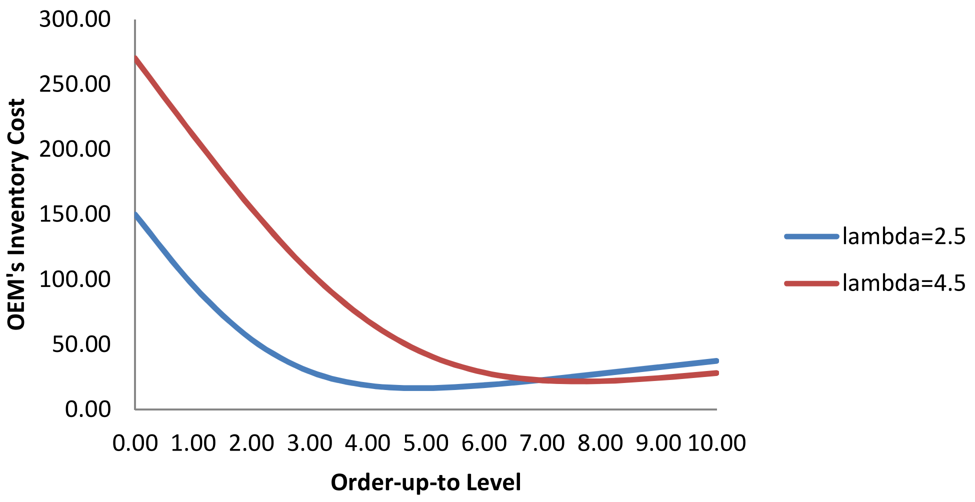


Figure 16: The OEM's inventory cost vs. the order-up-to level

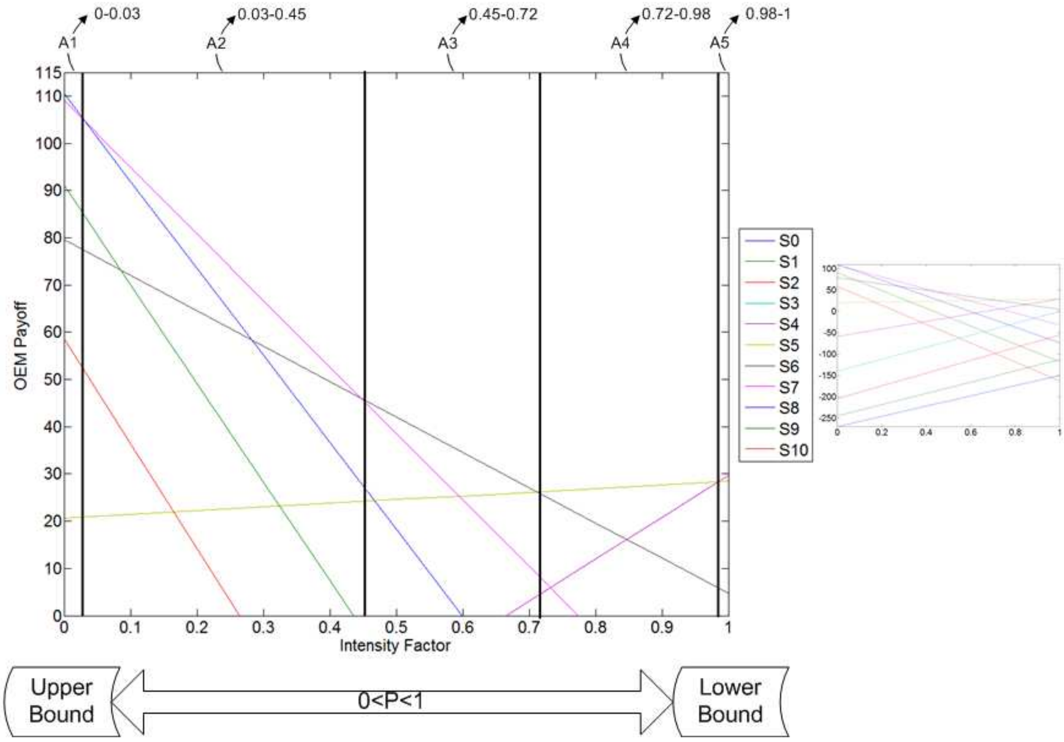


Figure 17: The OEM payoff distribution vs. the probability of the lower bound intensity factor

The expected utility of the OEM for 10 different levels of inventory as a function of the probability that the market chooses to play with the lower bound intensity factor in the aftermarket business $P(Market^{D_{lower}})$ has been calculated. The results are depicted in Figure 17 where the OEM payoff distribution is graphed vs. the probability of the lower bound intensity factor.

According to the results of the SEU, the optimal decision variables of the OEM are determined that maximizes his payoff in the aftermarket game. This decision-making is the inventory policy of the OEM that states the OEM should change his inventory level based on the probability of the market's intensity factor or demand:

- A1: If $0<P(Market^{D_{lower}})<0.03$ then select the inventory level of 8;
- A2: If $0.03< P(Market^{D_{lower}})<0.45$ then select the inventory level of 7;

- A3: If 0.45< $P(Market^{D_{lower}})$<0.72 then select the inventory level of 6;
- A4: If 0.72< $P(Market^{D_{lower}})$<0.98 then select the inventory level of 5;
- A5: If 0.98< $P(Market^{D_{lower}})$<1.00 then select the inventory level of 4;

The lowest point in the upper envelope of the expected payoff involves an inventory level of 6 and an inventory level of 5. According to the results of mixed strategy for that 2×2 matrix game, the optimal decision variables of the OEM are determined. The solution of the mixed strategy states that the OEM should switch between inventory levels of 6 and 5 with probability of 9% and 91% respectively. The resulting mixed strategy guarantees the payoff of 26 for the OEM in the long run.

On the other hand, the OEM has an opportunity to invest in performing a comprehensive market survey and precise data analysis to develop an accurate demand forecasting for the spare parts. Let us assume this investment costs the OEM $C_{forecasting}$. The method that is provided in our study helps the OEM to make decision to whether perform more precise demand forecasting. Equation (48) could evaluate the effort of the OEM to invest on extra demand forecasting:

$$\text{if } \max(K_B) - GT(K_B) > C_{forecasting} \text{ then invest on extra demand forecasting} \tag{48}$$

Where $GT(K_B)$ is the result of the mixed strategy solution for the OEM's payoff. In our proposed numerical study the $\max(K_B)$ is equal to 115 and $GT(K_B)$ equals to 26. Therefore, as long as $C_{forecasting}$ is less than 89, extra effort on demand forecasting would be a rational activity.

In order to examine the accuracy of the Game theoretical solution, a Monte Carlo simulation has been developed which relies on random sampling to obtain numerical results. The simulation runs many times to obtain the payoff of the OEM with respect to uncertainty of the market.

The simulation follows the following particular pattern:

1. Defining a domain of possible inputs;
2. Generating random inputs from a given probability distribution (uniform distribution) over the domain;
3. Implementing the spare part inventory policy and performing a deterministic computation over the inputs;
4. Aggregating the results;

The goal of the simulation is to show the comparison of the Game Theory approach and any other inventory and production policy.

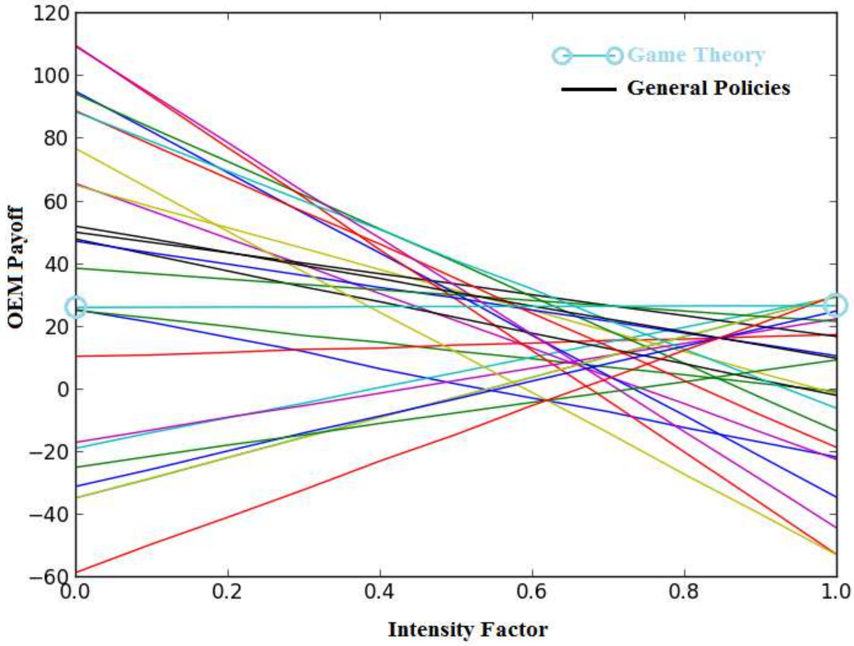


Figure 18: Simulation results - The OEM payoff vs. the probability of the lower bound intensity factor Comparison of General Policies & Game Theory

In figure 18, the payoff of the OEM vs. the probability of the lower bound intensity factor while implementing Game Theory solution and some other inventory policies (General Policies; including different levels of order-up-to level inventory with different strategies of keeping inventory for example setting order-up-to level to 4 and 8 and switching among them with probability of 30% and 70% respectively and etc.) is depicted. As we can see Game Theory approach guarantees the payoff of 26 for the OEM by switching among order-up-to levels of 6 and 5 with probability of 9% and 91% respectively.

The performance of the Game Theory approach is graphed in Figure 19 where general policies are considered as implementing order-up-to level of 5 and 6 with different strategies to keep the inventory such as 30% lower level and 70% upper level and etc. The results of the Monte Carlo simulation states that Game Theory approach allocates guaranteed payoff to the OEM in an uncertain market situation.

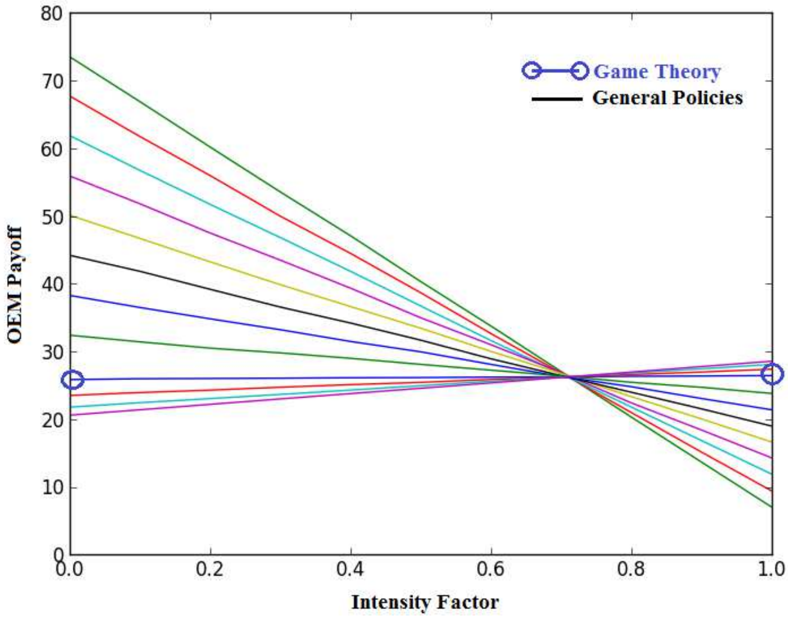


Figure 19: Simulation results - The OEM payoff vs. the probability of the lower bound intensity factor Comparison of General Policies (identical levels with different strategies) & Game Theory

5.3. OEM AGAINST WILL-FITTER

5.3.1. INTRODUCTION

This spare part inventory game is an N-person non-zero-sum single-shot game. The model is restricted to a non-cooperative three-person (two manufacturers and the market) game. The market is an unreasoning entity whose strategic choices including a bargain seeker or a price taker affect the payoff the manufacturers. The will-fitter has a fixed pricing strategy but the OEM can decide on his sale price to compete with the will-fitter. This interaction is modeled as the game against nature which means the manufacturers play against the market. The game is designed from the OEM viewpoint and it has no dominant strategy. A mixed-strategy solution that determines optimal strategies of the OEM to maximize his payoff in the aftermarket business is developed. An alternative scenario, where the OEM can implement re-manufacturing processes to manufacture more sustainable parts with cheaper costs, is also considered to determine the optimal re-manufacturing effort.

5.3.2. INVENTORY GAME, OEM AND WILL-FITTER AGAINST NATURE

The goal of this section is to investigate the inventory game in the case of an N-person non-zero-sum single-shot game. In order to achieve this goal, the problem is restricted to a non-cooperative three-person game. The game has three players: the OEM (who has a flexible sale-pricing strategy, and he can be a traditional manufacturer or a re-manufacturer), the will-fitter (who has a solid sale-pricing strategy, and he is only traditional manufacturer) and the market. The game has been set up from the OEM stand point which means the solution of the game gives him the maximum payoff or minimum loss.

We assume that the game is a non-cooperative game and the market is unkind and chooses hostile strategies. The manufacturers can only forecast the market base of spare parts. Moreover, they know that the market consists of two types of customers:

- Price takers, who purchase the parts with available prices in the market;
- Bargain seekers, who are searching for less expensive prices;

We consider the market as an unreasoning entity whose strategic choices affect the payoff the manufacturers, but which has no interest in the outcome of the game. The aforementioned characteristic of the market enables us to consider the spare parts inventory game as a game against nature. In our study we model the spare parts inventory management as a game against nature which means the manufacturers of the spare part compete with each other and meanwhile play against the market.

5.3.3. The Market Demand

In our spare part inventory game, we assume that the manufacturer has only limited information about the market’s expected demand. The market has two options, i.e. present itself as either a price taker or as a bargain seeker. These strategies can vary randomly, so there is a probability that the market chooses the price taker action or switches to the bargain seeker action. Demand for the products follows a general linear demand function (Trivedi, 1998, Wu, 2012). In other words, demand from the manufacturer i, (D_i) is a general linear demand function of his own selling price (p_i) and his competitor selling price (p_j) where i=1, 2 (number of the manufacturers) and j=3-i. Equation (51) develops the demand function, where parameter a_i is the market base for the product, the parameter (α) is the self-price elastic coefficient and the parameter (β) is the cross-price elastic coefficient. This formula represents that the demand for the products depends on its own sale price and its competitor's sale price. Unlike demands for finished goods, the total demand for spare parts will remain unchanged. We assume that the part's demand is mostly dependent on its own sale price, so we consider that $\alpha > \beta > 0$ which means that the demand for parts is more sensitive to changes of the self-price sale as Equation (49):

$$D_i = a_i - \propto (p_i - p_j) + \beta(p_j - p_i) \quad \forall\, i = 1,2 \text{ and } j = 3 - i \tag{49}$$

5.3.4. THE OEM COST FUNCTION-TRADITIONAL MANUFACTURER

The aim of this game is to develop an inventory control policy for the OEM. In the case of traditional manufacturing, the OEM manufactures parts directly from the raw material, so the control policy is the determination of the pricing strategy and levels of inventory. In this setup, the OEM is flexible to change his sale price to optimize his profit, while his competitor has a solid sale-pricing strategy. Spontaneously, this price change will affect his level of inventory. In order to develop the OEM pricing strategies, we consider three standard pricing strategies, which are listed as the following. The OEM can switch among them to increase his profit in the aftermarket business competition.

- Regular pricing (RP), in this strategy the OEM selects his sale price, regardless of his competitor's sale price;
- Matching price (MP), in this strategy the OEM matches his sale price with his competitor's sale price;
- Price guarantee (PG), in this strategy the OEM's sale price is $(n\%)$ cheaper than his competitor's sale price;

The payoff or profit of the OEM is the payoff of the game and it is the profit of the OEM or (K_B) which is equal to the difference between the cost of production and inventory or (K_M) and the revenue from selling parts or (K_R) which is acquired by selling products. The cost of production and inventory (K_M) is calculated based on a very basic inventory model known as the EOQ model which is formulated in Equation (1). Also, this formulation in an optimized solution provides the optimal lot-size (s^*). Equation (2) determines the optimal lot-size.

Equation (50) determines the revenue of selling products:

$$K_R = p \times D \tag{50}$$

And Equation (51) calculates the OEM payoff:

$$K_B = K_R - K_M \tag{51}$$

As we can see the OEM's payoff is a function of the demand and the optimal inventory level or lot-size, which are the strategic actions of the players. Moreover, there exist different parameters that affect the cost of production and inventory. Table 23 lists required parameters to calculate the payoff of the game:

Table 23: Cost function parameters and variables

Notation	Parameter definition
D	Demand (in units per period)
a	Market base demand for the product (in units)
s	Spare parts inventory level (in units)
c	Variable cost of production (in dollars per units)
h	Holding cost (in dollars per unit per period)
A	Setup cost (in dollars)
P	Variable sale price (in dollars per unit)
N	Price guarantee percentage

5.3.5. The OEM Cost Function for The Re-manufacturer

We consider the re-manufacturing as the process of re-using used products and use them to manufacture new parts. According to (Savaskan, Bhattacharya, & Van Wassenhove, 2004, Savaskan & Van Wassenhove, 2006), we assume that there is no difference between the re-manufactured and ordinary manufactured parts. Re-manufacturing requires the collection of the used products, which is known as reverse channels. We assume that the recycling processes insert a total collection cost, and the scaling parameter B can estimate it (Savaskan et al., 2004, Savaskan & Van Wassenhove, 2006). Furthermore, the OEM can decide to re-manufacture whole or some part of his production. This decision-making is known as re-manufacturing effort which is indicated as the re-manufacturing effort parameter (τ) that varies between zero and one; $0 < \tau < 1$.

The spare parts inventory control policy for the OEM in the case of re-manufacturing is the determination of pricing strategy, levels of inventory, and decision-making on re-manufacturing effort. By considering the re-manufacturing process, the cost of production and inventory will change. Equation (52) provides this cost.

$$K_M = \frac{hs^*}{2} + \frac{AD}{s^*} + (c(1-\tau) + c_r\tau)D + B\frac{\tau^2}{2} \tag{52}$$

Table 24 lists additional required parameters to calculate the payoff of the game in case of re-manufacturing:

Table 24: Re-manufacturing parameters

Notation	**Parameter definition**
$\boldsymbol{\tau}$	Re-manufacturing effort
$\boldsymbol{c_r}$	Variable cost of re-manufacturing (in dollars per units)
$\boldsymbol{B}$	Collection scaling parameter

5.3.6. The Game Setup

Using Game Theory, we determine the spare parts inventory control policy for the OEM who manufactures a single part and sells it to the market, while competing with a will-fitter who manufactures the same part. This inventory game is set up from the OEMs viewpoint. The inventory game is set up between the OEM and the market and the solution of the game maximizes the profit of the OEM in the buyer-seller environment. Players choose their strategies simultaneously and the game is a static game and it is solved by finding Nash Equilibrium. The following procedure describes the interaction between two manufacturers:

- Will-fitter chooses his sale price regardless of the market;

- OEM has an option to decide on his decision variable to choose the right sale price;
- The resulting sale prices generate demand distribution among the manufacturers;

The assumptions are:

- Players play simultaneously;
- The OEM possesses only the information of the part's market base;
- The market as a nature has two options of acting as either price taker or a bargain seeker;
- The probability that the market plays as a price taker is P;
- Intuitively, the probability that the market plays as a bargain seeker would be 1-P;
- The OEM has three strategies of pricing (regular price (RP), matching price (MP) and price guarantee (PG));
- The OEM also can decide on his re-manufacturing effort percentage (GMEP);

The game setup can be shown in matrix form. Table 25 gives us the information of the game setup in the matrix form. This matrix is known as payoff matrix and the value of each cell is the payoff of the OEM in the aftermarket game:

Table 25: The payoff matrix of inventory game

			Market (Nature)	
		Pricing Method	**Price Taker**	**Bargain Seeker**
OEM	**GMEP**	**RP**	$K_B(S_{RP}, D_{PT})$	$K_B(S_{RP}, D_{BS})$
		MP	$K_B(S_{MP}, D_{PT})$	$K_B(S_{MP}, D_{BS})$
		PG	$K_B(S_{PG}, D_{PT})$	$K_B(S_{PG}, D_{BS})$

Regarding the probability of market decision-making (price taker probability), the expected utility of the OEM is calculated from Equation (53):

$$\mathrm{EX}(\mathrm{OEM}^{S_r}) = \mathrm{P}(\mathrm{M}^{D_{PT}})\mathrm{U}(\mathrm{OEM}^{S_r}, \mathrm{M}^{D_{PT}}) + \mathrm{P}(\mathrm{M}^{D_{BS}})\mathrm{U}(\mathrm{OEM}^{S_r}, \mathrm{M}^{D_{BS}}) \;\; \forall \; r: \mathrm{RP}, \mathrm{MP}, \mathrm{PG} \tag{53}$$

Equation (54) tells us that the total probability of market decision-makings equals to 1.

$$\mathrm{P}(\mathrm{M}^{D_{PT}}) + \mathrm{P}(\mathrm{M}^{D_{BS}}) = 1 \tag{54}$$

The investigation of the payoff matrix declares there is no dominant strategy in this game that leads to lack of a pure strategy. Therefore, the game has a mixed strategy solution, which depends on the probability of the market's choices (the price taker or the bargain seeker) that varies the allocated demands among manufacturers.

5.3.7. THE MIXED STRATEGY SOLUTION

The mixed strategy solution follows the mixed strategy solution description that has been discussed in the section 5.2.6. In order to illustrate the decision-making of the OEM based on the mixed strategy solution, we consider an inventory game with a single Stage demand, which has the following parameters (Table 26).

Table 26: Sample parameters to illustrate the mixed strategy solution of the game

Notation	Parameter value
a	10
c	12
c_r	9.5
B	50
h	3

Notation	Parameter value
A	50
$\mathbf{P_1}$	25
$\mathbf{P_2}$	20
n	10
α	0.5
β	0.3

Assume all the parameters are the same for both manufacturers except their sale prices. The goal of the OEM is to maximize his payoff based on his competition with the will-fitter and the market choices. The expected utility of the OEM as a function of the market decision-making probability to play with price taker action is calculated. In the discussion that follows, the OEM inventory control policy is explained for the re-manufacturing situation.

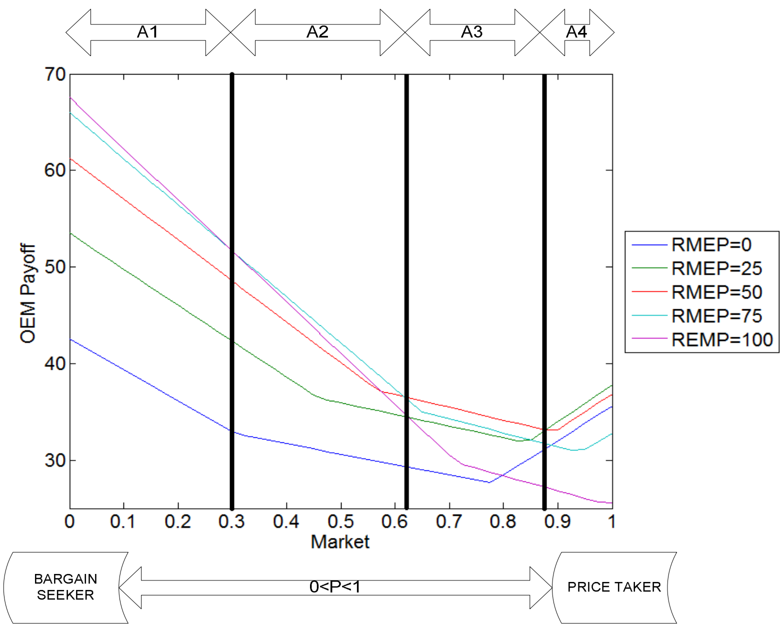


Figure 20: The OEM (Re-manufacturer) payoff distribution vs. The probability of the market's price taker action

Figure 20 shows the OEM payoff distribution vs. the probability of the market's price taker action or $P(M^{D_{PT}})$ in case of re-manufacturing. We assume that there are five possible re-manufacturing effort percentages (RMEP) available that the OEM can decide to select among them. For instance, if the OEM chooses RMEP of 10, it means he manufactures 10% of his production out of re-manufacturing processes. In this decision-making problem, the OEM makes a choice between different alternatives. If the payoff for each alternative is known, the decision is made under certainty. If not, the decision is made under uncertainty. The major solution for decisions under uncertainty and risks are expected utility (EU) and subjective expected utility (SEU) determines the pricing strategies and re-manufacturing efforts of the OEM as follows:

- A1: If $0<P(M^{D_{PT}})<0.30$ then select the price guarantee strategy with re-manufacturing effort of 100 and related inventory level;
- A2: If $0.30<P(M^{D_{PT}})<0.62$ then select the price guarantee strategy with re-manufacturing effort of 75 and related inventory level;
- A3: If $0.62<P(M^{D_{PT}})<0.88$ then select the matching price strategy with re-manufacturing effort of 50 and related inventory level;
- A4: If $0.88<P(M^{D_{PT}})<1$ then select the regular price strategy with re-manufacturing effort of 25 and related inventory level;

Based on the chosen strategies, the optimal payoff and inventory level for the OEM are derived which are depicted in Figure 21. The lowest point in the upper envelope of the expected payoff involves MP strategy with RMEP of 50 and RP strategy with RMEP of 25. According to the results of mixed strategy for that 2×2 matrix game, the optimal decision variables of the OEM is determined. The solution of the mixed strategy states that the OEM should switch randomly among MP strategy with RMEP of 50 and RP strategy with RMEP of 25 with probability of 74% and 26%

respectively. The resulting mixed strategy guarantees the payoff of 33.08 for the OEM in the long run.

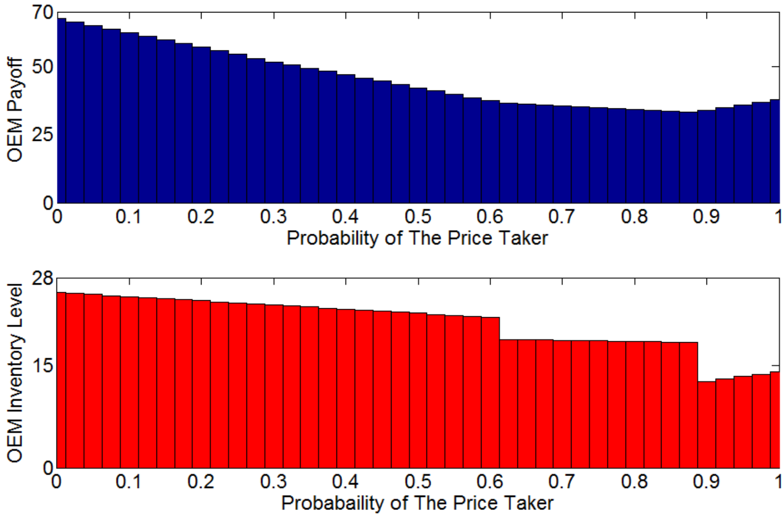


Figure 21: The OEM (Re-manufacturer) payoff and inventory level vs. The probability of the market's price taker action

5.3.8. Numerical Study

In this section the spare parts inventory management of the OEM during 1-year of production has been investigated. We assume that each year of production consists of 6 periods and the manufacturers possess the market’s base demand for the spare parts during these periods. It is assumed that the market’s base demand follows a uniform distribution which is known by the manufacturers. Figure 22 shows the market’s base expected demand in 1-year of production.

In order to generate the payoff matrix for the game in 6 periods, it is assumed that the production, inventory and market conditions are not changing during 1-year of production and the required parameters are consistent with the values that are listed in Table 26.

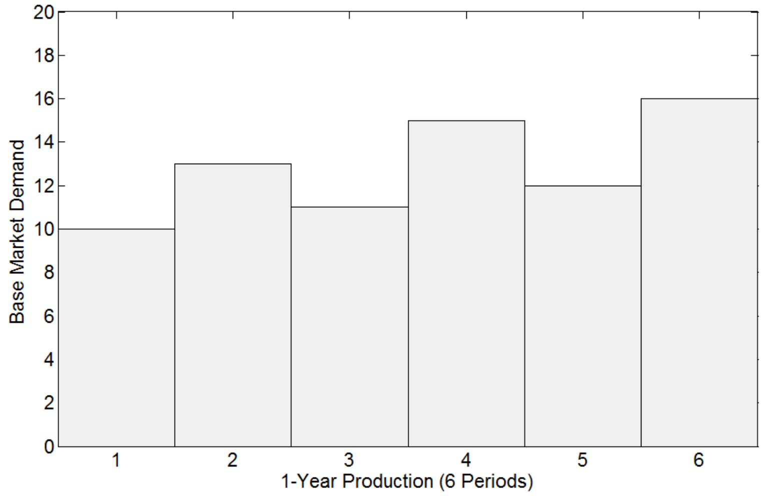


Figure 22: The base market demand in 1-year of production

The strategy of the OEM is to maximize his payoff by choosing the best sale price, re-manufacturing effort and inventory level. The expected utility of the OEM with respect to the probability of the market's price taker action P can be calculated in each period. Table 27 lists the optimal pricing strategies and re-manufacturing efforts of the OEM.

Table 27: The OEM (Re-manufacturer) strategies of the game

		1-Year Production Periods					
	Pricing Method	**1**	**2**	**3**	**4**	**5**	**6**
OEM	**RP**	0.88<P<1 GMEP=25	0.78<P<1 GMEP=50	0.85<P<1 GMEP=25	0.8<P<1 GMEP=50	0.8<P<1 GMEP=50	0.78<P<1 GMEP=50
	MP	0.62<P<0.88 GMEP=50	0.65<P<0.78 GMEP=75	0.65<P<0.85 GMEP=50	0.7<P<0.8 GMEP=75	0.65<P<0.7 GMEP=75 0.7<P<0.8 GMEP=50	0.7<P<0.78 GMEP=75
	PG	0<P<0.3 GMEP=100 0.3<P<0.62 GMEP=75	0<P<0.65 GMEP=100	0<P<0.45 GMEP=100 0.45<P<0.65 GMEP=75	0<P<0.7 GMEP=100	0<P<0.6 GMEP=100 0.6<P<0.65 GMEP=75	0<P<0.7 GMEP=100

This decision-making provides the OEM payoff distributions and inventory levels that have been depicted in Figure 23 (the results for re-manufacturer is provided). The strategies of the OEM during these periods are listed in Table 27.

As we can see the SEU and mixed strategy solutions can determine the OEM's decision-making on spare part pricing, inventory level and RMEP while the OEM implements the very basic inventory policy that is introduced as the EOQ. In next section we show that the same strategic problem solving is still useful for more complicated inventory control policies. Perhaps one of the most commonly used inventory policy for spare part management is the reorder point and order size policy known as (Q,r). We assume that demand for spare part arrives as a Poisson process, in batches of size one. The mean arrival rate is known by the manufacturers and they manage their inventory based on a (Q,r) policy, introduced in chapter 2.

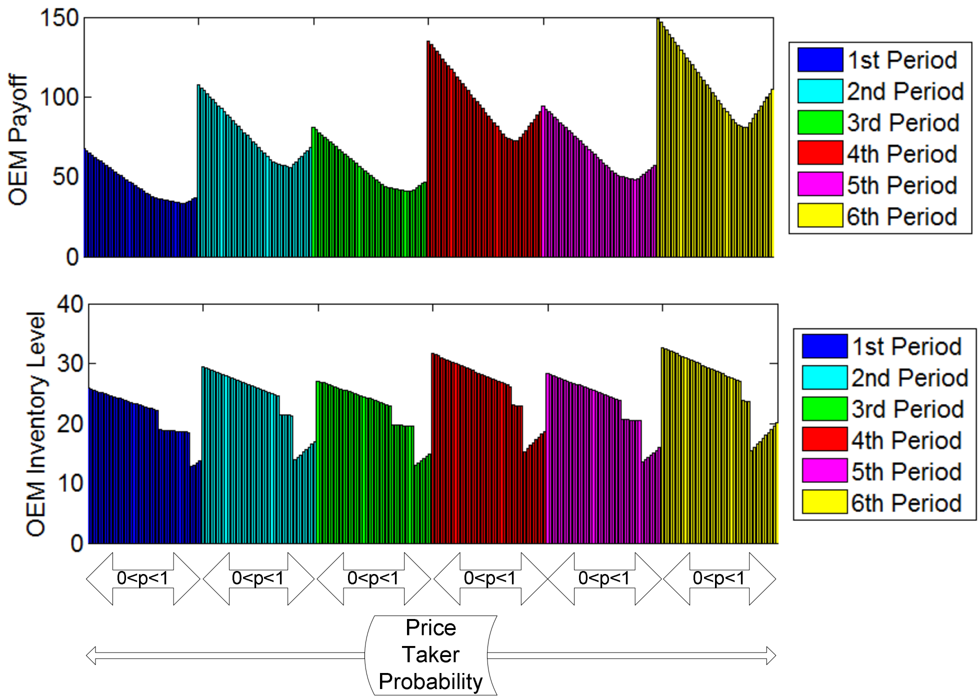


Figure 23: The OEM (Re-manufacturer) maximum expected utility vs. Price taker demand probability for each period

By consideration of the following parameters, the cost function of the OEM is calculated based on our proposed algorithm in the section 2.5.3.3. In our numerical study we assume that the annual demand rate for spare part is a=30, lead-time L=45 days, holding cost h=30, backorder cost b=100, setup cost=15, cost of production c=12, cost of re-manufacturing c_r=10.5 and cost of collecting used parts B=110.

Figure 24 shows the OEM payoff distribution vs. the probability of the market's price taker action or $P(M^{D_{PT}})$ in case of re-manufacturing. We assume that as before there are five possible re-manufacturing effort percentages (RMEP) available that the OEM can select them.

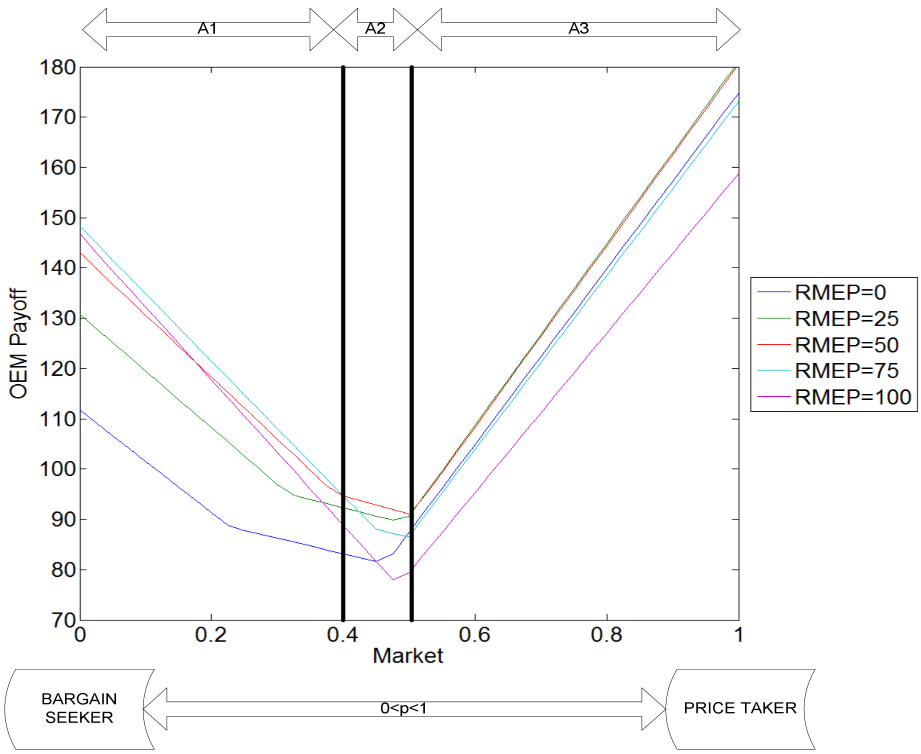


Figure 24: The OEM (Re-manufacturer) payoff distribution vs. The probability of the market's price taker action

The subjective expected utility (SEU) determines the pricing strategies and re-manufacturing efforts of the OEM as follows:

- A1: If 0<P($M^{D_{PT}}$)<0.40 then select the price guarantee strategy with re-manufacturing effort of 75 and related inventory level;
- A2: If 0.40<P($M^{D_{PT}}$)<0.5 then select the matching price strategy with re-manufacturing effort of 50 and related inventory level;
- A3: If 0.5<P($M^{D_{PT}}$)<1 then select the regular pricing strategy with re-manufacturing effort of 25 and related inventory level;

Based on the chosen strategies, the optimal payoff and re-order point and order lot-size for the OEM are derived and are depicted in Figure 25.

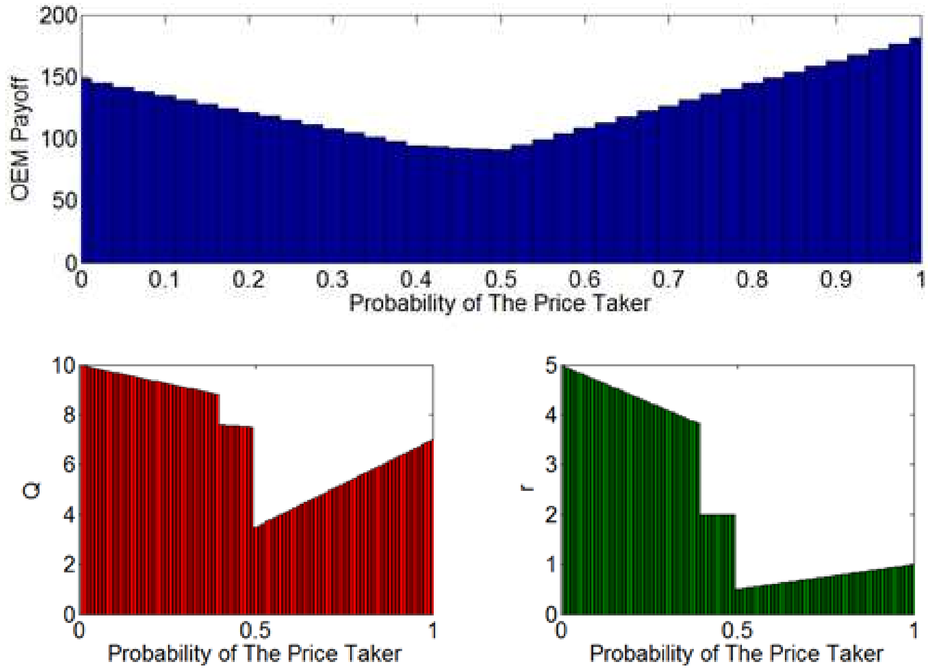


Figure 25: The OEM (Re-manufacturer) payoff and inventory level vs. The probability of the market's price taker action

The lowest point in the upper envelope of the expected payoff involves MP strategy with RMEP of 50 and RP strategy with RMEP of 25. According to the results of mixed strategy for that 2 × 2 matrix game, the optimal decision variables of the

OEM is determined. The solution of the mixed strategy states that the OEM should switch randomly among MP strategy with RMEP of 50 and RP strategy with RMEP of 25 with probability of 0.5 and 0.5 respectively. The resulting mixed strategy guarantees the payoff of 91 for the OEM in the long run.

In order to examine the accuracy of the Game theoretical solution, a Monte Carlo simulation has been developed which relies on random sampling to obtain numerical results. The simulation runs many times to obtain the payoff of the OEM with respect to uncertainty of the market.

The simulation follows the following particular pattern:

1. Defining a domain of possible inputs;
2. Generating random inputs from a given probability distribution (uniform distribution) over the domain;
3. Implementing the spare part inventory policy and performing a deterministic computation over the inputs;
4. Aggregating the results;

The goal of the simulation is to show the comparison of the Game Theory approach and any other inventory and production policy. In figure 26, the payoff of the OEM vs. the probability of the market's price taker action while implementing Game Theory solution and some other inventory policies (General Policies; including different pricing strategies, inventory level and re-manufacturing effort) is depicted. As we can see Game Theory approach guarantees the payoff of 33.08 for the OEM by switching randomly among MP strategy with RMEP of 50 and RP strategy with RMEP of 25 with probability of 74% and 26% respectively.

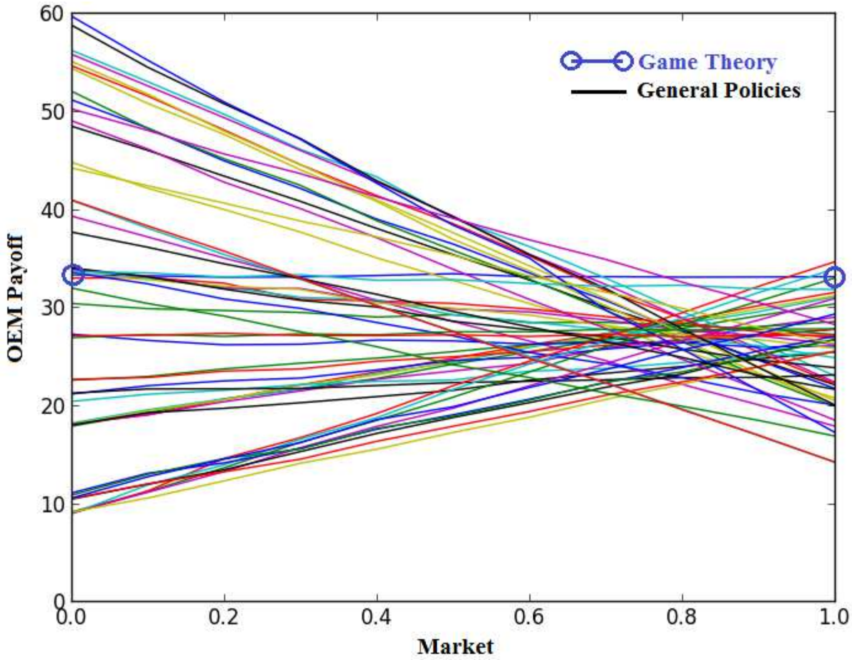


Figure 26: Simulation results - The OEM payoff vs. The probability of the market's price taker action

Comparison of General Policies & Game Theory

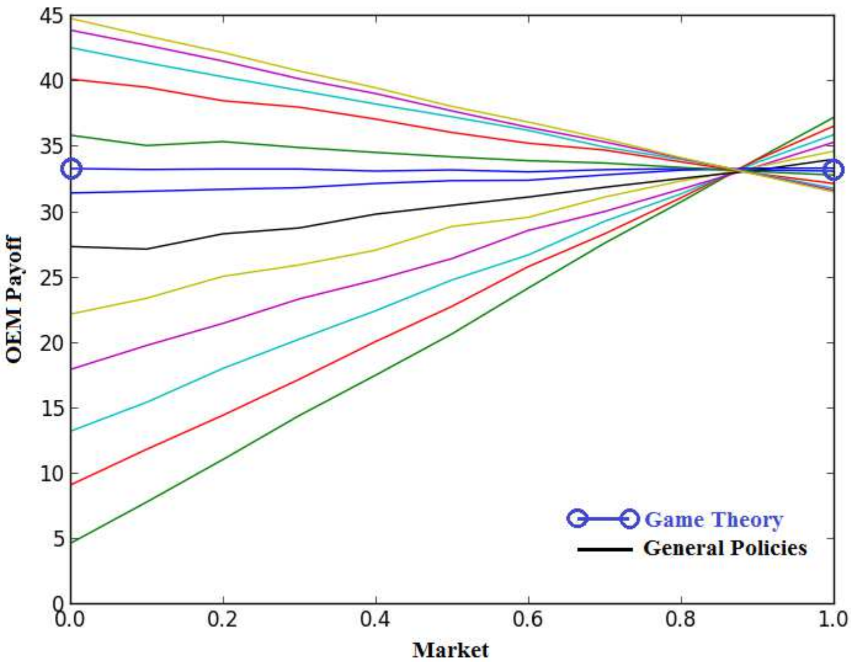


Figure 27: Simulation results - The OEM payoff vs. the probability of the lower bound intensity factor

Comparison of General Policies (identical prices with different strategies) & Game Theory

The performance of the Game Theory approach is graphed in Figure 27 where general policies are considered as implementing the same pricing strategies, same re-manufacturing efforts with different strategies to keep the inventory. The results of the Monte Carlo simulation states that Game Theory approach allocates guaranteed payoff to the OEM in an uncertain market situation.

5.4. EVOLUTIONARY SPARE PARTS INVENTORY GAME

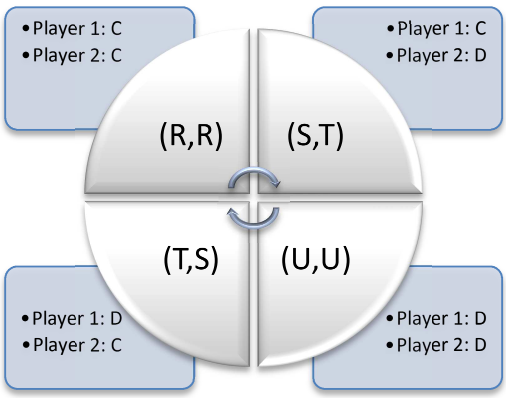
5.4.1. INTRODUCTION

The OEM, in a competitive and uncertain aftermarket, can benefit from Game Theory to manage his spare parts inventory. We study the spare parts inventory game in the case of an N-person non-zero-sum repeated game where players play simultaneously. This game is restricted to a two-person (the OEM and the will-fitter), cooperative and non-cooperative game setup. The game has two players (manufacturers) who manufacture the same spare parts. Based on their sale prices, they have an option to design a contract and cooperate with each other or compete with each other in the aftermarket without creating any agreements. The pricing strategies have been investigated through repeated Prisoners' Dilemma spare parts inventory game and the stability of the cooperation or defect in sale price determination has been studied through evolutionary stable strategy analysis of the two famous games of Prisoners' Dilemma and Stag Hunt. Moreover, the implementation of the re-manufacturing in manufacturing processes to increase the profitability of the spare parts inventory games is investigated.

5.4.2. Repeated Inventory Game

Every game with finite numbers of players and actions has at least one Nash Equilibrium. For all the games, the Nash Equilibrium is Pareto optimal except for the Prisoners' Dilemma. In 1950 Melvin Dresher and Merrill Flood at the RAND Corporation devised a game known as the Prisoners' Dilemma that is a non-zero-sum game with an equilibrium which is unique but fails to be Pareto optimal. In the years since 1950 this game has become known as the Prisoners' Dilemma and it is the most widely used and studied game in the social science.

The general form of the Prisoners' Dilemma has been depicted in Figure 28.



$$Conditions: T > R > U > S \ \& \ R > \frac{S+T}{2}$$

Figure 28: The general form of Prisoners' Dilemma

There are two players and they can decide to cooperate or defect. According to their chosen strategies, there are four different possible payoffs of the game including:

- Both players cooperate (CC): both players will get the reward of the cooperation (R);
- Player 1 cooperates and player 2 defects (CD): player 1 will get the sucker payoff (S) and player 2 will get the temptation payoff (T);
- Player 1 defects and player 2 cooperates (DC): player 1 will get the temptation payoff (T) and player 2 will get the sucker payoff (S);
- Both players defect: both players will get the uncooperative payoff (U);

Many social phenomena seem to have the Prisoners' Dilemma at their core. In the case of our inventory game, there are two spare parts manufacturers, deciding to cut their sale prices or not. If the will-fitter does not cut the prices, the OEM can attract more customers by cutting prices. If the will-fitter cuts its prices, the OEM had better cut prices in order not to lose its own customers. If both manufacturers cut prices, both will get lower benefits than if neither of them had cut prices.

In this inventory game, we assume that manufacturers have to decide on their sale prices and inventory levels as long as there is a demand for spare parts. In other words, they have to repeat playing the inventory game to satisfy demands for spare parts. In repeatedly play, the hope of arriving at the mutually beneficial outcome (CC) rather than reaching to a less profitable outcome (DD), encourages the manufacturers to cooperate.

In fact, this idea is under influence of a logical domino-type argument. Suppose players know this game lasts for 100 times, in the last game the strategy (D) dominates the strategy (C) because there is no future to induce mutual cooperation. The plays fall backwards like dominos, and affect previous games and even the first game must be (DD).

Now, let's assume because of the intermittent and uncertain characteristic of the demand for spare parts, manufacturers do not know how many games will be played. According to (Martin Shubik, 1970), after each play of Prisoners' Dilemma, the next play will occur with probability (p).

Manufacturers follow the strategy of Grim Trigger, means both players cooperate until one player defects, and then both players defect. If manufacturers never choose the strategy (D), the payoff for each player is calculated from Equation (55):

$$R + pR + p^2R + p^3R + \cdots = \frac{R}{1-p} \tag{55}$$

If one player decides to defect in the mth game, the resulting payoff for each player can be calculated from Equation (54).

$$R + pR + p^2R + \cdots + p^{m-1}R + p^mT + p^{m+1}U + \cdots = \frac{R - p^mR + (1-p)p^mT + p^{m+1}U}{1-p} \tag{56}$$

Hence, manufacturers should never choose the strategy (D) as long as Equation $(55) \geq$ Equation (56) for all values of (m). In other words, it makes sense for the manufacturers to choose the strategy (C), if the probability of the playing the next game (p) is larger than a threshold value.

The threshold can be calculated from Equation (57):

$$p > \frac{T-R}{T-U} \tag{57}$$

5.4.3. Evolutionary Stability and Bounded Rationality

In Game Theory it has been assumed that players are capable of unlimited acts of reasoning. In such a situation, once they find the solution to a game, they play that strategy from then on. In real world players will not find the solution immediately and they spend a lot of time and energy to find the solution. These types of players, who have to make mistakes to find the optimal solution, are considered bounded rational. Making mistakes and the trial-and-error procedure is part of their learning how to play, and while they are in learning phase they are out of the equilibrium. The game during out of equilibrium situation can be described via a dynamic system known as the replicator dynamic. Dynamic replicators are used to describe the evolution of systems and evolution of players' behavior in games. Bounded rational players, who obeying replicator dynamics, find the equilibrium called an evolutionary stable strategy (ESS).

Replicator dynamics says that if a player earns above-average payoff, its percentage in the whole population increases and if a player earns below-average its population will decrease. In our evolutionary game we assume that manufacturers, who are bounded rational players, have two strategies:

- Cooperate: Selecting the sale prices according to an agreement with the other player to increase the total payoff;
- Defect: Selecting the sale prices according to an individual better off payoff, or cutting sale prices to increase the resulting payoff;

The investigation of the game will determine the stable strategies of the players.

5.4.4. THE MARKET DEMAND

The life cycle of the spare parts is affected by finished goods life cycle. Their life cycle can be divided into three phase: initial, normal or repetitive and final (Fortuin, 1980). Hence demands for spare parts depend on finished goods, and following factors would affect the demand (Fortuin & Martin, 1999):

- Size and age of the final products (sales, running fleet, installation base, etc.);
- Products maintenance characteristics (preventive, corrective, etc.);
- Parts characteristics and their defects (wear, accident, aging, etc.);

Spare parts demand is intermittent or lumpy which means it occurs after a long variable periods without demand. The lack of parts leads to high losses, and demand forecasting can decrease the loss. The demand forecasting is very important, although it has some errors (Love, 1979). A classical method for demand forecasting has been done by (Wheelwright & Hyndman, 1998) and a related literature review has been provided by (Boylan et al., 2006) for last fifty years.

In some related literatures, the demand for the manufacturers follows a general linear demand function (Trivedi, 1998)(Wu, 2012). In other words, demand for the supplier i, (X_i) is a general linear demand function of his own sales price (P_i) and his competitor sales price (P_j) where i=1,2 (number of the manufacturers) and j=3-i.

In our study we suppose that the manufacturers can forecast the market's expected demand for spare parts over the normal or repetitive phase of the product life cycle (Fortuin, 1980) which is the market base for the products, that is parameter (X).

The total market demand equals to the summation of the market demand for both manufacturers as it is shown in Equation (58).

$$X = \sum_{i=1}^{2} X_i \quad \forall i = 1,2 \tag{58}$$

In order to simulate stochastic behavior of the demand, we assume that the demand distribution is Poisson. In other words, the mean of total market demand is known and the standard deviation of the demand σ can be calculated from Equation (59).

$$\sigma_i = \sqrt{X_i} \tag{59}$$

We assume that the market is a bargain seeker that means she purchases the products with lower prices. This characteristic will determine the demand allocation for the manufacturers.

5.4.5. The Manufacturer Cost Function-Traditional Manufacturer

The aim of our game is to develop a pricing strategy and an inventory control policy for the manufacturer of the spare parts. In case of traditional manufacturing, the OEM and will-fitter both manufacture parts directly from the raw material, so the control policy would be the determination of the pricing strategy and inherently the level of inventories. The payoff of the manufacturers is the payoff of the game and it would be the profit of the manufacturers (π^i) which is the difference between the cost of production and inventory $\left(Y^i\right)$ and the revenue $\left(R^i\right)$ which attained by parts sales. We suppose that the manufacturers follow the Newsvendor inventory control policy. Based on Newsvendor system we follow the listed assumptions:

- Products are separable (in this study we consider a single-item inventory);
- Planning is done for a single period;
- Demand is random;
- Deliveries are made in advance of demand;
- Costs of overage or shortage are linear;

To develop the model, manufacturers produce (Q_i) units and the demand is (X_i) units. Based on Newsvendor inventory policy, introduced in the section 2.5.1, the cost of production and inventory $\left(Y^i\right)$ is calculated. Equation (60) shows the derived formula to calculate the cost of inventory.

$$Y(Q)^i = C_{oi} \int_0^{Q_i} (Q_i - X_i)\, g(X_i) dX_i + C_{si} \int_{Q_i}^{\infty} (X_i - Q_i)\, g(X_i) dX_i \tag{60}$$

The optimal production quantity (Q_i^*) is derived from Equation (61). Equation (62) gives us the revenue of parts sales R^i and Equation (63) gives us the manufacturers' payoffs $\left(\pi^i\right)$:

$$G(Q_i^*) = \frac{C_{si}}{C_{oi} + C_{si}} \tag{61}$$

$$R^i = P^i X_i \tag{62}$$

$$\pi^{i} = R^{i} - Y^{i} \tag{63}$$

As we can see the manufacturers' payoffs are functions of demand, production level (optimal inventory level) and sales prices which are the strategic actions of the players. Moreover, there are different parameters that affect the cost of production and inventory and sales revenues as listed in Table 28:

Table 28: Cost function parameters and variables

Notation	Parameter definition
Q	Production rate (in units)
$\mathbf{C_p}$	Cost of production (in dollars per units)
P	Sale price (in dollars per units)
$\mathbf{C_s}$	Shortage cost (P-$\boldsymbol{C_p}$) (in dollars per unit)
$\mathbf{C_o}$	Overage cost or holding cost (I$\boldsymbol{C_p}$) (in dollars per unit)
ir	Interest rate (percentage per year)
X	Mean of demand (in units per period)
σ	Standard deviation of demand (in units)
G(X)	CDF of demand
g(X)	PDF of demand

5.4.6. THE MANUFACTURER COST FUNCTION FOR THE RE-MANUFACTURER

We consider Green manufacturing as the process of re-manufacturing used products and use them as new ones as it was discussed in the section 5.3.5. By considering re-manufacturing process, the shortage cost will be changed and can be calculated from Equation (62). The cost of production and inventory will change which is written as Equation (63).

The cost of shortage in the case of re-manufacturing:

$$C_{si} = P - \left(C_{ri}\tau + C_p(1-\tau)\right) \tag{64}$$

The cost of production and inventory Y^i:

$$Y(Q)^i = C_{oi}\int_0^{Q_i} (Q_i - X_i)\, g(X_i)dX_i + C_{si}\int_{Q_i}^{\infty} (X_i - Q_i)\, g(X_i)dX_i + B\frac{\tau^2}{2} \tag{65}$$

Parameters are being used to calculate the cost of production and inventory while using re-manufacturing are listed in Table 29:

Table 29: Green manufacturing parameters

Notation	Parameter definition
τ	remanufacturing effort
C_r	variable cost of remanufacturing (in dollars per units)
B	Collection scaling parameter

5.4.7. THE GAME SETUP

In this setting, Game Theory is applied to determine the inventory control policy for manufacturers who manufacture a single-item spare part and sell it to the market while they can cooperate or compete with each other. Our spare part inventory game has two players who have two different strategies: cooperation and defect. Decision-making on sale price (and re-manufacturing effort in the Green manufacturing situation) is/are their strategies which determine their production quantities and inventory levels. The market has a cyclic demand which is stochastic, and in each period the mean of demand is known. The total demand has to be distributed among players and manufacturers' pricing strategies affect their allocated demand. We assumed that the market is a bargain seeker which means it purchases from the supplier who offers lower prices. In this environment manufacturers can cooperate with each other or compete with each other that means each of them has two strategies:

- Strategy 1: Cooperate, manufacturers set their sale prices in such a way that gives them the highest profit;
- Strategy 2: Defect, manufacturers cut their sale prices in such a way that attracts more demand share;

The inventory game is a two-person non-zero-sum static game, so we are looking for the Nash Equilibrium as the solution of the game. Based on the players strategies (cooperate or defect) different payoffs would be assigned for each supplier. The payoff matrix is depicted in Table 30:

Table 30: The payoff matrix of inventory game

		Will-fitter	
		COOPERATE (C)	**DEFECT (D)**
OEM	**COOPERATE (C)**	$\pi^1 CC, \pi^2 CC$	$\pi^1 CD, \pi^2 CD$
	DEFECT (D)	$\pi^1 DC, \pi^2 DC$	$\pi^1 DD, \pi^2 DD$

Evolutionary Game Theory is used to study the evolutionary stability of strategies followed by two manufacturers. The concept of ESS was proposed by (Maynard Smith, 1974, J Maynard Smith & Price, 1973, John Maynard Smith, 1993). Later on (Taylor & Jonker, 1978) proposed a dynamic equation known as the dynamic replicator that reflects the dynamics and interactions between players in the game.

Assuming that the probability that the OEM cooperates is (α) and (β) is the probability that the will-fitter cooperates, replicator dynamics are given in Equations (66) and (67):

$$\begin{bmatrix} \frac{d\alpha}{dt} = \alpha[U^{OEM}(\text{cooperate}) - \text{Average OEM}] \\ \text{Average OEM} = \alpha U^{OEM}(\text{cooperate}) + (1-\alpha)U^{OEM}(\text{defect}) \end{bmatrix} \tag{66}$$

$$\begin{bmatrix} \frac{d\beta}{dt} = \beta\left[U^{will-fitter}(\text{cooperate}) - \text{Average will} - \text{fitter}\right] \\ \text{Average will} - \text{fitter} = \beta U^{will-fitter}(\text{cooperate}) + (1-\beta)U^{will-fitter}(\text{defect}) \end{bmatrix} \tag{67}$$

Where $U^{\text{manufacturer}_i}(\text{cooperate})$ is the expected utility of the manufacturer_i (i=1 OEM & i=2 will-fitter) when he cooperates and Average manufacturer_i is the average payoff for manufacturer_i. The stable states of the replicator dynamic equations are the Nash Equilibrium known as evolutionary equilibriums (EE). When $d\alpha/dt = 0$ and $d\beta/dt = 0$ the EE are pure strategies of E1=(1,1), E2=(1,0), E3=(0,1), E4=(0,0) and the fifth EE point is the mixed strategy solution that is driven as Equation (68):

$$E_5 = \left(\frac{\pi^2 DC - \pi^2 DD}{(\pi^2 CD - \pi^2 DD) + (\pi^2 DC - \pi^2 CC)}, \frac{\pi^1 CD - \pi^1 DD}{(\pi^1 DC - \pi^1 DD) + (\pi^1 CD - \pi^1 CC)}\right) \tag{68}$$

According to (Friedman, 1991) the stability of EE can be analyzed by the Jacobi matrix which can be derived from Equation (69). The stability of the EE depends on the sign of Jacobi matrix eigenvalues. If both eigenvalues are negative the EE is the stable strategy otherwise that would be an unstable strategy.

$$J = \begin{bmatrix} \frac{\partial}{\partial\alpha}\left(\frac{d\alpha}{dt}\right) & \frac{\partial}{\partial\beta}\left(\frac{d\alpha}{dt}\right) \\ \frac{\partial}{\partial\alpha}\left(\frac{d\beta}{dt}\right) & \frac{\partial}{\partial\beta}\left(\frac{d\beta}{dt}\right) \end{bmatrix} \tag{69}$$

5.4.8. NUMERICAL STUDY

In this section the inventory management of the manufacturers during 1-year of production has been investigated. We assume that the manufacturers possess the market base demand for the product during the production year. In order to generate the payoff matrix for the game, it is assumed that the production, inventory and marketing conditions are not changing during the production horizon and the required parameters are consistent with Table 31. We suppose all the parameters are the same for both manufacturers.

Table 31: Sample parameters used to generate the payoff matrix traditional manufacturer

Notation	Parameter value
C_p	10
P	Decision variable
ir	25
C_o	0.048
X	50

We assumed that the market is a bargain seeker, so based on strategies on sale prices in cooperation and competition the total demand will be distributed among manufacturers. This can be listed as below:

- CC or DD: Demands for spare parts are distributed among manufacturers equally;
- CD or DC: Demands for spare parts are distributed among manufacturers unequally, the manufacturer who cuts the prices will attract all the customers to himself;

In other words, when both players cooperate or defect, the annual mean of demand for each of them is 25 units and once each of them cut prices, he attracts the whole demand which means the annual mean of demand for him would be 50 units. Figure 29 depicts the distribution of the demand in case of cooperation and defect.

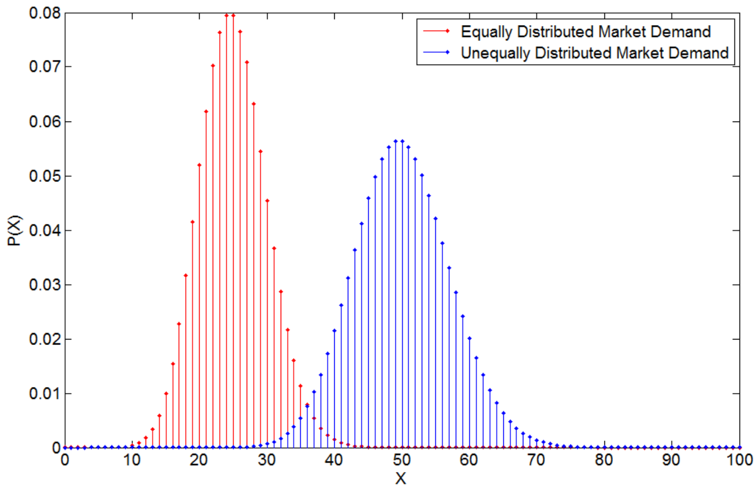


Figure 29: PDF of market demand

The determination of the sale prices generates the demand distribution among manufacturers. Based on resulting demands, manufacturers decide on their inventory levels which allocate their profits in the aftermarket game.

We first need to determine rational pricing strategy for the manufacturers. The minimum sale price that makes the spare parts inventory game profitable for the manufacturers is equal to 15.13, which is defined as the marginal cutting sale price. Next, we need to find the minimum cooperative sale price that establishes the spare parts inventory game as a Prisoners' Dilemma. The minimum cooperative sale price is equal to 16.72. However, in order to maintain the Prisoners' Dilemma condition, the reward of the cooperation should be less than the temptation payoff (R<T). Hence, the

maximum cooperative sale price is equal to 18.68. Now, we can setup a repeated Prisoners' Dilemma spare parts inventory game.

According to Equation (57), we can graph the threshold of the possibility of the future game vs. the maximum cooperative sale price which states that manufacturers should never defect as long as the probability of the future game is greater than the calculated thresholds (Figure 30).

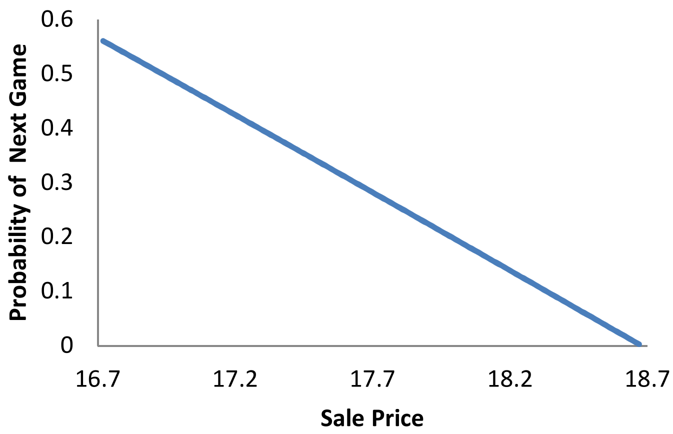


Figure 30: Probability of the next play vs. Cooperative Sale Price

The resulting payoff of the spare parts inventory game for each manufacturer is depicted in Figure 31.

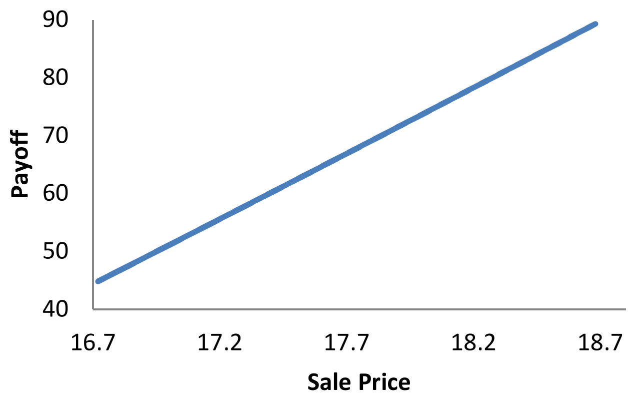


Figure 31: Payoff vs. Cooperative Sale Price

Now we can investigate the evolutionary spare parts inventory game. For the traditional manufacturing, the resulting payoff matrix is symmetrical because we assumed that the production and inventory costs are the same for both manufacturers.

The symmetrical payoff matrix is:

$$\pi^1 CC = \pi^2 CC$$

$$\pi^1 CD = \pi^2 DC$$

$$\pi^1 DC = \pi^2 CD$$

$$\pi^1 DD = \pi^2 DD$$

Stability of the evolutionary equilibrium (EE) points depends on the sign of the $\frac{\partial}{\partial \beta}\left(\frac{d\beta}{dt}\right)$ as the stability term, the result of the analysis is represented as Table 32:

Table 32: The local stability of EE

EE	Stability term
C	$-(\pi^1 CC - \pi^1 DC)$
D	$(\pi^1 CD - \pi^1 DD)$
Mixed	$-\frac{(\pi^1 CC - \pi^1 DC)(\pi^1 CD - \pi^1 DD)}{(\pi^1 CC - \pi^1 DC) - (\pi^1 CD - \pi^1 DD)}$

To investigate the evolutionary stable strategies, two experiments have been considered (the first experiment is the experiment with minimum acceptable sale price and the second one is the experiment with maximum acceptable sale price). The simulation software (Sandholm & Dokumaci, 2007) is used to study the evolutionary dynamics of two manufacturers' strategies in the market. The experiments are logistic systems based on decision-making to cooperate or defect by selecting related sale prices. By using the software the phase diagram of each experiment has been obtained as Figure 32. The colors in the contour plot represent speeds of motion under the dynamic: red is fast and blue is slow. In the first experiment, the tendency to leave the cooperation is fast and as sale price increases this movement becomes slower till it reaches to its slowest speed in the second experiment.

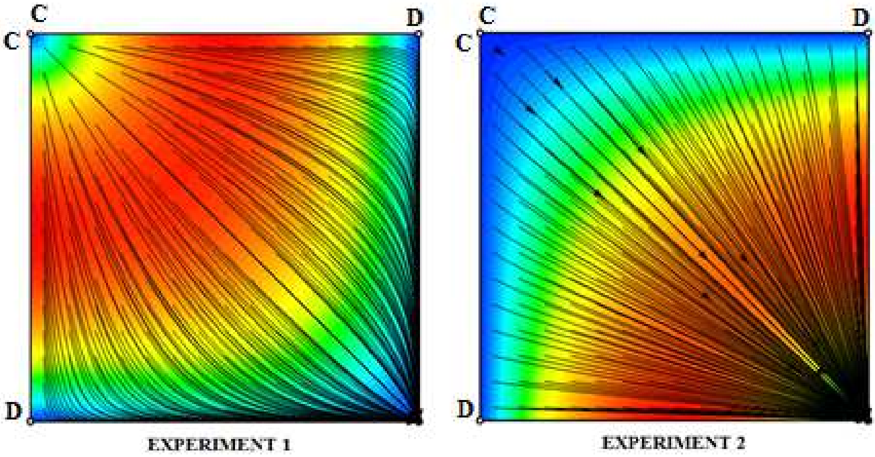


Figure 32: Phase diagram of experiments (1 & 2)

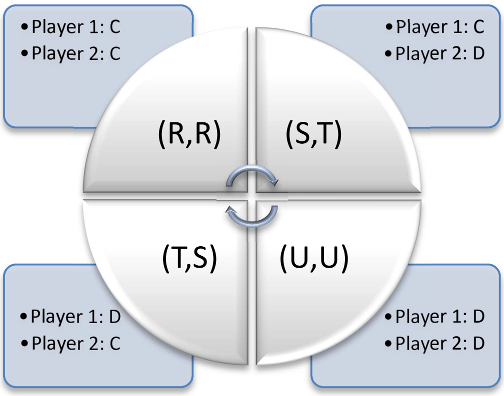
Also, the characterizations of the stability of EE for each experiment are listed in Table 33. The results of the ESS analysis are consistent with the results of the repeated Prisoners' Dilemma stating that as the sale prices increase, manufacturers stay in cooperation with less future play probability. However, the results of the ESS state that the Nash Equilibrium of the game is (DD), which means both players defect, and the equilibrium point is evolutionary stable (ES).

Table 33: The ESS analysis of the experiments (1 & 2)

S:Stable - U:Unstable - NE:Nash Equilibrium

	EXPERIMENTS			
	1		**2**	
P	D=15.13 & C=16.72		D=15.13 & C=18.68	
Payoff	10		10	
EE	**NE**	**S/U**	**NE**	**S/U**
E1		U		U
E2		U		U
E3		U		U
E4	•	S	•	S
E5	NA		NA	

The increase of the sale price above the value of the maximum cooperative sale price would change the game type from Prisoners' Dilemma to Stag Hunt game. Basically, Stag Hunt studies the conflicts between safety and cooperation. The general form of the Stag Hunt game is depicted in Figure 33.



$Conditions: R > T \geq U > S$

Figure 33: The general form of Stag Hunt

This game has two Nash Equilibriums and it has a mixed strategy solution. In other words, this game has two ES equilibriums (including cooperation and defect) and it has unstable mixed strategy equilibrium. The midpoint sale price is the sale price that leads to a mixed strategy with 50% chance of cooperation or defect, in other words, it is the sale price that makes the value of α and β equal to 50%. Two different Stages of sale prices can be considered for this spare parts inventory game including:

- If maximum cooperative sale price ≤ sale price < midpoint sale price: manufacturers should cooperate with each other with probability range of 0^+ to 50%;
- If midpoint sale price ≤ sale price: manufacturers should cooperate with each other with probability range of 50 to 100^-%;

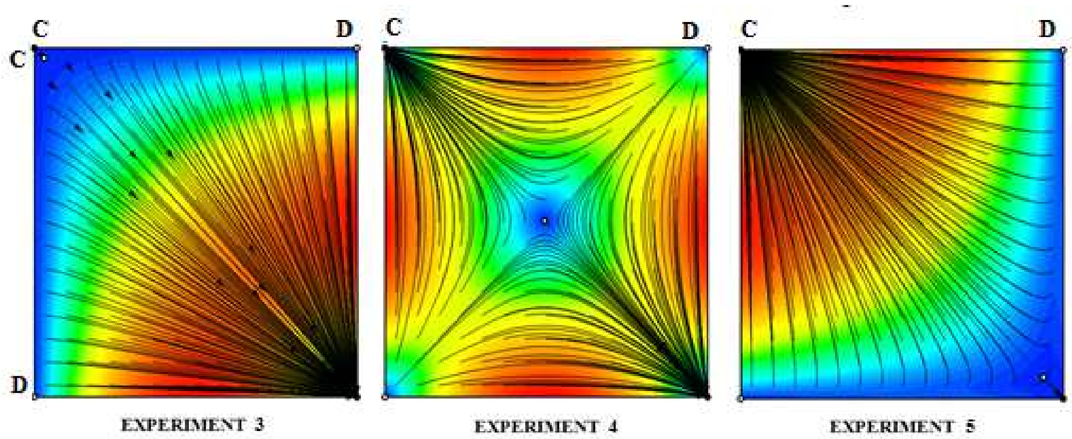


Figure 34: Phase diagram of experiments (3 – 5)

In the following, three different sale prices have been provided that can be investigated via ESS analysis. By using the software the phase diagram of each experiment has been obtained as Figure 34. The tendency to join the cooperation has its slowest speed in the third experiment and as the sale price increases the speed of this tendency becomes faster and it reaches to the fastest movement in the fifth experiment. Also, the characterizations of the stability of EE for each experiment are listed in Table 34.

Table 34: The ESS analysis of the experiments (3 – 5)

S:Stable - U:Unstable - NE:Nash Equilibrium

	EXPERIMENTS					
	3		**4**		**5**	
P	D=15.13 & C=18.69		D=15.13 & C=19.1		D=15.13 & C=25	
payoff	11.37		49.54		215.35	
EE	**NE**	**S/U**	**NE**	**S/U**	**NE**	**S/U**
E1	•	S	•	S	•	S
E2		U		U		U
E3		U		U		U
E4	•	S	•	S	•	S
E5	(0.02,0.98)	U	(0.50,0.50)	U	(0.94,0.06)	U

Now, we can study the situation where the manufacturers can implement re-manufacturing strategy. This game is also symmetrical. The minimum sale price that makes the spare parts inventory game profitable for the manufacturers is calculated as 14.76, which is equal to the marginal cutting sale price. In Figure 35 the effect of the re-manufacturing effort on minimum profit is depicted.

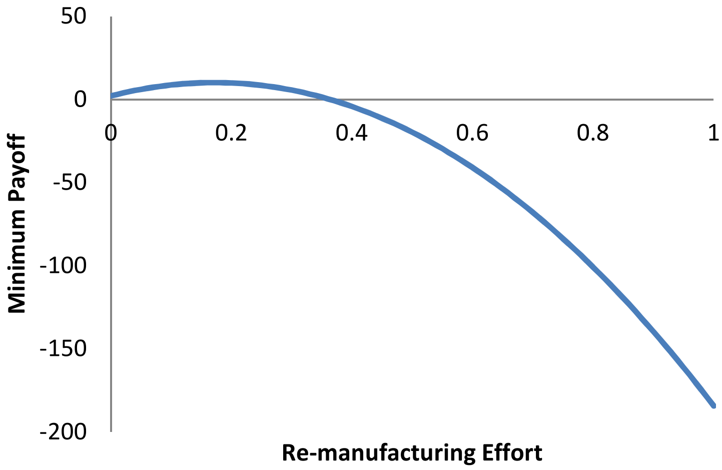


Figure 35: Minimum payoff vs. Re-manufacturing effort

The result shows us that implementing re-manufacturing effort of 17% can guarantee the minimum payoff with value 10 with sale price of 14.76.

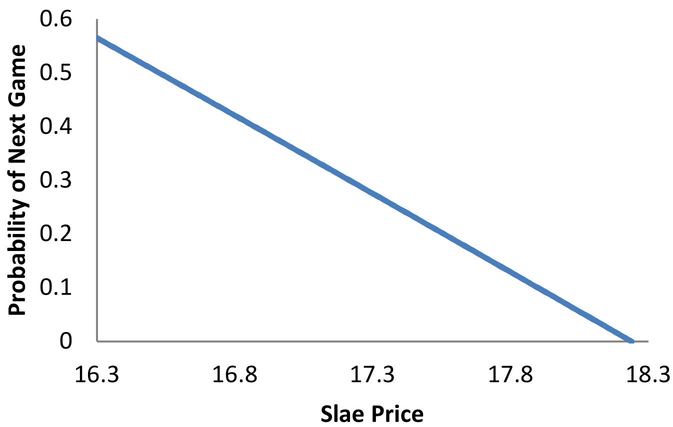


Figure 36: Probability of the next play vs. Cooperative Sale Price

In order to satisfy the Prisoners' Dilemma constraints, the minimum cooperative sale price changes to 16.3 and the maximum cooperative sale price changes to 18.24. Now, we can setup a repeated Prisoners' Dilemma spare parts inventory game. According to Equation (57), we can graph the threshold of the possibility of the future game vs. the maximum cooperative sale price which states that manufacturers should never defect as long as the probability of the future game is greater than the calculated thresholds as Figure 36. The resulting payoff of the spare parts inventory game for each manufacturer is depicted in Figure 37.

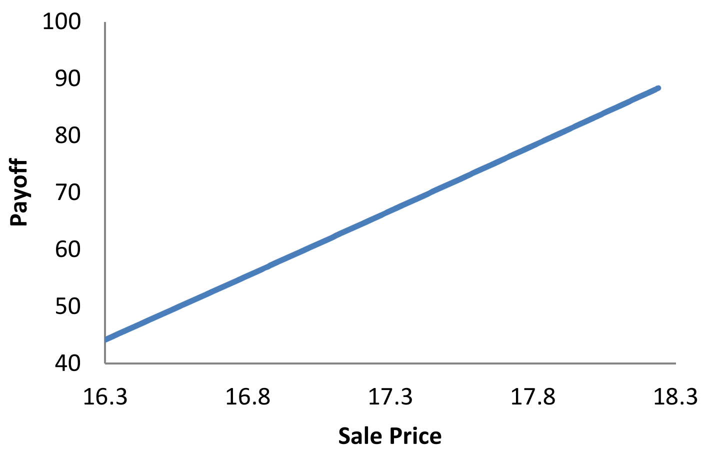


Figure 37: Payoff vs. Cooperative Sale Price

To investigate the evolutionary stable strategies, two experiments have been considered (the first experiment is the experiment with minimum acceptable sale price and the second one is the experiment with maximum acceptable sale price). The phase diagram of each experiment has been obtained as Figure 38. In the sixth experiment, the tendency to leave the cooperation is fast and as sale price increases this movement becomes slower till it reaches to its slowest speed in the seventh experiment.

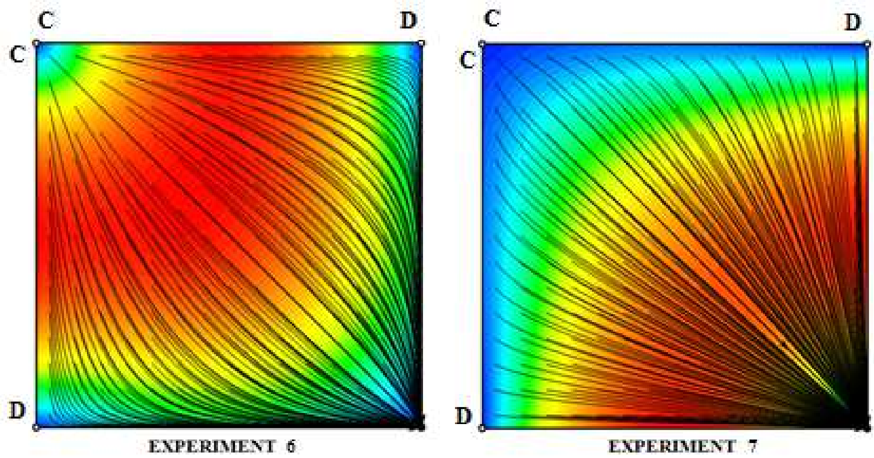


Figure 38: Phase diagram of experiments (6 & 7)

Also, the characterizations of the stability of EE for each experiment are listed in Table 35.

Table 35: The ESS analysis of the experiments (6 & 7)

S:Stable - U:Unstable - NE:Nash Equilibrium

	EXPERIMENTS			
	6		**7**	
P	D=14.76 & C=16.3		D=14.76 & C=18.24	
Payoff	10		10	
EE	**NE**	**S/U**	**NE**	**S/U**
E1		U		U
E2		U		U
E3		U		U
E4	•	S	•	S
E5	NA		NA	

The increase of the sale price above the value of maximum cooperative sale price would change the game type from Prisoners' Dilemma to Stag Hunt game. In the following, three different sale prices have been provided that can be investigated via ESS analysis. The phase diagram of each experiment has been obtained as Figure 39. The tendency to join the cooperation has its slowest speed in the eighth experiment and as the sale price increases the speed of this tendency becomes faster and it reaches to the fastest movement in the tenth experiment

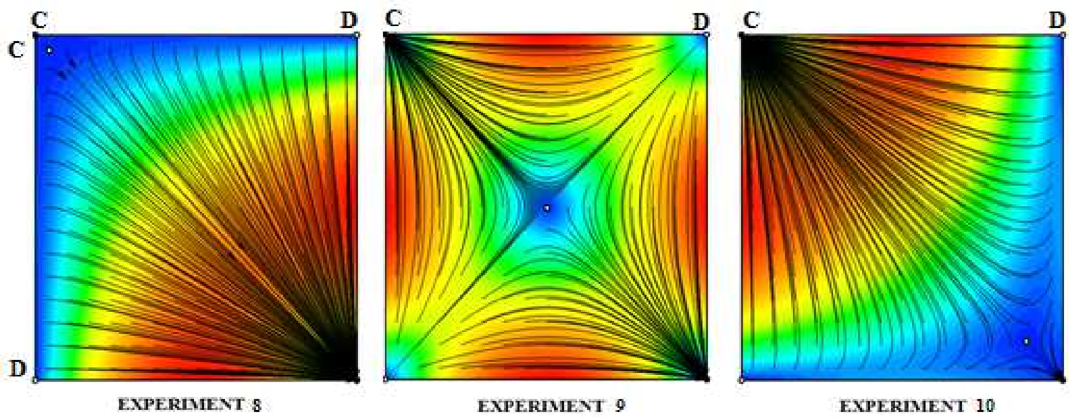


Figure 39: Phase diagram of experiments (8 – 10)

Also, the characterizations of the stability of EE for each experiment are listed in Table 36.

Table 36: The ESS analysis of the experiments (8 – 10)

S:Stable - U:Unstable - NE:Nash Equilibrium

	EXPERIMENTS					
	8		**9**		**10**	
P	D=14.76 & C=18.25		D=14.76 & C=18.66		D=14.76 & C=25	
Payoff	11.49		49.65		224.14	
EE	**NE**	**S/U**	**NE**	**S/U**	**NE**	**S/U**
E1	•	S	•	S	•	S
E2		U		U		U
E3		U		U		U
E4	•	S	•	S	•	S
E5	(0.02,0.98)	U	(0.50,0.50)	U	(0.94,0.06)	U

5.5. Cooperative Spare Parts Inventory Game

5.5.1. Introduction

In this spare part inventory game, we study the inventory game in case of an N-person and non-zero-sum game where players play simultaneously. Our game is restricted in three-person (there are three manufacturers who can manufacture a substitutable spare part), cooperative game setup. In our problem, we investigate the cooperation of manufacturers while there is a cost asserted as a binding agreement cost. Determination of decision-making on cooperation or defect depends on spare part sale price variations and the cost of binding agreement which are investigated as the Prisoners' Dilemma and Stag Hunt game. Meanwhile different methods of benefit allocation among cooperative manufacturers are investigated. Moreover, a centralized inventory configuration for manufacturers who decide to rely on a cooperative inventory system is designed and a comparison between inventory levels and cost of inventory for two different cases of centralized and decentralized inventory configuration is studied. To investigate the effect of the Green manufacturing on cost of inventory we assume that one of the manufacturers can implement re-manufacturing to produce spare parts and the role of re-manufacturing on payoffs of the manufacturers in cooperative inventory games is investigated and the optimal level of re-manufacturing effort is calculated.

5.5.2. Coalition and Benefit Allocation

In spare parts management, the variability of the demand affects the safety stock and increase the average inventory cost. In this situation, risk pooling as a method to protect against demand variability can decrease the average inventory. In spare parts business when there exists several companies that provide the same parts who may have different high and low demands can take advantage of cooperation. Risk pooling benefits from the aggregating demand across different manufacturers that results in less volatile demand size and decrease the inventory cost. The profit of reduction in cost of inventory should be allocated among the participants while the companies' profit share may vary in relation to their demands behavior (Guajardo & Rönnqvist, 2012). The cost allocation in general parts inventory systems has been studied adequately (Dror & Hartman, 2007, Gerchak & Gupta, 1991, Gupta & Srinivasa Rao, 1996, Bruce C Hartman et al., 2000, Robinson, 1993). However, the body of research for the cost allocation in spare parts inventories is not extensive. The first study has been carried out by (Wong et al., 2007) and recently series of related research has been provided by (F. Karsten et al., 2012, FJP Karsten et al., 2009).

The goal of this section is to investigate the spare parts inventory problem as an N-person non-zero-sum game where players (manufacturers) can cooperate or compete with each other through cooperative or competitive sale prices strategies. Manufacturers must play simultaneously, and in the case of cooperation, they have to decide how to allocate the profit of the resulting coalition. In order to achieve this goal, the problem is restricted to three-person game, and it is investigated through cooperative game setup. The game has three players including three manufacturers who manufacture a single-item substitutable spare part and compete with each other in the market. Also, in another consideration, it is possible for manufacturers to implement re-manufacturing effort into the manufacturing processes to increase their profit.

5.5.3. THE MARKET DEMAND

In this study it has been assumed that the OEM only has limited information about the market's expected demand. Original parts may face failure because of defects and aging, and once they fail, demands for spare parts will arise. Therefore, demands for spare parts depend on the following three factors:

- Quality of parts;
- Usage rate of products;
- Maintenance quality of products;

Due to the nature of characteristics of spare parts demand, spare parts demand arrival is considered as a Poisson process with a constant intensity factor or rate of (λ). The demand for the manufacturers follows a general linear demand function as discussed in the section 5.3.3 where i=1,2,3 (number of the manufacturers) and j=4-i. So the demand function can be written as Equation (68), where parameter (λ_i) is the market base for the product, the parameter α is the self-price elastic coefficient and the parameter (β) is the cross-price elastic coefficient.

This formula represents that the demand for the products depends on its own sale price and its competitor sale price. We assume that the product's demand is mostly dependent on its own sale price, so we consider that $\alpha > \beta > 0$ which means that the demand for products is more sensitive to changes of the self-price sale.

We suppose that the manufacturers can only forecast the original products failure rate or intensity factor that determines the expected demand for spare parts, which is parameter (λ). In other words the mean of the market demand for spare parts is derived based on Equation (70).

$$D_i = \lambda_i - \propto \left((N-1)p_i - \left(\sum_{1}^{N} p_k \middle| k \neq i \right) \right) + \beta \left(\left(\sum_{1}^{N} p_k \middle| k \neq i \right) - (N-1)p_i \right) \quad \forall\, i,j = 1,2,3 \text{ and } i \neq j \text{ and } N = 3 \tag{70}$$

5.5.4. The Manufacturer Cost Function-Traditional Manufacturer

The aim of our game is to develop an inventory control policy for the manufacturers and study the cooperation between them and discuss about the profit allocation among cooperative players. In case of traditional manufacturing, the manufacturers manufacture products directly from the raw material, so the control policy is the determination of the sale price strategies and inherently the level of inventory. The payoff of the manufacturers is the profit of the manufacturer $\left({K_B}^i\right)$ which is the difference between the cost of production and inventory $\left({K_M}^i\right)$ and the revenue $\left({K_R}^i\right)$ which attained by selling products.

The goal is to reach a low level of the backorder or a high level of fill rate with minimum investment on inventory as it was discussed in the section 5.2.4. We must calculate the cost of production and inventory, as well as the revenue from selling the products. Equation (42) gives us the cost of production and inventory, Equation (43) gives us the revenue of selling products and Equation (44) gives us the revenue of selling products.

As we can see the cost of the production and inventory is a function of demand and inventory level, which are the strategic actions of the players. Moreover there are different parameters that affect the cost of production and inventory. The parameters in Equation (42) are listed in Table 37.

Table 37: Cost of production and inventory parameters

Notation	Parameter definition
S	Spare parts inventory level (in units)
$\mathbf{c_p}$	Variable cost of production (in dollars per unit)
p	Penalty cost (in dollars per unit per period)
h	Holding cost (in dollars per unit per period)

Similarly the revenue of selling products is also a function of demand and inventory level. The parameters in Equation (43) are listed in Table 38:

Table 38: Revenues of selling products parameters

Notation	Parameter definition
D	Demand (in units per period)
$\mathbf{c_s}$	Variable cost of sells (in dollars per unit)

5.5.5. THE MANUFACTURER COST FUNCTION FOR THE RE-MANUFACTURER

We consider Green manufacturing as the process of re-manufacturing as it was discussed in the section 5.3.5. By considering re-manufacturing process, Equation (71) gives us the cost of production and inventory:

$$K_M = \left(c_p(1-\tau) + c_r\tau\right)S + p \times EBO(S) + h \times EI(S) + B\frac{\tau^2}{2} \tag{71}$$

Parameters are being used to calculate the cost of production and inventory while using Green manufacturing are listed in Table 39:

Table 39: Green manufacturing parameters

Notation	Parameter definition
τ	remanufacturing effort
C_r	variable cost of remanufacturing (in dollars per units)
B	Collection scaling parameter

5.5.6. Centralized vs. Decentralized

Inventory pooling known as lateral transshipments and direct deliveries can help companies to maintain high service levels with low cost. The effectiveness of the spare parts inventory pooling or resupply has been investigated by (Muckstadt, 2004) who provides examples of systems using centralized and decentralized strategies. The author declared that in centralized arrangements companies can decrease the amount of inventory and safety stocks to one third of the decentralized situation.

In our study, we assume that companies can cooperate with each other in two different inventory systems coordination:

- Decentralized;
- Centralized;

In the decentralized coordination, each manufacturer has his own production and inventory system, while in the centralized coordination, cooperative manufacturers run a single inventory system to meet their cumulative demand.

5.5.7. Cooperation and Benefit Allocation

Companies who use complex machines need to stock low-demand expensive spare parts. Those companies can cooperate with each other to meet their demands. A diminished total cost of spare parts inventory is achievable and the distribution of inventory costs among companies would be determined by using cooperative Game Theory models (F. Karsten et al., 2006).

We assume that manufacturers can cooperate with each other based on their product sale price. Also, in the case of centralized inventory system they can rely on a single inventory system for their coalition. Hence, basically manufacturers have two strategies to take:

- Cooperate: Make a coalition and establish the product sale price based on the coalition agreement to have an equally distributed market demand;
- Defect: Decrease the sale price to encourage the market to purchase the product from them;

In a cooperative game, communication between players is allowed, so they can agree to reach a better outcome than Nash Equilibrium. Cooperative games can be studied in characteristic function form. Our game in characteristic function form is a set of (N) players and a function (π)which assigns a number $\pi(\mathrm{S})$ to any subset $\mathrm{S} \subseteq \mathrm{N}$.

The number $\mathrm{K_B(S)}$ assigned to the coalition (S) is interpreted as the amount that players in the set (S) could win if they formed a coalition. A game in characteristic form is said to be super-additive when $\mathrm{K_B(S \cup T) \geq K_B(S) + K_B(T)}$ for any two disjoint coalition (S) and (T).

For a coalition $\mathrm{S} \subseteq \mathrm{N}$, we refer $\mathrm{K_B(S)}$ to the optimal expected payoff if all players in coalition (S) would implement cooperation. (N) is the grand coalition (N=3) where all 3 players cooperate with each other and $K_B(N)$ is its optimal expected payoff. For a given benefit allocation we refer (V_i) to the benefit or payoff allocated to player i.

5.5.8. The Core of The Game

A benefit allocation vector $V = (V_1, \dots, V_n)$ is said to be in the core of the game if it satisfies constraints formulated in Equations (72 – 74) (Rapoport, 1970, Martin Shubik, 1985)(Guajardo & Rönnqvist, 2012).

$$V_i \geq K_B(i) \quad \forall i \in N \tag{72}$$

$$\sum_{i \in S} V_i \geq K_B(S) \quad \forall S \in N \tag{73}$$

$$\sum_{i \in N} V_i = K_B(N) \tag{74}$$

Constraint 1 or Equation (72) corresponds to the individually rational condition, which says that the benefit allocated to each player i must not be greater than its stand-alone payoff. Constraint 2 or Equation (73) corresponds to a stability condition (coalition rationality), which states that there is no subsets of players such that if they would form a coalition separate from the rest they would perceive less benefit than the allocation (V). Constraint 3 or Equation (74) corresponds to the efficiency condition (collective rationality), which states that the sum of the benefits allocated to all the players equals the optimal payoff of the grand coalition, and thus it takes full advantage of cooperation. The core of the game is the set of all vectors (V) satisfying three aforementioned constraints. In other words, a payoff allocated vector in the core assures that the savings of cooperation are achieved and makes all players to stay in the grand coalition, without incentives for a player to stay alone or within a smaller coalition.

5.5.9. BENEFIT ALLOCATION METHODS

In the case of coalition players can increase their payoffs or benefits. In this section we introduce two different methods for the benefit allocation among players of the game.

5.5.9.1. SHAPLEY VALUE

Shapley (Shapley, 1952) suggested a solution concept for cooperative games which provides a unique imputation and represents payoffs distributed fairly by an outside arbitrator. The Shapley value is determined based on three axioms:

- The symmetries in payoffs (Axiom 1);
- Irrelevance of a dummy player (Axiom 2);
- The sum of two games (Axiom 3);

Axiom 1 implies that if some players have symmetric roles in payoff then the Shapley values to these players should be the same. From Axiom 2, the Shapley value to the player who adds nothing to any coalition should be determined as zero. Axiom 3 says that if two games have the same player set, then the characteristic value of the sum game for any coalition should be the sum of the characteristic values of two games. Based on the three Axioms, Shapley determines the unique values that is derived from Equation (75) in which S denotes a coalition and |S| is the size of S (Leng & Parlar, 2005).

$$V_i = \sum_{i \in S} \frac{(|S| - 1)!\,(N - |S|)!}{N!} [K_B(S) - K_B(S - i)] \tag{75}$$

5.5.9.2. BARGAINING SET

In a cooperative game if players desire the stability offered by the core, they will be unable to reach an agreement, so they have no choice but to relax their stability requirements. The bargaining set is a solution that allows players to reach an agreement while guaranteeing some stability (Aumann & Maschler, 1961, Davis & Maschler, 1962).

In a game with coalition structure, an objection of player i against player j is a pair (P,Y) where:

- $P \subseteq N$ is a coalition such that $i \in P$ and $j \notin P$;
- $Y_P \leq K_B(P)$ (Y is a feasible payoff distribution for the players in P);
- $\forall k \in P, Y_k \geq V_k$ and $Y_i > V_i$ (player i strictly benefits from Y, and the other members of P do not do worse in Y than in V);

An objection (P,Y) of player (i) against player (j) is a potential threat by coalition (P), which contains (i) but not (j), to deviate from (V). The goal is not to change (S), but to obtain a side payment from (j) to (i) to modify (V).

An counter-objection to (P,Y) is a pair (Q,Z) where:

- $Q \subseteq N$ is a coalition such that $i \in Q$ and $j \notin Q$;
- $Z_Q \leq K_B(Q)$ (Y is a feasible payoff distribution for the players in Q);
- $\forall k \in Q, Z_k \geq V_k$ (the members of Q get at least the value in V);
- $\forall k \in Q \cap P, Z_k \geq Y_k$ (the members of Q which are also members of P get at least the value promised in the objection);

In a counter-objection, player (j) must show that it can protect its payoff $K_B(j)$ in spite of the existing objection of (i).

A game with coalition structure the vector $V = (V_1, \ldots, V_n)$ is stable if for each objection at (V) there is a counter-objection. The pre-bargaining set preBS is the set of all stable members of $V = (V_1, \ldots, V_n)$, so $\text{core}(N, K_B, S) \subseteq \text{preBS}(N, K_B, S)$.

Let $I(N, K_B, S) = \{V \epsilon V_{(N,\pi,S)} | V_i \geq K_B(\{i\}) \forall i \in N\}$ be the set of individually rational payoff vector in $V_{(N,K_B,S)}$. The bargaining set BS is defined by Equation (76):

$$BS(N, \pi, S) = I(N, \pi, S) \cap \text{preBS}(N, \pi, S) \quad (76)$$

5.5.10. The Game Setup

Game Theory is applied to determine the inventory control policy for manufacturers who manufacture a single part and sell it to the market while they can cooperate or compete with each other. Our inventory game has two players who have two different strategies: coalition or competition. Decision-making on sale price (and re-manufacturing efforts in a Green manufacturing situation) involves strategies which determine their production lot-size and inventory levels. The market has a cyclic demand which is stochastic, and in each period the market base for the spare part for each manufacturer is known and the distribution of the demand is Poisson. The total demand has to be distributed among players and manufacturers' sale price strategies affect their allocated demand. In this environment manufacturers can cooperate with each other or compete with each other that means each of them has two strategies:

- Strategy 1: Coalition, manufacturers set their sale prices in such a way that gives them the highest profit;
- Strategy 2: Competition, manufacturers set their sale prices in such a way that attracts more demand share;

The inventory game is a three-person non-zero-sum game, so we are looking for Nash Equilibrium as the solution of the game. Based on the players strategies (cooperate or defect) different payoffs would be assigned for each manufacturer. The payoff matrix is depicted in Table 40.

Table 40: The payoff matrix of inventory game

		MANUFACTURER 3			
		(C)		**(D)**	
		MANUFACTURER 2		**MANUFACTURER 2**	
		(C)	**(D)**	**(C)**	**(D)**
MANUFACTURER 1	**(C)**	${K_B}^1CCC, {K_B}^2CCC, {K_B}^3CCC$	${K_B}^1CDC, {K_B}^2CDC, {K_B}^3CDC$	${K_B}^1CCD, {K_B}^2CCC, {K_B}^3CCD$	${K_B}^1CDD, {K_B}^2CDD, {K_B}^3CDD$
	(D)	${K_B}^1DCC, {K_B}^2DCC, {K_B}^3DCC$	${K_B}^1DDC, {K_B}^2DDC, {K_B}^3DDC$	${K_B}^1DCD, {K_B}^2DCD, {K_B}^3DCD$	${K_B}^1DDD, {K_B}^2DDD, {K_B}^3DDD$

Moreover we assume that in the case of cooperation, each manufacturer must invest on the cooperation agreement with the value (K_{CO}).

Also, it has to be mentioned that the payoff matrix has been provided for two different scenarios of centralized and decentralized inventory management. In the centralized strategies, cooperative manufacturers run a single inventory system to meet their cumulative demand while in decentralized situation each manufacturer has his own inventory system.

Investigation of the payoff matrix tells us, changes in the value of the cooperation agreement or (K_{CO}) determines the type of the spare parts inventory game. In other words, the aforementioned value changes the type of the game from Stag Hunt game to Prisoners' Dilemma and non-emptiness of the core of the game which affects the cooperation of the manufacturers. The following procedure determines the type of the game and the non-emptiness of the core:

- If $K_{CO} \leq \operatorname{Min}\begin{pmatrix} K_{B_{CCC}^{1}} - K_{B_{DDD}^{1}} \\ K_{B_{CCC}^{2}} - K_{B_{DDD}^{2}} \\ K_{B_{CCC}^{3}} - K_{B_{DDD}^{3}} \end{pmatrix}$ the cooperative strategy is feasible for manufacturers, otherwise there is no motivation for cooperation;
- If $K_{CO} \leq \operatorname{Max}\begin{pmatrix} K_{B_{CDD}^{1}} - K_{B_{DDD}^{1}} \\ K_{B_{DCD}^{2}} - K_{B_{DDD}^{2}} \\ K_{B_{DDC}^{3}} - K_{B_{DDD}^{3}} \end{pmatrix}$ the cooperate strategy dominates the defect strategy, or the game has a Nash Equilibrium which is the cooperate;
- If $\operatorname{Max}\begin{pmatrix} K_{B_{CDD}^{1}} - K_{B_{DDD}^{1}} \\ K_{B_{DCD}^{2}} - K_{B_{DDD}^{2}} \\ K_{B_{DDC}^{3}} - K_{B_{DDD}^{3}} \end{pmatrix} \leq K_{CO} \leq \operatorname{Min}\begin{pmatrix} K_{B_{CCC}^{1}} - K_{B_{DCC}^{1}} \\ K_{B_{CCC}^{2}} - K_{B_{CDC}^{2}} \\ K_{B_{CCC}^{3}} - K_{B_{CCD}^{3}} \end{pmatrix}$ the game has two Nash Equilibriums which are the cooperate and the defect or the

game is a Stag Hunt game, but according to the cooperation agreement manufacturers can benefit more through cooperation;

- If $K_{CO} \geq Min\begin{pmatrix} K_{B_{CCC}}^{1} - K_{B_{DCC}}^{1} \\ K_{B_{CCC}}^{2} - K_{B_{CDC}}^{2} \\ K_{B_{CCC}}^{3} - K_{B_{CCD}}^{3} \end{pmatrix}$ the game has a Nash Equilibrium which is the defect or the game is a Prisoners' Dilemma game, but according to the cooperation agreement manufacturers can benefit more through cooperation;

- If $K_{CO} \leq Min\begin{pmatrix} \frac{(K_{B_{CCD}}^{1}+K_{B_{CCD}}^{2})-(K_{B_{DDD}}^{1}+K_{B_{DDD}}^{2})}{2} \\ \frac{(K_{B_{CDC}}^{1}+K_{B_{CDC}}^{3})-(K_{B_{DDD}}^{1}+K_{B_{DDD}}^{3})}{2} \\ \frac{(K_{B_{DCC}}^{2}+K_{B_{DCC}}^{3})-(K_{B_{DDD}}^{2}+K_{B_{DDD}}^{3})}{2} \end{pmatrix}$ the essential constraint of the core of the game is satisfied;

5.5.11. Numerical Study

In this section the inventory management of the manufacturers during 1-year of production has been investigated. We assumed that the manufacturers possess the market base demand for the product during the production year. In order to generate the payoff matrix for the game, it is assumed that the production, inventory and marketing conditions are not changing during the production horizon and the required parameters are consistent with Table 41.

We suppose all the parameters are the same for all manufacturers except fill rates and the lead-time (T) is 1-year.

Table 41: Sample parameters used to generate the payoff matrix
Traditional manufacturer

Notation	Parameter value
λ	3
c_p	40
p	70
h	2
c_s	Cooperate:150 & Defect:100
α	0.005
β	0.003
EFR	M1= 0.79, M2=0.89, M3=0.99

First, we start with the decentralized coordination where manufacturers are traditional manufacturers. The resulting payoff matrix is depicted in Table 42. The core of the game is non-empty and in the normalized form the resulting imputation ratios for the manufacturers are [0.5386 0.5293 0.5138].

Table 42: The payoff matrix of inventory game - Decentralized coordination & Traditional manufacturer

		MANUFACTURER 3			
		(C)		***(D)***	
		MANUFACTURER 2		**MANUFACTURER 2**	
		(C)	***(D)***	***(C)***	***(D)***
MANUFACTURER 1	**(C)**	153.2,162.6,114.3	132.1,54.4,57	132.1,122.3,-35	99,31,-35
	Demand	3,3,3	2.6,3.8,2.6	2.6,2.6,3.8	2.2,3.4,3.4
	Inventory	5,6,8	5,7,8	5,6,10	4,7,9
	(D)	54.3,122.3,57	N:0 44,30,-71	N:0 44,98,-34.3	31,25,-34
	Demand	3.8,2.6,2.6	3.4,3.4,2.2	3.4,2.2,3.4	3,3,3
	Inventory	6,6,8	6,7,7	6,5,9	5,6,8

Investigation of the payoff matrix provides the following information:

- If $K_{CO} \leq 122.2$ the cooperative strategy is feasible for manufacturers, otherwise there is no motivation for cooperation;
- If $K_{CO} \leq 73$ the cooperate strategy dominates the defect strategy, or the game has a Nash Equilibrium which is the cooperate;

- If $73 \leq K_{CO} \leq 98.9$ the game has two Nash Equilibriums which are (i) cooperate, and (ii) defect, or the game is a Stag Hunt game, but by a cooperation agreement manufacturers, can gain additional benefits;
- If $K_{CO} \geq 98.9$ the game has a Nash Equilibrium which is the defect or the game is a Prisoners' Dilemma game, but according to the cooperation agreement manufacturers can benefit more through cooperation;
- If $K_{CO} \leq 94.15$ the essential constraint of the core of the game is satisfied;

In Figure 40 the maximum expected payoff of the game for each manufacturer based on bargaining set and Shapley value has been depicted.

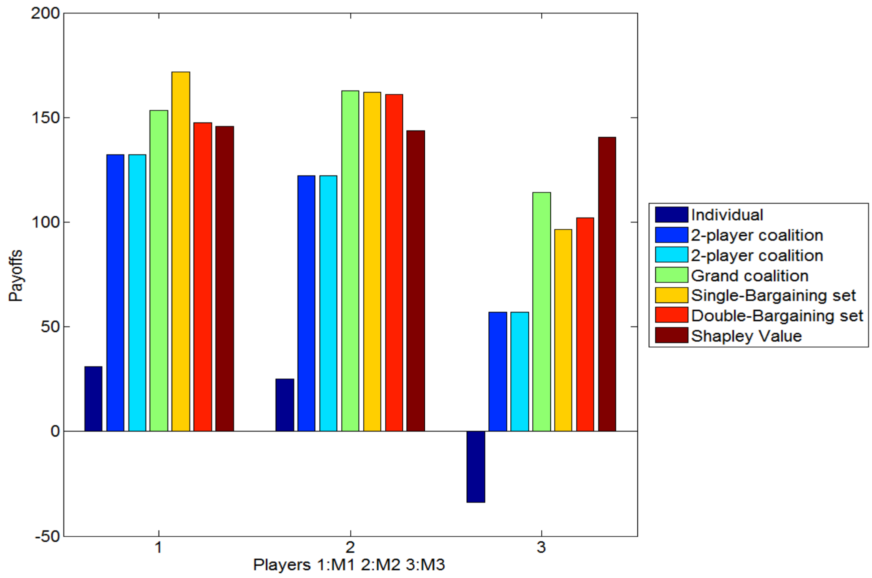


Figure 40: Manufacturers' maximum expected payoffs - Decentralized coordination Traditional manufacturers

Then, we investigate the centralized coordination where manufacturers are traditional manufacturers. The resulting payoff matrix is depicted in Table 43. The core of the game is non-empty and in the normalized form the resulting imputation ratios for the manufacturers are [0.5304 0.5402 0.5335].

Table 43: The payoff matrix of inventory game - Centralized coordination & Traditional manufacturer

<table>
<tr><td colspan="2" rowspan="4"></td><td colspan="4">MANUFACTURER 3</td></tr>
<tr><td colspan="2">(C)</td><td colspan="2">(D)</td></tr>
<tr><td colspan="2">MANUFACTURER 2</td><td colspan="2">MANUFACTURER 2</td></tr>
<tr><td>(C)</td><td>(D)</td><td>(C)</td><td>(D)</td></tr>
<tr><td rowspan="6">MANUFACTURER 1</td><td>(C)</td><td>N: 1&2&3 638.3</td><td>N:1&3 280.4,54.4</td><td>N:1&2 343.5,-34.8</td><td>N:0 99,31,-34.3</td></tr>
<tr><td>Demand</td><td>9</td><td>5.2,3.8</td><td>5.2,3.8</td><td>2.2,3.4,3.4</td></tr>
<tr><td>Inventory</td><td>17</td><td>12,7</td><td>9,10</td><td>4,7,9</td></tr>
<tr><td>(D)</td><td>N:2&3 280.4,54.3</td><td>N:0 44,30,-71</td><td>N:0 44,98,-34.3</td><td>N:0 31,25,-34</td></tr>
<tr><td>Demand</td><td>5.2,3.8</td><td>3.4,3.4,2.2</td><td>3.4,2.2,3.4</td><td>3,3,3</td></tr>
<tr><td>Inventory</td><td>12,6</td><td>6,7,7</td><td>6,5,9</td><td>5,6,8</td></tr>
</table>

Investigation of the payoff matrix provides the following information:

- If $K_{CO} \leq 181.76$ the cooperative strategy is feasible for manufacturers, otherwise there is no motivation for cooperation;
- If $K_{CO} \leq 73$ the cooperate strategy dominates the defect strategy, or the game has a Nash Equilibrium which is the cooperate;
- If $72 \leq K_{CO} \leq 158.36$ the game has two Nash Equilibriums which are (i) cooperate, and (ii) defect, or the game is a Stag Hunt game, but by a cooperation agreement manufacturers, can gain additional benefits;
- If $K_{CO} \geq 158.36$ the game has a Nash Equilibrium which is the defect or the game is a Prisoners' Dilemma game, but according to the cooperation agreement manufacturers can benefit more through cooperation;
- If $K_{CO} \leq 141.7$ the essential constraint of the core of the game is satisfied, so this game does not match the Prisoners' Dilemma, and it remains as a Stag Hunt game while $73 \leq K_{CO} \leq 141.7$;

In Figure 41 the maximum expected payoff of the game for each manufacturer based on bargaining set and Shapley value has been depicted.

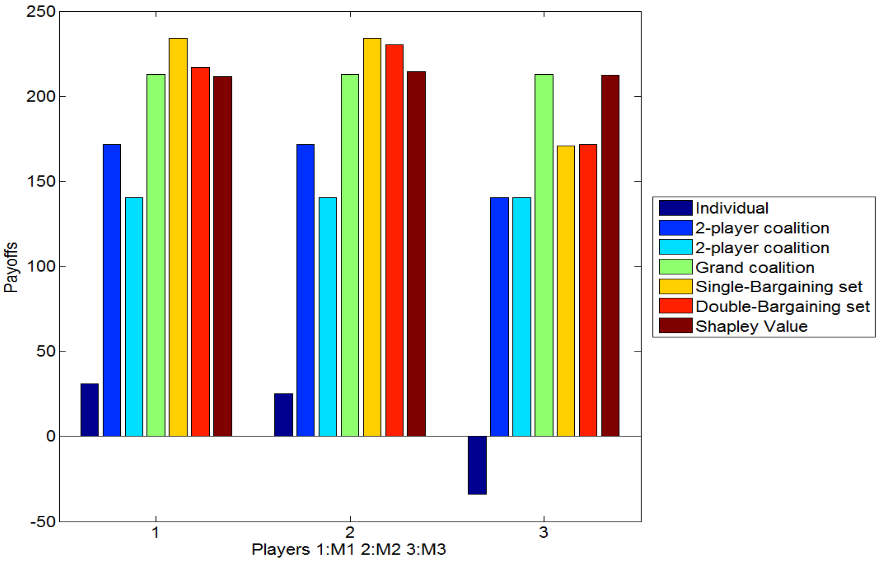


Figure 41: Manufacturers' maximum expected payoffs - Centralized coordination
Traditional manufacturers

Now, we can consider the effect of re-manufacturing. It is assumed that only one of the manufacturers can switch to re- manufacturing to benefit more in their payoffs. The required parameters are listed in the Table 47:

Table 44: Sample parameters used to generate the payoff matrix
Green manufacturer

Notation	Parameter value
τ	$0<\tau<1$
C_r	25
B	50

First, we study the decentralized coordination where there is a Green manufacturer. The resulting payoff matrix is depicted in Table 45. The optimal level of GMEP is 30%. The core of the game is non-empty and in the normalized form the resulting imputation ratios for the manufacturers are [0.5386 0.5293 0.5138].

Table 45: The payoff matrix of inventory game - Decentralized coordination & Green manufacturer

		MANUFACTURER 3			
		(C)		(D)	
		MANUFACTURER 2		MANUFACTURER 2	
		(C)	(D)	(C)	(D)
MANUFACTURER 1	(C)	153.2,162.6,132.3	132.1,54.4,75	132.1,122.3,-12.3	99,31,-14
	Demand	3,3,3	2.6,3.8,2.6	2.6,2.6,3.8	2.2,3.4,3.4
	Inventory	5,6,8	5,7,8	5,6,10	4,7,9
	(D)	54.3,122.3,75	44,30,-55.7	44,98,-14	31,25,-16
	Demand	3.8,2.6,2.6	3.4,3.4,2.2	3.4,2.2,3.4	3,3,3
	Inventory	6,6,8	6,7,7	6,5,9	5,6,8

Investigation of the payoff matrix provides the following information:

- If $K_{CO} \leq 122.2$ the cooperative strategy is feasible for manufacturers, otherwise there is no motivation for cooperation;
- If $K_{CO} \leq 73$ the cooperate strategy dominates the defect strategy, or the game has a Nash Equilibrium which is the cooperate;
- If $73 \leq K_{CO} \leq 98.9$ the game has two Nash Equilibriums which are (i) cooperate and, (i) defect, or the game is a Stag Hunt game, but by a cooperation agreement manufacturers, can gain additional benefits;
- If $K_{CO} \geq 98.9$ the game has a Nash Equilibrium which is the defect or the game is a Prisoners' Dilemma game, but according to the cooperation agreement manufacturers can benefit more through cooperation;
- If $K_{CO} \leq 94.15$ the essential constraint of the core of the game is satisfied, so this game does not match the Prisoners' Dilemma, and it remains as a Stag Hunt game while $73 \leq K_{CO} \leq 94.15$;

In Figure 42 the maximum expected payoff of the game for each manufacturer based on bargaining set and Shapley value has been depicted.

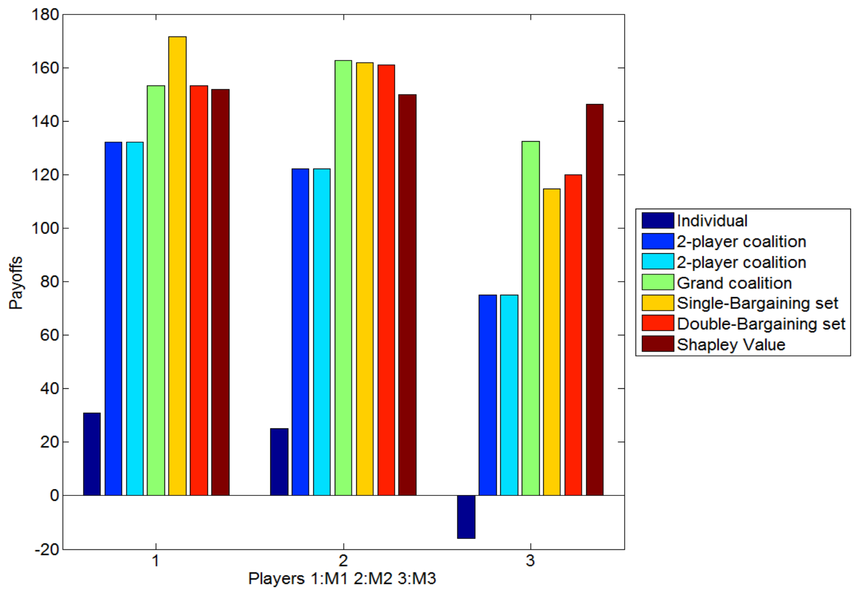


Figure 42: Manufacturers' maximum expected payoffs - Decentralized coordination Green manufacturer (M3)

Then, we investigate the centralized coordination where there is a Green manufacturer. The resulting payoff matrix is depicted in Table 46. The optimal level of GMEP is 30%. The core of the game is non-empty and in the normalized form the resulting imputation ratios for the manufacturers are [0.5338 0.5436 0.5295].

Table 46: The payoff matrix of inventory game - Centralized coordination & Green manufacturer

		MANUFACTURER 3			
		(C)		**(D)**	
		MANUFACTURER 2		**MANUFACTURER 2**	
		(C)	**(D)**	**(C)**	**(D)**
MANUFACTURER 1	**(C)**	N: 1&2&3 651.1	N:1&3 293.9,54.4	N:1&2 343.5,-12.3	N:0 99,31,-14
	Demand	9	5.2,3.8	5.2,3.8	2.2,3.4,3.4
	Inventory	17	12,7	9,10	4,7,9
	(D)	N:2&3 293.9,54.3	44,30,-55.7	44,98,-14	N:0 31,25,-16
	Demand	5.2,3.8	3.4,3.4,2.2	3.4,2.2,3.4	3,3,3
	Inventory	12,6	6,7,7	6,5,9	5,6,8

Investigation of the payoff matrix provides the following information:

- If $K_{CO} \leq 186$ the cooperative strategy is feasible for manufacturers, otherwise there is no motivation for cooperation;
- If $K_{CO} \leq 73$ the cooperate strategy dominates the defect strategy, or the game has a Nash Equilibrium which is the cooperate;
- If $73 \leq K_{CO} \leq 162.6$ the game has two Nash Equilibriums which are the cooperate and the defect or the game is a Stag Hunt game, but according to the cooperation agreement manufacturers can benefit more through cooperation;
- If $K_{CO} \geq 162.6$ the game has a Nash Equilibrium which is the defect or the game is a Prisoners' Dilemma game, but according to the cooperation agreement manufacturers can benefit more through cooperation;
- If $K_{CO} \leq 139.5$ the essential constraint of the core of the game is satisfied, so this game has no Prisoners' Dilemma type and it remains as a Stag Hunt game while $73 \leq K_{CO} \leq 139.5$;

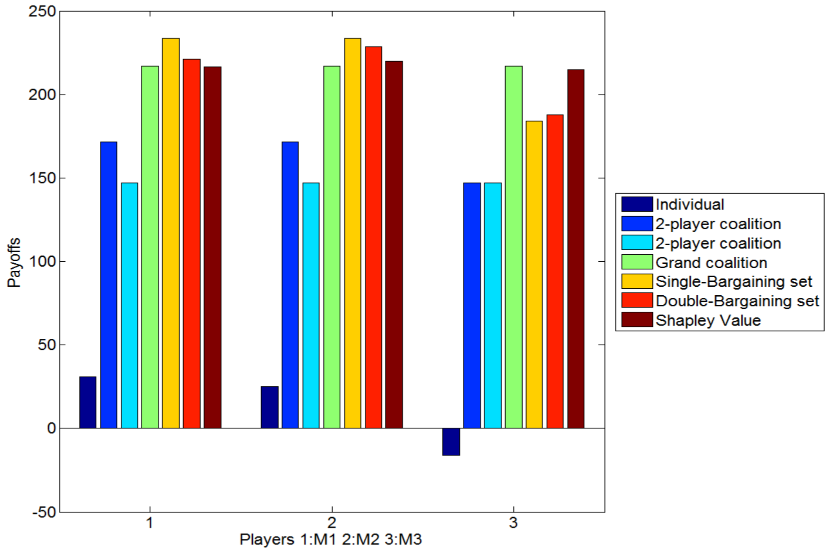


Figure 43: Manufacturers' maximum expected payoffs - Centralized coordination Green manufacturer

In Figure 43 the maximum expected payoff of the game for each manufacturer based on bargaining set and Shapley value has been depicted.

In Table 47 the cumulative payoff, the cumulative inventory level and GMEP for the Green manufacturer of the game for different combinations of decentralized, centralized, traditional and Green manufacturing are listed.

- Decentralized, traditional manufacturer main manufacturer (DC-T);
- Decentralized, Green manufacturer main manufacturer (DC-G);
- Centralized, traditional manufacturer main manufacturer (C-T);
- Centralized, Green manufacturer main manufacturer (C-G);

Table 47: The cumulative payoff

	DC-T	**DC-G**	**C-T**	**C-G**
TOTAL PAYOFF	430.1	448.1	638.3	651.1
TOTAL INVENTORY LEVEL	19	19	17	17
GMEP	0	30	0	30

5.6. Spare Parts' Price Leadership

5.6.1. Introduction

In the aftermarket business, the supply of spare parts can be investigated through the life span of the parent product. In other words, the spare part supply can be categorized into two periods of the time span before end-of-production of parent product and the time span after that period. In first period, OEMs have a monopolistic market for their aftersales services but in the second period there is a competition among OEMs and will-fitters. Game Theory can improve or clarify interactions between different manufacturers who are competing against each other based on determination of sale prices to attract more market demand share. In our study, we consider the OEM as the price leader and the will-fitters as fringe and the spare part inventory game is setup as a competitive fringe game. The solution of the game determines the OEM sale price decision-making and inventory level.

5.6.2. Competitive Fringe Spare Part Inventory Game

Spare parts are manufactured along with the original or parent product. The manufacturer keeps the inventory of spare parts for after sale services during warranty period, rendering period, and post warranty period. Availability of the spare parts during the life span of the product affects the competitiveness of the OEM in the market, meanwhile there are some legal obligations for the OEM to supply spare parts during that period in some countries. In other words, the OEM should supply spare parts for the parent product during product life cycle and the time span between end-of-production and end-of-service. For instance, in automobile industry in Germany, the life span of the products is 15 years (Inderfurth & Mukherjee, 2008).

Normally there is a monopoly for the OEM to supply spare parts during the product life cycle. By the beginning of the end-of-production, other manufacturers who can produce the same parts, known as will-fitters, enter to the aftermarket and absorb some share of the demand for spare parts. When a monopoly ends, the OEM maintains a cost advantage over later manufacturers. The OEM becomes a dominant firm: a price-setting firm that competes with price taking firms. The small price taking firms that compete with a dominant firm are called the competitive fringe.

In other words, the dominant-firm model of price leadership assumes that there is a main manufacturer known as OEM and many small manufacturers known as will-fitters whose production rates are not large enough to affect the sale price. According to (Scherer & Ross, 1970), “Dominant firm price leadership occurs when an industry consists of one firm dominant in the customary sense of the world i.e. controlling at least 50% of the total industry output plus a competitive fringe of firms, each too small to exert a perceptible influence on price through its individual output decisions”. When will-fitters in the fringe acting as price takers, the OEM is left as the only player who is able to set price and maximize the profit subject to its residual demand curve.

5.6.3. THE MARKET DEMAND

In many markets, the successful manufacturer has the skill to plan its production/inventory in advance to take benefit of the predicted demand conditions. In fact, a considerable amount of time and money is spent to forecast the demand. In a competitive market, the transmission of the information from demanders to suppliers is not efficient. The dominant firm has an incentive to invest in demand information, and the size of the competitive fringe depends on information costs and demand variability. In other words, the competitive fringe shrinks when the information cost and demand variability increase.

During the product life cycle, the OEM manages its spare parts production and inventory economically in a monopolistic market. Moreover, the demand for spare parts is predictable because of existence of the customer linkage, current information on performance of the market demand for the parent product and up-to-date time series data (Inderfurth & Mukherjee, 2008). Once the OEM stops the production of the parent product, the time span between end-of-production and end-of-service, there is no demand for the parent product but spare parts should be supplied as the replacement parts for the existing products. Spare parts management in this phase become challenging because of the demand uncertainty and emergence of the will-fitters into the market.

Spare parts demand is intermittent and due to the nature of characteristics of spare parts demand, we can consider the demand for spare parts as a Poisson process as it was discussed in the section 5.2.3. The OEM can forecast the parent products failure rate or intensity factor that determines the expected overall demand for spare parts, which is parameter (λ). We suppose that the sale price affects the overall market demand. Once the OEM raises the price, some customers will leave the market and their demand is met from refurbished parts by third parties. The overall market demand is a function of sale price (c_s) and the parent product failure rate (λ) as it is formulated in Equation (77) that provides the market demand curve. The OEM differs from a monopolist in one respect. If the monopolist raises the sale price, some customers will leave the market. However, if the OEM raises the price, there is a possibility that a price increase encourages some customers to start buying from the will-fitters. So the OEM must factors in the reaction of the will-fitters.

$$Q_D = f(c_s, \lambda) \tag{77}$$

5.6.4. THE MANUFACTURER COST FUNCTION

The manufacturers cost function includes cost of production, holding cost and backorder cost. The OEM strategies are determination of the sale price and the spare parts stock level in the order-up-to level inventory that it was discussed in the section 5.2.4. In this section, implementation of the order-up-to level inventory policy is explained and the marginal costs of production and inventory for the manufacturers ($MC_i \; \forall i \in \{OEM \; \& will - fitters\}$) are provided.

We must calculate the cost of production and inventory. Equation (78) gives us the cost of production and inventory in which expected backorder (EBO) and expected inventory (EI) are calculated from Equations (40 & 41):

$$MC_i = c_{pi} \times S_i + p_i \times EBO(S_i) + h_i \times EI(S_i) \tag{78}$$

The profit or the payoff the manufacturers is calculated as the difference between revenue from selling parts and cost of production and inventory which is described in Equation (79) in which expected fill rate (EFR) is calculated from Equation (39):

$$\Pi_i = c_s \times Q_i \times EFR(S_i) - MC_i \tag{79}$$

As we can see the cost of the production and inventory is a function of demand and order-up-to level which is the strategic actions of the players. Moreover there are different parameters that affect the cost of production and inventory. The required parameters are listed in Table 48.

Table 48: Cost of production and inventory parameters

Notation	Parameter definition
$\mathbf{S}$	Spare parts inventory level (in units)
$\mathbf{c_p}$	Variable cost of production (in dollars per unit)
$\mathbf{p}$	Penalty cost (in dollars per unit per period)
$\mathbf{h}$	Holding cost (in dollars per unit per period)
$\mathbf{c_s}$	Sale price (in dollars per unit)
$\boldsymbol{Q}$	Demand

We suppose that both the OEM and will-fitters are applying one-for-one policy for their spare parts inventory management. According to the formulation and parameters that are introduced in this section, the marginal costs of the manufacturers are calculated.

5.6.5. THE GAME SETUP

In our study, the spare part inventory control is modeled as a dominant firm with a competitive fringe game where the OEM plays as the dominant firm and the will-fitters are playing as the competitive fringe. This game is the combination of the monopoly and perfect competition games. As in perfect competition, it is rational to assume that the small firms or will-fitter are price takers. However, it is not rational to neglect the impact of the OEM price setting. Therefore, the OEM sets the market price and the will-fitters form their inventory decision-making considering this market price. The OEM is Strategic, means it takes into account the impact that its actions have on the will-fitters' actions. The will-fitters are Non-strategic.

The equilibrium of the dominant firm-competitive fringe determines the manufacturers spare part inventory policy including the OEM price setting and allocation of the market demand among the manufacturers and consequently the level of inventory.

The solution of the game can be described in three steps as follows:

- Find the residual demand;
- Find the OEM's optimal quantity supply;
- Find price and will-fitters quantity supply;

The residual demand equals to the difference between market demand and will-fitters marginal cost when price is:

- Below the intersection of market demand and will-fitters marginal cost;
- Above the vertical intercept of will-fitters marginal cost;

Otherwise the residual demand equals to the market demand. Intersection of the market demand and will-fitters marginal cost is calculated from Equation (80):

$$Q_D(c_s) = Q_{MC_{wf}}(c_s) \;\therefore \text{solve for } c_s \rightarrow c_s^* = c_{s1} \tag{80}$$

Vertical intercept of the will-fitters marginal cost is derived from Equation (81):

$$Q_{MC_{wf}}(c_s) = 0 \;\therefore \text{solve for } c_s \rightarrow c_s^* = c_{s2} \tag{81}$$

So the final residual demand is calculated from Equation (82):

$$\text{Residual Demand} = \begin{Bmatrix} Q_D(c_s) - Q_{MC_{wf}}(c_s) & \text{if } c_{s2} \leq c_s < c_{s1} \\ Q_D(c_s) & \text{otherwise} \end{Bmatrix} \tag{82}$$

The OEM's optimal quantity supply $(\mathrm{Q_{OEM}})$ is determined by $(\mathrm{MR} = \mathrm{MC_{OEM}})$. (MR), is the inverse residual demand with double the slope. Therefore, the OEM will allocate $(\mathrm{Q_{OEM}})$ units out of the overall market demand to itself. The price setting $(\mathrm{c_s^*})$ is given by substituting $(\mathrm{Q_{OEM}})$ into the inverse residual demand (MR). Finally, the will-fitters quantity supply $(\mathrm{Q_{WF}})$ is given by the will-fitters marginal curve at the setting price $(\mathrm{c_s^*})$.

5.6.6. NUMERICAL STUDY

In this section the spare part inventory management of a single-item for the manufacturers during 1-year of production has been investigated. We assume that the OEM possesses the market demand intensity rate during the time span after product life cycle. The game has two types of players: the OEM and will-fitters and the solution of the dominant firm-competitive fringe game provides price setting and inventory policy for the manufacturers.

We suppose that will-fitters are acting as fringe altogether, and each manufacturer implements its own inventory and production settings. These required parameters for the OEM and will-fitters are listed in Table 49. Both manufacturers have the same lead-time and fill rates.

Table 49: Sample parameters used to generate the marginal costs

Notation	OEM	Will-fitter
c_p	8	10
p	80	100
h	1.6	2
c_s	Decision variable	Price taker

Notation	OEM	Will-fitter
T	1	1
EFR	>0.8	>0.8

The overall market demand arrives as a Poisson process and the intensity rate of the demand is a function of the sale price that is given as an arbitrary function (Equation 83):

$$Q_D = \frac{17 - c_s}{0.5} \quad (83)$$

The cost of production and inventory for the OEM and will-fitters are calculated based on implementing order-up-to level inventory policy and the results are listed in Table 50:

Table 50: The manufacturers' costs

S	MC_{OEM}	$MC_{will-fitt}$
Initial cost	9.83	12.29
1	29.1	36.38
2	41.33	51.66
3	54.19	67.63
4	67.72	84.65
5	78.76	98.46
6	89.96	116.03
7	101.23	126.53
8	112.53	140.66
9	123.83	154.78
10	133.65	167.07

In figure 44 the trend of the suppliers' inventory costs vs quantity supply is depicted. As we can see, it is practical to consider this trend linear, which results in a constant marginal cost. The solution of the game is represented in figure 45 and explained as the three following steps:

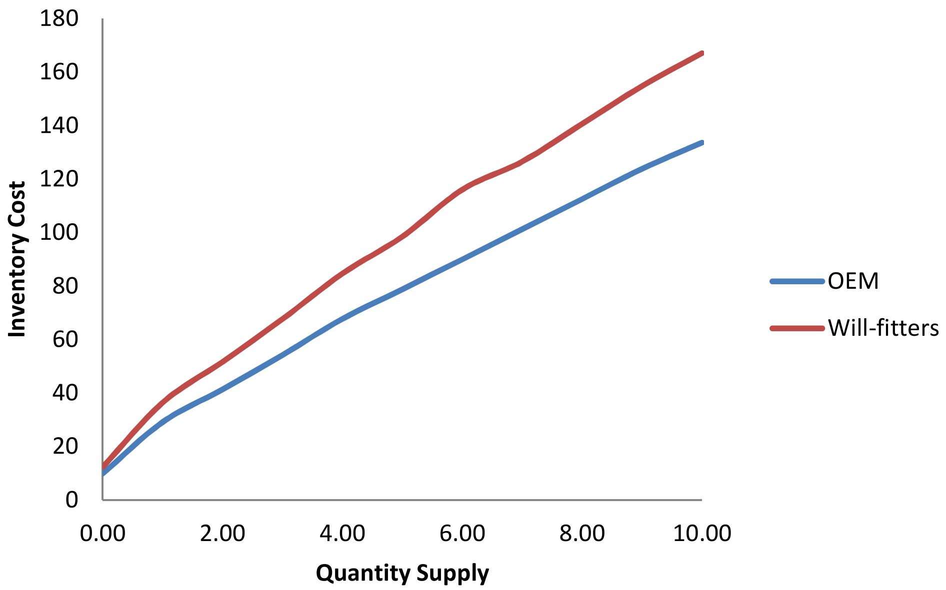


Figure 44: The Dominant firm-competitive fringe inventory costs vs. quantity supply

Step1: Find the residual demand

$$\text{Residual Demand} = \begin{Bmatrix} [0,4] & \text{if } c_s \geq 15 \\ \dfrac{17 - c_s}{0.5} & \text{otherwise} \end{Bmatrix}$$

Step 2: Find the OEM's optimal quantity supply

$$\text{Inverse Residual Demand} = \begin{Bmatrix} 15.124 & \text{if } 0 \leq Q < 4 \\ -0.5Q + 17 & \text{otherwise} \end{Bmatrix}$$

$$\text{MR} = \begin{Bmatrix} 15.124 & \text{if } 0 \leq Q < 4 \\ -Q + 17 & \text{otherwise} \end{Bmatrix}$$

$$\text{MR}(Q) = \text{MC}_{\text{OEM}}(Q) \therefore \text{solve for } Q \rightarrow Q^*_{\text{OEM}} = 5$$

Step 3: Find price and will-fitters quantity supply

$$\text{Replace } Q^*_{\text{OEM}} \text{ in Inverse Residual Demand } \therefore 17 - 0.5Q^*_{\text{OEM}} \rightarrow c^*_s = 14.5$$

$$\text{Replace } c^*_s \text{ in } \text{MC}_{\text{wf}} \therefore Q^*_{\text{wf}} = 0$$

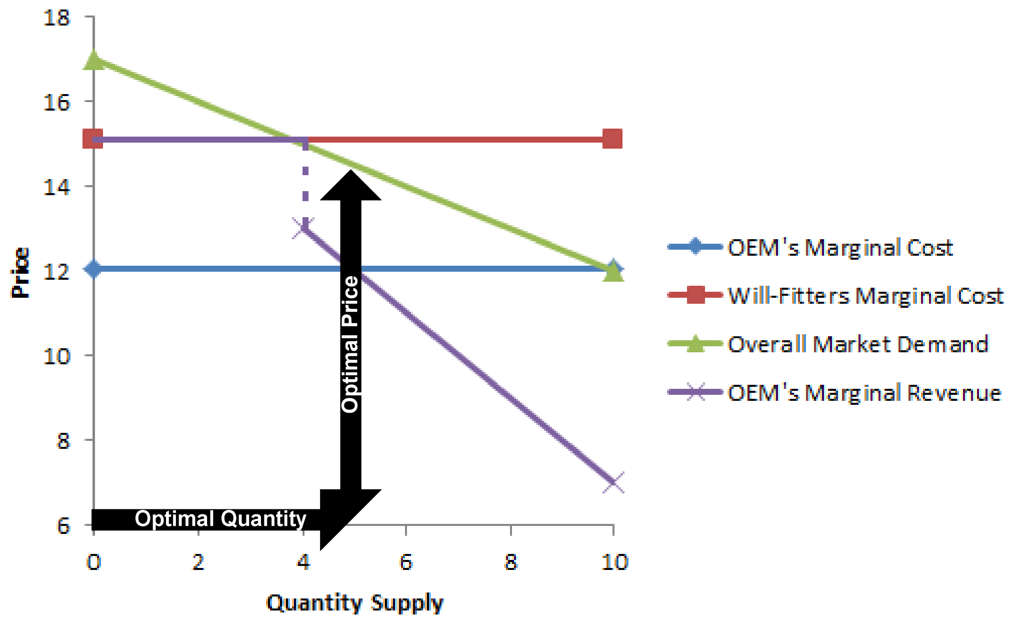


Figure 45: Price-supply diagram

After determination of the price setting, and the optimal quantity of the manufacturers we can refer to the marginal cost function and determine the allocated order-up-to levels for each manufacturer. Table 51 lists the optimal sale price, quantity supply and levels of inventory for the OEM:

Table 51: The optimal spare part inventory policy

Notation	OEM
Q	5
c_s	14.5
S	8
Π	12.5

So the OEM enters the parts to the market with a price that makes the will-fitters to drop out of the market and leave all of the demand to the OEM. The potential supply of fringe is irrelevant and the OEM becomes a monopoly. In another scenario we assume that the OEM still follows order-up-to level inventory policy and fringe firms supply spare parts to the market with a given quantity-price function or supply curve that is known by the OEM. In other words, the overall market demand rate is $(Q_D = 30 - c_s)$ and the will-fitters supply curve is $\left(Q_{wf} = \frac{c_s - 14}{0.8}\right)$ which are predicted by the OEM. The solution of the game is represented in figure 46 that results in $(Q^*_{OEM} = 10.26)$,$(Q^*_{wf} = 3.23)$,$(c^*_s = 16.59)$ and the demand rate that is satisfied through refurbishment is 16.51.

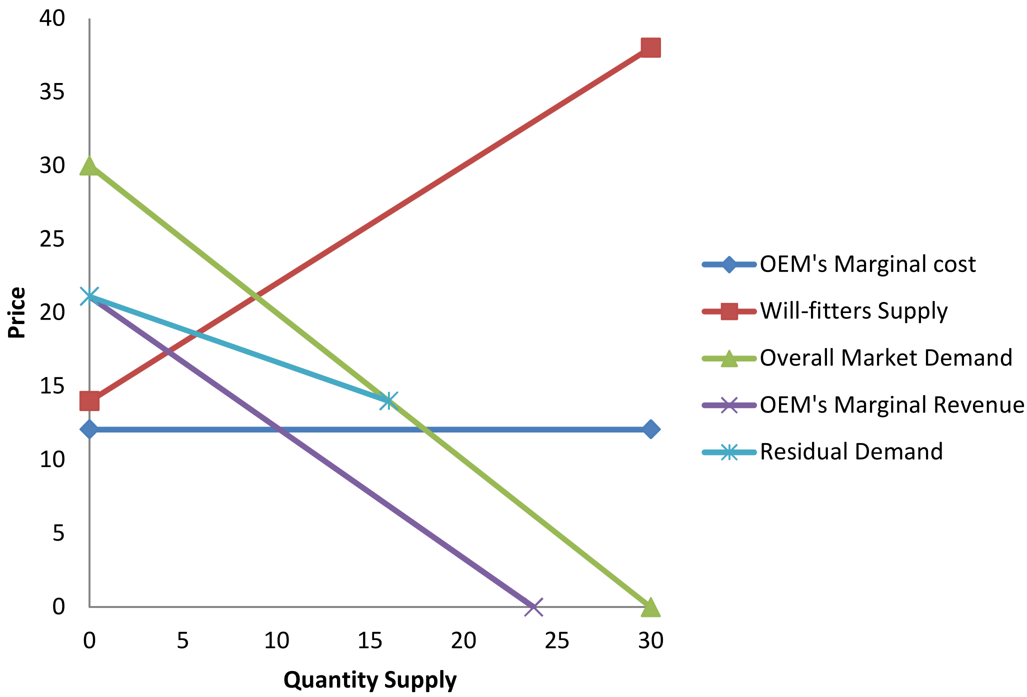


Figure 46: The competitive fringe price-supply diagram

In next section we investigate the effect of holding cost on pricing strategy. In table 52, the changes of the OEM's holding cost and its effect on the price setting, OEM's marginal cost, allocated demand rate, optimal sale price, average inventory level, cost and profit are listed.

Table 52: The effect of holding cost on pricing strategy

h	MC	Q_{OEM}	Q_{WF}	Price	AIL	Cost	Profit
0.40	11.77	10.60	3.04	16.44	3.67	147.87	5.79
0.80	11.87	10.49	3.11	16.49	3.77	147.49	6.98
1.20	11.97	10.38	3.17	16.54	3.87	147.29	7.90
1.60	12.07	10.26	3.23	16.59	3.97	147.28	8.53
2.00	12.17	10.15	3.29	16.64	4.08	147.45	8.89
2.40	12.27	10.03	3.36	16.69	4.18	147.80	8.98
2.80	12.37	9.92	3.42	16.74	4.29	148.33	8.78
3.20	12.47	9.81	3.48	16.79	4.40	149.04	8.31
3.60	12.57	9.69	3.54	16.84	4.51	149.93	7.58
4.00	12.67	9.58	3.61	16.89	4.62	150.99	6.57

CHAPTER 6

6. CONCLUSION

The comparison of the renewal cost and replacement cost of different products with variety of prices and parts declares that the cost of the renewal of the products is significantly higher than the replacement cost. This implies that the price of the spare parts is much more than the cost of the parent products and the ratio between the renewal cost and the replacement cost follows a certain pattern. The consistency of this ratio states that most OEMs implement cost-based or mark-up pricing to retrieve the final price of spare parts. The main disadvantages of cost-based pricing are under-pricing and over-pricing that leads to lower-than-average profitability, and ignoring competitiveness of the parts in the market. These factors are against the spare parts prices sustainability. This issue has been addressed as a competition-based pricing or strategic pricing by setting up the spare parts inventory games that are listed in Table 53.

Table 53: List of spare parts inventory games

	Game	**Players**	**Cooperation**
1	OEM Against Market	OEM & Market	Non-cooperative e
2	OEM Against Will-fitter	OEM, Will-fitter & Market	Non-cooperative
3	Evolutionay Spare Parts Inventory Game	OEM and Will-fitter	Cooperative/Non-cooperative
4	Cooperative Spare Parts Inventory Game	3-Manufacturer	Cooperative
5	Spare Parts‘ Price Leadership	OEM & Will-fitters	Non-cooperative

In the first game, Game Theory is applied to determine the sale price and the spare part stock level for an OEM who manufactures single-item spare parts, keeps them in the inventory with order-up-to level inventory policy and sells them to the market. This non-cooperative game has two players: the OEM and the market. In the aftermarket business, other than the OEM as an original manufacturer, there are other low cost manufacturers, known as will-fitters, and they compete with each other on their sale prices to absorb more customers. The OEM possesses the information of the original parts failure rates and can predict the allocated demand rates including: the upper bound intensity factor and the lower bound intensity factor with respect to its selected sale price. The market can be considered as an unreasoning entity whose strategic choices affect the payoff the OEM, but which has no interest in the outcome of the game. In this game there is no dominant level of inventory for the OEM, so the game has the mixed strategy solution. The solution of the mixed strategy provides the OEM's optimal sale price and the OEM's expected payoff distribution in relation to the market's expected demand. The OEM chooses his optimal level of inventory with respect to the probability of intensity factors that the market can choose among them. Furthermore, the comparison of the maximum attainable payoff and guaranteed payoff in the uncertain situation would justify the OEM's extra investment on the demand forecasting efforts.

The second game studies the spare parts inventory problem as an N-person non-zero-sum single-shot game. This non-cooperative game has three players: the OEM, the will-fitter and the market. The manufacturers can only forecast the market's base demand for the parts during the production horizon. The game has been modeled as a game against nature which means the manufacturers play against the market. In this game there is no dominant level of inventory for the OEM, so the game has the mixed strategy solution. The solution of the mixed strategy determines the strategy of the OEM to maximize his payoff in the aftermarket business. The strategies of the OEM are the determination of the level of inventory and sale prices in traditional manufacturing system. Furthermore, in another consideration, we implement the re-manufacturing processes in the inventory game in which the OEM has an ability to produce the whole or some part of his production out of the recycling process or re-manufacturing. In this environment, the solution of the game provides the re-manufacturing effort and the cost of collection of reusable products for the OEM. In other words, the theory of games in our spare parts inventory problem provides the expected payoff distribution in relation to the probability of the market's expected actions. The OEM chooses his optimal level of inventory, pricing strategy and re-manufacturing effort with respect to the probability of the demand distribution.

The third game investigates the spare parts inventory problem as an N-person, non-zero-sum and repeated game. This cooperative/non-cooperative game has two players: the OEM and his competitor known as the will-fitter. It has been assumed that the original product is in its normal or repetitive phase. In this phase, demands for spare parts are stochastic and repetitive which arrive as a Poisson process. The manufacturers are implementing the Newsvendor inventory policy to stock spare parts for the upcoming demands over the production horizon. Manufacturers must decide on their pricing strategy and respectively their level of inventory to optimize their payoff in the aftermarket. The pricing strategies have been investigated through repeated

Prisoners’ Dilemma game and ESS analysis where there is a transition from Prisoners’ Dilemma to Stag Hunt games. Moreover, the effect of the re-manufacturing is investigated as an option for the manufacturers to manufacture more sustainable parts and increase the profitability of the spare parts business. Based on the results that have been provided in previous sections we can present the following conclusions:

1. The minimum sale price that makes the spare parts inventory game profitable for the manufacturers is derived which is defined as the marginal cutting sale price.
2. The minimum cooperative sale price that establishes the spare parts inventory game as a Prisoners’ Dilemma is derived.
3. The maximum cooperative sale price is derived which states that manufacturers should never defect as long as there is a probability for the future game.
4. The results of the ESS state that the Nash Equilibrium of the Prisoners’ Dilemma spare parts inventory game is (DD), which means both players defect, and the equilibrium point is evolutionary stable (ES).
5. The increase of the sale price above the value of the maximum cooperative sale price changes the game type from Prisoners’ Dilemma to Stag Hunt game.
6. The Stag Hunt spare parts inventory game has two Nash Equilibriums and it has a mixed strategy solution. In other words, this game has two ES equilibriums (including cooperation and defect) and it has unstable mixed strategy equilibrium.

7. In Stag Hunt game, as sale price increases the chance of cooperation increases.
8. The optimal re-manufacturing effort is derived and implementation of the re-manufacturing states that manufacturers can reach to similar payoffs as the traditional manufacturing processes by inserting lower sale prices. In other words, implementation of the re-manufacturing can guarantee more sustainable parts both environmentally friendly wise and price wise while satisfying the expected payoff for the manufacturers.

The fourth game investigates the cooperation of the spare parts' manufacturers in a three-person (there are three manufacturers who can manufacture a substitutable spare part) cooperative game setup. In this game, manufacturers can decide to cooperate with each other on sale prices and acting in a centralized inventory system while there is a cost asserted as a binding agreement cost. Determination of decision-making on cooperation or defect depends on spare part sale price variation and the cost of binding agreement which are investigated as the Prisoners' Dilemma and Stag Hunt game. Two different methods of Shapley value and Bargaining set are implemented to allocate benefits of cooperation among cooperative manufacturers. Moreover, a centralized inventory configuration for manufacturers who decide to rely on a cooperative inventory system is designed and a comparison between inventory levels and cost of inventory for two different cases of centralized and decentralized inventory configuration is studied. To investigate the effect of the Green manufacturing on cost of inventory, it has been assumed that one of the manufacturers can implement re-manufacturing to produce spare parts and the role of re-manufacturing on payoffs of the manufacturers in cooperative inventory games is investigated while the optimal level of re-manufacturing effort is calculated. The results of the game can be listed as follows:

1. The variation of sale prices and cost of cooperation agreement changes the type of the inventory game from Prisoners' Dilemma to Stag Hunt game which changes the Nash Equilibrium of the game.
2. Our method first checks the non-emptiness of the core of the game. Then it determines whether manufacturers should cooperate or defect for given sale prices and variation of the cost of cooperation agreement.
3. The centralized inventory system configuration provides more profit for the manufacturers with less inventory level.
4. In the centralized configuration there will be no Prisoners' Dilemma game and the game stays as the Stag Hunt game.
5. Re-manufacturing improve the total profit of the manufacturers while the inventory level stays the same.

In the last game, the competition of the OEM and will-fitters in the aftermarket business during the time span between end-of-production and end-of-service of the original or parent product is investigated. Unlike the period during the product life cycle, there is no monopoly for the OEM to supply spare parts after the end-of-production cycle. In this period other competitors enter to the market and compete with the OEM to absorb more market demand share for the spares. Spare parts demand is intermittent and the demand arrival is considered as a Poisson process. The OEM can forecast the parent products failure rate or intensity factor that determines the expected overall demand for spare parts. The sale price affects the overall market demand. Once the OEM raises the price, some customers will leave the market and their demand is met from refurbished parts by third parties and the rest of customers' demands are allocated among the OEM and will-fitters. In this environment the OEM is considered as the dominant firm and will-fitters are considered as fringes. The interaction of the spare parts suppliers is studied as the competitive fringe game. In first scenario, both groups of manufacturers implement order-up-to level inventory policy to manage their spare parts stock levels that results in constant values marginal costs. Hence, the OEM enters the parts to the market with a price that makes the will-fitters to drop out of the market and leave all of the demand to the OEM. The potential supply of fringe is irrelevant and the OEM becomes a monopoly. In another scenario it has been assumed that the OEM still follows order-up-to level inventory policy and fringe firms supply spare parts to the market with a given quantity-price function or supply curve that is known by the OEM. The price leadership solution determines the optimal sale price and inventory level for the OEM.

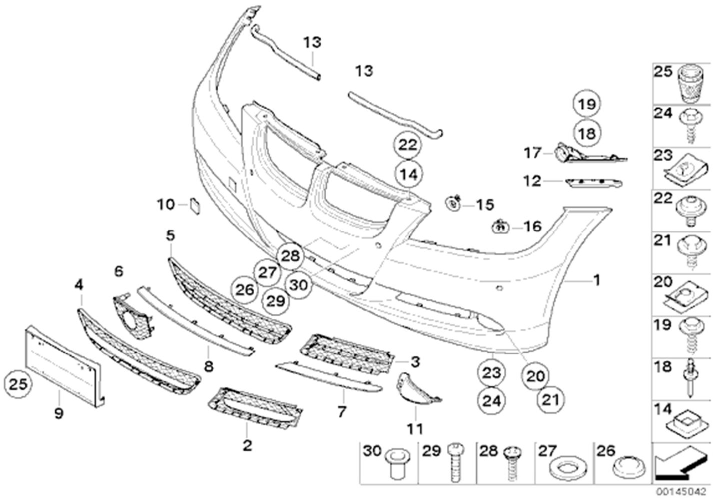
This research has introduced several game theoretical approaches to study OEM's decision-making on inventory levels, Green manufacturing and pricing strategies. The suggested strategic spare parts pricing methods factor in the customers' willingness to purchase the spare parts, the demand uncertainty, the market uncertainty, the competitiveness of the parts in the market, the stability of the cooperation or competition in price setting, the marginal costs of designing an agreement for cooperation, and the marginal cost of production and inventory. The consideration of aforementioned factors in spare parts price setting is a convincing reason for the

OEMs to replace cost-based pricing with strategic pricing to gain more profits in the aftermarket business. However, the ratio between the renewal cost and the replacement cost of the products can be distinguished as a factor to count the fairness of the pricing. Because of the high ratio for the selected products, it is evident that the spare parts pricing is unfair. Hence, it is possible to add this ratio to the price sustainability description and include it into strategic pricing formulations as a factor that affects the demand and supply curves.

APPENDIX

7. Data Acquisition

Online Database: The main source of gathering information is an unofficial online database which is RealOEM.com. Despite being an unofficial website, there is very accurate and up-to-date parts information in this website. It is possible to find a specific car on this website via two ways, one is selecting a car using its model and other detailed specification, and the other way is to search a car through its vehicle identification number (VIN) which is used in this study.



No.	Description	Supplement	Qty	From	Up To	Part Number	Price	Notes	Photo
01	PRIMED FRONT BUMPER TRIM		1			51117140859	$403.52		
	For vehicles with Headlight cleaning system	S502A=Yes							
01	PRIMED FRONT BUMPER TRIM		1			51117170052	$423.97		
	For vehicles with NATIONAL VERSION CANADA **and** Park Distance Control (PDC)	L838A=Yes S508A=Yes							
01	PRIMED FRONT BUMPER TRIM		1			51117170051	$423.97		
	For vehicles with NATIONAL VERSION CANADA **and** Headlight cleaning system **and** Park Distance Control (PDC)	L838A=Yes S502A=Yes S508A=Yes							

Figure 47: Subassembly diagram and corresponding parts list

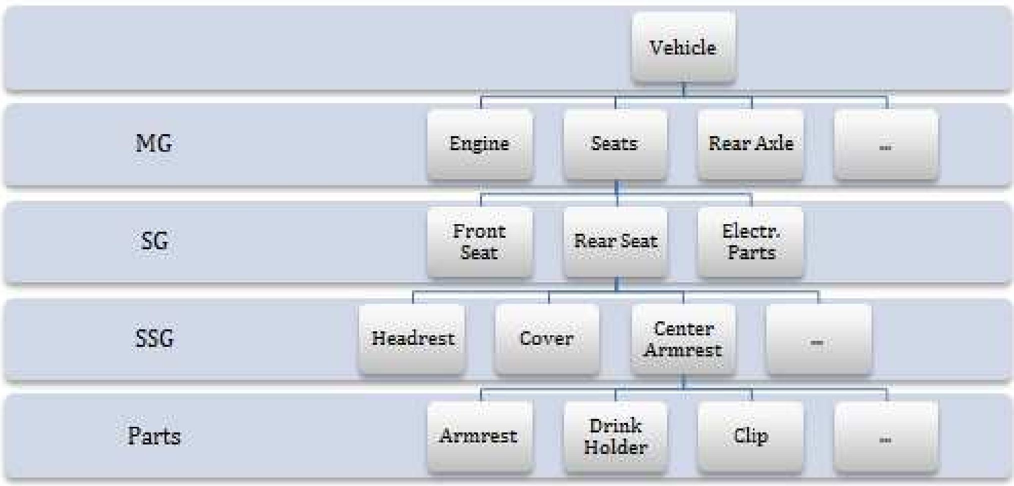
After the vehicle has been identified in the catalogue, the desired subassembly diagrams and parts lists can be accessed. However, this process does not include the identification of additional features in the vehicle such as seat heating or sunroof. These features have to be screened in the parts list manually. Figure 47 shows how subassembly diagrams and the corresponding parts lists are illustrated in the database. The website provides a diagram and corresponding catalogue with the following information for each part:

- Number (No.) – A number for identifying a part from the diagram in the list and vice versa;
- Description – Name of the part, e.g. "Support Fender Left";
- Supplement – Additional information about usage criteria. For example, if a part comes only in combination of a certain feature;
- Quantity (Qty) – The used quantity of this part in this subassembly;
- Production period (From, Up To) – Indicates in which time period a certain part has been used;
- Part Number – Unique serial number for every BMW part;
- Price;
- Notes;
- Photo;

STRUCTURE OF BMW PARTS LISTS: BMW uses a structure for arranging subassemblies. This structure shall be illustrated in this section. The whole vehicle is divided into Main Groups (MG) such as Engine, Transmission, Front Axle etc. These

MG are further divided into so called Sub Groups (SG) and Sub Sub Groups (SSG). This structure has been illustrated in Table 54.

Table 54: BMW subassembly structure



PRESENTATION OF THE VEHICLES: For this study, two models of BMW cars including 328i and X6 have been selected. The 328i is chosen by the criteria of highest market presence of BMW cars. The 3 series is the best-selling model of BMW in recent years. The X6 is chosen as a high price and luxury car among BMW series. The exact vehicles which have been chosen for this study are presented as the following:

1. Models: As it was mentioned two BMW models are selected and the detailed information of the vehicles is listed in Tables 55.

Table 55: BMW 328i Sedan & BMW X6 SUV model information

	BMW 328i Sedan (E90)	**BMW X6 3.5i (E71)**
Exterior	Jet Black	Mineral silver metallic (A14)
Interior	Leather Dakota Gray	Leather Nevada (LUSW)
Transmission	Automatic	Automatic
Fuel Type	Gas	Gas
Mileage (03/05/2012)	37,000	-
Production Date	04/17/2008	11/09/2008
VIN	WBAVA37588NL54270	5UXFG43529L222179

	BMW 328i Sedan (E90)	BMW X6 3.5i (E71)
Type Code	VA37	SC
Steering	Left	Left
Doors	4	4
Engine	N52K	N54

2. Features: The detailed features of two vehicles have been identified and listed in Tables 56.

Table 56: BMW 328i Sedan & BMW X6 SUV features information

BMW 328i		BMW X6	
Code	**Description**	**Code**	**Description**
S205A	Automatic transmission	**S1CAA**	Dummy-SALAPA
S249A	Multifunction steering wheel	**S248A**	Steering wheel heater
S2BGA	BMW alloy wheel, double spoke 161	**S2VBA**	Tyre pressure control (TPC)
S2VBA	Tire pressure control (TPC)	**S316A**	automatic trunk lid mechanism
S2XAA	Sport leather wheel + shift paddles	**S319A**	Integrated universal remote control
S319A	Integrated universal remote control	**S322A**	Comfort access
S403A	Glass roof, electrical	**S3AGA**	Reversing camera
S430A	Interior/outside mirror with auto dip	**S430A**	Interior/outside mirror with auto dip
S431A	Interior mirror with automatic-dip	**S431A**	Interior mirror with automatic-dip
S441A	Smoker package	**S441A**	Smoker package
S459A	Seat adjuster, electric, with memory	**S459A**	Seat adjuster, electric, with memory
S465A	Through-loading system	**S464A**	Ski bag
…	…	**…**	…

PROCEDURE FOR COLLECTING DATA: This section provides a quick overview about the procedure that has been developed and executed for downloading the information of thousands parts from the database while maintaining a uniform data structure.

1. Downloading Data: Before the actual downloading process was started, all relevant MGs, SGs and SSGs were identified. This was necessary since

not all of them are included in the vehicles which were analyzed in this study. By using a VBA Excel program, it was possible to download all parts lists of a MG into an Excel file. In these individual files the first sheet represents the overall results for this MG. This sheet is followed by sheets for every SG containing all SSGs accordingly.

2. Revising Data: As mentioned before, after the data had been downloaded from the database, it had to be revised. The following measurements have been performed:
    - Deleting irrelevant parts such as:
        - Parts that do not fit the production period of the vehicle;
        - Parts that only come in combination with features which are not included in the vehicle;
    - Sample comparisons of prices with different websites;
    - Entering missing prices by searching the part number on websites such as:
        - http://parts.bmwofsouthatlanta.com/
        - http://www.ecstuning.com/
        - http://www.online-teile.com/bmw/
    - Entering missing quantities;

BIBLIOGRAPHY

Abad, P. L. (1988). Determining Optimal Selling Price and Lot Size When the Supplier Offers All-Unit Quantity Discounts*. Decision Sciences, 19(3), 622–634. Wiley Online Library.

Alfredsson, P. & Verrijdt, J. (1999). Modeling emergency supply flexibility in a two-echelon inventory system. Management Science, 45(10), 1416–1431. INFORMS.

Allen, S. G. & D'Esopo, D. A. (1968). An ordering policy for repairable stock items. Operations Research, 16(3), 669–674. INFORMS.

Anily, S. & Haviv, M. (2007). The cost allocation problem for the first order interaction joint replenishment model. Operations Research, 55(2), 292–302. INFORMS.

Anupindi, R. & Bassok, Y. (1999). Centralization of stocks: Retailers vs. manufacturer. Management Science, 45(2), 178–191. INFORMS.

Anupindi, R., Bassok, Y. & Zemel, E. (2001). A general framework for the study of decentralized distribution systems. Manufacturing \& Service Operations Management, 3(4), 349–368. INFORMS.

Aronis, K. P., Magou, I., Dekker, R. & Tagaras, G. (2004). Inventory control of spare parts using a Bayesian approach: A case study. European Journal of Operational Research, 154(3), 730–739. Elsevier.

Ashayeri, J., Heuts, R., Jansen, A. & Szczerba, B. (1996). Inventory management of repairable service parts for personal computers: A case study. International

Journal of Operations \& Production Management, 16(12), 74–97. MCB UP Ltd.

Audy, J. F., D'Amours, S. & Rönnqvist, M. (2012). An empirical study on coalition formation and cost/savings allocation. International Journal of Production Economics, 136(1), 13–27. Elsevier.

Aumann, R. J. & Maschler, M. (1961). The bargaining set for cooperative games. Defense Technical Information Center.

Avsar, Z. M. & Baykal-Gürsoy, M. (2002). Inventory control under substitutable demand: A stochastic game application. Naval Research Logistics (NRL), 49(4), 359–375. Wiley Online Library.

Axsäter, S. (1990). Modelling emergency lateral transshipments in inventory systems. Management Science, 36(11), 1329–1338. INFORMS.

Bailey, G. & Helms, M. (2007). MRO inventory reduction—challenges and management: a case study of the Tennessee Valley Authority. Production Planning and Control, 18(3), 261–270. Taylor \& Francis.

Beckenkamp, M. (2008). Playing strategically against nature? Decisions viewed from a game-theoretic frame.

Ben-Daya, M., Duffuaa, S. O. & Raouf, A. (2000). Maintenance, modeling, and optimization. Kluwer Academic Pub.

Bian, J., Guo, L., Yang, Y. & Wang, N. (2013). Optimizing Spare Parts Inventory for Time-Varying Task. CHEMICAL ENGINEERING, 33.

Botter, R. & Fortuin, L. (2000). Stocking strategy for service parts-a case study. International Journal of Operations \& Production Management, 20(6), 656–674. MCB UP Ltd.

Boylan, J., Syntetos, A. & Karakostas, G. (2006). Classification for forecasting and stock control: a case study. Journal of the Operational Research Society, 59(4), 473–481. Nature Publishing Group.

Bridgman, M. S. & Mount-Campbell, C. A. (1993). Determining the number of spares in an inventory/repair system which supports equipment with scheduled usage. International journal of production economics, 30, 501–518. Elsevier.

Bruggeman, W. & Van Dierdonck, R. (1985). Maintenance resource planning an integrative approach. Engineering Costs and Production Economics, 9(1-3), 147–154. Elsevier.

Burgin, T. (1975). The gamma distribution and inventory control. Operational Research Quarterly, 507–525. JSTOR.

Burgin, T. & Wild, A. (1967). Stock control-experience and usable theory. OR, 35–52. JSTOR.

Cachon, G. P. & Netessine, S. (2006). Game theory in supply chain analysis. Tutorials in Operations Research: Models, Methods, and Applications for Innovative Decision Making. Informs.

Cachon, Gerard P. (2003). Supply chain coordination with contracts. Handbooks in operations research and management science, 11, 227–339. Elsevier.

Chan, L. M., Shen, Z. M., Simchi-Levi, D. & Swann, J. L. (2004). Coordination of pricing and inventory decisions: a survey and classification. International

Series in Operations Research and Management Science, 335–392. Kluwer Academic Publishers.

Chang, P.-L., Chou, Y.-C. & Huang, M.-G. (2005). A (r, r, Q) inventory model for spare parts involving equipment criticality. International Journal of Production Economics, 97(1), 66–74. Elsevier.

Chen, F. Y., Yan, H. & Yao, L. (2004). A newsvendor pricing game. Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on, 34(4), 450–456. IEEE.

Chen, X. & Zhang, J. (2007). Duality approaches to economic lot-sizing games. Operations Management Working Papers Series, Vol., pp.-.

Cheng, T. (1990). An EOQ model with pricing consideration. Computers \& Industrial Engineering, 18(4), 529–534. Elsevier.

Chinchuluun, A., Karakitsiou, A. & Mavrommati, A. (2008). Game theory models and their applications in inventory management and supply chain. Pareto Optimality, Game Theory And Equilibria, 833–865. Springer.

Cobbaert, K. & Van Oudheusden, D. (1996). Inventory models for fast moving spare parts subject to “sudden death” obsolescence. International Journal of Production Economics, 44(3), 239–248. Elsevier.

Cohen, M. A., Agrawal, N. & Agrawal, V. (2006). Winning in the aftermarket. Harvard business review, 84(5), 129.

Cohen, M. A., Kleindorfer, P. R. & Lee, H. L. (1986). Optimal stocking policies for low usage items in multi-echelon inventory systems. Naval Research Logistics Quarterly, 33(1), 17–38. Wiley Online Library.

Cohen, M. A., Kleindorfer, P. R., Lee, H. L. & Pyke, D. F. (1992). Multi-item service constrained (s, S) policies for spare parts logistics systems. Naval Research Logistics (NRL), 39(4), 561–577. Wiley Online Library.

Cohen, M. A. & Lee, H. L. (1990). Out of touch with customer needs? Spare parts and after sales service. Sloan management review, 31(2), 55–66.

Cohen, M. A., Zheng, Y. S. & Agrawal, V. (1997). Service parts logistics: a benchmark analysis. IIE transactions, 29(8), 627–639. Springer.

Cohen, M., Kamesam, P. V., Kleindorfer, P., Lee, H. & Tekerian, A. (1990). Optimizer: IBM's multi-echelon inventory system for managing service logistics. Interfaces, 20(1), 65–82. INFORMS.

Cohen, Morris A, Kleindorfer, P. R. & Lee, H. L. (1989). Near-optimal service constrained stocking policies for spare parts. Operations Research, 37(1), 104–117. INFORMS.

Croston, J. (1972). Forecasting and stock control for intermittent demands. Operational Research Quarterly, 289–303. JSTOR.

Croston, J. (1974). Stock levels for slow-moving items. Journal of the Operational Research Society, 25(1), 123–130. Nature Publishing Group.

Cullbrand, M. & Levén, L. (2012). Spare parts pricing-Setting the right prices for sustainable profit at Atlet. Chalmers University of Technology.

Dai, Y., Chao, X., Fang, S. C. & Nuttle, H. L. W. (2005). Game theoretic analysis of a distribution system with customer market search. Annals of Operations Research, 135(1), 223–228. Springer.

Davis, M. D. & Maschler, M. (1962). Existence of stable payoff configurations for cooperative games. Princeton University, Econometric Research Program.

Dennis, M. J. & Kambil, A. (2003). SERVICE MANAGEMENT: BUILDING PROFITS AFTER THE SALE. SUPPLY CHAIN MANAGEMENT REVIEW, V. 7, NO. 3 (JAN./FEB. 2003), P. 42-48: ILL.

Diaz, A. (2003). Modelling approaches to optimise spares in multi-echelon systems. International Journal of Logistics, 6(1-2), 51–62. Taylor \& Francis.

Diaz, A. & Fu, M. C. (1997). Models for multi-echelon repairable item inventory systems with limited repair capacity. European Journal of Operational Research, 97(3), 480–492. Elsevier.

Docters, R. G. (2003). Winning the profit game: smarter pricing, smarter branding. McGraw Hill Professional.

Dror, & Hartman. (2007). Shipment consolidation: who pays for it and how much? Management Science, 53(1), 78–87. INFORMS.

Dror, M. & Hartman, B. C. (2010). Survey of cooperative inventory games and extensions. Journal of the Operational Research Society, 62(4), 565–580. Nature Publishing Group.

Duchessi, P., Tayi, G. K. & Levy, J. B. (1988). A conceptual approach for managing of spare parts. International Journal of Physical Distribution \& Logistics Management, 18(5), 8–15. MCB UP Ltd.

Eaves, A. & Kingsman, B. (2004). Forecasting for the ordering and stock-holding of spare parts. Journal of the Operational Research Society, 55(4), 431–437. Nature Publishing Group.

Federgruen, A. & Heching, A. (1999). Combined pricing and inventory control under uncertainty. Operations research, 47(3), 454–475. INFORMS.

Federgruen, A. & Zheng, Y.-S. (1992). An efficient algorithm for computing an optimal (r, Q) policy in continuous review stochastic inventory systems. Operations research, 40(4), 808–813. INFORMS.

Feeney, G. & Sherbrooke, C. (1964). The (sl, s) Inventory Policy Under Compound Poisson Demand: A Theory of Recoverable Item Stockage, The Rand Corporation.

Fiestras-Janeiro, M. G., Garcia-Jurado, I., Meca, A. & Mosquera, M. A. (2011). Cooperative game theory and inventory management. European Journal of Operational Research, 210(3), 459–466. Elsevier.

Fleischmann, M., Van Nunen, J. A. E. E. & Gräve, B. (2003). Integrating closed-loop supply chains and spare-parts management at IBM. Interfaces, 33(6), 44–56. INFORMS.

Flores, B. E. & Whybark, D. C. (1987). Implementing multiple criteria ABC analysis. Journal of Operations Management, 7(1), 79–85. Elsevier.

Foote, B. (1995). On the implementation of a control-based forecasting system for aircraft spare parts procurement. IIE transactions, 27(2), 210–216. Taylor \& Francis.

Fortuin, L. (1980). The all-time requirement of spare parts for service after sales—theoretical analysis and practical results. International Journal of Operations \& Production Management, 1(1), 59–70. MCB UP Ltd.

Fortuin, L. & Martin, H. (1999). Control of service parts. International Journal of Operations \& Production Management, 19(9), 950–971. MCB UP Ltd.

Friedman, D. (1991). Evolutionary games in economics. Econometrica: Journal of the Econometric Society, 637–666. JSTOR.

Friend, J. (1960). Stock control with random opportunities for replenishment. OR, 130–136. JSTOR.

Frisk, M., Göthe-Lundgren, M., Jörnsten, K. & Rönnqvist, M. (2010). Cost allocation in collaborative forest transportation. European Journal of Operational Research, 205(2), 448–458. Elsevier.

Gajpal, P. P., Ganesh, L. & Rajendran, C. (1994). Criticality analysis of spare parts using the analytic hierarchy process. International Journal of Production Economics, 35(1), 293–297. Elsevier.

Gallagher, D. J. (1969). Two periodic review inventory models with backorders and stuttering Poisson demands. AIIE Transactions, 1(2), 164–171. Taylor \& Francis.

Gallagher, T., Mitchke, M. D. & Rogers, M. C. (2005). Profiting from spare parts. The McKinsey Quarterly, 2. HarperBusiness Press.

Gerchak, Y. & Gupta, D. (1991). On apportioning costs to customers in centralized continuous review inventory systems. Journal of Operations Management, 10(4), 546–551. Elsevier.

Ghobbar, A. A. & Friend, C. H. (2003). Evaluation of forecasting methods for intermittent parts demand in the field of aviation: a predictive model. Computers \& Operations Research, 30(14), 2097–2114. Elsevier.

Gomes, A. V. P. & Wanke, P. (2008). A model for spare parts stock management using Markov chains. Gestão \& Produção, 15(1), 57–72. SciELO Brasil.

Grahovac, J. & Chakravarty, A. (2001). Sharing and lateral transshipment of inventory in a supply chain with expensive low-demand items. Management Science, 47(4), 579–594. INFORMS.

Granot, D. & Sosic, G. (2003). A three-Stage model for a decentralized distribution system of retailers. Operations Research, 51(5), 771–784. INFORMS.

Gross, D. & Ince, J. F. (1978). Spares provisioning for repairable items: Cyclic queues in light traffic. AIIE Transactions, 10(3), 307–314. Taylor \& Francis.

Guajardo, M. & Rönnqvist, M. (2012). RISK POOLING AND COST ALLOCATION IN SPARE PARTS INVENTORIES.

Guardiola, L. A., Meca, A. & Puerto, J. (2009). Production-inventory games: A new class of totally balanced combinatorial optimization games. Games and Economic Behavior, 65(1), 205–219. Elsevier.

Gupta, U. & Srinivasa Rao, T. (1996). On the M/G/1 machine interference model with spares. European journal of operational research, 89(1), 164–171. Elsevier.

Gutierrez, R. S., Solis, A. O. & Mukhopadhyay, S. (2008). Lumpy demand forecasting using neural networks. International Journal of Production Economics, 111(2), 409–420. Elsevier.

Haneveld, W. K. K. & Teunter, R. (1997). Optimal provisioning strategies for slow moving spare parts with small lead times. Journal of the Operational Research Society, 184–194. JSTOR.

Hart, S. & Mas-Colell, A. (1997). Cooperation: game-theoretic approaches. Springer.

Hartman, B. C. & Dror, M. (1996). Cost allocation in continuous-review inventory models. Naval Research Logistics (NRL), 43(4), 549–561. Wiley Online Library.

Hartman, Bruce C. (1994). Cooperative games and inventory cost allocation. The University of Arizona.

Hartman, Bruce C, Dror, M. & Shaked, M. (2000). Cores of inventory centralization games. Games and Economic Behavior, 31(1), 26–49. Elsevier.

Hegde, G. & Karmarkar, U. S. (1993). Engineering costs and customer costs in designing product support. Naval Research Logistics (NRL), 40(3), 415–423. Wiley Online Library.

Heuvel, W, & van den P, B. (2007). H Hamers (2007). Economic lot-sizing games. European Journal of Operational Research, 176, 1117–1130.

Heuvel, Wilco van den, Borm, P. & Hamers, H. (2007). Economic lot-sizing games. European Journal of Operational Research, 176(2), 1117–1130. Elsevier.

HILL, R. M. (1999). Bayesian decision-making in inventory modelling. IMA Journal of Management Mathematics, 10(2), 147–163. IMA.

Hill, R. M., Omar, M. & Smith, D. K. (1999). Stock replenishment policies for a stochastic exponentially-declining demand process. European Journal of Operational Research, 116(2), 374–388. Elsevier.

Hinterhuber, A. (2004). Towards value-based pricing—An integrative framework for decision making. Industrial Marketing Management, 33(8), 765–778. Elsevier.

Hinterhuber, A. (2008). Customer value-based pricing strategies: why companies resist. Journal of business strategy, 29(4), 41–50. Emerald Group Publishing Limited.

Hopp, W. J. & Spearman, M. L. (2008). Factory physics. McGraw-Hill Irwin Irwin.

Hopp, Wallace J, Spearman, M. L. & Zhang, R. Q. (1997). Easily implementable inventory control policies. Operations Research, 45(3), 327–340. INFORMS.

Hua, Z., Zhang, B., Yang, J. & Tan, D. (2006). A new approach of forecasting intermittent demand for spare parts inventories in the process industries. Journal of the Operational Research Society, 58(1), 52–61. Nature Publishing Group.

Huiskonen, J. (2001). Maintenance spare parts logistics: Special characteristics and strategic choices. International journal of production economics, 71(1), 125–133. Elsevier.

Inderfurth, K. & Mukherjee, K. (2008). Decision support for spare parts acquisition in post product life cycle. Central European Journal of Operations Research, 16(1), 17–42. Springer.

Inger, R., Braithwaite, A. & Christopher, M. (1995). Creating a manufacturing environment that is in harmony with the market—the “how”of supply chain management. Production Planning \& Control, 6(3), 246–257. Taylor \& Francis.

Van Jaarsveld, W. & Dekker, R. (2011). Estimating obsolescence risk from demand data to enhance inventory control—A case study. International Journal of Production Economics, 133(1), 423–431. Elsevier.

Jin, T. & Liao, H. (2009). Spare parts inventory control considering stochastic growth of an installed base. Computers \& Industrial Engineering, 56(1), 452–460. Elsevier.

Johnston, F. (1980). An interactive stock control system with a strategic management role. Journal of the Operational Research Society, 1069–1084. JSTOR.

José Roberto do Regoa, M. A. de M. (2011). Spare parts inventory control: a literature review. International Journal of production economics, 21(4), 656–666.

Kaio, N. & Osaki, S. (1981). Optimum planned maintenance policies with lead time. Reliability, IEEE Transactions on, 30(1), 79–79. IEEE.

Kalchschmidt, M., Zotteri, G. & Verganti, R. (2003). Inventory management in a multi-echelon spare parts supply chain. International journal of production economics, 81, 397–413. Elsevier.

Karmarkar, U. S. (1981). Policy Structure in Multi-state Production-Inventory Problems: An Application of Convex Analysis. Management Science.

Karsten, F., Slikker, M., Houtum, G. G. J. & others. (2006). Cost allocation and cooperation stability in spare parts inventory pooling. Technische Universiteit Eindhoven.

Karsten, F., Slikker, M. & van Houtum, G. J. (2012). Inventory pooling games for expensive, low-demand spare parts. Naval Research Logistics (NRL). Wiley Online Library.

Karsten, FJP, Slikker, M. & van Houtum, G. J. (2009). Spare parts inventory pooling games. Eindhoven, The Netherlands: School of Industrial Engineering, Eindhoven University of Technology.

Kennedy, W., Wayne Patterson, J. & Fredendall, L. D. (2002). An overview of recent literature on spare parts inventories. International Journal of production economics, 76(2), 201–215. Elsevier.

Kim, B. & Park, S. (2008). Optimal pricing, EOL (end of life) warranty, and spare parts manufacturing strategy amid product transition. European Journal of Operational Research, 188(3), 723–745. Elsevier.

Kohlas, J. & Pasquier, J. (1981). Optimization of spare parts for hierarchicall decomposable systems. European Journal of Operational Research, 8(3), 294–300. Elsevier.

Kotler, P. & Armstrong, G. (2010). Principles of marketing. Pearson Education.

Kranenburg, A. & Van Houtum, G. (2009). A new partial pooling structure for spare parts networks. European Journal of Operational Research, 199(3), 908–921. Elsevier.

Krever, M., Wunderink, S., Dekker, R. & Schorr, B. (2005). Inventory control based on advanced probability theory, an application. European Journal of Operational Research, 162(2), 342–358. Elsevier.

Kukreja, A., Schmidt, C. P. & Miller, D. M. (2001). Stocking decisions for low-usage items in a multilocation inventory system. Management Science, 47(10), 1371–1383. INFORMS.

Kumar, S. (2004). Parts Management Models and Applications: A Supply Chain System Integration Perspective. Springer.

Kunreuther, H. & Richard, J. F. (1971). Optimal pricing and inventory decisions for non-seasonal items. Econometrica: Journal of the Econometric Society, 173–175. JSTOR.

Kwan, H. W. (1991). On the demand distributions of slow-moving items.

Lee, H. L. (1987). A multi-echelon inventory model for repairable items with emergency lateral transshipments. Management Science, 33(10), 1302–1316. INFORMS.

Lee, H. L., Padmanabhan, V. & Whang, S. (1997). The bullwhip effect in supply chains1. Sloan management review, 38(3), 93–102.

Leng, M. & Parlar, M. (2005). GAME THEORETIC APPLICATIONS IN SUPPLY CHAIN MANAGEMENT: A REVIEW12.

Li, S. & Kuo, X. (2008). The inventory management system for automobile spare parts in a central warehouse. Expert Systems with Applications, 34(2), 1144–1153. Elsevier.

Li, S. X., Huang, Z. & Ashley, A. (1996). Improving buyer-seller system cooperation through inventory control. International Journal of Production Economics, 43(1), 37–46. Elsevier.

Li, Susan X, & Huang, Z. (1995). Managing buyer-seller system cooperation with quantity discount considerations. Computers \& operations research, 22(9), 947–958. Elsevier.

Li, Susan X, Huang, Z. & Ashley, A. (1995). Seller—buyer System Co-operation in a Monopolistic Market. Journal of the Operational Research Society, 46(12), 1456–1470. Nature Publishing Group.

Liao, H., Wang, P., Jin, T. & Repaka, S. (2008). Spare parts management considering new sales. Reliability and Maintainability Symposium, 2008. RAMS 2008. Annual (pp. 502–507).

Lonardo, P., Anghinolfi, D., Paolucci, M. & Tonelli, F. (2008). A stochastic linear programming approach for service parts optimization. CIRP Annals-Manufacturing Technology, 57(1), 441–444. Elsevier.

Love, S. F. (1979). Inventory control. McGraw-Hill New York.

Mamer, J. W. & Smith, S. A. (1982). Optimizing field repair kits based on job completion rate. Management Science, 28(11), 1328–1333. INFORMS.

Marn, M. V. & Rosiello, R. L. (1992). Managing price, gaining profit. Harvard Business Review, 70(5), 84–94. Kluwer.

Maynard Smith, J. (1974). The theory of games and the evolution of animal conflicts. Journal of theoretical biology, 47(1), 209–221. Elsevier.

Meca, A. & Timmer, J. (2007). Supply chain collaboration. CIO (Centro de Investigación Operativa), Universidad Miguel Hernández de Elche.

Meca, A., Timmer, J., García-Jurado, I. & Borm, P. (2004). Inventory games. European Journal of Operational Research, 156(1), 127–139. Elsevier.

Michael, J. A. & Derek, R. A. (1996). Joint optimization of maintenance and inventory policies for a simple system. IIE transactions, 28(5), 415–424. Taylor \& Francis.

Mileff, P. & Nehéz, K. (2006). Applying Game Theory in Invenory Control Problems.

Montrucchio, L. & Scarsini, M. (2007). Large newsvendor games. Games and Economic Behavior, 58(2), 316–337. Elsevier.

Moore, R. (1996). Establishing an inventory management program. Plant Engineering, 50(3), 113–117. [Barrington, Ill., etc., Technical Pub. Co.].

Muckstadt, J. A. (1973). A model for a multi-item, multi-echelon, multi-indenture inventory system. Management science, 20(4-Part-I), 472–481. INFORMS.

Muckstadt, J. A. (2004). Analysis and algorithms for service parts supply chains. Springer.

Müller, A., Scarsini, M. & Shaked, M. (2002). The newsvendor game has a nonempty core. Games and Economic Behavior, 38(1), 118–126. Elsevier.

Naddor, E. (1982). Inventory systems. Krieger.

Nagle, T. T. & Holden, R. K. (2002). The strategy and tactics of pricing—a guide to profitable decision marketing. Prentice Hall/New York.

Nahmias, S. (1981). Managing repairable item inventory systems: a review. TIMS Studies in the Management Sciences, 16, 253–277.

Von Neumann, J. & Morgenstern, O. (1953). Theory of Games and Economic Behavior (Commemorative Edition). Princeton university press.

Noble, P. M. & Gruca, T. S. (1999). Industrial pricing: Theory and managerial practice. Marketing Science, 18(3), 435–454. INFORMS.

Noeuvéglise, S. & Chevenement, N. (2011). How Accenture Parts Optimization Services can boost operating profit and brand image. Accenture.

Özen, U., Fransoo, J., Norde, H. & Slikker, M. (2008). Cooperation between multiple newsvendors with warehouses. Manufacturing \& Service Operations Management, 10(2), 311–324. INFORMS.

Papadopoulos, H. (1996). A field service support system using a queueing network model and the priority MVA algorithm. Omega, 24(2), 195–203. Elsevier.

Parker, J. (1999). TNT, Fiat strengthen ties. Traffic World, 259(7), 18.

Parlar, M. (1988). Game theoretic analysis of the substitutable product inventory problem with random demands. Naval Research Logistics (NRL), 35(3), 397–409. Wiley Online Library.

Petrovi'c, D. & Petrovi'c, R. (1992). SPARTA II: further development in an expert system for advising on stocks of spare parts. International journal of production economics, 24(3), 291–300. Elsevier.

Petrovic, D., Petrovic, R., Senborn, A. & Vujoševi'c, M. (1990). A microcomputer expert system for advising on stocks in spare parts inventory systems. Engineering Costs and Production Economics, 19(1), 365–370.

Pierskalla, W. P. & Voelker, J. A. (1976). A survey of maintenance models: the control and surveillance of deteriorating systems. Naval Research Logistics Quarterly, 23(3), 353–388. Wiley Online Library.

Popovi'c, J. B. (1987). Decision making on stock levels in cases of uncertain demand rate. European Journal of Operational Research, 32(2), 276–290. Elsevier.

Prof. Maurizio Faccio, F. S. A. C. (2010). Forecasting Methods for Spare Parts Demand. Thessis.

Quenouille, M. H. (1949). A relation between the logarithmic, Poisson, and negative binomial series. Biometrics, 5(2), 162–164. JSTOR.

Rapoport, A. (1970). N-person game theory: Concepts and applications. DoverPublications. com.

Robert, H. H. (1980). The distribution of spare parts. International Journal of Production Research, 18(6), 665–675. Taylor \& Francis.

Robinson, L. W. (1993). A comment on Gerchak and Gupta's "On apportioning costs to customers in centralized continuous review inventory systems." Journal of Operations Management, 11(1), 99–102. Elsevier.

Roundy, R. (1985). 98\%-effective integer-ratio lot-sizing for one-warehouse multi-retailer systems. Management science, 31(11), 1416–1430. INFORMS.

Roundy, R. (1986). A 98\%-effective lot-sizing rule for a multi-product, multi-Stage production/inventory system. Mathematics of Operations Research, 11(4), 699–727. INFORMS.

Rustenburg, W., van Houtum, G. J. & Zijm, W. (2001). Spare parts management at complex technology-based organizations: An agenda for research. International Journal of Production Economics, 71(1), 177–193. Elsevier.

Sandholm, W. & Dokumaci, E. (2007). Dynamo: Phase diagrams for evolutionary dynamics. software suite.

Santoro, M. C., Freire, G. & others. (2008). Análise comparativa entre modelos de estoque. Revista Produção, 18(1), 89–98. SciELO Brasil.

Savaskan, R. C., Bhattacharya, S. & Van Wassenhove, L. N. (2004). Closed-loop supply chain models with product remanufacturing. Management science, 50(2), 239–252. INFORMS.

Savaskan, R. C. & Van Wassenhove, L. N. (2006). Reverse channel design: the case of competing retailers. Management Science, 52(1), 1–14. INFORMS.

Scherer, F. M. & Ross, D. (1970). Industrial market structure and economic performance.

Scudder, G. D. (1984). Priority scheduling and spares stocking policies for a repair shop: the multiple failure case. Management Science, 30(6), 739–749. INFORMS.

Seidel, K. (1983). Optimization of the availability of complex manufacturing systems—methods and examples. THE INTERNATIONAL JOURNAL OF PRODUCTION RESEARCH, 21(2), 153–162. Taylor \& Francis.

Shapley, L. S. (1952). A value for n-person games.

Sherbrooke, C. C. (1992). Multiechelon inventory systems with lateral supply. Naval Research Logistics (NRL), 39(1), 29–40. Wiley Online Library.

Sherbrooke, C. C. (2004). Optimal inventory modeling of systems: multi-echelon techniques (Vol. 72). Springer.

Shubik, M. (1981). Game theory models and methods in political economy. Handbook of Mathematical Economics, 1, 285–330. Elsevier.

Shubik, M. (1984). A game-theoretic approach to political economy (Vol. 2). MiT Press Cambridge, MA.

Shubik, M. (2006). Game theory in the social sciences: Concepts and solutions. $\$MIT Press$\$.

Shubik, M. & Levitan, R. (1980). Market structure and behavior. Harvard University Press Cambridge, MA.

Shubik, Martin. (1970). Game theory, behavior, and the paradox of the Prisoners' dilemma: Three solutions. Journal of Conflict Resolution, 14(2), 181–193. Sage Publications.

Shubik, Martin. (1985). Game Theory (Guillermo Owen). SIAM Review, 27(2), 303–305. SIAM.

Silver, E. A. (1972). Inventory allocation among an assembly and its repairable subassemblies. Naval Research Logistics Quarterly, 19(2), 261–280. Wiley Online Library.

Silver, E. A., Pyke, D. F., Peterson, R. & others. (1998). Inventory management and production planning and scheduling (Vol. 3). Wiley New York.

Simpson, V. P. (1978). Optimum solution structure for a repairable inventory problem. Operations Research, 26(2), 270–281. INFORMS.

Slikker, M., Fransoo, J. & Wouters, M. (2001). Joint ordering in multiple news-vendor problems: a game-theoretical approach. BETA Research School for Operations, Management and Logistics.

Slikker, M., Fransoo, J. & Wouters, M. (2005). Cooperation between multiple news-vendors with transshipments. European Journal of Operational Research, 167(2), 370–380. Elsevier.

Smith, J Maynard, & Price, G. (1973). lhe Logic of Animal Conflict. Nature, 246, 15.

Smith, John Maynard. (1993). Evolution and the Theory of Games. Springer.

Spengler, T. & Schröter, M. (2003). Strategic management of spare parts in closed-loop supply chains—a system dynamics approach. Interfaces, 33(6), 7–17. INFORMS.

Straffin, P. D. (1993). Game theory and strategy (Vol. 36). MAA.

Strijbosch, L., Heuts, R. & Van der Schoot, E. (2000). A combined forecast-inventory control procedure for spare parts. Journal of the Operational Research Society, 1184–1192. JSTOR.

Suomala, P., Sievänen, M. & Paranko, J. (2002). The effects of customization on spare part business: a case study in the metal industry. International Journal of Production Economics, 79(1), 57–66. Elsevier.

Syntetos, A. A. & Boylan, J. E. (2001). On the bias of intermittent demand estimates. International Journal of Production Economics, 71(1), 457–466. Elsevier.

Syntetos, Aris A, Babai, M. Z. & Altay, N. (2012). On the demand distributions of spare parts. International Journal of Production Research, 50(8), 2101–2117. Taylor \& Francis.

Taylor, P. D. & Jonker, L. B. (1978). Evolutionary stable strategies and game dynamics. Mathematical biosciences, 40(1), 145–156. Elsevier.

Tedone, M. J. (1989). Repairable part management. Interfaces, 19(4), 61–68. INFORMS.

Teunter, R. & Duncan, L. (2008). Forecasting intermittent demand: a comparative study. Journal of the Operational Research Society, 60(3), 321–329. Nature Publishing Group.

Teunter, R. H. & Klein Haneveld, W. K. (2002). Inventory control of service parts in the final phase. European Journal of Operational Research, 137(3), 497–511. Elsevier.

Thonemann, U. W., Brown, A. O. & Hausman, W. H. (2002). Easy quantification of improved spare parts inventory policies. Management Science, 48(9), 1213–1225. INFORMS.

Tijs, S. H. & Driessen, T. S. H. (1986). Game theory and cost allocation problems. Management Science, 32(8), 1015–1028. INFORMS.

Trivedi, M. (1998). Distribution channels: An extension of exclusive retailership. Management science, 44(7), 896–909. INFORMS.

Vereecke, A. & Verstraeten, P. (1994). An inventory management model for an inventory consisting of lumpy items, slow movers and fast movers. International Journal of Production Economics, 35(1), 379–389. Elsevier.

Vigoroso, W. M. (2005). Maximizing Profitability with Optimized Service Parts Pricing - A Benchmark Study. Report.

Vrat, S. N. P. (1984). A location-allocation model for a two-echelon repair-inventory system. IIE transactions, 16(3), 222–228. Taylor \& Francis.

Vujossevi'c, M., Petrovi'c, R. & Senborn, A. (1990). Spare parts inventory planning for a redundant system subject to a phased mission. Engineering Costs and Production Economics, 19(1), 385–389. Elsevier.

Wagner, S. & Lindemann, E. (2008). A case study-based analysis of spare parts management in the engineering industry. Production Planning and Control, 19(4), 397–407. Taylor \& Francis.

Wang, Q. (1991). Game theory models and their applications in some inventory control and new product management problems.

Wang, W. & Syntetos, A. A. (2011). Spare parts demand: Linking forecasting to equipment maintenance. Transportation Research Part E: Logistics and Transportation Review, 47(6), 1194–1209. Elsevier.

Ward, J. (1978). Determining reorder points when demand is lumpy. Management Science, 24(6), 623–632. INFORMS.

Watson, R. B. (1987). The effects of demand-forecast fluctuations on customer service and inventory cost when demand is lumpy. Journal of the Operational Research Society, 75–82. JSTOR.

Wheelwright, S. C. & Hyndman, R. J. (1998). Forecasting: methods and applications. John Wiley \& Sons Inc.

Whitin, T. M. (1955). Inventory control and price theory. Management science, 2(1), 61–68. INFORMS.

Willemain, T. R., Smart, C. N. & Schwarz, H. F. (2004). A new approach to forecasting intermittent demand for service parts inventories. International Journal of Forecasting, 20(3), 375–387. Elsevier.

Williams, T. (1982). Reorder levels for lumpy demand. Journal of the Operational Research Society, 185–189. JSTOR.

Williams, T. (1984). Stock control with sporadic and slow-moving demand. Journal of the Operational Research Society, 939–948. JSTOR.

Wong, H., Cattrysse, D. & Van Oudheusden, D. (2005). Stocking decisions for repairable spare parts pooling in a multi-hub system. International journal of production economics, 93, 309–317. Elsevier.

Wong, H., Oudheusden, D. V. & Cattrysse, D. (2007). Cost allocation in spare parts inventory pooling. Transportation Research Part E: Logistics and Transportation Review, 43(4), 370–386. Elsevier.

Wu, C.-H. (2012). Price and service competition between new and remanufactured products in a two-echelon supply chain. International Journal of Production Economics. Elsevier.

Zhang, J. (2009). Cost allocation for joint replenishment models. Operations research, 57(1), 146–156. INFORMS.

Zhang, R. Q., Hopp, W. J. & Supatgiat, C. (2001). Spreadsheet implementable inventory control for a distribution center. Journal of Heuristics, 7(2), 185–203. Springer.

Zinoecker, R. (2006). How to Price Spare Parts More Profitably. Report.

Zipkin, P. H. (2000). Foundations of inventory management (Vol. 2). McGraw-Hill New York.

Zohrul Kabir, A. & Al-Olayan, A. S. (1996). A stocking policy for spare part provisioning under age based preventive replacement. European journal of operational research, 90(1), 171–181. Elsevier.