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09. Electric currents. Resistor, resistance, and resistivity

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Equilibrium:

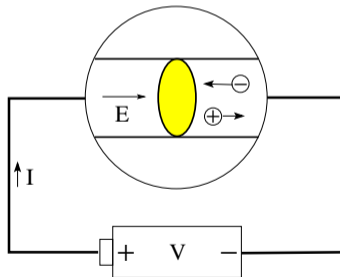
$\vec{E} = 0$ inside conductor. Mobile charge carriers undergo random motion.

Nonequilibrium:

$\vec{E} \neq 0$ inside conductor. Mobile charge carriers undergo random motion and drift. Positive charge carriers drift from high toward low electric potential and negative charge carriers from low toward high electric potential.

Electric current:

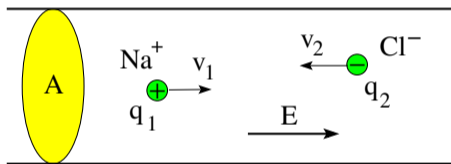
- Net charge flowing through given cross-sectional area per unit time.
- $I = \frac{dQ}{dt}$.
- SI unit: $1\text{C/s} = 1\text{A}$ (one Ampère)





Consider drift of Na^+ and Cl^- ions in a plastic pipe filled with salt water.

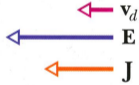
- $v_1 > 0, v_2 < 0$: drift velocities [m/s]
- $q_1 > 0, q_2 < 0$: charge on ions [C]
- n_1, n_2 : number of charge carriers per unit volume [m^{-3}]



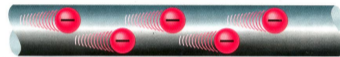
- Net charge flowing through area A in time dt : $dQ = n_1 q_1 v_1 A dt + n_2 q_2 v_2 A dt$ [C]
- Electric current through area A : $I \equiv \frac{dQ}{dt} = A(n_1 q_1 v_1 + n_2 q_2 v_2)$ [A]
- Current density: $\vec{J} = n_1 q_1 \vec{v}_1 + n_2 q_2 \vec{v}_2$ [A/m^2]
- Current equals flux of current density: $I = \int \vec{J} \cdot d\vec{A}$ [A]



positive charge carriers



negative charge carriers



positive current I flowing toward left in both cases

negative current I flowing toward right in both cases



- **Resistor:** device (material object with two terminals)
- **Resistance:** attribute of device
- **Resistivity:** attribute of conducting material

A voltage V provided by some source is applied to the terminals of a resistor and a current I is observed flowing through the resistor.

- Resistance: $R = \frac{V}{I}$ [1 Ω =1V/A] (1 Ohm)

The current density \vec{J} in a resistor depends on the local electric field \vec{E} and on the resistivity ρ of the resistor material.

- Resistivity: $\rho = \frac{E}{J}$ $\left[\frac{1\text{V/m}}{1\text{A/m}^2} = 1\Omega\text{m} \right]$
- Conductivity: $\sigma = \frac{1}{\rho}$ [1(Ωm)⁻¹]
- Vector relations: $\vec{E} = \rho\vec{J}$, $\vec{J} = \sigma\vec{E}$



TABLE 25-1

Resistivities and Temperature Coefficients

Material	Resistivity ρ at 20°C, $\Omega \cdot \text{m}$	Temperature Coefficient α at 20°C, K^{-1}
Silver	1.6×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Aluminum	2.8×10^{-8}	3.9×10^{-3}
Tungsten	5.5×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Lead	22×10^{-8}	4.3×10^{-3}
Mercury	96×10^{-8}	0.9×10^{-3}
Nichrome	100×10^{-8}	0.4×10^{-3}
Carbon	3500×10^{-8}	-0.5×10^{-3}
Germanium	0.45	-4.8×10^{-2}
Silicon	640	-7.5×10^{-2}
Wood	$10^8 - 10^{14}$	
Glass	$10^{10} - 10^{14}$	
Hard rubber	$10^{13} - 10^{16}$	
Amber	5×10^{14}	
Sulfur	1×10^{15}	

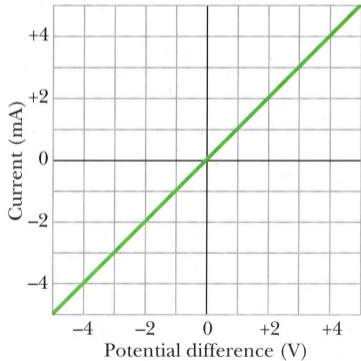
- $\alpha = \frac{(\rho - \rho_{20}) / \rho_{20}}{t_C - 20^\circ\text{C}}$
- α : temperature coefficient at 20°C in K^{-1}
- ρ : resistivity near 20°C
- ρ_{20} : resistivity at 20°C
- t_C : temperature in °C



$$V = RI \text{ with } R = \text{const}$$

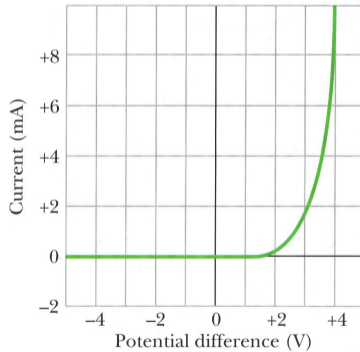
satisfied

RESISTOR 



violated

DIODE 



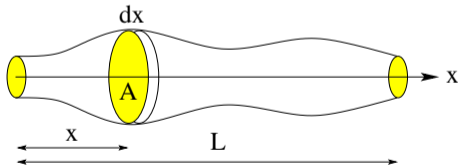


Uniform cross section

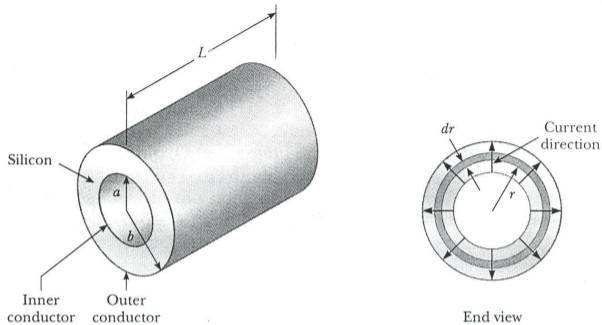
- Length of wire: L
- Area of cross section: A
- Resistivity of material: ρ
- Current density: $J = \frac{E}{\rho}$ [A/m^2]
- Current: $I = JA$ [A]
- Voltage: $V = EL$ [V]
- Resistance: $R \equiv \frac{V}{I} = \frac{\rho L}{A}$ [Ω]

Variable cross section

- Cross-sectional profile: $A(x)$
- Resistance of slice: $dR = \frac{\rho dx}{A(x)}$
- Resistance of wire: $R = \rho \int_0^L \frac{dx}{A(x)}$



Radial Current in Coaxial Cable

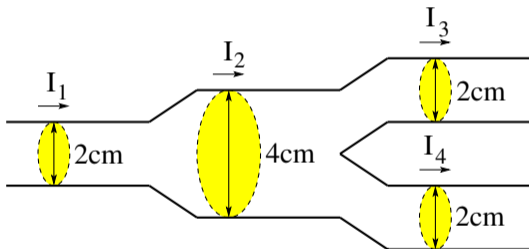


$$dR = \frac{\rho dr}{A}, \quad A = 2\pi rL$$

$$R = \frac{\rho}{2\pi L} \int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L} \ln \frac{b}{a}$$



A steady current I is flowing through a wire from left to right. The wire first doubles its diameter and then splits into two wires of the original diameter. Both branches on the right carry the same current.



Rank the current densities $J_1, J_2, J_3 = J_4$ in the three segments.



Consider three wires made of the same material.

Wire 1 of length 2m and diameter 2mm has a resistance 18Ω .

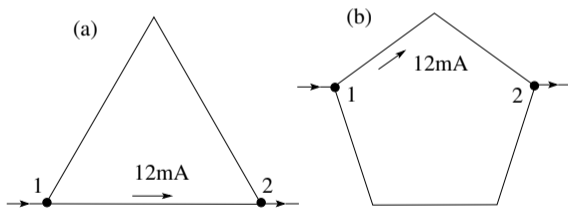
- (a) What resistance does wire 2 of length 4m and diameter 4mm have?
- (b) How long is wire 3 of diameter 6mm with a resistance of 18Ω ?



Two wires are formed into

- (a) an equilateral triangle,
- (b) a regular pentagon.

A voltage between points 1 and 2 produces a current of 12mA along the shorter path.

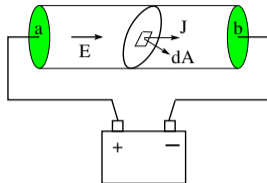


What is the current along the longer path in each case?



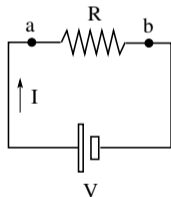
Material

- \vec{E} : electric field
- \vec{J} : current density
- ρ : resistivity
- $\sigma = 1/\rho$: conductivity
- $\vec{E} = \rho\vec{J}$, $\vec{J} = \sigma\vec{E}$



Device

- V : voltage
- I : electric current
- R : resistance
- $V = RI$



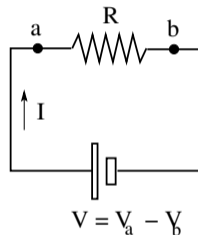
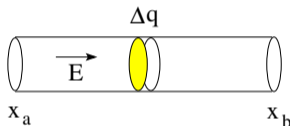
Current from current density: $I = \int \vec{J} \cdot d\vec{A}$

Voltage from electric field: $V = - \int_a^b \vec{E} \cdot d\vec{s}$



Consider a resistor in the form of a uniform wire.

- Voltage between ends: $V \equiv V_a - V_b = E(x_b - x_a)$
- Displaced charge: $\Delta q = I \Delta t$



- Work done by electric field \vec{E} on displaced charge Δq :

$$W_E = F(x_b - x_a) = E \Delta q (x_b - x_a) = V \Delta q = VI \Delta t$$

- Power dissipated in resistor: $P = \frac{W_E}{\Delta t} = VI = I^2 R = \frac{V^2}{R}$
- SI unit: $1\text{V} \cdot 1\text{A} = (1\text{J/C}) \cdot (1\text{C/s}) = 1\text{J/s} = 1\text{W}$ (Watt)

Resistor Problem (1)



A heating element is made of a wire with a cross-sectional area $A = 2.60 \times 10^{-6} \text{m}^2$ and a resistivity $\rho = 5.00 \times 10^{-7} \Omega\text{m}$.

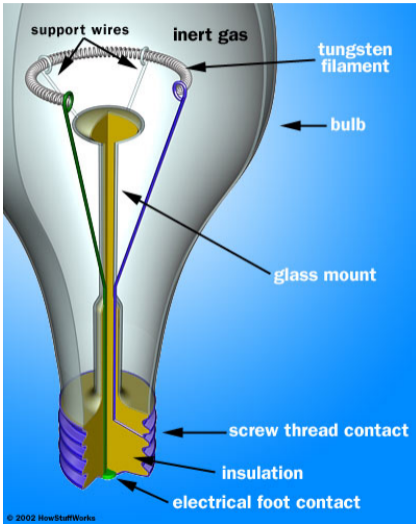
- (a) If the element dissipates 5000W when operating at a voltage $V_1 = 75.0\text{V}$, what is its length L_1 , its resistance R_1 , and the current I_1 running through it?
- (b) What must be the voltage V_2 , the resistance R_2 , and the length L_2 of a heating element made of the same wire if the same power should be generated with half the current?



A 1250W radiant heater is constructed to operate at 115V.

- (a) What will be the current in the heater?
- (b) What is the resistance of the heating coil?
- (c) How much thermal energy is generated in one hour by the heater?

Incandescent Light Bulb



- voltage drives electric current through filament
- electric current heats up filament
- hot filament emits radiation
- radiation has significant intensity in visible range around 4000 degrees Fahrenheit.
- most energy is emitted in the form of heat (infrared radiation)
- inert gas (e.g. argon) regulates heat transport and prevents oxidation
- tungsten has high melting point