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11. RC circuits. Charging and discharging of capacitors

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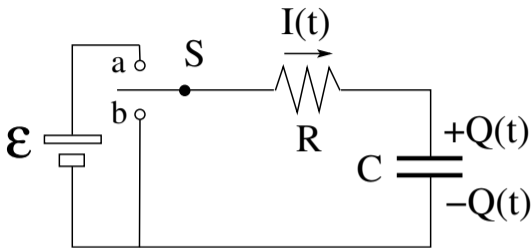


Specifications:

- \mathcal{E} (emf)
- R (resistance)
- C (capacitance)

Switch S :

- a: charging
- b: discharging



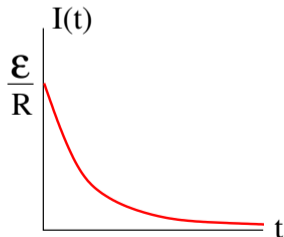
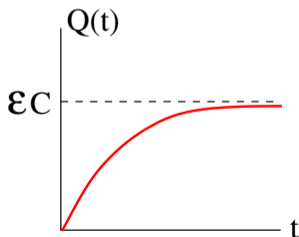
Time-dependent quantities:

- $Q(t)$: instantaneous charge on capacitor
- $I(t) = \frac{dQ}{dt}$: instantaneous current
- $V_R(t) = I(t)R$: instantaneous voltage across resistor
- $V_C(t) = \frac{Q(t)}{C}$: instantaneous voltage across capacitor

RC Circuit: Charging the Capacitor



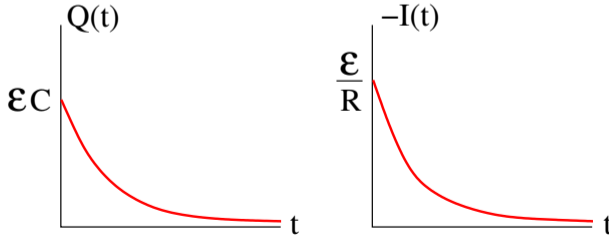
- Loop rule: $\mathcal{E} - IR - \frac{Q}{C} = 0$
- Differential equation: $R \frac{dQ}{dt} + \frac{Q}{C} = \mathcal{E} \Rightarrow \frac{dQ}{dt} = \frac{\mathcal{E}C - Q}{RC}$
 $\int_0^Q \frac{dQ}{\mathcal{E}C - Q} = \int_0^t \frac{dt}{RC} \Rightarrow -\ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right) = \frac{t}{RC} \Rightarrow \frac{\mathcal{E}C - Q}{\mathcal{E}C} = e^{-t/RC}$
- Charge on capacitor: $Q(t) = \mathcal{E}C \left[1 - e^{-t/RC}\right]$
- Current through resistor: $I(t) \equiv \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$



RC Circuit: Discharging the Capacitor



- Loop rule: $IR + \frac{Q}{C} = 0$
- Differential equation: $R \frac{dQ}{dt} + \frac{Q}{C} = 0 \Rightarrow \frac{dQ}{dt} = -\frac{Q}{RC}$
 $\Rightarrow \int_{\mathcal{E}C}^Q \frac{dQ}{Q} = -\int_0^t \frac{dt}{RC} \Rightarrow \ln\left(\frac{Q}{\mathcal{E}C}\right) = -\frac{t}{RC} \Rightarrow \frac{Q}{\mathcal{E}C} = e^{-t/RC}$
- Charge on capacitor: $Q(t) = \mathcal{E}C e^{-t/RC}$
- Current through resistor: $I(t) \equiv \frac{dQ}{dt} = -\frac{\mathcal{E}}{R} e^{-t/RC}$



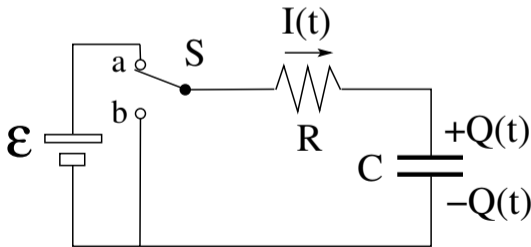
RC Circuit: Energy Transfer While Charging



Loop rule: $IR + \frac{Q}{C} = \mathcal{E}$ (I is positive)

- $I\mathcal{E}$: rate at which emf source delivers energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_C = \frac{IQ}{C}$: rate at which energy is stored in capacitor

Balance of energy transfer: $I^2R + \frac{IQ}{C} = I\mathcal{E}$



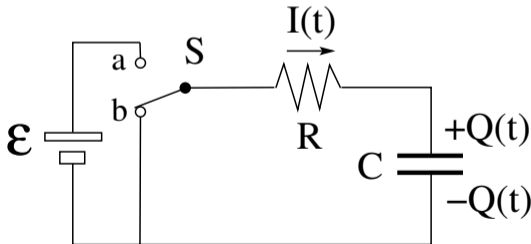
RC Circuit: Energy Transfer While Discharging



Loop rule: $IR + \frac{Q}{C} = 0$ (I is negative)

- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_C = \frac{IQ}{C}$: rate at which capacitor releases energy

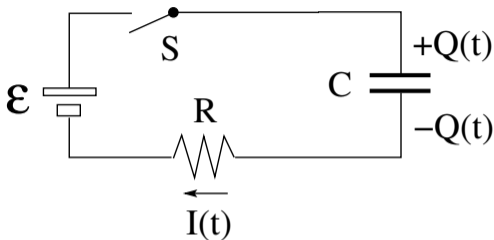
Balance of energy transfer: $I^2R + \frac{IQ}{C} = 0$





Specification of RC circuit
by 3 device properties:

- \mathcal{E} [V] (emf)
- R [Ω] (resistance)
- C [F] (capacitance)



Physical properties of RC circuit during charging process determined by 3 combinations of the device properties:

- $\mathcal{E}/R = I(t = 0)$: rate at which charge flows onto capacitor initially
- $\mathcal{E}C = Q(t = \infty)$: total charge placed on capacitor ultimately
- $RC = \tau$: time it takes to place 63% of the charge onto the capacitor
[$1 - e^{-1} = 0.632 \dots$]

RC Circuit: Application (3)



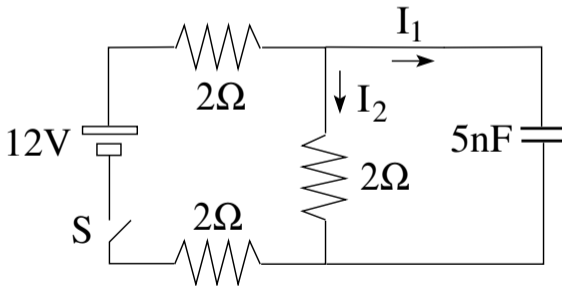
At time $t = 0$ the capacitor in this circuit is discharged and the switch is being closed.

Find the current I_1

- (a) at $t = 0$,
- (b) at $t = \infty$.

Find the current I_2

- (c) at $t = 0$,
- (d) at $t = \infty$.

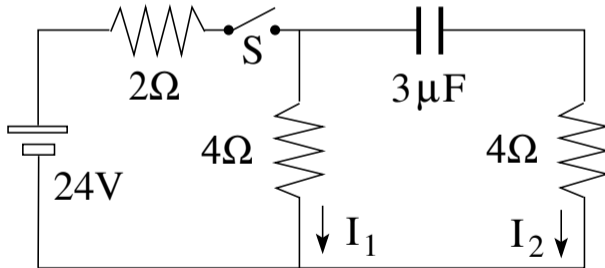


RC Circuit: Application (5)



In the RC circuit shown, the switch S has been open for a long time.

- Find the currents I_1 and I_2 immediately after the switch has been closed.
- Find the currents I_1 and I_2 a very long time later.

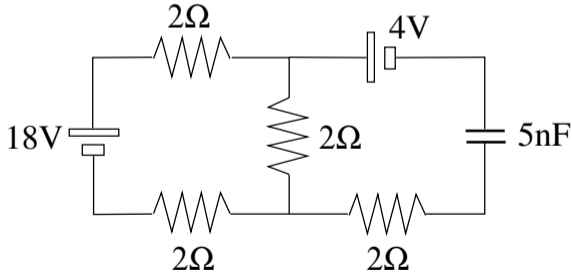


RC Circuit: Application (1)



This circuit has been running for a very long time.

- (a) Find the current through the 18V battery.
- (b) Find the total power dissipated in the resistors.
- (c) Find the charge stored on the capacitor.

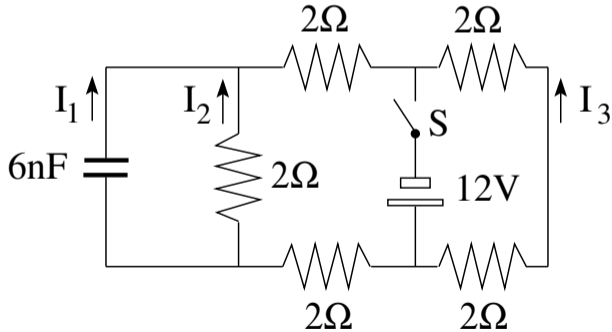


RC Circuit: Application (4)



In this 3-loop RC circuit, the switch S is closed at time $t = 0$.

- (a) Find the currents I_1, I_2, I_3 just after the switch has been closed.
- (b) Find the currents I_1, I_2, I_3 a very long time later.

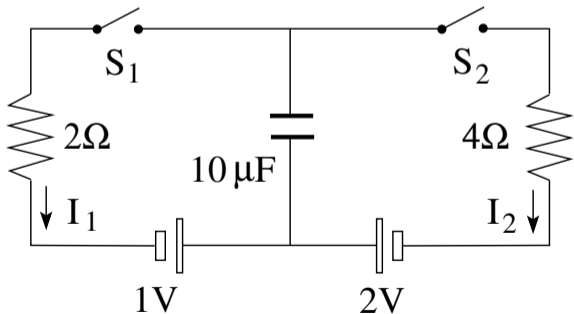


RC Circuit: Application (6)



In the RC circuit shown, both switches are initially open and the capacitor is discharged.

- Close switch S_1 and find the currents I_1 and I_2 immediately afterwards.
- Find the currents I_1, I_2 and the charge Q on the capacitor a very long time later.
- Now close switch S_2 also and find the currents I_1 and I_2 immediately afterwards.
- Find the currents I_1, I_2 and the charge Q on the capacitor a very long time later.



RC Circuit: Application (2)



The switches are closed at $t = 0$. This begins the charging process in each RC circuit.

Name the circuit in which...

- (i) the charge flows into the capacitor at the highest rate initially,
- (ii) the capacitor has the most charge ultimately,
- (iii) the capacitor is 63% full in the shortest time.

