2020

02. Motion of charged particles in electric field. Electric dipole

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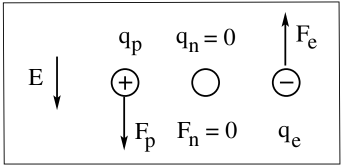
particle	charge	mass
electron	$q_e = -e$	$m_e = 9.109 \times 10^{-31}\text{kg}$
proton	$q_p = +e$	$m_p = 1.673 \times 10^{-27}\text{kg}$
neutron	$q_n = 0$	$m_n = 1.675 \times 10^{-27}\text{kg}$

Elementary charge:
$e = 1.602 \times 10^{-19}\text{C}.$

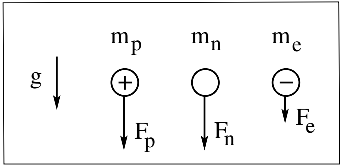
Electric field

- equation of motion: $\vec{F} = m\vec{a}$
- force law: $\vec{F} = q\vec{E}$
- acceleration: $\vec{a} = (q/m)\vec{E}$



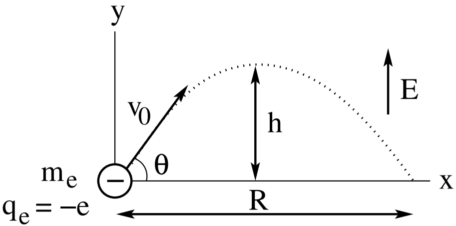
Gravitational field

- equation of motion: $\vec{F} = m\vec{a}$
- force law: $\vec{F} = m\vec{g}$
- acceleration: $\vec{a} = \vec{g}$



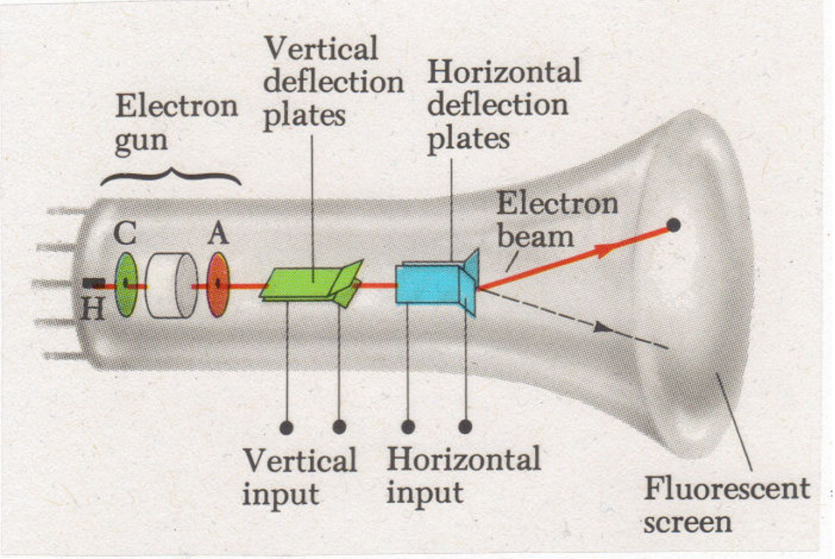
Projectile Motion in Electric Field

- electrostatic force: $F_x = 0 \quad F_y = -eE$
- equation of motion: $\vec{F} = m_e\vec{a}$
- acceleration: $a_x = 0 \qquad a_y = -\frac{e}{m_e}E \equiv -a$
- velocity: $v_x(t) = v_0 \cos\theta \qquad v_y(t) = v_0 \sin\theta - at$
- position: $x(t) = v_0[\cos\theta]t \qquad y(t) = v_0[\sin\theta]t - \frac{1}{2}at^2$



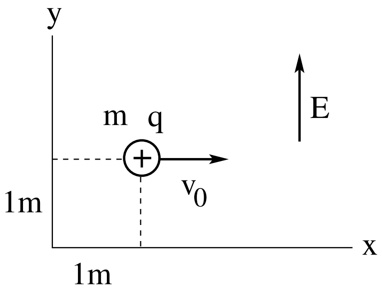
- height: $h = \frac{v_0^2}{2a}\sin^2\theta$
- range: $R = \frac{v_0^2}{a}\sin(2\theta)$

Cathode Ray Tube



Particle Projected Perpendicular to Uniform Electric Field

A charged particle $(m = 3\text{kg},\ q = 1\mu\text{C})$ is launched at $t_0 = 0$ with initial speed $v_0 = 2\text{m/s}$ in an electric field of magnitude $E = 6 \times 10^6 \text{N/C}$ as shown.



(a) Find the position of the particle at $t_1 = 3\text{s}$.

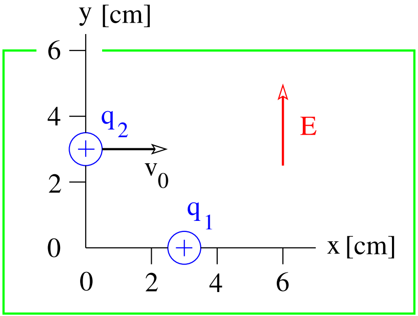
(b) By what angle does the velocity vector turn between $t_0 = 0$ and $t_1 = 3\text{s}$?

Particles Accelerated by Uniform Electric Field

A uniform electric field $E = 0.75 \times 10^3$N/C exists in the box.

(a) A charged particle of mass $m_1 = 1.9 \times 10^{-9}$kg is released from rest at $x = 3$cm, $y = 0$. It exits the box at $x = 3$cm, $y = 6$cm after a time $t_1 = 5.7 \times 10^{-5}$s. Find the charge q_1.

(b) A second charged particle of mass $m_2 = 2.7 \times 10^{-14}$kg is projected from position $x = 0$, $y = 3$cm with initial speed $v_0 = 3.2 \times 10^4$m/s. It exits the box at $x = 3.9$cm, $y = 6$cm. Find the charge q_2.

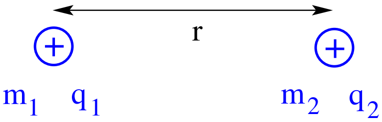


Action and Reaction due to Coulomb Interaction

Two particles with masses m_1, m_2 and charges q_1, q_2 are released from rest a distance r apart.

We consider the following four distinct configurations:

(a) $m_1 = 1\text{kg}, m_2 = 1\text{kg}, q_1 = 1\text{C}, q_2 = 1\text{C}$
(b) $m_1 = 1\text{kg}, m_2 = 1\text{kg}, q_1 = 1\text{C}, q_2 = 2\text{C}$
(c) $m_1 = 1\text{kg}, m_2 = 2\text{kg}, q_1 = 1\text{C}, q_2 = 1\text{C}$
(d) $m_1 = 1\text{kg}, m_2 = 2\text{kg}, q_1 = 1\text{C}, q_2 = 2\text{C}$



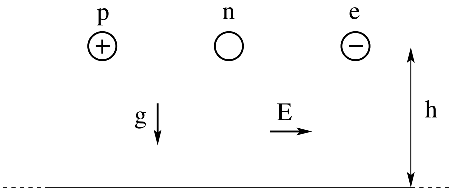
Anwer the following questions for each configuration:

(1) Is the force experienced by particle 1
smaller than or **equal to** or **larger than** the force experienced by particle 2?

(2) Is the acceleration of particle 1
smaller than or **equal to** or **larger than** the acceleration of particle 2?

Particle in Uniform Electric and Gravitational Field (1)

A proton, a neutron, and an electron are dropped from rest in a vertical gravitational field $\vec{g}$ and in a horizontal electric field $\vec{E}$ as shown. Both fields are uniform.



(a) Which particle travels the shortest distance?

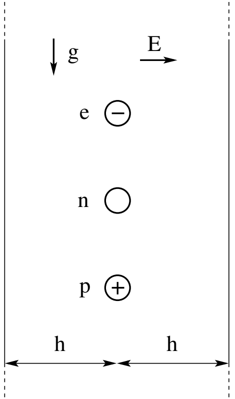
(b) Which particle travels the longest distance?

(c) Which particle travels the shortest time?

(d) Which particle reaches the highest speed?

Particle in Uniform Electric and Gravitational Field (2)

A proton, a neutron, and an electron are dropped from rest
in a vertical gravitational field $\vec{g}$
and in a horizontal electric field $\vec{E}$ as shown.
Both fields are uniform.



(a) Which particle travels the shortest distance?

(b) Which particle travels in a straight line?

(c) Which particle travels the shortest time?

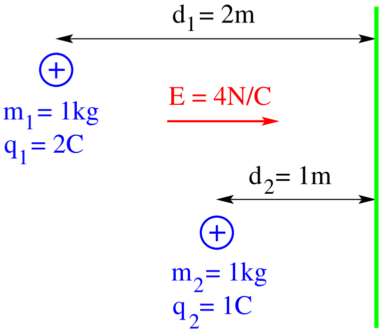
(d) Which particle reaches the highest speed?

Is the Faster also the Quicker?

Charged particles 1 and 2 are released from rest in a uniform electric field.

(a) Which particle moves faster when it hits the wall?

(b) Which particle reaches the wall more quickly?



Intermediate Exam I: Problem #3 (Spring '06)

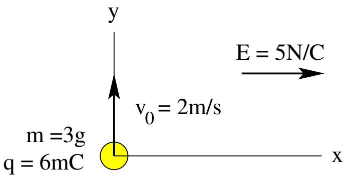
Consider a region of uniform electric field as shown. A charged particle is projected at time $t = 0$ with initial velocity as shown. Ignore gravity.

(a) Find the components a_x and a_y of the acceleration at time $t = 0$.

(b) Find the components v_x and v_y of the velocity at time $t = 0$.

(c) Find the components v_x and v_y of the velocity at time $t = 1.2$s.

(d) Find the components x and y of the position at time $t = 1.2$s.



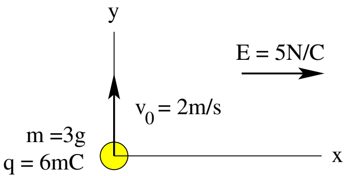
Consider a region of uniform electric field as shown. A charged particle is projected at time $t = 0$ with initial velocity as shown. Ignore gravity.

(a) Find the components a_x and a_y of the acceleration at time $t = 0$.

(b) Find the components v_x and v_y of the velocity at time $t = 0$.

(c) Find the components v_x and v_y of the velocity at time $t = 1.2\text{s}$.

(d) Find the components x and y of the position at time $t = 1.2\text{s}$.



Solution:

(a) $a_x = \frac{q}{m}E = \frac{6 \times 10^{-3}\text{C}}{3 \times 10^{-3}\text{kg}}(5\text{N/C}) = 10\text{m/s}^2, \quad a_y = 0.$

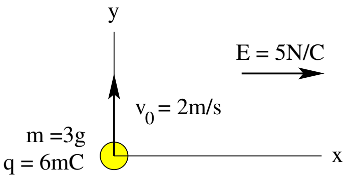
Consider a region of uniform electric field as shown. A charged particle is projected at time $t = 0$ with initial velocity as shown. Ignore gravity.

(a) Find the components a_x and a_y of the acceleration at time $t = 0$.

(b) Find the components v_x and v_y of the velocity at time $t = 0$.

(c) Find the components v_x and v_y of the velocity at time $t = 1.2\text{s}$.

(d) Find the components x and y of the position at time $t = 1.2\text{s}$.



Solution:

(a) $a_x = \frac{q}{m}E = \frac{6 \times 10^{-3}\text{C}}{3 \times 10^{-3}\text{kg}}(5\text{N/C}) = 10\text{m/s}^2, \quad a_y = 0.$

(b) $v_x = 0, \quad v_y = v_0 = 2\text{m/s}.$

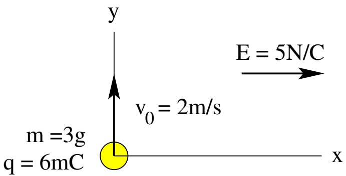
Consider a region of uniform electric field as shown. A charged particle is projected at time $t = 0$ with initial velocity as shown. Ignore gravity.

(a) Find the components a_x and a_y of the acceleration at time $t = 0$.

(b) Find the components v_x and v_y of the velocity at time $t = 0$.

(c) Find the components v_x and v_y of the velocity at time $t = 1.2\text{s}$.

(d) Find the components x and y of the position at time $t = 1.2\text{s}$.



Solution:

(a) $a_x = \frac{q}{m}E = \frac{6 \times 10^{-3}\text{C}}{3 \times 10^{-3}\text{kg}}(5\text{N/C}) = 10\text{m/s}^2, \quad a_y = 0.$

(b) $v_x = 0, \quad v_y = v_0 = 2\text{m/s}.$

(c) $v_x = a_x t = (10\text{m/s}^2)(1.2\text{s}) = 12\text{m/s}, \quad v_y = v_0 = 2\text{m/s}.$

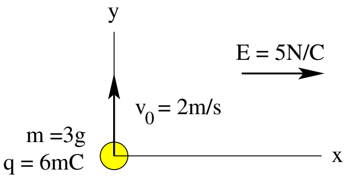
Consider a region of uniform electric field as shown. A charged particle is projected at time $t = 0$ with initial velocity as shown. Ignore gravity.

(a) Find the components a_x and a_y of the acceleration at time $t = 0$.

(b) Find the components v_x and v_y of the velocity at time $t = 0$.

(c) Find the components v_x and v_y of the velocity at time $t = 1.2\text{s}$.

(d) Find the components x and y of the position at time $t = 1.2\text{s}$.



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(b) $v_x = 0, \quad v_y = v_0 = 2\text{m/s}.$

(c) $v_x = a_x t = (10\text{m/s}^2)(1.2\text{s}) = 12\text{m/s}, \quad v_y = v_0 = 2\text{m/s}.$

(d) $x = \frac{1}{2}a_x t^2 = 0.5(10\text{m/s}^2)(1.2\text{s})^2 = 7.2\text{m}, \quad y = v_y t = (2\text{m/s})(1.2\text{s}) = 2.4\text{m}.$

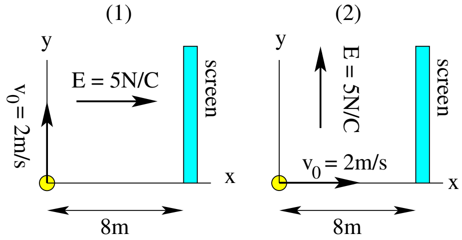
Consider two regions of uniform electric field as shown. Charged particles of mass $m = 2\text{kg}$ and charge $q = 1\text{C}$ are projected at time $t = 0$ with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

(a) At what time t_1 does the particle in region (1) hit the screen?

(b) At what height y_1 does the particle in region (1) hit the screen?

(c) At what time t_2 does the particle in region (2) hit the screen?

(d) At what height y_2 does the particle in region (2) hit the screen?



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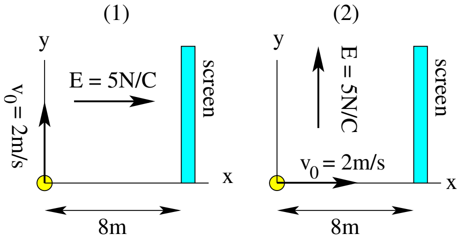
(b) At what height y_1 does the particle in region (1) hit the screen?

(c) At what time t_2 does the particle in region (2) hit the screen?

(d) At what height y_2 does the particle in region (2) hit the screen?

Solution:

(a) $x_1 = \frac{1}{2}at_1^2 \quad \text{with} \quad a = \frac{q}{m}E = 2.5\text{m/s}^2,$
$x_1 = 8\text{m} \quad \Rightarrow \; t_1 = 2.53\text{s}.$



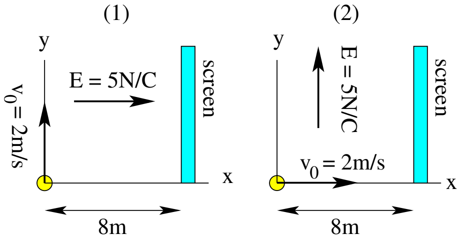
Consider two regions of uniform electric field as shown. Charged particles of mass $m = 2\text{kg}$ and charge $q = 1\text{C}$ are projected at time $t = 0$ with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

(a) At what time t_1 does the particle in region (1) hit the screen?

(b) At what height y_1 does the particle in region (1) hit the screen?

(c) At what time t_2 does the particle in region (2) hit the screen?

(d) At what height y_2 does the particle in region (2) hit the screen?



Solution:

(a) $x_1 = \frac{1}{2}at_1^2 \quad \text{with} \quad a = \frac{q}{m}E = 2.5\text{m/s}^2,$
$x_1 = 8\text{m} \quad \Rightarrow \; t_1 = 2.53\text{s}.$

(b) $y_1 = v_0 t_1 = 5.06\text{m}.$

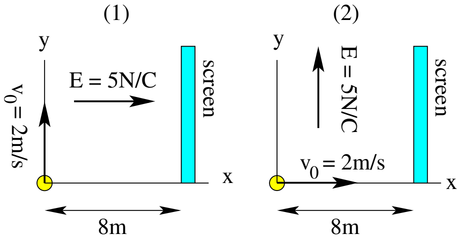
Consider two regions of uniform electric field as shown. Charged particles of mass $m = 2\text{kg}$ and charge $q = 1\text{C}$ are projected at time $t = 0$ with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

(a) At what time t_1 does the particle in region (1) hit the screen?

(b) At what height y_1 does the particle in region (1) hit the screen?

(c) At what time t_2 does the particle in region (2) hit the screen?

(d) At what height y_2 does the particle in region (2) hit the screen?



Solution:

(a) $x_1 = \frac{1}{2}at_1^2$ with $a = \frac{q}{m}E = 2.5\text{m/s}^2$,
$x_1 = 8\text{m} \Rightarrow t_1 = 2.53\text{s}.$

(b) $y_1 = v_0 t_1 = 5.06\text{m}.$

(c) $x_2 = v_0 t_2 \Rightarrow t_2 = \frac{8\text{m}}{2\text{m/s}} = 4\text{s}.$

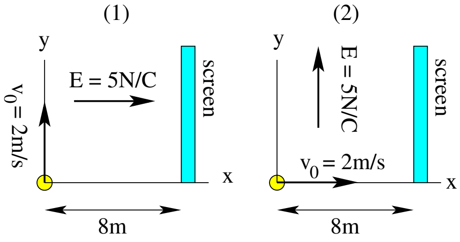
Consider two regions of uniform electric field as shown. Charged particles of mass $m = 2$kg and charge $q = 1$C are projected at time $t = 0$ with initial velocities as shown. Both particles will hit the screen eventually. Ignore gravity.

(a) At what time t_1 does the particle in region (1) hit the screen?

(b) At what height y_1 does the particle in region (1) hit the screen?

(c) At what time t_2 does the particle in region (2) hit the screen?

(d) At what height y_2 does the particle in region (2) hit the screen?



Solution:

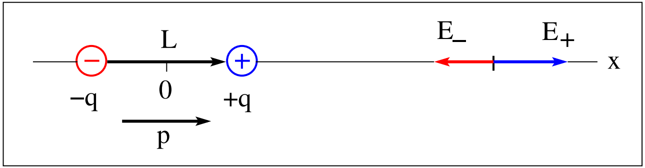
(a) $x_1 = \frac{1}{2}at_1^2$ with $a = \frac{q}{m}E = 2.5\text{m/s}^2$,
$x_1 = 8\text{m} \quad \Rightarrow \quad t_1 = 2.53\text{s}.$

(b) $y_1 = v_0 t_1 = 5.06\text{m}.$

(c) $x_2 = v_0 t_2 \quad \Rightarrow \quad t_2 = \frac{8\text{m}}{2\text{m/s}} = 4\text{s}.$

(d) $y_2 = \frac{1}{2}at_2^2 = 20\text{m}.$

Electric Dipole Field

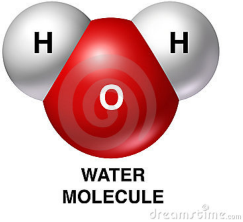


$$E = \frac{kq}{(x-L/2)^2} - \frac{kq}{(x+L/2)^2} = kq\left[\frac{(x+L/2)^2-(x-L/2)^2}{(x-L/2)^2(x+L/2)^2}\right] = \frac{2kqLx}{(x^2-L^2/4)^2}$$

$$\simeq \frac{2kqL}{x^3} = \frac{2kp}{x^3} \qquad (\text{for } x \gg L)$$

Electric dipole moment: $\vec{p} = q\vec{L}$

- Note the more rapid decay of the electric field with distance from an electric dipole ($\sim r^{-3}$) than from an electric point charge ($\sim r^{-2}$).
- The dipolar field is not radial.



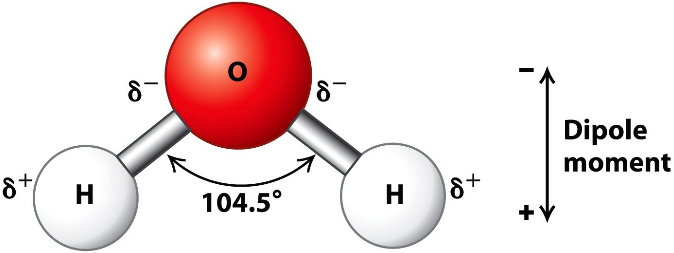
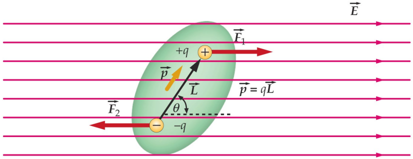
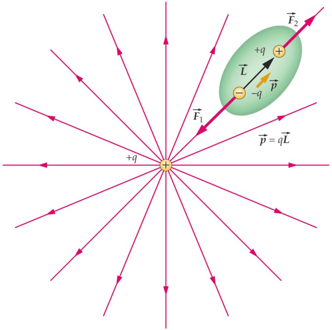


Figure 2-5
Molecular Cell Biology, Sixth Edition

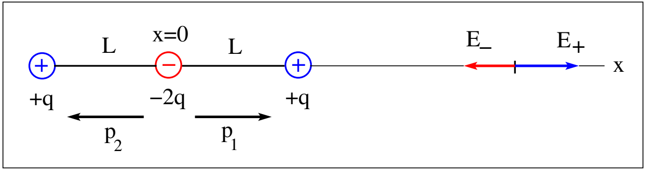
Force and Torque on Electric Dipole

- The net force on an electric dipole in a *uniform* electric field vanishes.
- However, this dipole experiences a torque $\vec{\tau} = \vec{p} \times \vec{L}$ that tends to align the vector $\vec{p}$ with the vector $\vec{E}$.
- Now consider an electric dipole that is already aligned (locally) with a *nonuniform* electric field. This dipole experiences a net force that is always in the direction where the field has the steepest increase.





Electric Quadrupole Field



$$
\begin{aligned}
E &= \frac{kq}{(x-L)^2} + \frac{kq}{(x+L)^2} + \frac{k(-2q)}{x^2} = \frac{kq}{x^2}\left[\left(1-\frac{L}{x}\right)^{-2} + \left(1+\frac{L}{x}\right)^{-2} - 2\right] \\
&= \frac{kq}{x^2}\left[\left(1+\frac{2L}{x}+\frac{3L^2}{x^2}+\cdots\right) + \left(1-\frac{2L}{x}+\frac{3L^2}{x^2}-\cdots\right) - 2\right] \\
&\simeq \frac{6kqL^2}{x^4} = \frac{3kQ}{x^4} \qquad (\text{for } x \gg L)
\end{aligned}
$$

Electric quadrupole moment: $Q = 2qL^2$

Different quadrupole configuration: