Environmental Policy with Collective Waste Disposal

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1. Introduction

Nearly every economic activity produces waste. In the case of solid waste alone, the United Nations Environmental Program (UNEP) estimates that 1.84 billion tons of municipal waste and over 400 million tons of hazardous waste were produced globally in 2004. Waste released directly into the environment without treatment has the potential to cause significant damage to land, water and air resources, making the proper disposal of residual and frequently toxic waste material a significant public policy concern.

Collective waste disposal systems are an important tool for controlling environmental damage from the release of untreated waste residues. Collective disposal systems, which exploit economies of scale in waste treatment and disposal costs among decentralized polluters, involve the spatial transportation of waste material from regions where it is produced to regions where facilities can be designed to assimilate it. Collective disposal systems are currently used to manage household garbage, public sewage, and a significant portion of hazardous waste material from industrial and manufacturing processes. Collective disposal systems are also emerging as a potentially important component of management strategies for disposing of gaseous waste, for instance carbon capture and storage systems for “clean coal”. In many cases, collective disposal systems are used in conjunction with traditional environmental policies, as in the case of deposit-refund systems for used lubricating oil [10].

This paper characterizes the socially optimal environmental policy in a setting with coordinated waste disposal activities among producers of solid, liquid and gaseous waste. Despite the importance of collective waste disposal in modern waste management systems, it is surprising to note that the optimal composition of waste networks and the implications of
centralized waste collection for environmental policy are subjects that have received little attention to date.\(^1\)

Absent market prices for waste disposal, it is well-known that the incentive structure of markets favors the discharge of untreated waste material directly into the environment. Vernon Smith [11] considers the dynamics of waste accumulation under circumstances where waste can be either disposed or recycled and shows that policies that implement the optimal solution involve unit incentives for waste disposal.

But levying prices for waste disposal, for instance the use of Pigouvian taxes on air and water pollution, provides incentives for agents to make illicit waste discharges through “midnight dumping”. Monitoring waste discharges among a potentially large set of non-point sources can be prohibitively costly. Several papers [2], [5], [8] consider optimal waste disposal policy under circumstances where monitoring the waste discharges of individual polluters is not possible and demonstrate that deposit-refund systems can be used to align private and social incentives for waste collection. The optimal policy involves a tax (deposit) on the good whose use results in residual waste combined with a subsidy (refund) on waste collection, which together act as an implicit Pigouvian tax on uncollected waste. This outcome is generalized in [6] from the case of solid waste materials to any activity that generates solid, liquid or gaseous waste residue as a by-product of production or consumption.

Our paper extends the literature on deposit-refund policies by explicitly modeling collective waste disposal facilities as part of an optimal waste management system that involves multiple methods of control. We show that centralized waste collection leads to network effects that fundamentally alter the structure of the optimal environmental policy among polluting firms.

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\(^1\) Previous papers [3],[4],[9] have considered policies that facilitate optimal waste disposal across a mix of production and abatement activities; however, to our knowledge our paper is the first to examine these measures in a context that includes the possibility of spatially reallocating waste residues through collective disposal.
Absent collective disposal, the optimal environmental policy imposes more stringent controls in hazardous regions where the local environmental damage function is high than in regions where the local environmental damage function is low; however, when a centralized waste system is introduced, the optimal environmental policy level is monotonically increasing in distance from the disposal facility, an outcome that holds irrespective of regional differences in environmental damage functions. That is, in an optimal environmental policy with collective disposal, members of the waste disposal network face higher environmental policy controls (e.g., greater deposits and refunds) as the distance increases between the polluter and the disposal site.

We investigate the optimal mix of waste management strategies under circumstances where spatially distributed polluters select among three alternative methods of waste disposal. One option available to waste generators is to invest in abatement equipment to reduce or eliminate waste residues, for instance by installing “scrubbers” on smokestacks. A second option is to deliver waste residues through a conveyance system to a collective disposal facility, which can exploit scale economies in treatment and disposal costs. The third option is to deposit waste residue directly into the environment as local pollution.\(^2\) Discharging untreated waste into the environment may be illegal; however, those engaging in the practice may be hard to catch.

We frame our analysis of collective waste disposal around a spatial distribution of polluters. To focus the model on the role of collective waste disposal in environmental policy, we treat waste residue as a homogenous product and suppress issues of toxic concentration.\(^3\)

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\(^2\) Another option is to control pollution directly by adopting conservation technologies that displace waste residues, for instance the use of precision technology to increase water use efficiency and reduce chemical drift [1], [7], [12]. Our model abstracts from the issue of technology investment to focus on the disposal of residual waste from a given waste generation function, but our results generalize readily to the case of conservation inputs.

\(^3\) In principle, interaction effects may exist when damages from heterogeneous pollutants depend on the composition of the “slurry”. We clarify the exposition by focusing on a single pollutant.
We consider the case of environmentally-damaging waste that is a by-product of production activities at each location. Waste residue that is not abated or collected must ultimately return to the environmental medium as pollution by the Law of Conservation of Mass. We allow pollution resulting from waste deposits to create both local environmental damage and transboundary damage, where transboundary damage from locally discharged waste material is equally shared as common property among all agents, for instance in cases where the environmental medium is “well-mixed”.

Our principle findings are as follows. First, the ability to dispose of waste in a collective system introduces network effects in waste disposal cost functions that lead to increasing prices for waste disposal over distance from the disposal facility. In the case of solid waste disposal, network effects can be driven by rising transportation costs over distance, while in the case of liquid and gaseous waste, network effects can arise when the cost of moving a given volume of pollution through a pipeline or drainage canal depends on the cumulative stock of waste.

Second, as disposal costs rise over distance from the disposal facility, the optimal mix of waste disposal activities must also change over distance to equate the marginal value of each activity with the unit cost of waste disposal. Relative to producers in close proximity to the disposal facility, producers more distant from the disposal facility, who face relatively higher disposal costs, must engage in greater abatement effort and incur greater local pollution damages to equalize margins in the socially optimal resource allocation.\footnote{It should be noted that these comparisons are relative to other producers. Individual producers (at least weakly) reduce pollution releases as the result of participating in a collective waste disposal system.}

Third, as a consequence of these outcomes, the policy control on uncollected waste, whether in the form of Pigouvian taxes or market prices in cap-and-trade systems, must rise over distance from the disposal facility in the social optimum. This feature of an optimal
environmental policy, which holds even when “hotspots” exist in regions with relatively high local environmental damage functions, results in monotonically decreasing environmental quality levels over distance from the disposal facility for all members of the disposal network.

The remainder of the paper is structured as follows. In the following section, we develop a model of waste disposal that contains a mix of alternative waste disposal options including the dumping of untreated waste residues into the local environment. In Section 3, we describe the socially optimal resource allocation. In Section 4, we characterize environmental policies that align the market outcome with the social optimum and consider the issue of optimal network size. Section 5 discusses the implications of spatial heterogeneity in environmental damage functions on the optimal disposal network and Section 6 concludes.

2. A Model of Collective Waste Disposal

Consider a waste disposal network (e.g., a pipeline or drainage canal) in which the consumers or producers generating waste residue (hereafter “waste production units”) are arrayed laterally along a line. We order waste production units in terms of distance from the origin according to the index, \( j = 0, ..., J \). Waste material generated at location \( j \) can be deposited locally or delivered to a collective waste disposal facility, for instance a landfill or sewage treatment plant, which is situated at the origin of the line. Larger values of \( j \) in the index refer to producers at successively greater distances from the collective disposal site, and we define the location of each production unit in terms of distance \( l_j \) from the disposal point, where \( l_{j+1} > l_j \).

To facilitate our analysis of the environmental policy implications of collective waste disposal, we suppress the input choice problem of firms and assume the output of waste production unit \( j \), which we denote by \( Y_j \), has unit value in the private market and social benefit
function. That is, we implicitly assume demand for the output of waste production units is infinitely elastic.

Production at each location generates waste material. Specifically, production unit \( j \) generates \( Q_j = Q(Y_j) \) units of waste residue as a by-product of production, where the residue function, \( Q(Y_j) \), is an increasing and convex function of output \( (Q' > 0, Q'' > 0) \).

Each producer faces three options for managing waste residues: (i) abate waste residues in-house; (ii) contribute waste residues to a disposal network that delivers the waste to a collective disposal site; or (iii) release waste residues as pollution into the local environment. We denote the abatement cost function as \( c^A(A_j) \), which is increasing, \( \frac{\partial c^A(A_j)}{\partial A_j} > 0 \), and convex, \( \frac{\partial^2 c^A(A_j)}{\partial A_j^2} > 0 \), in the abatement level \( A_j \).

Net residue after abatement may either be transferred or disposed locally at site \( j \). Let \( M_j \) denote the waste residue removed from location \( j \) through the collective disposal system and let \( Z_j \) denote the local pollution arising from waste residue retained at location \( j \). Local waste disposal at site \( j \) contributes to regional pollution damages that spill over to harm other locations through transboundary pollution effects that are equally shared by all regions.

The material balance equation at location \( j \) is

\[
Q_j = A_j + Z_j + M_j. \tag{1}
\]

Collective waste disposal among production units serves to reallocate waste residue over space, facilitating economies of scale in waste disposal cost functions. We denote the cost of waste disposal at the collective disposal facility by \( c^D(S) \) where \( S = \sum_{j=1}^{J} M_j \) is the stock of

\[ ^5 \text{It is relatively straightforward to extend our analysis to include input choices by production units and the potential to substitute towards the use of conservation inputs that reduce residual waste per unit of output. Details from a more general model that includes input choices by firms are available from the authors on request.} \]
waste disposed at location 0. We assume sufficient scale economies exist that collecting waste material from spatially-distributed waste production units for centralized disposal at a waste treatment plant is (at least potentially) part of the optimal waste management system.

The returns to collective waste disposal are offset to some degree by transportation costs involved in hauling waste. This is a common element of solid, liquid and gaseous waste disposal problems, although the mechanics involved in relocating waste can differ according to whether the disposal system involves waste transportation by pipelines (liquid and gaseous waste) or by trucks and railcars (solid waste). In general, the stock of waste conveyed across location \( j \) is represented by the equation of motion

\[
S_j = S_{j+1} + M_j = \sum_{k=j}^{j} M_k.
\]

In expression (2), the stock of waste that migrates through the \( j \)th production unit, \( S_j \), is the sum of the waste disposed by unit \( j \) and the waste disposed by all production units at greater distances from the disposal facility than location \( j \) (i.e., \( j+1, j+2, ..., J \)). The cost of moving this total amount of waste through the \( j \)th segment (from the \( j \)th unit to the \( j-1 \)th unit) is given by

\[
c^M_j = \Delta l_j c^M(S_j),
\]

where \( \Delta l_j = (l_j - l_{j-1}) \) is the length of production segment \( j \) and \( c^M(S_j) \) is the total cost of moving \( S_j \) units of waste residues one unit of distance. The marginal cost of moving waste across segment \( j \) is positive and increasing in the stock of waste: \( \partial c^M(S_j)/\partial S_j > 0 \) and \( \partial^2 c^M(S_j)/\partial S_j^2 > 0 \).

The total transportation cost associated with moving all waste residues from production location \( j \) to the disposal facility is \( T_j = \sum_{k=1}^{j} c^M_k \), which we write in terms of the waste residue
contributed by each production unit as \( T_j = T_j(M_1,\ldots,M_J) \). The transportation cost function is increasing in the disposal amount of production unit \( j \):

\[
\frac{\partial T_j}{\partial M_j} = \sum_{k=1}^i \Delta l_k \frac{\partial c^M(S_k)}{\partial M_j} > 0.
\]

Our specification for the disposal cost function encompasses the standard interpretation of rising unit transportation costs over distance in urban economics models, as \( \frac{\partial T_j}{\partial M_j} \) is increasing for pollution generating units at greater distances from the disposal site. Formally,

\[
\Delta T_M(j) := \frac{\partial T_j}{\partial M_j} - \frac{\partial T_{j-1}}{\partial M_{j-1}} = \Delta l_j \frac{\partial c^M(S_j)}{\partial M_j} > 0,
\]

where the inequality holds, because the marginal effect of an additional unit of waste disposal on all waste production units downstream from location \( j \) are equal (and thus cancel) in equilibrium. Consequently, the marginal cost of disposal rises over distance from the disposal facility.

Waste generated by production unit \( j \) either can be discharged to the collective disposal system or can be retained at location \( j \). Retaining waste at location \( j \) creates two types of environmental damage: (i) local environmental damage at pollution site \( j \), which we denote \( c^L(Z_j) \); and (ii) transboundary pollution damage, which we denote \( c^T(Z) \). To facilitate the comparison of outcomes with and without collective waste disposal, we confine attention to cases where transboundary pollution damages depend only on the aggregate level of pollution, \( Z = \sum_{j=1}^J Z_j \). This setting corresponds with the case of “common pool” damages in the sense that all regions share equally in the social cost of transboundary waste. The environmental damage functions for local pollution, \( c^L(Z_j) \), and transboundary pollution, \( c^T(Z) \), are positive, increasing and convex in pollution, and zero if no pollution is released.
3. The Social Optimization Problem

The social optimization problem involves selecting inputs, abatement, disposal and local pollution release to maximize social benefit (SB), which is defined as aggregate profits net of abatement costs, waste disposal costs and environmental damage. The social problem is

\[
\max_{(Y_j, A_j, M_j, Z_j)} \sum_{j=1}^{J} (Y_j - c^A(A_j) - c^L(Z_j)) - T_j - c^D(S) - c^T(Z)
\]

subject to the material balance equation (1), the equation of motion (2), and non-negativity constraints for the decision variables. A sufficient condition for a concave social benefit function is that disposal costs at the facility are at least weakly convex in the level of aggregate waste disposal, \( \frac{\partial^2 c^D(S)}{\partial S^2} \geq 0 \); however, disposal costs at the waste treatment facility can be concave (i.e., exhibiting returns to scale) in the level of aggregate waste disposal under the regularity condition that the overall social benefit function is concave. For the remainder of the paper, we assume this is so.

For expositional clarity, we characterize the solution to the social optimization problem (6) by examining the optimal waste disposal strategy at location \( j \) in the case where an interior solution holds for all decision variables. Implicitly, this assumes that a waste transportation system is available for use at all locations \( j \) and that the optimal waste mitigation strategy involves a mix across activities for waste production units participating in the disposal network. In Section 4, we extend this analysis by considering environmental policies and the equilibrium size of the waste disposal network in the general context where the non-negativity constraints on the decision variables are binding at various points in space.

To clarify the interpretation of our main results, it is helpful to define two terms. Let
denote the marginal social damage from pollution by production unit \( j \). The marginal social damage of pollution generated by production unit \( j \) sums the marginal cost of local and transboundary environmental damages from untreated waste residues at location \( j \). To simplify notation, we drop the arguments of this function and denote the total marginal cost of pollution for waste production unit \( j \) as \( d^Z_j \).

Let the marginal cost of waste disposal for the \( j \)th production unit be denoted by

(8)  
\[
d^M_j (S_j, S) = \frac{\partial c^D(S)}{\partial M_j} + \frac{\partial T_j}{\partial M_j},
\]

where we drop the arguments of the function \( d^M_j \) hereafter. Notice that the marginal cost of waste disposal at location \( j \), \( d^M_j \), rises over distance from the disposal facility. There are at least two reasons why this is true. First, the unit cost of transporting waste may rise over distance, as in the case when hazardous waste solids are trucked or hauled by rail to a landfill. Second, the cost of moving liquid or gaseous waste through a disposal network may be rising in the cumulative stock of waste conveyed through a canal or pipeline, which implies that waste disposed by upstream units imposes a negative network externality on downstream users in the disposal cost function. Under either interpretation, unit waste disposal costs rise for production units that are farther away from the disposal facility relative to more proximate production units (i.e., \( \Delta T^M_j (j) > 0 \)). Likewise, unit waste disposal costs rise with the length of segment, \( l_j \):

\[
\frac{\partial d^M_j}{\partial l_j} = \frac{\partial c^D(S_j)}{\partial M_j} > 0.
\]
Both $d_j^Z$ and $d_j^M$ are simultaneously determined within the model as a result of choices by production units regarding output $Y_j$, abatement $A_j$, waste disposal $M_j$, and local pollution $Z_j$. To develop intuition for the model outcomes, we start by formally evaluating the case in which an interior solution exists for all decision variables.

The problem for waste production unit $j$ is to choose an output level, $Y_j$, and to select the allocation of waste residue between abatement, disposal, and local pollution releases. We focus the model on waste disposal choices by tying production decisions to disposal options through the material balance constraint (1). Formally, we have a degree of freedom in characterizing optimal choices, because residues, $Q_j$, are a smooth, monotonically increasing function of $Y_j$, which ensures that every output-residue combination is unique and $Q_j^{-1}$ exists conditional on the selection of $A_j$, $M_j$, and $Z_j$. Accordingly, we recast the problem of waste production unit $j$ as choosing $Y_j = Q_j^{-1}(A_j + M_j + Z_j)$ implicitly through the selection of $A_j$, $M_j$, and $Z_j$.

We write the social optimization problem in terms of the material balance constraint as:

$$\max_{\{A_j, M_j, Z_j\}} \begin{array}{c} SB \\
\end{array} = \sum_{j=1}^{J} \left( Y_j (A_j + M_j + Z_j) - c^A(A_j) - c^L(Z_j) \right) - T_j - c^D(S) - c^T(Z).$$

The $3 \times J$ first-order necessary conditions are:

$$\frac{\partial SB}{\partial A_j} = \frac{\partial Y_j}{\partial Q_j} - \frac{\partial c^A}{\partial A_j} = 0 \quad \text{for } j = 1,...,J$$

$$\frac{\partial SB}{\partial M_j} = \frac{\partial Y_j}{\partial Q_j} - d^M_j = 0 \quad \text{for } j = 1,...,J$$

$$\frac{\partial SB}{\partial Z_j} = \frac{\partial Y_j}{\partial Q_j} - d^Z_j = 0 \quad \text{for } j = 1,...,J.$$
The social optimality conditions (10) - (12) are intuitive. Each waste production unit equates marginal abatement cost, marginal disposal cost, and the total marginal damage from pollution to the marginal production profit in the socially optimal resource allocation:

\[
\frac{\partial c^A}{\partial A_j} = d^M_j = d^Z_j = \frac{\partial Y_j}{\partial Q_j}.
\]

Waste production units faced with incentives to reduce pollution can increase waste disposal, invest in abatement equipment, or reduce production, and the optimal way to do so involves choosing a combination of waste management activities to minimize social cost.

The key to understanding the role of collective waste disposal in an optimal policy is that collective waste disposal provides an alternative to decentralized abatement investment by individual firms. Collective disposal allows waste residues to be diverted away from the local environmental medium and collected at a centralized location for treatment, thereby reducing the net environmental damage from pollution. Because waste diverted to the disposal system displaces waste residues released into the environment by the material balance equation, the marginal benefit of diverting pollution into the disposal network is equal to the marginal environmental damage of pollution at the social optimum. When waste from location \( j \) enters the disposal network, disposal costs depend on the level of waste disposed at location \( j \), the distance of location \( j \) from the collective waste disposal facility, and the total volume of waste delivered to the facility among all members of the waste disposal network.

Let \( d^*_j = d^M_j = d^Z_j \) denote the marginal damage of pollution from the selection of abatement, disposal, and local pollution release that simultaneously solve equations (10)-(12). The socially optimal mix of production, abatement, waste disposal, and pollution are determined to equate the marginal cost of a unit of pollution across activities, an observation that applies to
each waste production unit for which an interior solution exists to the social benefit maximization problem (9).

There is no guarantee that the private market equilibrium results in optimal waste disposal, and this is true both for the size of the disposal network and the volume of waste disposal contributed by individual members. In the following section, we identify policies that result in the socially optimal resource allocation in a decentralized market equilibrium.

4. Policy Outcomes

In this section, we decentralize the socially optimal resource allocation by considering policies that are capable of achieving the social optimum in the competitive market equilibrium.

Consider the case in which waste collection services are provided by a competitive, price-taking supplier who sets price equal to marginal cost. The supplier’s cost function is given by

\[ C(M_1, \ldots, M_J) = c^D(S) + T_j(M_1, \ldots, M_J) + F \]

where \( F \) denotes fixed costs of maintaining the waste disposal system. The competitive supplier charges each productive location a disposal fee, \( p_j \), per unit of waste disposal. The unit price of waste disposal is set equal to the marginal cost, so that

\[ p_j = \frac{\partial C}{\partial M_j} = \frac{\partial c^D}{\partial S} + \frac{\partial T_j}{\partial M_j}, \]

and a competitive market equilibrium for waste disposal service is defined as the outcome in which disposal markets clear at all locations.

We look for cases in which the competitive market equilibrium coincides with the socially optimal resource allocation under environmental regulation on waste production units. The appropriate policy approach in a given pollution setting depends both on the policy variables

\[^6\] Qualitatively similar results to those described below emerge as well in the case of a monopoly supplier.
available to the regulator and on the institutional framework for policy design. We consider two cases delineated according to whether the policymaker can observe the amount of waste residue released into the environment at each location. Under circumstances in which the policymaker cannot observe the amount of waste released into the environment, we assume the regulator can observe output at location $j$, and hence can ascertain the total level of waste generation, $Q_j$.\(^7\)

Consider, first, the case in which the regulator can observe the amount of waste residue released into the environment at each location. The private optimization problem for waste production unit $j$ is

$$\max_{A_j, M_j, Z_j} \pi_j = Y_j (A_j + M_j + Z_j) - c^A (A_j) - p_j M_j - t_j Z_j,$$

where $t_j$ is a tax on waste residue released as pollution into the environment. Differentiating this expression results in the first-order necessary conditions

$$\frac{\partial \pi_j}{\partial A_j} = \frac{\partial Y_j}{\partial Q_j} - \frac{\partial c^A}{\partial A_j} = 0 \quad \text{for } j = 1, \ldots, J$$

$$\frac{\partial \pi_j}{\partial M_j} = \frac{\partial Y_j}{\partial Q_j} - p_j = 0 \quad \text{for } j = 1, \ldots, J$$

$$\frac{\partial \pi_j}{\partial Z_j} = \frac{\partial Y_j}{\partial Q_j} - t_j = 0 \quad \text{for } j = 1, \ldots, J.$$

The socially optimal resource allocation can be decentralized in the competitive market equilibrium as follows:

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\(^7\) This latter case involves the regulator having full information on the production technology, but does not require monitoring pollution releases to the local environment at each site.
**Proposition 1.** If the policymaker can observe net waste residues directly at location $j$, the socially optimal resource allocation can be obtained by assessing a tax of $t_j^* = d_j^*$ per unit of waste released into the environment.

*Proof.* The private market equilibrium is completely characterized on substitution of equation (15) into equation (18) for $j = 1, ..., J$. Substituting $t_j^* = d_j^*$ into equation (19), and combining equations yields $d_j^* = p_j^* = \frac{\partial c^A}{\partial A_j} = \frac{\partial Y_j}{\partial Q_j}$, which completes the proof, where $p_j^*$ is the equilibrium price of waste disposal at location $j$ that equates equations (15) and (18) at $t_j^* = d_j^*$.

In the competitive market equilibrium, unit prices for waste disposal equate with marginal disposal cost at each location, which ensures that the disposal market clears only at the socially optimal resource allocation when $t^* = d_j^*$. Any deviation in waste disposal from the socially optimal waste disposal level of production unit $j$, $M_j^*$, leads either to excess supply or excess demand for disposal.

The outcome in Proposition 1 extends readily to the case in which the regulator cannot observe releases of waste residues into the environment. To see this, consider the problem of waste production unit $j$ facing a deposit-refund system comprised of a tax on total waste generation of $t_j$ and a subsidy on collected waste of $s_j$:

$$\max_{A_j, M_j, Z_j} \pi_j = Y_j \left( A_j + M_j + Z_j \right) - c^A \left( A_j \right) - p_j M_j - t_j Q_j + s_j \left( A_j + M_j \right).$$

**Proposition 2.** If the policymaker cannot observe waste residues, but can observe total waste generation, $Q_j$, abatement $A_j$, and waste disposal $M_j$, at location $j$, the social optimum can be obtained by:
(i) taxing total waste generation at a unit rate of \( t_j^* = d_j^* \), and

(ii) providing rebates equal to \( s_j^* = d_j^* \) on units of collected waste.

Proof. Noting that \( t_j = s_j \) in the optimal deposit-refund policy, the optimization problem of waste production unit \( j \) reduces upon use of the material balance constraint (1) to

\[
\max_{A_j, M_j, Z_j} \pi_j = Y_j (A_j + M_j + Z_j) - c^A (A_j) - p_j M_j - t_j Q_j + t_j (A_j + M_j)
\]

\[
= Y_j (A_j + M_j + Z_j) - c^A (A_j) - p_j M_j - t_j Z_j
\]

By inspection, this problem is identical to expression (16).

In the case of unobservable waste releases, the optimal policy takes the form of a deposit-refund system [2], [5]. The optimal policy levies a tax on all waste residue created by production units and then rebates the tax on all units disposed legally through abatement and collective disposal. The net cost of releasing waste into the environment under such a policy is \( Z_j d_j^* \) and the net cost of waste disposal is \( M_j p_j^* \). Aligning the tax with a comparable refund on legal disposal and abatement activities thus serves as an implicit tax on the discharge of untreated waste into the environment.

Under either policy, production units at location \( j \) are levied a (net) tax of \( d_j^* \) per unit for waste released into the environment and pay waste disposal prices of \( p_j^* = d_j^* \) in the private market equilibrium. The optimal tax on pollution, whether levied directly or indirectly on local pollution, is equal to the unit cost of waste disposal at each location.

To gain a better understanding of the patterns of resource use over space, we compare the socially optimal resource allocation for production units located at adjacent geographic positions, and then characterize policy implications for special cases of the model.
4.1. *Spatial Allocation of Resources*

Consider the case of spatially homogeneous waste production units. Absent a waste disposal system, homogeneous production units face identical taxes, generate the same amount of output, pollution, and abatement, and produce identical output levels in the socially optimal resource allocation. Introducing a collective disposal facility fundamentally alters this outcome. As the marginal cost of waste disposal rises at locations more distant from the disposal facility in the waste disposal network, the volume of waste generated and the amount of waste material diverted to collective disposal decreases, while the amount of pollution released into the local environment increases in the social optimum.

**Proposition 3.** For spatially homogeneous firms making use of a collective waste disposal system, firms located at successively farther locations from the waste treatment plant:

(i) produce lower output;

(ii) exercise greater abatement effort;

(iii) divert a smaller quantity of waste to the disposal facility; and

(iv) release a larger quantity of waste as pollution in the local environment.

**Proof.** Spatially differencing equation (13) between the $j$th and $j-1$th production units gives:

\[
\frac{\partial Y(Q_j)}{\partial Q_j} - \frac{\partial Y(Q_{j-1})}{\partial Q_{j-1}} = \frac{\partial c^A(A_j)}{\partial A_j} - \frac{\partial c^A(A_{j-1})}{\partial A_{j-1}} = d_j^M - d_{j-1}^M = d_j^Z - d_{j-1}^Z = \Delta T^*_m(j) > 0 ,
\]

where the inequality holds because the marginal cost of collective disposal rises over distance. Thus, transportation of waste to a collective disposal facility causes the marginal damage of pollution (and the marginal abatement cost) to increase over distance from the disposal facility, such that $A^*_j$ and $Z^*_j$ are increasing with distance in the socially optimal resource allocation. To
show that disposal is strictly decreasing, suppose not, so that \( M_j^* \geq M_{j-1}^* \). From above, \( A_j^* > A_{j-1}^* \) and \( Z_j^* > Z_{j-1}^* \) since pollution and abatement costs are convex. Thus, if \( M_j^* \geq M_{j-1}^* \) then total production residues increase by the material balance constraint: \( Q_j^* > Q_{j-1}^* \). However, combining equations (13) and (20) leads to a contradiction:

\[
\frac{\partial Y(Q_{j-1})}{\partial Q_j} \geq \frac{\partial Y(Q_j)}{\partial Q_j} = \frac{\partial c^4(A_j)}{\partial A_j} > \frac{\partial c^4(A_{j-1})}{\partial A_j} = \frac{\partial Y(Q_{j-1})}{\partial Q_j}
\]

where the first inequality arises because \( Q_j(Y_j) \) is convex, so \( Y_j = Q_j^{-1} \) is concave on the relevant domain. Thus, it follows that disposal is decreasing over distance: \( M_j^* < M_{j-1}^* \). The same contradiction arises from assuming \( Q_j^* \geq Q_{j-1}^* \), which confirms that output is strictly decreasing over distance when firms are homogeneous: \( Q_j^* < Q_{j-1}^* \).

Proposition 3 summarizes the implications of collective waste disposal for environmental policy. The marginal environmental damage from uncollected waste rises over distance in the market equilibrium, because the marginal cost of disposal increases over distance from the collective disposal facility. This is true both in the case where physical transportation costs rise over distance, for instance in the delivery of solid waste to landfills, as well as in cases where disposal costs rise with the cumulative stock of waste in a drainage canal or pipeline. Because the unit cost of disposing waste in the network rises over distance from the waste disposal site, the marginal damage of pollution must rise over distance at the social optimum, as waste production units equalize margins across all available waste disposal activities. In the case of homogeneous local environmental damage functions and interior solutions for the choice
variables of all producers, both abatement levels and pollution levels, $A^*_j$ and $Z^*_j$ respectively, increase with distance from the disposal facility as the marginal cost of collective disposal rises.

A similar outcome holds for heterogeneous waste production units, as long as they all participate in the disposal network. Collective disposal introduces network effects in waste management that lead to rising disposal costs over distance from the disposal facility. By equation (13), this implies that marginal abatement cost and marginal environmental damage from locally deposited waste material must rise over distance to equalize margins in the socially optimal resource allocation.

4.2. Corner Solutions and Optimal Network Size

We now relax the assumption that all production units have interior solutions for all waste management activities. When a collective disposal system is introduced, there are two cases where binding non-negativity constraints can arise. First, there is the case where a waste production unit is sufficiently far away from the disposal facility that joining the waste disposal network is not worthwhile, so that $M^*_j = 0$ for some producers. Producers who do not participate in the waste disposal network continue to operate as if no network existed, and pollute at the baseline levels characterized by equations (10) and (12), $A^*_j$ and $Z^*_j$. Second, there is the case where waste disposal is sufficiently cheap that introducing collective disposal leads to a corner solution with respect to abatement and/or pollution, so that $M^*_j > 0$ while $A^*_j = 0$ and/or $Z^*_j = 0$.

For expositional clarity, we consider the case in which interior solutions emerge for $A^*_j$ and $Z^*_j$, with corner solutions potentially arising with respect to the participation by individual
waste production units in the collective disposal network. We start the discussion by developing two corollaries to Proposition 3 (for the case of homogeneous polluters).

**Corollary 1.** If it is optimal for any location, \( j \leq J \), not to participate in collective disposal, then it will also be optimal for all locations more distant from the origin \( (k > j) \) not to participate.

This corollary follows directly from Proposition 3, because optimal disposal amounts, \( M_j^* \), are strictly decreasing with \( j \).

**Corollary 2.** Suppose some firms do not participate in collective disposal. Then, the optimal size of the waste disposal network \( (J^*) \) is determined by the greatest \( j \) such that the non-negativity constraint does not bind on \( M_j \), i.e., such that \( M_j^* > 0 \).

Corollaries 1 and 2 are intuitive. With homogeneous firms, optimal waste collection fees, \( p_j^* \), are set equal to marginal pollution taxes, \( d_j^* \), in the optimal environmental policy, both of which are strictly increasing with \( j \). This implies that the socially optimal level of collected waste, \( M_j^* \), must be strictly decreasing with \( j \). Moving from users in close proximity to the waste disposal site to more distant users, \( M_j^* \) may decrease to zero as the marginal social cost of waste disposal rises over distance. If \( M_j^* \) does decrease to zero for any production unit, then all production units further away from the disposal site also refrain from using the disposal system. Thus, all waste production units \( j \) that satisfy \( M_j^* > 0 \) utilize the collective disposal system, while the remaining waste production units that satisfy \( M_j^* = 0 \) do not contribute waste for
collective disposal. In the event that $M_j^* = 0$ for all $j$, then collective waste disposal is not an element of optimal environmental policy.

Suppose a collective waste disposal network emerges in which a subset of firms do not participate in the disposal network. In this case, firms not participating in the disposal network operate as if there was no collective disposal option available, resulting in strictly greater abatement and pollution levels (and strictly lower output levels) than waste production unit $J^*$. Thus, the collective disposal system reduces pollution and abatement for all firms who join the disposal network. Collective disposal also serves to increase the output levels of participating firms, with the largest output effects arising for firms located in close proximity to the waste disposal site.

5. Extensions to the Model

Extending our results to include the possibility of corner solutions is particularly important when waste production units are heterogeneous. In this Section, we address the case of heterogeneous producers by considering the possibility that the firms have heterogeneous local environmental damage functions, $c_j^L$. Without any restrictions on the spatial pattern of local damages, our findings in Proposition 3 and the resulting corollaries require some qualification. Specifically, it may be the case that the social optimum involves a set of firms closer to the origin that do not participate in the disposal network while firms farther away make use of collective disposal. It is still possible in this case to recover the optimal $J^*$ by identifying the greatest $j$ network members associated with positive $M_j^*$ at the social optimum; however, it is no longer necessarily the case that waste production units outside the network pollute more than firms within the network.
Our model of collective waste disposal is sufficiently general that it encompasses outcomes with negative disposal for waste production unit \( j \). Allowing for the possibility of “negative waste disposal” for waste production unit \( j \) raises the possibility that one or more locations become net repositories of untreated waste; that is, they become waste dumps for producers located farther upstream in the disposal system. Production locations that convert to waste dumps may even shut down production to enhance their value as disposal sites.

To see how our model generalizes to encompass the possibility of waste dumps, note that allowing negative values of \( M_j^* \) results in a real-valued interior solution for waste disposal for all firms. Next, suppose producer \( j \) becomes a waste dump (\( M_j^* < 0 \)) for a contiguous set of upstream producers \( j + 1, \ldots, k \) that all optimally make use of the disposal network. It follows immediately that our previous ordering results in Proposition 3 remain in force, so that firms at locations \( j + 1, \ldots, k \) have increasing, interior solutions for \( A_j^* \) and \( Z_j^* \). Thus, introducing the possibility of downstream locations that act as waste dumps preserves the ordering results derived for the case of homogeneous producers, where waste production units face increasing pollution levies (taxes or deposits and refunds) over distance from any disposal site.

Corollary 1 ensures that waste dumps can arise only when producing firms are heterogeneous in some fashion. Consider the limiting case in which one region can become a repository for untreated waste without producing local or transboundary environmental damage. In this case, creating a waste dump at this location entails no social cost, while providing positive network benefits to waste production units upstream from the waste dump by reducing disposal costs. Introducing waste dumps thus allows the total size of the disposal network to increase. In the event that local environmental damage occurs at the regional waste dump, but is simply less damaging than waste deposited elsewhere, a trade-off exists between increasing local
environmental damage at the site of the waste dump and reducing disposal costs for remaining waste production units further upstream in the disposal network.

6. Concluding Remarks

The optimal spatial allocation of waste residues in a waste disposal network creates network externalities among users that substantially alters the optimal design of environmental policy. We have shown that collective disposal systems for waste cause the optimal environmental policy level to increase monotonically over distance from the disposal facility. Moreover, we have demonstrated that this outcome occurs as a feature of the optimal policy for all members of a collective disposal network, irrespective of the degree of heterogeneity in local environmental damage functions across regions.

While our observations on collective waste disposal have the most bearing on the design of prevailing systems for solid and liquid waste disposal, our analysis generalizes readily to air pollutants. The development of waste collection systems for gaseous waste is central to emerging proposals for carbon capture and storage. Provided that significant economies of scale exist in the construction of storage systems, collective disposal of gaseous waste material can reduce total disposal costs. In this case, the optimal tax (or permit price) for a given unit of waste must be adjusted across polluting locations to account for each firm’s location in proximity to the storage facility, as rising policy controls over distance are necessary equalize pollution damages and abatement costs with disposal costs on the margin.

An interesting area for future research is to examine how the optimal size of a waste disposal network varies with pollution toxicity and disposal technology. Intuitively, it would seem that more toxic pollutants and more efficient disposal technology would lead to expansion in the size of the waste disposal network; however, these outcomes are not conclusive without
imposing further structure in the model. For example, toxicity may not directly impact how
disposal costs rise over distance, so the rate at which waste production units located at greater
distances from the facility reduce their waste disposal levels may be unaffected by changes in
pollution toxicity. Similarly, superior disposal technology may lead to greater waste disposal by
existing members of the network without providing excess capacity for users at more distant
locations.

Another potentially fruitful direction for future research is to consider the role of
collective waste disposal systems in addressing non-point source pollution problems. Given the
construction of a waste disposal network, it is possible to address non-point source pollution
problems through deposit-refund systems that levy taxes on polluting inputs and provide rebates
for the waste material returned through waste collection. Such policies might be particularly
effective in reducing the runoff of nitrites, phosphates and pesticides from agricultural land.
References


