2020

18. RL circuits. Current buildup and shutdown

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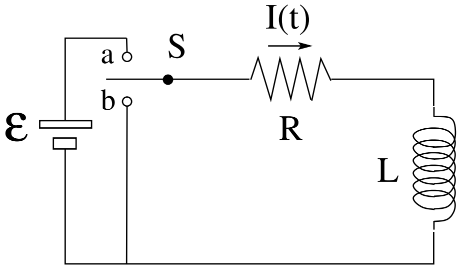
RL Circuit: Fundamentals

Specifications:

- $\mathcal{E}$ (emf)
- R (resistance)
- L (inductance)

Switch S:

- a: current buildup
- b: current shutdown



Time-dependent quantities:

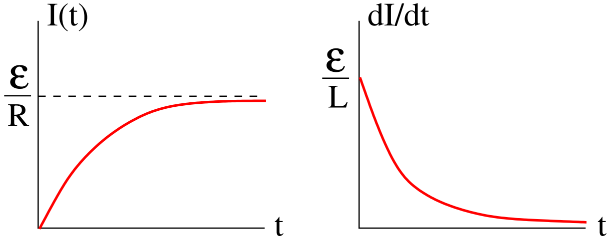
- $I(t)$: instantaneous current through inductor
- $\frac{dI}{dt}$: rate of change of instantaneous current
- $V_R(t) = I(t)R$: instantaneous voltage across resistor
- $V_L(t) = L\,\frac{dI}{dt}$: instantaneous voltage across inductor

RL Circuit: Current Buildup in Inductor

- Loop rule: $\mathcal{E} - IR - L\dfrac{dI}{dt} = 0$
- Differential equation: $L\dfrac{dI}{dt} = \mathcal{E} - IR \quad \Rightarrow \dfrac{dI}{dt} = \dfrac{\mathcal{E}/R - I}{L/R}$

$$\int_0^I \frac{dI}{\mathcal{E}/R - I} = \int_0^t \frac{dt}{L/R} \quad \Rightarrow \; -\ln\left(\frac{\mathcal{E}/R - I}{\mathcal{E}/R}\right) = \frac{t}{L/R} \quad \Rightarrow \frac{\mathcal{E}/R - I}{\mathcal{E}/R} = e^{-Rt/L}$$

- Current through inductor: $I(t) = \dfrac{\mathcal{E}}{R}\left[1 - e^{-Rt/L}\right]$
- Rate of current change: $\dfrac{dI}{dt} = \dfrac{\mathcal{E}}{L}\, e^{-Rt/L}$

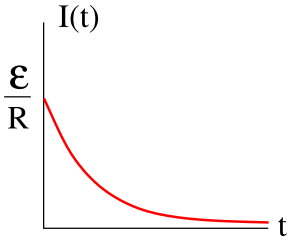


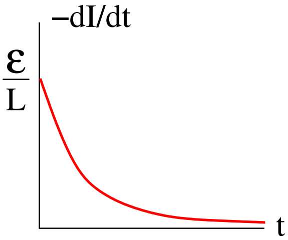
RL Circuit: Current Shutdown in Inductor

- Loop rule: $-IR - L\frac{dI}{dt} = 0$
- Differential equation: $L\frac{dI}{dt} + IR = 0 \quad \Rightarrow \frac{dI}{dt} = -\frac{R}{L}I$

$$\Rightarrow \int_{\mathcal{E}/R}^{I} \frac{dI}{I} = -\frac{R}{L}\int_0^t dt \quad \Rightarrow \ln\frac{I}{\mathcal{E}/R} = -\frac{R}{L}t \quad \Rightarrow \frac{I}{\mathcal{E}/R} = e^{-Rt/L}$$

- Current: $I(t) = \frac{\mathcal{E}}{R}\, e^{-Rt/L}$
- Rate of current change: $\frac{dI}{dt} = -\frac{\mathcal{E}}{L}\, e^{-Rt/L}$



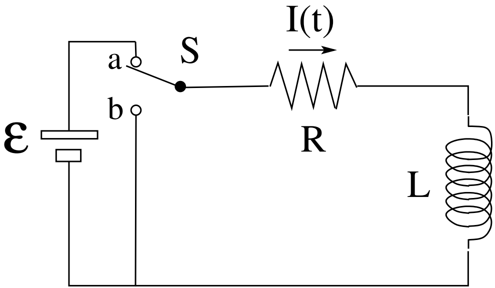


RL Circuit: Energy Transfer During Current Buildup

Loop rule: $IR + L\frac{dI}{dt} = \mathcal{E} \quad (I > 0,\ \frac{dI}{dt} > 0)$

- $I\mathcal{E}$: rate at which EMF source delivers energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_L = LI\frac{dI}{dt}$: rate at which energy is stored in inductor

Balance of energy transfer: $I^2R + LI\frac{dI}{dt} = I\mathcal{E}$

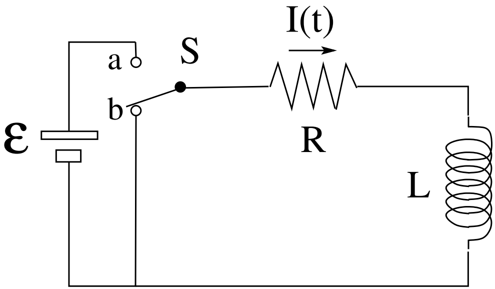


RL Circuit: Energy Transfer During Current Shutdown

Loop rule: $IR + L\frac{dI}{dt} = 0 \quad \left(I > 0,\ \frac{dI}{dt} < 0\right)$

- $IV_L = LI\frac{dI}{dt}$: rate at which inductor releases energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor

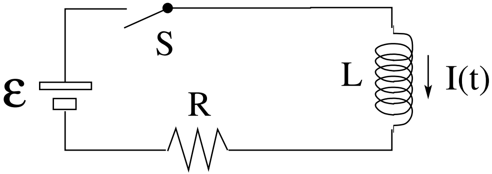
Balance of energy transfer: $I^2R + LI\frac{dI}{dt} = 0$



RL Circuit: Some Physical Properties

Specification of RL circuit by 3 device properties:

- $\mathcal{E}$ [V] (emf)
- R $[\Omega]$ (resistance)
- L [H] (inductance)



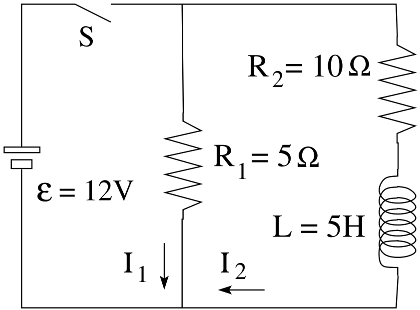
Physical properties of RL circuit during current buildup determined by 3 combinations of the device properties:

- $\dfrac{\mathcal{E}}{L} = \left.\dfrac{dI}{dt}\right|_{t=0}$: initial rate at which current increases
- $\dfrac{\mathcal{E}}{R} = I(t=\infty)$: final value of current
- $L/R = \tau$: time it takes to build up 63% of the current through the circuit
  $[1 - e^{-1} = 0.632\ldots]$

RL Circuit: Application (8)

In the circuit shown the switch has been open for a long time.
Find the currents I_1 and I_2

- just after the switch has been closed,
- a long time later,
- as functions of time for $0 < t < \infty$.

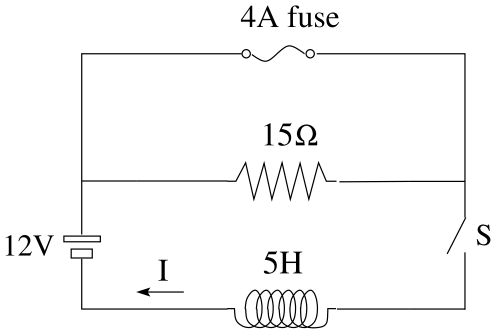


RL Circuit: Application (7)

In the circuit shown the switch S is closed at time $t = 0$.

(a) Find the current I as a function of time for $0 < t < t_F$, where t_F marks the instant the fuse breaks.

(b) Find the current I as a function of time for $t > t_F$.



RL Circuit: Application (6)

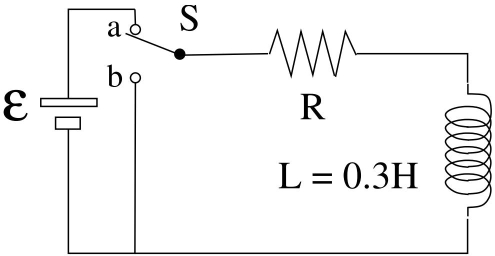
In the RL circuit shown the switch has been at position a for a long time and is thrown to position b at time $t = 0$. At that instant the current has the value $I_0 = 0.7\text{A}$ and decreases at the rate $dI/dt = -360\text{A/s}$.

(a) Find the EMF $\mathcal{E}$ of the battery.

(b) Find the resistance R of the resistor.

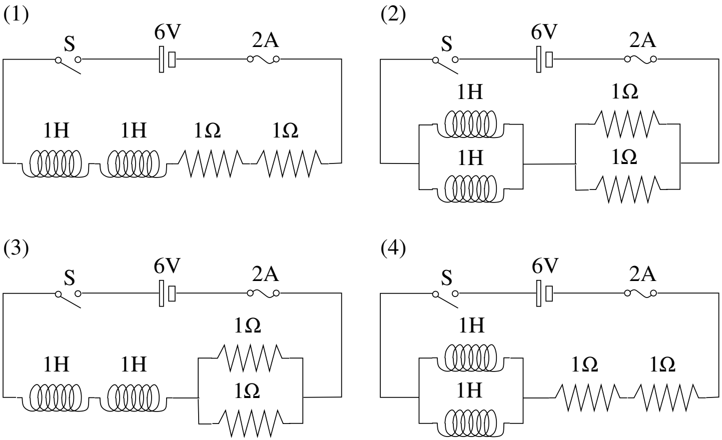
(c) At what time t_1 has the current decreased to the value $I_1 = 0.2\text{A}$?

(d) Find the voltage across the inductor at time t_1.



Each RL circuit contains a 2A fuse. The switches are closed at $t = 0$.

- In what sequence are the fuses blown?



RL Circuit: Application (1)

Each branch in the circuit shown contains a 3A fuse. The switch is closed at time $t = 0$.

(a) Which fuse is blown in the shortest time?

(b) Which fuse lasts the longest time?