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Multivariate Ewma Models and Monitoring Health Surveillance during a Pandemic

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Research Article,

Multivariate Ewma Models and Monitoring Health Surveillance during a Pandemic**Jeffrey E. Jarrett, Ph.D.^{1*}, Xia Pan, Ph.D.**

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Abstract:

We examine a common problem is biological analytics and surveillance in health care. These methods can improve greatly the process of monitoring health data to assess changes in the likelihood of Pandemics and disease incidence in a world where medical knowledge is still largely in an embryonic period. Based on an illustration, we suggest that multivariate exponential moving-average (MEWMA) control charts are suitable in many cases where detection and inspection of several or more variables over a lengthy period of testing provide for the best analysis of data leading to pre-diagnostic and diagnostic therapy. Though these methods came from the control of quality and continuous improvement in lean manufacturing and service operations, these methods are useful if not a vital application in the analysis of health care and therapeutic data. The indications from this study corroborate earlier findings by others that MEWMA methods fit the diagnostic activity under study. Unfortunately Pandemic Analysis is using oversimplified techniques in analyzing data secure by diagnostic tests which can easily be improved especially in the use modern day analytics based on quality control methods used in other disciplines.

Keywords: Multivariate Quality Control (MPC), Multivariate Exponentially Weighted Moving Average (MEWMA), Diagnostic Testing in a Pandemic..., Correlation among variables, Serial Correlation

Biological Surveillance with Multivariate Methods:

Modern bio surveillance involves the monitoring large number and wide range of data from samples of diagnostic and pre-diagnostic data for the purpose of giving health care professional to recognize, detect, investigate and respond to the outbreaks of disease and pandemics. A central tool in this monitoring in classical disease surveillance migrated to biological surveillance is the use of multivariate quality control methods. Fricker

(2007) applied multivariate statistical control methods with an application of multivariate quality control (MQC) to syndromic surveillance. Fricker *et al.* (2008) continued the earlier study by focusing on directionally sensitive procedure in bio surveillance. Joner *et al.* (2008) produced a one sided multivariate exponentially weighted moving-average (MEWMA) control chart for the analysis of health data. Niaki and Ershadi (2012)

used a solution to solve a statistically constrained economic model of a MEWMA control chart in which external intangible costs were considered. Shen and Cooper (2012) produced an MPC Decision Analytical model for bio surveillance. Last, Yahav and Schmueli (2013) introduced in practice directionally sensitive MPC charts to bio surveillance methods. They examined four such techniques and came to conclusions based on simulated data, but suggested further research in the application of these methods. Last, Jarrett (2016) indicated the need and motivation for implementing data analytical methods in the health care environment.

MEWMA Modeling and Quality Control:

Previously Ord, Koehler, Snyder and Hyndman 2009; hereafter, (OKSH) recognized the usefulness of monitoring social or economic processes is a clear application of the notion of statistical process control (SPC). They extended the notions of control by Shewhart Control Charting to that of monitoring univariate time series. Furthermore, OKSH suggested the use of EWMA charts for residuals, which will be effective in detecting level shifts and suggest their use in detecting shifts in variability. This improved process could also be explored by expanding the analysis to the multivariate case. OKSH suggested that we examine the ideas explored by Lowry, Woodall, Champ and Rigdon (1992, hereafter LWCR), Pan and Jarrett (2004) and Rungar, Barton, Del Castillo and Woodall (2007). The multivariate form of the EWMA control chart simultaneously monitors two or more related processes in an exponentially weighted moving-average control chart. For example, if one applies a MEWMA chart to monitor temperature and pressure in a plastic injection molding process. Each MEWMA point incorporates information from all the previous subgroups or observations in combination with a user defined weighting factor. MEWMA charts can help you detect small process shifts quicker than other multivariate charts, such as the T^2 control chart. Another advantage of

MEWMA charts is that they are not greatly influenced when a small or large value enters the calculation. Also, MEWMA charts can be custom tailored to detect any size shift in the process. Because of this, they are often used to monitor in-control processes for detecting small shifts away from the target. Multivariate process control simultaneously monitors several processes in combination. Previous, work on multivariate monitoring since LWCR include a large number of papers in the quality monitoring literature [i.e., Lowry and Montgomery, 1995; Sullivan and Woodall, 1996; Djauhari, 2005; Khoo and Quah, 2003; Kruegel, Valuer and Vigna, 2005; Ye and Chen, 2001; Ye, Chen and Borrer, 2004; Ye, Giardano and Feldman, 2001; Ye, Vilbert and Chen, 2003; Bersimis, Psakaris and Panaretos, 2006; Khoo, 2003, Yeh, Wang and Wu, 2004; Pan and Jarrett, 2004; Yang and Rahim, 2005, Jarrett and Pan, (2007a, 2007b, 2014) , and finally Pan and Jarrett, 2014]. These monitoring methods employ the Hotelling T^2 statistic for current samples. Exponentially weighted moving average (EWMA) charts which are more sensitive to moderate shifts in parameters than Univariate charts are widely used in univariate cases (Crowder, 1989; Lucas and Saccucci, 1990). LWCR extended the Univariate EWMA control chart to the multivariate case by simulation. They noted that the multivariate EWMA (hereafter MEWMA) chart has greater sensitivity to shifts in the mean than more traditional Hotelling T^2 control methods. An alternative MEWMA scheme is Pan (2005), which builds the Hotelling T^2 of the variables before the formation of the EWMA of the T^2 s. Lui (1996) presented an improvement for MEWMA. Runger and Prah (1996) used Markov chain analysis to calculate the ARL for MEWMA and Prah and Runger (1997) discussed the design of the same scheme. However, all these studies assumed the processes to be serially independent. Others chose to study the usefulness of MEWMA methods as well. Stoumbus and Sullivan (2002) investigated the effects of non-normality on the performance of

the MEWMA control chart, and its special case, the Hotelling's Chi-Square control chart when applied to individual observations. The purpose in this case was to monitor the mean vector of a multivariate process variable. Khoo studied the sensitivity of MEWMA control charts under other circumstances. In addition, Lee and Khoo (2006) explored a method for optimally designing multivariate EWMA charts based on the measures of average run length (ARL) and median run length (MRL). In this study, we utilize the concept of sensitivity ratios based on the works of Hanson, Eskridge, Steadman and Madisa (2009) and Väisänen and Hyttinen (2007) who argued that sensitivity ratios are a superior method to assess quality in the areas of bioelectric measurements, plant disease - screening methods and others involving new technology. The sensitivity ratio is a statistic specifically developed for comparing for different measuring methods and is not based on any particular assumption about how the measuring methods or scales are related. Hence, our purpose is to share new research in the evolution of monitoring processes by comparing results of experiments. The Pan MEWMA scheme builds the Hotelling T^2 of the variables before order of construction steps is the statistic of MEWMA chart. Pan (2005) used integral equation method to compute the ARL's of MEWMA charts for in-control and out-of-control situations without the presence of serial correlation. All MEWMA method variations are multivariate EWMA schemes. The above schemes have a common problem, that is, they cannot be directly employed when the processes are serially correlated. An indirect way to apply the MEWMA schemes for serially correlated processes is to adopt Alwan and Roberts' (1988) approach. They suggest estimating the residuals, i.e., one-step-ahead forecasting errors, of the Autocorrelated process. In turn, they apply traditional control charts for the residuals. Extending this approach to multivariate cases, one can apply the above MEWMA scheme to the residuals of the serially correlated multivariate processes, until the processes are modeled properly

and the initial number of observations is sufficiently large and the residuals are asymptotically independent over time. At this point, we determine the sensitivity of these approaches to changes in process parameters in the presence of serial correlation. Since the process parameters are usually unknown, the appropriate estimation and use of the covariance matrix is vital for correct execution of MEWMA. This may occur if the direct sample variance is a biased estimate of the population variance for a serially correlated process. The main reason for utilizing multivariate quality control charts occurs in the situation where the collected data for two or more variables show cross-correlation. In this event a multivariate control chart should utilize a better result than studying independent control charts for each variable and is currently available in quality control software. We will in the next section, consider applications of MEWMA charts to monitor bio surveillance data to understand the meaning and application of these charts in a simulated experiment. The data for the analysis came from a hospital source collected over a sufficiently long time to produce enough data for analysis by the methods used in this study. Patient information is not available but the sample size produces enough data for MEWMA analysis and the results are such that the methods used produced valuable results and interpreted through a series of multivariate control charts easily produced by standard quality control and improvement software. The user needs only to learn the meaning the control charts illustrated in the next sections.

An illustration of the mewma control chart:

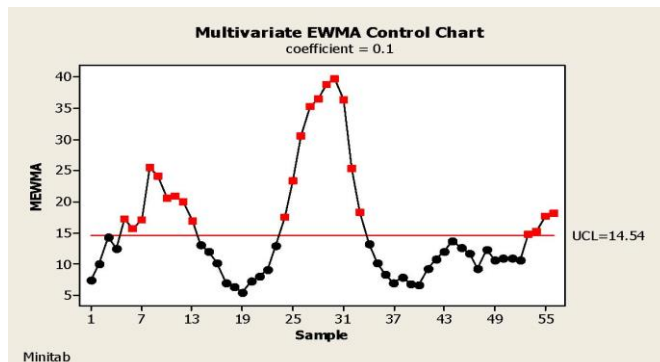
We consider a bio surveillance procedure where data is collected on five variables (A, B, C, D and E). Based on the results of Table 1 where we exhibit the correlation coefficients and their "**p-values**" or the cross correlations of the five variables denoted before. Five of the correlation coefficients are large enough to have produces p-values which are small enough to reject the null hypothesis that $\rho_i = 0$ [$\alpha =$

0.06 or less]. The remaining cross-correlation coefficients do not have small p-values; hence, we cannot make the same conclusion. With the mixed results, we will consider that here is enough evidence that cross-correlation exist among enough of the interactions to warrant the use of multivariate methods. To observe the sensitivity of the process, we construct using Minitab® six MEWMA control charts based on the dampening coefficient being equal to 0.1 to 0.6 with increments of Recall that the dampening coefficient in MEWMA refers to relationship between the prediction and observed values of the variables. The larger the value of the dampening coefficient indicates the greater the influence of the error in the previous prediction. Obviously if the dampening coefficient was 1, the prediction would be prediction in the previous time period. Figures 1 through 6 provide for the results of the MEWMA charts yield different upper control limits (UCL) for each of the control charts. For example Figure 1 where the coefficient is 0.1, one find a UCL of 14.54 and out of control points of 5 through 13, 24 through 33 and 53 through 56. Note when applying this method the value of the dampening coefficient is crucial.

Table 1: Cross-Correlations and P-Values; i.e. the probability of rejecting a null hypothesis

| Correlations: Var A, Var B, Var C, Var D, Var E | | | | |
|---|---------------------|--------|--------|--------|
| | Var A | VAR B | VAR C | VAR D |
| Var B | -0.032 | | | |
| | 0.593 | | | |
| Var C | 0.199 | -0.061 | | |
| | 0.001 | 0.309 | | |
| Var D | -0.181 | 0.064 | -0.832 | |
| | 0.003 | 0.288 | 0.000 | |
| Var E | -0.029 | 0.006 | 0.115 | -0.186 |
| | 0.630 | 0.915 | 0.056 | 0.002 |
| Cell Contents: | Pearson correlation | | | |
| | P-Value | | | |
| Minitab® | | | | |

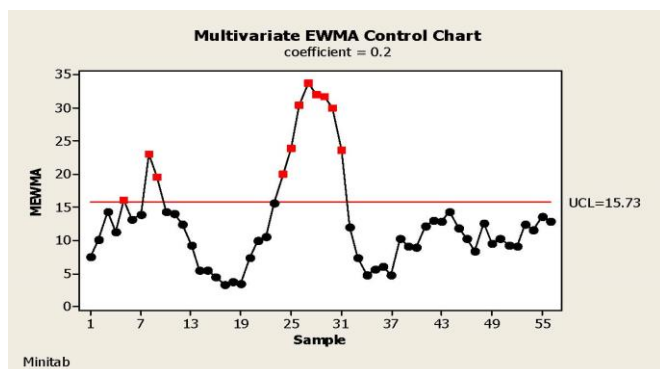
Figure 1:
Test Results for MEWMA Chart of Variable A, B, C, D and E



TEST. One point beyond control limits. Test Failed at points: 5, 6, 7, 8, 9, 10, 11, 12, 13, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 53, 54, 55, 56

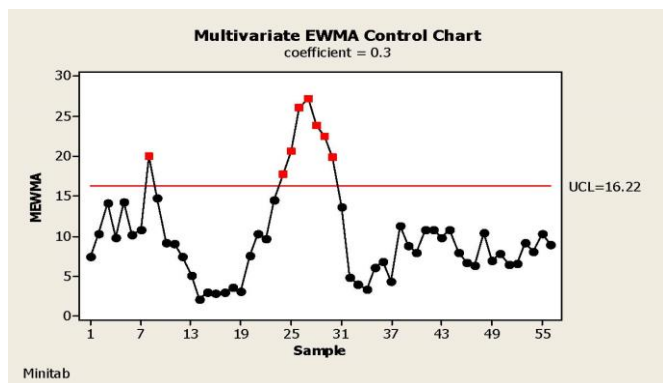
Observe in Figure 2, the UCL equals 15.73 but the number of points out of control becomes smaller. Only points 5, 8, 9 and 24 through 31 are out of control. Figure 3 the UCL increases to 16.22 and points 8 and 24 through 30 are out of control. For Figure 4 (coefficient of 0.4), the UCL equals 16.46 and points 8 and 25 through 28 are out of control. Figure 5 (coefficient equals 0.5), the UCL increases again to 16.60 and the points out of control 26 and 27. Finally, Figure 6 (coefficient equals 0.6) the UCL is now 16.67 and nothing is out of control.

Figure 2:
Test Results for MEWMA Chart of Variable A, B, C, D, and E



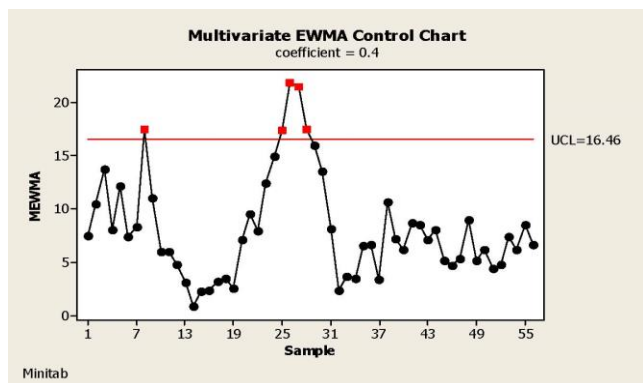
TEST. One point beyond control limits. Test Failed at points: 5, 8, 9, 24, 25, 26, 27, 28, 29, 30, 31

Figure 3:
Test Results for MEWMA Chart of Variable A, B, C, D and E



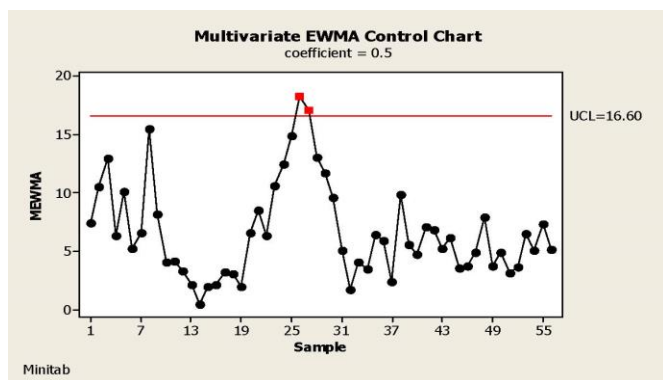
TEST. One point beyond control limits. Test Failed at points: 8, 24, 25, 26, 27, 28, 29, 30

Figure 4:
Test Results for MEWMA Chart of Variable A, B, C, D and E



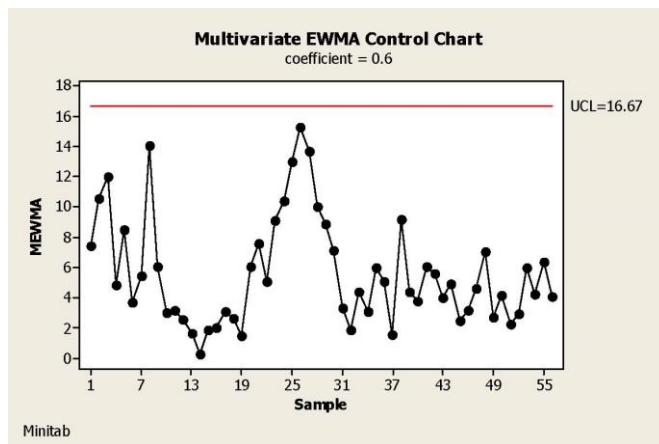
TEST. One point beyond control limits. Test Failed at points: 8, 25, 26, 27, 28

Figure 5:
Test Results for MEWMA Chart of Var A, B, C, D and E.



TEST. One point beyond control limits. Test Failed at points: 26, 27

Figure 6:
Test Results for MEWMA Chart of Variable A, B, C, D and E



TEST. Nothing is beyond upper control limits. From the analysis, we see the selection of the dampening coefficient is crucial in the decision to conclude when the process is out of control or not. The process being the surveillance indicates that for MEWMA control charts, the coefficient is a crucial parameter for assessing whether that each multivariate sample observation is in-control or out of control. Some solutions to this is to determine the optimal coefficient. In the MEWMA method of analysis, one would seek to find that value for the dampening coefficient that minimizes the sum of squares of the error term for the predictions resulting from the applications of MEWMA to the data observed in bio surveillance. This would be a multivariate least squares solution.

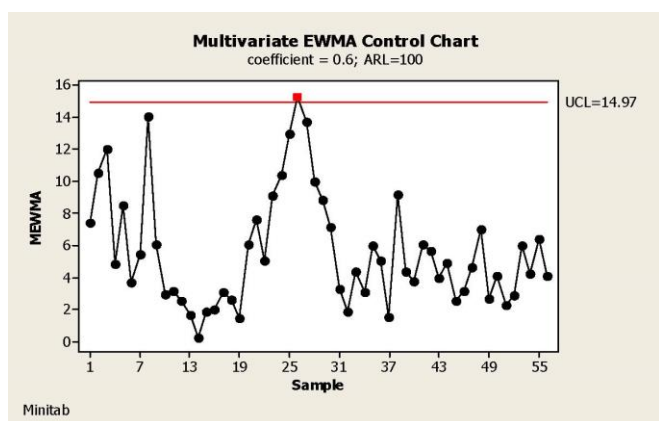
The Robustness of Average Run Length (ARL):

Average Run Length (ARL) is the number of data points before the MEWMA process is stopped. Prabhu and Runger (1997) provide some results on finding the optimal MEWMA control charts based on comparisons of values for the ARL and the initial number of variables in the multivariate process. Using the same simulated bio surveillance data as in the previous illustrations, we shall now consider looking at the sensitivity of our results to changes in assumed value of the ARL leaving the dampening coefficient at one value. If we reexamine Figure 6, where the coefficient is 0.6 and

ARL is 200 and compare it to the results of two additional figures. In Figure 7, the coefficient is still 0.6 but the ARL is now 100. The UCL is now 14.97 and there is one point out of control at 26. Figure 8 presents the same analysis when the ARL equals 50. The UCL is now constructed at 13.22 and three points are out of control at 8, 26 and 27.

Figure 7:

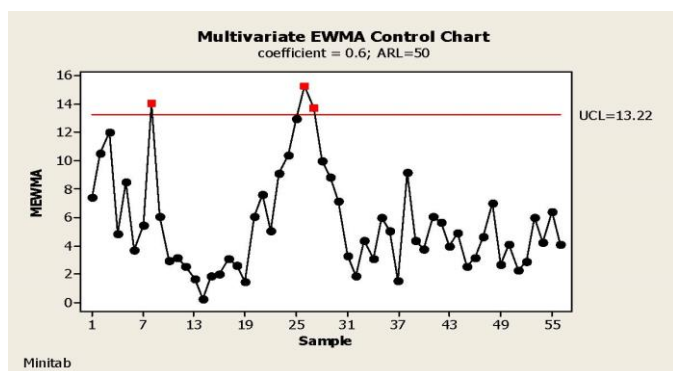
Test Results for MEWMA Chart of Variable A, B, C, D and E



TEST. One point beyond control limits. Test Failed at points: 26

Figure 8:

Test Results for MEWMA Chart of Variable A, B, C, D and E



TEST. Three points beyond control limits. Test Failed at points: 8, 26, 27

Based on the experience of these three constructed MEWMA control charts, we note that decreasing the assumed ARL will decrease the values of the UCL and increase the number of points where the observations will be out of control. These same or similar results occur when a sensitivity analysis is

done by varying values of the dampening coefficients.

Summary and Conclusions:

We studied the construction of MEWMA process control as it applies in bio surveillance. Our purpose was to indicate the usefulness of constructing MEWMA control charts under the condition where the cross-correlations among of set of observations of five variables produce results whereby these coefficients are often positive and shown to be significantly different from zero at low levels of the probability of a Type I error (significance level). When the construction of control charts are suggested, we find the results of such construction is that the dampening coefficient in the MEWMA process produces varying results. Further, these results are sensitive to the dampening coefficient employed with both the UCL varying and number of out of control points changing and becoming more diminutive as the coefficient increases as the ARL increases. Finally, we suggest a solution to the problem when then data also tend to Autocorrelated. We suggest a new way of construction multivariate charts under these conditions. Additional research is necessary to indicate the robustness and sensitivity of this alternative method to changes in the model parameters. Note, that although ARL is often the usual measure for multivariate chart performance, it is not the only criterion, and may have shortcomings. There is much to learn from using MEWMA in bio surveillance especially in the light of health and medical diagnostic processes which may have much great numbers of variables to consider when applications are merited. Alternatively, studies using other criteria such AD (average delay) or MRL (median run length) may prove superior in establishing the MQC decision point. In the future, Rare Event control charts may improve analysis and diagnosis which are now standard on quality control and improvement software. Last, standard office software such as Excel do not include these methods, hence,

specialized software programs such as demonstrated in this study will be employed in practice. To summarize, the elementary use of diagnostic analysis will not easily prevent and cure Covid-19. Thorough use of sophisticated analyses utilizing often quality control and continuous improvement will enable a better future for controlling the environment of the Coronavirus era that we are living under. The examples utilized in this document suggest better use of medical strategies to solve the problems of the current Pandemic. The response by the current administration in preparing the nation for the Pandemic has been unremarkable.

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