

2020

## 02. Motion of charged particles in electric field. Electric dipole

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### Recommended Citation

Müller, Gerhard and Coyne, Robert, "02. Motion of charged particles in electric field. Electric dipole" (2020). *PHY 204: Elementary Physics II -- Slides*. Paper 27.  
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# Particle in Uniform Electric or Gravitational Field



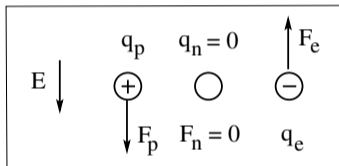
particle	charge	mass
electron	$q_e = -e$	$m_e = 9.109 \times 10^{-31} \text{kg}$
proton	$q_p = +e$	$m_p = 1.673 \times 10^{-27} \text{kg}$
neutron	$q_n = 0$	$m_n = 1.675 \times 10^{-27} \text{kg}$

Elementary charge:

$$e = 1.602 \times 10^{-19} \text{C.}$$

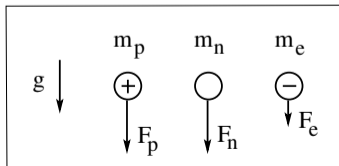
## Electric field

- equation of motion:  $\vec{F} = m\vec{a}$
- force law:  $\vec{F} = q\vec{E}$
- acceleration:  $\vec{a} = (q/m)\vec{E}$



## Gravitational field

- equation of motion:  $\vec{F} = m\vec{a}$
- force law:  $\vec{F} = m\vec{g}$
- acceleration:  $\vec{a} = \vec{g}$

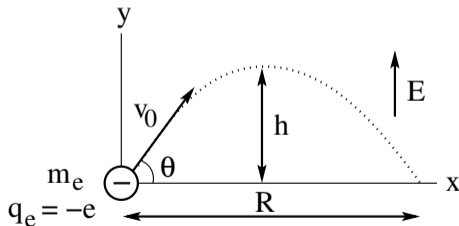


# Projectile Motion in Electric Field

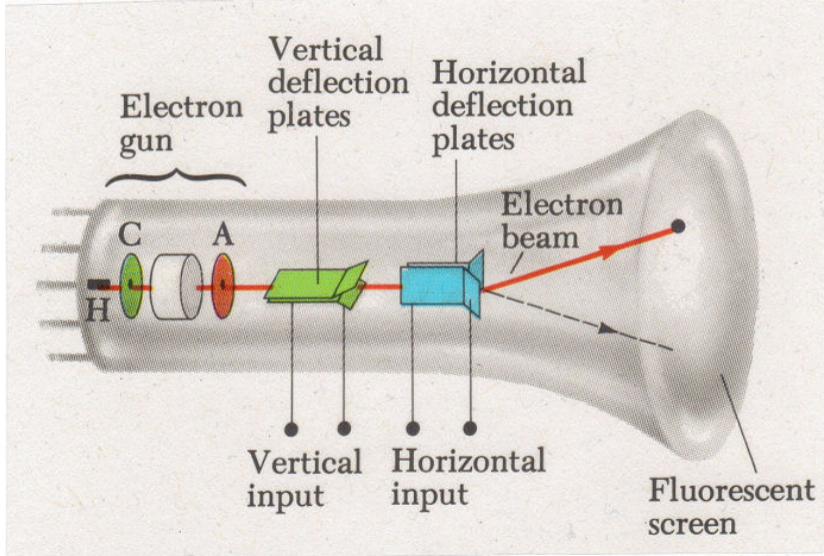


- electrostatic force:  $F_x = 0$   $F_y = -eE$
- equation of motion:  $\vec{F} = m_e \vec{a}$
- acceleration:  $a_x = 0$   $a_y = -\frac{e}{m_e} E \equiv -a$
- velocity:  $v_x(t) = v_0 \cos \theta$   $v_y(t) = v_0 \sin \theta - at$
- position:  $x(t) = v_0 [\cos \theta] t$   $y(t) = v_0 [\sin \theta] t - \frac{1}{2} at^2$

- height:  $h = \frac{v_0^2}{2a} \sin^2 \theta$
- range:  $R = \frac{v_0^2}{a} \sin(2\theta)$



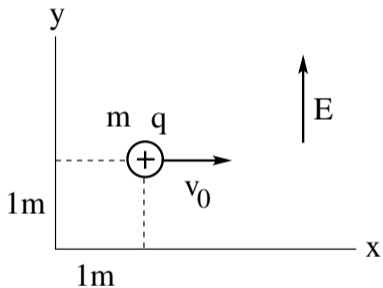
# Cathode Ray Tube



## Particle Projected Perpendicular to Uniform Electric Field



A charged particle ( $m = 3\text{kg}$ ,  $q = 1\mu\text{C}$ ) is launched at  $t_0 = 0$  with initial speed  $v_0 = 2\text{m/s}$  in an electric field of magnitude  $E = 6 \times 10^6\text{N/C}$  as shown.



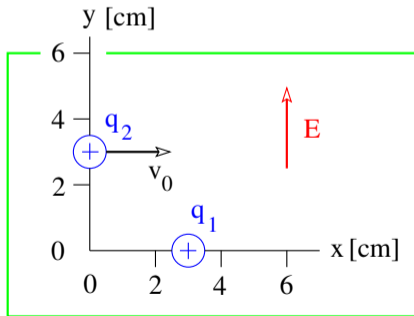
- Find the position of the particle at  $t_1 = 3\text{s}$ .
- By what angle does the velocity vector turn between  $t_0 = 0$  and  $t_1 = 3\text{s}$ ?

# Particles Accelerated by Uniform Electric Field



A uniform electric field  $E = 0.75 \times 10^3 \text{N/C}$  exists in the box.

- (a) A charged particle of mass  $m_1 = 1.9 \times 10^{-9} \text{kg}$  is released from rest at  $x = 3 \text{cm}$ ,  $y = 0$ . It exits the box at  $x = 3 \text{cm}$ ,  $y = 6 \text{cm}$  after a time  $t_1 = 5.7 \times 10^{-5} \text{s}$ . Find the charge  $q_1$ .
- (b) A second charged particle of mass  $m_2 = 2.7 \times 10^{-14} \text{kg}$  is projected from position  $x = 0$ ,  $y = 3 \text{cm}$  with initial speed  $v_0 = 3.2 \times 10^4 \text{m/s}$ . It exits the box at  $x = 3.9 \text{cm}$ ,  $y = 6 \text{cm}$ . Find the charge  $q_2$ .



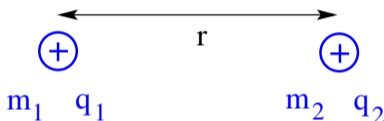
# Action and Reaction due to Coulomb Interaction



Two particles with masses  $m_1, m_2$  and charges  $q_1, q_2$  are released from rest a distance  $r$  apart.

We consider the following four distinct configurations:

- (a)  $m_1 = 1\text{kg}, m_2 = 1\text{kg}, q_1 = 1\text{C}, q_2 = 1\text{C}$
- (b)  $m_1 = 1\text{kg}, m_2 = 1\text{kg}, q_1 = 1\text{C}, q_2 = 2\text{C}$
- (c)  $m_1 = 1\text{kg}, m_2 = 2\text{kg}, q_1 = 1\text{C}, q_2 = 1\text{C}$
- (d)  $m_1 = 1\text{kg}, m_2 = 2\text{kg}, q_1 = 1\text{C}, q_2 = 2\text{C}$



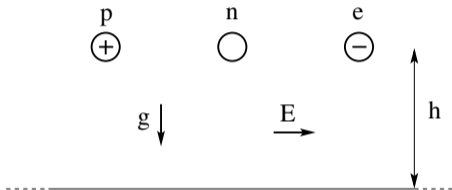
Answer the following questions for each configuration:

- (1) Is the force experienced by particle 1 **smaller than** or **equal to** or **larger than** the force experienced by particle 2?
- (2) Is the acceleration of particle 1 **smaller than** or **equal to** or **larger than** the acceleration of particle 2?

# Particle in Uniform Electric and Gravitational Field (1)



A proton, a neutron, and an electron are dropped from rest in a vertical gravitational field  $\vec{g}$  and in a horizontal electric field  $\vec{E}$  as shown. Both fields are uniform.



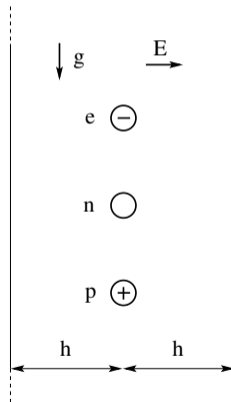
- (a) Which particle travels the shortest distance?
- (b) Which particle travels the longest distance?
- (c) Which particle travels the shortest time?
- (d) Which particle reaches the highest speed?



## Particle in Uniform Electric and Gravitational Field (2)



A proton, a neutron, and an electron are dropped from rest in a vertical gravitational field  $\vec{g}$  and in a horizontal electric field  $\vec{E}$  as shown. Both fields are uniform.



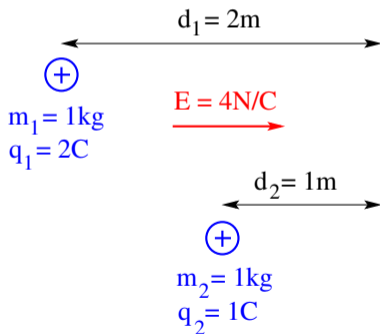
- Which particle travels the shortest distance?
- Which particle travels in a straight line?
- Which particle travels the shortest time?
- Which particle reaches the highest speed?

# Is the Faster also the Quicker?



Charged particles 1 and 2 are released from rest in a uniform electric field.

- (a) Which particle moves faster when it hits the wall?
- (b) Which particle reaches the wall more quickly?

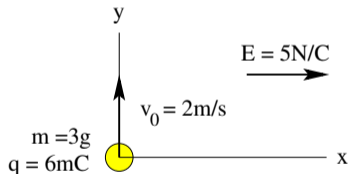


## Intermediate Exam I: Problem #3 (Spring '06)



Consider a region of uniform electric field as shown. A charged particle is projected at time  $t = 0$  with initial velocity as shown. Ignore gravity.

- (a) Find the components  $a_x$  and  $a_y$  of the acceleration at time  $t = 0$ .
- (b) Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 0$ .
- (c) Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 1.2\text{s}$ .
- (d) Find the components  $x$  and  $y$  of the position at time  $t = 1.2\text{s}$ .



## Intermediate Exam I: Problem #3 (Spring '06)

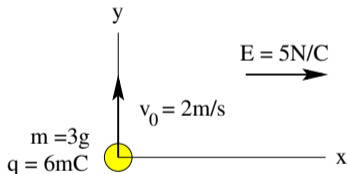


Consider a region of uniform electric field as shown. A charged particle is projected at time  $t = 0$  with initial velocity as shown. Ignore gravity.

- Find the components  $a_x$  and  $a_y$  of the acceleration at time  $t = 0$ .
- Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 0$ .
- Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 1.2\text{s}$ .
- Find the components  $x$  and  $y$  of the position at time  $t = 1.2\text{s}$ .

### Solution:

$$(a) \quad a_x = \frac{q}{m} E = \frac{6 \times 10^{-3}\text{C}}{3 \times 10^{-3}\text{kg}} (5\text{N/C}) = 10\text{m/s}^2, \quad a_y = 0.$$



## Intermediate Exam I: Problem #3 (Spring '06)



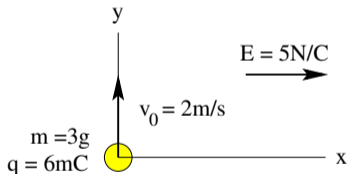
Consider a region of uniform electric field as shown. A charged particle is projected at time  $t = 0$  with initial velocity as shown. Ignore gravity.

- Find the components  $a_x$  and  $a_y$  of the acceleration at time  $t = 0$ .
- Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 0$ .
- Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 1.2\text{s}$ .
- Find the components  $x$  and  $y$  of the position at time  $t = 1.2\text{s}$ .

### Solution:

$$(a) a_x = \frac{q}{m} E = \frac{6 \times 10^{-3}\text{C}}{3 \times 10^{-3}\text{kg}} (5\text{N/C}) = 10\text{m/s}^2, \quad a_y = 0.$$

$$(b) v_x = 0, \quad v_y = v_0 = 2\text{m/s}.$$



## Intermediate Exam I: Problem #3 (Spring '06)



Consider a region of uniform electric field as shown. A charged particle is projected at time  $t = 0$  with initial velocity as shown. Ignore gravity.

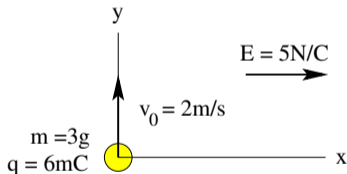
- Find the components  $a_x$  and  $a_y$  of the acceleration at time  $t = 0$ .
- Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 0$ .
- Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 1.2\text{s}$ .
- Find the components  $x$  and  $y$  of the position at time  $t = 1.2\text{s}$ .

### Solution:

$$(a) \quad a_x = \frac{q}{m} E = \frac{6 \times 10^{-3}\text{C}}{3 \times 10^{-3}\text{kg}} (5\text{N/C}) = 10\text{m/s}^2, \quad a_y = 0.$$

$$(b) \quad v_x = 0, \quad v_y = v_0 = 2\text{m/s}.$$

$$(c) \quad v_x = a_x t = (10\text{m/s}^2)(1.2\text{s}) = 12\text{m/s}, \quad v_y = v_0 = 2\text{m/s}.$$



## Intermediate Exam I: Problem #3 (Spring '06)



Consider a region of uniform electric field as shown. A charged particle is projected at time  $t = 0$  with initial velocity as shown. Ignore gravity.

- Find the components  $a_x$  and  $a_y$  of the acceleration at time  $t = 0$ .
- Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 0$ .
- Find the components  $v_x$  and  $v_y$  of the velocity at time  $t = 1.2\text{s}$ .
- Find the components  $x$  and  $y$  of the position at time  $t = 1.2\text{s}$ .

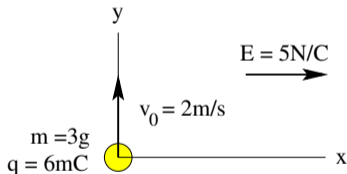
### Solution:

$$(a) \quad a_x = \frac{q}{m} E = \frac{6 \times 10^{-3}\text{C}}{3 \times 10^{-3}\text{kg}} (5\text{N/C}) = 10\text{m/s}^2, \quad a_y = 0.$$

$$(b) \quad v_x = 0, \quad v_y = v_0 = 2\text{m/s}.$$

$$(c) \quad v_x = a_x t = (10\text{m/s}^2)(1.2\text{s}) = 12\text{m/s}, \quad v_y = v_0 = 2\text{m/s}.$$

$$(d) \quad x = \frac{1}{2} a_x t^2 = 0.5(10\text{m/s}^2)(1.2\text{s})^2 = 7.2\text{m}, \quad y = v_y t = (2\text{m/s})(1.2\text{s}) = 2.4\text{m}.$$

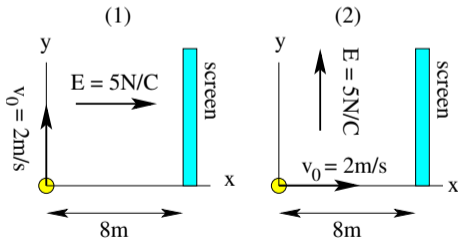


## Unit Exam I: Problem #3 (Spring '07)



Consider two regions of uniform electric field as shown. Charged particles of mass  $m = 2\text{kg}$  and charge  $q = 1\text{C}$  are projected at time  $t = 0$  with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

- (a) At what time  $t_1$  does the particle in region (1) hit the screen?
- (b) At what height  $y_1$  does the particle in region (1) hit the screen?
- (c) At what time  $t_2$  does the particle in region (2) hit the screen?
- (d) At what height  $y_2$  does the particle in region (2) hit the screen?





## Unit Exam I: Problem #3 (Spring '07)

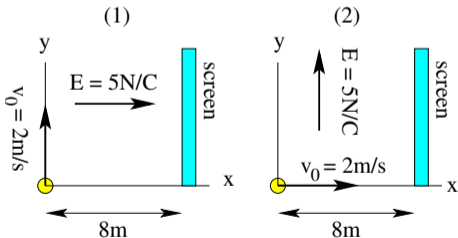


Consider two regions of uniform electric field as shown. Charged particles of mass  $m = 2\text{kg}$  and charge  $q = 1\text{C}$  are projected at time  $t = 0$  with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

- (a) At what time  $t_1$  does the particle in region (1) hit the screen?
- (b) At what height  $y_1$  does the particle in region (1) hit the screen?
- (c) At what time  $t_2$  does the particle in region (2) hit the screen?
- (d) At what height  $y_2$  does the particle in region (2) hit the screen?

### Solution:

$$\begin{aligned} \text{(a)} \quad x_1 &= \frac{1}{2}at_1^2 \quad \text{with} \quad a = \frac{q}{m}E = 2.5\text{m/s}^2, \\ x_1 &= 8\text{m} \quad \Rightarrow \quad t_1 = 2.53\text{s}. \end{aligned}$$



## Unit Exam I: Problem #3 (Spring '07)



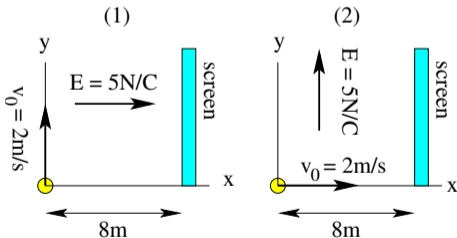
Consider two regions of uniform electric field as shown. Charged particles of mass  $m = 2\text{kg}$  and charge  $q = 1\text{C}$  are projected at time  $t = 0$  with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

- (a) At what time  $t_1$  does the particle in region (1) hit the screen?
- (b) At what height  $y_1$  does the particle in region (1) hit the screen?
- (c) At what time  $t_2$  does the particle in region (2) hit the screen?
- (d) At what height  $y_2$  does the particle in region (2) hit the screen?

### Solution:

(a)  $x_1 = \frac{1}{2}at_1^2$  with  $a = \frac{q}{m}E = 2.5\text{m/s}^2$ ,  
 $x_1 = 8\text{m} \Rightarrow t_1 = 2.53\text{s}$ .

(b)  $y_1 = v_0t_1 = 5.06\text{m}$ .



## Unit Exam I: Problem #3 (Spring '07)

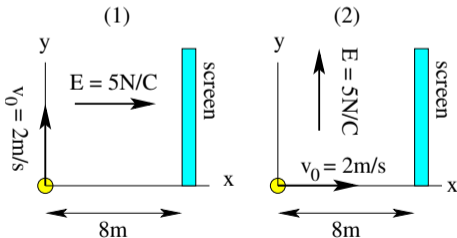


Consider two regions of uniform electric field as shown. Charged particles of mass  $m = 2\text{kg}$  and charge  $q = 1\text{C}$  are projected at time  $t = 0$  with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

- (a) At what time  $t_1$  does the particle in region (1) hit the screen?
- (b) At what height  $y_1$  does the particle in region (1) hit the screen?
- (c) At what time  $t_2$  does the particle in region (2) hit the screen?
- (d) At what height  $y_2$  does the particle in region (2) hit the screen?

### Solution:

- (a)  $x_1 = \frac{1}{2}at_1^2$  with  $a = \frac{q}{m}E = 2.5\text{m/s}^2$ ,  
 $x_1 = 8\text{m} \Rightarrow t_1 = 2.53\text{s}$ .
- (b)  $y_1 = v_0t_1 = 5.06\text{m}$ .
- (c)  $x_2 = v_0t_2 \Rightarrow t_2 = \frac{8\text{m}}{2\text{m/s}} = 4\text{s}$ .



## Unit Exam I: Problem #3 (Spring '07)



Consider two regions of uniform electric field as shown. Charged particles of mass  $m = 2\text{kg}$  and charge  $q = 1\text{C}$  are projected at time  $t = 0$  with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

- (a) At what time  $t_1$  does the particle in region (1) hit the screen?
- (b) At what height  $y_1$  does the particle in region (1) hit the screen?
- (c) At what time  $t_2$  does the particle in region (2) hit the screen?
- (d) At what height  $y_2$  does the particle in region (2) hit the screen?

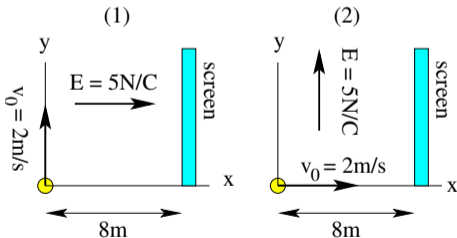
### Solution:

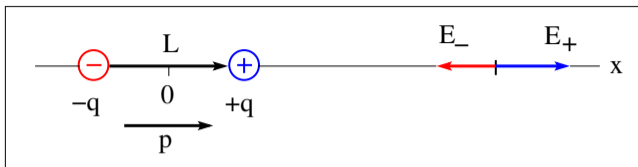
(a)  $x_1 = \frac{1}{2}at_1^2$  with  $a = \frac{q}{m}E = 2.5\text{m/s}^2$ ,  
 $x_1 = 8\text{m} \Rightarrow t_1 = 2.53\text{s}$ .

(b)  $y_1 = v_0t_1 = 5.06\text{m}$ .

(c)  $x_2 = v_0t_2 \Rightarrow t_2 = \frac{8\text{m}}{2\text{m/s}} = 4\text{s}$ .

(d)  $y_2 = \frac{1}{2}at_2^2 = 20\text{m}$ .





$$E = \frac{kq}{(x - L/2)^2} - \frac{kq}{(x + L/2)^2} = kq \left[ \frac{(x + L/2)^2 - (x - L/2)^2}{(x - L/2)^2(x + L/2)^2} \right] = \frac{2kqLx}{(x^2 - L^2/4)^2}$$
$$\simeq \frac{2kqL}{x^3} = \frac{2kp}{x^3} \quad (\text{for } x \gg L)$$

Electric dipole moment:  $\vec{p} = q\vec{L}$

- Note the more rapid decay of the electric field with distance from an electric dipole ( $\sim r^{-3}$ ) than from an electric point charge ( $\sim r^{-2}$ ).
- The dipolar field is not radial.

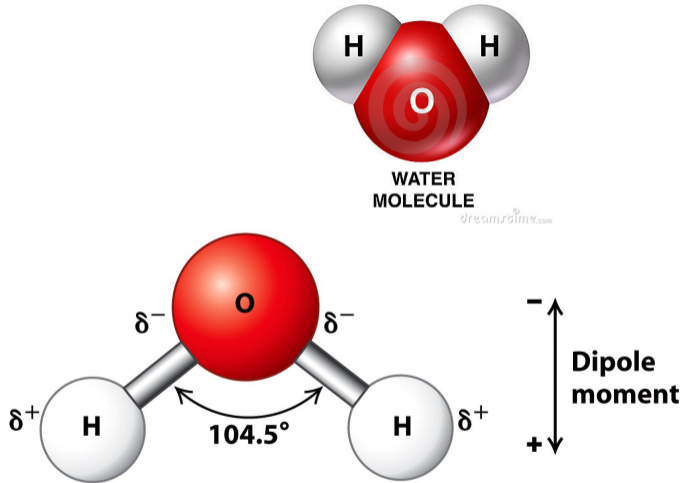
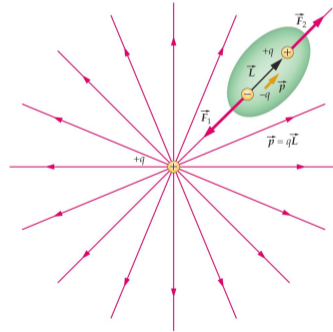
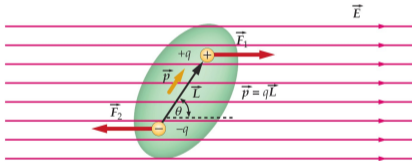


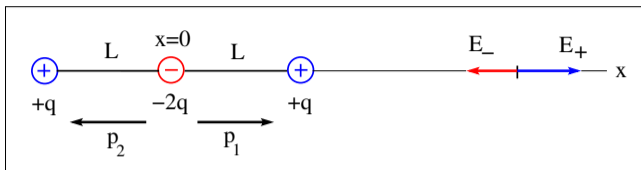
Figure 2-5  
*Molecular Cell Biology, Sixth Edition*  
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- The net force on an electric dipole in a *uniform* electric field vanishes.
- However, this dipole experiences a torque  $\vec{\tau} = \vec{p} \times \vec{E}$  that tends to align the vector  $\vec{p}$  with the vector  $\vec{E}$ .
- Now consider an electric dipole that is already aligned (locally) with a *nonuniform* electric field. This dipole experiences a net force that is always in the direction where the field has the steepest increase.



# Electric Quadrupole Field



$$\begin{aligned}
 E &= \frac{kq}{(x-L)^2} + \frac{kq}{(x+L)^2} + \frac{k(-2q)}{x^2} = \frac{kq}{x^2} \left[ \left(1 - \frac{L}{x}\right)^{-2} + \left(1 + \frac{L}{x}\right)^{-2} - 2 \right] \\
 &= \frac{kq}{x^2} \left[ \left(1 + \frac{2L}{x} + \frac{3L^2}{x^2} + \dots\right) + \left(1 - \frac{2L}{x} + \frac{3L^2}{x^2} - \dots\right) - 2 \right] \\
 &\simeq \frac{6kqL^2}{x^4} = \frac{3kQ}{x^4} \quad (\text{for } x \gg L)
 \end{aligned}$$

Electric quadrupole moment:  $Q = 2qL^2$

Different quadrupole configuration:

