2020

12. Effects of a magnetic field: magnetic force

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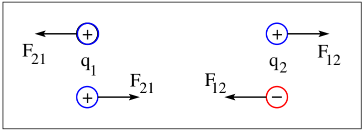
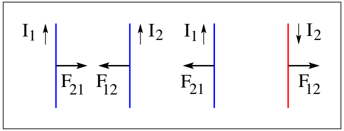
Electricity

- Electric charges generate an electric field.
- The electric field exerts a force on other electric charges.



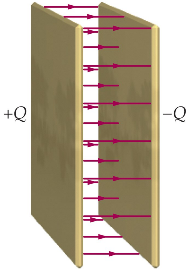
Magnetism

- Electric currents generate a magnetic field.
- The magnetic field exerts force on other electric currents.



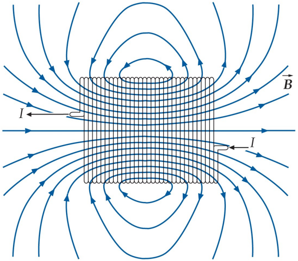
Capacitor

The parallel-plate capacitor generates a near uniform electric field provided the linear dimensions of the plates are large compared to the distance between them.

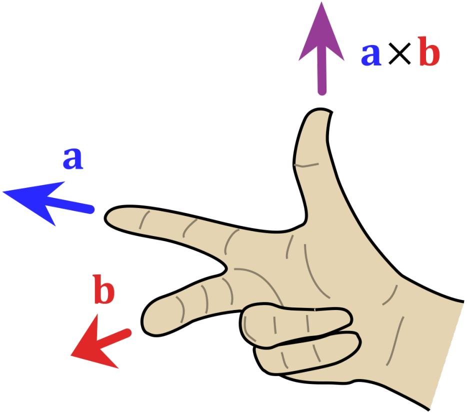


Solenoid

The solenoid (a tightly wound cylindrical coil) generates a near uniform magnetic field provided the length of the coil is large compared to its radius.



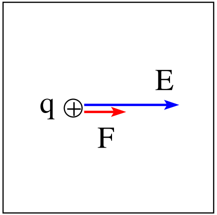
Right-Hand Rule



Electric and Magnetic Forces on Point Charge

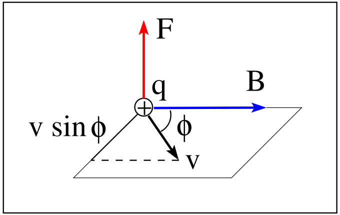
Electric Force

- $\vec{F} = q\vec{E}$
- electric force is parallel to electric field
- SI unit of E: 1N/C=1V/m



Magnetic Force

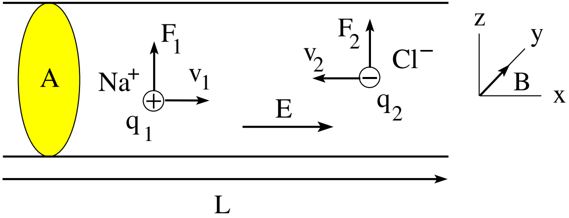
- $\vec{F} = q\vec{v} \times \vec{B}$, $\quad F = qvB\sin\phi$
- magnetic force is perpendicular to magnetic field
- SI unit of B: 1Ns/Cm=1T (Tesla)
- 1T=10^4G (Gauss)



Magnetic Force on Current-Carrying Conductor

Consider drift of Na^+ and Cl^- ions in a plastic pipe filled with salt water.

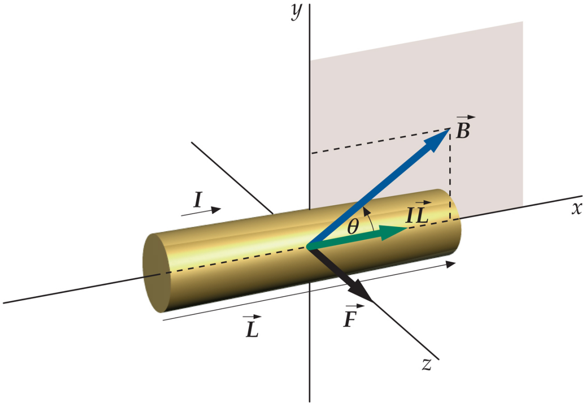
- $v_{1x} > 0,\ v_{2x} < 0$: drift velocities; $q_1 > 0,\ q_2 < 0$: charge on ions
- $n_1,\ n_2$: number of charge carriers per unit volume



- Electric current through A: $I = A(n_1 q_1 v_{1x} + n_2 q_2 v_{2x})$
- Force on Na^+: $\vec{F}_1 = q_1 \vec{v}_1 \times \vec{B} \ \Rightarrow\ F_{1z} = q_1 v_{1x} B_y$
- Force on Cl^-: $\vec{F}_2 = q_2 \vec{v}_2 \times \vec{B} \ \Rightarrow\ F_{2z} = q_2 v_{2x} B_y$
- Force on current-carrying pipe: $F_z = (n_1 q_1 v_{1x} + n_2 q_2 v_{2x}) A L B_y = I L B_y$
- Vector relation: $\vec{F} = I\vec{L} \times \vec{B}$

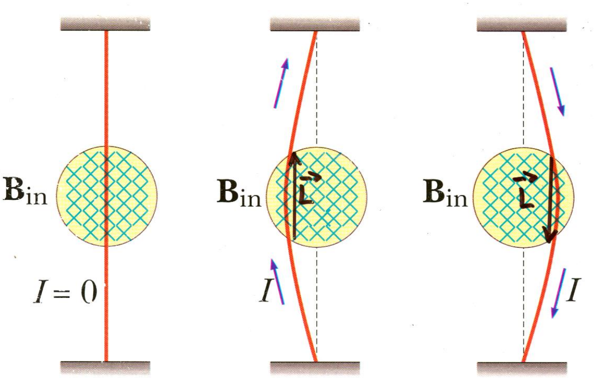
Direction of Magnetic Force (1)

$$\vec{F} = I\vec{L} \times \vec{B}$$



Direction of Magnetic Force (2)

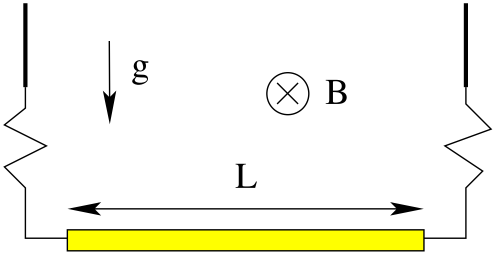
$$\vec{F} = I\vec{L} \times \vec{B}$$



Magnetic Force Application (1)

A wire of length $L = 62\text{cm}$ and mass $m = 13\text{g}$ is suspended by a pair of flexible leads in a uniform magnetic field $B = 0.440\text{T}$ pointing in to the plane.

- What are the magnitude and direction of the current required to remove the tension in the supporting leads?



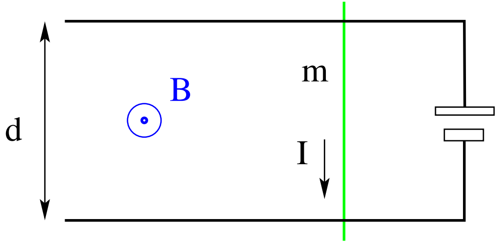
Magnetic Force Application (2)

A metal wire of mass $m = 1.5$kg slides without friction on two horizontal rails spaced a distance $d = 3$m apart.

The track lies in a vertical uniform magnetic field of magnitude $B = 24$mT pointing out of the plane.

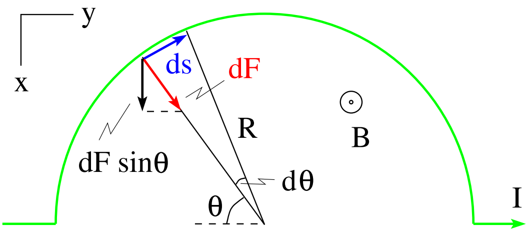
A constant current $I = 12$A flows from a battery along one rail, across the wire, and back down the other rail. The wire starts moving from rest at $t = 0$.

- Find the direction and magnitude of the velocity of the wire at time $t = 5$s.



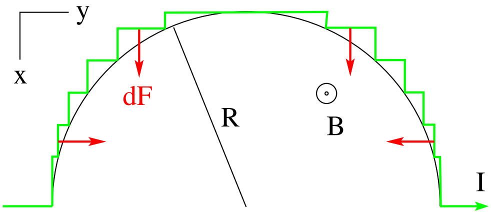
Fancy solution:

- Uniform magnetic field $\vec{B}$ points out of the plane.
- Magnetic force on segment ds: $dF = IBds = IBRd\theta$.
- Integrate $dF_x = dF\sin\theta$ and $dF_y = dF\cos\theta$ along semicircle.
- $F_x = IBR\int_0^\pi \sin\theta d\theta = 2IBR, \quad F_y = IBR\int_0^\pi \cos\theta d\theta = 0.$



Clever solution:

- Replace the semicircle by symmetric staircase of tiny wire segments.
- Half the vertical segments experience a force to the left, the other half a force to the right. The resultant horizontal force is zero.
- All horizontal segments experience a downward force. The total length is $2R$.
  The total downward force is $2IBR$.
- Making the segments infinitesimally small does not change the result.

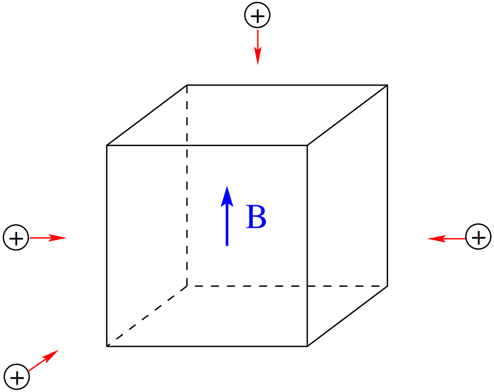


Magnetic Force Application (5)

Inside the cube there is a magnetic field $\vec{B}$ directed vertically up.

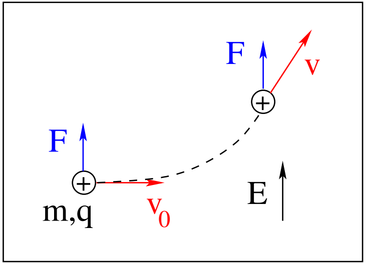
Find the direction of the magnetic force experienced by a proton entering the cube

(a) from the left,
(b) from the front,
(c) from the right,
(d) from the top.



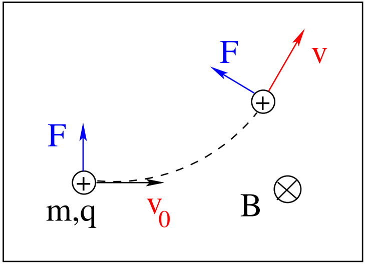
Charged Particle Moving in Uniform Electric Field

- Electric field $\vec{E}$ is directed up.
- Electric force: $\vec{F} = q\vec{E}$ (constant)
- Acceleration: $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{q}{m}\vec{E} = $ const.
- Horizontal motion: $a_x = 0 \;\Rightarrow\; v_x(t) = v_0 \;\Rightarrow\; x(t) = v_0 t$
- Vertical motion: $a_y = \dfrac{q}{m}E \;\Rightarrow\; v_y(t) = a_y t \;\Rightarrow\; y(t) = \dfrac{1}{2}a_y t^2$
- The path is parabolic: $y = \left(\dfrac{qE}{2mv_0^2}\right)x^2$
- $\vec{F}$ changes direction and magnitude of $\vec{v}$.



Charged Particle Moving in Uniform Magnetic Field

- Magnetic field $\vec{B}$ is directed into plane.
- Magnetic force: $\vec{F} = q\vec{v} \times \vec{B}$ (not constant)
- $\vec{F} \perp \vec{v} \;\Rightarrow\; \vec{F}$ changes direction of $\vec{v}$ only $\;\Rightarrow\; v = v_0$.
- $\vec{F}$ is the centripetal force of motion along circular path.
- Radius: $\dfrac{mv^2}{r} = qvB \;\Rightarrow\; r = \dfrac{mv}{qB}$
- Angular velocity: $\omega = \dfrac{v}{r} = \dfrac{qB}{m}$
- Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi m}{qB}$



Velocity Selector

A charged particle is moving horizontally into a region with "crossed" uniform fields:

- an electric field $\vec{E}$ pointing down,
- a magnetic field $\vec{B}$ pointing into the plane.

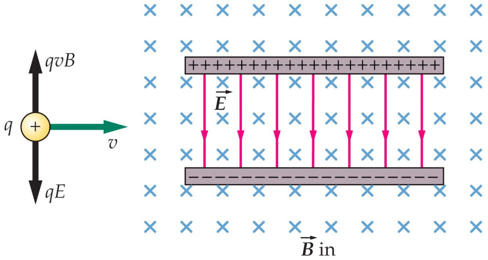
Forces experienced by particle:

- electric force $F = qE$ pointing down,
- magnetic force $B = qvB$ pointing up.

Forces in balance: $qE = qvB$.

Selected velocity: $v = \dfrac{E}{B}$.

Trajectories of particles with selected velocity are not bent.



Measurement of e/m for Electron

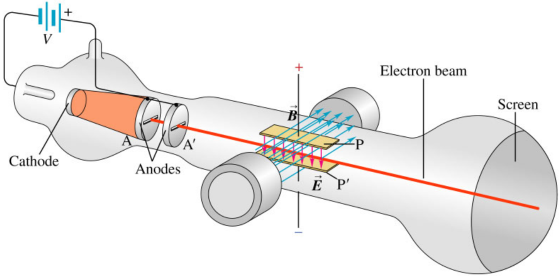
First experiment by J. J. Thomson (1897)

Method used here: velocity selector

Equilibrium of forces: $eE = evB \quad \Rightarrow \; v = \dfrac{E}{B}$

Work-energy relation: $eV = \dfrac{1}{2}mv^2 \quad \Rightarrow \; v = \sqrt{\dfrac{2eV}{m}}$

Eliminate v: $\dfrac{e}{m} = \dfrac{E^2}{2VB^2} \simeq 1.76 \times 10^{11}\mathrm{C/kg}$



Measurement of e and m for Electron

First experiment by R. Millikan (1913)

Method used here: balancing weight and electric force on oil drop

Radius of oil drop: $r = 1.64\mu\mathrm{m}$

Mass density of oil: $\rho = 0.851\mathrm{g/cm}^3$

Electric field: $E = 1.92 \times 10^5\mathrm{N/C}$

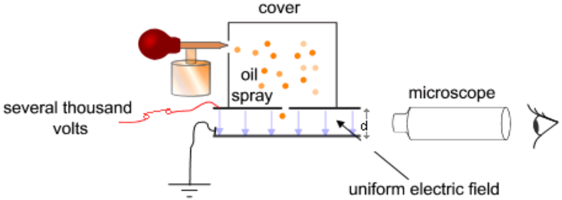
Mass of oil drop: $m = \dfrac{4\pi}{3} r^3 \rho = 1.57 \times 10^{-14}\mathrm{kg}$

Equilibrium of forces: $neE = mg$

Number of excess elementary charges (integer): $n = 5$

Elementary charge: $e = \dfrac{mg}{nE} \simeq 1.6 \times 10^{-19}\mathrm{C}$

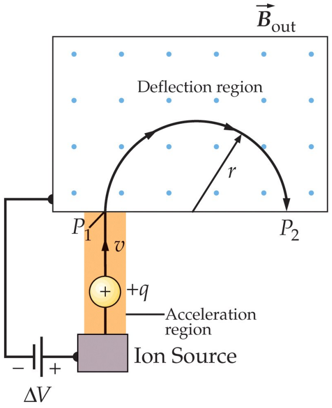
Mass of electron: $m \simeq 9.1 \times 10^{-31}\mathrm{kg}$



Mass Spectrometer

Purpose: measuring masses of ions.

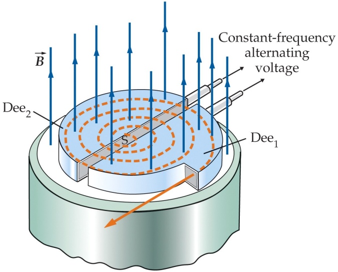
- Charged particle is accelerated by moving through potential difference $|\Delta V|$.
- Trajectory is then bent into semicircle of radius r by magnetic field $\vec{B}$.
- Kinetic energy: $\frac{1}{2}mv^2 = q|\Delta V|$.
- Radius of trajectory: $r = \frac{mv}{qB}$.
- Charge: $q = e$
- Mass: $m = \frac{eB^2r^2}{2|\Delta V|}$.



Cyclotron

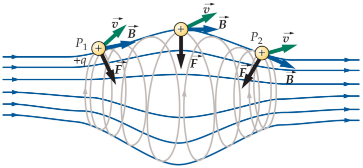
Purpose: accelerate charged particles to high energy.

- Low-energy protons are injected at S.
- Path is bent by magnetic field $\vec{B}$.
- Proton is energized by alternating voltage ΔV between Dee_1 and Dee_2.
- Proton picks up energy $\Delta K = e\Delta V$ during each half cycle.
- Path spirals out as velocity of particle increases: Radial distance is proportional to velocity: $r = \dfrac{mv}{eB}$.
- Duration of cycle stays is independent of r or v: cyclotron period: $T = \dfrac{2\pi m}{eB}$.
- Cyclotron period is synchronized with alternation of accelerating voltage.
- High-energy protons exit at perimeter of $\vec{B}$-field region.

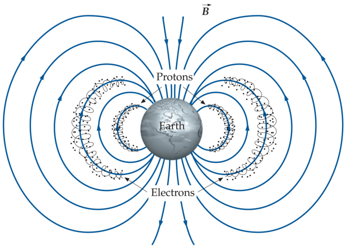


Magnetic Bottles

Moving charged particle
confined by
inhomogeneous magnetic field.

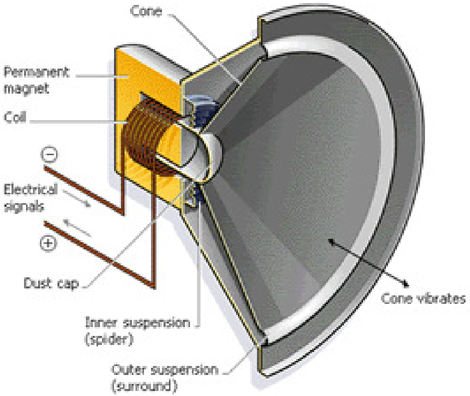


Van Allen belt:
trapped protons and electrons
in Earth's magnetic field.



Loudspeaker

Conversion of electric signal into mechanical vibration.

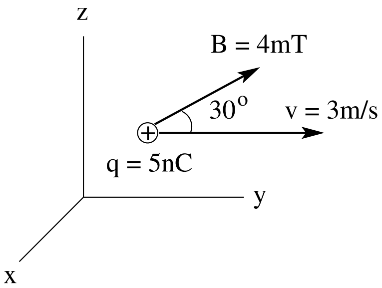


Intermediate Exam II: Problem #4 (Spring '05)

Consider a charged particle moving in a uniform magnetic field as shown. The velocity is in y-direction and the magnetic field in the yz-plane at 30° from the y-direction.

(a) Find the direction of the magnetic force acting on the particle.

(b) Find the magnitude of the magnetic force acting on the particle.



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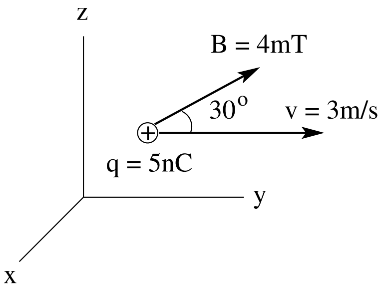

Solution:

(a) Use the right-hand rule: positive x-direction (front, out of page).

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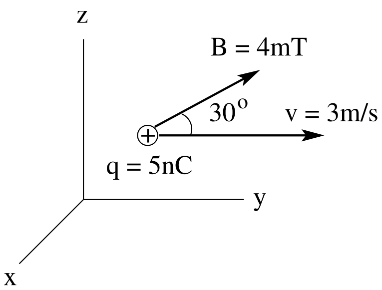


Solution:

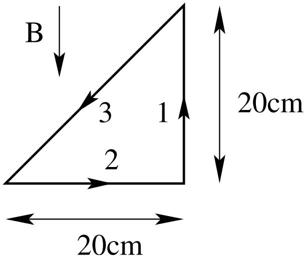
(a) Use the right-hand rule: positive x-direction (front, out of page).
(b) $F = qvB\sin 30^\circ = (5\times10^{-9}\text{C})(3\text{m/s})(4\times10^{-3}\text{T})(0.5) = 3\times10^{-11}\text{N}.$

Intermediate Exam II: Problem #4 (Spring ’06)

A current loop in the form of a right triangle is placed in a uniform magnetic field of magnitude $B = 30\text{mT}$ as shown. The current in the loop is $I = 0.4\text{A}$ in the direction indicated.

(a) Find magnitude and direction of the force $\vec{F}_1$ on side 1 of the triangle.

(b) Find magnitude and direction of the force $\vec{F}_2$ on side 2 of the triangle.



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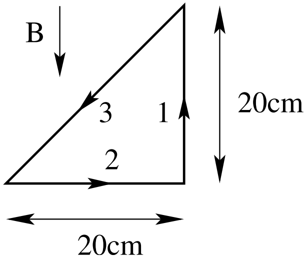

Solution:

(a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between $\vec{L}$ and $\vec{B}$ is 180°).

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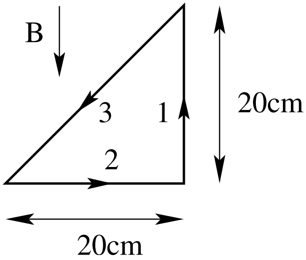

Solution:

(a) $\vec{F}_1 = I\vec{L} \times \vec{B} = 0$ (angle between $\vec{L}$ and $\vec{B}$ is 180°).

(b) $F_2 = ILB = (0.4\text{A})(0.2\text{m})(30 \times 10^{-3}\text{T}) = 2.4 \times 10^{-3}\text{N}$.
Direction of $\vec{F}_2$: $\otimes$ (into plane).

Magnetic Force Application (3)

The dashed rectangle marks a region of uniform magnetic field $\vec{B}$ pointing out of the plane.

- Find the direction of the magnetic force acting on each loop with a ccw current I.