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# Deterministic discrete dynamical systems: Advances in regular and chaotic behavior with applications

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### **Editorial Deterministic Discrete Dynamical Systems: Advances in Regular and Chaotic Behavior with Applications**

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A dynamical system is characterized by three major components: phase space, evolution operator(s), and time scale. Discrete dynamical systems, in particular, are governed by difference equations or iterative processes. They may result from discretizing continuous dynamical systems or modeling evolution systems for which the time scale is discrete. Indeed, discrete dynamical systems are prevalent in signal processing, population dynamics, numerical analysis and scientific computation, economics, health sciences, and so forth. Furthermore, investigating the long-term behavior is important in its own right. However, it also helps engineers, scientists, and policy makers to come up with informed decisions.

In this special issue of the journal, Discrete Dynamics in Nature and Society, we focused our attention on the longterm behavior of deterministic dynamical systems for which the underlying phase space is a continuum. Specifically, the contributors investigated special classes of bilinear rational difference equations of order two, first-order systems of bilinear rational difference equations, discrete-time periodic models with bang-bang feedback control, recurrent and discontinuous two-state dynamical systems involving nonnegative bifurcation parameters, and three-step computational family of iterative schemes for solving single-variable nonlinear equations.

Difference equations are abundant in the field of numerical analysis, particularly iterative techniques for solving linear and nonlinear equations. With that in mind, F. Soleimani et al. presented a novel three-step computational family of iterative schemes for solving single-variable nonlinear equations. This family can be viewed as a generalization of the wellknown Jains derivative-free method with optimal order four and the efficiency index 1.587. It is also a collection of eighthorder methods with optimal efficiency index 1.682. In addition, the authors provided the basins of attraction for some methods.

Bifurcation pertains to the change in qualitative behavior of a dynamical system when a change in governing parameters incurred. C. Hou and S. S. Cheng studied a recurrent and discontinuous two-neuron dynamical neural network system involving a nonnegative bifurcation parameter. By elementary but novel arguments, they managed to give a complete analysis on its asymptotic behavior when the parameter varies from 0 to  $\infty$ .

Competitive and anticompetitive systems received much attention among researchers in mathematical biology, particularly population dynamics, and other fields. The work of M. Dipippo and M. R. S. Kulenovic fits in this line of research. They investigated three first-order anticompetitive systems of bilinear rational (i.e., fractional linear) difference equations with positive real parameters and nonnegative real conditions. They managed to find the basins of attraction of all attractors of these systems.

S. Atawna et al. studied a second-order bilinear rational difference equation of order two with positive real parameters and nonnegative real conditions. They established necessary and sufficient conditions for the existence of period-two solutions. Furthermore, they proved that the existence of the aforementioned period-two solutions implies its local

stability. By doing so, the authors give positive confirmation of an open problem attributed to Ladas and Kulenovic posted in 2002.

Difference equations can be found in the fields of system control and signal processing. For instance, C. Hou and S. S. Cheng investigated a discrete-time periodic model with bang-bang feedback control. They showed that each solution tends to one of four different types of limit two cycles and determined the basin of attraction for each type of solutions. Furthermore, when a threshold parameter is introduced in the bang-bang function, their results form a complete bifurcation analysis of the control model under study. This, in turn, implies that their model can be used in the design of a control system where the state variable fluctuates between two state values with decaying perturbation.

In conclusion, we hope that these papers will enrich our readers and stimulate researchers to extend, generalize, and apply the established results.

> Raghib Abu-Saris Fathi Allan Mustafa Kulenovic Alfredo Peris











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