2020

04. Gauss's law for the electric field with applications

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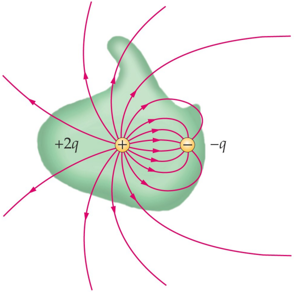
Gauss's Law for Electric Field

The net electric flux Φ_E through any closed surface is equal to the net charge Q_{in} inside divided by the permittivity constant ϵ_0:

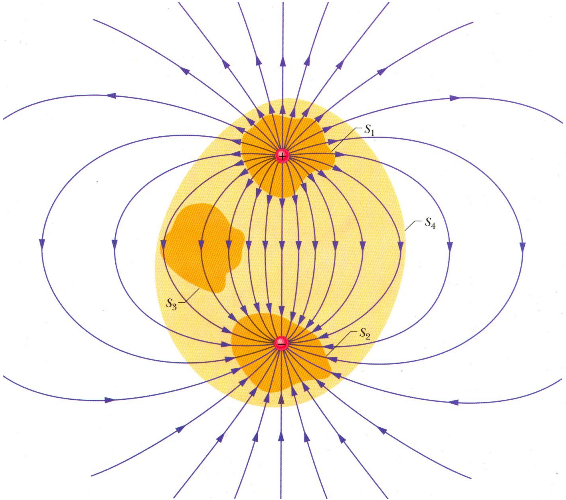
$$\oint \vec{E} \cdot d\vec{A} = 4\pi k Q_{in} = \frac{Q_{in}}{\epsilon_0} \qquad \text{i.e.} \qquad \Phi_E = \frac{Q_{in}}{\epsilon_0} \qquad \text{with} \quad \epsilon_0 = 8.854 \times 10^{-12} \mathrm{C}^2 \mathrm{N}^{-1} \mathrm{m}^{-2}$$

The closed surface can be real or fictitious. It is called "Gaussian surface".
The symbol $\oint$ denotes an integral over a closed surface in this context.

- Gauss's law is a general relation between electric charge and electric field.
- In electrostatics: Gauss's law is equivalent to Coulomb's law.
- Gauss's law is one of four Maxwell's equations that govern cause and effect in electricity and magnetism.



Gauss' Law for Electric Field (Illustration)



Gaussian surface problem (1)

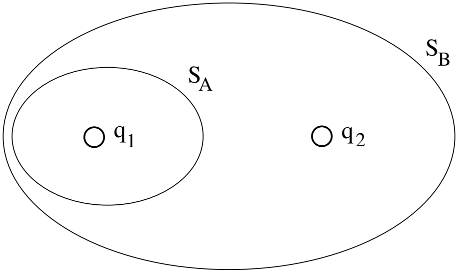
Two Gaussian surfaces S_A and S_B are shown in cross section.

Charge q_1 is on the inside of S_A and S_B.

Charge q_2 is on the inside of S_B only.

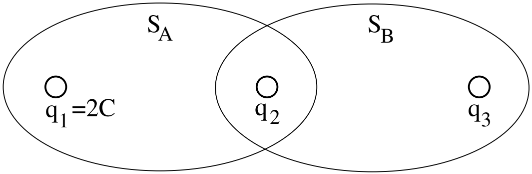
The electric fluxes produced by charges q_1 and q_2 through S_A and S_B are $\Phi_E^{(A)} = 5C/\epsilon_0$ and $\Phi_E^{(B)} = 3C/\epsilon_0$.

Find the electric charges q_1 and q_2.



Gaussian surface problem (2)

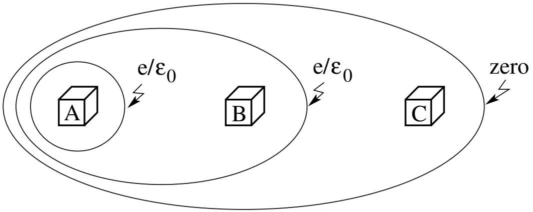
The electric fluxes through the Gaussian surfaces S_A and S_B are $\Phi_E^{(A)} = 1C/\epsilon_0$ and $\Phi_E^{(B)} = 3C/\epsilon_0$, respectively.



Find the electric charges q_2 and q_3.

Gaussian surface problem (3)

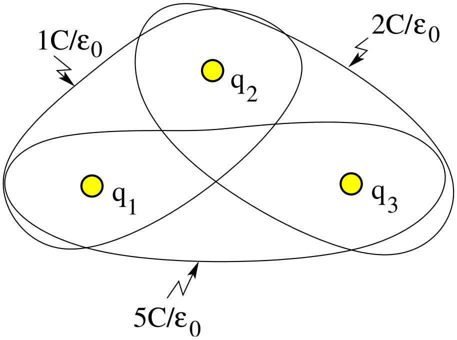
A proton, a neutron, and an electron are placed in different boxes. The electric fluxes through the three Gaussian surfaces are as indicated, where e stands for the elementary charge.



Name the particle in each box.

Gaussian surface problem (4)

Three point charges q_1, q_2, q_3 produce electric fluxes through the three Gaussian surfaces as indicated.

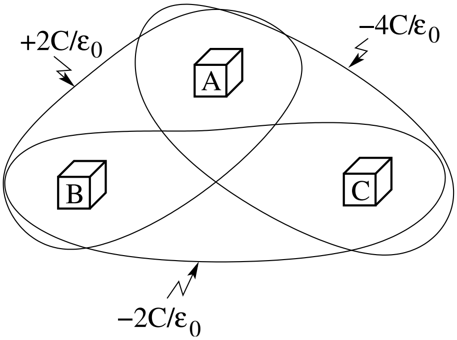


(a) Find the net charge $Q = q_1 + q_2 + q_3$.

(b) Find the individual charges q_1, q_2, q_3.

Gaussian surface problem (5)

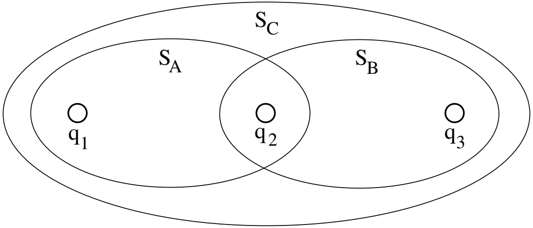
A positive charge and a negative charge are placed in different boxes. One box remains empty. The electric fluxes through the three Gaussian surfaces are as indicated.



(a) Which box contains the positive charge?
(b) Which box contains the negative charge?

Gaussian surface problem (6)

The electric fluxes through the Gaussian surfaces S_A, S_B, and S_C are $\Phi_E^{(A)} = 4C/\epsilon_0$, $\Phi_E^{(B)} = 1C/\epsilon_0$, and $\Phi_E^{(C)} = 2C/\epsilon_0$, respectively.



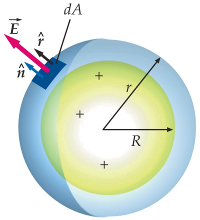
Find the electric charges q_1, q_2, and q_3.

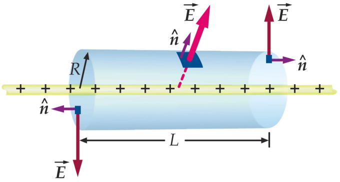
Calculating $\vec{E}$ from Gauss's Law: Strategy

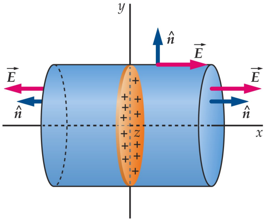
Design the Gaussian surface such that it reflects the symmetry of the problem at hand.

- **Use concentric Gaussian spheres in problems with spherically symmetric charge distributions.**
  The electric field is perpendicular to the Gaussian sphere ($\vec{E} \parallel d\vec{A}$).
- **Use coaxial Gaussian cylinders in problems with cylindrically symmetric charge distributions.**
  The electric field is perpendicular to the curved surface ($\vec{E} \parallel d\vec{A}$) and parallel to the flat surfaces ($\vec{E} \perp d\vec{A}$).
- **Use Gaussian cylinders with axis perpendicular to planar charge distributions.**
  The electric field is parallel to the curved surface ($\vec{E} \perp d\vec{A}$) and perpendicular to the flat surfaces ($\vec{E} \parallel d\vec{A}$).

Since the magnitude of the electric field $\vec{E}$ is constant along both curved surfaces, the integral $\oint \vec{E} \cdot d\vec{A}$ reduces to $\pm EA$, where $A = 4\pi r^2$ (sphere) or $A = 2\pi RL$ (cylinder).

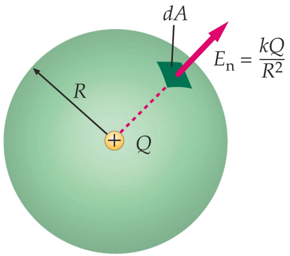






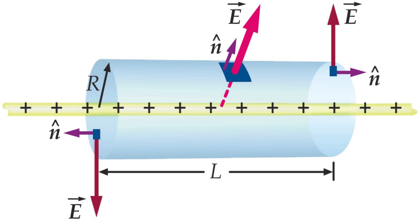
Calculating $\vec{E}$ from Gauss's Law: Point Charge

- Consider a positive point charge Q.
- Use a Gaussian sphere of radius R centered at the location of Q.
- Surface area of sphere: $A = 4\pi R^2$.
- Electric flux through Gaussian surface: $\Phi_E = \oint \vec{E} \cdot d\vec{A} = E(4\pi R^2)$.
- Net charge inside Gaussian surface: $Q_{in} = Q$.
- Gauss's law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$ becomes $E(4\pi R^2) = \frac{Q}{\epsilon_0}$.
- Electric field at radius R: $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} = \frac{kQ}{R^2}$.



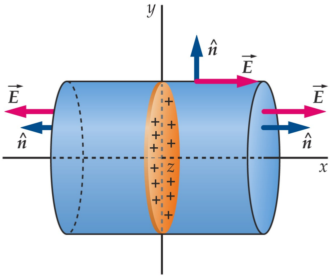
Calculating $\vec{E}$ from Gauss's Law: Charged Wire

- Consider a uniformly charged wire of infinite length.
- Charge per unit length on wire: λ (here assumed positive).
- Use a coaxial Gaussian cylinder of radius R and length L.
- Electric flux through Gaussian surface: $\Phi_E = \oint \vec{E} \cdot d\vec{A} = E(2\pi RL)$.
- Net charge charge inside Gaussian surface: $Q_{in} = \lambda L$.
- Gauss's law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$ becomes $E(2\pi RL) = \frac{\lambda L}{\epsilon_0}$.
- Electric field at radius R: $E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{R}$.



Calculating $\vec{E}$ from Gauss's Law: Charged Plane Sheet

- Consider a uniformly charged plane sheet.
- Charge per unit area on sheet: σ (here assumed positive).
- Use Gaussian cylinder with cross-sectional area A placed as shown.
- Electric flux through Gaussian surface: $\Phi_E = \oint \vec{E} \cdot d\vec{A} = 2EA$.
  Net charge charge inside Gaussian surface: $Q_{in} = \sigma A$.
- Gauss's law $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{in}}{\epsilon_0}$ becomes $2EA = \dfrac{\sigma A}{\epsilon_0}$.
- Electric field at both ends of cylinder: $E = \dfrac{\sigma}{2\epsilon_0} = 2\pi k\sigma$
  (pointing away from sheet).
- Note that E does not depend on the distance from the sheet.



Calculating $\vec{E}$ from Gauss's Law: Charged Slab

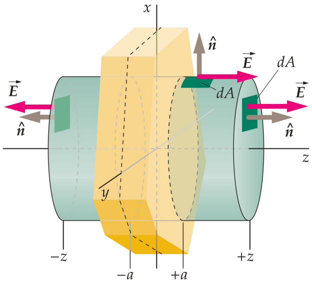
- Consider a uniformly charged slab.
- Charge per unit volume on slab: ρ.
- Use Gaussian cylinder as shown.

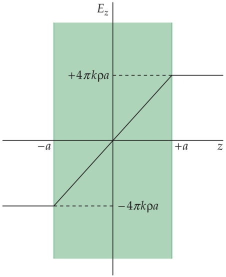
- Total electric flux: $\Phi_E = 2|E_z|A$.

- Net charge inside: $Q_{in} = \begin{cases} 2\rho A|z| & (|z| \leq a) \\ 2\rho Aa & (|z| \geq a) \end{cases}$

- Gauss's law: $2|E_z|A = \begin{cases} \dfrac{2\rho A|z|}{\epsilon_0} & (|z| \leq a) \\ \dfrac{2\rho Aa}{\epsilon_0} & (|z| \geq a) \end{cases}$

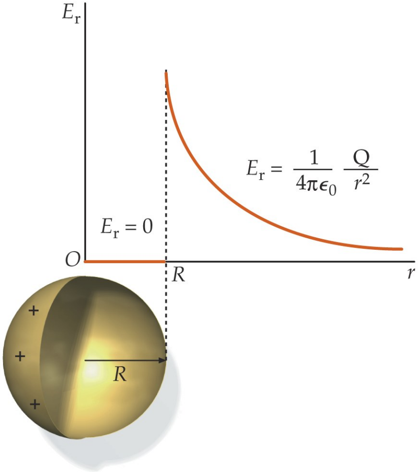
- Electric field: $E_z = \begin{cases} -\dfrac{\rho a}{\epsilon_0} & (z \leq -a) \\ \dfrac{\rho z}{\epsilon_0} & (-a \leq z \leq a) \\ \dfrac{\rho a}{\epsilon_0} & (z \geq a) \end{cases}$



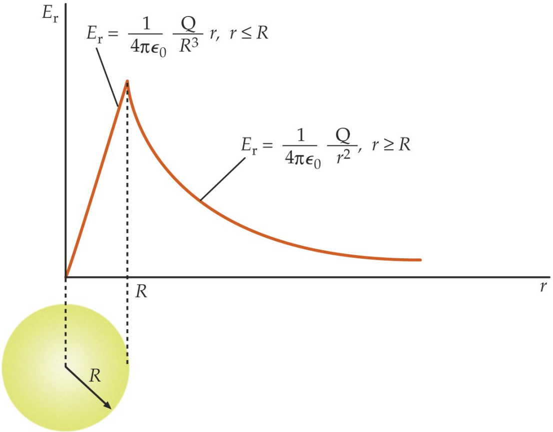


Electric Field of Uniformly Charged Spherical Shell

- Radius of charged spherical shell: R
- Electric charge on spherical shell: $Q = \sigma A = 4\pi\sigma R^2$.
- Use a concentric Gaussian sphere of radius r.
- $r > R$: $E(4\pi r^2) = \dfrac{Q}{\epsilon_0}$
  $\Rightarrow \; E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2}$
- $r < R$: $E(4\pi r^2) = \dfrac{Q_{in}}{\epsilon_0} = 0$
  $\Rightarrow \; E = 0$

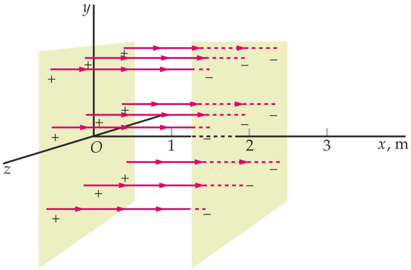


- Radius of charged solid sphere: R
- Electric charge on sphere:
  $Q = \rho V = \frac{4\pi}{3}\rho R^3.$
- Use a concentric Gaussian sphere of radius r.
- $r > R$: $E(4\pi r^2) = \frac{Q}{\epsilon_0}$
  $\Rightarrow \; E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$
- $r < R$: $E(4\pi r^2) = \frac{1}{\epsilon_0}\left(\frac{4\pi}{3}r^3\rho\right)$
  $\Rightarrow \; E(r) = \frac{\rho}{3\epsilon_0}\, r = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}\, r$



Electric Field of Oppositely Charged Infinite Sheets

- Consider two infinite sheets of charge with charge per unit area $\pm\sigma$, respectively.
- The sheets are positioned at $x = 0$ and $x = 2\text{m}$, respectively.
- Magnitude of field produced by each sheet: $E = \dfrac{\sigma}{2\epsilon_0}$.
- Electric field at $x < 0$: $E_x = E_x^{(+)} + E_x^{(-)} = -\dfrac{\sigma}{2\epsilon_0} + \dfrac{\sigma}{2\epsilon_0} = 0$.
- Electric field at $0 < x < 2\text{m}$: $E_x = E_x^{(+)} + E_x^{(-)} = +\dfrac{\sigma}{2\epsilon_0} + \dfrac{\sigma}{2\epsilon_0} = \dfrac{\sigma}{\epsilon_0}$.
- Electric field at $x > 2\text{m}$: $E_x = E_x^{(+)} + E_x^{(-)} = +\dfrac{\sigma}{2\epsilon_0} - \dfrac{\sigma}{2\epsilon_0} = 0$.

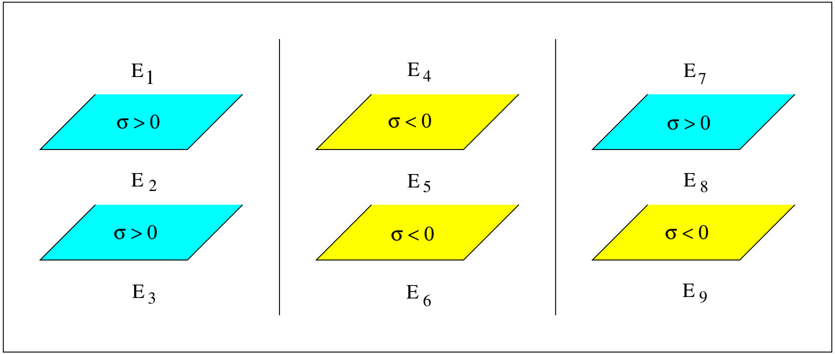


Electric Field Near Charged Infinite Sheets

Consider three pairs of parallel, infinite, uniformly charged sheets.

The charge per unit area is equal in magnitude on all sheets.

Find the direction $(\uparrow, \ \downarrow, \text{none})$ of the electric field at the nine locations indicated.

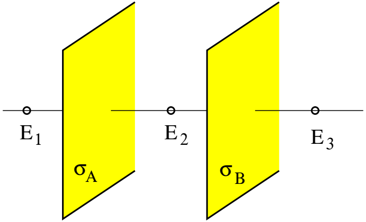


Unit Exam I: Problem #2 (Spring '09)

Consider two very large uniformly charged parallel sheets as shown.

The charge densities are $\sigma_A = +7 \times 10^{-12}\text{Cm}^{-2}$ and $\sigma_B = -4 \times 10^{-12}\text{Cm}^{-2}$, respectively.

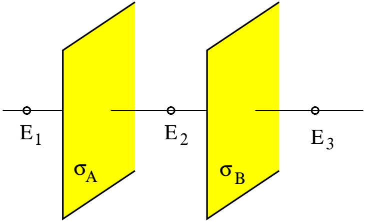
Find magnitude and direction (left/right) of the electric fields $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$.



Consider two very large uniformly charged parallel sheets as shown.

The charge densities are $\sigma_A = +7 \times 10^{-12}\mathrm{Cm}^{-2}$ and $\sigma_B = -4 \times 10^{-12}\mathrm{Cm}^{-2}$, respectively.

Find magnitude and direction (left/right) of the electric fields $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$.



Solution:

$$E_A = \frac{|\sigma_A|}{2\epsilon_0} = 0.40\mathrm{N/C} \quad \text{(directed away from sheet A).}$$

$$E_B = \frac{|\sigma_B|}{2\epsilon_0} = 0.23\mathrm{N/C} \quad \text{(directed toward sheet B).}$$

$$E_1 = E_A - E_B = 0.17\mathrm{N/C} \quad \text{(directed left).}$$

$$E_2 = E_A + E_B = 0.63\mathrm{N/C} \quad \text{(directed right).}$$

$$E_2 = E_A - E_B = 0.17\mathrm{N/C} \quad \text{(directed right).}$$

Unit Exam I: Problem #2 (Spring '12)

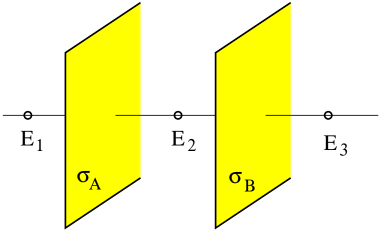
Two very large, thin, uniformly charged, parallel sheets are positioned as shown.

Find the values of the charge densities, σ_A and σ_B, if you know the electric fields $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$.

Consider two situations.

(a) $E_1 = 2\text{N/C}$ (directed left), $E_2 = 0$, $E_3 = 2\text{N/C}$ (directed right).

(b) $E_1 = 0$, $E_2 = 2\text{N/C}$ (directed right), $E_3 = 0$.



Unit Exam I: Problem #2 (Spring '12)

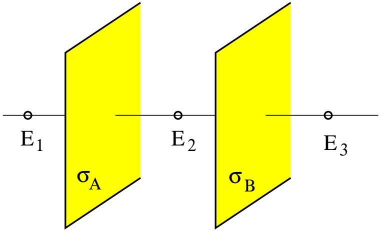
Two very large, thin, uniformly charged, parallel sheets are positioned as shown.

Find the values of the charge densities, σ_A and σ_B, if you know the electric fields $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$.

Consider two situations.

(a) $E_1 = 2\mathrm{N/C}$ (directed left), $E_2 = 0$, $E_3 = 2\mathrm{N/C}$ (directed right).

(b) $E_1 = 0$, $E_2 = 2\mathrm{N/C}$ (directed right), $E_3 = 0$.



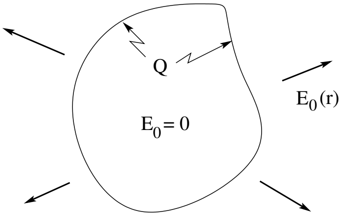
Solution:

(a) The two sheets are equally charged:
$\sigma_A = \sigma_B = 2\epsilon_0(1\mathrm{N/C}) = 1.77 \times 10^{-11}\mathrm{C/m^2}$.

(b) The two sheets are oppositely charged:
$\sigma_A = -\sigma_B = 2\epsilon_0(1\mathrm{N/C}) = 1.77 \times 10^{-11}\mathrm{C/m^2}$.

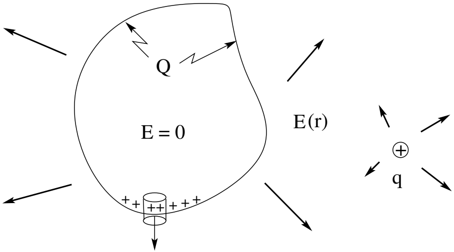
Charged Conductor at Equilibrium (1)

- Consider a conductor with excess charge Q in isolation.
- The mobile charges (electrons) are rearranged spontaneously until we have $\vec{E}_0 = 0$ everywhere inside the conductor.
- If $\vec{E}_0 = 0$ inside the conductor, then Gauss's law implies that there can be no net flux through any Gaussian surface that is inside the conductor.
- Hence there can be no net charge in any region inside the conductor.
- Hence all excess charge must be at the surface, where it produces an electric field $\vec{E}_0(\vec{r})$ on the outside only.



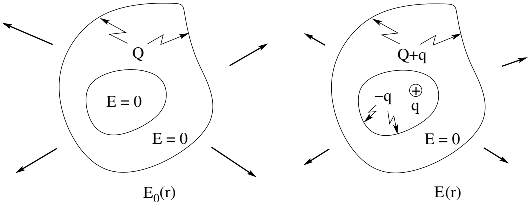
Charged Conductor at Equilibrium (2)

- Now place a point charge q near the charged conductor.
- The electric field produced by q causes a further rearrangement of mobile surface charges until we have again $\vec{E} = 0$ in the interior.
- Locally, the electric field $\vec{E}$ is perpendicular to the surface of the conductor, and its magnitude is proportional to the charge per unit area: $E = \sigma/\epsilon_0$.



Charged Conductor at Equilibrium (3)

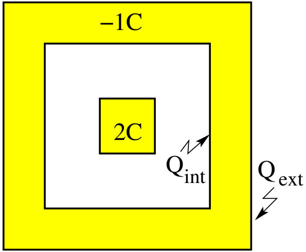
- Consider a conductor with a cavity and excess charge Q.
- Gauss's law implies that there is no net charge on the surface of the cavity.
- The external field is $\vec{E}_0(\vec{r})$. There is no field in the cavity.
- Now place a point charge q inside the cavity.
- Gauss's law implies that there is a charge $-q$ on the surface of the cavity.
- Charge conservation implies that there is a charge $Q + q$ on the outer surface of the conductor.
- The external field changes to $\vec{E}(\vec{r})$. There is a nonzero electric field field inside the cavity.



Charged Conductor Problem (1)

Consider a metal cube with a charge 2C on it positioned inside a cubic metal shell with a charge -1C on it.

- Find the charge Q_{int} on the interior surface and the charge Q_{ext} on the exterior surface of the shell.



Charged Conductor Problem (2)

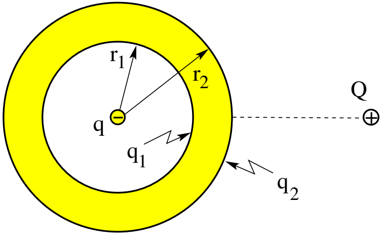
A conducting spherical shell of inner radius $r_1 = 4\text{cm}$ and outer radius $r_2 = 6\text{cm}$ carries no net charge. Now we place a point charge $q = -1\mu\text{C}$ at its center.

(a) Find the surface charge densities σ_1 and σ_2.

(b) Find the electric fields E_1 and E_2 in the immediate vicinity of the shell.

(c) What happens to the electric fields inside and outside the shell when a second point charge $Q = +1\mu\text{C}$ is placed a distance $d = 20\text{cm}$ from the center of the shell?

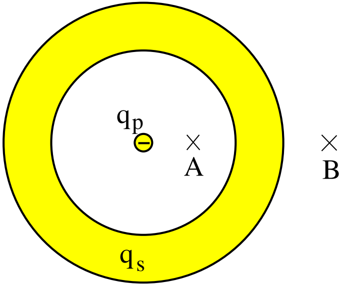
(d) Which objects exert a force on the second point charge?



Charged Conductor Problem (3)

A point charge $q_p = -7\mu$C is positioned at the center of a conducting spherical shell with a charge $q_s = +4\mu$C on it.

- Find the direction (inward/outward) of the electric field at the points A and B.
- Find the charge q_s^{int} on the inner surface and the charge q_s^{ext} on the outer surface of the shell.

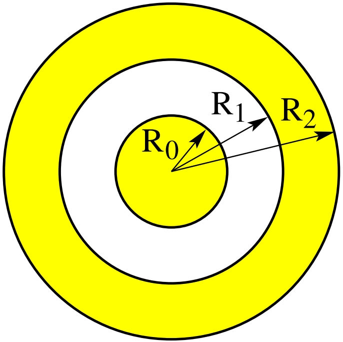


Charged Conductor Problem (4)

A long conducting cylinder of radius $R_0 = 3\text{cm}$ carries a charge per unit length $\lambda_c = 5.0\mu\text{C/m}$. It is surrounded by a conducting cylindrical shell of radii $R_1 = 7\text{cm}$ and $R_2 = 11\text{cm}$. The shell carries a charge per unit length $\lambda_s = -8.0\mu\text{C/m}$.

(a) Find the linear charge densities λ_1, λ_2 on the inner and outer surfaces of the shell.

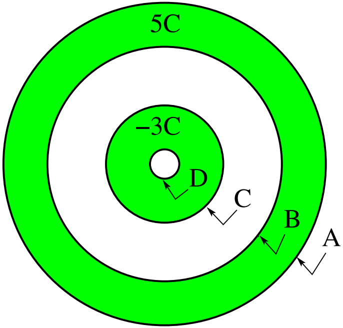
(b) Find the electric fields E_0, E_1, E_3 in the vicinity of the three conducting surfaces.



Charged Conductor Problem (5)

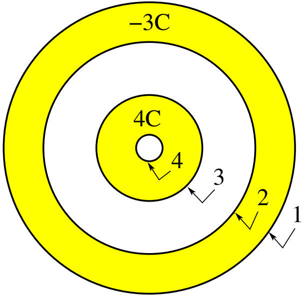
Consider two concentric shells with charges on them as indicated.

- Find the charges q_A, q_B, q_C, q_D on each of the four surfaces.



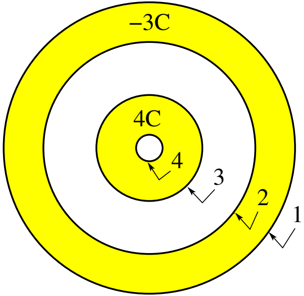
Intermediate Exam I: Problem #4 (Spring '05)

Consider two concentric conducting spherical shells. The total electric charge on the inner shell is 4C and the total electric charge on the outer shell is -3C. Find the electric charges q_1, q_2, q_3, q_4 on each surface of both shells as identified in the figure.



Intermediate Exam I: Problem #4 (Spring '05)

Consider two concentric conducting spherical shells. The total electric charge on the inner shell is 4C and the total electric charge on the outer shell is -3C. Find the electric charges q_1, q_2, q_3, q_4 on each surface of both shells as identified in the figure.



Solution:

Start with the innermost surface.
Note that any excess charge is located at the surface of a conductor.
Note also that the electric field inside a conductor at equilibrium vanishes.

- Gauss's law predicts $q_4 = 0$.
- Charge conservation then predicts $q_3 + q_4 = 4$C. Hence $q_3 = 4$C.
- Gauss's law predicts $q_2 = -(q_3 + q_4) = -4$C.
- Charge conservation then predicts $q_1 + q_2 = -3$C. Hence $q_1 = +1$C.

Intermediate Exam I: Problem #2 (Spring '06)

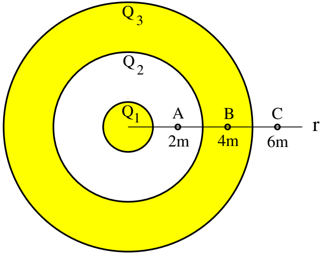
Consider a conducting sphere of radius $r_1 = 1\text{m}$ and a conducting spherical shell of inner radius $r_2 = 3\text{m}$ and outer radius $r_3 = 5\text{m}$. The charge on the inner sphere is $Q_1 = -0.6\mu\text{C}$. The net charge on the shell is zero.

(a) Find the charge Q_2 on the inner surface and the charge Q_3 on the outer surface of the shell.

(b) Find magnitude and direction of the electric field at point A between the sphere and the shell.

(c) Find magnitude and direction of the electric field at point B inside the shell.

(d) Find magnitude and direction of the electric field at point C outside the shell.



Intermediate Exam I: Problem #2 (Spring '06)

Consider a conducting sphere of radius $r_1 = 1$m and a conducting spherical shell of inner radius $r_2 = 3$m and outer radius $r_3 = 5$m. The charge on the inner sphere is $Q_1 = -0.6\mu$C. The net charge on the shell is zero.

(a) Find the charge Q_2 on the inner surface and the charge Q_3 on the outer surface of the shell.

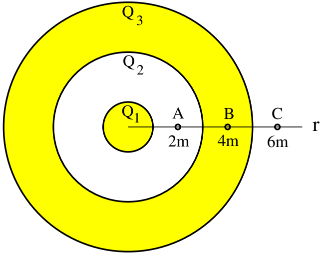
(b) Find magnitude and direction of the electric field at point A between the sphere and the shell.

(c) Find magnitude and direction of the electric field at point B inside the shell.

(d) Find magnitude and direction of the electric field at point C outside the shell.

Solution:

(a) Gauss's law implies that $Q_2 = -Q_1 = +0.6\mu$C.
Given that $Q_2 + Q_3 = 0$ we infer $Q_3 = -0.6\mu$C.



Intermediate Exam I: Problem #2 (Spring '06)

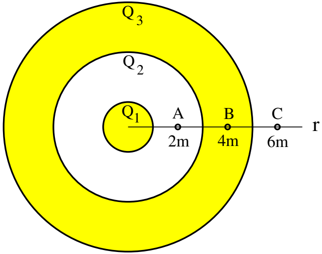
Consider a conducting sphere of radius $r_1 = 1\text{m}$ and a conducting spherical shell of inner radius $r_2 = 3\text{m}$ and outer radius $r_3 = 5\text{m}$. The charge on the inner sphere is $Q_1 = -0.6\mu\text{C}$. The net charge on the shell is zero.

(a) Find the charge Q_2 on the inner surface and the charge Q_3 on the outer surface of the shell.

(b) Find magnitude and direction of the electric field at point A between the sphere and the shell.

(c) Find magnitude and direction of the electric field at point B inside the shell.

(d) Find magnitude and direction of the electric field at point C outside the shell.



Solution:

(a) Gauss's law implies that $Q_2 = -Q_1 = +0.6\mu\text{C}$.
Given that $Q_2 + Q_3 = 0$ we infer $Q_3 = -0.6\mu\text{C}$.

(b) $E_A = k\dfrac{0.6\mu\text{C}}{(2\text{m})^2} = 1349\text{N/C}$ (inward).

Consider a conducting sphere of radius $r_1 = 1\mathrm{m}$ and a conducting spherical shell of inner radius $r_2 = 3\mathrm{m}$ and outer radius $r_3 = 5\mathrm{m}$. The charge on the inner sphere is $Q_1 = -0.6\mu\mathrm{C}$. The net charge on the shell is zero.

(a) Find the charge Q_2 on the inner surface and the charge Q_3 on the outer surface of the shell.

(b) Find magnitude and direction of the electric field at point A between the sphere and the shell.

(c) Find magnitude and direction of the electric field at point B inside the shell.

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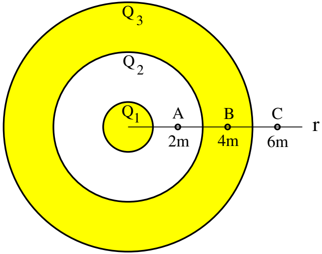


Solution:

(a) Gauss's law implies that $Q_2 = -Q_1 = +0.6\mu\mathrm{C}$.
Given that $Q_2 + Q_3 = 0$ we infer $Q_3 = -0.6\mu\mathrm{C}$.

(b) $E_A = k\dfrac{0.6\mu\mathrm{C}}{(2\mathrm{m})^2} = 1349\mathrm{N/C}$ (inward).

(c) $E_B = 0$ inside conductor.

Consider a conducting sphere of radius $r_1 = 1\text{m}$ and a conducting spherical shell of inner radius $r_2 = 3\text{m}$ and outer radius $r_3 = 5\text{m}$. The charge on the inner sphere is $Q_1 = -0.6\mu\text{C}$. The net charge on the shell is zero.

(a) Find the charge Q_2 on the inner surface and the charge Q_3 on the outer surface of the shell.

(b) Find magnitude and direction of the electric field at point A between the sphere and the shell.

(c) Find magnitude and direction of the electric field at point B inside the shell.

(d) Find magnitude and direction of the electric field at point C outside the shell.

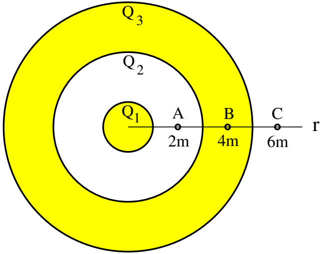
Solution:

(a) Gauss's law implies that $Q_2 = -Q_1 = +0.6\mu\text{C}$.
Given that $Q_2 + Q_3 = 0$ we infer $Q_3 = -0.6\mu\text{C}$.

(b) $E_A = k\dfrac{0.6\mu\text{C}}{(2\text{m})^2} = 1349\text{N/C}$ (inward).

(c) $E_B = 0$ inside conductor.

(d) $E_C = k\dfrac{0.6\mu\text{C}}{(6\text{m})^2} = 150\text{N/C}$ (inward).



Unit Exam I: Problem #2 (Spring ’07)

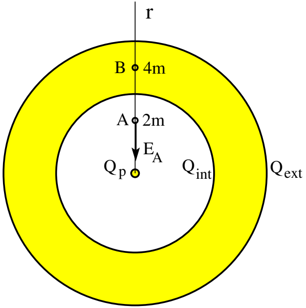
A point charge Q_p is positioned at the center of a conducting spherical shell of inner radius $r_2 = 3.00\text{m}$ and outer radius $r_3 = 5.00\text{m}$. The total charge on the shell $Q_s = +7.00\text{nC}$. The electric field at point A has strength $E_A = 6.75\text{N/C}$ and is pointing radially inward.

(a) Find the value of Q_p (point charge).

(b) Find the charge Q_{int} on the inner surface of the shell.

(c) Find the charge Q_{ext} on the outer surface of the shell.

(d) Find the electric field at point B.



Unit Exam I: Problem #2 (Spring '07)

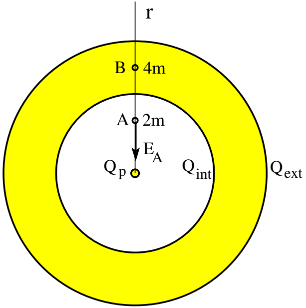
A point charge Q_p is positioned at the center of a conducting spherical shell of inner radius $r_2 = 3.00\text{m}$ and outer radius $r_3 = 5.00\text{m}$. The total charge on the shell $Q_s = +7.00\text{nC}$. The electric field at point A has strength $E_A = 6.75\text{N/C}$ and is pointing radially inward.

(a) Find the value of Q_p (point charge).

(b) Find the charge Q_{int} on the inner surface of the shell.

(c) Find the charge Q_{ext} on the outer surface of the shell.

(d) Find the electric field at point B.



Solution:

(a) Gauss's law implies that $-E_A(4\pi r_A^2) = \dfrac{Q_p}{\epsilon_0}$

$\Rightarrow \; Q_p = -3.00\text{nC}.$

Unit Exam I: Problem #2 (Spring '07)

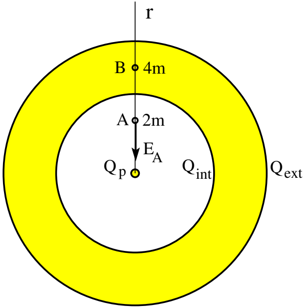
A point charge Q_p is positioned at the center of a conducting spherical shell of inner radius $r_2 = 3.00\text{m}$ and outer radius $r_3 = 5.00\text{m}$. The total charge on the shell $Q_s = +7.00\text{nC}$. The electric field at point A has strength $E_A = 6.75\text{N/C}$ and is pointing radially inward.

(a) Find the value of Q_p (point charge).

(b) Find the charge Q_{int} on the inner surface of the shell.

(c) Find the charge Q_{ext} on the outer surface of the shell.

(d) Find the electric field at point B.



Solution:

(a) Gauss's law implies that $-E_A(4\pi r_A^2) = \dfrac{Q_p}{\epsilon_0}$

$\Rightarrow \; Q_p = -3.00\text{nC}.$

(b) Gauss's law implies that $Q_{int} = -Q_p = +3.00\text{nC}.$

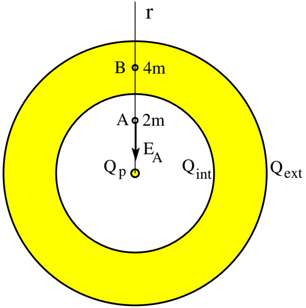
A point charge Q_p is positioned at the center of a conducting spherical shell of inner radius $r_2 = 3.00\text{m}$ and outer radius $r_3 = 5.00\text{m}$. The total charge on the shell $Q_s = +7.00\text{nC}$. The electric field at point A has strength $E_A = 6.75\text{N/C}$ and is pointing radially inward.

(a) Find the value of Q_p (point charge).

(b) Find the charge Q_{int} on the inner surface of the shell.

(c) Find the charge Q_{ext} on the outer surface of the shell.

(d) Find the electric field at point B.



Solution:

(a) Gauss's law implies that $-E_A(4\pi r_A^2) = \dfrac{Q_p}{\epsilon_0}$
$\Rightarrow \; Q_p = -3.00\text{nC}$.

(b) Gauss's law implies that $Q_{int} = -Q_p = +3.00\text{nC}$.

(c) Charge conservation, $Q_{int} + Q_{ext} = Q_s = 7.00\text{nC}$, then implies that $Q_{ext} = +4.00\text{nC}$.

A point charge Q_p is positioned at the center of a conducting spherical shell of inner radius $r_2 = 3.00$m and outer radius $r_3 = 5.00$m. The total charge on the shell $Q_s = +7.00$nC. The electric field at point A has strength $E_A = 6.75$N/C and is pointing radially inward.

(a) Find the value of Q_p (point charge).

(b) Find the charge Q_{int} on the inner surface of the shell.

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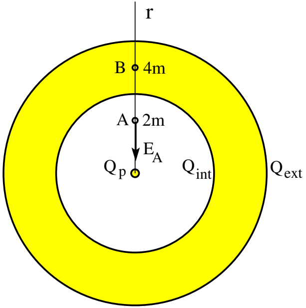

Solution:

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$\Rightarrow \; Q_p = -3.00$nC.

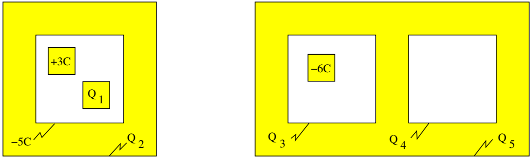
(b) Gauss's law implies that $Q_{int} = -Q_p = +3.00$nC.

(c) Charge conservation, $Q_{int} + Q_{ext} = Q_s = 7.00$nC, then implies that $Q_{ext} = +4.00$nC.

(d) $E_B = 0$ inside conductor.

(a) Consider a conducting box with no net charge on it. Inside the box are two small charged conducting cubes. For the given charges on the surface of one cube and on the inside surface of the box find the charges Q_1 on the surface of the other cube and Q_2 on the outside surface of the box.

(b) Consider a conducting box with two compartments and no net charge on it. Inside one compartment is a small charged conducting cube. For the given charge on the surface of the cube find the charges Q_3, Q_4, and Q_5 on the three surfaces of the box.



(a) (b)

(a) Consider a conducting box with no net charge on it. Inside the box are two small charged conducting cubes. For the given charges on the surface of one cube and on the inside surface of the box find the charges Q_1 on the surface of the other cube and Q_2 on the outside surface of the box.

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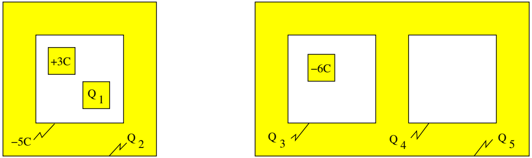


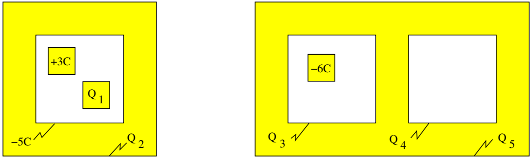
(a) (b)

Solution:

(a) Gauss's law implies $Q_1 + 3\mathrm{C} + (-5\mathrm{C}) = 0 \quad \Rightarrow \; Q_1 = +2\mathrm{C}$.
Net charge on the box: $Q_2 + (-5\mathrm{C}) = 0 \quad \Rightarrow \; Q_2 = +5\mathrm{C}$.

(a) Consider a conducting box with no net charge on it. Inside the box are two small charged conducting cubes. For the given charges on the surface of one cube and on the inside surface of the box find the charges Q_1 on the surface of the other cube and Q_2 on the outside surface of the box.

(b) Consider a conducting box with two compartments and no net charge on it. Inside one compartment is a small charged conducting cube. For the given charge on the surface of the cube find the charges Q_3, Q_4, and Q_5 on the three surfaces of the box.



(a) (b)

Solution:

(a) Gauss's law implies $Q_1 + 3\text{C} + (-5\text{C}) = 0 \quad \Rightarrow \; Q_1 = +2\text{C}.$
Net charge on the box: $Q_2 + (-5\text{C}) = 0 \quad \Rightarrow \; Q_2 = +5\text{C}.$

(b) Gauss's law implies $Q_3 + (-6\text{C}) = 0 \quad \Rightarrow \; Q_3 = +6\text{C}.$
Gauss's law implies $Q_4 = 0.$
Net charge on box: $Q_3 + Q_4 + Q_5 = 0 \quad \Rightarrow \; Q_5 = -6\text{C}.$

Unit Exam I: Problem #2 (Spring '11)

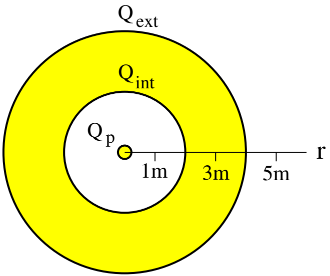
The charged conducting spherical shell has a 2m inner radius and a 4m outer radius. The charge on the outer surface is $Q_{\text{ext}} = 8\text{nC}$. There is a point charge $Q_{\text{p}} = 3\text{nC}$ at the center.

(a) Find the charge Q_{int} on the inner surface of the shell.

(b) Find the surface charge density σ_{ext} on the outer surface of the shell.

(c) Find the electric flux Φ_E through a Gaussian sphere of radius $r = 5\text{m}$.

(d) Find the magnitude of the electric field E at radius $r = 3\text{m}$.



Unit Exam I: Problem #2 (Spring '11)

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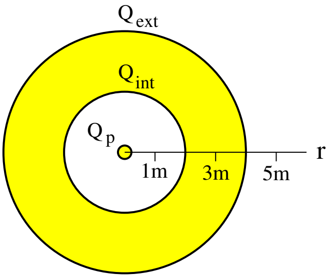


Solution:

(a) $Q_{\text{int}} = -Q_{\text{p}} = -3\text{nC}.$

The charged conducting spherical shell has a 2m inner radius and a 4m outer radius. The charge on the outer surface is $Q_{\rm ext} = 8\rm nC$. There is a point charge $Q_{\rm p} = 3\rm nC$ at the center.

(a) Find the charge $Q_{\rm int}$ on the inner surface of the shell.

(b) Find the surface charge density $\sigma_{\rm ext}$ on the outer surface of the shell.

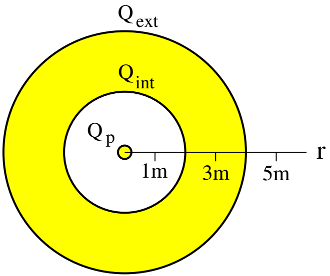
(c) Find the electric flux Φ_E through a Gaussian sphere of radius $r = 5\rm m$.

(d) Find the magnitude of the electric field E at radius $r = 3\rm m$.

Solution:

(a) $Q_{\rm int} = -Q_{\rm p} = -3\rm nC$.

(b) $\sigma_{\rm ext} = \dfrac{Q_{\rm ext}}{4\pi(4\rm m)^2} = 3.98 \times 10^{-11}\rm C/m^2$.



The charged conducting spherical shell has a 2m inner radius and a 4m outer radius. The charge on the outer surface is $Q_{\rm ext} = 8\text{nC}$. There is a point charge $Q_{\rm p} = 3\text{nC}$ at the center.

(a) Find the charge $Q_{\rm int}$ on the inner surface of the shell.

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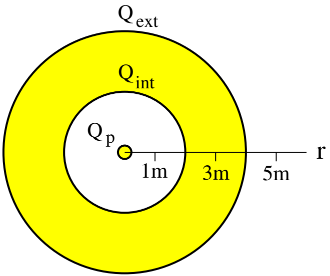
(c) Find the electric flux Φ_E through a Gaussian sphere of radius $r = 5\text{m}$.

(d) Find the magnitude of the electric field E at radius $r = 3\text{m}$.



Solution:

(a) $Q_{\rm int} = -Q_{\rm p} = -3\text{nC}.$

(b) $\sigma_{\rm ext} = \dfrac{Q_{\rm ext}}{4\pi(4\text{m})^2} = 3.98 \times 10^{-11}\text{C/m}^2.$

(c) $\Phi_E = \dfrac{Q_{\rm ext}}{\epsilon_0} = 904\text{Nm}^2/\text{C}.$

Unit Exam I: Problem #2 (Spring '11)

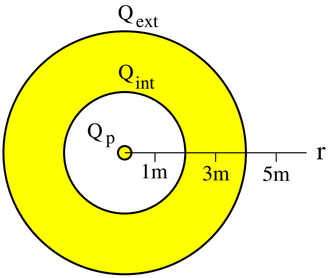
The charged conducting spherical shell has a 2m inner radius and a 4m outer radius. The charge on the outer surface is $Q_{\rm ext} = 8\rm nC$. There is a point charge $Q_{\rm p} = 3\rm nC$ at the center.

(a) Find the charge $Q_{\rm int}$ on the inner surface of the shell.

(b) Find the surface charge density $\sigma_{\rm ext}$ on the outer surface of the shell.

(c) Find the electric flux Φ_E through a Gaussian sphere of radius $r = 5\rm m$.

(d) Find the magnitude of the electric field E at radius $r = 3\rm m$.



Solution:

(a) $Q_{\rm int} = -Q_{\rm p} = -3\rm nC.$

(b) $\sigma_{\rm ext} = \dfrac{Q_{\rm ext}}{4\pi(4\rm m)^2} = 3.98\times 10^{-11}\rm C/m^2.$

(c) $\Phi_E = \dfrac{Q_{\rm ext}}{\epsilon_0} = 904\rm Nm^2/C.$

(d) $E = 0$ inside conductor.