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Quasi-2D Flows of ^{3}He – ^{4}He Mixtures

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Transport in degenerate $^3He - {}^4He$ mixtures in quasi-2D flow channels is discussed. The quasiparticle mean free path combines particle-wall and particle-particle collisions including the interference between them. The temperature, concentration, and polarization dependences of the transport coefficients allow easy extraction of the correlation parameters of random surface roughness from transport data. PACS numbers: 51.20.+d, 67.60.FP, 67.65.+z.

1. INTRODUCTION

Wall scattering is an integral part of almost any physical process in ultraclean and nanoscale systems. The convoluted nature of surface processes often makes the understanding of surface scattering difficult. In contrast to many other liquids and solids, ^3He allows a direct study of the effects of wall scattering on particle transport over a wide range of parameters.[1] Low-temperature $^3He - {}^4He$ mixtures provide even more flexibility by allowing one to scan particle mean free paths and wavelengths by simply changing the temperature, ^3He concentration, and degree of spin polarization. Since the formation of thin ^4He-rich layers near the walls can prevent energy and magnetic accommodation on the walls, one gets experimental access to a unique system with locally specular scattering of ^3He quasiparticles with a practically quadratic energy spectrum. This makes $^3He - {}^4He$ mixtures one of the most convenient tools for studying of the effects of random wall roughness on transport.

Below we report theoretical results of a comprehensive joint project on helium transport in ultrathin channels. The experiments are ongoing.[2]

2. SURFACE AND BULK RELAXATION

We consider degenerate $^3He-{}^4He$ mixtures, $T \ll T_F$, where $T_F = p_F^2/2m^* = \left(3\pi^2 N_3\right)^{2/3}/2m^*$ and p_F are the Fermi energy and momentum of ^3He quasiparticles, N_3 is the number of ^3He particles per unit volume, and m^* is the effective mass of a single ^3He quasiparticle (for values of parameters, see reviews[3]). We are interested in transport through a thin quasi-$2D$ flow channel or film of the (average) thickness L. The random inhomogeneities of the walls are characterized by the correlation function of thickness fluctuations $\zeta(y,z)$ (for definitions, see Ref. 4).

The motion of ^3He quasiparticles across the ultrathin films is quantized,

$$\epsilon(\mathbf{p}) = p^2/2m^* \to \epsilon_j(\mathbf{q}) = (1/2m^*)\left[(\pi j\hbar/L)^2 + q^2\right], \tag{1}$$

where $\mathbf{q}$ is the momentum along the film. Such quantization has been observed in recent experiments.[5] According to our recent results on transport in quantized systems with both bulk and boundary scattering,[4] the effective relaxation time in each of the minibands ϵ_j is given by the equation :

$$\begin{aligned} 1/\tau_j^{(eff)}(\mathbf{q}) &= 1/\tau_j^{(b)}(\mathbf{q}) \\ &+ \sum_{j'=1}^{S} \int \frac{W_{jj'}(\mathbf{q},\mathbf{q}')/\tau_{j'}^{(b)}(\mathbf{q}')}{\left(\epsilon_{j'}(\mathbf{q}') - T_F\right)^2/\hbar^2 + \left(1/2\tau_{j'}^{(b)}(\mathbf{q}')\right)^2} \frac{d\mathbf{q}'}{(2\pi\hbar)^2}, \end{aligned} \tag{2}$$

where S is the total number of occupied or energetically accessible minibands $\epsilon_j(\mathbf{q})$, and $\tau_j^{(b)}(\mathbf{q})$ is the bulk relaxation time in miniband ϵ_j. The wall-induced transition probability $W_{jj'}(\mathbf{q},\mathbf{q}')$ between the states $\epsilon_j(\mathbf{q})$ and $\epsilon_{j'}(\mathbf{q}')$ is determined by the Fourier image of the correlation functions of surface inhomogeneities $\zeta(\mathbf{q}-\mathbf{q}')$,

$$W_{jj'}(\mathbf{q},\mathbf{q}') = \frac{\pi^4\hbar^2}{M^2L^6} j^2 j'^2 \zeta(\mathbf{q}-\mathbf{q}'). \tag{3}$$

The interference between bulk and boundary scattering, which is one of the most important features of Eq.(2), has been thoroughly analyzed in Ref. 4. The application of these results to helium mixtures is straightforward since in dilute mixtures the bulk relaxation time does not depend on momenta:

$$\tau_j^{(b)}(\mathbf{q}) = \tau_b = \frac{5\hbar^3}{4\pi m^*}\left(\frac{1}{aT}\right)^2, \tag{4}$$

where $a \sim -0.9$ Å is the s-wave scattering length of two ^3He quasiparticles (for consistency, we use the viscous relaxation time).

Since extensive transport measurements in quantized films are not feasible at present, one should be more interested in quasiclassical transport in slightly thicker, but still ultrathin quasi-$2D$ systems. Then the analog of Eq.(2) is

$$\frac{1}{\tau^{(eff)}(\mathbf{p})} = \frac{1}{\tau_b}\left(1 + \int \frac{W(\mathbf{p},\mathbf{p}')}{(\epsilon(\mathbf{p}')-\mu)^2/\hbar^2 + 1/4\tau_b^2}\frac{d\mathbf{p}'}{(2\pi\hbar)^3}\right), \tag{5}$$

with

$$W(\mathbf{p},\mathbf{p}') = \frac{4\pi}{L}\left(\frac{\ell R}{\hbar M}\right)^2 p_x^2 p_x'^2 \exp\left(-(\mathbf{q}-\mathbf{q}')^2 R^2/2\hbar^2\right), \tag{6}$$

where we assume that the wall inhomogeneities are Gaussian with the amplitude (height) and correlation radius (size) ℓ and R respectively.

Eq.(5) can be drastically simplified in two limiting cases when the bulk mean free path is either much larger or much smaller than the wall-restricted mean free path for ballistic particles. At ultralow temperatures,

$$(T_F/T)^2 \gg (R/a)^2 X^{4/3} \tag{7}$$

(X is the molar concentration of ^3He in the mixture), the bulk mean free path is larger than the wall-driven one, and the bulk and surface scattering processes are independent from each other. Then the effective relaxation obeys Matthiessen's rule,

$$1/\tau^{(eff)} = 1/\tau_b + 1/\tau_w. \tag{8}$$

The expression for the wall contribution τ_w is especially simple for atomic- or nano-scale roughness in the low-concentration limit

$$\left(3\pi^2 N_3\right)^{2/3} R^2 \ll 1 \tag{9}$$

when

$$\frac{1}{\tau_w} = 4\pi^2 \frac{\ell^2 R^2 N_3}{L}\frac{T_F}{\hbar}. \tag{10}$$

Numerically, Eqs.(7) – (10) correspond at SVP to

$$\begin{aligned} (T/2.64)^2 &\ll 0.81\times 10^{-20}/R^2,\ 7.5\times 10^{19} R^2 X^{2/3} \ll 1, \\ \frac{1}{\tau_w} &= 3.16\cdot 10^{34}\frac{\ell^2 R^2}{L} X_3^{5/3}\ s^{-1},\ \frac{1}{\tau_b} = 5.35\cdot 10^{10} T^2\ s^{-1} \end{aligned} \tag{11}$$

with T in Kelvin and R, ℓ, and L in metres.

At higher temperatures or ^3He concentrations, when the bulk mean free path is much shorter than the wall-driven contribution, the effective relaxation time is a convoluted combination of the bulk and surface scattering parameters. This strong interference between bulk and surface scattering is explained by the fact that the particles return to the wall not after a reflection from the opposite wall, but after a series of bulk scattering processes. The value of the effective relaxation time at $(T_F/T)^2 \ll (R/a)^2 X^{4/3}$ is

$$\frac{1}{\tau^{(eff)}} = \frac{1}{\tau_b} + 6\sqrt{\frac{2}{5}}\pi^{5/2}\frac{|a|\,\ell^2 N_3 \hbar}{L p_F}\frac{T}{\hbar}. \tag{12}$$

Numerically, this equation corresponds to

$$\frac{1}{\tau^{(eff)}} = \frac{1}{\tau_b} + 4.5\cdot 10^{15}\frac{\ell^2}{L}X^{2/3}T\ s^{-1}. \tag{13}$$

3. SPIN-POLARIZED MIXTURES

Spin polarization of $^3He-{}^4He$ mixtures provides a useful experimental tool. The polarization results in change in particle wavelengths and bulk mean free paths.[3] In addition, in spin-polarized mixtures, spin-up and spin-down quasiparticles have different dynamic and kinetic properties. The polarization dependence of the *bulk* viscous relaxation times of the spin-up and spin-down particles is given by the equation

$$\tau_\pm = \frac{\hbar^3}{2\pi m^*}\left(\frac{1}{aT}\right)^2 \frac{5}{5-3\left(c_\mp/c_+\right)^{2/3}}\frac{c_+}{c_\mp},\ c_\pm \equiv \frac{N_\pm}{N_3} \tag{14}$$

where $c_\pm$ are the "concentrations" of spin-up and spin-down particles.

The above results can be easily expanded to polarized mixtures. Eqs.(7) – (11) for the limit of large bulk mean free path yield

$$\frac{1}{\tau_\pm^{(eff)}} = \frac{1}{\tau_\pm} + 4\pi^2\left(2c_\pm\right)^{8/3}\frac{\ell^2 R^2 N_3}{L}\frac{T_F}{\hbar} \tag{15}$$

$$\approx \frac{1}{\tau_\pm} + 3.16\cdot 10^{34}\frac{\ell^2 R^2}{L}\left(2c_\pm\right)^{8/3}X_3^{5/3}\ s^{-1}. \tag{16}$$

In the opposite case of not very low temperatures (or large-scale inhomogeneities) and moderate polarizations,

$$(T_F/T)^2 \ll (R/a)^2 X^{4/3} c_\mp, \tag{17}$$

the relaxation time becomes

$$\frac{1}{\tau_{\pm}^{(eff)}} = \frac{1}{\tau_{\pm}} + 6\pi^{5/2} q_{\pm} \frac{a\ell^2 N_3 \hbar}{L p_F} \frac{T}{\hbar} \tag{18}$$

$$\simeq \frac{1}{\tau_{\pm}} + 4.5 \cdot 10^{15} \frac{\ell^2}{L} X^{2/3} T q_{\pm} \ s^{-1}, \tag{19}$$

$$q_{\pm} = (2c_{\pm})^2 \left(\frac{5}{2} - \frac{3}{2} \left(\frac{c_{\mp}}{c_+} \right)^{2/3} \right)^{1/2} \left(\frac{c_{\mp}}{c_+} \right)^{1/2}. \tag{20}$$

All the polarization dependence is contained in the factors $q_{\pm}$, Eq.(20).

Interestingly, the wall contributions to the inverse relaxation time $1/\tau_{\pm}^{(eff)}$, Eq.(18), go to zero when the mixture becomes completely polarized, $q_{\pm}(c_- \to 0) \to 0$. In reality, the wall contribution does not disappear, at least for spin-ups: at high enough polarization one always leaves the hydrodynamic regime, Eq.(17), and gets into the opposite, ballistic regime, Eq.(15). Note, that q_- goes to zero at $c_- \to 0$ much faster than q_+.

At high spin polarization one can observe a peculiar "mixed" low-temperature regime between the limiting cases (15) and (18) when c_- is so small that the majority spins (spin-ups) are in the ballistic regime (15) while the spin-downs are in the hydrodynamic regime (18) with a relatively small bulk mean free path.

4. CONCLUSIONS

In summary, we present a theory of wall effects in transport of superfluid $^3He - {}^4He$ mixtures in narrow quasi-$2D$ flow channels with random rough walls. The low-temperature scattering of ^3He quasiparticles from the walls is almost completely specular locally and is a sensitive probe of the correlation function of surface inhomogeneities. Temperature, concentration, and spin polarization dependences of the transport coefficients allow one to monitor the system in a wide range of bulk mean free paths and particle wavelengths.

The results are especially simple and easily distinguishable in the limiting cases of ballistic and hydrodynamic regimes (Eqs. (10), (12)), when the bulk collisions are either less or more effective than the wall scattering. The condition (7) that separates the ballistic and hydrodynamic limits is determined solely by the temperature and the correlation radius of surface inhomogeneities. High polarization can result in a peculiar regime in which the spin-up quasiparticles are ballistic while the spin-downs remain hydrodynamic and the wall contributions from spin components of the quasiparticle system are distinctly different from each other.

The above results can be applied to pure liquid ^3He or metal films

with some caution. The main constraints are the requirement of quasielastic scattering by the walls and the use of constant bulk relaxation time.

Experiments on thermal conductivity and viscosity of helium in ultrathin channels are currently underway at Kamerlingh Onnes Laboratory. One of the goals is to extract parameters of wall roughness directly from the transport data. The wall contribution becomes noticeable at temperatures below 10 mK. At even lower temperatures it may be possible to observe roughness-driven localization. Measurements are being done in arrays of micro- and nanochannel glass capillaries. The material is fabricated at Naval Research Laboratory[6] by a glass fiber drawing process that can provide channel glass arrays having channel diameters ranging from about 100 nm to 10 μ. The experimental cell design and nanochannel glass fabrication were discussed at QFS2001.[2] Additional experimental details will be published later. More detailed theory will appear in Ref. 7.

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