2020

10. Resistor circuits. Kirchhoff's laws

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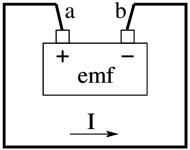
Direct Current Circuit

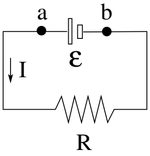
Consider a wire with resistance $R = \rho\ell/A$ connected to a battery.

- **Resistor rule**: In the direction of I across a resistor with resistance R, the electric potential drops: $\Delta V = -IR$.
- **EMF rule**: From the $(-)$ terminal to the $(+)$ terminal in an ideal source of emf, the potential rises: $\Delta V = \mathcal{E}$.
- **Loop rule**: The algebraic sum of the changes in potential encountered in a complete traversal of any loop in a circuit must be zero: $\sum \Delta V_i = 0$.

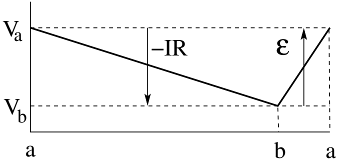
physical system



circuit diagram

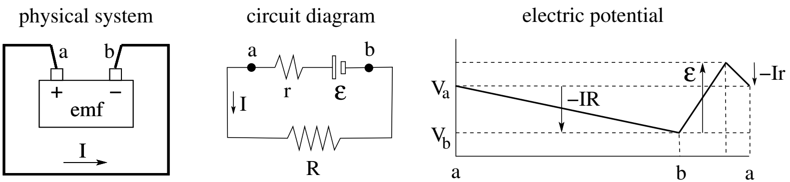


electric potential



Battery with Internal Resistance

- Real batteries have an internal resistance r.
- The terminal voltage $V_{ba} \equiv V_a - V_b$ is smaller than the emf $\mathcal{E}$ written on the label if a current flows through the battery.
- Usage of the battery increases its internal resistance.
- Current from loop rule: $\mathcal{E} - Ir - IR = 0 \quad \Rightarrow I = \dfrac{\mathcal{E}}{R+r}$
- Current from terminal voltage: $V_{ba} = \mathcal{E} - Ir = IR \quad \Rightarrow I = \dfrac{V_{ba}}{R}$



Resistor Circuit (4)

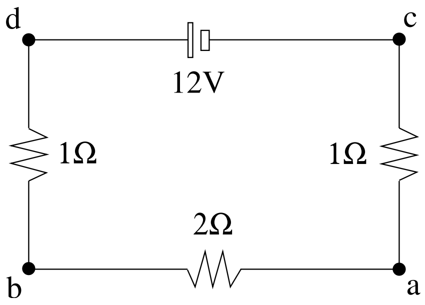
Consider the resistor circuit shown.

(a) Find the direction of the positive current (cw/ccw).

(b) Find the magnitude of the current.

(c) Find the voltage $V_{ab} = V_b - V_a$.

(d) Find the voltage $V_{cd} = V_d - V_c$.



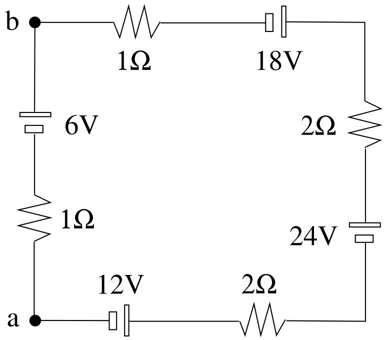
Resistor Circuit (6)

Consider the resistor circuit shown.

(a) Choose a current direction and use the loop rule to determine the current.

(b) Name the direction of positive current (cw/ccw).

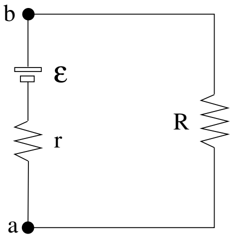
(c) Find $V_{ab} \equiv V_b - V_a$ along two different paths.



Power in Resistor Circuit

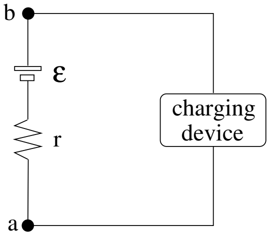
Battery in use

- Terminal voltage: $V_{ab} = \mathcal{E} - Ir = IR$
- Power output of battery: $P = V_{ab}I = \mathcal{E}I - I^2r$
    - Power generated in battery: $\mathcal{E}I$
    - Power dissipated in battery: I^2r
- Power transferred to load: $P = I^2R$



Battery being charged:

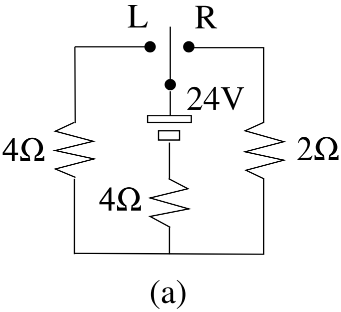
- Terminal voltage: $V_{ab} = \mathcal{E} + Ir$
- Power supplied by charging device: $P = V_{ab}I$
- Power input into battery: $P = \mathcal{E}I + I^2r$
    - Power stored in battery: $\mathcal{E}I$
    - Power dissipated in battery: I^2r



Resistor Circuit (7)

Consider two 24V batteries with internal resistances (a) $r = 4\Omega$, (b) $r = 2\Omega$.

- Which setting of the switch (L/R) produces the larger power dissipation in the resistor on the side?

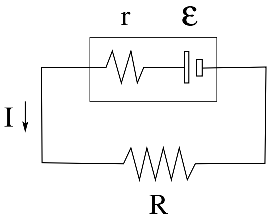


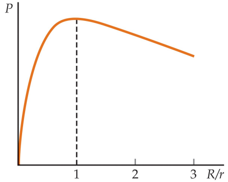
Impedance Matching

A battery providing an emf $\mathcal{E}$ with internal resistance r is connected to an external resistor of resistance R as shown.

For what value of R does the battery deliver the maximum power to the external resistor?

- Electric current: $\mathcal{E} - Ir - IR = 0 \quad \Rightarrow \; I = \dfrac{\mathcal{E}}{R+r}$
- Power delivered to external resistor: $P = I^2 R = \dfrac{\mathcal{E}^2 R}{(R+r)^2} = \dfrac{\mathcal{E}^2}{r} \dfrac{R/r}{(R/r+1)^2}$
- Condition for maximum power: $\dfrac{dP}{dR} = 0 \quad \Rightarrow \; R = r$





Resistor Circuit (5)

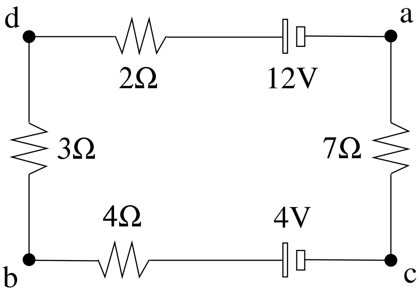
Consider the resistor circuit shown.

(a) Choose a current direction and use the loop rule to determine the current.

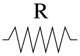
(b) Name the direction of positive current (cw/ccw).

(c) Find the potential difference $V_{ab} = V_b - V_a$.

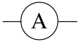
(d) Find the voltage $V_{cd} = V_d - V_c$.



Symbols Used in Cicuit Diagrams

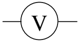


resistor

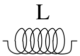


ammeter (connect in series)

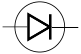
capacitor

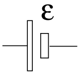


voltmeter (connect in parallel)

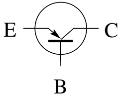


inductor


diode



emf source

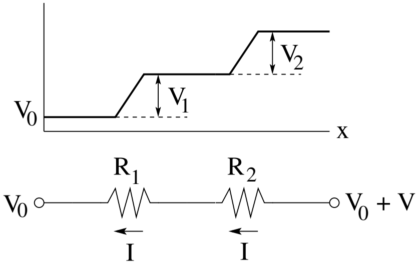


transistor

Resistors Connected in Series

Find the equivalent resistance of two resistors connected in series.

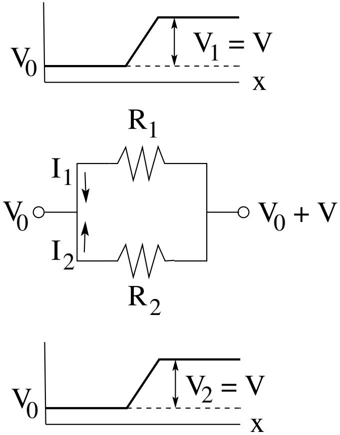
- Current through resistors: $I_1 = I_2 = I$
- Voltage across resistors: $V_1 + V_2 = V$
- Equivalent resistance: $R \equiv \dfrac{V}{I} = \dfrac{V_1}{I_1} + \dfrac{V_2}{I_2}$
- $\Rightarrow \; R = R_1 + R_2$



Resistors Connected in Parallel

Find the equivalent resistance of two resistors connected in parallel.

- Current through resistors: $I_1 + I_2 = I$
- Voltage across resistors: $V_1 = V_2 = V$
- Equivalent resistance: $\dfrac{1}{R} \equiv \dfrac{I}{V} = \dfrac{I_1}{V_1} + \dfrac{I_2}{V_2}$
- $\Rightarrow \dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$



Resistor Circuit (1)

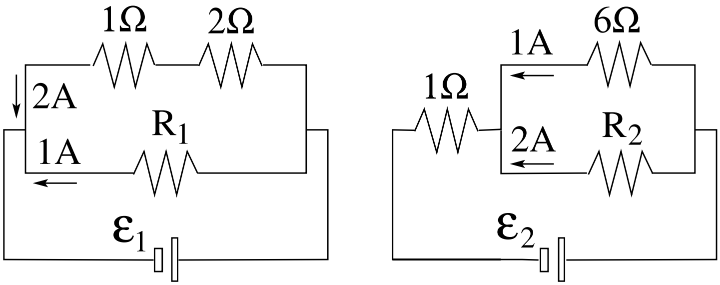
Consider the two resistor circuits shown.

(a) Find the resistance R_1.

(b) Find the emf $\mathcal{E}_1$.

(c) Find the resistance R_2.

(d) Find the emf $\mathcal{E}_2$.



Resistor Circuit (2)

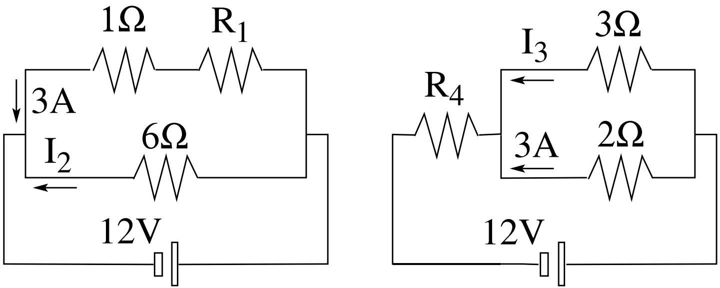
Consider the two resistor circuits shown.

(a) Find the resistance R_1.

(b) Find the current I_2.

(c) Find the current I_3.

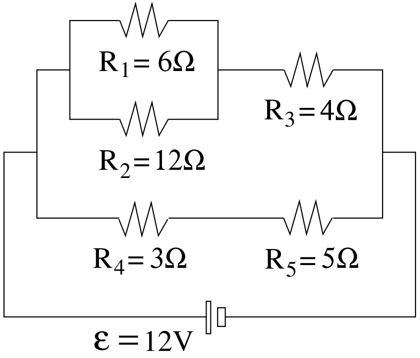
(d) Find the resistance R_4.



Resistor Circuit (8)

Consider the circuit of resistors shown.

- Find the equivalent resistance R_{eq}.
- Find the currents $I_1, \ldots, I_5$ through each resistor and the voltages $V_1, \ldots, V_5$ across each resistor.
- Find the total power P dissipated in the circuit.



Loop Rule

- When any closed-circuit loop is traversed, the algebraic sum of the changes in electric potential must be zero.

Junction Rule

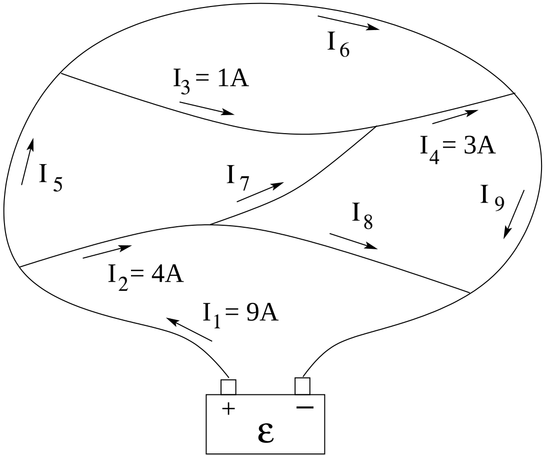
- At any junction in a circuit, the sum of the incoming currents must equal the sum of the outgoing currents.

Strategy

- Use the junction rule to name all independent currents.
- Use the loop rule to determine the independent currents.

Applying the Junction Rule

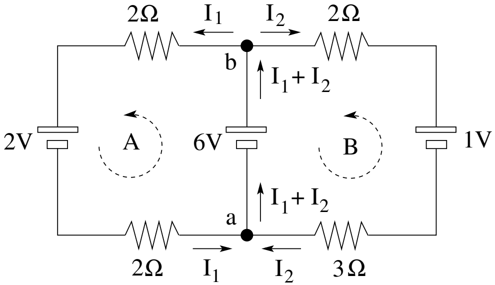
In the circuit of steady currents, use the junction rule to find the unknown currents $I_5, \ldots, I_9$.



Applying Kirchhoff's Rules

Consider the circuit shown below.

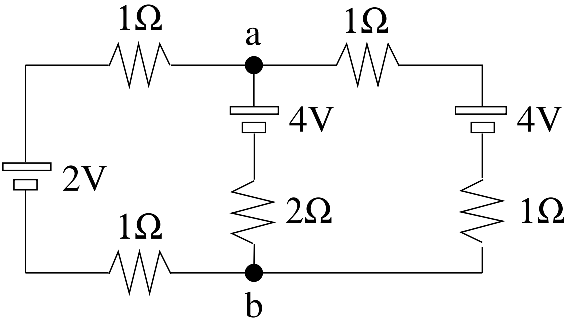
- Junction a: I_1, I_2 (in); $\quad I_1 + I_2$ (out)
- Junction b: $I_1 + I_2$ (in); $\quad I_1, I_2$ (out)
- Two independent currents require the use of two loops.
- Loop A (ccw): $6\mathrm{V} - (2\Omega)I_1 - 2\mathrm{V} - (2\Omega)I_1 = 0$
- Loop B (ccw): $(3\Omega)I_2 + 1\mathrm{V} + (2\Omega)I_2 - 6\mathrm{V} = 0$
- Solution: $I_1 = 1\mathrm{A}, \quad I_2 = 1\mathrm{A}$



Resistor Circuit (11)

Consider the electric circuit shown.

- Identify all independent currents via junction rule.
- Determine the independent currents via loop rule.
- Find the Potential difference $V_{ab} = V_b - V_a$.



Resistor Circuit (9)

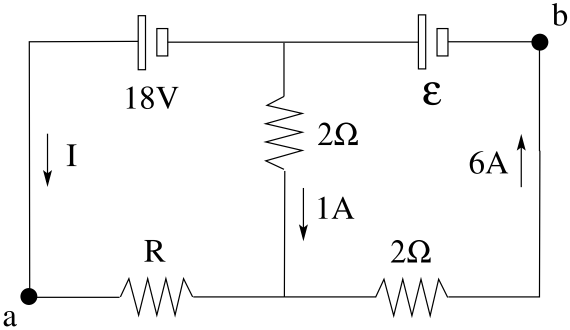
Use Kirchhoff's rules to find

(a) the current I,

(b) the resistance R,

(c) the emf $\mathcal{E}$,

(d) the voltage $V_{ab} \equiv V_b - V_a$.



Resistor Circuit (10)

Consider the electric circuit shown.

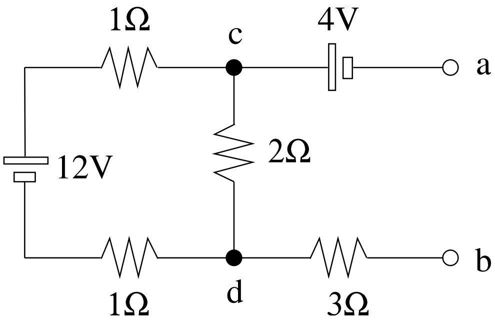
(a) Find the current through the 12V battery.

(b) Find the current through the 2Ω resistor.

(c) Find the total power dissipated.

(d) Find the voltage $V_{cd} \equiv V_d - V_c$.

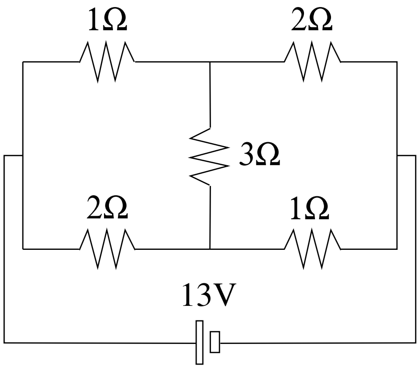
(e) Find the voltage $V_{ab} \equiv V_b - V_a$.



Resistor Circuit (12)

Consider the electric circuit shown.

- Find the equivalent resistance R_{eq} of the circuit.
- Find the total power P dissipated in the circuit.



Consider the rsistor and capacitor circuits shown.

(a) Find the equivalent resistance R_{eq}.

(b) Find the power P_2, P_3, P_4 dissipated in each resistor.

(c) Find the equivalent capacitance C_{eq}.

(d) Find the energy U_2, U_3, U_4 stored in each capacitor.