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Critical Exponents of the Heisenberg Spin Chain

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Critical Exponents of the Heisenberg Spin Chain

Recent work by Schlottmann[1] evaluates the thermodynamic Bethe-*Ansatz* equations numerically for the spin-$\frac{1}{2}$ Heisenberg ferromagnetic chain (HBFM), obtaining values for the critical exponents $\alpha = -0.49 \pm 0.02$ and $\gamma = 2.00 \pm 0.02$. We here discuss his very interesting results in a broader context of $T_c = 0$ critical behavior of quantum systems. The reader gains the impression from Ref. 1 that equality of critical exponents for classical and quantum HB chains may be a general result. This is not so, the values obtained by Schlottmann reflecting an exceptional situation. Furthermore, Schlottmann believes that he presents the first example where critical behavior for a quantum system is extracted via a thermodynamic Bethe-*Ansatz* approach. In fact, such results are already available for spin-$\frac{1}{2}$ planar *XXZ* and HB antiferromagnetic (HBAFM) chains.[2,3] Table I shows the scaling powers, a_T and a_H, for these models. The corresponding critical exponents are found from the $T_c = 0$ scaling and hyperscaling relations $\alpha = -1/a_T$, $\gamma = (2a_H + a_T - 1)/a_T$, $\delta = a_H/(1-a_H)$, $\eta = 2 + D(1 - 2a_H)$, and $\nu = 1/Da_T$. Only the Schlottmann values agree with the values for the classical chain. Critical exponents for the spin-$\frac{1}{2}$ HBFM from various calculations are displayed in Table II. Note that the Schlottmann susceptibility results agree with calculations (a), (b), and (d) as far down in temperature as these calculations extend. At lower temperatures, Schlottmann finds that his data enter a very low-T asymptotic regime, and tend to the classical result. Similar crossover behavior occurs in the case of calculation (e), but here results from the fact that when a Green's-function decoupling is employed, the asymptotic critical behavior is expected to be classical. We note that all calculated exponents (a) through (g) violate the $T_c = 0$ scaling and hyperscaling hypothesis.[11] The only set of exponents which does satisfy scaling and hyperscaling ($a_T = 2$, $a_H = 1$, $\alpha = -\frac{1}{2}$, $\gamma = \frac{3}{2}$, $\eta = 1$, $\nu = \frac{1}{2}$, and $\delta = \infty$) is given in (h). Hence from the calculations of Schlottmann, whose conclusions are paralleled by work of Takahashi and Yamada,[10] we must conclude that scaling relations are violated for the spin-$\frac{1}{2}$ HBFM. One might infer that this interesting result reflects the fact that this critical point has significant first-order character.

TABLE I. Scaling powers for various 1D models. The parameter μ is defined via the relation $\cos\mu = -\Delta$, where Δ is the anisotropy parameter.

	Model	a_T	a_H
$S = \infty$	Classical	1	1
$S = \frac{1}{2}$, $\lvert\Delta\rvert < 1$	Planar *XXZ* [a]	1	$\frac{1}{2} + \mu/2\pi$
$S = \frac{1}{2}$, $\Delta = 0$	*XY*	1	$\frac{3}{4}$
$S = \frac{1}{2}$, $\Delta = -1$	HBAFM [b]	1	$\frac{1}{2}$
$S = \frac{1}{2}$, $\Delta = 1$	HBFM [c]	1	1

[a]References 3 and 4. [c]Reference 1.
[b]References 2 and 3.

TABLE II. Critical exponents for the HBFM resulting from various calculations.

	Reference	α	γ
(a)	5	...	1.67 ± 0.07
(b)	6	$-\frac{1}{2}$	~ 1.80
(c)	7	...	~ 1.32
(d)	8	-0.3 ± 0.1	1.75 ± 0.02
(e)	9	$-\frac{1}{2}$	2.00
(f)	1	-0.49 ± 0.02	2.00 ± 0.02
(g)	10	$-\frac{1}{2}$	2
(h)	Scaling	$-\frac{1}{2}$	1.5

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Note added after receipt of Schlottmann's response: In Ref. 6, exponents of $\frac{3}{2}$ for the energy and $\frac{1}{2}$ for the entropy are explicitly given, corresponding to $\alpha = -\frac{1}{2}$. The quoted entropy amplitude differs by 24% and 20% from the leading amplitude values of Takahashi-Yamada and Schlottmann, respectively. It is further stated that this ". . . would imply a divergence of . . . $\chi(T)$ like $1/T^{9/5}$ as $T \rightarrow 0$. It is quite possible, however, that the true asymptotic behavior sets in only below $kT/J = 0.2$."

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