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34. Applications of ac circuits. Transformer. Amplitudes versus rms value

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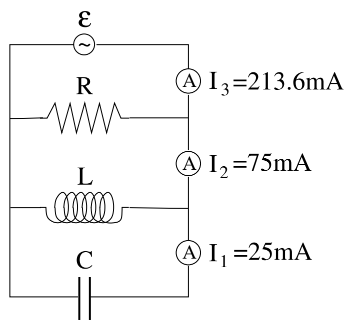
PHY204 Lecture 34 [rln34]

AC Circuit Application (4)

Given the current amplitudes I_1, I_2, I_3 through the three branches of this RLC circuit, and given the amplitude $\mathcal{E}_{max} = 100$V and angular frequency $\omega = 500$rad/s of the ac source, find the device properties R, L, C.



tsl310

Given in this application of an RLC parallel circuit are, in addition to the angular frequency, the EMF amplitude and three current amplitudes. We are being asked to find the device properties R, L, and C.

We get the device properties if we know the single-device impedances. Those we get from the ratio of voltage amplitude and current amplitude. The voltage amplitude is equal to the EMF amplitude in this circuit.

Not all currents measured by the three ammeters are currents that flow though individual devices. For the determination of the amplitudes I_R, I_L, and I_C through the devices, it us useful to have the phasor diagram for the RLC parallel circuit in front of us. Then we can reason as follows:

$$I_1 = I_C = 25\text{mA} \quad \Rightarrow \; X_C = \frac{100\text{V}}{25\text{mA}} = 4000\Omega \quad \Rightarrow \; C = 0.5\mu\text{F}.$$

$$I_2 = |I_L - I_C| = 75\text{mA} \quad \Rightarrow \; I_L = I_2 + I_C = 100\text{mA}.$$

$$\Rightarrow \; X_L = \frac{100\text{V}}{100\text{mA}} = 1000\Omega \quad \Rightarrow \; L = 2\text{H}.$$

$$I_3^2 = I_2^2 + I_R^2 \quad I_R = \sqrt{I_3^2 - I_2^2} = 200\text{mA}.$$

$$\Rightarrow \; R = \frac{100\text{V}}{200\text{mA}} = 500\Omega.$$

Power in AC Circuits

Voltage of ac source: $\mathcal{E} = \mathcal{E}_{max} \cos \omega t$
Current through circuit: $I = I_{max} \cos(\omega t - \delta)$

Instantaneous power supplied: $P(t) = \mathcal{E}(t)I(t) = [\mathcal{E}_{max} \cos \omega t][I_{max} \cos(\omega t - \delta)]$

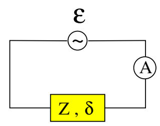
Use $\cos(\omega t - \delta) = \cos \omega t \cos \delta + \sin \omega t \sin \delta$

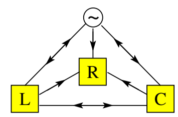
$$\Rightarrow \; P(t) = \mathcal{E}_{max} I_{max} [\cos^2 \omega t \cos \delta + \cos \omega t \sin \omega t \sin \delta]$$

Time averages: $[\cos^2 \omega t]_{AV} = \frac{1}{2}$, $\quad [\cos \omega t \sin \omega t]_{AV} = 0$

Average power supplied by source: $P_{AV} = \frac{1}{2} \mathcal{E}_{max} I_{max} \cos \delta = \mathcal{E}_{rms} I_{rms} \cos \delta$

Power factor: $\cos \delta$





tsl311

Here we take a closer look at power transfer in a general ac circuit connected to an outlet. The instantaneous power is stated, in the third line, as the product of voltage and current. Both factors are oscillatory functions of time.

The product is also oscillatory in time, positive and negative, back and forth. When $P(t)$ is positive, power is transferred from the outlet into the circuit, The circuit returns some power to the outlet when $P(t)$ is negative.

Using a trignometric identity, we can express the function $P(t)$ as the sum of two terms, with distinct attributes. The first term is non-negative and the second oscillates about zero symmetrically.

When we take a time average (as the meters of utility companies do), then only the first term contributes. The average power is one half times EMF amplitude times current amplitude times power factor.

In a typical appliance, when it is turned off, there is current flowing, but it is out of phase with the EMF by a phase angle close to $\delta = \pi/2$, implying a very small power factor, i.e. a very small power consumption even if the current is not small. Turning on the appliance changes the current but, more importantly, makes the phase angle smaller, which increases the power factor.

The interaction between an ac power source and an appliance is complex power exchange. We know that capacitors and inductors are places of energy storage. These devices store and release electromagnetic energy at the rate of ac current oscillations (see boxes labeled L and C). Some of that energy is exchanged between different parts of the appliance, some is exchanged with the power-source.

The box labeled R stands for those parts of the appliance that convert electromagnetic energy into different forms of energy (heat, radiation, mechanical work). Here the energy transfer is, for the most part, a one-way street. Little if anything is given back.

The last expression for average power on the slide uses *root-mean-square* (rms) values for EMF and current instead of amplitudes. They are related as follows:

$$\mathcal{E}_{rms} = \frac{\mathcal{E}_{max}}{\sqrt{2}}, \quad I_{rms} = \frac{I_{max}}{\sqrt{2}}.$$

The motivation for the use of rms values is related to the measurement of ac signals such

$$I(t) = I_{max}\sin(\omega t),$$

via time average. The time average of $I(t)$, expressed as an integral over one period, $T = 2\pi/\omega$, vanishes:

$$I_{ave} \doteq \frac{1}{T}\int_0^T I(t)dt = \frac{I_{max}}{T}\int_0^T \sin(\omega t)dt = -\frac{I_{max}}{\omega T}\left[\underbrace{\cos(\omega T)}_{1} - \underbrace{\cos(0)}_{1}\right] = 0.$$

Taking the rms average instead of just the average implies the following steps, first the *square*, then the *mean* (time average), and, finally, the *root*:

$$[I(t)]^2 = I_{max}^2\sin^2(\omega t) = \frac{1}{2}I_{max}^2[1 - \cos(2\omega t)].$$

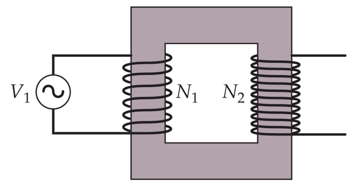
$$I_{ave}^2 = \frac{I_{max}^2}{2T}\int_0^T [1 - \cos(2\omega t)]dt = \frac{I_{max}^2}{2T}\left[T - \underbrace{\frac{\sin(2\omega T)}{2\omega}}_{0} + \underbrace{\frac{\sin(0)}{2\omega}}_{0}\right] = \frac{1}{2}I_{max}^2.$$

$$I_{rms} = \sqrt{I_{ave}^2} = \frac{I_{max}}{\sqrt{2}}.$$

Note that the often quoted value, 120V, for voltage in US household outlets is, in fact, the rms value. The amplitude is higher, $\sim$ 170V.

Transformer

- Primary winding: N_1 turns
  $V_1(t) = V_1^{(rms)} \cos(\omega t)$, $\quad I_1(t) = I_1^{(rms)} \cos(\omega t - \delta_1)$
- Secondary winding: N_2 turns
  $V_2(t) = V_2^{(rms)} \cos(\omega t)$, $\quad I_2(t) = I_2^{(rms)} \cos(\omega t - \delta_2)$
- Voltage amplitude ratio: $\dfrac{V_1^{(rms)}}{V_2^{(rms)}} = \dfrac{N_1}{N_2}$
- Power transfer: $V_1^{(rms)} I_1^{(rms)} \cos\delta_1 = V_2^{(rms)} I_2^{(rms)} \cos\delta_2$



tsl418

Perhaps the greatest advantage of ac power over dc power is that the former can transformed from high voltage to low voltage or from low voltage to high voltage with no significant losses.

Transporting electric power over long distance involves currents through conducting wires with, say, resistance R. The (average) power transported and the power lost (i.e. converted into heat and radiation) are, respectively,

$$P_{trans} = \mathcal{E}_{rms} I_{rms} \cos\delta, \quad P_{loss} = I_{rms}^2 R.$$

For a given value of P_{trans} the value of P_{loss} can be made smaller by increasing the voltage and decreasing the current. High voltages are beneficial for overland transport but dangerous to users. Hence the need of transformers that step up and step down voltages as needed.

The schematic of a transformer is shown on the slide. An iron core (magnetizable material) has two windings with different numbers of turns. The power source on the left generates an alternating magnetic flux through its (primary) winding. The iron core guides the flux through the secondary winding with a different number of turns, N_2, where it induces an EMF as dictated by Faraday's law.

It can be shown that the voltage amplitude ratio is simply equal to the ratio of turns in the primary and secondary windings.

Power transfer through the transformer conserves energy. This is regulated by the currents and the power factors left and right.

AC Circuit Application (6)

Consider an RLC series circuit with inductance $L = 88m$H, capacitance $C = 0.94\mu$F, and unknown resistance R.

The ac generator $\mathcal{E} = \mathcal{E}_{max}\sin(\omega t)$ has amplitude $\mathcal{E}_{max} = 24$V
and frequency $f = 930$Hz. The phase angle is $\delta = 75°$.

(a) Find the resistance R.
(b) Find the current amplitude I_{max}.
(c) Find the maximum energy U_L^{max} stored in the inductor.
(d) Find the maximum energy U_C^{max} stored in the capacitor.
(e) Find the time t_1 at which the current has its maximum value I_{max}.
(f) Find the time t_2 at which the charge on the capacitor has its maximum value Q_{max}.

tsl313

We conclude this lecture with a series of ac-circuit applications. All applications are worked out explicitly. Every step should be self-explanatory.

(a) The phase angle δ depends on R, L, C, and $\omega = 2\pi f$:

$$\tan\delta = \frac{\omega L - 1/\omega C}{R} \quad \Rightarrow \; R = 89.0\Omega.$$

(b) The impedance is the ratio of EMF amplitude and current amplitude:

$$Z \doteq \frac{\mathcal{E}_{max}}{I_{max}} = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = 344\Omega \quad \Rightarrow \; I_{max} = 69.8\text{mA}.$$

(c) The energy stored on the inductor is highest when the current has its maximum value:

$$U_L^{max} = \frac{1}{2}LI_{max}^2 = 214\mu\text{J}.$$

(d) The energy stored on the capacitor is highest when the charge on it has its maximum value:

$$Q_{max} = \frac{I_{max}}{\omega} = 11.95\mu\text{C} \quad \Rightarrow \; U_C^{max} = \frac{Q_{max}^2}{2C} = 76.0\mu\text{J}.$$

(e) Start from the expression for the current as a function of time:

$$I(t) = I_{max}\sin(\omega t_1 - \delta) = I_{max} \text{ if } \omega t_1 - \delta = \frac{\pi}{2} \quad \Rightarrow \; t_1 = 493\mu\text{s}.$$

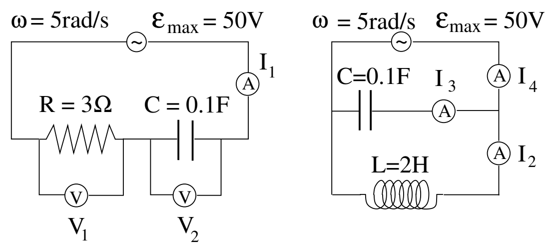
(f) Integrate $I(t)$ to get $Q(t)$ then reason analogously:

$$Q(t) = -Q_{max}\cos(\omega t_2 - \delta) = Q_{max} \text{ if } \omega t_2 - \delta = \pi \quad \Rightarrow \; t_2 = 762\mu\text{s}.$$

Consider the two ac circuits shown.

(a) In the circuit on the left, determine the current amplitude I_1 and the voltage amplitudes V_1 and V_2.

(b) In the circuit on the right, determine the current amplitudes I_2, I_3, and I_4.



tsl384

(a) RLC series circuit with $L = 0$ (no inductor in series):

$$Z = \sqrt{(3\Omega)^2 + (2\Omega)^2} = 3.61\Omega, \quad I_1 = \frac{50\text{V}}{3.61\Omega} = 13.9\text{A},$$

$$V_1 = (3\Omega)(13.9\text{A}) = 41.7\text{V}, \quad V_2 = (2\Omega)(13.9\text{A}) = 27.8\text{V}.$$

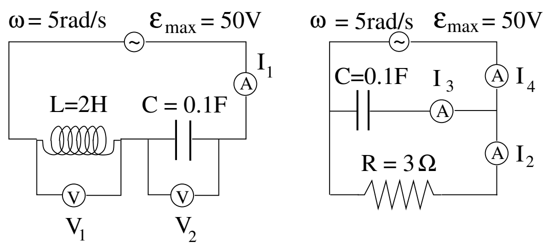
(b) RLC parallel circuit with $R \to \infty$ (no resistor in parallel).

$$I_2 = \frac{50\text{V}}{10\Omega} = 5\text{A}, \quad I_3 = \frac{50\text{V}}{2\Omega} = 25\text{A}, \quad I_4 = I_3 - I_2 = 20\text{A}.$$

Consider the two ac circuits shown.

(a) In the circuit on the left, determine the maximum value of current I_1 and the maximum value of voltages V_1 and V_2.

(b) In the circuit on the right, determine the maximum value of currents I_2, I_3, and I_4.



tsl385

(a) RLC series circuit with $R = 0$ (no resistor in series):

$$Z = 10\Omega - 2\Omega = 8\Omega, \quad I_1 = \frac{50\text{V}}{8\Omega} = 6.25\text{A},$$

$$V_1 = (10\Omega)(6.25\text{A}) = 62.5\text{V}, \quad V_2 = (2\Omega)(6.25\text{A}) = 12.5\text{V}.$$

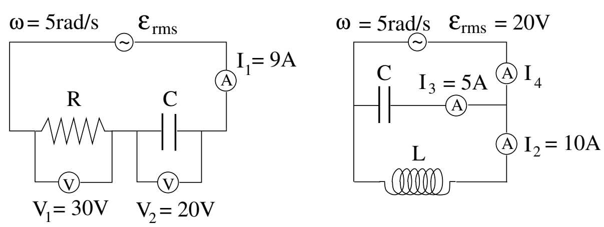
(b) RLC parallel circuit with $L \to \infty$ (no inductor in parallel).

$$I_2 = \frac{50\text{V}}{3\Omega} = 16.7\text{A}, \quad I_3 = \frac{50\text{V}}{2\Omega} = 25.0\text{A}, \quad I_4 = \sqrt{(I_3)^2 + (I_2)^2} = 30.1\text{A}.$$

In the two ac circuits shown the ammeter and voltmeter readings are rms values.

(a) In the circuit on the left, find the resistance R of the resistor, the capacitance C of the capacitor, the impedance Z of the two devices combined, and the voltage $\mathcal{E}_{rms}$ of the power source.

(b) In the circuit on the right, find the capacitance C of the capacitor, the inductance L of the inductor, the impedance Z of the two devices combined, and the rms value of the current I_4.



tsl386

(a) RLC series circuit with $L = 0$ (no inductor in series):

$$R = \frac{V_1}{I_1} = \frac{30\text{V}}{9\text{A}} = 3.33\Omega,$$

$$X_C = \frac{1}{\omega C} = \frac{V_2}{I_1} = \frac{20\text{V}}{9\text{A}} = 2.22\Omega \quad \Rightarrow \; C = 90\text{mF},$$

$$Z = \sqrt{R^2 + X_C^2} = 4.00\Omega \quad \Rightarrow \; \mathcal{E}_{rms} = I_1 Z = 36\text{V}.$$

(b) RLC parallel circuit with $R \to \infty$ (no resistor in parallel).

$$X_L = \omega L = \frac{\mathcal{E}_{rms}}{I_2} = \frac{20\text{V}}{10\text{A}} = 2\Omega \quad \Rightarrow \; L = 0.4\text{H}.$$

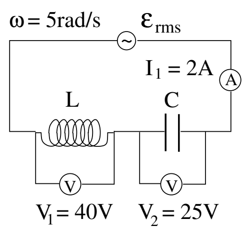
$$X_C = \frac{1}{\omega C} = \frac{\mathcal{E}_{rms}}{I_3} = \frac{20\text{V}}{5\text{A}} = 4\Omega \quad \Rightarrow \; C = 50\text{mF}.$$

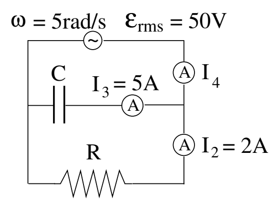
$$I_4 = I_2 - I_3 = 5\text{A}. \quad Z = \left|\omega C - \frac{1}{\omega L}\right|^{-1} = 4\Omega.$$

In the two ac circuits shown the ammeter and voltmeter readings are rms values.

(a) In the circuit on the left, find the capacitance C of the capacitor, the inductance L of the inductor, the impedance Z of the two devices combined, and the voltage $\mathcal{E}_{rms}$ of the power source.

(b) In the circuit on the right, find the capacitance C of the capacitor, the resistance R of the resistor, the impedance Z of the two devices combined, and the rms value of the current I_4.





tsl387

(a) RLC series circuit with $R = 0$ (no resistor in series):

$$X_L = \omega L = \frac{V_1}{I_1} = \frac{40\text{V}}{2\text{A}} = 20\Omega \quad \Rightarrow \quad L = 4\text{H}.$$

$$X_C = \frac{1}{\omega C} = \frac{V_2}{I_1} = \frac{25\text{V}}{2\text{A}} = 12.5\Omega \quad \Rightarrow \quad C = 16\text{mF},$$

$$Z = |X_L - X_C| = 7.5\Omega \quad \Rightarrow \quad \mathcal{E}_{rms} = I_1 Z = 15\text{V}.$$

(b) RLC parallel circuit with $L \to \infty$ (no inductor in parallel).

$$R = \frac{\mathcal{E}_{rms}}{I_2} = \frac{50\text{V}}{2\text{A}} = 25\Omega.$$

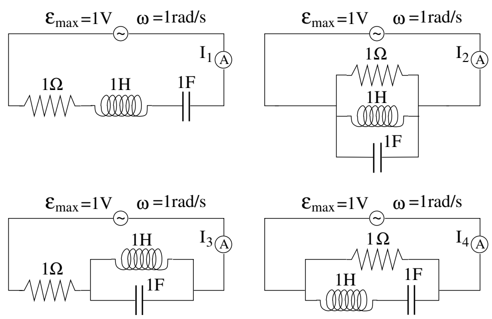
$$X_C = \frac{1}{\omega C} = \frac{\mathcal{E}_{rms}}{I_3} = \frac{50\text{V}}{5\text{A}} = 10\Omega \quad \Rightarrow \quad C = 20\text{mF}.$$

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}} \quad \Rightarrow \quad Z = 9.3\Omega.$$

$$I_4 = \sqrt{I_2^2 + I_3^2} = 5.4\text{A}.$$

AC Circuit Application (5)

Find the current amplitudes I_1, I_2, I_3, I_4 in the four *RLC* circuits shown.



tsl312

This is the quiz for lecture 34.

The top two circuits are familiar, the bottom two are unfamilar. In the case of the latter two, you have to think outside the box.

Hint: Two of the currents are finite and nonzero, one is infinite, and one is zero.