Review of Discrete Models of Financial Markets

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Books on finance with their mathematical underpinnings have created a new field of mathematical study entitled “Mathematical Finance.” Like Mathematical Economics, Mathematical Finance begins to explore how decisions are made by practitioners in a world where hunches and intuition from past behavior are explained in a manner that they can be understood without the personalities of financial decision makers. Stated differently, financial decisions are able to be analyzed scientifically and without the noise associated with the execution of decisions in financial markets. This book, by Marek Capinski and Ekkehard Kopp, is a product of two very wise mathematicians who possess prestigious backgrounds and are able to apply mathematics to solve difficult and complicated problems in financial markets whereby financial models are discrete.

The level of mathematics is at a high standard and, thus, would not be easily understood by graduate students in financial economics and business statistical modeling. This book is designed to be read by mathematicians at the masters level in mathematics whose goals are to solve serious and complicated problems not solved easily by the standard methods of statistics and econometrics. These conclusions are based on my consultation with various colleagues, including mathematicians at my university, who state that the book is written at a high level.

The future will decide how this book is used in master’s level mathematics programs or perhaps Ph.D. programs in Financial Economics and Statistics, which has become a much richer program in the past half century. The publisher and authors claim that their “simple” approach permits the parsing of many elements of financial decision making to account for viability, completeness, self-financing and replicating strategies, arbitrage, and equivalent martingale
measures. All of these are very practical and are related to the pricing and hedging options of both European and American types. Although the claim is made that the mathematics requirements include only undergraduate calculus, linear algebra, and elementary probability theory, the reader should be well endowed with these scholarship disciplines. I suggest that some background in financial economics would be appropriate so that the reader better understands the background of these problems which would motivate him/her to read further.

The book contains six chapters taking the reader from a short, but interesting, introduction to the intricate modeling of bonds and interest rates. The introduction states specifically that the nature of financial markets has become far more complex in the past four decades due to the introduction of derivative securities. Such securities are far more difficult to study than that of the simpler securities of the bygone era. We know now how the lack of regulation of derivatives and their associated methods of pricing led to the financial crisis in the United States and developed world. Although the authors do not speak of regulation of these securities, to create a level playing field for investors, as one reads the manuscript, he/she will acknowledge that the lack of transparency and the inclusion of imperfect markets led to the financial disasters of recent years. The second chapter introduces the mathematics of the binomial tree, options prices, derivatives, underlying securities, the trinomial tree, and the general single-step model. The underlying proofs are given if wanted by the reader. A casual reader, if there is such a person, may wish to overlook the proofs since they are there largely to complete the mathematical education. Chapter 3 begins to bring complexity into the financial decision making process by considering multistep binomial models. Remember, there will be no discussion of continuous models, which are left to another publication. The
discussion of the Cox–Ross–Rubenstein model extends the analysis to an n-step model. By the time one has finished studying the pricing of options, he/she will be introduced to the “Delta” coefficient for hedging problems.

In Chapter 4, the authors formulate a general market model based on the developments made in the previous chapters. They begin with a single-step binomial model in a finite sample space and move directly into finite multistep models with partitions of the sample space and other factors to bring the reader to multistock problems where they derive general theorems for asset pricing. These other factors were considered in the earlier chapters but are now used to describe the general problem associated with pricing a complex asset having many parts with different categories of investments. As one learns from reading the chapter, derivatives can and do become very complex and difficult to totally understand by the casual market investor. Proofs again are produced to indicate the complexities associated with the market models.

Chapter 5 discusses the methods by which one can compare the American options with the European options proposed in the earlier chapter (Chapter 4). The main difference between the European and American options concerns the time one can exercise the options. The European options prescribe the time one may implement the options; however, there is no time requirement for exercising the American options. Further research in this area will determine the costs and benefits of the difference in the two methods for exercising options. However, one now has the models to determine if differences have real results or not.

Chapter 6 no longer assumes that the money market contains the funds necessary to accomplish the financial strategies described in the earlier chapters. Since investors may hold funds in other instruments such as bond and similar assets, the authors explore the mathematical problems associated with such holding strategies, that is, they consider modeling bonds and interest rates.

Financial modeling in recent years erred greatly in predicting the results for the inefficient and inaccurate models designed to estimate and predict the
financial risks associated with newly developed financial investments. There are so many financial instruments today designed to satisfy the appetite of an extremely wide variety of investors with widely varying goals. Standard statistics coupled with financial instruments designed to ignore and not even consider all the complexities of investing were a significant cause of the financial collapse in 2007–2008. Regulation of the financial instruments often referred to as “swaps” and “derivatives” with values based on averages was misunderstood by inept buyers and sellers. Hence, the goal of mathematical finance to extend and explore our knowledge of financial markets and their associated decision-making contains elements that increase our knowledge of economic events. Social good is produced when we understand decision making better. Hence, the subject of this book is valuable to those beyond the field of mathematical finance. Although this book requires a high level of mathematics typically beyond the ability of academics in the field of finance studies in schools of business, the book would be very useful for mathematicians who are interested in applying mathematics to financial economics. Much would be learned concerning issues in the oversight of financial and insurance regulations that, in turn, can be legislated to avoid financial calamities. Obviously, we all gain by the increased knowledge of mathematical finance as presented in this book.

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