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17. Magnetic induction: inductor and inductance. Self/mutual induction

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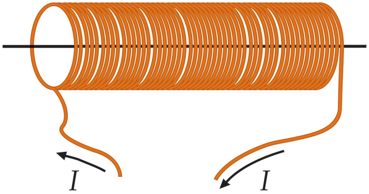
Inductor (device):

- A wire that is wound into N turns of some shape and area.
- The current I flowing through the wire generates a magnetic field $\vec{B}$ in its vicinity.
- The magnetic field $\vec{B}$, in turn, produces a magnetic flux Φ_B through each turn.

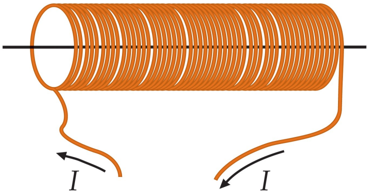
Inductance (device property):

- Definition: $L = \dfrac{N\Phi_B}{I}$
- SI unit: 1H = 1Wb/A (one Henry)



Inductance of a Solenoid

- A: cross-sectional area
- ℓ: length
- n: number of turns per unit length
- $N = n\ell$: total number of turns
- $B = \mu_0 nI$: magnetic field inside solenoid
- $\Phi_B = BA$: magnetic flux through each turn
- $\Rightarrow$ Inductance of solenoid: $L \equiv \dfrac{N\Phi_B}{I} = \mu_0 n^2 A\ell$



Inductance of a Toroid

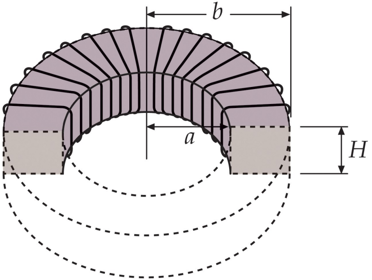
- Total number of turns: N
- Magnetic field inside toroid: $B = \dfrac{\mu_0 I}{2\pi r}$
- Magnetic flux through each turn (loop):

$$\Phi_B = \int_a^b BH\,dr = \frac{\mu_0 INH}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 INH}{2\pi}\ln\frac{b}{a}$$

- Inductance: $L \equiv \dfrac{N\Phi_B}{I} = \dfrac{\mu_0 N^2 H}{2\pi}\ln\dfrac{b}{a}$
- Narrow toroid: $s \equiv b - a \ll a$

$$\ln\frac{b}{a} = \ln\left(1 + \frac{s}{a}\right) \simeq \frac{s}{a}$$

- Inductance: $L = \dfrac{\mu_0 N^2 (sH)}{2\pi a}$

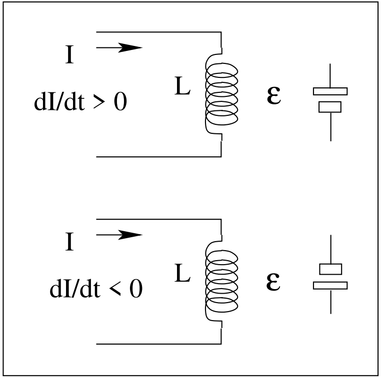


Self-Induction

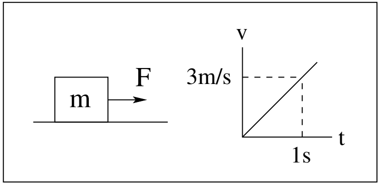
The induced electric EMF acts such as to oppose the change in the current that causes it (Lenz's law).

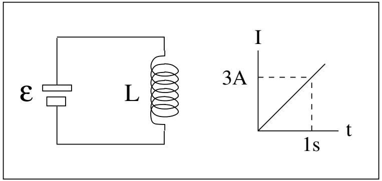
The presence of an inductance makes the electric current sluggish (resistant to change).

- Faraday's law: $\mathcal{E} = -\frac{d}{dt}(N\Phi_B)$
- Inductance: $L = \frac{N\Phi_B}{I}$
- Self-induced EMF: $\mathcal{E} = -L\frac{dI}{dt}$



Inertia: Mechanical vs Electrical





$$F = 6\text{N}, \quad m = 2\text{kg}$$

$$\mathcal{E} = 6\text{V}, \quad L = 2\text{H}$$

Newton's second law

$$F - m\frac{dv}{dt} = 0$$

$$\frac{dv}{dt} = \frac{F}{m} = 3\text{m/s}^2$$

Loop rule

$$\mathcal{E} - L\frac{dI}{dt} = 0$$

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L} = 3\text{A/s}$$

Energy Stored in Inductor

Establishing a current in the inductor requires work.
The work done is equal to the potential energy stored in the inductor.

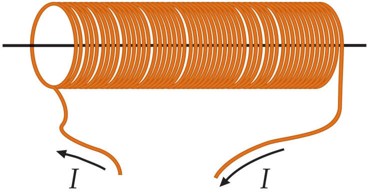
- Current through inductor: I (increasing)
- Voltage induced across inductor: $|\mathcal{E}| = L\dfrac{dI}{dt}$
- Power absorbed by inductor: $P = |\mathcal{E}|I$
- Increment of potential energy: $dU = Pdt = LIdI$
- Potential energy of inductor with current I established:
  $U = L\displaystyle\int_0^I IdI = \frac{1}{2}LI^2$

Q: where is the potential energy stored?
A: in the magnetic field.

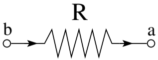
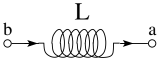
Energy Density Within Solenoid

Energy is stored in the magnetic field inside the solenoid.

- Inductance: $L = \mu_0 n^2 A\ell$
- Magnetic field: $B = \mu_0 n I$
- Potential energy: $U = \frac{1}{2}LI^2 = \frac{1}{2\mu_0}B^2(A\ell)$
- Volume of solenoid interior: $A\ell$
- Energy density of magnetic field: $u_B = \frac{U}{A\ell} = \frac{1}{2\mu_0}B^2$

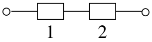
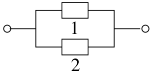
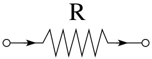
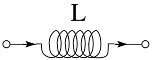


Response of Device to Electric Current

device	symbol	voltage
resistor		$V_{ab} = RI$
inductor		$V_{ab} = L\dfrac{dI}{dt}$
capacitor		$V_{ab} = \dfrac{1}{C}Q \qquad \dfrac{dQ}{dt} = I$

Note that that the response of each device to a current I is a voltage $V_{ab} \equiv V_b - V_a$.

- resistor: response proportional to current itself
- inductor: response proportional to derivative of current
- capacitor: response proportional to integral of current

		in series	in parallel
			
resistors		$R_{eq}=R_1+R_2$	$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}$
inductors		$L_{eq}=L_1+L_2$	$\frac{1}{L_{eq}}=\frac{1}{L_1}+\frac{1}{L_2}$
capacitors		$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}$	$C_{eq}=C_1+C_2$

Mutual Induction

Φ_{12}: magnetic flux through each loop of coil 2 caused by current I_1 through coil 1

Φ_{21}: magnetic flux through each loop of coil 1 caused by current I_2 through coil 2

$M_{12} = \dfrac{N_2\Phi_{12}}{I_1}$ (mutual inductance)

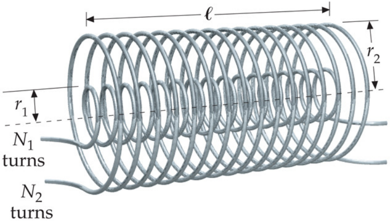
$M_{21} = \dfrac{N_1\Phi_{21}}{I_2}$ (mutual inductance)

$\mathcal{E}_2 = -M_{12}\dfrac{dI_1}{dt}$ (emf induced in coil 2 due to current in coil 1)

$\mathcal{E}_1 = -M_{21}\dfrac{dI_2}{dt}$ (emf induced in coil 2 due to current in coil 1)

$M_{12} = M_{21} = M$ (symmetry property)

$M = \mu_0 \dfrac{N_1}{\ell}\dfrac{N_2}{\ell}(\ell\pi r_1^2)$ (present configuration)



(a)

Magnetic Induction: Application (12)

Consider two conducting loops (i) and (ii) (indicated by green lines in cubes of sides $L = 2$m). Each loop is placed in a region of uniform magnetic field with linearly increasing magnitude, $B(t) = bt, b = 2$T/s, and one of the five directions indicated.

(a) Find the magnetic flux through each loop as produced by each field.

(b) Find the magnitude and direction of the emf induced by each field in each loop.