2020

08. Capacitors with dielectrics

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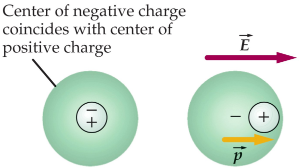
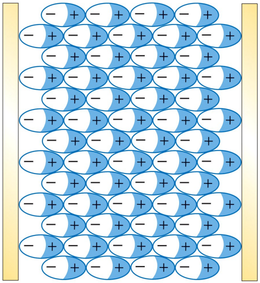
Capacitor with Dielectric

Most capacitors have a dielectric (insulating solid or liquid material) in the space between the conductors. This has several advantages:

- Physical separation of the conductors.
- Prevention of dielectric breakdown.
- Enhancement of capacitance.

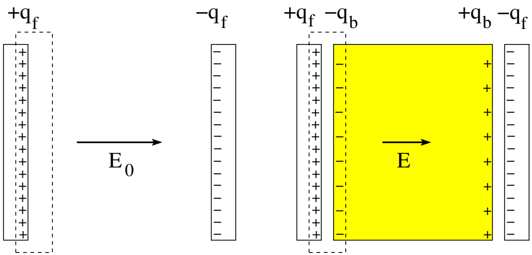
The dielectric is polarized by the electric field between the capacitor plates.





Parallel-Plate Capacitor with Dielectric (1)

The polarization produces a bound charge on the surface of the dielectric.

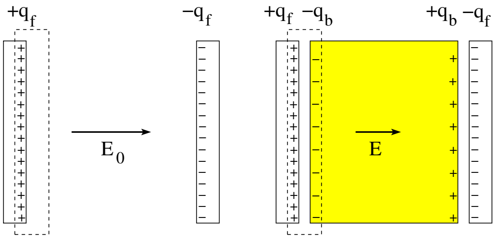


The bound surface charge has the effect of reducing the electric field between the plates from $\vec{E}_0$ to $\vec{E}$.

- A: area of plates
- d: separation between plates
- $\pm q_f$: free charge on plate
- $\pm q_b$: bound charge on surface of dielectric
- $\vec{E}_0$: electric field in vacuum
- $\vec{E}$: electric field in dielectric

Parallel-Plate Capacitor with Dielectric (2)

Use Gauss' law to determine the electric fields $\vec{E}_0$ and $\vec{E}$.



- Field in vacuum: $E_0 A = \frac{q_f}{\epsilon_0} \quad \Rightarrow \quad E_0 = \frac{q_f}{\epsilon_0 A}$
- Field in dielectric: $EA = \frac{q_f - q_b}{\epsilon_0} \quad \Rightarrow \quad E = \frac{q_f - q_b}{\epsilon_0 A} < E_0$
- Voltage: $V_0 = E_0 d$ (vacuum), $V = Ed = \frac{V_0}{\kappa} < V_0$ (dielectric)

Dielectric constant: $\kappa \equiv \frac{E_0}{E} = \frac{q_f}{q_f - q_b} > 1.$ Permittivity of dielectric: $\epsilon = \kappa\epsilon_0$.

Dielectric Materials

TABLE 24-1

Dielectric Constants and Dielectric Strengths of Various Materials

Material	Dielectric Constant κ	Dielectric Strength, kV/mm
Air	1.00059	3
Bakelite	4.9	24
Glass (Pyrex)	5.6	14
Mica	5.4	10–100
Neoprene	6.9	12
Paper	3.7	16
Paraffin	2.1–2.5	10
Plexiglas	3.4	40
Polystyrene	2.55	24
Porcelain	7	5.7
Transformer oil	2.24	12

- Dielectrics increase the capacitance: $C/C_0 = \kappa$.
- The capacitor is discharged spontaneously across the dielectric if the electric field exceeds the value quoted as dielectric strength.

Impact of Dielectric (1)

What happens when a dielectric is placed into a capacitor with the **charge on the capacitor** kept constant?

	vacuum	dielectric
charge	Q_0	$Q = Q_0$
electric field	E_0	$E = \frac{E_0}{\kappa} < E_0$
voltage	V_0	$V = \frac{V_0}{\kappa} < V_0$
capacitance	$C_0 = \frac{Q_0}{V_0}$	$C = \frac{Q}{V} = \kappa C_0 > C_0$
potential energy	$U_0 = \frac{Q_0^2}{2C_0}$	$U = \frac{Q^2}{2C} = \frac{U_0}{\kappa} < U_0$
energy density	$u_E^{(0)} = \frac{1}{2}\epsilon_0 E_0^2$	$u_E = \frac{1}{2}\epsilon E^2 = \frac{u_E^{(0)}}{\kappa} < u_E^{(0)}$

Impact of Dielectric (2)

What happens when a dielectric is placed into a capacitor with the **voltage across the capacitor** kept constant?

	vacuum	dielectric
voltage	V_0	$V = V_0$
electric field	E_0	$E = E_0$
capacitance	$C_0 = \dfrac{Q_0}{V_0}$	$C = \dfrac{Q}{V} = \kappa C_0 > C_0$
charge	Q_0	$Q = \kappa Q_0 > Q_0$
potential energy	$U_0 = \dfrac{1}{2} C_0 V_0^2$	$U = \dfrac{1}{2} C V^2 = \kappa U_0 > U_0$
energy density	$u_E^{(0)} = \dfrac{1}{2} \epsilon_0 E_0^2$	$u_E = \dfrac{1}{2} \epsilon E^2 = \kappa u_E^{(0)} > u_E^{(0)}$

Stacked Dielectrics

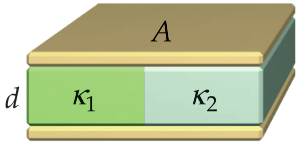
Consider a parallel-plate capacitor with area A of each plate and spacing d.

- Capacitance without dielectric: $C_0 = \dfrac{\epsilon_0 A}{d}$.

- Dielectrics stacked in parallel: $C = C_1 + C_2$

  with $C_1 = \kappa_1 \epsilon_0 \dfrac{A/2}{d}$, $C_2 = \kappa_2 \epsilon_0 \dfrac{A/2}{d}$.

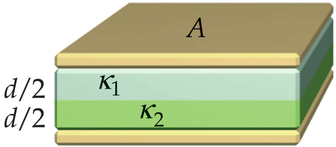
  $\Rightarrow\ C = \dfrac{1}{2}(\kappa_1 + \kappa_2) C_0$.



- Dielectrics stacked in series: $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

  with $C_1 = \kappa_1 \epsilon_0 \dfrac{A}{d/2}$, $C_2 = \kappa_2 \epsilon_0 \dfrac{A}{d/2}$

  $\Rightarrow\ C = \dfrac{2\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} C_0$.



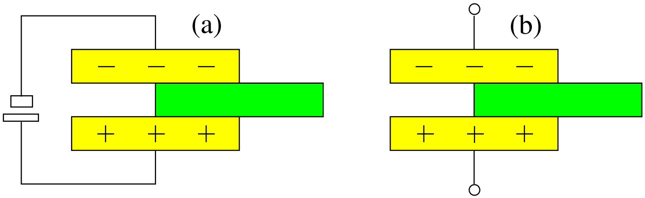
Lateral Force on Dielectric

Consider two charged capacitors with dielectrics only halfway between the plates.

In configuration (a) any lateral motion of the dielectric takes place
at **constant voltage** across the plates.

In configuration (b) any lateral motion of the dielectric takes place
at **constant charge** on the plates.

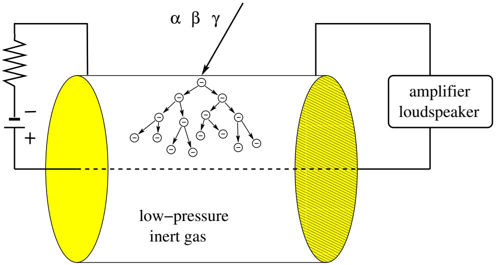
Determine in each case the direction (left/zero/right) of the lateral force experienced by the dielectric.



Geiger Counter

Radioactive atomic nuclei produce high-energy particles of three different kinds:

- α-particles are ^{4}He nuclei.
- β-particles are electrons or positrons.
- γ-particles are high-energy photons.



- Free electrons produced by ionizing radiation are strongly accelerated toward the central wire.
- Collisions with gas atoms produce further free electrons, which are accelerated in the same direction.
- An avalanche of electrons reaching the wire produces a current pulse in the circuit.

Capacitor Circuit (4)

Connect the three capacitors in such a way that the equivalent capacitance is $C_{eq} = 2\mu$F. Draw the circuit diagram.