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R. A. Ashton

J. A. Northby
University of Rhode Island, jnorthby@uri.edu

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ence of the squares of the correlation lengths in two directions. With the appropriate ξ_n and ξ_t the Landau result is, of course, recovered.

A three-parameter fit of the corrected values of R for MBBA by the function $C(T - T_c)^{-2\nu}$ gives $\nu = 0.57 \pm 0.09$ and $T_p - T_c = 1.27 \pm 0.36°C$. Again the range of errors is the 95% confidence limit. This fit is shown in Fig. 3 as line *b*. The multiple-scattering correction does not affect the best value of ν and only increases T_c. Examination of Fig. 3 suggests that the departure of ν from 0.5 might be significant. However in view of the error bar it is difficult to ascertain a real departure from mean-field theory.[11] The peculiar behavior in OBBA suggests further study of the coupling between nematic and smectic ordering in the isotropic phase. The coupling leads to a departure from the simple phenomenological theory already observed for the scattered intensity and the linewidth,[12] but our results indicate that $\xi_n^2 - \xi_t^2$ is for OBBA a much more sensitive quantity than the scattered intensity itself.

In conclusion, the first measurement of L_2/a in the isotropic phase of a liquid crystal has been obtained. The sensitive method described here is expected to find applications in the study of liquid crystals, homologous series, and other depolarized scattering studies.

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*Present address: Physics Department, Technion, Haifa, Israel.

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Vortex Velocity in Turbulent He II Counterflow*

R. A. Ashton and J. A. Northby
Department of Physics, University of Rhode Island, Kingston, Rhode Island 02881
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We have used the ion-vortex interaction to measure the drift velocity of the vorticity present in turbulent counterflow, and find that it moves in the direction of normal-fluid flow. The result is in direct conflict with a central assumption of the presently accepted model of the turbulent state.

Recent work on turbulence in He II counterflow has shown that the model first proposed by Vinen[1] to account for second-sound attenuation is also able to give a reasonable description of critical velocities,[2] ion trapping,[3] and noise on second-sound signals.[4] The principal assumptions of this model are (1) the superfluid state consists of an homogeneous, isotropic, tangled mass of vortex lines. (2) The vorticity is geometrically similar at all times and therefore may be characterized by L_0, the length of line per unit volume. (3) The state is maintained by a dynamic balance between growth and annihilation processes; dimensional arguments then serve to pre-

dict their dependence on the heat current, $\dot{q}$.
(4) The vorticity moves on the average with the
superfluid (toward the heater). This last assump-
tion is needed in order to relate axial tempera-
ture gradients to transverse second-sound atten-
uation. Direct experimental evidence of vortex
motion is lacking, but early studies[5] of the de-
flection of negative-ion currents in turbulent
counterflow did indicate a strong additional drag
in the direction of the normal-fluid motion. This
could mean that vortex motion is away from the
heater.

We report here the first measurement of the
sign and magnitude of the vortex velocity. In
addition to resolving the conflict, we feel it pro-
vides a valuable clue to the detailed nature of the
turbulent state. Our method, which also utilizes
the ion-vortex interaction, indicates that in con-
trast to the model, vorticity drifts away from the
heater. Furthermore, the magnitude of the ve-
locity proves to be comparable to that of a vor-
tex ring whose diameter is equal to the mean
intervortex spacing.

Our experimental counterflow channel is shown
schematically in Fig. 1. It is 10 cm long, with a
0.5×1.0 cm^2 rectangular cross section, and is
open to the bath at the top. A wire heater resis-
tor, R, initiates the counterflow. The electrodes
are gold plated, and inlaid along the Lucite chan-
nel walls. A 200-mCi 1×0.6-cm^2 tritium β source,
S, is recessed in the wall behind a grid, G. The
constant bias voltage, V_0, applied across the
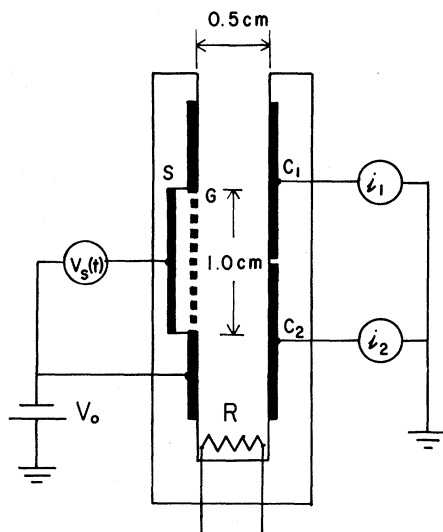channel, permits a negative-ion current to flow

FIG. 1. Schematic of counterflow channel (not to
scale).

from G to a pair of collectors, C_1 and C_2, placed
symmetrically on the opposite wall. This current
may then be switched on and off by modulating
the voltage $V_s(t)$ applied to the source.

The interaction between the negative-ion beam
and the vorticity present in turbulence has been
discussed previously.[3] Negative ions are trapped
on vorticity by the same mechanism as in ro-
tating helium,[6] but in turbulence the escape proc-
ess is not thermal excitation of the ion off the
line. Instead it is related to the annihilation of
the line segment on which the trapped charge re-
sides.[7] In the presence of constant ion and heat
currents, competition between these two process-
es leads to the buildup of a uniform distribution
of trapped charge in front of the grid. If the
charging current is then switched off, after a
transient comparable in length to the free-ion
transit time, the total current reaching the two
collectors will decay exponentially as the initial-
ly trapped charge is released through vortex an-
nihilation. The decay time constant is a measure
of the annihilation probability for an element of
vorticity in the flow. If the vorticity is drifting
with a velocity v_L, the plug of trapped charge
must be drifting also. Since the fraction of the
total current which reaches each individual col-
lector will depend on the fraction of the remain-
ing trapped charge in front of it, the downstream
collector should receive an increasing fraction
of the total current. We can obtain a quantitative
measurement of v_L by analyzing the time devel-
opment of the differences in these currents in the
following simplified model. We assume that the
turbulence is homogeneous, and that the initial
distribution of trapped charge is a uniform plug
whose length, L, is equal to the length of the
source. We also assume that the trapping cen-
ters drift toward C_1 with a uniform velocity, v_L.
We presume the applied voltage V_0 to be large
enough so that we may neglect retrapping of re-
leased charges, as well as their deflection by the
normal fluid and other effects arising from a
finite transit time. Finally we assume the re-
lease process is governed by an escape proba-
bility per unit time, p_e. In this model, currents
arise from two processes; transverse motion of
released charges, i^t, and axial motion of trapped
charges, i^a. Consider first transverse motion.
Each charge released in front of a collector will
contribute an average charge $(-e/2)$ to the cur-
rent on that collector. Consequently

$$i_i{}^t = (-e/2)(p_e N_T f_i), \tag{1}$$

where $N_T(t)$ is the total number of trapped charg-
es and $f_i(t)$ is the fraction in front of the ith col-
lector. Given our postulates we can write

$$f_i(t) = (1 \pm \delta)/2 \pm (v_L/L)t, \qquad (2)$$

where the positive sign refers to the downstream
collector, C_1. The time-independent parameter
δ accounts for any initial displacement of the
plug. Next consider axial motion. For each
trapped charged which crosses the boundary be-
tween the two collectors an average induced sur-
face charge of $(+e/2)$ will be transferred from
C_2 to C_1. Since the rate at which charges cross
the boundary is $(N_T v_L/L)$ we have

$$i_i{}^a = (\pm e/2)N_T v_L/L, \qquad (3)$$

where the minus sign applies to C_1. Defining
$\Delta i \equiv i_1 - i_2$ and $i_T \equiv i_1 + i_2$ we can combine these
results into the convenient expression

$$(\Delta i/i_T) = (\delta + 2v_L/p_e L) + (2v_L/L)t. \qquad (4)$$

We therefore expect the difference in the currents
divided by their sum, to be a linear function of
time whose slope is a direct measure of v_L. This
model is valid only until the charge plug drifts
beyond C_2. After this time $t_1 \equiv (L/2v_L)$, $i_2 = 0$, and
$\Delta i/i_T = 1$. The result (4) can be extended to the
case of a distribution of velocities by replacing
v_L and p_e by their average values, provided that
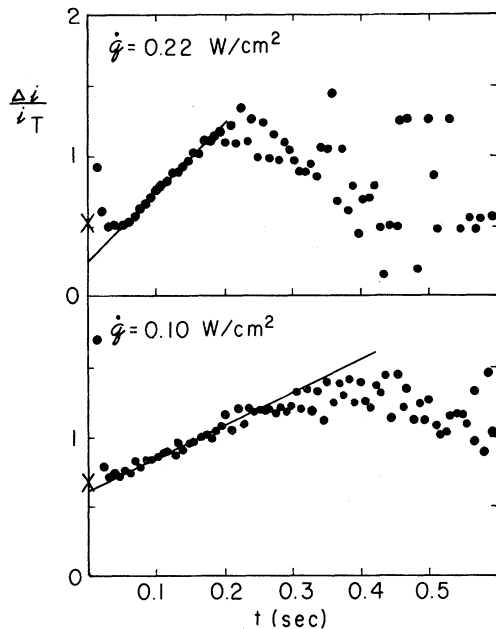they are uncorrelated.

Figure 2 shows typical experimental plots of
$\Delta i/i_T$ versus time at $T = 1.31$ K, for two differ-
ent heat currents. After an initial transient the
curve is an increasing function of time, which is
unambiguous evidence of motion of the trapped
charges in the direction of normal-fluid flow.
In fact, the linear behavior predicted in our sim-
ple model is clearly in evidence. The straight
lines are least-squares fits to the region or ap-
parent linearity. Their slopes give us $(2v_L/L)$.
They are terminated at the time t_1 calculated
from this slope, since beyond that point the mod-
el requires $\Delta i/i_T = 1$. It is encouraging that t_1
characterizes fairly well the end of linear be-
havior.

In Fig. 3 we show values of v_L as a function of
$\dot{q}$, for three different temperatures. The depen-
dence on $\dot{q}$ is roughly linear and the magnitude
ranges from about $\frac{1}{5}$ of the calculated normal-
fluid velocity at 1.31 K, to about $\frac{1}{3}$ at 1.59 K.
We have also obtained data at three intermediate
temperatures which are qualitatively similar,
but which have been omitted for reasons of clar-
ity. These data were taken with a transverse
electric field of 280 V/cm. Reduction to 140
V/cm produced no noticeable change in our re-
sults, however. Since retrapping depends on
field strength, this insensitivity supports our
assumption that it is negligible.

One conclusion which must follow from a non-
zero value of $v_L - v_s$ is that the vortex array is
almost certainly anisotropic. Since Vinen's ar-
gument relating axial temperature gradients to
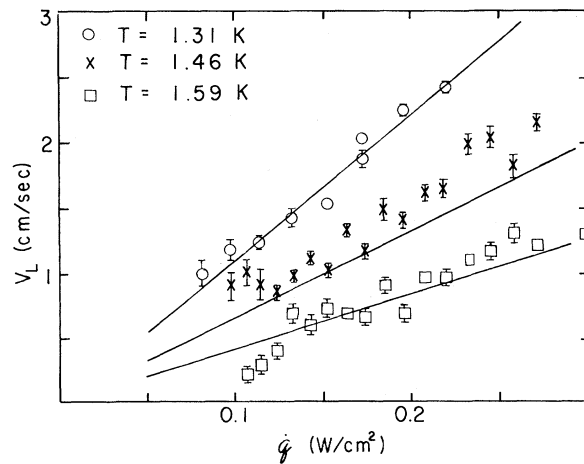transverse second-sound attenuation assumes

FIG. 2. Difference current divided by total current
versus time at $T = 1.31$ K, for two heat currents.

FIG. 3. Vortex velocity versus heat current at three
different temperatures. Indicated errors represent
± 1 standard deviation.

isotropy as well as that $v_L = v_s$, that aspect of his model requires modification. Other than this, however, none of the postulates underlying his dimensional analysis are in direct conflict with these new results. We can, in fact, predict the dependence of v_L on $\dot{q}$ and on T by the following simple extension of that analysis. In the localized self-induction approximation[8] valid for the motion of vortex filaments, the velocity of an element of vorticity relative to the superfluid is inversely proportional to the radius of curvature. An average drift velocity must then imply the existence of an average radius of curvature,[9] $\bar{R}$. If we accept the hypothesis of geometrical similarity, the only relevant length in the problem is $L_0^{-1/2}$, and we must have $\bar{R} \propto L_0^{-1/2}$. Consequently, if we define v_L as the average vortex velocity in the direction of normal-fluid flow measured in the lab frame, we have

$$v_L = CL_0^{1/2} - v_s, \qquad (5)$$

where C is independent of both $\dot{q}$ and T, and v_s is the superfluid velocity. The solid curves in Fig. 3 are calculated using Vinen's experimental values of L_0, with C adjusted to equal 2.4κ, where κ is the quantum of circulation. The agreement here is quite good, as it is for other intermediate temperatures. It must be emphasized that a single adjustable constant generates all of these curves—the dependence of $(v_L - v_s)$ on $\dot{q}$ and T is entirely contained in $L_0^{1/2}$.

These arguments do not allow us to predict the magnitude of the constant C. It is suggestive however that a plot of the velocity in the lab frame of a vortex ring whose diameter $D = (0.8)L_0^{-1/2}$ produces essentially the same curves shown in Fig. 3.

Expression (4) indicates that the intercept depends on v_L also. Unfortunately, quantitative analysis is prevented by our lack of knowledge of δ. This parameter accounts principally for initial axial displacement of the trapped-charge plug, which can result from several processes. In any case, however, we expect it to be small. The other term, $(2v_L/p_e L)$, may be determined

experimentally, since p_e^{-1} is just the decay constant for i_T. This quantity which is indicated by the crosses at $t = 0$ in Fig. 2, should equal the experimental intercept if δ is indeed negligible. In general, agreement is good for small $\dot{q}$ but gets progressively poorer at larger $\dot{q}$, particularly at the highest temperatures. It is tempting to attribute the difference to a temperature and heat current dependent δ, but we are presently unable to account for even the qualitative nature of these dependences. This difficulty may arise from a fundamental limitation of our model, but we feel it is more likely to be a manifestation of some second-order effect such as turbulence in the source region or channel-induced inhomogenieties. We are presently planning further experiments in different geometries which should help to clarify these points. In addition we expect to extend the method to study turbulence in pure-superfluid flow.

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[9]More precisely, if $\hat{n}$ is the unit vector normal to the line and $\vec{\kappa}$ the circulation, the average of $(\vec{\kappa} \times \hat{n}/R)$ must be axial, with magnitude $\kappa/\langle \bar{R} \rangle$.