Bivariate Autoregressive State-Space Modeling of Psychophysiological Time Series Data

Daniel M. Smith
Mohammadreza Abtahi

See next page for additional authors

Follow this and additional works at: https://digitalcommons.uri.edu/ele_facpubs

The University of Rhode Island Faculty have made this article openly available. Please let us know how Open Access to this research benefits you.

This is a pre-publication author manuscript of the final, published article.

Terms of Use
This article is made available under the terms and conditions applicable towards Open Access Policy Articles, as set forth in our Terms of Use.
Authors
Daniel M. Smith, Mohammadreza Abtahi, Amir Mohammad Amiri, and Kunal Mankodiya
Bivariate Autoregressive State-Space Modeling of Psychophysiological Time Series Data

Daniel M. Smith\textsuperscript{1,3}, Mohammadreza Abtahi\textsuperscript{2}, Amir Mohammad Amiri\textsuperscript{2}, and Kunal Mankodiya\textsuperscript{2,3}

Abstract—Heart rate (HR) and electrodermal activity (EDA) are often used as physiological measures of psychological arousal in various neuropsychology experiments. In this study, we analyze HR and EDA data collected from four participants during a cognitive task known as the Paced Auditory Serial Addition Test (PASAT). Using a state space modeling approach to time series analysis, we evaluate the effect of an exogenous input, i.e., the stimulus presentation rate which was increased systematically during the experimental task. Participants differed in several parameters characterizing the way psychological arousal was experienced during the task. Increasing the stimulus presentation rate was associated with an increase in EDA in participants 2 and 4. The effect on HR was positive for participant 2 and negative for participants 3 and 4.

I. INTRODUCTION

Psychological arousal, behavior, and emotion are related to responses in the sympathetic autonomic nervous system. Specific effects of sympathetic activity include increased heart rate and sweating, dilation of lungs, blood vessels, and pupils, decreases in digestion and salivation, and adrenaline release. Of particular relevance here is the relationship between arousal, heart rate (HR), and electrodermal activity (EDA). Perhaps due to the heart's salience to many body functions, studies investigating HR and arousal have been mixed. Some have reported an increase in HR, while others have observed the opposite [1]. EDA, which reflects the activity of sweat glands in the skin, is a physiological indicator of sympathetic arousal. Many studies have found an increase in EDA associated with the presentation of arousing stimuli [2]–[4]. Some investigators have found EDA to be more closely linked to arousal than HR [5]–[6], and among studies aiming to classify emotions EDA is one of the most common physiological signals used [7]. Also important to the current study, which includes the analysis of EDA data from persons with suicidal tendencies, is a review by Thorell [8], which demonstrated the potential for electrodermal hyporeactivity to be a useful instrument for identifying suicide risk in depressed patients.

II. METHODS

A. Participants and Data Collection

Four participants gave informed consent to the protocol approved by the Butler Hospital Institutional Review Board. Physiological data including pulse oximetry, EDA, and respiration rate were collected from participants experiencing suicidal tendencies as they performed a cognitive task known as the Paced Auditory Serial Addition Test (PASAT). The PASAT is a neuropsychological test used to assess capacity and speed of information processing, including in investigations like the current study that use psychophysiological assessment. The PASAT requires participants to add numbers within a set time period. As the task progresses, participants are required to answer simple one-digit addition math problems as digits (stimuli) appeared on a screen over a 250-second period (see Fig. 1). The frequency of stimulus presentation was increased as follows. One stimulus was presented every 3.0 seconds for the first 150 seconds, every 1.5 seconds for the next 75 seconds, and every 1.0 second for the remaining 25 seconds. Alternatively, these presentation rates can be expressed as 0.33, 0.67, and 1.00 stimuli per second. Each stimulus appeared on the screen for 500 milliseconds regardless of presentation frequency. HR and EDA were sampled at rate of 1 kHz. These data were decimated to 1 Hz for analysis.

B. Modeling Technique

The aim of the study is to assess whether there are patterned responses that characterize the way each individual experiences psychological arousal during the cognitive task. To that end, the analytical objectives of the study are (1) to examine the dynamics of HR and EDA using state space time series analysis and (2) to investigate whether increasing the stimulus presentation rate is associated with increases in HR and EDA.

Figure 1. PASAT experimental setup.
Bivariate time series of HR and EDA (see Fig. 2 and 3) were analyzed for each participant using a first-order autoregressive model with the stimulus presentation rate included as an exogenous input. The state and observation sub-models are represented by equations 1 and 2, respectively.

\[ X_t = AX_{t-1} + Bu_t + \varepsilon_t; \varepsilon_t \sim N(0, Q) \]  
\[ y_t = CX_t + \mu + \omega_t; \omega_t \sim N(0, R) \]  

- \( X_t \) and \( X_{t-1} \) are 2 \( \times \) 1 vectors of state variables for HR and EDA at times \( t \) and \( t-1 \)
- \( A \) is a 2 \( \times \) 2 transition matrix reflecting the auto-dependence of the state variables
- \( u_t \) is a single exogenous input, the stimulus presentation rate at time \( t \)
- \( B \) is a 2 \( \times \) 1 vector of regression coefficients relating the input \( u_t \) to the state vector \( X_t \)
- \( Q \) is a 2 \( \times \) 2 covariance matrix of the dynamic errors, \( \varepsilon_t \)

Figure 2. HR time series data for all participants. Vertical dashed lines show the time points at which the stimulus presentation rate increased.

Figure 3. EDA time series data for all participants. Vertical dashed lines show the time points at which the stimulus presentation rate increased.
• $y_t$ is a $2 \times 1$ vector of HR and EDA observations at time $t$

• $C$ is a $2 \times 2$ identity matrix relating the state variables, $X_t$, to the observed variables, $y_t$

• $\mu$ is a $2 \times 1$ vector reflecting the level (analogous to linear intercept) of HR and EDA

• $R$ is a $2 \times 2$ covariance matrix of the measurement errors, $\omega_t$

The software used in this analysis was the \texttt{MARSS} (multivariate autoregressive state space) package in R. MARSS uses the Kalman [9] filter and smoother to generate state variable estimates, which are outputted to a maximum likelihood algorithm for computation of parameter estimates and their standard errors. The optimization algorithm used in these analyses was the quasi-Newton Broyden–Fletcher–Goldfarb–Shanno (BFGS) method.

### III. RESULTS

Parameter estimates and 95% confidence intervals are shown in Table I. Parameters whose confidence interval does not span zero are statistically significant at a type I error rate of 0.05. The A parameters reflect each variable’s first-order autoregressive process. All but one of these is positive and some of these reflect very strong autoregression (i.e., close to 1, such as $A_{EDA}$ for participant 4). The B parameters reflect substantively interesting relationships, i.e., the change in HR and EDA per unit increase in the stimulus presentation rate. Fig. 4 illustrates the changes in HR and EDA predicted by the B estimates that were statistically significant. These parameters were not significant for participant 1, suggesting that her/his HR and EDA were unchanged across the increasing demands of the cognitive task. By contrast, both B parameters were positive and significant for participant 2, which indicates that this person’s HR and EDA increased as

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Participant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{HR}$</td>
<td>0.57$^a$</td>
<td>0.26</td>
<td>0.24</td>
<td>-0.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.47, 0.67)$^b$</td>
<td>(0.13, 0.38)</td>
<td>(0.11, 0.38)</td>
<td>(-0.4, -0.04)</td>
<td></td>
</tr>
<tr>
<td>$A_{EDA}$</td>
<td>0.94</td>
<td>0.90</td>
<td>0.65</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.90, 0.97)</td>
<td>(0.85, 0.94)</td>
<td>(0.60, 0.69)</td>
<td>(0.94, 0.98)</td>
<td></td>
</tr>
<tr>
<td>$B_{HR}$</td>
<td>-4.56</td>
<td>5.63</td>
<td>-0.75</td>
<td>-5.42</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-9.76, 0.64)</td>
<td>(0.28, 11.0)</td>
<td>(-0.9, -0.6)</td>
<td>(-8.6, -2.2)</td>
<td></td>
</tr>
<tr>
<td>$B_{EDA}$</td>
<td>-0.04</td>
<td>0.06</td>
<td>-0.06</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.09, 0.02)</td>
<td>(0.01, 0.11)</td>
<td>(-0.15, 0.02)</td>
<td>(0.02, 0.06)</td>
<td></td>
</tr>
<tr>
<td>$\xi_{HR}$</td>
<td>72.9</td>
<td>67.5</td>
<td>75.2</td>
<td>98.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(66.4, 79.5)</td>
<td>(63.7, 71.3)</td>
<td>(74.5, 76.0)</td>
<td>(97.3, 100)</td>
<td></td>
</tr>
<tr>
<td>$\xi_{EDA}$</td>
<td>0.70</td>
<td>0.05</td>
<td>2.43</td>
<td>-0.73</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.29, 1.12)</td>
<td>(-0.16, 0.26)</td>
<td>(2.30, 2.57)</td>
<td>(-1.0, -0.5)</td>
<td></td>
</tr>
</tbody>
</table>

Error Variances

| $\hat{\sigma}_{HR}$ | 86.6 | 87.4 | 18.5 | 0 |
|                       | (72.0, 102) | (72.7, 103) | (15.4, 21.9) | (NA, NA) |
| $\hat{\sigma}_{EDA}$ | 0.007 | 0.0048 | 0.025 | 0.001 |
|                       | (6e-3, 5e-3) | (4e-3, 5e-3) | (2e-2, 3e-2) | (1e-3, 1e-3) |
| $\gamma_{HR}$ | 0 | 0 | 21.4 | 21.4 |
|           | (NA, NA) | (NA, NA) | (17.8, 25.3) | (17.8, 25.3) |
| $\gamma_{EDA}$ | 0 | 0 | 0 | 0 |
|           | (NA, NA) | (NA, NA) | (NA, NA) | (NA, NA) |

Figure 4. Changes in HR or EDA predicted by the models that had significant B parameters. Vertical dashed lines show the time points at which the stimulus presentation rate increased.

---

a. Estimates in bold are statistically significant at the 0.05 Type I error rate.

b. Values shown in parentheses are 95% confidence limits.
the stimulus presentation rate increased. The same can be said of EDA for participant 4. Unexpectedly, $B_{HR}$ was negative and significant for participants 3 and 4, from which it can be concluded that HR decreased for these participants as stimulus presentation rate increased. The $\varepsilon$ parameters are not intended to have substantive value, but they are reflective of individual differences in participants' mean HR and EDA. For example, from these we can see that the mean HR is higher for participant 4 than the others. The error variances also differed greatly between the four participants. In most cases, the measurement error variance was very close to 0.

IV. DISCUSSION AND CONCLUSION

In this study, we used a state-space modeling framework to execute autoregressive time series analysis on bivariate time series of HR and EDA. Performing four separate analyses enabled the estimation of participant-specific parameter estimates that reflected each person's unique patterns of change in HR and EDA across the 250-second cognitive task. Assessing the significance and direction of the B parameters, it is apparent that each participant experienced the cognitive task differently in terms of physiological response. While the task was anticipated to induce psychological arousal manifested in increases in HR and EDA, there was a significant, positive effect in only three cases out of eight (i.e., participant 2 HR and EDA and participant 4 EDA). Similar to others who have reported mixed results with arousal and HR [1], in our study there were two cases of negative effects on HR, in participants 3 and 4. In sum, the state space modeling framework is a convenient and advantageous approach for handling many aspects of psychophysiological data analysis, including auto-dependence, exogenous inputs, multiple sources of error, and multiple observed variables.

ACKNOWLEDGMENT

We would like to convey our thanks to Dr. Michael Armey and his research group for helping us collect the data.

REFERENCES