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06. Electric potential and electric field

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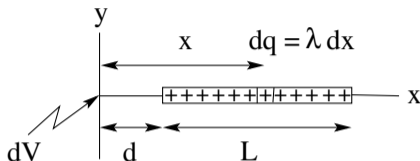
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Electric Potential of Charged Rod



- Charge per unit length: $\lambda = Q/L$
- Charge on slice dx : $dq = \lambda dx$



- Electric potential generated by slice dx : $dV = \frac{k dq}{x} = \frac{k \lambda dx}{x}$
- Electric potential generated by charged rod:

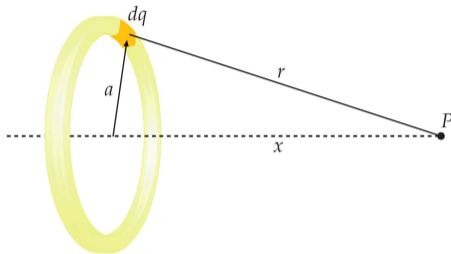
$$V = k\lambda \int_d^{d+L} \frac{dx}{x} = k\lambda \left[\ln x \right]_d^{d+L} = k\lambda [\ln(d+L) - \ln d] = k\lambda \ln \frac{d+L}{d}$$

- Limiting case of very short rod ($L \ll d$): $V = k\lambda \ln \left(1 + \frac{L}{d} \right) \simeq k\lambda \frac{L}{d} = \frac{kQ}{d}$

Electric Potential of Charged Ring



- Total charge on ring: Q
- Charge per unit length: $\lambda = Q/2\pi a$
- Charge on arc: dq



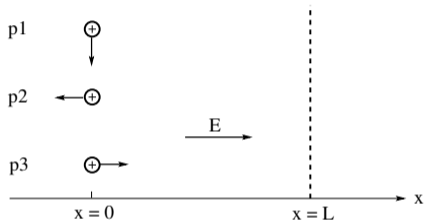
Find the electric potential at point P on the axis of the ring.

- $dV = k \frac{dq}{r} = \frac{k dq}{\sqrt{x^2 + a^2}}$
- $V(x) = k \int \frac{dq}{\sqrt{x^2 + a^2}} = \frac{k}{\sqrt{x^2 + a^2}} \int dq = \frac{kQ}{\sqrt{x^2 + a^2}}$

Electric Potential and Potential Energy: Application (6)



Three protons are projected from $x = 0$ with equal initial speed v_0 in different directions. They all experience the force of a uniform horizontal electric field \vec{E} . Ultimately, they all hit the vertical screen at $x = L$. Ignore gravity.



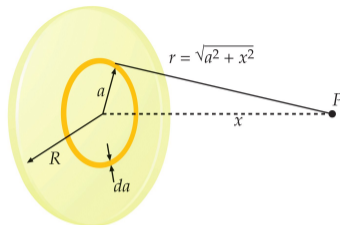
- (a) Which proton travels the longest time?
- (b) Which proton travels the longest path?
- (c) Which particle has the highest speed when it hits the screen?

Two of the questions are easy, one is hard.

Electric Potential of Charged Disk



- Area of ring: $2\pi ada$
- Charge on ring: $dq = \sigma(2\pi ada)$
- Charge on disk: $Q = \sigma(\pi R^2)$



Find the electric potential at point P on the axis of the disk.

- $dV = k \frac{dq}{\sqrt{x^2 + a^2}} = 2\pi\sigma k \frac{ada}{\sqrt{x^2 + a^2}}$
- $V(x) = 2\pi\sigma k \int_0^R \frac{ada}{\sqrt{x^2 + a^2}} = 2\pi\sigma k \left[\sqrt{x^2 + a^2} \right]_0^R = 2\pi\sigma k \left[\sqrt{x^2 + R^2} - |x| \right]$

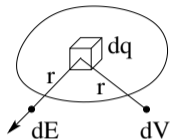
Electric potential at large distances from the disk ($|x| \gg R$):

$$V(x) = 2\pi\sigma k|x| \left[\sqrt{1 + \frac{R^2}{x^2}} - 1 \right] \simeq 2\pi\sigma k|x| \left[1 + \frac{R^2}{2x^2} - 1 \right] = \frac{k\sigma\pi R^2}{|x|} = \frac{kQ}{|x|}$$



Determine the field or the potential from the source (charge distribution):

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$



$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

Determine the field from the potential: $\vec{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}$

Determine the potential from the field: $V = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{s}$

- Systems with $\vec{E} = E_x(x)\hat{i}$: $E_x = -\frac{dV}{dx} \Leftrightarrow V(x) = -\int_{x_0}^x E_x dx$
- Application to charged ring: $E_x = \frac{kQx}{(x^2 + a^2)^{3/2}} \Leftrightarrow V = \frac{kQ}{\sqrt{x^2 + a^2}}$
- Application to charged disk (at $x > 0$): $E_x = 2\pi\sigma k \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right] \Leftrightarrow V = 2\pi\sigma k \left[\sqrt{x^2 + R^2} - x \right]$

Electric Potential and Electric Field in One Dimension (1)

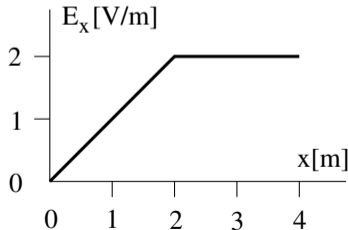
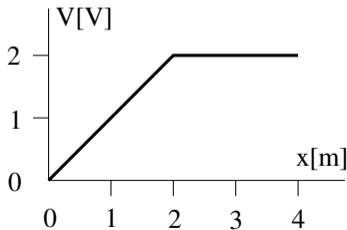


For given electric potential $V(x)$ find the electric field

- (a) $E_x(1\text{m})$,
- (b) $E_x(3\text{m})$.

For given electric field $E_x(x)$ and given reference potential $V(0) = 0$ find the electric potential

- (c) $V(2\text{m})$,
- (d) $V(4\text{m})$.



Electric Potential and Electric Field in One Dimension (2)

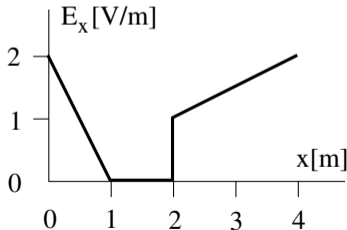
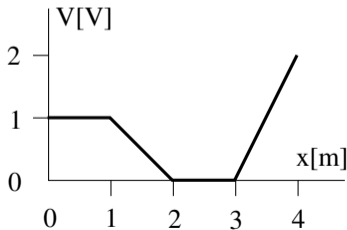


For given electric potential $V(x)$ find the electric field

- (a) $E_x(0.5\text{m})$, (b) $E_x(1.5\text{m})$,
(c) $E_x(2.5\text{m})$, (d) $E_x(3.5\text{m})$.

For given electric field $E_x(x)$ and given reference potential $V(0) = 0$
find the electric potential

- (e) $V(1\text{m})$, (f) $V(2\text{m})$, (g) $V(4\text{m})$.

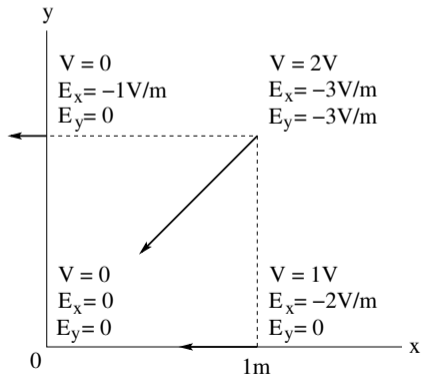


Electric Field from Electric Potential in Two Dimensions



- Given is the electric potential: $V(x, y) = ax^2 + bxy^3$ with $a = 1\text{V/m}^2$, $b = 1\text{V/m}^4$.
- Find the electric field: $\vec{E}(x, y) = E_x(x, y)\hat{i} + E_y(x, y)\hat{j}$ via partial derivatives.

$$E_x = -\frac{\partial V}{\partial x} = -2ax - by^3, \quad E_y = -\frac{\partial V}{\partial y} = -3bxy^2$$



Electric Potential from Electric Field in Two Dimensions



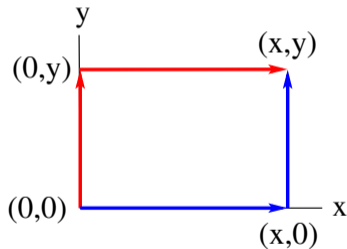
- Given is the electric field: $\vec{E} = -(2ax + by^3)\hat{i} - 3bxy^2\hat{j}$ with $a = 1\text{V/m}^2$, $b = 1\text{V/m}^4$.
- Find the electric potential $V(x,y)$ via integral along a specific path:

Red path $(0,0) \rightarrow (0,y) \rightarrow (x,y)$:

$$\begin{aligned}V(x,y) &= -\int_0^y E_y(0,y)dy - \int_0^x E_x(x,y)dx \\ &= 0 + \int_0^x (2ax + by^3)dx = ax^2 + bxy^3\end{aligned}$$

Blue path $(0,0) \rightarrow (x,0) \rightarrow (x,y)$:

$$\begin{aligned}V(x,y) &= -\int_0^x E_x(x,0)dx - \int_0^y E_y(x,y)dy \\ &= \int_0^x (2ax)dx + \int_0^y (3bxy^2)dy = ax^2 + bxy^3\end{aligned}$$



Electric Potential of a Charged Plane Sheet



Consider an infinite plane sheet perpendicular to the x -axis at $x = 0$.

The sheet is uniformly charged with charge per unit area σ .

- Electric field (magnitude): $E = 2\pi k|\sigma| = \frac{|\sigma|}{2\epsilon_0}$
- Direction: away from (toward) the sheet if $\sigma > 0$ ($\sigma < 0$).

- Electric field (x -component):

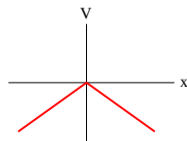
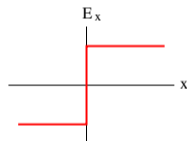
$$E_x = \pm 2\pi k\sigma.$$

- Electric potential:

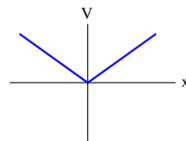
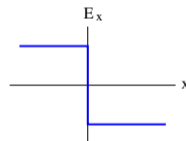
$$V = - \int_0^x E_x dx = \mp 2\pi k\sigma x.$$

- Here we have used $x_0 = 0$ as the reference coordinate.

positively charged sheet



negatively charged sheet



Electric Potential of a Uniformly Charged Spherical Shell



- Electric charge on shell: $Q = \sigma A = 4\pi\sigma R^2$

- Electric field at $r > R$: $E = \frac{kQ}{r^2}$

- Electric field at $r < R$: $E = 0$

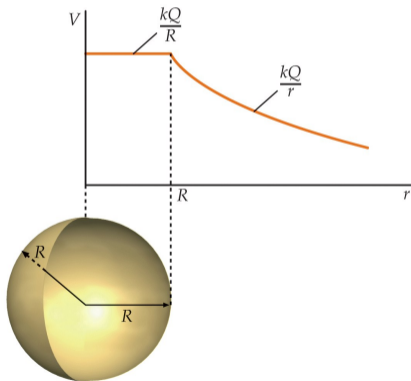
- Electric potential at $r > R$:

$$V = - \int_{\infty}^r \frac{kQ}{r^2} dr = \frac{kQ}{r}$$

- Electric potential at $r < R$:

$$V = - \int_{\infty}^R \frac{kQ}{r^2} dr - \int_R^r (0) dr = \frac{kQ}{R}$$

- Here we have used $r_0 = \infty$ as the reference value of the radial coordinate.



Electric Potential of a Uniformly Charged Solid Sphere



- Electric charge on sphere: $Q = \rho V = \frac{4\pi}{3} \rho R^3$

- Electric field at $r > R$: $E = \frac{kQ}{r^2}$

- Electric field at $r < R$: $E = \frac{kQ}{R^3} r$

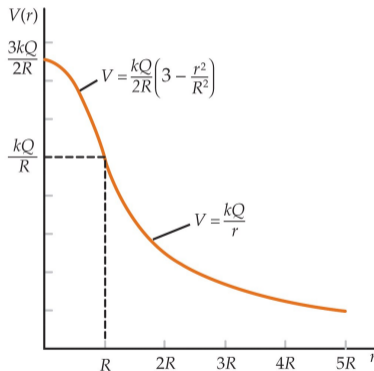
- Electric potential at $r > R$:

$$V = - \int_{\infty}^r \frac{kQ}{r^2} dr = \frac{kQ}{r}$$

- Electric potential at $r < R$:

$$V = - \int_{\infty}^R \frac{kQ}{r^2} dr - \int_R^r \frac{kQ}{R^3} r dr$$

$$\Rightarrow V = \frac{kQ}{R} - \frac{kQ}{2R^3} (r^2 - R^2) = \frac{kQ}{2R} \left(3 - \frac{r^2}{R^2} \right)$$



Electric Potential of a Uniformly Charged Wire



- Consider a uniformly charged wire of infinite length.
- Charge per unit length on wire: λ (here assumed positive).

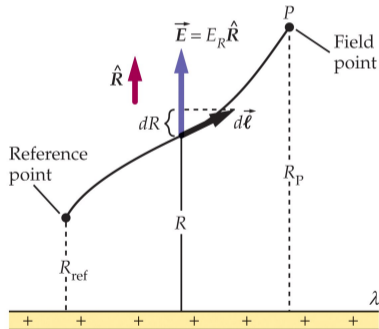
- Electric field at radius r : $E = \frac{2k\lambda}{r}$.

- Electric potential at radius r :

$$V = -2k\lambda \int_{r_0}^r \frac{1}{r} dr = -2k\lambda [\ln r - \ln r_0]$$

$$\Rightarrow V = 2k\lambda \ln \frac{r_0}{r}$$

- Here we have used a finite, nonzero reference radius $r_0 \neq 0, \infty$.
- The illustration from the textbook uses R_{ref} for the reference radius, R for the integration variable, and R_p for the radial position of the field point.

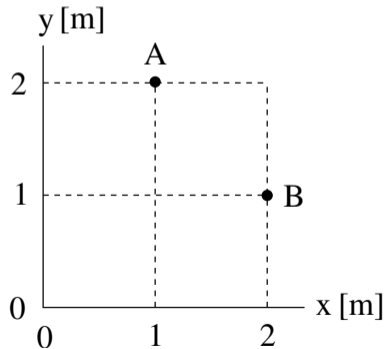


Electric Potential and Electric Field in Two Dimensions



Given is the electric potential $V(x, y) = cxy^2$ with $c = 1\text{V/m}^3$.

- (a) Find the value (in SI units) of the electric potential V at point A .
- (b) Find the components E_x, E_y (in SI units) of the electric field at point B .

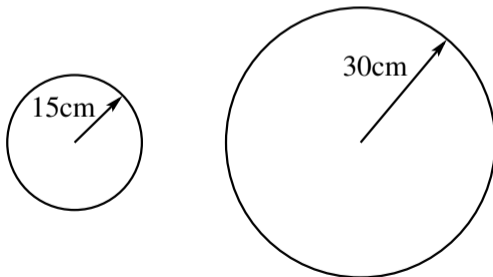


Electric Potential of Conducting Spheres (2)



Consider a conducting sphere with radius $r = 15\text{cm}$ and electric potential $V = 200\text{V}$ relative to a point at infinity.

- Find the charge Q and the surface charge density σ on the sphere.
- Find the magnitude of the electric field E just outside the sphere.
- What happens to the values of Q, V, σ, E when the radius of the sphere is doubled?

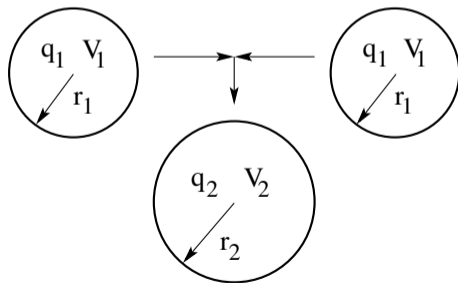


Electric Potential of Conducting Spheres (3)



A spherical raindrop of 1mm diameter carries a charge of 30pC.

- Find the electric potential of the drop relative to a point at infinity under the assumption that it is a conductor.
- If two such drops of the same charge and diameter combine to form a single spherical drop, what is its electric potential?

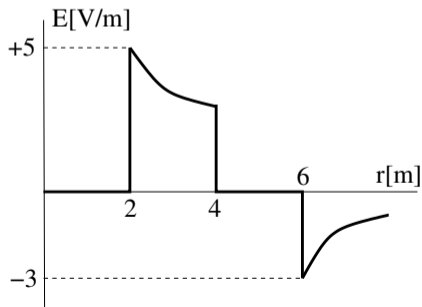


Electric Potential of Conducting Spheres (1)



A conducting sphere of radius $r_1 = 2\text{m}$ is surrounded by a concentric conducting spherical shell of radii $r_2 = 4\text{m}$ and $r_3 = 6\text{m}$. The graph shows the electric field $E(r)$.

- Find the charges q_1, q_2, q_3 on the three conducting surfaces.
- Find the values V_1, V_2, V_3 of the electric potential on the three conducting surfaces relative to a point at infinity.
- Sketch the potential $V(r)$.



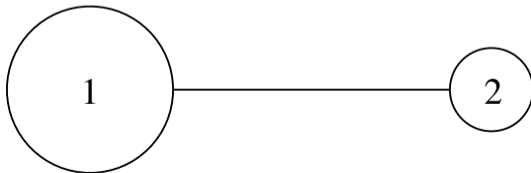
Electric Potential of Conducting Spheres (4)



A positive charge is distributed over two conducting spheres 1 and 2 of unequal size and connected by a long thin wire. The system is at equilibrium.

Which sphere (1 or 2)...

- (a) carries more charge on its surface?
- (b) has the higher surface charge density?
- (c) is at a higher electric potential?
- (d) has the stronger electric field next to it?



Electric Potential Energy of Two Point Charges



Consider two different perspectives:

#1a Electric potential when q_1 is placed: $V(\vec{r}_2) \doteq V_2 = k \frac{q_1}{r_{12}}$

Electric potential energy when q_2 is placed into potential V_2 : $U = q_2 V_2 = k \frac{q_1 q_2}{r_{12}}$

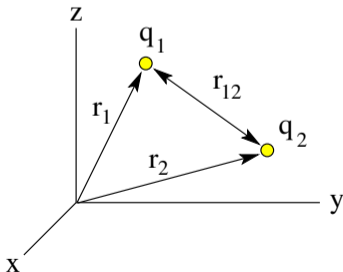
#1b Electric potential when q_2 is placed: $V(\vec{r}_1) \doteq V_1 = k \frac{q_2}{r_{12}}$

Electric potential energy when q_1 is placed into potential V_1 : $U = q_1 V_1 = k \frac{q_1 q_2}{r_{12}}$

#2 Electric potential energy of q_1 and q_2 :

$$U = \frac{1}{2} \sum_{i=1}^2 q_i V_i,$$

where $V_1 = k \frac{q_2}{r_{12}}$, $V_2 = k \frac{q_1}{r_{12}}$.



Electric Potential Energy of Three Point Charges



#1 Place q_1 , then q_2 , then q_3 , and add all changes in potential energy:

$$U = 0 + k \frac{q_1 q_2}{r_{12}} + k \left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right).$$

#2 Symmetric expression of potential energy U in terms of the potentials V_i experienced by point charges q_i :

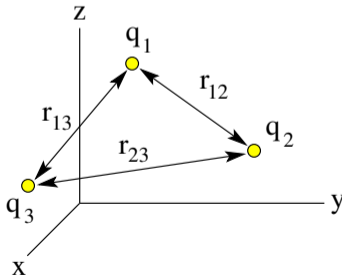
$$U = \frac{1}{2} \sum_{i=1}^3 q_i V_i = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right),$$

where

$$V_1 = k \left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}} \right),$$

$$V_2 = k \left(\frac{q_1}{r_{12}} + \frac{q_3}{r_{23}} \right),$$

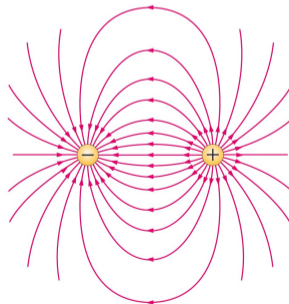
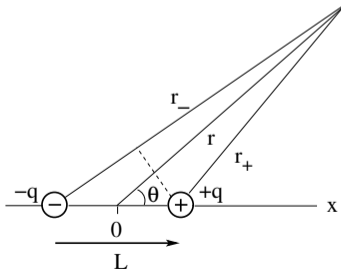
$$V_3 = k \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right).$$



Electric Dipole Potential



- Use spherical coordinates: $V = V(r, \theta)$ independent of azimuthal coordinate ϕ .
- Superposition principle: $V = V_+ + V_- = k \left(\frac{q}{r_+} + \frac{(-q)}{r_-} \right) = kq \frac{r_- - r_+}{r_- r_+}$
- Large distances ($r \gg L$): $r_- - r_+ \simeq L \cos \theta$, $r_- r_+ \simeq r^2 \Rightarrow V(r, \theta) \simeq k \frac{qL \cos \theta}{r^2}$
- Electric dipole moment: $p = qL$ (magnitude)
- Electric dipole potential: $V(r, \theta) \simeq k \frac{p \cos \theta}{r^2}$

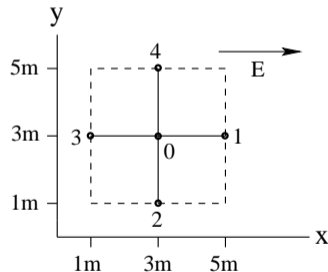


Unit Exam I: Problem #3 (Spring '11)



Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\text{V/m}\hat{i}$. Ignore gravity.

- If the electric potential vanishes at point 0, what are the electric potentials at points 1 and 2?
- If an electron ($m = 9.11 \times 10^{-31}\text{kg}$, $q = -1.60 \times 10^{-19}\text{C}$) is released from rest at point 0, toward which point will it start moving?
- What will be the speed of the electron when it gets there?



Unit Exam I: Problem #3 (Spring '11)

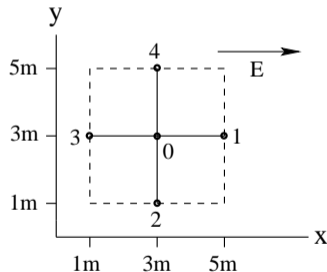


Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\text{V/m}\hat{i}$. Ignore gravity.

- (a) If the electric potential vanishes at point 0, what are the electric potentials at points 1 and 2?
- (b) If an electron ($m = 9.11 \times 10^{-31}\text{kg}$, $q = -1.60 \times 10^{-19}\text{C}$) is released from rest at point 0, toward which point will it start moving?
- (c) What will be the speed of the electron when it gets there?

Solution:

(a) $V_1 = -(0.5\text{V/m})(2\text{m}) = -1\text{V}$, $V_2 = 0$.



Unit Exam I: Problem #3 (Spring '11)



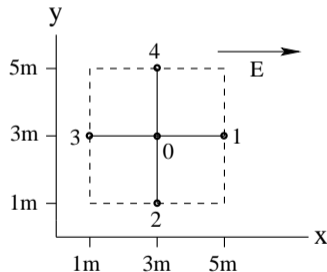
Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\text{V/m}\hat{\mathbf{i}}$. Ignore gravity.

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- What will be the speed of the electron when it gets there?

Solution:

(a) $V_1 = -(0.5\text{V/m})(2\text{m}) = -1\text{V}$, $V_2 = 0$.

(b) $\mathbf{F} = q\mathbf{E} = -|qE|\hat{\mathbf{i}}$ (toward point 3).



Unit Exam I: Problem #3 (Spring '11)



Consider a region of space with a uniform electric field $\mathbf{E} = 0.5\text{V/m}\hat{\mathbf{i}}$. Ignore gravity.

- If the electric potential vanishes at point 0, what are the electric potentials at points 1 and 2?
- If an electron ($m = 9.11 \times 10^{-31}\text{kg}$, $q = -1.60 \times 10^{-19}\text{C}$) is released from rest at point 0, toward which point will it start moving?
- What will be the speed of the electron when it gets there?

Solution:

(a) $V_1 = -(0.5\text{V/m})(2\text{m}) = -1\text{V}$, $V_2 = 0$.

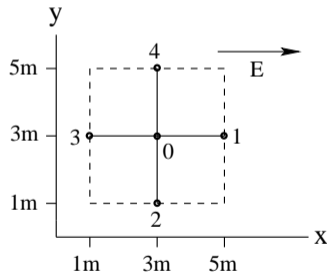
(b) $\mathbf{F} = q\mathbf{E} = -|qE|\hat{\mathbf{i}}$ (toward point 3).

(c) $\Delta V = (V_3 - V_0) = 1\text{V}$, $\Delta U = q\Delta V = -1.60 \times 10^{-19}\text{J}$,
 $K = -\Delta U = 1.60 \times 10^{-19}\text{J}$, $v = \sqrt{\frac{2K}{m}} = 5.93 \times 10^5\text{m/s}$.

Alternatively:

$$F = qE = 8.00 \times 10^{-20}\text{N}, \quad a = \frac{F}{m} = 8.78 \times 10^{10}\text{m/s}^2,$$

$$|\Delta x| = 2\text{m}, \quad v = \sqrt{2a|\Delta x|} = 5.93 \times 10^5\text{m/s}.$$

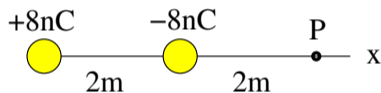


Unit Exam I: Problem #1 (Spring '09)



Consider two point charges positioned on the x -axis as shown.

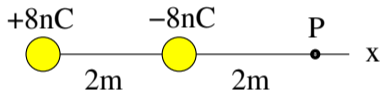
- Find magnitude and direction of the electric field at point P.
- Find the electric potential at point P.
- Find the electric potential energy of an electron (mass $m = 9.1 \times 10^{-31}$ kg, charge $q = -1.6 \times 10^{-19}$ C) when placed at point P.
- Find magnitude and direction of the acceleration the electron experiences when released at point P.





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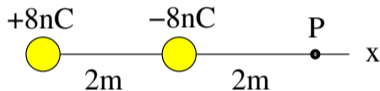
Solution:

$$(a) E_x = +k \frac{8nC}{(4m)^2} + k \frac{(-8nC)}{(2m)^2} = 4.5N/C - 18N/C = -13.5N/C \quad (\text{directed left}).$$



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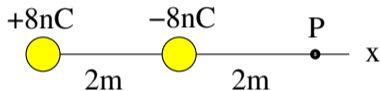
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$$(b) V = +k \frac{8nC}{4m} + k \frac{(-8nC)}{2m} = 18V - 36V = -18V.$$



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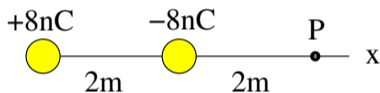
$$(b) V = +k \frac{8nC}{4m} + k \frac{(-8nC)}{2m} = 18V - 36V = -18V.$$

$$(c) U = qV = (-18V)(-1.6 \times 10^{-19}C) = 2.9 \times 10^{-18}J.$$



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$$(b) V = +k \frac{8nC}{4m} + k \frac{(-8nC)}{2m} = 18V - 36V = -18V.$$

$$(c) U = qV = (-18V)(-1.6 \times 10^{-19}C) = 2.9 \times 10^{-18}J.$$

$$(d) a_x = \frac{qE_x}{m} = \frac{(-1.6 \times 10^{-19}C)(-13.5N/C)}{9.1 \times 10^{-31}kg} = 2.4 \times 10^{12}ms^{-2} \quad (\text{directed right}).$$

Electric Potential and Potential Energy: Application (9)



Consider four point charges of equal magnitude positioned at the corners of a square as shown. Answer the following questions for points A , B , C .

- (1) Which point is at the highest electric potential?
- (2) Which point is at the lowest electric potential?
- (3) At which point is the electric field the strongest?
- (4) At which point is the electric field the weakest?

