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Experimental and Numerical Study of a Wave Eneergy Harvesting Buoy

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EXPERIMENTAL AND NUMERICAL STUDY OF A WAVE ENERGY
HARVESTING BUOY

BY
JOHN MONTGOMERY

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
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2013

ABSTRACT

Since 2006, as part of projects funded by the Office of Naval Research (SBIRs and STTRs) and the State of Rhode Island (STAC, RI Alliance), the Department of Ocean Engineering at the University of Rhode Island (OCE) and Electro Standard Laboratories (ESL) have worked on the design, modeling (both computer and laboratory), construction, and testing of a series of small point absorber, multi-directional buoys for wave energy conversion. At full scale, these systems target up to 1kW of power per unit, mostly to power ocean instrumentation systems. Initially 4 design concepts were proposed. From these, two design concepts were selected for further examination. Design concept 2 (DC2) has a spherical float, to which a cylindrical canister is rigidly attached, and which houses a Linear Electric Generator (LEG; made of a permanent magnet, suspended to a spring, oscillating within a (two-phase) coil). A rod, attached to the generators magnetic armature, exits through the bottom of the canister and connects to a resistance platform. Differential movement between the float and the platform drives the generator oscillations. Design concept 3 (DC3) is comprised of a self-contained resonating multiple-spar buoy. A long central spar contains an LEG and is surrounded by four shallower satellite spars (satellite spars reduce draft, increase stability and help achieve proper resonance characteristics). The LEG has a large ballast attached to its bottom which oscillates as a result of buoy heave through coupled resonance. Hence, LEG oscillations are maximized by matching buoy heave and LEG natural periods to the targeted sea state peak spectral period. Successful prototype testing at the 1:10 scale (in the URI-OCE wavetank) and the 1:4 scale (in field tests performed in Narragansett Bay) have been performed for both buoys. These scale model experiments are used to calibrate parameters of a pre-existing numerical model (viscous drag coefficients) and select buoy characteristics to maximize energy

production.

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CHAPTER 1

Introduction

URI and ESL have collaborated on a project to design, numerically model and test a wave energy capturing system since 2006. Funded by the Office of Naval Research (ONR) and the State of Rhode Island, the project goal is to implement a wave energy harvesting buoy that is easily deployable and storm resilient to power marine surveillance and instrumentation systems. The system was designed with a target of extracting 1 KW of power in the 20 year average sea state in Rhode Island shelf waters. This design sea state has a significant wave height $H_s = 1.2m$ and peak period $T_p = 4.5s$. These spectrum parameters result in an energy density $J = 3.2KW/m$ of wave crest ([1]).

To accomplish the project requirements, four initial design concepts were created. Two of the initial concepts were selected for additional study (referred to as DC2 and DC3; Figs. 1, 2). 1 : 10 scale models were designed, built and tested in the URI wave tank. Numerical models were created to predict buoy behaviour in different sea conditions and model scales. Larger, 1:4 scale, models of both design concepts were constructed and tested in Narragansett bay in order to achieve the necessary scaled sea state of $H_s = 0.3$ m and $T_p = 2.25$ s. Unfortunately, sea conditions (H_s and T_p) during the larger model field tests that were performed prior to the start of this work were not appropriate for the models being tested. Therefore, using testing results to validate the numerical model could not be initially accomplished.

Both DC2 and DC3 capture energy through wave induced heave motion coupled to oscillations of a linear electric generator (LEG). DC3 is a resonating design in which the differential movement between the generator, rigidly mounted within

Figure 1. 1:4 scale DC2 buoy assembled in URI laboratory.

the buoy, and magnetic armature is driven by the inertia of the armature. The buoy is fully sealed and targets a specific wave frequency at which heave resonance is achieved. DC2 is a direct drive system in which a spherical float approximately follows the the sea surface. A generator is rigidly mounted within the canister attached to the float. A magnetic armature is attached to a resistance platform at the bottom of the buoy and connected to a system of springs within the canister. The platform's resistance to the vertical motion of the spherical float drives the power generating differential movement between the generator and magnetic armature. The spring system restores the system to equilibrium. The DC2 design targets a broader range of wave frequencies than DC3.

An appropriately calibrated numerical model will provide the ability to predict buoy dynamics, power production in the target sea state and power production at different buoy scales. The time and cost associated with design, construction and

Figure 2. Solidworks drawing of DC-3

testing makes the ability to predict performance numerically an invaluable asset in optimizing the design of future iterations of the buoy.

Calibration of the numerical model requires an accurate measurement of testing conditions. These sea state parameters are used as inputs for the model so that outputs can be compared to relevant field conditions. To determine the sea state, URI developed a waverider buoy to measure waves. The waverider buoy had not yet been proven to accurately determine sea conditions. Our industrial partner ESL purchased a commercially available sea state measuring buoy, from Datawell, that will be used to measure the sea state in future field tests. Additionally, the purchased buoy will be used to compare to sea conditions measured by URI's waverider and therefore used to validate previous testing results.

1.1 DC2 design concepts

The DC2 buoy is designed to generate power in a wide range of sea conditions. However, buoy design is tailored to achieve maximum power extraction in a design sea state (design sea state at varying buoy scales is determined by Froude scaling). Important design considerations include float diameter, total system mass, internal spring selection/resistance platform diameter and depth of the resistance platform. Specifically:

- The diameter of spherical surface float D_b is selected to maximize buoy heave motion at the peak spectral period of the design sea state T_p . This is achieved by treating the buoy as a rigid system (no internal spring allowing for differential movement between the resistance platform and float). Buoy resonance period T_b is calculated using a simple spring mass equation $T_b = 2\pi\sqrt{M_t/C_{33}}$ in which total mass M_t is the sum of buoy mass and heave added mass (frequency dependent) of both the float and resistance platform. C_{33} is the buoyancy restoring force and is calculated with Eq. (1) and assumes that the buoy has constant cross-sectional area A for small displacements near the buoy waterline and therefore has a buoyant force which is linearly proportional to displacement (comparable to a spring).

$$C_{33} = \rho g A \tag{1}$$

- Total buoy mass is selected so that at static equilibrium the waterline of the buoy is at the centreline of the spherical float. This allows for the maximum cross-sectional area to occur at the waterline and therefore the greatest change in buoyant force with oscillations from equilibrium.
- The resistance platform/internal LEG spring system is designed to have

a natural period T_s equal to the design sea state natural period. The spring/platform natural period is calculated with Eq. (2),

$$T_s = 2\pi\sqrt{\frac{M_p + M_{pa}}{K_s}} \quad (2)$$

with platform mass M_p , frequency independent platform added mass M_{pa} , and LEG spring constant K_s . Numerical simulations have shown that matching the natural period of the resistance platform/spring system to the wave forcing period can cause the resistance platform to move out of phase from the float. This increases the power generating stroke range, improving buoy efficiency compared with a bottom anchored system. Additionally, the static position of the resistance plate/armature must be in the center of the generator stroke range. This allows for maximum oscillations of the magnetic armature to be used in energy harvesting.

- The depth of the resistance platform must be sufficient to be below, or near the bottom of, the influence of wave induced water particle motions. Hydrodynamic forcing, driven by wave actions, on the spherical float and resistance platform are always in phase. Therefore, the two buoy sub-systems are forced in phase with each other which will tend to decrease the power generating differential movement between the float and resistance platform. Sufficiently submerging the resistance platform minimizes this in phase forcing between the float and resistance platform.

1.2 DC3 design concepts

The multi-spar buoy (referred to as DC3) is designed to resonate in heave at a target wave spectral peak period and therefore achieve significant power extraction over only a narrow range of wave frequencies. The buoy is comprised of

two subsystems: (i) the buoy body/hydrodynamic subsystem; and (ii) the internal spring/magnetic armature subsystem. Since the cross-sectional area of the buoy is constant near the waterline, the change in the buoyancy restoring force is linearly proportional to buoy heave displacement and thus is treated as a “spring”. The DC3 system is therefore represented in design as two coupled spring-mass oscillators. Hence, the main design consideration of the buoy includes matching the parameters of the two sub-systems to achieve the desired coupled system natural resonance period (the mass of the buoy, cross-sectional area of the buoy at the waterline, internal spring and armature/driving mass). This requires modeling the hydrodynamic efficiency of the buoy, and designing armature mass, armature/generator length and static displacement of the internal spring/armature. Specifically:

- Resonance of the coupled spring-mass system must occur at the peak spectral period T_p of the design sea state to achieve maximum power extraction under ideal conditions. Eqs. (3) and (26) predict the coupled system resonance ω_p based on buoy mass m_t (including added mass), buoyancy spring constant c_{33} , armature mass m_a and LEG spring constant c_g . Natural frequency of the buoy sub-system $\omega_b = \sqrt{c_{33}/m_t}$ and armature sub-system $\omega_a = \sqrt{c_g/m_a}$ are notations used to simplify Eq. 26,

$$c_{33} = \rho g S_{\perp} \quad (3)$$

$$\omega_p = \frac{\sqrt{(\omega_b + \omega_a)^2 + \frac{c_g}{m_t}} - \sqrt{(\omega_b - \omega_a)^2 + \frac{c_g}{m_t}}}{2} \quad (4)$$

where ρ is water density, g is gravitational acceleration and $S_{\perp}$ is buoy cross-sectional area.

- Hydrodynamic efficiency (i.e., reduced drag and added mass) is essential to achieving maximum buoy oscillations. The spars of DC3 were thus designed to maximize their length to width ratio and fitted with streamlined end cones to obtain the most efficient shape possible. The radial spars provide a mechanism to increase the inertia of the waterplane of the buoy while maintaining a hydro dynamically streamlined shape (the resulting increase in roll stability reduces drag that occurs when the buoy rolls and the flow is across buoy spars rather than along). The buoy is designed to move purely in heave; buoy roll dramatically increases hydrodynamic damping forces. A secondary function of the radial spars on DC3 is to reduce the buoy equivalent draft, thus allowing for a longer central spar that accommodates the LEG. Additionally, due to the buoyancy of the satellite spars, the central spar bottom nose cone is heavily ballasted, which lowers the center of gravity and increases weight, also further minimizing buoy roll.
- Maximizing armature mass of the DC3 buoy is important because the harvested mechanical power, transformed to electrical power, is proportional to it. Therefore, the maximum armature mass that is compatible with other design constraints of the buoy is utilized.
- Static displacement (i.e., initial displacement) of the armature/driving mass needs to be in the center of the generator in order to maximize the useful stroke of DC3.
- The length of the magnetic LEG armature and generator (as well as reasonable oscillation range of the internal spring) must be selected to ensure sufficient stroke range within the constraint of the length of the buoy central spar. The DC3 buoy is designed to achieve resonance of the internal

spring/armature system. Therefore, the armature needs to be able to oscillate with a range greater than the target wave height. Practical stroke range of the armature is the difference in length between the magnetic armature and generator. End course “bumpers” are included in the design to absorb shocks and limit armature course in case of large wave trains.

List of References

- [1] S. T. Grilli, A. R. Grilli, S. P. Bastien, R. B. Sepe, and M. L. Spaulding, “Small buoys for energy harvesting: Experimental and numerical modeling studies,” in *International Society of Offshore and Polar Engineers (ISOPE)*, 2011, pp. 598–605.

CHAPTER 2

Methods

2.1 Froude Scaling

The full scale wave energy harvesting buoys targets extracting about 1kW of power from a sea state with peak period of 4.5 seconds and significant wave height of 1.2 m ([1]). Previous work focused on 1:10 scale model testing. The current work on the wave energy harvesting buoys is concentrating on 1:4 scale models (a.k.a. mini-prototypes). To appropriately scale the buoy designs, Froude scaling is utilized. The Froude number is the ratio of inertial to gravitational forces (Eq. (5)) and is utilized here due to the buoy interaction with the air-water interface which is a floating body dynamics problem.

$$Fr = \sqrt{\frac{Inertia}{Gravity}} = \frac{V}{\sqrt{gl}} \quad (5)$$

where V is velocity, g is gravity and l is the length scale of the problem. To maintain proper scaling, the Froude number of the full scale prototype and model prototypes must be equivalent.

$$\frac{V_m}{\sqrt{gl_m}} = \frac{V_p}{\sqrt{gl_p}} \quad (6)$$

$$l_{\star} = \frac{l_p}{l_m}; t_{\star} = l_{\star}^{\frac{1}{2}}; m_{\star} = l_{\star}^3 \quad (7)$$

Therefore, the design sea state for the full scale, $\frac{1}{4}$ scale and $\frac{1}{10}$ scale buoys is given by Table 1.

Power scales with Eq. (8). Therefore, if the targeted power harvesting for the full scale buoy is 1 kW, the targets for the 1:10 and 1:4 scale buoys are .32 W and 7.8 W respectively.

Table 1. Design sea states for full scale, 1:4 scale and 1:10 scale models

Scale	H_s (m)	T_p (s)
1:1	1.2	4.5
1:4	.3	2.25
1:10	.12	1.42

$$P_{\star} = \frac{P_p}{P_m} = \ell_{\star}^{3.5} \quad (8)$$

2.2 Sea state prediction

As indicated, the 1:4 scale wave energy harvesting buoys are designed to target a significant wave height of .3 m and a peak period of 2.25 s. The wave tank at URI cannot reproduce this sea state and therefore the buoys require field testing. The complexity of organizing a field test of a large buoy necessitates the ability to predict sea conditions several days prior to the desired testing window. To find when/where the required conditions are likely to occur, a fetch limited sea state model was programmed utilizing Eq.'s (9, 10 and 11) (??). Where significant wave height H_{m_0} , peak period T_p and duration of wind conditions required to obtain the fully developed sea $t_{x,u}$ are predicted based on fetch X and wind speed at 10 m elevation (U_{10}) (friction velocity $u_{\star}$ is a function of U_{10} (Eq. 12)). Wind speed estimates were obtained using local weather forecasts. Fetch length was found through a combination of testing location selection and wind direction. Note that all length and time scales are in meters and seconds respectively.

$$H_{m_0} = 0.0413 \frac{u_{\star}^2}{g} * \left(\frac{gX}{u_{\star}^2} \right)^{\frac{1}{2}} \quad (9)$$

$$T_p = 0.751 \frac{u_{\star}}{g} \left(\frac{gX}{u_{\star}^2} \right)^{\frac{1}{3}} \quad (10)$$

$$t_{x,u} = 77.23 \frac{X^{0.67}}{u_{\star}^{0.34} g^{0.33}} \quad (11)$$

$$u_{\star} = U_{10}^2 \sqrt{0.001(1.1 + .035U_{10})} \quad (12)$$

It was determined that given the geometry of the west passage of Narragansett bay a wind from the north or south with an approximately velocity of 12 mph and a duration of at least 10 hours will achieve the desired sea state for testing the 1:4 scale buoy models.

2.3 Power

The goal of the wave energy harvesting buoys is to convert wave energy to stored electrical energy. This is accomplished by first converting wave energy to mechanical energy in the form of a magnetic armature moving relative to a generator. The total buoy efficiency is the ratio of generated electrical power P_e to captured wave power P_w available in the sea state over the width of the buoy. However, the mechanical efficiency of the buoy is the ratio of harnessed mechanical power P_m in the armature to the available wave power. The electrical efficiency of the generator can similarly be computed as the ratio of electrical power captured to the mechanical power available at the generator.

Wave power

According to linear wave theory, the mean power per meter of wave crest P_w is given by Eq. (78) and is equal to the product of mean wave energy E and wave group velocity C_g .

$$P_w = E C_g ; \quad E = \frac{\rho g H_s^2}{16} ; \quad C_g = \frac{c}{2} \left(1 + \frac{2kh}{\sinh 2kh} \right) ; \quad c = \frac{gT}{2\pi} \tanh kh \quad (13)$$

with $k = 2\pi/L$ (with wavelength L).

Symbol	Definition
μ_{LEG}	Generator coefficient
γ_{eff}	Speed dependant generator efficiency
D_{coil}	Average coil diameter
N_c	Number of magnetic pairs covered by stator
N_{turns}	Number of windings per slot
b_1	Magnetic flux density
γ_{slot}	Fraction of slot width containing windings
R_s	Internal resistance

Table 2. Definition of Linear Electric Generator (LEG) parameters

Assuming deep water wave conditions (i.e., $kh \geq \pi$ or $h \geq L/2$), which is acceptable for the relatively short wavelength considered here, we find for narrow banded wave spectra a power given by Eq. (79). The average wave power available to the harvesting buoy is the product of average wave power per meter of wave crest P_{w0} and the device width w (or capture width).

$$P_{w0} \simeq \frac{\rho g^2 H_s^2 T_p}{64\pi} \quad (14)$$

Mechanical power

The force exerted on the magnetic armature by the generator is linearly proportional to armature velocity $\dot{z}_a$. The ratio of electromagnetic force exerted by the generator to the velocity of the armature is the generator coefficient μ_{LEG} . The generator coefficient can be theoretically calculated based on parameters of the generator (Eq. (69)). The definition of the different generator parameters are summarized in Table 2. For each generator used in buoy field tests, a summary of the relevant generator parameters and resulting generator coefficient will be presented.

$$\mu_{LEG} = \frac{32 \gamma_{eff} (D_{coil} N_c N_{turns} b_1)^2 (1 + \cos(\pi - \frac{\pi \gamma_{slot}}{2}))}{\gamma_{slot}^2 (R_s + R_L)} \quad (15)$$

The mechanical power harnessed by the buoy is given by Eq. (75).

$$P_m = \mu_{LEG} \dot{z}_a^2 \quad (16)$$

Electrical power

Harnessed electrical power is computed with Joule’s law (Eq. (17)). For the December 15, 2011 test of DC2 and the June 5, 2012 test of DC3 both current and voltage were directly measured to compute harnessed power. Measuring both current I and voltage V was necessary because advance circuitry developed by ESL loaded the generator with a variable resistance. The May 1, 2012 test of DC2 utilized a known fixed resistive load on the generator and therefore only a voltage measurement was required to compute electrical power harvested (Eq. (18)).

$$P_e = IV \quad (17)$$

$$P_e = \frac{V^2}{R_L} \quad (18)$$

2.4 1:4 Scale DC2

The 1:4 scale DC2 buoy designed, constructed, and field tested (May 2011) as part of the STTR Phase II base period was retested in December 2011. In preparation for this test, the buoy’s instrumentation was re-calibrated (accelerometers, gyroscopes, current and voltage rectifiers) and some aspects of the data acquisition and transmission board fabricated by ESL were improved.

Sea state during the December 2011 test, however, turned out to be much more than expected and significantly beyond what the buoy had been designed for. This led to various overstressing and damaging of the buoy and some of its equipment and instrumentation. While 5 minutes of good data, confirming the

expected buoy performance, were acquired before some of the instrumentation failed, the bottom platform detached (being permanently lost) and the buoy float sustained permanent damage upon buoy retrieval (due to collisions with the ship A-frame; Fig. 3).

The buoy float was repaired (Fig. 4) and ruggedized by coating it in a stronger shell of polyester resin with fiberglass reinforcement. A new platform was designed and fabricated, as well as a platform stem (Fig. 5); a new stronger U-joint was fitted, with a system preventing rotation around the axis (and unscrewing, the likely reason for the loss of the previous platform) (Fig. 7). The armature-rod connection was similarly redesigned, ruggedized, and equipped with a system preventing rotation and unscrewing (Fig. 6).

To aid in deployment of the DC2 buoy, the lifting attachment point on the buoy was modified. The newly designed lifting connection effectively shortens the buoy and facilitated an easier deployment and recovery process (Fig. 9). Additionally, following the December 2011 testing, a new, more powerful and larger diameter Linear Electric Generator (LEG) designed by ESL was retrofitted into the DC2 buoy. This required modifying several design aspects of the DC2 buoy, including fabricating new generator support mounts (Fig. 10).

Figure 3. DC2 buoy damaged float following the December 2011 experiments.

Figure 4. Repair (left) and ruggedization (right) of the DC2 buoy following the December 2011 experiments.

Figure 5. Redesigned platform of the DC2 buoy following the December 2011 experiments.

Figure 6. Redesigned armature-rod connection of the DC2 buoy following the December 2011 experiments.

Figure 7. Redesigned U-joint of the DC2 buoy following the December 2011 experiments.

Figure 8. Redesigned generator support plates to accommodate larger LEG.

Figure 9. Redesigned lifting attachment point.

Figure 10. New generator support plates installed on the chassis.

Prior to fabrication and assembly, detailed Solidworks drawings were prepared for all the new or redesigned buoy parts. Note, a number of aspects of the new design were verified regarding maximum stresses and/or deformation, using standard mechanics of material. Figure 11 shows the full buoy assembly in Solidworks and Fig. 12 shows a picture of the actual assembled buoy. Figs. 13 to 16 show details of redesigned parts in Solidworks.

Figure 11. Solidworks of fully assembled DC2 buoy.

Figure 12. Fully assembled DC2 buoy during lab testing prior to field test.

Figure 13. Solidworks of Float assembly for DC2 buoy.

Figure 14. Solidworks of Spring assembly for DC2 buoy.

Figure 15. Solidworks of Generator assembly for DC2 buoy.

Figure 16. Solidworks of Plate assembly for DC2 buoy.

DC2 Instrumentation/Measurements

Buoy Rotation

To measure rotation rate about three orthogonal axis, gyroscopes have been mounted to the instrumentation board. The instrumentation board records ADC values for each of the gyroscope axis. The recorded ADC values are converted to meaningful units of rate of rotation (deg/second) through a calibration process (described in reference [2]). The angle the buoy is pitched relative to equilibrium (vertical) is obtained through integrating the rate of rotation. This measured roll/pitch/yaw angle can be used to transform the measured accelerations into true accelerations relative to a non-rotating frame of reference with the same process described in the URI waverider section of this document.

Buoy Acceleration

Buoy accelerations are measured with accelerometers in three orthogonal axis. The accelerometers are mounted to an instrumentation board installed inside the spherical float. The instrumentation board records an analog-to-digital converter (ADC) value which is linearly related to the acceleration. A calibration must be performed to convert the raw ADC value to a meaningful acceleration value (described in reference [2]). The measured accelerations are relative to the buoy and corrections must be accomplished using the angle of roll/pitch/yaw to determine accelerations relative to a non-rotating frame of reference similar to the process described in the URI waverider processing section presented previously.

Power Captured

The primary objective of this buoy project is to capture and store water wave energy. For the December 15th, 2012 DC2 test, quantifying the captured power is achieved through measuring voltage and current harvested and applying Joule's Law ($P = IV$). Both current and voltage are required measurements to compute

harvested power because the generator was subjected to a variable resistive load for the test. During the May 1st, 2012 test the generator was loaded with a constant resistance. Therefore, only voltage measurements were necessary for each generator phase to compute power. Power harvested by each generator phase ($i = 1, 2$) was then calculated with the equation $P_i = V_i^2/R_L$, where R_L is the resistive load on the generator.

Voltage The LEG housed inside DC2 is comprised of a two phase generator. For the December test, a voltage rectifier records an ADC value from the combined output voltage of both generator windings. During the May field test, each generator voltage was measured independently. Converting the recorded ADC values to volts requires a calibration process detailed in reference [2].

Current The current rectifier combines the current produced by the two phase LEG and records an ADC value proportional to current. The calibration to transform the raw ADC number to current in Amperes is achieved through a calibration process described in reference [2]. Current measurements were only obtained for the December field test.

Platform Position The movement of the resistance platform is determined through the use of two Hall effect sensors. These sensors measure changes in magnetic field caused by the movement of the magnetic armature past the sensors (embedded in the LEG). Hall effect sensors do not require calibration. Instead, both Hall effect sensor measurements (σ_1 and σ_2) are used in Eq. (29) to determine armature position z_a . The length of each magnetic pair l_p is 3 cm for DC2. N_c is the number of magnetic poles covered by the LEG stator, passing by the sensors. The armature is rigidly connected to the resistance platform and therefore

determining position of the armature fully describes the location of the platform relative to the buoy.

$$z_a = l_p \left(2 N_c + \frac{1}{\pi} \text{atan} \left\{ \frac{\sigma_1}{\sigma_2} \right\} \right) \quad (19)$$

2.5 1:4 Scale DC3

During a previous field test conducted in May 2011, the buoy behaved as expected during short bursts of proper excitation by incident waves near its resonant period. However, the sea state was not similar to the desired conditions and performance of the 1:4 scale buoy could not be evaluated. Additionally, the internal moving parts as well as some streamlining elements of the DC3 buoy (i.e., end nose cones) did not appear to be sufficiently ruggedized to endure repeated sea tests, as they suffered minor damage and leaks, respectively.

Accordingly, the work presented here has two main goals: (i) improving and ruggedizing the initial 1:4 scale DC3 design; and (ii) field testing the buoy in its target sea state (Ref. [3]). To do so, work during this project’s performance period focused on the following tasks:

- Redesign of buoy components:
 - Stiffen internal generator chassis.
 - Modify nose cones to provide structural reinforcement, allow for routing of accelerometer cables and support the antennae for wireless communication.
 - Add fendering to prevent damage from collisions due to extreme armature movement in the LEG.
 - Improve mounting of instrumentation systems and extend central spar to provide the necessary space for the electronics.

- Redesign the internal spring-mass system to better achieve the two-oscillator resonance.
- Field test the 1:4 scale DC3 buoy in its target sea state.

A rendering of the 1:4 scale DC3 buoy is shown in Fig. 17, with dimensions of major components given in Table 3. Mass and other properties of the buoy are detailed later in this report.

Figure 17. DC3 buoy with new chassis insert.

Buoy redesign

Internal chassis

The existing DC3 internal chassis (Fig. 18) design did not have sufficient stiffness to support the required loads, and hence easily twisted. This problem was attributed to inadequate stiffness of the longitudinal chassis rods resulting both from the small diameter of the threaded rods and their inadequate lateral stiffening,

Parameter	Value
Central spar length	351.8 cm
Central spar draft	229.5 cm
Central spar diameter	15.2 cm
Radial spar length	161.2 cm
Radial spar draft	80.6 cm
Radial spar diameter	15.2 cm
Nose cone length	26.7 cm
Cross-sectional area at water line	912 cm ²
Device total width	97.3 cm

Table 3. DC3 1:4 scale dimensions. Note: all spar lengths/drafts include nose cone(s).

using an insufficient number of plastic discs (to shorten the unsupported column length of the threaded rods). Additionally, the chassis rods were constructed of regular steel which was subject to corrosion and strongly attracted the magnetic armature, resulting in transverse loading of the rods.

Figure 18. Previous chassis assembly and close-up of stiffening plastic plates. Note the heavy corrosion of the regular steel rods in the former design.

The improved chassis was designed with larger diameter stainless steel rods (3/8th inch diameter stainless steel rods compared 1/4th inch diameter regular steel rods), which increases the area moment of inertia (I) and, therefore, increases the maximum critical buckling load. Increasing the diameter of the longitudinal rods increased the critical buckling load by a factor of 5. Stiffer aluminium discs

(i.e., non-ferromagnetic; Figs. 19 and 20) shorten the unsupported column length of the rods and restrain deflections of the threaded rods at each connection point. This significantly decreases the effective length of the column (L) and the column length factor (K), dramatically increasing the chassis critical buckling load (F_c). Based on its characteristics and Eq. (20), where E is Young's modulus, the new chassis is able to support an axial load of 4.86 kN without buckling, compared to an estimated 680 N for the earlier chassis. Additionally, the high grade stainless steel rods will have significantly less magnetic attraction to the armature than the previous mild steel components and will have increased corrosion resistance.

$$F_c = \frac{\pi^2 EI}{(KL)^2} \quad (20)$$

Figure 19. Internal chassis comprised of aluminium stiffening discs and stainless steel threaded rods.

Figure 20. (Left) Solidworks rendering of new DC3 Chassis. (Right) Fully assembled DC3 chassis ready for installation in the buoy.

Figure 21. Solidworks drawing of generator chassis.

Nose cone alteration

The lateral satellite spar nose cones were built out of plastic polymers, using a three-dimensional printer (via thermo-lithography). The resulting cones have a precise streamlined shape but are quite fragile and porous. To improve water tightness, the nose cones had previously been painted with a waterproof coating. During the May 2011 field tests, one of the bottom nose cones had been cracked, causing leaks and tilt of the buoy.

This design was improved by first using a two part epoxy system to seal up any previous damage done to the nose cones. Additionally, to provide structural reinforcement and protection against water leakage, if damage of the cone shells should still occur during testing (e.g., cracking), the bottom nose cones were filled with polyurethane foam (Fig 22).

Figure 22. Lower radial nose cone filled with foam.

Additionally, holes were drilled in two of the upper nose cones to allow for the installation of accelerometers in the lateral spars. Excess space surrounding the accelerometers cables was patched with silicone to provide a water resistant seal.

Finally, a hole was drilled in the top of the central nose cone to allow for a wireless antennae to protrude and maximize wireless communication range during

testing.

Figure 23. Upper radial nose cone with accelerometer cable prior to silicon waterproofing.

Armature fendering

The fragile nature of the LEG's magnetic armature results in the need for protection against collisions at both limits of the armature range of motion (corresponding to the maximum contraction and extension of the internal springs). In Fig. 24, we see that the spring connection cap would collide with the generator if the magnetic armature moved downward relative to the generator (causing increased springs system extension). To increase the robustness of the system, a neoprene pad was installed at the top of the central nose cone ballast, to provide fendering at the maximum extension of the spring system.

At maximum compression of the spring system, the armature driving mass was previously not protected from a collision with the generator mounting plate (a spring was in place, but did not have an initial static length, which was sufficient to prevent collision). A properly designed compression spring was selected to prevent a potentially damaging collision between the armature mass and generator mounting plate (Fig. 25).

Figure 24. Generator and spring connection cap

Figure 25. Compression spring to prevent collision between the top of the armature driving mass and generator mounting plate.

Instrumentation mounting/housing

A new instrumentation control board was designed and constructed by ESL to allow for wireless communication and data transmission with the buoy during field testing (Fig. 26). The previous electronics system was loosely placed inside the top central spar nose cone and was not rigidly attached to the buoy. In the new design, the instrumentation board is mounted to a foam block which is installed with Velcro to the inside of the central spar (Fig. 26). This Velcro installation provides a durable mounting connection for the electronics, as well as a convenient method for installation and removal, to facilitate any needed work on the system. As the new electronic board requires additional space, as compared with the previous

design iteration, a 6 inch aluminum cylinder extension was welded to the central spar tube (Fig. 27).

Figure 26. (Left) Instrumentation board mounted to a foam support. (Right) The instrumentation, battery pack, accelerometer and extra cables installed in the top of the central spar. The top nose cone fits over the top of the central spar to provide water protection and aid in mounting the antennae for wireless communication.

Redesign of internal spring

The 1:4 scale DC3 buoy is designed for the two-oscillator (i.e., buoy and LEG) system to have maximum resonance in heave at a period $T = 2.25$ s. Heave resonance for a spar buoy system depends on draft. As a result of the resonance period constraint and since the geometry of the existing buoy system is essentially fixed, so must be its total mass. Slight changes in total mass, however, are allowable as it will be shown that these will have only a slight effect on buoy draft and resonance period.

Figure 27. Welded central spar extension provides additional space for larger electronic system.

Additionally, the same LEG is used in the redesign, i.e., with an existing magnetic armature (and generator coil), with a connected driving mass. Because of the redesign (such as extension of the central spar and thickening of the internal chassis rods) that will slightly change mass, the driving mass may also be adjusted slightly in the re-design, as well as the internal spring stiffness, to achieve the desired power production and overall system resonance. The full system design process is outlined in reference [3]. The problem presented here is simplified, compared to the full design process, due to the constraints of using an existing buoy; however, an overview of the procedure is presented below.

Resonant Conditions

The DC3 buoy is a two-resonator heaving body, comprised of an internal spring-mass system coupled to the motion of the buoy body float. Neglecting frictional and hydrodynamic damping, this system can be idealized as two coupled spring-mass oscillators. The buoy can be treated as a spring-mass system due to the constant cross-sectional area over the expected range of buoy drafts, providing a constant buoyancy restoring force per unit draft (equivalent to a spring stiffness): the constant cross-section causes the change in buoyant force to be linearly proportional to change in draft. The buoy "spring" constant relating the change

in draft to the change in buoyant force (c_{33}) is given in Eq. (21) (where ρ is water density, g is gravitational constant and $S_{\perp} = S_0 + 4S_1$ is the total cross-sectional area (including satellite spars) (S_0 is the cross-sectional area of the central spar and S_1 is the cross-sectional are of each lateral spar).

$$c_{33} = \rho g S_{\perp} \quad (21)$$

$$F = c_{33} z_b \quad (22)$$

where z_b denotes the buoy heave motion. The natural heave frequency of the buoy sub-system is calculated using a general spring-mass resonant equation (Eq. (23)).

$$\omega_b = \sqrt{\frac{c_{33}}{m_t}} \quad (23)$$

where the total buoy mass (m_t) is the sum of the buoy mass (m_b) and heave added mass (m_{33}) ($m_t = m_b + m_{33}$), including half of the internal spring mass, but excluding the magnetic armature. The added mass is assumed to be very small and nearly frequency independent for a spar buoy. The natural frequency of the internal spring mass system is ω_a is given by,

$$\omega_a = \sqrt{\frac{c_g}{m_a}} \quad (24)$$

in which c_g is the LEG spring constant and m_a is armature mass (including half the internal spring mass). Hence, we have,

$$\frac{\omega_a}{\omega_b} = \sqrt{\frac{m_t c_g}{m_a c_{33}}} \quad (25)$$

It is desirable to match the full system resonance period to that of the expected driving force, corresponding to the peak wave spectral period of the target sea state ($T_p = 2.25$ s), to maximize vertical movement of the internal magnetic armature

Table 4. Fixed Buoy Parameters

T_p	2.25 s
ω_p	2.7925 $\frac{\text{radians}}{\text{s}}$
c_{33}	908 $\frac{\text{N}}{\text{m}}$
m_{33}	4.6 kg

and therefore optimize power production. The coupled system resonance (ω_p) is given by Eq. (26) (see Eq. 11.2d in reference [3]).

$$\omega_p = \frac{\sqrt{(\omega_b + \omega_a)^2 + \frac{c_g}{m_t}} - \sqrt{(\omega_b - \omega_a)^2 + \frac{c_g}{m_t}}}{2} \quad (26)$$

Design Considerations

Armature spring constant (c_g), armature mass (m_a), and buoy mass (m_b) are parameters which can be varied during redesign to achieve the desired system resonance characteristics. Since the buoy external dimensions could not be changed, armature mass can only be slightly modified and, most of all, must fit within the space afforded by buoy geometry. Total buoy mass is restricted by the requirement of maintaining a buoy draft very close to the previous value. Besides achieving the proper heave resonance, the buoy draft was such that the waterline with respect to the lateral spars allowed for maximum buoy oscillations, while maintaining constant cross-sectional area (i.e., lateral spar tips did not exit the water). Eq. (24) shows that the LEG natural armature frequency is dependent on spring constant and system mass.

In redesigning the buoy, varying armature masses were plotted against different ratios of spring/buoy subsystem resonance frequencies (ω_a/ω_b). The plots were generated using an approximate buoy mass of $m_b = 98$ kg. This mass estimate was computed by measuring the buoy components which are existing and reused, and estimating the components, which have not been constructed yet (using a

Solidworks model of the buoy). As a result, an additional 4 kg were added to the estimated buoy mass to account for buoy design aspects, which were still unknown at the time of spring selection (i.e., spar extension, thicker chassis rods, mass of instrumentation, radial spar foam plugs, etc...).

Figure 28. Complete system Resonance Frequency vs armature mass for different ratios of internal spring-mass subsystem to buoy natural frequencies. $\omega_a/\omega_b = 1$ (solid blue), 1.1 (solid green), 1.2 (solid red), 1.3 (solid teal), 1.4 (solid purple) and 1.5 (solid yellow). Note that the horizontal black line represents points at which the coupled system natural frequency matches the target sea state ($\omega = 2.79$ rad/s or $T=2.25$ s)

Based on Fig. (28), an armature mass $m_a = 6.7$ kg and $\omega_a/\omega_b = 1.3$ were selected to achieve the desired system resonance frequency ($\omega_p = 2.79$ rad/s). From the fixed buoy parameters (and Eq. (23)), the natural frequency of the buoy subsystem is $\omega_b = 2.98$ rad/s. This results in an internal LEG subsystem natural frequency of $\omega_a = 3.87$ rad/s. Therefore, for the selected buoy parameters (and Eq. (24)), the spring constant needs to be $c_g = 100.3$ Nm. Note, the existing DC3

Table 5. Desired Buoy Parameters

m_b	98 kg
ω_b	2.98 rad/s
T_b	2.11 s
ω_a	3.87 rad/s
T_a	1.62 s
m_a	6.7 kg
c_g	100.3 N/m

buoy had an armature mass $m_a = 5.8$ kg and therefore only a slight modification of armature mass is required. A summary of desired system parameters is presented in Table (5).

It is difficult to locate a commercially available spring with the desired spring constant. Therefore, an iterative procedure is required to modify flexible buoy dimensions/properties to achieve the desired system resonant period with the an available spring system. Multiple springs were used in series to achieve system parameters close to the desired spring constant $c_g = 100.3$ N/m. Four springs in series available through mcmastercarr.com were thus combined to nearly achieve the desired spring constant value. The spring constant for four springs in series is given by,

$$c_{g_{eff}} = \frac{c_{g1}c_{g2}c_{g3}c_{g4}}{c_{g1}c_{g2}c_{g3} + c_{g1}c_{g2}c_{g4} + c_{g1}c_{g3}c_{g4} + c_{g2}c_{g3}c_{g4}} \quad (27)$$

The combination of four springs in series (Table (6)) yields a $c_{g_{eff}} = 102.5$ N/m (i.e., very close to the target spring constant). Individual spring tensions (T_i) and lengths (L_i) fare also given in Table (6). Note that the spring system is comprised of two sets of identical springs (Fig. 29).

Despit being close, the combined spring system does not have an identical spring constant to the desired condition ($c_{g_{eff}} = 102.5$ N/m compared to $c_g =$

Table 6. Spring System

c_{g1}	359 N/m
L_1	0.2032 m
T_1	17.1 N
c_{g2}	359 N/m
L_2	0.2032 m
T_2	17.1 N
c_{g3}	478 N/m
L_3	0.2032 m
T_3	22.5 N
c_{g4}	478 N/m
L_4	0.2032 m
T_4	22.5 N
$c_{g_{eff}}$	102.5 N/m

Figure 29. Four main springs to be connected in series.

100.3 N/m). Hence the buoy free parameters were adjusted this effective spring constant, i.e., buoy and armature mass were slightly modified to achieve the desired resonant conditions. The final (slightly altered) parameters and resulting resonant conditions are summarized in Table (7). Modification of the existing armature driving mass was accomplished with the addition of a small steel plate (Fig. 30). Minimal alteration of the buoy was required to match the desired mass.

Finally, an additional system requirement is for the spring system to have an initial (static) deformation, which will fit into the central spar of the buoy while allowing for the expected full armature range of motion. Also, the spring

Table 7. Buoy Parameters

c_{geff}	102.5 N/m
m_a	6.83 kg
ω_a	3.87 rad/s
T_a	1.62 s
m_b	97.75 kg
m_{add}	4.6 kg
c_{33}	908 N/m
ω_b	2.98 rad/s
T_b	2.11 s
T_{sys}	2.25 s
ω_{sys}	2.79 rad/s

Figure 30. Original (brass) armature driving mass with additional steel plate to increase mass.

system should be in tension at the upper limit of the armature range to prevent spring entangling. To obtain a static deformation (z_{a0}) of the spring/armature system, armature mass, spring initial tension (T_i), spring constant and spring initial length (L_i) are required (see Table (6)). The static deformation is computed using Eq. (28) for a multiple spring system. For the selected parameters, the static deformation of the spring system under the armature weight is $z_{a0} = 1.27$ m. As the existing configuration of the buoy geometry allows for a maximum static displacement of 1.4 m, this selected spring system fits within the spatial limits of the buoy. Currently, the armature stroke length is 0.62 m, which indicates that the spring system must be under tension at 0.98 m spring length in order to utilize

the full range of armature movement. The unloaded length of the four springs is 0.81 m, which confirms that the spring system will be in tension at full spring contraction.

$$z_{a0} = \sum_{i=1}^4 \left\{ L_i + \frac{m_a g - T_i}{c_{g_i}} \right\} \quad (28)$$

Instrumentation and measurements

DC3's instrumentation system was designed and constructed to measure buoy movement, orientation, armature/driving mass movement and parameters of the generated electrical power. Buoy accelerations are measured with three accelerometers each with three orthogonal axes. Accelerations are measured relative to the orientation of the buoy. It was anticipated that differential acceleration between the accelerometers (combined with the separation of the accelerometers) would be used to determine buoy rotations about its three principal axes. However, the slight errors present in the accelerometer data made the measured differences in accelerations between the instruments unreliable when double integrating (to get angles of rotation). Additionally, video from testing revealed that buoy roll/pitch was insignificant. Therefore, the analysis of DC3 buoy movement is restricted to translational movement (primarily in heave).

Quantifying the harvested power is achieved through measuring the current I and voltage V produced by the combined two phases of the generator (with $P = IV$). To maximize generator efficiency, the DC3 instrumentation system provides a variable resistive load on the generator and therefore requires both current and voltage measurements to calculate power and resistance ($R = V/I$). Armature/driving mass position is measured through the use of 2 Hall effect sensors.

Figure 31. Plan view of location of DC3 accelerometers. Location of accelerometers are given in Table 8 relative to the central accelerometer

DC3 instrumentation

The three accelerometers (labelled J6, J7 and J8 due to the pin connections on the instrumentation board) mounted on the DC3 instrumentation board record ADC values, which are linearly proportional to acceleration. Following instrument calibration (reported elsewhere) coefficients are applied to convert these values to acceleration in g . For the three accelerometers on the DC3 buoy the calibration coefficients are given in Table 9, where the calibration is given for each accelerometer relative to the local orientation of the instrument (x, y, z) (e.g., A_x^{J6} , A_y^{J6} and A_z^{J6} , for the J6 accelerometer).

The instrumentation system combines the current and voltage output for both phases of the generator, so calibration of only a single equivalent voltage (V) and

Accelerometer	x (m)	y (m)	z (m)
A^{J6}	0.46	0.46	-0.42
A^{J7}	0	0	0
A^{J8}	0.46	-0.46	-0.66

Table 8. Location of DC3 accelerometers. All dimensions are given relative to the central accelerometer (A^{J7})

current (I) is required. Hall effect sensors do not require calibration. Instead, both Hall effect sensor measurements (σ_1 and σ_2) are used in Eq. (29) to determine the armature position z_a . In the equation, l_p is the length of each pair of pole in the armature (1 cm for DC3), N_c is the number of pair of poles covered by the LEG stator, passing by the sensors.

$$z_a = l_p \left(2 N_c + \frac{1}{\pi} \text{atan} \left\{ \frac{\sigma_1}{\sigma_2} \right\} \right) \quad (29)$$

Instrument	Scale	Offset
A_x^{J6}	.000114339	-2.53341
A_y^{J6}	.00011572	-2.60285
A_z^{J6}	.000112616	-2.5596
A_x^{J7}	.0001169954	-2.63451
A_y^{J7}	.000114797	-2.65159
A_z^{J7}	.000113791	-2.60763
A_x^{J8}	.00011469958	-2.605674
A_y^{J8}	.00011467478	-2.601119
A_z^{J8}	.0001125301	-2.558296
V	-.00814765	25.52
I	.000809588	-.0027

Table 9. Calibration coefficients for DC3 instruments. Coefficients for acceleration measurements (A_k^{Ji}) provide results in g 's, voltage outputs from the LEG circuits (V) in Volts and current outputs from the LEG circuits (I) in Amps. The equation to calibrate each instrument is given by Output = Input*Scale - Offset.

2.6 Sea State Measuring Buoys

URI developed a waverider buoy to quantify sea state during field tests of the wave energy conversion devices. The data analysis procedure, relevant equations and practical considerations for processing data collected by the waverider buoy will be discussed.

2.6.1 Spectral Parameters

The sea conditions are quantified by three parameters: significant wave height H_s , peak period T_p and JONSWAP peak enhancement factor γ . For deep water, significant wave height is calculated through a relationship with the zero-th order moment of the energy spectrum m_0 , Eq. (30).

$$H_s = 4\sqrt{m_0} \quad (30)$$

The zero-th order moment of the energy spectrum is calculated by way of integrating the energy spectrum $S(\omega)$ (Eq. (31)). The JONSWAP peak enhancement factor is estimated by visually comparing the measured sea state spectrum with a JONSWAP spectrum having the same H_s and T_p .

$$m_0 = \int_0^\infty S(\omega) d\omega \quad (31)$$

2.6.2 URI Waverider Buoy

Waverider buoy acceleration and rates of rotation are measured using a 3-axes accelerometer and 3 axes gyroscope rigidly mounted on the instrumentation board within the buoy. The recorded data was processed using a Matlab function previously developed at URI. Converting the measured buoy accelerations and angular velocities to a wave spectrum requires the following steps:

- Convert measured buoy acceleration to true acceleration relative to a non-

rotating frame of reference. Correct for measured accelerations induced by the effect of buoy rotation.

- Determine buoy orientation from gyroscope angular velocity data.
 - Remove rotation induced accelerations from accelerometer data.
 - Calculate vertical acceleration of buoy by projecting measured accelerations to true vertical
- Determine and apply a linear transfer function to buoy acceleration spectrum to obtain wave spectrum.

Determine True Vertical Buoy Acceleration

The waverider buoy measures and records accelerations and rates of rotation (angular velocity) of the buoy with a data acquisition system comprised of a three axes accelerometer and a three axes gyroscope. The recorded data is relative to the instantaneous orientation of the buoy. This necessitates converting the measured accelerations to a non-rotating frame of reference. Additionally, because the instruments are not located at the center of buoy rotation, any rotation of the buoy induces accelerations in the accelerometers. To isolate heave acceleration it is necessary to remove these rotation induced accelerations.

Buoy Orientation Buoy orientation is conveniently described by Euler angles. Euler angles are a system of three angles which describe the orientation of a rotated reference frame to an inertial frame of reference. Many different conventions/notations exist in selecting Euler angles, order of operation and naming convention. In this paper the instantaneous frame of reference of the buoy will be described by xyz and the fixed reference frame by XYZ . Buoy rotations will be initially defined by α for rotations about z , β for rotations about y and γ for

rotations about x (see figure (32)). Rotations about frame of reference xyz are easily accomplished by way of simple time integration of angular velocity about each respective axes $(\omega_x, \omega_y, \omega_z)$.

Figure 32. Euler Angles

$$\alpha = \int \omega_z dt \quad (32)$$

$$\beta = \int \omega_y dt \quad (33)$$

$$\gamma = \int \omega_x dt \quad (34)$$

The process of projecting accelerations from the xyz coordinate system onto the Z axis is simplified if an additional angle, ψ , is introduced. Due to the radial symmetry of the waverider buoy, combining roll and pitch into an angle of declination is useful. Therefore, ψ is introduced as the angle of declination from the vertical axis, Z , to the buoy axis, z .

The sequence in which the Euler angles are processed produces a unique solution except at points of singularity. Computing rotations about the z (α) axis followed by simultaneously computing rotations about the y (β) and x (γ) axes (sequence presented above) produces singularities at angles of $\beta, \gamma > \frac{\pi}{2}$. A rotation

of $\pi/2$ would indicate that the buoy had rolled/pitched to horizontal. It is assumed that due to the inherent stability of the buoy that such rotations will not occur.

Project Buoy Accelerations to Inertial Reference Frame

The projection of buoy accelerations measured relative to the rotated frame of reference, xyz , to the Z axes is accomplished with equation (35), where g is gravitational constant and measured accelerations in the rotating frame of reference (xyz) are denoted by a_x, a_y, a_z .

$$A_{Z_{measured}} = [-\cos(\theta)\sin(\psi)a_x + \sin(\theta)\sin(\psi)a_y + \cos(\psi)a_z]g \quad (35)$$

Remove Effect of Buoy Rotation on Measured Accelerations

The instruments (accelerometers and gyroscopes) are rigidly mounted to the waverider buoy within the spherical float. The calculation of center of rotation ($\bar{AC}$) is accomplished later in this report. However, a spatial separation between center of rotation and location of the instruments exists. This separation does not effect the measurement of angular velocity; however, separating the accelerometers from the center of rotation causes accelerations to be measured which are induced by buoy rotation and not heave. To isolate buoy heave motion, these rotation induced measured accelerations can be removed from the vertical component of acceleration by equation (36). Additional heave accelerations are induced in the buoy due to the separation of center of buoyancy and center of rotation. The effect of this separation is not accounted for in data processing and results in the waverider buoy being unsuitable for high frequency/low period wave measurements. This analysis/discussion will be presented in a later section of this document.

$$A_{Z_{induced}} = \bar{RC}\cos(\psi)(\sin(\alpha)\dot{\gamma}) + \bar{RC}\sin(\psi)(\sin(\alpha)\ddot{\gamma} + \cos(\alpha)\ddot{\beta}) \quad (36)$$

$$A_{Z_{true}} = A_{Z_{measured}} - A_{Z_{induced}} \quad (37)$$

Linear Acceleration Transfer Function

The power spectral density (PSD) of true buoy acceleration $A_{Z_{true}}$ is computed using Welch's method. Reasonable period/frequencies at which the spectrum should be truncated to achieve the most reliable results will be discussed below. Note that additional frequency resolution of the PSD can be obtained by lengthening the window of data used in generating each periodogram. However, lengthening the window of data used to compute each periodogram results in increased variance in the final power spectrum. It is therefore recommended that the windowing of data in Welch's method be carefully selected to achieve sufficient frequency resolution to obtain a clear indication as to reasonable cut-off frequencies for spectral truncation while minimizing variance.

The true vertical acceleration spectrum is converted to a wave spectrum by way of a linear heave transfer function (Fig. (33)). This linear transfer function is computed by solving a complex linear equation for buoy motion in heave assuming periodic incident waves with amplitude A and frequency ω (Eq.(38)) (see reference [1] for additional details). Where $(M + a_{33})$ is buoy effective mass (buoy mass plus added mass), $(b_{33} + b_{f,33}^l)$ is radiative damping plus linearized friction, (c_{33}) is buoyancy coefficient and (χ_3) is complex buoy heave.

$$[-\omega^2(M + a_{33}) + i\omega(b_{33} + b_{f,33}^l) + c_{33}]\chi_3 = Ar e^{i\psi_3} \quad (38)$$

$$\overline{W_{b_{f,33}^l}} = \int_0^T b_{f,33}^l \dot{z} = \overline{W_{b_{f,33}}} = \int_0^T b_{f,33} \dot{z} |\dot{z}| \quad (39)$$

Buoy position, velocity and acceleration are given by z , $\dot{z}$ and $\ddot{z}$ respectively.

$$z = z_0 e^{i\omega t}; \dot{z} = i\omega z_0 e^{i\omega t}; \ddot{z} = -\omega^2 z_0 e^{i\omega t} \quad (40)$$

The resulting relationship between non-linear and linear friction terms are shown in equation

$$b_{f,33}^l = \frac{8\omega z_0 b_{f,33}}{3\pi} \quad (41)$$

Solving for χ_3/A produces the frequency dependant heave response amplitude operator H_{tran} (equation (42)). The frequency dependant coefficients are calculated using WAMIT; float added mass a_{33} , radiative damping coefficient b_{33} , buoyancy coefficient c_{33} . The linearized friction coefficient $b_{f,33}^l$ is calculated by setting the average dissipated power of linearized friction over one wave period equal to the average dissipated power of non-linear friction over the same time interval (one wave period)(see equation (39)).

$$H_{tran} = [c_{33} - \omega^2(M + a_{33})^2 + \omega^2(b_{33} + b_{f,33}^l)^2]^{-\frac{1}{2}} \quad (42)$$

It should be noted that the determined transfer function is linear. Therefore, the buoy response is over predicted near the buoy natural frequency. In the relevant application this result in an under prediction of waves in the determined sea state around 1.4 second period (.71 Hz). Natural frequency occurs at a frequency (ω) given by Eq. (43).

$$\omega = \sqrt{\frac{c_{33}}{M + a_{33}}} \quad (43)$$

Converting the heave transfer function to an acceleration transfer function is accomplished by multiplying the heave transfer function by the square of angular frequency (Eq. (44)). This is due to the relationship between position and acceleration (see Eq. (40)).

Figure 33. Heave Acceleration Transfer Function

$$A_{tran} = -\omega^2 H_{tran} \quad (44)$$

This results in an amplification of errors at high frequency. Therefore, the spectrum should be truncated to remove the high frequency components. A specific cut-off frequency at which the spectrum should be truncated is unknown. For practical purposes spectral truncation should occur at frequencies with insignificant buoy accelerations or inherent error. An additional source of error at high frequency is the heave induced by buoy rotation. The separation between center of rotation and center of buoyancy causes the buoy to bob at a frequency double the natural roll frequency of the buoy. This is caused by the buoyant force which acts to restore the buoy to equilibrium orientation (vertical). The dynamics of the buoy rolling cause the spherical float to partially submerge, this induces a greater buoyant force than at equilibrium. This increased buoyant force is cre-

ated at both positive and negative angles of roll/pitch and therefore has double the natural roll frequency of the buoy (occurs twice for each complete roll cycle). Note that the natural roll frequency is not the same as natural heave frequency. Additional information on buoy heave accelerations caused by roll are presented later in this document. The frequency of roll induced vertical heave accelerations are higher than the natural heave frequency of the buoy. Therefore, the practical high frequency/low period limit that the waverider buoy is suitable for measuring is caused by the inaccuracies of the linear transfer function and not the dynamic coupling between heave and roll movement. The high period/low frequency limit for the waverider buoy is caused by the small value of the transfer function at long periods. This results that any energy at these low frequencies/high periods will be amplified by the transfer function. It is therefore desirable to truncate the energy spectrum at periods above which significant buoy accelerations occur.

Roll test

To verify that waverider rotations are inducing heave accelerations, roll tests were performed in the University of Rhode Island acoustic tank. The buoy was rolled to different small angles (5° and 8°) and allowed to freely oscillate in the tank. Fig. 35 displays the angle of roll for one of the 8° roll tests. The power spectral density of true vertical acceleration was computed (see Fig. 36). The dominant period of buoy heave for the roll test is around 1 second (results for all test repetitions are comparable) indicating that roll is the mechanism responsible for inducing heave accelerations in the 1 Hz frequency range. The period of roll was also used in the computation of center of rotation.

Figure 34. Acceleration PSD for URI waverider buoy during December 15, 2012 field test. Note large energy concentrations near 1 second (due to heave induced by buoy roll).

Calculate center of rotation of the waverider buoy

The location of the center of rotation of the waverider buoy is a needed dimension in order to remove rotation induced accelerations during data processing. The separation between the location of the accelerometers and the point about which the buoy rotates, in conjunction with the rate of rotation, enables the removal of the induced accelerations. See Fig. (37) for geometry of waverider buoy and Table (10) for description of labels in the figure. Note that the center of buoyancy was found with a Solidworks model of the waverider buoy. Center of mass/gravity was determined by hanging the buoy horizontally to find the longitudinal balance point. Instrument location was measured. The center of added mass was determined by the assumption that only the spherical float of the buoy would contribute hydrodynamic added mass. Therefore, the center of added mass was estimated as

Figure 35. Waverider angle of declination for 8° roll test

the center of buoyancy for the submerged hemisphere (assuming the buoy floats at the mid line of the sphere).

An equation to estimate the natural roll period was developed beginning with the rotational formulation of Newton's second law (Eq. (45)). I is the rotational inertia of the buoy about the center of rotation (including the hydrodynamic effect of added mass) and ψ is the buoy angle of declination from vertical.

$$\tau = I\ddot{\psi} \quad (45)$$

The torque (τ) on the body acting to restore the body to equilibrium (upright position) is given by Eq. (46). The restoring torque on the body is treated as a coupling between gravitational and buoyant forces. The two forces are equal in magnitude (due to no net vertical acceleration) and opposite in direction. The

Figure 36. Acceleration PSD of waverider during 5° roll test

gravitational force acts through the buoy center of gravity. The buoyant force acts through the buoy metacenter. Location of the buoy metacenter relative to the center of buoyancy $\overline{BM}$ is computed with Eq. (47), in which V_{sub} is the submerged volume of the waverider and I_{WL} is the second moment of area of the water plane. The second moment of area for a body with a circular cross-sectional at the water plane is given by Eq. (48). This equation for the location of the metacenter is only valid for small angles of roll/pitch. The small angle simplification is also made to the formulation for torque in that $\sin(\psi) = \psi$.

$$\tau = m_b g \overline{GM} \sin(\psi); = m_b g \overline{GM} \psi \quad (46)$$

$$\overline{BM} = \frac{I_{WL}}{V_{sub}} \quad (47)$$

Figure 37. Schematic of URI developed waverider buoy.

$$I_{WL} = \frac{\pi D^4}{64} \quad (48)$$

$$\overline{GM} = \overline{GB} - \overline{BM} \quad (49)$$

The formulation for torque is substituted into Eq. (45) resulting in the differential equation presented in Eq. (50). This is an equation for a harmonic oscillator with no damping. The resulting solution for the angle of roll/pitch (ψ) is given in Eq. (51).

$$0 = I\ddot{\psi} - m_b g \overline{GM} \psi \quad (50)$$

$$\psi = \psi_{max} \cos(\omega_n t); \dot{\psi} = -\psi_{max} \omega_n \sin(\omega_n t); \ddot{\psi} = -\psi_{max} \omega_n^2 \cos(\omega_n t) \quad (51)$$

Table 10. Waverider properties

Buoy mass	m_b	119 kg
Submerged volume	V_{sub}	.119 m^3
Spherical float diameter	D	.763 m
Second moment of neutral water plane	I_{WL}	$\frac{\pi D^4}{64} m^4$
Buoyant/gravitational force	$m_b g$	1167 N
Reference location (origin)	O	center of buoy
Location of instruments	S	.19 m
Center of added mass	M_a	-.14 m
Center of buoyancy	B	-.23 m
Center of gravity	G	-.99 m
Center of gravity to center of buoyancy	$\overline{GB}$	.76 m
Center of buoyancy to metacenter	$\overline{BM}$	.14 m
Metacenter	M	-.09 m
Center of gravity to metacenter	$\overline{GM}$	.9 m
Rotational axis	R	unknown

Arranging the differential equation to solve for natural angular frequency (ω_n) results in Eq. (52).

$$\omega_n = \sqrt{\frac{m_b g \overline{GM}}{I}} \quad (52)$$

An estimate of rotational inertial is necessary to allow for computation of center of rotation. Rotational inertia is separated into two components: rotational inertia due to real mass distribution of the buoy I_{m_R} and rotational inertia due to virtual mass $I_{m_{added}}$. Estimation of rotational inertia about the center of rotation for real mass is determined by the parallel axis theorem shown in Eq. (54) and is the sum of rotational inertia about the center gravity I_{G_m} and the product of buoy mass and the square of the separation between the center of mass (center of gravity) and center of rotation. The contribution of virtual mass to the rotational inertia is also formulated in Eq. (54) and is the sum of rotational inertia about the center of added mass I_{55} and the product of the frequency dependant added mass of the sphere in sway $I_{11}(\omega)$ and square of the separation between the center of added

mass and center of rotation $\overline{RM}_a$. The rotational inertial of added mass about the center of added mass I_{55} is neglected because there is no significant hydrodynamic resistance to a sphere rotating about the center of the sphere. The contribution of virtual mass to rotational inertia is caused by the separation between the center of added mass and the center of rotation. This separation is estimated as the distance between the center of buoyancy of the submerged hemisphere and the center of rotation $\overline{RM}_a$.

$$I_R = I_G + m\overline{RG}^2 \quad (53)$$

$$I = I_{R_m} + I_{R_{added}}; I_{R_m} = I_{G_m} + m(\overline{RG})^2; I_{R_{added}} = I_{55} + m_{11}(\omega)(\overline{RM}_a)^2 \quad (54)$$

The final equation for natural roll angular frequency ω_n is given by Eq. (55). The parameters used in this equation are summarized in Table 11. The component of rotational inertia due to real mass was determined using a Solidworks model. The virtual sway added mass is frequency/period dependant and was found using Wamit. A plot of sway added mass at varying periods is shown in Fig. (38). Only sway added mass and not the tangential components of sway and heave added mass were used due to the symmetry of the buoy and small angle approximation. To validate the approximation of sway added mass, the added mass for a sphere was calculated at infinite period. Added mass for a sphere is given as $m_{a_{sphere}} = (2/3)\rho\pi r^3$. Therefore, for a hemisphere (submerged portion of waverider spherical float) the sway added mass in fresh water is 1/2 this value and is 58 kg. The wamit plot of sway added mass (Fig. 38) converges on this value at long period and is therefore considered to be a reasonable tool for obtaining the frequency dependant added mass.

Figure 38. Frequency/period dependant sway added mass for submerged hemisphere.

Natural roll period is determined from free decay roll tests to be 2.03 seconds (see Fig. (35)), therefore from Fig. (38) the sway added mass is determined to be 70 kg.

$$\omega_n = \sqrt{\frac{m_b g \overline{GM}}{I_{m_G} + m(\overline{RG}^2) + m_{11}(\overline{M_a R})^2}} \quad (55)$$

The distance between the center of rotation and center of mass $\overline{RG}$ and the distance between the center of rotation and center of added mass $\overline{M_a R}$ can be formulated in terms of a single unknown, the center of rotation (because the center of mass and center of added mass are known). Therefore, Eq. (55) can be solved using the quadratic equation. The center of rotation is therefore either -0.365 m or -0.985 m. It is assumed that the center of rotation is at -0.365 m as it is reasonable for the center of rotation to have moved towards the center of effect mass (instead

Table 11. URI Waverider dimensions

Real mass rotational inertia about G	I_{m_G}	59.8 kgm^2
Sway added mass	m_{11}	70 Kg
Natural roll period	T_n	2.03 s
Natural angular roll frequency	ω_n	3.10 $\frac{radians}{s}$
Center of rotation	R	-.365 m
Separation between center of rotation and sensors	$\overline{SR}$	.55 m

of further from the center of effective mass). Therefore, the separation between the center of rotation and the sensors (accelerometers and gyroscopes) in the waverider buoy is .55 m.

$$[\omega_n^2(m+m_{11})]R^2 - [2\omega_n^2(mG+m_{11}M_a)]R + [\omega_n^2(I_{mg}+mG^2+m_{11}M_a^2) - m_bg\overline{GM}] = 0 \quad (56)$$

2.6.3 Datawell Buoy

Specifications of the Datawell buoy indicate, it is able to resolve peak spectral periods from 1-30 seconds and significant wave heights at centimeter resolution. As indicated, testing the 1:4 scale DC2 buoy, however, required measuring a scaled sea state made of low period waves (~ 0.5 -5 seconds, with target peak period $T_p \simeq 2.25$ s), with hopefully centimeter resolution on wave height and determining the corresponding wave energy spectrum parameters (e.g., JONSWAP spectrum significant wave height, peak period, and peakedness factor).

Since this was at the lower end of periods that can be measured with the Datawell buoy, prior to field testing, a series of wave tank experiments of the buoy were carried out in the URI Ocean Engineering wave tank facility, to determine its suitability to obtain the necessary measurements. On March 3, 2012 the buoy was successfully tested in periodic waves, but failure of some of tank instrumentation occurred before the buoy could be tested in irregular waves. This led us to deploy

Figure 39. Wave tank test of Datawell buoy on March 3, 2012

both waverider buoys during the May 1st, 2012 field testing, to compare and cross-validate their sea state predictions.

To test the Datawell buoy in the wave tank, the buoy was deployed in the tank (Fig. 39) with a loose tether, to prevent drifting during experiments. Periodic or irregular waves were generated using the paddle wavemaker (actuated by a computer controlled hydraulic jack) at one end of the tank and partly absorbed at the other end, by a parabolic shape sloping beach (Fig. 40). Capacitance wave gages were deployed in the tank during experiments, to independently measure the sea state during the time window, the Datawell waverider buoy was acquiring data.

The method used by the Datawell instrument to compute significant wave height, peak period, spectral energy densities, buoy displacements, individual wave heights and individual wave periods (output parameters from the buoy) are provided by the manufacturer's website (www.datawell.nl). The buoy outputs four different data files: (i) a spectral density text file; (ii) a buoy displacement file; (iii) an individual wave statistics file; and (iv) a hexadecimal file. The spectral density

Figure 40. Experimental set up of Datawell wave tank test

output file contains a calculated significant wave height, peak spectral period, and spectral density as a function of frequency, from $f = 0.635 - 0.025$ Hz (period $T = 1.57 - 40$ s). The spectral densities are computed based on 30 minutes of collected data. The buoy displacement file provides buoy displacement as a function of time, sampled at 2.57 Hz (0.39 s intervals). The displacement file can be manually separated into individual waves using a zero-up-crossing method. Significant wave height and mean period can then be manually calculated based on the individual wave components. The wave statistics file automatically separates the displacement file into individual waves. The data in the wave statistic files contain wave displacement above mean water level, wave displacement below mean water level and period for each wave. It should be noted that no transfer function is applied to buoy motion to estimate sea surface displacement (it is assumed that the buoy will exactly track the sea surface over the frequencies of interest which may be a severe approximation for shorter sea states). The hexadecimal file is comprised of all the information from the other 3 data files. Every 30 min, all file types are generated and recorded by the buoy.

The wave spectrum data is computed every 30 minutes and represents averaged spectral parameters over that time frame. the data cannot be windowed to select a more specific time frame. For short deployments, the spectral files are unreliable due to the the possible inclusion of data from before or after buoy deployment. The result is the inclusion of data recorded while the buoy is still on the boat/in the process of being deployed. An example of this is evident from field test data recorded during the May1st, 2012 deployment (see Fig. 41). The figure is generated from data recorded 20 minutes after the Datawell buoy was deployed. The spectral peak centered around 2.5 seconds is due to the sea state. The spectral peak at 6 seconds is from accelerations imparted on the Datawell buoy from ship

roll prior to buoy deployment. This effect occurs unless a suitably long deployment (at least 1 hour to ensure a single uncorrupted spectral file) is accomplished. Additionally, the fetch limited locally generated sea states, in which the 1:4 scale wave energy harvesting buoys are tested, are highly variable and dependent on local wind conditions. If a significant change in conditions occurs during the test (as happened during the May 1 deployment) the spectral parameters are averaged over a 30 minute window will therefore not accurately reflect the true conditions during different segments of the test.

Figure 41. Wave spectrum obtained from the Datawell buoy spectrum file, for the May 1st, 2012 deployment, illustrating the inclusion of ship roll in the Datawell spectrum file.

All 4 file types are recorded every half hour. The only file type which time stamps the data is the buoy displacement file. [In field testing, this means, it is possible to select the time window of the displacement file corresponding to the exact time frame of useful DC2 data.]

Details of wavetank testing of the Datawell buoy are provided below.

Periodic wave testing

The Datawell buoy was excited by a series of short (2-5 min) periodic wave trains in the URI wavetank. A summary of periodic wave tank test accomplished are presented in Table 12. For periods larger than the Datawell buoy spectral cut-off period (1.57 seconds), the peak period measured and outputted in the spectral files closely matched the specified period of the generated waves.

Inputs to wave maker		Spectrum file		Displacement file		Wave statistics file	
H_s (cm)	T_p (s)	H_s (cm)	T_p (s)	H_s (cm)	T_p (s)	H_s (cm)	T_p (s)
7	3	3.3	2.99	6.02	3.00	6.0	2.44
7	2.5	2.9	2.50	6.24	2.50	6.24	2.50
5	2	3.1	2.01	4.03	2.01	4.05	3.14
5	1.75	4.6	1.764	6.0	1.75	6.0	1.75
5	1.5	N/A	N/A	6.0	1.502	6.0	1.92
5	1	N/A	N/A	2.21	1.37	2.38	2.27

Table 12. Datawell buoy periodic wave tank test. Comparison of significant wave height and peak period for different file types, relative to target values of the wavemaker. [Note that no spectral data exists for periods shorter than 1.575 seconds.]

Fig. 42 shows an example of spectrum measured with the Datawell buoy, for a periodic wave test with $T = 2$ s and $H = 5$ cm. Unfortunately, due to various equipment failure, no reliable wave gauges were operational during the laboratory testing to quantify the error in the calculated significant wave height determined by the Datawell files. However, the wave parameters input into the wavemaker can be assumed to be similar to the generated wave conditions. The spectrum files drastically erred in estimating significant wave height. It could not be determined from the brief periodic wave tests if this error in significant wave height measurement is due to the short period of waves tested, unusual spectral shape of the wave forcing, Datawell buoy inaccuracies or insufficient durations of the tests. However, the spectral files accurately measured wave period for all

periods for which spectral data is recorded (periods greater than 1.575 seconds).

The buoy displacement files were used, in combination with a zero up-crossing method, to parse the data into individual waves. Applying a direct application of the definition of significant wave height (average of highest one-third of waves) to the separated displacement data provided an estimate for wave height within 1 cm for the majority of periodic wave tests. The exception is the short 1 second period wave test for which the displacement file significantly under predicted wave height (likely because of the large discrepancy between buoy heave and surface displacement). The buoy displacement files accurately measured wave period for all regular wave tests except the shortest, 1 s period wave test. Similarly, the wave statistic file had good agreement for measured wave heights for all but the 1 s periodic wave test. The wave statistic file was less accurate for estimating wave periods than both the displacement and spectral files.

Figure 42. Datawell buoy testing in wave tank for periodic wave forcing, with $T = 2$ s and $H = 5$ cm. Output spectral energy density.

Random wave testing

The Datawell buoy was tested in the wavetank in random waves on June 4, 2012. The wave gauges were fully operational and therefore the Datawell buoy output parameters can be compared to the known wave conditions encountered. A wave gauge was placed next to the Datawell buoy so that all waves encountered by the buoy were measured by the gauge (including reflection). The sea surface displacement recorded by the wave gauges was used to compute a wave spectrum. A JONSWAP spectrum was then fit to the data (see Fig. 43). A single long duration (80 minute) random wave test was performed. The input spectral parameters to the wave tank were $H_s = 7$ cm, $T_p = 2$ s and JONSWAP peak enhancement factor = 3.

Figure 43. Wave spectrum measured in the wave tank using wave gages, during random sea state tank test of Datawell buoy, with $H_s = 7.73$ cm, $T_p = 1.90$ s and JONSWAP peak enhancement factor = 1.7.

The random wave spectral parameters measured by the wave gauges and determined by the different Datawell output files are summarized in Table 13. Note that the presented values for significant wave height obtained from the wave gauges were found using the relationship $H_s = 4\sqrt{m_0}$. This value was compared to a significant wave height obtained by parsing the wave gauge data files into individual wave components (using a zero-up-crossing method) and averaging the largest one-

Figure 44. Wave spectrum measured by the Datawell waverider buoy during the June 4th, 2012 random wave tank test (solid blue) and corresponding JS spectrum (solid green), for $H_s = 0.13$ m and $T_p = 1.95$, and $\gamma = 2.5$.

third waves. The two methods had less than a 4% difference.

Wave gauge parameter		Spectrum file		Displacement file		Wave statistics file	
H_s (cm)	T_p (s)	H_s (cm)	T_p (s)	H_s (cm)	T_p (s)	H_s (cm)	T_p (s)
7.73	1.90	6.52	1.95	8.35	1.55	8.34	1.53

Table 13. Datawell buoy random wave tank test. Comparison of significant wave height and peak period for different Datawell file types and wave gauge measured values.

The agreement between the actual significant wave height during the random wave test and the significant wave height obtained from using the displacement/wave statistic files is better than 8%. The error in the significant wave height calculated using the spectral file is 15%. It is not surprising that spectral files under predict significant wave height because the spectral files truncate energy occurring at a frequency greater than 0.635 Hz (less than 1.57 seconds) (see Fig. 44). In a

high frequency/low period sea state, such as the wavetank test, with peak period of 1.9 s, a significant amount of energy exists beyond the frequency cut-off limit. To estimate the peak enhancement factor, a JONSWAP spectrum was fit to the spectral data (Fig. 44). In order to achieve a good fit between the JONSWAP spectrum and Datawell spectrum a significant wave height of 0.13 m was required. The result was a JONSWAP peak enhancement factor of 2.5, compared to the 1.7 which provided a good fit to for the wave gauge data.

The ability to window the displacement files to any desired time frame indicates that these file types should be used to measure significant wave height during field tests. However, the spectral files measured peak period to within 2% of the actual peak period of the random wave test and provided good agreement with peak period predictions for all the periodic wave test. Therefore, the spectral data will be used to measure peak period and the displacement files to determine significant wave height during field tests of the wave energy harvesting buoys.

2.7 DC2 Numerical Modeling

Development of the numerical model to predict DC2 buoy motion and power harvesting performance was initially accomplished and detailed in reference [4]. The model was developed to predict heave motion (assuming roll effects are negligible). 1:10 scale wave tank testing indicated the importance of roll effects at that scale. The numerical model was extended to include these coupled heave-roll effects and reported in reference [1]. Current 1:4 scale field test results have demonstrated that roll effects are minimal at larger buoy scales. Therefore, the model presented in this work only predicts heave motion and assumes roll effects are negligible. The numerical model was initially validated and calibrated using the 1:10 scale tank testing.

LEG dynamics Displacement of the LEG magnet from equilibrium z_a , relative to the coil/stator (rigidly attached to the buoy), is described by a 2nd order Ordinary Differential Equation(ODE)(Eq. (57)). Armature mass M_g (including half the mass of the LEG springs and ballast mass), suspended by a spring of stiffness K_s and with damping $\mu = \mu_{LEG} + m u_{friction}$ (both electromagnetic damping μ_{LEG} and frictional damping $\mu_{friction}$) proportional to armature speed $\dot{z}_a$ is forced by inertia of the buoy accelertion $\ddot{z}_b$ and by a platform excitation force $\Delta F_p = F_p(t) - F_{p0}$ (where F_{p0} is platform weight).

$$M_g \ddot{z}_a + \mu \dot{z}_a + K_s z_a = \Delta F_p - M_g \ddot{z}_b \quad (57)$$

Float dynamics A 2nd -order nonlinear ODE describes DC2 buoy motion with inertia, radiative wave damping, viscous damping, gravity and buoyancy forces as,

$$(M_b + a_{33}(\infty)) \ddot{\xi}_3 + \int_0^t K_{b3}(t - \tau) \dot{\xi}_3(\tau) d\tau + b_{f,33} |\dot{\xi}_3| \dot{\xi}_3 + F_{bs} = F_3 + F_g \quad (58)$$

with buoy mass M_b , float added mass at very large period $a_{33}(\infty)$ and vertical float acceleration $\ddot{\xi}_3$. Additionally, the heave viscous damping coefficient $b_{f,33} = (1/2)\rho S_{\perp} C_d$ (where cross sectional area of the float at the instantaneous water line is $S_{\perp}$ and C_d is the heave drag coefficient) and velocity of the surface float relative to water particle velocity $\dot{\xi}_3' = \dot{\xi}_3 - \tilde{w}$ ($\tilde{w}$ is wave particle velocity at draft of buoy float d). Heave drag coefficient C_d is a parameter which will be experimentally determined by selecting a value which allows the model to best match experimental results. Additional forces affecting the float are a buoyancy restoring force $F_{bs}(t) = \rho_w g (\nabla_b(\xi_3(t)) - \nabla_{b0})$, wave excitation force $F_3(t)$ (Eq. (61)) and generator reaction force $F_g(t) = \mu \dot{z}_a + K_s z_a$ (z_a is armature displacement and K_s is LEG spring constant).

The sea-state is assumed to consist of the superposition of N linear periodic waves of amplitude A_n , frequency ω_n and phase ψ_n . Sea state elevation is given as,

$$\eta(t) = \sum_{n=1}^N A_n \cos(\omega_n t + \psi_n) \quad (59)$$

and vertical water particle velocity,

$$\tilde{w} = \sum_{n=1}^N -A_n \omega_n \frac{\sinh k_n(h - \tilde{d})}{\sinh k_n h} \sin(\omega_n t + \psi_n) \quad (60)$$

Wave excitation force is expressed as,

$$F_3(t) = \rho g \sum_{n=1}^N A_n r_{3n} \cos(\omega_n t + \phi_{3n} + \psi_n) \quad (61)$$

where $r_{3n}(\omega_n)$ and $\phi_{3n}(\omega_n)$ are the module and phase of the heave excitation force.

Radiative wave damping is expressed as the convolution integral in Eq. (58), where the heave impulse response function, K_{b3} , is determined with either of the inverse Fourier transforms,

$$K_{b3} = -\frac{2}{\pi} \int_0^\infty (a_{33}(\omega) - a_{33}(\infty)) \omega \sin \omega t d\omega = \frac{2}{\pi} \int_0^\infty b_{33}(\omega) \cos \omega t d\omega \quad (62)$$

which are a function of the frequency dependent heave added mass $a_{33}(\omega)$ or wave radiative damping $b_{33}(\omega)$ coefficients (frequency dependant coefficients are determined for a given buoy geometry using WAMIT). Computation of the convolution integral in Eq. (58) is accomplished with the Prony method, transforming the convolution into an additional system of ODEs. Details of this formulation are presented in reference [1].

Platform dynamics The DC2 resistance platform is connected to the magnetic armature by way of a universal joint. Numerically this is treated as a separate rigid body with displacement z_p , relative to the buoy heave displacement ξ_3 , connected

to the magnetic armature with a large but finite stiffness K_u . The force exerted on the connecting rod is,

$$F_p = (M_p - \rho V)g + K_u(z_a - z_p) \quad (63)$$

It is assumed that the resistance platform is sufficiently submerged to be unaffected by direct wave excitation; therefore, the resistance platform force balance reads,

$$M'_p \ddot{z}_p + b_{fp} |\dot{z}_p| \dot{z}_p + K_u Z_p = K_u z_a - M'_p \ddot{\xi}_3 \quad (64)$$

with the effective mass of the platform $M'_p = M_p + M_{pa}$. Heave added mass of the platform $M_{pa} = (1/3)\rho D_p^3$. The heave viscous drag coefficient $b_{fp} = (1/2)\rho S_p C_{dp}$, with drag coefficient $C_{dp} = 8/(KC_p)^{(1/3)}$, a function of Keulegan Carpenter number $KC_p = 2\pi A_s/D_p$. Note that this formulation for drag coefficient is presented as an approximation in reference [5]. A value for platform drag coefficient will be selected to achieve the best agreement between experimental test results and the numerical model. Additionally, the apparent vertical velocity of the resistance platform $\dot{z}_p = \dot{\xi}_3 + \dot{z}_p - w_p$ (w_p is the vertical water particle velocity at the mean platform depth h_p).

See reference [1] for details of analytic solutions of the presented equations under periodic forcing, as well as the method used to numerically solve the set of equations for random wave forcing.

List of References

- [1] S. T. Grilli, A. R. Grilli, S. P. Bastien, R. B. Sepe, and M. L. Spaulding, “Small buoys for energy harvesting: Experimental and numerical modeling studies,” in *International Society of Offshore and Polar Engineers (ISOPE)*, 2011, pp. 598–605.
- [2] B. Stapleton, “Instructions for wave buoy adc calibration,” ESL, Tech. Rep. 11 pps, 2011.

- [3] S. Bastien, S. Grilli, A. Grilli, R. Sepe Jr., and M. Spaulding, “Ocean energy extraction for sensor applications,” ESL, ONR STTR Phase I No. N00014-08-M-0277 164 pps, 2009.
- [4] A. R. Grilli, J. Merrill, S. Grill, M. Spaulding, and J. Cheung, “Experimental and numerical study of spar buoy-magnet/spring oscillators used as wave energy absorbers,” in *Proc. 17th International Conf. on Offshore and Polar Engineering (ISOPE07)*, 2007, pp. 489–496.
- [5] O. Faltinsen, *Sea Loads on Ships and Offshore Structures*. Cambridge, United Kingdom: Cambridge University Press, 1990.

CHAPTER 3

Results

3.1 Field Tests of 1:4 Scale DC2

3.1.1 December 15, 2011

The 1 : 4 scale DC2 wave energy harvesting buoy was field tested on December 15, 2012 in the West Passage of Narragansett Bay. Testing occurred between noon and 12 : 42 pm. Several problems encountered during testing restricted the useful DC2 data to approximately 300 seconds. An unknown error occurred with the data acquisition system which prevented applying a load to the generator for the first 32 minutes of testing. Buoy testing was halted when catastrophic failure of the universal joint connecting the resistance platform to the magnetic armature occurred. The following analysis is based on the 5 minutes of useful data collected between applying a load to the generator and structural failure.

The URI developed waverider buoy was used to quantify the sea conditions during the DC2 test. Although the waverider buoy was operational during the entire duration of testing, only the data collected while the DC2 buoy was fully functioning is used. Restricting the waverider analysis to the same time window as DC2 will provide the best estimate of sea conditions during the relevant DC2 data.

Sea state measurements

The URI waverider buoy was deployed from the R/V Shana Rose for the duration of the DC2 field test. The method used by the waverider buoy to compute sea state parameters is presented elsewhere in this document. The relevant sea state parameters are summarized in Table (14) and the wave spectrum shown in Fig. 45.

H_s (m)	T_p (s)	γ
0.63	3.33	2

Table 14. Sea state parameters calculated based on URI waverider data.

Figure 45. Wave Spectrum determined with URI waverider buoy for December 15, 2011 DC2 field test (solid blue). Significant wave height H_s is .63 m and peak period T_p is 3.33 s. Best fit JONSWAP spectrum with a peak enhancement factor γ of 2 (solid green).

Linear Electric Generator

The DC2 buoy is equipped with a Linear Electric Generator (LEG) whose characteristics are provided in Table 15. The generator is made of a series of slots with copper windings. Each slot has an average of 42 windings with a diameter of .039 m. The magnetic rod which comprises the armature of the generator is made of a series of magnets each of which is 3 cm in length. 12 of these magnetic poles are covered by the generator N_c .

The generator was loaded with variable resistive load which is represented by

Table 15. Generator parameters		
μ_{LEG}	Generator coefficient	99
γ_{eff}	Speed dependant generator efficiency	1
D_{coil}	Average coil diameter	.039 m
N_c	Number of magnetic pairs covered by stator	12
N_{turns}	Number of windings per slot	42
b_1	Magnetic flux density	.23 T
γ_{slot}	Fraction of slot width containing windings	1
$R_{emul}(t)$	Emulated load resistance	10.75 Ohms

an equivalent (i.e., emulated) resistance. The average emulated resistive load on the generator R_{emul} was 10.75Ω , which resulted in a generator coefficient μ_{LEG} of $99kg/s$ (see previous section for relevant equations).

Buoy Movement

Measured DC2 buoy accelerations and rotation rates are used to determine buoy orientation and true acceleration using the same method detailed in this report for the URI waverider buoy. The PSD of the float/buoy is presented in Fig. (46). The dominant frequency of acceleration is at the wave peak forcing period of 3.33 seconds. Additional concentrations of energy exist at periods of .9 and 1.6 seconds. The natural frequency of the float sub-system is at 2.15 seconds.

Hall effect sensors data is processed (using method presented previously) to obtain the movement of the magnetic armature relative to the float canister. The buoy design constrains the movement of the armature to be co-linear with the canister of the buoy. Therefore, the linear position of the armature relative to the float fully defines armature position (Fig. 47).

The available range of armature displacement is $\pm 20cm$. For the testing period root mean square (RMS) armature displacement is $6.8cm$. Much of the armature motion did not utilize a majority of the power generating stroke length, despite the

Figure 46. PSD of DC2 Float Acceleration during December 15 field test

large wave heights during testing (significant wave height is more than double the design wave height). Several factors contribute to the overall low buoy efficiency in harvesting wave power. The peak forcing period of the wave spectrum is significantly longer than the design/optimal forcing period. This longer period forcing will cause the response of the float to be less significant compared with optimal frequency forcing (see figure (49)). Also, the natural period of the armature/spring system is at a lower period than the forcing period. Similar to the buoy, this results in less armature response than at the design wave period.

Additionally, the mechanical power available in the armature movement for the generator to harvest is proportional to the square of armature velocity (equation (75)). The longer period forcing will result in smaller armature velocities compared to shorter period forcing provided net armature motion is similar (armature moves the same distance in more time). Compared with ideal conditions,

Figure 47. DC2 armature motion $z_a(t)$ during December 15th test. Dashed lines indicate maximum/minimum allowable armature motion.

the testing sea state results in smaller float response, which produces a smaller forcing amplitude on the armature/spring system at a frequency at which the armature/spring system does not achieve maximum response. The smaller amplitude armature response occurs at a decreased velocity. These factors all contribute to decreased energy harvesting efficiency.

The long forcing waves during the DC2 field test would result in less depth decay of the water particle movement. According to linear dispersion a 3.33 second wave has a length of 17.7 meters in 7.5 meter water depth (approximate testing depth), compared to waves with a 2.25 second period having a length of 7.9 meters. Equation (66) describes the vertical water particle displacement (ξ), for a single wavelength, normalized by wave height (H). The water particle displacement is dependant on water depth (h), wavelength (L) and depth of interest (z).

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi h}{L}\right) \quad (65)$$

$$\frac{\xi}{H} = \frac{1}{2} \frac{\sinh\left[\frac{2\pi}{L}(h+z)\right]}{\sinh\left(\frac{2\pi}{L}h\right)} \quad (66)$$

At a depth of interest of the resistance platform of DC2 (3.5 meters) the vertical displacement of water particles decays by 97% for the desired peak wave period. For the testing condition peak period, vertical particle motion decays by only 87%. This indicates that there is significant vertical particle motion forcing the resistance platform for the testing conditions. Although water particle motion decays with depth, the particle motion does not have any phase lag compared to the surface. Therefore, any component of wave motion which is present at depth will act to force the resistance platform and float in phase, lessening the differential movement between resistance plate/armature and float. This effectively shortens the power generating stroke length and negatively effects the mechanical efficiency of DC2.

Armature velocity and acceleration are calculated by differentiation of displacement. Velocity rms for the test was 0.21m/s and acceleration was 1.01m/s^2 . The armature power spectral density is shown in Fig. (48).

Figure 48. (Left) PSD of armature acceleration $z_a''(t)$. (Right) PSD of armature displacement $z_a(t)$.

The difference in the PSD plot of armature acceleration and displacement is a factor ω^2 which inflates high frequency/short period data relative to low frequency/long period data. Therefore, the dominant period of armature displacement PSD is at the peak period of the forcing wave spectrum (3.33 seconds), but the PSD of armature acceleration does not display this same trend. The natural period of the spring/armature/resistance plate system is at 2.15 seconds and a spectral peak in the acceleration PSD is present. It would not be surprising to have a significant contribution of acceleration at the natural frequency of the system. The large spectral peaks at shorter period in the acceleration PSD are attributed to artefacts amplified by the ω^2 factor in converting from acceleration to position.

The peak frequency of the sea state during the DC2 field test is at 3.33 seconds. From the heave acceleration transfer function (figure (49)) it can be seen that the maximum buoy response is at a period of 2.15 seconds. The transfer function has a value of 47 at the peak period and only 5 at the wave spectrum peak forcing period. This indicates that the buoy will experienced significantly greater response if the forcing wave spectrum peak period more closely matches the natural frequency of the buoy.

Power

The current and voltage harvested during the approximately 300 seconds of testing are shown in Fig. 50. The resulting extracted power is presented in Fig. 51. The mean electrical power captured during the test was 0.423 W.

The mean mechanical power extracted during the field test was 2.4 Watts resulting in a generator efficiency of 17.9%. Equations to calculate the generator coefficient and mechanical extracted are presented previously in this report.

The calculation of power per meter of wave crest P_w is presented in a previous section of this report. The wave length corresponding to the peak period and water

Figure 49. DC2 heave acceleration transfer function if buoy was treated as a rigid system (no internal spring).

Figure 50. (Left) Voltage harvested during December test. (Right) Current extracted by LEG during field test.

depth for the December field test result in intermediate water depth and therefore the full wave power equation must be used. The power per meter of wave crest for a peak period of 3.33 s, significant wave height of 0.63 m and water depth (h) of 7.5 m is 670 W/m . With a device width of 0.92 m this results in a mechanical efficiency of 0.4% and an overall device efficiency of 0.07%. A power summary as

Figure 51. Extracted Power during December 15, 2012 field test.

well as relevant buoy efficiencies are presented in Table (3.1.1).

P_w	Wave mean power	616 W
$P_{m_{mean}}$	Available mean mechanical power	2.4 W
P_{mean}	Mean harvested power	0.42 W
	Mechanical efficiency	0.4%
	Generator Efficiency	17.9%
	Total Buoy Efficiency	0.04%

3.1.2 May 1, 2012

Overview of field experiment

The 1:4 scale prototype of the DC2 wave energy harvesting buoy was field tested on May 1st, 2012 in the West Passage of Narragansett Bay. The buoy was deployed at noon, for 45 min, off of the R/V Cap'n Bert by way of the vessel's A-frame (Fig. 52; Table 16). Additionally, both the Datawell and URI's waverider buoys were deployed to measure sea state during the experiment. The DC2 buoy instrumentation collected useful data for the first 20 min of deployment until failure

of the instrumentation system occurred.

Event	Time
Datawell buoy started	10 : 43 am
Vessel leaves harbor	11 : 40 am
DC2 buoy deployed	12 : 05 pm
URI's waverider deployed	12 : 17 pm
Datawell buoy deployed	12 : 19 pm
DC2's instrumentation failure	12 : 20 pm
DC2 buoy retrieved	12 : 45 pm
Datawell buoy retrieved	1 : 01 pm
URI's waverider retrieved	1 : 02 pm

Table 16. Excerpts from May 1st, 2012, field testing Log

As indicated, sea state was measured during field testing using the waverider buoy designed by URI as well as ESL's Datawell buoy, a commercial grade wave measurement device; the latter was rated by its vendor for wave spectral peak periods $T_p = 1-30$ s. Testing the 1:4 scale DC2 buoy however required measuring a scaled sea state, made of low period waves ($\sim 0.5-5$ seconds, with peak period $T_p \simeq 2.25$ s), at centimeter resolution on wave height and determining the corresponding wave energy spectrum (e.g., JONSWAP spectrum significant wave height, peak period and peakedness factor).

Hence, prior to field testing, a series of wave tank experiments of the Datawell buoy were carried out in the URI Ocean Engineering wave tank facility (detailed previously in this report), to determine the suitability of the instrument to obtain the necessary measurements in its lower period (higher frequency) region of rated performance. As these tests had not been fully completed prior to the May 1st field testing, both waverider buoys were deployed during the test to compare and cross-validate their sea state predictions.

The observed sea state varied significantly during buoy deployment. Initially, the sea state contained significantly more energy than in the latter part of the 40

Figure 52. DC2 buoy field test on May 1, 2012: (a) buoy assembled and ready for deployment using the R/V Cap'n Bert; (b) buoy deployed in target sea state.

minute test window. Data closest to the target 1:4 scale sea state for DC2 testing ($H_s \simeq 0.3$ m and $T_p \simeq 2.25$ s) was collected during the first 15 minutes of buoy deployment, during which all of the instrumentation was also operational. Table 16 is an excerpt from the testing log, which can be used to synchronize the useful test segments of both wave rider and the DC2 buoys, and in particular the relevant spectra computed based on the data from URI's and Datawell's waverider buoys.

Sea state measurements

The spectrum file generated by the Datawell buoy for the entire field test is shown in Fig. 53. The peak period $T_p = 2.5$ s and significant wave height $H_s = 0.32$ m. The peakedness factor of the best fit JS spectrum was $\gamma = 2.3$.

As indicated before, however, the Datawell buoy uses all buoy motions recorded during a 30 minute window to estimate a single sea state for that time frame. Since sea state varied significantly during testing, one elected to recreate the spectrum using the buoy displacement file data, which allowed using the proper time window during which the DC2 buoy instrumentation was operational (first 20 min of the 45 min field testing).

Based on the buoy displacement file the Datawell buoy measured a significant wave height of $H_s = 0.29$ m for the time segment during which the DC2 collected useful data. This is reduced from the 0.33 m significant wave height measurements made using URI's waverider buoy for this same time period. The peak period determined by the URI waverider buoy was 2.14 s and the Datawell buoy calculated a peak period of 2.5 s for the first 20 minutes of the deployment. The over estimation of significant wave height by the URI waverider buoy is expected due to the damage to the pitch gyroscope. The source of the discrepancy between the two values for peak period is unknown. However, due to the accuracy of the period measurements of the Datawell spectrum files during wave tank tests it is expected

that the error is with the URI device.

Figure 53. Wave spectrum measured by the Datawell waverider buoy during the May 1st, 2012 entire field testing (solid blue) and and corresponding JS spectrum (solid green), for $H_s = 0.29$ m and $T_p = 2.5$, and $\gamma = 2.3$.

Linear Electric Generator

DC2 is equipped with a Linear Electric Generator (LEG), whose characteristics are provided in Table 17. The generator is comprised of a series of slots with copper windings. The wire has an average of $N_{turns} = 184$ windings per slot with a diameter of $D_{coil} = 0.038$ m; $N_c = 10$ magnet pairs of the magnet are covered by the generator windings. The internal resistance of the generator is $R_s = 4.8$ ohms per phase.

The field testing of the DC2 buoy on May 1st, 2012 produced approximately 15 min of reliable data, prior to a malfunction of the instrumentation system. During the test, each phase of DC2's LEG was loaded with a fixed resistance $R_L = 10$ ohms, which is unlike the previous design where the load was variable and only

Symbol	Definition	Value
μ_{LEG}	Generator coefficient	559 Kg/s
γ_{eff}	Speed dependant generator efficiency	1
D_{coil}	Average coil diameter	0.038 m
N_c	Number of magnetic pairs covered by stator	10
N_{turns}	Number of windings per slot	184
b_1	Magnetic flux density	0.23 T
γ_{slot}	Fraction of slot width containing windings	1
R_s	Internal resistance	4.8 Ohms

Table 17. DC2's Linear Electric Generator parameters

represented an equivalent (i.e., emulated) resistance.

The buoy was equipped with the same sensor package as in the previous Dec. 15 tests, which was detailed before. Because of the fixed resistance, however, the generated electric power is calculated based on a voltage measurement V_i across the load for each phase of the LEG (instead of both a voltage and a current measurement). Therefore, power produced during the test is calculated using (for phase $i = 1, 2$),

$$P_i = \frac{V_i^2}{R_L} \quad (67)$$

for each phase of the generator, where R_L is the external load of each phase. The total power generated being the sum of each phase: $P = P_1 + P_2$.

The mechanical power captured by the LEG is given by Eq. (75), where μ_{LEG} is the generator coefficient and $\dot{z}_a(t)$ is the magnetic armature velocity. Coefficient μ_{LEG} can be calculated based on the generator geometry, winding and external resistor with Eq. (69) using parameters whose values are summarized in Table 17. For the May field testing (fixed generator load $R_L = 10$ Ohms or each phase), we find $\mu_{LEG} = 559$ kg/s.

$$P_m = \mu_{LEG} \dot{z}_a^2 \quad (68)$$

$$\mu_{LEG} = \frac{32 \gamma_{eff} (D_{coil} N_c N_{turns} b_1)^2 (1 + \cos(\pi - \frac{\pi \gamma_{slot}}{2}))}{\gamma_{slot}^2 (R_s + 2R_L)} \quad (69)$$

Buoy Movement

As detailed above, absolute buoy motion (i.e., heave $z_b(t)$ and roll/pitch $\alpha_b(t)$) were computed based on measurements made with a 3-axis accelerometer and 3-axis gyroscope located on ESL's instrumentation board above the buoy float. The platform motion (i.e., armature and rod displacement) $z_a(t)$ with respect to the buoy was measured by way of Hall effect sensors located in the LEG.

The wave conditions in which the DC2 buoy was tested were detailed above. These testing conditions are very similar to the target sea state used in designing the energy harvesting buoy ($H_s = 0.3$ m, $T_p = 2.25$ s).

The Power Spectral Density (PSD) of the float heave acceleration $\ddot{z}_b(t)$ is shown in Fig. 54, during the first testing segment (first 13 min after 12:17 pm). The natural heave period of the buoy treated as a rigid system (no internal spring) is 2.06 s. This value is calculated using equation (70) where the spring constant is given by $k = \rho g \pi (D/2)^2$ and effective mass is the sum of the buoy mass, platform mass, buoy added mass and platform added mass ($m_e = m_b + m_p + m_{ba} + m_{pa}$). Therefore, the majority of the energy in the system is near this natural period, which can also be seen in the buoy heave acceleration response, where maximum PSD is located near 2 s. Fig. 55 similarly shows the PSD of the armature/platform acceleration $\ddot{z}_a(t)$, which is also maximum in this same period range (2-3 seconds). Maximum armature motion is expected when the wave forcing is around the natural period of the spring/resistance plate sub-system. The natural period T_n of this sub-system is given by Eq. (70) where the spring constant k is 3940 N/m, m_e is the

effective mass (sum of platform mass (118.1 kg) and added mass $((1/3)\rho D^3 = 340$ kg)). The resulting natural period of the spring/platform is 2.14 seconds.

$$T_n = 2\pi\sqrt{\frac{m_e}{k}} \quad (70)$$

Figure 54. PSD of DC2 float acceleration $\ddot{z}_b(t)$ during first testing segment on May 1st, 2012

The DC2 armature/resistance platform is designed to move out of phase with the buoy float. The expected result of this out of phase movement is to get amplified armature motions with respect to buoy float movement compared to buoy float movement relative to an inertial frame of reference. Large motion, however, is only expected when the system is driven near its design period range. Armature velocity was measured using the Hall effect sensors as $\dot{z}_a(t)$, which is shown in Fig. 56.

We find the armature motion $z_a(t)$ shown in Fig. 57, which is mostly in the range ± 0.1 m, while the maximum range of allowable armature motion, by design,

Figure 55. PSD of DC2 armature/platform acceleration $\ddot{z}_a(t)$ during first testing segment on May 1st, 2012

is ± 0.21 m. The root mean square of observed armature motion is $z_{aRMS} = 0.04$ m and for the velocity, it is $\dot{z}_{aRMS} = 0.12$ m/s. A significant wave height of $H_s = 0.26$ m corresponds to a root mean square surface elevation $\eta_{RMS} = H_s/4 = 0.065$ m. Hence, the observed armature motion is clearly not an amplification of surface elevation motion. Therefore, it is likely that the system was over damped, due to a combination of frictional and electromagnetic damping forces, to allow for optimal armature movement and therefore maximum power harvesting. A larger resistive load, and therefore less electromechanical damping, could have been applied in the conditions of this test, to allow for larger armature displacements under the testing conditions.

Figure 56. Armature velocity of DC2 $\dot{z}_a(t)$ during first testing segment on May 1st, 2012

Power

The root mean square of total electric power extracted during the May 1st, 2012 test is $P_{RMS} = 2.6$ W (from Eq. (67) with a resistance of $R_L = 10$ Ohms). Mean power harvested was 1.29 W for this test period.

Based on Eq. (75) and using the measured $\dot{z}_{aRMS}$, the root mean square mechanical power capture during the field test was $P_{m_{rms}} = 13.7$ W. The resulting generator efficiency is thus 16%. The mean mechanical power captured was 7.8 W.

According to linear wave theory, the mean power per meter of wave crest P_w is given by Eq. (78) and is equal to the product of mean wave energy E and wave group velocity C_g . Assuming nearly deep water wave conditions in water depth $h = 12$ m (i.e., $L < 6$ m or $T < 2$ s), the power P_{w0} is given by Eq. (79). According to this equation, the average power per meter of wave crest for a peak period of $T_p = 2.5$ s, significant wave height of $H_s = 0.29$ m is $P_{w0} = 206$ W/m. Therefore,

Figure 57. DC2 armature motion $z_a(t)$ during first testing segment on May 1st, 2012. Dashed lines indicate the maximum range of allowable armature motion, by design, of ± 0.21 m.

the average power available in the tested sea state over the width of the buoy (.92 m) is 189 W. This results in a mechanical efficiency of 4% and an overall device efficiency of 0.7%.

$$P_w = E C_g ; \quad E = \frac{\rho g H_s^2}{16} ; \quad C_g = \frac{c}{2} \left(1 + \frac{2kh}{\sinh 2kh} \right) ; \quad c = \frac{gT}{2\pi} \tanh kh \quad (71)$$

with $k = 2\pi/L$.

Assuming deep water wave conditions (i.e., $kh \geq \pi$ or $h \geq L/2$), which is acceptable for the short wavelength considered here, we find for narrow banded wave spectra is given by Eq. 79.

$$P_{w0} \simeq \frac{\rho g^2 H_s^2 T_p}{64\pi} \quad (72)$$

P_w	Wave mean power	189 W
P_{rms}	Available RMS mechanical power	13.7 W
P_{mean}	Available mean mechanical power	7.8 W
P_{RMS}	Root mean square harvested power	2.6 W
P_{mean}	Mean harvested power	1.3 W
	Mechanical efficiency	4%
	Generator Efficiency	16%
	Total Buoy Efficiency	0.7%

Table 18. Power summary for May 1, 2012 DC2 field test.

3.2 Field Test of 1:4 Scale DC3

3.2.1 June 5, 2012

The 1:4 scale prototype of the DC3 wave energy buoy was field tested on June 5, 2012 in the West Passage of Narragansett Bay. The buoy was deployed at 10 : 46 am, for 34 minutes, from the R/V Shanna Rose by way of the vessel's A-frame (Fig. 58). During data processing it was learned that the instrumentation system failed after about 30 minutes of testing (approximately 11 : 15 am). A Datawell sea state measuring buoy was also deployed during the field test to quantify sea state parameters. Table 19 is an excerpt from the June 5th test log. It is used to synchronize data collected using the DC3 instrumentation system, with relevant sea state measurements from the Datawell waverider buoy.

Event	Time
Datawell buoy started	9 : 52 am
Data collection started on DC3	10 : 20 am
Vessel leaves dock	10 : 24 am
DC3 buoy deployed	10 : 46 am
Datawell buoy deployed	10 : 51 am
DC3's Hall effect sensor ground wire disconnected	11 : 15 am
DC3 buoy retrieved	11 : 20 am
Datawell buoy retrieved	11 : 55 am

Table 19. Excerpts from June 5th, 2012, field testing log

Figure 58. Deployment of the DC3 buoy on June 5th, 2012 from R/V Shanna Rose.

Figure 59. DC3 buoy deployed in target sea state on June 5th, 2012.

Sea state measurements

The Datawell waverider buoy was deployed from the R/V Shanna Rose (Fig. 60) to determine sea state during the test. Significant wave height was determined using buoy displacement files and was windowed to include only the time for which the DC3 buoy was collecting usable data (see Table 19). Peak wave period was found using the spectral files calculated by the Datawell buoy. Determining the peak enhancement factor was done by visually fitting a JONSWAP spectrum to the Datawell buoy spectrum for the relevant time period (Fig. 61).

Figure 60. Deployment of the Datawell waverider buoy from the R/V Shanna Rose.

Linear Electric Generator

DC3 is equipped with a Linear Electric Generator (LEG), whose characteristics are provided in Table 21. The generator was purchased from LinMot and is comprised of a series of copper windings and is two phases. There is an average

Figure 61. Wave energy density spectrum computed during the June 5th, 2012 field test, using the Datawell buoy spectrum file (solid blue) and equivalent JS spectrum fit (solid green) for $H_s = 0.31$ m, $T_p = 2.52$ s and $\gamma = 6$.

of 170 wire windings per slot with a diameter of 0.0152 m. Four magnetic pairs of the magnet are covered by the generator windings at any time. The internal resistance of the generator is 9.85 ohms per phase.

The field testing of the DC3 buoy on June 5th, 2012 produced approximately 30 minutes of reliable data prior to a disconnection of a Hall effect sensor wire. During the test, the generator was loaded with an emulated resistance of approximately $R_{emul} \simeq 8$ ohms. The emulated load is not constant and therefore extracted electrical power is computed with Eq. (73). Although the system was designed to be able to remotely change generator resistance in order to optimize power extraction, range/speed limitations of the wireless communication system prevented any manipulation of the emulated resistive generator load during field testing.

H_s (m)	T_p (s)	γ
0.31	2.52	6

Table 20. Sea state parameters calculated based on Datawell waverider data.

Symbol	Definition	Value
μ_{LEG}	Generator coefficient	9 Kg/s
γ_{eff}	Speed dependant generator efficiency	1
D_{coil}	Average coil diameter	0.0152 m
N_c	Number of magnetic pairs covered by stator	4
N_{turns}	Number of windings per slot	170
b_1	Magnetic flux density	0.21764 T
γ_{slot}	Fraction of slot width containing windings	1
R_s	Internal resistance	9.85 Ohms
R_{emul}	Emulated resistive load	8 Ohms

Table 21. DC3's LinMot Linear Electric Generator (LEG) parameters

$$P = IV \quad (73)$$

$$\mu_{LEG} = \frac{32 \gamma_{eff} (D_{coil} N_c N_{turns} b_1)^2 (1 + \cos(\pi - \frac{\pi \gamma_{slot}}{2}))}{\gamma_{slot}^2 (R_s + R_{emul})} \quad (74)$$

The mechanical power captured by the LEG is given by Eq. (75), where μ_{LEG} is the generator coefficient and $\dot{z}_a(t)$ is the magnetic armature velocity. Coefficient μ_{LEG} can be calculated based on the generator geometry and external resistive load with Eq. (74) using parameters with values summarized in Table 21. For the May field testing (emulated generator load $R_{emul} = 8$ Ohms), we find $\mu_{LEG} = 9$ kg/s.

$$P_m = \mu_{LEG} \dot{z}_a^2 \quad (75)$$

Armature-generator frictional considerations

The mechanical power extracted by the buoy is proportional to the LEG damping coefficient as presented in Eq. (75). In addition to the electromechanical damping forces affecting armature movement, frictional damping is also influential. To approximate the frictional damping coefficient (frictional damping is assumed to also be linearly proportional to armature velocity) a series of free decay tests were accomplished in the laboratory. The free decay testing was accomplished by displacing the magnetic armature and releasing it with no resistive load applied to the generator (removing all electromechanical damping forces and isolating frictional forces). Four free decay tests were accomplished to provide an estimate for the frictional damping coefficient μ_{fr} . Eq. (76) is the equation for an under damped oscillator where x is position, A is initial displacement, ω is natural frequency and t is time. Eq. 77 is a solution to Eq. 76.

$$m\ddot{x} + \mu\dot{x} + kx = 0 \quad (76)$$

$$x = e^{-\frac{\mu_{fr}}{2A}t} A \cos(\omega t) \quad (77)$$

A theoretical best fit damped oscillator curve was found for each free decay test by visual comparison with recorded armature position. The resulting damping coefficient is a reasonable approximation for the frictional damping and will act as an additional dampening force when the generator has a resistive load (such as during field testing). Fig. 62 compares a theoretical damped oscillator to the armature position from a free decay test.

The theoretical damped oscillator curve fits the armature movement reasonably well while the armature has significant movement; however, the armature quickly returns to equilibrium after only a few oscillations instead of continuing

Figure 62. Free decay test of unloaded generator. Armature position (solid red) was visually compared to theoretical damping curve (solid blue) to determine a good fit between theoretical and experimental damping curves.

with increasingly diminished amplitudes as the theoretical curve predicts. This indicates that a larger frictional force (stiction) affects the armature motion at small velocities. From the armature displacement plot (Fig. 64) and velocity plot (Fig. 65) it can be seen that wave forcing on the armature was able to overcome stiction forces and achieve significant armature movement during the field test. The average damping coefficient for the free decay tests was 5 kg/s, which is slightly more than 50% of the electromechanical damping coefficient ($\mu_{LEG} = 9 \text{ kg/s}$ for LEG damping coefficient during DC3 field test). This indicates that a significant amount of energy was extracted from armature motion by friction and is wasted (as frictional heat loss). While assembling the DC3 buoy, significant effort was made to minimize friction between the generator and armature (the source of the

majority of frictional damping) including alignment of generator/armature, lubrication of armature and straightening of armature. Note that future discussion of power extraction and buoy efficiency excludes the increased damping caused by friction.

Buoy movement

As detailed above, absolute buoy motion was computed based on measurements made with three separate 3-axis accelerometers located within the buoy. Magnetic armature motion z_a was measured with respect to the buoy by way of Hall effect sensors located in the LEG.

Wave conditions in which the DC3 buoy was tested were very similar to the target sea state used in designing the buoy ($H_s = 0.31$, $T_p = 2.52$ compared with $H_s = 0.3$, $T_p = 2.25$ for the design sea state).

The buoy and armature sub-system natural frequencies are given by Eqs. 23 and 24 and are 0.474 Hz (2.11 s) and 0.617 Hz (1.62 s), respectively (given in Table 7). The combined system natural frequency is determined with Eq. 26 and is 0.44 Hz (2.25 s). The Power Spectral Density (PSD) of buoy and armature accelerations are presented in Fig. 63 and show movement centered about 0.45 Hz (2.22 s) for both sub-systems. It is expected that responses of both buoy sub-systems (buoy and armature) when forced at or near the design frequency would have movement at that same frequency. It should be noted that buoy and armature acceleration PSD are of similar shape, but buoy spectral values are larger. This indicates that the buoy experienced larger accelerations over the same period, and therefore larger oscillations, than the armature, which is expected due to armature damping.

Armature position for the field test is shown in Fig. 64. The apparent drift in armature motion throughout the test is likely due to insufficient data sampling rate. When insufficient data samples are recorded to process armature position with the

Figure 63. PSD of DC3 buoy vertical acceleration $\ddot{z}_b(t)$ (solid blue) and PSD of armature acceleration $\ddot{z}_a(t)$ (solid red) during June 5th, 2012 field test

Hall effect sensors, the result is a drift in armature location. The root mean square of armature position is $z_{aRMS} = 0.14$ m and for velocity, it is $\dot{z}_{aRMS} = 0.40$ m/s. The testing significant wave height $H_s = 0.31$ m corresponds to a RMS of sea surface elevation of $\eta_{RMS} = H_s/4 = 0.0775$ m. Root mean square armature position is nearly double that of sea surface displacement and indicates that overall resonant response of the internal spring/mass system did occur as expected. The armature position plot (Fig. 64) shows that much of the allowable armature range was utilized. Additional power may have been able to be extracted if significant armature movement could have been maintained with increased electromagnetic damping (a smaller resistive load), and also less frictional damping. Unfortunately, due to unreliability of the wireless communication system the emulated load on the generator could not be altered to optimize power extraction in this way.

Figure 64. Armature motion ($z_{a(t)}$) during June 5th field test

Power

The current and voltage harvested during the June 5th field test are shown in Figs. 3.2.1 and 67. The resulting power captured ($P = IV$) is shown in Fig. 68.

The root mean square of total electric power extracted during the June 5th, 2012 test is $P_{RMS} = 0.74$ W (from Eq. (73)). Mean power harvested was 0.41 W. Based on Eq. (75) the root mean square mechanical power captured during the field test was 2.45 W with a mean mechanical power captured of 1.47 W. The generator efficiency is therefore 28%.

According to linear wave theory, the mean power per meter of wave crest P_w is given by Eq. (78) and is equal to the product of mean wave energy E and wave group velocity C_g . Assuming nearly deep water wave conditions in water depth $h = 12$ m (i.e., $L < 6$ m or $T < 2$ s), the power P_{w0} is given by Eq. (79). According to this equation, the average power per meter of wave crest for a peak

Figure 65. Armature velocity ($\dot{z}_a(t)$) for June 5th field test.

Figure 66. Current harvested during DC3 field test.

period of $T_p = 2.52$ s and significant wave height of $H_s = 0.31$ m is $P_{w0} = 237$ W/m. Therefore, the average power available in the tested sea state over the width of the buoy (1 m) is 237 W. This results in a mechanical efficiency of 0.60% and an overall device efficiency of 0.17%.

Figure 67. Voltage harvested by LEG during June 5th field test.

Figure 68. Power captured by the LEG.

$$P_w = E C_g ; \quad E = \frac{\rho g H_s^2}{16} ; \quad C_g = \frac{c}{2} \left(1 + \frac{2kh}{\sinh 2kh} \right) ; \quad c = \frac{gT}{2\pi} \tanh kh \quad (78)$$

$$P_{w0} \simeq \frac{\rho g^2 H_s^2 T_p}{64\pi} \quad (79)$$

P_w	Wave mean power	237 W
$P_{m_{rms}}$	Available RMS mechanical power	2.45 W
$P_{m_{mean}}$	Available mean mechanical power	1.47 W
P_{RMS}	Root mean square harvested power	0.74 W
P_{mean}	Mean harvested power	0.41 W
	Mechanical efficiency	0.60%
	Generator Efficiency	28%
	Total Buoy Efficiency	0.17%

3.3 DC2 Model Calibration

The DC2 numerical model was initially validated and drag coefficients calibrated using a series of 1:10 scale random and periodic sea state tests accomplished in the URI Ocean Engineering wavetank. For both DC2 field tests, December 2011 and May 2012, model outputs are compared to test data to select the float heave drag coefficient C_d and platform heave drag coefficient C_{dp} which allow the model to best predict performance.

The output model parameters which will be compared to field test results are Root Mean Square (RMS) vertical buoy acceleration $\ddot{\xi}_{3_{RMS}}$, RMS vertical platform velocity $\dot{z}_{p_{RMS}}$, RMS vertical platform displacement from equilibrium $z_{p_{RMS}}$, mean mechanical power harvested $\overline{P_{mech}}$ and RMS mechanical power harvested $P_{mech_{RMS}}$. Buoy vertical acceleration is examined as opposed to velocity or position data due to the introduction of error associated with integrating (or double integrating) accelerometer data to get velocity (or position). Both resistance platform position and velocity relative to the buoy float position (and therefore magnetic armature position and velocity due to the rigid connection of the platform to the armature) are compared to the relevant model outputs due to the Hall effect sensors measuring position and the importance of platform velocity to harvested mechanical power. Mechanical power is examined instead of electrical power because of the uncertainty

of generator inefficiency required to convert mechanical to electrical power.

December 2011 Field Test			
Parameter	Field Test Result	Numerical Model Result	Percent Error
$\ddot{\xi}_{3_{RMS}}$	0.93	0.73	22%
$z_{p_{RMS}}$	0.07	0.13	85%
$\dot{z}_{p_{RMS}}$	0.22	0.27	23%
P_{mech}	2.35	2.47	2%
$P_{mech_{RMS}}$	5.36	5.21	3%

May 2012 Field Test			
Parameter	Field Test Result	Numerical Model Result	Percent Error
$\ddot{\xi}_{3_{RMS}}$	0.36	0.34	6%
$z_{p_{RMS}}$	0.040	0.050	25%
$\dot{z}_{p_{RMS}}$	0.12	0.14	17%
P_{mech}	7.8	7.9	1%
$P_{mech_{RMS}}$	13.7	12.2	11%

Utilizing a buoy heave drag coefficient C_d of 2 and resistance platform heave drag coefficient C_{dp} formulated as a function of Keulegan Carpenter number KC_p (Eq. 80) as described in reference [1] (where $KC_p = 2\pi A_s/D_p$) resulted in optimum performance of the numerical model for both the December 2011 and May 2012 field tests.

$$C_{dp} = \frac{8}{KC_p^{(1/3)}} \quad (80)$$

It is theorized that the under prediction of buoy float acceleration $\ddot{\xi}_{3_{RMS}}$ in the numerical model compared to the December 2011 test data is a result of non-linear wave effects present due to the severe wave conditions during the test. The sharp wave crests created by the high wind would result in narrow, steeper wave peaks and wide, flatter wave troughs. This results in the surface float encountering larger spikes in acceleration compared to purely sinusoidal wave forcing. The root mean square statistical property is strongly influenced by large data points, skewing the collected data towards these large data points.

The over prediction of RMS platform position relative to the LEG z_{pRMS} is attributed to the long wave period during the December field test. The numerical model was formulated assuming that the resistance platform is submerged sufficiently to avoid direct wave forcing. The long peak period $T_p = 3.33$ s during the test resulted in 28% of vertical surface wave particle motion occurring at the resistance platform depth. This is determined with Eq. (81) where ξ is water particle vertical oscillation amplitude which is a function of wave height H , gravity g , wave number $k = 2\pi/L$, angular frequency $\sigma = 2\pi/T_p$ and depth of interest z (in this case the depth of the resistance platform, 3.5 m). Similarly, the slight over prediction of platform movement in RMS platform movement for the May 2012 test can be at least partly attributed to direct wave forcing of the resistance platform. For the May field test peak period $t_p = 2.5$ s, 10% of wave particle motion still exists at the mean platform depth. Failure to include this direct wave effect on the resistance platform will result in a consistent under prediction of platform displacement by the numerical model as was experienced for the two available data sets. The longer the peak period of the sea state, the longer the wave length and the greater amount of vertical particle motion existing at the resistance platform depth.

$$\xi = \frac{H g k}{2\sigma^2} \frac{\sinh(h+z)}{\sinh(kh)} \quad (81)$$

List of References

- [1] O. Faltinsen, *Sea Loads on Ships and Offshore Structures*. Cambridge, United Kingdom: Cambridge University Press, 1990.

CHAPTER 4

Discussion/Conclusion

Many possibilities exist for advancement/improvement of both the wave energy harvesting buoy design concepts presented in this work. The considerations utilized in the design of DC2 and DC3 are outlined in a previous section of this paper. Presented here are suggestions for improvements to the design process as well as methods for maximizing the energy harvesting ability of the buoys.

4.1 DC2 Improvements Buoy shape

The spherical float shape utilized for the 1 : 4 scale DC2 buoy was selected because it is relatively easy to build and numerically model. However, other float shapes offer the possibility of increased efficiency in harnessing wave energy. A flattened spheroid shaped surface float would allow for a greater cross-sectional area at the waterline (increasing the effective buoyant force c_{33}) and minimizing total buoy mass, compared with a spherical float shape. The smaller submerged volume of the spheroid, compared with the spherical float, allows for the buoy to have less total mass and maintain the desired waterline. Additionally, a flattened spheroid shape will increase buoy stability, decreasing roll.

A conical shape (cone with point facing downward) is another possible improvement on the spherical float currently utilized by DC2. This shape would provide for less submerged volume for a given cross-sectional area at the waterline (and therefore less total buoy mass), a greater equilibrium restoring buoyancy force at both increased and decreased float submergence and potentially less energy loss to radiative damping. The conical shape would allow for both a smaller cross sectional area at the waterline as the float became emerged from the water

(increased buoyancy restoring force) as well as a larger cross-sectional area at the instantaneous waterline as the buoy is more submerged (increased buoyancy restoring force). Other proposed float shapes have a smaller cross-sectional area at the waterline when the float is displaced in either direction. A cone shaped float also has the potential to have less energy loss to radiative wave damping, compared to a spheroid shape, due to the hydro dynamically efficient shape entering the water. The slope of the cone would need to be optimized to determine the most efficient shape for energy harvesting.

Depth of resistance platform

As discussed previously, the resistance platform needs to be below as much direct wave influence as practical. Although in the 1 : 4 scale DC2 design the platform was sufficiently submerged to avoid significant influence from waves at the design sea state, both field tests had longer than desired wave periods, resulting in wave forcing directly on the resistance platform. Unfortunately, increasing the depth of the resistance platform makes the buoy more difficult to construct and deploy. An additional benefit of more deeply submerging the platform is increased buoy stability.

Optimize generator damping

Ideal generator loading for a given sea state and buoy design must be determined to achieve maximum energy extraction. This optimum balance between electromagnetic damping and magnetic armature movement must be achieved to maximize energy extraction (these quantities are linked as increased electromagnetic damping decreases armature motion, etc.). The ideal generator damping is dependant on sea state and therefore the possibility of electrically optimizing buoy performance in varying conditions exists.

Multiple LEG spars

In the 1:4 scale DC2 buoy the universal joint connecting the resistance platform to the magnetic armature was necessary to avoid a large bending moment being exerted on the magnetic armature by the platform. Another solution would be to install multiple LEGs between the float and resistance platform. This would reinforce the connection between float and resistance platform and prevent the platform from rotating relative to the float, providing increased buoy stability and ensuring that the platform is always perpendicular to the power generating LEG movement. It is proposed that 3 LEGs would be ideal as this would constrain rotation in every plane without extraneous spars which would necessitate more precise machining/construction of the buoy.

Mathematical model of DC2

In the original design of DC2, the float was sized so that the natural frequency of the entire buoy (assuming no armature/platform movement relative to the surface float) was at the design sea state peak period. Similarly, the internal LEG spring/resistance platform was treated as a separate sub-system and designed with a natural period similar to the design sea state peak period. A more accurate mathematical representation of DC2 is to treat the two buoy sub-systems as a coupled oscillator (similar to DC3) and size buoy components accordingly.

4.2 DC3 Improvements

Optimize generator damping

Similar to DC2, ideal LEG electromagnetic damping for any sea state and buoy configuration must be determined to maximize energy extraction. The generator coefficient (achieved by altering the resistive load on the generator) can be altered to optimize energy extraction and would be a form of tuning to adapt the buoy to variable environmental conditions.

Magnetic armature/generator connection details

In the current design of DC3 the internal LEG springs are connected to the top of the magnetic armature, which passes through the generator, and connects to the armature driving mass. This system does not provide for as large a stroke range as desired. If the spring system attached directly to the armature driving mass (passing through holes in the generator mounting plate), the initial length of the spring system could be utilized for other components. This connection method would require at least 2 identical spring systems in parallel to achieve the desired spring constant to avoid eccentric loading on the armature. An alternate approach would be to have the magnetic armature fixed in the buoy and connect the spring system directly to the generator (generator becomes driving mass). Additional mass could be attached to the generator as needed. This would allow for the maximum amount of stroke length (no driving mass on the magnetic rod to limit oscillations), but presents additional design challenges. These challenges mostly involve difficulty in connecting the moving generator electrically (most likely to batteries or instruments mounted to the buoy hull) to the buoy. Alternate arrangements of these components maximize the use of the interior central spar and allow for a combination of the following: longer LEG spring system/heavier driving mass (while maintaining similar internal armature/spring frequency), longer magnetic armature (increase stroke length), longer/more powerful generator.

Maximize armature mass

The redesign of the 1:4 scale DC2 was constrained by the existing buoy structure (design process is detailed previously). Ideally, the armature mass should be a larger proportion of total buoy mass. Armature mass was approximately 1/20 of total buoy mass for the 1:4 scale buoy. Additionally, the two buoy subsystems (buoy/hydrodynamic system and internal LEG spring/magnetic armature subsys-

tem) should have the same natural frequency (in previous design these subsystem frequencies were split to increase armature mass) to maximize internal armature movement. To accomplish this for the existing buoy structure the resonant period would need to be longer than the desired 2.25 s for the 1:4 scale. To increase armature mass and maintain the 2.25 s target period a new buoy would need to be constructed.

BIBLIOGRAPHY

- Bastien, S., Grilli, S., Grilli, A., Sepe Jr., R., and Spaulding, M., “Ocean energy extraction for sensor applications,” ESL, ONR STTR Phase I No. N00014-08-M-0277 164 pps, 2009.
- Bastien, S., Grilli, S., Grilli, A., Spaulding, M., and Sepe Jr., R., “Small buoy for energy harvesting,” ESL, ONR SBIR Phase I No. N00039-07-C-0076 85 pps, 2008.
- Bastien, S., Sepe Jr., R., Grilli, A., Grilli, S., and Spaulding, M., “Ocean wave energy harvesting buoy for sensors,” Tech. Rep., 2007.
- Faltinsen, O., *Sea Loads on Ships and Offshore Structures*. Cambridge, United Kingdom: Cambridge University Press, 1990.
- Grilli, A. R., Merrill, J., Grill, S., Spaulding, M., and Cheung, J., “Experimental and numerical study of spar buoy-magnet/spring oscillators used as wave energy absorbers,” in *Proc. 17th International Conf. on Offshore and Polar Engineering (ISOPE07)*, 2007, pp. 489–496.
- Grilli, S. T., Grilli, A. R., Bastien, S. P., Sepe, R. B., and Spaulding, M. L., “Small buoys for energy harvesting: Experimental and numerical modeling studies,” in *International Society of Offshore and Polar Engineers (ISOPE)*, 2011, pp. 598–605.
- Headquarters, *Coastal Engineering Manual*. Washington,DC, United States of America: U.S. Army Corps of Engineers, 2002.
- Stapleton, B., “Instructions for wave buoy adc calibration,” ESL, Tech. Rep. 11 pps, 2011.