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A Hike Through the Forest: The Knapsack Problem in Graph Theory

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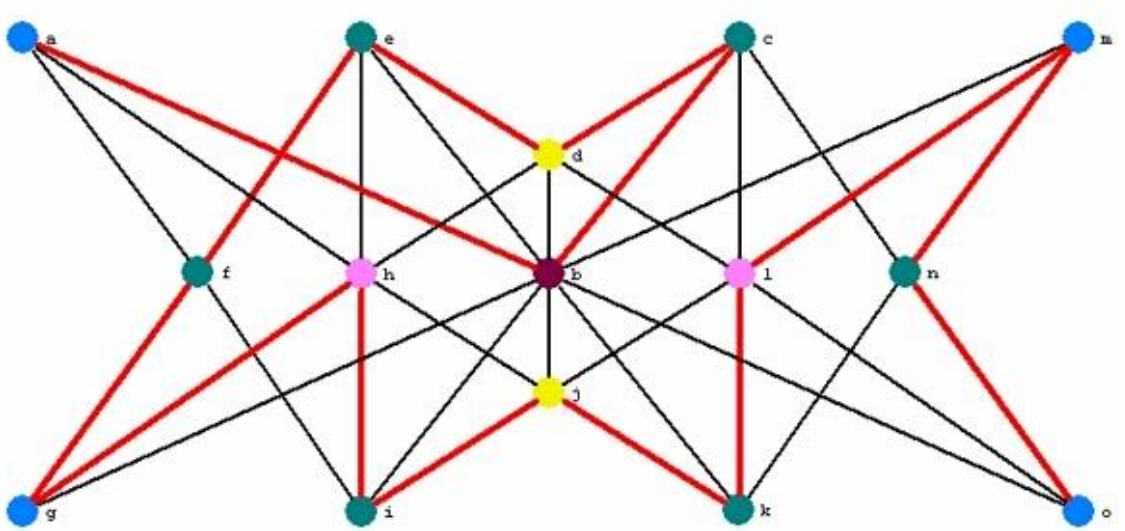
Graph Theory

Introduction

What is Graph Theory?

- Graph theory the study of graphs
- Graph - mathematical structures used to model pairwise relations between objects from a certain collection.



Applications of Graph Theory

Introduction

What is Graph Theory?

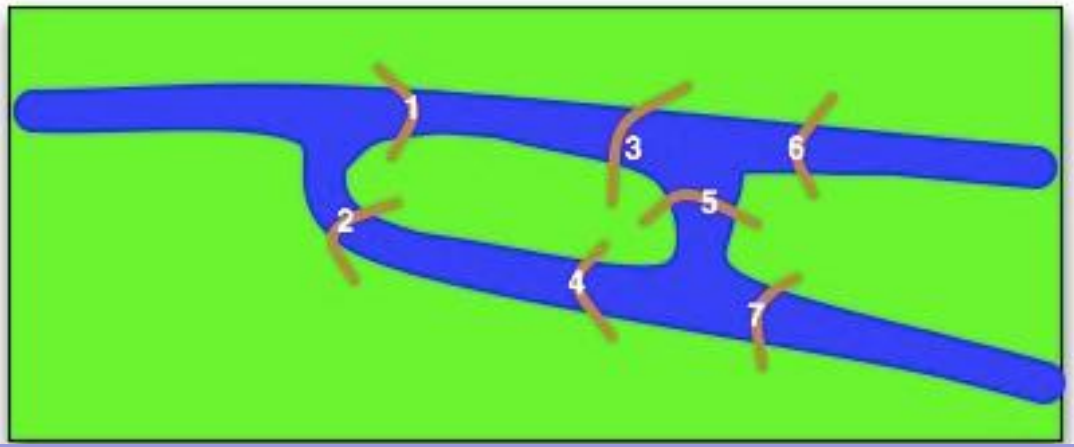
- Example: Six Degrees of Kevin Bacon trivia game started in 1994 by three college students. States that any actor can be linked to Kevin Bacon through their film roles. Goal of the game is to link the quickest as possible with the fewest links as possible.
- Take Elvis Presley. Elvis Presley was in Change of Habit 1969 with Eddie Albert. Eddie Albert was in The Big Picture 1989 with Kevin Bacon. Hence Eddie Albert has a Bacon Number 1 and Elvis Presley has Bacon Number 2.

The Beginning: The Seven Bridge Problem

History

What is Graph Theory?

- Pregel River flows through the city of Konigsberg in Eastern Prussia.
- People of Konigsberg would entertain themselves by trying to find a way to cross each of the seven bridges exactly once.



The Seven Bridge Problem

History

Cross each of the seven bridges exactly once?

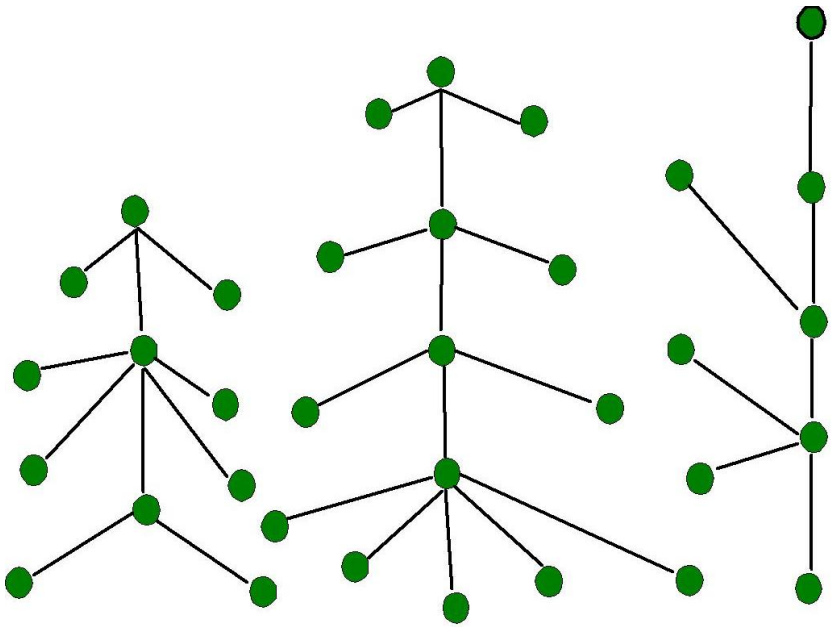
- Leonhard Euler, 1707-1783, Swiss mathematician and physicist
- Wrote an article addressing the problem and its solution
- He generalized into this question: whatever be the arrangement and division of the river into branches, and however many bridges there be, can one find out whether or not it is possible to cross each bridge exactly once?

Definitions

Information

Definitions of graph theory terms

- A *tree* is a connected graph that contains no cycles
- A *forest* is a collection of trees



Math Research?

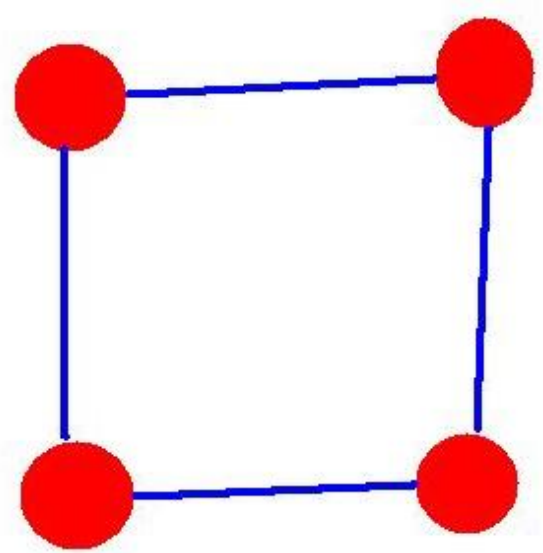
Research

- Read papers & problems
- Learn background information - MTH 447
- Select a problem or conjecture and see where it goes
- Programs such as Matlab and LaTex
- Graduate research

Along the trail

Research

There is no interval representation of the cycle on four vertices, C4



The Knapsack Problem

Research

The Knapsack Problem - combinatorial optimization problem
Given a set of items, each with a value and weight, maximize value of the backpack subject to a weight limit

$$\textit{maximize} \quad \sum_{j=1}^{n} p_j x_j \quad \textit{subject} \quad \textit{to} \quad \sum_{j=1}^{n} w_j x_j$$

$$x_j = 0 \quad \textit{or} \quad 1 \quad j = 1, 2, ..., n$$

Results in Matlab

Results

3 Matlab programs created in order to see how accurate my method is

- Binomial - n choose k objects, finds all backpacks possible for small n values
- Greedy Algorithm - keep taking most valuable items until maximum weight is reached
- Not-So-Greedy Algorithm - Greedy Algorithm, then throw out heaviest item, continue Greedy Algorithm

Matlab Program

Results

Imput for each program

1. Value vector
2. Weight vector
3. Maximum weight limit

Output

1. For binomial program, all possible outcomes
2. For Greedy and Not-So-Greedy, maximum value and its corresponding weight

Matlab Code

Results

Greedy Algorithm

```
[ sorted value, initial index ] = sort( value,'descend') ;
total W = 0;
for i = 1:n
        diff = MaximumWeight - total W;
        new value = sorted value(i) ;
        W = weight( initial index(i)) ;
        if W ¡= diff
                backpack(i) = new value;
                Weight(i) = W;
                total W = sum(Weight);
                last n = i;
        end
end
```

Matlab Experiments

Results

- Ran each of three programs with values, weights, and maximum capacities
- Recorded the backpacks & array of backpacks
- Compared Not-So-Greedy Algorithm vs. Greedy Algorithm

Matlab Experiments

Results

v - total value of backpack
w - total weight of backpack

Trial	Binomial	Greedy	Not-So-Greedy
1.) 5 items	v-20 , w-20	v-20 , w-20	v-20 , w-20
2.) 5 items	v-24 , w-20	v-24 , w-20	v-24 , w-20
3.) 5 items	v-59 , w-17	v-59 , w-17	v-59 , w-17
4.) 5 items	v-69 , w-25	v-60 , w-24	v-69 , w-25
5.) 12 items	v-78 , w-30	v-78 , w-30	v-78 , w-30
6.) 18 items	v-112 , w-45	v-96 , w-49	v-98 , w-50

Future Trails

Results

1. More effecient Matlab program
2. Repeat experiments with specific controls and variables, i.e. value, weight, and maximum capacity; number of items; average, standard deviation, variance
3. Create and compare other algorithms
4. Statistical methods to analyze data

Sources & Thanks

Sources

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